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Unit linked Insurance Plans and their applications in India

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Fondidega seotud elukindlustuspoliisid ja nende rakendused Indias

Lühikokkuvõte. Töös uuritakse elukindlustuse kaasaegseid mudeleid, kus erinevalt traditsioonilisest juhust, kus kindlustushüvitise suurus on garanteeritud, mingi osa preemiamakseid investeeritakse, ehk kindlustushüvitised on mittegaranteeritud suurusega. Sellised mudeleid nimetatakse fondidega seotud mudeliteks ning töös tuvustatakse nii mittestohhastilist kui stohhastilist lähenemist ja esitatakse arvutuslikud näited kasumi hindamiseks. Lisaks antakse ülevaade India elukindlustusturust ning fondidega seotud elukindlustustoodetest.
CERCS kood: P160 Statistika, operatsioonanalüüs, programmeerimine, finants- ja kindlustusmatemaatika
Märksõnad: fondidega seotud poliisid, India kindlustusturg, tekkekulud, kasumi hindamine

Unit Linked Insurance Plans and their Applications in India

Abstract. This paper focuses on a modern insurance plan, called unit linked insurances, where differently from the traditional life insurance, the policyholder gets guaranteed insurance benefit, whereas in case of unit linked insurance policies, the policyholder gets both guaranteed insurance and non-guaranteed investment benefits. Both deterministic and stochastic approaches for unit linked insurances are considered with examples. Apart from the theory, an overview of the insurance industry in India and different unit linked insurance products is given.

CERCS codes: P160 Statistics, operation research, programming, actuarial mathematics.
Keywords: Indian insurance market, emerging costs, profit testing, unit linked life insurance.

DEDICATION

This thesis is dedicated to my parents, my sister who always loved and cared me and my teachers at University of Tartu who taught me mathematics and finance.

Contents

1 Introduction

An insurance is a means of protection against any kind of financial loss. In this paper we consider the models of life insurance. A life insurance is a contract between an insurance policy holder (an insured person) and an insurer (an insurance company) in which the insurer promises to pay a benefit in exchange for a premium upon the death of the policy holder or upon completion of a period of time or before a period of time.

There are different types of life insurances. The traditional products are the whole life, term and endowment insurances. However, recently the design of products has changed. In this paper we focus on a more modern product, a **unit linked insurance** (e.g., [2], [5], [11]). It is a modern life insurance contract which combines both insurance and investment benefits. In this paper we first discuss the traditional life insurance models and then introduce some of the modern ones while the main focus is on deterministic and stochastic profit testing for unit linked insurances. In addition we give an overview of the insurance industry in India and discuss the features of the different unit linked insurance products offered there.

The paper is organized as follows. In Section 2 we give an overview and notation of traditional life insurance and briefly discuss modern life insurance models, including the unit linked insurance. In Section 3 three we discuss the theories of deterministic and stochastic profit testing for unit linked insurance and give examples. In Section 4 we give an overview of the Indian insurance market and discuss the features of different unit linked insurance products offered there.

2 Life insurance models

A life insurance is a contract between an insurance policy holder (an insured person) and an insurer (an insurance company) in which the insurer promises to pay a benefit (a fixed sum of money) in exchange for a premium (a sum of money) upon the death of the policy holder or upon completion of a period of time or before a period of time. Life insurance mathematics is a branch of actuarial science in which concepts from different fields of mathematics and science like probability theory, statistics, finance, economics and computer science are used to analyse the key features of insurance contracts like benefits, premiums, risk and uncertainties. The professionals who work on such problems in the insurance companies are known as actuaries.

2.1 Traditional life insurances

Traditional life insurance contracts are the most basic types of insurance contracts where there are predetermined premiums and benefits. There are mainly five types of such policies offered in most of the countries - whole life insurance plans, term insurance plans, endowment plans and pure endowment plans. In this section we will discuss in brief about the different traditional life insurance models.
Let (x) denote a person aged x years.
A **whole life insurance** policy is a traditional life insurance policy which provides for a benefit following the death of the insured (x) at any time in the future. On the other hand, a **term insurance** policy of duration n years provides for a benefit only if the insured (x) dies within the n-year term. The third type of insurance policy, a **pure endowment** policy of duration n years provides for a benefit at the end of the n years only if the insured (x) survives at least n years. Next, an **endowment** policy of duration n years provides for a benefit either following the death of the insured or upon the survival of the insured to the end of the n-year term, whichever occurs first. Hence, an endowment contract can always be expressed as a combination of a term insurance contract and a pure endowment contract. The last type of traditional insurance contract, a u-year **deferred whole life insurance** contract is very similar to the whole life insurance contract but does not begin to offer death benefit cover until the end of a deferred period, u-years.

As in our examples in Section 3, (e.g., [5], [6]) we will show some detailed theoretical results, then we give more specific overview of the calculations of term and endowment insurances.

Let i be the annual effective interest rate used for calculating the present value of the benefit payable to the policyholder, v be the annual effective discount factor which can also be denoted as a function of annual effective discount rate, $v = \frac{1}{1+i}$ and δ be the force of interest which is again a function of i, $\delta = \ln(1+i)$.
Let $T(x)$ be a continuous random variable denoting future lifetime of a person aged x years, i.e., (x) and let $K(x)$ be a discrete random variable denoting the number of completed future years lived by (x) and $R(x)$ be the continuous random variable denoting the fractional part of $T(x)$. We can also say that $K(x)$ is the integer part of $T(x)$, $K(x) = \lfloor T(x) \rfloor$, $R(x) = \{T(x)\}$ and $T(x) = K(x) + R(x)$.

Definition 1. *We denote by ${}_tq_x$ the probability that (x) does not survive beyond age $x+t$. This is the distribution function of $T(x)$,*

$$ {}_tq_x = P[T(x) \leq t]. \tag{1} $$

Definition 2. *We denote by ${}_tp_x$ the probability that (x) does survive beyond age $x+t$, in other words, it is the survival probability for (x),*

$$ {}_tp_x = P[T(x) > t] \tag{2} $$

.

In addition we denote the probability that (x) dies between ages $x+u$ and $x+u+t$ by ${}_{u|t}q_x = P[u < T(x) \leq u+t]$. We also have the following relations.

$$ {}_{n|}q_x = {}_np_x - {}_{n+1}p_x \tag{3} $$

$$ {}_{u|t}q_x = {}_tq_{x+u} \cdot {}_up_x \tag{4} $$

$$ {}_1p_x \cdot {}_{n-2}p_{x+1} = {}_{n-1}p_x \tag{5} $$

In our examples later we use term and endowment insurances. For that we discuss the calculation of benefits for such policies.

The expected present value of n-year term insurance, issued to (x), with benefit of 1 unit payable at the end of year of death is denoted $A^{1}_{x:\overline{n}|}$ and calculated as

$$A^{1}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1}\,{}_{k}p_{x}q_{x+k}. \tag{6}$$

The expected present value of n-year increasing term insurance (i.e the level of benefit increases in arithmetic progression 1 unit per year) with benefit payable at the end of year of death is denoted by $(IA)^{1}_{x:\overline{n}|}$ and calculated as

$$(IA)^{1}_{x:\overline{n}|} = \sum_{k=0}^{n-1}(k+1)v^{k+1}\,{}_{k}p_{x}q_{x+k}. \tag{7}$$

The expected present value of n-year pure endowment with benefit of 1 unit payable at time n is denoted by $A_{x:\overset{1}{\overline{n}|}}$ and calculated as

$$A_{x:\overset{1}{\overline{n}|}} = v^{n}\,{}_{n}p_{x}. \tag{8}$$

Finally, the expected present value of an endowment policy with benefit of 1 unit payable at the end of year of death is denoted by $A_{x:\overline{n}|}$ and is the sum of $A^{1}_{x:\overline{n}|}$ (term insurance) and $A_{x:\overset{1}{\overline{n}|}}$ (pure endowment). So it is calculated as

$$A_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1}\,{}_{k|}q_{x} + v^{n}P(K_{x} \geq n) \tag{9}$$

