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# Design of educational technology platform for acquiring mathematics and physics concepts by basic and middle school students

Master thesis

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## ABSTRACT

Design of educational technology platform for acquiring mathematics and physics concepts by basic and middle school students

This master thesis describes the endeavour undertaken to create an innovative Educational Technology platform for teaching and learning mathematics and physics in basic and middle school. The platform consists of tangible manipulatives and simulation software. Methodological and didactical approach to design of this platform is described, whereas special attention is paid to student's misconceptions analysis. Design and implementation of the platform are described, showing main development stages. Early results from the school pilot are given.

Keywords: educational technology, mathematics education, physics education, concept formation, misconceptions

## KOKKUVÕTE

Haridustehnoloogia platvorm matemaatika ja füüsika mõistete omandamiseks põhi ja gümnaasiumiastme õpilastele

Käesolev magistritöö kirjeldab haridustehnoloogia platvormi loomist. Platvorm on mõeldud matemaatika- ja füüsika õpetamiseks ja õppimiseks põhi- ja keskkooli astmetes ning koosneb füüsilisest õppevahendist ja simulatsioonitarkvarast. Kirjeldatud on metodoloogiline ja didaktiline alus, kusjuures eriline tähelepanu on pööratud õpilaste väärarusaamade analüüsile. Põhilised projekteerimise ja ehituse etapid on kirjeldatud. Varajased tulemused mis on saadud koolides toimunud pilootprojekti raames (7. klassi matemaatika) on samuti toodud.

Võtmesõnad: haridustehnoloogia, matemaatikaharidus, füüsikaharidus, mõistete kujunemine, väärarusaamad

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# 1 Research problem and target of thesis

## 1.1 Introduction to research problem

The quality of science education depends on the context, in which it happens. Importance of meaningful and relevant real-life context in science education is highlighted at national (HTM, TLÜ, TÜ, 2017), pan-European (European Commission, 2015) and worldwide highest political level (Fensham, 2008).

Among various ways, educational technology platforms (e.g. educational robotics, virtual reality environments, etc.) try to bring this context into the classroom. However, analysis of state-of-the-art platforms reveal their significant drawbacks. While students are generally enthusiastic about usage of technology in the classroom and of certain game factor that appear in technology-based learning (Drijvers, 2015), such classes generally facilitate obtaining only content knowledge, while conceptual learning and problem-solving abilities of students participating in such classes remain below optimal (Williams et al., 2007), (Clark-Wilson et al., 2014), (Drijvers, 2015), (Savard & Highfield, 2015), (Wang et al., 2017).

Research shows that didactic and methodological support for fundamental disciplines (e.g. mathematics, physics) provided by the state-of-the-art educational technology platforms is not sufficient. This weakness is recognized by the research community, and there is growing recognition that science didactics must define the design of educational technology – not vice versa (Altin & Pedaste, 2013), (Drijvers, 2015).

In addition to the above, usage of technology in the classes is often a challenging task for teachers due to non-negligible administrative and organizational overhead (Leoste & Heidmets, 2019).

## 1.2 State-of-the-art educational technology overview

### 1.2.1 <u>Introduction</u>

There are a plenty of educational technology development attempts around the globe: from both research and business domains. The author' intention is to create an Educational Technology (EdTech) platform for usage in real educational settings, and therefore following analysis concentrates on the solutions, covering a significant amount of curriculum topics and school grades. This is an important aspect which shall be explained further.

Analysis of educational research literature shows a considerable number of attempts to create learning materials mediated via educational technology for contextualizing specific curriculum topics. For example Savard et al., (2013) describes creation of gambling simulator for contextualization of teaching probability, Hwang et al., (2019) proposed an app for teaching fractions in an authentic environment. Without denying the importance and usefulness of these attempts whatsoever, author mostly limits following analysis with EdTech **platforms** – continuously maintained (digital) platforms that cover substantial amount of curriculum disciplines, topics – as well as school grades. It should be also mentioned, that authenticity of the contextualization (Vos, 2018) delivered by platforms is taken into account for the analysis. This aspect is further addressed in the following sub-chapters, which describes four main groups of platforms: apps, science kits, virtual environments and robotics.

### 1.2.2 Math/physics learning apps

These solutions are usually available for desktop or mobile devices and mainly enable access to digitized curricula content.

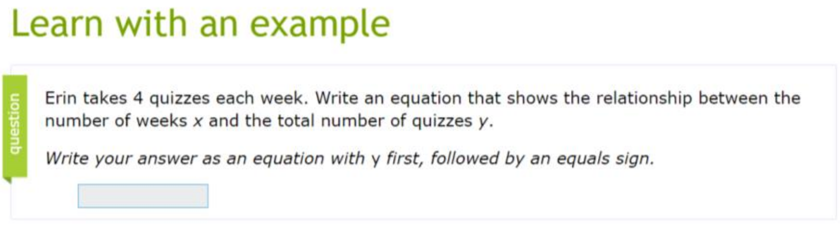



**Figure 1.** Typical example of digitized math curricula content via mobile app (screenshot from IXL webpage))

Notable commercial vendors include IXL (*IXL Webpage*, s.a.) and Matific (*Matific Webpage*, s.a.). Price range of these solutions lies in 10-30€ per student per month. Also the non-profit sector is present for this product segment. PHET interactive simulations (*PHET Webpage*, s.a.) have to be pointed out as a widely recognized environment for both mathematics and physics. The solution is free for users (desktop version) and provides very solid simulation environment for various scientific concepts.

#### *1.2.2.1 Advantages*

Many studies conducted over the last decade report on the positive influence of apps on students' achievement and engagement (Chang et al., 2015), (Fabian et al., 2018), (Hwang et al., 2019). Ubiquitous spread of mobile devices along with availability of desktop computers at schools make usage of apps relatively easy. Many apps provide access to usual math/physics exercises in digitized form – which makes its use quite habitual for teachers. Apps often provide access to learning analytics, which simplify teacher work in determining and prioritizing of topics.

#### *1.2.2.2 Disadvantages*

Claims given by vendors of commercial platforms are often misleading (*Online Programs...* s.a.). Some objective analyses (e.g. (Ben-Haim et al., 2019)) reported on negative effects of too detailed interface of learning app as student attention distractor. In the author's view, the biggest disadvantage of many apps is their straightforward approach of simply digitizing textbook content – an approach that was criticized 5 decades ago by Seymour Papert (Papert, 1972). To the best of author knowledge, there is no available app that would bring any significant amount of authentic context to mathematics and physics. Also, the majority of teachers interviewed by the author confirmed that they want to follow the mental reasoning of students via pen-and-paper exercise solving, which is not possible with screen-only apps.

### 1.2.3 Science kits

Typical representative of these products is IOLab (*IOLab Webpage*, s.a.) – miniature (30 mm x 75 mm x 130 mm) all-in-one physical measurement suite (force, velocity, sound, pressure etc. and corresponding data analysis) – see Figure 2. In addition to IOLab, notable suppliers include Quality Science Labs (*Quality Science Labs Webpage*, s.a.) and PocketLab (*PocketLab Webpage*, s.a.). Their price range is 150-300€.


**Figure 2.** IOLab miniature science kit for physics (screenshot from IOLab webpage)

#### *1.2.3.1 Advantages*

These products propose excellent ways for physical measurements, have good precision and enable a wide variety of experimentations. Convenience of use and pedagogical value were observed by Bodegom et al. (2019) and Leblond & Hicks (2021).

#### *1.2.3.2 Disadvantages*

To the best of my knowledge, these kits are available only for earth sciences in targeted students' age (grades from 6th onwards), no support is provided for mathematics. Moreover, while proposing very good phenomena observation possibilities, such kits have rather weak connection to "beyond-school" context, and don't necessarily result in positive change of students' attitudes (Bodegom et al., 2019). My personal experience shows also certain teacher overhead (device troubleshooting, charging batteries, etc.).

#### 1.2.4 Virtual science labs and VR/AR (Virtual/Augmented Reality) environments

These environments allow students to perform virtual experiments, for example for earth sciences (physics, chemistry, biology – see Figure 3):

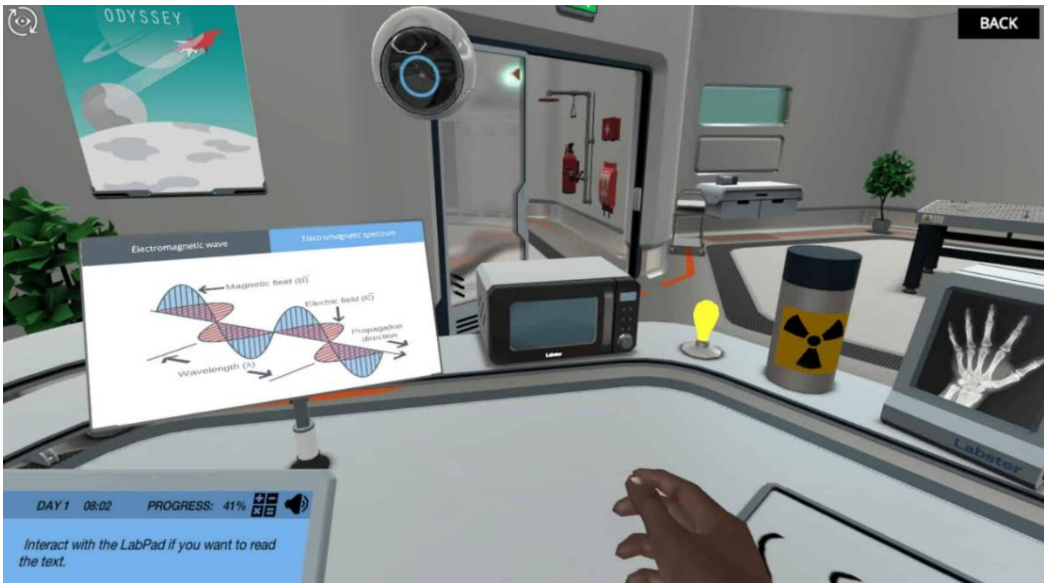



**Figure 3.** Virtual experimentation environment from Labster (screenshot from Labster webpage))

Notable suppliers of these solutions include CLASSVR (*ClassVR Webpage*, s.a.), Labster (*Labster Webpage*, s.a.), VRLabAcademy (*VRLab Academy Webpage*, s.a.) and others. Price range varies from ca. 20 to 100€ per student per year. Virtual Reality glasses costs shall be added for virtual reality solutions (from 200 to 500€ per set).

##### *1.2.4.1 Advantages*

While these labs and environments are designed mostly for earth sciences, their unbeatable advantage is the ability to support a very wide range of experiments, including virtual experimentation with radioactive materials. High computational capabilities of these solutions can be used for visualization of complex concepts, e.g. from geometry (*VR MATH*, s.a.).

##### *1.2.4.2 Disadvantages*

While providing visualization and interaction opportunities, support provided by these solutions to mathematics is still moderate, and often goes the path of simple digitizing of the textbook content. I wasn't able to find any substantial and well maintained platform for algebra/calculus, however there are several design attempts: e.g. Kaufmann (2011), Shipulina et al. (2013), Civelek et al. (2014). It shall be noted that advanced visualization capabilities might not be advantageous for learning (Erkek & Işıksal-Bostan, 2015). The design of VR environments is often technology-push, therefore learning objectives may remain unachievable (Zhang et al., 2019). It shall be also noted, that technical complexity of the solution and authoring of the content still limits adaptation of these solutions (Kaufmann, 2011). Finally, there is evidence, that fundamental experimentation skills (e.g. using real equipment) are not obtained during virtual experiments. For example, students learning chemistry with virtual laboratories may lack basic skills, like holding a pipette (*COVID-19 Forces Science Labs Online and in Homes*, s.a.). Also, high cost is an essential barrier.

