On the tracks of Félix-Marie Delastelle

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Abstract

"Can not find any info on Delastelle — Nothing on record in this country." (William Friedman, 18 Jan 1955, NSA Archives A63734) Following these words, the then-director of the US National Security Agency hailed contacts in Europe, hoping that someone would fill in this missing information. The initial inquiry was sent to Friedman by amateur American cryptographer William Maxwell Bowers; in 1963, Bowers would publish under a pseudonym all that he could find on the matter (The Cryptogram 1963, preserved under reference VF 54-30 at the US National Cryptologic Museum). Since this document, which had a very limited audience, almost no new information on Delastelle was published, and indeed most of the information available widely today on Delastelle is at best fragmentary.

In this paper we reopen that case, reviewing information about the life and work of Félix-Marie Delastelle, establishing data overlooked by earlier historians, correcting several oft-repeated errors and bringing novel documents to public awareness.

Félix-Marie Delastelle is briefly mentioned in Kahn's monograph (Kahn, 1996, Chap. 8) as one of the great French cryptographers of the late 19th century — along the likes of Kerckhoffs, de Viaris, Valerio, and Bazeries — and "the only major writer on cryptology of the time who was not in the military". A single paragraph sketches the biography of that unusual individual. Kahn drew the (partially incorrect) information verbatim from Bowers' *Cryptogram* article[1] (Bowers, 1963), which is based "on documents in the Mairie of Saint-Malo and on recollections of Delastelle's niece (Kahn, 1996, Note 242)."

As part of his personal interest in the matter, Bowers contacted NSA's director William Friedman, in the hope that he would know additional details — he did not. This triggered an investigation from the NSA[2,3] (Figure 1) that seems to have culminated in a different, but similarly laconic summary[4]. While both sources point out that Delastelle was a complete outsider in the field of cryptography, and highlight his detailed knowledge of the techniques of his time, neither manage to peer into any details of his life or works besides his famous "bifid" and "trifid" ciphers. Delastelle died in 1902 (more on that later).

Our investigation followed the tracks that these sleuths had initiated. We obtained documents from the Tobacco Administration where Delastelle worked, got access to his civil status registry, found published and unpublished work of his, followed his formative years and early career, tracked down his family properties and finances, and scoured various archives to collect evidence and crumbs of context. This information allows us to draw a much more precise, though still incomplete, portrait of this evasive and famous amateur cryptographer. At the very least, it makes it possible to correct mistakes in earlier accounts that are (at the time of writing) uncritically repeated by multiple sources, including Wikipedia[5].

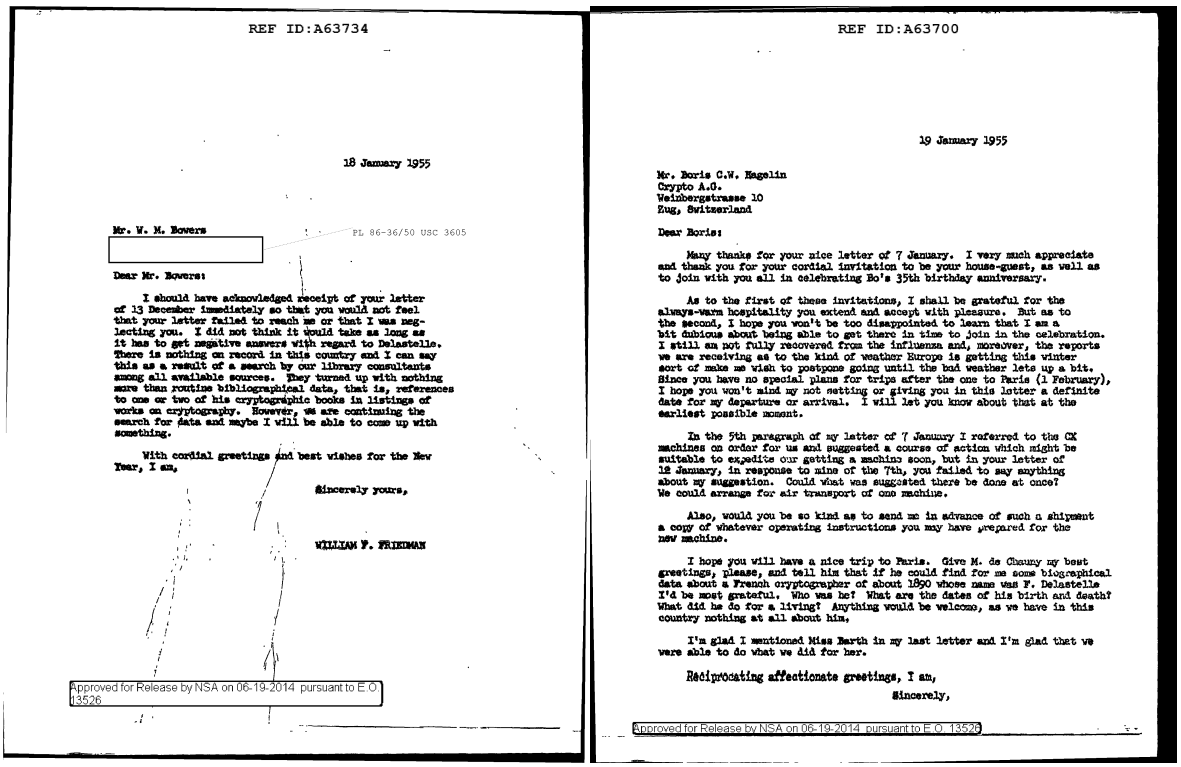
[1] Bowers also wrote a series of earlier leaflets, *Practical cryptanalysis*, edited by the American Cryptogram Association. Volume 2 (1960) and 3 (1961) discuss Delastelle's ciphers. His later *Cryptogram* article added some information and focuses on biographical aspects that appear in Kahn's account.

