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**On estimating open position risk at the
electricity forward market**

Master thesis (15 ECTS)

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On estimating open position risk on electricity forward market

Abstract:

The aim of the thesis is to study the question of estimating the risks of electricity retailers when they offer clients contracts with fixed electricity prices for future periods and the question of choosing prices for such contracts so that the level of risk is acceptable for the company. In the theoretical part of the master thesis details of such contracts and derivation of prices of those contracts under no arbitrage condition are discussed as well as a brief electricity market description and a few models proposed for forward price modelling are given. In the practical part open position risk problem in electricity market is explored by using simulations based on historical data for forward prices and various probability distributions for the number of clients accepting the offers, their desired quantities of electricity and decision making times. In the result the risk premiums for given risk levels under various assumptions are found and conclusions about which of the model parameters are affecting the risk premium most strongly are made.

Keywords: financial mathematics, time series analysis, risk management, futures contracts, forward contracts, electricity market

Avatud positsiooni riski hindamisest “Forward” tüüpi elektrilepingute müümisel jaeklientidele.

Lühikokkuvõte:

Magistritöö eesmärgiks on uurida elektrimüügifirma riske ning valitud riskitasemele vastavate hindade leidmist fikseeritud hinnaga elektrilepingu pakkumisega kliendile mingiks tulevikuperiodiks. Töö teoreetilises osas antakse ülevaade erinevatest tulevikuhinna fikseerimisega seotud lepingutest ning selliste lepingute teoreetilistest hindadest teatud lihtsustavatel eeldustel, samuti kirjeldatakse mõningaid mudeliteid, mida on elektri tulevikulepingute hindade modelleerimisel kasutatud. Töö praktilises osas uuritakse mainitud lepinguga seotud riske arvutisimulatsioonide abil, kasutades ajaloolisi andmeid elektri tulevikulepingute hindade kohta ning mitmesugused töenäosusjaotusi klientide arvu, nende soovitud elektrienergia koguste ja otsuse vastuvõtmiseks kuluva aja kirjeldamiseks. Tulemusena leitakse erinevatele riskitasemetele vastavad hinnad mitmesugustel eeldustel ning tehakse kindlaks, millised mudeli parameetrid mõjutavad neid hindu kõige tugevamini.

Võtmesõnad: finantsmatemaatika, aegridade analüüs, riskihaldus, futuurid, elektritururg

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INTRODUCTION

A rational person producing something wants to be sure that when product is ready there will be someone who will buy it. A rational consumer also wants to be sure that for all products needed there will be someone from whom those products can be purchased. Even in Ancient Greece people wanted to be sure that there will be possibility to purchase products they were growing. Aristotle was the first who wrote about futures trading. In his trade description he gave an example of olive grower and his agreements with local olive-press owners about trading in future. [7]¹

Nowadays this type of trading is well-developed and takes its place in futures markets, which are central financial exchanges where people can trade standardized contracts for obtaining the underlying asset in the future for a fixed price. However, in futures markets are some regulations what and how can be traded. For example, not all the customers can trade in those futures markets which deal with trading in large quantities. Therefore various companies can intermediate this market. These intermediary companies offer to a client a fixed price for underlying asset/obtaining an asset in a fixed future date or time period, which is based on current market price. If a client accepts it, company makes trading in futures markets for agreed quantity. However usually client cannot give an answer in the same day when an offer has been made – they need some time for considering it or just because of formalities. During this period the price of the corresponding contracts in the market could change and, consequently intermediary company takes some risk. If the price in the market at the time when client accepts an offer has increased comparing with the day when an offer was made, it results in loss due to this difference. The company can avoid this risk, if it is measured before and included in the initial price offered. The practical part of these thesis will be focused on finding this risk premium in electricity futures market.

In the first chapter of theoretical part of this thesis two main types of contracts for fixing the price for a future date, namely, futures and forward contracts and their differences are described and the formulas for the prices of those contracts under no arbitrage condition are derived. Further the focus will be directed more on electricity market and a few proposed models for forward price modelling and risk premium will be presented.

Nowadays futures markets and trading with forward contracts for electricity have developed and become a major financial activity of the energy commodity industry. Usually most of the electricity is delivered on basis of forward contracts for some fixed time period.

¹ Article: Futures exchange (access time 15.04.2014.)

However, electricity has non-storable feature, so the pricing of risk in forward trading becomes more complicated.

In practical part electricity market will be explored focusing on questions:

- 1) what would be universal, constant over time risk premium, which company should add to the market price making an offer to a client;
- 2) what would be the risk premium for current market situation;
- 3) how does risk premium depends on amount of electricity.

Calculations are based on historical prices which had been offering to clients for electricity for some specific time period.

1. FUTURES AND FORWARD CONTRACTS

1.1. General description of futures and forward contracts

Futures contract is an agreement between two parties to buy or sell a stated quantity of a specified asset with a pre-agreed price (the futures price) on a specific future date (delivery date). Futures price is fixed such that entering the contract does not cost any money to the parties of the contract. Futures contracts are widely used for commodities such as sugar, wool, gold and financial assets – currencies, bonds, stock indices.

There are two largest exchanges on which the futures contracts are traded – the Chicago Board of Trade and the Chicago Mercantile Exchange (CME). The Chicago Board of Trade is the world's oldest futures and options exchange, it was founded in 1848, while the Chicago Mercantile Exchange (CME) was established in 1898. Nowadays it is focused on trading several types of financial instruments (interest rates, equities, currencies, and commodities) and also has the largest options and futures contracts open interest (number of contracts outstanding) of any futures exchange.

The party which agrees to buy the asset in the futures contract is said to be “long” and the party which agrees to sell the asset in future, is called “short”. To govern positions in futures contracts specified daily agreement procedure is used which is known as marking to market. Initial deposit of investor is called initial margin and it is daily adjusted to reflect the losses and profits which are related to movements of the futures price. For example, if we consider the party which has a long position (agrees to buy), when the futures price decreases, then the margin account is reduced by appropriate amount of money and the broker has to pay it to the exchange. Exchange passes this money to the broker of party which has a short position. Similarly, in case when futures price increases, the broker from short position party needs to pay appropriate sum of money to exchange and the broker from long position party receives it from exchange. Thus the trade is marked in market at the end of each trading day. If delivery date is attained, party with a short position has to make a delivery to party with a long position. The price which party with a short position receives is the futures price at the time when the last marked to market has been made.

The concept for forward prices is similar, just trading is less formal and regulated. The forward contract is an agreement to buy or sell an asset at a certain time in the future for a certain price which is agreed upon current time moment. The buyer (or seller) in a forward contract acquires a legal obligation to buy (or sell) an asset (known as the underlying asset) at some specific future date (the expiration date) for a price (the forward or delivery price)

which is fixed today. Party which has long position of forward contract agrees to buy the underlying asset on a specified future date for a delivery price. Holder of short position of forward contract agrees to sell the asset on the same future date for the same price. At the time when the forward contract is made, the delivery price is chosen so that the value of this contract for both sides is zero. However, later forward contract value can become positive or negative. Some features for forward contracts are similar to those in futures contracts. However there are differences, for example, forward contracts are not usually traded on exchange, it is settled only once – at the maturity date. The party of short position delivers the asset to the holder of the long position, getting the amount of money which is equal to the delivery price in return. [7]² [11] [12]

1.2. Main differences between the futures and forward contracts

Contract specification and delivery

Delivery dates and delivery procedures are standardized to a limited number of specific dates per year, at approved location. In reality delivery is often not taking place, since it is usually not the aim of transaction. According to [11] just less than 2% are delivered. In forward contracts the range of instruments is almost unlimited, with individual contract prices. Delivery can take place at any negotiated place and date, and it is the goal of transaction. More than 90% of forward contracts ended up with delivery.

Prices

In futures contracts the price is the same for all participants, not depending on the transaction size. Prices are disseminated publicly. Each transaction is made with the best price which is available at that time. In forward contracts the price depends on different factors like the size of transaction, the credit risk and others. Trading takes place between individual seller and buyer and prices are not spread publicly. There is no confidence that the available price is the best one.

Security deposit and margin

For futures contracts there are specific rules from exchange – for requiring initial margin and the daily settlement of variation margin. The daily reassessment of open position cash payments and delivery procedures for each exchange is done by a central clearing house. In forward contracts the amount of money which the seller gets even if other party breaks the contract is negotiable, with no adjustment for daily price variations. There is no separate clearing house function. If one party defaults, all market participants take this risk.

² Article: Futures contracts, Forward contract (access time 05.03.2014.)

Market place and trading hours

Trading with futures contracts is centralized on the trading floor of exchange, with worldwide communications during the hours which are fixed by the exchange. On the contrary – it depends on agreement between individual buyer and seller, where trading with forward contract is organized. Trading goes on around the clock over-the-counter world-wide.

Volume and market liquidity

The information about volume of futures contracts is freely available. The liquidity for futures contracts is very high, what makes it easy to sell the contracts to other market participants. It is achieved mainly due to standardization of contracts. However, the information about volume of forward contracts is not openly available. The liquidity for forward contracts is limited, because some terms in contract is specified to the owner of the contract. [11]

1.3. Forward price

Forward contract is signed at initial date ($t = 0$). T denotes delivery date and K – delivery price. The forward price is the delivery price at the current time moment for fixed future date T which is offered to clients (for which it is possible to enter new forward contract at time t without any cost). So at the time when the forward contract is entered into it is the same as delivery price. Usually for different maturities the forward prices vary. For example, forward price for a 3 months contract is different than that for a 9 months contract. The party which has long position is obligated to buy an asset at delivery date with spot price S_T , for delivery price K . The payoff function from the long position in a forward contract on one unit of an asset is

$$X = S_T - K,$$

where X is contingent claim at time T . And similarly payoff function in case of short position is

$$X = K - S_T.$$

As from both sides it is not needed to pay anything to enter into the forward contract, the total gain or loss is the payoff from the contract. There are payoff functions for forward contracts in case of long position and short position shown in Figure 1. [6] [7]³

³ Article: Forward contract, Forward price (access time 08.03.2014.)

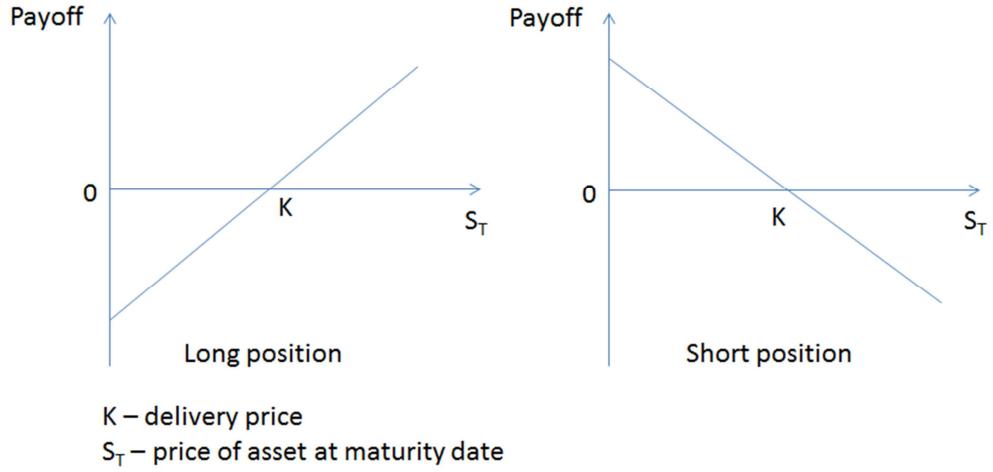


Figure 1. Payoffs from Forward Contracts.

If underlying asset is freely tradable and can be bought earlier and stored for later use, then the delivery price for which it is possible to enter forward contract can be obtained from the arbitrage principle. Arbitrage principle states that almost surely (with probability 1) one side will get a profit. If arbitrage profit exists and it is quite high, more of arbitrage opportunities will be used, but in this case the profits will fall. As in idealized market information spreads quickly, arbitrage opportunities become evident and more investors want to use it and get some profit. It results that arbitrage cannot last for a long time. Therefore it is assumed that arbitrage is not possible.

At first let us consider the case of asset that can be held without any storage costs and which does not provide any income (dividends). Let S_0 be the spot price of an asset, $F_{0,T}$ be forward price (at time $t = 0$ normally the delivery price set equal to the forward price $K = F_{0,T}$) and r as the risk-free interest rate. Then in no-arbitrage case the price of a forward contract is

$$F_{0,T} = S_0 e^{rT}. \quad (1)$$

Let us show that formula (1) cannot hold in arbitrage case. Firstly assume that $F_{0,T} > S_0 e^{rT}$. Then one can construct an arbitrage strategy: at time $t = 0$ borrow S_0 with interest rate r , buy one unit of asset and take short position into a forward contract with – sell the asset for $F_{0,T}$ at delivery date. At time $t = T$ needs to sell a unit of asset for $F_{0,T}$ and must repay the loan $S_0 e^{rT}$. So this leads to riskless profit $F_{0,T} - S_0 e^{rT} > 0$. But it is contrary to the no-arbitrage. Secondly, assume that $F_{0,T} < S_0 e^{rT}$. Then the arbitrage strategy is reverse: at time $t = 0$ the owner sells one unit of asset for S_0 and invests this at interest rate r for the time period T and takes long position into forward contract. At time $t = T$ receives $S_0 e^{rT}$, but need

to buy an asset for $F_{0,T}$. Again this leads to a certain profit $S_0 e^{rT} - F_{0,T} > 0$, which is contrary to the no-arbitrage principle - $S_0 e^{rT} - F_{0,T} \leq 0$. If combine both cases of no-arbitrage principle, get (1). [4] [10] [12]

In general for any time moment equality (1) transfers by

$$F_{t,T} = S_t e^{r(T-t)}, \quad (2)$$

where $F_{t,T}$ – forward price at time t ,

S_t – price of underlying asset in forward contract at time t .

The equation (2) changes if consider forward contracts on security that provides known cash income. Denote the present value of income which holder receives during the life of the forward contract in case of risk-free discount rate by I . Then under no-arbitrage condition, the price of forward contract must be

$$F = (S - I)e^{r(T-t)}. \quad (3)$$

To prove that previous formula cannot hold in arbitrage principle, at first, assume $F > (S - I)e^{r(T-t)}$. Arbitrage strategy is such: at time t borrow money, buy the asset and take short position in forward contract. At maturity, sell the asset for F and need to repay the loan which is $(S - I)e^{r(T-t)}$, assuming that a part of loan is covered by incomes which are received. Then the profit becomes $F - (S - I)e^{r(T-t)}$, which obviously is positive. And on the contrary – assume $F < (S - I)e^{r(T-t)}$. At time t sell the asset for S_t and invest the money and go long in forward contract. If we are using short-selling, then we have to provide the owner of the share we sold, if we sell our own asset then, compared to not selling we are losing during time interval $[t, T]$ the income that is equivalent to $Ie^{r(T-t)}$ at time T , so the profit is $(S - I)e^{r(T-t)} - F$. It follows that (3) no-arbitrage condition.