### 1.2.5 Education robotics

Education robotics (Figure 4) are usually designed for teaching programming basics to students. Usually having two or more wheels, these robots are reacting to programmed movement commands and may have various sensors. However, there is a lot of attempts taken by schools and academic researchers to adapt this technology for curriculum mathematics and physics. Prominent vendors of educational robotics include Lego (*LEGO Mindstorms Webpage*, s.a.), Edison (*Meetedison Webpage*, s.a.), Engino (*Engino Robotics Webpage*, s.a.) and others. Their price range lies in the range of 50-500€.

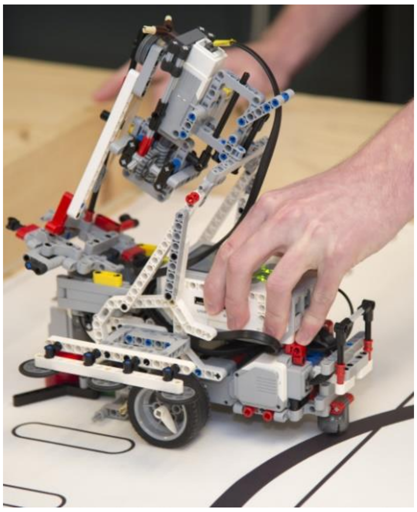


**Figure 4.** Lego Mindstorms educational robot

#### *1.2.5.1 Advantages*

From my experience with extracurricular lessons – as well as from relevant literature (Drijvers, 2015) it is known that robotics are usually well met by students. Educational robotics can be used for giving math challenge an authentic context (e.g. space exploration mission (Tuchscherer, 2010). Nowadays simpler robots are commercially available at already ca. 50€, which makes this way of science contextualization relatively affordable for schools.

#### *1.2.5.2 Disadvantages*

As mentioned above, educational robotics are designed primarily for teaching programming, and this constitutes the main barrier for adoption. This barrier often becomes insurmountable, since turning a digital tool into a didactical tool wholly depends on the robotic platform used in the classroom (Alimisis, 2012). Peculiarity of the situation, is that it is often industrial hardware that defines (actually restricts!) teaching methodology used by teacher. For example, the engine for the programming environment of Lego Mindstorms is based on

LabVIEW, which is a proprietary software for industrial automation developed by National Instruments - US-based multinational company (*LabVIEW – LEGO Engineering*, s.a.). Pedagogical and didactic value of such an approach is questionable. In addition, it shall be noted that real-life context mediated by robotics is limited to robotics itself – i.e. the number of applications from other domains is substantially limited.

## 1.3 Design research questions

The analysis reveals a lack of EdTech platforms for teaching and learning of mathematics and physics in authentic real-life context, which would combine several characteristics essential for education. These characteristics include cognitive, methodological and practical aspects and form design research questions of this master thesis. These questions are as follows:

1. Which fundamental aspects of the cognitive psychology of learning and teaching can be applied to the design of the EdTech platform?
2. What is the proper combination of curricula topics and real-life context for their integration into the EdTech platform? Particularly:
    - Could math/physics curricula topics be in principle integrated into anticipated EdTech Platform?
    - Is it feasible to integrate a critical mass of curricula topics into the platform?
3. What is the most suitable design configuration of EdTech platform, so that it is:
    - usable in the classroom environment (e.g. has proper size, simple in use, etc.),
    - produce minimum disruption of the existing teaching practices (e.g. well aligned with traditional exercises, doesn’t need excess time for performing experimentation/demonstration, etc.),
    - affordable for usage in basic and middle school (e.g. in terms of costs)?

## 1.4 The purpose of the master thesis

Main target of this thesis was to create EdTech Platform, that would:

a) bring relevant real-life context for basic and secondary school science curriculum (particularly mathematics and physics);
b) be defined (as much as possible) by methodological and didactical aspects of science teaching;
c) be aligned with formal curricula, and
d) provide minimum administrative and organizational overhead for teachers.

Author has initially targeted only design and development of the platform; evaluation of results was planned via expert feedback. However, as the reader will see later, the latest platform design attempt underwent testing in real classroom settings as well.

# 2 Main methodological aspects of targeted EdTech platform*

* This section is based on author' unpublished work conducted under subject „SVHI.01.029 Adult Learning and Teaching " – which has defined main methodological pillars, further developed over last year.

## 2.1 Introduction

First research question assumes analysis of various aspects of cognitive science. Main cognitive aspects relevant for EdTech Platform are related to the scientific concept formation, whereas I approach the latter via analysis of relevant students' misconceptions.

Misconception is faulty interpretation of a scientific concept (i.e. when student' interpretation of a concept differs from the one widely accepted by experts). Such formulation seem to be well accepted by number of researchers, including both foreign– e.g. Mayer (1987), N. Voudoukis & Pagiatakis (2015), as well as domestic authors – e.g. Kikas & Saul (2003).

## 2.2 Role of everyday experience in concept formation

Research identifies students' previous experience as a source of misconceptions. The wording given to this previous experience vary (e.g. „common sense beliefs" (Halloun & Hestenes, 1985), „intuitive conceptions" (Clement et al., 1981), „everyday concepts" (Reif, 2008), etc.), but they are common in essence:

> "Students do not come to the classroom as "blank slates" (…). Instead, they come with theories constructed from their everyday experiences. They have actively constructed these theories, an activity crucial to all successful learning. Some of the theories that students use to make sense of the world are, however, incomplete half-truths (…). They are "misconceptions." " (Mestre, 1989, p. 2).

Also absence of previous experience can play significant role in concept formation. N. Voudoukis & Pagiatakis (2015) describe misconceptions possessed by telecommunication students, caused by inability of students to rely on everyday experience:

> "Telecommunications is a subject that requires students to integrate advanced mathematical concepts into their understanding of physical systems with significant portions covering topics that are dealt with as abstract mathematical constructs. Furthermore, much of the material is not known from daily experience at least for students. This unfamiliarity is in contrast to some courses in the physical sciences, such as introductory Newtonian mechanics, in which some understanding of the concepts can be demonstrated without reference to the corresponding mathematical formalism." (p. 193)

This last aspect, in my opinion, even more highlights the importance of everyday experience in concept formation. Next chapter will explain it in more detail.

## 2.3 Supporting (scaffolding) everyday experience of a student

There is a solid amount of research that draws recommendations on how to address students' misconceptions. Bardini et al. (2014) recommends exposure of certain concept to many examples that shall be followed by formalization of concept. Inductive approach has been proposed by Mestre (1989), that instructs teacher to ask students a set of qualitative, quantitative and conceptual questions, promoting discussion and collective resolving of misconceptions. Similar approach is recommended by Reif (2008) and by Rittle-Johnson et al. (2017), who propose several procedures of concept interpretation. These recommendations generally tend to propose improved instructional practices to fix these misconceptions.

Without doubting the value of these recommendations, I'd like to highlight the fact that these techniques pay little (if any) to in-depth analysis of misconceptions (or concept) genesis (apart from declaring its origin in everyday experience). No attention is paid to supporting students' everyday experience. Below I explain further this statement.

Constructivist approach to learning stresses the importance of connecting newly taught knowledge to the previous experience of the student (Krull, 2000). This approach generally calls to induce a cognitive conflict in order to immediately propose appropriate (scientific) explanation to the observed phenomena. I fully agree with this approach, however argue that these recommendations rarely specify **how** to connect any significant number of scientific concepts to previous (everyday) experience of the students – especially in II and III school stages. It is not an easy task - to find meaningful everyday situations, which could be connected to, say, teaching algebra or calculus.

I propose to reconsider what we used to call "everyday experience" (which seem to be mostly perceived as something happening outside of school context (Vos, 2018)). I propose to **create** everyday experiences for students with the help of EdTech Platform, so that these experiences can be further used for teaching science. Perhaps I could characterize this approach as broadening the concept of scaffolding introduced by J. Bruner - but instead of supporting only student learning, I propose also scaffolding of everyday experience.

Before I describe the vision of this EdTech Platform, I first must further explain some theoretical foundations, upon which this approach relies. These foundations are based on works of Russian psychologist Lev Vygotsky, who proposed a relatively clear structure of concept genesis and explicitly outlined the role of everyday experience in the latter.

## 2.4 Vygotsky vision on mutual development of scientific and everyday concepts

Vygotsky proposed to analyse “the concept” as a unit that is built on two main pillars: scientific concept and everyday concept. Such a distinction was made before Vygotsky, however he assumed that scientific and everyday concepts are not “conflicting” neither displacing each other (as argued by e.g. J. Piaget) – but developing in a joint way.

Scientific concept can be characterised by its inclusion to a strict hierarchical system (among other scientific concepts), and can be expressed via other scientific concepts in a variety of ways (“ratio of concept generalization”). Vygotsky exemplifies this via explaining the concept of a number: for a child, “One” (1) is a number that can be expressed only as a natural number, which is only related to a physical object. However, as the child is educated, the arithmetic concept of number evolves to algebraic and the child is becoming able to abstract from physical object. From there, “one” can be expressed as the difference between two adjacent natural numbers, as a ratio of any number to itself, etc. (Vygotsky, 1934).

The subjective development of scientific concept can be characterized by the cognitive ability of the student to generalize concepts and create a kind of inner “mental conceptual grid”. In this grid, each new concept learned by the student is not simply memorized – but is finding its place among other concepts (Yasjukova, 2005). Fundamental aspect of this theory, is that in order to be able to interiorise certain scientific concept, corresponding everyday concept of student shall be mature enough. Vygotsky highlights, that development of both scientific and everyday concepts is inseparable: everyday concept is paving the way to development of scientific concept, and vice versa:

> “Development of scientific concepts begins in the arbitrary domain, in the domain of awareness and continues growing downwards to the domain of personal experience and specificity. The development of everyday concepts begins in the empiric and concrete domain, and grows upwards – to the higher functions of arbitrary and awareness (…). Scientific concepts grow downwards through everyday concepts. Everyday concepts grow upwards through scientific concepts.” (Vygotsky, 1934, p. 232).

Looking through the prism of Vygotsky theory, I assume that science misconceptions are prone to arise because in the contemporary school system, the development of the scientific concept of student only “grows downwards” – but doesn’t “meet” enough basis of everyday concept to rely upon. Thus, the approach of “scaffolding of everyday experience” along with “ratio of concept generalization”, described in chapters 2.3 and 2.4 above is an attempt to mature everyday concepts of students, in order to create a basis for what Vygotsky calls “spontaneous experience”. Next chapter will describe my attempts to create EdTech Platform which would integrate this methodological approach.

# 3 Development attempts

## 3.1 First attempt – didactic vehicle (July 2019 – March 2020)

First attempt to build EdTech Platform proposed so called "didactic vehicle" - an artefact similar to passenger car, but smaller in size (Figure 5). Purpose of the vehicle was to support teaching science and STEM topics to 11-18 years old students via extracurricular activities.

**Figure 5.** 3D model of didactic vehicle and comparison of vehicle frame with passenger car

The methodology behind this didactic vehicle was based on widening spontaneous experience of the student - as described above. It allowed physical manipulation of vehicle using learned scientific concepts – in what Vygotsky called "empiric and concrete domain". Following example describe how this manipulation could have been happening.

### 3.1.1 The concept of function derivative with didactic vehicle

The objective of this exercise is principal design of steering system for autonomous vehicle, whereas trajectory that didactic vehicle needs to follow is described with the function $f(x)$ (e.g. $f(x) = \sqrt{x}$). One approach to keep vehicle in the path is keeping the projection of one of the front wheels aligned with the tangent line of a curvature. In this exercise, students are guided to find a tangent line equation corresponding to the projection of a vertical plane of one wheel and calculate turning angle. Figure 6 demonstrates mathematical model for this calculation using k-coefficient of tangent line at $x = 0.1$ (corresponds to α) and tangent line equation $y = 1.581x + 0.1579$ which can be used to align vehicle wheel projection.

