

NOTES ON THE INTERPRETATION OF AEROSOL ELECTRICAL DENSITY

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Concept of aerosol electrical density

The term "aerosol electrical density" has been proposed in [1] to denote the coefficient g in the balance equation of small air ions

$$\frac{dn}{dt} = q - \alpha n^2 - gn, \quad (1)$$

where n - the concentration of small air ions,

q - the ionization rate, and

α - the coefficient of recombination of small air ions.

Equation (1) is written on the assumption of charge symmetry $n_+ = n_- = n$.

The unit of measurement of aerosol electrical density is s^{-1} . For the ground layer of the atmosphere the typical values are $g = 0.01-0.1 s^{-1}$. In tabulating measurements it is useful to apply the unit ks^{-1} , then the typical numerical values will be 10-100 and can be written as integers.

The interpretation of aerosol electrical density from the point of view of atmospheric electricity and the study of electrical processes in the air is given directly by the determination of this value. Aerosol electrical density is the measure of adsorption of small air ions by the aerosol. For the interpretation of aerosol electrical density from the point of view of other possible applications of the notion, it is necessary to determine the weight function $w_m(r)$ that would make it possible to define aerosol electrical density as a particular case of the generalised aerosol density

$$g = \int w_m(r) f(r) dr, \quad (2)$$

where $f(r)$ is the size distribution function of particles normed to the number concentration of aerosol particles $N = \int f(r) dr$.

Equation of the weight function of aerosol electrical density

To calculate the function $w_m(r)$ the summand gn in equation (1) is to be expressed through the coefficients of attachment of small air ions by particles of a certain size and preliminary charge.

Let us use $\beta_{\gamma}(r)$ to denote the partial attachment coefficient of a positive small air ion to a particle with the radius r and charge γe , where e is the elementary charge. On the assumption of charge symmetry, the coefficients of the attachment of a negative small air ion by a positively charged particle, and of a positive small air ion by a negatively charged particle $\beta_{-\gamma}(r)$ are equal. The attachment coefficients are determined so that the flow of positive small air ions to a particle with the charge γe is $\beta_{\gamma} n$.

Let us use $p_{\gamma}(r)$ to denote the probability of the charge γe on a particle with the radius r .

The concentration of particles with the charge γe and the radius $r \dots r + dr$ will be $p_{\gamma}(r)f(r)dr$. The adsorption of small air ions by these particles is $n\beta_{\gamma}(r)p_{\gamma}(r)f(r)dr$. To find total adsorption gn , the resulting expression is to be summarized by all charges and integrated over all sizes: $gn = \int \sum n\beta_{\gamma}(r)p_{\gamma}(r)f(r)dr$. Cancelling n , we obtain the equation for calculating g . Comparing this equation with the expression of generalised aerosol density (2) it can be noticed that

$$w_{\Sigma}(r) = \sum_{\gamma} \beta_{\gamma}(r)p_{\gamma}(r). \quad (3)$$

Approximation of the function $\beta(r)$

In the region of Knudsen small numbers ($r > 100\text{nm}$) the first formula of Fuchs [2] has a good theoretical and experimental foundation

$$\beta_{\gamma}(r) = 4\pi r D \frac{\alpha}{e^{\alpha} - 1}, \quad (4)$$

where

$$\alpha = \gamma \frac{e^2}{4\pi\epsilon_0 r K T},$$

K - the Boltzmann constant and T - the temperature. The diffusion coefficient of small air ions is calculated by the Einstein formula

$$D = \frac{K T k}{e}, \quad (5)$$

where k - air ion mobility. At $k = 1.2 \cdot 10^{-4} \text{ m}^2/\text{V}\cdot\text{s}$ and $T = 290 \text{ K}$ we have $\alpha = \gamma(57.6 \text{ nm} / r)$.

The inadequacy of formula (4) in the region of Knudsen large numbers is demonstrated by the fact that instead of the experimentally confirmed limiting value $\beta_{-1}(0) = \alpha =$

$= 1.4-1.6 \cdot 10^{-12} \text{ m}^3/\text{s}$, it gives a higher value $2.2 \cdot 10^{-12} \text{ m}^3/\text{s}$. The simplified molecular kinetic theory of White also gives an incorrect limit $\beta_{-1}(0) = 0$. Pui's well-known semi-empirical formula is at $r \rightarrow 0$ reduced to White's formula and gives the same result.

A correct molecular kinetic adsorption theory has been proposed by Natanson [5]. Unfortunately, Natanson has not specified the methods of calculating certain variables, the values of which are necessary for concrete calculations. This gap in Natanson's theory was filled by Hoppel [6] who also carried out concrete numerical calculations. Due to the amount of computations involved the results are limited to a table of four coefficients $\beta_1, \beta_0, \beta_{-1}, \beta_{-2}$ for a certain set of sizes. On a different basis, the adsorption theory was developed by Fuchs [7] who proposed another fairly complex formula. Kojima [8] demonstrated that the discrepancy between Hoppel's results and the second formula of Fuchs does not exceed experimental error, and that experiment agrees with general theoretical results. He also proposed a simplified empirical formula which sufficiently well approximates experimental data of measuring $p_1(r)$. However, it is not satisfactory for approximating the function $\beta_f(r)$ in the whole region of definition. In fact, Kojima's improvement strengthens, rather than weakens the conflict of formula (4) with experiment in the case of determining $\beta_{-1}(0)$.

