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UNIPOLAR CHARGING OF INITIALLY CHARGED AEROSOLS

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The general pattern of unipolar charging has received a fairly detailed treatment already in [1]. For initially neutral aerosols the equations of kinetics are written as (with minor alterations in symbols):

$$\frac{dN_0}{dt} = - I_0 N_0 ,$$

$$\frac{dN_1}{dt} = I_0 N_0 - I_1 N_1 ,$$

$$\frac{dN_2}{dt} = I_1 N_1 - I_2 N_2 ,$$

.....

$$\frac{dN_q}{dt} = I_{q-1} N_{q-1} - I_q N_q ,$$

where N_q is the partial concentration of aerosol particles of a given radius and charge number q ,
 t is time,
 I_q is air ion flux for a particle of a given charge number.

The initial conditions for charging of initially uncharged aerosol are:

$$N_0(t = 0) = a_0 ,$$

$$N_q(t = 0) = 0 , \quad q \geq 1 .$$

This system of differential equations is also known as the equations of generation. They can also be used for the description of radioactive transformations of elements [2].

The solution of the system is written as:

$$N_q(t) = a_0 \left(\prod_{k=0}^{q-1} I_k \right) \sum_{L=0}^q \frac{\exp(-I_L t)}{\prod_{m=0}^{q-L} (I_m - I_L)} ,$$

where 1) Π^* is product with one exception: in the case of equality of indexes the respective factor is always equal to unity;
 2) in the case $q = 0$ the product

$$\prod_{k=0}^{q-1} I_k = 1 .$$

Thus

$$N_0(t) = a_0 \exp(-I_0 t) ,$$

$$N_1(t) = a_0 I_0 \frac{\exp(-I_0 t)}{I_1 - I_0} + \frac{\exp(-I_1 t)}{I_0 - I_1} ,$$

$$N_2(t) = a_0 I_0 I_1 \frac{\exp(-I_0 t)}{(I_1 - I_0)(I_2 - I_0)} + \frac{\exp(-I_1 t)}{(I_0 - I_1)(I_2 - I_1)} + \frac{\exp(-I_2 t)}{(I_0 - I_2)(I_1 - I_2)} ,$$

.....

Actually atmospheric aerosol is never completely neutral, therefore it is better to allow for an initial charge distribution of partial concentrations. Let us suppose that in the initial moment we have both negatively and positively charged aerosol particles with the concentrations a_q , where q is the charge number.

In normal atmospheric conditions the charge distribution is probably close to the stationary distribution, i.e. a_q differs from zero in a certain interval $q_{min} \leq q \leq q_{max}$ symmetrically in relation to the point $q = 0$. For instance, in the case of the smallest particles only the initial concentrations a_{-1}, a_0, a_1 are taken into account. With the growth of the radius a_{-2}, a_2 , etc. appear. In any case, there is some minimum charge number q_{min} , below which $a_q = 0$.

In such a case the system of equations of charge kinetics looks like as follows:

$$\frac{dN_{q_{min}}}{dt} = - I_{q_{min}} N_{q_{min}} ,$$

$$\frac{dN_{q_{min}+1}}{dt} = I_{q_{min}} N_{q_{min}} - I_{q_{min}+1} N_{q_{min}+1} ,$$

$$\frac{dN_{q_{\min}+2}}{dt} = I_{q_{\min}+1}N_{q_{\min}+1} - I_{q_{\min}+2}N_{q_{\min}+2},$$

.....

$$\frac{dN_q}{dt} = I_{q-1}N_{q-1} - I_q N_q$$

with the initial conditions:

$$N_q(t=0) = a_q, \quad q_{\min} \leq q \leq q_{\max};$$

$$N_q(t=0) = 0, \quad q_{\min} > q > q_{\max}.$$

This system of equations consists of linear differential equations in relation to time. The solution of the first equation can be found when the initial conditions are specified, and it is possible to try to write the solution in a general shape for an arbitrary equation:

$$1) q = q_{\min}$$

$$N_{q_{\min}} = a_{q_{\min}} \exp(-I_{q_{\min}} t).$$

$$2) q = q_{\min} + 1$$

$$N_{q_{\min}+1} = a_{q_{\min}+1} \exp(-I_{q_{\min}+1} t) + a_{q_{\min}} I_{q_{\min}} \sum_{L=q_{\min}}^{q_{\min}+1} \frac{\exp(-I_L t)}{q_{\min}+1} \prod_{n=q_{\min}}^L (I_n - I_L).$$

$$3) q = q_{\min} + 2$$

$$N_{q_{\min}+2} = a_{q_{\min}+2} \exp(-I_{q_{\min}+2} t) + a_{q_{\min}+1} I_{q_{\min}+1} \sum_{L=q_{\min}+1}^{q_{\min}+2} \frac{\exp(-I_L t)}{q_{\min}+2} \prod_{n=q_{\min}+1}^L (I_n - I_L) + a_{q_{\min}} I_{q_{\min}} I_{q_{\min}+1} \sum_{L=q_{\min}}^{q_{\min}+2} \frac{\exp(-I_L t)}{q_{\min}+2} \prod_{n=q_{\min}}^L (I_n - I_L).$$

Using mathematical induction the solution can be written for an arbitrary partial concentration

$$N_q(t) = \sum_{j=q_{\min}}^q a_j \left(\prod_{k=j}^{q-1} I_k \right) \sum_{L=j}^q \frac{\exp(-I_L t)}{q} \prod_{n=j}^L (I_n - I_L).$$

Analogously to what was presented above: 1) Π^* denotes the

product in which the factor with equal indexes is equal to unity,

$$2) \text{ the product } \prod_{k=j}^{q-1} I_k = 1, \text{ if } q = q_{\min}.$$

Thus, proceeding from the initial distribution a_j and the air ion fluxes I_L , it is possible to find the distribution of partial concentrations at any arbitrary moment of time $N_q(t)$. To be more precise, an explicit two-dimensional distribution by charges and radii $N_{q,r}(t)$ is obtained proceeding from the initial distribution $a_{j,r}$ and the fluxes $I_{L,r}$. The radius r enters all the expressions as a supplementary argument (parameter).

According to the above suppositions it is necessary for the calculation of charge distributions to know the air ion fluxes for particles of different charges and radii.

References

1. Boisdron, M., Brock, J.R. On the stochastic nature of the acquisition of electric charge and radioactivity by aerosol particles // Atmos. Envir. - 1970. - V. 4. - P. 35-50.
2. Гольданский В.И., Куценко А.В., Подгорецкий М.И. Статистика отсчетов при регистрации ядерных частиц. - М., 1958.

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