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SOME REMARKS ON THE PHOTOMETRY OF COMETS

С РЕЗЮМЕ:
О ФОТОМЕТРИИ КОМЕТ

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Some Remarks on the Photometry of Comets.

In his articles about the physical processes in comets Wurm [1] explains the different chemical composition of the coma and the tail by photodissociation or -ionisation of molecules. According to his opinion the main molecules in the coma CN and C_2 are unstable in the radiation field of the sun, and disappear before they can reach the great distances from the nucleus into the tail; which consists of more stable molecules like CO^+ and N_2^+. Although Wurm's treatments are only of qualitative character, it ought to be possible to obtain from the suitably arranged photometric observations on the basis of this hypothesis some quantitative results also. The present note aims at pointing out some possibilities in this direction.

1. The theoretical distribution of brightness in the coma.

The mathematical treatment below is based on the following scheme of the structure of the coma. Gas molecules flow out from the nucleus, apparently under the influence of solar heat. Emitting during some time in the solar radiation field resonance bands in the limits of observable spectrum, these molecules disappear by photodissociation or -ionisation caused by the solar ultraviolet light quanta, since the new molecules evidently have no strong emission line in the observable region of the spectrum. Supposing that the molecules leave the nucleus in all directions uniformly with constant velocity v, the density n of the molecules, emitting observable light, at the distance R from the nucleus is

$$n = \frac{K}{4\pi R^2 v} e^{-\frac{R}{l}} \quad \dots\dots\dots\dots (1),$$

where K is the amount of molecules which leave the nucleus during a unit of time interval and the quantity $l = v \cdot \tau$ may be called the mean pathlength of molecules, if τ is their mean lifetime in the solar radiation field. For the sake of simplicity the velocity of molecules is assumed to be constant. The influence of nuclear gravitation and solar repulsive force upon the movement of the molecules is neglected also. The almost spherical shape of the coma seems to confirm partly this assumption. As W u r m [1] has pointed out, the molecules CN, C_2 and also CO^+ and N_2^+ observed in comets may be dissociation products or heavier molecules like C_2N_2 and CO_2, which the nucleus may originally contain. In this case the formula (1) holds good only, if the mean lifetime of the original molecules in the solar radiation field is short enough so that they would be practically completely dissociated comparatively near to the nucleus.

Assuming that the density of the matter in the comets is very small, the light emitted by the molecules is proportional to their amount and to the density of solar exciting radiation. Thus the apparent surface brightness i in the coma at the distance a from the nucleus may be written according to the mathematical treatment of K e e n a n [2 p. 7.]

$$i = \frac{1}{2\pi} \frac{C}{r^2} \frac{K}{v} \frac{1}{a} \int_0^{\frac{\pi}{2}} e^{-\frac{a}{l \sin \chi}} d\chi \quad . \quad . \quad . \quad . \quad . \quad (2),$$

where $R = a/\sin \chi$, χ is the phase angle, C is a constant, and r is the distance between the comet and the sun. It is not possible to express the integral in the formula (2) with simple analytical functions, only in the case when there is no dissociation of molecules, thus $l = \infty$.

$$i = \frac{1}{4} \frac{C}{r^2} \frac{K}{v} \frac{1}{a} \quad . \quad . \quad . \quad . \quad . \quad (2a).$$

But the general formula (2) may be written as follows, to express the brightnesses in the stellar magnitude scale.

$$H = -2{,}5 \log \alpha\, i = H_0 + S_1 \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where H_0 is a constant. In this formula the true distances are replaced by the apparent angular distances α, which may be directly measured. Whereas $a = \Delta\alpha$, if Δ is the distance of the comet from the earth, the quantity

$$H_0 = -2{,}5 \log \frac{1}{4} \frac{C}{r^2} \frac{K}{v} + 2{,}5 \log \Delta \quad . \quad . \quad . \quad . \quad . \quad (4).$$

By S_1 is denoted the following quantity

$$S_1 = -2{,}5 \log \frac{2}{\pi} \int_0^{\frac{\pi}{2}} e^{-\frac{a'}{\sin \chi}} d\chi \quad . \; . \; . \; . \; . \; . \quad (5).$$

In the formula (5) $a' = a/l$. The values of S_1 are computed by numerical integration and represented in the table 1, column 2 as function of a'.

Table 1.

a' \ $\frac{\alpha_2}{\alpha_1}$	values of ΔS_1: 0.00	0.25	0.50	0.75
0.0	0.00	0.00	0.00	0.00
0.2	0.43	0.31	0.20	0.10
0.4	0.80	0.57	0.37	0.18
0.6	1.13	0.80	0.51	0.24
0.8	1.43	1.00	0.63	0.30
1.0	1.71	1.18	0.74	0.35
1.2	1.98	1.36	0.85	0.41
1.4	2.24	1.53	0.96	0.46
1.6	2.50	1.70	1.07	0.52
1.8	2.75	1.86	1.18	0.58
2.0	3.00	2.03	1.29	0.63

