DISSERTATIONES PHYSICAE UNIVERSITATIS TARTUENSIS

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HANNES LIIVAT

Probing new physics in e^+e^- annihilations into heavy particles via spin orientation effects

TARTU UNIVERSITY
PRESS

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Supervisor: Dr. Ilmar Ots, Institute of Physics, University of Tartu, Tartu, Estonia

Opponents: Dr. Katri Huitu, University of Helsinki, Finland
Dr. Margus Saal, Tartu Observatory, Estonia

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Contents

List of Publications

Publications considered in the thesis:

I S. Groote, H. Liivat, I. Ots and T. Sepp, *"Probing scalar particle and unparticle couplings in $e^+e^- \to t\bar{t}$ with transversely polarized beams"*, Eur. Phys. J. C **66** pp. 271–281, 2010.

II I. Ots, H. Uibo, H. Liivat, R.-K. Loide and R. Saar, *"Possible anomalous $ZZ\gamma$ and $Z\gamma\gamma$ couplings and Z boson spin orientation in e^+e^- annihilation"*, Nucl. Phys. B **702** pp. 346–356, 2004.

III I. Ots, H. Uibo, H. Liivat, R.-K. Loide and R. Saar, *"Possible anomalous $ZZ\gamma$ and $Z\gamma\gamma$ couplings and Z boson spin orientation in e^+e^- annihilation: The role of transverse polarization"*, Nucl. Phys. B **740** pp. 212–221, 2006.

IV I. Ots, R. Saar, R.-K. Loide, and H. Liivat, *""Dynamical" non-minimal higher-spin interaction and gyromagnetic ratio $g = 2$"*, Europhys. Lett. **56** pp. 367–371, 2001.

Author's contribution

The author has given an essential contribution to the Publications I-III. He participated in development of mathematical apparatus, calculations and in the analysis of the results. He is one of the authors of the idea in Publication IV.

Other related Publications:

1. I. Ots, H. Uibo, H. Liivat, R. -K. Loide and R. Saar, *"Spin polarization and alignment of the single Z boson from e^+e^- annihilation"*, Nucl. Phys. B **588** pp. 90–100, 2000.

2. I. Ots and H. Liivat, *"General spin density matrix formalism and spin orientation of gauge bosons in e^+e^- annihilation"*, Hadronic J. **23** pp. 341-352, 2000.

3. G. Moortgat-Pick, A. Bartl, K. Hidaka, T. Kernreiter, H. Liivat, R. -K. Loide, I. Ots, W. Porod, R. Saar and H. Uibo, *'New physics searches at a linear collider with polarized beams*, Nuc. Phys. B – Proc. Suppl. **117** pp. 803–806, 2003.

4. R.-K. Loide, I. Ots, R. Saar and H. Liivat, *"Higher-spin equations with varible mass and spin and causality"*, Hadronic J. **26** pp. 193-202, 2003.

Chapter 1

Introduction

Since its detection in 1925 the spin as a fundamental characteristic of elementary particles has been in the forefront of research in particle physics. Starting with low-energy experiments like the determination of the gyromagnetic ratio of an electron, applications reach through colliders like the synchrotron into present time where the linear colliders have become important for the determination of the structure of fundamental interactions.

The current thesis studies some aspects of spin physics in the domain of electroweak interactions of the Standard Model, taking into account possible new physics interactions. Applying the formalism of the general relativistic spin density matrix, the analytical calculation of the spin effects in e^+e^- annihilation processes with heavy final particles in $e^+e^- \to t\bar{t}$ and $e^+e^- \to Z\gamma$ have been carried out and analyzed. New measurable spin-dependent quantities sensible to the new physics manifestations have been constructed. These quantities turn out to be helpful in disentangling new physics contributions from the Standard Model ones. Beside the investigation of possible new physics manifestations in the given processes, some problems of higher spin physics are analyzed in the thesis as well. The obtained results are helpful in understanding of the Standard Model and setting limits to new theories that may exist at energy scales unavailable at present time.

The thesis consists of 8 chapters followed by the four research publications. With a slight exceptions, Chapters 2-7 cover all the contents of Publications I-IV.

Chapter 2 gives a short introduction to the Standard Model and new physics effects under consideration.

In Chapter 3 the mathematical apparatus for the description of the spin orientations, given in Publications I-III in one or another form, is also presented as a whole but more generally and at the same time also more detailed as it is done in the Publications.

The content of Chapter 4 where spin orientation effects in the process $e^+e^- \to t\bar{t}$ are considered together with possible anomalous scalar type interactions, is practically identical to the one of Paper I without the introductory part and the given mathematical apparatus for describing the spin states.

In Chapter 5 the spin physics in the process $e^+e^- \to Z\gamma$ is investigated and clarified in the case of the presence of anomalous $Z\gamma Z$ and $Z\gamma\gamma$ couplings. This chapter covers in a slightly more general form the main contents of papers II and III.

In Chapter 6 the spin polarization and alignment of the Z boson in $e^+e^- \to Z\gamma$ are compared with those of $e^+e^- \to ZH$. In this chapter the results of Papers II and III are also partly used.

In Chapter 7 the problem of the gyromagnetic ratio for charged spin-1 particles is considered. As compared to Publication IV, a short review of the difficulties in higher-spin physics is given here as well.

Except for Chapter 7, in all chapters the different notations of the kinematical parameters and the usage of them for expressing the final results are made uniform. The same is done for references to equations and to the literature.

Chapter 2

Standard Model of elementary particles and new physics

The present theory of the fundamental interactions – the Standard Model – is phenomenologically successful at energies up to some hundred giga-electronvolt (GeV). At present the Standard Model (SM) is in consistence with all accelerator-based experiments. Despite that quite impressive phenomenological success the SM as a theory is far to be satisfactory. There are several fundamental questions that remain unanswered by the SM. It does not explain clearly the mechanisms and the scale of the electroweak symmetry breaking. The origin of flavours, the spectrum of fermion masses and the CP-violation also remain beyond the scope of the SM. Though, the SM contains higher-spin ($s = 1$) massive gauge bosons, the problem of building consistent higher-spin interaction theory is not yet solved in the SM. The SM does not answer also the questions which are needed for complete understanding of Big Bang cosmology: the dark matter, the dark energy as well the inflation. This list can be continued. To these theoretical shortcomings one can add also the first experimental discrepancy. At present there is strong experimental evidence for neutrino oscillations [1, 2, 3, 4] whose most obvious and natural explanation is that, contrary to the SM conception, the neutrinos are massive.

Due to all that, there are strong reasons to expect that there is a great deal of (new) physics beyond the SM, with characteristic mass scale. The present good agreement between the accelerator-based experimental data and the SM predictions suggests that the energy scale associated with any new physics (NP) model should be high as compared to the electroweak scale (≈ 246 GeV).

The search for NP can be proceeded in various ways. The most straightforward method for searching NP would be the production of NP particles. Up to now the energy of the existing accelerators is not high enough to produce such particles. It is believed that the new accelerator, Large Hadron Collider (LHC), started in 2009, is able to produce new physics particles, first of all new on-shell resonances or a single heavy new particle in association with a SM particle [5]. However, such a scenario for visualizing NP experimentally is obviously beyond the reach of future e^+e^- colliders. Fortunately, there are ways to probe NP at the energies below the NP mass scale. These more indirect scenarios are based on observations of small deviations from the SM predictions in processes where the external particles are ordinary SM ones, and NP effects can arise only from non-standard interactions. The price to pay for such possibilities to measure NP effects is, of course, the need of higher sensitivity, both theoretical and experimental.

One of the possible sources of NP may be the existence of anomalous scalar-tensor type couplings. Such couplings arise in many extensions of the Standard Model. Most of all the possible manifestations of scalar-tensor couplings are investigated in the top-antitop pair production in e^+e^- annihilation.

The top quark is by far the heaviest fundamental particle. Because of this, couplings including the top quark are expected to be more sensitive to new physics manifestations than couplings to other particles. This is why the top quark physics is a very fascinating field of investigations and has been developed actively for a long time. During the last decades theoretical investigations have been connected closely to the physics of future colliders like the LHC at CERN and the International Linear Collider (ILC). As already stated above, the LHC is no longer a future collider. The setup has been completed and first useful scientific information will be available in near future. The center-of-mass energy of 14 teraelectronvolt (TeV) and the very large statistics allows to determine top quark properties accurately. On the other hand, the future of the ILC is presently unknown. Nevertheless, we use it as an example of future e^+e^- linear collider and its possibilities.

The proposed ILC designed for a center-of-mass starting energy of 500 GeV (later up to 1 TeV) and about three orders less statistics as compared to the LHC is still considered as a perspective tool for complementary investigations of new physics manifestations. The reason is that compared to LHC, the ILC has two distinctive advantages: a very clean experimental environment and the possibility to use both longitudinally polarized (LP) and transversely polarized (TP) beams. In the baseline design of the ILC, electron beams with LP around 0.8 and positron beams with LP about 0.6 are foreseen. By using spin rotators these polarizations can be converted almost without losses into the TP ones. Especially the use of TP beams gains more and more attention. By using LP one can enhance the sensitivity for different parts of the coupling which, at least in principle, can be measured also for unpolarized beams. However, TP provides new directions which allow to analyze interactions beyond the Standard Model (SM) more efficiently. This facility should be available at the ILC or other colliders of the same type.

One of the areas where the advantage of TP beams can be used is just the investigation of anomalous scalar-tensor type couplings. More than thirty years ago Dass and Ross [6] and later Hikasa [7] showed that for TP e^+e^- beams the amplitudes of such couplings interfere with the SM ones. Due to the helicity conservation this is not the case when using unpolarized or LP beams. For vanishing initial state masses the scalar and tensor-type couplings at the e^+e^- vertex are helicity violating, whereas the SM containing vector and axial vector couplings are helicity conserving. Therefore, in the limit of massless initial particles there are no non-zero interference terms for unpolarized and LP beams. However, as the argument of helicity conservation fails for TP beams, for TP initial beams the scalar-tensor coupling amplitudes interfere with the SM ones. Ananthanarayan and Rindani [8] demonstrated how TP beams can provide additional means to search for CP violation via interference between SM and anomalous, scalar-tensor type coupling contributions in $e^+e^- \to t\bar{t}$. Therefore, the use of TP beams enables to probe new physics appearing already in first order contributions. In addition, the additional polarization vector allows to analyze CP violation asymmetries without the necessity for final state top or antitop polarizations.

The aforementioned advantages can be used also in analysing (pseudo) scalar unparticle manifestations via their virtual effects. The unparticle is a new concept proposed by Georgi [9] based on the possible existence of a nontrivial scale invariant sector with an energy scale much higher than that of the SM. At lower energies this sector is assumed to couple to the SM fields via nonrenormalizable effective interactions involving massless objects of fractional scale dimension d_u coined as unparticles. Using concepts of effective theories one can calculate the possible effects of such a scale-invariant sector for TeV-scale colliders. The existence of unparticles could lead to measurable deviations from SM predictions as well as from the predictions of various models beyond SM. The experimental signals of unparticles might be of two kinds. If unparticles are produced, they manifest themselves as missing energy and momentum. On the other hand, unparticles can cause virtual effects in processes of SM particles.

Since Georgi's significant publications the study of unparticle physics has gained a lot of attention, shedding light on both theoretical and phenomenological aspects. The most interesting theoretical developments of unparticle physics are listed in [10].

From these theoretical studies it follows that unparticle physics features a very rich phenomenology which may be radically different from particle theory. The phenomenological studies consider mainly possible unparticle manifestations in LHC and ILC processes. Since unparticle physics has a very rich phenomenology, the number of papers in this sector is much greater than in the theoretical sector. A significant part of the phenomenological studies in particle physics are related to the top quark, especially to top quark pair production processes in e^+e^- collisions (see *e.g.* [11] and references therein). A unique feature of virtual unparticle exchanges is the complex phase of the unparticle propagator for timelike momenta. If this feature could be identified, it would be a conclusive device for the existence of unparticles. One way to capture the feature is again to use TP initial beams at linear e^+e^- collider processes.

In this thesis it is studied how TP initial beams can be used to disentangle scalar particle and unparticle contributions from SM contributions in the process $e^+e^- \to t\bar{t}$. The analytic expressions for the differential cross section of the process with anomalous scalar particle and virtual scalar unparticle coupling corrections are presented in the case where the top (antitop) quark polarization is measured. The main features of the SM, anomalous particle and unparticle contributions and the methods to isolate signatures for different contributions are presented and analyzed.

Already for a long time it is believed that a possible source of NP may be also the existence of anomalous gauge boson self-couplings which can reveal themselves through the gauge boson production processes with nonstandard gauge boson self-couplings vertices. Motivated by this possibility, such couplings have been theoretically extensively investigated and experimentally tested. Though to date no evidence of anomalous gauge boson self-couplings have been established, the bounds obtained at the CERN e^+e^- collider and Fermilab Tevatron are comparatively weak [22, 23]. It is also clear that the current colliders cannot provide sensitivities which would be considerably better than the ones already achieved. Obviously, if the anomalous gauge boson self-interactions altogether exist they are too small to be established at current colliders. However, the study of gauge boson anomalous self-interactions is an important item in the physics programs of the planned next generation colliders. As soon as the proposed high energy colliders start running, a dramatic improvement of the sensitivity of the experiments to non-standard couplings is expected [24, 25]. It has been shown that one of the most sensitive probes of NP is provided by the couplings of three neutral gauge bosons [26]. Among the processes to which such couplings could contribute are the neutral gauge boson production processes in e^+e^- collisions ($e^+e^- \to \gamma\gamma$, $Z\gamma$, ZZ).

After putting into operation the planned next generation electron-positron colliders, new prospects for testing aforementioned anomalous couplings in these processes will be open. A future e^+e^- International Linear Collider (ILC) operating in the wide energy range up to 1 TeV and designed with high luminosity, with an additional advantage to have both initial beams longitudinally and transversely polarized provides an excellent discovery potential. Due to these possibilities, the role of the initial particle polarizations, especially the role of the transverse polarization in the processes with possible anomalous parts of couplings have attracted noteworthy attention in recent years [27, 28, 30]. A considerable part of these investigations is constituted by those considering the LP and TP effects in the process $e^+e^- \to Z\gamma$. A lot of initial beams polarization effects in this process which would be helpful in the experimental testing of anomalous couplings have been found and analyzed. However, the studies connected with the role of initial beam polarizations in searching for anomalous gauge boson self interactions are not yet exhaustive.

In this thesis the possible anomalous gauge boson self-couplings corrections to the Z boson spin orientation in the process $e^+e^- \to Z\gamma$ are studied with an accent on the role of the longitudinal and transverse polarization of the initial beams in disentangling the SM and anomalous couplings ($Z\gamma Z$ and $Z\gamma\gamma$).

The methods for searching non-standard physics manifestations below the NP mass scale at high-sensitive colliders cannot be successful without knowing the SM predictions with needed precision. Maybe most expressively this requirement is given in the paper of Lykken [5]. He writes that "... to first approximation LHC experimenters do not need to know anything about BSM [Beyond the SM] models in order to make discoveries – but they need to know a lot about Standard Model physics!" For detecting NP manifestations through the spin orientation effects in the processes presented in this thesis, one needs to know a lot about spin effects within the SM. One can say that the knowledge of all possible spin effects within the framework of the SM forms a basis for rejecting or limiting various nonstandard couplings. Hence, the studies of spin effects in the SM are still worthwhile. In the thesis the spin orientation effects in $e^+e^- \to Z\gamma$ in the SM near the threshold energy of the process is investigated in detail and compared with those of the process $e^+e^- \to ZH$.

Beside the possible existence of anomalous gauge boson self-couplings another problem connected with the possible non-standard behaviour of the gauge bosons exists. It is the problem of higher-spin interaction theory. With the massive gauge bosons introduced into the SM, also the known difficulties of higher-spin interaction theories have been incorporated into the SM. Indeed, the charged spin-1 $W^\pm$ bosons cause serious troubles when coupled minimally to electromagnetic field, among others the bad high-energy behavior of scattering amplitudes. Weinberg showed [31] that non-minimal couplings, especially those with gyromagnetic ratio $g = 2$, can (partly) cure these difficulties.

The fact that in the case of higher spin particles the minimal electromagnetic coupling leads to serious inconsistencies was known more than 40 years ago [32, 33]. During the long-time investigations it became clear that a promising way to get the consistent higher-spin interaction theory is to introduce non-minimal interaction into the theory. The question is how to find the true non-minimal coupling. In the case of spin-1 $W^\pm$ gauge bosons, a suitable non-minimal coupling term linear in the field strength tensor $F_{\mu\nu}$ was added in fact by hand in order to overcome the bad high-energy behaviour. Obviously the true form of the non-minimal coupling with $g = 2$ lays on some theoretical grounds. Such grounds have been looked for [34, 35]. However, the models proposed seem to be not general enough. More generally, the search for theoretical grounds for the non-minimal coupling which gives $g = 2$ is also the search for the dynamical principle for building a consistent higher-spin electromagnetic interaction theory.

One of the theoretical models that determines a special "dynamical" nonminimal coupling is based on using the field-dependent invariant representation of the Poincaré algebra [36]. In this thesis it is shown that the "dynamical" coupling leads to the gyromagnetic ratio $g = 2$. In such a manner, using a "dynamical" interaction may be one of the ways to reach closer to the consistent higher-spin interaction theories.

Chapter 3

Description of spin orientations of particles

In processes the particles with non-zero spins are as a rule in a mixed spin state. Contrary to the pure states, which can be described by a single wave function, for describing the mixtures an incoherent mixture of $2s+1$ orthogonal pure state wave functions are needed. The most natural way to describe the mixtures is to use the spin density matrix formalism. However, since in the higher-spin ($s \geq 1$) cases the calculations using general relativistic spin density matrix formalism are not very simple and the expressions obtained are often quite cumbersome, usually other ways to describe spin physics in processes are used. In most papers the spin-related studies are carried out by employing the helicity bases. This is reasonable because the method of summing the Feynman diagram amplitudes with definite helicities of initial and final particles has proved to be a powerful tool for describing the spin orientation phenomena. However, the helicity basis will not always be the most suitable one for spin effects analyses, especially when particles are only moderately relativistic [42, 43, 44]. Therefore, since the massive gauge bosons and the top quark, participating in the processes under consideration in the thesis are very heavy, there is no reason to believe that the helicity bases will be the best choice to describe their spin orientations. In this thesis it is demonstrated that one can reach the desired results comparatively easily also by using the relativistic density matrix formalism in a general form. The essence of the method is explained in the following. When interested in the spin orientation of certain particle in the process, one replaces the $u\bar{u}$ (spin-1/2 case) or $\varepsilon_\mu \varepsilon^*_\nu$ (spin-1 case) in the squared amplitude of the process by relativistic spin density matrix describing the mixed state.

3.1 General properties of spin density matrices

To calculate the processes under consideration in the thesis the relativistic spin density matrices for massive spin-1/2 and spin-1 particles are needed. Thus, one must construct these density matrices.

We start with presenting the definition of the spin density matrix and its main properties in a general case. Let us have a pure spin ensemble, which is described by the wave function (state vector) that is generally a coherent mixture of the eigenstates, *i.e.*

$$|\psi\rangle = \sum_n |n\rangle\langle n|\psi\rangle = \sum_n a_n|n\rangle. \tag{3.1}$$

The probability of finding the state in the eigenstate $|n\rangle$ is

$$p_n = \langle n|\psi\rangle \langle\psi|n\rangle = |a_n|^2. \tag{3.2}$$

Denoting the observable belonging to ensemble $|\psi\rangle$ by Q, one can give the mean value of this operator as

$$\langle Q\rangle = \langle\psi|Q|\psi\rangle = \sum_{n,n'} a_n^* a_{n'} \langle n|Q|n'\rangle. \tag{3.3}$$

Instead of the wave function $|\psi\rangle$ the pure ensemble can be described by an operator

$$\rho = |\psi\rangle \langle\psi|. \tag{3.4}$$

Indeed, if one has an observable Q, it is a simple task to show that

$$\mathrm{Tr}\,\rho Q = \langle\psi|Q|\psi\rangle. \tag{3.5}$$

Hence, multiplying operator (3.4) by the operator Q and tracing the result, one gets the same formula as in finding mean values with the help of the wave function. By using the ρ-operator, one can for instance also find the probability of the appearance some eigenvalue. For this one has to multiply the operator ρ by the operator $\hat{p}(a_k) = |a_k\rangle \langle a_k|$ and then take the trace from the result to get

$$\mathrm{Tr}\,\rho\hat{p}(a_k) = |a_k|^2 = p(a_k). \tag{3.6}$$

Thus, it can be shown that pure ensembles may be described either by the wave function, which is the vector in n-dimensional Hilbert space or by the second-rank tensor in the same space. The operator ρ is called the density or statistical operator but more often it is called the density matrix. One gets no advantage from using the density matrix instead of the wave function. It is only an alternative possibility to describe pure ensembles. Nevertheless, for the further aims we present here the general features of the density matrix (3.4):

1. The density matrix is hermitian. If $|\psi\rangle = \sum_n |n\rangle \langle n|\psi\rangle = \sum_n a_n |n\rangle$, then $\langle n|\rho|n^{'}\rangle = \rho_{nn'} = \langle n|\psi\rangle \langle \psi|n^{'}\rangle = a_n a^*_{n^{'}}$ and $\rho_{n^{'}n} = a_{n^{'}} a^*_n$. Hence, $\rho_{n^{'}n} = \rho^*_{nn^{'}}$ or
$$\rho = \rho^{\dagger}. \tag{3.7}$$

2. The trace of the density matrix in any matrix representation is equal to unity. Indeed, if the wave function is normalized, then
$$\mathrm{Tr}\, \rho = \sum_n a_n a^*_n = 1. \tag{3.8}$$

3. As every hermitian matrix, the density matrix can be reduced to the diagonal form by a unitary transformation.

4. Every diagonal element of ρ in any representation must be non-negative. The diagonal element $\rho_{nn} = a_n\, a^*_n$ is connected with the probability of the ensemble being in some definite eigenstate, therefore
$$\rho_{nn} \geq 0. \tag{3.9}$$

Let us consider the diagonal form of pure state density matrix in more detail. The operator ρ defined in (3.4) is in fact a projection operator, which projects out the state $|\psi\rangle$. If the operator is the projection operator, the squared operator has to be the same operator again. Thus,

$$\rho^2 = |\psi\rangle \langle\psi|\psi\rangle \langle\psi| = |\psi\rangle \langle\psi| = \rho \tag{3.10}$$

and
$$\mathrm{Tr}\, \rho^2 = 1. \tag{3.11}$$

These features are conserved also in the diagonal form. Denoting the eigenvalues of ρ by ρ_n, one can write $\rho_n^2 = \rho_n$ from which one gets $\rho_n = 0$ or $\rho_n = 1$. As $\sum_n \rho_n = 1$ (from (3.8)), one gets that the diagonal elements of ρ are equal to zero except for one of them, which is equal to unity.

Let us now turn to mixtures. Suppose that the ensembles, which cannot be described by a single wavefunction can be described by some operator ρ, which is called the density matrix. Naturally, it is not the density matrix defined by (3.4), but one supposes that formula (3.5) which in the pure state determines the mean values of the operators is valid here as well. We rewrite this formula with the new operator ρ:

$$\langle Q\rangle = \mathrm{Tr}\,\rho Q. \tag{3.12}$$

One may take (3.12) (and most of experts really take) as definition of the density matrix ρ. Let us now present the properties of this density matrix [45].

1. The condition that $\langle Q\rangle$ is real for every hermitian operator requires ρ to be hermitian:
$$\rho_{nn'} = \rho^*_{n'n} \quad (\rho = \rho^\dagger). \tag{3.13}$$

2. The condition that the unit operator $\hat{I}$ has the mean value 1, requires
$$\mathrm{Tr}\,\hat{I}\rho = \mathrm{Tr}\,\rho = \sum_n \rho_{nn} = 1. \tag{3.14}$$

3. The hermitian matrix ρ can be reduced to the diagonal form by a unitary transformation U:
$$\rho_d = U\rho U^\dagger. \tag{3.15}$$
Since ρ is no longer a projection operator, all its diagonal elements can have nonzero values.

4. The condition that every operator with non-negative eigenvalues has a non-negative mean value requires ρ to be positively definite. This means that every diagonal element of ρ in any matrix representation has to be non-negative:
$$\rho_{nn} \geq 0. \tag{3.16}$$

The values of diagonal elements are restricted by the properties (3.14) and (3.16) in the following way:

$$\mathrm{Tr}\,\rho^2 = \sum_n \rho_n^2 \leq (\mathrm{Tr}\,\rho)^2\,. \tag{3.17}$$

However, this formula restricts not only the diagonal but also the non-diagonal elements of ρ. Indeed,

$$\mathrm{Tr}\,\rho^2 = \mathrm{Tr}\,\rho\,\rho = \sum_{n\,n'} \rho_{n\,n'}\,\rho_{n'\,n} \leq 1 \tag{3.18}$$

that limits the value of every single element of the density matrix.

