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Capital Asset Pricing Model (CAPM) and Fama-French Three Factor Model (FF3)

Financial Mathematics

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Master thesis by Parviz Piriyev

Abstract. The Capital Asset Pricing Model (CAPM) is a financial model that interprets the any possible relationship between the systematic risk of an individual stock or portfolio and its expected return. The model first introduced by William Forsyth Sharpe in 1964 as an extension of the earlier research done by Harry Markowitz about modern portfolio theory. CAPM theory considers the Beta as the only important factor that affects the expected return of an asset. Despite being simple to use and depict, the model is disapproved by many scientists because of its unrealistic assumptions and lack of ability to explain the risk-return connection. However later in 1993 Eugene Fama and Kenneth French came up with the adjusted version of model (FF3) which has 2 more factors that would affect the required return of an investment. In this paper we will test both of those models on 30 individual stocks on monthly returns in New York Stock Exchange (NYSE) to see how strong is the beta and 2 other factors are in terms ofexplaining the average returns.

Keywords: Capital Asset Pricing Model (CAPM), Fama-French 3 factor model, Asset Pricing.

CERCS research specialization: P160 Statistics, operation research, programming, actuarial mathematics.

Finantsvara hinnastamise mudel (CAPM) ja Fama-French kolme faktoriga mudel (FF3)

Magistritöö

Parviz Piriyev

Lühikokkuvõte. Finantsvara hinnastamise mudel (CAPM) on finantsmudel, mis selgitab üksiku aktsia või portfelli süstemaatilise riski seost oodatava tulususega. CAPM mudeli pakkus välja W.F. Sharpe 1964.a. H. Markowitzi portfelliteooria edasiarendusena. CAPM mudeli kohaselt nn beeta kordaja on ainus factor, mis mõjutab finantsvara tulusust. Vaatamata oma lihtsusele pole CAPM mudel siiski leidnud paljude teadlaste poolt tunnustust ja seda eeskätt ebareaalste eelduste pärast ning suutmatuse tõttu seletada piisavalt riski ja tulususe vahelist seost. E. Fama ja K. French pakkusid seepärast 1993.a. välja mudeli täiendatud versiooni (FF3) , kus on 2 täiendavat faktorit, mis mõjutavad investeeringu oodatavat tulusust. Antud töös on testitud mõlemat mudelit NYSE 30 aktsia tulususte statistika baasil, et selgitada välja, kui hästi beeta ja 2 ülejäänud faktorit seletavad tulususi.

Märksõnad: finantsvara hinnastamise mudel, Fama-French 3 faktoriga mudel, varade hinnastamine.

CERCS teaduseriala: P160 Statistika, operatsioonianalüüs, programmeerimine, finants-ja

kindlustusmatemaatika.

Dedication

To my family, friends

and

my lovely Margaret

Who always supported me.

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Contents

Introduction

In financial world, one of the main concerns is how the asset risk and expected return are correlated. The Capital Asset Pricing Model (CAPM) is simple and strong way to demonstrate the risk-award relationship of the financial investment. From first look it seems like an ordinary approach, but it is still a hot topic in economic world and combines decades of research in itself.

CAPM has been developed by William Sharpe (1964), Jack Treynor (1962), John Lintner (1965) and Jan Mossin (1966). Shortly the model supposes that non-diversifiable risk (in other words, Beta (β)) is the only factor that affects the expected return on individual stock or portfolio. For individual stocks, Security Market Line (SML) can be used to visualize relationship between expected return and systematic risk. Also, the model is built on some assumptions which are shortly touched on theoretical part of the paper. Despite being criticized and blamed by scientists, model still keeps its power in terms of explaining the expected return of the investment in modern financial sphere.

However more advanced version of the model is introduced by Eugene Fama and Kenneth French in 1993 to explain the stock returns. Unlike in standard CAPM, Fama and French version of model introduces 2 more factors, size-SMB (Small Minus Big) and value-HML (High Minus Low) which are going to be explained in following chapters. According to three factor model, Beta (β) itself is not enough to estimate the expected stock returns.

Many researches have been conducted to examine the capability of the Fama French three factor model on expected rate of returns, at the same time some other have been implemented to check if model outperforms the standard version of the model.

The purpose of the thesis is to test both of the models on 30 chosen individual stocks in US stock market, between 2014 - 2019 time frame on monthly basis to see how well Beta (β) and other 2 factors explain the outcomes and compare the expected return obtained by models.

In 1st and 2nd chapters some theoretical background is given to have more detailed idea about the models. In 3rd and 4th chapters testing methodology and conclusion part is covered.

1.Theoretical Background of CAPM

1.1.Standard Capital Asset Pricing Model

The prediction of the price of financial instruments is one of the main topics in Finance. The main issue is how much risk investors would consider for the given amount of expected return. In this sense relationship between the risk and return gives us the value of the investment. According to classical financial theory the risk that investors are faced can be eliminated by diversification but in Modern finance theory the relationship between the assets themselves should be used to eliminate the risk.

The CAPM is the most essential and favored in asset pricing models. This model clarifies the relation between the return of an asset and the risk required for that return. In other words, model investigates whether or not the return of an invested asset is optimum for the given risk amount. Also, the CAPM can estimate the future price of an asset which has not even started to operate in the market yet. [14]

In order to be able to explain this relationship between the risk and return William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966) have developed the standard version of the model. According to the model the only factor that can affect the final result is estimated beta. Beta is the most important element of CAPM. In the model, the tendency of a stock value to move together with the market value is reflected by the beta coefficient, which measures the variability of the stock according to an average value of the market. This was the first established equilibrium model evolved and it is found on the most rigid set of assumptions. The reason why the model is so widespread and famous is because it is theoretically easy to understand and implement in real life. The actual world is adequately complicated to understand it and establish the models of how it performs. In order to eliminate the effects of those complexities some assumptions have to be made. Below some of the most important assumptions are mentioned: [1]

- The first assumption is that there is no fee for the transactions. If there would be a fee for the transaction, the return of an investment would be the function which in turn explains whether or not the investment has been owned before the decision period.

- The second assumption is that the investments are infinitely divisible. It means that an investor can take any action about the investment, regardless of his /her financial situation. For instance, an investor is allowed to buy as small as one Euro worth of Apple stock.

- Third assumption is that, only an investor by himself cannot affect the price of the investment by his buying or selling decisions. This is similar to the assumption of ideal competition, where no single individual can alter the prices.

- Fourth assumption is that investors are expected to determine their decisions only based on the risk – return relationship, in other words, expected return and standard deviation.

- According to the fifth assumption the investors are allowed to make unlimited short sales.

- In the sixth assumption it says that all the investors can lend and borrow any number of stocks at risk free rate.

- The seventh assumption says that the personal income taxes are kept out of the CAPM model.

- The eighth assumption is about homogeneity of expectations. It is the assumption from Harry Markowitz's Modern Portfolio Theory and states that all the investors have the identical expectations and they will have the same decisions under the given conditions. For example, if investors have been offered several investment options which have same risk but different returns then they will choose the one with highest return and similarly if they are being offered with the investment plans which have the same returns but different risks then they will choose the one with the lowest risk.

- The ninth assumption says that unlimited free borrowing and lending is possible in the market. The investor can make unlimited investments on a risk-free rate and lend money. The investor is interested in the returns and risks of financial assets, and this has made the curve of efficient frontier in Modern Portfolio Theory a linear efficient frontier. By investing in a risk-free financial asset, the risk can be reduced or decreased.

These assumptions mean that each investor has the same knowledge and opinion about the future financial expectations of the financial asset and that they analyze the information by the same methods. Due to the common behavior of all investors in the market, the structure of the balance relationship between risk and return of each financial asset is being developed.

In real world many studies have examined the effectiveness of the model, but it is still broadly used in financial world. In spite of the fact that diversity of the individual stocks is complicated process to estimate through Beta (β) value, but it is still believed that an investment portfolio with bigger β value has bigger volatility than the market itself regardless of market value goes up or down while it has the same properties for vice versa condition in terms of β value. [1]

1.2.Definition of Beta

In this work, monthly rate of returns for individual stocks and market portfolio are calculated with the following formula:

$$r_{it} = \frac{p_{it} - p_{it-1}}{p_{it-1}} * 100\,, \qquad (1)$$

where,

r_{it} is the rate of return,

p_{it} is the adjusted close price at time t,

p_{it-1} is the adjusted close price at time t-1.