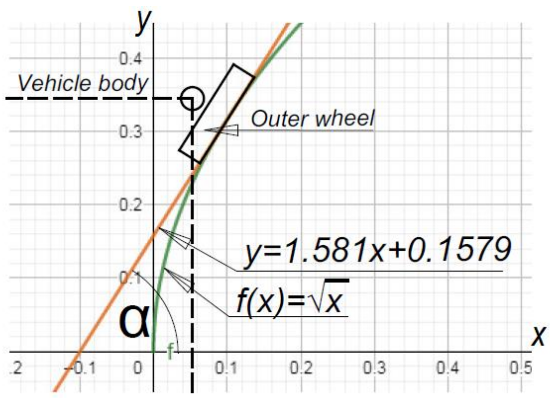



**Figure 6.** Mathematical model for calculation of wheel turning angle using concept of derivative of a function

As can be seen, this exercise exposes student to simultaneous use of multiple concepts (Cartesian coordinate system, function, tangent line, plane, projection) – in order to establish generalization ratio between all these concepts – and express all these via the overwhelming concept of function derivative. Valuable aspect of this approach was in training students with mathematical description of real-world situations. It is meaningful that school age youngsters would have been enabled with powerful didactic tool, allowing them to solve very complex engineering problem using only curricular knowledge.

## 3.2 Second attempt – didactic artefact (April 2020 – September 2020)

### 3.2.1 Pedagogical experience

After building the first prototype, before proceeding with more advanced development, the author decided to obtain pedagogical practice in one basic school, with extracurricular activities related to science and robotics for $6^{th}$-$7^{th}$ grades. 6-8 students were attending my classes from September 2019 until March 2020 (classes were terminated due to COVID-19).

Some of the methodological aspects described above were applied in these classes, however the main outcome of these were pedagogical experience and discussions with pedagogical staff of the school. Experience with students' motivation on attending extracurricular classes – as well as analysis of some literature related to extracurricular education (e.g. (SA Eesti Teadusagentuur, 2019)) showed that proposed EdTech platform shall be oriented towards formal education. Therefore, proposed form-factor (didactic vehicle) was no longer considered, and a new design attempt was made. In this next attempt, two other people joined the author for technical implementation. For that reason, hereafter I will use the pronoun „we", because technical development work was performed as a joint effort.

### 3.2.2 Didactic artefact

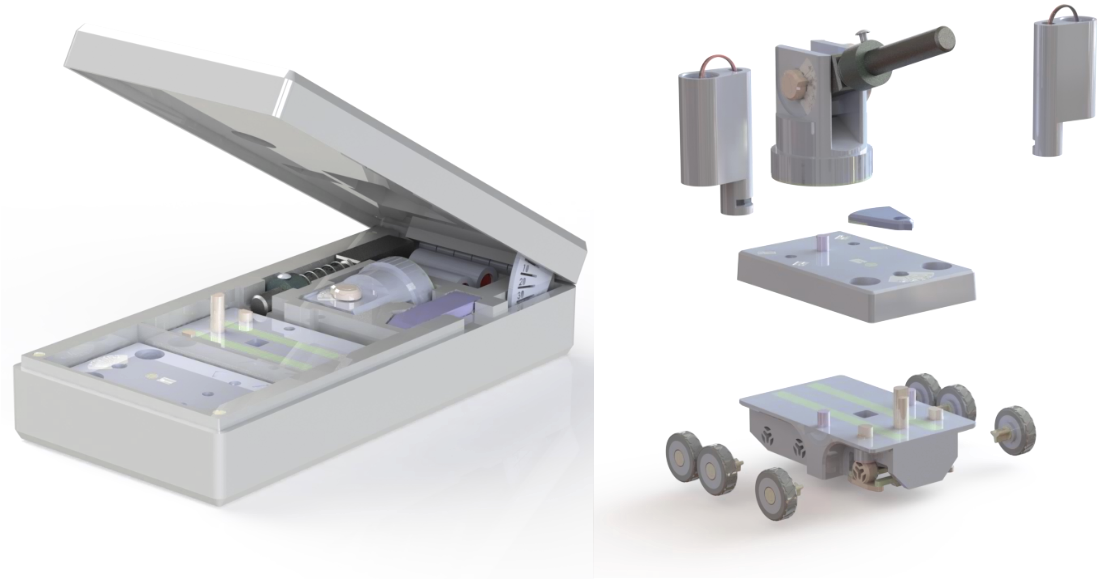


**Figure 7.** CAD models of didactic artefact in enclosure (left) and exploded view (right)

On a basis of pedagogical experience, the next design attempt resulted in reducing the size of the vehicle. Principal way of operating artefact remained the same – experiencing and manipulation of the scientific concept via physical artefact (Figure 7), however software became a considerable add-on. Figure 8 below illustrates how the concept of linear equations – normally studied in 7th grade is applied via both artefact manipulation and software simulation. We anticipated that students would use the concept of linear equations to model (or approximate) car path trajectory – whereas this would be done via both physical experimentation – as well as with the help of computer simulation.

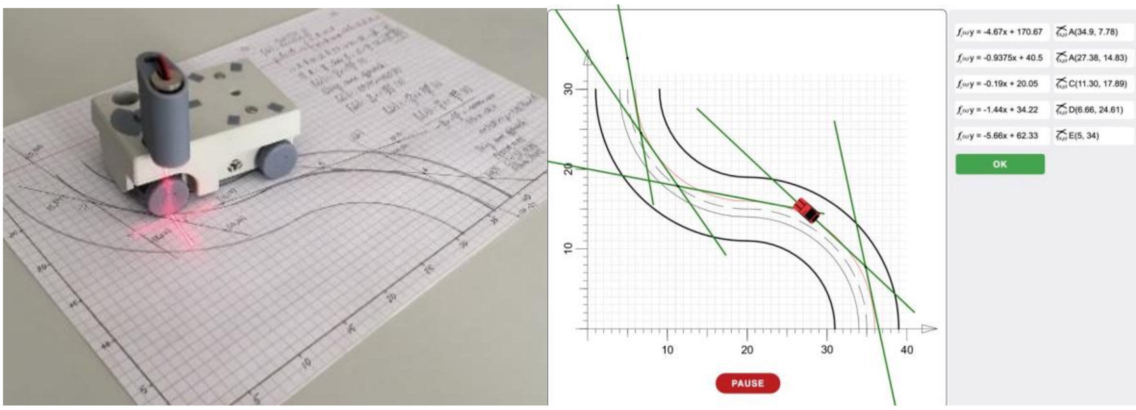



**Figure 8.** Experimentation with artefact on A4 paper (left) and corresponding computer simulation.

This second design attempt was demonstrated to three basic school teachers (two mathematics teachers and one physics teacher). Teachers found that anticipated exercises might be too complicated for students, however said that methodological approach itself might be of interest for them. Classroom pilot was agreed with two of these teachers for January 2021; however, it didn't eventually happen.

## 3.3 Third attempt – abandoning vehicle modality (October 2020 – March 2021)

First two design attempts were guided by overarching vehicle modality, whereas creation of second attempt happened mostly due to certain "thinking inertia" that the author had after the didactic vehicle. After more tests with software, we realized that vehicle modality limits our application scope – which is mainly lying in the domain of engineering.

Third design was an attempt to bring a significantly richer experience to students. The methodology remained largely intact, but we started to look into very heterogenous real-world domains. This attempt was mainly guided by trial-and-error strategy, which finally resulted in a set of 5 artefacts (Figure 9) and 8 software pieces (Figure 10), able to bring to student manipulation of scientific concepts in different contexts: cultural, medical, engineering, etc.

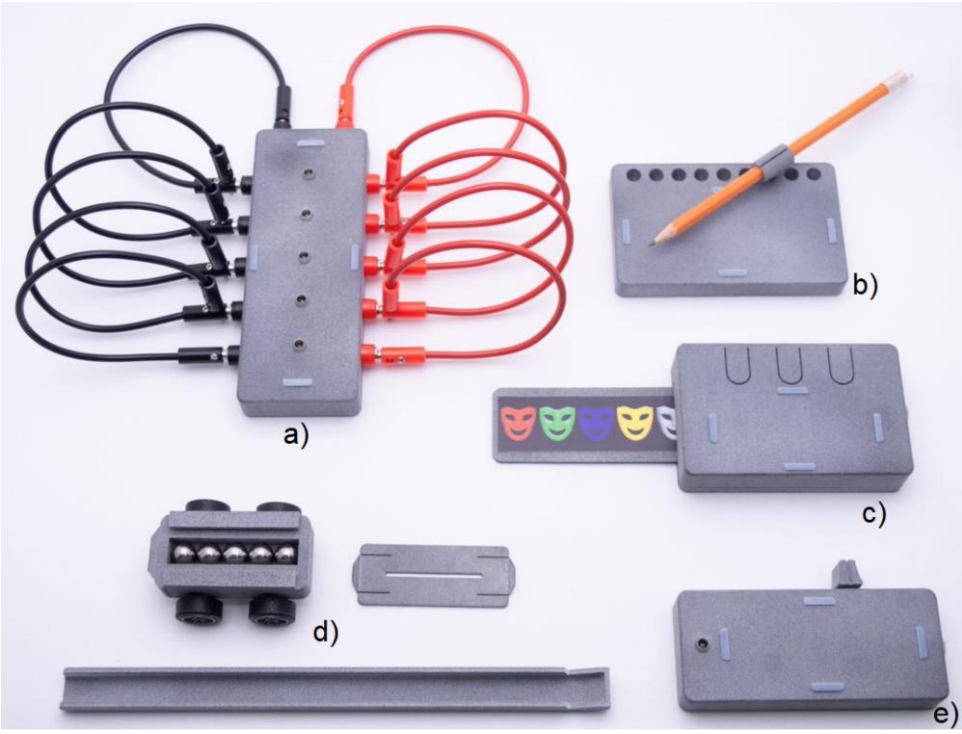



**Figure 9.** Artefacts of 3rd design attempt: a) Series and parallel connection; b) Trigonometry of acute angle; c) Light and colour; d) Newton’s $2^{nd}$ Law of motion; e) Functional relations

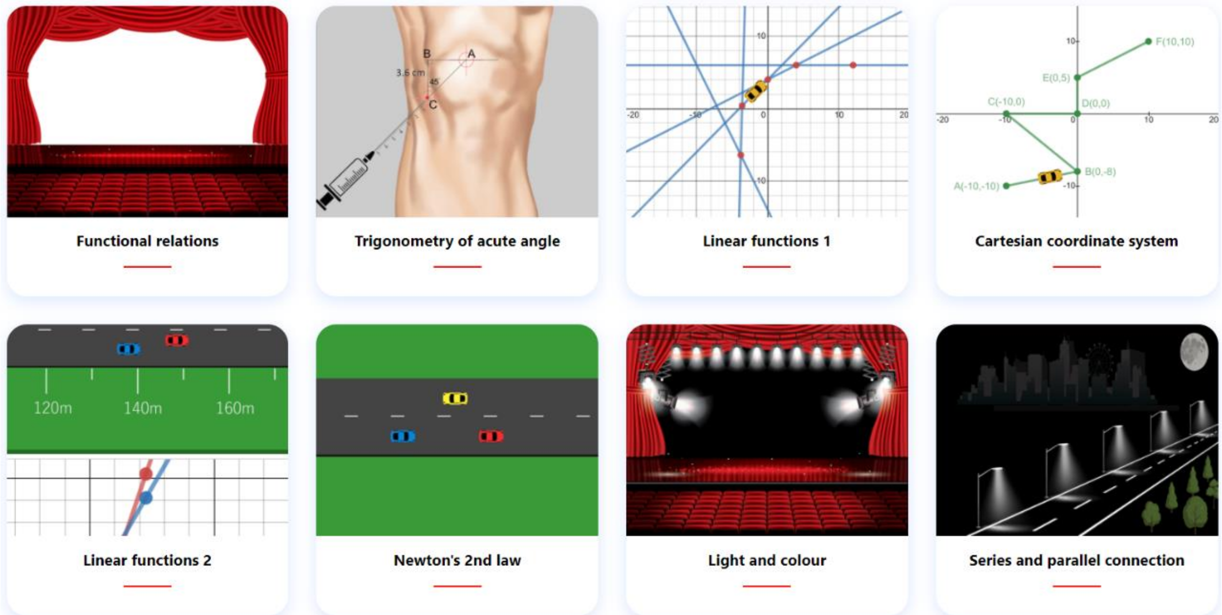



**Figure 10.** Software from 3rd design attempt. Software environment can be accessed via https://dartef-education.web.app/ (English) or https://mat.dartef.com/ and https://phy.dartef.com/ (Estonian)

Design of artefacts and software was significantly driven by analysis of students’ misconceptions. Next chapter gives an overview of our overall approach to design of artefacts and/or software – having a topic of functional relations (mathematics) as a reference sample.

