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Natural catastrophe modeling for pricing in insurance

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Natural catastrophe modeling for pricing in insurance

Abstract

Catastrophe modeling is an untraditional branch of property and casualty insurance. Although, Baltic States recently faced some catastrophe events such as storms and floods, there is no accurate storm or flood model present which can provide assistance to insurance companies to underwrite premium and risk management in catastrophe prone areas.

This thesis presents an analysis of natural catastrophe events in Estonia. Due to lack of historical data one can use three approaches. First, take the scenario of rest Baltic States, Scandinavia and Finland to present some accurate picture of historical losses. Second, analyze windstorm and flood event and their distributions. Third, by combining windstorm and floods together what potential damage may occur. This thesis also gives light to mathematical and statistical modeling of vulnerability function, damage ratio, average annual loss and exceedance probability which are used for natural catastrophic perils to estimate financial losses.

Keywords: Cat modeling, insurance, vulnerability function, damage ratio, exceedance probability, storm, flood

Looduslike katastroofide modelleerimine kindlustuse tarbeks

Lühiülevaade

Katastroofide modelleerimine on kahjukindlustuse ebatraditsiooniline haru. Kuigi Balti riikides on hiljuti toimunud mitmeid looduskatastroofe nagu tormid ja üleujutused, pole mudeleid, mida kindluskompaniid saaks kasutada kindlustuspreemiate määramisel ja riskide juhtimisel.

Käesolev magistritöö analüüsib katastroofe Eestis. Kuna vastavad kindlustusega seotud ajaloolised andmed puuduvadm, siis on võimalikud kolm lähenemist. Eiteks, kasutatakse teiste Balti riikide, Skandinaavia ja Soome ststenaariumeid ja ajaloolisi andmeid. Teiseks, analüüsime tuuletormide ja üleujutuste juhtumeid ja nendega seotud jaotusi. Kolmandaks, huvi pakub tuuletormide ja üleujutuste koosesinemine ja sellega kaasnev kahju. Töös vaadeldakse ka matemaatilisi ja statistilisi mudeleid purustusfunktsiooni, kahjusuhte, keskmise aastakahju ja läveületustõenäosuse jaoks, mida kasutatakse finantskahjude hindamisel.

Märksõnad: katastroofide modelleerimine, kindlustus, purustusfunktsioon, kahjusuhe, läveületustõenäosus, torm, üleujutus.

Preface

I would like to give special thanks to Dr. Kalev Pärna suggesting and encouraging me to write a thesis on this topic. This is an untraditional topic of actuarial science and there is no other thesis written on this topic in Estonia before. However, he keeps giving me good ideas and tips about this thesis. His advice and suggestion are valuable for me.

I would like to thank Professor Raul Kangro and Meelis Käärik for their valuable inputs and for supporting me. Their time and guidance have helped me to accomplish this thesis work.

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Kapil Sharma

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Chapter 1

Introduction, history and recent development in natural catastrophe modeling

1.1 Natural catastrophe modeling

Catastrophe modeling is widely known as cat modeling and natural catastrophe is usually called Nat cat. A programmed system that able to simulate catastrophe events and

• Determines the insured loss

• Estimates the magnitude or intensity and location

• Calculates the amount of damage

Cat models are efficient to provide the following answers:

• What can be the location of future events and the size

• How frequent can be the events in the future

• Severity of insured loss and damage and

Basically, cat modeling is a confluence of actuarial science, civil engineering, hydrology, meteorology, seismology and it is quite often used for simulating risk for insurance and reinsurance company.

It is also used for various purposes:

- For pricing purpose of cat bonds, most of the investment banks, cat bond investors and bond agencies use cat modeling.
- Insurer use cat modeling for risk management and deciding how much reinsurance treaties it should buy from the reinsurer.
- Rating agencies (e.g. Fitch ratings, Moody's) use cat modeling to rate the score for insurer against catastrophe risk.
- Insurer and reinsurer use cat modeling to underwrite its business in catastrophe-prone areas.

1.2 History of cat risk industry

Catastrophe modeling originated from civil engineering and spatial analysis somewhere around 1970s, there were published some papers on the frequency of natural hazard events. Development in measuring natural hazards scientifically inspired to U.S researcher to determine the loss studies from Nat cat perils (e.g. earthquakes, floods).

Initially, a group of insurance companies started using the approach to estimate the losses from individual cat events taking account of the worst case scenarios for a portfolio on the basis of deterministic loss models and what could be the probabilities in future historical loss occur. Almost at the same duration two companies had launched their own software by collecting the data from university researchers to estimate the losses from Nat cat events. First, cat risk service Provider Company was founded in 1987 in Boston named AIR Worldwide but now it is a part of Verisk Analytics. Next year in 1988 Risk Management Solutions (RMS) was also launched its software at Stanford University. Third, cat modeling company began in San Francisco in 1994 named EQE International. However, in 2001 EQE International was acquired by ABS Consulting and in 2013 it was again acquired by CoreLogic [2, p 24].

In the beginning, no Insurance or reinsurance companies were interested in cat risk providers. In 1989, two big disasters occurred that caused a stir in insurance and reinsurance industry. On September 21, 1989, Hurricane Hugo hit the coast of South Carolina and shocking insured losses calculated $4 billion. In the next month only on October 17, 1989, the Loma Prieta earthquake occurred at the San Francisco peninsula and insured losses were calculated $6 billion. These two events made the insurance companies think about seriously about cat risk service providers.In 1992 Hurricane Andrew hit Southern Florida and within an hour after occurring it AIR Worldwide issued a fax to its clients and it calculated losses surprising amount of $13 billion. When actual losses were calculated, it exceeded the amount of $15.5 billion. Hurricane Andrew made eleven insurance companies insolvent. At last, insurer and reinsurer company made their mind, if they want to run their businesses they needed to follow cat models and required to take service from cat service providers. Today all the insurer, reinsurer and cat risk provider use only software of these three companies.

Chapter 2

The recent impact of Nat cat events in Baltic states and Scandinavia

2.1 The storm Gudrun

January 2005, proved to be one of the worst month for insurance and reinsurance business in the Baltic States and Scandinavia. Total estimated losses in Nordic and Baltic countries created by the storm approximately €1 billion [1]. The Guy Carpenter explanation was, the jet stream took air upwards from the low pressure and due to this it created moisture to condense and as a result it formed clouds and precipitation. Contrary to it, the dried air moved towards downwards and created sting jet, an upper level wind descending to the ground. When it was compared country wise to gusts, it was found that the highest wind speed was estimated in Denmark 46 m/s and Estonia (37.5 m/s).

Maximum wind speed measured in different countries during Gudrun (Erwin)

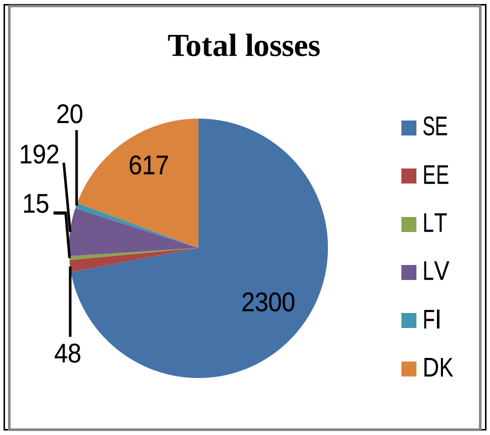
Country	Maximum wind speed (gusts, m/s)	Maximum wind speed (sustained, m/s)
Denmark	41-46 (on the coast), 30-33 (over the whole country)	28-34 m/s (mean values)
Sweden	42 (Hanö), 33 (Ljungby & Växjö, worst hit areas)	33 (Hanö)
Poland	34	20
Lithuania	32	26
Estonia	37.5 (Kihnu, Sorve)	28 (Sorve)
Finland	30, Hanko Tulliniemi (Southern coast)	24, Lemland Nyhamn, Rauma Kylmäpihlaja (Southern coast)

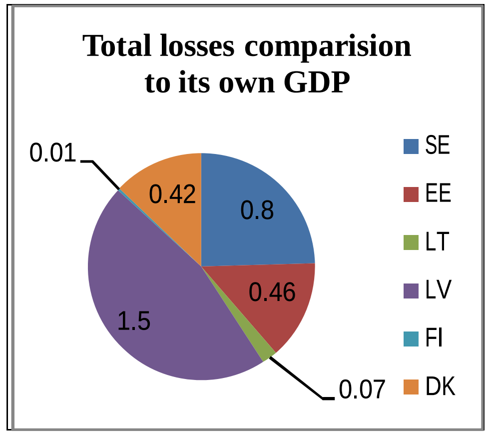
This table is taken from European Union funded research project named Astra

In Estonia, due to the storm maximum sea level reached up to +275 cm in Pärnu and in Tallinn 152 cm. Heavy wind reached in Pärnu, Haapsalu and Matsalu Bays. Total property damaged in Estonia was €9 m but at that time only 1/3 population was insured. Flood water damaged 300 cars, agricultural and outdoor equipment, firewood stocks, heaps of movables which leads to total loss of €48 m. Rest of the Baltic and Nordic countries faced the same problem of access flooding.

Total damage in Baltic and Nordic countries (in million EUR)

Sweden	2 300	Latvia	192
Estonia	48	Finland	20
Lithuania	15	Denmark	617





2.2 St. Jude storm

The St. Jude storm, also named Cyclone Christian, It is the most recent and worst windstorm hit in Northwestern Europe on 27 and 28 October 2013.

The highest wind speed was measured in Denmark where a gust of 54 m/s (120.8 mph) was recorded in the south part of the country it was the strongest wind speed ever recorded in Denmark. Then the storm turned towards north and east, it hit northern Germany, Sweden, and Russia. However, it got slow across the Baltic Sea to Latvia and Estonia. It caused damage and disruption the Northern coastal nations of Europe, including Denmark, Sweden, Estonia, and Latvia. Total insured loss was estimated between € 1.5 billion and € 2.3 billion by AIR Worldwide. Nevertheless, overall atmospheric conditions were favorable for storms to impact Baltic States and Northern Europe.

Chapter 3

Main modules and financial perspectives of cat modeling

3.1 Information required for cat modeling

To know how to model cat events, its input, output and definitions are essential to know. In this chapter brief overview of inputs, outputs, definitions are presented and further, statistical derivation of its financial perspectives has been done.