In order to calculate the β coefficient for a stock i the following regression model is used:

$$R_{it} = \alpha_i + \beta_i \cdot r_m + \varepsilon \qquad (2)$$

where,

α_i is the intercept term,

β_i is the stocks sensitivity coefficient to the market portfolio,

r_m is the market return (return of the market portfolio, for example Dow Jones Index),

ε is the model error.

By standard linear regression theory, the Beta coefficient can be estimated as:

$$\beta_i = \frac{\mathrm{Cov}(r_i, \mathrm{r_m})}{\mathrm{Var}\,(\mathrm{r_m})}$$

where

r_i is the return on a specific stock *i*,

$\mathrm{r_m}$ is market return,

$\mathrm{Cov}(r_i, \mathrm{r_m})$ is the covariance (how modifications in returns of a stock are related to the modifications in the returns of a market)

$\mathrm{Var}\ (\mathrm{r_m})$ is the variance (how far the data points of a market spread out form their value that is average).

Despite of being relatively easy to calculate the β, there are some issues and questions are rising before calculation in order to get more accurate results.

1. Time period: Mostly when β coefficient is calculated the taken time frame is approximately between 2 and 9 years. Time period is very important variable in calculation as it directly affects the accuracy of the result to a big extent. If the taken time period is short, then it more indicates the current stock change of the company. For instance, for an entity that has done some structural and managerial changes, then it is better to use short time frame to calculate the beta, otherwise for bigger and more stable companies, longer estimation period is acceptable.

2. Yield frequency: While making the regression, it is possible to consider daily, weekly and monthly return of security but which interval to choose it is debatable and individual preference. Commonly taking low frequency data will result less accuracy with β coefficient and vice versa however, high frequency data holds big economical and financial "noise". Also, other effecting factor here is the time period. For longer time frames e.g. 5 or more years it is more suitable using low frequency.

3. Small-cap securities: Generally small cap securities bear more risk and therefore more potential return. Thus, analysts may want to adjust beta of small cup securities upwards. [21]

Taking into consideration above variables, it is possible to calculate the beta of publicly traded entities, as it is possible to find all the relevant stock price data. But there is a problem with non-public companies for which there is no available stock yield information. In this case, analysts might check the accounting data in order to obtain beta value.

After considering all those affecting variables for calculation of β for 30 stocks from US stock market have been chosen. Companies are chosen by their average beta value in recent years from smallest being 0.2 to the biggest being 3 and the time frame is chosen from 2014 March till 2019 March. The reason for taking the most recent data is to avoid the last world financial crisis effects. Lastly, as a market proxy Standard & Poor 500 index is chosen.

1.3.Unsystematic and Systematic risk

Risk is defined as objective uncertainty or the likelihood of unintended consequences from any occurring event. In finance, companies are mainly faced with non-systematic risks and systematic risk. Non-systematic risks are the risks that are specific to a single sector or company and only effect those who are in the same operational field. Operational risk, financial risk, managerial risk can be given as an example of non-systematic risk. Companies have the chance and ability to control and eliminate such kind of risks. On the other hand, systematic risks are arising from overall economic, political and other environmental conditions affecting all the companies in the economy and it is impossible to eliminate that risk no matter what actions companies take against it.

In terms of portfolio theory, investors must also consider the risks that companies are exposed to. Investors who have diversified the total risks of companies (systematic risk and non-systemic risk) can reduce non-systemic risks. However, because systematic risk affects all firms, investors will not be able to reduce such risks through diversification within the same economy.

Next we show mathematically, how the total risk of a stock *i* can be decomposed into two parts, systematic and unsystematic risk.

Recall, first, that the error term of linear regression is not correlated with the regressor. In our case it means that

$$\mathbf{Cov}(\boldsymbol{r_m}, \varepsilon) = 0.$$

Therefore, the risk associated with the asset *i* is (since α_i is constant)

$$\begin{aligned} \sigma_i^2 = \mathrm{Var}\,(\mathrm{r_i}) &= Var(\alpha_i + \beta_i r_m + \varepsilon) \\ &= Var(\beta_i r_m + \varepsilon) \\ &= \beta_i^2 \sigma_m^2 + \sigma_\varepsilon^2 \end{aligned} \tag{4}$$

Thus, the risk of the asset *i* consists of two components – quantity $\beta_i^2 \sigma_m^2$, called systematic risk and quantity σ_ε^2 called unsystematic or unique or idiosyncratic risk. Systematic risk is proportional to the market risk, with the proportionality factor β_i^2

1.4.Security Market Line

We first explain what is risk free rate of return. It can be said that the risk-free rate of return is referred to as a yield of an investment with almost no risk. Interest rate is represented by the risk-free rate that an investor is expecting from the investment that has no risk involved for the given time frame. Theoretically, the rate that is free of risk is the least possible yield that is expected by an investor for almost any investment as more risk is not taken by him until return's potential rate is bigger than the risk-free rate. But practically speaking, it seems that there is no risk-free rate in economy as the known least risky investments such as U.S Treasury Bill still holds very small amount of risk. But as U.S Treasury Bills are the least risky investment in overall economy, their 3-month rate is considered as risk free rate of return. [22]

Following is the formula for the calculation of predicted return of specific assets while considering its risk [12]:

$$Er_i = r_f + \beta_i(r_m - r_f) \quad (5)$$

Where,

Er_i is Investment's expected return,

r_f is Risk-free rate,

(r_m - r_f) is Market risk premium.

Generally, investors expect to have some type of compensation for the time value and risk of the money. In the formula of CAPM, risk-free rate is equivalent to the money's time value. Other components of the formula of CAPM, account for investors taking additional risks. The beta of potential investment is actually a measure of just how many risks will be added to a portfolio by an investment. If a certain stock is riskier than markets, its beta will be more than one. On the other hand, if beta is less than one, it is assumed by the formula that risk of a portfolio will be reduced by it.

Moving on, beta of a stock is multiplied by the premium market risk which is the return predicted from the market over the rate that is risk-free. An investor should be given the discount rate or required rate by the outcome that they can utilize for finding the value of a specific asset. The objective of the formula of CAPM is to determine whether a stock is value fairly when its time and risk of money are compared to its predicted returns. For instance, let's consider an investor who is thinking about a stock that is a hundred dollars per share and pays a three percent dividend on a yearly basis. It has a beta which is compared to the 1.3 market, meaning that it is riskier than a portfolio of market. In addition, let's say that 3% is the risk-free rate and this investor believes that the market will increase by almost 8 percent in value per year [16].