In order to understand more about the term and endowment insurances we first show that

$$A_{x:\overline{n}|} = \sum_{k=0}^{n-2} v^{k+1}\,{}_{k|}q_{x} + v^{n}\,{}_{n-1}p_{x}\,. \tag{10}$$

Formula (10) easily follows from (3)

$$A_{x:\overline{n}|} = \textstyle\sum_{k=0}^{n-1} v^{k+1}\,{}_{k|}q_{x} + v^{n}P(K_{x} \geq n)$$

$$= \sum_{k=0}^{n-2} v^{k+1}{}_{k|}q_x + v^n{}_{n-1|}q_x + v^n{}_n p_x$$

$$= \sum_{k=0}^{n-2} v^{k+1}{}_{k|}q_x + v^n({}_{n-1|}q_x +_n p_x)$$

$$= \sum_{k=0}^{n-2} v^{k+1}{}_{k|}q_x + v^n{}_{n-1}p_x \,.$$

Now we compare last formula with formula

$$\sum_{k=0}^{n-1} v^{k+1}{}_{k|}q_x + v^n P(K_x \geq n) = A^{1}_{x:\overline{n}|} + v^n{}_n p_x$$

where $K(x)$ is the integer part of $T(x)$. Formula (9) splits expected present value of the benefit of 1 unit payable at the end of year of death (endowment) into expected present value of the benefit of 1 unit payable at the end of year of death (term insurance i.e covering n year term) and expected present value of the benefit of 1 unit payable at the end of year of death (pure endowment i.e a guaranteed benefit of 1 unit on survival to age $x+n$ at time n) whereas (10) splits expected present value of the benefit of 1 unit payable at the end of year of death (endowment) into expected present value of the benefit of 1 unit payable at the end of year of death (term insurance covering $n-1$ year term) and expected present value of the benefit of 1 unit on survival to age $x+n-1$ at time n.

Now we derive a recursion formula for an n-year increasing term insurance which is given as

$$(IA)^{1}_{x:\overline{n}|} = vq_x + vp_x\Big((IA)^{1}_{x+1:\overline{n-1}|} + A^{1}_{x+1:\overline{n-1}|}\Big). \tag{11}$$

By using (4), (5), (6) and (7) we get

$$(IA)^{1}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1}(k+1)_{k|}q_x$$

$$= v_{0|}q_x + 2v^2{}_{1|}q_x + 3v^3{}_{2|}q_x + ... + nv^n{}_{n-1|}q_x$$

$$= vq_x + 2v^2 p_x q_{x+1} + 3v^3{}_2 p_x q_{x+2} + ... + nv^n{}_{n-1}p_x q_{x+n-1}$$

$$= vq_x + vp_x(2vq_{x+1} + 3v^2 p_{x+1}q_{x+2} + ... + nv^{n-1}{}_{n-2}p_{x+1}q_{x+n-1})$$

$$= vq_x + vp_x(vq_{x+1} + 2v^2 p_{x+1}q_{x+2} + ... + v^{n-1}{}_{n-2}p_{x+1}q_{x+n-1})$$

$$+vp_x(vq_{x+1} + v^2 p_{x+1}q_{x+2} + ... + v^{n-1}{}_{n-2}p_{x+1}q_{x+n-1})$$

$$= vq_x + vp_x(IA)^1_{x+1:\overline{n-1}|} + vp_x A^1_{x+1:\overline{n-1}|}$$

$$= vq_x + vp_x\Big((IA)^1_{x+1:\overline{n-1}|} + A^1_{x+1:\overline{n-1}|}\Big).$$

Formula (11) splits the expected present value (EPV) of the increasing benefit of 1 unit into two parts - the first part gives the EPV if the person (x) dies in the first policy year, the second part gives the EPV terms (both of the benefit and increasing benefit) provided that the person survived the first policy year which means the benefit of 1 unit is issued to a person aged $x+1$ years with $n-1$ term insurance.

2.2 Unit linked life insurances

The modern life insurance contracts are the successors of traditional life insurance contracts. These type of policies started being issued to the general public very recently. They have become very popular because unlike in case of traditional insurance which only offer insurance benefits, they offer both insurance and investment benefits. There are mainly three types of modern life insurance policies - unitized with-profit life insurance, universal life insurance and unit linked insurance.

The **unitized with-profit life insurance** [5] introduces variability in the cash flows of whole life or endowment insurance through a profit sharing agreement. This type of policies are popular in United Kingdom. On the other hand a **universal life insurance** [5] is a form of whole life or endowment insurance, with some profit sharing incorporated in the design, and which also has more flexible payment schedules than traditional insurance. These type of policies are popular in North America.

A **unit linked insurance** policy [2], [5], [11] is very similar to unitized-with life and universal life insurance policies. The main difference is in unit linked insurance contracts the policyholder's assets and the insurer's assets are kept separate from each other whereas in unitized-with life and universal life insurance contracts the assets are combined.
A unit linked insurance contract is a type of insurance contract which combines both insurance and investment benefits. Such products are also known as **equity linked insurance** [5] contracts. In the United States it is known by the name **variable annuities** (here the name annuity can be misleading because there can be no actual annuity component) and in Canada it is known as **segregated fund policies**.
There are mainly two approaches to analyse unit linked insurance. One is the deterministic approach and other is the stochastic approach. The main difference between those two approaches is that in deterministic approach we use a fixed/deterministic interest rate earned on the policyholder's fund (money invested by the policyholder) whereas in stochastic approach we treat the interest rate on the policyholder's fund as a random variable. As a result of that, in case of deterministic approach, the insurer's profit is a number whereas in case of stochastic approach, the insurer's profit is a random variable.
Different authors have given slightly different approaches in understanding unit linked insurance. Dickson et al. [5] discuss both deterministic and stochastic approaches and give nice introduction to the cash flow analysis. A more general approach, with more details about the stochastic pricing theory and its use in determining the premiums is given by Bacinello [2], Koller [11]. Schreiber [17] gives a general example of unit-linked insurance using Brownian motion and compound Poisson process.

2.3 Multiple state models

While considering profit testing for unit linked insurances in next chapter we use multiple decrement insurance models. Those are special case of multiple state models which describe the random movements of a subject among various states. A general type multiple state model (i.e., [5], [11], [12]) includes a finite set of states, that represent different conditions for an individual (x), labeled $0, 1, ..., n$ (the natural starting state - alive, healthy, employee, . . . is labelled by 0) and between selected pairs of states (and in selected directions) the immediate transitions are possible. We give in Figure 1 an example of a multiple state model with 5 states where State 1 denotes temporary disabled, State 2 denotes permanently disabled, State 3 denotes dead, State 4 denotes critically ill and the arrows represent the possible transitions between states.

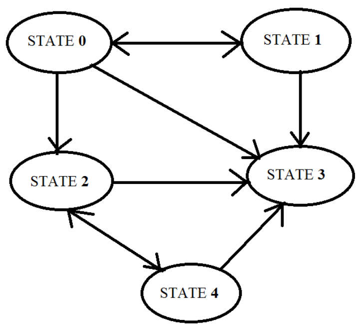

Figure 1: An example of multiple state life insurance model

For each $t \geq 0$, the random variable $Y(x+t)$ takes one of the values 0, 1, ... , n and the event $\{Y(x + t) = i\}$ means, that the individual is in state i at age $x + t$. The set of random variables $\{Y(x + t)\}_{t\geq 0}$ is a continuous time stochastic process.
We will also assume that for any states i and j and any times t and $t + s$, where $s \geq 0$, the conditional probability $P[Y(t + s) = j|Y(t) = i]$ does not depend on any information about process before time t.

Definition 3. *The probability that* (x)*, currently in state* i *will be in state* j *after* *time* $t \geq 0$ *, at age* $(x + t)$ *is defined as* ${}_tp_x^{ij} = P[Y(x + t) = j|Y(x) = i]$.

The probability that a life aged (x) in state i is in state j at age $x+1$ is the transition probability from state i to state j. We denote the probability $P[Y(x+1) = j|Y(x) = i]$ also as p_x^{ij}.

A **multiple decrement model** is a special case of multiple state model where transitions only take place from the initial state to the end state(s). The previously introduced notation for multiple state models still holds with $i = 0$.