We now proceed to analyze the relations between the density matrix and mixture. The latter is described by the incoherent superposition of the pure states. Incoherent superposition means by definition that to calculate the probability of finding a certain mixed state, one must calculate the probability for each pure state and then take an average attributing to each of pure states an assigned weight. The pure state density matrix expressed through the pure state wave function is given by (3.4). It can be shown that the spin density matrix expressed through the incoherent mixture of pure state wave functions takes the form [46]:

$$\rho = \sum_{\lambda} \rho_\lambda |\psi_\lambda\rangle \langle\psi_\lambda|. \tag{3.19}$$

Comparing this outcome to the pure state density matrix one finds that the density matrix describing mixed state is given as a sum of several density matrices of different pure states each of which taken with its weight. One can take also (3.19) as the definition of the density matrix. However here a problem arises. This representation is not unique. Many different incoherent mixtures leading to the same density matrix can be constructed. As an example, the mixtures

$$\begin{cases} \rho_1 = \frac{1}{2} \\ \\ \rho_2 = \frac{1}{2} \end{cases}, \quad \begin{cases} |\phi_1\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \\ |\phi_2\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle \end{cases} \tag{3.20}$$

and

$$\begin{cases} \rho_1 = \frac{1}{2} \\ \\ \rho_2 = \frac{1}{2} \end{cases}, \quad \begin{cases} |\phi_1\rangle = |+\rangle \\ |\phi_2\rangle = |-\rangle \end{cases} \tag{3.21}$$

lead to the same density matrix

$$\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ \\ 0 & \frac{1}{2} \end{pmatrix},$$

describing unpolarized spin-1/2 beams.

Generally, in arbitrary spin case, if among the eigenvalues of the ρ there exist nonzero degenerate ones, several different mixtures correspond to the same ρ. It is the lack of one to one correspondence between the mixtures and the density matrices because the definition of the ρ by (3.12) is preferred as compared to the definition through the mixture.

3.2 Parametrization of spin density matrices

There are lot of possibilities to parametrize the density matrices. However, not all ways of parametrization are convenient for solving most problems, including the ones considered in this thesis. The two methods for the parametrization of the spin density matrices mainly used are parametrization by Cartesian tensors and parametrization by spherical tensors. In the thesis we use the first of them.

In nonrelativistic physics the spin-s density matrix is a hermitian $(2s+1)(2s+1)$ matrix, which can be described maximally by $4s(s+1)$ real parameters. To expand such a matrix a complete set of $(2s+1)^2 = 4s(s+1)+1$ basis matrices is needed. If we constitute the basis by the unit matrix, the spin matrices S_x, S_y, S_z and certain combinations of the products of the spin matrices whose mean values transform like Cartesian tensors, we have parametrized spin density matrices by Cartesian tensors.

Nonrelativistic spin-1/2 density matrix

We begin with the spin-1/2 case. To expand 2×2 density matrix four basis matrices are needed. As the basis matrices may serve the unit matrix and three spin matrices. Since the product of two spin matrices in the case of $s = 1/2$ can be given again by the unit matrix and spin matrices

$$S_i\, S_j = \delta_{ij} I/4 + \mathrm{i}\epsilon_{ijk} S_k/2, \tag{3.22}$$

where ϵ_{ijk} is totally antisymmetric tensor, this choice is unique. Hence,

$$\rho = cI + \xi_x S_x + \xi_y S_y + \xi_z S_z. \tag{3.23}$$

Since $\mathrm{Tr}\, S_i = 0$, and $\mathrm{Tr}\, I = 2$, the trace condition $\mathrm{Tr}\, \rho = 1$ gives at once $c = 1/2$ and one can write

$$\rho = \frac{1}{2}(1 + 2\xi_i S_i), \tag{3.24}$$

where we have used the summation convention over repeated indices.

More often, instead of the spin matrices S_i, the Pauli matrices $\sigma_i = 2\, S_i$ are used. This gives a familiar form of nonrelativistic spin-1/2 density matrix

$$\rho = \frac{1}{2}(1 + \xi_i\, \sigma_i). \tag{3.25}$$

The expansion coefficients ξ_i as parameters of the ρ, are the mean values of the basis matrices:

$$\xi_i = \mathrm{Tr}\, \rho \sigma_i. \tag{3.26}$$

They are the components of the polarization vector. The basis matrices I and σ_i have some very good properties, which make their usage for spanning the ρ matrix extremely convenient. The first property is the trace condition, the second property is the orthogonality property in the sense of

$$\mathrm{Tr}\, S_i\, S_j = \frac{1}{2}\delta_{ij}, \quad (\mathrm{Tr}\, \sigma_i\, \sigma_j = 2\delta_{ij}). \tag{3.27}$$

The third property is the advantage that if expanding the ρ matrix in terms of I and σ_i one has used the scalar and vector representation of the basis. These representations are irreducible. This means that various-rank tensors do not mix under the space rotation: the components of tensors of any rank are given by the components of the same rank tensors. If certain rank tensors are equal to zero in some coordinate system, they must be zero in any system.

The last good property of the basis used above is its hermiticity. From this property the reality of the mean values of the basis follows.

Nonrelativistic spin-1 density matrix

Here our task is to construct a spin-1 density matrix expansion on the basis with the same good properties as in spin-1/2 case. For describing the spin-1 density matrix one needs maximally 8 real parameters and to expand it 9 basis matrices are needed. For the first four matrices one can take here, similarly to the spin-1/2 case a unit 3×3 matrix and three spin matrices S_x, S_y, S_z. Therefore we need five independent second rank irreducible tensor components to span the whole basis. There are six different products of two spin matrices: $S_x\, S_y$, $S_x\, S_z$, $S_y\, S_z$, $S_x\, S_x$, $S_y\, S_y$, $S_z\, S_z$, but these products do not satisfy any of the four good properties of the spin-1/2 case. Their traces are not equal to zero, they are not irreducible, hermitian or orthogonal. We begin with making this part of basis matrices to be hermitian. In order to convert the basis to a hermitian one we will use the fact that only the symmetrized product of two hermitian matrices is hermitian. The second rank tensor can be divided into a symmetrical and antisymmetrical part:

$$S_i\, S_j = \frac{1}{2}(S_i\, S_j + S_j\, S_i) + \frac{1}{2}(S_i\, S_j - S_j\, S_i) = \frac{1}{2}(S_i\, S_j + S_j\, S_i) + \frac{\mathrm{i}}{2}\epsilon_{ijk} S_k. \tag{3.28}$$

Since the antisymmetrized part is a tensor one rank lower, we are interested in the symmetric part only. According to group theory it reduces to a scalar and a second rank tensor with zero trace. Since $\mathrm{Tr}\, S_i S_j = 2\delta_{ij}$, one may define

$$S_{ij} = \frac{3}{2}\left(S_i\, S_j + S_j\, S_i - \frac{4}{3}\delta_{ij} I\right) \tag{3.29}$$

that are hermitian irreducible zero trace matrices. Therefore, one can take S_{ij} together with I and S_i as the needed basis elements.

Then the density matrix takes the form

$$\rho = \frac{1}{3}\left(I + \frac{3}{2}\, t_i S_i + \frac{1}{3}\, t_{ij} S_{ij}\right). \tag{3.30}$$

However, this basis is overdetermined. Instead of 9 matrices needed for expanding the density matrix it contains ten matrices: It can be readily seen when one expands (3.30) as

$$\begin{aligned} \rho &= \frac{1}{3}\Big[I + \frac{3}{2}(t_x S_x + t_y S_y + t_z S_z) \\ &+ \frac{2}{3}(t_{xy} S_{xy} + t_{xz} S_{xz} + t_{yz} S_{yz}) \\ &+ \frac{1}{3}(t_{xx} S_{xx} + t_{yy} S_{yy} + t_{zz} S_{zz})\Big]. \end{aligned} \tag{3.31}$$

But not all of the elements of the basis are linearly independent. There exists a linear relation between three elements:

$$S_{xx} + S_{yy} + S_{zz} = 0. \tag{3.32}$$

Indeed, because of the square angular momentum formula

$$\vec{S}^{\,2} = S(S+1),$$

which leads to $\vec{S}^{\,2} = 2$ in the spin-1 case, one has

$$\frac{3}{2}\left[2(S_x^2 + S_y^2 + S_z^2) - 4\right] = \frac{3}{2}(2\vec{S}^{\,2} - 4) = 0.$$

Now the reduction of the number of the parameters from 9 to 8 can be conveniently done by taking

$$t_{xx} + t_{yy} + t_{zz} = 0. \tag{3.33}$$

Keeping in the mind these restrictions one can calculate the mean values of all basic elements. These mean values ought to be equal to the corresponding expanding parameters. Using the trace formulas in the spin-1 case

$$\mathrm{Tr}\, S_i S_j = 2\delta_{ij} \tag{3.34}$$

and

$$\mathrm{Tr}\, S_{ij} S_{lm} = \frac{3}{2}(-2\delta_{ij}\delta_{lm} + 3\delta_{il}\delta_{jm} + 3\delta_{im}\delta_{jl}), \tag{3.35}$$

one can readily find that

$$\langle S_i \rangle = \mathrm{Tr}\, \rho S_i = t_i \tag{3.36}$$

and for $i \neq j$

$$\langle S_{ij} \rangle = \mathrm{Tr}\, \rho S_{ij} = t_{ij}. \tag{3.37}$$

For the mean values of the rest three basis matrices one gets

$$\begin{aligned}
\langle S_{xx} \rangle &= \mathrm{Tr}\, \rho S_{xx} = \frac{2}{3} t_{xx} - \frac{1}{3} t_{yy} - \frac{1}{3} t_{zz}, \\
\langle S_{yy} \rangle &= \mathrm{Tr}\, \rho S_{yy} = \frac{2}{3} t_{yy} - \frac{1}{3} t_{xx} - \frac{1}{3} t_{zz}, \\
\langle S_{zz} \rangle &= \mathrm{Tr}\, \rho S_{zz} = \frac{2}{3} t_{zz} - \frac{1}{3} t_{xx} - \frac{1}{3} t_{yy}.
\end{aligned} \tag{3.38}$$

However, by using the restriction (3.33), one gets

$$\langle S_{xx} \rangle = \frac{2}{3} t_{xx} - \frac{1}{3} t_{yy} - \frac{1}{3} t_{zz} - \frac{1}{3} t_{xx} + \frac{1}{3} t_{xx} = t_{xx} \tag{3.39}$$

and similarly for $\langle S_{yy} \rangle$ and $\langle S_{zz} \rangle$. Hence, the overcomplete nonorthogonal basis with the supplementary condition (3.32) is equivalent to the orthogonal basis. Also, due to restriction (3.33), one gets

$$\rho_{ij} = \frac{1}{3}\delta_{ij} + \frac{1}{2}\, t_k\, (S_k)_{i,j} + \frac{1}{6} t_{kl} (S_k\, S_l + S_l\, S_k)_{ij}, \tag{3.40}$$

with $t_k = \mathrm{Tr} \rho\, S_k$ and $t_{kl} = \mathrm{Tr} \rho\, S_{kl}$. The first is called the polarization vector and the second is the orientation tensor describing the alignment of spins. Note here, that the coefficient 3/2 in (3.29) is introduced to guarantee that the parameters t_k and t_{kl} would be exactly the mean values of the basic matrices. Taking the representation $(S_k)_{ij} = -i\epsilon_{ijk}$, one gets

$$(S_k\, S_l + S_l\, S_k)_{ij} = 2\delta_{ks}\delta_{ij} - \delta_{ki}\delta_{lj} - \delta_{kj}\delta_{li}$$

and (3.40) can be written in a more simple form:

$$\rho_{ij} = \frac{1}{3}(\delta_{ij} - \frac{3}{2} i\, t_k \epsilon_{ijk} - t_{ij}). \tag{3.41}$$

3.3 Restrictions on spin density matrix parameters by positivity conditions

We begin with presenting physical boundaries (extremal values) of the spin density matrix parameters. The knowledge of the boundaries of the parameters is useful when determining experimentally the spin states of the particles. That is why there exist several papers devoted to the problem of finding extremal values of spin density matrix parameters [46, 47]. The problem itself is not complicated and in spin-1/2 and spin-1 cases one can solve it in a straightforward way. As the parameters are equal to the mean values of basis operators, one must find the expressions of these mean values and then evaluate their extrema. As a rule the parameters take their extremal values when the spin ensemble is in some pure state. The boundaries for spin-1/2 and spin-1 are given in the Table 3.1.

Parameter\ Spin	1/2	1
ξ_i resp. t_i	1	1
t_{ij} $(i \neq j)$	-	3/2
t_{ii}	-	$1,\ -2$

Table 3.1: The extremal values of spin-1/2 and spin-1 density matrix parameters.

But physical boundaries are not the only restriction to the parameters of the density matrix. In their physical region parameters cannot take values separately from each other. Parameters are linearly independent. However, they depend on each other nonlinearly, which is due to the positivity (non-negativity) condition of the density matrix. It appears that the positivity requirement restricts substantially all density matrix elements and with this also the parameters of the density matrix. In order to become aware of this, note that the positivity requirement is equivalent to the statement that for every complex vector in a $2s+1$ dimensional space the condition $\langle x|\rho|x\rangle \geq 0$ is valid. From this it follows that all principal minors of ρ are non-negative. Beginning from the lower-rank principal minors, one gets:

$$\rho_{mn} \geq 0, \tag{3.42}$$

$$\begin{vmatrix} \rho_{mm} & \rho_{mn} \\ \rho_{nm} & \rho_{nn} \end{vmatrix} = \rho_{mm}\rho_{nn} - |\rho_{mn}|^2 \geq 0, \tag{3.43}$$

$$\begin{vmatrix} \rho_{mm} & \rho_{mn} & \rho_{ml} \\ \rho_{nm} & \rho_{nn} & \rho_{nl} \\ \rho_{lm} & \rho_{ln} & \rho_{ll} \end{vmatrix} = \rho_{mm}\rho_{nn}\rho_{ll} + \rho_{mn}\rho_{nl}\rho_{lm} + \rho_{nm}\rho_{ln}\rho_{ml} - \rho_{lm}\rho_{nn}\rho_{ml} - \rho_{nm}\rho_{mn}\rho_{ll} - \rho_{ln}\rho_{nl}\rho_{mm} \geq 0. \tag{3.44}$$

The restrictions on the elements of ρ coming from these minors can be given as linear combinations of various powers of the density matrix. The equation (3.42) is the statement that every diagonal element of ρ is non-negative.

In the case of $s = 1/2$ one has

$$\mathrm{Tr}\rho^2 = \rho_{mm}^2 + \rho_{nn}^2 + 2|\rho_{nm}|^2 = 1 + 2(|\rho_{nm}|^2 - \rho_{mm}\rho_{nn}) \tag{3.45}$$

and instead of (3.43) one can write

$$\mathrm{Tr}\rho^2 \leq 1. \tag{3.46}$$

This formula can be deduced from (3.43) also in the case of spin-1 (actually in the case of any spin). This is already familiar formula, which does not restrict substantially the ρ parameters. However, one can show that (3.44) is equivalent to the inequality [46]

$$2\mathrm{Tr}\rho^3 - 3\mathrm{Tr}\rho^2 + 1 \geq 0. \tag{3.47}$$

This new formula restricts substantially the parameters in the spin-1 case. If one expresses the density matrix ρ through its parameters, one gets the restriction to the parameters:

$$\frac{2}{9} - \frac{1}{2}t_i\, t_i + \frac{1}{2}t_i t_j t_{ij} - \frac{1}{9}t_{ij}t_{ij} - \frac{2}{27}t_{ij}t_{jk}t_{ki} \geq 0. \tag{3.48}$$

From this expression one can clearly see how the vector and tensor parameters of spin-1 density matrix depend nonlinearly on each other. The expression is useful when analyzing the possibilities of tuning the final spin-1 particle (Z-boson) orientation by varying the polarizations of the initial beams of electrons and positrons.

3.4 Relativistic spin-1/2 and spin-1 density matrices

In order to calculate the processes under consideration in this thesis the relativistic spin density matrices are needed. One can expect that the relativistic density matrices can be easily obtained by applying the Lorentz boost transformation to the nonrelativistic ones deduced above. This is indeed the case for the spin-1, however, for the spin-1/2 case difficulties arise. The solutions of the Dirac equations describe both particles and antiparticles and in these solutions one can separate the particle and antiparticle ones and with it one can also build for them different spin density matrices. On the contrary, the nonrelativistic density matrix (3.25) does not distinguish between particle and antiparticle. Due to this it is better to construct the relativistic pure state density matrix (the spin projection operator) by boosting the rest frame density matrix built from rest frame Dirac solutions. One can find such a procedure for building relativistic density matrices in many textbooks (see, for example [70]) and this is why we give here only the final result. It is

$$\rho_{\mp} = \frac{1}{2}(\not{k}_{\mp} \pm m)(1 + \gamma_5 \not{s}_{\mp}), \tag{3.49}$$

where the upper sign refers to the particle (electron) and the lower one to the antiparticle (positron). Here and afterwards we use the Feynman slash notation $\not{A} = A_\mu \gamma^\mu$. The polarization four-vectors are given as

$$s^{\mu}_{\mp} = (s^0,\, \vec{s}_{\mp}) = \left(\frac{\vec{k}_{\mp} \cdot \vec{\xi}_{\mp}}{m},\, \vec{\xi}_{\mp} + \frac{(\vec{k}_{\mp} \cdot \vec{\xi}_{\mp})\vec{k}_{\mp}}{m(k_0 - m)} \right), \tag{3.50}$$

where $\vec{\xi}_{\mp}$ are polarization vectors in the rest frames of particles (antiparticles). It turns out that for construction of relativistic spin-1/2 density matrices one does not need the theory of parametrization of non-relativistic spin-1/2 density matrices. However, we hope that presenting such a theory is helpful for a better understanding of the corresponding spin-1 theory.

Density matrices like (3.49) are usually substituted into the squared amplitude instead of $u\bar{u}$ and $v\bar{v}$. Since u and v describe the pure states, the $\rho_{\mp}$ are pure state density matrices and due to this the restriction

$$|\vec{\xi}_{\mp}| = 1 \tag{3.51}$$

is used. However, since the particles are really in mixed spin states, it is more natural to substitute mixed state density matrix into the expression of the squared amplitude. When doing so the density matrix preserves its form. Only the rest frame polarization vector module can now take any values between zero and one ($0 \leq |\vec{\xi}| \leq 1$).

If the initial electron and positron beams have both LP and TP components, it is useful to divide the polarization vector $s_{\mp}$ into LP and TP parts. The limit $m/k^0_{\mp} \to 0$, which is used for the calculations in the thesis, can be conveniently taken by making use of the approximation

$$s^\mu_{\mp} \approx h_{\mp} \frac{k^\mu_{\mp}}{m} + \tau^\mu_{\mp}, \tag{3.52}$$

and subsequently setting $m = 0$. In this equation $h_{\mp}$ is the measure of the LP and $\tau^\mu_{\mp} = (0, \vec{\tau}_{\mp})$ is the TP four-vector with $\vec{\tau}_{\mp}$ as transverse ($(\vec{k}_{\mp} \cdot \vec{\tau}_{\mp}) = 0$) component of the polarization vector. When substituting $s^\mu_{\mp}$ in the form (3.52) into (3.49), after using the limit $m \to 0$, the latter takes the form convenient for the calculations [6]:

$$\rho = \frac{1}{2}(1 \pm h_{\mp}\gamma_5 + \gamma_5 \not{\tau}_{\mp}) \not{k}_{\mp}. \tag{3.53}$$

In our calculations it is assumed that both initial beams, the electron and the positron ones are arbitrarily polarized and this formula is always used. It is assumed that the polarization state of only one of the final particle in the process $e^+e^- \to t\bar{t}$ is observed. When the top quark polarization is measured, one replaces $u(p_t)\bar{u}(p_t)$ in the squared amplitude by the density matrix

$$u(p_t)\bar{u}(p_t) \to \frac{1}{2}(\not{p}_t + M_t)(1 + \gamma_5 \not{s}_t) \tag{3.54}$$

and sums over the spin states of the antitop, *i.e.*

$$v(p_{\bar{t}})\bar{v}(p_{\bar{t}}) \to (\not{p}_{\bar{t}} - M_t). \tag{3.55}$$

If the antitop polarization is measured, one uses the replacements

$$v(p_{\bar{t}})\bar{v}(p_{\bar{t}}) \quad \to \quad \frac{1}{2}(\not{p}_{\bar{t}} - M_t)(1 + \gamma_5 \not{s}_{\bar{t}}) \tag{3.56}$$

$$u(p_t)\bar{u}(p_t) \quad \to \quad \not{p}_t + M_t. \tag{3.57}$$

Contrary to the spin-1/2 case, there are no problems when boosting the nonrelativistic spin-1 density matrix deduced above to get the relativistic one. Since the anti-Z boson coincides with the Z itself, there are no difficulties connected with the particle-antiparticle problem and the relativistic density matrix for spin-1 Z boson can be obtained by simply boosting the nonrelativistic one given by (3.41)

$$\rho_{\mu\nu} = \Lambda_\mu{}^i \rho_{ij} (\Lambda^{-1})^j{}_\nu = \frac{1}{3} \Lambda_\mu{}^i \Lambda_\nu{}^j (\delta_{ij} - \frac{3}{2} \mathrm{i} t_k \epsilon_{ijk} - t_{ij}), \tag{3.58}$$

where the boost operator is given as

$$\Lambda^\kappa{}_\lambda = \begin{pmatrix} \frac{E}{M} & -\frac{p_l}{M} \\ \frac{p_k}{M} & \delta^k_l - \frac{p^k p_l}{M(E+M)} \end{pmatrix}. \tag{3.59}$$

3.5 Polarization of final particles

In the thesis it is supposed that the spin orientation of one of the final particles is observed. If the other final particle has non-zero spin, the summation over its orientation states is performed. Under these conditions the squared amplitude of the processes $e^+e^- \to t\bar{t}$ and $e^+e^- \to Z\gamma$ can be given respectively in the forms

$$|M|^2 \sim S + V_i s_i \tag{3.60}$$

and

$$|M|^2 \sim S + V_i t_i + T_{ij} t_{ij}, \tag{3.61}$$

where S, V_i and T_{ij} are the scalar, vector and tensor built from the polarization parameters of the initial particles (h_-, h_+, $\vec{\tau}_-$, $\vec{\tau}_+$), kinematical parameters of all the particles participating in the process and the coupling constants.

In the processes analyzed in this thesis the kinematical parameters in the CM system are $\vec{k}$, $\vec{p}$ and M, where $\vec{k}$ is the electron momentum and $\vec{p}$, M accordingly the momenta and masses of the top quark or Z boson. At the threshold energies the processes are described only by one vector ($\hat{k}$).

We have used the symbols S_i and V_i for both processes not assuming that they have the same values. The squared amplitudes given above determine the probability that the processes produce the final particles with the spin orientation characterized by the density matrix parameters ξ_i (for t and $\bar{t}$) and t_i, t_{ij} (for Z). On the other hand, the same probabilities can be expressed also as the traced production of two density matrices:

$$\mathrm{Tr}\, \rho^r \rho \sim (1 - s^r s) = 1 + \vec{\xi}^r \vec{\xi} \tag{3.62}$$

in the spin-1/2 case and

$$\mathrm{Tr}\rho^r \, \rho \sim 1 + \frac{3}{2} t_i^r t_i + \frac{1}{3} t_{ij}^r \, t_{ij} \tag{3.63}$$

in the spin-1 case, where the real (actual) density matrices (and their parameters) of the final particles are denoted by the index r. The unindexed symbols are the density matrices and their parameters, which are substituted into the squared amplitude instead of $u\bar{u}\,(v\bar{v})$ or $(\varepsilon_\mu^Z \varepsilon_\nu^{Z\,*})$. By comparing the calculated squared amplitudes (3.60) and (3.61) with the expressions (3.62) and (3.63) one can find the actual polarization vectors and alignment tensors.

As a result one obtains

$$\xi_i^r = \frac{V_i}{S} \tag{3.64}$$

for the process $e^+e^- \to t\bar{t}$ and

$$t_i^r = \frac{2}{3S} V_i, \tag{3.65}$$

$$t_{ij}^r = \frac{3}{S} T_{ij} \tag{3.66}$$

for the process $e^+e^- \to Z\gamma$. Such a method of finding the polarization parameters of the final particles in the spin-1/2 case was first given in [48] and generalized for the spin-1 case in [12, 13].

Chapter 4

Anomalous scalar-type couplings in $e^+e^- \to t\bar{t}$

In this chapter we consider the role of the polarization of the initial beams in the process $e^+e^- \to t\bar{t}$ in searching for indications of possible anomalous scalar particle and unparticle couplings. The main accent in these investigations has been put to the role of transversely polarized initial beams. We find and analyze the analytical expressions for the differential cross section of the process with top or antitop polarization observed and show how the differences between SM and anomalous particle or unparticle coupling contributions provide means to search for anomalous coupling manifestations at future linear colliders.

In our calculations we assume that the amplitudes for the anomalous couplings are much smaller than the amplitudes of SM couplings. Because of this, the squared amplitude of the SM process can be supplemented by the interference of SM and anomalous couplings. The electron mass is taken to be zero. The calculations have been performed in the center-of-mass system without specifying the coordinate system and spin polarization axes.

4.1 Amplitudes of the couplings

In the SM there are two tree-level s-channel Feynman amplitudes, one with γ-exchange and the other with Z-exchange, describing the process

$$e^+(k_+) + e^-(k_-) \to t(p_t) + \bar{t}(p_{\bar{t}}). \tag{4.1}$$

In the approximation used one can write these as

$$\mathcal{M}_\gamma = \frac{2e^2}{3s}\bar{v}(k_+)\gamma^\mu u(k_-)\bar{u}(p_t)\gamma_\mu v(p_{\bar{t}}), \tag{4.2}$$

$$\mathcal{M}_Z = -\frac{g^2}{4\cos^2\theta_W(s-M_Z^2)}\bar{v}(k_+)\gamma^\nu(g_V - g_A\gamma_5)u(k_-)\bar{u}(p_t)\gamma_\nu(c_V - c_A\gamma_5)v(p_{\bar{t}}), \tag{4.3}$$

where $g_V = -\frac{1}{2} + 2\sin^2\theta_W$, g_A and $c_V = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W$, c_A are correspondingly the vector and axial-vector coupling constants of the Z-boson to the electrons and the top quarks and the θ_W is the Weinberg angle. $s = (k_+ + k_-)^2$ is the first Mandelstam variable.

