JOANNIS GUILIELMI PFAFF

A CONSILIIS RUSSICIS AULICIS MATHESEOS IN UNIVERSITATE

DORPATENSI PROFESSORIS P. O.

COMMENTATIO ASTRONOMICA

DE

CALCULO TRAJECTORIARUM.

SECTIO I.

MITAVIAE, MDCCCV.

TYPIS JOH. FRIEDR. STEFFENHAGEN ET FILII.

Praefatio.

Problema de inveniendis trajectoriis corporum coelestium ex tribus observationibus geocentricis, vel serie observationum geocentricarum, et calculi molestia, complicatione et difficultate famosum, a Neutono Princ. Lib. 3. Pr. 41 sqq. primum solutum, postea varie ab astronomis tentatum, cum solutionem rigidam, directam atque absolutam haud admittat, duabus tractatur partibus, altera approximante, praevio calculo ope aequationum rigidarum vel vero accedentium orbitae elementa eruente, altera rectificante, hypothesium approximantium, atque observationum ipsarum errores emendante, corpusque coeleste, adhibitis perturbationibus, ad systema reliquorum referente.

Scriptione hac aequationes plurima ex parte attuli, quibus prima pars solutionis nititur. Supersunt multae, quas e Lamberti libro, Insigniores orbitarum proprietates, Olbersio, la Landio Pingréi cometographia, du Sejourii libro Traité des mouvemens apparens, Vince system of Astronomy repetas; vel ex Euleri libro notisquae

Paccassii elicias: et ipsa la Placii methodus varias admittit solutionis formas. Hypotheses de orbita rectilinea vel circulari muttas suppeditant aequationes notissimas.

Exempla quae calculavi monstrant solutionis possibilitatem quibusdam casibus hac forma proposita, sola Neutoni hypothesi approximante chordam in ratione temporum secari, admissa. Ceterum licet sententiae accedam viri de Astronomia meritissimi illustr. de Zach, qui Olbersii methodum pro cometis expeditissimam censet, cujus librum ipse tabulis formulisque calculum levantibus adornavit, licet ista Methodus, in usum adhibendo la Placii theoremata Mechanic des Himmels II. Buch, 4. Cap. §. 27 Lamberti universaliora; ad corpora coelestia quaevis extendatur — tamen confido, methodum vulgarem, quam methodum ex analysi indeterminata nuncupes, a la Caille Lecons elementaires expositam, ad quam rediit ipse Eulerus, quamque concinnam exposuit ill. Schubert Astronomie II. Th. 8. Abschn. 2 Cap. ad majorem perfectionis gradum tolli posse, adhibitis aequationibus de limitibus, de maximis et minimis [quarum aliquas demonstrat Lambert, Gauss calculata Cereris, Palladis, Junonisque orbita insignis] atque ex analysi forsan indeterminata repetitis quibusdam.

Rigorem calculi approximantis aestimandi partemque alteram solutionis tractandi Sectione altera occasio tempusque erit.

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§. 1.

Designent

a_1, a_2, a_3, longitudines Cometæ seu Planetæ geocentricas tempore primæ, secundæ ac tertiæ observationis; similiter

b_1, b_2, b_3, latitudines geocentricas,

A_1, A_2, A_3, longitudines Solis

r_1, r_2, r_3, radios vectores Cometæ; R_1, R_2, R_3, radios vectores terræ

D_1, D_2, D_3, distantias ejusdem a terra,

k_1, k_2, k_3, chordas a primo ad secúndum Cometæ in orbita locum ductas, a secundo ad tertium, a primo ad tertium: t, tempus inter primam et secundam observationem elapsum, t_2, inter secundam ac tertiam.

§. 2.

Aequationes Solutioni inservientes demonstratu faciles offert: Sejour Traité anal. II. Chap. 8. §. 575 la Grange Memoires de Berlin 1778.

Aequatio 1. $S_2\,[R_1 \text{ Cos. } A_1 - D \text{ Cos. } b \text{ Cos. } a] - S_3\,(R_2 \text{ Cos. } A_2 - D_2 \text{ Cos. } b_2 \text{ Cos. } a_2) + S_1\,(R_3 \text{ Cos. } A_3 - D_3 \text{ Cos. } b_3 \text{ Cos. } a_3) = 0.$

Aequatio 2. $S_2 D \text{ Sin. } b - S_3 D_2 \text{ Sin. } b_2 + S D_3 \text{ Sin } b_3 = 0$

Aequatio 3.
$$\left.\begin{array}{l} S_2\,(R \text{ Sin. } a - D \text{ Cos. } b \text{ Sin. } a) \\ - S_3\,(R_2 \text{ Sin. } A_2 - D_2 \text{ Cos. } b_2 \text{ Sin } a_2) \\ + S\,(R_3 \text{ Sin. } A_3 - D_3 \text{ Cos. } b_3 \text{ Sin. } a_3) \end{array}\right\} = 0.$$

§. 3.

His tribus æquationibus sex insunt quantitates incognitæ; verum S_1, S_2, S_3, sunt triangula radiis vectoribus et chordis intercepta: S_1 scil. r_1, r_2, et chorda K_1; S_2 radiis vectoribus r_2, r_3 et chorda K_2, denique S_3 radiis vectoribus r_1, r_3, et K_3 intercipitur: æquationes vero determinandis chordis et radiis victoribus ita se habent

$$\overline{r_1}^2 = \overline{R_1}^2 + \overline{D_1}^2 - 2 R_1 D_1 \text{ Cos. } b_1 \text{ Cos. } (a_1 - A_1)$$
$$\overline{r_2}^2 = R_2^2 + D_2^2 - 2 R_2 D_2 \text{ Cos. } b_2 \text{ Cos. } (a_2 - A_2)$$

Similiter r_3^2 determinatur;

Chordæ his obtinentur formulis

$$\overline{K_1}^2 = \overline{r_1}^2 + \overline{r_2}^2 - 2 P_1$$
$$\overline{K_2}^2 = \overline{r_2}^2 + \overline{r_3}^2 - 2 P_2$$

similiter $\overline{K_3}^2$ exprimitur

determinatio P_1, P_2, P_3 æquationibus hisce;

$$P_1 = D_1{}^2 D_2 [\text{Cof. b Cof. b}_2 \text{ Cof.} (a_2 - a_1) + \text{Sin. b}_1 \text{ Sin. b}_2] - R_2 D_1 \text{ Cof. b}_1 \text{ Cof.} (a_1 - A_2) - R_1 D_2 \text{ Cof. b}_2 \text{ Cof.} (a_2 - A_1) + R_1 R_2 \text{ Cof.} (A_2 - A_1)$$

in qua aquatione R_1, R_2, D_1 b_1, mutantur cum istis quæ respondent secundæ ac tertiæ chordæ, ut proclive est judicare, si P_2, P_3 quæruntur.

Triangulor. istorum S itaque 3 latera sunt determinata, area itaque eorum ita innotescit

$$S_1 = \sqrt{(r_1\ r_2)^2 - P_1^2}$$

$$S_2 = \sqrt{(r_2\ r_3)^2 - P_2^2}$$

$$S_3 = \sqrt{(r_1\ r_3)^2 - P_3^2} = S_1 + S_2 - \tfrac{1}{4}\sqrt{(k_1 + k_2)^2 - k_3^2)(k_3^2 - (k_2 - k_1)^2}$$

§. 4.

