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**ON THE PARALLELISM BETWEEN THE RATE OF
CHANGE IN ELECTRIC RESISTANCE AT FUSION
AND THE DEGREE OF CLOSENESS OF PACKING
OF METALLIC ATOMS IN CRYSTALS**

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Abstract: On examining the data for the rate of change of electric resistance at fusion it is found that this rate increases as the atoms in crystals are packed more closely (1—9); a restriction implied by this relation on 3 previously given rules on this subject is stated (10); according to the rates of change of resistance lattices are suggested for Ga, Rb, Cs, Te- β , Tl- β , Sn- γ and Hg (11—20). Conclusion (21).

1. It was shown in a previous paper¹⁾ that as a rule:—
- “[1] Metallic elements, the resistances of which at fusion change in the proportion of 1:2, have rhombohedral hexagonal lattices of the antimony type.
 - [2] Metallic elements, the resistances of which at fusion change in the proportion of 3:2, have cubic body-centred lattices.
 - [3] Metallic elements, the resistances of which at fusion change in the proportion of 2:1, have lattices of the close-packed type.”

2. Then the crystal was conceived, from a purely geometrical point of view, as an assemblage of mathematical points arranged in a regular lattice. Now we shall think of it “as an assemblage of spheres packed tightly together, the centres of the spheres coinciding with those of the atoms²⁾”. On inspecting the lattices referred to in the three rules, we observe that they differ in regard to the closeness of the packing of the atoms. In regard to this point they may be divided into 3 groups:—

- [1] The rhombohedral hexagonal lattice of the antimony type with the loosest packing of atoms;
- [2] The cubic body-centred lattice with a closer packing of atoms;
- [3] The close-packed type, both the cubic and hexagonal one, with a still closer packing of atoms;

and this leads us to relate the rate of change of electric resistance at fusion to the closeness of the packing of the atoms, because on comparing the respective statements in paragraphs

1) Harald Perlitz, *Phil. Mag.* vii. p. 1148 (1926).

2) W. Lawrence Bragg, *Phil. Mag.* xl. p. 170 (1921).

1 and 2, we find that the rate of change of electric resistance at fusion increases with the closeness of packing.

3. To bring out this parallelism the following table was compiled:—

TABLE 1.

| Element | Rate in change of resistance at fusion | Mean rate | Degree of closeness of packing |
|--------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|--------------------------------|
| Bismuth . . | ·466 ¹ ; ·465 ² ; ·45 ³ ; ·466 ⁴ ; ·478 ⁵ ; ·451 ⁶ ; ·45 ⁷ ; ·43 ⁸ | ·458 | ·616 |
| Antimony . | ·703 ¹ ; ·754 ⁹ ; ·729 ⁶ ; ·67 ⁸ | ·714 | ·646 |
| Sodium . . | 1·35 ¹⁰ ; 1·34 ¹¹ ; 1·45 ¹² ; 1·6 ¹³ ; 1·45 ¹⁴ ; 1·53 ¹⁵ | 1·54 | ·680 |
| Potassium . | 1·39 ¹⁶ ; 1·39 ¹⁷ ; 1·52 ¹³ ; 1·5 ¹² ; 1·53 ¹⁸ ; 1·60 ¹⁹ | | |
| Lithium . . | 2·51 ²⁰ ; 1·68 ²¹ ; 1·27 ²² | | |
| Aluminium | 1·64 ⁸ | | |
| Silver . . . | 1·98 ⁷ ; 1·74 ⁸ | | |
| Cadmium . . | 1·79 ¹ ; 1·96 ³ ; 1·98 ²³ ; 1·97 ²⁴ | 1·99 | ·740 |
| Lead . . . | 1·90 ¹ ; 1·95 ³ ; 1·92 ²⁴ ; 2·07 ⁸ | | |
| Copper . . . | 2·09 ²⁵ ; 2·04 ⁸ | | |
| Zinc | 2·00 ¹ ; 2·17 ²⁶ ; 2·19 ²⁷ ; 2·09 ⁸ | | |
| Gold | 2·28 ²⁸ | | |

¹) Computed from L. de La Rive, *Compt. Rend.* lvii. p. 702 (1863).