3.1.1 Definitions

These all definitions are important to have basic knowledge of cat modeling

- **Average annual loss (Pure premium) -** The mean value of a loss distribution or expected annual loss is known as average annual loss. It is estimated the requirement of annual premium to cover losses from the modeled perils over time.
- **Probable maximum loss (PML) -** The value of the largest loss that occurred from a catastrophe event is to be called probable maximum loss. Which assumes the failure of all active protective features (e.g. - In earthquake failure of sprinkler linkage may cause a bigger loss rather than in its availability).
- **Return period –** In very common term return period is an inverse of probability and explain that the event will be exceeded in any one year. It is a statistical measure of historic data denoting the average recurrence interval over an extended period of time.). For example, a 10 year flood has a 1/10=0.1 or 10% chance of being exceeded in any one year and a 50 year flood has a 0.02 or 2% chance of being exceeded in any one year.

$$T = 1/p = (n+1)/m$$

Where, T= return period, p= probability of occurrence of event

n = number of years on record, m= number of recorded occurrences of the event

- **Exceedance probability (EP) –** It explains that the probability of different levels of losses will be exceeded. An exceedance probability curve is called EP curve. For example - windstorm has an exceedance probability of 2%. So it means that there is a 2% probability, a certain level of loss will exceed.
- **Aggregate exceedance probability (AEP) -** The AEP shows the probability of seeing aggregate annual losses of a particular amount or greater.
  - It gives the information of losses assuming one or more occurrences in a year.
  - It is useful for aggregate based structures like stop loss, reinstatements etc.

  AEP(>=OEP)
- **Occurrence exceedance probability (OEP) -** The OEP shows the probability of seeing any single event within a given period and with a particular loss size or greater.
  - It gives the information on losses assuming a single event occurrence in a given year.
  - It is useful for occurrence based structures like quota share, working excess, etc.
- **Event loss tables (ELT) –** The ELT generates the raw data that is useful to build up EP curves and calculate other measures of risk. In general ELT is a set of events along with the modeled losses estimated to occur from each event.
- **Deductible -** The part of an insurance claim to be paid by the insured is called deductible or it is an insured retention.
- **Ground up loss -** The total amount of loss before taking account of any retention, deductibles, or reinsurance. A ground up loss is the loss to the policyholder.
- **Gross loss –** Total financial loss to the insurer.

3.1.2 Inputs and Outputs

3.1.2.1 Input (Exposure Data)

This input data of the building is required to estimate its losses. These given information is shown limited, as data requirement may vary risk to risk (e.g. flood, storm, earthquakes).

- **Geocoding data** - Street address, postal code, county/CRESTA zone, etc.
- **Primary attribute information** (physical characteristics of the exposures) - Construction, occupancy, year built, number of stories
- **Secondary Attribute**- Roof type, square footage (area) of building

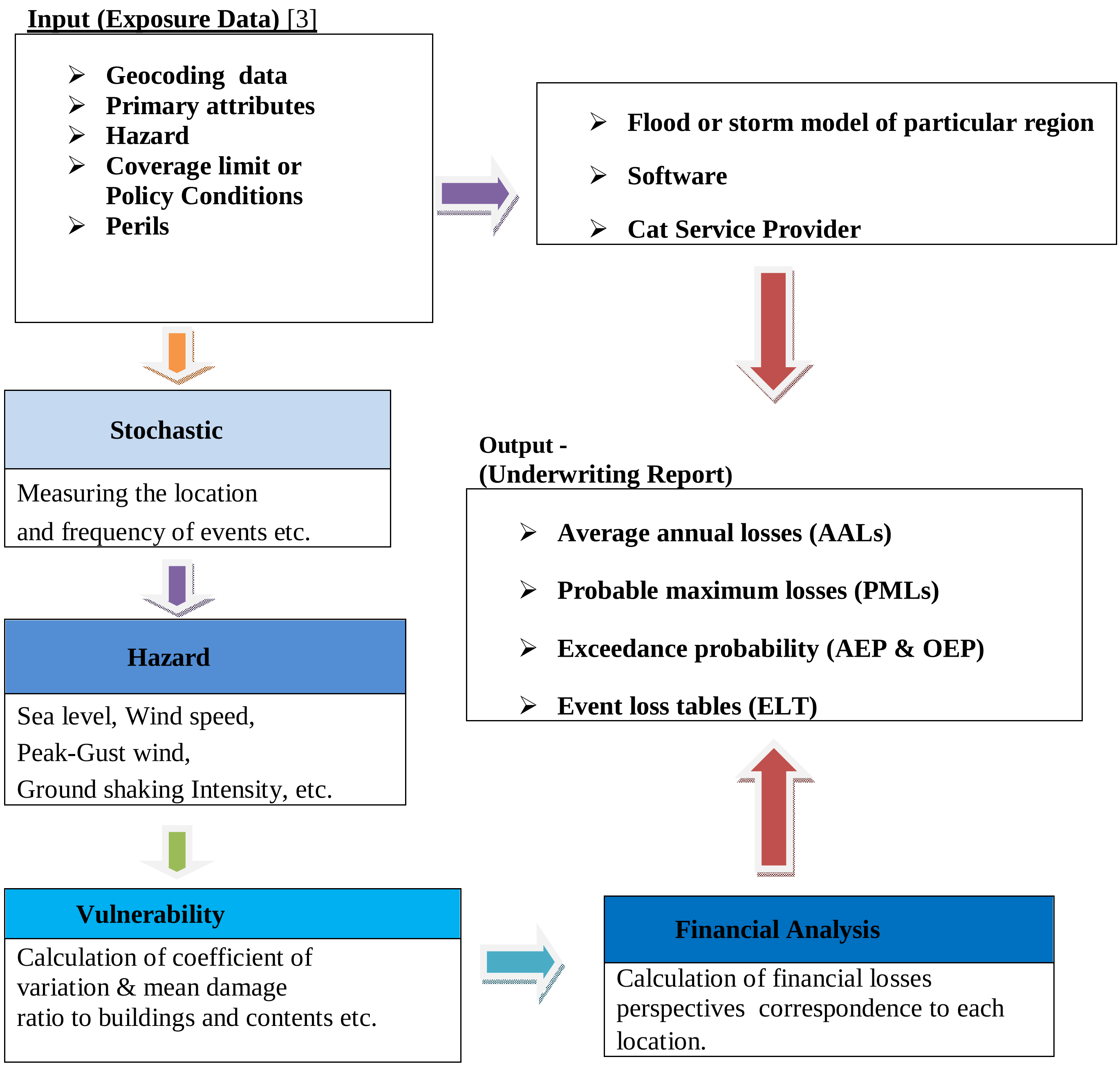
- **Hazard –** E.g. - Soil type, distance to coast(for flood insurance)
- **Coverage limit or Policy Conditions–** Deductible, sum insured, layers, limit and reinsurance treaties
- **Coverage -** Buildings, contents, time elements (business interruption and expense coverage)
- **Perils –** Flood, storm (hurricane or windstorm), earthquake, tornado, winter storms (snow, ice, freezing rain), wild fire, tsunami
- Man made catastrophes:
  - Terrorism
- **Line of business (LOB) -** E.g. - Private properties or commercial properties.

3.1.2.2 Output (Financial Prospective)

Insurance and reinsurance companies are interested in exceedance probability (AEP & OEP) and event loss tables to compute different perspectives of level (e.g.- ground up , gross ,net pre cat and net post cat). They also want to know about probable maximum losses (PMLs) and average annual losses (AALs) of their portfolio. It can be calculated from the loss distributions. It helps insurance company to charge their premium in risk prone areas, underwrite its premium.

3.2 Working process of cat modeling for pricing purpose

3.2.1 Basic concept for pricing



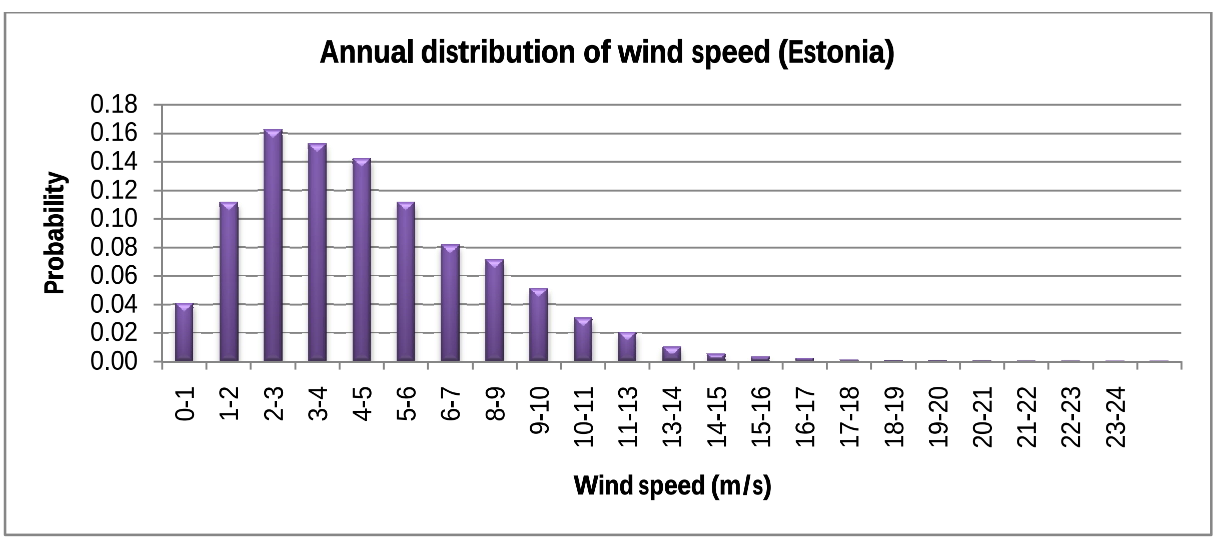
3.3 Cat modeling main modules

Cat modeling is composed three main modules hazard, vulnerability and financial module. In this section, this thesis is going to explain all the three modules

3.3.1 Hazard module

The hazard module estimates the potential disasters and their frequency. Whenever wind speed reach to its heavy level and getting (≥ 33 m/s or ≥ 74 mph) a form of hurricanes. In this case intensity parameters (wind speed, pressure, forward velocity, radius of maximum wind etc.) are modeled using complex mathematical equations. Windstorm model will not simulate just only historical windstorms already occurred but also simulate a much larger number of storms. Mostly windstorm models are derived from 10,000 of stochastic storms. Each event is modeled using the exposure data. Basically, it depends on the location of building (i.e. a hurricane occurring in Pärnu does not impact on the building situated in Harju county) no impact in this case. However, it may have some impact if the building is close to the hurricane path. The windstorm model equations allow the model to estimate the wind speed and a frequency at the building location, for each windstorm and its intensity parameters. Intensities from all computed events give the probabilistic distribution of wind speeds at the structure location. This is sent to the engineering module where the probability distribution of the corresponding damage will be derived

Distribution of wind speed probability in Estonia



Output of hazard module

3.3.2 The vulnerability module

Next module explains how the damaged can be calculated which is done with building by an event. However, there are many factors which can cause damage to the building but the main feature of a building proves to be a good indicator of its vulnerability and damage ratio. The ratio of the cost to repair a building or content, to the cost of rebuilding it, is known as damage ratio. Damage ratio of a building is a function of wind speed (v).