# 4 Example of design approach (topic of functional relations)

## 4.1 Introduction

The topic of functional relations is standing at a special place in our overall strategy. During interviews with teachers, it became clear that functions create significant difficulties for students, which is also confirmed by academic research. Initially, we planned to integrate only two types of functions in our platform: reciprocal function and linear function. However, during early software tests with reciprocal functions it became clear that our technical approach is very suitable for all functions studied in the primary and secondary school curriculum.

## 4.2 Relevant students' misconceptions and errors as development driver

Development of the concept of "function" throughout history, and its applications in the real-world cause difficulties to understand this abstract construct. These difficulties result in misconceptions that are relatively well studied in the literature (Bardini et al., 2014). According to Sfard (1991), misconceptions stem from the dual nature of the function. In fact, the function can be understood in two significantly different ways: (i) structurally – as a set of ordered pairs; and (ii) operationally – as a computational process. These two aspects have to complete each other and to constitute in a coherent unity as they are always present. Consequently, while teaching the concept of the function both aspects should be equally stated. Otherwise, many students may develop a pseudo-structural (neither operational nor structural) concepts.

The empirical evidence on the structural nature shows that students tend to perceive the graph of the function as a picture and not as a set of ordered pairs. They often visually memorise graphs without being capable to connect, for example, the coefficients of a linear function to the scope of its graph and overall, translate graphical representation of function to its algebraic form (Birgin, 2012). Ocal (2017) also highlights visual memorising of graphs, where essential asymptote characteristics are being neglected. This finding is confirmed by Katalenić et al. (2020) as well. Furthermore, Oehrtman et al. (2008) discovered that students frequently mismatch the visual characteristics of a real-world context with similar features regarding the graph of a function that map the situation. The authors argue that:

> "… students are thinking of the graph of a function as a picture of a physical situation rather than as a mapping from a set of input values to a set of output values. Developing an understanding of function in such real-world situations that model dynamic change is an important bridge for success in advanced mathematics" (p. 154).

The research on the operational nature shows that many students reject to see the constant function (e.g. $y = 5$) as a function, giving the justification that a function must always vary (Leinhardt et al., 1990), (Oehrtman et al., 2008). The authors also found that many of the questioned students could not calculate $f(x+a)$ given the function $f(x)$. 43 % of them just added $a$ onto the end of the expression for $f(x)$ (i.e. $f(x)+a$). Knuth (2000) states that students indicating a deep understanding of the concept of function know which representation is most suitable for applying in different contexts and they are able to switch simultaneously between different representations. Dreyfus (2002) claims that students need to reach many linked aspects of the function concept in order to be successful in mathematics.

Given this evidence – and targeting preventing or correction of misconceptions, our design attempt was initially targeted at preventing students from perceiving function graph as a picture. In particular, if a student has simply visually memorised a function graph, he/she most probably will not be able to draft a particular functional relation using our artefact/software. Usage of the latter requires conceptual understanding of function as relation – not memorisation of function graphs or simple application of procedural knowledge.

## 4.3 Operational relation between time and light intensity in the context of theatre

The approach behind the designed artefact-software includes three representations of a functional relation given a real-world situation – narrative, artefact and software.

### 4.3.1 Narrative of the real-world situation

Maria is technician in charge of the theatre lighting system. Her responsibility is to guarantee the lighting of the stage during the whole performance, according to the scenario trained during the rehearsals. For example, there is a requirement to dim the stage from 100 % to 0 % during 10 seconds, or from 0 % to 50 % during 5 seconds.

Our expectation using the artefact/software is that the student is provided (scaffolding – see chapter 2.3) with familiar context (cinema, theatre, concert hall, or opera house). Consequently, while visiting the theatre or the cinema next time, the student might have an associative connection between the event and the mathematics taught in the classroom.

### 4.3.2 Description of the artefact

Artefact (Figure 11 left) is a plastic box (120mm x 50mm x 18mm) with sliding knob and electrical bulb and represents physical replica of the theatre stage. The box can be powered up by any USB-A power source. The knob enables regulating the bulb (stage) illumination over time according to the studied mathematical function. On the front side of the box, a space for a paper holder is designed that is aligned with the knob trajectory. The student is required to draft both the x- and the y-axes in a very uncommon form, that is, the axes are parallel to each other (Figure 11 right). Accordingly, if the student has only partial or no understanding of functional relations, we expect the missing knowledge will be discovered by this task.

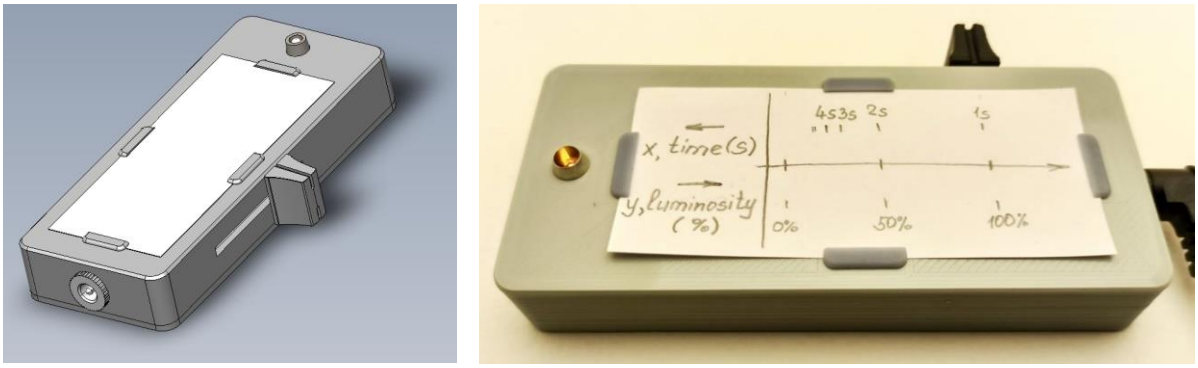



**Figure 11.** Artefact for study of mathematical functions (left) and representation of the function f(x)=100/x via artefact, when x and y axes are parallel to each other (right)

### 4.3.3 Description of the software

The software can be accessed via https://dartef-education.web.app/SNA_1 (English) and https://mat.dartef.com/SNA_1 (Estonian). It represents a theatre stage that can be lightened or darkened depending on the mathematical function selected for study. The software combines three different representations of the same function: symbolic, graphical and real-world representation of the function in terms of lightening or darkening the stage according to the mathematical function applied (see Figure 12). The simultaneous application of different representations shows a positive effect with respect to understanding of the function concept (Knuth, 2000), (Dreyfus, 2002). Currently the software supports all functions studied in primary and secondary school: linear, reciprocal, quadratic, cubic, trigonometric, exponential etc.

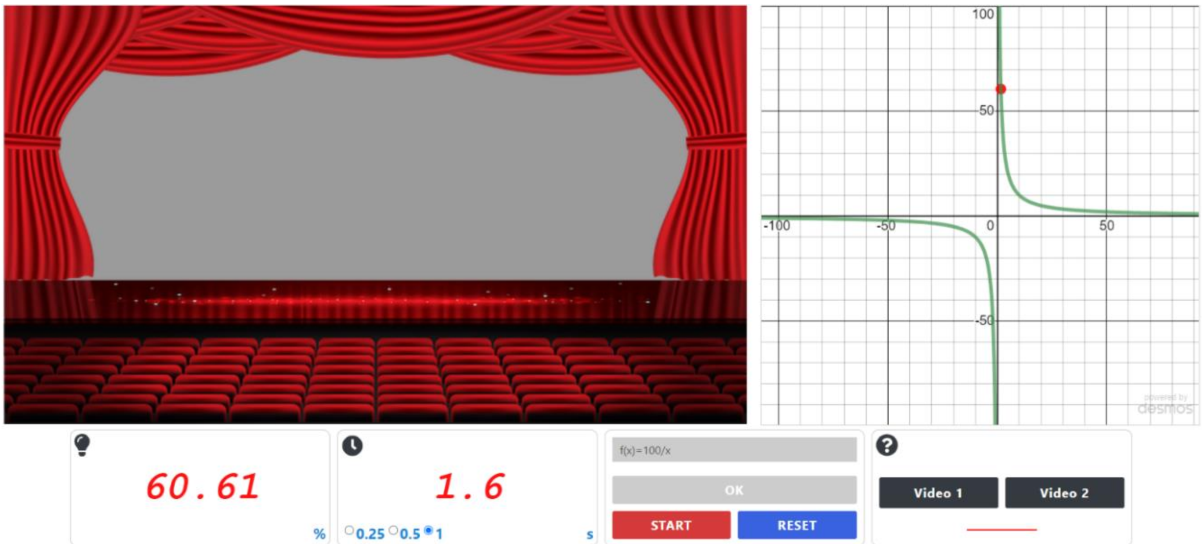



**Figure 12.** Software illustration for the function f(x)=100/x

#### 4.3.4 Samples of classroom or homework tasks

At the time of designing this artefact/software pair we were at the opinion that this approach is suitable for accomplishing homework activities at first. Thus, we formulated samples below (for linear functions) with the assumption, that these are used as homework guidance.

Sample A:

- Define the linear function by symbolic representation that describes luminosity change from 0 % to 100 % in a period of 10 seconds.
- Draft the graph of the defined function on the artefact paper using the parallel lines as a coordinate plane. Use $t$ for the time component and $f(t)$ for the light intensity.
- Check the behaviour of the function in real-time using the software.
- Explain verbally the function behaviour based on the graphical, symbolical and real-world representation of the defined function.