It is evident that from the distribution of the surface brightness in the coma it is possible to determine the mean pathlength l of the molecules. Really it is necessary to know only for two points in the coma their distances from the nucleus $a_1 = \Delta \cdot \alpha_1$ and $a_2 = \Delta \cdot \alpha_2$ and the surface brightnesses i_1 and i_2. Thus the quantities $\alpha_1 \cdot i_1$ and $\alpha_2 \cdot i_2$ and also the difference

$$(-2{,}5 \log \alpha_2 i_2) - (-2{,}5 \log \alpha_1 i_1) = S_1\,(a'_2) - S_1\,(a'_1) = \Delta S_1$$

may be calculated. If S_1 is given there as the function of a', it is possible to find, for example by trials a'_1 and a'_2, which correspond to the known quantity ΔS_1 so that $\frac{a'_2}{a'_1} = \frac{a_2}{a_1} = \frac{\alpha_2}{\alpha_1}$. As it has been pointed out above, the mean pathlength of molecules is determined by the differences of S_1 at separate points, the zero point of the stellar magnitude scale of the surface brightnesses may be arbitrary. Thus for the present purpose it is necessary to know only the distribution of relative surface brightnesses in the coma. In order to simplify the calculation of the mean pathlength of molecules, there are given in the table 1, the values of ΔS_1 at some concrete values of $\frac{\alpha_2}{\alpha_1}$ as functions of a'_1. From a known distribution curve of the surface brightnesses in the coma it is always possible to choose the pairs of points, between which the difference ΔS_1 is to be calculated, so that the quotient $\frac{\alpha_2}{\alpha_1}$ equals some value given in the table 1. Comparing the values ΔS_1 calculated from the observation with those in the table, it is possible to find easily the corresponding value of a' and from it the mean pathlength of molecules.

On the other hand H_0 contains besides other quantities the number of molecules K, which leave the nucleus during a unit of time. A characteristic of this production of molecules may be defined as follows

$$Q = -2{,}5 \log C \frac{K}{v},$$

which can be obtained from the value of H_0, deduced directly from the observations, without further assumptions, while the constants C and v are not generally known. The quotient K/v denotes the number of molecules that leave the nucleus in the time interval during which a molecule moving with the velocity v would cover a unit distance. Thus Q is the brightness in stellar magnitudes of this amount of molecules, reduced to a unit distance from the sun and from the earth. If the apparent distance α from the nucleus is measured in minutes of arc and the surface brightness in stellar magnitudes per square minutes, then Q may be found from the following equation:

$$Q = H_0 - 5 \log 2r - 2{,}5 \log \Delta \quad . \quad . \quad . \quad . \quad . \quad (6).$$

Quantities r and Δ are measured in A. U. and the unit of the distance from the nucleus is the length ca 43500 km, that would have the parallax $1'$ at the distance of one A. U.

In the following the possibilities offered by the extrafocal photometry for examining the comets according to the scheme proposed above are treated. If not to take into account the imperfectness of the photographic objective, the brightness at a certain point of the extrafocal image of some object with perceivable areal extent corresponds to the intensity of the light integrated over the circular area around this point with the diameter equal to that of the extrafocal image of a pointlike lightsource (star). Thus at the centre of an extrafocal comet image the intensity of light is

$$I = 2\pi \int_0^{\alpha_0} i\alpha \, d\alpha - \frac{2\pi}{\Delta^2} \int_0^{a_0} i \, a \, da \quad . \quad . \quad . \quad . \quad . \quad . \quad (7),$$

where i is the apparent surface brightness as defined by the formula (2), α and a are respectively the apparent and true distance from the nucleus and α_0 the semidiameter of the extrafocal point image. If there is no dissociation of molecules, the integrated light I appears

$$I = \frac{\pi}{2} \frac{C}{r^2 \Delta^2} \frac{K}{v} \quad a_0 \quad . \quad . \quad . \quad . \quad . \quad (7a).$$

While I and $a_0 = \Delta\, \alpha_0$ may be measured directly, it is possible to find the rate of the production of molecules Q, if the mean pathlength l is known.

$$Q = M - 5 \log r\Delta + 2{,}5 \log \Delta\alpha_0 - S_2 + 0^{m}.49 \quad . \quad . \quad . \quad . \quad . \quad (8),$$

where $M = -2{,}5 \log I$ is the light intensity expressed in stellar magnitudes. The correction S_2 caused by the dissociation of molecules is a function of $a'_0 = \frac{a'_0}{l}$ only

$$S_2 = -2{,}5 \log \frac{2}{\pi} \frac{1}{a'_0} \int_0^{a'_0} da \int_0^{\frac{\pi}{2}} e^{-\frac{a'}{\sin \chi}} d\chi \quad . \quad . \quad . \quad . \quad . \quad (9).$$

The values of S_2 are given in table 2. column 2. If $a'_0 > 2$, the following approximative formula may be used

$$S_2 \approx 2{,}5 \log \frac{\pi}{2} a'_0.$$

It means that the molecules beyond the distance $a'_0 = 2$ from the nucleus add practically nothing to the integral light of the comets. In this case the formula (8) is simplified to

$$Q = M - 5 \log r \cdot \Delta + 2{,}5 \log l \quad . \quad . \quad . \quad . \quad . \quad (8a).$$

If there are two measurements of brightnesses of an extrafocal comet image received at different focal settings during a sufficiently short time interval from each other, that Q may be assumed constant, it is possible to determine the mean pathlength of molecules also. Denoting the

Table 2.

α_{02}/α_{01} → a'_{01} ↓	The values of ΔS_2			
	0.0	0.2	0.4	0.6
0.0	$0^m.00$	$0^m.00$	$0^m.00$	$0^m.00$
0.1	0 .07	0 .06	0 .04	0 .03
0.2	0 .16	0 .13	0 .10	0 .07
0.3	0 .28	0 .24	0 .19	0 .14
0.4	0 .37	0 .32	0 .25	0 .16
0.5	0 .46	0 .39	0 .30	0 .18
0.6	0 .54	0 .45	0 .33	0 .21
0.7	0 .62	0 .51	0 .36	0 .23
0.8	0 .69	0 .56	0 .39	0 .25
0.9	0 .76	0 .61	0 .42	0 .27
1.0	0 .82	0 .66	0 .45	0 .28
1.2	0 .93	0 .72	0 .49	0 .30
1.4	1 .03	0 .77	0 .52	0 .32
1.6	1 .13	0 .83	0 .56	0 .34
1.8	1 .22	0 .88	0 .59	0 .36
2.0	1. 31	0 .94	0 .62	0 .38
∞	∞	1 .75	1 .00	0 .56