²) Computed from C. L. Weber, *Wied. Ann.* xxxiv. p. 580 (1888).

³) G. Vicentini & D. Omodei, *Atti R. Acad. Torino*, xxv. p. 35 (1889) 90 *Nuovo Cim.* xxvii. p. 209 (1890).

⁴) Giuseppe Vassura, *Nuovo Cim.* xxxi. p. 46 (1892).

⁵) Edwin F. Northrup & V. A. Suydam, *Journ. Franklin Inst.* clxxv. p. 160 (1913).

⁶) Deduced from E. F. Northrup & R. G. Sherwood, *Journ. Franklin Inst.* clxxx. p. 502 (1916).

⁷) P. W. Bridgman. Quoted from Landolt-Börnstein, *Phys.-chem. Tabellen*, p. 1052 (1923).

⁸) Hidéo Tsutsumi, *Sc. Rep. Tôhoku Univ.* vii. p. 101 (1918).

⁹) Deduced from E. F. Northrup & R. G. Sherwood, *Journ. Franklin Inst.* clxxv. fig. facing p. 158 (1913).

¹⁰) Deduced from Matthiesen, *Pogg. Ann.* c. p. 190 (1857).

¹¹) Arciero Bernini, *Nuovo Cim.* vi. p. 30 (1903); *Phys. ZS.* v. p. 245 (1904). — In N. C. the rate is stated as 1·342, in agreement with the values for the relative resistances in the molten and solid state as quoted in table IX (i. c. p. 29); whilst in P. Z. the rate is stated as 1·337 again in agreement with the values for the relative resistances in the molten and solid state as quoted in "Tabelle IX" (i. c. p. 244).

¹²) Edwin F. Northrup, *Trans. Amer. Electrochem. Soc.* xx. p. 201 (1911).

¹³) L. Hackspill, *Thèse*, p. 80 (Paris, 1912).

¹⁴) P. W. Bridgman, *Proc. Amer. Acad.* lvi. p. 82 (1921).

Continued at the bottom of the next page.

4. In the 1st column of the table are listed all those metals with known rates of change of resistance at fusion, the structures of which seem to be well established from X-ray diffraction data³⁾: in all 12 metals out of 19. Their order of arrangement is determined by the averages (not listed) from the respective individual values stated in the 2nd column.

5. The 2nd column displays the rate of change of the electric resistance at fusion, *i. e.* the ratio of the resistance of the molten metal to the solid one, both taken at the temperature of fusion. These are arranged in the horizontal lines in chronological order and have been taken, computed or deduced from the literature on the subject. The term "deduced" here means that they have been determined by the author of this paper from curves, in which resistances were plotted against temperatures. This course was used in cases, where the resistances at fusion are given in the form of curves only, and in the case of Matthiesen⁴⁾, where these curves were plotted in accordance with his tabulated values. In all these cases due allowance was made for the rounding of the curves at the point of fusion. "Computed" means that the values cited have been obtained by simple division of the figures for the resistance in the liquid state by those for the resistance in the solid state. This method was used in cases where lack

3) International Critical Tables, i. pp. 340—341 (1926).

4) Footnotes 10 & 16 to Table 1.

Continued from the bottom of the preceding page.

15) Deduced from Charles C. Bidwell, Phys. Rev. xxiii. p. 368 (1924).

16) Deduced from Matthiesen, Pogg. Ann. c. p. 189 (1857).

17) Arciero Bernini, Nuovo Cim. vi. p. 297 (1903); Phys. ZS. v. p. 410 (1904).

18) P. W. Bridgman, Proc. Amer. Acad. lvi. p. 87 (1921).

19) Deduced from Charles C. Bidwell, Phys. Rev. xxiii. p. 367 (1924).