Definition 4. *The probability that* (x)*, currently in state* 0 *will be in some end state* j *after time* $t \geq 0$ *, at age* $(x + t)$ *is defined as* ${}_tp_x^{0j} = P[Y(x + t) = j|Y(x) = 0]$.

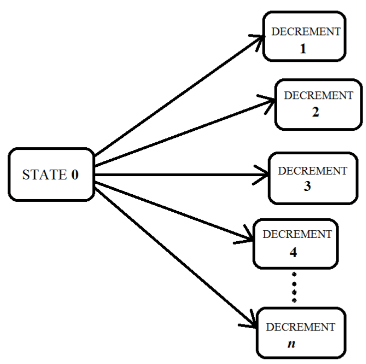

Figure 2: Multiple decrement model

In Figure 2 is a multiple decrement model with n states while arrows represent the possible transitions.
In the next section in our examples we present a special type of term and endowment insurance by defining them as a multiple decrement model.

3 Profit testing for unit linked insurance

In this chapter we discuss the cash flow analysis of unit linked insurance. There are two approaches for analysing cash flows - deterministic profit testing approach and the stochastic profit testing approach. The main difference between those two approaches is that in deterministic approach we use a fixed/deterministic interest rate earned on the policyholder's fund (money invested by the policyholder) whereas in stochastic approach we treat the interest rate on the policyholder's fund as a random variable. As a result of that, in case of deterministic approach, the insurer's profit is a number whereas in case of stochastic approach, the insurer's profit is a random variable. We will discuss both the approaches for calculating insurer's profit vector in the next sections. First we give the main definitions and then illustrate the theory with deterministic and stochastic example.

3.1 Definitions and notations

In case of traditional life insurance policies, the policyholder pays premium to the insurance company and in return gets insurance benefit, whereas in case of **unit linked insurance** policies, the policyholder pays premium to the insurance company and in return gets both insurance and investment benefit. This investment benefit comes into picture because the premium paid by the policyholder, after deducting expenses, is invested on behalf of the policyholder in investment funds also known as the policyholder's fund.

Definition 5. ***Policyholder's fund*** *denoted by* F_t *is defined as the amount of money kept in the policyholder's account at time* t*.*

Next we define premium and allocated premium.

Definition 6. ***Premium*** *denoted by* P_t *is defined as the amount of money paid by the policyholder to the insurer at time* $t-1$.

Definition 7. ***Allocated premium*** *denoted by* AP_t *is a part of the premium* P_t *which is invested in the policyholder's fund.*

The most important assumption in deterministic approach of understanding unit linked insurance is that the interest rate earned on the policyholder's fund is fixed/deterministic.

Definition 8. ***Interest on policyholder's assets*** *denoted by* i_t^f *is the rate of interest earned on the policyholder's fund at time t.*

Every month, a certain amount in the name of management charges is deducted from the policyholder's fund and added to the insurer's fund. These management charges are used to cover expenses and insurance charges.

Definition 9. ***Management charge*** *denoted by* MC_t *is an amount of money deducted from the policyholder's fund at time t and deposited into the insurer's fund.*

Definition 5-9 projects the policyholder's fund which gives the equation

$$F_t = (F_{t-1} + AP_t)(1 + i_t^f) - MC_t \tag{12}$$

Though the assets of the policyholder are always kept separate from the assets of the insurer, cash flows in the insurer's fund is always dependent on the policyholder's fund. The next definitions will project the insurer's fund.

Sometimes the policyholder's fund is not sufficient to cover the policyholder's benefits. That is when insurer's reserve comes into help.

Definition 10. ***Reserve*** *denoted by* ${}_{t-1}V$ *is the insurer's reserve brought forward at time* $t - 1$ *i.e. at the start of tth year. In addition to the policyholder's fund, it is required only when there are future liabilities which need reserves in advance.*

Next we define unallocated premium which is dependent on the policyholder's fund.

Definition 11. ***Unallocated premium*** *denoted by* UAP_t *is the difference between the premium paid by the policyholder and the allocated premium invested in the policyholder's fund at time* $t - 1$. *It is deposited into the insurer's fund.*

$$UAP_t = P_t - AP_t \tag{13}$$

Next we define expenses.

Definition 12. ***Expenses*** *denoted by* E_t *are the projected incurred expenses. Pre-contract expenses are incurred at time 0 and other expenses are incurred at time* $t-1$.

Now we define the interest income on the insurer's assets.

Definition 13. ***Interest*** *denoted by* I_t *is the interest income on the insurer's assets invested through the tth year.*

As we have discussed, in unit linked insurance contract one gets both investment and insurance benefit. So, it also serves the purpose of a traditional insurance contract.

Definition 14. ***Death benefit*** *denoted by* DB_t *is the amount of money paid to the beneficiary at the end of year of death of the policyholder.*

Next we consider a multiple decrement model with 3 states. We denote by 0 the state where the policyholder is alive and is currently under insurance contract, by d the state where policyholder is dead and by w the state where the policyholder is alive but has withdrawn himself/herself from the contract.

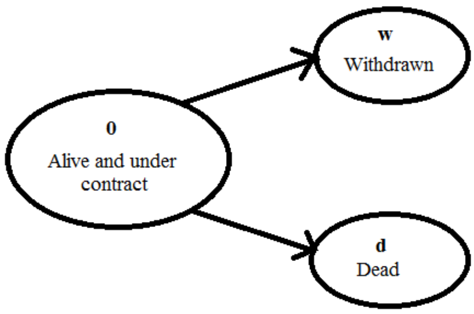

Figure 3: Multiple decrement model for profit testing

Using the notation from section 2.3 we define the expected cost of death benefit which is defined using death benefit and policyholder's fund.

Definition 15. ***Expected cost of death benefit*** *denoted by* EDB_t *covers the additional death benefit which is not covered by the policyholder's fund. The additional* *death benefit is $DB_t - F_t$. If the mortality probability for the tth year (i.e. a person aged $(x + t - 1)$ in state 0 will be in state d at age $x + t$) is p^{0d}_{x+t-1}, then*

$$EDB_t = p^{0d}_{x+t-1} \cdot (DB_t - F_t) \tag{14}$$

Next we define the payment at maturity.

Definition 16. *CV_t is the **payment at maturity** (at time t) to the policyholder.*

Now we define the expected cost of surrender which is defined using payment at maturity and policyholder's fund.

Definition 17. ***Expected cost of surrender or maturity cash value** denoted by ECV_t covers additional cash value which is not covered by the policyholder's fund. If there is a surrender penalty such that the surrendering policyholder receives less than F_t at maturity time t, then ECV_t is negative which implies an income for the insurer. If the probability of surrender at time t (i.e. a person aged $(x + t - 1)$ in state 0 will be in state w at age $x + t$) is p^{0w}_{x+t-1}, then*

$$ECV_t = p^{0w}_{x+t-1} \cdot (CV_t - F_t) \tag{15}$$

Next we define the expected cost of year end reserve which is defined using insurer's reserve.

Definition 18. ***Expected cost of year end reserve** denoted by E_tV is the expected value of insurer's reserve ${}_tV$ brought forward at time t. If the probability that a policy in force at time $t - 1$ is still in force at time t (i.e. a person aged $(x + t - 1)$ in state 0 will remain in state 0 at age $x + t$) is p^{00}_{x+t-1}, then*

$$E_tV = p^{00}_{x+t-1} \cdot {}_tV \tag{16}$$

Thus from the above definitions, we get the **profit emerging at time t for a policy in force at time $t - 1$**,

$$Pr_t = {}_{t-1}V + UAP_t - E_t + I_t + MC_t - EDB_t - ECV_t - E_tV \tag{17}$$

Till now we have not considered any uncertainty from the investment returns. We assumed that the **interest on policyholder's assets** is fixed which gives the **profit emerging at time t for the policy in force at time** $t-1$ a number. Practically this is not true as there is always some uncertainty in investment. So now we will consider the interest on policyholder's assets as a random variable which will give the profit emerging at time t for the policy in force at time $t-1$ as a random variable thus making the insurance contract more realistic.