semidiameters of the extrafocal point images at the two measurements α_{01} and α_{02}, the difference

$$2{,}5 \; log \frac{I_1}{\alpha_{01}} - 2{,}5 \; log \frac{I_2}{\alpha_{02}} = S_2(a'_{02}) - S_2(a'_{01}) = \Delta S_2$$

is determined by direct photometric measurements, as it follows from the equation (8). In the table 2 are given the values of ΔS_2 corresponding to the different quotients $\frac{a_{02}}{a_{01}} = \frac{\alpha_{02}}{\alpha_{01}}$ as functions of a'_{01}. While the quantities ΔS_2 and $\frac{\alpha_{02}}{\alpha_{01}}$ are known from observations, it is easy to find from the table 2 the corresponding value of a'_{01} and thus the mean pathlength $l = \frac{\alpha_{01}}{a'_{01}}$.

2. The distribution of the surface brightness in the comet's tail.

Contrary to the coma molecules, the dissociation of the molecules in the comet's tail may be neglected, but on the other hand the influence of the repulsive force of the sun must be accounted for. Since the present paper is confined to the parts of the tail near to the comet's head only, it is sufficient to assume the repulsive force of the sun as constant. It ought to be mentioned that this problem has been treated already by Mocknatch [3] but it is found neccessary to do that once more in a different way in order to pay special attention to some interesting circumstances. Now according to the so-called "fountain theory" the trajectories of the molecules in the tail are paraboles, and the envelope of the family of all possible trajectories is a paraboloid of revolution, if the molecules leave the nucleus with equal velocity in all directions. As Eddington [4. p. 457] and Mocknatch [3. p. 73] have shown, the space dencity n^* of molecules in this parabolic conoid may be written

$$n^* = \frac{k^*}{(vt^2) \cdot (gt \cos \varphi - v)} \quad . \quad . \quad . \quad . \quad . \quad (10),$$

where k^* is the amount of molecules leaving the nucleus during a unit of the time interval per unit of the solid angle in the given direction v — the initial velocity of the molecules, g — the acceleration caused by the solar repulsive force, φ — the angle between the radius-vector of the comet and the direction of ejection ($\varphi = o$ if the molecules are ejected towards the sun) and t — the time during which the molecules have been moving. Let there be a system of rectangular coordinates with the origin in the nucleus and the positive x axes coinciding with the direction of the prolonged radius vector. Further it is assumed that the direction of vision coincides with the direction of z axes. Thus the surface brightness at any point of the comet's tail appears

$$i^* = \frac{C^*}{r^2} \int n^* \, dz \quad . \quad . \quad . \quad . \quad . \quad (11),$$

where C^* is a constant. According to the "fountain theory" the particles ejected at the same moment from the nucleus are also at any other moment upon a spherical surface. The radius of the sphere is $R = vt$, as if the particles had been moving rectilinearly and uniformly with their initial velocity relatively to the center of the sphere, which lying on the x axes moves relatively to the nucleus as a particle under the solar repulsive acceleration g with zero initial velocity; thus its distance from the nucleus varies as $\frac{1}{2} gt^2$. On the basis of this geometrical conception it is possible to express the quantities t, $\sin\varphi$, $\cos\varphi$ and also n^* and z as functions of R, if the coordinates of a point x_0 and y_0, for which the surface brightness is to be determined, are given. Thus

$$z^2 = R^2\left(1 + 2\frac{g}{2v^2}x_0\right) - \left(\frac{g}{2v^2}\right)^2 R^4 - x_0^2 - y_0^2 \quad \ldots\ldots \quad (12)$$

and also the formula (11) may be expressed

$$i^* = 2\frac{C^*}{r^2}\int \frac{k^*}{v}\frac{dR}{R\sqrt{R^2\left(1+2\frac{g}{2v^2}x_0\right) - \left(\frac{g}{2v^2}\right)^2 R^4 - x_0^2 - y_0^2}} \quad \ldots\ldots \quad (11)$$

In order to simplify the mathematical treatment, the quantities k^* and v are assumed as constant. The coefficient 2 in the formula (11 a) is caused as follows: since the conoid is symmetrical relative to the xy-plane, it is necessary to take the intgral over one half of it only and to multiply the result by 2. The formula (11 a) may be written more simply also, if we take the new unit of length as $2v^2/g$. On the other hand it is known that the equation of the meridian curve of the parabolic conoid in the xy-plane is $y^2 = \frac{2v^2}{g}x + \frac{v^4}{g^2}$, where the new unit $2v^2/g$ equals the length of the double parameter of the parabola. If we introduce a new variable $u = R^2$ also, the final expression for the surface brightness appears

$$i^* = -\frac{C^*}{r^2}\frac{k^*}{v}\int_{u_1}^{u^2}\frac{du}{u\sqrt{u(1+2x_0) - u^2 - x_0^2 - y_0^2}} =$$

$$= 2\frac{C^*}{r^2}\frac{k^*}{v}\frac{1}{\sqrt{x_0^2+y_0^2}}\arctan\left.\sqrt{\frac{2(x_0^2+y_0^2) - u\left(2x_0+1-\sqrt{4x_0+1-4y_0^2}\right)}{u\left(2x_0+1+\sqrt{4x_0+1-4y_0^2}\right) - 2(x_0^2+y_0^2)}}\,\right|_{u_1}^{u_2} \quad (13).$$