20) Arciero Bernini, Nuovo Cim. viii. p. 272 (1904); Phys. ZS. vi. p. 78

1905).

21) P. W. Bridgman, Proc. Amer. Acad. lvi. p. 75 (1921).

22) Deduced from Charles C. Bidwell, Phys. Rev. xxiii. p. 370 (1924).

23) Giuseppe Vassura, Nuovo Cim. xxxi. p. 49 (1892).

24) Edwin F. Northrup & V. A. Suydam, Journ. Franklin Inst. clxxv. p. 158 (1913).

25) Edwin F. Northrup, Journ. Franklin Inst. clxxvii. p. 21 (1914).

26) Deduced from A. A. Somerville, Phys. Rev. xxxiii. p. 79 (1911).

27) Edwin F. Northrup & V. A. Suydam, Journ. Franklin Inst. clxxv. p. 159 (1913).

28) Edwin F. Northrup, Journ. Franklin Inst. clxxvii. p. 291 (1914).

or scarcity of data prevented the construction of reliable curves, but where data for the resistances just before and after fusion were available. Computation and deduction were resorted to only in cases, in which the authors themselves did not state rates of change of resistance at fusion; where they did so, their own rates have been listed. In none of the three cases, however, have more than three figures been retained.

6. The 3rd column contains the arithmetical means computed from the preceding one. In computing them, bismuth and antimony were treated separately, whilst sodium, potassium and lithium were united into one group, that of the face-centred cubic type, and the rest of the elements were grouped together into that of the close-packed cubic and hexagonal type. The reason for proceeding thus lies in the fact that the unit cells of individual elements of the face-centred group, as well as those of the close-packed groups, differ only in size, so that by a mere change of the scale all cells of the same group can be made to coincide, whereas a reduction of this kind does not transform the unit cell of bismuth into that of antimony. Specially mentioned should be the distinction of the rates of change of resistance of aluminium and gold from the rates of other metals belonging to the close-packed group.

7. The last column gives the degree of closeness of packing, as defined by the relation

degree of closeness of packing

$$= \frac{\text{volume of atoms contained in the unit cell}}{\text{volume of the unit cell}},$$

the atoms being treated as spheres, in contact with one another.

8. For bismuth the closeness of packing was computed from the data given by O. Hassel and H. Mark⁵⁾ and for antimony from those given by R. W. James and Norman Tunstall⁶⁾. In both cases the spherical atoms were supposed to have radii equal to the greater of the radii of combinations and the atoms consequently treated as partly intersecting spheres⁷⁾. The corresponding coefficients for the face-centred and the close-packed

5) ZS. f. Physik, xxiii. p. 269 (1924).

6) Phil. Mag. 1. p. 233 (1927).

7) Sir W. H. Bragg and W. L. Bragg, X Rays and Crystal Structure, 4-th edition, plate viii, facing p. 172.

lattices were not computed from the data for the individual metals, but were arrived at by purely geometrical considerations.

9. A comparison of the data in the two last columns shows the dependence of the rate of change of resistance at fusion on the closeness of packing of supposedly spherical atoms. The main feature of this relation is *the increase of the rate of change of resistance with increasing closeness of packing*, but the available data are not yet sufficient to warrant a deduction of a quantitative relation. This relation, however, is easily interpreted by the "gap" theory of electrical resistance⁸⁾. On melting metals undergo a "polymorphic" transition which transforms all of them into the same "modification": the liquid state, characterized by a proper mean degree of closeness of packing. This signifies that the closely packed structures are loosened, their gaps accordingly widened, their resistance increased; in the contrary case the packing of loosely packed structures is tightened, their gaps narrowed and their resistance decreased.