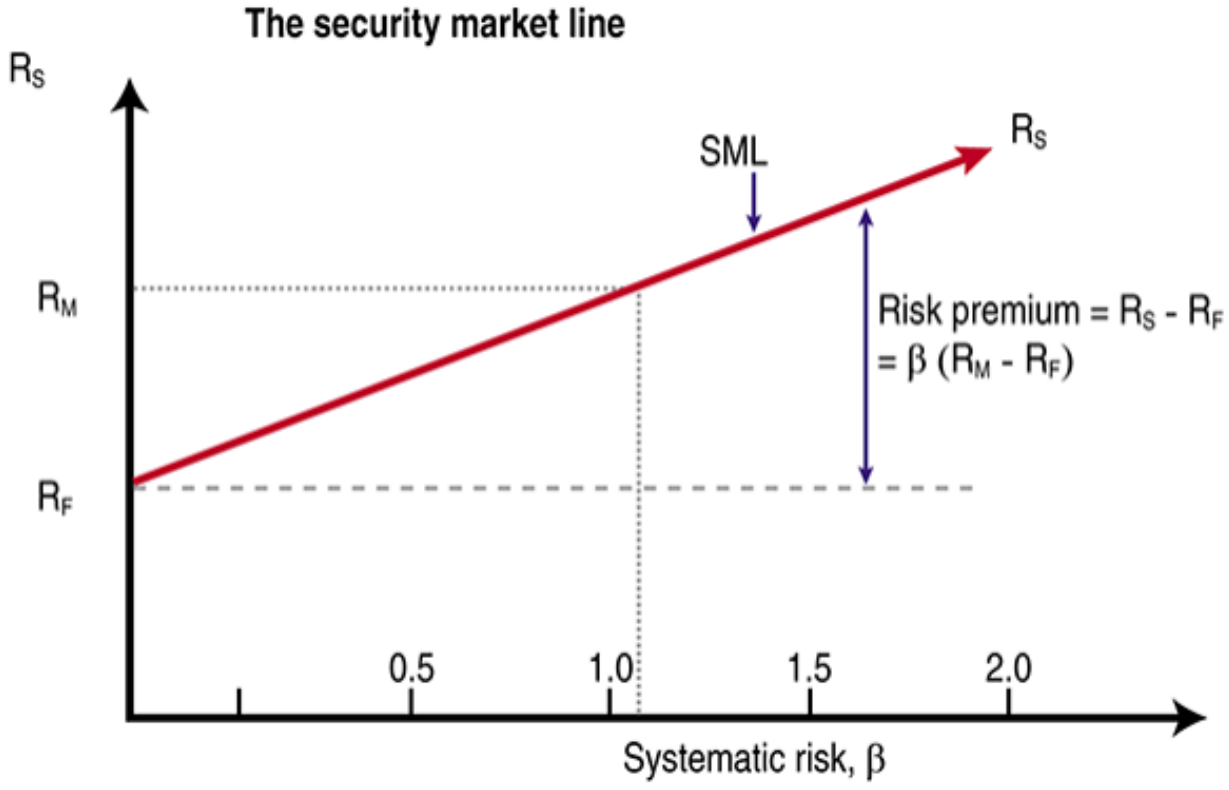
The stocks' expected return based on the formula of CAPM is 9.5 %.

$$9.5\% = 3\% + 1.3(8\% - 3\%)$$

The expected return of the formula of CAPM is utilized for discounting the capital appreciation and expected return of the stock over the expected period of holding. Moreover, if the discounted value of cash flows in the future is equivalent to 100 dollars then the formula of CAPM highlights that the stock is valued fairly relative to the risk.

The Security Market Line or SML is a graphical picture of Capital Asset Pricing Model. It is based on the formula (5), which illustrates distinct levels of systematic risk for several stocks which are vendible and plotted against the entire market's expected return at a given time. Security Market Line is also known as the characteristic line and is a visual of CAPM where the chart's x-axis is regarded as beta risk and the y-axis states the expected return. The market risk premium of an individual stock is evaluated by the location where it is plotted on the graph in association with SML. [20]

Figure 1**:** SML line



SML is often used by analysts and investors in order to evaluate the stock's value in terms of whether it gives the expected return regarding the risk level that stock is holding. Generally, if the security is plotted under the SML then it can be said that this stock is overvalued and vice versa is holding true.

1.5.Empirical tests of CAPM

According to CAPM predictions investors have the securities in terms of standard deviation and expected return. That is why whole market portfolio is considered to be the most efficient.

In very first tests of the model researchers did not actually examine the prediction of the model. Instead they had an assumption that if there is a positive linear relationship between yield of portfolio and systematic risk then the Market portfolio is the most efficient.

The model of capital asset pricing explains that the predicted return from risky asset is linearly and positively associated with its beta. Even though many of the extended models of capital asset pricing seems to feature the linear relationship between the beta and expected return, other models like model of multi-period asset pricing suggests that systematic risks other than the beta are required to describe the return. Other models also suggest that a beta is not the only source of a systematic risk. In addition, predicted return might be related to the standard deviation or with some specific firm characteristics like financial rations or firm size. These measures of alternative risk develop the need of testing how well the model of capital asset pricing fits its empirical data. There are actually 2 approaches for checking the authenticity of models of asset pricing: cross-sectional regression and time-series tests. [5]

The test of CAPM is not about testing the predicted return obtained by the model but to examine whether the beta coefficient of a single stock or portfolio is positively correlated with the expected return.

Since the model gives concise explanation of the relationship between returns and systematic risks of different securities and portfolios, if the model holds true in real world then, undoubtedly it is one of the most important tools for investors. In fact, there have been many empirical tests regarding the model in different times.

The main matter about empirically experiment the model is market portfolio to decide which market portfolio is needed. That is kind of portfolio which holds all the assets which are operating in stock market. For instance, most of the time when researchers want to use US market for their test, they usually refer to S&P 500 index to substitute the market portfolio. Apparently, when the stock index is not able to adequately express the market portfolio, then the test and result of CAPM

cannot be accurately concluded. The CAPM consider the systematic risk as the only effecting factor and according to theory non-systematic risk is zero. However, when the stock index which is taken as the market index is not efficient enough then we cannot say the above information is holding true.

The CAPM empirical studies are full of distinct content. They are mostly consisting of 3 approaches: Initial one is to test the correlation between risk and return. The second one is the time series analysis of CAPM. The last one is CAPM cross-sectional test. [15]

The very first example of test is done by American academic Sharpe (1964). In order to make this test he has chosen 34 mutual fonds from United States market and measured their annual average return and standard deviation for the period of 1954-1963. His primary result was that for the test period US stock market outperformed the risk-free rate of return. The average return and standard deviation had the correlation coefficient of 0.8 which obviously can be interpreted as there was a distinct linear relationship between the variables. [19]

In order to test the theory, early tests are following a two-step regression. According to the first step, time series regression analysis has to be done to get the β value of the stock or portfolio. The second one is cross-sectional regression to make the regression test between obtained β values and excess average returns of stocks or portfolios. [15]

1.6.Black Jensen and Scholes method (BJS)

Black, Jensen and Scholes conducted the research in 1972. The main direction of the test was to check if there is positive linear relationship between the expected return and the β of the portfolio. Testing period was chosen as 1926-1965 time frame for all stocks that are operating in NYSE for chosen period.

Below their approach is explained. [11]

1) The regression model made for stocks with the chosen market proxy in order to obtain the β values and the stocks are divided into 10 groups regarding the size of their β value. Portfolios are made according the mentioned values.

$$r_i - r_f = \alpha + \beta_i(r_m - r_f) + \varepsilon, i = 1, 2, 3, \dots, n \quad (6)$$

2) After that, portfolio returns are calculated and regressed against market portfolio in order to obtain the portfolio β values.

$$r_{pi} - r_f = \alpha + \beta_{pi}(r_m - r_f) + \varepsilon_j,\ i = 1, 2, \dots, 10 \quad (6)$$

Where,

r_{pi} is the rate of return on portfolios,

β_{pi} is the portfolio β

3) After obtaining the portfolio betas and arranging portfolios according their beta values, average returns and beta values have been regressed in order to see the picture of relationship.

The addition of BJS test is that they created portfolios instead of using individual stocks and as a result, they had a well-diversified portfolio and more accurate beta value.

After all, the results show that there is a positive linear relationship between the regressed variables and this connection is powerful enough. [11]

1.7.Fama and MacBeth (1973) method

In Fama and MacBeth method they adopted the same data as in previous BJS test. The main objective of this test was to form a non-linear regression and see any possible relationship between the systematic risk (β) and portfolio yield. The following regression equation was formed by Fama and MacBeth for testing purposes: [24]

$$r_{pi} = y_{0i} + y_{1i}\,\beta_{pi} + y_{2i}\,\beta_{pi}^2 + y_{3i}\,\sigma_{epi} + u_i \quad (7)$$

Where,

r_{pi} is the average portfolio return for the test period,

β_{pi} is the portfolio beta which also was calculated in BJS test,

β_{pi}^2 is the square of the systematic risk and it is non-linear variable. In case, CAPM is true the y_{2i} coefficient claimed to be very small,

σ_{epi} is the standard deviation which refers to the non-systematic risk of portfolios.