Sample B:

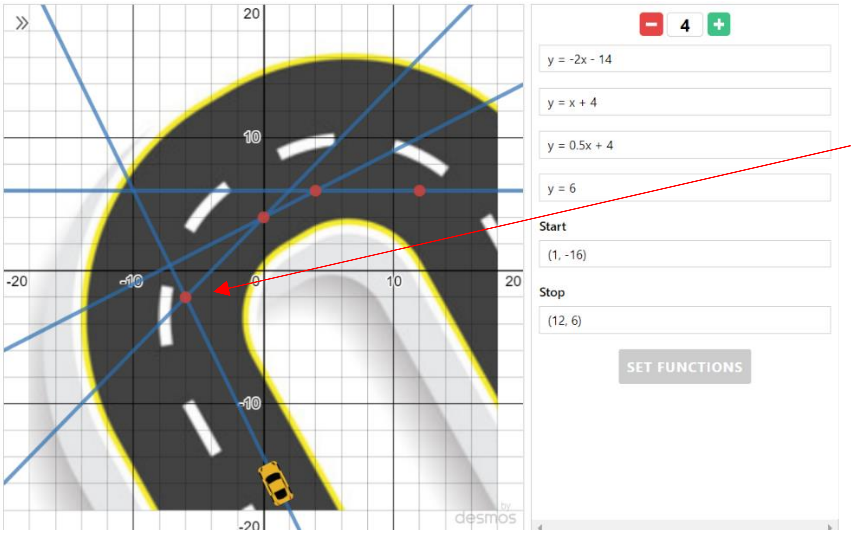
- Draft both the x- and the y-axes on the artefact paper for the functions: $(1)\ f(x) = 10x; (2)\ f(x) = -10x + 100; (3)\ f(x) = 10x + 50.$
- Describe verbally the meaning of the drafted function in the real-world context, e.g. lighting vs. darkening process at stage without using the software.
- Check the correctness of your discretion applying the designed software.

These classroom/homework exercise samples were our first attempt to approach the classroom realm. These evolved into homework worksheets, used for school pilot (Chapter 5).

## 4.4 Structural relation between pairs of coordinates in the context of self-driving car

#### 4.4.1 Narrative of the real-world situation

Albert is an engineer in a start-up company and his task today is to design an algorithm for self-driving car navigation using linear functions. More detailed, he is observing the behaviour of mathematical model of autonomous car on a specific road segment – curve. His task is to evaluate whether the car trajectory in the curve is smooth enough, or additional trajectory corrections might be needed. Albert has just built a mathematical model of a road curve shown on Figure 13 below. However, during the simulation it can be seen that at one specific place, the car trajectory changes too rapidly. Consequently, one more function graph is needed “between” two adjacent functions y = -2x – 14 and y = x + 4.




Trajectory at this point is too rapid, and one more function „between“ two adjacent functions (i.e. „between“ y = -2x – 14 and y = x + 4) must be introduced in order to smoothen the movement of the autonomous car.

**Figure 13.** Modelling autonomous car trajectory by linear functions

### 4.4.2 Description of the software

The software can be accessed via the following link: [https://mat.dartef.com/SNA_3](https://mat.dartef.com/SNA_3). It represents the Cartesian coordinate plane, where an image of any road segment can be uploaded. The student can insert any linear functions of the form $y = ax + b$ in order to approximate the road path. When linear functions are defined, the car model is moving according to the defined trajectory. The Figure 14 below depicts an example for function $y = 36x + 130$ inserted "between" the two previously mentioned $y = -2x - 14$ and $y = x + 4$ for smoothing the car trajectory.

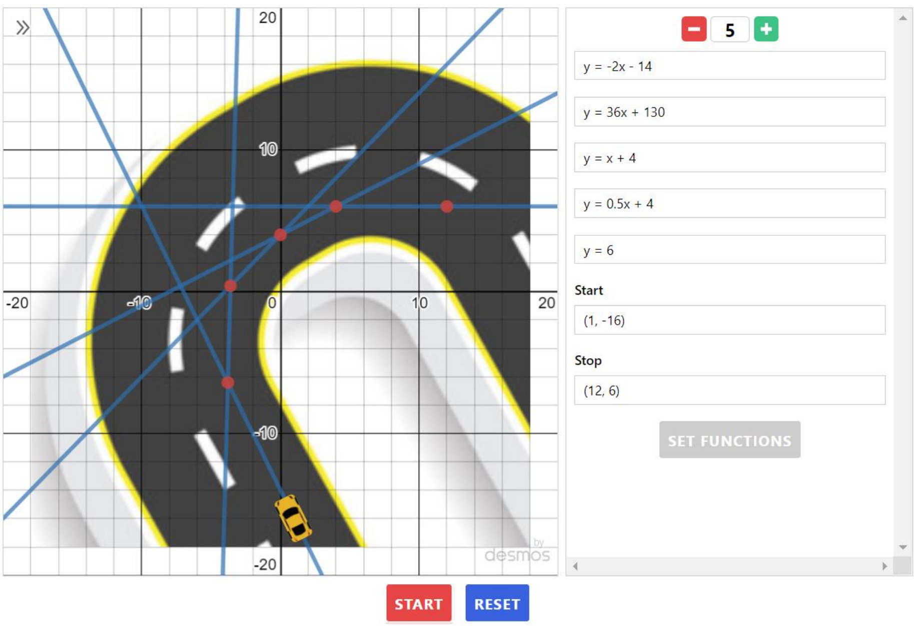



**Figure 14.** Example of "smoothened" car trajectory in the software

#### 4.4.3 Samples of classroom or homework tasks

The specific target of tasks to be performed with the help of this software is to either prevent or correct students' mistakes in defining the slope of a function (expressed as $a$ in $y = ax + b$) and in computing intersection points with axes. As mentioned above, one more function is needed "between" two adjacent functions $y = -2x - 14$ and $y = x + 4$. Obviously, this question doesn't assume only one correct answer, because an endless number of linear functions can be defined to be graphically placed between two adjacent functions.

Thus, students are asked to propose any equation that would "fit" between two adjacent functions. To make it, students must know fundamentals of linear equations (at least the notion of slope, like "$a$ coefficient in $y = ax + b$ shows how many $y$-s it would be for each $x$"). Thus, samples of tasks for students are as follows:

a) Make a proposal of any random linear function, that would "fit between" $y = -2x - 14$ and $y = x + 4$.
b) Propose any random linear function with positive $k$, that would "fit between" $y = -2x - 14$ and $y = x + 4$.
c) Propose any random linear function with negative $k$, that would "fit between" $y = -2x - 14$ and $y = x + 4$.
d) How many functions might "fit" between $y = -2x - 14$ and $y = x + 4$?

## 4.5 EdTech expert evaluation of third design attempt

Third design attempt was timely co-located with attracting external funding via the IMPACT EdTech project (*IMPACT EdTech Webpage*, s.a.), which gave the possibility to hire development stuff and get access to educational mentoring and evaluation services. This design attempt was evaluated by 4 EdTech experts from Finland, Estonia, Romania and the United Kingdom, involved in the IMPACT EdTech project with the help of European Schoolnet (*EUN Webpage*, s.a.). These specialists were introduced to the platform via means of video conference and then were given an opportunity to the test platform for one week (artefacts were sent beforehand to them). After one week, another web-conference took place, where specialists gave comprehensive feedback regarding their opinion of platform usage. Following aspects were highlighted by experts as strengths of the platform:

- Overall high educational value and credibility of methodological approach;
- Convincing and modern real-world scenarios;

- Ease of use of artefacts and software (however necessity of specific guidance was pointed out);
- Possibility of hand-on activities.

Several recommendations regarding further improvement of the platform were given. These included various aspects – concerning some specific artefacts/software and even colour of the artefacts. However, one strong and overwhelming recommendation, critical to successful reception of the platform was given: thorough teacher and students instructions must be supplied, preferably in the form of lesson plans or worksheets.

### 4.6 Change of priorities during third design attempt

During our third design attempt (October 2020 – March 2021), I was interviewing math and physics teachers (23 teachers were interviewed). Also, design work was co-located with my master studies, and these two aspects (teachers interviews and further studies) guided us more and more towards the idea that we need to pay even more attention on student' motivation. While conceptual understanding of science still plays a decisive role for our artefact/software design, primary attention in our next design attempt is paid to students' motivation.

## 5 School pilot

### 5.1 Introduction

While the 3rd design attempt was reaching its end, we have discovered an opportunity to pilot it in schools. Particularly - the topic of linear functions was well developed in our platform (3 software and one artefact), and this topic was taught in Estonian schools during March-April.

I'd like to highlight the fact that main target of this master thesis was the design and technical implementation of the EdTech platform. Pilot was never meant to be conducted in the scope of the thesis. However, a lucky coincidence of circumstances (that is: thorough implementation of linear functions into the platform by March 2021 and teaching of this topic in schools just during March-April 2021) allowed us to perform this pilot with minimal preparations, and get early feedback from teachers and students.

### 5.2 Pilot description

Altogether, eight 7th grades under guidance of 6 teachers have tested our solution with the topic of linear functions. In addition, one of those teachers expressed a wish to test same software also with 10th grade (topic: straight line equation). Total number of students was 166.

Due to COVID-19 pandemic restrictions pilot took place entirely during distant learning, and therefore we didn't have chance to test usage of artefacts as homework aid. Only one artefact per teacher was produced and delivered. Main testing was therefore done using software. Altogether, 3 different versions of software were proposed to teachers and students:

- Controlling of theatre stage lighting with the help of linear functions (https://mat.dartef.com/SNA_1);
- Modelling of autonomous car path using linear functions (https://mat.dartef.com/SNA_3);
- Prediction of car overtake time using linear functions (https://mat.dartef.com/SNA_5).

Teachers were asked to teach the topic in their usual manner, and in addition to typical textbook exercises, give also tasks proposed by our software – either for classroom work or for homework (and perhaps to demonstrate artefact usage via web camera). Detailed exercise descriptions were produced for each of these software environments and teachers were instructed on how to use software via means of videoconferencing. Example of worksheet can be found in Appendix I. Questionnaire measuring students' attitude (Appendix II) was designed and distributed by the teacher to students after completing the topic of linear functions. Altogether 141 students accomplished the questionnaire.

No personal data was collected during the testing and questionnaire. Software was installed at Google Firebase services, and we haven't installed any tracking tool (Google Analytics or alike) on these websites. No login/authorization was necessary for operating software, therefore no geo-tracking of users was possible.

## 5.3 Pilot results

### 5.3.1 Teachers' feedback

After accomplishment of piloting in particular school, unstructured interviews took place with every teacher, in order to understand their overall impression. Although there were certain aspects that interested us, I tried not to ask any explicit questions that could bias teacher answers. For example, the reader will see below that all teachers perceive artefact as in-class learning aid, rather than as homework aid. However, I never asked questions like "Do You see artefact as homework aid or as an in-class solution?" Instead, only general questions were asked and following chapters describe summary of these interviews, structured according to main categories, that appeared during discussions.

##### *5.3.1.1 Overall teachers' impression*

All teachers highlighted a very different approach to teaching mathematics in the case of our EdTech platform and confirmed that they never saw such an approach before. All of them expressed their positive perception of proposed approach of putting math topics in real-life context and highlighted the authenticity of the proposed contexts. Overall, the context delivered by the platform was very unusual for them, and they were positively surprised about the applicability of mathematics for such situations (e.g. control of theatre stage lighting). Three teachers said that they were rather sceptical in the beginning and thought that proposed exercises would be too difficult for students, however the opinion changed during piloting. These three teachers also said that they were very curious themselves, what would happen in a software simulation environment, if they would change some parameters of the linear functions. All 6 teachers expressed readiness to contribute to further platform development and to participate in the next piloting stage.

##### *5.3.1.2 Teachers' opinions on students' perception*

As mentioned above, three teachers initially anticipated that exercises delivered via platform would be too difficult for students. We witnessed this attitude also before (during interviews with teachers). However, the reader will see later, that these fears were not confirmed. All teachers said that due to distant learning they couldn't get a full picture on student perception (e.g. due to inability to track emotional state of students), but overall perception is rather positive. One teacher said that she was very positively surprised, that during solving exercises valuable and meaningful mathematical discussions were started by students – this phenomenon is rather rare to arise in classes.

##### *5.3.1.3 Opinion about artefact (i.e. embodied manipulation of mathematical concept)*

Four teachers couldn't express any strong opinion regarding usage of artefact since they couldn't give it to students during distant learning, two others said that artefact would be very interesting for students. While we were anticipating usage of artefacts as homework aid (fearing teachers' overhead), all 6 teachers expressed see artefacts rather for work in pairs during classes, in order to be able to track students' reasoning process and promote discussions in class. One teacher told, that artefact is rather not necessary, however still would give it to students to see their reaction.