Considering previous situation with two portfolios, in case of known cash income, the portfolio 1 remains the same, but in the portfolio 2 – one unit of the security plus amount I , which comes from loan at a risk-free rate. This means that the loan can be repaid from the income of the security, so that at time T the value of the portfolio would be one unit of security. From previous portfolio 1 value is the same, therefore

$$f + K e^{-r(T-t)} = S - I.$$

As forward price F is the same as delivery price K initially and it is chosen so that the value of forward contract is zero. Thus (3).

Let us now consider forward contract in a security what provides a known dividend yield, this means that the income which is expressed as percentage form the security price is known. Assume that the dividend yield is paid continuously with an annual rate q . Looking at

previous case with two portfolios, this time portfolio 1 remains the same, but as regards portfolio 2 – buy $e^{-q(T-t)}$ units of underlying asset and reinvest all income in the security. In portfolio 2 the amount of security grows, because of paid dividends and at the time T it reaches one unit of security. So at maturity date both portfolios are worth the same value

$$f + Ke^{-r(T-t)} = Se^{-q(T-t)}.$$

In time $t = 0$ the forward price is equal with delivery price and set so that value of forward contract is zero, so

$$F = Se^{(r-q)(T-t)}. \quad (4)$$

1.4. The forward price relationship with the futures price

It can be shown that if interest rate is nonstochastic and forward contracts can be traded, then forward price is equal to futures price. [6] Trading happen with futures contract as it would be a forward contract and mark-to-market has been ignored.

Let us provide a proof that in the case of constant interest rate and when futures and forward contracts can be traded for any quantity of assets, the futures and forward prices are the same. Consider the futures contract which lasts for n days and the futures price at the end of day i is F_i ($0 < i < n$). Denote a risk-free rate per day by ρ and assume that it is constant. Consider the strategy where at the initial time one buys futures contract for e^{ρ} assets, at the end of the first day increases the long position to $e^{2\rho}$, at the end of the second day increases long position to $e^{3\rho}$ and so on. Hence, at the beginning of day i investor has long position of $e^{i\rho}$ and the profit (or loss) from the position is $(F_i - F_{i-1})e^{i\rho}$. This amount will compound at risk-free rate until the end of day n and becomes

$$(F_i - F_{i-1})e^{i\rho}e^{(n-i)\rho} = (F_i - F_{i-1})e^{n\rho}.$$

The value of this strategy at the end of day n is

$$\sum_{i=1}^n (F_i - F_{i-1})e^{n\rho} = ((F_1 - F_0) + (F_2 - F_1) + \dots + (F_n - F_{n-1}))e^{n\rho} = (F_n - F_0)e^{n\rho}.$$

As final future price F_n is equal to asset price at the delivery date S_T , then

$$\sum_{i=1}^n (F_i - F_{i-1})e^{n\rho} = (S_T - F_0)e^{n\rho}.$$

An investment of F_0 in a risk-free bond at time $t = 0$ and together with previous strategy yield at time T

$$F_0e^{n\rho} + (S_T - F_0)e^{n\rho} = S_Te^{n\rho}.$$

Observe that no initial investment is required for the strategy.

Denote forward price at the end of initial day as G_0 . In strategy where invest G_0 into riskless bond and take long position of $e^{n\rho}$ forward contracts, at maturity date amount $S_T e^{n\rho}$ is guaranteed. So there are two strategies which at time T give the same return $S_T e^{n\rho}$. For the first initial expenses F_0 are required and for the second initial expenses are G_0 . It follows that if no-arbitrage principle holds, then futures price and forward price are identical:

$$F_0 = G_0.$$

However, in case when interest rate is not deterministic – it can depend on time, but cannot be stochastic, futures and forward prices are not theoretically identical. [6] [10]

1.5. Futures on commodities and the cost of carry

Commodities could be divided in two main types for which futures contracts are used: in the first commodities are held primarily for investment purposes, such as, gold and silver, in the second, commodities mainly are held for consumption.

In case commodities are held solely for investment if the storage costs are zero, then the previous case can be considered that underlying asset provide no income (2). Denote the present value of all the storage costs which occur during the life of a futures contract by U . If there exist storage cost, then it can be assumed as negative income and (3) could be used to describe the relationship between F and S , as result $F = (S + U)e^{r(T-t)}$. However, if u denotes the storage costs which are proportional of the price for commodity, then the case when dividend yield is known could be used. As storage costs provide a negative dividend yield, form (3) get $F = S e^{(r+u)(T-t)}$.

From commodities which primarily are held for consumption it is possible to get some benefit from holding the physical commodity, so it is logical that those who keep the commodity do not want to sell it therefore

$$F \leq (S + U)e^{r(T-t)}, \quad F \leq S e^{(r+u)(T-t)}.$$

The benefit from ownership of commodity is measured by convenience yield, which is denote by y such that

$$F e^{y(T-t)} = (S + U)e^{r(T-t)}, \quad F e^{y(T-t)} = S e^{(r+u)(T-t)}.$$

The relationship between futures price and spot price describe so-called cost of carry. It measures the storage costs plus the interest costs paid by carrying the asset, but it should be less for the income earned on the asset. For stock which pays no dividend, the cost of carry is r , because there are no storage costs and it does not give any income. If the cost of carry is denoted by c , then for an investment asset $F = S e^{c(T-t)}$ and for a consumption asset $F = S e^{(c-y)(T-t)}$. [6] [10]

2. ELECTRICITY MARKET

2.1. Basic physical and economical aspects of electricity market

The main difference of electricity market and traditional commodity market is the fact that electricity cannot be stored like goods in warehouses in such quantities that can make impact on market processes. It is impossible to produce larger amount of electricity at the moments when price of electricity production is lower and sell it in market when electricity price is higher. According to the laws of physics and electrical aspects, the quantity of produced electricity at any moment is balanced with power consumption. Production volume is determined by customer load size and it is continuously variable. On basis of physics it is not possible to sell electricity in queue order (serve customers as in the store) or make any other priorities of customers to provide them with electricity. All grid connected customer appliances receive electricity simultaneously. Supply and demand balance influences not only market conditions but also physical processes. The compliance of load and producing power determines the frequency of electrical system that combined with voltage quality characterizes the overall quality of electricity supply. If load exceeds producing power system frequency is decreasing, but if producing power is larger than load, system frequency is rising. Therefore in electricity power system permanently is maintained generating power capacity that can ensure power for sudden load rise and there are some power stations the main mission of which is to control system frequency and take sudden load. These are processes that customer and market stakeholders often do not know – they are like invisible part. For covering the peak loads in Baltic states most commonly hydro power plants are used, but in Europe also gas power plants are used due to their shorter start-up time – only a few minutes. [13]

Specific feature, characterizing the electricity market is the fact that the market price of electricity can vary in large range within a relatively short period. During peak hours, when electricity consumption is the highest, electricity price can be several times larger than at the moments when the consumption is the lowest. According to the characteristics of the power consumption during the day the highest electricity consumption is in the morning peak hours from 7 till 11 a.m. and in the evening peak hours from 17 till 23 p.m.. The lowest consumption is during the night hours. During a year the highest consumption and electricity prices are in winter and summer months (especially in countries where conditioning appliances are developed), but the lowest consumption and prices are in autumn and spring. In spring, especially in Baltic and Scandinavian countries, lower prices are because of high water level in rivers. During these periods comparatively cheaper electricity from hydro power

plants is available. The electricity price is also influenced by meteorological forecasts, planned network outages (maintenance works), availability of production units etc. On basis of meteorological situation varies production in hydro power plants and in wind farms. Planned network maintenance works reduces carrying capacity of network, so during this time there is limited offer of electricity. Recently existing power carrying capacity limitation on the border between Estonia and Latvia is a reason why on JSC "Nord pool spot" market in Latvian bidding area, that was made in June, 2013 electricity price is higher than in Estonian bidding area. [8]

Baltic states joined to electricity market quite late. Monopolistic system is changed to free market with electricity trader competition. Electricity prices are not determined by public utility regulator but by the market situation. Electricity market in Europe was opened in the year 2007 and the first countries joining to the market were Sweden, Finland and United Kingdom. According to the strategic document of Baltic states "Baltic Energy Market Interconnection Plan" (BEMIP) was started opening process of electricity markets of Baltic states. Liberalization of electricity market in Latvia was started on July 1, 2007 when from holding company "Latvenergo" transmission and distribution system operators were separated. JSC "Latvenergo" now is electricity producer and trader. Now electricity market in Latvia is fully opened for legal persons but for household market it will be opened on January 1, 2015. In Latvian electricity market in spring, 2014 are 10 active electricity trading companies. In Estonia electricity market is opened since April 1, 2010 when legal persons joined, but on January 1, 2013 households joined. Since electricity trading in the free market is a relatively new thing, electricity operators have to acquire new knowledge and to analyze the experience acquired in order to operate successfully in the market. [9]

In the electricity market electricity producers, power transmission and distribution system operators, customers, traders and electricity exchange are involved. In the electricity exchange traders and producers submit their offers. Customers evaluate offers in the electricity market and choose electricity trader from which they will purchase electricity. Power transmission and distribution operators provide physical network infrastructure to deliver electricity from producer to customer appliances. Bill for electricity basically consists of two parts – charge for electricity and for system services.

Today electricity exchange participants are prompted to use current spot market and forward market and that is why the market has great liquidity, more efficient use of electricity capacity and ensures transparent price formation. All the participants of electricity market are responsible for adequate purchased amount of electricity and the delivered in system to

amount of electricity received from transmission system. That is why one organization that controls (balances) these values is necessary. It is a special power balancing service and usually it is done by power transmission operator. [9]

As mentioned before electricity cannot be stored in reasonable quantity and that is why it cannot be considered as regular commodity. No-arbitrage principle is based on buy-and-hold strategy, but because of electricity non-storable feature, it cannot be used for deriving the price of forward contracts. In literature commonly two ways of models for price are described: the price could be modeled in traditional way – as stochastic process and derive prices of assets from equilibrium principle (modeling and balancing demand and supply). [5] [3]

2.2. Some proposed models for electricity forward price

Next we are going to consider two models for electricity forward price: the first, model for forward price dynamics, the second, model for forward price using equilibrium principle.

2.2.1. Model for electricity forward curve dynamics

There has been suggested a model for price dynamics in Nordic electricity market elaborated by N. Audet, P. Heiskanen, J. Keppo, and I. Vehviläinen in year 2004. [2] Consider that forward contracts are traded continuously within finite time horizon $[0, \tau]$. Denote $F(t, T_1, T_2)$ by forward price for the period $[T_1, T_2]$ at time t and for this period it is constant. To get the forward price which would depend just on one maturity date, consider the following theoretical forward price

$$F(t, T) = \lim_{T_2 \rightarrow T} F(t, T, T_2), \quad \forall t \in [0, T], T \in [0, \tau].$$

It means that the forward price at time t is for the time period $[T, T + dt]$.

In the paper the model is

$$dF(t, T) = F(t, T)e^{-\alpha(T-t)}\sigma(T)dB_T(t), \quad \forall t \in [0, T], T \in [0, \tau],$$

where α is a constant ($\alpha > 0$), σ is a bounded and deterministic spot volatility curve, and B_T is a Brownian motion corresponding to the T -maturity forward price on the probability space (Ω, \mathcal{F}, P) along with the standard filtration $\{F_t: t \in [0, \tau]\}$. The correlation structure of the Brownian motions is given by

$$dB_{T^*}(t)dB_T(t) = e^{-\rho|T-T^*|}dt, \quad \forall T, T^* \in [0, \tau],$$

where ρ is a constant ($\rho > 0$). This model implies that logarithmic returns of forward prices are independent and normally distributed and that the variance of the changes over fixed time intervals is increasing as the maturity date is approaching. [2]

2.2.2. Model for the electricity forward price using equilibrium principle

Another model to satisfy non-storable property has been suggested model for the equilibrium electricity forward price by Hendrik Bessembinder and Michael L. Lemmon in year 2002. [3] The model is based on assumptions that most of market participants are interested to sell or buy electricity not to make speculations, so the price is set by industry participants. It is assumed that the all power producers and all retailers are similar to each other and specific forms for their objective functions and operating cost functions are assumed. Based on equilibrium principle the following formula for the price of one period ahead forward price is derived:

$$F = E(S_W) - \frac{N_P^x}{Na^x c} (cS_R cov(S_W^x, S_W) - cov(S_W^{x+1}, S_W)),$$

where F – forward price,

S_W – wholesale spot power price at next period,

S_R – fixed retail price,

N_P, N_R – power producers and power retailers,

c – constant which is greater than or equal to two ($x = \frac{1}{c-1}$),

a – variable cost parameter,

$N = \frac{N_R + N_P}{A}$, where $\frac{A}{2}$ is the coefficient on the variance of profit in the objective functions

of power retailers and producers.

To use this model information about power producers and retailing firms is needed, however, as we do not have such information then this model would not be practically analyzed. [3]

3. ESTIMATING RETAILER'S RISK IN ELECTRICITY MARKET

3.1. General description of the problem

As in electricity futures market there are regulations for participants, therefore all clients cannot take part in futures market, so there are companies which operates as intermediaries between the futures markets and the clients. However, acting as intermediary a company takes the open position risk. Open position is a financial status, where counterparty has an option to enter into a contract to buy a certain amount of electricity during specified future period for a fixed price. Open position risk is the problem when a company offers to client one price which is related to the electricity futures market price in the day the offer is made. Client usually needs more than one day to make the decision, but meanwhile the company cannot change the offered price. However, during this period price in the market could change. If the market price in the day when client makes a decision has decreased compared with the day when the company has made an offer, it results in a profit due to this difference, but if the price has increased, it is accordingly a loss for the company. Our aim is to estimate how much should the offered price be larger than the current market price in order to get the probability of loss of specified level.

3.2. Assumptions and model components for finding the risk premium

To find the price difference which can be used in the future contracts and to be quite sure that the company would end up without big losses, we have to find the distribution of the gains/losses per MWh. Therefore we form a model for trading outcomes that consists of the following components:

1. model for forward price process (F_t) for a fixed delivery period;
2. clients arrival – how many clients who eventually accept the offer arrive in one day, denoted by C_t ;
3. quantities of electricity for a client – how much MWh of electricity each of the clients want to buy, denoted by $Q_i, i = 1, 2, \dots, C_t$;
4. how long each client is considering an offer, denoted by $T_i, i = 1, 2, \dots, C_t$.

In this thesis we assume that all model components are independent. It means that it does not depend on forward prices, how many clients arrive in one day and how much electricity they want to buy. And the length of time needed to clients for making up their minds also does not depend on amount of electricity they want to buy and on what happens with the prices at the market while a client is thinking.

As we do not have data about client arrivals and their thinking times and electricity quantities they are going to buy, we just choose some reasonable distributions for those random variables. For client arrival in one day we consider two cases: $C_t \sim 1 + X$, $X \sim Po(4)$ and $C_t \sim 1 + X$, $X \sim Geom\left(\frac{1}{5}\right)$, where the values of the geometric distribution are assumed to start from 0. For quantity of MWh of electricity exponential distribution was chosen $(Q_i \sim Exp\left(\frac{1}{5}\right))$. It was considered that the client thinking time is measured in days and T_i has discrete uniform distribution with values $\{1, 2, \dots, T\}$ for some fixed maximal thinking time T . First examples use $T = 3$, but later we consider also cases $T = 10, 20, 30$.