Spem omnem æquationes istas §. 2, factis substitutionibus quas §. 3 suppeditas, revera et rigore solvendi abjecerunt dudum, qui hisce meditationibus operam dederunt: æquationes novas partim ex theoria virium centralium, partim ex natura et proprietatibus curvarum quas corpora describunt viribus centralibus agitata, parabolæ, ellipseos, hyperbolæ petitas potius, in usum et calculi expeditioris commodum in auxilium vocantes, aut graphicis quibusdam artibus, vel hypothesibus et tentaminibus, ex regula falsi, calculum moderantes.

§. 5.

Simpliciores quædam æquationes secundariæ derivari possunt ex fundamentalibus, scilicet Eliminata S_3 ope æquationis

$$S_3 = \frac{S_2 D \text{ sin. b} + S D_3 \text{ sin. b}_3}{D_2 \text{ sin. b}_2}$$

et D_2 ope duarum reliquarum

Obtinetur æquatio:

$$S_1 \text{ Sin.} (A_3 - A_2) + S_2 [\text{Cof. b Sin.} (A_2 - a) - \frac{\text{Cof. b}_2 . \text{Sin. b.}}{\text{Sin. b}_2} \text{ Sin.} (A_2 - a_2)]\ D - S_2 R \text{ Sin.} (A_2 - A) - S_1 [\text{Cof. b}_3 \text{ Sin.} (A_2 - a_3) - \frac{\text{Cof. b}_2 . \text{Sin. b}_3}{\text{Sin. b}_2} . \text{Sin.} (A_2 - a_2)]\ D_3 = 0$$

quam hujus formæ esse apparet

Aequatio 4. $K_1 + L_1 D + M_1 D_3 = 0 = K_1 + S_2 l_1 D + S_1 m_1 D_3$

Eliminata similiter S_2 ope æquationis 2, et D_1 ope duarum reliquarum, orietur

$$S^1 R_3 \text{ Sin.} (A''' - A) + S D_3 \text{ Cof. b}_3 \text{ Sin.} (a_3 - A) + D_2 \text{ Sin. b}_2 S_3 \frac{\text{Cof. b}}{\text{Sin. b}} \text{ Sin.} (a - A) - S_3 R_2 \text{ Sin.} (A_2 - A) - S D_3 \text{ Sin. b}_3 \frac{\text{Cof. b Sin.} (a - A)}{\text{Sin. b}} - D_2 \text{ Cof. b}_2 . S_3 . \text{ Sin.} (a_2 - A)$$

cujus forma

Aequatio 5. $K_2 + L_2 D_3 + M_2 D_2 = 0$

Eliminata S_1, ope æquationis 2, it: D_3 ope duarum reliquarum, obtinebitur æquatio

$$- S_2 R \text{ Sin.} (A_3 - A) + D_1 S_2 \text{ Cof. b Sin.} (a - A_3) + D_2 \text{ Sin. b}_2 \frac{S_3 \text{ Cof. b}_3}{\text{Sin b}_3} \text{ Sin.} (a_3 - A_3) + S_3 R_3 \text{ Sin.} (A_3 - A_2) - D S_2 \frac{\text{Sin. b Cof. b}_3}{\text{Sin. b}_3} \text{ Sin} (a_3 - A_3) - D_2 \text{ Cof. b}_2 S_3 \text{ Sin.} (a_2 - A_2)$$

quam ita exprimere licet

Aquatio 6. $K_3 + L_3 D + N_3 D_2 = 0$

§. 6.

Ope æquationis 5, 6, 4 ipsæ D_3, D_1, D_2, determinantur scilicet

Aeq. 7. $D_1 (L_1 L_2 L_3 - M_1 M_2 M_3) + K_1 L_2 L_3 - K_2 L_3 M + M_1 M_2 K_3 = 0$

Aeq. 8. $D_3 (M_1 M_2 M_3 - L_1 L_2 L_3) + K_1 M_2 M_3 - K_3 L M_2 + L L_3 K_2 = 0$

Aeq. 9. $D_2 (L_1 L_2 L_3 - M_1 M_2 M_3) + K_3 L_1 L_2 - K_1 L_2 M_3 + M_3 M_1 K_2 = 0$

§. 7.

Solutionem æquationum 1, 2, 3 generalem alia methodo dedit la Grange. l. c. æquationes 4, 5, 6 generaliores, rigidiores simplicioresque, sunt istis, quas du Sejour. proposuit, calculoque ut infra videbimus adaptavit; æquationem 4 Schulze Mem. Berlin 1782 ex Lamberti methodo geometrica satis prolixe deduxit; eandem et aliis insistens principiis, proposuit Olbers Bestimmung der Bahn eines Cometen §. 58.

§. 8.

Aequationes 4, 5, 6 primi gradus incognitas $\frac{S''}{S'}\ \frac{S'''}{S'}$ continent; Verum dantur æquationes secundi gradus; scilicet eliminata S_3 reliquæ sunt duæ scilicet

$$\text{Aeq. 10.}\quad - D \text{ Sin. b} S_2 . \text{ Sin. A}_2 R_2 + S D_2 D_3 [\text{Sin. b}_3 \text{ Sin. a}_2 \text{ Cof. b}_2 - \text{Sin. b}_2 \text{ Cof. b}_3 \text{ Sin. a}_3] + D_2 \text{ Sin. b}_2 (S_2 \text{ Sin. A. R.} + S R_3 \text{ Sin. A}_3) - S_3 D_2 D [\text{Sin. b}_2 \text{ Cof. b Sin. a} - \text{Sin. b Cof. b}_2 \text{ Sin. a}_2] - D_3 \text{ Sin. b}_3 S . {}_1 R_2 \text{ Sin. A}_2$$

Similis huic obtinetur ex æquatione 1, mutando scilicet Sinus longitudinum solis et Cometæ in 10 ubique cum Cosinu.

Quæ æquatio ope 4tæ in hanc mutatur

Aeq. 11. $D_2 = \frac{N + T D}{O + Q D}$; altera similem producit

Similis obtinetur ope 5tæ (eliminatione S_2 facta)

Aeq. 12. $D_1 = \frac{N_2 + T_2 D_2}{O_2 + Q_2 D_2}$; denique eliminata S_1, ex quadratica et æquatione 6 obtinetur

Aeq. 13. $D_3 = \frac{N_3 + T_3 D_2}{O_4 + Q_3 D_2}$

§. 9.