We use the effective anomalous scalar[1] coupling amplitude (particle case) in the form

$$\mathcal{M}_p = K_p\,\bar{v}(k_+)(g_S + ig_P\gamma_5)u(k_-)\;\bar{u}(p_t)(c_S + ic_P\gamma_5)v(p_t), \tag{4.4}$$

where g_S, g_P and c_S, c_P are the scalar and pseudoscalar coupling constants of the electron and the top quark, respectively, $K_p = g_p^2/\Lambda_p^2$ with g_p as a dimensionless coupling constant and Λ_p is the scale of the anomalous coupling.

The propagator for the scalar unparticle has the general form [9, 14]

$$\Delta = \frac{A_{d_u}}{2\sin(d_u\pi)}(-P^2)^{d_u-2}, \tag{4.5}$$

where d_u is the scale dimension and the factor A_{d_u} is given by

$$A_{d_u} = \frac{16\pi^{5/2}\Gamma(d_u+1/2)}{(2\pi)^{2d_u}\Gamma(d_u-1)\Gamma(2d_u)}. \tag{4.6}$$

In the process under consideration mediated by the s-channel unparticle exchange, the propagator features a complex phase,

$$(-P^2)^{d_u-2} = |P^2|^{d_u-2}\,e^{-id_u\pi}. \tag{4.7}$$

The Feynman rules for the interaction of the virtual scalar unparticle with SM fermionic fields can be found in [14]. We use the general case with different coupling constants for scalar and pseudoscalar interactions as well

[1] For simplicity, we use the term "scalar" to refer the combination of scalar and pseudoscalar couplings used in what follows.

as for different flavours. In this case the virtual exchange of a scalar unparticle between two fermionic currents can be expressed by the four-fermion interaction

$$\mathcal{M}_u = \frac{g_u^2\, A_{d_u}\, |P^2|^{d_u-2}\, e^{-i\, d_u \pi}}{2\sin(d_u\pi)(\Lambda_u^2)^{d_u-1}} \bar{v}(k_+)(g_S{+}ig_P\gamma_5)u(k_-)\ \bar{u}(p_t)(c_S{+}ic_P\gamma_5)v(p_{\bar{t}}). \tag{4.8}$$

In this expression we also use the same symbols g_S, g_P, c_S and c_P for the scalar and pseudoscalar coupling constants without assuming that they take the same values as in (4.4). In the CM system one takes $\vec{k}_- = \vec{k}$, $\vec{k}_+ = -\vec{k}$, $\vec{p}_t = \vec{p}$ and $\vec{p}_{\bar{t}} = -\vec{p}$.

After substituting the needed spin density matrices (3.53) and (3.54)-(3.55) or (3.56)-(3.57) into the squared amplitudes, after fairly routine calculations one gets the analytical expressions for the squared amplitude of the process with possible anomalous scalar particle or unparticle corrections.

4.2 The expressions for the differential cross section

Here we present the analytical expressions for the differential cross section contributed from the three sources: from the SM couplings and from the interference of the SM couplings with the anomalous scalar (particle) coupling and scalar unparticle coupling. Each of these three expressions describes two cases – when the top polarization and when the antitop polarization is measured. All these contributions will be considered in the following.

The SM couplings

$$\frac{d\sigma_{SM}}{d\Omega}\Big|_{cm} = \frac{p}{256\pi^2 k^3}|\mathcal{M}_{SM}|^2, \tag{4.9}$$

where

$$\begin{aligned} |\mathcal{M}_{SM}|^2 \ &= \ \mathcal{M}_{\gamma\gamma}^2 + \mathcal{M}_{ZZ}^2 + 2\,\mathrm{Re}\,\mathcal{M}_\gamma \mathcal{M}_Z^* \\ &= \ 8k^2 N_C \Bigg\{ A_1\left(E^2 + p^2\cos^2\theta\right) + A_2 M^2 + 4A_3 Ep\cos\theta \\ &\quad - 2M\left[(A_4\,E + A_6\,p\cos\theta)\,\hat{k}\cdot\vec{s} + (A_5\,\frac{p}{E}\cos\theta + A_6)\,\vec{p}\cdot\vec{s}\right] \end{aligned}$$

$$+ A_7 \left(-\vec{\tau}_- \cdot \vec{\tau}_+ \, p^2 \sin^2\theta + 2\,\vec{p}\cdot\vec{\tau}_- \, \vec{p}\cdot\vec{\tau}_+\right)$$

$$- 2A_8 \, M \left[\vec{\tau}_- \cdot \vec{s}\,\vec{p}\cdot\vec{\tau}_+ + \vec{\tau}_+ \cdot \vec{s}\,\vec{p}\cdot\vec{\tau}_- + \vec{\tau}_- \cdot \vec{\tau}_+ \left(-\vec{p}\cdot\vec{s} + \hat{k}\cdot\vec{s}\,p\cos\theta\right)\right] \Bigg\}, \quad (4.10)$$

$N_C = 3$ is the number of quark colours and

$$\begin{aligned}
A_1 &= K_\gamma^2(1-h_-h_+) + K_Z^2(c_V^2+c_A^2)[(g_V^2+g_A^2)(1-h_-h_+) + 2g_Vg_A(h_+-h_-)] \\
&\quad + 2K_\gamma K_Z c_V[g_V(1-h_-h_+) + g_A(h_+-h_-)], \\
A_2 &= K_\gamma^2(1-h_-h_+) + K_Z^2(c_V^2-c_A^2)[(g_V^2+g_A^2)(1-h_-h_+) + 2g_Vg_A(h_+-h_-)] \\
&\quad + 2K_\gamma K_Z c_V[g_V(1-h_-h_+) + g_A(h_+-h_-)], \\
A_3 &= K_Z^2 c_V c_A[(g_V^2+g_A^2)(h_+-h_-) + 2g_Vg_A(1-h_-h_+)] \\
&\quad + K_\gamma K_Z c_A[g_V(h_+-h_-) + g_A(1-h_-h_+)], \\
A_4 &= K_\gamma^2(h_+-h_-) + K_Z^2 c_V^2[(g_V^2+g_A^2)(h_+-h_-) + 2g_Vg_A(1-h_-h_+)] \\
&\quad + 2K_\gamma K_Z c_V[g_V(h_+-h_-) + g_A(1-h_-h_+)], \\
A_5 &= K_Z^2 c_A^2[(g_V^2+g_A^2)(h_+-h_-) + 2g_Vg_A(1-h_-h_+)], \\
A_6 &= K_Z^2 c_V c_A[(g_V^2+g_A^2)(1-h_-h_+) + 2g_Vg_A(h_+-h_-)] \\
&\quad + K_\gamma K_Z c_A[g_V(1-h_-h_+) + g_A(h_+-h_-)], \\
A_7 &= K_\gamma^2 + K_Z^2(c_V^2+c_A^2)(g_V^2-g_A^2) + 2K_\gamma K_Z c_V g_V, \\
A_8 &= K_Z^2 c_V c_A(g_V^2-g_A^2) + K_\gamma K_Z c_A g_V \qquad (4.11)
\end{aligned}$$

with

$$K_\gamma = \frac{Q_f e^2}{4k^2}, \qquad K_Z = -\frac{e^2}{\sin^2(2\theta_W)(4k^2-M_Z^2)}. \qquad (4.12)$$

We use the three LP-dependent coefficients A_i $(i = 1,2,3)$ for the unpolarized final state and the three LP-dependent coefficients A_i $(i = 4,5,6)$ for the polarized final state. The two coefficients A_i $(i = 7,8)$, which do not depend on the LP parameters $h_\pm$ are used for contributions which depend on the initial state transverse polarization for unpolarized (A_7) and polarized final state (A_8). The coefficients are used to disentangle the coupling constants and LP parameters from the kinematical parts as much as possible. $Q_f = +2/3$ is the electric charge of the top quark. $\hat{k} = \vec{k}/k$ is the unit vector given by the momentum $\vec{k}$, and $k = |\vec{k}|$ is the energy of the electron. $E = k$ is the top quark energy, and $\vec{p}$ is the momentum of the top quark $(p = \sqrt{E^2 - M^2})$. Finally, θ and $\vec{s}$ are the scattering angle (with $\cos\theta = \hat{k}\cdot\vec{p}/p$) and the polarization vector of the top quark. Both the top and antitop polarization measured cases have been described in terms of the top momentum and scattering angle. We have also used the same notation $\vec{s}$ for the top and antitop polarization vectors $\vec{s}_t$ and $\vec{s}_{\bar{t}}$. As a result the expressions in top and antitop polarization measured cases entirely coincide. If one would like to describe the antitop case in terms of antitop parameters, one has to take $\vec{p}$ and $\cos\theta$ with the opposite signs $(-\vec{p},\ -\cos\theta)$. This procedure changes the signs in front of a part of the terms in (4.10) and one must use upper and lower signs to distinguish top and antitop cases.

We have not used the Mandelstam variables because this makes the expressions cumbersome and less clear for their further analysis. For the same reason we have not expressed the top quark's energy and momentum by the energy and momentum of the electron.

<u>The interference of the SM and anomalous scalar particle coupling</u>

In taking into account anomalous scalar and pseudoscalar contributions, we can assume that the factor K_p is small. Therefore, we can skip the contribution $|\mathcal{M}_p|^2$ and obtain

$$\frac{d\sigma_{SM+p}}{d\Omega}\Big|_{cm} \approx \frac{d\sigma_{SM}}{d\Omega}\Big|_{cm} + \frac{p}{256\pi^2 k^3} K_p \mathcal{B}_a, \tag{4.13}$$

where

$$\begin{aligned}
\mathcal{B}_a &= 2\mathrm{Re}\mathcal{M}_{SM}\mathcal{M}_a^*/K_p = \\
&= -16k^2 N_C \Big\{ (K_\gamma + K_Z g_V c_V)\Big[g_S\,\hat{k}\times(\vec{\tau}_- + \vec{\tau}_+) - g_P\,(\vec{\tau}_- - \vec{\tau}_+)\Big]\cdot \\
&\quad \cdot\Big[c_S E\vec{p}\times\vec{s} \pm c_P(\vec{p}\cdot\vec{s}\,\vec{p} - E^2\vec{s})\Big] + K_Z g_A\Big[g_S(\vec{\tau}_- + \vec{\tau}_+) + g_P\hat{k}\times(\vec{\tau}_- - \vec{\tau}_+)\Big]\cdot \\
&\quad \cdot\Big[c_V c_S\, M\vec{p} \pm c_A c_P\, E\vec{p}\times\vec{s} - c_A c_S(\vec{p}\cdot\vec{s}\,\vec{p} - p^2\vec{s})\Big] + \\
&\quad + \Big[g_S(h_+\vec{\tau}_- - h_-\vec{\tau}_+) + g_P\hat{k}\times(h_+\vec{\tau}_- + h_-\vec{\tau}_+)\Big]\cdot \\
&\quad \cdot\Big[(K_\gamma + K_Z g_V c_V)c_S M\vec{p} + K_Z g_V c_A(\pm c_P E\vec{p}\times\vec{s} - c_S(\vec{p}\cdot\vec{s}\,\vec{p} - p^2\vec{s}))\Big] + \\
&\quad + K_Z g_A c_V\Big[g_S\hat{k}\times(h_+\vec{\tau}_- - h_-\vec{\tau}_+) - g_P(h_+\vec{\tau}_- + h_-\vec{\tau}_+)\Big]\cdot \\
&\quad \cdot\Big[c_S E\vec{p}\times\vec{s} \pm c_P(\vec{p}\cdot\vec{s}\,\vec{p} - E^2\vec{s})\Big]\Big\}
\end{aligned} \tag{4.14}$$

<u>The interference of the SM and anomalous scalar unparticle coupling</u>

Apart from the different overall constants, the real part of the complex phase in the unparticle amplitude in (4.8) leads to the same expression $\mathcal{B}_a$ as the scalar particle coupling amplitude. Therefore, one can write

$$\frac{d\sigma_{SM+u}}{d\Omega}\Big|_{cm} \approx \frac{d\sigma_{SM}}{d\Omega}\Big|_{cm} + \frac{p}{256\pi^2 k^3} K_u \left(\cos(d_u\pi)\mathcal{B}_a + \sin(d_u\pi)\mathcal{B}_b\right), \tag{4.15}$$

where $\mathcal{B}_a$ is given in (4.14) and

$$\begin{aligned}
\mathcal{B}_b \; = \; 2\mathrm{Re}\mathcal{M}_{SM}\mathcal{M}_u^*/K_u \; = \; -16k^2 N_C \Big\{ & \Big[g_S \hat{k}\times(\vec{\tau}_- + \vec{\tau}_+) - g_P(\vec{\tau}_- - \vec{\tau}_+)\Big] \cdot \\
& \cdot \Big[(K_\gamma + K_Z c_V g_V) c_S M\vec{p} + K_Z c_A g_V \left(\pm c_P E\vec{p}\times\vec{s} - c_S(\vec{p}\cdot\vec{s}\,\vec{p} - p^2\vec{s})\right)\Big] + \\
& - K_Z c_V g_A \Big[g_S(\vec{\tau}_- + \vec{\tau}_+) + g_P\hat{k}\times(\vec{\tau}_- - \vec{\tau}_+)\Big]\cdot\Big[c_S E\vec{p}\times\vec{s} \pm c_P(\vec{p}\cdot\vec{s}\,\vec{p} - E^2\vec{s})\Big] + \\
& - (K_\gamma + K_Z g_V c_V)\Big[g_S(h_+\vec{\tau}_- - h_-\vec{\tau}_+) + g_P\hat{k}\times(h_+\vec{\tau}_- + h_-\vec{\tau}_+)\Big]\cdot \\
& \cdot \Big[c_S E\vec{p}\times\vec{s} \pm c_P(\vec{p}\cdot\vec{s}\,\vec{p} - E^2\vec{s})\Big] + \\
& + K_Z g_A \Big[g_S\hat{k}\times(h_+\vec{\tau}_- - h_-\vec{\tau}_+) - g_P(h_+\vec{\tau}_- + h_-\vec{\tau}_+)\Big]\cdot \\
& \cdot \Big[c_V c_S M\vec{p} \pm c_A c_P E\vec{p}\times\vec{s} - c_A c_S(\vec{p}\cdot\vec{s}\,\vec{p} - p^2\vec{s})\Big]\Big\}
\end{aligned} \tag{4.16}$$

and

$$K_u = \frac{g_u^2\, A_{d_u}\, |P^2|^{d_u-2}}{2\sin(d_u\pi)(\Lambda_u^2)^{d_u-1}} \tag{4.17}$$

(note that $|P^2| = 4k^2$). In the following we consider the region $1 \leq d_u < 2$. For $d_u = 1$ the unparticle contribution is given by $\mathcal{B}_a$ which is already included in the contribution of the anomalous scalar and pseudoscalar interactions. On the other hand, for $d_u = 3/2$ the contribution is given purely by $\mathcal{B}_b$. However, we do not restrict to these two values but consider the whole interval.

4.3 The main features of the contributions

Using the approximation in (4.13) or (4.15) where the squared amplitude $|\mathcal{M}_p|^2$ resp. $|\mathcal{M}_u|^2$ is neglected, the process $e^+e^- \to t\bar{t}$ is fully described by the analytical expressions for the SM and the anomalous scalar particle and unparticle coupling contributions. In this section we report about observations on the SM and anomalous coupling contributions. We present and analyze the main features of the contributions and the differences between the SM and anomalous coupling contributions as well as between the scalar particle and unparticle coupling contributions. These differences are helpful in disentangling the different contributions at future e^+e^- colliders. Part of the features given below are already known. We present them for completeness only.

Standard Model versus anomalous coupling contributions

In comparing the SM contribution with the contribution from the scalar particle or unparticle coupling, we come to the following conclusions:

1. The SM contributions depend on the longitudinal polarization of the initial beams through the coefficients A_i ($i = 1, \ldots, 6$) which contains the LP parameters h_- and h_+ as well as the coupling constants (g_V, g_A, c_V and c_A). The coefficients A_i contain both linear and quadratic terms in the LP parameters. By changing the values of h_- and h_+ one can substantially increase or decrease the coefficients A_i and by this selected parts of the coupling. However, one cannot form observables different from those of the unpolarized beams. The anomalous scalar (particle and unparticle) coupling contributions depend linearly on the longitudinal polarization. However, the LP-depending terms cannot occur without the existence of TP vectors: the LP parameters h_- and h_+ are always multiplied by the vectors $\vec{\tau}_-$, $\vec{\tau}_+$ in combinations $h_-\vec{\tau}_+$ and $h_+\vec{\tau}_-$.

2. In the SM contributions the TP dependent terms depend quadratically on the TP vectors. Due to this they are different from zero only when both of the initial beams have TP components. In the anomalous coupling contributions all the terms have to be and are TP dependent. They depend linearly on the TP vectors without or with the multiplicative LP parameters and, as a consequence, can be different from zero also in the case where only one of the initial beams is transversely polarized. The linear dependence provides a crucial tool at future e^+e^- linear colliders for isolating signatures of anomalous scalar couplings from the SM ones.

3. In the SM contributions all the terms depending on the final state polarizations are proportional to the final state fermion mass while the terms independent of the final state polarizations for the most part are independent of this mass. On the other hand, for the anomalous coupling contributions the term independent of the final state polarization is proportional to the final state fermion mass which is not the

case for most of the terms depending on the final state polarization. This fact stresses the advantages of investigating final state polarization effects in e^+e^- annihilation just for top-antitop pair productions.

4. The SM contributions are invariant with respect to the interchange $\vec{\tau}_- \rightleftarrows \vec{\tau}_+$. Since applying the CP transformation to the process causes the same changes, the above given invariance once more reflects the CP conservation at that level. On the other hand, both the scalar particle and unparticle contributions depend on the TP vectors through the four combinations
$$\left[g_S\hat{k}\times(\vec{\tau}_- + \vec{\tau}_+) - g_P(\vec{\tau}_- - \vec{\tau}_+)\right], \tag{4.18}$$
$$\left[g_S(\vec{\tau}_- + \vec{\tau}_+) + g_P\hat{k}\times(\vec{\tau}_- - \vec{\tau}_+)\right], \tag{4.19}$$
$$\left[g_S(h_+\vec{\tau}_- - h_-\vec{\tau}_+) + g_P\hat{k}\times(h_+\vec{\tau}_- + h_-\vec{\tau}_+)\right], \tag{4.20}$$
$$\left[g_S\hat{k}\times(h_+\vec{\tau}_- - h_-\vec{\tau}_+) - g_P(h_+\vec{\tau}_- + h_-\vec{\tau}_+)\right]. \tag{4.21}$$
The fact that under CP the TP vectors $\vec{\tau}_-$ and $\vec{\tau}_+$ interchange suggests that there have to be CP-odd terms in the anomalous coupling contributions and that CP invariance is violated in the process.

5. Expressing the results in terms of the momentum and scattering angle of the top quark, the SM contributions to the differential cross section is independent on whether the top or antitop polarization $\vec{s}$ is measured. This is not the case for the anomalous contributions. Here the terms containing the coupling constant c_P have opposite signs for the case of top and antitop polarization measurement. As we will see later, this leads to different CP-odd parts in the c_P- and c_S-depending terms.

6. The TP dependent terms of the SM contributions vanish at the threshold of the process. On the other hand, in the anomalous coupling contributions there exist terms that survive at the threshold. This gives an additional tool for separating anomalous coupling contributions from the SM ones.

Scalar particle versus unparticle coupling contributions

In comparing the contributions including scalar particle and unparticle couplings, we obtain the following conclusions:

1. The scalar particle and unparticle coupling contributions in (4.14) and (4.16) depend on the same combinations of TP vectors $\vec{\tau}_-$ and $\vec{\tau}_+$ as given in (4.18)–(4.21). This makes it difficult to separate these contributions.

2. However, in the scalar particle and the unparticle coupling contributions the TP-dependent combinations in (4.18)–(4.21) are multiplied by different final state expressions. In principle, this enables to disentangle the different contributions by measuring the final state polarizations.

4.4 CP violation analysis

CP violation in weak interactions was first reported for the neutral K-meson system [15]. Further examples were found for D and B meson systems [16, 17]. Apart from this, the CP violation due to SM interactions is predicted to be unobservably small [8, 18]. Hence, one of the important indications of new physics would be the observation of CP violation outside the aforementioned systems.

In this section we demonstrate that due to anomalous scalar particle or unparticle coupling corrections to the SM contribution the CP symmetry in $e^+e^- \to t\bar{t}$ is violated. We investigate how the interference between SM and anomalous couplings gives rise to CP-odd quantities in case of transversely polarized initial beams and construct the CP-odd asymmetries sensitive to CP violation. For testing CP violation in the process it is not sufficient to measure only the momenta $\vec{k}$ and $\vec{p}$ because the only scalar observable which can be constructed from these vectors is $\vec{k}\cdot\vec{p}$ which is CP-even. Therefore, either initial or final state polarization vectors are needed. In the case under consideration the TP initial beams are mandatory: the interference between SM and scalar anomalous couplings are non-vanishing only with TP initial beams. The possibility to test CP violation in $e^+e^- \to t\bar{t}$ with TP beams in the presence of scalar- and tensor-type anomalous couplings was first demonstrated in [8] without measured final state polarization.

The SM and anomalous contributions given by (4.10), (4.14) and (4.16) enable to construct CP-odd asymmetries for TP initial beams both in the case of observed and non-observed final top (antitop) polarization for scalar particle and unparticle interactions. For both initial beams transversely polarized, we take $h_- = h_+ = 0$.

<u>CP violation for unpolarized final state quarks</u>

Let us first consider the case where the final particle spin states are not observed. In this case both the scalar particle and unparticle coupling contributions in (4.14) and (4.16) do not depend on the final state top (antitop) polarization vector $\vec{s}$. In taking $h_- = h_+ = 0$ as proposed, only a single term remains in both contributions $\mathcal{B}_a$ and $\mathcal{B}_b$. For the particle coupling contribution this term contains the TP-depending factor (4.19),

$$\mathcal{B}_a = -16k^2 N_C K_Z c_V g_A [g_S(\vec{\tau}_- + \vec{\tau}_+) + g_P \hat{k} \times (\vec{\tau}_- - \vec{\tau}_+)] \cdot c_S M \vec{p}. \quad (4.22)$$

The CP transformation interchanges the TP vectors of the electron and the positron whereas the momenta $\vec{k}$ and $\vec{p}$ remain unchanged. As a consequence, the second part depending on the difference $(\vec{\tau}_- - \vec{\tau}_+)$ in (4.22) changes sign under the CP transformation (*i.e.* it is CP-odd). Therefore, CP is violated in the process. One can construct an asymmetry which is sensitive to CP violation in case where the TP vectors $\vec{\tau}_-$ and $\vec{\tau}_+$ of the electron and positron have opposite directions. If we use a coordinate system where the z-axis is determined by the electron momentum $\vec{k}$, we can direct the x-axis along the electron and opposed to the positron TP vectors. The situation is illustrated in Fig. 4.1(a). Such a choice leads to the CP-odd quantity

$$\frac{\hat{k} \times (\vec{\tau}_- - \vec{\tau}_+) \cdot \vec{p}}{|\vec{\tau}_- - \vec{\tau}_+| p} = \sin\theta \sin\phi \quad (4.23)$$

in the differential cross section, where ϕ is the azimuthal angle of the process.

In the unparticle case the contribution $\cos(d_u\pi)\mathcal{B}_a + \sin(d_u\pi)\mathcal{B}_b$ with

$$\mathcal{B}_b = -16k^2 N_C (K_\gamma + K_Z c_V g_V)[g_S \hat{k} \times (\vec{\tau}_- + \vec{\tau}_+) - g_P(\vec{\tau}_- - \vec{\tau}_+)] \cdot c_S M \vec{p} \quad (4.24)$$

contains both TP dependent factors (4.18) and (4.19), mixed by the angle $d_u\pi$. The part in (4.18) causing CP violation in the process is $g_P(\vec{\tau}_- - \vec{\tau}_+) \cdot \vec{p}$. If the unparticle dimension is given by a specific model, the CP-odd quantity in the differential cross section corresponding to (4.23) is achieved when vectors $\vec{\tau}_-$ and $\vec{\tau}_+$ are taken to be opposite and directed along an axis which is rotated by α with

$$\tan\alpha = -\frac{\mathcal{A}_a}{\mathcal{A}_b} \tan(d_u\pi), \qquad \mathcal{A}_a := K_\gamma + K_Z c_V g_V, \quad \mathcal{A}_b := K_Z c_V g_A \quad (4.25)$$

starting from the positive and negative direction of the x-axis, respectively (cf. Fig. 4.1(b)). In this case we obtain a CP-odd quantity

$$\frac{[\hat{k} \times (\vec{\tau}_- - \vec{\tau}_+) \cos\alpha + (\vec{\tau}_- - \vec{\tau}_+) \sin\alpha] \cdot \vec{p}}{|\vec{\tau}_- - \vec{\tau}_+| p} = \sin\theta \sin\phi. \quad (4.26)$$

In both cases one can construct the CP-odd asymmetry

$$\mathcal{A}(\theta) = \frac{\displaystyle\int_0^{\pi} \frac{d\sigma}{d\Omega} d\phi - \int_{\pi}^{2\pi} \frac{d\sigma}{d\Omega} d\phi}{\displaystyle\int_0^{\pi} \frac{d\sigma}{d\Omega} d\phi + \int_{\pi}^{2\pi} \frac{d\sigma}{d\Omega} d\phi}, \tag{4.27}$$

where $\sigma = \sigma_{SM+p}$ or σ_{SM+u}, resp. Such a quantity for the scalar- and tensor-type (particle) couplings was first constructed and analyzed by Ananthanarayan and Rindani [8]. They estimated the sensitivity of planned future colliders to new physics CP violation in $e^+e^- \to t\bar{t}$ and showed the possibility to put a bound of approx. 7 TeV on the new-physics scale.