#### *5.3.1.4 Contribution of platform to learning goals*

All teachers confirmed that authentic context delivered via platform and corresponding worksheets are fully aligned with (their) mathematics teaching methodology, fully contribute to learning goals and students' motivation and significantly diversify the learning process. Very valuable aspect for us was already mentioned stimulation of mathematical discussions, as it gives us early indication that our approach is in line with contemporary learning paradigm (HTM, TLÜ, TÜ, 2017). Moreover, 4 teachers said that using the platform resulted in certain positive shift in their usual way of thinking about mathematics.

#### *5.3.1.5 Time for lessons preparation*

3 teachers mentioned that time for lesson preparation was shorter than usually due to availability of ready-made timesheets. None of the teachers mentioned any significant overhead related to lesson preparation.

### 5.3.2 Student's feedback

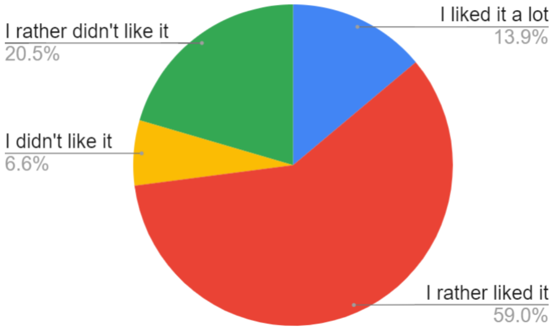
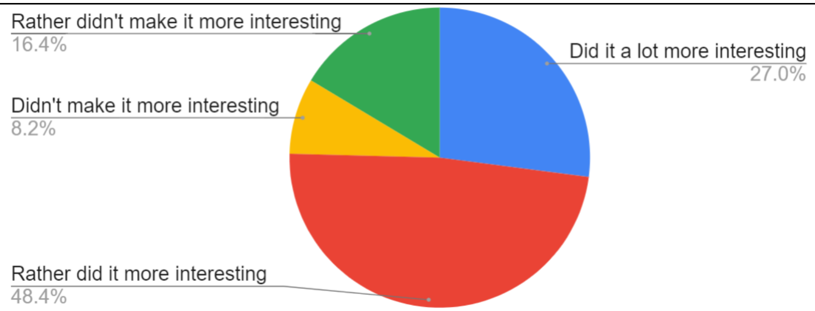
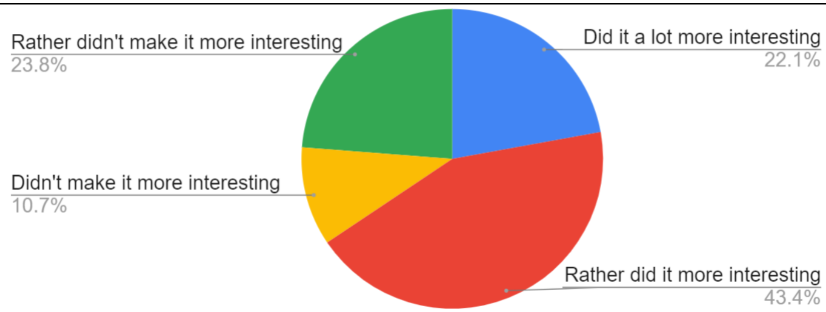
As mentioned in Chapter 4.6 above, we had to change the priorities of our development approach towards motivation of students. Therefore, during pilot testing we paid most attention to students' perception of learning content and methodology delivered via platform. Thus, questionnaire (Appendix II) was designed in order to evaluate students' attitude and motivation.

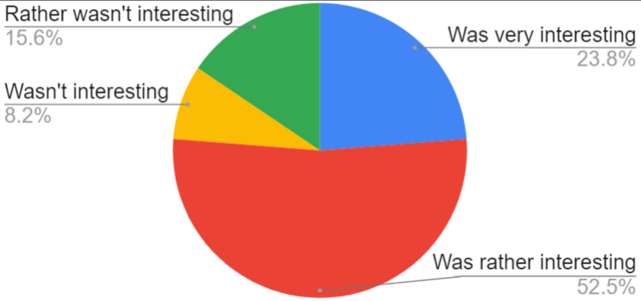
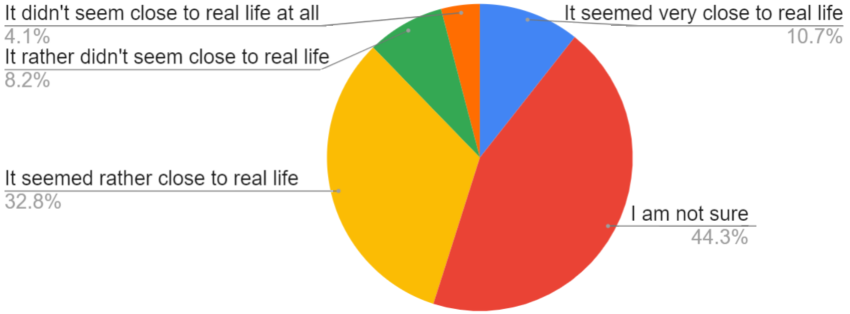
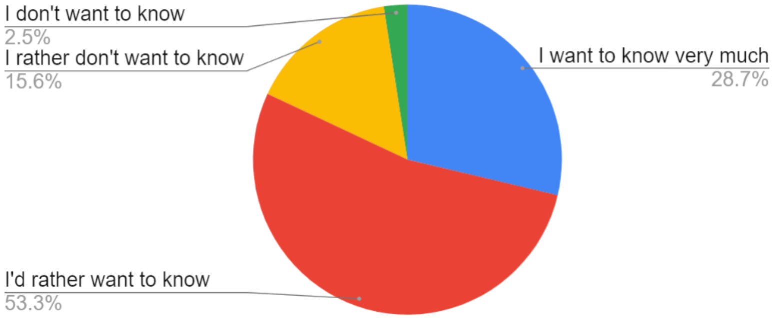
141 out of 166 students have answered the questionnaire, however I will report here on answers of 122 students. The reason for that, is that after receiving the first questionnaire (19 responses) we saw some logical inconsistencies in response options, and slightly modified these for all future questionnaires. Therefore, first 19 responses were removed.

Altogether, 8 questions were asked from students. Information below is limited to 6 questions which are the most important for us (from further EdTech Platform development point of view). Table 1 below shows students' responses, augmented with author comments.

We perceive obtained results in a positive light. Students' motivation to learn math and science is decreasing in the whole world (European Commission, 2015) and my interviews with teachers confirmed this fact. The reader will see in a while that we were able to attract significant interest with a short piloting cycle, that gives us certain hope for credibility of proposed approach.

**Table 1.** School pilot results

<table>
<tr><th>Question, as in questionnaire</th><th>Responses in a form of pie chart (N=122)</th><th>Comments</th></tr>
<tr><td>Did You like to solve exercises with the help of given software?</td><td>
</td><td>73% of students showed good reception of our software, which is – undoubtedly – a very good sign. For further reliability of results, comparison with test group using other educational technology would be beneficial (it is possible, that students would answer similarly to any other Educational Technology, that bring some diversity to their learning)</td></tr>
<tr><td>Did this software make math lessons more interesting than usual?</td><td>
</td><td>¾ of students indicated positive perception, whereas the percentage of those who gave the highest possible score is significantly larger than in the previous question.</td></tr>
<tr><td>Did this software make math homework more interesting than usual?</td><td>
</td><td>All groups delivered very similar results – the percentage of students who evaluated this aspect positively is smaller than in the previous question. Interviewed teachers assumed that this is related to overall perception of homework by students as not a very attractive activity. Nevertheless, the result is still very good in our opinion.</td></tr>
</table>

| | | |
|---|---|---|
| Was it interesting to investigate how You can control something (for example stage lighting or vehicles) with the help of mathematics? |  | More than ¾ of students responded positively to the interactive nature of our software, which enables control of virtual objects with the help of their mathematical descriptors. Such response gives us certain reasons to think that our approach to software design is valid. |
| Did this way of teaching seemed to be close to real life? |  | This response is most surprising – not only for us, but also for teachers: 44% of students are not sure if the context delivered by our software is "real-life" context. We don't yet have any substantial explanation for this phenomenon. In private conversations math teachers confirmed that questions like "Why do we need math in real-life?" are very frequent from students, thus further investigation on how students perceive "real-life" is needed. We have first hints regarding this phenomenon; however, it is yet early to make any conclusions. |
| Would You like to know how school mathematics is used in real life? |  | This response must be received, undoubtedly, very positively. None of 122 students said s(he) don't want to know about mathematics real-life application. It is still somewhat disappointing to see 18% of students who are (rather) not interested to know math applications, however overall attitude is positive and indicates students' curiosity. |

# 6 Fourth development attempt

Taking into account experience that we got during three design iterations – as well as school pilot, a relatively clear vision on next development steps has been formed. Fourth development attempt will not substantially differ from the third one, and the approach of artefact-software pairs will be kept. Two major changes from the third design attempt will be dedicated to creation of universal artefacts and introducing some career guidance elements into our platform. Next chapters describe it in more detail.

## 6.1 Further outlook towards artefact-software pairs development

In the beginning of the third design attempt, the default option for us was producing a specific artefact-software pair, corresponding to a particular real-world context. For example, the artefact-software pair described in Chapters 4.3.2 and 4.3.3 respectively corresponds to a specific real-world context of theatre stage lighting. However, during development of this pair we saw very clearly that both artefact and software actually allow integration of a very wide variety of mathematical functions (2nd research question of this thesis). Moreover, we prepared two additional software pieces for the topic of functional relations (the context of autonomous car and car overtake simulation), which didn't have any artefact behind it. Therefore, our current intention is to abandon producing fixed artefact-software pair for each and every real-world context. Instead, our next design iteration is concentrated on producing a kind of "universal" artefacts (i.e. not related to some specific real-world context) and a multiple software pieces for different mathematical and physical topics. Topics are prioritized via teachers' feedback and analysis of students' misconceptions. As of now, this approach is already in implementation, Figure 15 below shows design for two future artefacts.

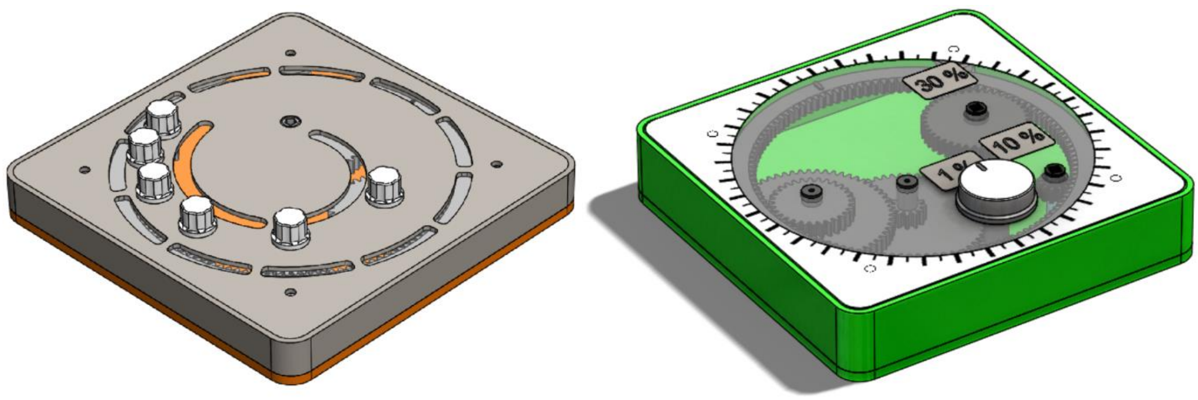



**Figure 15.** 3D models of artefacts for studying fractions (left) and percentages (right)

## 6.2 Real-life narrative with the help of career theories

During school pilot usage of our platform was preceded by small narrative – explaining the real-life context (e.g. lighting of theatre stage) and learning goals. It was assumed that this narrative would be introduced by the teacher, also pilot worksheets delivered these written narratives. However, we were searching also for others – more realistic methods to deliver narratives. One of the possibilities we were looking at was related to career guidance practices, and our idea was to integrate career guidance with mathematics/physics lessons.