Totum itaque negotium eo redit, ut determinentur coefficientes, K, L, M, N, T, O, Q: quibus insunt incognitæ, S_1, S_2, S_3: cum vero æquationes 4, 5, 6 hujus sint formæ

$$X S_1 + Y S_2 = 0$$
$$X_2 S_1 + Y_2 S_3 = 0$$
$$X_3 S_2 + Y_3 S_3 = 0$$

Sequitur æquatio hujus formæ

$$Y_2 X_1 X_3 + Y Y_3 X_2 = 0 \qquad (K + M_2 D_2)(p_2\ m\ D_3)(t + l_3 D_1) + (m_1 + L_1 D_1)(p_1 m_3 D_2)(t_1 + l_2 D_3) = 0$$

Problema itaque de inveniendis trajectoriis Cometarum datis tribus observationibus, æquationibus du Sejour et la Grange insistendo ad æquationem Cubicam formæ

Aeq. 14. $$A_1 + B_1 D_1 + C_1 D_1 D_2 + E D_1 D_2 D_3 = 0$$
$$+ B_2 D_2 + C_2 D_1 D_3$$
$$+ B_3 D_3 + C_3 D_2 D_3$$

deducit: Coefficiens $E = M_1 M_2 L_3 + L_1 L_2 M_3$; Simili modo coefficientes reliqui determinantur:

Aequationem cubicam ex eo deduxit du Sejour. §. 553 orbitam Cometæ in uno plano sitam esse §. 553 quam recenset Olbers.

§. 10.

Quando æquationes 1, 2, 3 resolvuntur, ut incognitæ 3 S_1, S_2, S_3 ex iis eruantur, obtinentur 3 æquationes biquadraticæ: quarum alteram saltim adducam: quæ oritur eliminata S_2 ex æquationibus 1. 2. 3. hujus formæ.

Aeq. 15. $$(mD + nD_3 + pDD_3)\ (kD + tD_2 + oDD_2) = 0$$

ad tres æquationes biquadraticas problema tali modo reducitur:

Coefficientes aliquos afferam

$m = R_3$ Cof. A_3. Sin. b

$n = R_1$ Cof. A_1. Cof. b_3 Sin. a_3

$p = -$ CSin. b_3 Cof. a_3 Sin. b_2 + Cof. b Cof. a Cof. b_3 Sin. a_3

Aequationes istæ, indirecte solvuntur, sumto D_2 ex hypothesibus noto; tum enim fit secunda

$$(a + bD_3)(c + eD_1) = 0$$

atque tertia $(o + q D_3)(m_3 D_1 + n_3 D_3 + p_3 D_3) = 0$

Ex quibus D_3 ope æquationis cubicæ determinatur: quis vero earum solutioni ita operam dabit?

§. 11.

Aequationes 7, 8, 9 manent invariatæ, quando observator in cometa seu planeta positus, orbitam calculabit terræ: positis loco geocentricis cometæ et solis locis, locis terræ ac Solis cometocentricis.

Sumatur initium longitudinum ex Nodo; Cogitetturque triangulum tribus punctis junctis, Sole, Cometæ in orbita loco, atque perpendiculo ex terra in orbitam cometæ dejecto; sint S, C, T tria ista puncta; apparet fore

SC = r; Angulus interceptus SC et ST lateribus determinari potest, nec non latus ST: scilicet obtinentur tandem tres æquationes determinantes R, R_2, R_3, per radios vectores cometæ, atque logitudinem nodi inclinationem, locumque in orbita. Aequationes vero incognitas ipsas continent, maximopere implicitas: ita ut in solutionem ipsam hac methodo inversa vix utilitatis aliquid vel compendii redundet.

§. 12.

Quas la Grange æquationes deduxit pro D, D_2, D_3, si explicentur, et considerentur seu æquationes pro determinandis in cognitis tribus S_1, S_2, S_3, quorum forma adest

$$S_1\, a + S_2\,(b + lD) + S_3\, c,$$

in uno coefficiente tantum occurrnete D, vel D_1, vel D_2, facta eliminatione Incognitæ S_1, S_2, S_3; oriuntur tres æquationes cubicæ, quarum forma

Aeq. 16. $$A + BD + cD_2 + ED_3 + FD_1 D_2 + bD_1 D_3 + HD_2 D_3 + L D_1 D_2 D_3 = 0$$

coefficientibus A, B, C... omnibus cognitis atque ex observationibus eruendis: Coefficientium determinationi non inhæreo: si opus foret alia forma æquationum commodior foret.

Ita determinatio incognitarum D, D_2, D_3 fit ope trium æquationum cubicarum; quæ solutio omnium ut videtur, analytice simplicissima est: Calculi magna ut videtur foret molestia.

§. 13.

Ex iis quæ hactenus demonstrata sunt, satis manifestum est: solutionem problematis ope æquationum 1, 2, 3, et reliquarum, quas analysis finita suppeditat, quam maxime in commodam, inpeditam atque fere inpossibilem fore. Adde, quam plurimis casibus coefficientes æquationum hujus fore naturæ, ut rigide calculari nequeant, et solutionem quæ ex iis pendet, incertam reddant atque fallacem.

Hisce incommodis medela petitur ex theoria virium centralium, et quidem communiter ac primario ex Prop. I. Libri I. operis immortalis: Principia philosophiæ Naturalis mathematica, qua demonstratur: areas, quas corpora in gyros acta radiis ad immobile centrum virium ductis describunt, et in planis immobilibus consistere, et esse temporibus proportionales.

Quando itaque licet sumere triangula ista S_1, S_2, in eadem ratione esse ac areas, sequitur, approximando tanto proprius ad veritatem, quanto minora temporum intervalla:

Aeq. 18. $$t\ S_2 = t_2 S_1$$

$$\frac{t}{t_2} = \frac{S_1}{S_2}; \quad \text{ita ut } \frac{S_1}{S_2} \text{ sit data ex observationibus}$$

vel $$t_1^2\ [(\overline{r_2 . r_3})^2 - P_2^2) = t_2^2\ [(rr_2)^2 - P_1^2]$$

sive $$\frac{t_1^2}{t_2^2}(r_2^2\ r_3^2 - P_2^2) - \overline{rr_2}^2 + P_1^2 = 0$$

Areas in eodem plano fore hanc æquationem suppeditat facilem demonstratu.

$$\text{Aeq. 19.}\ P_3 = P_1 P_2 - \sqrt{\left(r_1^2 - \frac{P_1^2}{r_2}\right)\left(r_3^2 - \frac{P_2^2}{r_2}\right)} = \frac{P_1 P_2}{r_2^2} - \frac{t_1}{t_2}\left(r_1^2 - \frac{P_1^2}{r_2^2}\right) \text{ ope æquation. 18)}$$

Aequationibus hisce si jungatur æquatio quarta: calculatis ope æquationibus fundamentalibus §. 3. adductis quantitatibus r, P &c.; obtinebitur $D \ldots D_2,\ D_3$:

Exemplum calculi sumam idem quod Olbers proposuit: 2) §. 49 solutio æquationum indirecte fit, aliis nimis inplicata:

Calculus Aequationis 4.