At first we shall treat of the case where the molecules are ejected from the nucleus uniformly in all directions. It is known that at certain x_0, y_0, the spheres, mentioned before, with the smallest and also with the greatest radius R go through the point where $z = o$. Taking into account the denotations and assumptions made above, the relation

$$z^2 = u(1+2x_0) - u^2 - x_0^2 - y_0^2 = 0$$

as it follows from the formula (12) enables us to determine the limits of integration

$$\left.\begin{aligned} u_{01} &= \frac{1}{2}(2x_0 + 1 + \sqrt{4x_0 + 1 - 4y_0^2}) \\ u_{02} &= \frac{1}{2}(2x_0 + 1 - \sqrt{4x_0 + 1 - 4y_0^2}) \end{aligned}\right| \quad . \; . \; . \; . \; . \; . \; (14).$$

If these limits are placed in the formula (13), the result will be

$$i_0^* = \frac{C^*}{r^2}\,\frac{k^*}{v}\,\frac{\pi}{\sqrt{x_0^2 + y_0^2}} = \frac{1}{4}\,\frac{C^*}{r^2}\,\frac{K^*}{v}\,\frac{1}{a} \quad . \; . \; . \; . \; . \; . \; (15),$$

where $K^* = 4\pi k^*$ is the total production of the tail molecules and $a = \sqrt{x_0^2 + y_0^2}$ is the distance from the nucleus regardless of the direction. This relation holds good up to the boundary of the conoid. Besides the expression (15) is wholly identical with the formula (2a) derived for non-dissociating molecules, if there is no solar repulsive force. Since according to the formula (15) the surface brightness at the border of the parabolic conoid should not be zero, it should be defined quite clearly against the sky background, if the molecules leave the nucleus uniformly in all directions with the same constant velocity v. But as it follows from the observations, this is not generally the case. It seems as if the molecules are ejected mostly in separate streamers from the nucleus, which cause the rays in the tail.

It has appeared that the flow of molecules is directed towards the sun mainly. In the following a solution of the formula (13) is treated, which at least qualitatively represents the distribution of surface brightness in this case. So it is assumed that the molecules are ejected from the nucleus inside a circular cone, symmetrical to the x axes and directed towards the sun. But as it has been supposed previously when deriving the formula (13), the outflux k^* of molecules is constant throughout the cone, regardless of the direction. The cone of the outflux is determined by the limiting trajectories, which are the trajectories of molecules, whose initial velocities make the greatest possible angle φ_0 with the x axes. Since all the trajectories outside the cone are absent, the enveloping paraboloid is built up only partly near its vertex. Further the borders of the tail are determined by the limiting trajectories. The values of u at the smallest value of $z = o$ are given in the formula (14). In the present case the greatest value of z at the certain x_0, y_0, is given by

$$z^2 = R^2 \sin^2 \varphi_0 - y_0^2 = u \sin^2 \varphi_0 - y_0^2$$

and taking into account the formula (12) the equation for other limiting values of u is obtained as follows, if we use the new units of length

$$u^2 - (2x_0 + \cos^2 \varphi_0)\, u + x_0^2 = 0$$

$$\left.\begin{aligned} u_1 &= \frac{1}{2}\left[2x_0 + \cos^2 \varphi_0 - \cos \varphi_0 \sqrt{4x_0 + \cos^2 \varphi_0}\right] \\ u_2 &= \frac{1}{2}\left[2x_0 + \cos^2 \varphi_0 + \cos \varphi_0 \sqrt{4x_0 + \cos^2 \varphi_0}\right] \end{aligned}\right| \quad \ldots (16)$$

The combination of these four values of u as limits of integration in the formula (13) depends on the place of the given point in the tail. As it appears the distribution of the surface brightness must be treated separately for the three parts in the tail. Thus the surface brightness in the so called cone of outflux contains the light emitted by the molecules moving from the nucleus towards the sun, but to this is added the light of molecules, which have been turned back already by the solar repulsive force. The first quantity is accounted for if we take the limits of the integration u_1 and u_{02} and for the second correspondingly u_{01} and u_2. The total surface brightness is the sum of the two. Thus if we denote by $f(u)$ the value of *arctan* member in the formula (13) at a certain value of u then

$$i_I^* = 2 \frac{C^*}{r^2} \frac{k^*}{v} \frac{1}{\sqrt{x_0^2 + y_0^2}} \left[\frac{\pi}{2} - f(u_1) + f(u_2)\right]$$

in the adjacent area the flow of the molecules directed away from the sun is present only. Thus the integral must be taken within the limits u_{01} and u_2 and so

$$i_{II}^* = 2 \frac{C^*}{r^2} \frac{k^*}{v} \frac{1}{\sqrt{x_0^2 + y_0^2}} f(u_2)$$

But inside the area at the border of the tail, where the enveloping paraboloid is built up, all the possible trajectories are present. This part coincides therefore entirely with the case treated above, where all the molecules leave the nucleus uniformly in every direction and thus the surface brightness is given similarly to the formula (15) as

$$i_{III}^* = \frac{C^*}{r^2} \frac{k^*}{v} \frac{\pi}{\sqrt{x_0^2 + y_0^2}}.$$

As it follows, the surface brightness in the area *I* is considerably greater than in the adjacent part of the area *II* and must be plainly visible separately. That is fully in accordance with the observations, since the cone of outflux is frequently found in the comets' heads mainly at the time of great cometary activity. According to the theory and observations, also the enveloping area *III* around the vertex of the paraboloid is brighter than its neighbourhood.

There are still some interesting features in the theoretical distribution of the surface brightness in the area *II*. On the border of the tail, where it is not enveloped by the parabolic conoid, the surface brightness is zero. Towards the axes of the tail, which coincides with the x axes, the surface brightness increases at first, but then decreases to a minimum at the axes. The minimum is deepest directly behind the nucleus, but as the distance from the nucleus increases the minimum becomes more and more flat till it disappeares wholly. Accordingly there must be some kind of "shadow" behind the nucleus and indeed for example on some photographs of Halley's comet (1910 II) reproduced by Bobrovnikov [5] such a "shadow" is plainly perceivable.

