Modulating Axial Resolutions of Scenes Recorded Incoherently Using Cubic Phase Masks by Chaos-Engineering

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***Abstract*—Axial and lateral resolutions form the cornerstones of any imaging system including coded aperture imaging systems. The above characteristics are intertwined and it is not possible to change one characteristic without affecting the other using conventional means i.e., by changing the numerical aperture. Recently, using computational imaging concepts, the interdependency between lateral and axial resolutions was broken and axial resolution was tuned without affecting the lateral resolution. However, the above capability is possible only by engineering of coded phase masks before recording and impossible after completing the recording process. In this study, we propose and demonstrate a novel computational imaging method that allows to tune axial resolution with a constant lateral resolution after completing the recording process. A total of four unique cubic phase masks (CPMs) were designed for generating Airy patterns with different 3D paths relatively chaotic with respect to one another. The 4 CPMs were used to record the point spread and object intensity patterns. The axial resolution was tuned by creating synthetic point spread and object intensity patterns by summing the patterns recorded using different CPMs. The axial resolution improved with an increase in chaos that is proportional to the number of patterns that were summed to create the synthetic patterns. The object is reconstructed using Lucy-Richardson-Rosen algorithm. Experimental studies were carried out to confirm the capability to tune axial resolution of pictures post recording.**

Keywords*—incoherent digital holography, cubic phase mask, axial resolution, lateral resolution, Lucy-Richardson-Rosen algorithm, computational imaging, chaos engineering.

I. Introduction

Fresnel incoherent correlation holography (FINCH) is an incoherent digital holography (IDH) technique developed based on self-interference principle in 2007 [1]. In FINCH, minimum three phase shifted holograms of the object were recorded and combined to reconstruct the image of the object by Fresnel back propagation without twin image and bias terms. FINCH exhibits a higher lateral resolution and lower axial resolution compared to those of direct imaging system. In 2016, a new IDH method was developed to record incoherent digital holograms of 3D scenes using quasi-random coded phase masks called as coded aperture correlation holography (COACH) [1]. COACH involves two stages of recording holograms. A point spread hologram (PSH) is recorded in the first stage and the object hologram (OH) is recorded in the second stage. By cross correlating the PSH and OH, the image of the object is reconstructed. COACH exhibits a high axial resolution but a low lateral resolution than those of FINCH. Later in 2017, COACH was upgraded to interferenceless COACH (I-COACH) which enables to record holograms without two wave interference [1]. In I-COACH, light from an object is scattered by a randomly coded phase mask and recorded by a camera without two-beam interference as needed in COACH and FINCH. The reconstruction procedure of I-COACH involves a cross-correlation between PSH and OH, which is similar to COACH. I-COACH exhibits axial and lateral resolutions are the same as those of a direct imaging system with the same numerical aperture (NA). The initial version of I-COACH used cross-correlation and requires minimum three recorded camera shots to reconstruct the object information. Later, single shot functionality in I-COACH was made possible by the development of a unique reconstruction technique known as non-linear reconstruction (NLR) [1]. Hybrid imaging systems were developed by combining FINCH and COACH and reconstructing using the methods of COACH. In 2017, the first hybridization approach allowed tuning the axial and lateral resolutions of the hybrid system between the limits of FINCH and COACH [2]. However, there was no crossing over of characteristics, i.e., with a high axial resolution, there was a low lateral resolution and vice versa. In 2021 a new IDH method called coded aperture with FINCH intensity responses (CAFIR) was developed by combining FINCH and COACH to obtain the lateral resolution of FINCH and axial resolution of I-COACH [3]. In the above two methods, when axial resolution was changed, lateral resolution changed. In 2022 a new method was developed to tune the axial resolution with a constant lateral resolution using scattered Bessel beams. In this method, the lateral resolution remains the same, when the axial resolution was tuned by increasing the number of scattered Bessel beams [4]. Recently, in 2023 two other methods were reported to tune axial resolution with a constant lateral resolution by scattering Airy beams and self-rotating beams [5, 6]. The above two methods involving scattered Bessel, Airy and self-rotating beams used random multiplexing method to combine the phase masks which resulted in substantial reconstruction noise. So, when the