10. Let us review from this point of view the previously given rules (*cf.* paragraph 1). According to them a definite crystal lattice corresponds to a given rate of change of electric resistance at fusion. We now conclude that this may be so, but need not necessarily always be so. Our reason is that according to the assumed relation between the closeness of packing and the rate of change of resistance, metallic elements with definite lattices should change their resistance at fusion in definite proportions, according to their relative closeness of packing. But the reverse need not to be always true, for more than one lattice may correspond to a given degree of closeness, and they should all exhibit the same rate of change of resistance at fusion. Therefore the previous rules cannot be maintained unconditionally, and due regard has to be paid to the ambiguity involved in deriving a crystal lattice from the degree of closeness of packing.

11. We shall now turn to the remaining 6 metallic elements for which data are available for the rate of change of resistance at fusion, but no data or no definitely established data for their structures, and let us see what hints can be obtained regarding

8) P. W. Bridgman, Phys. Rev. ix. p. 269 (1917) & xvii. p. 161 (1921).

their structures. For this purpose the following table was compiled in analogy to Table 1:—

TABLE 2.

| Element | Rate of change in resistance at fusion | Mean rate |
|---------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|
| Gallium . | .455 ¹ ; .577 ² | .516 |
| Rubidium (1.81 ³); 1.58 ⁴ ; 1.5 ⁵ ; 1.61 ⁶ | | 1.56 ²⁹ |
| Caesium. 1.7 ⁷ ; 1.7 ⁵ ; 1.79 ⁸ ; 1.66 ⁹ | | 1.71 |
| Thallium 2.0 ¹⁰ | | 2.0 |
| Tellurium 2.1 ¹¹ | | 2.1 |
| Tin . . . | 2.07 ¹² ; 2.0 ¹³ ; 1.92 ¹⁴ ; 2.21 ¹⁰ ; 2.13 ¹⁵ ; 2.21 ⁶ ; 2.15 ¹⁷ ; 1.92 ¹⁸ ; 2.03 ¹⁹ . | 2.07 |
| Mercury . | 4.06 ²⁰ ; 4.09 ²¹ ; 1.5 ²² ; 4.11 ²³ ; 2.5 ²⁴ ; 3.92 ²⁵ ; 4.90 ²⁶ ; 5 ²⁷ ; 3.22 ¹⁹ ; 3.8 ²⁸ | 3.71 ³⁰ |

¹) Deduced from A. Guntz & W. Broniewski, J. d. Chém. phys. vii. p. 478 (1907). — A. G. & W. B. state the rate to be $\frac{1}{2}$ (l. c. 481).

²) P. W. Bridgman, Proc. Amer. Acad. lvi. p. 111 (1921).

³) Deduced from Louis Hackspill, Thèse, p. 51 (Paris, 1907). — This figure has not been used in computing the average, because L. Hackspill himself states in his later investigation — Thèse, p. 71 (Paris, 1912) — that the rubidium in this case was contaminated by dissolved oxydes. In the table this figure is retained only as an illustrative example of discrepancies that might occur in the rate of change in resistance at fusion, even if deduced from manipulations of the same investigator.

⁴) A. Guntz & W. Broniewski, J. d. Chém. phys. vii. p. 476 (1907).

⁵) L. Hackspill, Thèse, p. 80 (Paris 1912).

⁶) P. W. Bridgman, Proc. Amer. Acad. lx. p. 474 (1907).

⁷) A. Guntz & W. Broniewski, J. d. Chém. phys. vii. p. 474 (1907).

⁸) Deduced from Charles G. Bidwell, Phys. Rev. xxiii. p. 363 (1924).

⁹) P. W. Bridgman, Proc. Amer. Acad. lx. p. 407 (1925).

¹⁰) G. Vicentini & D. Omodei, Atti Acad. di Torino, xxv. p. 35 (1889/91); Nuovo Cim. xxvii. p. 209 (1890).

¹¹) Deduced from A. Guntz & W. Broniewski, J. d. Chém. phys. vii. p. 485 (1907).

¹²) Deduced from Werner Siemens, Pogg. Ann. xxiii. p. 99 (1861). — W. S. states the rate to be *ca.* $2\frac{1}{2}$ (l. c. p. 100).