As mentioned before to model the spot price and also forward price in electricity market is a difficult task, because of electricity's non-storable property as well as the fact the price depends on lots of other factors, such as weather and oil price. We consider two approaches of forward prices according to risk preferences: the first is adding a constant premium to current price, the second is increasing the current price by fixed percentage. The first approach:

$$V_t = F_t + \delta,$$

where F_t – price in the market at time t ;

V_t – price which have been offered to the client;

δ – constant premium.

The second:

$$V_t = F_t(1 + \gamma),$$

where γ – fixed percentage.

At time t company takes an obligation to sell Q_i MWh electricity for price V_t to client i during delivery period, but at time $t + T_i$ buys it from the market with price F_{t+T_i} . In the first case the total money what the company earns/loses during the delivery period based on price offered at time t is

$$\sum_{i=1}^{C_t} Q_i(F_t + \delta) - \sum_{i=1}^{C_t} Q_i F_{t+T_i} = \delta \sum_{i=1}^{C_t} Q_i + \sum_{i=1}^{C_t} Q_i(F_t - F_{t+T_i}).$$

If a measure of risk is specified, it is possible to compute the risk premium so that the risk level is acceptable. In this thesis measure the risk by probability of losing moneys. So if risk level α is specified, we find premium α form the condition:

$$P\left(\delta - \sum_{i=1}^{C_t} \frac{Q_i}{Q} (F_{t+T_i} - F_t) > 0\right) = q,$$

where $Q = \sum_{i=1}^{C_t} Q_i$ and $q = 1 - \alpha$.

It means that interpretation of the result will be, how much the company should add to the current market price, so that risk would be at acceptable level. Two levels of q will be considered: 80% and 90%.

Denote

$$Z = \sum_{i=1}^{c_t} \frac{Q_i}{Q} (F_{t+T_i} - F).$$

Then we find the risk premium δ from the equation

$$P(Z < \delta) = q.$$

Thus δ is the q -quantile of Z .

If considering the second case for the offered price, the problem is different. The total earnings/losses of the company per one day are:

$$\sum_{i=1}^{c_t} Q_i F_t (1 + \gamma) - \sum_{i=1}^{c_t} Q_i F_{t+T_i} = \gamma \sum_{i=1}^{c_t} Q_i F_t + \sum_{i=1}^{c_t} Q_i (F_t - F_{t+T_i}).$$

Similarly to the previous case we can transform the condition that risk is less than α to the condition

$$P \left(\sum_{i=1}^{c_t} \frac{Q_i}{Q} \left(\frac{F_{t+T_i} - F_t}{F_t} \right) < \gamma \right) = q,$$

where $q = 1 - \alpha$.

If current time t and model for forward market price is fixed, it does not matter which approach to use. Which of the two approaches is more meaningful, depends on the model for forward prices – for some models δ does not depend on current time (so is the same for all time moments), for other models γ does not depend on time.

3.3. Characteristics of the analyzed data

In practical part historical prices which the company X had been offering to clients for electricity for year 2012, 2013 and 2014 will be used in the role of market prices of corresponding contracts. In Figure 2 it can be seen that for years 2012 and 2013 in general electricity price decreases, but for year 2014 it is more stable.



Figure 2. Electricity prices.

The first plot of Figure 3 shows historical price differences for the year 2013. Similar plots for year 2012 and 2014 data could be found in Appendix Figure 9 and Figure 10. It can be seen that price movements are different, some of them are relatively small, but some are quite large, so one could say that one day price difference is quite random.

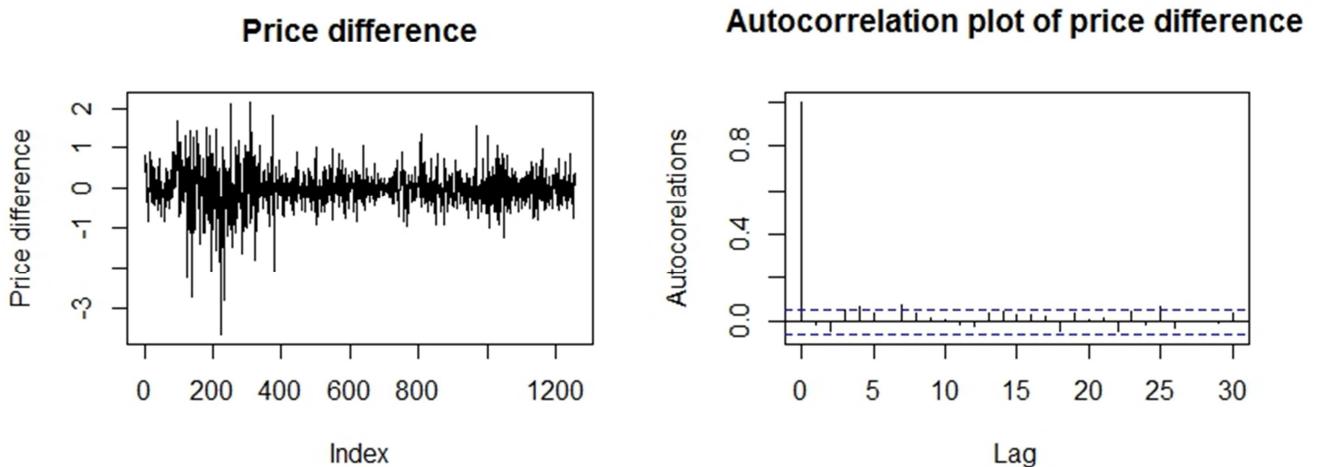


Figure 3. Price differences and their autocorrelation plot.

Let us first try to fit an Autoregressive Integrated Moving Average (ARIMA) type model for forward prices. It is statistical method which uses time series data for predicting future trends. The Figure 1 of prices shows that the process is not stationary, so taking a difference is needed. From the second plot of Figure 3 and Appendix 1. Figure 9 and Figure

10 could be seen that autocorrelations and partial autocorrelations are practically within error bounds, so we can say that ARIMA(0,1,0) models (random walk) may be appropriate:

$$F_{t+1} = F_t + x_{t+1},$$

where x_{t+1} is price difference at time $t + 1$.

Firstly it will be assumed that all one day historical price differences are with the same distribution and, as autocorrelations are close to zero, then independent identically distributed (IID) increments will be considered, so in order to simulate forward prices according to the model, there are two common approaches. The first one is to find a suitable distribution for the changes and generate random variables from this distribution. The second approach is to consider historical differences as possible outcomes from sampling of the distribution and for generating a new value of x_t a random historical value is chosen. As historical data is available, in these thesis the second approach is chosen. So at first will be simulated the simple model for forward price movements with independent identically distributed (IID) increments.

However usually in finance it is often reasonable to assume that changes of asset values are proportional to the current value. It was also assumed in model for electricity forward curve dynamics from theoretical part. This means that in modelling it may be reasonable to use either returns or to model logarithm of prices (which differences are logarithmic returns).

Return at time t is the relative change of the stock price, if S_t is a stock price at time t , then return is

$$r_t = \frac{S_t - S_{t-1}}{S_{t-1}}.$$

Sometimes logarithmic return is analysed. It is defined by

$$R_t = \ln(1 + r_t) = \ln\left(\frac{S_t}{S_{t-1}}\right).$$

If r_t is small, then $r_t \approx R_t$, so which is used depends on the problem. Look for ARIMA model for logarithm of forward price, then looking at autocorrelations of differences (or logarithmic returns) are shown in Figure 4 for data of year 2013 and for data of years 2012 and 2014 in Appendix 1. Figure 11 and Figure 12 and it could be seen that they are close to zero, so it could be assumed that logarithmic returns are IID. Therefore again the model ARIMA(0,1,0) can be considered to be a reasonable one. This gives us the model:

$$\ln(F_{t+1}) = \ln(F_t) + x_{t+1} \leftrightarrow F_{t+1} = F_t e^{x_{t+1}},$$

where x_{t+1} is IID logarithmic returns ($x_{t+1} = \ln\left(\frac{F_{t+1}}{F_t}\right)$), for the market prices of forward contracts available for retailers.

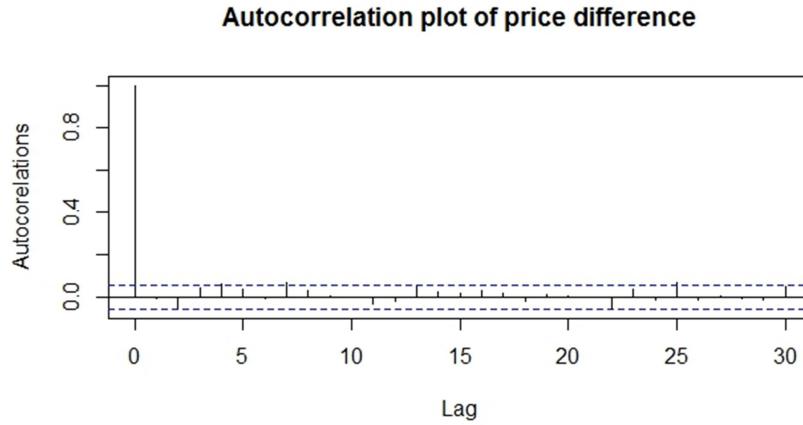


Figure 4. Autocorrelation plot from logarithmic returns.

It is easy to see that if the first model is used, the value of parameter delta (reference) does not depend on the value of the current market price F_t and if the model for the logarithm of the market price is used, the computation of the parameter gamma does not depend on the current value F_t . If we assume that changes in prices are proportional to the current prices, then it is reasonable also to ask, what percent of the current price should be added to it in order to have risk at acceptable level.

In previous model we assumed that differences between consecutive working days behave similarly but it is definitely possible that changes over longer periods (like weekends) behave differently, so it would not be reasonable to use previous assumptions about one day historical price differences. In Figure 5 one may see the historical price difference histogram for working days and weekends for year 2013 data. Price differences for year 2012 data and year 2014 could be seen in Appendix 1. Figure 13 and Figure 14 and they are similar to those of Figure 5 for year 2013.

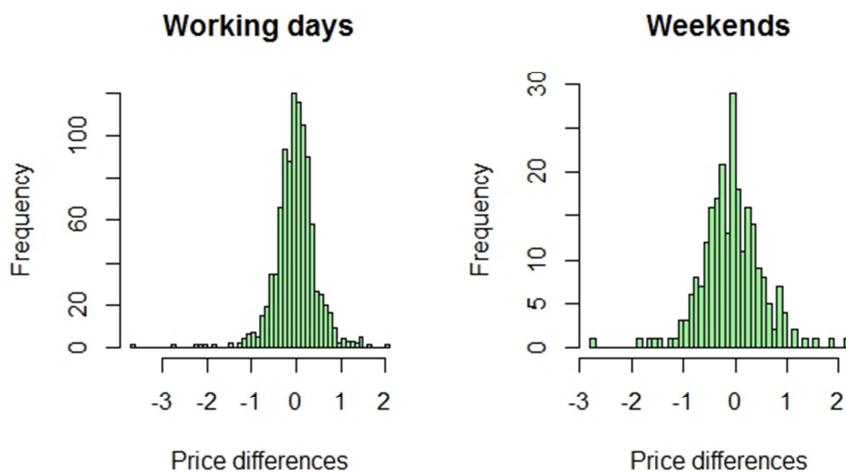


Figure 5. Price differences in working days and weekends for year 2013 data.

Mean value from both samples are around zero, for example, from data of year 2013 mean value from working days is -0.000485 and mean value from weekends is -0.057066 . With Kolmogorov-Smirnov test it was checked, if historical price differences in working days and weekends has the same distribution. It rejected this hypothesis with p – value 0.006212 for data of year 2012, 0.001489 – in year 2013 and 0.0254 – in year 2014. It means that it would be logical to consider two different samples and make calculations depending on which day of the week client has come, then price difference depends on the fact whether client decision-making time crosses Friday or not – if it does then also weekend data are included. For example, when client thinking time is limited to three days, then if entry day is Monday, price difference consists just from randomly picked one day increments from working day sample, but if entry day is Wednesday, then price difference is the sum of two randomly picked values from working day sample and one from weekend day sample. So the model when increments are identical, but the distribution for weekends is different also will be analyzed.

3.4. Numerical results

Further the results of numerical calculation will be represented. In each case considered, the distribution of losses was estimated by simulating outcome of our model 100000 times.

3.4.1. Simple model for forward price movements with IID increments

Computation results are possible loss per share if clients are offered the market prices, so Figure 6 describes the distribution of losses per share if clients are offered the current market price in case if client arrival per one day has Poisson distribution for year 2013 data. It can be seen that 90% of all 100000 times these losses are less than $E = 0.7046(EUR)$ and each colour (from light green to dark orange) represents 5% of results. Looking at numbers (from A to E) we can see that their differences are quite small – difference from A to B is approximately 0.08 of one euro, if comparing with one MWh price (for example 50 (EUR)), it means that for lower confidence level like 70% and 75% even small changes of the price can have an effect on the level that actual price differences (offered price and market price) would not be larger than estimated. Similar figures for year 2012 and 2014 can be found in Appendix 1. Figure 15 and Figure 16.

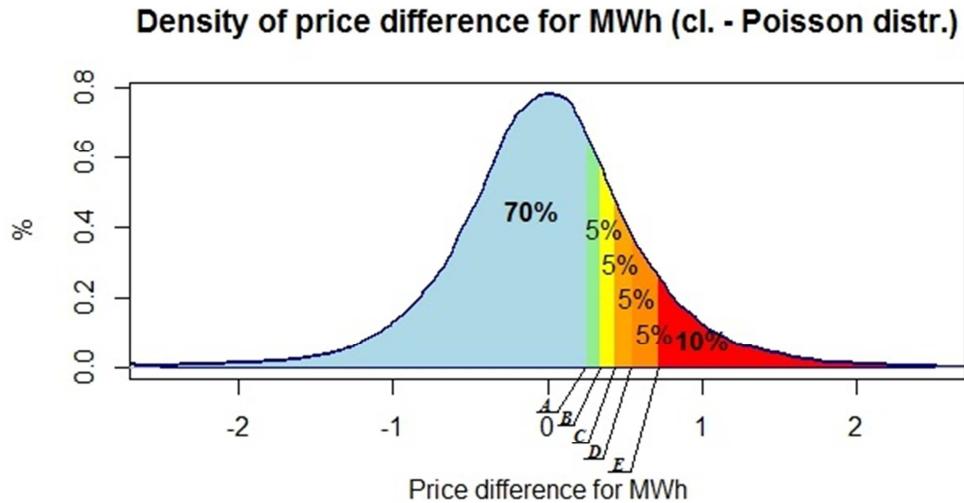


Figure 6. $A = 0.2524$; $B = 0.3349$; $C = 0.4253$; $D = 0.5466$; $E = 0.7046$.