Posito, temporum intervallo sat parvo, $K = 0$, dat $\text{Log.}\,\frac{L}{M} = 0{,}137652$

Calculus radiorum vectorum simplicissimus

$$r_1^2 = 0{,}96754 - 0{,}59292\ D\ \mathrm{Cos.}\ b + 1{,}24328\ D_1^2\,\mathrm{Cos.}\ b_1^2$$
$$r_2^2 = 0{,}96842 - 0{,}433243\ D_2\ \mathrm{Cos.}\ b_2 + 1{,}20222\ D_2^2\,\mathrm{Cos.}\ b_2^2$$
$$r_3^2 = 0{,}96941 - 0{,}40185\ D_1\ \mathrm{Cos.}\ b_1 + 2{,}20087\ D_1^2\,\mathrm{Cos.}\ b_1^2$$

Calculus cordarum sive quantitatum $P_1,\ P_2,\ P_3$

$$2P_1 = 1{,}9309 - 0{,}726238\,D\,\mathrm{Cos.}\,b - 0{,}26114\,D_2\,\mathrm{Cos.}\,b_2 + 2{,}413788\,D\,D_2\,\mathrm{Cos.}\,b\,\mathrm{Cos.}\,b_2$$
$$2P_2 = 1{,}9320 - 0{,}536575\,D_2\,\mathrm{Cos.}\,b_2 - 0{,}119324\,D_3\,\mathrm{Cos.}\,b_3 + 2{,}35273\,D_2\,D_3\,\mathrm{Cos.}\,b_2\,\mathrm{Cos.}\,b_3$$
$$P_3 = 0{,}958622 - 0{,}43598\,D_1\,\mathrm{Cos.}\,b_1 + 1{,}15761\,D_1\,D_3\,\mathrm{Cos.}\,b_1\,\mathrm{Cos.}\,b_3$$

Calculus æquationum 18, 19 indirecta methodo,

Hypothesis 1, $D_2\,\mathrm{Cos.}\,b_2 = 1$

$\overline{r_2}^{\,2} = 1{,}737391$ addendo tres numeros obtinetur;

$2P_2 = 1{,}395425 + 2\ 233406\ D_3\ \mathrm{Cos.}\ b_3$; addendo numeros

$2P_1 = 1{,}66976 + 2{,}152648\ D\ \mathrm{Cos.}\ b$

$-P_3 =$ sub finem calculatur

Quærantur logarithmi horum coefficientium, ita ut quadrari possit $P_2,\ P_1$, erit resolvendo logarithmos 6

$$\frac{t_1^2}{t_2^2}\overline{P_2}^{\,2} = 0{,}4929945 + 158609\,D_3\,\mathrm{Cos.}\,b_3 + 1{,}269\,D_3\,\mathrm{Cos.}\,b_3^2$$
$$P_1^2 = 0{,}6970245 + 1{,}71371\,D\,\mathrm{Cos.}\,b + 1{,}05334\,D\,\mathrm{Cos.}\,b_2$$

quæratur logarithmus $\frac{t_1^2}{t_2^2}\,\overline{r_2}^{\,2}$; ohtinebitur resolvendo 6 logarithmos

$$\frac{t_1^2}{t_2^2}\,r_2^2\,\overline{r_3}^{\,2} = 1{,}71432 - 0{,}710638\,D_1\,\mathrm{Cos.}\,b_1 + 3{,}89383\,D_1\,\mathrm{Cos.}\,b_1^2$$
$$r_2^2\,r_1^2 = 1{,}68100 - 1{,}0299\,D_1\,\mathrm{Cos.}\,b_1 + 2{,}16007\,D_1\,\mathrm{Cos.}\,b_1^2$$

Quare obtinetur æquatio 18) quadratica, $\frac{t_1^2}{t_2}(r_2 r_3)^2 - r_2^2 r_1^2 - \frac{t_1^2}{t_2^2}P_2^2 + P_1^2$

$$\text{Aeq. 18.}\ 0 = 0{,}2372490 - 0\,13283\,D_1\,\mathrm{Cos.}\,b_1 + 0{,}39607\,D_1\,\mathrm{Cos.}\,b_1^2;$$ valor D_1 impossibilis

Sumatur itaque nova hypothesis

$D_2\,\mathrm{Cos.}\,b_2 = 0{,}5$;

$\text{log.}\,D_2\,\mathrm{Cos.}\,b_2 = 9,\ 6989700.$

$r_2^2 = 1{,}05235$; $\text{log.}\ r_2^2 = 0{,}0221601.$

$P_1 = 0{,}90016 + 0{,}24033\ D\ \mathrm{Cos.}\ b$

$P_2 = 0{,}83185 + 0{,}52852\ D_3\ \mathrm{Cos.}\ b_3$

Ex his Sequitur æquatio quadratica. Aeq. 18.

		$D_1\,\mathrm{Cos.}\,b_1$	$D_1\,\mathrm{Cos.}\,b_1^2$
P_1^2	$= +\,0{,}81028$	$+\,0{,}43267$	$+\,0{,}05775$
$-\frac{t_1^2 P_2^2}{t_2}$	$= -\,0{,}70433$	$-\,1{,}22852$	$-\,0{,}53571$
$-\,r_2\,\overline{r_1}^{\,2}$	$= -\,1{,}01820$	$+\,0{,}62396$	$-\,1{,}30837$
$+\frac{t_1^2}{t_2^2}(r_3 r_2^2)$	$= +\,1{,}03337$	$-\,0{,}48043$	$+\,2{,}35851$

Scilicet

$$\overline{D_1\,\mathrm{Cos.}\,b_1}^{\,2} - \frac{0{,}60232}{0{,}57218}\,D_1\,\mathrm{Cos.}\,b_1 = -\frac{0{,}14612}{0{,}57218}$$

Quæ resoluta ope trigonometriæ quærendo tres logarithmos et arcum, dat $\text{Log.}\,D_1\,\mathrm{Cos.}\,b_1 = 9,\ 5788398$ Similiter $= 9,\ 8283806$; inde

$$\text{Aeq. 19.}\quad 0 = r_2^2\,P_3 - P_1 P_2 + \frac{t_2}{t_1}\left(\overline{r_2 r_1}^{\,2} - P_1^2\right)$$ facillime calculatur,

Scilic. Aequatio 19 ex valore

majore	minore
1,06042	1,39582
— 1,32246	— 1,04390
+ 0,05988	+ 0,28037
fit. + 0,39784	+ 0,63229

Hypothesis tertia

$D_2\,\mathrm{Cos.}\,b_2 = 0,\ 6$

$r_2^2 = 1{,}14127$

		$D_1\,\mathrm{Cos.}\,b_1$	$\overline{D_1\,\mathrm{Cos.}\,b_1}^{\,2}$
P_1^2	$=\ 0{,}78696$	$+\,0{,}81774$	$+\,0,\ 21253$
$\frac{t_1}{t_2}\overline{P_2}^{\,2}$	$= -\,0{,}65959$	$-\,1{,}43128$	$-\,0,\ 77612$
$-\,r_1 r_1 1^2$	$= -\,1{,}10422$	$+\,0{,}67668$	$-\,1{,}418916$
$\frac{t_1}{t_1}\,r_2 r_3^2$	$= -\,1{,}12611$	$-\,0{,}46681$	$-\,2,\ 55780$

Hinc oritur æquatio quadratica

$$\overline{D_1\,\mathrm{Cos.}\,b_1}^{\,2} - \frac{0{,}40367}{0{,}57530}\,D_1 + \frac{0{,}14926}{0{,}57530} = 0$$

quæ est inpossibilis: cum quadratum termini secundi minus quadruplo tertii.