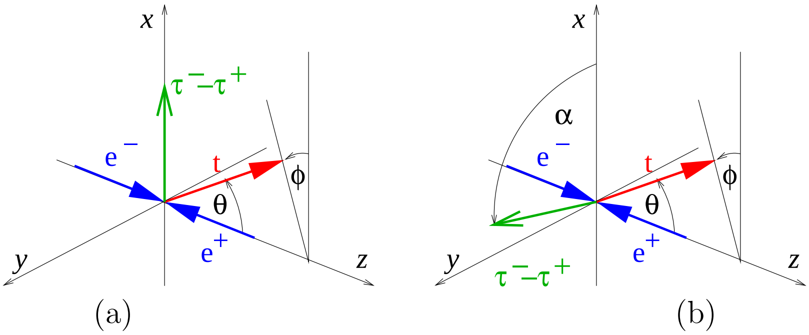

Figure 4.1: Choice of the kinematics for the analysis of scalar particle (a) and unparticle interactions (b) with $\tan\alpha = -(\mathcal{A}_a/\mathcal{A}_b)\tan(d_u\pi)$, where $\mathcal{A}_{a,b}$ are given in (4.25).

CP violation and final state polarization

CP-odd contributions are also observed in the terms which depend on the final state top or antitop polarizations. These polarizations can be determined by analysing the distributions of the final state charged leptons from the top (or antitop) decay. This method is viable in the top quark case because the top quark is so massive that it decays before it can hadronize, therefore avoiding masking nonperturbative effects. Of course, the observation of the CP violation through the measurement of the final state polarization means a loss of statistics. On the other hand, this shortage might be partly softened by the fact that most of the polarization depending terms are not proportional to the top mass and can be quite large as compared to terms independent of the top polarization.

If we divide the polarization vector $\vec{s}$ of the final top or antitop quark into a longitudinal and a transverse part,

$$\vec{s} = \vec{s}_L + \vec{s}_T = \frac{E}{Mp} h\vec{p} + \vec{\tau}, \tag{4.28}$$

there is only a single term in both $\mathcal{B}_a$ and $\mathcal{B}_b$ that depends on $\vec{s}_L$. While $\vec{p}\cdot\vec{s}_L\vec{p} - p^2\vec{s}_L = 0$, the factor

$$\vec{p}\cdot\vec{s}_L\vec{p} - E^2\vec{s}_L = -\frac{EMh}{p}\vec{p} \tag{4.29}$$

is proportional to the top mass. This factor is multiplied by the $\vec{\tau}_+$- and $\vec{\tau}_-$-depending expressions (4.18) in the particle and both (4.18) and (4.19) in the unparticle case. Besides this, the terms containing these factors are c_P-dependent and therefore, as mentioned in point 5 of Sec. 3.1, have different signs if both the contributions from top and antitop polarization measurements are given by the top parameters $(\vec{p}, \theta)$. Due to this the CP-odd terms depend on $(\vec{\tau}_- + \vec{\tau}_+)$ in the combinations $h\hat{k}\times(\vec{\tau}_- + \vec{\tau}_+)\cdot\vec{p}$ for the particle case and in addition on $h(\vec{\tau}_- + \vec{\tau}_+)\cdot\vec{p}$ for the unparticle case.

The terms depending on the transverse polarization of the final top (antitop) are not proportional to the top mass. One can divide these terms into c_S-depending and c_P-depending parts. In the c_S-depending terms the CP-odd parts depend on the difference of the $\vec{\tau}_-$ and $\vec{\tau}_+$ vectors while in the c_P-depending parts they depend on the sum of these vectors. However, the CP-odd parts in the corresponding terms of scalar particle and scalar unparticle contributions depend differently on these vectors. If the CP-odd part of some scalar particle contribution term contains the factor $\vec{\tau}_- - \vec{\tau}_+$ (or $\vec{\tau}_- + \vec{\tau}_+$), the corresponding term in unparticle case depends in addition on $\hat{k}\times(\vec{\tau}_- - \vec{\tau}_+)$ (or $\hat{k}\times(\vec{\tau}_- + \vec{\tau}_+)$) and *vice versa*. This circumstance might enable, at least in principle, to separate CP-odd asymmetries in scalar particle and unparticle cases.

4.5 Final state polarizations

In this section we consider the actual polarizations of the final top or antitop quarks. The final quark polarizations provide additional tools for studying the mechanisms of the process and for separating the anomalous coupling contributions from the SM ones. It is well known that in the Born approximation the process $e^+e^- \to t\bar{t}$ with unpolarized or longitudinally polarized initial beams produces final quarks with polarization vector lying in the scattering plane [19]. When using TP beams the TP vectors $\vec{\tau}_-$ and $\vec{\tau}_+$ move the final top or antitop polarization vectors out of scattering plain. Therefore, in the approximation used the deviation of the final quark polarization vectors from the reaction plain is only due to the TP initial beams. SM contributes to TP-dependent terms only if both of the initial beams are transversely polarized. If only one of the initial beams is transversely polarized, such a deviation would indicate the presence of anomalous couplings.

Top polarization for the SM

Let us consider the final state polarization in more detail at the threshold of the process. At threshold the analytical expressions for the differential cross sections in (4.10), (4.14) and (4.16) simplify considerably and the polarization properties of the quarks are displayed more clearly. We start our investigations from the SM sector considering the polarization properties of the top (antitop) quarks more generally. Since the TP-dependent terms vanish at the threshold, the main question will be how much one can tune the top (antitop) quark polarization by varying the LP parameters h_+ and h_- of the initial beams. The result for the polarization turns out to depend effectively on the parameter

$$\chi = \frac{h_+ - h_-}{1 - h_+ h_-}. \tag{4.30}$$

At threshold the squared SM amplitude takes the form

$$|\mathcal{M}_{SM}|^2|_{\text{thres}} = 24M^4 \left[A_1 + A_2 - 2A_4 \hat{k} \cdot \vec{\xi}\right], \tag{4.31}$$

where $\vec{\xi}$ is the top quark polarization vector and K_γ and K_Z have their threshold forms

$$K_\gamma = \frac{e^2}{6M^2}, \qquad K_Z = -\frac{e^2}{\sin^2(2\theta_W)(4M^2 - M_Z^2)}. \tag{4.32}$$

Using the method given in [13] one can find the magnitude and direction of the actual polarization vector of the top quark,

$$\vec{\xi}_{SM} = -\frac{B(\chi)\hat{k}}{A(\chi)}, \tag{4.33}$$

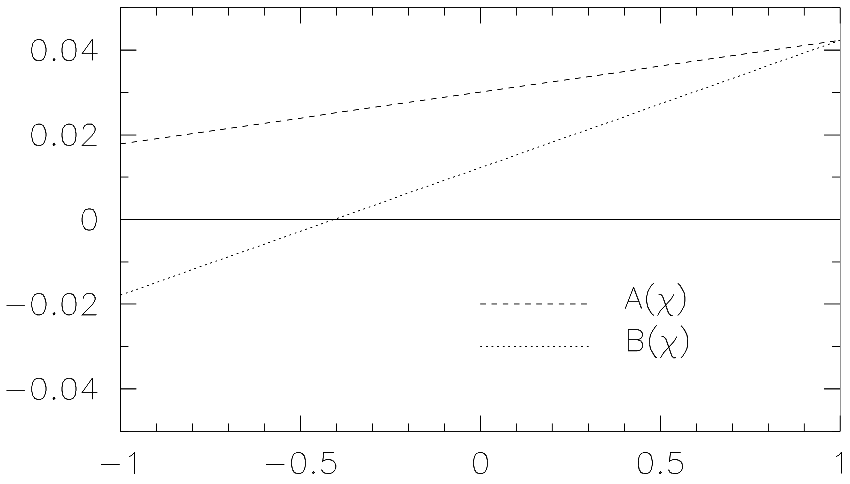

Figure 4.2: The dependence of $A(\chi)$ and $B(\chi)$ on χ.

where

$$A(\chi) = \frac{A_1 + A_2}{2(1 - h_+h_-)} = a_1 + a_2\chi,$$
$$B(\chi) = \frac{A_4}{1 - h_+h_-} = a_1\chi + a_2 \tag{4.34}$$

with

$$a_1 = K_\gamma^2 + K_Z^2 c_V^2 (g_V^2 + g_A^2) + 2K_\gamma K_Z c_V g_V,$$
$$a_2 = 2K_Z c_V g_A (K_\gamma + K_Z c_V g_V). \tag{4.35}$$

In Fig. 4.2 the dependence of $A(\chi)$ and $B(\chi)$ on χ is given.

For the SM sector we use the values of the coupling constants and other parameters as given by the Particle Data Group [20], $g_V = -0.037$, $g_A = -0.5$, $c_V = 0.191$, $c_A = 0.5$, $g = e/\sin\theta_W$, $\sin^2\theta_W = 0.2415$, $M_t = 171.2\,\mathrm{GeV}$, and $M_Z = 91.2\,\mathrm{GeV}$. We draw the attention to the fact that for $\chi_0 = -0.408$ we obtain $B(\chi_0) = 0$. Therefore, at this value $\chi = \chi_0$ the top polarization in the process appears only due to the anomalous coupling contributions. At the same time $A(\chi_0)$ is smaller than at the point $\chi = 0$

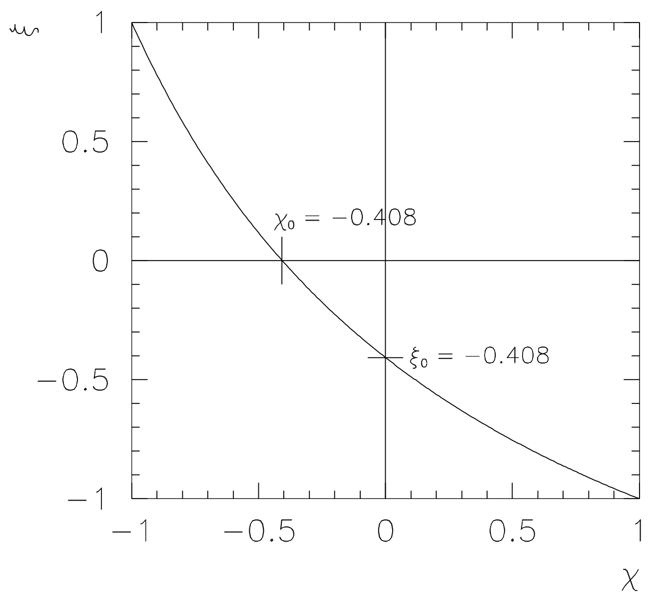

Figure 4.3: The top polarization in SM at threshold as a function of χ.

and as a consequence the top polarization from anomalous couplings is larger than in the case of unpolarized initial beams.

Fig. 4.3 shows how much the top polarization vector can be tuned by χ as compared to the case $\chi = 0$, where the polarization is given by [21]

$$\vec{\xi}_{SM}|_{\chi=0} = -0.408\hat{k}. \tag{4.36}$$

The fact that the magnitude of the top polarization vector at $\chi = 0$ given in (4.36) is equal to the value of χ at which $\vec{\xi}_{SM}$ vanishes is not an occasional coincidence but a consequence of the special shape of the structure functions $A(\chi)$ and $B(\chi)$ in (4.34). The polarization function ξ_{SM} in (4.33) is of the same shape as the reciprocal function

$$\chi(\xi_{SM}) = -\frac{a_1 + a_2\xi_{SM}}{a_1\xi_{SM} + a_2}. \tag{4.37}$$

As a consequence, $\xi_{SM}(\chi = 0) = \chi(\xi_{SM} = 0) = -a_1/a_2 = -0.408$. One can write the functions $\xi_{SM}(\chi)$ and $\chi(\xi_{SM})$ in a more compact forms

$$\xi_{SM}(\chi) = -\frac{\chi + a}{a\chi + 1} \tag{4.38}$$

and

$$\chi(\xi_{SM}) = -\frac{\xi_{SM} + a}{a\xi_{SM} + 1} \tag{4.39}$$

where $a = a_1/a_2$.

Anomalous coupling corrections to the SM top polarization

The anomalous scalar particle coupling corrections to the SM contribution at threshold are given by

$$\begin{aligned} \mathcal{B}_a|_{\text{thres}} &= \pm 48 M^4 c_P \vec{\xi} \Big\{ (K_\gamma + K_Z c_V g_V)[g_S \hat{k} \times (\vec{\tau}_- + \vec{\tau}_+) - g_P(\vec{\tau}_- - \vec{\tau}_+)] + \\ &\quad + K_Z c_V g_A [g_S \hat{k} \times (h_+ \vec{\tau}_- - h_- \vec{\tau}_+) - g_P(h_+ \vec{\tau}_- + h_- \vec{\tau}_+)] \Big\}. \end{aligned} \tag{4.40}$$

For the anomalous scalar unparticle corrections one obtains in addition

$$\begin{aligned} \mathcal{B}_b|_{\text{thres}} &= \pm 48 M^4 c_P \vec{\xi} \Big\{ (K_\gamma + K_Z c_V g_V)[g_S (h_+ \vec{\tau}_- - h_- \vec{\tau}_+) + g_P k \times (\hat{h}_+ \vec{\tau}_- + h_- \vec{\tau}_+)] + \\ &\quad + K_Z c_V g_A [g_S(\vec{\tau}_- + \vec{\tau}_+) + g_P \hat{k} \times (\vec{\tau}_- - \vec{\tau}_+)] \Big\}. \end{aligned} \tag{4.41}$$

For $h_- = h_+ = 0$, the corresponding corrections to the top polarization vector are

$$\vec{\xi}_p = K_p \vec{\xi}_a, \qquad \vec{\xi}_u = K_u(\cos(d_u\pi)\vec{\xi}_a + \sin(d_u\pi)\vec{\xi}_b) \tag{4.42}$$

with

$$\begin{aligned} \vec{\xi}_a &= \frac{\mathcal{A}_a c_P}{A(0)}[g_S \hat{k} \times (\vec{\tau}_- + \vec{\tau}_+) - g_P(\vec{\tau}_- - \vec{\tau}_+)], \\ \vec{\xi}_b &= \frac{\mathcal{A}_b c_P}{A(0)}[g_S(\vec{\tau}_- + \vec{\tau}_+) + g_P \hat{k} \times (\vec{\tau}_- - \vec{\tau}_+)], \end{aligned} \tag{4.43}$$

where $\mathcal{A}_{a,b}$ are defined in (4.25). In calculating values for the polarizations, we have to give values to the anomalous coupling constants g_S, g_P, c_S and c_P. Scalar particle couplings arise in many extensions of the SM. However, up to now there exist no definite predictions about their values [8]. On the other hand, the unparticle phenomenology stands beyond the other SM extension models. Therefore, one has to make here quite voluntary presumptions that do not lay on definite theoretical grounds. Here we use the "SM-connected" setting $g_S = g_V$, $g_P = g_A$, $c_S = c_V$, $c_P = c_A$, and $g_p = g_u = g$.

The corrections to the top or antitop quark polarization due to anomalous couplings are transverse to the top (antitop) quark polarizations due to the SM which is antiparallel to the direction $\hat{k}$ of the initial beams. The angle between these two components is

$$\tan\varphi_{p,u} = \frac{|\vec{\xi}_{p,u}|}{|\vec{\xi}_{SM}|}. \tag{4.44}$$

Using $h_\pm = 0$, $\tau_+ = 0$ and $\tau_- = 0.8$ in order to eliminate the TP dependent SM terms also close to the exact threshold, the vector $\vec{\xi}_u$ is a vector in the plane spanned by $\vec{\tau}_-$ and $\hat{k}\times\vec{\tau}_-$ orthogonal to $\hat{k}$. However, a better reference frame to consider is the one spanned by the orthonormal basis

$$\hat{e}_a = \frac{g_S\hat{k}\times\vec{\tau}_- - g_P\vec{\tau}_-}{\sqrt{g_S^2+g_P^2}|\vec{\tau}_-|}, \qquad \hat{e}_b = \frac{g_S\vec{\tau}_- + g_P\hat{k}\times\vec{\tau}_-}{\sqrt{g_S^2+g_P^2}|\vec{\tau}_-|} \tag{4.45}$$

The situation is illustrated in Fig. 4.4. For different values of d_u the vector runs on a ellipse with half axis of length $\mathcal{A}_a\sqrt{g_S^2+g_P^2}|\vec{\tau}_-|$ along $\hat{e}_a$ and half axis of length $\mathcal{A}_b\sqrt{g_S^2+g_P^2}|\vec{\tau}_-|$ along $\hat{e}_b$. The angle in negative mathematical order with respect to $\hat{e}_a$ is given by α', where $\tan\alpha' = (\mathcal{A}_b/\mathcal{A}_a)\tan(d_u\pi)$. Together with the d_u-dependence given by K_u in (4.44) we can calculate the dependence of the deviation angle φ_u on d_u in the region $1 \leq d_u < 2$ for different values of the scale Λ_u. The result is shown in Fig. 4.5. Apparently, close to $d_u = 1$ the value of the angle does no longer depend on the scale Λ_u but takes a constant value because, using $\lim_{d_u\to 1}\sin(d_u\pi)\Gamma(1-d_u) = -\pi$, $K_{d_u=1}$ is given by $K_1 = -g_u^2/4k^2$. In Fig. 4.6 we show the dependence of the scale Λ_u on the deviation angle φ_u for the values $d_u = 1.3$, 1.5, 1.7, and 1.9. Finally, in Fig. 4.7 we show the dependence of the scale Λ_p on the deviation angle φ_p for the anomalous scalar particle.

Our studies once more demonstrate the utility of using TP initial beams for searching new physics indications. The additional directions provided by TP vectors can be successfully used for constructing new measurable quantities both in the presence of final top (antitop) polarization and its

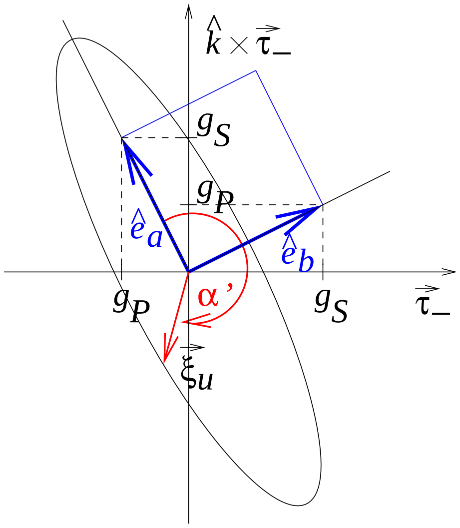

Figure 4.4: The vector $\vec{\xi}_u$ in the plane spanned by $\vec{\tau}_-$ and $\hat{k} \times \vec{\tau}_-$ for exemplary values for c_S and c_P and arbitrary scale for the coefficients, where $\tan\alpha' = (\mathcal{A}_b/\mathcal{A}_a)\tan(d_u\pi)$.

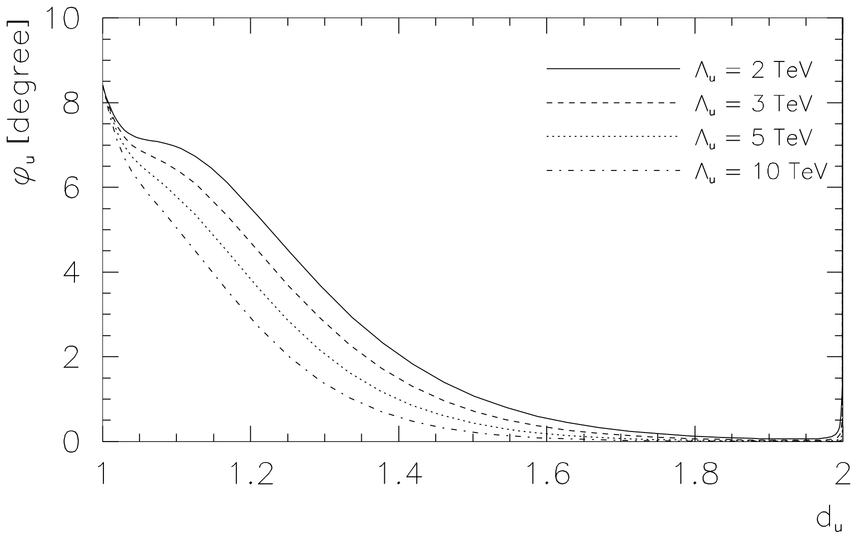

Figure 4.5: Deviation angle φ_u in dependence on d_u for scales $\Lambda_u = 2$, 3, 5, and 10 TeV

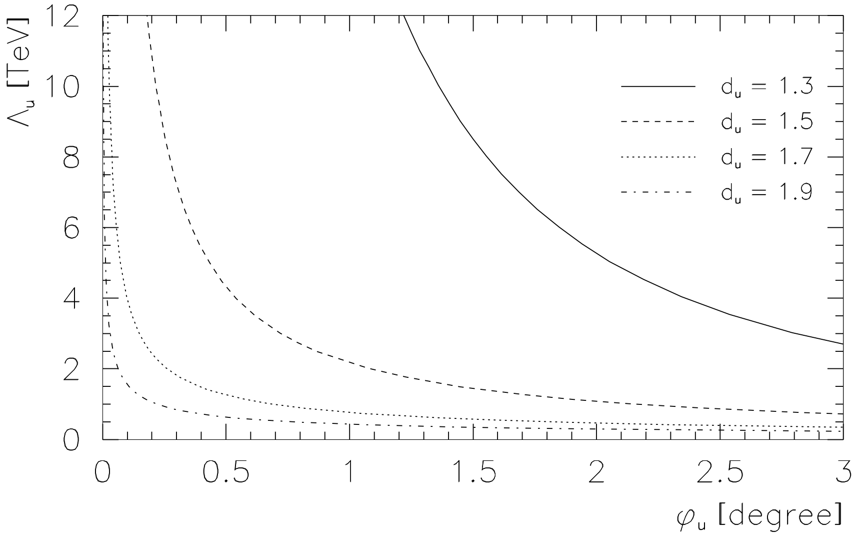

Figure 4.6: Scale Λ_u in dependence on the deviation angle φ_u for unparticle dimensions $d_u = 1.3$, 1.5, 1.7, and 1.9

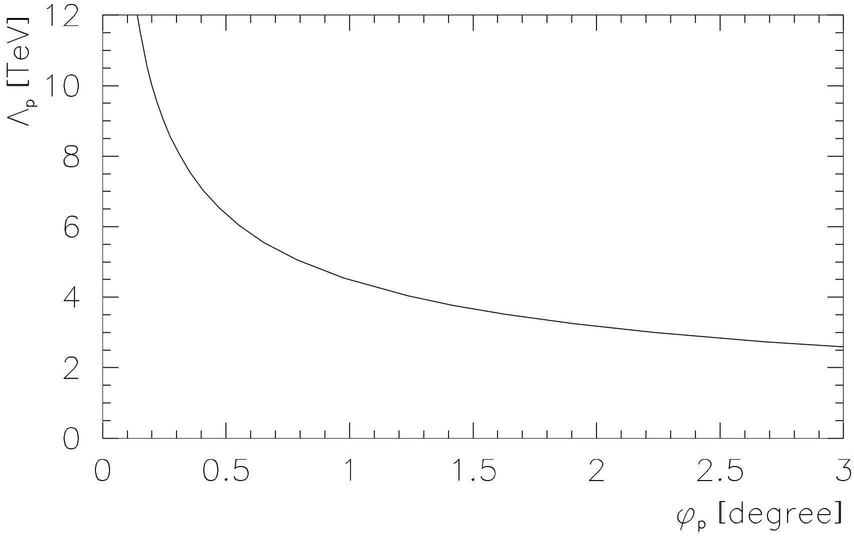

Figure 4.7: Scale Λ_p in dependence on the deviation angle φ_p

absence. In the previous case one looses statistics but gains other advantages in separating anomalous coupling signals from the SM contributions. The anomalous coupling contributions depend linearly on the transverse polarization vectors. This circumstance enables one to take only one of the initial beams to be transversely polarized. Such a choice eliminates the transverse polarization depending SM contributions. As an illustrative example we showed how to estimate the anomalous scalar particle and unparticle coupling manifestations through the measurement of the top quark polarization near the threshold of the process.

