MODEL CALCULATION OF GLOBAL COMPONENTS IN TROPOSPHERIC ELECTRIC FIELD VARIATION

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ABSTRACT: A simple mathematical model is used to study the dependence of atmospheric electric field on ionospheric potential variations. The model takes into account the inertia of redistribution of space charge after the change of ionospheric potential. The three-term Schweidler-Gish formula is used to describe air resistivity in a vertical column of the atmosphere. Response of a ground-based antenna to ionospheric potential variation is calculated. Phase shift between the signals of two distant antennas is demonstrated in examples. The shift is expected to be in correlation with the ratio of vertical current densities in the neighbourhoods of the antennas.

INTRODUCTION

Ruhnke (1969) has proposed to look for short-period global variations in the data series of simultaneous observations of the Maxwell current density in the atmosphere. The Maxwell current is the sum of the conductivity current and the displacement current which appears if the electric field changes. The Maxwell current is independent of altitude and that is the motivation for the proposal by Ruhnke. The Maxwell current density can be measured by means of a ground-based antenna which has the zero time constant. In an experiment of simultaneous measurements of the signals of two widely spaced antennas in Waldorf (USA) and Vilsandi (Estonia) (Ruhnke et al., 1983), the antennas had a time constant of 1 s. A synchronous response of ionospheric potential variations in the signals of both antennas was expected. The synchronism was detected in records but the signal of one antenna was shifted more than ten seconds from the signal of the other antenna in some cases. This result has not been explained. It would be necessary to repeat this experiment in conditions of improved reliability of clock synchronization. On the other hand, it would be essential to examine the design and techniques of the experiment for the possible effects of signal shift in time.

A simple mathematical model is used to explain the transmission of ionospheric potential variations to lower atmosphere and ground-based antennas. The model is based on Wilson's classical model of atmospheric electricity. Magnetospheric and ionospheric dynamo effect are neglected and the following assumptions are made:

1) electric potential at the Earth's surface is zero and the ionosphere is equipotential,
2) horisontal layers of the atmosphere are uniform in the range of several hundreds of kilometers,
3) the atmosphere is free from mechanical transport of charges,
4) the electrode effect near the ground is negligible,
5) the relaxation process of the antenna is equivalent to

the relaxation process of a simple RC circuit,

6) the air resistivity is time independent.

In addition, we will apply the three-term Schweidler-Gish formula in computing the numerical examples:

$$r = \sum_{i=1}^{3} G_i e^{-h/H_i} , \qquad (1)$$

where r is the resistivity of the air and h is the altitude over the sea level.

The values of parameters G_i and H_i depend on local conditions in the atmosphere. The following values are chosen to determine the standard distribution of resistivity in a vertical column of the atmosphere:

G_1 = 50 TΩ·m G_2 = 20 TΩ·m G_3 = 2 TΩ·m

H_1 = 0.5 km H_2 = 4 km H_3 = 12 km.

The determined standard distribution represents an approximation to the results of recent measurements (Markson, 1985; Кихтенко, Брагин, 1986).

The simplifications result in inconsistencies between model calculations and the reality. However, we hope the generalities of the global effects derived from the model are not cancelled by the disagreement between the model and the nature.

THE MATHEMATICAL RELATIONS

The mathematical model of the atmospheric electric field could be derived from the Ohm's law, the Gauss' law and the principle of charge conservation. The standard way is to express the model as a differential equation. An alternative way to determine the equivalent circuit was advanced by Kasemir and Ruhnke. We have preferred the last one which has enabled us to simplify the interpretation and calculations.

A vertical column of the atmosphere could be replaced by a set of parallel RC-circuits. Every RC-circuit corresponds to one thin horizontal layer of air in the vertical column. All RC-circuits connected in long series make up an electrical equivalent of the vertical air column. This model has a specific advantage for our problem. It enables us to link the mathematical models of the vertical column and the ground-based measurement antenna by using a straightforward procedure. The antenna could be replaced by an extra RC-circuit added to the lower end of the circuit which corresponds to the air column. The area of the column cross-section must be chosen so that it would equal the effective area of the antenna to match the parts of the joint circuit. The effective area of the long wire antenna could be calculated by the formula:

$$S = \frac{2\pi h_a l_a}{\ln(2h_a/r_a)} , \qquad (2)$$

where h_a is the height of the wire from the ground, l_a is the length and r_a the radius of the wire.

The Fourier transform and the Laplace transform are two common techniques for circuit analysis. The mathematical model of the vertical column based on the Laplace transform was developed by Ruhnke (1969). In the present paper the Fourier transform technique is used. This technique enables us to learn the ionospheric effects in lower atmosphere using amplitude-frequency and phase-frequency characteristics.