Afterwards, they examined the significance of the factor coefficients. According to the results obtained from test y_{2i} coefficient was not strong enough and it means that β_{pi}^2 does not have relationship with r_{pi}.

As a summary, the aim of the method primarily is to test 4 hypotheses:

1) $E(y_{0i}) = r_f$, $y_{0i}>0$
2) $E(y_{1i}) > 0$
3) $E(y_{2i}) = 0$, and in this case β_{pi} will be significant enough to effect r_{pi}
4) $E(y_{3i}) = 0$, non-systematic risk has no influence on r_{pi}

As a conclusion, the results achieved from this method says that there is a linear relationship between the systematic risk (β) and the return of security or the portfolio. [10]

2.Theoretical Background of Fama French 3 Factor Model

2.1.Fama French 3 Factor Model

After testing the Capital Asset Pricing Model for many years, overall results were not supporting the model that much and this led to the establishment of similar but at the same time more wide range model. The model developed in 1992 by Eugene Fama and Kenneth French who were professors in Chicago Business School. According to their findings, the classical asset pricing model (CAPM) is not performing good enough and the beta itself is not significant enough to affect the stock or portfolio returns. Being different than CAPM, they suggest that there are two more variables SMB and HML which will be explained more detailed in following paragraphs. [9]

According to the model the expected yield of the security or the portfolio is calculated with the following formula: [25]

$$r_{it} - r_{ft} = \alpha_{it} + \beta_1(r_{mt} - r_{ft}) + \beta_2 SMB \quad + \beta_3 HML \tag{8}$$

Where,

r_{it} indicates the overall return of a portfolio or stock, i at the time of t,

r_{ft} indicates the rate of return that is risk-free at the time t,

r_{mt} indicates the overall portfolio return of market at the time t,

$r_{it} - r_{ft}$ is indicating the predicted excess return,

$r_{mt} - r_{ft}$ is indicating the excess return on the portfolio of market in the formula,

SMB indicates the size premium and is equivalent to (small minus big),

HML is indicating the value premium (high minus low),

β_1, β_2, β_3 are indicating the factor coefficients in the formula.

In their 1992 study, Fama and French examined the time dependent change in average stock returns and stated that the yield of the market portfolio as well as the size and book to market ratios could be effective on stock returns. Thus, as with most researches in the financial literature, Fama and

French's study in 1992 found that the CAPM was insufficient to explain the change in stock returns. Fama and French made the following conclusion in 1993.

In the study, CAPM was added these two new factors and made three-factor asset pricing model as an alternative to CAPM and three risk factors in the model have explained that they can describe the change in yield.

Moreover, from statistical point of view in case of addition more independent variables to the regression equation definitely increases the strength of the model.

A primary implication of this model is that an investor can simply choose to weight their different portfolios so that they have a lesser or greater exposure to each and every risk factor. Therefore, they can target levels of return in a precise way. It is quite interesting to realize that Fama and French perceive high returns as rewards for taking a high risk. It means that with an increment in the returns with the price, then stocks with a large ratio of book/price will be riskier. This view is different from the beliefs of a traditional analyst. [11]

2.2.SMB (Small Minus Big) and HML (High Minus Low)

Small minus Big or in other words, SMB is one of the three factors in Fama French 3 factor model which is based on the size factor of companies' stocks. To be more precise, the idea behind that factor is mainly because it is assumed that the companies with smaller market capitalization are outperforming and earning more than the ones with bigger market capitalization. Moreover, if we make a portfolio of stocks that belong to companies of small size then in long term it should earn more than the market itself.

A positive SMB seems to indicate in a month that small-cap stocks outperform the large ones. In a particular month, a negative SMB indicates the outperformed large caps

High minus Low or shortly HML is the last factor that added to Fama French 3 factor model. The main assumption behind adding this factor is that there are companies which can be grouped by their book to market ratio. In other words, value stocks are the ones with high book to market ratio and the growth stocks are vice versa and it is believed that the value stocks are earning more than growth stocks. Furthermore, it can be said that HML factor established to calculate the value premium which is offered to investors that are investing to value stocks.

In a month, a positive HML seems to indicate that growth stocks have been outperformed by value stocks in that specific month. In a specific month, a negative HML indicates the outperformed growth stocks. BA seems to measure the exposure that an asset has to the risk of market while SA evaluates the exposure level to size risk.

2.3.Prior Studies in Developed Economies

Studies on the Three-Factor Model have been applied primarily on developed markets. The findings of the studies on the American stock market, European stock exchanges and the stock market in Japan are listed in the following lines.

In a study conducted by Fama and French in 1993, the effect of firm size and book to market ratio on stock returns was investigated (Fama and French, 1993). As a result of the study, it was determined that the shares with high book to market ratio yielded a higher return than the stocks with smaller book to market ratios and the shares of small-scale companies compared to the stocks of large-scale companies. Fama and French reported that the resulting normal return was due to the high risk of book to market ratio and the share of small-scale companies. [7]

Fama and French, in their study in 1995, investigated the risks that make these stocks risky (Fama and French, 1995). In order to determine the causes, the relationship between the three-factor model and the normal returns of NYSE, AMEX, NASDAQ-traded stocks in the 1963-1992 period were investigated. In the study, it was found that the firms with high book to market ratio had higher returns compared to the firms with low book to market ratio, and that small-scale firms gained more returns than large-scale firms. [8]

Barber and Lyon conducted studies on the stocks of financial and non-financial firms traded in NYSE, AMEX and NASDAQ between 1973 and 1994 (Barber and Lyon, 1997). In their work including financial sector companies, they determined that the book to market ratio and the size of the company have a significant effect on the stock returns. At the end of their research, they concluded that the strongest factors predicting return were firm size and book to market ratio. Moreover, this effect does not differ for real and financial sector companies. [2]

Liang (2003) compared CAPM with Fama and French's three-factor model. In his study, Liang examined the American stock market data for the period 1933-2003. It divided the long-term into four sub-periods and used Fama and French (1993) to update 25 of their portfolios of NYSE, AMEX and NASDAQ shares. At the end of the study, both models were rejected for the data set used. However, CAPM performed better during 1933-1963 and 1994-2003. The Three-Factor Model was valid in 1963-1993 period. [18]

2.4.Prior Studies in Developing Economies

After Fama and French's three factor models were introduced to finance literature in 1993, research was conducted to test the validity of this model in emerging markets. The findings of these studies in detail in the following paragraphs:

First, Chui and Wei (1998) examined the effect of market beta, book to market ratio and firm size on stock returns in their study using data from stocks traded in Hong Kong, Korea, Malaysia, Taiwan and Thailand. According to the results of the research covering the period 1977 - 1993, it was found that the relationship between average stock returns and market betas was weak in all markets examined. In addition, the firm size effect is observed in markets other than Taiwan, while book to market ratio explains the change in stock returns only in Hong Kong, Korea and Malaysia. As a result, it is concluded that returns are more related to firm size and book to market ratio and the results are consistent with Fama and French's (1993) findings. [3]

Another study for Malaysia was done by Drew and Veeraraghavan (2002). In this study, they examined whether there is a firm size effect in Malaysia and whether the multifactor model of Fama and French explains the stock returns. According to the findings of the study covering the period 1992 to 1999, the three-factor model explains the change in stock returns in Malaysia. Small-cap or high book to market ratio stocks have a higher return than large-cap or low book to market ratio stocks. These findings show that there is a size and book to market effect in Malaysia. [4]

In its 2009 study, Şakar tested the validity of the three-factor asset pricing model by using panel data analysis, using annual data of the stocks traded on the Istanbul Stock Exchange. Panel data analysis allowed us to examine both time and horizontal cross-sectional dimensions. In the analysis, the shares traded on the ISE during the period of 1996 - 2008 and during the said period of 13 years. At the end of the month, all non-negative equity shares (87) were used. In the study, fixed effects panel regression results showed that the coefficients of the equation were valid (significant). The findings of this study consistently explain that the variability of the returns of the 87 stocks traded in the said period is explained well by Fama and French 3 factor model. [17]

3.Methodology, Data and Empirical Results

3.1.Methodology

The aim of this study is to investigate the validity of Capital Asset Pricing Model and Fama French three factor model, which are being widely used since the end of last century. For this purpose, the analysis of three variables Beta, SML and HML has been carried out to determine the effects of those factors on stock returns in New York Stock Exchange (NYSE). First, we are going to test the CAPM and draw the SML line to see how well model performs compared to actual values. If there is a positive linear relationship as it is stated in theory, then the actual returns should not be significantly far from SML line.