3. **The observed surface brightness in the coma.**

The sufficiently precise photometric data about the comets are at the present moment too few in order to enable us to draw more detailed conclusions from them. Thus the aim of the present paper is only to show the possibility of some practical applications of the theory proposed above, while the numerical results possess less significance. The only suitable measurements about the distribution of the surface brightness in the coma, available in the present work, are those made photographically by Šternberk [6] at Finsler's comet (1937 f). The instrument used is a reflector with the aperture 60 cm and the focal ratio f : 5. The results of the photometric measurements are given by Šternberk [6. p. 20] in his table 1., where the distribution of the surface brightness along the direction sun — comet is represented. From the given surface brightness i and the distance from the nucleus, the quantities $H = -2{,}5 \log i \cdot a$ are calculated here, where a is measured in minutes of arc and the unit of the surface brightness is a $9^m.0$ star per a circle with the diameter $1'.04$. The results are represented in the table 3., where the calculated values are given as functions of a.

It appears, that from the nucleus in the direction opposite to the sun (in the table the lower half) the surface brightness is systematically

Table 3.

α	308/19		309/15		312/21		313/22		317/24		318/16		320/17	
	H	H_0	H	H_0	H	H_0	H	H_0	H	H_0	H	H_0	H	H_0
3'.87	0.57	—0.87												
3 .72	0.55	—0 82												
3 .57	0.52	—0.81												
3 .42	0.45	—0.83												
3 .28	0.43	—0.80	0.57	—0.66	0.43	—0 80								
3 .12	0.42	—0.77	0.51	—0.68	0.39	—0.80								
2 .98	0.42	—0.73	0.48	—0.66	0.38	—0.76								
2 .83	0.39	—0.71	0.41	—0.69	0.33	—0.77								
2 .68	0.31	—0.74	0.36	—0.69	0.31	—0.74							1.04	(—0.01)
2 .53	0.27	—0.73	0.28	—0.72	0.26	—0.74							0.92	(—0.13)
2 .38	0.19	—0.75	0.25	—0.69	0.19	—0.75							0.78	(—0.22)
2 .24	0.15	—0.73	0.21	—0.67	0.13	—0.75	0.23	—0.65					0.69	(—0.26)
2 .08	0.05	—0.78	0.12	—0.71	—0.01	—0.84	0.15	—0.68			0.52	—0.38	0.64	(—0.26)
1 .94	—0.04	—0.82	0.06	—0.72	—0.02	—0.80	0.11	—0.67			0.40	—0.44	0.50	(—0.34)
1 .79	—0.10	—0.82	—0.03	—0.75	—0.07	—0.79	0.03	—0.69			0.30	—0.48	0.37	—0.41
1 .64	—0.17	—0.83	—0.11	—0.77	—0.13	—0.79	—0.03	—0.69			0.12	—0.60	0.20	—0.53
1 .49	—0.17	—0.78	—0.15	—0.76	—0.13	—0.74	—0.08	—0.69			—0.02	—0.68	0.14	—0.52
1 .34	—0.19	—0.74	—0.23	—0.78	—0.19	—0.74	—0.15	—0.70			—0.08	—0.68	0.07	—0.53
1 .19	—0.27	—0.77	—0.30	—0.80	—0.22	—0.72	—0.22	—0.72	—0.02	—0.56	—0.19	—0.73	—0.04	—0.58
1 .04	—0.28	—0.73	—0.35	—0.80	—0.25	—0.70	—0.30	—0.75	—0.14	—0.62	—0.22	—0.70	—0.13	—0.61
0 .89	—0.31	—0.70	—0.40	—0.79	—0.33	—0.72	—0.38	—0.77	—0.25	—0.68	—0.30	—0.73	—0.20	—0.63
0 .74	—0.35	—0.68	—0.40	—0.73	—0.33	—0.66	—0.46	—0.79	—0.28	—0.65	—0.30	—0.67	—0.24	—0.61
0 .60	—0.42	—0.68	—0.41	—0.67	—0.42	—0.68	—0.53	—0.79	—0.29	—0.59	—0.33	—0.63	—0.27	—0.57
0 .45	—0.36	—0.55	—0.34	—0.53	—0.51	—0.70	—0 51	—0.70	—0.27	—0.49	—0.38	—0.60	—0.25	—0.47
0 .30	—0.54	—0.66	—0.57	—0.69	—0.64	—0.76	—0.59	—0.71	—0.41	—0.54	—0.39	—0.52	—0.42	—0.53
0 .15	—0.07		0.00		—0.08		0.00		0.11		—0.02		—0.02	