Next we discuss stochastic profit testing introduced in [5] and [11]. Our aim is again to determine the profit earned by the insurer.

Let R_1, R_2,... be a sequence of random variables where R_t denotes the accumulation at time t of a unit amount invested in the policyholder's fund at time $t-1$. The accumulation factor is R_t. Then $R_t - 1$ is the **interest on policyholder's assets** i.e. the rate of interest earned on the policyholder's fund at time t. We assume a possibility that $\{\ln R_t\}$ is a sequence of **iid** (independent and identical) **normal** random variables with parameters mean μ and standard deviation σ which gives $\{R_t\}$ a sequence of **iid lognormal** random variables with parameters μ and σ.

Using the fact that the moment generating function of $\ln R_t$,

$$M_{\ln R_t}(t) = Ee^{t \ln R_t} = \exp\{\mu t + \frac{1}{2}\sigma^2 t^2\},$$

we get the following terms [1],[4]

Expectation of R_t,

$$ER_t = Ee^{\ln R_t} = \exp\{\mu + \frac{1}{2}\sigma^2\} \tag{18}$$

which is the **expected accumulation factor** each year.

Variance of R_t,

$$VarR_t = Var(e^{\ln R_t}) = Ee^{2\ln R_t} - (Ee^{\ln R_t})^2 = \exp\{2\mu + \frac{4}{2}\sigma^2\} - (\exp\{\mu + \frac{1}{2}\sigma^2\})^2$$

$$= e^{2\mu+2\sigma^2} - e^{(\mu+\frac{\sigma^2}{2})2} = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2} = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1).$$

For any $n \in \mathbb{R}/\mathbb{C}$, the nth raw moment of R_t is

$$ER_t{}^n = Ee^{n \ln R_t} = \exp\{\mu n + \frac{1}{2}\sigma^2 n^2\}.$$

We must note that the definitions and concepts discussed before hold true for stochastic profit testing as well except for the fact that the interest rate on the policyholder's fund and the terms associated with it and not numbers any more.
Next we will give examples of calculation of profit vector $Pr = (Pr_1, ..., Pr_T)$ for a T-year unit linked insurance contract.

3.2 Example of deterministic profit vector of unit linked insurance

In this section we give an example of deterministic profit testing. The example is a modification of the Example 14.1 in [5]. Our aim is to determine the profit earned by the insurer.
Consider a 20-year term unit linked insurance contract issued to a life aged 40. The policyholder pays an annual premium of 3000 euros. The insurer deducts a 6% expense allowance from the first premium and a 2% allowance from subsequent premiums. The remainder is invested into the policyholder's fund. At the end of each year a management charge of 0.5% of the policyholder's fund is transferred from the policyholder's fund to the insurer's fund. If the policyholder dies during the contract term, a benefit of 105% of the value of the policyholder's year end fund (after management charge deductions) is paid at the end of the year of death. If the policyholder surrenders the contract, he receives the value of the policyholder's fund at the year end, after management charge deductions. If the policyholder holds the contract to the maturity date, he receives the greater of the value of the policyholder's fund and the total of the premiums paid.
For the survival model, we will assume the probability of dying in any year is 0.004. Next we consider lapses of 12% of lives in force at year end surrender in the first year of the contract, 7% in the second year and none in subsequent years. All surrenders occur at the end of a year immediately after the manage-

ment charge deduction. We will also assume the initial expenses of 9% of the first premium plus 120 euros, incurred before the first premium payment and renewal expenses of 0.4% of the second and subsequent premiums. Consider that the insurer's funds earn interest at 5% per year and the insurer holds no reserves for the contract. Also assume that the policyholder's fund earns interest at 8% per year.

First we will project the year end fund values for a contract that remains in force for 20 years. As the annual premium is assumed to be $P_t = 3000 \quad \forall t = 1, 2, ..., 20.$ then allocated premiums for the first year is calculated as

$$AP_1 = 3000 - \frac{6}{100} \times 3000 = 2820$$

and after that as

$$AP_t = 3000 - \frac{2}{100} \times 3000 = 2940 \quad \forall t = 2, 3, ..., 20.$$

As the interest rate of policyholder's assets is assumed to be $i_t^f = 0.08 \quad \forall t = 1, 2, ..., 20.$ then the management charge can be expressed as

$$MC_t = 0.005 \times (F_{t-1} + AP_t) \times (1 + i_t^f)$$

$$= 0.005 \times (F_{t-1} + AP_t) \times 1.08$$

$$= 0.0054\,(F_{t-1} + AP_t) \quad \forall t = 1, 2, ..., 20$$

Following equation (12) we can calculate policyholder's fund as

$$F_t = (F_{t-1} + AP_t)\,(1 + i_t^f) - MC_t$$

$$= (F_{t-1} + AP_t) \times 1.08 - MC_t$$

$$= (F_{t-1} + AP_t) \times 1.08 - 0.005 \times (F_{t-1} + AP_t) \times 1.08$$

$$= 0.995 \times (F_{t-1} + AP_t) \times 1.08$$

$$= 1.0746\,(F_{t-1} + AP_t) \quad \forall t = 1, 2, ..., 20$$

F_ts are the year end fund values for the 20 year unit linked insurance contract issued to the person aged 40. The fund value calculations are given in Appendix A and we illustrate it in Figure 4.

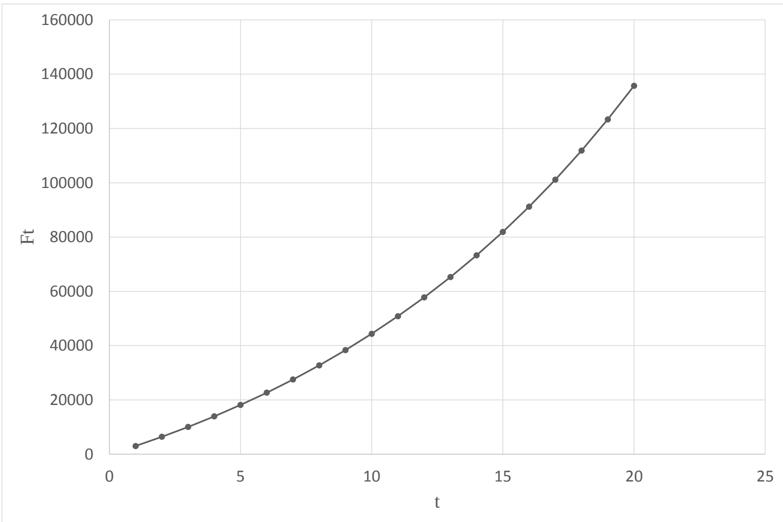

Figure 4: Time (t) vs Policyholder's fund (F_t)

Figure 4 shows that in our example the policyholder's fund value increases with increasing year till the end of the contract.

Next we will calculate the profit vector.

First the unallocated premium can be written as

$$UAP_t = 3000 - AP_t$$

Next the expenses are calculated as
Pre-contract expense E_0

$$E_0 = 0.09 \times 3000 + 120 = 390$$

$$E_1 = 0$$

$$E_t = 0.004 \times 3000 = 12 \quad \forall\, t = 2, 3, ...20$$

Now the Interest income is given as

$$I_t = 0.05(UAP_t - E_t) \quad \forall\, t = 1, 2, ..., 20$$

Next the death benefit is given as

$$DB_t = \frac{105}{100} \times F_t = 1.05F_t \quad \forall\, t = 1, 2, ..., 20$$

Now the expected cost of death benefit

$$EDB_t = p^{0d}_{x+t-1} \cdot (DB_t - F_t) = 0.004 \times (1.05F_t - F_t) = 0.0002\, F_t \quad \forall\, t = 1, 2, ..., 20$$

From equation (17), we get the profit vector for the contract

$$Pr_t = UAP_t - E_t + I_t + MC_t - EDB_t \quad \forall\, t = 1, 2, ..., 20$$

The **Guaranteed Minimum Maturity Benefit** (GMMB) is $3000 \times 20 = 60000$ in this case and $F_{20} = 135707.1$ is much more than 60000.
The profit vector $Pr = (Pr_1, ..., Pr_{20})$ is calculated and given in Appendix B.