Chapter 5

Anomalous neutral gauge boson self-couplings

5.1 Non-standard gauge boson couplings

Since the formulation of the $SU(2)_L \otimes U(1)_Y$ gauge theory of the electroweak interaction [71, 72, 73] its predictions have been investigated and tested with increasing precision. The discovery of the massive gauge bosons [74, 75] confirmed the correctness of the basic ideas of the theory. To date the couplings of W and Z bosons to leptons and quarks have been tested with a good accuracy [20]. At the same time the interactions of W, Z and γ bosons with each other have been tested with much lower precision. The existence of gauge bosons self-couplings is an immediate consequence of the non-Abelian local gauge symmetry and there are at least two reasons for precise verification of such couplings. First, an experimental confirmation of the existence of gauge boson self-couplings strengthens the belief that the electroweak interaction is indeed governed by a non-Abelian gauge theory. Second, in the case of establishing the deviations from the SM predictions, an accurate measurement of the gauge boson self-couplings could act as a pointer to the existence of new physics beyond the SM. In such a way anomalous self-couplings provide an opportunity to infer NP at energies lower than the production threshold of NP particles. Such an opportunity adds, of course, importance to the precision test of the gauge boson self-interactions and that is why these couplings have been extensively investigated during the last two decades or so, both theoretically and experimentally. At the more early stage of investigations, the charged boson self-couplings received more attention as compared to the neutral boson ones. This is probably because charged boson self-couplings already receive tree level SM contributions. In order to establish the non-Abelian structure of the SM, the investigation of the processes with charged boson vertices was more suitable. Later, as the investigations of the possible anomalous self-couplings became more actual, the neutral gauge boson self-couplings sector received more attention. Differently from the charged bosons, the neutral boson self-couplings do not receive SM contributions at the lowest order of perturbation theory: the gauge invariance dictates that within the SM the trilinear neutral gauge boson vertices vanish at tree level. At the same time the loop level SM contributions are too small to be measured [38]. Due to these circumstances any experimentally established neutral gauge boson self-couplings may be considered as purely anomalous. The possible existence of anomalous neutral boson self-couplings can be tested through the production of pair of neutral gauge bosons in $p\bar{p}$ or e^+e^- collisions. It is believed that one of the most sensitive probes of new physics beyond the SM is provided by the trilinear couplings of the neutral gauge bosons in the processes $e^+e^- \to Z\gamma$, ZZ, $\gamma\gamma$. The final gauge bosons here are easy to detect experimentally, while their theoretical structure provides a clean and unambiguous test of the SM electroweak interactions. Therefore, without ambiguities the precision measurements of these processes can be compared with the SM predictions and any deviation from these can be considered as the contributions from non-standard couplings [37].

Aforegiven features make the potential of the given processes to probe anomalous trilinear neutral boson self-couplings considerably high. Therefore, these processes, especially the process $e^+e^- \to Z\gamma$, have been extensively studied both theoretically [37, 38] and experimentally [39, 40], though without success in observing anomalous couplings. If new physics arises near the TeV scale, one expects on rather general grounds that the deviations from the SM predictions are in order $O(10^{-3} - 10^{-2})$. The sensitivities provided by current colliders are of orders lower (beyond these borders). However, the study of anomalous neutral gauge boson self-couplings is an important item also in the physics programs of future colliders. A future e^+e^- International Linear Collider (ILC) designed with very high luminosity ($L = 3.4 \times 10^{34}\, m^{-2} s^{-1} \mathrm{at} \sqrt{\mathrm{s}} = 500\,\mathrm{GeV\,and\,L} = 5.8 \times 10^{34}\,\mathrm{m^{-2}s^{-1}at}\sqrt{\mathrm{s}} = 800\,\mathrm{GeV}$) can provide a much better discovery potential.

As already stated above, an additional advantage of the ILC is the possibility to have both initial beams longitudinally polarized. By using spin rotators the longitudinal polarization can be converted into the transverse polarization. Therefore, another powerful tool of the ILC will be the use of TP beams alongside with LP ones. Due to these possibilities, the role of polarization, especially the role of TP in processes with possible anomalous couplings has attracted noteworthy attention in recent years [27, 41, 29]. A considerable part of these investigations is constituted by the ones considering the TP effects in the process $e^+e^- \to Z\gamma$. The possible use of TP as a tool for non-standard couplings searches has been ignored for a long time. However, recent years studies have clearly shown that the use of TP beams would significantly enhance the potential for testing SM physics and possible non-standard interactions. The use of LP and TP initial beams complement each other. By using LP beams one can substantially enhance the sensitivities to one or the other part of couplings, which however, at least in principle, are measurable also in the case of unpolarized beams. At the same time the use of TP beams enables the measurements of the parts of couplings which are not accessible with unpolarized or LP beams.

A lot of work in clarifying the roles of LP and TP beams for searching possible anomalous gauge boson self-couplings has been done. However, these studies are yet not exhaustive. In this thesis the role of longitudinal and transverse polarization of the initial beams in disentangling SM contributions and anomalous $Z\gamma Z$, and $Z\gamma\gamma$ couplings corrections to the Z boson spin orientation in $e^+e^- \to Z\gamma$ are considered and analyzed.

5.2 Description of anomalous triple neutral gauge boson couplings

As already written above, the anomalous couplings between three gauge bosons can be divided into charged ($WW\gamma$, WWZ) and neutral (ZZZ, $Z\gamma Z$, $Z\gamma\gamma$) sectors. For several reasons the description of neutral gauge boson self-couplings are less simple than for charged bosons. One of these reasons is that one should add here the constraints due to Bose statistics. Since there are always at least two identical particles in triple neutral boson interactions, the self-couplings vanish identically if all three particles are on-shell. The appearance of neutral boson self-coupling vertices is only possible if at least one of the gauge boson involved is off-shell.

The most general expressions for anomalous neutral gauge boson triple self-coupling vertices restricted by Bose statistics, Lorentz and $U_{em}(1)$ invariance were first given in [52]. From these general considerations the general form of the couplings of a single off-shell boson ($V = Z$, γ) to the final pair of on-shell $Z\gamma$ bosons follows. It is [37]

$$\Gamma^{\alpha\beta\mu}_{Z\gamma V}(q_1, q_2, q) = \frac{i(q^2 - m_V^2)}{m_Z^2}\Bigg\{ h_1^V (q_2^\mu g^{\alpha\beta} - q_2^\alpha g^{\mu\beta})$$
$$+ \frac{h_2^V}{m_Z^2} q^\alpha [(q \cdot q_2) g^{\mu\beta} - q_2^\mu q^\beta] - h_3^V \epsilon^{\mu\alpha\beta\sigma} q_{2\sigma} \; - \; \frac{h_4^V}{m_Z^2} q^\alpha \epsilon^{\mu\beta\rho\sigma} q_\rho q_{2\sigma} \Bigg\}, \quad (5.1)$$

where the four-momenta are defined as given in Figure 5.1

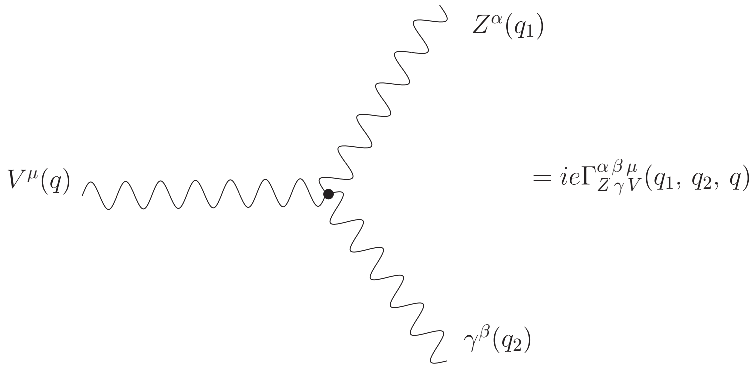

Figure 5.1: Triple gauge boson vertex.

As already stated, one of the processes, where NP interactions can induce $Z\gamma Z$ and $Z\gamma\gamma$ vertices (5.1) is the process $e^+e^- \to Z\gamma$.

The anomalous neutral gauge boson self-coupling between two Z bosons and γ occurs also in the process $e^+e^- \to ZZ$. As compared to this process the $Z\gamma$ variant seems to be more promising in a sence of searching NP effects. Its cross section is larger than the ZZ one (by a factor 2 at large scattering angles and larger at smaller angles).

In addition, the detection of $Z\gamma$ mode is more efficient than the ZZ one. As a whole, the number of events for the $Z\gamma$ mode is estimated to be an order of magnitude larger than for the ZZ mode [37]. As a consequence, the sensitivities achievable by investigating the process $e^+e^- \to Z\gamma$ have to be better as well.

The vertices like (5.1) can also be considered as generated by an effective Lagrangian [53]. The main assumption of the Effective Lagrangian Theory (ELT) is that the mass scale Λ of the NP is so heavy that it is beyond the reach of present and near future colliders. If so, the only observable NP effects should be due to anomalous interactions of usual SM particles. Under these conditions, one can integrate out heavy NP states and the observable effects can be described by an effective Lagrangian depending on operators involving only SM fields. In other words, the NP effects below Λ can be described by effective interaction terms in the Lagrangian constructed only with the help of light fields. The situation here is (and must be) similar to the one in Fermi's effective weak interaction theory.

As we can see from (5.1), in the general case both the $Z\gamma Z$ and $Z\gamma\gamma$ couplings can be parametrized in terms of four couplings: $h_1^{Z,\gamma}$, $h_2^{Z,\gamma}$, $h_3^{Z,\gamma}$ and $h_4^{Z,\gamma}$ (h_i^Z for $Z\gamma Z$ and h_i^γ for $Z\gamma\gamma$ coupling). The $h_1^{Z,\gamma}$ and $h_2^{Z,\gamma}$ coupling terms violate CP invariance and the $h_3^{Z,\gamma}$ and $h_4^{Z,\gamma}$ conserve it. Due to the fact that anomalous couplings have a non-renormalized nature, a constant value of the couplings $h_i^{Z,\gamma}$ leads to the cross-sections rapidly growing with energy ($\sqrt{s}$) and, therefore, to an unreasonable unitarity violating size. To cure such unreasonable behaviour, one must take $h_i^{Z,\gamma}$ to be form factors decreasing with increasing CM energy. One can choose between various forms of form factors. Ordinarily they are given as [54]

$$\left(1+\frac{s}{\Lambda^2}\right)^{-n} \tag{5.2}$$

with $n = 3$ for $h_{1,3}^{Z,\gamma}$ and $n = 4$ for $h_{2,4}^{Z,\gamma}$. Here the parameter Λ should be regarded as a physical cutoff, where the effective theory does not work any more and has to be replaced by a more fundamental theory.

Actually, as pointed out by several authors (see [50, 51, 37]), the introduction of form factors is somewhat in contradiction with the basic assumption of the ELT that the NP mass scale must be very much higher than the energy level, where the Effective Lagrangian formalism is tested. Only in this case one can work with effective Lagrangians keeping only the lowest dimension operators. The additional s-dependence introduced by the form factors corresponds to the presence of special (but arbitrary) higher dimensional operators. Therefore, it would be generally better to work without form factors keeping the energies far from the unitary limit.

Fortunately, when analyzing the experimental results obtained at e^+e^- colliders at a given energy, it is not very important whether one uses form factors or not. It is because one can unambiguously translate the results obtained with form factors to those obtained without them. In this thesis the form factors in their explicit form are not introduced.

For calculating the spin orientation effects with possible anomalous $Z\gamma Z$ and $Z\gamma\gamma$ couplings corrections, the following assumptions have been made.

First, we use the approximation linear to the anomalous couplings. Similarly to the investigations of the NP effects in $e^+e^- \to t\bar{t}$ in the previous chapter, anomalous couplings are considered to be so small that when calculating the NP corrections one can take into account only the contributions from the interference of the SM and anomalous couplings amplitudes.

Second, we limit ourselves to the CP-conserving anomalous couplings. Since the CP-violating couplings in (5.1) cannot be generated if NP interactions of γ and Z conserve CP, we suppose that this is indeed the case. There are two lowest order diagrams in the SM (Fig. 5.2) responsible for the process

$$e^+(k_+) + e^-(k_-) \to Z(p) + \gamma(p_\gamma). \tag{5.3}$$

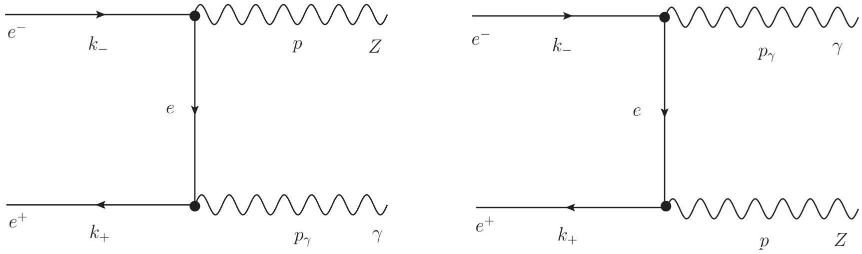

Figure 5.2: The lowest order diagrams of the process $e^+e^- \to Z\gamma$ in the SM.

The corresponding amplitude is

$$\begin{aligned}
\mathcal{M} &= -\frac{eg}{2\cos\theta_{\rm w}}\left\{\frac{1}{t-m^2}\,\overline{v}(k_+)\gamma^\lambda(\not{k}_- - \not{p} + m)\gamma^\nu(g_V - g_A\gamma_5)u(k_-)\right.\\
&+ \left.\frac{1}{u-m^2}\,\overline{v}(k_+)\gamma^\nu(g_V - g_A\gamma_5)(\not{p} - \not{k}_+ + m)\gamma^\lambda u(k_-)\right\}\\
&\times\ \varepsilon^*_{(Z),\nu}(p)\,\varepsilon^*_{(\gamma),\lambda}(p_\gamma)\ .
\end{aligned} \tag{5.4}$$

The anomalous $Z\gamma Z$ and $Z\gamma\gamma$ CP-conserving amplitudes (Fig. 5.3) are given respectively

$$\mathcal{M}_{Z\gamma Z} = \frac{ieg}{2\cos\theta_{\rm w}}\frac{1}{M^2}\overline{v}(k_+)\gamma^\alpha(g_V - g_A\gamma_5)u(k_-)$$

$$\times \quad \left\{ h_3^Z \epsilon_\alpha{}^{\nu\lambda\rho} p_\rho^{(\gamma)} + \frac{h_4^Z}{M^2} q^\nu \epsilon_\alpha{}^{\lambda\rho\sigma} q_\rho p_\sigma^{(\gamma)} \right\} \varepsilon^*_{(Z),\nu}(p) \varepsilon^*_{(\gamma),\lambda}(p_\gamma) \;, \quad (5.5)$$

$$\mathcal{M}_{Z\gamma\gamma} = -\mathrm{i}e^2 \frac{1}{M^2} \overline{v}(k_+) \gamma^\alpha u(k_-)$$

$$\times \quad \left\{ h_3^\gamma \epsilon_\alpha{}^{\nu\lambda\rho} p_\rho^{(\gamma)} + \frac{h_4^\gamma}{M^2} q^\nu \epsilon_\alpha{}^{\lambda\rho\sigma} q_\rho p_\sigma^{(\gamma)} \right\} \varepsilon^*_{(Z),\nu}(p) \varepsilon^*_{(\gamma),\lambda}(p_\gamma) \;. \quad (5.6)$$

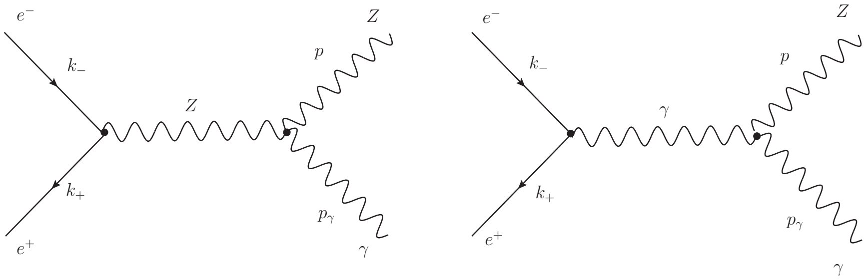

Figure 5.3: The anomalous diagrams of the process $e^+e^- \to Z\gamma$.

In these expressions $t = (k_- - p)^2$ and $u = (p - k_+)^2$ are invariant Mandelstam variables, $q^\mu = k_+^\mu + k_-^\mu$ is the four-momentum transfer, M and m stand for the Z boson and electron masses. In our calculation the latter is taken to be negligible.

5.3 Anomalous $ZZ\gamma$ and $Z\gamma\gamma$ couplings and Z boson spin orientation in $e^+e^- \to Z\gamma$

Using the formalism given above, one can in a general manner calculate the Z boson spin polarization and alignment in $e^+e^- \to Z\gamma$ with the correction of possible anomalous $Z\gamma Z$ and $Z\gamma\gamma$ couplings. This is done in the CM system and in the limit of vanishing electron mass as in Chapter 4. The calculations needed are fairly routine. They are based on using the standard trace technique for Dirac spinors with both the longitudinal and transverse components of the polarization vectors in combination with Lorentz boosts.

The general structure of the polarization vector $\vec{t}$ and the alignment tensor is found to be

$$\vec{t} = \frac{2}{3S'} \left\{ [-]\vec{V} + \left([+]h_k^Z - \frac{1}{2} <+> \sin 2\theta_\mathrm{w} h_k^\gamma \right) \vec{V}_k \right.$$

$$+3\left[g_L\, g_R\; h_k^{\rm Z} - \frac{1}{4}(g_L + g_R)\sin 2\theta_{\rm w} h_k^{\gamma}\right]\vec{V}_{k,{\rm T}}\Bigg\}, \tag{5.7}$$

$$\begin{aligned} t_{ij} = \frac{3}{S'}\Bigg\{ & [+]T_{ij} + \left([-]h_k^{\rm Z} - \frac{1}{2} <-> \sin 2\theta_{\rm w} h_k^{\gamma}\right)T_{k,\,ij} \\ & +2\left[g_L\, g_R\, T_{ij,\,\rm T} - \frac{1}{4}\sin 2\theta_{\rm w}(g_L - g_R)h_k^{\gamma}\, T_{k,\,ij,\,\rm T}\right]\Bigg\}, \end{aligned} \tag{5.8}$$

where

$$S' = [+]S + ([-]h_k^Z - \frac{1}{2} <-> \sin 2\theta_{\rm w}\, h_k^{\gamma})S_k + g_L\, g_R\, S_T + \sin 2\theta_{\rm w}(g_L - g_R)h_k^{\gamma}\, S_{k,T}. \tag{5.9}$$

We have used the shorthand notations

$$\begin{aligned} [\pm] &= g_L^2(1-h_-)(1+h_+) \pm g_R^2(1+h_-)(1-h_+), \\ <\pm> &= g_L(1-h_-)(1+h_+) \pm g_R(1+h_-)(1-h_+). \end{aligned} \tag{5.10}$$

In order to write the expressions in a more symmetric form, we have used the chiral coupling constants

$$g_L = \frac{1}{2}(g_V + g_A), \;\; g_R = \frac{1}{2}(g_V - g_A). \tag{5.11}$$

The contributions from the anomalous couplings have been denoted by the index k with the summation over repeated indices ($k = 3,\, 4$), the other ones are the SM contributions. The contributions from the transverse polarizations of the initial beams are denoted by an additional index T. The analytical expressions for the scalars (S, $S_{\rm T}$, S_k, $S_{k,\,\rm T}$), vectors ($\vec{V}$, $\vec{V}_k$, $\vec{V}_{k,\,\rm T}$) and tensors (T_{ij}, $T_{ij,\,\rm T}$, $T_{k,\,ij}$, $T_{k,\,ij,\,\rm T}$) are:

$$S = 2k^2[M^2 + p^2(1+\cos^2\theta)], \tag{5.12}$$

$$\vec{V} = 3k^2\left\{E\,M\hat{k} + [2\,p^2 - M(E-M)\cos\theta\,\hat{p}]\right\}, \tag{5.13}$$

$$\begin{aligned} T_{ij} &= k^2\Big\{M^2(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}) + M(E-M)\cos\theta(\hat{k}_i\hat{p}_j + \hat{k}_j\hat{p}_i - \frac{2}{3}\cos\theta\,\delta_{ij}) \\ &+ [p^2 + (E-M)^2\cos^2\theta](\hat{p}_i\hat{p}_j - \frac{1}{3}\delta_{ij})\Big\}, \end{aligned} \tag{5.14}$$

$$S_3 = \frac{4}{M^2}k^3p^2E\sin^2\theta, \tag{5.15}$$

$$S_4 = -\frac{8}{M^4}k^5p^3\sin^2\theta, \tag{5.16}$$

$$
\begin{aligned}
\vec{V}_3 &= \frac{6}{M^3}k^2p^2\Bigg\{\left[k(E+p\cos^2\theta)+\frac{M^2}{2}\sin^2\theta\right]\,k\,\hat{k} \\
&- \left[k^2\,E+\frac{M^2}{4}(2k-M)+k(E-M)(k+M)\cos^2\theta\right]\cos\theta\,\hat{p}\Bigg\} \qquad (5.17)\\
\vec{V}_4 &= \frac{12}{M^5}k^5p^3(E+p\cos^2\theta)(\hat{p}\cos\theta-\hat{k}), \qquad (5.18)\\
T_{3,\,ij} &= \frac{4\,k^3\,p^2}{M^3}\Bigg\{k\,M(\hat{k}_i\hat{k}_j-\frac{1}{3}\delta_{ij})+k(k-M)\cos\theta(\hat{k}_i\hat{p}_j+\hat{k}_j\hat{p}_i-\frac{2}{3}\cos\theta\,\delta_{ij}) \\
&- \frac{1}{2}\left[M\,p+(E-M)(4k+M)\cos^2\theta\right]\left(\hat{p}_i\hat{p}_j-\frac{1}{3}\delta_{ij}\right)\Bigg\}, \qquad (5.19)\\
T_{4,\,ij} &= \frac{8k^5p^3}{M^5}\Bigg\{-k\cos\theta\left(\hat{k}_i\hat{p}_j+\hat{k}_j\hat{p}_i-\frac{2}{3}\cos\theta\delta_{ij}\right) \\
&+ \left[M+(2k-M)\cos^2\theta\right]\left(\hat{p}_i\hat{p}_j-\frac{1}{3}\delta_{ij}\right)\Bigg\}, \qquad (5.20)\\
S_{\mathrm{T}} &= -4k^2\,p^2\,[\sin^2\theta(\vec{\tau}_-\cdot\vec{\tau}_+)-2(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)], \qquad (5.21)\\
S_{3,\,\mathrm{T}} &= \frac{2\,k^3\,p^3}{M^2}[\sin^2\theta(\vec{\tau}_-\cdot\vec{\tau}_+)-2(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)] \qquad (5.22)\\
S_{4,\,\mathrm{T}} &= \frac{4\,k^5\,p^3}{M^4}[\sin^2\theta(\vec{\tau}_-\cdot\vec{\tau}_+)+2(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)], \qquad (5.23)\\
\vec{V}_{3,\,\mathrm{T}} &= -\frac{4\,k^2\,p^2}{M^3}\Bigg\{k\,\hat{k}\cos\theta\left[-\frac{M^2}{2}(\vec{\tau}_-\cdot\vec{\tau}_+)+2kp(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)\right] \\
&+ \hat{p}\Bigg[\frac{M^2}{4}[M+(2k-M)\cos^2\theta](\vec{\tau}_-\cdot\vec{\tau}_+) \\
&- 2k(k+M)(E-M)(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)\Bigg] \\
&+ [(\hat{p}\cdot\vec{\tau}_+)\vec{\tau}_-+(\hat{p}\cdot\vec{\tau}_-)\vec{\tau}_+]k\left(k\,p\,\sin^2\theta-\frac{M^2}{2}\right)\Bigg\}, \qquad (5.24)\\
\vec{V}_{4,\,\mathrm{T}} &= -\frac{8\,k^4\,p^3}{M^5}\Bigg\{\hat{k}\cos\theta\left[\frac{M^2}{2}(\vec{\tau}_-\cdot\vec{\tau}_+)-2k\,E\,(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)\right] \\
&+ \hat{p}\left[-\frac{M^2}{2}\cos^2\theta(\vec{\tau}_-\cdot\vec{\tau}_+)+2k\,p\,(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)\right] \\
&+ k[\vec{\tau}_-(\hat{p}\cdot\vec{\tau}_+)+\vec{\tau}_+(\hat{p}\cdot\vec{\tau}_-)](E\cos^2\theta-p)\Bigg\}, \qquad (5.25)\\
T_{ij,\,\mathrm{T}} &= k^2\Bigg\{M^2(\vec{\tau}_-\cdot\vec{\tau}_+)\left(\hat{k}_i\hat{k}_j-\frac{1}{3}\delta_{ij}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad M(E-M)(\vec{\tau}_- \cdot \vec{\tau}_+)\cos\theta\left(\hat{k}_i\hat{p}_j - \hat{k}_j\hat{p}_i - \frac{2}{3}\cos\theta\delta_{ij}\right) \\
&- \quad \left(\hat{p}_i\hat{p}_j - \frac{1}{3}\delta_{ij}\right)\Big[(p^2-(E-M)^2\cos^2\theta)(\vec{\tau}_- \cdot \vec{\tau}_+) \\
&- \quad 2(E-M)^2(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)\Big] \\
&+ \quad M(E-M)\Big[(\hat{p}\cdot\vec{\tau}_+)\left(\hat{p}_i\tau_{-\,j} + \hat{p}_j\tau_{-\,i} - \frac{2}{3}(\hat{p}\cdot\vec{\tau}_-)\delta_{ij}\right) \\
&+ \quad (\hat{p}\cdot\vec{\tau}_-)\left(\hat{p}_i\tau_{+\,j} + \hat{p}_j\tau_{+\,i} - \frac{2}{3}(\hat{p}\cdot\vec{\tau}_+)\delta_{ij}\right)\Big] \\
&+ \quad M^2\left(\tau_{-\,i}\tau_{+\,j} + \tau_{-\,j}\tau_{+\,i} - \frac{2}{3}(\vec{\tau}_- \cdot \vec{\tau}_+)\delta_{ij}\right)\Big\}, \qquad (5.26)
\end{aligned}
$$

$$
\begin{aligned}
T_{3\,ij,\,\mathrm{T}} \quad &= \quad -\frac{4k^2p^2}{M^2}\Big\{k^2\left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right)(\vec{\tau}_- \cdot \vec{\tau}_+) - \frac{k}{M}\cos\theta\left(\hat{k}_i\hat{p}_j + \hat{k}_j\hat{p}_i - \frac{2}{3}\cos\theta\delta_{ij}\right) \\
&\times \quad \left[M\left(k-\frac{M}{4}\right)(\vec{\tau}_- \cdot \vec{\tau}_+) - k(E-M)(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)\right] \\
&+ \quad \frac{k}{M}\left(\hat{p}_i\hat{p}_j - \frac{1}{3}\delta_{ij}\right)\Big[\frac{M}{2}(p+(E-M)\cos^2\theta)(\vec{\tau}_- \cdot \vec{\tau}_+) \\
&+ \quad (2k-M)(E-M)(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)\Big] \\
&+ \quad k^2\sin^2\theta\left[\tau_{-\,i}\tau_{+\,j} + \tau_{-\,j}\tau_{+\,i} - \frac{2}{3}(\vec{\tau}_- \cdot \vec{\tau}_+)\delta_{ij}\right] \\
&+ \quad \frac{k^2}{2}\left[(\hat{p}\cdot\vec{\tau}_+)(\hat{k}_i\tau_{-\,j} + \hat{k}_j\tau_{-\,i}) + (\hat{p}\cdot\vec{\tau}_-)(\hat{k}_i\tau_{+\,j} + \hat{k}_j\tau_{+\,i})\right] \\
&+ \quad \frac{k}{2M}\left[k(E-M)\sin^2\theta - \frac{M}{2}(2k-M)\right] \\
&\times \quad \Big[\left(\hat{p}_i\tau_{-\,j} + \hat{p}_j\tau_{-\,i} - \frac{2}{3}(\hat{p}\cdot\vec{\tau}_-)\delta_{ij}\right)(\hat{p}\cdot\vec{\tau}_+) \\
&+ \quad \left(\hat{p}_i\tau_{+\,j} + \hat{p}_j\tau_{+\,i} - \frac{2}{3}(\hat{p}\cdot\vec{\tau}_+)\delta_{ij}\right)(\hat{p}\cdot\vec{\tau}_-)\Big]\Big\}, \qquad (5.27)
\end{aligned}
$$

$$
\begin{aligned}
T_{4\,ij,\,\mathrm{T}} \quad &= \quad -\frac{4k^4p^3}{M^5}\Big\{\left(\hat{k}_i\hat{p}_j + \hat{k}_j\hat{p}_i - \frac{2}{3}\cos\theta\delta_{ij}\right)\cos\theta \\
&\times \quad \left[-\frac{M^2}{2}(\vec{\tau}_- \cdot \vec{\tau}_+) + 2kp(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)\right] \\
&+ \quad 2\left(\hat{p}_i\hat{p}_j - \frac{1}{3}\delta_{ij}\right)\Big[M\left(k-\left(k-\frac{M}{2}\right)\cos^2\theta\right)(\vec{\tau}_- \cdot \vec{\tau}_+)
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 2k(E-M)(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)\Big] \\
&- \quad k(E-p\cos^2\theta)\Bigg[\left(\hat{p}_i\tau_{-\,j}+\hat{p}_j\tau_{-\,i}-\frac{2}{3}(\hat{p}\cdot\vec{\tau}_-)\delta_{ij}\right)(\hat{p}\cdot\vec{\tau}_+) \\
&+ \quad \left(\hat{p}_i\tau_{+\,j}+\hat{p}_j\tau_{+\,i}-\frac{2}{3}(\hat{p}\cdot\vec{\tau}_+)\delta_{ij}\right)(\hat{p}\cdot\vec{\tau}_-)\Bigg]\Bigg\}. \qquad (5.28)
\end{aligned}
$$