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number of the beams increased, the axial resolution improved but the reconstruction noises also increased. To avoid random multiplexing a novel algorithm based on Gerchberg-Saxton algorithm (GSA) called transport of amplitude into phase based on GSA (TAP-GSA) was developed [7]. This TAP-GSA approach improved the SNR for the above cases. All the above methods discussed, have the capability to tune the axial resolution in real time but impossible to change axial resolution after completing the recording process. To enable tuning axial resolution post recording, incoherent hybrid imaging systems (INCHIS) were developed [8]. To tune the axial resolution after recording, INCHIS requires only two simultaneous collinear camera shots of the same scene using two passive refractive elements - lens and axicon. By applying different weights to the recorded intensity patterns, the axial resolution can be tuned independent of lateral resolution between the limits of lens and axicon. Another IDH method was also reported that uses quartic coded apertures for real-time tuning of axial resolution but has the capability to tune axial resolution post recording by modification of the recording and reconstruction methods [9]. Now coming back to the previous approaches using scattering of Airy, Bessel and self-rotating beams, it was possible to tune axial resolutions only pre-recording. In this study, we propose and demonstrate a unique IDH method of recording using scattered Airy beams and reconstruction of static scenes that allows tuning axial resolution post-recording without affecting the lateral resolution. In this study, instead of NLR, a newly developed computational reconstruction algorithm called Lucy-Richardson-Rosen algorithm (LR^2A) was used for image reconstruction [10].

II. Methods

The optical configuration is shown in Fig. 1. Four different cubic phase masks namely CPM_1, CPM_2, CPM_3 and CPM_4 that can generate Airy beams with unique 3D paths were engineered [6]. The light from a point object enters the spatial light modulator (SLM) in which CPM_1, CPM_2, CPM_3 and CPM_4 were displayed one after another and the point spread holograms I_{PSH1}, I_{PSH2}, I_{PSH3} and I_{PSH4} were recorded by the image sensor at times T_1 to T_4 respectively. The object and image distances are denoted as z_s and z_h respectively. The above recording process was repeated for recording an object where I_{OH1}, I_{OH2}, I_{OH3} and I_{OH4} are the intensity patterns recorded using CPM_1, CPM_2, CPM_3 and CPM_4 respectively. Synthetic point spread and object holograms were generated as "(1)" and "(2)" respectively, where $M = 1$ to 4.

$$I_{SPSH} = \sum_{k=1}^{M} I_{PSH(k)} \qquad (1)$$

$$I_{SOH} = \sum_{k=1}^{M} I_{OH(k)} \qquad (2)$$

When $M = 1$, the synthetic pattern consists of only one Airy pattern and so has a low axial resolution. When $M = 4$, the synthetic pattern has a high axial resolution given as $\sim\lambda/NA^2$. Between the values of 1 and 4, intermediate values of axial resolution are obtained. The control of axial resolution using chaos engineering is shown in Fig. 2. The image reconstruction is carried out by LR^2A given as "(3)", where '$\circledcirc_n^{\alpha,\beta}$' is the LR^2A operator,

$$I_R = I_{SPSH} \circledcirc_n^{\alpha,\beta} I_{SOH} \qquad (3)$$

α and β control the magnitudes of spectra of I_{SPSH} and I_{SOH} respectively and n is the number of iterations. The schematic of LR^2A is shown in Fig. 3.