Computerized search yielded a simple empirical formula which at $r < 100 \text{ nm}$ satisfactorily approximates Hoppel's table and at $r > 100 \text{ nm}$ is quickly reduced to the first formula of Fuchs:

$$\beta_f(r) = 4\pi r D \frac{\alpha}{e^{\alpha} - 1} \left(1 - \frac{7}{7 + 10 \beta^{-2} + r : \text{nm}} \right). \quad (6)$$

Here α is determined by formula (4) and $r : \text{nm}$ denotes the numerical value of the radius in nm.

Calculation of the weight function of aerosol electrical density

If the function $\beta_f(r)$ is known, then the function $p_f(r)$ for uniform distribution can be calculated according to the known procedure. Let us denote partial concentrations of particles with the charge q_e and the radius $r \dots r + \Delta r$ with N_f . The sum of flows proceeding from a fraction as a result of

air ion adsorption is equal to the sum of flows entering this fraction. The balance of the zero fraction gives $\beta_0 N_0 = \beta_{-1} N_1$ and makes it possible to express N_1 through N_0 . The balance of the i -th fraction makes it possible to express N_{i+1} through N_i . In practical calculations it is necessary to select an arbitrary value of N_0 , to calculate recurrently all significant

$$N_i = \frac{\beta_{i-1}}{\beta_i} N_{i-1}$$

and then

$$P_\delta(r) = N_\delta / (N_0 + 2 \sum_{\delta=1}^{\infty} N_\delta). \quad (7)$$

The results of concrete calculations are depicted in the Figure by dots.

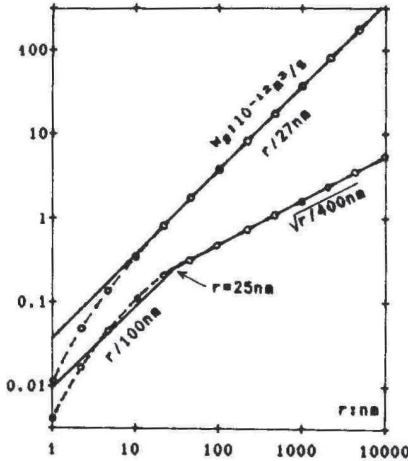


Fig. Approximation of the functions $w_w(r)$ and $w_s(r)$.

The function $w_w(r)$ in the region of large values of the radius approaches the straight line $w_w(r) = (r / 27 \text{ nm}) \cdot 10^{-12} \text{ m}^3/\text{s}$. This follows from the fact that all significant coefficients β_i are simultaneously approaching the diffusion limit $\beta = 4\pi rD$ which is independent of γ , and that the value w_w in accordance to formula (3) is viewed as a weighted average of β_i by γ . In the region $r < 10 \text{ nm}$, w_w is approximately proportional to $r^{1.15}$. If the particles with $r < 5 \text{ nm}$ are negligible, then the function is approximated by the formula

$$w_{\text{eff}}(r) \approx \frac{r}{27 \text{ nm}} 10^{-12} \text{ m}^3/\text{s}, \quad (8)$$

where the value of the coefficient 27 corresponds to the assumption $k = 1.2 \cdot 10^{-4} \text{ m}^2/\text{V}\cdot\text{s}$.

Comparative interpretation of large ion concentration

An effective concentration of large air ions N_{eff} where every particle is taken into account f times can also be represented as a generalised aerosol density:

$$N_{\text{eff}} = \int w_{\text{eff}}(r) f(r) dr. \quad (9)$$

As $N_{\text{eff}} = \int \sum f P_f(r) f(r) dr$, then

$$w_{\text{eff}}(r) = \sum_{f=1}^{\infty} f P_f(r). \quad (10)$$

Results of the calculations carried out according to formula (10) are also presented in the Figure. This function cannot be approximated as simply as the function w_{eff} . The simplest approximation is as follows

$$w(r) = \begin{cases} \frac{r}{100 \text{ nm}} & \text{at } r \leq 25 \text{ nm} \\ \sqrt{\frac{r}{400 \text{ nm}}} & \text{at } r \geq 25 \text{ nm}. \end{cases} \quad (11)$$

This approximation does not ensure a simple mechanical interpretation of the concentration of large air ions as an integral characteristic of aerosol.

Mechanical interpretation of aerosol electrical density

In approximation (8) aerosol electrical density is proportional to size concentration of aerosol particles. The size concentration v_d is determined as a generalised aerosol density at the weight function $w = 2r$. It has an imaginable mechanical interpretation - the value of v_d indicates the length of the chain composed of all particles in a volume unit. In approximation (8) we obtain the conversion formulae

$$g \approx (0.0185 \frac{\text{m}^2}{\text{ks}}) v_d, \quad v_d \approx (54 \frac{\text{ks}}{\text{m}^2}) g. \quad (12)$$

The values of $v_d = 540-5400 \text{ m}^{-2}$ correspond to the values $g = 10-100 \text{ ks}^{-1}$ typical for the ground layer atmospheric air. $v_d = 1000 \text{ m}^{-2}$ means that the length of the chain composed of particles in 1 m^3 will be 1 km.

The concepts of particle size concentration and aerosol electrical density are useful everywhere where the effect of aerosol particles is approximately proportional to their size.

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Comment:

The improved calculation of $w_{\text{ж}}(r)$ is published in paper:

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