							—		—		—		—	
0'.00	—		—		—		0.02		0.00		0.06		—0.07	
0 .15	—0.12		—0.03		—0.36		—0.58		—0.53		—0.51		—0.58	
0 .30	—0.55		—0.63		—0.76		—0.42		—0.37		—0.32		—0.38	
0 .45	—0.39		—0.41		—0.57		—0.45		—0.38		—0.37		—0.39	H_0^*
0 .60	—0.41		—0.45		—0.51		—0.41		—0.31		—0.36		—0.40	1.56
0 .74	—0.37		—0.50		—0.46		—0.39		—0.30		—0.33		—0.39	1.61
0 .89	—0.38		—0.48		—0.41		—0.33		—0.22		—0.27		—0.29	1.87
1 .04	—0.36		—0.43		—0.35		—0.28		—0.17		—0.24		—0.25	1.65
1 .19	—0.35		—0.41		—0.35		—0.23		—0.13		—0.16		—0.24	1.26
1 .34	—0.32		—0.36		—0.32		—0.16		—0.15		—0.05		—0.15	1.42
1 .49	—0.28		—0.29	H_0^*	—0.27		—0.14				0.03		—0.10	1.44
1 .64	—0.29		—0.28	1.82	—0.27		—0.07				0.21		0.02	1.42
1 .79	—0.23		—0.21	1.84	—0.21		—0.03				0.34		0.13	1.48
1 .94	—0.18	H_0^*	—0.14	1.80	—0.17		0.06						0.16	1.28
2 .08	—0.13	1.92	—0.11	1.69	—0.14	H_0^*	0.10						0.24	1.41
2 .24	—0.11	1.56	0.01	1.85	—0.11	1.64							0.28	1.36
2 .38	—0.07	1.60	0.02	1.82	—0.04	1.76							0.33	1.27
2 .53	—0.04	1.46	0.04	1.79	0.00	1.67							0.40	1.26
2 .68	—0.01	1.47	0.12	1.87	0.02	1.59								
2 .83	0.02	1.39	0.14	1.78	0.04	1.61								
2 .98	0.07	1.47	0.20	1.80	0.08	1.62								
3 .12	0.08	1.50	0.23	1.83	0.09	1.63								
3 .28	0.15	1.75	0.26	1.76	0.11	1.59								
3 .42	0.17	1.77	0.28		0.12									
3 .57	0.20	1.68	0.35		0.13									
3 .72	0.23	1.71	0.37		0.15									
3 .87	0.24	1.69	0.43		0.18									
4 .02					0.19									
4 .17					0.20									
4 .32					0.21									

greater than at the same distance towards the sun. That may be expected, because in the former case to the light of the coma is added the radiation of the tail also. On the other hand it seems quite reasonable to assume that the surface brightness from the nucleus towards the sun is due to the coma molecules only. On this assumption the mean pathlength l is derived for the coma molecules, as it has been described above in Chap. 1.

The numerical data are given in the table 4. The successive columns contain the denotations of the plates by Šternberk, the dates of the observations, the distances from the sun and from the earth and the apparent mean pathlength l found from the observations. In the following column are given the values of the true mean pathlength of molecules $l_1 = \Delta \cdot l$ reduced to the unit distance from the earth. But $l_1 = v \cdot \tau$ depends on the mean lifetime τ of the molecules and on the velocity of ejection v. Since the mean lifetime is inversely proportional to the density of the dissociating radiation, it is proportional to the square of the distance from the sun. On the other hand, according to Wurm's theory [1], as the observable molecules are photodissociation products of some more complicate molecules, and their initial velocities are caused by the residual energy of light quanta absorbed at the dissociation, the velocity v is independent of the distance from the sun. But if we should assume that the velocity were due to the termal motion of molecules, and the temperature of the nucleus equals the equilibrium temperature of a "black body" in the solar radiation field, the velocity of ejection would be inversely proportional to the fourth root of the distance from the sun only, and so its value would remain practically constant too in the limits of the present observations. In conclusion it may be expected that the quanti-

Table 4.

Plate	Date	r	Δ	l	l	l_1	H	Q	H_0^*
	1937								
308/19	Aug. 7.01	0.878	0.556	5′.78	3′.20	4′.15	—0m.75±0.02	7m.48	1m.61±0.05
309/15	„ 8.88	0.872	0.550	5′.00	2′.75	3′.65	—0 .71+0.02	7 .55	1 .80 ±0.02
312/21	„ 9.88	0.869	0.551	5′.47	3′.01	4′.02	—0 .75±0.01	7 .52	1 .64±0.02
313/22	„ 9.93	„	„	4′.25	2′.35	3′.15	—0 .71±0.02	7 .56	
317/24	„ 12.85	0.864	0.573				—0 .59+0.03	7 .64	
318/16	„ 12.86	„	„				—0 .60+0.03	7 .63	
320/17	„ 12.95	„	„	3′.50	2′.00	2′.70	—0 .55±0.03	7 .68	1 .45±0.05
					Mean	3′.53±0′.30			

ties $l_0 = l_1/r^2$ given further in the table 4. must be constant. As it appears, the dispersion of the value of l_0 is quite considerable, but it seems very likely that the deviations are caused by errors in the observations. The values of the mean pathlength are very sensitive to the errors in the slope of the distribution of the surface brightness. The slope is especially influenced by the errors of a systematic character, like the errors in the intensity scale and some photographic effects (Eberhard effect). Their influence may be different for separate plates. As it follows, the present "photometric" measurement of the size of coma is not much more precise than the direct measurements with the micrometer, but it has the advantage of being free from the systematic errors of the latter caused by the visibility conditions. It seems that the errors in the values of the mean pathlength are more or less accidental, unless there is no incontinuity in the output of molecules. On this account the mean pathlength as determined above is more suitable for the study of the excitation conditions of the molecules in the comets than the directly measured diameters of the coma. In order to receive sufficiently precise photometric data from a comet, it is necessary to use photographic instruments with a large linear scale, and of great light-gathering power such as reflectors. Although the instrument used in the present case satisfies these conditions, the observations are too few and cover a short time interval only. Therefore no further conclusion may be obtained beside the mean value of the pathlength of the coma molecules reduced at the unit distances from the sun and from the earth $l_0 = 3'.53$ or $1.5 \cdot 10^5$ km.