Figure 7 represents risk premium for one MWh in case when number of clients arrive in one day follows geometric distribution with parameter $p = \frac{1}{5}$. 90% of repeated times risk premium was smaller than $E = 0.7046$ (EUR). Therefore if a company under those assumptions would ask by 0.7 EUR more than the market price, then the company could be 90% confident that they would not end up with big loss. One can compare results of both Figure 6 and Figure 7 – as they are very similar, then probably all differences are caused by random effects of simulation. This type of figures for risk premium using year 2012 and 2014 data could be seen in Appendix 1. Figure 17 and Figure 18 and they also are similar for clients arrival with Poisson and geometric distributions.

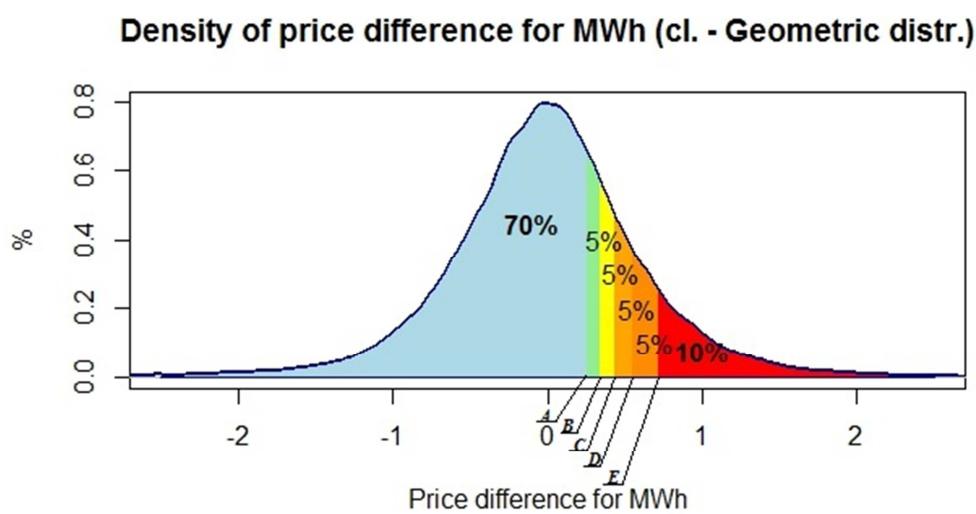


Figure 7. $A = 0.2535$; $B = 0.3383$; $C = 0.43$; $D = 0.55$; $E = 0.7074$.

3.4.2. Logarithmic model for forward prices, short maximal thinking time

Table 1 represents results for all three year data using regular differences and logarithmic returns. Risk premium for logarithmic returns is calculate as the product of mean value of the price and obtained percentage, so in this case percentages are random. Comparing results for regular differences and logarithmic returns it can be seen that they are quite close to each other. For example, difference between year 2013 90th percentile for assumptions that clients are coming with Poisson distribution is just approximately 0.0005. There is no information available how the company is calculating its forward price and as results under assumptions are very similar, then in the further calculations regular differences will be used.

Table 1. Results for previous calculations.

		2012		2013		2014	
		Regular difference	Log-return	Regular difference	Log-return	Regular difference	Log-return
Poisson distr.	80% of differences less than	0.5559	0.5610	0.4253	0.4326	0.2997	0.3036
	90% of differences less than	0.914	0.9170	0.7046	0.7051	0.5	0.5114
Geometric distr.	80% of differences less than	0.5535	0.5680	0.4376	0.4307	0.3	0.3053
	90% of differences less than	0.9192	0.9346	0.72	0.7097	0.5026	0.5164

In Table 2 one may see the percentage which should be added to the current market price for finding the price which the company should offer to the clients in order to have a risk at acceptable level: 20% and 10%.

Table 2. Percentage what of the current price should added.

	2012		2013		2014	
	Poisson d.	Geom. d.	Poisson d.	Geom. d.	Poisson d.	Geom. d.
Risk level 20%	0.0122	0.0123	0.0094	0.0094	0.0072	0.0072
Risk level 10%	0.0199	0.0203	0.0154	0.0155	0.0122	0.0122

In Table 1 it can be seen that comparing results for client arriving with Poisson and geometric distribution and results from using regular difference and log-returns they are more or less close to each other, but the comparative results during years are a bit different. For example, 90th percentile for assumptions that clients are coming with Poisson distribution are 0.9155, 0.7046 and 0.5 for year 2012, 2013 and 2014, the difference for each next year is around 0.2. Also Figure 8 shows the differences, however the views for both client arrival distributions are similar. Using data of the year 2014 more risk premiums are around zero and less values around 1 comparing with years 2012 and 2013, however the results using data of the year 2012 there are less values around zero and the heaviest tails, and the results of year 2013 are in between both others. As it can be seen in the Figure 2, this could happen because the data of year 2012 fluctuate the most, but the data of year 2014 are more stable. Also variation for one day price increments for year 2012 data is the largest (0.3854737), but for year 2014 data it is 0.1181933 and for year 2013 – 0.2344494.

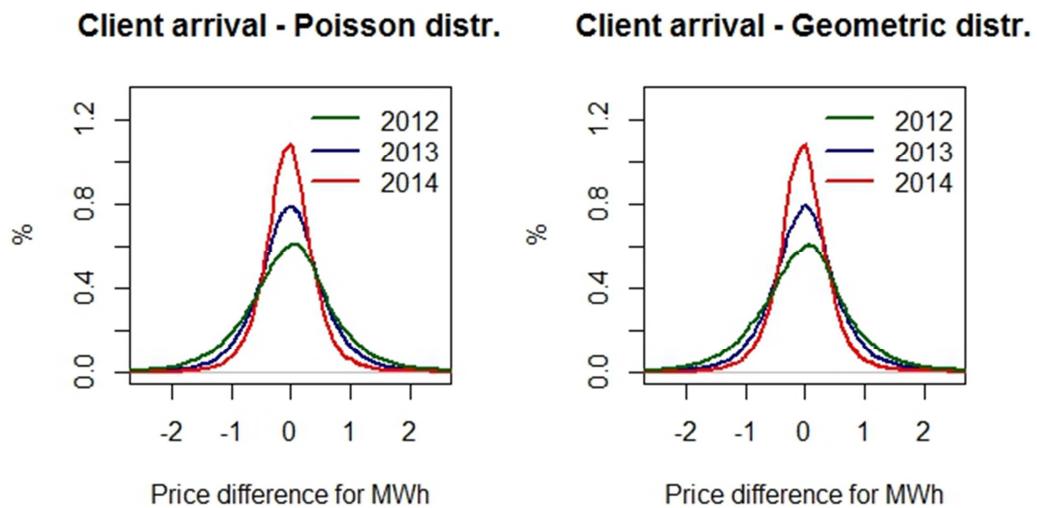


Figure 8. These are losses per share in the case of 0 risk premium (random variable Z from before).

In previous analysis risk premium is determined as fixed percentile from 100000 computed times, however we do not know how random are the obtained results, it means that it is necessary to study the question, how much the basis changes. The influence to results of increasing number of calculations is studied, if the differences between results using different number of calculations are small, then even calculating with smaller number of generations we get accurate results.

Table 3. Percentiles using different number of calculations ($n_1 = 100000$, $n_2 = 1000000$).

	2012		2013		2014	
	n1	n2	n1	n2	n1	n2
70%	0.3388	0.3397	0.2524	0.2559	0.17	0.1678
75%	0.4392	0.4408	0.3349	0.3372	0.2294	0.2279
80%	0.5559	0.5587	0.4253	0.431	0.2997	0.2992
85%	0.7063	0.7076	0.5466	0.55	0.3867	0.384
90%	0.914	0.9149	0.7046	0.7104	0.5	0.4986

Table 3 represents percentiles of the risk premiums using different number of calculations under assumption that client arrival follows Poisson distribution: $n_1 = 100000$, $n_2 = 1000000$. It could be seen that maximum absolute difference between all given percentiles for data of year 2013 is 0.0028, for data of year 2013 is 0.0058 and for data of the year 2014 is 0.0027, again the largest difference is approximately $\frac{3}{5}$ of EUR cent then it could be said that if repeating computation for 100000 times obtained results are sufficiently accurate.

In Appendix 1. Table 15 one may find results for case if client arrival follows geometric distribution. It can be seen that maximum absolute difference from all years is less than 0.01. If comparing results from Table 1 between using Poisson and geometric distribution for client arrival, the difference is relatively small and as the randomness error also is not significant, then it could be said that client arrival distribution (Poisson or geometric) does have major influence of the results.

3.4.3. Model for not identically distributed price increments

Table 4 represents the risk premiums from this model. It can be seen that results using Poisson and geometric distributions for how many clients are coming in one day are again quite close. However, examining results for using data from different years certain variations can be observed. Average difference between results of year 2012 and 2013 is approximately 0.13 and it remains the same between results of year 2013 and 2014, however average difference between risk premiums of year 2012 and 2014 is approximately twice as large – 0.26.

Table 4. 80th percentile of the second task risk premiums.

Entry day	2012		2013		2014	
	Poisson d.	Geom. d.	Poisson d.	Geom. d.	Poisson d.	Geom. d.
Monday	0.5533	0.5579	0.4155	0.4198	0.2991	0.3002
Tuesday	0.5565	0.5525	0.4112	0.4135	0.2936	0.3
Wednesday	0.5705	0.5710	0.4315	0.4333	0.3063	0.3073
Thursday	0.5702	0.5700	0.4460	0.45	0.3069	0.3029
Friday	0.5687	0.5792	0.48	0.4862	0.3195	0.3145

In Table 5 one can find the 90th percentile of 100000 simulation times. Comparing the same data as in the previous table, conclusions are similar. There are insignificant differences between results using Poisson and geometric distribution for clients arriving and comparing risk premiums between two years (2012 and 2013 as well as 2013 and 2014) the average difference is approximately 0.25 and differences between year 2012 and 2014 also are twice larger – 0.42.

Table 5. 90th percentile of second tasks risk premiums.

Entry day	2012		2013		2014	
	Poisson d.	Geom. d.	Poisson d.	Geom. d.	Poisson d.	Geom. d.
Monday	0.8835	0.8970	0.6729	0.6905	0.4897	0.4974
Tuesday	0.8879	0.8946	0.6754	0.6829	0.48	0.4997
Wednesday	0.9300	0.9417	0.72	0.73	0.5145	0.52
Thursday	0.9574	0.9577	0.7486	0.7664	0.5277	0.5261
Friday	0.9949	1.0144	0.8048	0.8279	0.55	0.55

In Table 4 and Table 5 it can be observed, that results for Monday and Tuesday as entry day should be more or less the same, because in calculations the same principle was used. Also results for Wednesday, Thursday and Friday as entry day also are more similar. The biggest risk premium is for Friday as entry day, it is because variance of price difference between weekend is bigger than that of working days, as it can be seen in Table 6, and regardless of client thinking time data from weekend sample will be included in price difference.

Table 6. Variance of historical one day price difference.

	2012	2013	2014
Working days	0.20754	0.34812	0.10447
Weekend	0.33953	0.50555	0.16808

From the results can be obtained that the risk premium for Monday, Tuesday as entry day is slightly lower as it is for Wednesday, Thursday or Friday as entry day. This proves that a client entry day should be taken into account in analysis as well as the fact that working days and weekend increments do not have the same distribution. From data with less fluctuations 90% of all 100000 repeated calculations average risk premium was smaller than approximately 0.525 *EUR*, and from data with wide fluctuations 90% of all simulations give results which was smaller than approximately 0.95 *EUR*.

3.4.4. Dependence of risk premium on clients thinking time distribution

All previous analysis were based on assumption that clients are thinking three or less days, but in the real life clients usually can make their decision for a longer time. In the following calculation we assume that clients' thinking time is from 1 to T days, all with equal probability, for several values of T . Also assumptions about client coming with Poisson and geometric distributions, client size follows exponential distribution are used. In the first part of analysis assumption about IID price differences is considered. Calculations are based on three cases – maximum client decision-making time is 10, 20 or 30.

Table 7 shows risk premiums 80th and 90th percentile of 100000 calculation times. Under different assumptions if maximum client thinking time is 10 days the risk premiums are less than for maximum – 20 and 30 – days, also results using maximum thinking for 20 days are less than those for 30 days. It is quite logical: the more days client thinks the bigger is a risk that the price in the market will change. And if market price increases during that time then company may lose a lot of money, so to avoid this situation the company should add bigger risk premium.

Table 7. Risk premiums depending on maximum client thinking time and IID price increment

	Max. thinking time	2012		2013		2014	
		Poisson d.	Geom. d.	Poisson d.	Geom. d.	Poisson d.	Geom. d.
80%	10d.	0.9267	0.9399	0.7169	0.7238	0.4823	0.4864
	20d.	1.2619	1.2802	0.9837	0.9851	0.654	0.6507

	30d.	1.4989	1.523	1.1516	1.1776	0.754	0.7722
90%	10d.	1.496	1.52	1.1649	1.1943	0.8155	0.83
	20d.	2.0435	2.0987	1.6088	1.612	1.117	1.1308
	30d.	2.4487	2.5051	1.8998	1.9477	1.3205	1.3606

In further calculations previous assumptions are used, just for calculation of the price differences it is taken into account that increments from working days and weekends have different distributions, so analysis depend on day of the week when client arrives and it is based on three cases – maximum client decision-making time is 10, 20 or 30 days.

The principle for calculations is the same as in the second model, however in this case client thinking time is longer. In Table 8, Table 9 and Table 10 one may find the results. In those tables 90th percentiles of loss distributions are presented for different maximum thinking time based on the weekday the offer is made, the distribution of client arrival and exercise period. It means that if the risk premium under certain assumptions is as in the table, the company with 90% can be sure that it will be sufficient to cover incurred price differences. Table 6 represents the results using 10 days as maximum client thinking time, it could be seen that result differences between years (2012 and 2013 as well as 2013 and 2014) are approximately 0.335.

Table 8. Risk premiums with maximum client thinking time 10 days.

Entry day	Poisson distribution			Geometric distribution		
	2012	2013	2014	2012	2013	2014
Monday	1.467	1.138	0.800	1.499	1.16	0.832
Tuesday	1.460	1.146	0.803	1.501	1.150	0.820
Wednesday	1.458	1.159	0.810	1.481	1.169	0.828
Thursday	1.470	1.166	0.804	1.515	1.180	0.822
Friday	1.493	1.181	0.815	1.514	1.193	0.829

In Table 9 and Table 10 results using maximum client thinking day 20 and 30 can be found. Again there is difference between years, with maximum decision making time 20 days it is approximately 0.45 and with maximum time 30 days – around 0.53. Compared to result presented in Table 9, the dependence of risk on the day the offer is made is very weak and not clearly visible in Table 10. It means if for maximum clients thinking time is relatively large (10, 20, 30), risk premium depends very little upon the fact on which day of week the client

arrives, but if maximum decision making time is small – like three days, then risk premium is influenced by entry day.

Table 9. Risk premiums with maximum client thinking time 20 days.

Entry day	Poisson distribution			Geometric distribution		
	2012	2013	2014	2012	2013	2014
Monday	1.995	1.559	1.106	2.052	1.590	1.132
Tuesday	2.014	1.559	1.097	2.023	1.604	1.114
Wednesday	1.975	1.558	1.109	2.042	1.590	1.114
Thursday	2.000	1.576	1.096	2.044	1.600	1.117
Friday	1.993	1.577	1.091	2.038	1.591	1.122

Table 10. Risk premiums for maximum client thinking time 30 days.