Damage ratio ($DR_B(v)$) = Cost to repair of damaged building/replacement cost of building (3.1)

Where, Replacement cost of building = Replacement value is the actual cost to replace an item or structure at its pre-loss condition

B = Building which we are analysis

v = Wind speed

As quiet often, all the buildings have small differences in construction, occupancy, number of stories and local site. So, when the same intensity of wind speed hit to two identical buildings. It faces different levels of damage and major differences in losses [4]. To find this variability in damage and losses, it is better to concentrate on the whole distribution of possible values of the damage ratio not only a single value. The mean of this distribution is called as the mean damage ratio. Mean damage ratio is expectation of damage Ratio ($DR_B(v)$).

Mean damage ratio ($MDR_B(v)$) = Average loss / Replacement value (3.2)

Uncertainty in building damage ratio is a reflection of the variance of Damage ratio [6, p 3.2]

$$[\sigma_{DR}(v)]^2 = Var[DR_v] \quad (3.3)$$

Where, $\sigma_{DR}(v)$ =standard deviation

For wind speed of windstorm, a graph of the mean damage ratio as a function of intensity is known as a vulnerability function and it can be shown in section 3.4 and Table 5.

3.3.3 Financial Module

To compute the loss distribution of damage, which is done to the building by windstorm, is a part of financial module. While doing all the calculation in this module all the policy conditions of insurance should be remembered because it is also incorporated in it. The damage ratio distribution calculated from the vulnerability module for a windstorm is multiplied by the building replacement value to compute the loss distribution. Sometimes convolution proves the key to compute financial loss distribution [4].. The combined loss distribution of all buildings can be calculated by convolution method. Let us assume that two locations A and B, for each event has loss distributions l_i and l_j respectively. So all the possible combinations of loss distributions $l_i + l_j$ to their correspondence probabilities, given the probability distributions of l_i and l_j separately can be calculated by convolution method. Let, L defines the total loss for two locations then probability distribution for two locations can be showed as

$$P(L) = \sum_{P_1(l_i) \times P_2(l_j)} P_1(l_i) \times P_2(l_j)$$

Where, $P(L)$ = Total Probability distribution of both the locations

$P_1(l_i)$ = Probability distribution of location A

$P_2(l_j)$ = Probability distribution of location B

In this way by using convolution method, if we find two loss distributions for the two locations then the range of the resulting loss distributions is equal to the sum of the ranges of loss distributions separately.

3.4 Estimation of mean damage ratio of building in respect of windstorm

In section 3.3.2, it has already discussed the damage ratio and mean damage ratio briefly. In this section it will be discussed more broadly.

How much damaged has been done and building is replaced. It depends on various factors we are discussing here three scenario of it. First case, due to minimum intensity of the wind, damage is done in roof covering and rest building is fine then only one element of building to be replaced. Second case, if wind speed is high and damage is done to many elements of the building then it can be seen that only those elements to be replaced which are damaged or it leads to whole building failure. Third case, if wind speed is extreme and damage is done to the whole building, then whole building will be replaced [6, p 3.6].

So if we are aware of three components of damage ratio, then model for mean damage ratio of building can be defined as

$$DR_B(v) = \lfloor \sum_{i=1}^{n} x_i P_i^{\alpha}(v) \rfloor^{1/\alpha} \tag{3.4}$$

Where, x_i = Weights and $\sum x_i$ =1 and

n = Number of components

α = Parameter which reflects how many component elements must fail before the building is replaced

P_i (v) = Probability of component i of the structure to be replaced which is function of wind speed v

Let us assume, random variable R_i which can be defined as the wind speed range over which i component can be replaced and v is called wind speed. Now introducing new random variable Y_i which can be described as

$$Y_i = R_i - v \tag{3.5}$$

If $R_i \leq$ v, then the component ith can be replaced and if r_i is the realization of changing variable R_i. The density function of R_i is $f_{R_i}(r_i)$, then component ith can be replaced and its probability can be defined as

$$P_i(v) = \int_0^v f_{R_i}(r_i) \, d r_i \tag{3.6}$$

We can collect some knowledge from historical data of the density function of R_i. So providing some estimated, a value in the range of $f_{R_i}(r_i)$ with some confidence interval [6, p 3.7]. As unavailability of accurate estimate, we can assume $f_{R_i}(r_i)$ is uniformly distributed r.v. and its distribution is

$$f_{R_i}(r_i) = \begin{cases} 0 & r_i < v_{i1} \text{ or } r_i > v_{i2} \\ \frac{1}{v_{i2} - v_{i1}} & v_{i1} < r_i \leq v_{i2} \end{cases} \tag{3.7}$$

Where, v_{i1} = The wind speed at which component i starts to be replaced

v_{i2} = The wind speed at which all components will be replaced

Using equations (3.6) and (3.7), we get the distribution such as

$$P_i(v) = \begin{cases} 0 & v \leq v_{i1} \\ \frac{v - v_{i1}}{v_{i2} - v_{i1}} & v_{i1} < v \leq v_{i2} \\ 1 & v > v_{i2} \end{cases} \tag{3.8}$$

By using (3.4) and (3.8) equations, we get the damage model such as

$$DR_B(v) = \left[\sum_{i=1}^{n} x_i \left(\frac{v - v_{i1}}{v_{i2} - v_{i1}}\right)^{\alpha}\right]^{1/\alpha} \tag{3.9}$$

In the beginning of this model, we already discussed about three cases of damaged due to wind (Low, medium and high). Further, if damaged is done then we can give preferences which element is to be replaced. Due to lack of data in the thesis, these assigning values are hypothetical only. We are providing rating according to its importance in building (i.e. first, second, third and so on….) and this rating according to its level of importance is denoted by M_i.The weights then can be calculated by following formula [6, p 3.9]

$$x_i = \frac{M_i}{\sum_{j=1}^{n} M_j} \tag{3.10}$$

3.4.1 Computation of mean damage ratio

Following problem and its explanation can explain properly how to estimate the mean damage ratio in Estonia correspondence to wind speed in case of windstorm.

Let us assume, a hypothetical class of building located in Pärnu county which consists 1-2 stories wooden buildings which are corresponding to all 10 levels of windstorms given in table 2 and components are shown in table 1 .If mean damage ratio is explained as equation (3.9). Calculate the mean damage ratio for each windstorm and plot the damageability curve for the building.

The Table (1) denotes most often components failure in buildings when building hit by wind and it is also divided correspondence to its relative Importance of Mode Mi. Estimation of v_{i1} and v_{i2} for 1-2 stories wooden buildings in Estonia. As it has already been discussed due to lack of information and data in thesis, these tables values are hypothetical [6, p 3.10].

Table 1

Categorize relative Importance of Mode Mi	Components	Thresholds of resistance(m/s)	
1	Roof covering replaced	10	35
2	Roof decking replaced	13	40
3	Roof framing replaced	17	43
3	Roof-wall anchorage replaced due to sunction	15	47
3	Roof wall anchorage replaced due to int pressure	17	50
3	Lat. Bracing system replaced	19	53
1	Openings replaced	22	57
1	Cladding replaced	25	60
3	Frame foundation connection replaced	27	62
3	Foundation replaced	33	65

Table 2

Categorize Windstorm	Windstorm Wind speed range(m/s)	Windstorm Wind speed (m/s)
1	14-18	16
2	18-22	20
3	22-26	24
4	26-30	28
5	30-34	32
6	34-38	36
7	38-42	40
8	42-46	44
9	46-50	48
10	50-54	52

Table 3

Building failure modes are shown by parameter α	
Number of cause leads to building failure	Approximate values of α
1 (Series system)	10
2 (Hybrid system)	5
3 (Hybrid system)	1
4 (Hybrid system)	-1
5 (Hybrid system)	-2
6 (Parallel system)	-5

Solution – As we are doing calculation for 1-2 stories wooden buildings and for wooden, private property and low rise combination can lead towards failure of the system. We know from equations (3.8), (3.9) and (3.10), if α = 1 then the mean damage ratio will be such as

$$DR_B(v) = \left[\frac{\sum_{i=1}^{n} M_i * P_i(v)}{\sum_{j=1}^{n} M_j}\right] \tag{3.11}$$

From equation (3.8), probability of damageability function for component i is defined as

$$P_i\ (v) = \frac{v - v_{i1}}{v_{i2} - v_{i1}}$$

The component damage function for roof covering replaced by using Table 1

$$P_1(v) = \frac{v-10}{35-10}$$

In the same way, the component damage function for roof decking replaced by using Table 1

$$P_2(v) = \frac{v-13}{40-13}$$

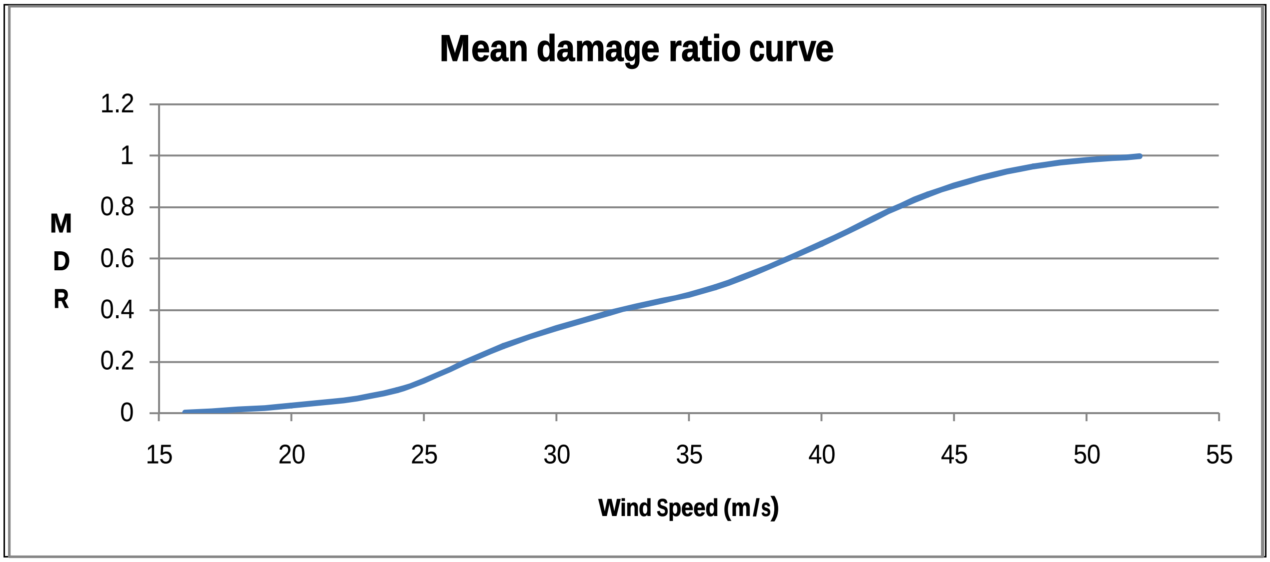
and so on. Therefore, the mean damage ratio for the building can be calculated

$$MDR_B(v)) = \{\frac{v-10}{35-10} + 2 * \frac{v-13}{40-13} + \ldots + 3 * \frac{v-33}{65-33}\}/(1+2+3+..3) \quad (3.12)$$

By putting v (wind speed) values from Table 2 in equation (3.12) mean damage ratio table can be found such as Table 4 and this table is quite useful and it can be used in creating mean damage ratio curve, vulnerability function and calculation of loss distribution. The damage ratio distribution for a specific event is multiplied by the building replacement value to obtain the loss distribution.