[2] Letter from William F. Friedman to William Bowers, 18 January 1955, Archives of the NSA, ref A63734.

[3] Letter from William F. Friedman to Boris Hagelin, 19 January 1955, Archives of the NSA, ref A63700.

[4] Letter DK 54-40, 5 May 1969, National Cryptologic Museum.

[5] At the time of writing, the Google search engine when queried for an image of "Félix-Marie Delastelle" only returns



REF ID:A63734

18 January 1955

Mr. W. M. Bowers

PL 86-36/50 USC 3605

Dear Mr. Bowers:

I should have acknowledged receipt of your letter of 13 December immediately so that you would not feel that your letter failed to reach me or that I was neglecting you. I did not think it would take as long as it has to get negative answers with regard to Delastelle. There is nothing on record in this country and I can say this as a result of a search by our library consultants among all available sources. They turned up with nothing more than routine bibliographical data, that is, references to one or two of his cryptographic books in listings of works on cryptography. However, we are continuing the search for data and maybe I will be able to come up with something.

With cordial greetings and best wishes for the New Year, I am,

Sincerely yours,

WILLIAM F. FRIEDMAN

Approved for Release by NSA on 06-19-2014 pursuant to E.O. 13526

REF ID:A63700

19 January 1955

Mr. Boris C.W. Hagelin
Crypto A.G.
Weinbergstrasse 10
Zug, Switzerland

Dear Boris:

Many thanks for your nice letter of 7 January. I very much appreciate and thank you for your cordial invitation to be your house-guest, as well as to join with you all in celebrating Bo's 35th birthday anniversary.

As to the first of these invitations, I shall be grateful for the always-warm hospitality you extend and accept with pleasure. But as to the second, I hope you won't be too disappointed to learn that I am a bit dubious about being able to get there in time to join in the celebration. I still am not fully recovered from the influenza and, moreover, the reports we are receiving as to the kind of weather Europe is getting this winter sort of make me wish to postpone going until the bad weather lets up a bit. Since you have no special plans for trips after the one to Paris (1 February), I hope you won't mind my not setting or giving you in this letter a definite date for my departure or arrival. I will let you know about that at the earliest possible moment.

In the 5th paragraph of my letter of 7 January I referred to the CX machines on order for us and suggested a course of action which might be suitable to expedite our getting a machine soon, but in your letter of 12 January, in response to mine of the 7th, you failed to say anything about my suggestion. Could what was suggested there be done at once? We could arrange for air transport of one machine.

Also, would you be so kind as to send me in advance of such a shipment a copy of whatever operating instructions you may have prepared for the new machine.

I hope you will have a nice trip to Paris. Give M. de Chauny my best greetings, please, and tell him that if he could find for me some biographical data about a French cryptographer of about 1890 whose name was F. Delastelle I'd be most grateful. Who was he? What are the dates of his birth and death? What did he do for a living? Anything would be welcome, as we have in this country nothing at all about him.

I'm glad I mentioned Miss Barth in my last letter and I'm glad that we were able to do what we did for her.

Reciprocating affectionate greetings, I am,

Sincerely,

Approved for Release by NSA on 06-19-2014 pursuant to E.O. 13526

Figure 1: Friedman's letters to Bowers and Hagelin, Archives of the NSA.

It was indeed difficult to find accurate and verifiable information about the man. Delastelle moved a lot across France, he often changed the way he himself spelled his name (seemingly on a whim), and he lived a very discreet life. While digitised databases and declassified documents are tremendous help and easily accessible, we cannot undo the damage of time and wars — archives were lost or destroyed, documents and witnesses did not survive. Bower's main source of information, Delastelle's niece, died in 1970.

One key result of this research[6] is that contrary to the established narrative, his interest and contributions to the field didn't come out of nowhere out of the boredom of retirement following a nondescript career: he was a lifelong inventor, tinkerer, and mathematician.