Entry day	Poisson distribution			Geometric distribution		
	2012	2013	2014	2012	2013	2014
Monday	2.398	1.876	1.318	2.422	1.912	1.353
Tuesday	2.383	1.878	1.310	2.440	1.917	1.360
Wednesday	2.375	1.872	1.330	2.440	1.923	1.345
Thursday	2.389	1.889	1.321	2.433	1.911	1.333
Friday	2.375	1.885	1.315	2.420	1.920	1.340

If the results between different entry days are compared one can see that for longer client decision making time the risk premiums should be larger. For example, results of the year 2012 with Poisson distribution as client arrival for entry day Wednesday are the following: risk premium with maximum client thinking time 10 days is 1.458, with maximum client thinking 20 days it is 1.975 and with 30 days – 2.375. Difference between maximum thinking time 10 and 20 is approximately 0.5, between maximum thinking time 20 and 30 it is 0.4. Table 11 shows the boundaries for differences of risk premium for various maximum client thinking days. It could be seen that results of year 2012 has the largest differences depending on maximum thinking time and the closest results for different number of maximum thinking days are for data of year 2014. It also could be explained with fluctuation from initial data, if the variation from historical prices is bigger it also appears in results and if variation is smaller than results varies less as well.

Table 11. Differences in results between different days.

		Poisson distribution			Geometric distribution		
		2012	2013	2014	2012	2013	2014
10d. to 20d.	Minimum	0.500	0.395	0.276	0.522	0.398	0.287
	Maximum	0.553	0.421	0.305	0.561	0.454	0.300
20d. to 30d.	Minimum	0.370	0.309	0.213	0.370	0.311	0.216
	Maximum	0.403	0.319	0.224	0.417	0.333	0.247

3.4.5. Effect of changing expected number of clients' arrival

The aim of following analysis is to find out whether mean number of clients' arrival per one day effects the risk premium. The cases when mean client arrival is 2, 3, 5, 7 and 10 are analysed. Table 12 shows percentiles of the results for data of the year 2013 and tables with results from data of years 2012 and 2014 are in Appendix 1. Table 16 and Table 17. Comparing results from those tables one may see that most of them are quite close, however maximum difference between different average numbers of clients arrival is 0.033 for data of year 2013 and for data of the years 2012 and 2014: 0.0405, 0.0183. For data of the year 2012 this maximum difference of the risk Premium is approximately 4 eurocents, it means that if change the mean number of client arrival, it could have an effect to the risk premium which the company should add to the current market price.

Table 12. Percentiles of the results using different mean client arrival.

	2 clients	3 clients	5 clients	7 clients	10 clients
70%	0.2564	0.2575	0.2524	0.2561	0.2563
75%	0.3408	0.3383	0.3349	0.3367	0.3356
80%	0.4392	0.4315	0.4253	0.4281	0.4274
85%	0.5633	0.55	0.5466	0.5426	0.5459
90%	0.73	0.7101	0.7046	0.697	0.7011

3.4.6. Effect of changing given expected order size

Next we want to find out how much client size effects the results of analysis. In previous analysis clients size follows exponential distribution with parameter $\frac{1}{5}$, which means

that clients want to buy in average 5 MWh of electricity. Further is studied how much changes result of risk premiums if change average client size to 2 MWh , 3 MWh , 5 MWh , 7 MWh and 10 MWh . Table 13 represents percentiles from 100000 computed times of risk premiums in those cases for data of the year 2013 and results from data of years 2012 and 2014 one may find in Appendix 1. Table 18 and Table 19. There are no significant differences between results, maximum difference within data of the year 2012 is 0.0074, in data of the year 2013 it is 0.0134 and between data of the 2014 – 0.0075. As the differences are small, then it could cause by randomness.

Table 13. Percentiles of risk premiums for data using different average client size.

	2 MWh	3 MWh	5 MWh	7 MWh	10 MWh
70%	0.2535	0.2577	0.2524	0.2549	0.26
75%	0.3342	0.3406	0.3349	0.3367	0.3384
80%	0.43	0.4359	0.4253	0.4324	0.434
85%	0.55	0.5543	0.5466	0.5512	0.5552
90%	0.7129	0.7169	0.7046	0.7113	0.718

CONCLUSION

The aim of the master thesis was to analyse data from electricity market estimating the risks of the company when it offered to clients contracts with fixed prices for future periods. For studying this question historical electricity prices which the company had been offering to clients for year 2012, 2013 and 2014 were used. Before starting practical part the theoretical background was prepared. These thesis includes the futures and forward contract descriptions and comparison, as well as derivation of prices of the contracts under no arbitrage condition are included. Information about electricity market was collected and discovered because of electricity's non-storable character, the pricing of risk in forward trading become more complicated, so also a few models for forward price modelling were described.

In the master thesis practical part data analysis was based on ARIMA(0,1,0) model (random walk) for market price of wholesale futures contracts. For comparison a model using logarithmic return was analyzed. As a result the risk premiums for given risk levels under various assumptions were found. The questions were studied, whether the risk premium depended on maximum client thinking time, average client arrival per one day and average amount of electricity which they wanted to buy. At first it was assumed that all price differences and logarithmic returns were IID.

From the obtained results it was concluded that client arrival distribution (Poisson or geometric distribution) did not have an effect, however the results depend on mean number of clients arrival for one day. Major differences between results using regular differences and logarithmic returns was not observed. If changed the expected number of client size analyzed results did not show any significant effect from it. In practical part the question that price differences between weekdays and weekends were not from the same distribution was studied. Results showed dependence on client entry day per week showed in case if maximum client thinking time were three days, however if the maximum client thinking time was larger – like 10, 20 and 30 – then results did not have significant differences. Differences of data results from different years of all analyzed cases were obtained. From the data which fluctuated the most the results were the largest, but from data which were more stable the smallest results were obtained.

As there are some unexpected factors which could influence processes in the future and as obtained results depend on quite many assumptions, then one universal, constant overtime risk premium could not be concluded. It would be better to take results as advisory

information and data source. As the topic is very complicated, it is still opened to deeper analysis.

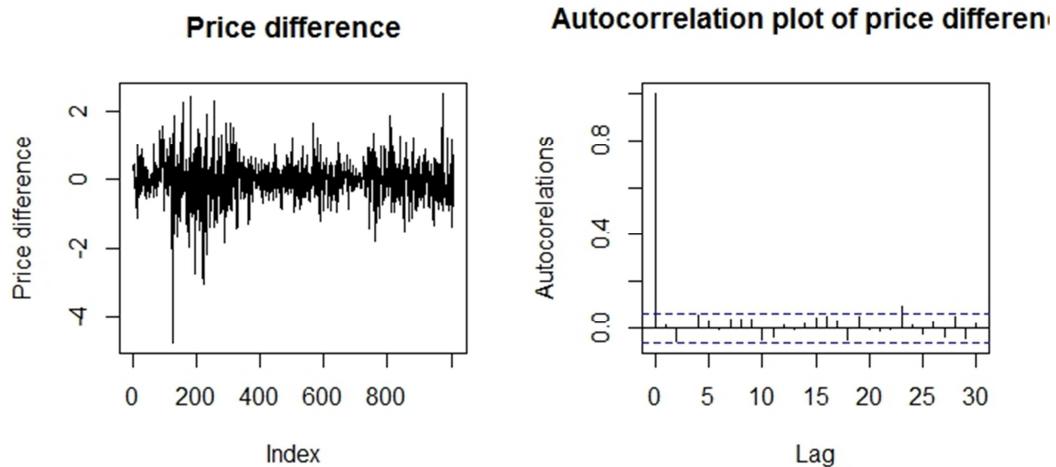
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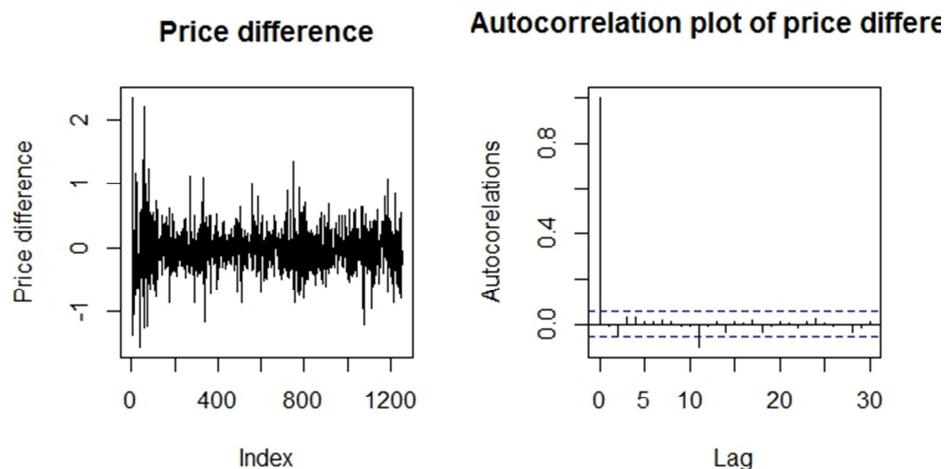
APPENDIX

1. appendix

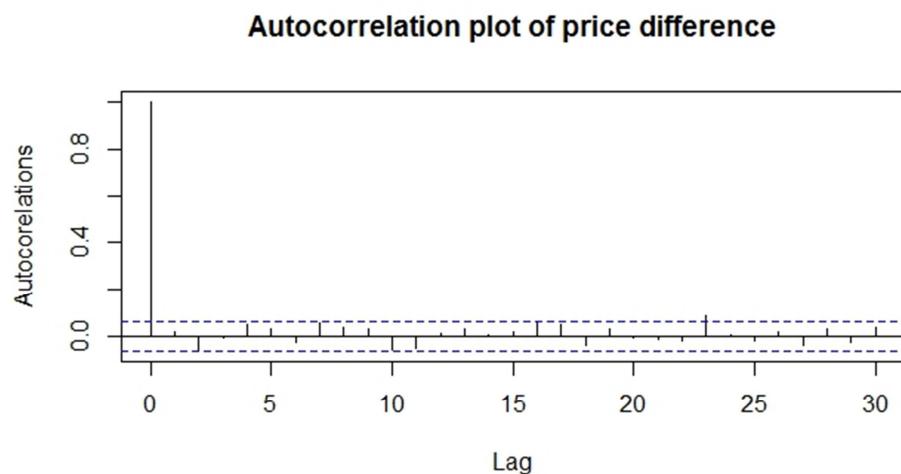
Figures and tables



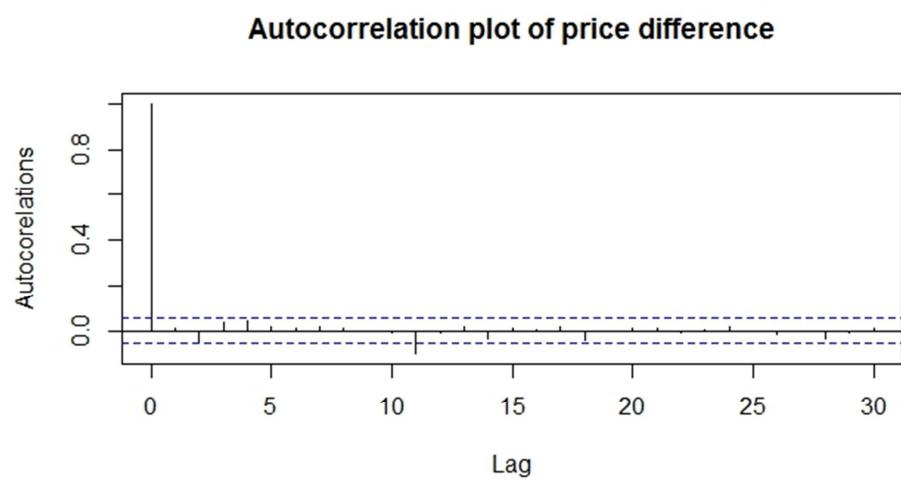
A.1. Figure 9. Price difference and their autocorrelation plot (2012).



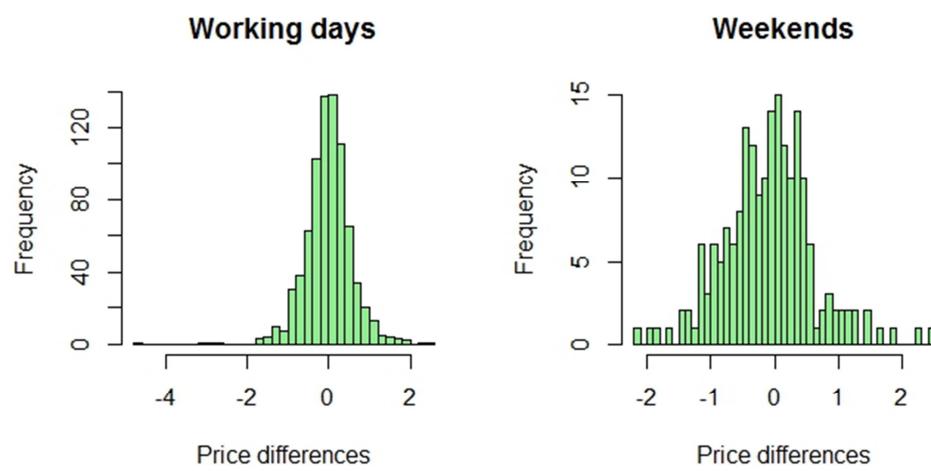
A.1. Figure 10. Price difference and their autocorrelation plot (2014).



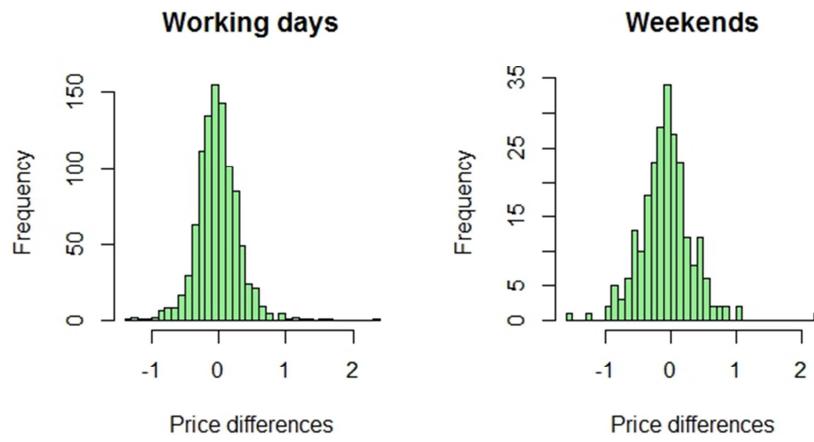
A.1. Figure 11. Autocorrelation plot from logarithmic returns. (2012).