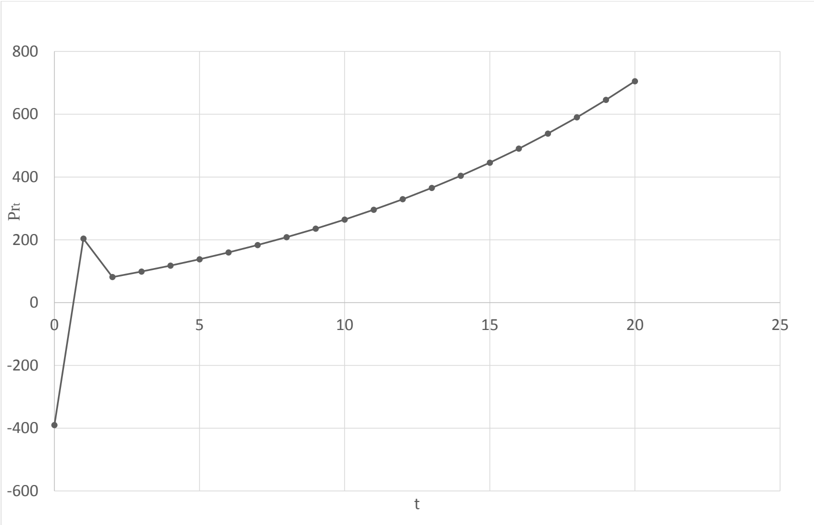

Figure 5: Time (t) vs Profit emerging at time t for the policy in force at time $t-1$ (Pr_t).

Figure 5 shows that at the start of the contract i.e. at time 0, the insurer's profit is theoretically negative (i.e. he/she is having a loss) which cannot be a concern for the insurer because the contract will always be for at least 1 year and the figure says that for positive time (for years 1,2,...,20), the insurer will always have some profit. The figure also says that at the end of year 1 the insurer's profit increases to a positive amount of money, then at the end of year 2 it decreases but with again a positive amount and then keeps on increasing with increasing year till the end of the contract.

3.3 Example of stochastic profit vector of unit linked insurance

We will revisit Example 3.2 which was discussed by considering the interest rate as a random variable.
The annual premium and the allocated premiums will remain the same.

$$P_t = 3000 \quad \forall\, t = 1, 2, ..., 20 \tag{19}$$

$$AP_1 = 2820$$

$$AP_t = 2940 \quad \forall\, t = 2, 3, ..., 20.$$

The management charge is now a random variable.

$$MC_t = 0.005 \times (F_{t-1} + AP_t) \times (1 + R_t - 1)$$
$$= 0.005\, R_t\, (F_{t-1} + AP_t)$$

Hence, the policyholder's fund can be expressed as

$$F_t = (F_{t-1} + AP_t)\,(1 + R_t - 1) - MC_t$$
$$= (F_{t-1} + AP_t)\, R_t - 0.005\, R_t\, (F_{t-1} + AP_t)$$
$$= 0.995\, R_t\, (F_{t-1} + AP_t)$$

F_ts are the year end fund values for the 20 year unit linked insurance contract issued to the person aged 40.
We generate normal random variables $\ln R_1, ..., \ln R_{20}$ (in R) and then find $R_1, ..., R_{20}$ by exponentiating. We will generate the values of $\ln R_t$ by Monte Carlo simulation using the function "rnorm" in R. At first assume that the parameter σ =15%. Then we can estimate the parameter μ from σ by equating $1 + i_t^f$ with ER_t where i_t^f was the fixed interest rate on the policyholder's fund used in deterministic profit testing approach.
Equation (18) gives

$$ER_t = 1 + i_t^f$$
$$\implies \exp\,(\mu + \frac{1}{2}\sigma^2) = 1.08$$
$$\implies \mu + \frac{1}{2}0.15^2 = \ln 1.08$$
$$\implies \mu = \ln 1.08 - \frac{0.15^2}{2}$$
$$= 0.06571104$$

The fund value calculations are given in Appendix C and we illustrate it in Figure

6.

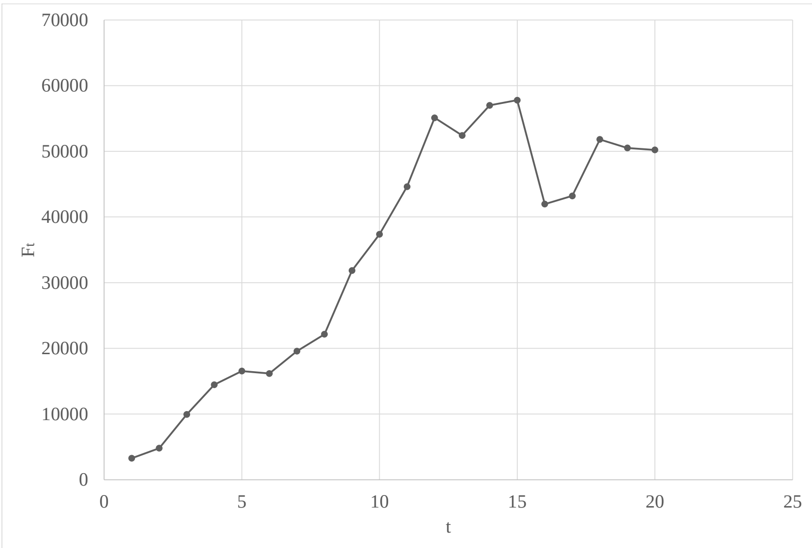

Figure 6: Time (t) vs Policyholder's fund (F_t)

Figure 6 shows that the policyholder's fund value increases with increasing year till year 5, then has a slight decrease at the end of year 6, then starts increasing again till year 12 and after that has random ups and downs till the end of the contract. These random ups and downs are due to the stochastic behaviour of policyholder's fund.

Next we will calculate the profit vector.

The unallocated premium, expenses and the Interest income will be the same as before.

$$UAP_t = 3000 - AP_t$$

$$E_0 = 0.09 \times 3000 + 120 = 390$$

$$E_1 = 0$$

$$E_t = 0.004 \times 3000 = 12 \quad \forall\, t = 2, 3, ...20$$

$$I_t = 0.05\,(UAP_t - E_t) \quad \forall\, t = 1, 2, ..., 20$$

The death benefit and the expected cost of death benefit will be different from the values obtained by deterministic approach because now they are random variables.

$$DB_t = 1.05F_t \quad \forall\, t = 1, 2, ..., 20$$

$$EDB_t = 0.0002F_t \quad \forall\, t = 1, 2, ..., 20$$

From equation (17), we get the profit vector for the contract which is a random variable in this case

$$Pr_t = UAP_t - E_t + I_t + MC_t - EDB_t \quad \forall\, t = 1, 2, ..., 19$$

Since $F_{20} = 50212.28$ is less than GMMB=60000, Pr_{20} is calculated as

$$Pr_{20} = p_{39}^{00}\ \max(GMMB - F_{20}, 0)$$

$$= 0.996 \times \max(60000 - 50212.28, 0)$$

$$= 0.996 \times 9787.72$$

$$= 9748.56912$$

The profit vector $Pr = (Pr_1, ..., Pr_{20})$ is calculated and given in Appendix D. We illustrate the behaviour of the profit vector with Figure 7.