III. Experiments

The snapshot of the optical experimental setup is shown in Fig. 4. The light from a high-power LED (Thorlabs, 940 mW, $\lambda = 660$ nm and $\Delta\lambda = 20$ nm) was passed through an iris and a diffuser. A refractive lens "L1" acquires the light from the diffuser and directed it through a polarizer that was positioned along the active axis of an SLM in which the CPMs are displayed. A pinhole with a diameter of 50 μm was critically illuminated by a refractive lens "L2" and the light from it was collimated by another refractive lens "L3" located at $z_s = 5$ cm. The collimated light entered the beam splitter and then reaches the SLM in which the designed CPMs - CPM_1, CPM_2, CPM_3 and CPM_4 shown in Fig. 5 are displayed one after the other and the corresponding PSHs I_{PSH1}, I_{PSH2}, I_{PSH3} and I_{PSH4} were recorded by the image sensor. The procedure was then repeated by shifting the location of the pinhole to $z_s = 5.6$ cm. Two test objects from USAF resolution target, digit '1' and digit '3' from Group 5 were mounted one after another and recorded in the same fashion from locations $z_s = 5$ cm and 5.6 cm respectively. The experimental results are shown in Fig. 6. On seeing the reconstruction results corresponding to $M = 1$, both objects '1' and '3' appear focused and strong exhibiting a low axial resolution. When a second Airy pattern corresponding to T_2 is summed with that of T_1 i.e., $M = 2$, it can be seen that the reconstructed result at the first depth z_s = 5 cm, the object '1' is focused and the object '3' is slightly blurred and weaker than object '1'. Similarly, at another depth $z_s = 5.6$ cm, the object '3' is focused and the object '1' is slightly blurred and weaker than object '3' which clearly indicates that the axial resolution is increased. When a greater number of unique Airy patterns were summed as seen for the cases $M = 3$ and 4 in Fig. 6, the axial resolution improves further indicated by an increase in blur and decrease in strength of objects at different depths. Another interesting behavior is the changes in location of the object with changes in M. With an increase in M the shift between objects decreases. By controlling the 3D propagation factors of the Airy patterns, it is possible to change the shift between objects. This is a desirable tunability especially when imaging objects that are overlapped in x and y but not in z.

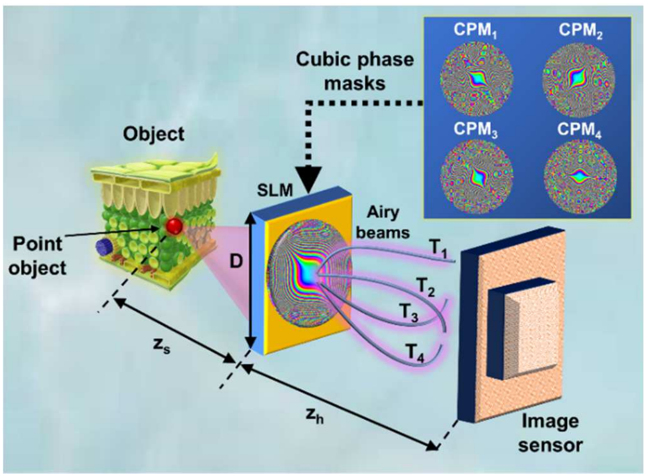

Fig. 1. Optical configuration. Engineered cubic phase masks are used one after the other to modulate the light coming from the object and holograms are captured by the camera.

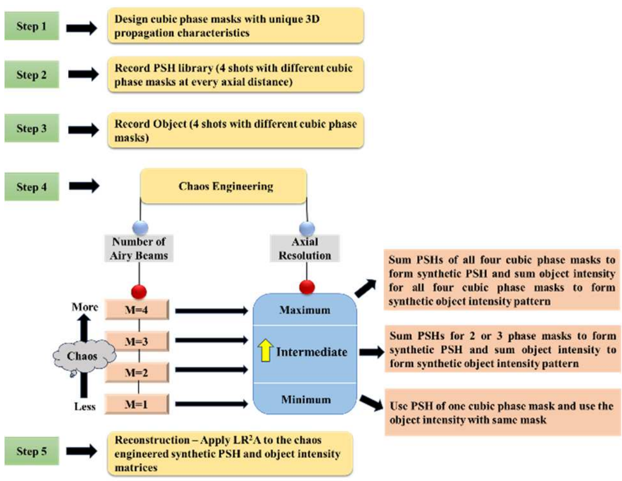

Fig. 2. Block diagram of the control of axial resolution by chaos engineering.