Ex quibus apparet, valores pro $D_2\,\mathrm{Cos.}\,b_2 = 1,\ 0,\ 6$ nimis magnos esse; verum ex æquationibus 19 apparet posita $D_2\,\mathrm{Cos.}\,b = 0{,}5$ majorem valorem pro $D_1\,\mathrm{Cos.}\,b_1$ minus a veritate distare, quam minorem: Itaque valor verus $D_2\,\mathrm{Cos.}\,b_2$ accedit ad 0, 6 majorque est 0, 5: Calculus itaque ab initio resumendus: Verum quam maxime laboriosus: utpote ut perveniatur ad æquationem quadraticam, quæ quatuor membris constat, 16 logarithmorum quærendi sunt numeri: nec pro repetito calculo logarithmorum constantium usus est.

§. 14.

Expeditior, attamen satis laboriosa, nec tam universalis methodus est; quæ jungit æquationes.

IV, XI, et 18 sive 19:

Olbersii exemplum ex hac methodo calculavi; proprie perducit ad æquationem 18) biquadraticam

Aequatio IV.

$$L D_1 \text{Cof. } b_1 + m D_3 \text{Cof. } b_3 = 0$$

$$\log. \frac{l}{m} = 0,137562$$

Aequatio XI. $$D_2 \text{Cof. } b_2 = \frac{0,891848}{0,757432 + 0,001997\, D_1 \text{Cof. } b_1} D_1 \text{Cof. } b_1$$

Posito	$D_1 = 1$	fit æquatio 18, =	+ 11,9813
	$= 0,57$		+ 3,6465
	$= 0,562$		+ 0,7536
	$= 0,561$		− 0,1789
	$= 0,56$		− 1,1245

Calculus multo expeditior, quam qui procedit ex æquationibus IV, 18 et 19.

Coefficientes æquationis undecimæ ita se habent $D_2 = \frac{T}{O + QD}$. D

Etenim Posito $K = 0$, Aeq. 4) evanescit N;

$$T = \frac{t_2}{t} R_2 \text{Sin. } A_2 (M \text{Sin. } b + L \text{Sin. } b_3);$$

$$O = \left(\frac{t_2}{t} R \text{Sin. } A + R_3 \text{Sin. } A_3\right) \text{Sin. } b$$

denique $$Q = M(\text{Sin. } b_2 \text{Cof. } b \text{Sin. } a - \text{Sin. } b \text{Cof. } b_2 \text{Sin. } a_2$$
$$- L(\text{Sin. } b_3 \text{Cof. } b_2 \text{Sin. } a_2 - \text{Sin. } b_2 \text{Cof. } b_3 \text{Sin. } a_3)$$

Ex quibus apparet Q altioris ordinis diffirentiam fore, quam L, M.; dum O, T eundem ordinem servant: quam plurimis itaque casibus, æquatio XI. plane inutilis incerta.

§. 15.

Aequationes 18, 19 calculum postulant operosissimum, cum 3 omnes incognitas involvant D, D_2, D_3, sive P, P_2, P_3.. æquatioque quadratica 18 membris 12 componatur:

Assumatur æquatio simplicior

20) $r_1 + r_3 = 2r_2$, res facilius conficietur ope æquationis 4) 19) et 20)

Scilicet determinata ratione inter D_1 et D_3 ope 4tæ; assumtaque D, innotescet r, r_3; atque r_2, ex æquatione 20), solutaque æquatione quadratica innotescet D_2; exinde P_1, P_2; qui valores satisfacere debent æquationi 19.

§. 16.

Melius junguntur æquationes 4, 5, 18;

assumta D_1; determinetur $S''' = \sqrt{rr_3^2 - P_3^2}$ ex qua deducitur ope æquationis

quintæ S, D_2; qui valor satisfacere debet æquationi 18; valor D_2 eruitur ope quadraticæ: omnium chordarum calculum supponit: adhibitis æquationibus 4, 5 et 20 calculus chordæ unius, atque P_2, cessat. Apparet male determinari S_1 ex æquatione s), cum coefficientes ordinis sunt multo minores quam magnitudo quæ ex istis eruitur.

§. 17.

Alia methodus consistit interpolationibus quærendo æquationem similem 4tæ, qua determinatur ratio inter D_2 et D_1. Inventis scilicet ope methodorum interpolandi hisce casibus usitatarum, longitudinibus atque latitudinibus cometæ, quæ respondent loco inter primam et secundam observationem, sumtaque denuo hypothesi, æquationis 18, triangula ut areas. f. Tempora; obtinetur formula

$$k + s_2 l_1 D_1 + s_1 m_1 D_2 = 0$$

Coefficientibus l_1, m_1, determinandis æquatione

$$l_1 = \text{Cof. } b. \text{Sin. } (X - a) - \text{Cof. } y. \frac{\text{Sin. } b}{\text{Sin. } y} \text{Sin. } (X - Z)$$

$$m_1 = \text{Cof. } b_2 \text{Sin. } (X - a_2) - \text{Cof. } y \frac{\text{Sin. } b_2}{\text{Sin. } y} \text{Sin. } (X - Z)$$

denotantibus X longitudinem Solis tempore, cui respondet interpolatio; y latitudinem Z longitudinem cometæ interpolatam: S_2, S_1 ex tempore innotescunt.

Determinatis ita rationibus inter D_1, D_2, D_3 eæ ipsæ vel ex 18 vel 19, vel 20 obtinentur.

§. 18.

Du Sejour l. c. novam introduxit æquationem, quæ differentialibus æquationibus pro orbita nititur, vero approximantem.

Aequ. 21. $$S_3 = \frac{t + t_2}{t} \left(1 - \frac{Ftt_2}{r_2^3}\right) S;$$

F exprimit vim Solis attractivam media terræ distantia.

Ita æquationes 4 et 5 plene innotescunt. — Proprie jam omnes æquationum 4, 5, 6
coefficientes dantur. Determinationem vero D_1, D_2, D_3 ope istarum incertam atque falla-
cem, imino sæpissime inpossibilem demonstrat earum forma. Etenim cum K_1, K_2, K_3
termini sint minimi, (qui communiter, ut Olbers sumit, negliguntur) quibus neglectis ter-
mini omnes evanescunt, solutioni haud inserviunt æquationes 7. 8. 9; quod luculenter appa-
ret ex §pho 6ta. — Determinata ratione inter distantias ope æquationum 4, 5, aut 6: di-
stantiæ ipsæ vel ope 18, 19 vel 20, vel nova æquatione, aut hypothesi eruuntur.

Sumta hypothesi, in circulo gyrare planetam, aderit $r = r_2 = r_3$; angulusque inter radios vectores innotescet prope ex angulo radiis vectoribus terræ intercepto:

Assumta linea recta æquatio aderit $S_3 = S_1 + S_2$.

Hypotheses hæ binæ sæpius vicem sustinere queant tentaminum, constructionum graphicarum, conjecturarum.

§. 19.