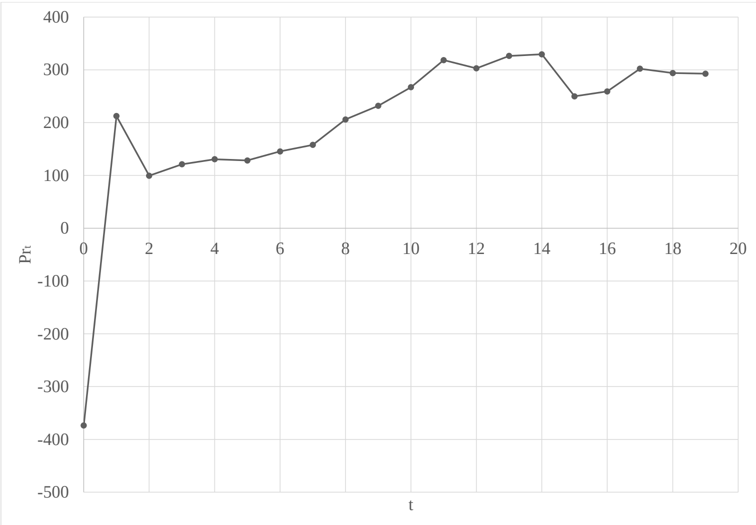

Figure 7: Time (t) vs Profit emerging at time t for the policy in force at time $t-1$ (Pr_t), t=1,2,...,19

Figure 7 shows that at the start of the contract i.e. at time 0, the insurer's profit is theoretically negative (i.e. he/she is having a loss) which cannot be a concern for the insurer because the contract will always be for at least 1 year and the figure says that for positive time (for years 1,2,...,19), the insurer will always have some profit. The figure also says that at the end of year 1 the insurer's profit increases to a positive amount of money, then at the end of year 2 it decreases but with again a positive amount, then starts increasing with a slight decrease in year 5 and after that keeps on increasing till year 11. Between year 11 and year 19, we observe random ups and downs which are due to the stochastic behaviour of the profit vector. However, at the end of the contract i.e at the end of year 20, we observe a steep rise of profit as visible in Figure 8.

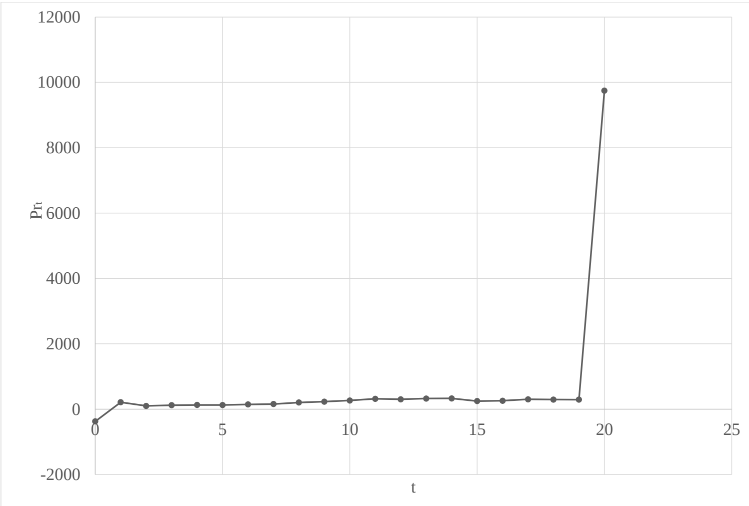

Figure 8: Time (t) vs Profit emerging at time t for the policy in force at time $t-1$ (Pr_t), t=1,2,...,20

Figure 8 shows the steep rise of profit at the the end of year 20 i.e. at the end of the contract. We show this steepness in a different figure which actually covers the entire contract to explain both the steepness behaviour at the end of the contract and the stochastic behaviour in between very clearly which is nicely shown in Figure 7.

Thus we spot on the differences between deterministic and stochastic profit testing approaches by simply looking at the graphs depicting the policyholder's fund and the insurer's profit.

4 Unit linked insurance in India

In this section we will discuss about the insurance industry in India. We will give a brief overview of the general insurance industry in India and then discuss in details about the life insurance industry there.

4.1 Overview of Indian insurance market

Modern insurance industry started in India with the foundation of the Oriental Life Insurance company in Kolkata in 1818. The Life Insurance Companies Act and the Provident Fund Act were passed in 1912 to regulate the insurance business. The oldest existing insurance company in India is the National Insurance Company which was founded in 1906. Nowadays **Insurance Regulatory and Development Authority of India** (IRDAI) is an autonomous body which regulates and develops the insurance industry in India. It was constituted by an act of the Parliament of India - *Insurance Regulatory and Development Authority Act* in 1999 [7], [9].

Currently Indian insurance industry is the 14th insurance market in the world. Its current size is 59 billion euros and is increasing annually at a rate of 17% [13].
The following figure compares the size of the world's 15 biggest insurance markets by premium volume percentage.
Though India is the second most populous country in the world, it ranks quite low in terms of size of insurance market. The main reason is India currently accounts for 1.94% of the world's total insurance premiums despite having 17.9% of the world's population whereas United States being the third most populous country having 4.4% of the world's population has 30.25% (918 billion euros) of the world's total insurance premiums. Following United States in terms of total insurance premium volume are Japan, United Kingdom, China, France and Germany. But with time, more and more people of India are becoming aware of the importance of being insured and as a result the insurance market is growing rapidly. In fact, the current growth rate of Indian insurance market is quite impressive - around 17% and it is projected that Indian insurance market will become the 10th largest in 2025 [13].
Life Insurance industry in India is one of the fastest growing sector in India. There are altogether 53 insurance companies in India of which 24 are life insurance companies and 29 are non-life insurance companies. Among these 53 companies, there are 6 public Non-life insurance companies and **1 public Life insurance**

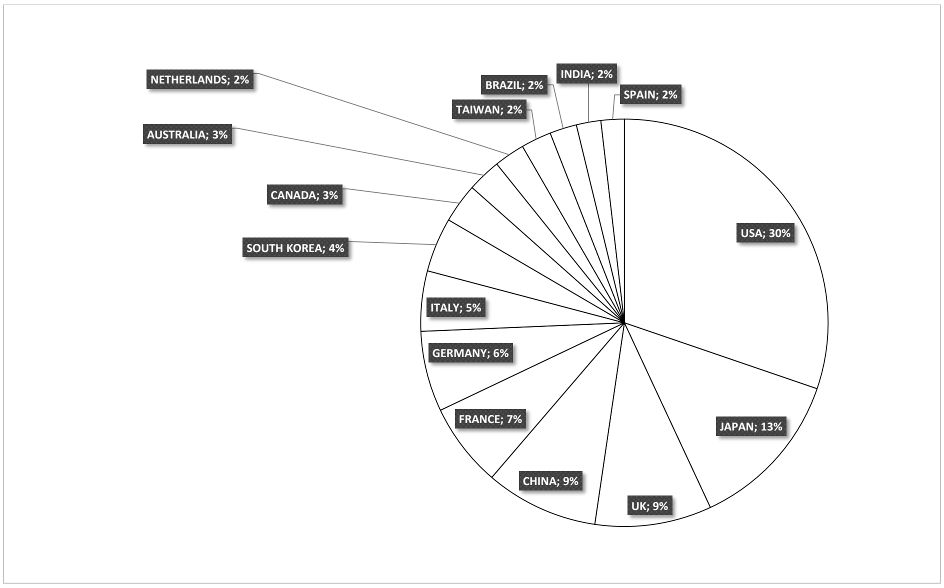

Figure 9: Primary insurance markets by premium volume percentage

company. A public company generally means that the ownership is distributed among the general public. In India, a public insurance company is owned by the government and it should also be noted that India is a democratic country. Democracy literally means that the government is run by the whole population. So, the term *public company* is justified here.
The table in Appendix E gives total funds of life insurers as on 31.03.2015 [8]. **Life Insurance Corporation of India** (LIC), which is the only public sector life insurance company is currently the largest life insurance company of India. It was established in 1956 after the Life Insurance Corporation Act passed by the Parliament of India. Among the private life insurance companies in India, **ICICI Prudential** is the largest [8].

In India there are mainly 8 types of insurance policies - Endowment plans, Pure Endowment plans, Term Insurance plans, Whole life Insurance plans, Unit Linked Insurance Product (ULIPs), Money back plans, Child Insurance plans and Pension

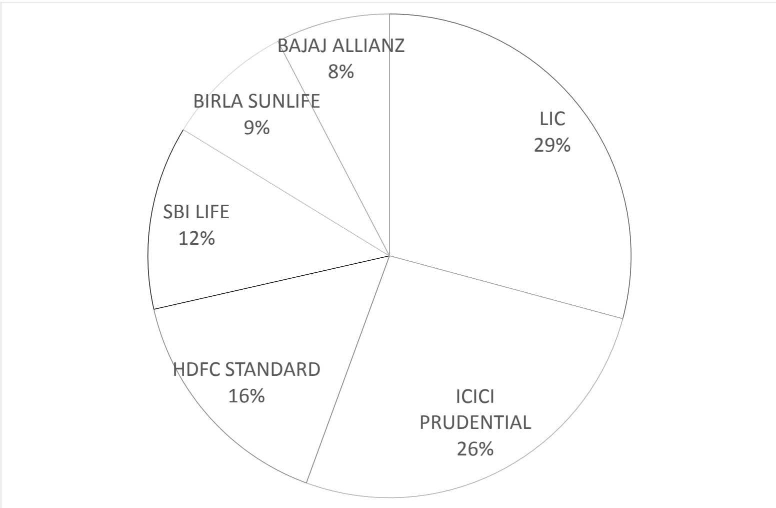

Figure 10: Percentages of total ULIP funds of Life Insurers as on 31.03.2015.

plans. In this paper we focus on unit linked insurance products offered in India.