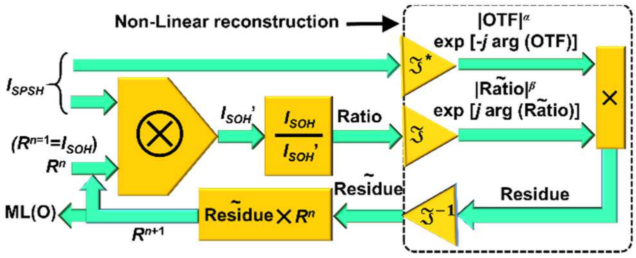

Fig. 3. Schematic of the Lucy-Richardson-Rosen algorithm. ML – Maximum Likelihood; n—number of iterations; OTF—Optical transfer function; $\mathfrak{F}^*$—refers to complex conjugate following a Fourier transform; $\otimes$—2D convolutional operator; $\mathfrak{F}^{-1}$- Inverse Fourier transform; R^n is the n^{th} solution and n is an integer, when n = 1, $R^n = I_{SOH}$; α and β are varied from -1 to 1.

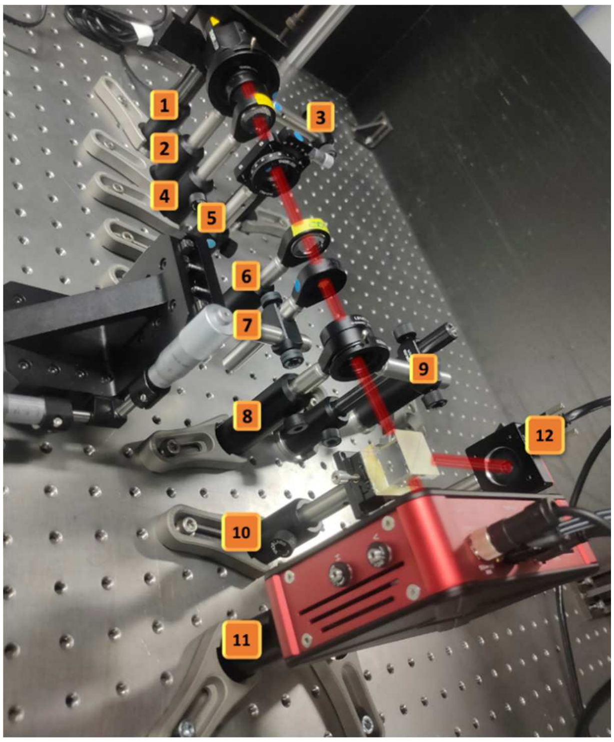

Fig. 4. Snapshot of the experimental configuration: (1) High power LED source, (2) iris, (3) diffuser, (4) refractive lens L1, (5) polarizer, (6) refractive lens L2, (7) pinhole/object, (8) refractive lens L3, (9) iris, (10) beam splitter, (11) spatial light modulator, (12) Monochrome camera.

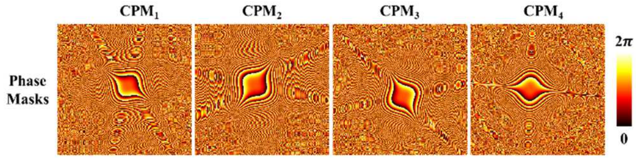

Fig. 5. Engineered cubic phase masks CPM_1, CPM_2, CPM_3 and CPM_4.

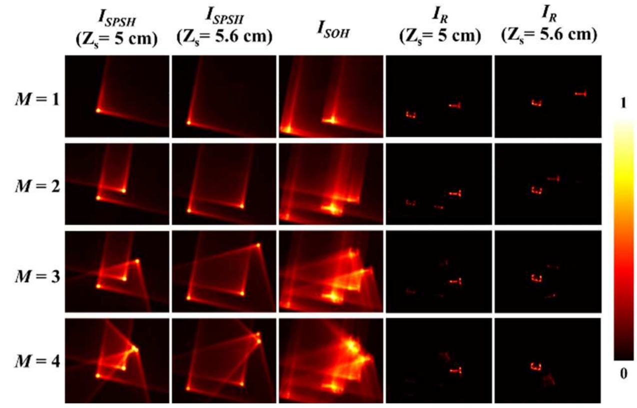

Fig. 6. Experimental results: The first column shows the I_{SPSH} at (z_s = 5 cm), the second column shows the I_{SPSH} at (z_s = 5.6 cm), the third column shows the I_{SOH} at (z_s = 5 cm) and (z_s = 5.6 cm), fourth column shows the I_R (z_s = 5 cm), and fifth column shows at I_R (z_s = 5.6 cm) by LR^{2}A for M = 1, M = 2, M = 3 and M = 4 respectively.