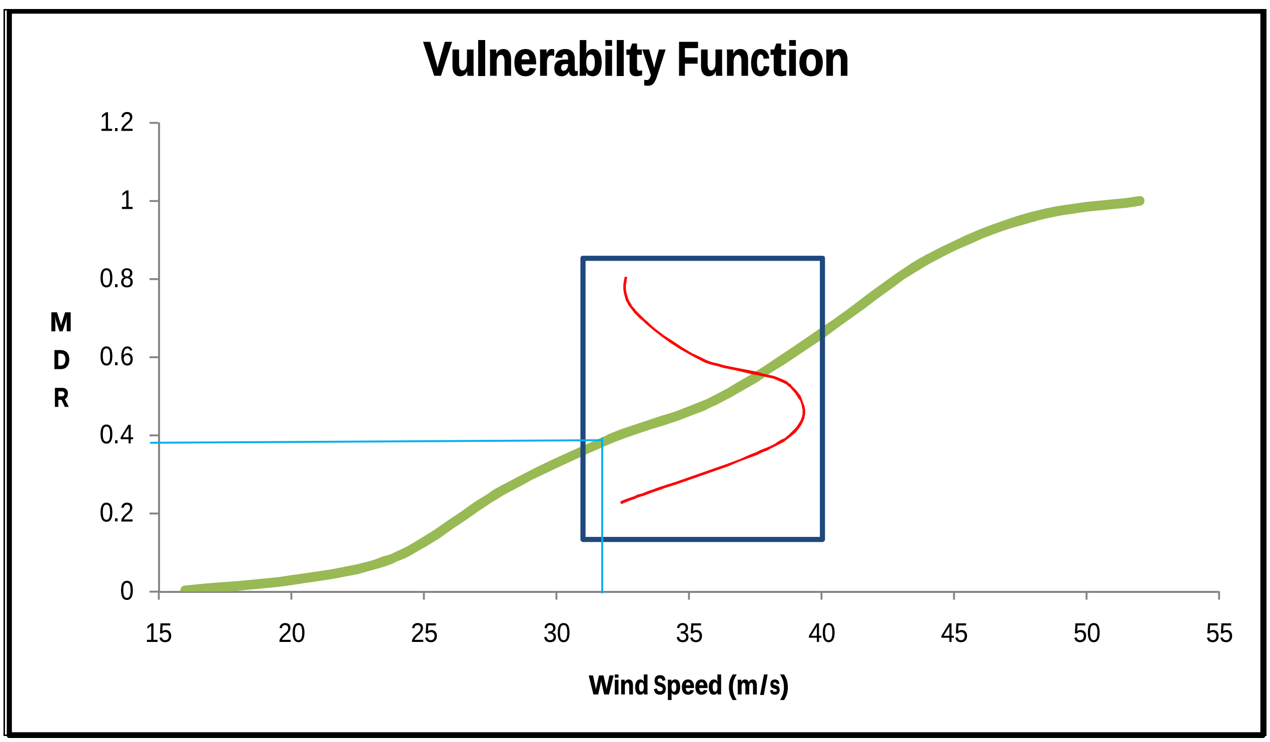
Categorize Windstorm	Windstorm Wind speed range(m/s)	Windstorm Wind speed (m/s)	Mean damage ratio of building
1	14-18	16	0.003
2	18-22	20	0.03
3	22-26	24	0.09
4	26-30	28	0.26
5	30-34	32	0.39
6	34-38	36	0.49
7	38-42	40	0.66
8	42-46	44	0.85
9	46-50	48	0.96
10	50-54	52	1.00

Table 4



Using Table 4 this mean damage ratio curve has been constructed

By using mean damage ratio Table 4 correspondence to wind vulnerability function of building can be shown such



Damage ratio of given intensity of wind speed

Damage ratio distribution taking into account of all the different values of damage ratio surrounding the mean damage

This graph represents as the intensity of wind speed increases, the mean damage ratio also increases.

3.5 Simulations of financial loss on the basis of cat modeling

3.5.1 Windstorm model to calculate exceedance probability (EP)

Exceedance probability plays vital role in cat modeling to see that loss can exceed to a certain limit with correspondence to return period. In windstorm modeling EP curve is very important. There is used a statistical approach to derive EP. For any given portfolio, in the course of the year of the maximal loss occurrence of EP distribution can be defined as

$$EP\ (L) = P\ \{\text{A loss exceeding L will occur during the year}\} \quad (3.13)$$

If we follow the standard actuarial approach then the exceedance probability of loss distribution generated from the event can be decomposed separately into frequency and severity components. Let us assume, severity of occurrence of a single random windstorm is corresponding to the cumulative distribution function (CDF) and express as

$$F_s\ (L) = P\ \{\text{The loss does not exceed L, given that the windstorm will occur}\} \quad (3.14)$$

Let us assume, windstorm mean frequency is Poisson distributed λ and it is independent of severity. In this way, it can be said that the number of windstorms, take place within the course of a year with loss amounts greater than L is also a Poisson distributed with mean $\lambda\ (1-F_s\ (L))$. So the exceedance probability per occurrence (OEP) can be expressed as

$$EP\ (L) = \{1 - \exp\ [-\lambda\ (1-F_s\ (L))]\} \quad (3.15)$$

If we assume that a single windstorm event can be denoted by a parameter vector $\boldsymbol{\omega}$, then set of historically storms events can be expressed as

$$\omega_H = (\omega_1, \omega_2 \ldots \ldots \ldots \ldots \ldots \ldots \omega_N) \tag{3.16}$$

Where, N = The number of windstorms which have occurred over the past 100 years.
$\boldsymbol{\omega_i}$ = Physical properties of the i^{th} historical Windstorms (i.e. translation speed, landfall location)
The dependence of the exceedance probability on the historical windstorms can be shown as

$$EP\ (L, X \mid \omega_H) = \{\ 1 - \exp[-\lambda(\boldsymbol{\omega}_H)\ \ (1 - F_s\ (L, X \mid \omega_H))]\} \tag{3.17}$$

Where, The mean annual frequency $\lambda(\boldsymbol{\omega}_H) = N / n$

N = Total number of historical windstorms events
n = The number of years of historical windstorms events
X = The matrix has all the information of certain portfolio (i.e. total insured value geocoding, construction, coverage)
In equation (3.17) it is a big challenge to calculate, the exact dependence of $F_s\ (L, X \mid \omega_H)$ on ω_H . Here, F_s has this sort of structure that it can reproduce the meteorological variability of the historical windstorms which took place. For example, a windstorm model will not simulate just only historical windstorms already occurred but also simulate a much larger number of windstorms. This type of procedure is dependent on model and provides a more smoother, varying geographical and intensity coverage.

3.6 Windstorm model methodology to calculate the statistics of losses (AAL)

As it is discussed, in 3.3 last section. A windstorm model will not simulate just only historical windstorms already occurred but also simulate a much larger number of windstorms somewhere around 10,000 years of simulation [4]. Following the same approach, let us assume during an interval of time t, the number of windstorms N (t) is a Poisson distributed [5, p 121]. The frequency of windstorms occurring has a single parameter, $(\lambda t) > 0$. So its means and variance will be such as

$$E[N(t)] = Var[N(t)] = (\lambda t) \qquad (3.18)$$

We can assume that losses occurred due to multiple windstorms will be summed up. Let us assume, the loss correspondence to ith event, i=1,2,……….,N(t) is denoted by Li>0 and the event loss is independent of event occurrence. In this way it can be said that windstorms losses (Li|e)s are independent and identically distributed [9]. So total loss can be represented as

$$L(t) = \sum_{i=1}^{N(t)} Li|e \qquad (3.19)$$

We are trying to calculate the average of total loss in unconditional form over time period t. So simply, we can find it by using expectation of loss conditioned on the occurrence of windstorms and average annual loss (AAL) can be calculated by taking expectation in equation (3.19), we get

$$\mu_L = E[L(t)] = E\{E[L(t)|N(t)]\} = E\left\{E\left[\sum_{i=1}^{N(t)} Li|e\right]\right\} \qquad (3.20)$$

As, the event loss is independent of event occurrence then from equation (3.20), we get

$$\mu_L = \{E[N(t)]*E(Li|e)\} = (\lambda t)\mu_{L|e} \qquad (3.21)$$

In the same unconditional variance of loss can be found such as

$$\sigma_L^2 = Var[L(t)] = \{E[N(t)]*Var(Li|e) + Var[N(t)]*[E(Li|e)]^2\} \qquad (3.22)$$

So equation (3.22) can be written such as

$$\sigma_L^2 = Var[L(t)] = (\lambda t)*(\sigma_{L|e}^2 + \mu_{L|e}^2) \quad (3.23)$$

Where, $\mu_{L|e}$ = The conditional mean loss of given the occurrence of windstorm

$\sigma_{L|e}$ = The conditional standard deviation of given the occurrence of windstorm

In equation (3.23) the conditional mean and standard deviation of windstorm can be calculated such as

$$\mu_{L|e} = \sum_{j=1}^{K} P_j * (L_j|e) \quad (3.24)$$

$$\sigma_{L|e} = \{ \sum_{j=1}^{K} P_j * \left(L_j|e - \mu_{L|e}\right)^2 \}^{1/2} \quad (3.25)$$

Where, P_j = The conditional probability for the jth event and $(\sum P_j = 1)$
K = The number of simulated windstorms in the set of event

As we are trying to calculate, average annual loss (AAL) so it means we want average loss for one year [5, p 122]. So we can find mean and variance for one year (t=1) from equation (3.21) and (3.23)

$$\mu_L = \lambda * \mu_{L|e} \quad (3.26)$$

$$\sigma_L^2 = \lambda * (\sigma_{L|e}^2 + \mu_{L|e}^2) \quad (3.27)$$

For each windstorm category, the conditional mean and standard deviation can be computed by using (3.26) and (3.27) formulas.