In case of Fama French 3 factor model the coefficients are calculated with multivariate regression in excel and after obtaining all three coefficients, the required rates of return are calculated by the model according to formula (8). Later those results are being plotted and then we see how efficient the model works compared to the real rates of return.

3.2.Data and Stock Selection

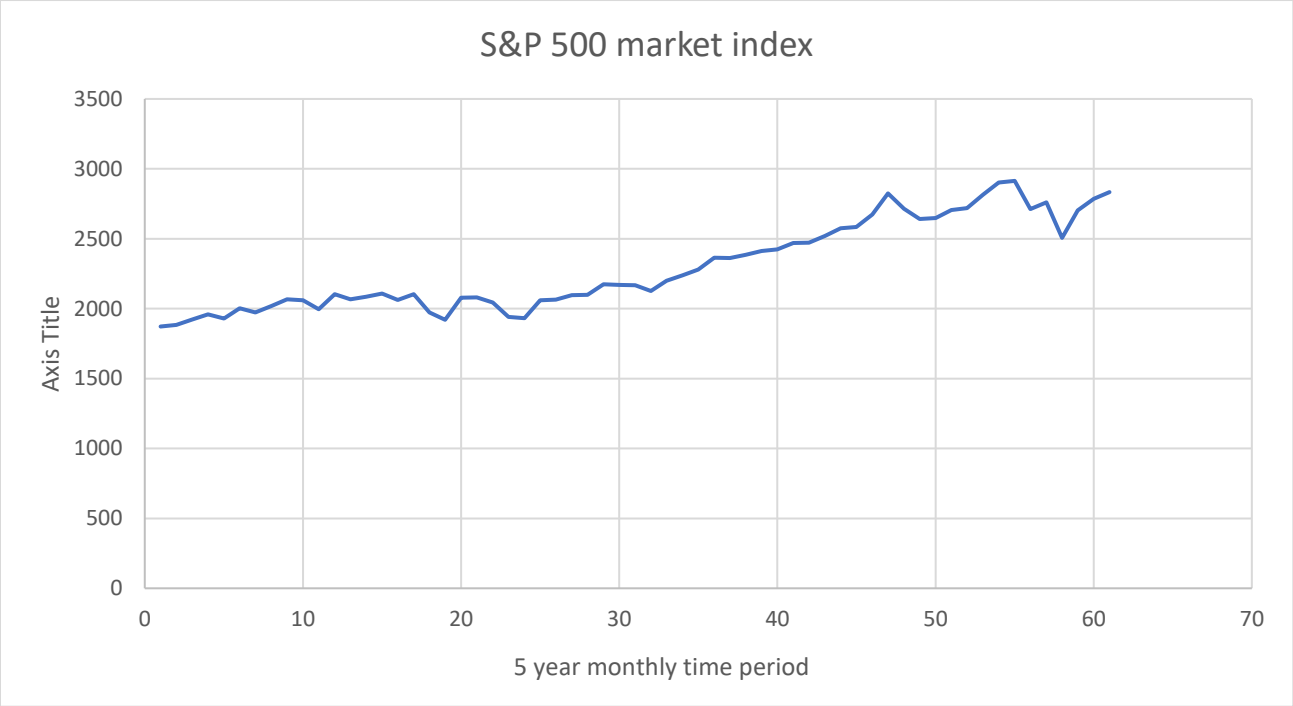
In order to accomplish the test, 30 stocks data have been chosen from US stock market for the period of 2014-2019 and downloaded from finance.yahoo.com. The criteria that stocks have been chosen is mainly for their beta value and plus those stocks have to operating in market for at least last 5 years. Beta values of stocks vary between 0.14 and 2.45. The important reason to make such a big beta range is to clearly see how the increase in beta values will affect the result.

Moreover, 5 years of time frame has been taken as the optimal number because the more years the less accurate our beta estimate will hold for the current period.

The same argument has been taken into consideration for choosing the frequency of returns. The more frequent the noisier and the less frequent the less accurate will be the beta coefficients. That is why optimally monthly rate of returns are chosen for sample period.

As a market proxy S&P 500 market index has been chosen. The reason for choosing S&P 500 is a better representation of the market, as it is one of the biggest market indexes in US stock market and as a result, it explains the overall economy and financial situation much more accurately. Moreover, as our sample stocks are holding bigger market capitalization, in that case S&P 500 index suits for our sample stocks. The monthly change of market index can be seen in following graph.

Graph 1: Market Index S&P 500 time series



For Fama and French model the data has been taken from Kenneth French's website which is widely used for the research purposes.

3.3.Empirical work

Initially, the individual stock returns and market index return rates are calculated according to formula (1). After obtaining those results, individual stock returns are regressed against market returns in order to calculate the beta coefficient with the formula (2). Using the calculated returns and beta estimates now it is possible to see what numbers CAPM model predicts with the given beta values. In below table actual rate of returns, Beta estimates and CAPM predicted rate of return are given for each stock and the CAPM results are calculated with according to formula (5).

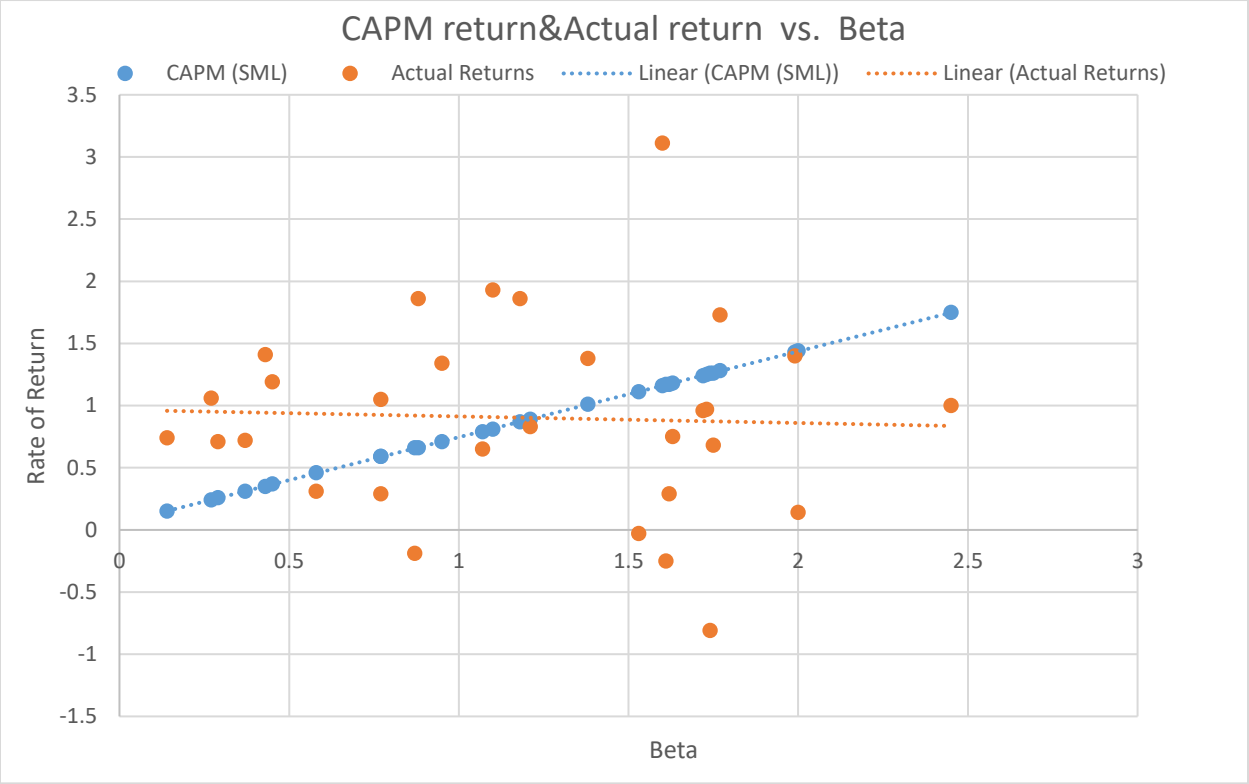
Table 1 : Stock returns, beta estimates and CAPM predicted rates of return