During last semester (September 2020 – January 2021), author gained some theoretical fundamentals during studies at SVHI.01.044 "Career Planning and Career Service Management", and then we made early attempts to create videos, where narrative or real-life situation and its connection with mathematics/physics is delivered via imaginary character (e.g. already mentioned Maria - lighting specialist in theatre).

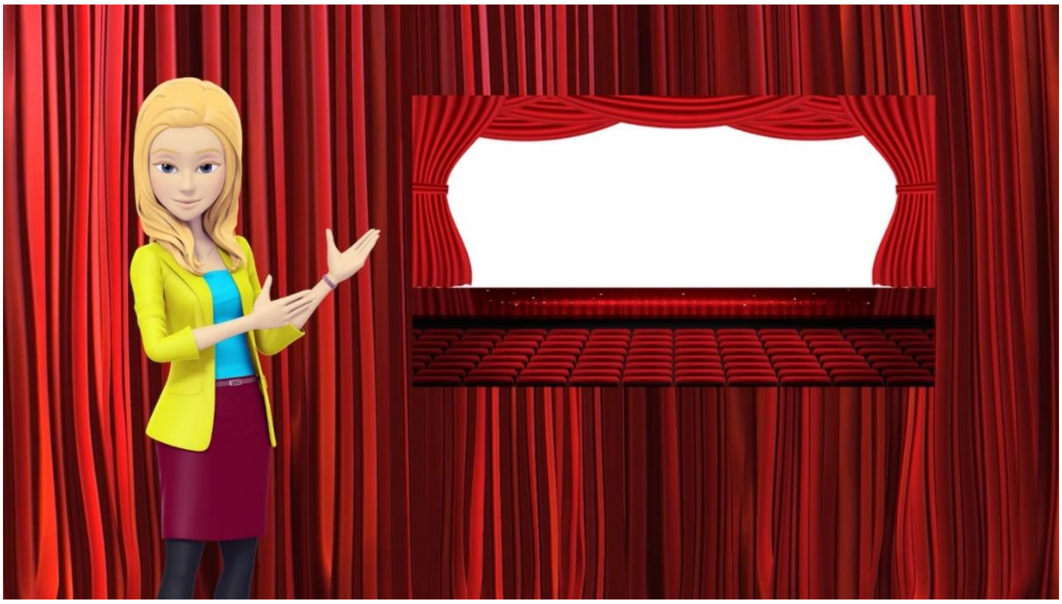


**Figure 16.** Maria, lighting specialist at theatre explaining connection between her job and school mathematics

This approach is based on some fundamental theories of career guidance from Edward S. Bordin, Donald E. Super and John D. Krumboltz. Unfortunately, the scope of the present master thesis doesn't allow me to describe the theoretical foundations behind these narratives in more detail. However, one of these foundations is important to be mentioned here, and is related to selecting female character. This wasn't accidental, and during accomplishment of course, we received positive feedback from both subject lecturers – as well as from math/physics teachers. Positive feedback was related to assigning female character to introduce lighting technician occupation, which is usually perceived in society as men's area.

Altogether, 4 videos with this imaginary character have been produced and integrated into software environments. These videos can be seen at https://mat.dartef.com/SNA_1 and https://phy.dartef.com/BSI_3 (Estonian voice over) and at https://dartef-education.web.app/SNA_1 and https://dartef-education.web.app/BSI_3 (Estonian voice-over with English subtitles), as shown on the figures below:

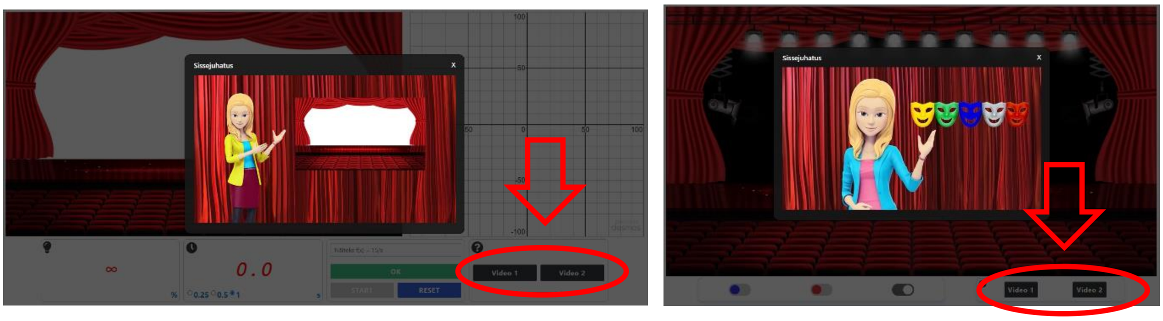


**Figure 17.** Two software environments with integrated narrative videos

Our expectation is that during the fourth design attempt we will further work on these narratives, however in a slightly modified way. This way is yet to be clarified.

# 7 Conclusion

Currently we tend to think that our third design attempt was in principle adequate. We think we have reasons for cautious optimism regarding positive acceptance of our EdTech Platform in schools – from both teachers and students. We couldn’t yet test changes in students’ conceptual understanding of math and science when using our platform, but we plan to do so in autumn 2021 – shall COVID-19 pandemic situation allow it. Nevertheless, we can already make conclusions, and return back to research questions:

1. Which fundamental aspects of the cognitive psychology of learning and teaching can be applied to the design of educational technology hardware?

Our EdTech Platform is largely based on Vygotsky theory on concept formation and takes analysis of students’ misconceptions for its design. Author have proposed the “scaffolding of everyday experience” approach, which is based on this theory. “Ratio of concept generalization” is another cognitive aspect that is well used in our platform. One more aspect we use for our design in simultaneous usage of declarative and procedural knowledge, and these pillars are currently fundamental for us.

2. What is the proper combination of curricula topics and real-life context for their integration into education technology hardware platform? Particularly:
    - Could math/physics curricula topics be in principle integrated into anticipated EdTech Platform?
    - Is it feasible to integrate a critical mass of curricula topics into the platform?

Experience showed that considerable variety of math/physics topics can be integrated in our EdTech platform. Currently all mathematical functions studied in school, trigonometry of acute angle, inverse trigonometric functions, Pythagoras theorem, Cartesian coordinate plane, straight line equation are integrated for mathematics; series and parallel connection, light and colour and Newton's 2nd Law of motion are integrated for physics. Our current strategy foresees integration of fundamental mathematical concepts for grades 6th to 9th (fractions, percentages, negative numbers, etc.) into our platform. We have also ideas for integration set of concepts from gymnasium grades mathematics (derivatives, integrals). We estimate, that ca. 60-70% of math and ca. 50% of physics concepts can be well integrated.

3. What is the most suitable design configuration of educational technology platform, so that it is:
    - adequately usable in the classroom environment (e.g. has proper size, simple in use, etc.),
    - produce minimum disruption of the existing teaching practices (e.g. well aligned with traditional exercises, doesn't need excess time for performing experimentation/demonstration, etc.),
    - affordable for usage in basic and middle school (e.g. in terms of costs)?

Teachers' feedback showed readiness to integrate our platform into classroom learning – at least its software part. Therefore, we make an early assumption that selected approach – hand-size artefact and digital simulation – is suitable for a classroom environment. However, usage of artefacts – either as classroom or as homework learning aid is not clear yet. Interviewed teachers confirmed good alignment of tasks delivered by EdTech Platform with formal curriculum. Cost analysis for artefacts shows that it's price would lie in the range of 10-40€ per piece.

# 8 Acknowledgements

The work described in this master thesis was performed as a joint effort of many people. Author is working together with Maksim Jemberdin and Sergei Pavlov on a daily basis, FTE (Full-Time-Equivalent) for all 3 of us. I am very grateful to both Maksim and Sergei. All software and hardware are technically implemented together with them.

The results presented in this master thesis are part of an IMPACT EdTech project that has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 871275. IMPACT EdTech (*IMPACT EdTech Webpage*, s.a.) is a programme funded through the European Commission's H2020 programme and run by a consortium of three project partners:

- European Schoolnet (*EUN Webpage*, s.a.) – a network of 34 European Ministries of Education, based in Brussels, leading the engagement of educational stakeholders and coordinating the consortium;
- Funding Box (*FundingBox Webpage*, s.a.) – cascade funding specialists who are managing the selection of innovative start-ups who participate in the programme and distribute their funding;
- ISDI Accelerator (*ISDI Accelerator Webpage*, s.a.) - develops acceleration & innovation programs for innovative start-ups and supports their growth.

IMPACT EdTech aims to support promising European start-ups in the educational sector to develop and commercialise their innovative educational solutions, by providing them with equity-free financial support and access to a range of services which aim to help companies make their products better fit the needs of the educational sector.

In this program our project is presented under the name of DARTEF (which stands for Didactic ARTEFact). This program enabled us to proceed with a third design attempt and gave our project whole new dimension. I am grateful to all mentors and managers of the IMPACT EdTech program.

Videos and voice-over for Maria – theatre lighting technician were produced by Suive Pahrt – professional video maker. I am very grateful to Suive for very precise work, numerous language corrections and fantastic animations.

I am extremely grateful to Dr. Nikolinka Fertala – math and physics lecturer at BHAK & BHAS Wien 10 (vocational school in Vienna, Austria). Dr. Fertala is our educational mentor in the IMPACT EdTech project. Her numerous comments related to the school realm – as well as her excellent knowledge of cognitive aspects made our platform better.

I am very grateful to Irene Álvarez Caro – our business mentor at IMPACT EdTech project. Her help in giving a market view on our platform helped us to give it specific shape and think more on sustainability of our approach.

And of course, I am very grateful to Dr. Karin Täht – supervisor of this thesis, for her constant feedback and help in organizing the school pilot.

# 9 Confirmation of authorship

Main author of this master thesis is Roman Kaurson. All theoretical and educational research (including school pilot) is performed solely by him. The methodology of everyday experience scaffolding is proposed by Roman Kaurson and principal design of described EdTech Platform (software and hardware) is performed by him as well.

Technical implementation of the platform (i.e. software development and mechanical design of artefacts) is performed with the help of Maksim Jemberdin and Sergei Pavlov respectively. Overall, technical implementation of EdTech Platform is performed jointly by Roman Kaurson, Maksim Jemberdin and Sergei Pavlov. This is the reason for repeatedly used pronoun „we“ throughout the text of the thesis.

Videos and voice-over for Maria – lighting technician in theatre are created by Suive Pahrt, on a basis of scripts produced by Roman Kaurson.

Chapter 4 was revised and commented on by Dr. Nikolinka Fertala, who provided a set of very valuable theoretical sources and made several methodological suggestions.

I, Roman Kaurson, author of this master thesis confirm, that I compiled the present thesis myself and have properly identified authorship and contribution of third parties. Thesis is compiled according to the requirements of the Institute of Education of University of Tartu and is in accordance with good practices of academic research.