Let us consider a vertical column having the cross-section area S and a thin horizontal layer in this column with thickness dh. This layer has the resistance rdh/S, the capacitance $\varepsilon S/dh$ and the complex impedance

$$dZ = \frac{r}{S}\,\frac{1-i\omega r\varepsilon}{1+(\omega r\varepsilon)^2}\,dh\ , \tag{3}$$

where ω is the frequency of a sinusoidal signal, r is the resistivity and ε the absolute permittivity of air.

The first problem is to learn the behaviour of the potential on the altitude h if the ground has an altitude h_0 over the sea level. Two impedances $Z_h = \int_{h_0}^{h} dZ$ and $Z_\infty = \int_{h_0}^{\infty} dZ$ must be calculated to solve the problem by analysing a potential divider which has impedances $Z_\infty - Z_h$ and Z_h. If the amplitude of the sinusoidal component in the ionospheric potential is U_∞, then the amplitude of this component on altitude h is $U = KU_\infty$, where

$$K = \frac{|Z_h|}{|Z_\infty|} \tag{4}$$

is the potential transfer coefficient from the ionosphere to the altitude h. The phase shift of this sinusoidal signal between the altitude h and the ionosphere is

$$\varphi = \mathrm{arctg}\,\frac{\mathrm{Im}\,Z_h}{\mathrm{Re}\,Z_h} - \mathrm{arctg}\,\frac{\mathrm{Im}\,Z_\infty}{\mathrm{Re}\,Z_\infty}\ . \tag{5}$$

If $\varphi > 0$, the potential variation on the altitude h leads the ionospheric potential variation. If $\varphi < 0$, the potential variation on the altitude h delays from the ionospheric potential.

The second problem is in learning the response of the ground-based antenna to ionospheric potential variations. The solution can be obtained by repeating the above procedure. The only difference is that the impedance Z_h must be replaced by the impedance of the antenna

$$Z_a = R_a\,\frac{1-i\omega\tau_a}{1+(\omega\tau_a)^2}\ , \tag{6}$$

where R_a and τ_a are the resistance and the time constant of the antenna.

VARIATIONS OF POTENTIAL IN ATMOSPHERE

The probe technique is commonly used for experimental determination of the atmospheric potential distribution. Tethered balloons make it possible to lift the probes to the heights of several kilometers. Special examples have been computed to learn the transmission of the ionospheric potential variation to the altitudes of the probes. In these examples the standard distribution of resistivity is assumed. The results are presented in Fig. 1. Rapid variations of the potential are damped at small altitudes. This effect could be interpreted as follows. The pure capacitive divider of the potential is in action for very quick variations in the lower atmosphere. The output of this divider is proportional to the altitude. Only 1-2% of ionospheric potential variations are transmitted to the altitude of 1 km in this case. The pure resistive divider is acting at very slow variations. About 30% of the ionospheric potential variations are transmitted to 1 km by the resistive divider. The redistribution of atmospheric volume charge occurs after the ionospheric potential change (Hansen, 1935). Hundreds of seconds are needed for this process

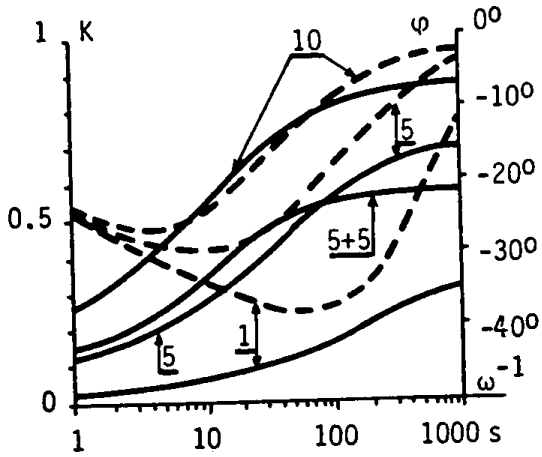

Fig. 1. Transmission of a sinusoidal potential to the altitudes 1, 5 and 10 km above the sea level. The dashed line represents the phase shift. Curve 5+5 corresponds to the height of 5 km above the mountain plateau with the altitude of 5 km.

which is accompanied by the transfer from the capacitive divider to the resistive divider. The facilities of the described process are illustrated in Fig. 1. An additional curve 5+5 in Fig. 1 is designed to estimate the effect of the high mountain plateau in the tethered balloon experiment. The standard resistivity distribution without any corrections was used. It is hoped that the resulting inaccuracy does not invalidate the general conclusion that highland conditions enable only a minor gain in the uniformity of frequency characteristics.