In these expressions, $\hat{k}$ and $\hat{p}$ are unit vectors along the electron and the Z boson momentum, respectively, and ϑ is the angle between them (the scattering angle). In the formulas one can express the Z boson energy E and momentum $p=|\vec{p}|$ through the initial electron energy $k=|\vec{k}|$

$$
E=\frac{1}{k}\left(k^2+\frac{M^2}{4}\right), \quad p=\frac{1}{k}\left(k^2-\frac{M^2}{4}\right). \qquad (5.29)
$$

minimizing in this way the number of final state variables. This has been done in [55] and [56]. However, to keep the formulas similar to the ones given in the previous chapter for the process $e^+e^-\to t\bar{t}$ and afterwards, we do not do it here.

5.4 Observation and analysis

The process $e^+e^-\to Z\gamma$ with a subsequent leptonic decay of the Z boson $Z\to l^+l^-$ is a self-analyzing process with respect of the Z boson spin orientation. The scattering process determines the Z boson spin orientation. The spin orientation (polarization and alignment) of the Z boson is transferred to the angular distribution of the Z boson lepton decay products. As a consequence, if the angular energy distribution of the final lepton in $Z\to l^+l^-$ has been measured, one can put limits to the anomalous $Z\gamma Z$ and $Z\gamma\gamma$ couplings. The expressions for the anomalous coupling contributions to the Z boson spin orientation, especially the ones describing the TP contributions to the alignment tensor, do not seem to be very enlightening. However, one must take into account that the calculations have been made in a general case without specifying spin quantization axes and coordinates. By introducing an appropriate coordinate system and by choosing suitable quantization axes one can considerably simplify the expressions. However, even in the aforegiven general form, the fully analytical expressions have the structure from which their general features can easily be learned.

We now make some observations on these SM and anomalous LP- and TP-dependent contributions to the Z boson spin orientation. We begin with presenting two common features of the LP- and TP-dependent contributions. First, it is not difficult to see, that due to the momentum factors $p^n \equiv |\vec{p}|^n$ ($n = 2$ or 3) in the expressions of all anomalous couplings these terms vanish at the threshold of the process. This is because the inability of the anomalous couplings to contribute at the threshold is coded already in the forms of these couplings: they are linear in the final photon four-momentum p_γ, which vanishes at the threshold of the process. However, being zero at the threshold of the process, the anomalous coupling corrections rapidly increase with the energy of colliding beams, which, as already stated, leads to the necessity of describing the anomalous couplings by the energy-dependent form-factors, which decrease when the energy increases. Second, from all anomalous coupling expressions, one can also learn that they do not give contribution to the process when the Z boson is emitted along the beamline ($\hat{p} = \pm\hat{k}$). Contrary to SM contributions, those of anomalous coupling grow with the increase of scattering angle. Thus, the ratio of anomalous contributions to the SM ones is larger when the angle between beamline and direction of Z boson momentum is larger.

As next we proceed to analyze the influence of the longitudinal polarization of the initial beams on the SM and NP contributions to the Z boson spin orientation. As one can see, in the absent of transversely polarized initial beams all contributions (*i.e.* the SM and the anomalous ones) to the process depend on the coupling constants (g_L, g_R) and LP parameters (h_+, h_-) only through the common factors (5.10). The scalar, vector and tensor quantities (5.12) – (5.20) depend neither on the coupling constants nor on the LP parameters. Just due to this, as emphasized earlier, LP does not enable measurements which are inaccessible by using unpolarized beams. But one can substantially change the SM and anomalous contributions to the Z boson orientations by choosing different values for the h_- and h_+ parameters in these polarization depending factors. However, this conclusion does not apply in the same way to the polarization vector (5.7) and the alignment tensor (5.8) of the Z boson. The contributions from the SM, $Z\gamma Z$ and $Z\gamma\gamma$ couplings to the polarization vector and the alignment tensor are accompanied by the ratios of different factors. From (5.7) and (5.8) one can easily see that the SM Z boson polarization vector depends on the polarization of the initial beams but the alignment does not. Examining from the same viewpoint the anomalous correction parts, one observes that the signs "+" and "−" in the polarization-depending factors belonging to anomalous contributions are reversed as compared to the signs belonging to the corresponding SM ones. Due to this the con-

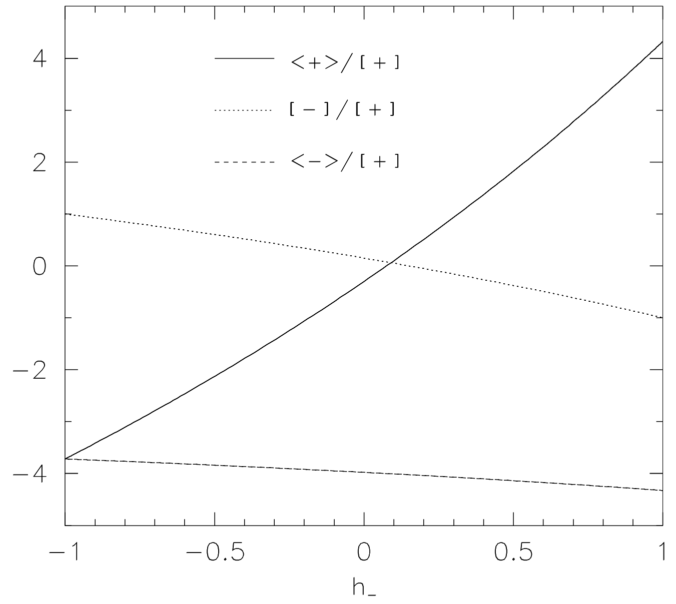

Figure 5.4: The ratios of polarization-dependent factors as the functions of the electron polarization.

tributions from the $Z\gamma Z$ couplings to the Z boson polarization and the alignment depend also "reversely" on the initial state polarizations. The anomalous $Z\gamma Z$ coupling corrections to the Z boson alignment tensor depend on the polarization parameters h_+ and h_-, whereas the corrections to the polarization vector do not depend on these parameters if one neglects the weak influence of the $[-]h_k^Z S_k$. Since the anomalous $Z\gamma\gamma$ coupling contributions are accompanied by the ratios $<\pm>/[+]$ (which differ from the unity), the $Z\gamma\gamma$ contributions both to the polarization and to the alignment depend on the longitudinal polarization of the initial beams. The actual magnitudes of the initial beams polarization effects in the contributions from different couplings are determined by the sensitivities of the ratios $[-]/[+]$ and $<\pm>/[+]$ to the variations of the longitudinal polarization parameters h_+ and h_-. In Fig. 5.4 these ratios are given as functions of the longitudinal polarization of the initial electron. Since one cannot enlarge here the theoretical range of the changes of the ratios with the use of simultaneously polarized beams, the investigations of the h_- dependence of the factors would be sufficient. One can see in Fig. 5.4 that the ratio $<+>/[+]$

(the factor in the $Z\gamma\gamma$ contribution to the polarization vector) most sensitively depends on the polarization of the initial electron. Taking this fact into account, one can state that the substantial initial-beam-polarization-dependent deviation from the SM contribution to the polarization vector (5.7) indicates the $Z\gamma\gamma$ coupling.

A different picture appears for the alignment tensor (5.8). Here the contribution from the $Z\gamma Z$ coupling is (via $[-]/[+]$) more sensitive to the variation of the beam polarization. The range of the changes of the $<-\!>/[+]$ factor due to the h_- variation is much narrower. This ratio also does not change the signs like the other ones. At the same time its module is large. Such a behaviour of the $Z\gamma Z$ and $Z\gamma\gamma$ anomalous coupling contributions may give a possibility to distinguishing them. Still one more observation is that $[-] = 0$ if $h_- = (g_L^2 - g_R^2)/(g_L^2 + g_R^2)$ and $<+>= 0$ if $h_- = (g_L + g_R)/(g_L - g_R)$. Thus, the SM does not give any measurable contribution to the polarization vector (5.7) and the $Z\gamma Z$ coupling to the alignment tensor (5.8) when the initial electron beam is about 15% polarized along its momentum. If the electrons are about 7.5% polarized in the same direction, the $Z\gamma\gamma$ coupling contribution to the polarization vector is also vanishingly small.

To illustrate the possible range of the anomalous coupling contributions and the role of the longitudinally polarized beam in it, we have chosen a few plots that demonstrate the angular dependence of the SM and SM+NP contributions to the Z boson polarization and the alignment with different values of the e^+ and e^- longitudinal polarizations (Figs. 5.5, 5.6).

For simplicity we have restricted ourselves to the case where only one of the anomalous couplings h_3^Z, h_4^Z, h_3^γ or h_4^γ differs from zero. The anomalous couplings $h_1^{Z,\gamma}$ and $h_3^{Z,\gamma}$ are accompanied by a M^{-2} factor and $h_2^{Z,\gamma}$ and $h_4^{Z,\gamma}$ by a M^{-4} factor (5.5), (5.6). Often these factors are reinterpreted in terms of the NP scale. We have not redefined them in such a manner. In all figures we have taken $h_3^{Z,\gamma} = 0.001$ and $h_4^{Z,\gamma} = 0.0001$. These values are larger than the ones to which the ILC observability limits are found to correspond. However, for these values the SM and SM+NP curves are still clearly visible in every given diagram.

We now focus ourselves to the main features of the TP contributions from both the SM and the anomalous parts of couplings. For a better understanding the reasons why by using TP one can get different information as compared to LP, we present a comparison of the TP contributions with those from LP.

For the beginning we remember the already well-known fact. Differently

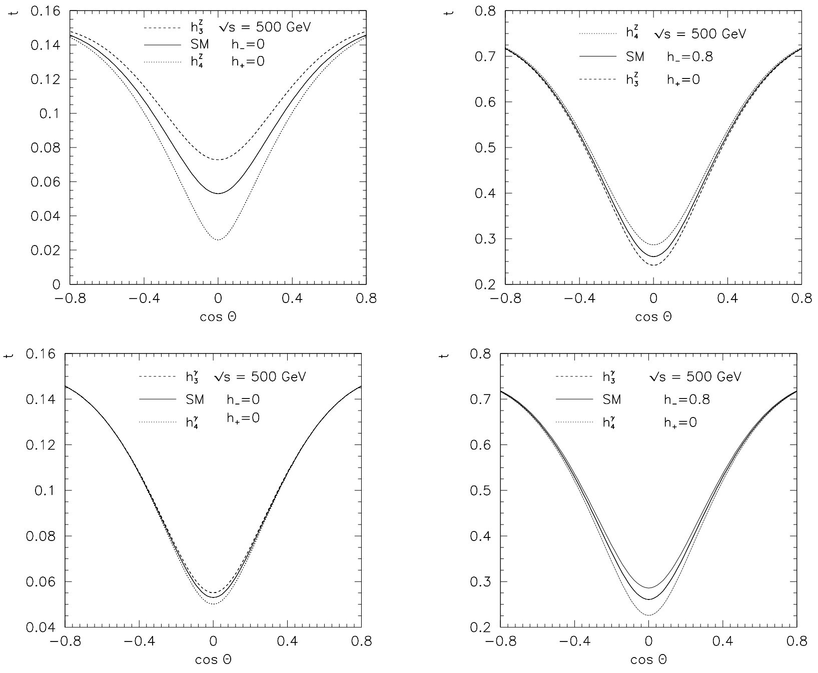

Figure 5.5: SM and anomalous contributions to the module of the Z boson polarization vector.

from the LP contributions, in the case of the vanishing electron mass the TP contributions are different from zero only if both the electron and the positron are simultaneously transversely polarized. The validity of this statement can also be observed from the expressions above.

Further, one can see from (5.21)-(5.28) that, unlike the LP contributions, the various couplings (SM, $Z\gamma Z$ and $Z\gamma\gamma$) do not give TP dependent contribution to all the quantities calculated, *i.e.* to S, $\vec{t}$ and t_{ij}. Only the anomalous $Z\gamma\gamma$ couplings contribute the TP-dependent part to all of these quantities. The SM does not contribute to $\vec{t}$ and, on the contrary, the $Z\gamma Z$ couplings contribute only to $\vec{t}$. This feature could be helpful for disentangling the contributions from various couplings.

As already emphasized above, in the LP case the contributions to the process depend on the coupling constants and on the beam polarization

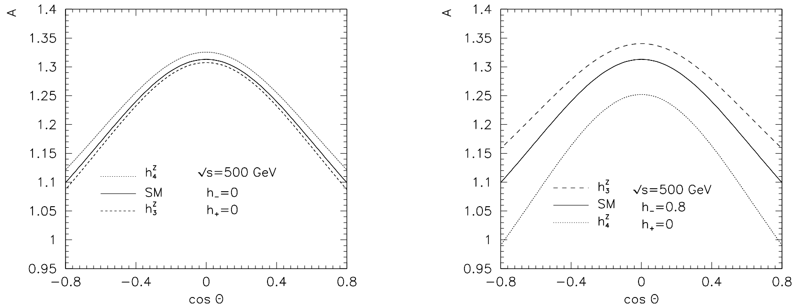

Figure 5.6: SM and anomalous contributions to the Z boson spin alignment with respect to the Z boson momentum line $(A(\hat{p}_i\hat{p}_j - 1/3\delta_{ij}))$.

parameters only through the factors $[\pm]$ or $<\pm>$. Contrary to the LP contributions, the TP contributions are not factorized. They depend on the polarization parameters $\vec{\tau}_-$ and $\vec{\tau}_+$ through the quantities $S_{\rm T}$, $S_{i,\rm T}$, $V_{i,\rm T}$ *etc.* in a manner given in (5.21)-(5.28). As a consequence, TP can enable the measurements which are not accessible with LP or unpolarized beams.

TP provide the theory with extra directions. Due to the vectors $\vec{\tau}_-$ and $\vec{\tau}_+$, the Z boson polarization vector $\vec{t}$ obtains additional components ((5.24)-(5.25)) and new alignment axes are added to the spin orientation tensor t_{ij} ((5.27)-(5.28)). The extra directions can also be used for constructing new asymmetries.

We now proceed to demonstrate some possibilities of using the TP contributions for testing the SM and anomalous couplings. We begin with the Z boson polarization vector. Since the SM generates no TP-dependent terms in $\vec{t}$, any such term has to come from the anomalous couplings contributions. As a characteristic feature of these contributions, we emphasize that due to the $Z\gamma Z$ and $Z\gamma\gamma$ TP-dependent contributions to the Z boson polarization vector, the latter obtains an additional part which generally lies not on the reaction plain (*cf.* the last terms in (5.24) and (5.25)). As a consequence, the whole Z boson polarization vector lies outside the reaction plain. Hence, the existence of the angle between the Z boson polarization vector and the reaction plain indicates the anomalous self-couplings contributions from the transversely polarized initial beams.

Let us consider a concrete example. For simplicity we restrict ourselves to the cases where only one of the anomalous couplings (h_3^γ or h_3^Z) differs from zero. We also assume that two TP vectors are perpendicular to each other and the Z boson momentum is perpendicular to one of these vectors. For choosing the degrees of polarization we take the values of the linear polarizations foreseen in the base line design of the ILC ($L_{e^-} = 0.8$, $L_{e^+} = 0.6$) and assume a 100% efficiency of spin rotators, *i.e.* we take the values of the transverse polarizations to be $|\vec{\tau}_-| = 0.8$ and $|\vec{\tau}_+| = 0.6$. In this case the values for the angle (α) between the reaction plain and the Z boson polarization vector for different values of h_3^γ and h_3^Z are given as functions of the scattering angle in Fig. 5.7.

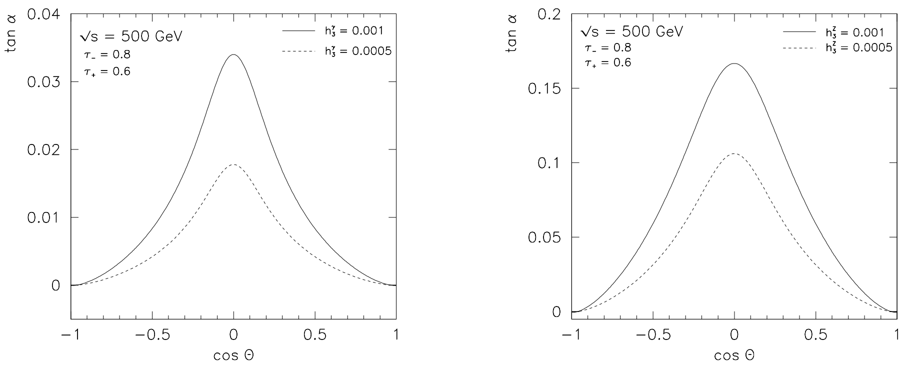

Figure 5.7: The dependence of $\tan\alpha$ on the scattering angle.

The same configuration for the vectors $\vec{\tau}_-$, $\vec{\tau}_+$, $\hat{\vec{k}}$ and $\hat{\vec{p}}$ simplifies considerably also the formulae of the TP contributions to the alignment tensor t_{ij} (5.8), (5.26) - (5.28). The axial vector coupling constant ($g_A = g_L - g_R$), which is at work in the anomalous TP-dependent contributions to the alignment tensor t_{ij}, is large as compared to the SM one ($g_L g_R = (g_V^2 - g_A^2)/4$). At the same time the anomalous $ZZ\gamma$ coupling does not contribute to the TP-dependent part of t_{ij} (5.8). Due to these factors the TP-dependent spin alignment can improve the sensitivity to anomalous $Z\gamma\gamma$ couplings.

When taking for concreteness the coordinate axes so that $\vec{\tau}_- = (|\vec{\tau}_-|, 0, 0)$, $\vec{\tau}_+ = (0, |\vec{\tau}_+|, 0)$, $\hat{\vec{k}} = (0, 0, 1)$, $\hat{\vec{p}} = (\sin\vartheta, 0, \cos\vartheta)$, one can readily see that only the terms with the alignment axes built with the help of TP vectors survive. Furthermore, in this particular case, the components t_{xy} and t_{yz} of the alignment tensor contain only TP-dependent tensor components. TP-independent contributions are equal to zero. In Fig. 5.8 we compare the SM and SM+$Z\gamma\gamma$ contributions to t_{xy}.

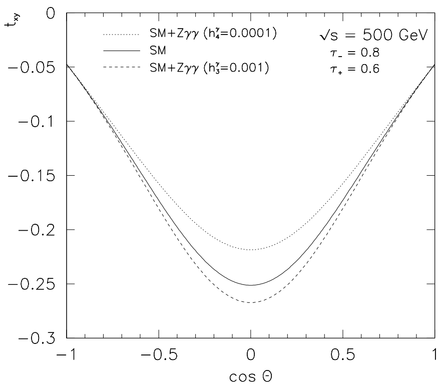

Figure 5.8: The dependence of t_{xy} on scattering angle.

We end the section with a comment on the possible additional asymmetries in the TP case. Here one must depart from the fact that the TP contributions are invariant under a simultaneous reversal of the directions of $\vec{\tau}_-$ and $\vec{\tau}_+$ and change the sign when only the direction of one of these is reversed.

Thus, by renoting $S^{'}$ in (5.9) as $S^{'} \equiv S^{'}(\vec{\tau}_-, \vec{\tau}_+)$, one can write

$$S^{'}(-\vec{\tau}_-, -\vec{\tau}_+) = S^{'}(\vec{\tau}_-, \vec{\tau}_+), \quad S^{'}(-\vec{\tau}_-, \vec{\tau}_+) = S^{'}(\vec{\tau}_-, -\vec{\tau}_+) = -S^{'}(\vec{\tau}_-, \vec{\tau}_+). \tag{5.30}$$

From (5.30) it follows that one cannot construct asymmetries at this point similar to the left-right asymmetry in the LP case [55]. Instead, one can define

$$A_{+,-} = \frac{S^{'}(\vec{\tau}_-, \vec{\tau}_+) - S^{'}(-\vec{\tau}_-, \vec{\tau}_+)}{S^{'}(\vec{\tau}_-, \vec{\tau}_+) + S^{'}(-\vec{\tau}_-, \vec{\tau}_+)}, \tag{5.31}$$

which due to (5.30) is the ratio of the TP-dependent and TP-independent parts of contributions.

Let us consider the SM case. For the vectors $\vec{\tau}_-$, $\vec{\tau}_+$ and $\hat{\vec{k}}$ we choose the same configuration as in the previous example. However, we do not restrict the direction of the Z boson momentum, *i.e.* $\hat{\vec{p}} = (\sin\vartheta\, \cos\phi, \sin\vartheta\, \sin\phi, \cos\vartheta)$. Then we get

$$A_{+,-} = \frac{2g_L g_R}{g_L^2 + g_R^2} |\vec{\tau}_-||\vec{\tau}_+| \frac{p^2 \sin^2\theta \sin 2\phi}{M^2 + p^2(1 + \cos^2\theta)}. \tag{5.32}$$

It is interesting to note that in the case $k \gg M$ the maximal value of $A_{+,-}$ (*i.e.* when $\vartheta = \pi/2$ and $\phi = \pi/4$) is approximately determined by a very simple formula

$$A_{+,-} = \frac{2g_L g_R}{g_L^2 + g_R^2} |\vec{\tau}_-||\vec{\tau}_+| \approx -|\vec{\tau}_-||\vec{\tau}_+|. \tag{5.33}$$

By using (5.9) and (5.21) – (5.23) one can easily construct the formulae for $A_{+,-}$ asymmetries in the presence of anomalous couplings. Since the anomalous $ZZ\gamma$ coupling does not generate TP-dependent contributions in $S^{'}(\vec{\tau}_-, \vec{\tau}_+)$, such asymmetries would be helpful in testing the existence of the anomalous $Z\gamma\gamma$ couplings. We do not present any of these formulae. Instead we illustrate the dependence of $A_{+,-}$ on the scattering angle in the cases of SM and SM+$Z\gamma\gamma$ in Fig. 5.9.