Roman Kaurson

20.05.2021

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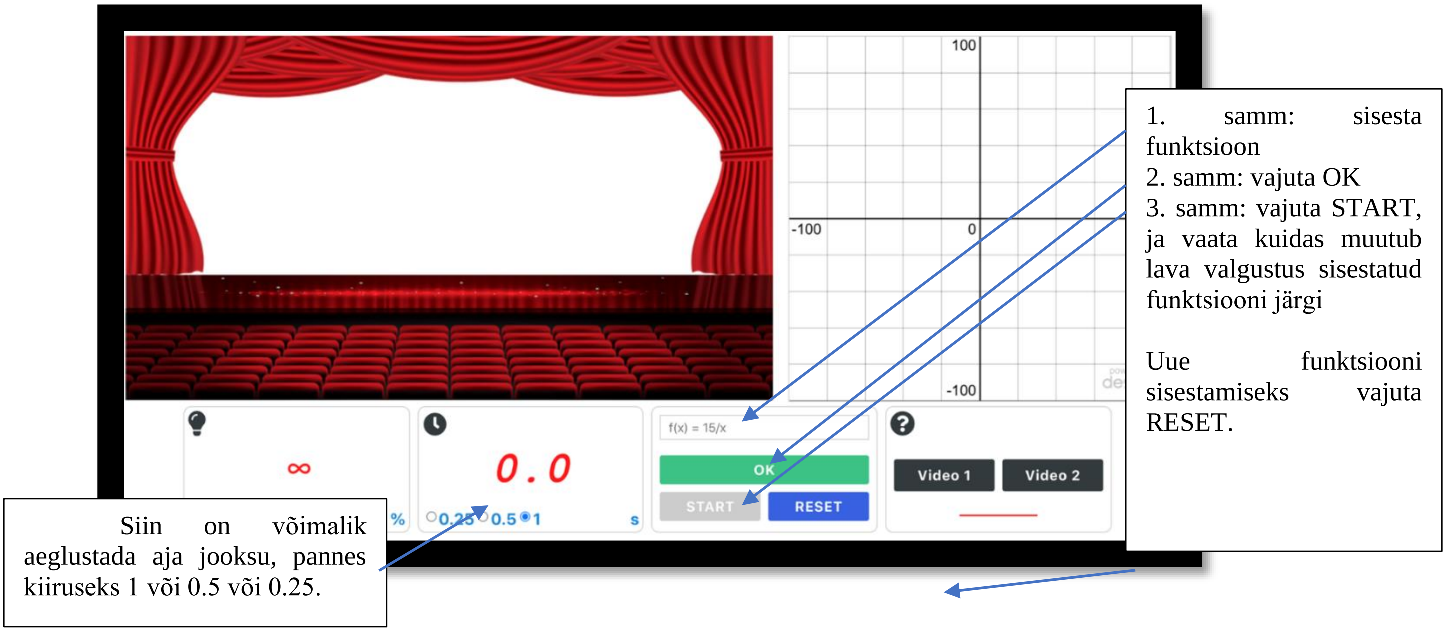
# Appendix I – Example of worksheet used during school pilot

## LINEAARFUNKTSIOON JA SELLE GRAAFIK

**Töö programmiga:**

Funktsioone võib kasutada erinevate protsesside juhtimiseks ja kontrollimiseks – näiteks valgustuse juhtimiseks. Järgnev lehekülg annab võimalust kasutada teadmisi lineaarfunktsioonidest teatris, lavavalgustuse juhtimisel: https://mat.dartef.com/SNA_1




Vaata videot, mis seletab lavavalgustuse juhtimist lineaarfunktsioonide abil (vajuta „Video 1“ nuppu programmis). Järgnevad ülesanded aitavad Sul paremini mõista funktsioonide olemust ja kasutusvõimalust:

1) Lava valgustus on juhitav funktsiooniga f(x)=10x. Mitu protsenti on lava valgustus ajahetkel x=2 sekundit? x=5 sekundit? Mitu sekundit kulub, et valgustada lava 100%-ni selle funktsiooni abil? Lahenda seda esmalt kirjalikult, seejärel sisesta funktsiooni f(x)=10x programmi, ja võrdle oma lahendust programmi tulemusega.

2) Lava valgustus on juhitav funktsiooniga f(x)=5x+50. Joonista funktsiooni graafik ja leia graafiku järgi:
   - Mitu protsenti on lavavalgustus kohe alguses (ajahetkel x=0)?
   - Mitu sekundit on vaja selleks, et lavavalgustus oleks 100% valgustatud? Leia graafiku järgi, seejärel võrdle lahendust programmi tulemusega.

3) Määra funktsiooni lavavalgustuse juhtimiseks, kus lineaarkordajaks on Sinu sünnikuupäev ja vabaliikmeks on Sinu sünnikuu number. Mitu sekundit kuulub, et valgustada lava 100%-ni selle funktsiooni abil? Lahenda kirjalikult, seejärel võrdle lahendust programmi tulemusega.

4) Lava valgustus on juhitav funktsiooniga f(x)=-10x+100.
   - Mitu protsenti on lavavalgustus kohe alguses (ajahetkel x=0)?
   - Mitu protsenti on lavavalgustus ajahetkel x=3? x=5?
   - Mitu sekundit kuulub, et pimendada lava 0%-ni selle funktsiooni abil? Lahenda kirjalikult, seejärel võrdle oma lahendust programmi tulemusega.

5) Lava valgustus on juhitav funktsiooniga f(x)=-5x+80.
   - Mitu protsenti on lavavalgustus kohe alguses (ajahetkel x=0)?
   - Mitu sekundit kuulub, et pimendada lava 0%-ni selle funktsiooni abil? Lahenda kirjalikult, seejärel võrdle oma lahendust programmi tulemusega.
   - Kuidas teha nii, et kohe alguses lava oleks valgustatud 60%?

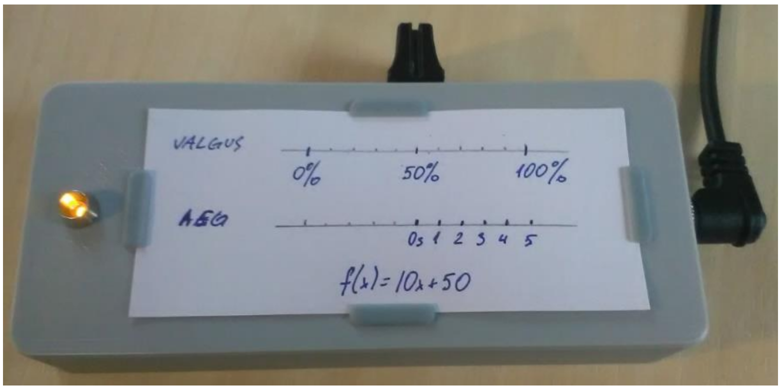
6) Sul on vaja muuta lava valgustust 50%-st 100-ni 5 sekundiga? Lahenda kirjalikult, seejärel võrdle oma lahendust programmi tulemusega.

7) Sul on vaja muuta lava valgustust 20%-st 90%-ni 14 sekundiga. Lahenda kirjalikult, seejärel võrdle oma lahendust programmi tulemusega.

**Töö õppevahendiga:**

1) Vahendi kasutamine teema seletamisel õpetaja poolt.
   - Näidata lihtsama funktsiooni (näiteks f(x)=10x) käitumist õppevahendi abil. See konkreetne juhtum on seletatud ka videos („Video 2“ nupp programmis).
   - Näidata f(x)=5x käitumist – selle funktsiooni muutus on kaks korda aeglasem kui f(x)=10x puhul.
   - Näidata f(x)=20x käitumist – selle funktsiooni muutus on kaks korda kiirem kui f(x)=10x puhul.

- Leppida, et näitame funktsiooni f(x)=10x. Seejärel panna õppevahendi nupp mingisse positsiooni, ja küsida õpilaselt, mis on x ja f(x) väärtused antud punktis. Samuti paluda õpilastel panna nupp mingisse positsiooni (näiteks „Näita nupu positsiooni f(x)=70% jaoks).
- Näidata vabaliikme olemust, näiteks funktsiooni f(x)=10x+50 puhul (50% ajahetkel x=0s):



# Appendix II – Questionnaire used during school pilot

## Lineaarfunktsiooni õppimine

Õppisid just koolis lineaarfunktsiooni. Kasutasid tundides ja kodutööde tegemisel erinevaid arvutiprogramme: lava valgustamine, isejuhtiva auto teekonna koostamine, autode möödasõit. Palun ütle meile nende programmide kohta oma arvamus.

Selleks vasta alljärgnevatele küsimustele.

* Required

Kas Sulle meeldis lahendada ülesandeid nende programmide abil? *

- ◯ Ei meeldinud
- ◯ Pigem ei meeldinud
- ◯ Pigem meeldis
- ◯ Väga meeldis

Kas need programmid tegid matemaatika tunde tavapärasest huvitavamaks? *

- ◯ Ei teinud huvitavamaks
- ◯ Pigem ei teinud huvitavamaks
- ◯ Pigem tegid huvitavamaks
- ◯ Tegid palju huvitavamaks

Kas need programmid tegid matemaatika kodutöid tavapärasest huvitavamaks? *

- ◯ Ei teinud huvitavamaks
- ◯ Pigem ei teinud huvitavamaks
- ◯ Pigem tegid huvitavamaks
- ◯ Tegid palju huvitavamaks

Kas Sul oli huvitav uurida kuidas Sa saad midagi juhtida (näiteks lavavalgustust või autosid) matemaatika abil? *

- Ei olnud huvitav
- Pigem ei olnud huvitav
- Pigem oli huvitav
- Oli väga huvitav

Kas sellisel viisil matemaatika õpetamine tundus päriselule lähedane? *

- Üldse ei tundunud lähedane päriselule
- Pigem ei tundunud lähedane päriselule
- Ma ei ole kindel
- See tundus suhteliselt lähedane küll
- See tundus väga lähedane päriselule

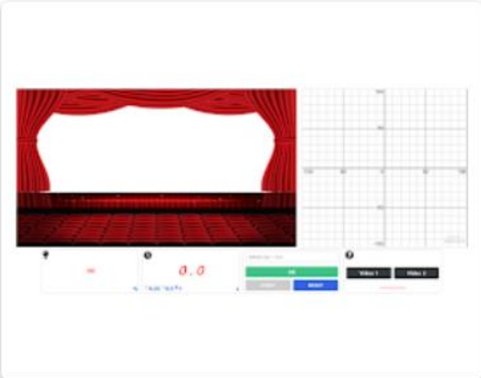
Kas Sa tahaksid teada milleks saab kasutada koolimatemaatikat päriselus? *

- Ei taha teada
- Pigem ei taha teada
- Pigem tahaksin teada
- Tahaksin väga teada

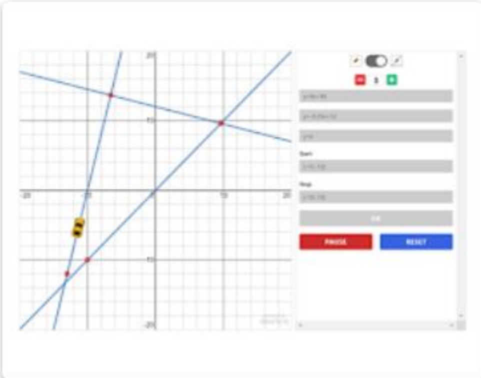
Kas Sa usud, et said hästi aru mis on lineaarfunktsioon? *

- Üldse ei saanud aru
- Sain aru vähesel määral
- Ma ei ole kindel
- Üsna hästi sain aru
- Sain täiesti aru

Milline programm Sulle kõige rohkem meeldis? Võid valida ka mitu. *




- [ ] Teatrilava valgustuse juhtimine



- [ ] Isejuhtiva auto teekonna koostamine


- [ ] Autode möödasõit

- [ ] Üksi ei meeldinud

Ütle meile rohkem - miks programmid meeldisid või ei meeldinud?

Your answer

Submit

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