THE RESPONSE OF ANTENNA TO IONOSPHERIC POTENTIAL VARIATIONS

The performance of a measurement antenna depends on its own time constant τ_a. The response of an antenna of which $\tau_a = 0$ is proportional to the vertical Maxwell current density for all frequencies. The antenna of which $\tau_a = \varepsilon r$ (where r denotes air resistivity near the ground) is equivalent to a thin layer of air near the ground. Such antennas are tools for the electric field strength measurements (Crozier, 1963).

The response of various antennas to ionospheric potential variations is illustrated in Fig. 2. The examples are computed under the assumptions that the standard resistivity distribution in the atmosphere is valid and the altitude of the ground is zero. The relative transfer coefficient K_ω/K_0 and the phase shift φ of the sinusoidal component are shown as dependent on the frequency. K_ω denotes the absolute transfer coefficient (4) at the frequency ω and K_0 is the absolute transfer coefficient at the zero frequency. Fig. 2 displays the main features of the antennas depending on the time constant. The performance of various antennas is considerably different in the case of rapid variations and would equalize in the case of slow variations.

The effect of the high mountain plateau on the response of the antenna is shown in Fig. 3. The performance of various antennas has a tendency to equalize with the altitude. The damping of local disturbances is known as the main advantage of high mountain location for the measurement antenna. However, this advantage has no expression in our model as it does not represent local disturbances.

The phase shift between the antenna response and the ionospheric potential variations depends on the parameters of the vertical distribution of air resistivity. The air parameters differ in different sites and this results in the phase shift between the signals measured by two distant antennas. This conclusion is true for the Maxwell current measurements as well. The ionosphere synchronizes the variations of its potential but not of the Maxwell current densities

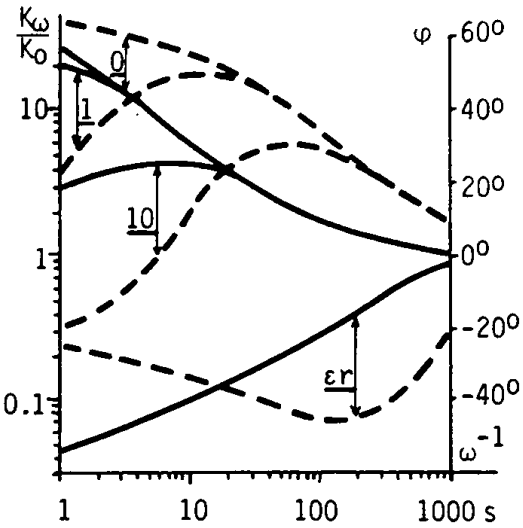

Fig. 2. Response of the antenna as a function of the frequency ω or variation time 1/ω. The curves are labelled with values of the time constant. The dashed lines represent the phase shift.

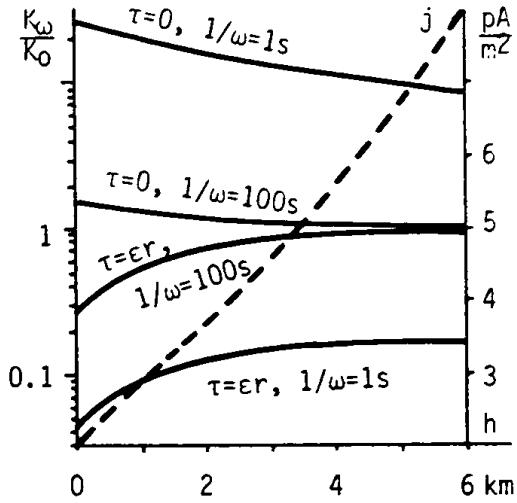

Fig. 3. The dependence of the response of the antenna on the ground altitude. The dashed line represents an average density of the vertical current.

in special regions. The Maxwell current densities in widely spaced sites have a phase shift of tens of seconds. Some examples are given in Table 1. The coefficient of resistivity of the troposphere middle layers G_2 has crucial effect on the phase shift in these examples.

Table 1

The shift between sinucoidal variations of antenna response and ionospheric potential depending on the coefficients of Formula 1 (h = 0 and 1/ω = 100 s).

G_1	G_2 (TΩ·m)	G_3	Shift: sec		Average current density: pA/m^2
			$\tau_a=0$	$\tau_a=\varepsilon r$	
50	40	2	77	-68	1.23
50	20	5	61	-82	1.56
100	20	2	64	-84	1.68
50	20	2	62	-80	2.00
50	20	1	63	-78	2.21
0	20	2	57	-53	2.48
0	10	2	39	-43	4.03

The average current density depends on the distribution of air resistivity. Therefore, the phase shift between the signals of two distant antennas is expected to be in correlation with the ratio of average current densities in the neighbourhoods of the antennas.

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