Félix-Marie Delastelle was born January 5, 1840 in the French city of Saint-Malo, at 8:00 AM, the son of 37-years old Captain Charles Delastelle and 27-years old Louise Cécile Delastelle (née Moison). The Delastelle family — sometimes written "de Lastelle"[7] — was a rather wealthy family of seafarers and merchants long associated with the city of Saint-Malo[8], and later with Madagascar (Micouin and Harel, 2008; Fontoynont, 1935). Félix-Marie was preceded by three children (see fig. 2): Charles Hyacinthe (named after his paternal grandfather, born 21 November 1833 and deceased 31 October 1834, before the age of one); Charles Louis (born in 1835); Auguste Michel (born in 1837), who shall play an important role model. Félix-Marie was followed by a younger sister Louise Marie Joséphine (born in 1843).

The father Charles is presumed lost at sea, with an official date of death of 16 November 1842[9,10]. It is unclear how the family dealt with this loss. Presumably, it put some financial pressure on them[11];

pictures of the completely unrelated French mathematician Camille Jordan!

[6] Which is extended in the full version of this article to include all new documents and additional biographic details.

[7] The name is absent from contemporary French nobiliaries (Dayre de Mailhol, 1843 1898).

[8] The name Delastelle is one of those to whom the famous French poet Chateaubriand, writing about his future sepulture in Saint-Malo, expresses gratitude in a 1831 letter to the then-Mayor of the city, Louis-François Hovius (de Chateaubriand, 1997 original publication 1849, Appendix 1).

[9] Civil registry for the birth of Louise Josephine, Saint-Malo Archives.

[10] Civil registry for the marriage of Louise Joséphine with Édouard Béhier, Saint-Malo Archives.

[11] Louise Cécile owned appartments that she later rented as barracks to the local soldiers dispatch; the other owner was Mayor Hovius. *Rapports et délibérations du Conseil général*

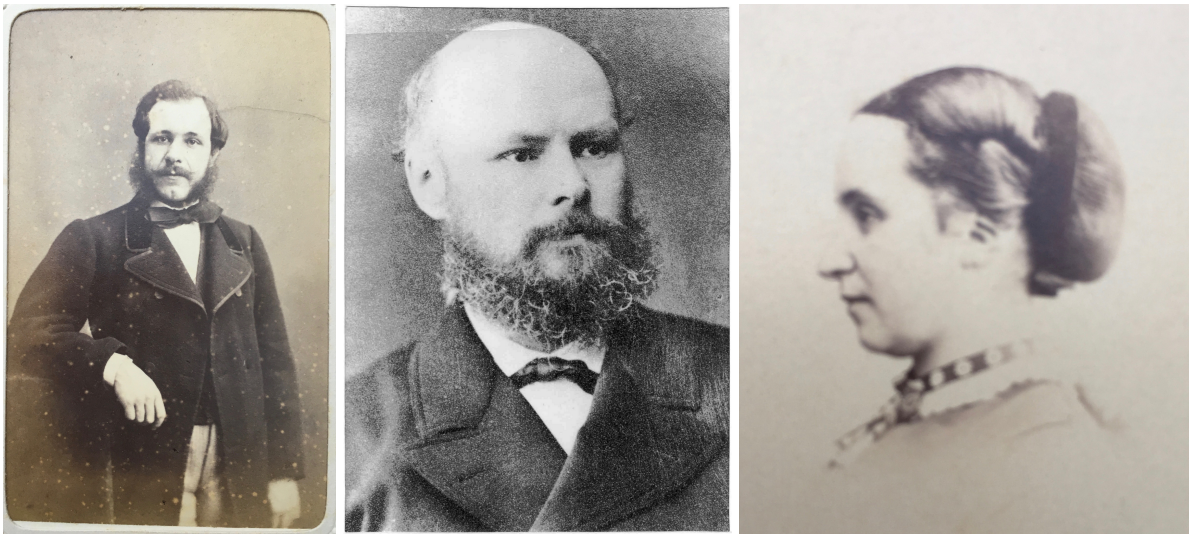


Figure 2: Left: Auguste Michel Delastelle. Middle: Félix-Marie Delastelle. Right: Louise Marie Joséphine. Dates unknown, family archives.

Louise Marie would be sent at age 10 to a girl-only boarding school (*Pensionnat de Mlle A. Egault*, Saint-Malo), where she stayed until at least 1858.

1 School years and first inventions (1852–1861)

Félix-Marie Delastelle entered the local *collège de Saint-Malo* in 1852 (aged 11), receiving multiple certificates of excellence (*Témoignage de complète satisfaction*). He applied to the *Lycée Impérial de Rennes* — the first *lycée* in Brittany, and the most prestigious at that time — to prepare for the national and very selective *baccalauréat* exam.