¹³) Computed from L. de La Rive, Compt. Rend. lvii. p. 702 (1863).

¹⁴) Deduced from C. L. Weber, Wied. Ann. xxxiv. plate IV, fig. 9 (1888).

¹⁵) Giuseppe Vassura, Nuovo Cim. xxxi. p. 45 (1892).

¹⁶) Deduced from A. A. Sommerville, Phys. Rev. xxxiii. p. 77 (1911).

¹⁷) Edwin F. Northrup & V. A. Suydam, Journ. Franklin Inst. clxxv. p. 159 (1913).

¹⁸) Deduced from E. F. Northrup & R. G. Sherwood, Journ. Franklin Inst. clxxii. p. 503 (1916).

¹⁹) Hidéo Tsutsumi, Sc. Rep. Tôhoku Univ. vii. p. 101 (1918).

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12. The data of this table should be commented upon in the same way as those of Table 1 (*cf.* paragraphs 5 and 7). Then we may conclude that gallium has a very loosely packed lattice; that the packing of rubidium is as close as that of the body-centred cubic lattices; that the packing of caesium is very near to the packing of body-centred cubic lattices; that the lattices of thallium, tellurium and tin are as closely packed as the close-packed cubic and hexagonal lattices; and that mercury seems to have an exceedingly close-packed structure.

13. The case of mercury needs special comment on account of the great differences in the data for its rate of change of resistance at fusion. These discrepancies have been explained in two ways: A. Baltruszaitis⁹⁾, and H. Kamerlingh Onnes and G. Holst¹⁰⁾ accept only the highest figures as guaranteeing an absence of pores in the conducting filament in its solid state, whilst P. W. Bridgman attributes the discrepancies to differences in "the position in which the crystals tend to separate out", produced by "the particular form of vessel in which the mercury is frozen", the crystals having different resistances in different directions¹¹⁾. Therefore in the case of mercury averageing seems to be out of question, and the degree of closeness to be ascribed to mercury depends on the explanation that is accepted.

9) Bul. Acad. Sc. Cracovie, p. 893 (1912, A).

10) Comm. Phys. Lab. Leiden № 142-a, p. 5 (1914).

11) Proc. Amer. Acad. xlvii. p. 397 (1911).

Continued from the bottom of the preceding page.

²⁰⁾ Deduced from C. L. Weber, Wied. Ann. xxv. pp. 248 & 250 (1885). — C. L. W. states the rate to be *ca.* 4 (l. c. p. 251).

²¹⁾ Cailletet & Bouty, Compt. Rend. c. p. 1189 (1885).

²²⁾ L. Grunmach, Ann. der Phys. & Chem. xxxv. p. 772 (1888).

²³⁾ Carl Ludwig Weber, Wied. Ann. xxxvi. p. 589 (1889).

²⁴⁾ L. Grunmach, Ann. der Phys. & Chem. xxxvii. p. 514 (1889).

²⁵⁾ Deduced from James Dewar & J. A. Fleming, Proc. Roy. Soc. lx. pp. 78 & 79 (1896). — According to J. D. & J. F. it would be about 4 (l. c. 79).

²⁶⁾ A. Baltruszaitis, Bul. Acad. Sc. Cracovie, p. 898 (1912, A).

²⁷⁾ H. Kamerlingh Onnes & G. Holst, Comm. Phys. Lab. Leiden № 142-a, p. 5 (1914).

²⁸⁾ Georg Gehlhoff & Friedrich Neumeier, Verh. d. Deut. Phys. Ges. xxi. p. 210 (1919).

²⁹⁾ The bracketed figure has not been used in computing the average — *cp.* the remark to footnote 3.

³⁰⁾ *cp.* paragraph 13.