Chapter 4

Windstorm and flood loss distribution of Estonia

4.1 Windstorm loss distribution

In this section, estimation is done to find the loss distribution of hurricane using wind speed data of seven wind speed stations of Estonia and insurance property data. It computes some foggy picture of losses. At last of this chapter, it is presented if we combined two perils windstorm and flood together in coastal areas then how much loss they both can create to properties and to insurance companies in Estonia. Although, it is difficult to correlate map information manually without any mapping software or any tool.

Wind data is used from seven given stations to its correspondence counties Jõhvi station - Ida-Viru county, Kihnu station – Pärnu county, Ruhnu station – Saare county, Sõrve station– Harju county, Viljandi station – Viljandi county, Virtsu station-Lääne county, Kunda station - Lääne-Viru county.

4.1.1 Relationship between wind and building

Whenever the wind hits with the building, there creates two pressures positive pressures (>ambient pressure) and negative pressure (<ambient pressure). The building should have good strength to resist the pressure of wind. In this condition the magnitude of the pressures is a function [11].

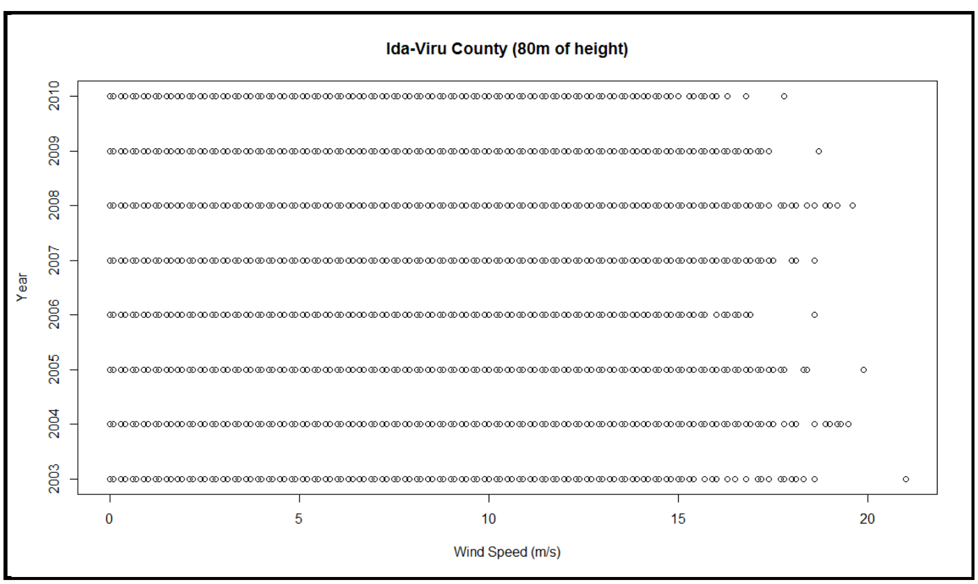
Exposure of the buildings can be defined and separated in zones e.g. –

Exposures (Zone)	Definition
A	Roughest terrain (Includes urban, suburban, and wooded areas)
B	It includes open flat terrain with scattered obstructions and areas adjacent to oceans in hurricane-prone regions
C	Smoothest (includes areas mud flats, salt flats, adjacent to large water surfaces outside hurricane-prone regions and unbroken ice)

Exposure zone shows that zone A, properties are more vulnerable to zone B. According to the historical wind speed data, Pärnu county can be put in more vulnerable storm zone A, then Harju county can be put in zone B.

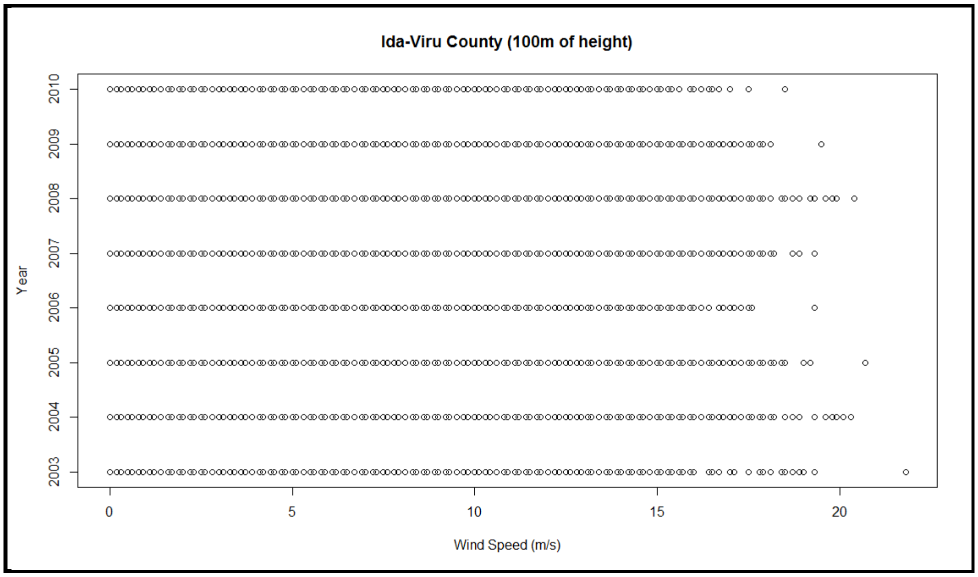
4.1.2 Windstorm loss distribution in Estonia

Further, we can think about to correlate properties with wind speed. Higher the height, higher the wind speed, in other words wind speed increases with height. The taller the building, the greater the wind speed [11]. We can see the same approach by using Jõhvi station, Ida- Viru county wind speed data.

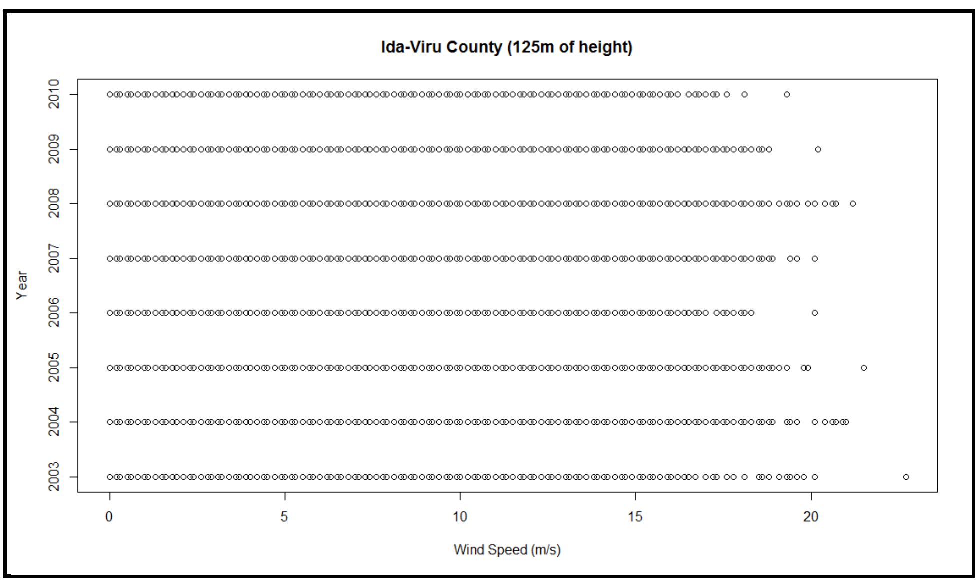


Graph- 4.1

Graph 4.1, 4.2, 4.3 shows maximum wind speed during days since 2003-2010 in Ida- Viru county



Graph- 4.2

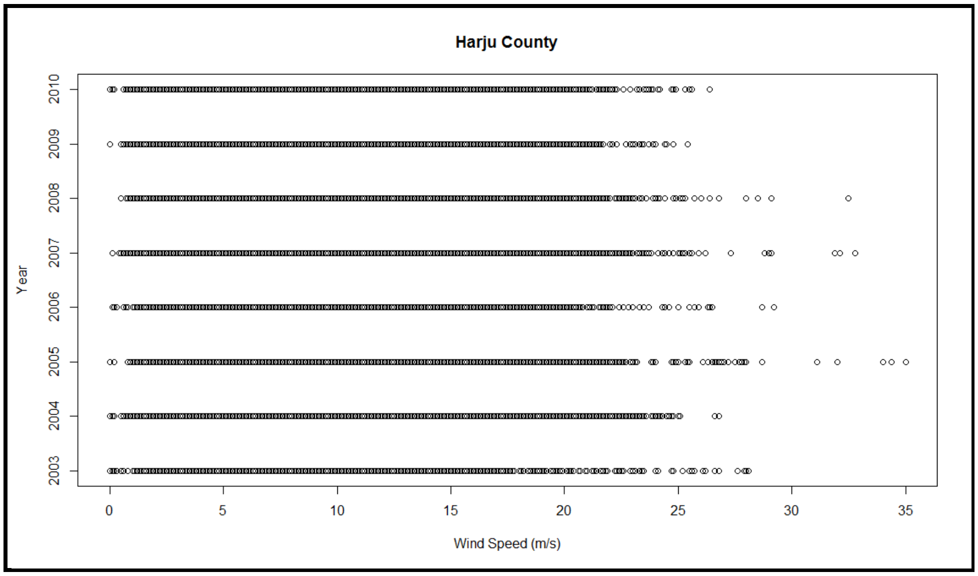


Graph- 4.3

By graphs 4.1, 4.2 and 4.3, it can be explained that as height increases from 80m, 100m, 125m respectively then wind speed is also increasing so buildings lying in Ida- Viru county which are high rises will be impacted more rather than low rises. However, this rule follows for every region.

Moving forward, we know if wind speed is at least 33 m/s or 74 mph then it takes the form of windstorm or hurricane and it can cause big amount of losses [10].