On 17 October 1858, as a 18 years old *lycée* student, Félix-Marie Delastelle submits an invention to the French Academy of Sciences (temporarily renamed *Institut Impérial de France*) — his first publication[12]. Spelling his name *Félix de Lastelle*, he describes the *chrono-barométrographe*: a precision weather instrument dedicated to recording variations of atmospheric pressure during a day. He hints at further similar inventions, such as a device to record temperature variations, which he hopes to describe in a further letter once some technical details are figured out, but there is no evidence that he went through with this promise.

Delastelle sent a second letter of invention to the French Academy of Sciences in December 1860, on the matter of chemistry, more specifically an algebraic notation appropriate for complex syntheses: *Note sur l'Annotation chimique*[13,14]. Here too, the letter concludes with a promise to pursue research in that field, a promise not followed through.

Delastelle would have attempted the baccalauréat in 1861, although we have no evidence that he did. That year, 2872 students obtained this most coveted diploma, including the first ever woman, Julie-Victoire Daubié. Delastelle was not one of them. An academic career was impossible without this diploma. Thus, contrary to an oft-reprinted myth, Félix-Marie Delastelle *did not* attend university, and in particular did not attend any Grande École.

2 Early Carreer at the Tobacco Administration (1861–1889)

Félix-Marie Delastelle joined the French Tobacco Administration on 1 December 1861 as unpaid intern (*surnuméraire des tabacs*) — two years after his brother Auguste did the same. Such a position was not easy to attain as it was required that the applicant be financially independent (hence a deposit of 4000 francs) and required that the applicant successfully passes multiple rounds of tests and checks

d'Ille-et-Vilaine, session held on 14 December 1878. Bibliothèque nationale de France, département Droit, économie, politique, 8-LK16-103.

[12] Archives of the French National Academy of Sciences, Letter 331, No 464, Session of 18 October 1858.

[13] Archives of the French National Academy of Sciences, Letter 341, No 449, Session of 2 December 1860.

[14] This letter is also signed *Félix de Lastelle* and indicates the same address as the 1858 communication, confirming that they were written by the same person.

(Block, 1877 1885)[15]. It is noted in his application that Delastelle could speak English and Italian.

After these two years of unpaid internship, in 1863, Félix-Marie Delastelle was accepted as a paid employee (*commis*), then promoted to *vérificateur* that same year. In theory, to reach higher positions (as he did), it was necessary to go through yet another series of exams, before the age of 35[16].

While his siblings start having families[17,18], Félix-Marie remains isolated, single and reclusive, applying himself to tasks unrelated to his professional obligations: on 22 September 1875 he applies for a French patent on candle manufacture and lighting[19] also submitted to the London patent office[20] and also attempted to obtain an Italian patent that same year[21]. He applied again, perhaps with a variation of his original idea, for a British patent in 1877 or 1878[22].

In 1879, Félix-Marie is promoted to *contrôleur de culture* and sent away from his hometown, in the city of Morlaix (Lot-et-Garonne department) working for Mr. Delestre[23]; he is then sent to Chambéry (Savoie department) in 1880. There, he applied for a British patent on lightweight ambulance beds on 7 December 1885[24]. Notably, he did not apply for a similar patent in France.

Promoted to *contrôleur principal* in 1886, he moved to Béthune (Pas-de-Calais), then to Dieppe (Normandy) in 1889, where he applied for a French patent on a new telescope model[25].

3 The Mathematical Turn (1890–1900)

Up to that point, it seems difficult to identify an underlying theme to Delastelle's various inventions, and they seem disconnected from his daily activity of overseeing tobacco transactions. But starting around 1889, his interest in mathematics and especially algebra solidified itself.

He sent a memoir to the French Academy of Sciences titled *Contributions à la théorie des équations algébriques* (contributions to the theory of algebraic equations). This memoir was read on 15 September 1890 by Hermite and Darboux[26,27]. He presents this work as motivated by some ongoing research of his where extant approaches due to Lagrange and Sturm are deemed too laborious.

The 32-page essay concerns itself with polynomials, extending a theorem of Budan and Fourier (Akritas, 1981). Its main result establishes a condition for a degree-m polynomial to have all its roots in $\mathbb{R}$, and locating them, together with an *algorithm* (called "algorithme des différences") to ease manual computations of the relevant invariant, which Delastelle claims is also helpful in obtaining continuous fraction approximation of roots.

The methods used in this work are not new, relying on a method of differences (à la Budan) and root symmetry considerations[28]. Perhaps for this reason, the Academy did not reply and made no public comment.

Delastelle would send a final unsolicited mathematical note to the Academy of Sciences in 1892, on Cryptography this time[29]. The note is prefaced by a letter insisting that the extreme simplicity of this method makes it available to "anyone who can read or write" in a matter of minutes. This is the

[15] *Programme des conditions d'admission à l'administration des tabacs, Moniteur du 28 fevrier 1865, Ministère des Finances.*

[16] *Examen des postulants de première série*, Moniteur, 28 February 1865; Le Temps, 14 October 1875, p. 3 ; Journal de la gendarmerie de France, 11 March 1877, p. 105.