Inclinatio plani orbitæ ad eclipticam, longitudo Nodi, angulus denique radiis vectoribus atque recta Nodos jungente interceptus Solis D ac D_3 determinatur: scilicet denotante I inclinationem, N, Nodum, U angulum radii et lineæ per nodos ductæ, notæ sunt æquationes

$$r \text{ Sin. } U \text{ Sin. } I. - D \text{ Sin. } b = o.$$
$$r_2 \text{ Sin. } U_2 \text{ Sin. } I - D_2 \text{ Sin. } b_2 = o$$
$$r_3 \text{ Sin. } U_3 \text{ Sin. } I - D_3 \text{ Sin. } b_3 = o$$

Ex quibus sequitur

$$2 \text{ Tang. } U = \frac{D \text{ Sin. } b\, S_3}{r_3\,(r)^2 D_3 \text{ Sin. } b_3 - r_3 P_3 D_1 \text{ Sin. } b_1}$$

Proinde I innotescit; cum vero

$$r\, r_2 \text{ Sin. } U_2 = \text{Sin. } U\, P_1 + \text{Cos. } U\, S_1$$

eruitur factis substitutionibus

Aequat. 22. $\text{Sin. I. Sin. } U\, P - r\, D_2 \text{ Sin. } b_2 + \text{Cos. } U \text{ Sin. } I \sqrt{rr_2{}^2 - p^2} = o$ qua D_2 determinatur: scilicet tres adsunt æquationes determinandis tribus D_1, D_2, D_3 inservientes.

Cum æquatio adsit, v. infra §. 27 Probl. 2.

$$S_2\, r_1 \text{ Sin. } (A + u)$$
$$- S_3\, r_2 \text{ Sin. } (A + u_2) = o$$
$$+ S_1\, r_3 \text{ Sin. } (A + u_3)$$

quæ ex §. secunda deducitur sumendo A quantitatem quæ a nodo tantum et inclinatione pendet, obtinetur ope æquat. 21 nova, denotante C coefficientem notum.

Aequat. 23. $\left(1 - \frac{Ftt_2}{r_2{}^3}\right)\left(\text{Sin. } (A + U)\, P + \text{Cos. } (A + U)\, S\right) = C\, r^2$

quæ quartæ ac quintæ jungi potest.

§. 20.

Calculus pro parabola novas suppeditat æquationes: æquatio determinans axem majorem a, evenit $\frac{1}{a} = o$.

Simplicissima omnium æquatio est deducta ex famoso Lamberti theoremate, quod analytice demonstrat la Grange l. c, scilicet, denotante m coefficientem notum.

Aeq. 24. $m\,(t + t_2) = (r + r_3 + K)^{\frac{3}{2}} - (r + r_3 - K)^{\frac{3}{2}}$

quæ juncta æquationi quartæ r, r_3 determinat, quibus et ipsa parabola, solenni methodo, invenitur. Methodum hanc in Germania divulgavit atque primus proposuit Olbers libro laudato.

Sumendo cum du Sejour approximationis caussa, velocitates corporum viribus centripetis in gyros actorum, in ratione Chordarum sat parvo tempore descriptarum, obtinebitur pro ellipsi

$$\frac{1}{a} = \frac{C^2 - K^2}{4\,(r + r_3)}$$

ita ut C sit chorda parabolæ, quæ ope reversionis Serierum innotescit ex æquatione 24, et K chorda ellipseos. Usus aliquis hujus æquationis in sequentibus demonstrandus est.

§. 22.

Du Sejour ex Neutoni solutione æquationem deduxit, eo propiorem veritati, quo propior $2\, r_2 = r_1 + r_3$;

Aequ. 25. $(r + r_3)\, K_3{}^2 = 4\, F (t + t_2)^2$

Methodum hanc prolixe explanavit, rigorem solutionis æstimavit, casus quos haud complectitur enumeravit, exempla denique attulit du Sejour l. c; Aequatio radicalibus liberata ad 8vum gradum ascendit. Indirecte solvitur.

§. 23.

Vulgaris pro parabola methodus ita fere procedit.

Angulus n_2 ab r et r_3 comprehensus, eruitur assumta D, et æquatione quarta, cum rr_3 Cos. $n_2 = P_3$; anomalia tum v obtinetur æquatione $\text{Tg } \frac{v}{2} = \text{Cotg. } \frac{n_2}{2} - \sqrt{\frac{r}{r_3}} \cdot \frac{1}{\text{Sin. } \frac{1}{2} n_2}$; quare T tempus perihelii

$$T = mr^{\frac{3}{2}} \text{ Sin. } \tfrac{1}{2} v\,(3 - 2 \text{ Sin. } \tfrac{1}{2} v^2);$$ Correctio Valoris assumti D fit ope æquationis

$$T + t + t_2 = mr_3{}^{\frac{3}{2}} \text{ Sin. } \frac{v + n_2}{2}\left(3 - 2 \text{ Sin. } \frac{v + n_2{}^2}{2}\right)$$

Methodus usitatissima quam recenset Olbers l. c., totus nixa positionibus falsis, et communibus parabolæ proprietatibus, a la Caille opere præstantissimo leçons elementaires d'astronomie §. 807 sqq. exposita, eo præcipue commendatur quod inventio atque correctio elementorum orbitæ uno calculi tenore absolvatur.

§. 24.

Primus ill. la Place problema ope æquationum pro orbibus differentialium, quas theoria virium centralium suppeditat solvit, ingeniosissima nixus hypothesi differentialia ex observationibus ipsis determinari posse: Posita scilicet vi Solis attractiva media terræ distantia = 1, sumtaque temporis mensura ex medio terræ motu, tribus æquationibus

$$o = \frac{D \text{ Cos. } b \text{ Cos. } a - R \text{ Cos. } A}{r^3} + \frac{d^2.\ D \text{ Cos. } b \text{ Cos. } a - R \text{ Cos. } A}{dt^2}$$

$$o = \frac{D \text{ Cos. } b \text{ Sin. } a - R \text{ Sin. } A}{r^3} + \frac{d^2.\ D \text{ Cos. } b \text{ Sin. } a - R \text{ Sin. } A.}{dt^2}$$

$$o = \frac{d^2\,(D \text{ Sin. } b)}{dt^2} + \frac{D \text{ Sin. } b}{r_3}$$

tres insunt incognitæ $D, \frac{dD}{dt}, \frac{d^2D}{dt^2}$, r ex æquationibus §. per D determinata: quando sumere licet $\frac{da}{dt}, \frac{d^2a}{dt^2}, \frac{db}{dt}, \frac{d^2b}{dt^2}$ ex observationibus deduci posse. Solutio æquationum offert

Aequ. 26. $2d\left(\frac{D\ \mathrm{Cos.}\ b}{dt}\right).\ \frac{da}{dt} = R\ \mathrm{Sin.}\ (A - a)\left(\frac{1}{R^3} - \frac{1}{r^3}\right) - D\ \mathrm{Cos.}\ b\ \frac{d^2a}{dt^2}$

Aequ. 27. $2.d\left(\frac{D\ \mathrm{Cos.}\ b}{dt}\right)\frac{db}{dt} = R\ \mathrm{Sin.}\ b\ \mathrm{Cos.}\ \mathrm{Cos.}\ (A-a)\left(\frac{1}{R^3} - \frac{1}{r^3}\right)$

$$+ \left[\frac{d^2b}{dt^2} + \frac{d^2a}{dt^3} - \mathrm{Sin.}\ b\ \mathrm{Cos.}\ b + \left(\frac{db}{dt}\right)^2\right] D\ \mathrm{Cos.}\ b$$

ex quibus innotescunt $\frac{d.D\ \mathrm{Cos.}\ b}{dt}$; $D\ \mathrm{Cos.}\ b$

Frequentius vel d^2, vel d^2b rigide ex observationibus interpolando vix obtinentur; Assumta pro parabola æquatione.