Stocks	Beta	Average returns	CAPM results	Stocks	Beta	Average returns	CAPM results
SO	0.14	0.74	0.15	MS	1.21	0.83	0.89
WXC	0.27	1.06	0.24	A	1.38	1.38	1.01
WMT	0.29	0.71	0.26	ARNC	1.53	-0.03	1.11
PG	0.37	0.72	0.31	AMZN	1.60	3.11	1.16
SBUX	0.43	1.41	0.35	WYNN	1.61	-0.25	1.17
MO	0.45	1.19	0.37	ALXN	1.62	0.29	1.17
T	0.58	0.31	0.46	C	1.63	0.75	1.18
BMY	0.77	0.29	0.59	CELG	1.72	0.96	1.24
ALL	0.77	1.05	0.59	APTV	1.73	0.97	1.25
CVS	0.87	-0.19	0.66	FLR	1.74	-0.81	1.26
FB	0.88	1.86	0.66	XRX	1.75	0.68	1.26
LMT	0.95	1.34	0.71	IPGP	1.77	1.73	1.28
ORCL	1.07	0.65	0.79	SIVB	1.99	1.4	1.43
AAPL	1.10	1.93	0.81	HES	2.00	0.14	1.44
CSCO	1.18	1.86	0.87	URI	2.45	1	1.75

In table 1, the beta values are sorted in an increasing order to see how actual returns and CAPM predicted returns are sensitive to the change in beta. In other words, with current way of sorting, it is possible to test CAPM and see the effectiveness of the model and influence of beta on stock returns.

In order to see how model works and how beta explains the required returns, the actual rate of returns and CAPM predicted returns are plotted in scatter plot on y-axis and Beta values in x-axis to see how different the returns are, in corresponding beta value. In below table results are plotted.

Graph 2: CAPM predicted rate of returns vs. Actual rate of return



In this graph SML line is formed from the predicted rate of returns with CAPM which is plotted with blue dots and actual rate of returns are plotted with red dots and the red regression line. According to CAPM theory if the model has a strong predictive power then all the red lines has to be on SML line or at least not significantly far. However, as seen from the scatter plot the values are quiet far from the SML line and actually there is small negative correlation with the beta and rate of return.

For second stage, initially the Fama French 3 factor model is calculated according to the formula (8). In order to calculate it firstly all three factors are put in multiple regression as the independent variable and the stock returns as the Y variable to get the factor coefficients. This approach is applied to individual stocks one by one and in the following table the results can be seen.

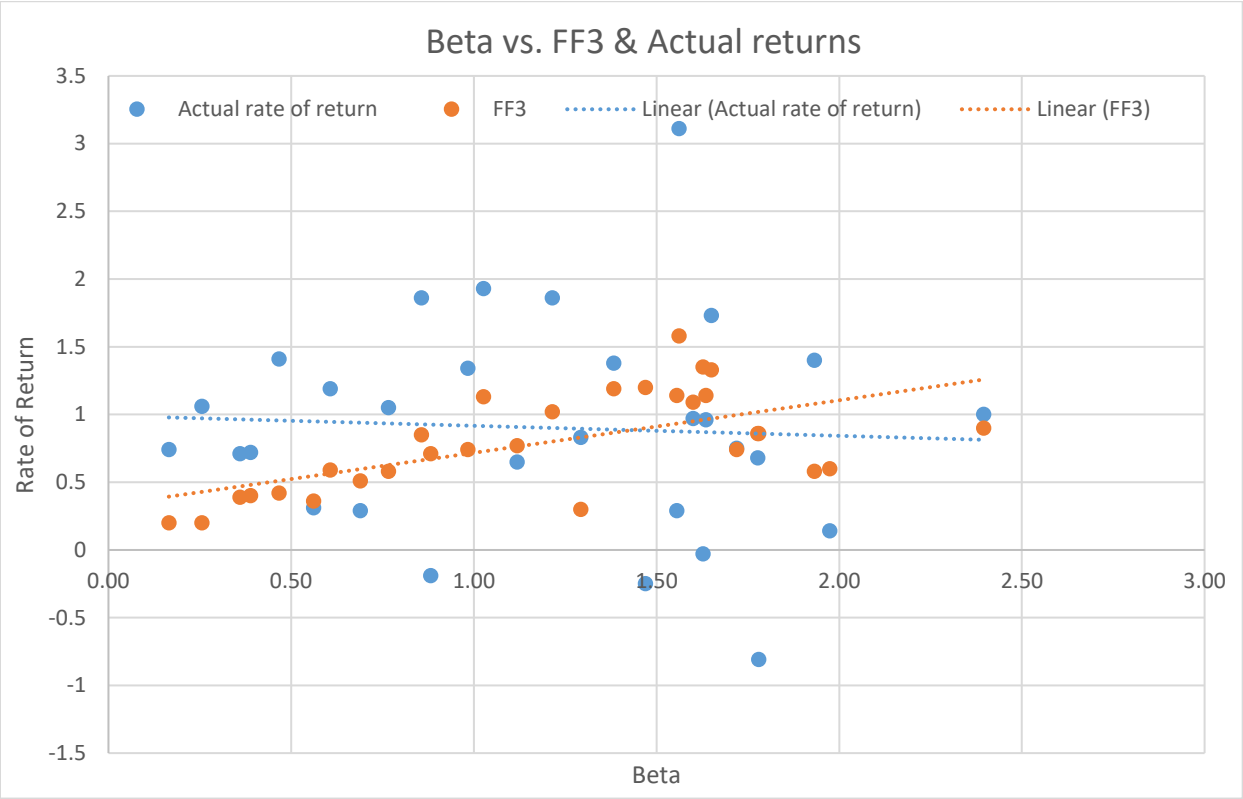
Table 2 : Multiple regression results and Fama French predicted rates of return

Stocks	β_1 (beta)	β_2 (SMB)	β_3 (HML)	FF3	Average returns
SO	0.17	-0.19	-0.15	0.20	0.74
EXC	0.26	0.06	-0.07	0.20	1.06
PG	0.39	-0.24	-0.23	0.40	0.72
WMT	0.36	-0.49	-0.19	0.39	0.71
All	0.77	-0.02	-0.11	0.58	1.05
BMY	0.69	0.41	-0.23	0.51	0.29
SBUX	0.47	-0.26	-0.15	0.42	1.41
LMT	0.98	-0.21	-0.08	0.74	1.34
CVS	0.88	-0.14	-0.20	0.71	-0.19
ORCL	1.12	-0.21	0.08	0.77	0.65
AAPL	1.03	-0.09	-1.00	1.13	1.93
FB	0.86	-0.13	-0.58	0.85	1.86
MS	1.29	0.31	1.34	0.30	0.83
A	1.38	-0.29	-0.48	1.19	1.38
ALXN	1.56	0.23	-0.22	1.14	0.29
AMZN	1.56	-0.36	-1.13	1.58	3.11
C	1.72	0.10	1.04	0.74	0.75
HES	1.97	1.06	1.50	0.60	0.14
MO	0.61	-0.95	-0.10	0.59	1.19
T	0.56	0.14	0.01	0.36	0.31
CSCO	1.21	-0.35	-0.33	1.02	1.86
CELG	1.64	0.45	-0.17	1.14	0.96
WYNN	1.47	0.52	-0.63	1.20	-0.25
XRX	1.78	-0.28	-0.28	1.38	0.68
APTV	1.60	0.68	-0.21	1.09	0.97
ARNC	1.63	-0.71	-0.31	1.35	-0.03
FLR	1.78	0.28	0.80	0.86	-0.81
IPGP	1.65	0.41	-0.61	1.33	1.73
SIVB	1.93	1.21	1.42	0.58	1.40
URI	2.40	1.18	1.43	0.90	1.00