Graph- 4.4 shows maximum wind speed during days since 2003-2010 for in Harju county



Graph- 4.4

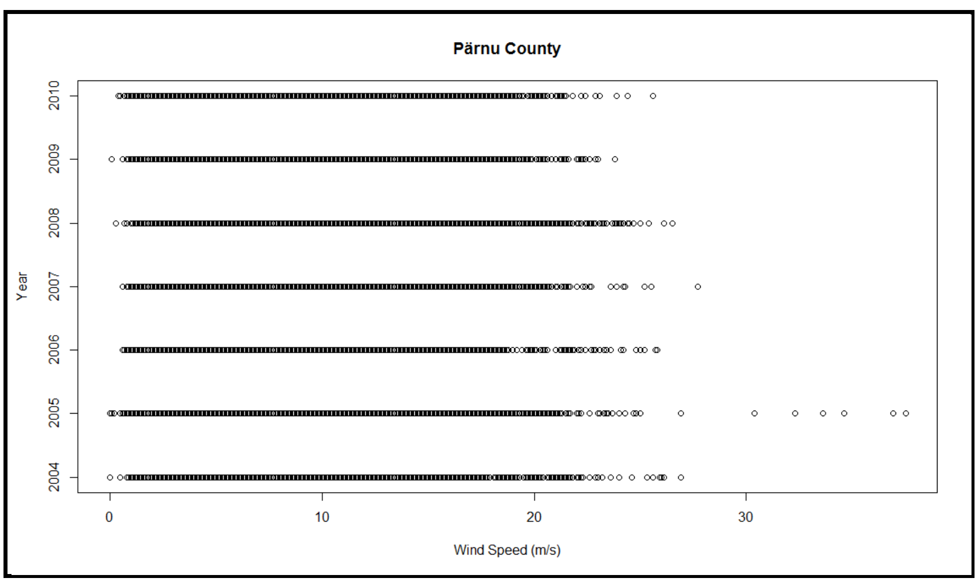
As it is explained, in chapter 3 in section 3.4.1 that mean damage ratio distribution can be useful to compute the loss distribution. If the mean damage ratio distribution for a particular event is multiplied by the building replacement value then it obtains the loss distribution. However, it should be remembered that mean damage ratio distribution generated for buildings in the chapter 3. That is about 1-2 stories wood buildings only otherwise if its construction, number of stories, year built, occupancy,geocoding, policy coverage and other factors may vary, then loss distribution will also vary a lot. Although same approach is used, to estimate Table 5, Table 6 and Table 7 for Harju, Pärnu and Ida- Viru counties respectively, by using ERGO properties insured data.

Harju County				
Categorize Windstorm	Windstorm Wind speed range(m/s)	Windstorm Wind speed (m/s)	Mean damage ratio of building	Maximum loss of total Insured value to portfolio(EUR)
1	14-18	16	0.003	13,900,479
2	18-22	20	0.03	139,004,793
3	22-26	24	0.09	417,014,378
4	26-30	28	0.26	1,204,708,204
5	30-34	32	0.39	1,807,062,306
6	34-38	36	0.49	2,270,411,615
7	38-42	40	0.66	3,058,105,440
8	42-46	44	0.85	3,938,469,127
9	46-50	48	0.96	4,448,153,368
10	50-54	52	1	4,633,493,091

Table 5

Table 5, expresses probable maximum loss correspondence to wind speed ranges. It other words, for example if the wind speed range is 30-34 then its correspondence mean damage ratio to the building is 0.39 and all the properties in the insured portfolio (to be assumed 1-2 stories wood only). Otherwise loss and mean damage ratio to building may vary a lot as discussed previously.

Graph- 4.5 shows maximum wind speed during days since 2004-2010 for in Pärnu county.



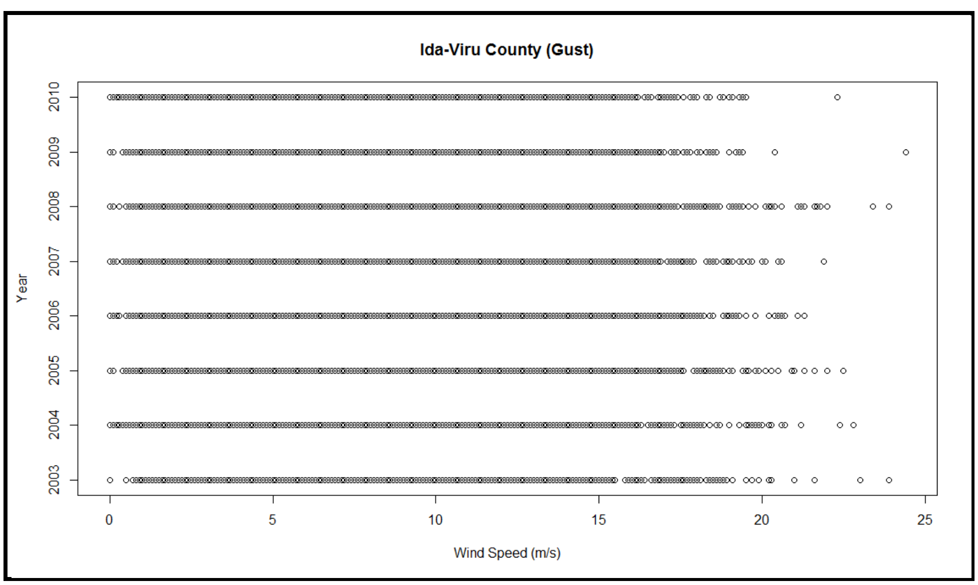
Graph- 4.5

Parnu county				
Categorize Windstorm	Windstorm	Windstorm	Mean damage ratio of building	Maximum loss of total Insured value to portfolio(EUR)
	Wind speed range(m/s)	Wind speed (m/s)		
1	14-18	16	0.003	1,325,336
2	18-22	20	0.03	13,253,359
3	22-26	24	0.09	39,760,078
4	26-30	28	0.26	114,862,447
5	30-34	32	0.39	172,293,670
6	34-38	36	0.49	216,471,535
7	38-42	40	0.66	291,573,904
8	42-46	44	0.85	375,511,846
9	46-50	48	0.96	424,107,496
10	50-54	52	1	441,778,642

Table 6

Table 6 and Table 7, express probable maximum loss correspondence to wind speed ranges. It other words, for example if the wind speed range is 34-38 then its correspondence mean damage ratio to buildings is 0.49 and all the properties in the insured portfolio (to be assumed 1-2 stories wood only). Otherwise loss and mean damage ratio to building may vary a lot as discussed previously.

Graph- 4.6 shows maximum wind speed during days since 2003-2010 for in Ida-Viru county

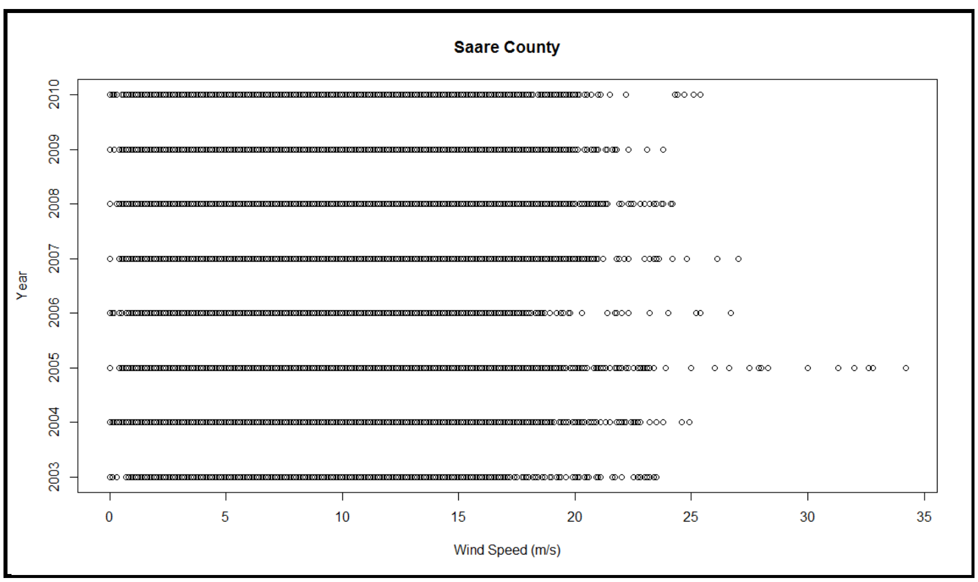


Graph- 4.6

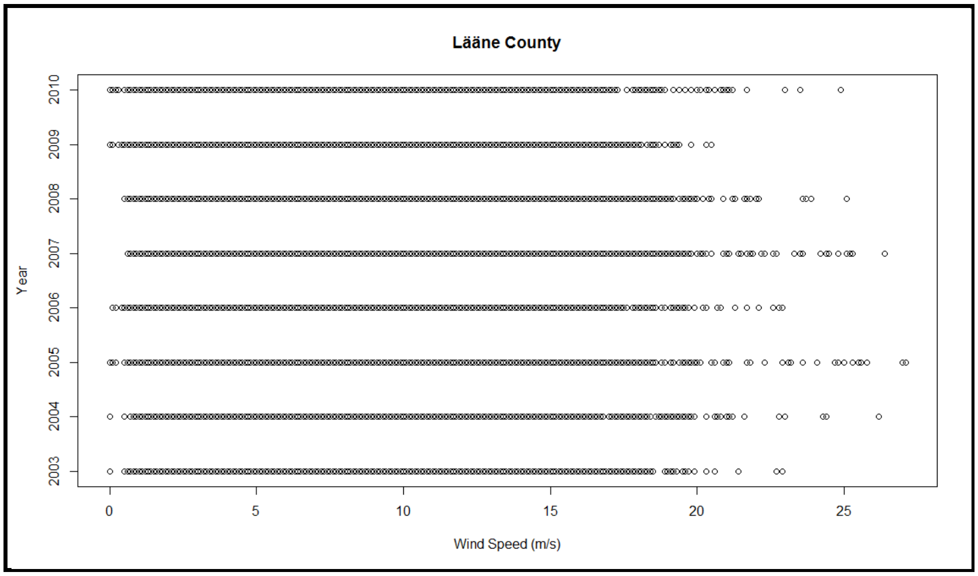
Ida- Viru County				
Categorize Windstorm	Windstorm	Windstorm	Mean damage ratio of building	Maximum loss of total Insured value to portfolio(EUR)
	Wind speed range(m/s)	Wind speed (m/s)		
1	14-18	16	0.003	1,775,774
2	18-22	20	0.03	17,757,738
3	22-26	24	0.09	53,273,215
4	26-30	28	0.26	153,900,399
5	30-34	32	0.39	230,850,598
6	34-38	36	0.49	290,043,060
7	38-42	40	0.66	390,670,244
8	42-46	44	0.85	503,135,920
9	46-50	48	0.96	568,247,627
10	50-54	52	1	591,924,612

Table 7

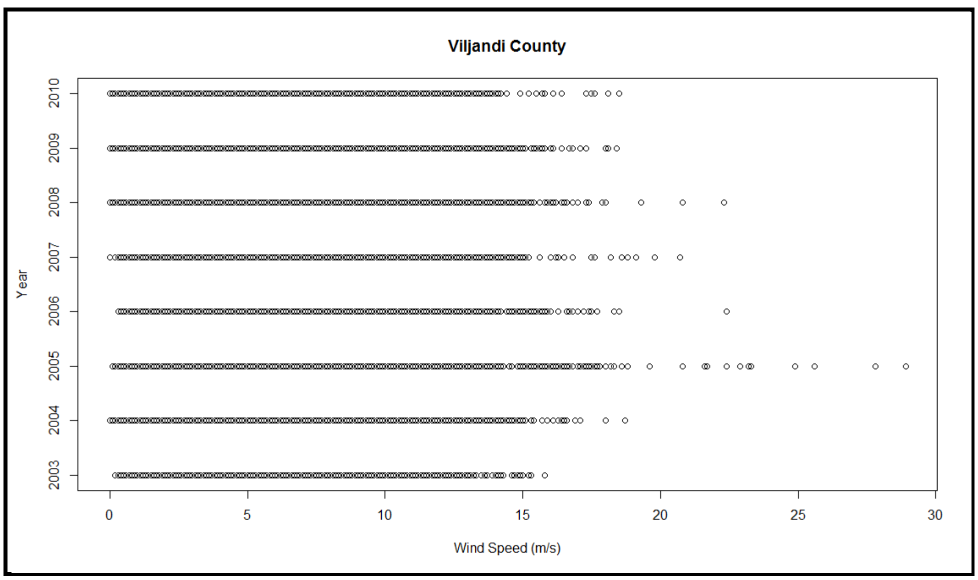
Graph- 4.7, 4.8, 4.9 show maximum wind speed during days since 2003-2010 for in Saare, Lääne, Viljandi counties respectively.