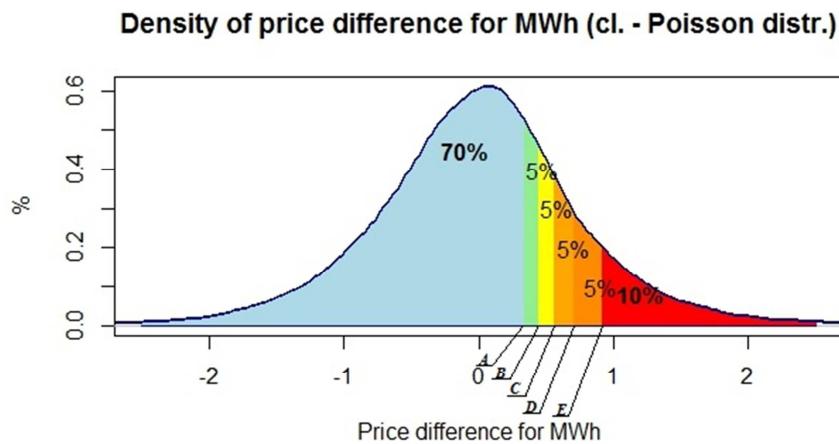
A.1. Figure 12. Autocorrelation plot from logarithmic returns. (2014).



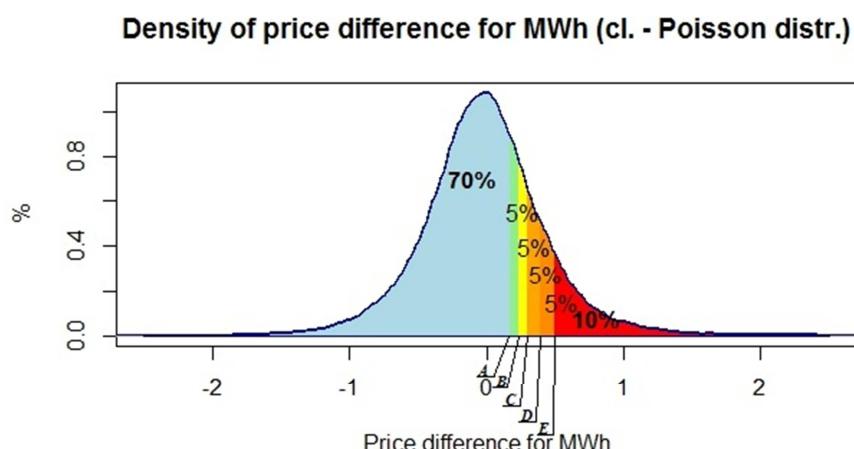
A.1. Figure 13. Price differences in working days and weekends for year 2012 data.



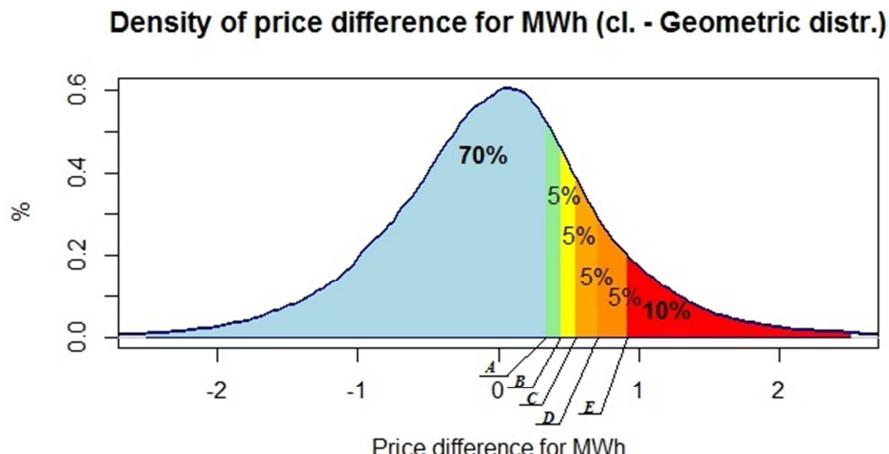
A.1. Figure 14. Price differences in working days and weekends for year 2014 data.



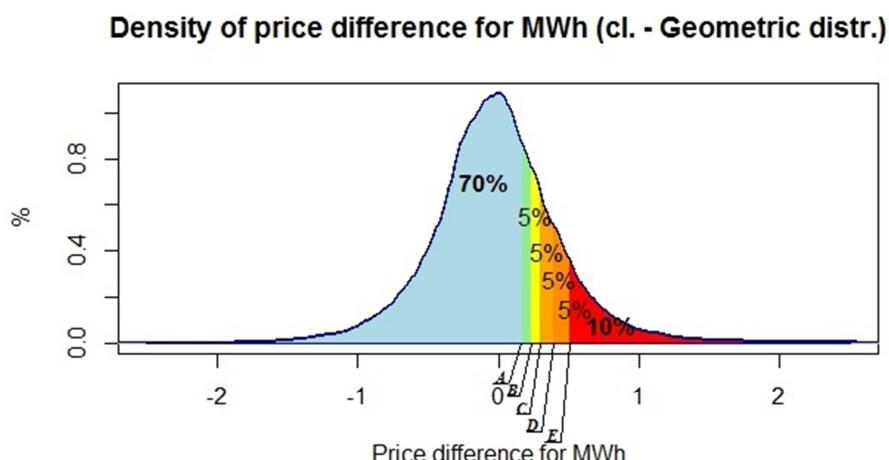
A.1. Figure 15. $A = 0.3388$; $B = 0.4392$; $C = 0.5559$; $D = 0.7063$;
 $E = 0.914$ (2012).



A.1. Figure 16. $A = 0.17$; $B = 0.2294$; $C = 0.2997$; $D = 0.3867$;
 $E = 0.5$ (2014).



A.1. Figure 17. $A = 0.3402; B = 0.4405; C = 0.5535; D = 0.7035;$
 $E = 0.9192$ (2012).



A.1. Figure 18. $A = 0.17; B = 0.2312; C = 0.3; D = 0.3929; E = 0.5026$ (2014).

A.1. Table 15. Percentiles using different number of calculations ($n_1 = 100000$, $n_2 = 1000000$) (client arrival - geometric distribution).

	2012		2013		2014	
	n1	n2	n1	n2	n1	n2
70%	0.3402	0.3442	0.2559	0.255	0.17	0.1698
75%	0.4405	0.4471	0.341	0.3396	0.2312	0.23
80%	0.5535	0.5604	0.4376	0.4336	0.3	0.3
85%	0.7035	0.7126	0.5588	0.555	0.3929	0.39
90%	0.9192	0.9274	0.72	0.7206	0.5026	0.5022

A.1. Table 16. Percentiles of the results using different mean client arrival (2012).

	2 clients	3 clients	5 clients	7 clients	10 clients
70%	0.3467	0.341	0.3388	0.3393	0.3369
75%	0.4497	0.445	0.4392	0.4382	0.4363
80%	0.5659	0.5624	0.5559	0.5531	0.5499
85%	0.7195	0.7138	0.7063	0.6974	0.6934
90%	0.9306	0.924	0.914	0.8966	0.8901

A.1. Table 17. Percentiles of the results using different mean client arrival (2014).

	2 clients	3 clients	5 clients	7 clients	10 clients
70%	0.1703	0.1698	0.17	0.1644	0.1661
75%	0.2315	0.23	0.2294	0.2236	0.2271
80%	0.3001	0.3	0.2997	0.2934	0.2972
85%	0.3931	0.3876	0.3867	0.3786	0.38
90%	0.5062	0.5	0.5	0.4879	0.4913

A.1. Table 18. Percentiles of risk premiums for data of the year 2012

	2 MWh	3 MWh	5 MWh	7 MWh	10 MWh
70%	0.3429	0.3386	0.3388	0.3402	0.3359
75%	0.4421	0.4382	0.4392	0.4431	0.4388
80%	0.5592	0.5566	0.5559	0.5607	0.5578
85%	0.7086	0.7046	0.7063	0.7081	0.7105
90%	0.9113	0.9108	0.914	0.9157	0.9182

A.1. Table 19. Percentiles of risk premiums for data of the year 2014

	2 MWh	3 MWh	5 MWh	7 MWh	10 MWh
70%	0.1688	0.1711	0.17	0.1642	0.1671
75%	0.23	0.2307	0.2294	0.2244	0.2271
80%	0.3	0.3	0.2997	0.296	0.2962
85%	0.3855	0.3883	0.3867	0.3808	0.3834
90%	0.4978	0.5	0.5	0.4952	0.4999

R code

```

Initial_data_send <- read.delim("~/Studijas EE/MT/Practical part/Initial_data_send.txt", dec=",")
Initial_data_send
dati_2013 <-Initial_data_send$Price_2013
dati_2012 <-Initial_data_send$Price_2012
dati_2014 <-Initial_data_send$Price_2014
dati_2012=dati_2012[complete.cases(dati_2012)]
dati_2014=dati_2014[complete.cases(dati_2014)]


dati_2013

time<-Initial_data_send$Date_2013
time_2013<-strptime(time, format="%d.%m.%Y", tz ="GMT")
time_2013=time_2013[complete.cases(time_2013)]
time_2013

time1<-Initial_data_send$Date_2012
time_2012<-strptime(time1, format="%d.%m.%Y", tz ="GMT")
time_2012=time_2012[complete.cases(time_2012)]
time_2012

time2<-Initial_data_send$Date_2014
time_2014<-strptime(time2, format="%d.%m.%Y", tz ="GMT")
time_2014=time_2014[complete.cases(time_2014)]
time_2014

#####for year 2012#####
Dati=dati_2012
D=diff(dati_2012)
D_date=diff(time_2012)
length(D)
mean(Dati)
var(Dati)

#####for year 2013#####
Dati=dati_2013
D=diff(dati_2013)
D_date=diff(time_2013)
length(D)
mean(Dati)
var(Dati)

#####for year 2014#####
Dati=dati_2014
D=diff(dati_2014)
D_date=diff(time_2014)
length(D)
mean(Dati)
var(Dati)
####.

par(mfrow=c(1,2))
plot(D, type="l", ylab="Price difference", main="Price difference")
acf(D,main='Autocorrelation plot of price difference', ylab="Autocorelations")
layout(1:1)

```

```

#####
1. task.#####

#write one complect for one day
#clients arrival - exponential distribution with rate 1/5
#amount of electricity - exponential distribution with rate 1/5
#clients thinking time:
#1 day with probability - 33%
#2 days with probability - 33%
#3 days with probability - 33%

#day differences as empirical data - random chosen one day difference form historical data

n=100000

task1=function(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,D){
  S_1task=rep(NA,n)
  Z_1task=rep(NA,n)
  for(k in 1:n) {
    cl_arrival=client_arrival_gen(1) #how many clients arrive in one day ("1+" is because this
distribution also gives 0 value, but here we consider case when clients actually are coming)
    cl_amount=client_size_gen(cl_arrival) #how much the clients want to buy
    #generates sample from historical price
    diff=diff_gen(D)
    ta_1task=rep(NA,cl_arrival) #vector for profit/loss from one clients
    for (i in 1:cl_arrival) { #cycle for one day's client arrival
      F_diff=0
      #generate how long client is thinking - one, two or three days (each day has the same probability
1/6)
      t=client_thinking_gen(1)
      F_diff=sum(diff[1:t])
      ta_1task[i]=F_diff*cl_amount[i] # amount of money which the company get/lose from one client
    }
    S_1task[k]=sum(ta_1task) #total amount which company get/lose for one day (from one
"complect")
    Z_1task[k]=S_1task[k]/sum(cl_amount) # amount which company should add to current price
  }
  return(Z_1task)}

client_thinking_gen=function(n){
  T=3
  p=rep(1/T,T)
  return(sample(T,n,replace=T,prob=p))}

client_size_gen=function(n){
  return(rexp(n,rate=1/5))}

diff_gen=function(D){
  return(sample(D, 3, replace = TRUE, prob = NULL) )}

#poisson distribution
client_arrival_gen=function(n){
  return(1+rpois(n,lambda=4))}

Z_1task_po=task1(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,D)
Z1_70_po<-round(quantile(Z_1task_po,0.70, names = FALSE), digits = 4)
Z1_75_po<-round(quantile(Z_1task_po,0.75, names = FALSE), digits = 4)

```

```

Z1_80_po<-round(quantile(Z_1task_po,0.80,names = FALSE),digits = 4)
Z1_85_po<-round(quantile(Z_1task_po,0.85,names = FALSE),digits = 4)
Z1_90_po<-round(quantile(Z_1task_po,0.90,names = FALSE),digits = 4)
perc_70_po <-Z1_70_po
perc_75_po <-Z1_75_po
perc_80_po <-Z1_80_po
perc_85_po <-Z1_85_po
perc_90_po <-Z1_90_po
plot(density(Z_1task_po), lwd=2.5,xlim=c(-2.5,2.5),col="darkblue",main="Density of price difference
for MWh (cl. - Poisson distr.)",xlab="Price difference for MWh",ylab="%",bg="red",font.main=2)
clip(-2.5,perc_70_po,0,30)
polygon(density(Z_1task_po), col="lightblue", border="darkblue")
clip(perc_70_po,perc_75_po,0,30)
polygon(density(Z_1task_po), col="lightgreen", border="darkblue")
clip(perc_75_po,perc_80_po,0,30)
polygon(density(Z_1task_po), col="yellow", border="darkblue")
clip(perc_80_po,perc_85_po,0,30)
polygon(density(Z_1task_po), col="orange", border="darkblue")
clip(perc_85_po,perc_90_po,0,30)
polygon(density(Z_1task_po), col="darkorange", border="darkblue")
clip(perc_90_po,2.5,0,30)
polygon(density(Z_1task_po), col="red", border="darkblue")
clip(-2,2,-1,30)
text(-0.1,0.45,"70%", font=2)
text(1,0.08,"10%", font=2)
text(0.37,0.40,"5%", font=1)
text(0.47,0.30,"5%", font=1)
text(0.58,0.20,"5%", font=1)
text(0.7,0.10,"5%", font=1)

#Geometric distribution
client_arrival_gen=function(n){
  return(1+rgeom(1, prob = 1/5))}

Z_1task_geo=task1(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,D)
#####
##### previous assumptions, just change average value of client size

client_size_gen=function(n){
  return(rexp(n,rate=1/7))
}

Z_1task_po_size7=task1(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,D)
Z1_70_po_size7<-round(quantile(Z_1task_po_size7,0.70,names = FALSE),digits = 4)
Z1_75_po_size7<-round(quantile(Z_1task_po_size7,0.75,names = FALSE),digits = 4)
Z1_80_po_size7<-round(quantile(Z_1task_po_size7,0.80,names = FALSE),digits = 4)
Z1_85_po_size7<-round(quantile(Z_1task_po_size7,0.85,names = FALSE),digits = 4)
Z1_90_po_size7<-round(quantile(Z_1task_po_size7,0.90,names = FALSE),digits = 4)

client_size_gen=function(n){
  return(rexp(n,rate=1/3))
}

Z_1task_po_size3=task1(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,D)
Z1_70_po_size3<-round(quantile(Z_1task_po_size3,0.70,names = FALSE),digits = 4)
Z1_75_po_size3<-round(quantile(Z_1task_po_size3,0.75,names = FALSE),digits = 4)
Z1_80_po_size3<-round(quantile(Z_1task_po_size3,0.80,names = FALSE),digits = 4)