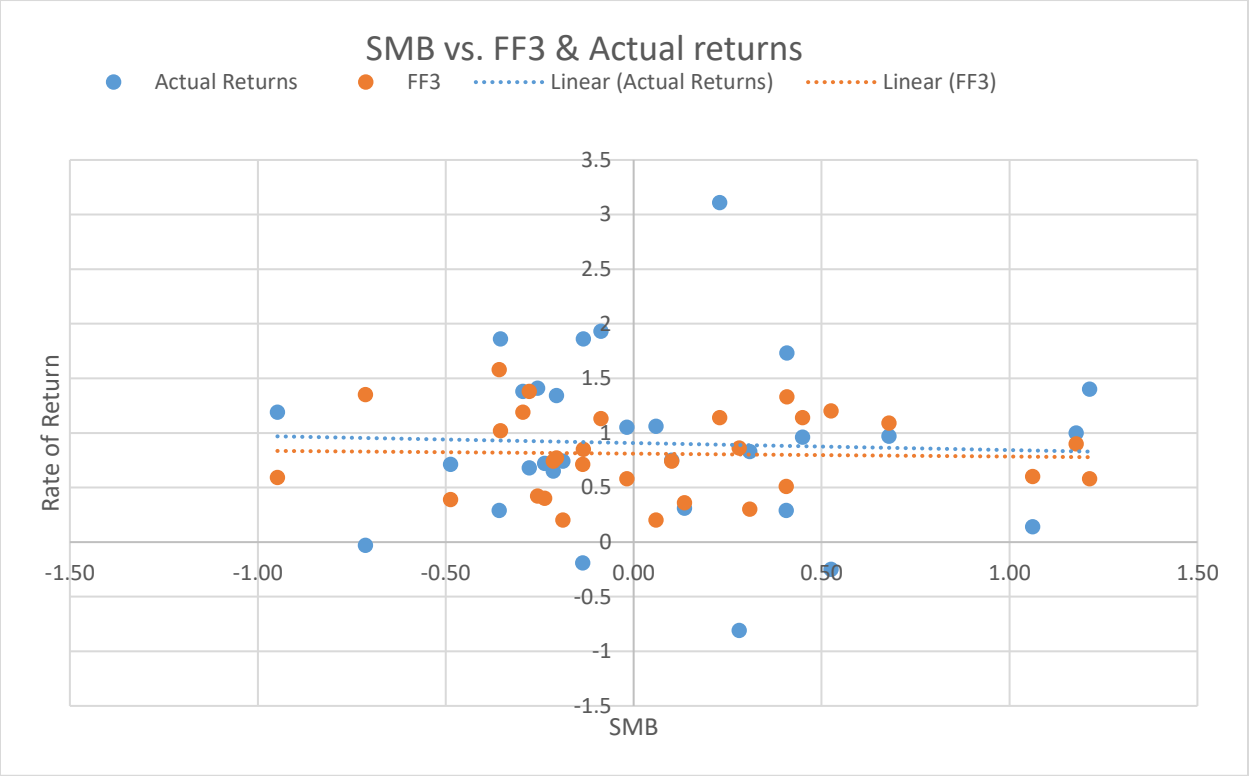
To see how efficient and accurate the model works it needs to be plotted as it is done with CAPM, but the same approach cannot be used here with Fama French three factor model because it consists of two more factors. In order to make test actual returns are regressed against the factors one by one and the results are plotted in three different graphs.

The main idea here with this approach is that, when we sort the factor values in an increasing order and regress them with the corresponding actual return values as the independent variable one by one, only then it can be seen if the actual values are affected by those values or not. The regression results can be seen in the following graphs.

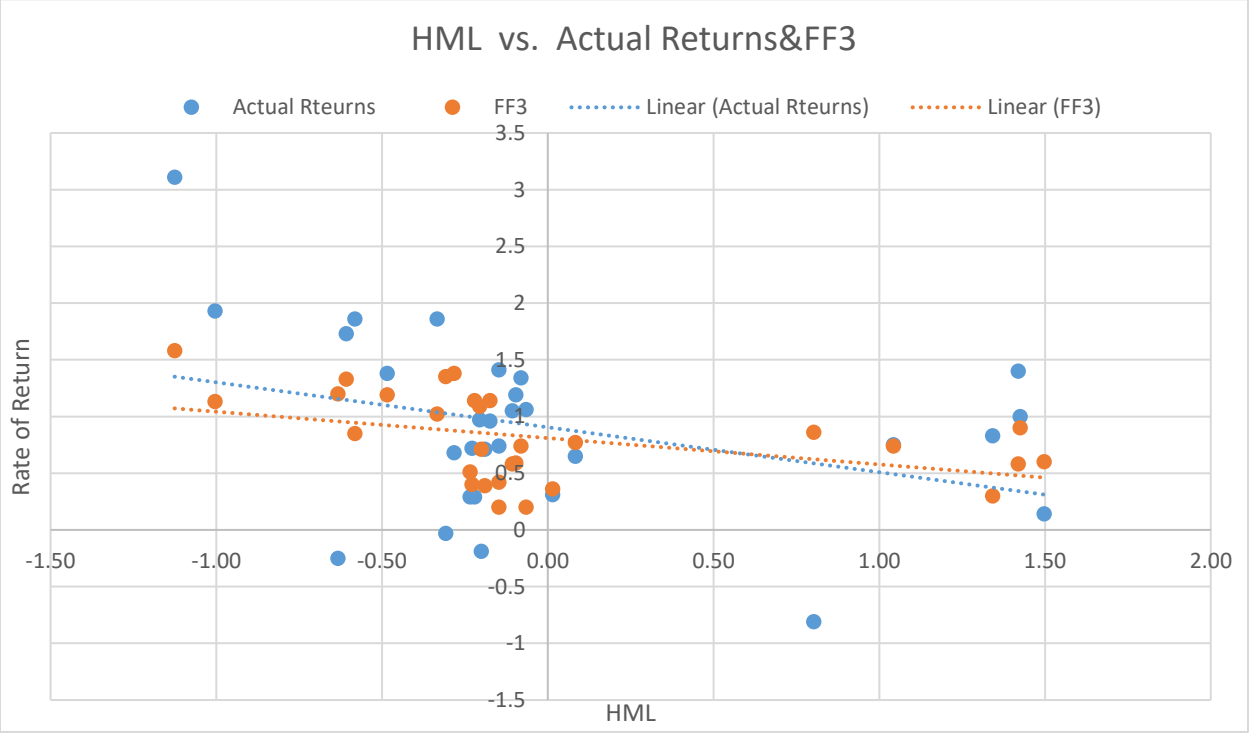
Graph 3: Regression plot FF3 and Actual rate of return (beta as an independent variable)



Graph 4: Regression plot of FF3 and Actual rate of return (SMB as an independent variable)



Graph 5: Regression plot of FF3 and Actual return (HML as an independent variable)