[17] On 24 February 1872 his sister Louise Joséphine marries in Saint-Malo tax inspector Édouard Pierre Béhier; Auguste and Félix-Marie are among the witnesses. Three days after, 27-29 February 1872, his older brother Auguste Michel gets married too, in Montreuil (Pas-de-Calais) to Georgina Francès Amam Reid Dalton, the daughter of a retired British General in the Royal Artillery. Félix-Marie is again a witness.

[18] Interestingly, Félix-Marie and Louise write their name "de Lastelle", including on the invitations — all other family members write "Delastelle".

[19] *Amélioration dans la fabrication des bougies*, Number 109753, under the name "Delastelle". Cited in The Commissioner of Patents' Journal, "Grants of provisional protection for six months", Entry 2641, p. 2460, 27 oct 1876; "Notice to proceed" Oct 31 1976, p. 2479; cited in The Engineer, Volume 42, p 316, Morgan-Grampian (Publishers), 1876.

[20] *Improvements in the manufacture of candles and improved apparatus connected therewith*, 14 June 1876, under the name "de Lastelle". Also mentioned in *The London Gazette*, issue 24377, p. 5806, 31 October 1876.

[21] Cite in: Subject matter index of patents for inventions (attestati di privative industriali) granted in Italy from 1848 to January 1, 1886. Reference 26 I). 1876. II, 7, 612 (18, 65), pi. 156.

[22] *Arranging wicks in candles for increasing light*, Applications to the Great Britain Patent Office, Reference 2361.

[23] *Almanach national*, 1879, p. 348.

[24] *Movable structure for ambulances &c.*, application number 15,017.

[25] *Catalogue complet des brevets français délivrés au 3 janvier 1891*, publication du cabinet Émile Barrault, S. (divers) 297,796 dépôt du 26 août 1890, Téléscopes.

[26] *Comptes rendus de l'Académie des Sciences*, tome 2, 1890, p. 412.

[27] Delastelle's letter is dated 12th September 1890.

[28] Delastelle himself considers them well-known, and refers to Joseph Serret (Serret, 1866), but it is unclear which edition. The first edition dates from 1849, and contains one of the first systematic expositions of Galois theory.

[29] *Comptes rendus de l'Académie des Sciences*, 1892, p. 344. Session of 16 August 1892.

first known description of the *bifid cipher*, which is traditionally dated to a decade later. Once again the Academy was uninterested, and Delastelle would stop attempting to catch their attention any further.

At about the same time, Delastelle becomes visibly active in the French mathematical community. He contributes questions, problems, solutions and articles to *Les Tablettes du chercheur*[30] and *L'Intermédiaire des mathématiciens* (edited by Gauthier-Villars), a form of early precursors to Internet forums. Both were hugely popular amongst mathematicians, and included many contributions by famous names (including e.g., Henri Poincaré, Camille Jordan, Jacques Hadamard, and Adolf Hurwitz).

Delastelle's contributions to *L'Intermédiaire* (see Appendix A for the original French) are varied. From a modern cryptographer's outlook, some questions suggest that he may have investigated the mathematics of exponentiation modulo primes, with an eye to statistical and combinatorial properties. However to the best of our knowledge this was never fleshed out and he likely dropped these ideas to favour simpler notions.

Delastelle also sent a series of longer, self-contained articles with title *Cryptographie nouvelle*. These articles span over two years and are scattered across multiple issues (1892 p. 308–310, 1893 p. 24, 37, 54, 69, 81, 130, 140, 147). His last contribution is found in volume 13.

4 Retirement, final years, and death (1900-1902)

In 1893, Félix-Marie lives in Dieppe, 12 rue des tribunaux[31], under the name "Delastel"[32]. He gets promoted to *entreposeur* and leaves for Marseille (Bouches-du-Rhône). That same year he publishes his first book on cryptography (Delastelle, 1893), building from his collection of earlier published articles. The retail price is 3 Francs. He then moves to Marmande (Lot-et-Garonne).

In 1896, his very successful brother Auguste — who reached, in record time, the position of *directeur de la culture* — is knighted in the Légion d'Honneur. Félix-Marie's highest position in the administration, reached in 1898, is merely *inspecteur*. While employed at the Tobacco Administration, Félix-Marie Delastelle was regularly evaluated by superiors: although they unanimously portray an educated and competent man, many point out with regret that his career was unavoidably limited by his being "*original*" — an expression that appears verbatim in multiple reports.

Félix-Marie Delastelle retired in 1900 and returned to his hometown of Saint-Malo, more specifically the nearby town of Paramé. There he would complete his final book (Delastelle, 1902) — finished on 15 May 1901.

He was awarded a pension of 2332 Francs, for 37 years, 1 month and 26 days of service — with no military service and no compensation for any widow or children[33].