IV. Summary and Conclusion

A new IDH method based on chaos engineering using Airy patterns has been proposed and demonstrated to modulate axial resolution with a constant lateral resolution post recording. The axial resolution increased as the number of Airy patterns increased, leading to an increase in chaos. The axial resolution is low when a single airy beam is used. The axial resolution increases in parallel with the number of airy beams, i.e., when M = 1, the axial resolution is low, and when M = 4, the axial resolution is high. When there is a single Airy beam, the intensity distribution does not vary with distance but only experiences a shift. Consequently, it is not possible to discriminate object information at different distances. However, when the number of beams increases, even though the intensity distributions do not vary for individual beams, the relational intensity information between them varies. This relational intensity information increases with the number of beams. This effect compresses the focal depth resulting in an improved axial resolution. The proposed method has a low temporal resolution and prolonged training and therefore not suitable for recording scenes that vary in time. We hope that the newly developed method will play a vital role in controlling imaging characteristics completely post recording. The developed technique will benefit holography, microscopy, cinematography and computer vision.

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References

[1] J. Rosen, A. Vijayakumar, M. Kumar, M. R. Rai, R. Kelner, Y. Kashter, A. Bulbul, and S. Mukherjee, “Recent advances in self-interference incoherent digital holography,” *Adv. Opt. Photon.*, vol. 11, no. 1, pp. 1-66, Mar. 2019.

[2] A. Vijayakumar, Y. Kashter, R. Kelner, and J. Rosen, “Coded aperture correlation holography system with improved performance [Invited],” *Appl. Opt.*, vol. 56, no. 13, pp. F67-F77, May 2017.

[3] A. Bulbul, N. Hai, and J. Rosen, "Coded aperture correlation holography (COACH) with a superior lateral resolution of FINCH and axial resolution of conventional direct imaging systems," *Opt. Express*, vol. 29, no. 25, pp. 42106-42118, Dec. 2021.

[4] A. Vijayakumar, "Tuning Axial Resolution Independent of Lateral Resolution in a Computational Imaging System Using Bessel Speckles," *Micromachines*, vol. 13, no. 8, p. 1347, Aug. 2022.

[5] R. Kumar, A. Vijayakumar, and J. Rosen, "3D single shot lensless incoherent optical imaging using coded phase aperture system with point response of scattered airy beams," *Sci. Rep.*, vol. 13, no. 1, pp. 2045-2322, Feb. 2023.

[6] A. Bleahu, S. Gopinath, T. Kahro, et al, "3D incoherent imaging using an ensemble of sparse self-rotating beams," *Opt. Express*, vol. 31, no. 16, pp. 26120-26134, Jul. 2023.

[7] S. Gopinath, A. Bleahu, T. Kahro, et al, "Enhanced design of multiplexed coded masks for Fresnel incoherent correlation holography," *Sci.Rep.*, vol. 13, no. 1, pp. 7390, May 2023.

[8] S. Gopinath, A. S. J. F. Rajeswary, and A. Vijayakumar, "Sculpting axial characteristics of incoherent imagers by hybridization methods," *Optics and Lasers in Engineering*, vol. 172, pp. 107837, Jan. 2024.

[9] M. R. Rai, and J. Rosen, "Depth-of-field engineering in coded aperture imaging," *Opt. Express*, vol. 29, no. 2, pp. 1634-1648, Jan. 2021.

[10] A. Vijayakumar, M. Han, et al, "Single-shot mid-infrared incoherent holography using Lucy-Richardson-Rosen algorithm," *Opto-Electron Sci*., vol. 1, no. 3, pp. 210006, Mar. 2022.