Graph- 4.7

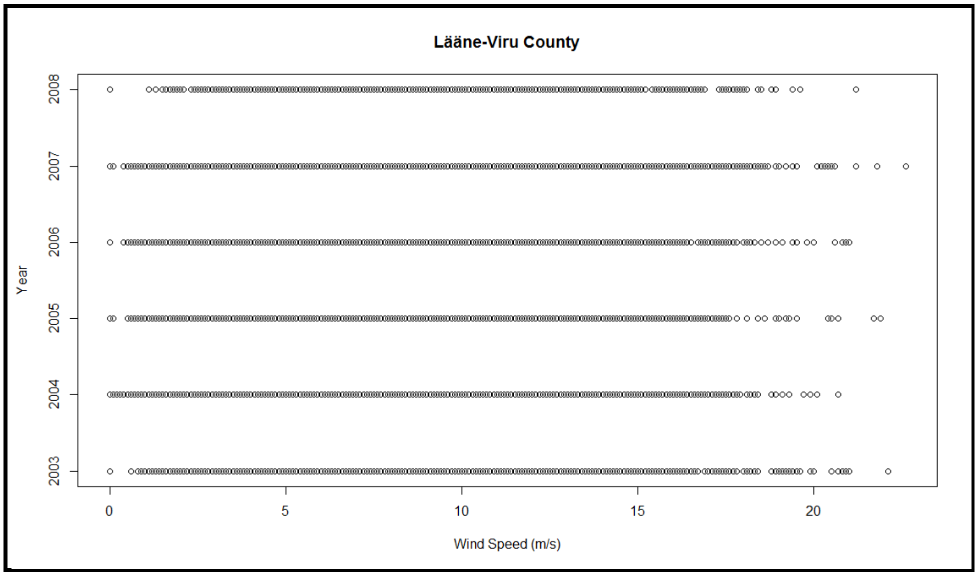


Graph- 4.8



Graph- 4.9

Graph- 4.10 shows maximum wind speed during days since 2003-2008 for in Lääne-Viru county.



Graph- 4.10

4.2 Flood loss distribution

If we discuss about Estonia then almost twenty areas in Estonia which are more vulnerable to flood it includes Tallinn, Pärnu and Tartu counties. Municipalities like Häädemeeste, Hanila and Haaslava. As coastal sea levels, snowmelt and rainfall are increasing due to climate change this is making many of these areas at risk [8]. Specially, we should think about the most vulnerable region for flood such as Pärnu and Lääne counties and South-West part of Estonia. The length of Lääne and Pärnu county coastline are 400 km and 242 km respectively.

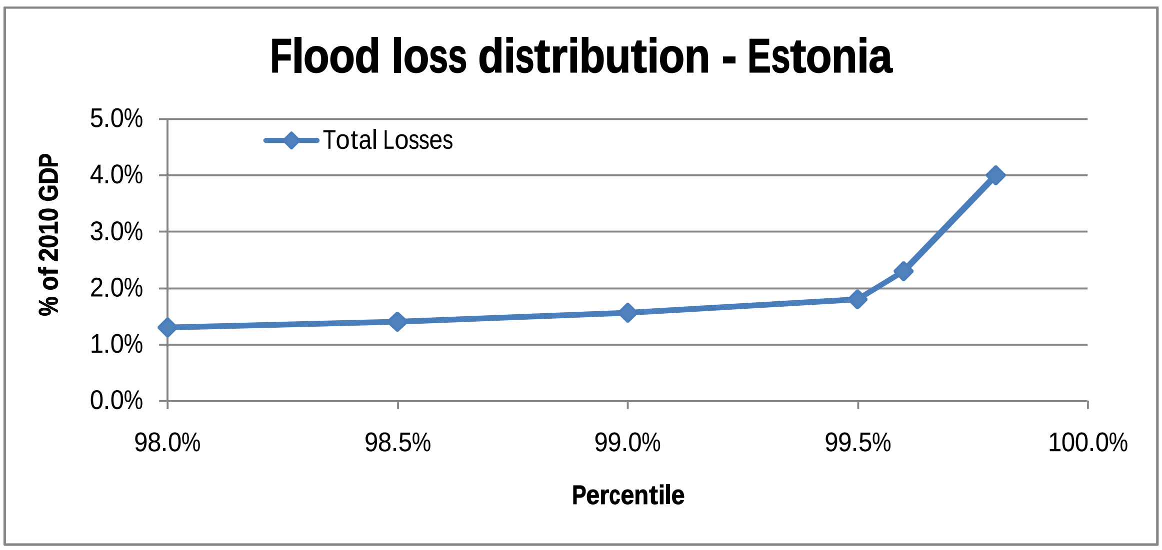
Flood loss distribution of Estonia, rest Baltic States and Scandinavia is presented in this part of the thesis. Baltic States and Scandinavia loss distribution are important as there is lack of historical data in Estonia. Hence, by creating such a scenario may help to show more appropriate estimation in case of Estonia.

4.2.1 Flood loss distribution in Baltic States and Nordic countries during 1990 -2010

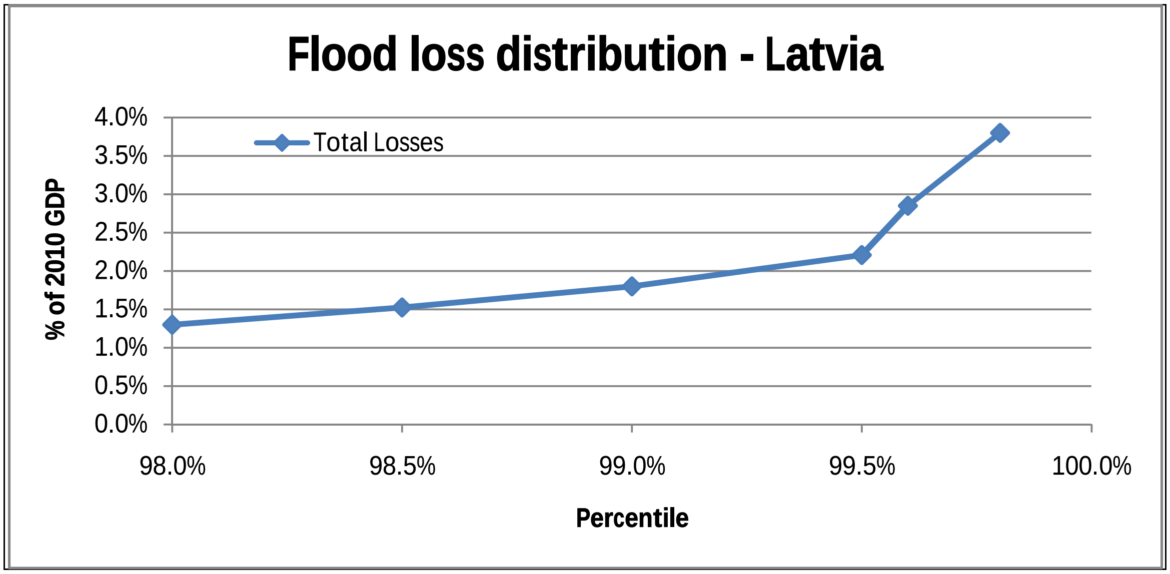
Flood loss distribution is estimated in the form of return period. For example, we can assume the 99th percentile, corresponding to the 100-year return period. The maximum loss occurred during

20-years time may underestimate the relevance of a given risk, but in many of the scenarios this is the only feasible solution to get an estimate of the risk.

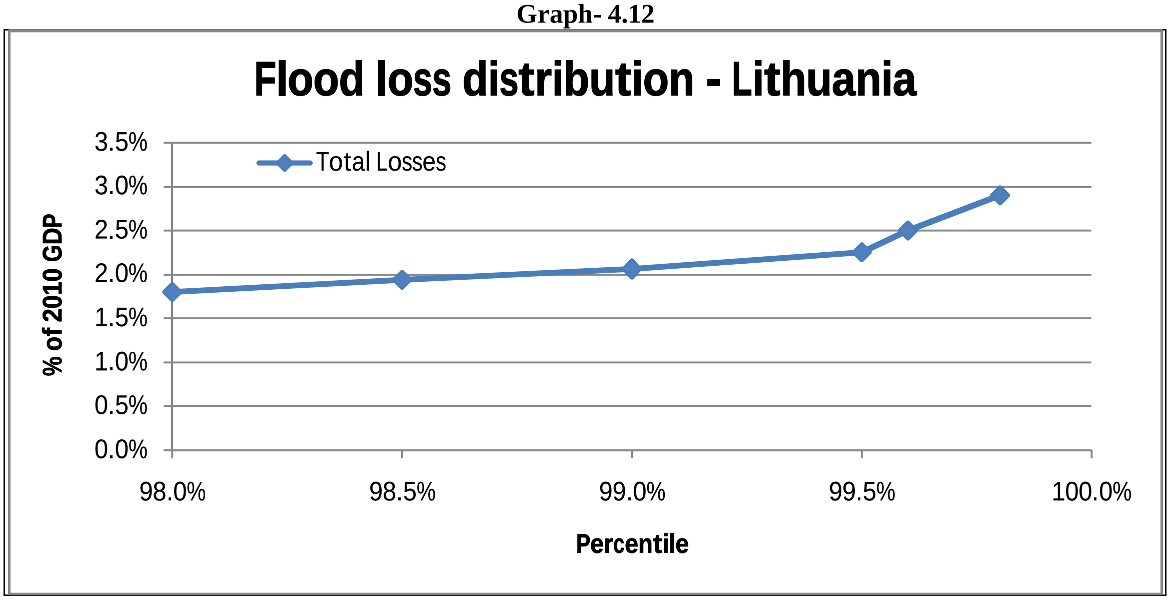
Graph 4.11-4.16, express the maximum historical total loss caused by flood in Estonia, Latvia, Lithuania, Finland, Sweden during 1990-2010 as a percentage of its own according to 2010 GDP respectively. These graphs also explains, exceedance probability of losses with respect to any event. For example in graph 4.11 flood loss distribution is reaching 4% of Estonian GDP (2010) at 99.8th percentile. It is corresponding to 500-year return period (1/500=0.2% => 100-0.2= 99.8%). So, chance of flood loss distribution to exceed 4% of Estonian GDP is 0.2% in any one year.