4.2 Comparison of unit linked products in India

We start we an comparing overview about unit linked insurance market in India Unit-linked insurance plans (ULIP) are quite popular in India because they give both insurance and investment opportunities. It was first launched in India by **Unit Trust of India** in the year 1971. As of 31.03.2015 in terms of total ULIP funds in billion euros , **LIC** (10.93) is the largest followed by **ICICI Prudential** (9.89), **HDFC Standard** (5.94), **SBI Life** (4.6), **Birla Sunlife** (3.23) and **Bajaj Allianz** (2.86) [8], Appendix F. This is nicely shown in Figure 10.

In India there are many private companies which compare different life insurance plans offered by different life insurance companies. Some of the well known companies are **Policybazaar**, **BankBazaar** and **MyInsuranceClub**.

We will discuss how they compare and offer different unit linked insurance contracts. From now on we use Indian rupee currency (1 euro ≈ 75.61 Indian rupees) to discuss the ULIP products offered in India.

4.2.1 PolicyBazaar

For comparing different insurance contracts, PolicyBazaar takes into account policyholder's age, yearly investment, term of contract and several features namely loyalty additions, free switches, automatic rebalancing, fund growth, growth rate and total Cost. Loyalty addition is the kind of bonus/rewards given by the plans of ICICI Prudential. Free switches is the facility where the policyholder can switch any time between different fund options. Automatic rebalancing is same as free switches, just that it is an inbuilt feature of a plan offered by SBI Life. The total cost includes all deductions made by the insurer on the policyholder's investment as well as returns.
For example if the policyholder's age is 25 years, yearly investment is 1 lakh Indian rupees (1 lakh = 10^5) with a pay term of 10 years and the money will be invested for 10 years, then **SBI Life**'s *eWealth Insurance* plan provides growth of 1 lakh into 14.4 lakh with fund growth rate of 8 %, total cost 1.51 % with automatic rebalancing but no loyalty additions and free switches.
PolicyBazaar use both the terms unit linked insurance and equity linked insurance simultaneously. Both the terms mean the same [16].

4.2.2 BankBazaar

BankBazaar takes into account minimum and maximum entry age of the policyholder, minimum premium, premium allocation charge, policy administration charge, number of funds, number of free switches in a year when comparing the insurance contracts.
For example **SBI Life**'s *eWealth Insurance* plan provides minimum entry age - 18 years, maximum entry age - 50 years, minimum premium - Yearly: 10000 Indian

rupees per annum ; Monthly: 1000 Indian rupees per month, no premium allocation charge, 45 Indian rupees policy administration charge, number of funds - 3 and no free switches in a year [3].

4.2.3 MyInsuranceClub

MyInsuranceClub takes into account similar things like that of Policybazaar and BankBazaar.
For example **Bajaj Allianz**'s *Future Gain* plan provides policyholder's age - 25 years, coverage amount - 1 lakh Indian rupees, minimum premium - 25000 Indian rupees , maximum premium allocation, choice of two investment portfolio strategies, choice of seven funds, option to take maturity benefit in instalments (settlement option), death benefit - higher of sum assured or regular premium fund value, maturity benefit - fund value, tax benefit on premiums and tax benefit on death benefit [14].

SUMMARY

In this paper we have studied a type of modern life insurance, which is the "unit linked insurance". We started with introducing the traditional life insurance models and went on to proceed with unit linked insurance by combining investment with the usual insurance cover. In this paper we have explored both the deterministic and stochastic profit testing approaches to understand such type of contracts. We introduced the main components of policyholder's fund and insurer's fund. Using these components, we expressed the insurer's profit vector which was one of our aims to derive. First we have expressed the cash flows of such contracts in a simpler scenario by not considering any uncertainty in investment. Then we have expressed them in a more general set-up by taking investment uncertainty into consideration. In order to understand the theory well enough, we discussed a real life example of a unit linked insurance contract and calculated the profit vector using both the approaches. We also discussed how the policyholder's fund values and insurer's profit values depend on time.

In addition we gave a brief overview of the Indian insurance market and more specifically of the unit linked insurance products by discussing different parameters which are used in comparing them.

Appendix A

Projection of policyholder's fund in Example 3.2 using deterministic profit testing

t	AP_t	F_{t-1}	MC_t	F_t
1	2820	0	15.228	3030.372
2	2940	3030.372	32.24001	6415.762
3	2940	6415.762	50.52111	10053.7
4	2940	10053.7	70.16599	13963.03
5	2940	13963.03	91.27637	18164
6	2940	18164	113.9616	22678.36
7	2940	22678.36	138.3391	27529.49
8	2940	27529.49	164.5352	32742.51
9	2940	32742.51	192.6855	38344.42
10	2940	38344.42	222.9359	44364.24
11	2940	44364.24	255.4429	50833.14
12	2940	50833.14	290.3749	57784.61
13	2940	57784.61	327.9129	65254.67
14	2940	65254.67	368.2512	73281.99
15	2940	73281.99	411.5988	81908.15
16	2940	81908.15	458.18	91177.83
17	2940	91177.83	508.2363	101139
18	2940	101139	562.0267	111843.3
19	2940	111843.3	619.8299	123346.1
20	2940	123346.1	681.9452	135707.1

Appendix B

Calculation of the profit vector in Example 3.2 using deterministic profit testing

t	UAP_t	E_t	I_t	MC_t	EDB_t	Pr_t
0	0	390	0	0	0	-390
1	180	0	9	15.228	0.606074	203.6219
2	60	12	2.4	32.24001	1.283152	81.35686
3	60	12	2.4	50.52111	2.01074	98.91037
4	60	12	2.4	70.16599	2.792606	117.7734
5	60	12	2.4	91.27637	3.6328	138.0436
6	60	12	2.4	113.9616	4.535672	159.8259
7	60	12	2.4	138.3391	5.505898	183.2332
8	60	12	2.4	164.5352	6.548502	208.3867
9	60	12	2.4	192.6855	7.668884	235.4166
10	60	12	2.4	222.9359	8.872848	264.4631
11	60	12	2.4	255.4429	10.16663	295.6763
12	60	12	2.4	290.3749	11.55692	329.218
13	60	12	2.4	327.9129	13.05093	365.262
14	60	12	2.4	368.2512	14.6564	403.9948
15	60	12	2.4	411.5988	16.38163	445.6172
16	60	12	2.4	458.18	18.23557	490.3444
17	60	12	2.4	508.2363	20.2278	538.4085
18	60	12	2.4	562.0267	22.36866	590.058
19	60	12	2.4	619.8299	24.66922	645.5607
20	60	12	2.4	681.9452	27.14142	705.2038