The narrative of Delastelle's death requires some corrections. On 1st April 1902, Félix-Marie Delastelle died age 62 in Saint-Ideuc, in the *Ker-Cadoc* villa. Remarkably, Bowers (and Kahn after him, and the NSA after them) misread the situation and thought that Félix-Marie died upon learning of his brother's death — the reality is much less poetic.

Indeed, the cause of death of Félix-Marie, at 5 PM, is unknown[34]. The next day, Auguste took his wife and the train to attend to his brother's funerary service. The train arrived at Le Mans at 11 PM, another train left for Saint-Malo at 12 PM — rushing to make this connection on time, 65 years old Auguste died of a heart attack[35].

Félix-Marie Delastelle was buried in the Cimetière Rocabey (Section 3 S–40) in Saint-Malo, besides other family members. His book, *Traité élémentaire de cryptographie*, was published posthumously by Gauthier-Villars, in 1902.

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[30] *Les Tablettes du chercheur : journal des jeux d'esprit et de combinaisons*, editor B. Decolombe, owner and publisher P. Dubreuil.

[31] The street is now named rue Victor Hugo.

[32] *Fonds Ancien et Local*, Médiathèque Jean Renoir, Dieppe.

[33] *Bulletin des lois de la République française*, décret N°59.972, p. 1332–1333, 12 October 1900. Bibliothèque nationale de France.

[34] Civil registry for the death of Félix-Marie Delastelle (Saint-Ideuc, Ille-et-Vilaine).

[35] Civil registry for the death of Auguste Delastelle (Le Mans, Sarthe); also cited in *L'Avenir de la Dordogne*, 10 April 1902; also cited in *La Croix de la Charente*, 13 April 1902.

References

Alkiviadis G Akritas. 1981. On the Budan–Fourier controversy. *ACM SIGSAM Bulletin*, 15(1):8–10.

Maurice Block. 1877–1885. *Dictionnaire de l'administration française*, volume 2.

William Bowers. 1963. All that you ever wanted to know about Felix Delastelle, French cryptologist. *The Cryptogram*, XXX:79–82, 85, 101, 106–109.

Camille Philippe Dayre de Mailhol. 1843–1898. *Dictionnaire historique et héraldique de la noblesse française, rédigé dans l'ordre patronymique, d'après les archives des anciens Parlements, les manuscrits de d'Hozier et les travaux des auteurs, contenant un vocabulaire du blason, la notice des familles nobles existant actuellement en France, avec la description et le dessin de leurs armes*.

François-René de Chateaubriand. 1997 (original publication 1849). *Mémoires d'Outre-tombe*. Gallimard.

Félix-Marie Delastelle. 1893. *Cryptographie nouvelle, assurant l'inviolabilité absolue des correspondances chiffrée*. P. Dubreuil (18 bis rue des Martyrs, Paris).

Félix-Marie Delastelle. 1902. *Mathématiques appliquées. Traité élémentaire de cryptographie*. Gauthier Villars.

Dr Fontoynont. 1935. Napoléon de Lastelle (1802–1856). *La Revue de Madagascar*, 11:91–107.

David Kahn. 1996. *The Codebreakers: The comprehensive history of secret communication from ancient times to the Internet*. Simon and Schuster.

Françoise Micouin and Patrick Harel. 2008. *De Saint-Malo à l'Isle de France*, volume I.

Joseph Alfred Serret. 1866. *Cours d'algèbre supérieure*, volume 1. Gauthier-Villars.

A Delastelle's contributions to *L'Intermédiaire*, original French

- [Vol 1 p. 285 (ref 624, Iia)] Étant donné l'énoncé de la question 330, il serait fort intéressant, notamment du point de vue de la décimation cryptographique, de pouvoir calculer l'ordre probable de sortie des boules, principalement pour $n = 25$ et $n = 27$. Ce calcul est-il possible ?

  - [For reference, question 330, by Émile Lemoine, is the following] n boules $A_1, A_2, \ldots, A_n$ sont rangées en cercle et se suivent dans l'ordre $A_1, A_2, \ldots, A_n$. On part de A_1 sur la circonférence, en marchant dans le sens $A_1, A_2, \ldots, A_n$; on compte 1 sur A_1, 2 sur A_2, 3 sur A_3, etc. On convient d'enlever la boule sur laquelle on compte p. Cela fait, on continue à marcher dans le même sens en recommençant à compter $1, 2, 3, \ldots, p$ sur chaque boule que l'on rencontre et en enlevant celle sur laquelle on compte p; on continue ainsi jusqu'à ce qu'il ne reste plus qu'une seule boule. Peut-on savoir d'avance quel numéro portera cette boule? (...) Y a-t-il des valeurs de n et de p pour lesquelles on puisse donner une solution si l'on ne peut trouver la solution générale?
  - A response to this question is given in Vol. 3, p. 109, by Adr. Akar.