On the other hand, the output of the coma molecules is directly dependent on the value H_0, defined according to the formula (3)

$$H_0 = H - S_1.$$

While H is given in the table 3., S_1 may be taken from the table 1, if the mean pathlength l is known. Since the dispersion in the individual values of l is considerable, as it appeared above, it was decided to use the mean value of $l_0 = 3'.53$ and to calculate the apparent pathlength for each plate according to the formula

$$l = \frac{l_0\, r^2}{\Delta} \quad \ldots\ldots\ldots \quad (17).$$

The values of H_0 found from the single measurements are given in the table 3. and their arithmetical means with the mean errors are briefly represented for each plate in the table 4. There are given also the equivalents Q to the output of the coma molecules, calculated from H_0 according to the formula (6). The latter are given in more common units

($0^m.0$ star per square minute) instead of those used by Šternberk. Since the observations include only a small time interval, no major change in the output of the coma molecules can be detected. Further observations of that kind would give valuable material for studying the physical processes in comets.

As it has been said above, the systematically greater surface brightness found in the direction along the tail, may be explained by the extra radiation of the tail molecules. In the following it is attempted to estimate the intensity of this radiation. Its separation from the measured total brightness is possible only, if the other component — the brightness of the coma — is known. For this purpose it was assumed that the distribution ot the surface brightness in the coma is the same in all directions, as it is measured in the direction from the nucleus towards the sun already. Practically not the surface brightness i^* itself, caused by the radiation of the tail molecules, but the value $H_0^* = -2.5 \log a i^*$ was calculated. This quantity was found directly from the difference between the values of H taken from the table 3. for the points which lie at equal distances from the nucleus but in opposite directions from each other. The results of the calculation are given in the same table 3. The computation was carried out for the points only where the separation of the radiation of the tail molecules from that of the coma was possible at least with some certainty. These points lie mainly at some distance from the nucleus, where the radiation of the coma is fainter.

The values of H_0^* appear constant, regardless of the distance from the nucleus. That seems to be in accordance with the formula (15) derived for the case if the tail molecules leave the nucleus uniformly in all directions, but the present data are too uncertain for such a conclusion. The presence of streamers in the tail of Finsler's comet (1937 f) at that time confirms quite an opposite opinion. In every case H_0^*, as calculated above, characterizes the actinic efficiency of the tail molecules in the comet's head. On this account in the table 4 are given the arithmetical mean values of H_0^* and their mean dispersions for each plate. Comparing the mean value of H_0^* with these belonging to the coma molecules, it is quite obvious that at least in the present case the radiation of the tail molecules has little influence upon the total brightness of the comet's head. During the period of observations no certain change in the brightness of the tail was found.

From the ordinary photographs measured surface brightnesses are treated above, as if they were belonging to one certain sort of molecules in the coma only, in which case the present theory holds good.

But really there are at least two different kinds of molecules, which have prominent emission bands in the photographic region, namely CN at $\lambda = 3880$ Å and C_2 at $\lambda = 4700$ Å. Thus for more detailed investigation it is surely necessary to separate these radiations from each other, but this can be done by the spectrophotometric method only. While such observations are as yet entirely lacking, some preliminary results may be obtained on the basis of the photometry in the integrated light also; only the numerical values of the mean pathlength and of the output of the molecules found thus are averages corresponding to the combined effect of the several kinds of coma molecules. As it appears, the scheme proposed in Chap. 1, seems to be in accordance with the observational data as a whole, although it is only a rough approximation to the true circumstances.

4. **Extrafocal photometry of comets.**

For the present purpose there were available only a few observations mady by the author upon the comets Jurlov-Achmarov-Hassel (1939 d) and Cunningham (1940 c) at the Tartu Observatory. The camera used has an apochromatic objective of Steinheil with the aperture 6 cm and the focal ratio $f:10$. The photographic density over the whole extrafocal image is very uniform and so this objective is suitable for extrafocal photometry. The photographic plates were all Agfa Isochrom used without any filter. Further data are given in the table 5. The successive columns contain the date of the observation in universal time, the distance of the comet from the sun r and from the earth Δ, the semidiameter a_0 of the extrafocal image and the brightness in stellar magnitudes M of the extrafocal comet image. The brightnesses of the comets are determined relative to the adjacent stars, the photographic density being measured with a microphotometer of Hartmann's type. Although the plates used are orthochromatic, the magnitudes of the comparison stars, chosen only of the spectral type B—F, are photographic; taken from the H. D. catalogue for the comet 1939 d and for the comet 1940 c from Harvard Mim. Ser. I № 2 and 3. In the table 5. the last value of M is quite uncertain, because it has been obtained by extrapolation.

There are two photographs of the comet 1940 c obtained at the same date with different focal settings, which are suitable for determining the mean pathlength of coma molecules. From the data given in the table 5. the values of $\Delta S_2 = 0^m.50$ and $a_2/a_1 = 0.225$ may be found, from which follows $a_1 = 0.76$ as taken from the table 2. Thus the apparent

Table 5.

	r	Δ	α_0	M	Q_1	Q_2
☄ 1939 d						
Apr. 25.8	0.629	0.803	$1'.01$	$5^{m}.96$		
„ 25.8			$1'.01$	$5^{m}.90$		
		Mean	$1'.01$	$5^{m}.93$	$7^{m}.67$	$7^{m}.17$
☄ 1940 c						
Dec. 16.65	0.859	0.932	$0'.365$	$7^{m}.58$	$7^{m}.38$	$7^{m}.29$
„ 16.66			$1'.62$	$6^{m}.47$	$7^{m}.88$	$7^{m}.37$
„ 27.65	0.640	0.758	$2'.12$	$5^{m}.00$	$7^{m}.58$	$6^{m}.70$

mean pathlength is obtained as $l = 2'.13$ and also the corresponding true and reduced ones respectively $l_1 = 1'.16$, $l_0 = 2'.65$. Comparing this value of the mean reduced pathlength with those found in the previous chapter, they are in full agreement within the limits of the observational errors. This extrafocal method is more suitable for small cameras. But in order to obtain the results with some precision, it is necessary to arrange the observations so that one photograph is taken with the smallest possible extrafocal image and the other with a sufficiently large extrafocal image, that the greater part of the total brightness of the comet's head is integrated. This method has a certain disadvantage compared with the former more detailed one, which was used in the previous chapter. Namely the ideal conditions assumed for the derivation of this method are disturbed by the radiation from the nucleus and the tail molecules. Since the influence of these factors cannot be easily eliminated, the extrafocal photometry enables us to get only preliminary results about the mean pathlength of the coma molecules.