Aequ. 28. $\frac{1}{a} = \frac{2}{r} + \left(d.\ \frac{(D\ \mathrm{Cos.}\ b\ \mathrm{Cos.}\ a - R\ \mathrm{Cos.}\ A)}{dt}\right)^2 = o$

$$+\left(d.\ \frac{(D\ \mathrm{Cos.}\ b\ \mathrm{Sin.}\ a - R\ \mathrm{Sin.}\ A)}{dt}\right)^2$$

$$+\left(d.\ \frac{D\ \mathrm{Sin.}\ b}{dt}\right)^2$$

quæ reductionem admittit, formamque simpliciorem nanciscitur, ac primi tantum ordinis differentialia continet — vel d^2a, vel d^2b non opus est.

§. 25.

Methodi hujus, quam nuperrime in *Mechanica coelesti* dilucidavit denuo Auctor,
principiis insistendo varias construes solutionis formas. theoremata *Tayloriano* datur

$$D2 = D1 + \frac{dD}{dt}t + \frac{d^2D}{2dt^2}t^2 + \frac{d^3D}{6.dt^3}t^3 + \dots$$

Similes æquationes obtinebis pro D2 Cos. b2 Cos. a2 — R2 Cos. A2,
pro D2 Cos. b2 Sin. a2 — R2 Sin. A2, pro D2 Sin. b2; mutandoque ubique 2 et 3, pro
tertia distantia D3 similes tres sunt æquationes: ita ut ex tribus observationibus 9 deducere
queas æquationes. Neglectis membris quarta t potestate affectis, eliminataque $\frac{dr}{dt}$, cum $\frac{d^2r}{dt^2}$ in
membris altioribus t potestatibus affectis tantum occurrat, obtinebis tres æquationes, quibus
incognitæ D2, D1, $\frac{dD}{dt}$, ac differentialia latitudinum ac longitudinum *primi tantum*
gradus insunt, quibus ipsas D2, D1, $\frac{dD}{dt}$ vel si placet, rationem inter D et D2 determina-
bis. Usus hujus æquationis qua ratio D et D2 determinatur ad æstimationem formularum ap-
proximantium aliquis esse poterit. Generaliter ope Tayloriani et æquationum §. 24 determi-
nabis rationem inter D2, D1 ope differentialium primi ordinis, ac differentialium secundi or-
dinis vel latitudinis solummodo vel longitudinis. Etenim ex æquatione $\frac{r^2\,d^3r^2}{dt3} + \frac{2\,dr}{dt} = o$
altiora differentialia radii vectoris eliminabis. Ita serie infinita ratio radiorum vectorum simi-
liter innotescit.

Aequationibus 26 et 27 in calculi approximantis usus addere possis varias, quæ vicem
28tæ sustinere possint. Pro Circulo æquatio $\frac{dr}{dt} = o$, novam relationem inter D ac $\frac{dD}{dt}$ per
primi ordinis differentialia suppeditat. Aequatio $\frac{d^2r}{dt^2} = o$, vel, quod eodem redit $r = a$
$\left(1 - r\frac{dr^2}{dt}\right)$, quæ pro parabola fit $r\frac{dr^2}{dt} = 1$, vel $\frac{d^3r}{dt^3} = o$ similiter calculo approximanti in-
servient, eliminabisque differentialia altiora.
Novem istas æquationes sub initium §. hujus allatas ope serierum infinitarum rigide ac
plene solvit *la Grange*, eliminando differentialia latitudinum, longitudinum, ac ad deter-
minationem trium incognitarum r, $\frac{dr}{dt}$, $\frac{d^2r}{dt^2}$ rem perducendo. Mem. Berlin 1783.

Calculi rigori consultum magis foret, formulas hujus et reliquarum methodorum per
quantitates istas, quas observatio ipsa suppeditat, rectascensionem scilicet et declinationem
exprimi.

§. 26.

Problemata ipsa, quibus orbitæ elementa determinantur, et quæ calculum emendant
ac recticificant ita fere solvuntur.
Considerentur tria plana ad quæ referatur methodo solenni tribus coordinatis invicem
perpendicularibus locus Cometæ: planum scilicet *eclipticæ*, planum æquatoris, et pla-
num ipsum *orbitæ*. Principium abscissarum e centro Solis; Positivæ sumantur perpendicu-
lares versus orientem ac polum borealem. Sint e, f, g coordinatæ respectu eclipticæ, axis
per puncta æquinoctiorum: f illi perpendicularis, g demissa a Cometæ loco ad eclipticam per-
pendicalaris; p, q, r coordinatæ respectu æquatoris; x, y coordinatæ in plano orbitæ, x
sumatur in linea per nodos transeunte. Designetur T longitudo terræ, E obliquitas Eclipticæ,
d declinatio, as ascensio Cometæ; servatis reliquis signis quæ §§. superioribus adhibita sunt,
habebis æquationes

$p = D\ \mathrm{Cos.}\ d\ \mathrm{Cos.}\ as + R\ \mathrm{Cos.}\ T;\quad q = D\ \mathrm{Cos.}\ d\ \mathrm{Sin.}\ as + R\ \mathrm{Sin.}\ T\ \mathrm{Cos.}\ E$

$t = D\ \mathrm{Sin.}\ d + R\ \mathrm{Sin.}\ T\ \mathrm{Sin.}\ E.$ porro

$e = D\ \mathrm{Cos.}\ b\ \mathrm{Cos.}\ a - R\ \mathrm{Cos.}\ A;\ f = \mathrm{Cos.}\ b\ \mathrm{Sin.}\ a - R.\ \mathrm{Sin.}\ A;\ g = D\ \mathrm{Sin.}\ b$

$e = x\ \mathrm{Cos.}\ N - y\ \mathrm{Cos.}\ I\ \mathrm{Sin.}\ N;\ f = x\ \mathrm{Sin.}\ N + y\ \mathrm{Cos.}\ N\ \mathrm{Cos.}\ I;\ g = y\ \mathrm{Sin.}\ I$

$f = q\ \mathrm{Cos.}\ E + t\ \mathrm{Sin.}\ E;\ g = q\ \mathrm{Sin.}\ E - t\ \mathrm{Cos.}\ E$

tandem

$p^2 + q^2 + t^2 = e^2 + f^2 + g^2 = x^2 + y^2 = r^2;\quad r\ \mathrm{Cos.}\ u = x;\quad r\ \mathrm{Sin.}\ u = y.$

§. 27.