Appendix C

Projection of policyholder's fund in Example 3.3 using stochastic profit testing

t	AP_t	$\ln R_t$	R_t	F_{t-1}	MC_t	F_t
1	2820	0.15224	1.164437968	0	16.41858	3267.296
2	2940	-0.252	0.77723374	3267.296495	24.1226	4800.398
3	2940	0.254943	1.290388596	4800.39767	49.9406	9938.18
4	2940	0.121288	1.128950184	9938.180275	72.69412	14466.13
5	2940	-0.04555	0.955467203	14466.12987	83.15493	16547.83
6	2940	-0.18138	0.834119017	16547.83129	81.27585	16173.89
7	2940	0.02852	1.028930383	16173.89484	98.33434	19568.53
8	2940	-0.0113	0.988754331	19568.53281	111.277	22144.13
9	2940	0.24349	1.275691737	22144.13225	159.9981	31839.62
10	2940	0.07691	1.079944683	31839.62212	187.8003	37372.27
11	2940	0.106418	1.112286526	37372.26764	224.194	44614.6
12	2940	0.15226	1.164462946	44614.59817	276.8778	55098.69
13	2940	-0.0969	0.907609617	55098.68963	263.3824	52413.09
14	2940	0.034232	1.034824958	52413.09051	286.4038	56994.36
15	2940	-0.0315	0.968990317	56994.35578	290.3791	57785.43
16	2940	-0.3644	0.694584037	57785.43135	210.8946	41968.02
17	2940	-0.0336	0.96695501	41968.0207	217.1202	43206.92
18	2940	0.12072	1.128313286	43206.91542	260.3409	51807.84
19	2940	-0.0756	0.927229195	51807.83687	253.819	50509.97
20	2940	-0.0575	0.944146476	50509.97376	252.323	50212.28

Appendix D

Calculation of the profit vector in Example 3.3 using stochastic profit testing

t	UAP_t	E_t	I_t	MC_t	EDB_t	Pr_t
0	0	390	0	16.41858	0	-373.581
1	180	0	9	24.1226	0.653459	212.4691
2	60	12	2.4	49.9406	0.96008	99.38052
3	60	12	2.4	72.69412	1.987636	121.1065
4	60	12	2.4	83.15493	2.893226	130.6617
5	60	12	2.4	81.27585	3.309566	128.3663
6	60	12	2.4	98.33434	3.234779	145.4996
7	60	12	2.4	111.277	3.913707	157.7633
8	60	12	2.4	159.9981	4.428826	205.9693
9	60	12	2.4	187.8003	6.367924	231.8324
10	60	12	2.4	224.194	7.474454	267.1195
11	60	12	2.4	276.8778	8.92292	318.3549
12	60	12	2.4	263.3824	11.01974	302.7626
13	60	12	2.4	286.4038	10.48262	326.3212
14	60	12	2.4	290.3791	11.39887	329.3802
15	60	12	2.4	210.8946	11.55709	249.7375
16	60	12	2.4	217.1202	8.393604	259.1266
17	60	12	2.4	260.3409	8.641383	302.0995
18	60	12	2.4	253.819	10.36157	293.8574
19	60	12	2.4	252.323	10.10199	292.621
20	60	12	2.4	681.9452	10.04246	9748.569

Appendix E

Total funds of life insurers as on 31.03.2015

(1 crore = 10^7, 1 billion = 10^9, 1 EUR ≈ 75.61 INR)

Name of the company	Total funds (in crore INR)	Total funds (in billion EUR)	Percentage of total funds
LIC	1786312.55	236.2534784	79.479%
ICICI PRUDENTIAL	97875.98	12.94484592	4.355%
SBI LIFE	70773.75	9.360368999	3.149%
HDFC STANDARD	67002.4	8.861579156	2.981%
BAJAJ ALLIANZ	43157.67	5.707931491	1.920%
MAX LIFE	30960.61	4.094777146	1.378%
BIRLA SUNLIFE	30045.03	3.973684698	1.337%
TATA AIA	19522.98	2.582063219	0.869%
RELIANCE	15827.76	2.093342151	0.704%
KOTAK MAHINDRA	15050.92	1.990599127	0.670%
PNB METLIFE	12736.2	1.684459728	0.567%
CANARA HSBC	9783.39	1.293928052	0.435%
AVIVA	9122.71	1.206548076	0.406%
EXIDE LIFE	8609.76	1.13870652	0.383%
INDIAFIRST	7997.93	1.05778733	0.356%
STAR UNION DAI-ICHI	5384.79	0.712179606	0.240%
IDBI FEDERAL	4260.24	0.563449279	0.190%
BHARTI AXA	2962.23	0.391777543	0.132%
FUTURE GENERALI	2653.94	0.351003835	0.118%
SHRIRAM LIFE	2350.03	0.310809417	0.105%
AEGON RELIGARE	1712.73	0.226521624	0.076%
DLF PRAMERICA	1542.8	0.204047084	0.069%
SAHARA	1125.65	0.14887581	0.050%
EDELWEISS TOKIO	750.13	0.099210422	0.033%

Appendix F

Assets under management of life insurers

(1 crore = 10^7, ₹ is the symbol for Indian Rupee (INR) and 1 EUR ≈ 75.61 INR)

Contd… **STATEMENT 8**

ASSETS UNDER MANAGEMENT OF LIFE INSURERS

(₹ Crore)

Insurer	UNIT LINKED FUND						TOTAL (ALL FUNDS)	
	Approved Investments		Other Investments		Total (ULIP Funds)			
	31.3.2015	31.3.2014	31.3.2015	31.3.2014	31.3.2015	31.3.2014	31.3.2015	31.3.2014
AEGON RELIGARE	1129.02	945.76	46.28	28.69	1175.31	974.45	1712.73	1335.46
AVIVA	5536.92	5219.73	80.94	114.91	5617.85	5334.64	9122.71	8163.39
BAJAJ ALLIANZ	20429.20	19963.29	1215.84	1324.31	21645.04	21287.60	43157.67	38612.83
BHARTI AXA	1741.65	1729.92	167.20	51.90	1908.85	1781.82	2962.23	2417.88
BIRLA SUNLIFE	23348.63	19330.88	1046.77	1220.00	24395.40	20550.87	30045.03	24676.59
CANARA HSBC	7239.68	6205.00	380.33	139.47	7620.01	6344.47	9783.39	8360.51
DLF PRAMERICA	249.54	229.93	1.91	5.46	251.44	235.39	1542.80	725.62
EDELWEISS TOKIO	54.69	26.05	4.92	1.21	59.61	27.26	750.13	650.64
EXIDE LIFE	2310.07	2351.29	158.71	149.76	2468.78	2501.05	8609.76	7707.43
FUTURE GENERALI	865.71	906.45	12.84	25.41	878.55	931.86	2653.94	2362.80
HDFC STANDARD	43332.60	33212.03	1587.72	701.47	44920.32	33913.51	67002.40	50253.39
ICICI PRUDENTIAL	73437.94	57861.06	1339.59	2449.37	74777.54	60310.43	97875.98	79399.46
IDBI FEDERAL	1743.13	1652.50	13.42	6.08	1756.55	1658.58	4260.24	3388.32
INDIAFIRST	3490.13	2765.26	105.36	68.13	3595.49	2833.39	7997.93	6160.30
KOTAK MAHINDRA	9048.67	7918.87	631.44	120.15	9680.11	8039.02	15050.92	12003.77
MAX LIFE	12742.31	10883.00	657.26	447.00	13399.57	11330.00	30960.61	24633.00
PNB METLIFE	7273.07	6370.63	30.54	161.67	7303.62	6532.30	12736.20	10929.21
RELIANCE	8293.98	9943.60	493.77	340.45	8787.75	10284.05	15827.76	18284.25
SAHARA	267.99	329.16	3.54	5.61	271.53	334.77	1125.65	1176.82
SBI LIFE	33994.08	28375.79	815.99	221.49	34810.07	28597.28	70773.75	58195.20
SHRIRAM LIFE	1027.09	956.22	36.59	66.23	1063.68	1022.45	2350.03	1936.11
STAR UNION DAI-ICHI	3389.13	2758.12	33.49	44.38	3422.62	2802.49	5384.79	4391.25
TATA AIA	10021.27	9375.43	238.42	179.46	10259.69	9554.90	19522.98	17405.01
PRIVATE TOTAL	**270966.49**	**229309.96**	**9102.88**	**7872.61**	**280069.37**	**237182.57**	**461209.63**	**383169.24**
LIC	81404.95	93146.03	1266.15	1332.57	82671.10	94478.59	1786312.55	1574296.34
INDUSTRY TOTAL	**352371.44**	**322455.98**	**10369.03**	**9205.18**	**362740.47**	**331661.16**	**2247522.18**	**1957465.57**

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