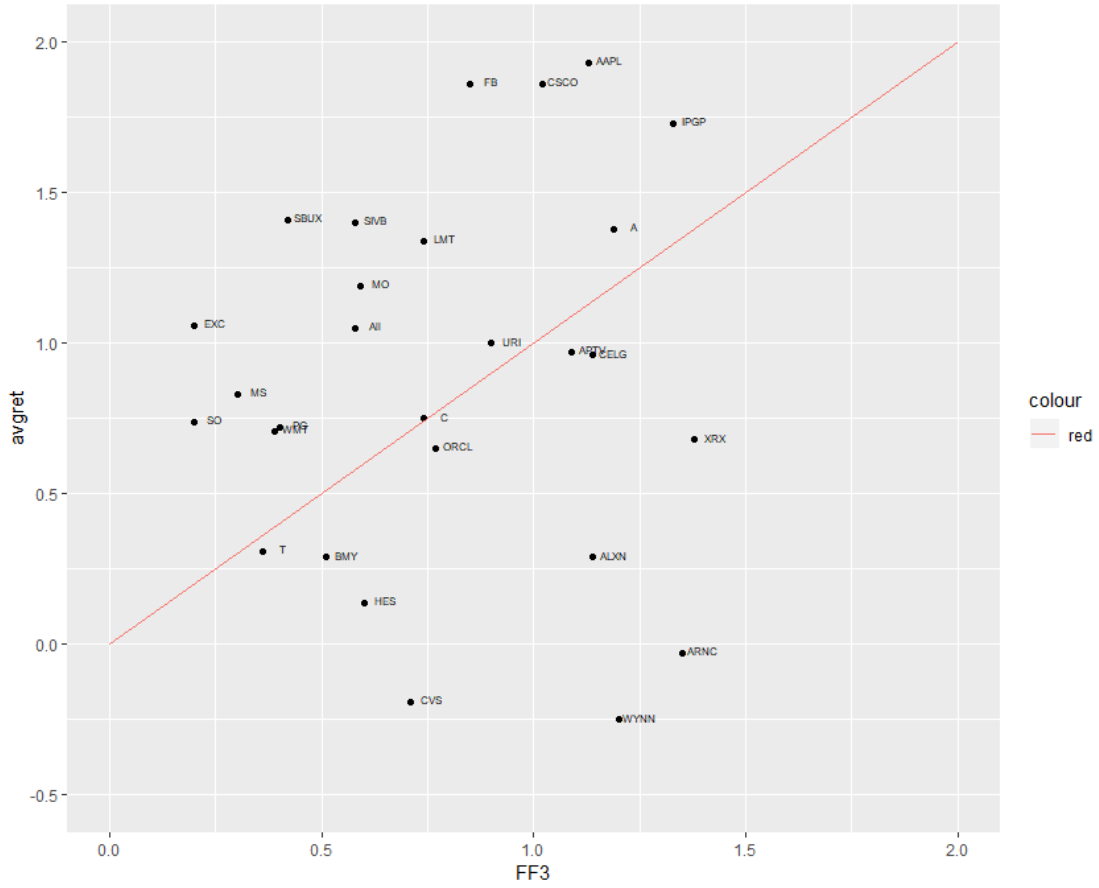
In graph 3, two sets of data with the common independent variable (beta) are plotted. y-axis represents the actual returns and FF3 returns based on the beta value. According to Table 5 it can be said that beta value is not significant to explain the actual rate of returns as the regression line of each data set is in different direction. Hence, FF3 beta stays weak in terms of explanatory power for the model.

In graph 4, the same data set as in graph 3 is used in y-axis, but this time the x-axis is the SMB factor. With the same approach and testing method It can be said there is almost no linear relationship with the SMB factor and actual returns.

Lastly, in table 7, the same method has been implemented and HML factor is plotted as independent variable on x-axis. According to results HML factor has shown slightly negative relationship with both FF3 and actual returns.

Eventually, to see how the FF3 predicted values correspond to the actual values, the following plot visually demonstrates the difference.

Figure 2: FF3 vs. Actual rate of return (comparison with the actual rates of return)



In figure 2, the diagonal line is the combination of values which means that if any dot is on this line then the model predicted value and actual value are same and model has strong explanatory

power. However, in this plot the dots are quiet far from the diagonal line, hence the predicted values do not correspond to the actual values.

4.Conclusion

This research is made to test the Capital Asset Pricing Model and Fama French 3 factor model in terms of explanatory power of their factors. According to test results none of the models were significantly strong to predict the rate of return close to the actual returns.

Additionally, to compare those models and to see how much they are spread from the actual returns, standard deviations of CAPM and FF3 are calculated. However, results were not significantly different as standard deviation of CAPM was 0.61 and 0.62 for FF3 model.

One should take into account that all our empirical results are only conditional since they are based on a sample of stocks and chosen data frame.

In conclusion, we have seen that although the Fama-French model contains more factors and is more complicated than the classical CAPM model, there are still significant differences between actual average returns and those assumed by the model.

References

1. Brad M. Barber, John D.Lyon: Detecting long run abnormal stock returns: The empirical power and specification of test statistics, Journal of Financial Ecnomics 43 (1997), 341-372. [2]

2. Chui, A. W. & Wei, K.J.C: Book-to-Market, Firm Size and the Turn-of-the-Year Effect: Evidence from Pacific–Basin Emerging Markets. Pasific–Basin Finance Journal (1998) Cilt.6, 275– 293. [3]

3. Drew M. E. & Veeraraghavan, M. A: Closer Look at the Size and Value Premium in Emerging Markets : Evidence from the Kuala Lumpur Stock Exchange. Asian Economic Journal (2002), Cilt.16, No.4, 337-351. [4]

4. Edwin J. Elton, Martin J. Gruber: Modern Portfolio Theory and Investment Analysis 5th edition (1995), 279-301. [12]

5. Eugene F. Fama and Kenneth R. French: The Capital Asset Pricing Model, Journal of Economic Perspectives - Volume 18, Number 3, Summer 2004, Pages 25-46. [5]

6. Eugene F. Fama; James D. MacBeth :The Journal of Political Economy, Vol. 81, No. 3. (May - Jun., 1973), 607-636. [24]

7. Eugene F.F, French K: Common Risk Factors in the Returns on Stocks and Bonds. Journal of Financial Economics (1993), 33:3–56. [7]

8. Eugene F.F, French K: Size and Book-to-Market Factors in Earnings and Returns. Journal of Finance (1995), 50(1):131–155. [8]

9. Eugene F.F. & FRENCH, K. R: The cross-section of expected stock returns. the Journal of Finance (1992), 47, 427-465. [10]

10. Eugene F.F., French, K.R : Multifactor Explanations of Asset-Pricing Anomalies, Journal of Finance (1996), 51/1: ss. 55-84 [9]

11. Gökgöz, F: Üç Faktörlü Varlık Fiyatlandırma Modelinin İstanbul Menkul Kıymetler Borsası'nda Uygulanabilirliği. Ankara Üniversitesi SBF Dergisi (2008), 63-2, 43-64. [11]

12. https://financetrain.com/calculating-beta-using-market-model-regression-slope/ [21]

13. https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html [23]

14. https://www.investopedia.com/terms/f/famaandfrenchthreefactormodel.asp [25]

15. https://www.investopedia.com/terms/s/sml.asp [20]

16. https://www.wallstreetmojo.com/risk-free-rate/ [22]

17. Jensen, M. C., Black, F. & Scholes, M. S. :The capital asset pricing model: Some empirical tests (1972). 6-24 [15]

18. Köseoğlu Sinem Derindere, Burcu Adıgüzel Mercangöz: Testing the Validity of Standard and Zero Beta Capital Asset Pricing Model in Istanbul Stock Exchange. International Journal of Business, Humanities and Technology Vol. 3 No. 7; September 2013, 58-67 [14]

19. Mustafa KIRLI: Halka Açık Olmayan Şirketlerde Sistematik Risk Ölçütü Beta Katsayısının Tahmin Edilmesi (2006), 122 - 125 [13]

20. Ross, S. A. :The current status of the capital asset pricing model (CAPM). The Journal of Finance (1978), 33(3), 885-901. [16]

21. Şakar, B. :Varlık Fiyatlamada Faktör Modelleri ve Üç Faktörlü Modelin İMKB'de Testi, Yayımlanmamış Yüksek Lisans Tezi. İstanbul Üniversitesi Sosyal Bilimler Enstitüsü (2009). 1-116 [17]

22. Sharpe F.William :The Journal of Business, Vol. 39, No. 1, Part 2: Supplement on Security Prices (January, 1966), 119-138 [19]

23. Shweta Bajpai, Anil K Sharma: An Empirical Testing of Capital Asset Pricing Model in India / Procedia - Social and Behavioral Sciences 189 (2015), 259 – 265 [1]

24. Steinberg M. Adrian, Wissenschaftlicher Forscher :European Journal of Economics, Finance and Administrative Sciences (2011), 30-37 [6]

25. Yuanyuan Liang: Cross Sectional and Multivariate tests of CAPM and Fama French 3 factor model (2003) [18]

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