```

```

Z1_85_po_size3<-round(quantile(Z_1task_po_size3,0.85,names = FALSE),digits = 4)
Z1_90_po_size3<-round(quantile(Z_1task_po_size3,0.90,names = FALSE),digits = 4)

client_size_gen=function(n){
  return(rexp(n,rate=1/2))
}

Z_1task_po_size2=task1(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,D)
Z1_70_po_size2<-round(quantile(Z_1task_po_size2,0.70,names = FALSE),digits = 4)
Z1_75_po_size2<-round(quantile(Z_1task_po_size2,0.75,names = FALSE),digits = 4)
Z1_80_po_size2<-round(quantile(Z_1task_po_size2,0.80,names = FALSE),digits = 4)
Z1_85_po_size2<-round(quantile(Z_1task_po_size2,0.85,names = FALSE),digits = 4)
Z1_90_po_size2<-round(quantile(Z_1task_po_size2,0.90,names = FALSE),digits = 4)

client_size_gen=function(n){
  return(rexp(n,rate=1/10))
}

Z_1task_po_size10=task1(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,D)
Z1_70_po_size10<-round(quantile(Z_1task_po_size10,0.70,names = FALSE),digits = 4)
Z1_75_po_size10<-round(quantile(Z_1task_po_size10,0.75,names = FALSE),digits = 4)
Z1_80_po_size10<-round(quantile(Z_1task_po_size10,0.80,names = FALSE),digits = 4)
Z1_85_po_size10<-round(quantile(Z_1task_po_size10,0.85,names = FALSE),digits = 4)
Z1_90_po_size10<-round(quantile(Z_1task_po_size10,0.90,names = FALSE),digits = 4)

#####using not regular differences, but logarithmic returns

task1_return=function(Dati,n,client_size_gen,client_thinking_gen,log_return_gen,client_arrival_gen){
  S_1task_return=rep(NA,n)
  Z_1task_return=rep(NA,n)
  for(k in 1:n) {
    cl_arrival=client_arrival_gen(1) #how many clients arrive in one day ("1+" is because this
    distribution also gives 0 value, but here we consider case when clients actually are coming)
    cl_amount=client_size_gen(cl_arrival) #how much the clients want to buy
    #generates sample from historical price
    log_return_sample=log_return_gen(log_return)
    ta_1task=rep(NA,cl_arrival) #vector for profit/loss from one clients
    for (i in 1:cl_arrival) { #cycle for one day's client arrival
      F_return=0
      #generate how long client is thinking - one, two or three days (each day has the same probability
      1/6)
      t=client_thinking_gen(1)
      F_return=sum(log_return_sample[1:t])
      ta_1task[i]=(exp(F_return)-1)*cl_amount[i] # amount of money which the company get/lose from
      one client
    }
    S_1task_return[k]=sum(ta_1task) #total amount which company get/lose for one day (from one
    "complect")
    Z_1task_return[k]=S_1task_return[k]/sum(cl_amount) # amount which company should add to
    current price
  }
  return(Z_1task_return)
}

client_thinking_gen=function(n){
  T=3

```

```

p=rep(1/T,T)
return(sample(T,n,replace=T,prob=p))
}

client_size_gen=function(n){
  return(rexp(n,rate=1/5))
}
log_return=log(Dati[-1]/Dati[-length(Dati)])
log_return

log_return_gen=function(log_return){
  return(sample(log_return,3, replace = TRUE, prob = NULL) )
}

log_return_gen(log_return)
acf(log_return)

#poisson distribution

client_arrival_gen=function(n){
  return(1+rpois(n,lambda=4))
}

result_po=task1_return(Dati,n,client_size_gen,client_thinking_gen,log_return_gen,client_arrival_gen)

Z1_80_po_return=quantile(result_po,0.80, names = FALSE)
Z1_90_po_return=quantile(result_po,0.90, names = FALSE)
mean(Dati)*Z1_90_po_return

#Geometric distribution

client_arrival_gen=function(n){
  return(1+rgeom(1, prob = 1/5))
}

result_geo=task1_return(Dati,n,client_size_gen,client_thinking_gen,log_return_gen,client_arrival_gen
)

Z1_80_geo_return=quantile(result_geo,0.80, names = FALSE)
Z1_90_geo_return=quantile(result_geo,0.90, names = FALSE)
mean(Dati)*Z1_90_geo_return

#####
#####2. task.#####
#check if price differences in working days are with the same distribution as in weekends

length(D_date)
l1=length(D_date[D_date==1])
l2=length(D_date[D_date==3])
set1=rep(NA,l1)
set2=rep(NA,l2)
set1=D[D_date==1]
set2=D[D_date==3]
mean(set1) # -0.0004848485
mean(set2) # -0.05706612
var(set1) # 0.2075353

```

```

var(set2) # 0.3395279

par(mfrow=c(1,2))
hist(set1, breaks=50, xlab="Price differences", main="Working days", col="lightgreen")
hist(set2, breaks=50, xlab="Price differences", main="Weekends", col="lightgreen")
layout(1:1)

ks.test(set1, set2)
#Two-sample Kolmogorov-Smirnov test
#data: set1 and set2
#D = 0.1361, p-value = 0.001489
#alternative hypothesis: two-sided

#write one complect for one day depending on which day client enter

#clients arrival: 1) Poisson distribution distribution with lambda=5
#           2) Geometric distributon with prob.=1/6
#amount of electricity - exponential distribution with rate 1/5
#clients thinking time:
#1 day with probability - 33.33%
#2 days with probability - 33.33%
#3 days with probability - 33.33%
#day differences as empirical data - random chosen one day difference form historical data, but
#consider different distributions for workdays and weekends price differences

#####using Poisson distribution for client arrival
task2=function(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,set1, set2){
  S_2task=rep(NA,n)
  Z_2task=rep(NA,n)
  for(k in 1:n) {
    cl_arrival=client_arrival_gen(1) #how many clients arrive in one day ("1+" is because this
    distributio also gives 0 value, but here we consider case when clients actually are coming)
    cl_amount=client_size_gen(cl_arrival) #how much the clients want to buy
    #generates sample from historical price
    diff1=diff_gen(set1,3) #generates sample from historical working days differences
    diff2=diff_gen(set2,1) #generates sample from historical weekend days differences
    ta_2task=rep(NA,cl_arrival) #vector for profit/loss from one clients
    for (i in 1:cl_arrival) { #cycle for one day's client arrival
      F_diff=0
      #generate how long client is thnking - one, two or three days (each day has the same probability
      1/6)
      t=client_thinking_gen(1)
      F_diff=F_diff_gen(diff1,diff2,t)
      ta_2task[i]=F_diff*cl_amount[i] # amount of money which the company get/lose from one client
    }
    S_2task[k]=sum(ta_2task) #total amount which company get/lose for one day (from one
    "complect")
    Z_2task[k]=S_2task[k]/sum(cl_amount) # amount which company should add to current price
  }
  return(Z_2task)
}

client_thinking_gen=function(n){
  T=3
  p=rep(1/T,T)
  return(sample(T,n,replace=T,prob=p))
}

```

```

client_size_gen=function(n){
  return(rexp(n,rate=1/5))
}
diff_gen=function(D,n){
  return(sample(D, n, replace = TRUE, prob = NULL) )
}
#poisson distribution

client_arrival_gen=function(n){
  return(1+rpois(n,lambda=4))
}
#####For Monday
F_diff_gen=function(diff1,diff2,t){
  return(sum(diff1[1:t]) )
}
Z_2task_po_1day=task2(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,set1, set2)
Z1_2task_80_po_1day=quantile(Z_2task_po_1day,0.80, names = FALSE)
Z1_2task_90_po_1day=quantile(Z_2task_po_1day,0.90, names = FALSE)
#####For Tuesday
F_diff_gen=function(diff1,diff2,t){
  return(sum(diff1[1:t]) )
}
Z_2task_po_2day=task2(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,set1, set2)
Z1_2task_80_po_2day=quantile(Z_2task_po_2day,0.80, names = FALSE)
Z1_2task_90_po_2day=quantile(Z_2task_po_2day,0.90, names = FALSE)
#####For Wednesday
F_diff_gen=function(diff1,diff2,t){
  if (t<3) {
    F_diff_g=sum(diff1[1:t])
  }else {F_diff_g=(sum(diff1)+diff2)}
  return(F_diff_g )
}
Z_2task_po_3day=task2(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,set1, set2)
Z1_2task_80_po_3day=quantile(Z_2task_po_3day,0.80, names = FALSE)
Z1_2task_90_po_3day=quantile(Z_2task_po_3day,0.90, names = FALSE)
#####For Thursday
F_diff_gen=function(diff1,diff2,t){
  if (t==1) {
    F_diff_g=diff1[1]
  }else if (t==2){
    F_diff_g=diff1[1]+diff2
  } else {F_diff_g=(sum(diff1)+diff2)}
  return(F_diff_g )
}
Z_2task_po_4day=task2(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,set1, set2)
Z1_2task_80_po_4day=quantile(Z_2task_po_4day,0.80, names = FALSE)
Z1_2task_90_po_4day=quantile(Z_2task_po_4day,0.90, names = FALSE)
Z1_2task_80_po_4day
Z1_2task_90_po_4day
#####For Friday
F_diff_gen=function(diff1,diff2,t){
  if (t==1) {
    F_diff_g=diff2
  }else if (t==2){
    F_diff_g=diff2+diff1[1]

```

```

} else {F_diff_g=(diff2+sum(diff1))}
return(F_diff_g )
}
Z_2task_po_5day=task2(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,set1, set2)
Z1_2task_80_po_5day=quantile(Z_2task_po_5day,0.80,names = FALSE)
Z1_2task_90_po_5day=quantile(Z_2task_po_5day,0.90,names = FALSE)

#####using geometric distribution for client arrival
client_arrival_gen=function(n){
  return(1+rgeom(1, prob = 1/5))
}
#####For Monday
F_diff_gen=function(diff1,diff2,t){
  return(sum(diff1[1:t]) )
}
Z_2task_geo_1day=task2(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,set1, set2)
Z1_2task_80_geo_1day=quantile(Z_2task_geo_1day,0.80,names = FALSE)
Z1_2task_90_geo_1day=quantile(Z_2task_geo_1day,0.90,names = FALSE)
#####For Tuesday
F_diff_gen=function(diff1,diff2,t){
  return(sum(diff1[1:t]) )
}
Z_2task_geo_2day=task2(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,set1, set2)
Z1_2task_80_geo_2day=quantile(Z_2task_geo_2day,0.80,names = FALSE)
Z1_2task_90_geo_2day=quantile(Z_2task_geo_2day,0.90,names = FALSE)
#####For Wednesday
F_diff_gen=function(diff1,diff2,t){
  if (t<3) {
    F_diff_g=sum(diff1[1:t])
  }else {F_diff_g=(sum(diff1)+diff2)}
  return(F_diff_g )
}
Z_2task_geo_3day=task2(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,set1, set2)
Z1_2task_80_geo_3day=quantile(Z_2task_geo_3day,0.80,names = FALSE)
Z1_2task_90_geo_3day=quantile(Z_2task_geo_3day,0.90,names = FALSE)
#####For Thursday
F_diff_gen=function(diff1,diff2,t){
  if (t==1) {
    F_diff_g=diff1[1]
  }else if (t==2){
    F_diff_g=diff1[1]+diff2
  } else {F_diff_g=(sum(diff1)+diff2)}
  return(F_diff_g )
}
Z_2task_geo_4day=task2(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,set1, set2)
Z1_2task_80_geo_4day=quantile(Z_2task_geo_4day,0.80,names = FALSE)
Z1_2task_90_geo_4day=quantile(Z_2task_geo_4day,0.90,names = FALSE)
#####For Friday
F_diff_gen=function(diff1,diff2,t){
  if (t==1) {
    F_diff_g=diff2
  }else if (t==2){

```

```

F_diff_g=diff2+diff1[1]
} else {F_diff_g=(diff2+sum(diff1))}
return(F_diff_g )
}
Z_2task_geo_5day=task2(n,client_size_gen,client_thinking_gen,diff_gen,client_arrival_gen,set1,
set2)
Z1_2task_80_geo_5day=quantile(Z_2task_geo_5day,0.80,names = FALSE)
Z1_2task_90_geo_5day=quantile(Z_2task_geo_5day,0.90,names = FALSE)

#####
#3. task.#####
#previous analysis if longest client thinking time is T
#####
#using 1. task assumptions (IID increments)

task3_1=function(T,n,client_size_gen, client_arrival_gen, client_thinking_gen){
  S_1task=rep(NA,n)
  Z_1task=rep(NA,n)
  for(k in 1:n) {
    cl_arrival=client_arrival_gen(1) #how many clients arrive in one day ("1+" is because this
distributio also gives 0 value, but here we consider case when clients actually are coming)
    cl_amount=client_size_gen(cl_arrival) #how much the clients want to buy
    diff=sample(D, T, replace = TRUE, prob = NULL) #generates sample from historical differences
    ta_1task=rep(NA,cl_arrival) #vector for profit/loss from one clients
    cl_thinking=client_thinking_gen(T,cl_arrival) #generate how long client is thinking
    for (i in 1:cl_arrival) { #cycle for one day's client arrival
      F_diff=0
      t=cl_thinking[i]
      F_diff=sum(diff[1:t]) # the sum of price differences for all client "thinking days"
      ta_1task[i]=F_diff*cl_amount[i] # amount of money which the company get/lose from one client
    }
    S_1task[k]=sum(ta_1task) #total amount which company get/lose for one day (from one
"complect")
    Z_1task[k]=S_1task[k]/sum(cl_amount) # amount which company should add to current price
  }
  return(Z_1task)
}
client_thinking_gen=function(T,n){
  p=rep(1/T,T)
  return(sample(T,n,replace=T,prob=p))
}
client_size_gen=function(n){
  return(rexp(n,rate=1/5))
}
#poisson distribution
client_arrival_gen=function(n){
  return(1+rpois(n,lambda=4))
}
Z_3task_1_po_10=task3_1(T=10,n,client_size_gen, client_arrival_gen, client_thinking_gen)
Z1_3task_1_80_po_10<-round(quantile(Z_3task_1_po_10,0.80,names = FALSE),digits = 4)
Z1_3task_1_90_po_10<-round(quantile(Z_3task_1_po_10,0.90,names = FALSE),digits = 4)
Z_3task_1_po_20=task3_1(T=20,n,client_size_gen, client_arrival_gen, client_thinking_gen)
Z1_3task_1_80_po_20<-round(quantile(Z_3task_1_po_20,0.80,names = FALSE),digits = 4)
Z1_3task_1_90_po_20<-round(quantile(Z_3task_1_po_20,0.90,names = FALSE),digits = 4)

Z_3task_1_po_30=task3_1(T=30,n,client_size_gen, client_arrival_gen, client_thinking_gen)
Z1_3task_1_80_po_30<-round(quantile(Z_3task_1_po_30,0.80,names = FALSE),digits = 4)