14. Assuming, for the sake of computation, for *gallium* a rhombohedral hexagonal lattice of the antimony type, as density at room temperature 5.91^{12}), and as the rate of change of resistance at fusion the mean value from Table 2, we obtain by graphical interpolation from the degrees of closeness of packing and from the radii of closest approaches of bismuth and antimony 1.43 \AA and 1.24 \AA for the radii of combinations of gallium. The last figure differs only little from 1.26 \AA , which is the interpolated atomic radius of gallium according to Maurice L. Huggins¹³) — Therefore a lattice of the looseness of the antimony type seems not to be at variance with the behaviour of gallium.

15. For *rubidium* the mean rate of change of resistance at fusion is almost the same as that in Table 1 obtained from data for the three body-centred elements: sodium, potassium and lithium. Therefore, and also because of the general similarity of the alkali elements, let us ascribe to rubidium a body-centred lattice. On this assumption and taking for the density of rubidium at room temperature 1.53^{14}), we obtain 2.47 \AA for the radius of combination, which is greater than 2.25 , W. Lawrence Bragg's figure derived from X-ray data for crystal structures of compounds¹⁵).

16. In the case of *caesium* the mean rate of change of resistance is 1.73 , and therefore caesium should be given a structure with the corresponding closeness of packing. But instead of doing so we shall allow ourselves to be guided by the great similarity of all alkali elements, and ascribe to it also the cubic body-centred lattice, and accordingly compute its radius of combination. Taking 1.90 as the density at room temperature¹⁶), we obtain 2.92 \AA for the radius of combination. This figure again exceeds Maurice L. Huggins's values, which range from 2.59 \AA to 2.78 \AA ¹⁷). But if we accept for rubidium the cubic body-centred lattice, we have to account for the difference in the rate of change of resistance between the mean rates established for the cubic body-centred lattice and that of caesium. If this difference were

12) International Critical Tables, i, p. 104 (1926).

13) Maurice L. Huggins, Phys. Rev. xxviii. p. 1094 (1926).

14) International Critical Tables, i, p. 104 (1926).

15) W. Lawrence Bragg, Phil Mag. xl. p. 180 (1920).

16) International Critical Tables, i, p. 104 (1926).

17) Maurice L. Huggins, Phys. Rev. xxviii. pp. 1094 & 1103 (1926).

calculated for rubidium from the corresponding values listed in Tables 1 and 2, we should certainly get an exaggerated value for it, as 1.54, the figure listed in Table 1 for the rate of change of resistance for the body-centred lattice, is rather low. This is indicated by the systematic increase of the individual rates of change of resistance of both potassium and sodium in recent observations, and observations of later origin certainly require more attention than earlier ones. Furthermore, of all the alkalis caesium is most liable to contamination, and not too much stress should be laid on the rates of change of resistance listed in Table 2.

17. According to the rate of change of resistance, the close packed structure should be ascribed to *tellurium*, and probably the hexagonal one, as hexagonal prisms are an observed form¹⁸). The recorded structure, which "is a threefold spiral, composed of three interpenetrating simple triangular lattices"¹⁹), belongs to the rhombohedral modification. But as there are two modifications of tellurium and as the point of transition lies between room temperature and the temperature of fusion, the assumption of an close packed lattice, derived from the rate in change of resistance, may be regarded as a possible one.

18. *Thallium*, according to the rate of change in resistance at fusion, should have a close packed lattice in the modification preceding fusion, and probably the hexagonal one, because thallium shows the hexagonal form in this modification²⁰). No determinations of this lattice have yet been made, although the transition has been verified by X-ray analysis²¹).

19. According to its rate of change of resistance, a close-packed structure should be ascribed to *tin*. Its crystal form in the modification preceding fusion, according to Trechmann and v. Foulon, is generally assumed to be rhombic²²). As it is impossible by a rhombic stowage to attain a closeness of packing high enough to correspond to the rate of change of resistance, the case of tin would be an evidence against the supposed parallelism, if the assumption of the rhombic form for the third modi-

18) P. Groth, Chem. Kristallographie, i. p. 35 (1906).