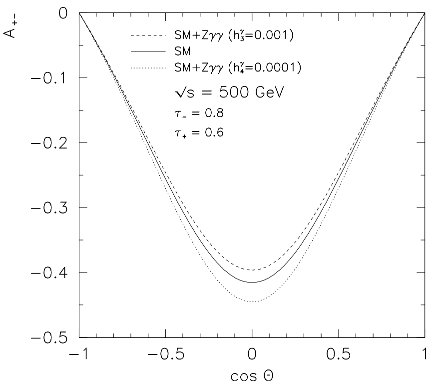

Figure 5.9: Asymmetries $A_{+,-}$ for SM and SM+$Z\gamma\gamma$.

Chapter 6

Processes $e^+e^- \to Z\gamma, ZH$ in the Standard Model

Besides the anomalous neutral gauge boson self-couplings analyzed above, there exists another trilinear neutral boson self-coupling, namely the self-coupling ZHZ important for testing the SM and searching for possible new physics implications. Such a coupling occurs in the process $e^+e^- \to ZH$ mediated by virtual Z boson exchange in the s-channel. In the SM at tree level this process is described by a Feynman diagramm with a point-like ZHZ vertex (Fig. 6.1). For investigating possible manifestations of inter-

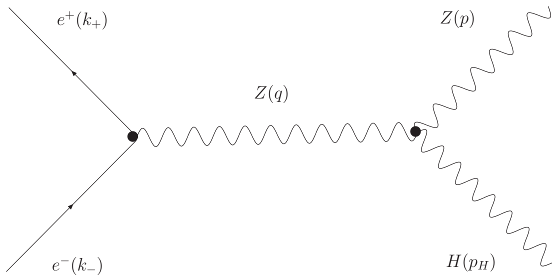

Figure 6.1: Higgs production with point-like ZHZ coupling.

actions beyond the SM one can modify the point-like vertex by means of a momentum-dependent form factor or by adding more complicated forms of anomalous couplings [76, 77]. A scalar boson with properties of the SM Higgs boson is likely to be discovered at the LHC. However, one cannot be sure that only one scalar Higgs doublet is sufficient for symmetry breaking as predicted by the SM. According to various models beyond the SM something more is needed. To discover possible manifestations of these models the ILC is more suitable than the LHC. Taking into account the possibility of finding the indications of physics beyond the SM changing the nature of the Higgs mechanism, one can say that the process $e^+e^- \to ZH$ provides an important mechanism for the production of the Higgs boson. However, it is clear that searching for non-standard physics effects at future colliders cannot be successful without knowing the SM predictions with sufficient precision. Therefore, the studies of spin effects within the SM are still worthwhile. The knowledge of all possible spin effects in some of the SM processes forms a basis for rejecting or limiting various anomalous couplings or any other deviations from the SM.

6.1 Similarities of the Z boson spin orientations

In this section we demonstrate that the spin orientation of the Z boson in $e^+e^- \to ZH$ and $e^+e^- \to Z\gamma$ calculated in the framework of the SM at tree level are quite similar. Even more, at threshold energies the expressions for the Z boson polarization vector t_i and alignment tensor t_{ij} in those processes coincide. One can find the expressions of these parameters for $e^+e^- \to Z\gamma$ in Chapter 5. For calculating corresponding expressions in $e^+e^- \to ZH$, one can start from the amplitude corresponding to the Feynman diagram in Fig. 6.1. Actually, at lowest order there are three Feynman diagrams corresponding to this process. However, the two diagrams in which the Higgs boson couples to the electron (or positron) line can be ignored due to the smallness of the coupling, which is proportional to the electron mass. The amplitude corresponding to the diagram in Fig. 6.1 is [13]:

$$\mathcal{M} = \frac{g^2}{2\cos^2\theta_W}\frac{M}{s-M^2}\bar{v}(k_+)\gamma^\mu(g_V - g_A\gamma_5)u(k_-)(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2})\varepsilon_Z^{\nu *}(p), \tag{6.1}$$

where $q^\mu = k_+^\mu + k_-^\mu$ is the four-momentum transfer and M stands for the Z boson mass. Note that the term $q_\mu q_\nu/M^2$ in (6.1) can also be ignored due to the small electron mass. With the approximation that the electron mass is negligible as compared to the typical energy scale of the process, for the CM system one can find the polarization vector and the alignment tensor of the Z boson by using traditional methods. The expressions for these parameters have been found in [13]. The polarization vectors and alignment tensors for the $Z\gamma$ and ZH processes are:

$$
\begin{aligned}
\vec{t}_{Z\gamma} &= \frac{2}{S_{Z\gamma}}[-]\,[EM\hat{k} + (2E+M)(E-M)\cos\theta\,\hat{p}], &(6.2)\\
\vec{t}_{ZH} &= \frac{2}{S_{ZH}}[-]\,[EM\hat{k} - M(E-M)\cos\theta\,\hat{p}], &(6.3)\\
t_{ij,\,Z\gamma,ZH} &= \frac{3}{S_{Z\gamma,\,ZH}}\Bigg\{[+]\,\Big[M^2(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}) + M(E-M)\cos\theta\,(\hat{k}_i\hat{p}_j + \hat{k}_j\hat{p}_i - \frac{2}{3}\cos\theta\delta_{ij})\\
&+ (\pm p^2 + (E-M)^2\cos^2\theta)(\hat{p}_i\hat{p}_j - \frac{1}{3}\delta_{ij})\Big] + 2g_Lg_R\Big[M^2(\vec{\tau}_-\cdot\vec{\tau}_+)(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij})\\
&+ M(E-M)\cos\theta(\vec{\tau}_-\cdot\vec{\tau}_+)(\hat{k}_i\hat{p}_j + \hat{k}_j\hat{p}_i - \frac{2}{3}\cos\theta\delta_{ij})\\
&- [(p^2 - (E-M)^2\cos^2\theta)(\vec{\tau}_-\cdot\vec{\tau}_+) - 2(E-M)^2(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)](\hat{p}_i\hat{p}_j - \frac{1}{3}\delta_{ij})\\
&+ M(E-M)[(\hat{p}\cdot\vec{\tau}_+)(\hat{p}_i\tau_{-\,j} + \hat{p}_j\tau_{-\,i} - \frac{2}{3}(\hat{p}\cdot\vec{\tau}_-)\delta_{ij})\\
&+ (\hat{p}\cdot\vec{\tau}_-)(\hat{p}_i\tau_{+\,j} + \hat{p}_j\tau_{+\,j} - \frac{2}{3}(\hat{p}\cdot\vec{\tau}_+)\delta_{ij})]\\
&+ M^2(\tau_{-\,i}\tau_{+\,j} + \tau_{-\,j}\tau_{+\,i} - \frac{2}{3}(\vec{\tau}_-\cdot\vec{\tau}_+)\delta_{ij})\Big]\Bigg\}, &(6.4)
\end{aligned}
$$

where

$$S_{Z\gamma} = 2[+][M^2 + p^2(1+\cos^2\theta)] - 4g_Lg_Rp^2[\sin^2\theta(\vec{\tau}_-\cdot\vec{\tau}_+) - 2(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)] \quad (6.5)$$

and

$$S_{ZH} = [+]\,[2M^2 + p^2\sin^2\theta] + 2g_Lg_Rp^2[\sin^2\theta(\vec{\tau}_-\cdot\vec{\tau}_+) - 2(\hat{p}\cdot\vec{\tau}_-)(\hat{p}\cdot\vec{\tau}_+)]. \quad (6.6)$$

The other notations have been already used before or are obvious. One can immediately see the similarities between the Z boson orientation parameters in two different processes. Note here that the numerators of the expressions for the $t_{ij\,Z\gamma}$ and $t_{ij\,ZH}$ are the same, except for the signs in front of a single term where the upper sign belongs to $Z\gamma$ and the lower sign to the ZH process.

As already stated, at threshold energies the expressions for both processes coincide:

$$\vec{t}_{\rm thres} = \frac{[-]}{[+]}\hat{k} = \frac{g_L^2(1-h_-)(1+h_+) - g_R^2(1+h_-)(1-h_+)}{g_L^2(1-h_-)(1+h_+) + g_R^2(1+h_-)(1-h_+)}\hat{k}, \quad (6.7)$$

$$t_{ij\,\text{thres}} = \frac{3}{2}(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}) + \frac{3\,g_L\,g_R}{g_L^2(1-h_-)(1+h_+) + g_R^2(1+h_-)(1-h_+)}$$
$$\times \left[(\vec{\tau}_- \cdot \vec{\tau}_+)(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}) + \tau_{-\,i}\tau_{+\,j} + \tau_{-\,j}\tau_{+\,i} - \frac{2}{3}(\vec{\tau}_- \cdot \vec{\tau}_+)\delta_{ij}\right]. \tag{6.8}$$

The coincidence of the Z boson spin orientations at the thresholds of two different processes, one of them being described by two (u- and t-channels) Feynman diagrams and the other by one (s-channel) diagram, may seem somewhat peculiar. However, this result can be expected. One can say that the formulas (6.7) and (6.8) give the spin orientation parameters of the real Z boson in the process $e^+e^- \to Z$. The Higgs boson is a spin-0 particle and therefore cannot affect the spin orientation of the Z boson at the threshold of $e^+e^- \to ZH$. On the other hand, at the limit $\vec{p} \to 0$ in $e^+e^- \to Z\gamma$ the photon can obviously be considered as a radiative correction to the main process $e^+e^- \to Z$. So the coincidence of the Z boson orientation parameters for the two different processes can be taken as a cross-check for our formulas.

6.2 Tuning the Z boson spin polarization and alignment

We proceed to analyze these threshold expressions in order to demonstrate the importance of using polarized initial beams in e^+e^- linear colliders. From (6.7) one can learn how much the Z boson polarization vector can be tuned by variation of the LP of the initial beams. In allowing the h_- and h_+ parameters to take values as large as possible, we suppose that the beams have no TP components. Like it was done when considering the process $e^+e^- \to t\bar{t}$ in Chapter 4, we also express $\vec{t}_{\text{thres}}$ through the effective polarization parameter

$$\chi = \frac{h_+ - h_-}{1 - h_-\,h_+}.$$

We obtain

$$\vec{t}_{\text{thres}} = t(\chi)\hat{k}, \tag{6.9}$$

where

$$t(\chi) = \frac{\chi + b}{b\chi + 1}$$

with

$$b = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2} = \frac{2\,g_V\,g_A}{g_V^2 + g_A^2} \approx 0.147.$$

In Fig. 6.2 the dependence of the polarization vector of the Z boson on the

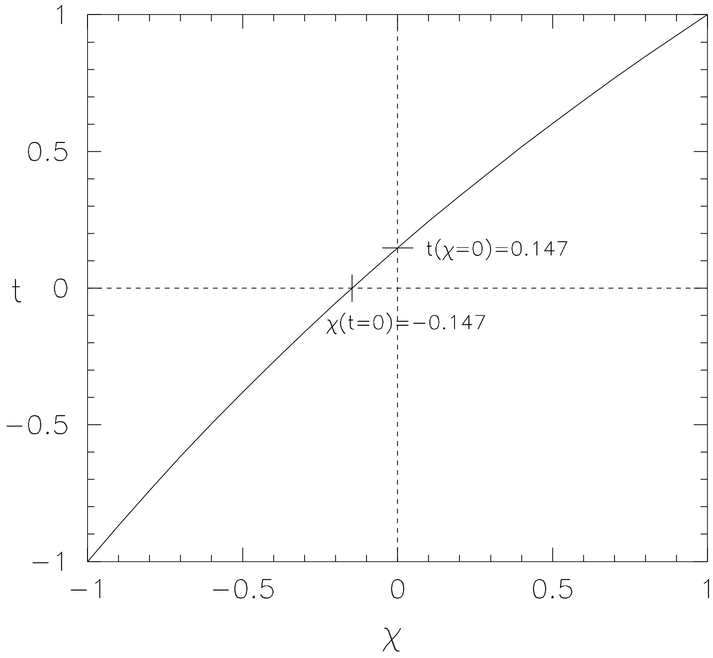

Figure 6.2: The Z boson polarization in SM at the thresholds of $e^+e^- \to Z\gamma$, ZH as a function of χ.

effective polarization of the beams is given. The fact that $t(\chi_0) = -\chi(t_0) = b = 0.147$ is again due to the similarity of $t(\chi)$ and

$$\chi(t) = \frac{-t+b}{bt-1}.$$

To clarify how much it is possible to tune the Z boson polarization vector as compared to the unpolarized ($\chi = 0$) case, one has to justify whether there exist some additional restrictions on this vector due to positivity conditions (3.48). Both the polarization vector and the alignment tensor depend only on the vector $\hat{k}$. When choosing the coordinate system with the z-axis along the vector $\hat{k}$, one gets

$$\begin{aligned} \vec{t} \to t_z &= \frac{\chi + b}{b\chi + 1}\hat{k}_z, \\ t_{ij} \to t_{zz} &= \frac{3}{2}(\hat{k}_z^2 - \frac{1}{3}\delta_{zz}) = 1, \end{aligned} \tag{6.10}$$

$$
\begin{aligned}
t_{xx} &= -\frac{1}{2}, \\
t_{yy} &= -\frac{1}{2}.
\end{aligned}
$$

All the other components are equal to zero. When substituting these components into the positivity conditions (3.48) it turns out that the condition is satisfied independently of the t_z and, therefore, also independently of the value of χ.

Hence, by varying the effective polarization χ one can theoretically force the Z boson polarization vector to take any value between -1 and 1. Practically achievable values are not far from the theoretical ones. When using the values $h_- = \pm\, 0.8$, $h_+ = \pm\, 0.6$, planned to be achieved at the ILC, one can reach the effective polarization of the initial beams $\chi = \pm\, 0.95$ which lead to

$$
t(\chi) = {}^{+0.96}_{-0.93}\;. \tag{6.11}
$$

If $h_- = h_+ = 0$ and $\vec{\tau}_- \neq 0$, $\vec{\tau}_+ \neq 0$, one can tune the alignment tensors by changing the TP vectors. One can show that also in this case the positivity condition (3.48) does not put additional restrictions to the components of the alignment tensor. If one uses the same reference frame as above then

$$
\begin{aligned}
t_z &= \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}, \\
t_{zz} &= 1, \\
t_{xx} &= -\frac{1}{2} + \frac{3\, g_L\, g_R}{g_L^2 + g_R^2} \tau_{-\,x} \tau_{+\,x}, \\
t_{yy} &= -\frac{1}{2} - \frac{3\, g_L\, g_R}{g_L^2 + g_R^2} \tau_{-\,x} \tau_{+\,x}.
\end{aligned} \tag{6.12}
$$

When inserting these parameters into the expression for the positivity condition, one finds that this condition is satisfied for any value of the TP components $\tau_{-\,x}$, $\tau_{+\,x}$ as well for the coupling constants g_L, g_R. As a consequence, one can tune the Z boson alignment by changing the TP vectors over all practically available values.

The studies given in this chapter demonstrate that the spin orientations of the final massive particles from the e^+e^- annihilation near threshold are quite similar.

Chapter 7

Higher-spin interaction theory and gyromagnetic factor

7.1 Difficulties of higher-spin field theory

Understanding the higher-spin interactions is a longstanding problem. However, in spite of its more than 70 years history, the main goal – construction of a consistent higher-spin interaction theory – has not been achieved yet. Higher spins start from the values spin one and higher. This concept is not universally accepted. For a part of investigators "higher-spin" means $s \geq 3/2$. The specialists in supergravity updated the convention of the higher spin to be even $s \geq 5/2$. [57] Nevertheless, at least in the Standard Model the troubles start already from the value $s = 1$. Therefore, it seems that the convention $s \geq 1$ as the higher-spin region is more justified than the other ones. The investigations of higher-spin fields started in last century thirties from papers by Dirac [58], Wigner [59], Fierz and Pauli [60] and followed by the works of Rarita and Schwinger [61], Bargmann and Wigner [62] and others. The difficulties in higher spin physics revealed themselves when one tried to couple higher-spin fields to an electromagnetic field. In the 1960s concrete defects of the higher-spin interaction theory were found. Federbush [32], Johnson and Sudarshan [33] and Schwinger [81] demonstrated that in the case of minimal electromagnetic coupling some of the anticommutation relations become indefinite. It appeared that the defects were also present at the classical level. Velo and Zwanziger [84] showed that in an external electromagnetic field there appeared acausal (superluminal) modes of propagation. Afterwards other defects (bad high-energy behaviour of the amplitudes, various algebraic problems *etc.*) revealed themselves. Since the sixties of the last century much work was done to solve the problems, but no satisfactory results have been obtained in the framework of ordinary field theory with minimal electromagnetic coupling.

The search for a consistent higher-spin interaction theory has been faced with various difficulties. The theory of the relativistic wave equations is based on the representations of the Poincaré group, which in the field theory are somewhat specific in their mathematical realization. In addition, the theory of higher-spin fields is altogether rather complicated and due to that the wave functions and Lagrangians proposed have not been always correct. Thus it has been sometimes difficult to understand whether the problems were technical or pertined to a principle. As a matter of fact, the difficulties in higher-spin physics are generic to all field theoretic descriptions of relativistic higher-spin particles. The difficulties are related to the fact that covariant higher-spin field has more components than it is necessary to describe the spin degrees of freedom of the physical particle. To get rid of redundant degrees of freedom one must set up constraints between the field components. Using the language of Lagrangians, one has to construct free Lagrangians, which in addition to the Dirac and Proca type higher-spin equations would yield also constraint equations that reduce the number of degrees of freedom to the physical values. The problem is how to introduce interactions under these conditions. If the interactions are introduced consistently with the free field theory, the number of independent field components remains unchanged. Otherwise the free theory constraints may be violated and unphysical degrees of freedom may be involved.

In order to put constraints on the field components it is reasonable to use the symmetry framework. To reduce the number of degrees of freedom of the free field to a physical value certain symmetries have to be imposed in formulating the action. Any free higher-spin action must be invariant under a transformation which leaves only the physical $(2s+1)$ spin degrees of freedom. Needless to say that not every interaction introduced into the theory will be consistent with the constructions of the free theory. The inconsistent, free theories symmetries violating forms of interactions violate also the degree of freedom counting of the free theory, which generally leads to acausal modes of propagation of particles, to indefinite norms of states, to bad, non-unitary high-energy behaviour of the amplitudes of processes *etc.* Hence the main promise of constructing consistent higher-spin theory must be that the interaction introduced into the theory should not introduce also spurious degrees of freedom. Therefore, the interacting theory has to obey similar symmetry requirements as the corresponding free theory or, even better, preserves the gauge symmetries of the free theory. The possibility to construct consistent higher-spin theories with gauge invariant couplings was first pointed out by Weinberg and Witten [63]. However, the realization of this scenario is beset with difficulties and to date no general prescription for constructing in such a way a consistent higher-spin field theory for any spin has been found. Even though certain progress in understanding of massless higher-spin interaction theory has been made during the last two decades [34, 64, 65, 66, 67, 68].

The higher-spin interaction theory is related to the Standard Model in several ways. With introducing the massive spin one gauge bosons into the theory, one also introduces higher-spins problems into the Standard Model. The difficulties reveal themselves, for example, in scattering processes with participation of a charged gauge bosons, $W^{\pm}$, in the initial or final state, or when constructing three-vertex gauge boson self-interactions. The consistent higher-spin interaction theory is also needed in quantum chromodynamics. Quantum chromodynamics does not yet allow to describe low-energy hadronic processes in terms of underlying quark-gluon dynamics. Due to this one has to use a more phenomenological approach in terms of hadronic fields. However, one of the basic problems here is the treatment of hadrons with $s \geq 1$ [86].

To understand better the problems of modern theories beyond the SM one also needs a better understanding of ordinary higher-spin field theory. For example, the string theory is free from many of aforementioned higher spin problems and due to this it is believed that it can consistently describe quantum gravity. A reason behind this consistent behaviour is that string theories contain an infinite tower of all spin states. But at the same time there exist serious troubles in the physical interpretation of string theories. The existence of consistent higher-spin interaction theory would help to understand better the physics behind the string theory. It is believed that if a breakthrough in understanding the basic problems of the ordinary higher-spin field theory would happen, it might become a fashionable topic [66].

7.2 Higher-spin physics and gyromagnetic factor

The gyromagnetic factor can be defined for systems possessing a charge, mass and angular momentum. In relativistic quantum mechanics a factor $g = 2$ is resulted from the Dirac ($s = 1/2$) equation. But what is the value of the gyromagnetic factor for the higher spin particles? Belinfante [93] calculated the g-factor for $s = 3/2$ in the case of minimal electromagnetic coupling and got $g = 2/3$. After comparing this result with Dirac $s = 1/2$ and $s = 1$ case he supposed that the g-factor for arbitrary spin s is given by the formula $g = 1/s$. A general proof of Belinfante's conjecture $g = 1/s$ for particles with arbitrary spin in the case of minimal electromagnetic coupling was given by Napsuciale and Vaguera-Arango [94].

This is a simple and nice formula. However, due to the serious inconsistences of the higher-spin minimal electromagnetic interaction theory it cannot be taken too seriously. The value $g = 1$ for $s = 1$ charged bosons $W^{\pm}$ for example leads to serious difficulties in description of γW -scattering, where the amplitude of the process increases rapidly with increasing energy, leading to a violation of unitarity. Weinberg [31] showed that when taking instead of the minimal coupling non-minimal ones, especially those which lead to $g = 2$ for arbitrary spin, one can (at least partially) cure the difficulties. And though the electrodynamics of charged particles of unit spin remains unrenormalizable for any value of $g \geq 1$, the higher order electromagnetic corrections in γW scattering amplitude ought to be small and the amplitudes behave well at infinity if $g = 2$. Currently it is almost a common agreement that the value for the gyromagnetic factor for all truly elementary charged particles of any spin is $g = 2$. This picture is also supported by the experimental value for the W boson gyromagnetic factor, which is $g_W = 2.22 \pm 0.20$ [20] and which definitely rules out value $g_W = 1$. To justify the choice $g = 2$ for any spin usually the following main arguments are given:

1. $g = 2$ value must hold to generate good high-energy behaviour of scattering amplitudes [31];

2. In the case $g = 2$ the Bargmann-Michel-Telegdi [80] equation of motion of the polarization vector takes its simplest form [34]:
$$\frac{dS_\mu}{d\tau} = \frac{e}{m} F_{\mu\nu} S^\nu; \tag{7.1}$$

3. The only massive higher-spin charged fundamental particle in the SM, the W boson with $s = 1$ has at tree level $g_W = 2$ [35].

The first two of these reasons can be classified as "practical needs" and the third one presumes in more general case the particle to be a gauge one [69].

Obviously, the value $g = 2$ must rest on some fundamental theoretical ground. Such grounds have been looked for in recent years [34]. In [69] another symmetry principle based on field-dependent invariant representation (the "dynamical" representation) of the Poincaré algebra given in [36] is used.

Since the gyromagnetic factor is determined by the form of interaction, the search for the symmetry principle that would generate the $g = 2$ value is also the search for the symmetry principle for building a consistent interaction theory. In relativistic particle theory the Poincaré group plays a fundamental role. However, when the interaction is introduced in the form of minimal electromagnetic coupling, in the case of higher-spin theories the Poincaré invariance is violated. Thus, to avoid the violation of the Poincaré invariance one needs a dynamical principle, which would result in a minimal coupling in lower spin cases ($s = 0,\ 1/2$) and a new, non-minimal Poincaré-invariant coupling in the higher spin ($s \geq 1$) cases.

We take the representation in [36] as a dynamical principle that determines the "dynamical" electromagnetic coupling. The "dynamical" coupling for arbitrary spin $s \geq 1$ contains a non-minimal term linear in the field strength tensor $F_{\mu\nu}$. It appears that due to this term the non-minimal coupling may lead to $g = 2$.

The "dynamical" algebra in the case of the spin-1/2 particles was first introduced by Chakrabarty [87] and further studied by Beers and Nickle [89]. In [36] the construction of the "dynamical" representation has been generalized to the arbitrary spin case. The representations are built by introducing a plane electromagnetic field into the free Poincaré algebra. The new, "dynamical" representations are constructed from the generators of the free Poincaré algebra and the external field in such a way that the new, field-dependent generators obey the commutation relations of the free Poincaré algebra.

In analogy to the free particle theory, from this starting point the wave equations with respect to the "dynamical" representation of the Poincaré algebra can be constructed. These equations describe the "dynamical" interactions of the particles with the external plane wave field. In spite of the external electromagnetic field,in this theory the particle behaves like a free particle. Since the free higher-spin theory has no defects there is hope that the troubles existing in the minimal coupling theory (or at least some of them) can be avoided in the "dynamical" interaction theory. This would be in accordance with the aforegiven statement that the interaction has to be introduced in such a way that the symmetries of the free theory are not violated.

As has been shown already by Chakrabarti [87], the simplest way to build the "dynamical" representation is to introduce the external field by a non-singular transformation. Consequently, the problem is to find a field-dependent transformation U, such that the transformed Poincaré generators

$$\begin{aligned} \pi_\mu &= U P_\mu U^{-1} \\ \mathfrak{M}_{\mu\nu} &= U M_{\mu\nu} U^{-1}, \end{aligned} \tag{7.2}$$

where $P_\mu = \mathrm{i}\partial_\mu$, $M_{\mu\nu} = L_{\mu\nu} + S_{\mu\nu}$ with $L_{\mu\nu} = x_\mu P_\nu - x_\nu P_\mu$ and $S_{\mu\nu}$ as the generators of the finite-dimensional representation of the Lorentz group, would obey the commutation relations of the free-particle theory, *i.e.*

$$\begin{aligned} [\mathfrak{M}_{\mu\nu}, \mathfrak{M}_{\rho\sigma}] &= \mathrm{i}(g_{\mu\sigma}\mathfrak{M}_{\nu\rho} + g_{\nu\rho}\mathfrak{M}_{\mu\sigma} - g_{\mu\rho}\mathfrak{M}_{\nu\sigma} - g_{\nu\sigma}\mathfrak{M}_{\mu\rho}), \\ [\mathfrak{M}_{\mu\nu}, \pi_\sigma] &= \mathrm{i}(g_{\nu\sigma}\pi_\mu - g_{\mu\sigma}\pi_\nu), \\ [\pi_\mu, \pi_\nu] &= 0. \end{aligned} \tag{7.3}$$

The concept of Lorentz covariance raises the requirement that the operator U has to be of Lorentz type for the generator $S^{\mu\nu}$ (local Lorentz transformation).