```

```

Z1_3task_1_90_po_30<-round(quantile(Z_3task_1_po_30,0.90,names = FALSE),digits = 4)
#Geometric distribution
client_arrival_gen=function(n){
  return(1+rgeom(1, prob = 1/5))
}
Z_3task_1_geo_10=task3_1(T=10,n,client_size_gen, client_arrival_gen, client_thinking_gen)
Z1_3task_1_80_geo_10<-round(quantile(Z_3task_1_geo_10,0.80,names = FALSE),digits = 4)
Z1_3task_1_90_geo_10<-round(quantile(Z_3task_1_geo_10,0.90,names = FALSE),digits = 4)
Z_3task_1_geo_20=task3_1(T=20,n,client_size_gen, client_arrival_gen, client_thinking_gen)
Z1_3task_1_80_geo_20<-round(quantile(Z_3task_1_geo_20,0.80,names = FALSE),digits = 4)
Z1_3task_1_90_geo_20<-round(quantile(Z_3task_1_geo_20,0.90,names = FALSE),digits = 4)
Z_3task_1_geo_30=task3_1(T=30,n,client_size_gen, client_arrival_gen, client_thinking_gen)
Z1_3task_1_80_geo_30<-round(quantile(Z_3task_1_geo_30,0.80,names = FALSE),digits = 4)
Z1_3task_1_90_geo_30<-round(quantile(Z_3task_1_geo_30,0.90,names = FALSE),digits = 4)

#####
#using 2. task assumptions (increments are not identically distributed)

task3_2=function(T,n,client_size_gen, client_arrival_gen, client_thinking_gen,enter_day){
  S_1task=rep(NA,n)
  Z_1task=rep(NA,n)
  for(k in 1:n) {
    cl_arrival=client_arrival_gen(1) #how many clients arrive in one day ("1+" is because this
distributio also gives 0 value, but here we consider case when clients actually are coming)
    cl_amount=client_size_gen(cl_arrival) #how much the clients want to buy
    diff=sample(D, T, replace = TRUE, prob = NULL) #generates sample from historical differences
    ta_1task=rep(NA,cl_arrival) #vector for profit/loss from one clients
    cl_thinking=client_thinking_gen(cl_arrival,T) #generate how long client is thinking
    S=Diff_vector(T,enter_day)
    for (i in 1:cl_arrival) { #cycle for one day's client arrival
      F_diff=0
      t=cl_thinking[i]
      F_diff=S[t] # the sum of price differences for all client "thinking days"
      ta_1task[i]=F_diff*cl_amount[i] # amount of money which the company get/lose from one client
    }
    S_1task[k]=sum(ta_1task) #total amount which company get/lose for one day (from one
"complect")
    Z_1task[k]=S_1task[k]/sum(cl_amount) # amount which company should add to current price
  }
  return(Z_1task_po)
}
client_size_gen=function(n){
  return(rexp(n,rate=1/5))
}
client_thinking_gen=function(n,T){
  p=rep(1/T,T)
  return(sample(T,n,replace=T,prob=p))
}
Diff_vector=function(T,enter_day){
  diff1=sample(set1, T-floor(T/5), replace = TRUE, prob = NULL) #generates sample from historical
working days differences
  diff2=sample(set2, ceiling(T/5), replace = TRUE, prob = NULL)
  S=rep(NA,T) #vector of all possible price differences
  if(enter_day<5){
    for (j in 1:T){
      if(j<(6-enter_day)){

```

```

        S[j]=sum(diff1[1:j])
    } else {
        S[j]=sum(diff1[1:(j-floor((j+(enter_day-1))/5))])+sum(diff2[1:floor((j+(enter_day-1))/5)])}
    }
else if(enter_day==5){
for (j in 1:T){
if(j==1){
    S[j]=sum(diff2[1:floor((j+4)/5)])
} else {
    S[j]=sum(diff1[1:(j-floor((j+4)/5))])+sum(diff2[1:floor((j+4)/5)])}
}
return(S)}

```

#poisson distribution

```

client_arrival_gen=function(n){
  return(1+rpois(n,lambda=4))
}
Z_3task_2_po_10_1day=task3_2(T=10,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=1)
Z1_3task_2_80_po_10_1day<-round(quantile(Z_3task_2_po_10_1day,0.80, names = FALSE),digits =
4)
Z1_3task_2_90_po_10_1day<-round(quantile(Z_3task_2_po_10_1day,0.90, names = FALSE),digits =
4)
Z_3task_2_po_20_1day=task3_2(T=20,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=1)
Z1_3task_2_80_po_20_1day<-round(quantile(Z_3task_2_po_20_1day,0.80, names = FALSE),digits =
4)
Z1_3task_2_90_po_20_1day<-round(quantile(Z_3task_2_po_20_1day,0.90, names = FALSE),digits =
4)

Z_3task_2_po_30_1day=task3_2(T=30,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=1)
Z1_3task_2_80_po_30_1day<-round(quantile(Z_3task_2_po_30_1day,0.80, names = FALSE),digits =
4)
Z1_3task_2_90_po_30_1day<-round(quantile(Z_3task_2_po_30_1day,0.90, names = FALSE),digits =
4)
Z_3task_2_po_10_2day=task3_2(T=10,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=2)
Z1_3task_2_80_po_10_2day<-round(quantile(Z_3task_2_po_10_2day,0.80, names = FALSE),digits =
4)
Z1_3task_2_90_po_10_2day<-round(quantile(Z_3task_2_po_10_2day,0.90, names = FALSE),digits =
4)
Z_3task_2_po_20_2day=task3_2(T=20,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=2)
Z1_3task_2_80_po_20_2day<-round(quantile(Z_3task_2_po_20_2day,0.80, names = FALSE),digits =
4)
Z1_3task_2_90_po_20_2day<-round(quantile(Z_3task_2_po_20_2day,0.90, names = FALSE),digits =
4)

Z_3task_2_po_30_2day=task3_2(T=30,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=2)
Z1_3task_2_80_po_30_2day<-round(quantile(Z_3task_2_po_30_2day,0.80, names = FALSE),digits =
4)
Z1_3task_2_90_po_30_2day<-round(quantile(Z_3task_2_po_30_2day,0.90, names = FALSE),digits =
4)

```

```

Z_3task_2_po_10_3day=task3_2(T=10,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=3)
Z1_3task_2_80_po_10_3day<-round(quantile(Z_3task_2_po_10_3day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_po_10_3day<-round(quantile(Z_3task_2_po_10_3day,0.90,names = FALSE),digits =
4)
Z_3task_2_po_20_3day=task3_2(T=20,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=3)
Z1_3task_2_80_po_20_3day<-round(quantile(Z_3task_2_po_20_3day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_po_20_3day<-round(quantile(Z_3task_2_po_20_3day,0.90,names = FALSE),digits =
4)

Z_3task_2_po_30_3day=task3_2(T=30,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=3)
Z1_3task_2_80_po_30_3day<-round(quantile(Z_3task_2_po_30_3day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_po_30_3day<-round(quantile(Z_3task_2_po_30_3day,0.90,names = FALSE),digits =
4)

Z_3task_2_po_10_4day=task3_2(T=10,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=4)
Z1_3task_2_80_po_10_4day<-round(quantile(Z_3task_2_po_10_4day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_po_10_4day<-round(quantile(Z_3task_2_po_10_4day,0.90,names = FALSE),digits =
4)
Z_3task_2_po_20_4day=task3_2(T=20,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=4)
Z1_3task_2_80_po_20_4day<-round(quantile(Z_3task_2_po_20_4day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_po_20_4day<-round(quantile(Z_3task_2_po_20_4day,0.90,names = FALSE),digits =
4)

Z_3task_2_po_30_4day=task3_2(T=30,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=4)
Z1_3task_2_80_po_30_4day<-round(quantile(Z_3task_2_po_30_4day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_po_30_4day<-round(quantile(Z_3task_2_po_30_4day,0.90,names = FALSE),digits =
4)

Z_3task_2_po_10_5day=task3_2(T=10,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=5)
Z1_3task_2_80_po_10_5day<-round(quantile(Z_3task_2_po_10_5day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_po_10_5day<-round(quantile(Z_3task_2_po_10_5day,0.90,names = FALSE),digits =
4)
Z_3task_2_po_20_5day=task3_2(T=20,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=5)
Z1_3task_2_80_po_20_5day<-round(quantile(Z_3task_2_po_20_5day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_po_20_5day<-round(quantile(Z_3task_2_po_20_5day,0.90,names = FALSE),digits =
4)
Z_3task_2_po_30_5day=task3_2(T=30,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=5)

```

```

Z1_3task_2_80_po_30_5day<-round(quantile(Z_3task_2_po_30_5day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_po_30_5day<-round(quantile(Z_3task_2_po_30_5day,0.90,names = FALSE),digits =
4)

#Geometric distribution
client_arrival_gen=function(n){
  return(1+rgeom(1, prob = 1/5))
}

Z_3task_2_geo_10_1day=task3_2(T=10,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=1)
Z1_3task_2_80_geo_10_1day<-round(quantile(Z_3task_2_geo_10_1day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_geo_10_1day<-round(quantile(Z_3task_2_geo_10_1day,0.90,names = FALSE),digits =
4)
Z_3task_2_geo_20_1day=task3_2(T=20,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=1)
Z1_3task_2_80_geo_20_1day<-round(quantile(Z_3task_2_geo_20_1day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_geo_20_1day<-round(quantile(Z_3task_2_geo_20_1day,0.90,names = FALSE),digits =
4)

Z_3task_2_geo_30_1day=task3_2(T=30,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=1)
Z1_3task_2_80_geo_30_1day<-round(quantile(Z_3task_2_geo_30_1day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_geo_30_1day<-round(quantile(Z_3task_2_geo_30_1day,0.90,names = FALSE),digits =
4)

Z_3task_2_geo_10_2day=task3_2(T=10,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=2)
Z1_3task_2_80_geo_10_2day<-round(quantile(Z_3task_2_geo_10_2day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_geo_10_2day<-round(quantile(Z_3task_2_geo_10_2day,0.90,names = FALSE),digits =
4)
Z_3task_2_geo_20_2day=task3_2(T=20,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=2)
Z1_3task_2_80_geo_20_2day<-round(quantile(Z_3task_2_geo_20_2day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_geo_20_2day<-round(quantile(Z_3task_2_geo_20_2day,0.90,names = FALSE),digits =
4)
Z_3task_2_geo_30_2day=task3_2(T=30,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=2)
Z1_3task_2_80_geo_30_2day<-round(quantile(Z_3task_2_geo_30_2day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_geo_30_2day<-round(quantile(Z_3task_2_geo_30_2day,0.90,names = FALSE),digits =
4)

Z_3task_2_geo_10_3day=task3_2(T=10,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=3)
Z1_3task_2_80_geo_10_3day<-round(quantile(Z_3task_2_geo_10_3day,0.80,names = FALSE),digits =
4)
Z1_3task_2_90_geo_10_3day<-round(quantile(Z_3task_2_geo_10_3day,0.90,names = FALSE),digits =
4)

```

```

Z_3task_2_geo_20_3day=task3_2(T=20,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=3)
Z1_3task_2_80_geo_20_3day<-round(quantile(Z_3task_2_geo_20_3day,0.80,names = FALSE),digits
= 4)
Z1_3task_2_90_geo_20_3day<-round(quantile(Z_3task_2_geo_20_3day,0.90,names = FALSE),digits
= 4)

Z_3task_2_geo_30_3day=task3_2(T=30,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=3)
Z1_3task_2_80_geo_30_3day<-round(quantile(Z_3task_2_geo_30_3day,0.80,names = FALSE),digits
= 4)
Z1_3task_2_90_geo_30_3day<-round(quantile(Z_3task_2_geo_30_3day,0.90,names = FALSE),digits
= 4)

Z_3task_2_geo_10_4day=task3_2(T=10,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=4)
Z1_3task_2_80_geo_10_4day<-round(quantile(Z_3task_2_geo_10_4day,0.80,names = FALSE),digits
= 4)
Z1_3task_2_90_geo_10_4day<-round(quantile(Z_3task_2_geo_10_4day,0.90,names = FALSE),digits
= 4)
Z_3task_2_geo_20_4day=task3_2(T=20,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=4)
Z1_3task_2_80_geo_20_4day<-round(quantile(Z_3task_2_geo_20_4day,0.80,names = FALSE),digits
= 4)
Z1_3task_2_90_geo_20_4day<-round(quantile(Z_3task_2_geo_20_4day,0.90,names = FALSE),digits
= 4)

Z_3task_2_geo_30_4day=task3_2(T=30,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=4)
Z1_3task_2_80_geo_30_4day<-round(quantile(Z_3task_2_geo_30_4day,0.80,names = FALSE),digits
= 4)
Z1_3task_2_90_geo_30_4day<-round(quantile(Z_3task_2_geo_30_4day,0.90,names = FALSE),digits
= 4)

Z_3task_2_geo_10_5day=task3_2(T=10,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=5)
Z1_3task_2_80_geo_10_5day<-round(quantile(Z_3task_2_geo_10_5day,0.80,names = FALSE),digits
= 4)
Z1_3task_2_90_geo_10_5day<-round(quantile(Z_3task_2_geo_10_5day,0.90,names = FALSE),digits
= 4)
Z_3task_2_geo_20_5day=task3_2(T=20,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=5)
Z1_3task_2_80_geo_20_5day<-round(quantile(Z_3task_2_geo_20_5day,0.80,names = FALSE),digits
= 4)
Z1_3task_2_90_geo_20_5day<-round(quantile(Z_3task_2_geo_20_5day,0.90,names = FALSE),digits
= 4)

Z_3task_2_geo_30_5day=task3_2(T=30,n,client_size_gen, client_arrival_gen,
client_thinking_gen,enter_day=5)
Z1_3task_2_80_geo_30_5day<-round(quantile(Z_3task_2_geo_30_5day,0.80,names = FALSE),digits
= 4)
Z1_3task_2_90_geo_30_5day<-round(quantile(Z_3task_2_geo_30_5day,0.90,names = FALSE),digits
= 4)

#####
#####plot for all.
par(mfrow=c(1,2))

```

```

plot(density(task_1_po_2013), xlim=c(-2.5,2.5), ylim=c(0,1.3),lwd=2.5,col="darkblue",main="Client
arrival - Poisson distr.",xlab="Price difference for MWh",ylab="%",bg="red",font.main=2)
lines(density(task_1_po_2014),lwd=2.5, col="red")
lines(density(task_1_po_2012),lwd=2.5, col="darkgreen")
legend("topright",
       c("2012","2013","2014"),
       lty=c(1,1,1),
       lwd=c(2,2,2),
       bty="n",
       col=c("darkgreen",'darkblue',"red" ))
plot(density(task_1_geo_2013), xlim=c(-2.5,2.5), ylim=c(0,1.3),lwd=2.5,col="darkblue",main="Client
arrival - Geometric distr.",xlab="Price difference for MWh",ylab="%",bg="red",font.main=2)
lines(density(task_1_geo_2014),lwd=2.5, col="red")
lines(density(task_1_geo_2012),lwd=2.5, col="darkgreen")
legend("topright",
       c("2012","2013","2014"),
       lty=c(1,1,1),
       lwd=c(2,2,2),
       bty="n",
       col=c("darkgreen",'darkblue',"red" ))
layout(1:1)

```

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