- [Vol 1 p. 314 (ref 639, 125b)] L'étude de la question n°400 m'a conduit à la notion, nouvelle pour moi, des nombres CIRCULAIRES. Les multiples de ces nombres jouissent de la propriété de reproduire toujours les *mêmes chiffres* disposés dans le *même ordre*. Quand le nombre des chiffres du produit surpasse celui du multiplicande, il suffit d'écrire le produit en *hélice* pour en trouver les chiffres primitifs. Certains nombres sont MULTICIRCULAIRES. Cette propriété est indépendante de la base du système de numération. Les nombres *circulaires* ont-ils déjà été étudiés ?

  - [For reference, question 400, by Charles-Ange Laisant, is the following] Si l'on multiplie le nombre $123456789 = N$, écrit dans le système décimal, par $2, 4, 5, 7$ ou 8, les différents produits

obtenus s'écrivent en permutant simplement les chiffres de N. Cette propriété, que j'ai constatée par hasard, m'a conduit à la proposition *empirique* suivante, que j'ai vérifiée sur un grand nombre d'exemples, et que je crois vraie, mais dont je n'ai cependant aucune démonstration : soit $123\ldots n = N$ un nombre écrit dans le système de numération de base $n+1$. Si l'on forme le produit de ce nombre par un mutiplicateur inférieur à n et premier avec n, ce produit s'écrira au moyen des n chiffres $1,2,3,\ldots,n$, pris chacun une seule fois et convenablement permutés. Quelque correspondant pourrait-il me donner une démonstration de ce théorème d'Arithmétique, ou en établir l'inexactitude?

  – [On this question, in Vol. 3, p. 100, Palmström says] J'ait fait en 1893 sur ces nombres que M. Delastelle nomme *circulaires* une Conférénce à la Société polytechnique de Bergen; mais je n'ai rien publié à ce sujet; ma question dans l'*Intermédiaire* avait pour but de m'informer auparavant de ce qui avait été fait sur le sujet.

- [Vol. 3, p. 128 (ref 846, A3d)] Je désirerais savoir si la proposition suivante est connue: soient $f(x) = 0$ une équation de degré m, ℓ et L les limites entre lesquelles les racines réelles sont toutes comprises, et h une quantité moindre que la différence des deux racines inégales les plus rapprochées. Substituons à x, dans le polynôme $f(x)$, une suite de nombres en progression arithmétique: $\ell, \ell+h, \ell+2h, \ldots, \ell+(n-1)h, \ell+nh$, dont le premier est *au plus* égal à la limite inférieure des racines et le dernier *au moins* égal à la limite supérieure de ces mêmes racines. Écrivons le résultat de ces substitutions en une colonne verticale, que nous désignerons par u; formons dans les colonnes suivantes le tableau des différences des divers ordres $\Delta_1, \Delta_2, \ldots, \Delta_n, \ldots, \Delta_m$; puis indiquons par V_n le nombre de variations de signes que renferme chaque colonne Δ_n du tableau ainsi formé.

  Ceci posé, si chaque colonne contient une variation de plus que la colonne suivante, on a pour chacune d'elles la relation $V_n = m - n$ et les racine sde la proposées sont toutes réelles et inégales. Si, pour un ecolonne quelconque, on trouve $V_n < m-n$, c'est-à-dire $V_n + d = m - n$, on aura, pour toutes les colonnes $u, \Delta_1, \ldots, \Delta_{n-1}$ qui précèdent Δ_n la relation $V_{n'} + d = m - n'$ et la proposée possèdera d racines imaginaires.

  Si, indépendamment des variations d perdues par une colonne quelconque Δ_n, une colonne de différences d'ordre inférieur Δ_ν, $\nu < n$, perd un nouveau nombre d' de variations, les racines imaginaires de la proposée s'élèveront à $d + d'$, et ainsi de suite.

  Si enfin la colonne Δ_1 présente plus de variations que la colonne u, la proposée a pour chaque signe (ou couple de variations) perdu deux racines doubles ou deux racines imaginaires, ce que l'on sait distinguer.

- [Vol. 5, p. 57] Après les multiples réponses à cette question, je crois qu'il n'est pas sans intérêt d'exposer brièvement la théorie des *nombres circulaires* [...] Les nombres circulaires me paraissent appelés à rendre d'utiles services dans beaucoup de calculs, notamment dans la recherche de *racines primitives*[36]. Leur détermination facile est encore simplifiée par la propriété dont ils jouissent d'avoir la seconde moitié de leurs chiffres formée du complément à 9 (ou, plus généralement, à $a-1$) de chacun des chiffres de la première moitié.