The measurements of the integrated brightnesses of the comets may be carried out from the extrafocal photographs available with sufficient precision, the probable error being below $\pm 0^{m}.1$. But as it appears from the formula (8), it is necessary to know the value of the mean pathlength in order to determine the equivalent to the output of the coma molecules, and since the data about the mean pathlength are only of a preliminary character, it introduces the main uncertainty into the value of Q. In the present case the quantities of Q are calculated according to two different suppositions. In the table 5. column 6. are given the values of Q_1, which are obtained on the assumption that there is no dissociation of the molecules and the values of Q_2 in the column 7 are found calculating the mean pathlength according to the formula (17), where the rough value

$l_0 = 3'.5$ was used. It is clear by itself, that the values of Q_2 are nearer to the truth than those of Q_1, but the difference between Q_1 and Q_2 characterizes the possible influence of the uncertainty in the mean pathlength. As it appears, the equivalent to the output of molecules is not highly sensitive to the concrete value of the mean pathlength. Its influence is the smaller, the smaller the semidiameter of the extrafocal image is. Thus it seems that the value of Q may be found exactly enough, for example even disregarding the dissociation of the molecules entirely, if we take the photographs with sufficiently small extrafocal images. But really the precise photometric measurement on too small extrafocal images is difficult, because the density distribution over the images becomes more uneven with the diminution of their size, due to the imperfectness of the photographic objective. Also the absolute error in the measurement of the diameters of extrafocal images remains the same, regardless of their size, but the relative error, on which depends the error in Q, consequently is greater in the smaller images. Thus practically there exists a lower limit of the size of the extrafocal images, dependant on the photographic objective used, beyond which no precise photometric work is possible. It must be pointed out that the mean pathlength has the greatest influence upon the zero point of Q, while the relative variation of the output of molecules is much less dependent on the error in the assumed value of the mean pathlength, especially if the conditions of the observations are uniform throughout the whole series. Since it is of great importance to know the relative variation of the output of the radiating molecules in the comets with the distance from the sun, the absolute value being of minor importance only, it may be expected that suitably arranged extrafocal photometry would give more information about the comets than the estimated so called total brightnesses, as they are in use up to the present. As it has been said above, the observational data available when prepaing the present paper are too few in order to draw from them further conclusions and thus they serve only as numerical examples to the application of the theoretical scheme proposed.

In the paper above it is supposed that the integrated brightness of the comet's head belongs to the coma molecules only, the radiation of the tail molecules and the nucleus being neglected entirely. As it appeared in the previous chapter, in most cases in the comet's head the radiation of the tail molecules is rather small compared with that of the coma molecules, and may be neclected, if not the highest precision is required. The same may be said of the radiation due to the nucleus, especially if the extrafocal images are not very small. In every case the extrafocal photometry does not enable detailed study of the comets, for which more refined measurements of the surface brightness are necessary, carried out if possible in the light of different emission bands separated spectrographically. But this requires powerful instrumental equipment, while extrafocal photometry can be carried out with small instruments. The theoretical scheme proposed in the present article must be regarded as a first approximation to the real condition in the comets, and thus using more detailed methods in observations it is necessary to improve the theory also. But it may be hoped that at the present stage of our knowledge about the comets this approximative method would enable to obtain some interesting results too.

Literature.

1. K. Wurm. Beitrag zur Deutung der Vorgänge in Kometen I; II. Zeitschr. f. Astrophys. $\mathbf{8}_{281}$ (1934); $\mathbf{9}_{62}$ (1935).
2. P. C. Keenan. Photometry of the Diffuse Nebula NGC 7023. Contrib. from the Perk ngs Obs. 5 = Astrophys. Journ. $\mathbf{84}_{600}$ (1935).
3. D. O. Mocknatch. On the Physical Processes in the Head of the Comets. Leningrad Univ. Ann. **22** Astr. Iss. $\mathbf{4}_{32}$ (1938).
4. A. S. Eddington. The Envelopes of Comet Morehouse (1908 c). Monthly Not. of R. A. S. $\mathbf{70}_{442}$ (1910).
5. N. T. Bobrovnikov. Halley Comet in Its Apparition of 1909—1910. Lick. Obs. Publ. XVII. Part II (1931).
6. B. Šternberk. Le profil photométrique de la tête de comète Finsler au voisinage du périhéle. Travaux dédiés a F. Nusl, pg. 17 (1938).

О фотометрии комет.

Резюме.

В настоящей работе рассматривается теоретически распределение поверхностной яркости в коме с учётом исчезания излучающих видимый свет молекул, вызванного фотодиссоциацией или фотоионизацией в поле излучения Солнца (как это допускается К. Вурмом).

Показано, как по наблюдаемому распределению поверхностной яркости можно вычислить среднюю длину пути молекул комы и число молекул, которые отделяются от ядра кометы в единицу времени. Далее показано, как экстрафокальную фотометрию комет можно применять для той же цели. Для некоторых простых случаев вычислено теоретическое распределение поверхностной яркости излучения молекул хвоста. Наконец, некоторыми численными примерами иллюстрировано применение теоретических формул.

Sisukord.

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