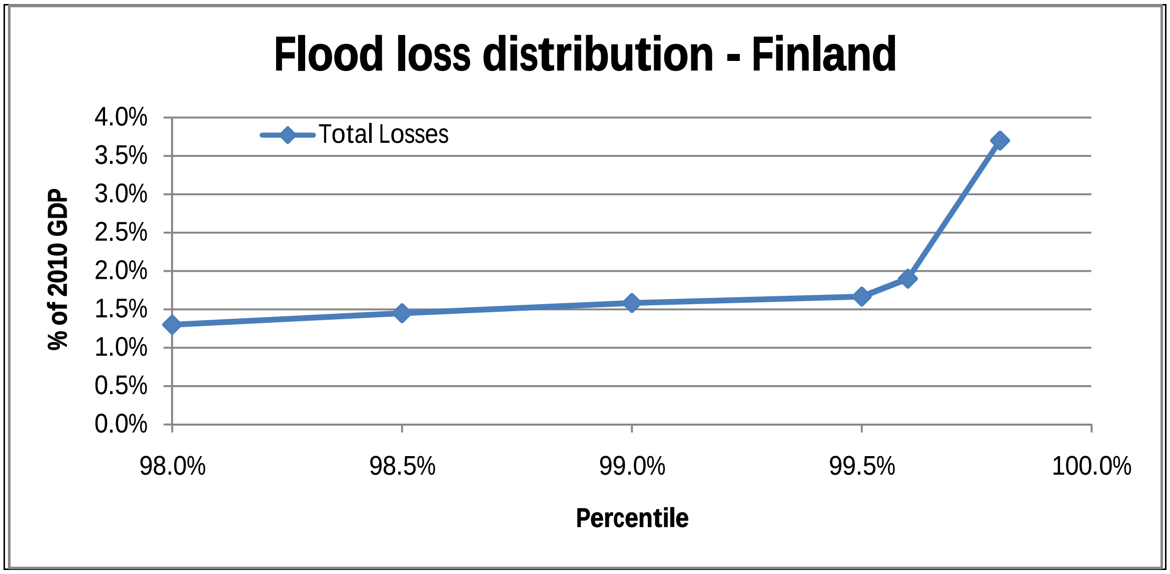
Graph- 4.11



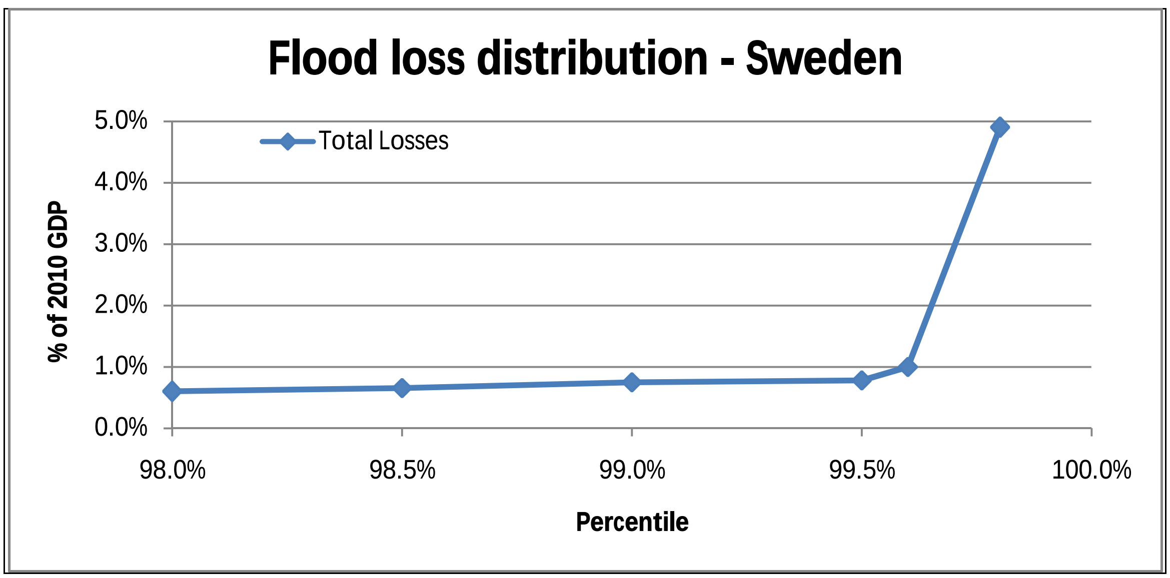
Graph- 4.12



Graph- 4.13

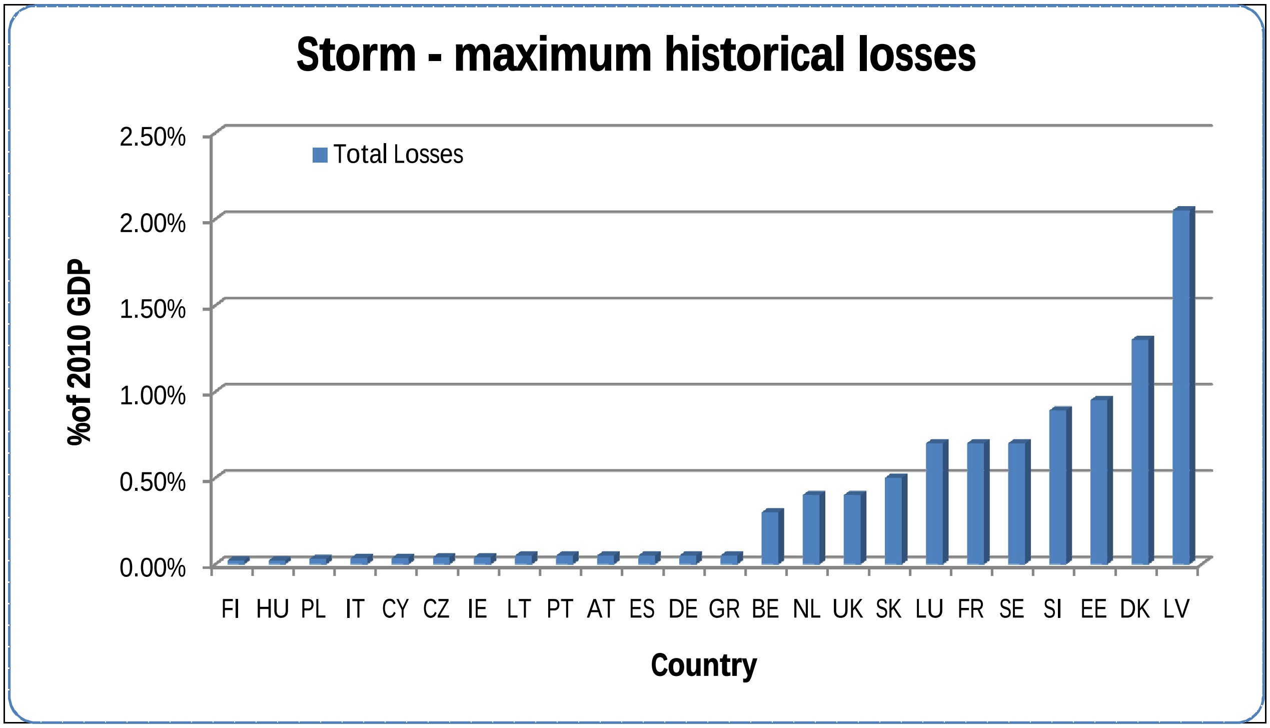


Graph- 4.14



Graph- 4.15

Graph 4.16 describes the maximum historical total loss due to storm in European countries during 1990-2010 as a percentage of its own according to 2010 GDP [7].



Graph- 4.16

Due to lack of data graphs from 4.11-4.16 are created by using information and data provided in:
Source: For historical total losses is the Emergency Events Database (EM-DAT) (Europe)
Source: Joint research centre- JRC- European Commission -Version September [7]

Think about Estonian flood distribution in graph 4.11, it shows that 1%, 0.5%, 0.2% probability that loss exceed 1.55%, 1.8%, 4% respectively of Estonian GDP in a single year according to 2010 GDP but now if we see Estonian storm loss distribution in graph 4.16, it is 0.8% of Estonian GDP in a single year. Why it is discussing over here reason is that if we are looking individual loss distributions of flood or storm then it seems not so worst but if we add up both the loss distribution due to flood and storm then loss distribution increases significantly and it can create potential loss to Estonian insurance companies.

Chapter 5

Conclusion

Estonia is considered to be less vulnerable to natural catastrophe. It is a sigh of relief for insurance companies over here. However, it cannot be ignored that there is no Nat cat risk. There have been seen several big and small storms; floods in recent years. According to climate change scenario, coastal sea level is rising and poles ice melting. So there exists some risk which may create big damage and loss.

In 2005, when the storm hit in Estonia, it created a big amount of loss. But insurance sector did not impact a lot, the reason being, at that time there were not so many people had insured their properties. Still there is no compulsory disaster insurance in Estonia, though we have to agree that the risks related to natural catastrophe and man-made catastrophe are covered relatively modestly by insurance contracts. People are being aware about catastrophe risk cover in insurance and they prefer to take a policy which provides coverage of catastrophe risk. So in future, if such a big event occurs then it may make significant losses to the insurance sector without having a proper catastrophe modeling strategy.

Estonia is the least populated country and population density plays an key role in in deciding the loss which occurs due to catastrophe events. Because of the scattered inhabitation of Estonia, the local windstorms can be damaging, but not on a huge scale. As it is already discussed individual risk is not exceeding to Nat cat retention, but if we combined two perils- windstorm and flood in coastal areas which can cause severe damage to the property and this can count potential losses to insurance companies. So it can be worked on this approach. This approach is so valuable for insurance companies in Estonia because in this case loss can exceed their paying capacity. So, insurance companies can be careful in future. In such a scenario insurance company can diversify its risk to reinsurer by buying treaties or ceding its exceeding paying limit to coinsurer.

It is true that the data is not available of Nat cat events in Estonia as there are no so many previous historic events. In this condition better to make possible approximation by creating some scenarios. In other words, it is really interesting to use neighboring countries like Scandinavia and Finland historical Nat cat events data. So it can assess the probability that something similar may occur in future here.

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