- [Vol. 5, p. 77] Soient p un nombre premier absolu, et $a, b, c, \ldots$ ses racines primitives. En écrivant la suite $a, a^2, a^3, a^4, \ldots, a^{p-1}$ et en retranchant de chaque terme tous les multiples de p qu'il peut contenir, on obtient la suite naturelle des nombres de 1 à $p-1$ disposés dans un ordre quelconque. Les autres racines $b, c, d, \ldots$ donneront cette même suite dans des ordres différents. Ceci posé : (1) La somme des puissances n-ièmes de $a, b, c, \ldots$ est congrue avec zéro, relativement au module p, lorsque n est diviseur ou multiple d'un diviseur de $p-1$:

$$a^n + b^n + c^n + \cdots = 0 \bmod p$$

  (2) La somme des n racines de a^n jouit de la même propriété, ainsi que toutes les puis-

[36] This can refer to two different notions in modern parlance: either the *multiplicative units* of $\mathbb{Z}/p\mathbb{Z}$, or the *discrete logarithm* of a number modulo p.

sances de ces racines, à l'exclusion de celles qui sont égales à n ou multiple de ce nombre. Ce théorème peut encore s'énoncer: dans toute suite calculée comme il est dit plus haut, la somme des nombres équidistants est toujours congrue à zéro pour le module p. Peut-on donner une démonstration élémentaire de ce théorème?

- [Vol. 5, p. 78] Lorsque le module est puissance d'un nombre premier p, la suite, qui contient $p^{\nu-1}(p-1)$ termes, jouit de propriétés analogues à celles de la question précédente; mais il faut alors que n ait un facteur commun avec $p^{\nu-1}(p-1)$; lorsque ce facteur est égal à $p-1$, $p(p-1)$, $p^2(p-1),\ldots$, la somme des $p^{\nu-1}, p^{\nu-2},\ldots$ termes, congrue avec zéeo pour le module p, ne l'est pas pour le module p^{ν}; elle a pour valeur les multiples $p, 2p, \ldots, p(p^{\nu-1})$ ou $p^2, 2p^2, \ldots, p^3, 2p^3, \ldots$ chacun augmenté du plus petit des termes trouvés. Peut-on démontrer ce théorème?

- [Vol. 5, p. 93] Delastelle solves a diophantine problem posed by Escott in the 1896 issue, ref 153.

- [Vol. 5, p. 111] Delastelle solves an interpolation problem posed by Cyp. Stephanos in the 1898 issue.

- [Vol. 5, p. 112] Delastelle solves a problem posed by A. Goulard in the 1897 issue.

- [Vol. 5, p. 148] L'étude des grands nombres, ou si l'on veut, des fractions décimales illimitées, paraît bien délaissée. Il me semble cependant qu'elle pourrait fournir des résultats intéressants. Pour en donner un aperçu, soit proposé de trouver la fraction génératrice du nombre illimité écrit dans le système B,

$$N = \frac{a}{B^{\alpha}} + \frac{b}{B^{2\alpha}} + \frac{c}{B^{3\alpha}} + \frac{d}{B^{4\alpha}} + \cdots$$

Si les quantités $a, b, c, d, \ldots$ sont liées par les relations

$$\begin{aligned} b &= a + \delta_1, \\ c &= a + 2\delta_1 + \delta_2, \\ d &= a + 3\delta_1 + 3\delta_2 + \delta_3, \\ e &= a + 4\delta_1 + 6\delta_2 + 4\delta_3 + \delta_4, \\ &\ldots \end{aligned}$$

on aura

$$N = \frac{a}{B^2-1} + \frac{\delta_1}{(B^2-1)^2} + \frac{\delta_2}{(B^2-1)^3} + \cdots + \frac{\delta_{n-1}}{(B^2-1)^n}$$

les différences $\delta_n, \delta_{n+1}, \delta_{n+2}, \ldots$ étant égales à zéro. Si

$$b = aq, \quad c = aq^2, \quad d = aq^3, \quad \cdots$$

on aurra $N = a/(B^2-q)$. Ces formules ont-elles été publiées?
La solution de l'équation

$$N = \frac{x}{B^y-1} + \frac{\delta_1}{(B^y-1)^2} + \frac{\delta_2}{(B^y-1)^3} + \cdots + \frac{\delta_{n-1}}{(B^y-1)^n}$$

où $x, y, \delta_1, \delta_2, \ldots, \delta_{n-1}$ sont inconnues ne semble pas offrir de sérieuses difficultés. Il n'en est pas de même de l'équation

$$N = \frac{x}{B^y - z}$$

et je serai reconnaissant à qui pourra m'en donner la solution (...). [Le problème a été résolu par Brocard, p. 11 du volume 13]

- [Volume 13, publié en 1906, p. 163 (ref 996 A31)] Je désirerais une solution *algébrique* de l'équation

$$mA^x - By^{f(x)} = C$$

que je sais seulement résoudre par l'arithmétique, lorsque les quantités $A, B, C < B$ et m étant remplacées par des nombres entiers, les inconnues x et y ont des valeurs entières et positives.

There are more contributions by Delastelle in *L'Intermédiaire*: in Vol. 8, p. 31 and Vol. 12, p. 218. Unfortunately we could not find these issues in archives.