It appears that such an operator can be found without problems for a plain wave (laser) field

$$A_\mu = A_\mu(\xi),\ \xi = k \cdot x. \tag{7.4}$$

For arbitrary spin s

$$U = U_0 \cdot U(s)\,, \tag{7.5}$$

where

$$U_0 = exp\Big\{\mathrm{i}\int \frac{d\xi}{2(k\cdot P)}\left[2e\,P\cdot A(\xi) - e^2 A^2(\xi)\right]\Big\}. \tag{7.6}$$

and

$$U(s) = exp\Big\{-\mathrm{i}\frac{e}{2(k\cdot P)}\left[k_\mu A_\nu - k_\nu A_\mu\right] S^{\mu\nu}\Big\}. \tag{7.7}$$

The details connected with the inverse operator $(kP)^{-1}$ can be found in [87]. Note that this operator is well-defined and successfully used in a lot of papers (see for instance [89, 90, 91]). By applying the transformation (7.5) to the operator P_μ one gets

$$\pi_\mu = P_\mu + \frac{e}{2(k\cdot P)}\, k_\mu(eA^2 - 2A\cdot P - F_{\sigma\rho}S^{\sigma\rho}). \tag{7.8}$$

Being supplied with the mathematical apparatus given above one is ready to transform a free Klein-Gordon equation

$$(P^2 - m^2)\psi = 0 \tag{7.9}$$

into the equation in the "dynamical" representation.

Equation (7.9) describes a spectrum of spins. As already described above, to get a spin-1 theory one must eliminate all superfluous spins by putting subsidiary conditions (constraint equations) to the equation. Since there exists no prescription how this can be done in arbitrary spin case, we will consider in the following the simplest and most familiar spin-1 and spin-3/2 cases.

In the massive spin-1 case the subsidiary condition is already present in the Proca equation

$$\Big\{(P^2 - m^2)g_{\mu\nu} - P_\mu P_\nu\Big\}\phi^\nu = 0. \tag{7.10}$$

This equation can equivalently be written as the equation and subsidiary condition

$$\begin{aligned}(P^2 - m^2)\phi_\mu &= 0,\\ P_\nu\phi^\nu &= 0.\end{aligned} \tag{7.11}$$

By applying the U-transformation to these free spin-1 particle equations one gets the equations in the "dynamical" representation:

$$\Big\{(D^2 - m^2)g_{\mu\nu} - D_\mu D_\nu - 2\mathrm{i}eF_{\mu\nu}\Big\}\phi_d^\nu = 0 \tag{7.12}$$

and

$$\begin{aligned}(D^2 - m^2)\phi_\mu - 2\mathrm{i}eF_{\mu\nu}\phi_d^\nu &= 0,\\ D_\nu\phi_d^\nu &= 0,\end{aligned} \tag{7.13}$$

where $D^2 = D_\sigma D^\sigma$, $D_\sigma = P_\sigma - e\,A_\sigma$ and $\phi_d = U\phi$.

The details of the deduction of these equations can be found in [36]. The non-minimal term linear in $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ guarantees a good high-energy behaviour of the scattering amplitudes and leads to a value $g = 2$ of the gyromagnetic factor. The equations (7.12) and (7.13) deduced in the framework of the "dynamical" theory coincide with well-known equations describing the coupling of a spin-1 particle with the charge e to the electromagnetic field. The non-minimal term linear in $F^{\mu\nu}$ was put into these equations as a practical need without reasonable theoretical foundations.

In spite of this, quite often it is stated that the coupling in (7.12) and (7.13) is the minimal one, *i.e.* it can be obtained by making the substitution $P_\mu \to D_\mu = P_\mu - eA_\mu$ in the free equations (7.10) and (7.11). Indeed, due to the fact that the replacement $P_\mu \to D_\mu$ is not unique in (7.10) one can use a trick here [92]:

$$-P_\mu P_\nu \to -P_\mu P_\nu + k\,[P_\mu, P_\nu] \to -D_\mu D_\nu + k\,[D_\mu, D_\nu] \xrightarrow{k=2} -D_\mu D_\nu - 2\mathrm{i}eF_{\mu\nu}. \tag{7.14}$$

However, the choice $k = 2$ is only one possibility among many. By a trick of such kind one can get the field strength term with an arbitrary numerical coefficient in front of $F_{\mu\nu}$. Besides, without adding the commutator term $2[P_\mu, P_\nu](= 0!)$ to the left hand side of the first equation in (7.11) one does not get from this equation the $F_{\mu\nu}$ term in (7.13) by the minimal coupling prescription $P_\mu \to D_\mu$ either. In the "dynamical" theory, the field strength term arises from the P^2 term ($UP^2U^{-1} = \pi^2 = D^2 - eF_{\mu\nu}S^{\mu\nu}$) and (7.13) follows uniquely from (7.11). Moreover, since π_μ and π_ν commute like P_μ and P_ν, by applying the U-transformation to (7.10) one gets also uniquely (7.12).

To get a unique minimal coupling theory one must depart from first-order equations, where the procedure $P_\mu \to D_\mu$ is unambiguous. However, the Kemmer-Duffin spin-1 equation with the minimal coupling leads in according with the $g = 1/s$ conjecture to $g = 1$. In the case of the first-order equations it is not difficult to verify that the "dynamical" interaction is introduced by a modified minimal coupling procedure [36]

$$P_\mu \to P_\mu - eA_\mu - \frac{e}{2(k\cdot P)}k_\mu F_{\rho\sigma}S^{\sigma\rho} = D_\mu - \frac{e}{2(k\cdot P)}k_\mu F_{\rho\sigma}S^{\rho\sigma}. \tag{7.15}$$

The last term in the equation does not give any contribution to the spin-0 and spin-1/2 equations. However, in the $s > 1/2$ cases the added spin-dependent term increases the gyromagnetic ratio as compared to the minimal coupling one.

It can be seen more clearly by examining the spin-dependent terms in the second-order equations. Since every "dynamical" first-order equation has the Klein-Gordon divisor (if such an operator exists for free equation), one can always find the corresponding second-order equation. By applying, for example, the Klein-Gordon divisor to the "dynamical" Rarita-Schwinger linear spin-3/2 equation, one obtains

$$\begin{aligned} \left[(D^2 - m^2 - eF_{\rho\sigma}s^{\sigma\rho})g_{\mu\nu} - 2\mathrm{i}eF_{\mu\nu}\right]\psi_d^\nu &= 0, \\ \gamma_\mu\psi_d^\mu &= 0, \end{aligned} \tag{7.16}$$

where $s^{\sigma\rho} = \frac{\mathrm{i}}{4}[\gamma^\sigma, \gamma^\rho]$ is the Lorentz spin-1/2 generator. Contrary to the minimal coupling case, where spin-3/2 Rarita-Schwinger equation leads to the gyromagnetic ratio $g = 2/3$, the spin dependent terms in (7.16) suggest the value $g = 2$.

Finally, the investigations have shown that the "dynamical" interactions can be developed also in two plain-wave cases or maybe even in the general case of n plain-waves [95].

Chapter 8

Summary

The Standard Model has been phenomenologically very successful. Up to now the predictions of the Standard Model have been in consistence with all accelerator-based experiments. However, the Standard Model as a theory is less impressive. There are a lot of fundamental questions that remain unanswered by the Standard Model. Due to this there are strong reasons to expect that beyond the Standard Model at the higher energy level there exists new physics, more fundamental than the Standard Model physics. The most straightforward method for searching new physics would be the production of new physics particles. However, the Large Hadron Collider (LHC) excluded, the energies needed for such a scenario are beyond the reach of present colliders and obviously also of the future International Linear Collider (ILC). But the possibilities to probe new physics at the energies below the new physics mass scale exist. These, more indirect scenarios are based on observations of small departures from the Standard Model predictions in the processes between Standard Model particles, and new physics indications can arise only from non-standard interactions. For such methods higher sensitivity, both experimental and theoretical is needed.

It is well known that spin effects as compared to the other ones, are more sensitive to new physics indications. The possibility to use at ILC both longitudinally and transversely polarized initial beams provides a powerful tool for searching new physics through the spin orientation effects in ILC processes. In this thesis the utility of using polarized and especially transversely polarized beams for searching new physics is once more demonstrated.

In the first publication the process $e^+e^- \to t\bar{t}$ is studied in the case of possible existence of non-standard (anomalous) scalar-type particle and unparticle couplings. The use of transversely polarized initial beams in this process enables to probe the appearance of such type anomalous couplings already in first order contributions making these studies altogether sensible. The analytic expressions of these contributions are found for the case when either the top or the antitop polarization is measured. It is shown that they contain CP-odd terms due to which the CP-invariance in the process is violated. Various asymmetries sensible to CP violation are constructed. It is also demonstrated how one can probe the anomalous couplings by measuring the angle between the reaction plain and the final state top (antitop) polarization vector.

In the second publication the process $e^+e^- \to Z\gamma$ with longitudinally polarized initial beams is studied in the case of existence of anomalous trilinear neutral gauge boson self-couplings $Z\gamma Z$ and $Z\gamma\gamma$. The analytical expressions for the final Z boson polarization vector and the alignment tensor are found and analyzed. The differences between the Standard Model and anomalous couplings contributions are presented which in principle enable to disentangle them. It appears that the influence of the longitudinal polarizations on the Z boson spin orientation is determined by the factors depending on longitudinal polarization that are different in the different contributions. By varying the longitudinal polarization parameters h_- and h_+ in these factors, one can increase or decrease the role of different contributions. This probability is helpful in separating anomalous coupling contributions from the Standard Model ones.

In the third publication the role of the transversely polarized initial beams in disentangling the Standard Model and anomalous $Z\gamma Z$ and $Z\gamma\gamma$ self-couplings in $e^+e^- \to Z\gamma$ is studied. The analytical expressions for the transverse-polarization-depending contributions for the Z boson spin orientation from the Standard Model and anomalous couplings are calculated and analyzed. The differences between the contributions depending on transverse and longitudinal polarization are presented. Differently from the longitudinal polarization the transverse polarization provides the theory with extra directions. Thanks to these directions determined by the transverse polarization vectors $\vec{\tau}_-$ and $\vec{\tau}_+$ the Z boson polarization vector obtains additional components. Since the Standard Model generates no terms to the Z boson polarization vector which depend on transverse polarization, any such term has to be from the anomalous couplings. Due to the $Z\gamma Z$ and $Z\gamma\gamma$ couplings, the Z boson polarization vector obtains an additional part and is moved outside the reaction plain. By measuring the angle between the reaction plain and the Z boson polarization vector, one can get information about the possible existence of the anomalous couplings.

The contributions depending on the transverse polarization change also the alignment tensors which can be measured in the experiment. The extra directions can also be used to construct new asymmetries.

In the fourth publication the "dynamical" interaction theory developed by Estonian scientists is used to support theoretically the value $g = 2$ of the gyromagnetic factor in spin-1 case. In the relativistic quantum mechanics the factor $g = 2$ is resulted from the Dirac ($s = 1/2$) equation. In the general case the minimal electromagnetic coupling being the part of the Standard Model, leads to g-factor $g = 1/s$. The value $g = 2$ from this well-known formula is consistent with the Dirac theory as well as with the experiment. However, the value $g = 1$ for the $s = 1$ charged gauge bosons $W^{\pm}$ leads to serious difficulties in description of γW scattering where the amplitude of the process in this case increases rapidly with increasing energy. Currently it is almost common agreement that the value for the gyromagnetic factor for all truly elementary charged particles of any spin is $g = 2$. This agreement is also supported by the experimental value of W boson gyromagnetic factor $g_W = 2.22 \pm 0.20$. However, theoretical arguments for supporting the $g = 2$ value emerge rather from the practical needs than from fundamental theoretical grounds. In the "dynamical" theory a term linear to the field strength tensor $F_{\mu\nu}$ is added to the minimal electromagnetic coupling part. This term is zero for the spin-0 and spin-1/2, but nonzero in higher-spin cases. In the publication it is shown that due to this term the "dynamical" theory leads to the value of $g = 2$ for spin-1 case. Such a result supports the value $g = 2$ and also suggests to the further developing of the "dynamical theory".

The thesis starts with the overview of the Standard Model and possibilities for searching new physics indications shedding light also at the concrete cases of anomalous scalar-type interactions and neutral gauge boson self couplings. In the next chapter the mathematical apparatus for describing spin orientation phenomena is presented in a general manner. The following chapters decribe the possibilies of probing new physics in the aforegiven processes as well some higher spin physics problems. The thesis ends with a short survey of the results.

Summary in Estonian

Uue füüsika otsingud raskete osakeste tekkel e^+e^--annihilatsiooniprotsessides spinnorientatsiooni efektide kaudu

Kuigi juba läinud sajandi 70-ndate alguses loodud elementaarosakeste füüsika teoreetiline mudel – Standardmudel – on olnud kooskõlas kõigi seniste kiirendikatsetega, ei ole ta teooriana siiski rahuldav. Paarikümmet katsest võetud parameetrit sisaldades on ta liialt empiiriline ega suuda vastust anda temast endast johtuvatele olulistele küsimustele. Seetõttu on põhjust arvata, et seniste kiirendite energiatest palju kõrgemal energiaskaalal eksisteerib hoopis fundamentaalsem, nn uus füüsika, millelt loodetakse saada vastuseid paljudele Standardmudelis vastamata jäänud küsimustele. Uue füüsika olemasolu võib katsest tuvastada kaheti – otseselt ja kaudsemalt. Uue füüsika otseseks avastamiseks oleks uue füüsika osakeste leidmine kiirendiprotsessidest. Usutakse, et kui 2009. aastal käivitatud Suur Hadronite Põrguti (LHC) elementaarosakese uurimiskeskuses CERNis maksimaalse põrkeenergiaga (14 teraelektronvolti) tööle hakkab, on ta võimeline uue füüsika osakesi tekitama. Aga plaanitud tulevikukiirendi, elektrone positronidega põrgatava Rahvusvaheline Lineaarpõrguti (ILC) energiast (algul 0.5, hiljem 1 teraelektronvolt) jääb uue füüsika osakeste tootmiseks ilmselt vajaka. Õnneks on teid uue füüsika ilmingute otsimiseks ka allpool uue füüsika energiaskaalat. Need põhinevad Standardmudeliga ennustatud tulemustest väikeste kõrvalekallete otsimisel Standardmudeli osakeste vahelistes protsessides, kus uue füüsika efektid võivad tekkida ainult mittestandardsetest interaktsioonidest. Taoline uue füüsika ilmingute otsimine nõuab nii katselt kui teoorialt uut tundlikkust.

Käesolev doktoritöö, mis võtab kokku autori nelja avaldatud publikatsiooni sisu, käsitleb valdavas osas uue füüsika ilmingute otsimist spinnefektide kaudu ILC protsessides $e^+e^- \to t\bar{t}$ ja $e^+e^- \to Z\gamma$. On ammune tõde, et võrreldes ülejäänutega on osakeste spinnide orientatsioonidest tingitud efektid tundlikumad uue füüsika ilmingute suhtes. See asjaolu annab spinnefektide kasutamise seisukohalt ILC-le LHC ees eelise – vaatamata suurele allajäämisele põrkeenergiates. Seda võimaluse tõttu kasutada temal nii piki- kui ristpolariseeritud algosakeste (elektronide ja positronide) kimpe. Eriti ristpolariseerituid, mis erinevalt pikipolariseeritutest toovad teooriasse täiendavaid suundi, mille abil saab konstrueerida spinnolekutest sõltuvaid uusi mõõdetavaid suurusi.

Esimeses publikatsioonis uuritakse uue füüsika ühe võimaliku allika – anomaalsete, skalaarset tüüpi interaktsioonide – mõju protsessile $e^+e^- \to t\bar{t}$. Ristpolariseeritud algkimpude korral on, erinevalt polariseerimata või pikipolariseeritud algkimpude juhust, skalaarset tüüpi anomaalsete interaktsioonide panused juba häiritusarvutuse esimest järku liikmetesse – Standardmudeli ja anomaalsete interaktsioonide interferentsliikmetesse – nullist erinevad. See asjaolu annabki anomaalsete skalaarset tüüpi interaktsioonide uurimisele toodud protsessis mõtte. Publikatsioonis arvutatakse nende panuste analüütilised avaldised juhul, kui mõõdetakse kas t- või $\bar{t}$-kvargi polarisatsiooni. Tulemustest selgub, et panuste hulgas on CP-paarituid liikmeid, mistõttu protsessis rikutakse CP-invariantsust. Konstrueeritakse mitmeid CP-rikkumise suhtes tundlikke asümmeetriaid nii lõppkvarkide polarisatsiooniolekute mõõtmise kui mittemõõtmise korral. Näidatakse ka, kuidas reaktsioonitasandi ja t- (või $\bar{t}$-)kvargi polarisatsioonivektori vahelise nurga mõõtmine võimaldaks tuvastada anomaalsete interaktsioonide olemasolu.

Teises publikatsioonis uuritakse neutraalsete kalibratsioonibosonite võimalike anomaalsete omainteraktsioonide $Z\gamma Z$ ja $Z\gamma\gamma$ avastamisvõimalusi pikipolariseeritud algkimpudega protsessis $e^+e^- \to Z\gamma$. Leitakse Z-bosoni polarisatsioonivektori ja reastustensori analüütilised avaldised. Näidatakse, et Standardmudeli interaktsioonide panuste erinevusi saab põhimõtteliselt kasutada nende panuste üksteisest eraldamiseks ILC eksperimentides. Selgub, et algkimpude pikipolarisatsioon mõjutab Z-bosoni spinnide orientatsioone erinevates panustes olevate erinevate faktorite kaudu. Vaid need faktorid sõltuvad elektroni ja positroni pikipolarisatsioonide parameetritest h_- ja h_+. Muutes neid parameetreid, võib suurendada või vähendada erinevate panuste osakaalu, mis avaldub omakorda Z-bosoni spinnide orientatsioonide muudatustes. See asjaolu aitab kaasa anomaalsete interaktsioonide eraldamisele Standardmudeli omadest ja ka omavahel.

Kolmandas publikatsioonis uuritakse algkimpude ristpolarisatsiooni rolli võimalike anomaalsete omainteraktsioonide $Z\gamma Z$ ja $Z\gamma\gamma$ ilmingute tuvastamisel protsessis $e^+e^- \to Z\gamma$. Leitakse analüütilised kujud Standardmudeli ja anomaalsete interaktsioonide ristpolarisatsioonidest sõltuvatele panustele Z-bosoni polarisatsioonivektorisse ja reastustensorisse. Erinevalt pikipolarisatsioonidest toovad ristpolarisatsioonid teooriasse täiendavad suunad ja tänu nendele genereeritakse Z-bosoni polarisatsioonivektorile täiendavaid komponente. Kuna leitud avaldistest nähtub, et Standardmudeli interaktsioonid ei genereeri Z-bosoni polarisatsioonivektorisse ristpolarisatsioonist sõltuvaid panuseid, siis iga Z-bosoni ristpolarisatsioonist sõltuv liige saab tulla vaid anomaalsetest panustest. Tänu anomaalsetele panustele saab polarisatsioonivektor täiendava osa, mis viib ta reaktsioonitasandist välja. Nurga olemasolu Z-bosoni polarisatsioonivektori ja reaktsioonitasandi vahel viitab anomaalsetele interaktsioonidele. Ristpolarisatsiooni vektorid $\vec{\tau}_-$ ja $\vec{\tau}_+$ lisavad ka orientatsioonitensoritele täiendavaid liikmeid. Samuti on näidatud, kuidas nende vektorite abil saab konstrueerida uusi asümmeetriaid.

Neljanda publikatsiooni temaatika kuulub kõrgemate spinnide ($s \geq 1$) füüsika valdkonda. Siin kasutatakse eesti teadlaste poolt arendatavat "dünaamilise" vastastikmõju teooriat tuletamaks güromagnetilise kordaja väärtust spinn-1 alusosakeste jaoks. Relativistlikus kvantmehhaanikas tuleneb güromagnetilise kordaja g väärtus $g = 2$ Diraci (spinn-1/2) võrrandist. Kasutades Standardmudelisse kuuluvat minimaalse elektromagnetilise vastastikmõju teooriat on näidatud, et suvalise spinni s korral on güromagnetiline kordaja avaldatav valemiga $g = 1/s$. See ammune valem annab $s = 1/2$ korral güromagnetilisele kordajale sama väärtuse mis Diraci võrrand ning see on kooskõlas ka eksperimendiga. Kuid valemist tuleneva güromagnetilise kordaja väärtus $g = 1$ spinn-1 osakeste jaoks viib sama spinniga laetud $W^\pm$ kalibratsioonibosonite juures tõsistele raskustele γW-hajumise kirjeldamisel, kus tõenäosusamplituudid energia kasvades kiiresti suurenevad. Sel ja muudelgi põhjustel on välja kujunenud peaaegu ühine arvamus, et iga alusosakese güromagneetiline kordaja võrdub sõltumata spinni suurusest kahega. Seda arvamust toetab ka W-bosoni güromagnetilise kordaja eksperimentaalne väärtus $g_W = 2.22 \pm 0.20$. Samas on seda väärtust toetavad senised teoreetilised argumendid liiga praktilised, vähesema toetusega fundamentaalsematele alustele. "Dünaamilise" vastastikmõju teoorias lisandub minimaalse elektromagnetilise vastastikmõju osale väljatugevustensorit $F_{\mu\nu}$ sisaldav liige. See liige on null spinnide 0 ja 1/2 korral, kuid nullist erinev kõrgemate spinnide juhul. Publikatsioonis on näidatud, et tänu väljatugevustensorit sisaldavale liikmele annab "dünaamiline" teooria spinn-1 korral güromagnetilise kordaja väärtuseks $g = 2$. Selline tulemus toetab mitte ainult teadlaste üldist arvamust, vaid omab tähtsust ka “dünaamilise” teooria edasise arendamise seisukohalt.

Doktoritöö algab ülevaatega Standardmudelist ja uue füüsika ilmingute võimalikest tuvastamismeetoditest, valgustades ka anomaalsete skalaarset tüüpi interaktsioonide ja kalibratsioonibosonite omainteraktsioonidega seotud küsimusi ning selles valdkonnas senitehtut. Järgnev peatükk annab osakeste spinnide orientatsioonide kirjeldamise üldise matemaatilise aparatuuri. Neile lisanduvad ülaltoodud protsesse analüüsivad peatükid ja kõrgemate spinnide füüsika raskusi käsitlev peatükk. Doktoritöö lõpeb töötulemuste põgusa kokkuvõttega.

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PUBLICATIONS

Curriculum Vitae

Hannes Liivat

Date and place of birth:	May 4, 1970, Tartu, Estonia
Citizenship:	Estonian
Address:	Institute of Physics, University of Tartu, Riia 142, 51014, Tartu, Estonia
Phone:	(+372) 737 4722
Fax:	(+372) 738 3033
E-mail:	liivat@hexagon.fi.tartu.ee

Education

1988:	Secondary School No 15 of Tartu
1996:	BSc in physics (fundamental physics), University of Tartu (*cum laude)*
1998:	MSc in physics (theoretical physics), University of Tartu

Employement

1998-2004:	University of Tartu, PhD student
2004-	Institute of Physics, University of Tartu, research associate

Scientific work

Main field of research: Particle phenomenology, spin in electroweak processes

Curriculum Vitae in Estonian

Hannes Liivat

Sünniaeg ja koht:	4. mai 1970, Tartu, Eesti
Kodakondsus:	Eesti
Aadress:	Füüsika Instituut, Tartu Ülikool, Riia 142, 51014, Tartu, Eesti
Telefon:	(+372) 737 4722
Faks:	(+372) 738 3033
E-mail:	liivat@hexagon.fi.tartu.ee

Haridus

1988:	Tartu 15. Keskkool
1996:	BSc füüsikas (fundamentaalfüüsika), Tartu Ülikool (*cum laude)*
1998:	MSc füüsikas (teoreetiline füüsika), Tartu Ülikool

Teenistuskäik

1998-2004:	Tartu Ülikool, doktorant
2004-	TÜ Füüsika Instituut, teadur

Teadustegevus

Peamine töösuund:	elementaarosakeste füüsika: spinnefektid, elektronõrga interaktsiooni protsessid

DISSERTATIONES PHYSICAE UNIVERSITATIS TARTUENSIS

1. **Andrus Ausmees.** XUV-induced electron emission and electron-phonon interaction in alkali halides. Tartu, 1991.
2. **Heiki Sõnajalg.** Shaping and recalling of light pulses by optical elements based on spectral hole burning. Tartu, 1991.
3. **Sergei Savihhin.** Ultrafast dynamics of F-centers and bound excitons from picosecond spectroscopy data. Tartu, 1991.
4. **Ergo Nõmmiste.** Leelishalogeniidide röntgenelektronemissioon kiiritamisel footonitega energiaga 70–140 eV. Tartu, 1991.
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6. **Tõnu Pullerits.** Primary energy transfer in photosynthesis. Model calculations. Tartu, 1991.
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13. **Tõnu Reinot.** The kinetics of polariton luminescence, energy transfer and relaxation in anthracene. Tartu, 1992.
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