19) A. J. Bradley, Phil. Mag. xlviii. p. 196 (1924).

20) W. Guertler, ZS. f. Elektrochem. xviii. p. 612 (1912).

21) Genshichi Asahara, Scient. Pap. Inst. Phys. & Chem. Res., Tokyo, ii. p. 125 (1924).

22) Max Werner, ZS. anorg. Chem. lxxxiii. p. 292 (1913).

fication of tin were a unanimous one. But as that is not the case, and even the hexagonal form is regarded by at least two investigators as a possible resp. a proved one²³), judgment ought to be suspended until further and conclusive evidence on the third modification of tin is available.

20. As was pointed out in paragraph 13, there are two alternatives in the case of *mercury*. If we accept the first explanation, the rate of change of resistance at fusion would be given by the highest figure *i. e.* 5, indicating an exceedingly close packing. But the degree of closeness of packing would still remain undetermined, as mercury has no counterpart in Table 1 in regard to the high value of its rate of change, and an extrapolation would be merely guesswork in the present case. If we accept the second explanation, we cannot calculate the rate of change of resistance that would correspond to a haphazard orientation of crystals, without making further assumptions. But we may assert that the packing in this case would be looser, and the coefficient of packing consequently less, but still above 2. Therefore in this case also a definitive value for the degree of closeness can not be obtained. On the other hand, none of the two different structures which have been derived for mercury from X-ray analysis²⁴) has a coefficient of packing approaching to that of the close-packed lattices. The case of mercury, therefore, is the most doubtful one.

21. In conclusion, we may state:—

That all the 12 metals, for which data for the rate of change of resistance at fusion and the closeness of packing in crystals are available, show a parallelism between these data;

That the 7 doubtful metals, with may be the exception of mercury, do not provide cases of contradiction;

That therefore a parallelism certainly exists between the rate of change of electric resistance at fusion and the degree of closeness of packing of metallic atoms in crystals, to the effect that the closer the packing, the greater is the rate of change.

23) Н. Пушкинъ, Журн. Русск. Физ.-Хим. О-ва xxxiv. стр. 896—897 (1902); ZS. anorg. Chem. xxxvi. pp. 245—246 (1903).

24) Ch. Mauguin, La structure des cristaux, pp. 221—222 (Paris, 1924).

22. *Supplementary note* to paragraph 14 (added on May 30th, 1927). Since the above was sent for insertion in the *Acta et Commentationes*, I received, through the courtesy of Dr. P. Terpstra, a paper on the crystal structure of gallium (F. M. Jaeger, P. Terpstra and H. G. Westenbrink, Proc. Acad. Amsterdam, xxix. pp. 1193—1217). There the crystal lattice of gallium is shown to be a simple tetragonal one with 8 atoms to the unit cell, and the closeness of packing may be deduced from the dimensions of the unit cell and the radii of combinations (the smallest radius of combination is found to be equal to 1.28 \AA). But an alternative way of computing the closeness of packing is suggested by the perfect agreement between the actually observed specific weight of liquid gallium and the specific weight deduced for liquid gallium from that of the solid metal as is done in the paper quoted above. Namely, not to assign to the spherical atoms of gallium radii equal to the greatest of the radii of combinations and to treat the atoms as partly intersecting spheres, as was done in the case of bismuth and antimony (*cf.* paragraph 8), but to attribute to them radii equal to the smallest radius of combination, and consequently treat the atoms as spheres which do not intersect. On this assumption we find $.466$ for the closeness of packing of gallium, and recalculating the closenesses of packing of bismuth and antimony on the same assumptions, we obtain $.438$ and $.419$ respectively. As these three figures are close enough to one other, and are far enough from $.680$ and $.740$ (*cf.* paragraph 3, Table 1), the case of gallium contributes an additional evidence in favour of the view upheld in the present paper.

(Read at a meeting of the Scientists' Society at Tartu, March 17, 1927.)

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