Problema 1. Innotescentibus a, b, N, I determinare D, x, y

Solutio: Adsunt tres æquationes

$$- D\ \mathrm{Cos.}\ b\ \mathrm{Cos.}\ a + R\ \mathrm{Cos.}\ A + x\ \mathrm{Cos.}\ N - y\ \mathrm{Cos.}\ I.\ \mathrm{Sin.}\ N = o$$
$$- D\ \mathrm{Cos.}\ b\ \mathrm{Cos.}\ a + R\ \mathrm{Sin.}\ A + x\ \mathrm{Sin.}\ N - y\ \mathrm{Cos.}\ I.\ \mathrm{Cos.}\ N = o$$
$$- D\ \mathrm{Sin.}\ b + y\ \mathrm{Sin.}\ N = o$$

quæ tribus incognitis determinandis sufficiunt; obtinendo

$$y = R\,\frac{\text{Sin.}\,(A - N).\,\text{Sin.}\,K.}{\text{Sin.}\,(I - K)}$$

$$x = R.\,[\text{Sin.}\,N\,\text{Cotg.}\,b\,\text{Sin.}\,(a - A) + \text{Cos.}\,I\,\text{Cos.}\,(N - A)]\,\frac{\text{Sin.}\,K}{\text{Sin.}\,(I - K}$$

$$D = y\,\frac{\text{Sin.}\,N}{\text{Sin.}\,b};$$

Posito $\text{Cotg.}\,K = \text{Sin.}\,(N + a)\,\text{Cotg.}\,b.\,\frac{\text{Sin.}\,N}{\text{Sin.}\,I.}$

Aliam Solutionem dedit Olbers l. c. §. 70.

Problema 2. Innotescentibus x, y, N, I, determinare p, q, t

Solutio: adsunt tres æquationes

$p = x\,\text{Cos.}\,N - y\,\text{Cos.}\,I\,\text{Sin.}\,N,$

$q = -x\,\text{Sin.}\,N\,\text{Cos.}\,E. - y\,[\text{Cos.}\,I\,\text{Cos.}\,N\,\text{Cos.}\,E - \text{Sin.}\,I\,\text{Sin.}\,E]$

$t = x\,\text{Sin.}\,N.\,\text{Sin.}\,E + y\,[\text{Cos.}\,N\,\text{Cos.}\,I\,\text{Sin.}\,E + \text{Sin.}\,I\,\text{Cos.}\,E]$

Politis loco x, y, valoribus r Cos. u, r Sin. u obtinetur

$p = r\,[\text{Cos.}\,u\,\text{Cos.}\,N - \text{Sin.}\,u\,\text{Cos.}\,I\,\text{Sin.}\,N]$

$= r\,\text{Cos.}\,N\,[\text{Cos.}\,u - \text{Sin.}\,u\,\text{Cos.}\,I\,\text{lg.}\,N]$

$= \frac{r\,\text{Cos.}\,N}{\text{Sin.}\,A}.\,\text{Sin.}\,(A - u)$

Sumendo $\text{Cos.}\,I\,\text{Tang.}\,N = \text{Cotg.}\,A$; denique

$= r\,\text{Sin.}\,a.\ \text{Sin.}\,(A - u)$

Sumendo $\text{Sin.}\,I\,\text{Sin.}\,N = \text{Cos.}\,a$

Porro

$$q = r\,\text{Sin.}\,N\,\text{Cos.}\,E\left(-\text{Cos.}\,u + \text{Sin.}\,u\,\frac{\text{Sin.}\,I\,\text{lg}\,E - \text{Cos.}\,I\,\text{Cos.}\,N}{\text{Sin.}\,N.}\right)$$

$$= \frac{r\,\text{Sin.}\,N\,\text{Cos.}\,E}{\text{Sin.}\,B}.\ \text{Sin.}\,(u - B)$$

Summendo $\text{Cotg.}\,B = \frac{\text{Sin.}\,I\,\text{lg}\,E - \text{Cos.}\,I\,\text{Cos.}\,N}{\text{Sin.}\,N}$; sive

$= r\,\text{Sin.}\,b.\ \ \text{Sin.}\,(u - B)$

Sumendo $\text{Cos.}\,b = \text{Cos.}\,I\,\text{Sin.}\,E + \text{Sin.}\,I\,\text{Cos.}\,N\,\text{Cos.}\,E$

Denique simili prorsusmodo obtinetur

$$t = r\,\frac{\text{Sin.}\,E\,\text{Sin.}\,N}{\text{Sin.}\,C}.\,\text{Sin.}\,(u + C),\ \text{sumto}\ \frac{\text{Cos.}\,N\,\text{Cos.}\,I + \text{Sin.}\,I\,\text{Cotg.}\,E}{\text{Sin.}\,N} = \text{Cotg.}\,C$$

$t = r\,\text{Sin.}\,c.\ \text{Sin.}\,(u + C)$; posito $\text{Cos.}\,c = \text{Cos.}\,I\,\text{Cos.}\,E - \text{Sin.}\,I\,\text{Sin.}\,E\,\text{Cos}\,N$

Formulas has ex ipsis triangulis sphæricis deduxitceleb. Gauss. (v. Zach. monatliche Correspondenz Mai 1804.)

Problema 3. Innotescentibus d, as, N, I determinare D, x, y

Adsunt tres æquationes

$D\,\text{Cos.}\,d\,\text{Cos.}\,as + R\,\text{Cos.}\,T = x\,\text{Cos.}\,N - y\,\text{Sin.}\,N\,\text{Cos.}\,I$

$D\,\text{Cos.}\,d\,\text{Sin.}\,as + R\,\text{Sin.}\,T\,\text{Cos.}\,E = y\,\text{Sin.}\,I\,\text{Sin.}\,E - y\,\text{Cos.}\,N\,\text{Cos.}\,I\,\text{Cos.}\,F - x\,\text{Sin.}\,N\,\text{Cos.}\,E$

$D\,\text{Sin.}\,d + R\,\text{Sin.}\,T\,\text{Sin.}\,E = y\,\text{Sin.}\,I\,\text{Cos.}\,E + y\,\text{Cos.}\,N\,\text{Cos.}\,I\,\text{Sin.}\,E + x\,\text{Sin.}\,N\,\text{Sin.}\,E$

Aequationes hæ transmutantur ita, Sumendo A et B negativis; scilicet ut Gausius, qui valores sint (A), (B)

$D\,\text{Cos.}\,d\,\text{Cos.}\,as + R\,\text{Cos.}\,T = x\,\text{Sin.}\,a\,\text{Sin.}\,(A) + y\,\text{Sin.}\,a\,\text{Cos.}\,(A)$

$D\,\text{Cos.}\,d\,\text{Sin.}\,as + R\,\text{Sin.}\,T\,\text{Cos.}\,E = x\,\text{Sin.}\,b\,\text{Sin.}\,(B) + y\,\text{Sin.}\,b\,\text{Cos.}\,(B)$

$D\,\text{Sin.}\,d + R\,\text{Sin.}\,T\,\text{Sin}\,E = x\,\text{Sin.}\,c\,\text{Sin.}\,C + y\,\text{Sin.}\,b\,\text{Cos.}\,C$

Ex quo sequitur

$\text{Sin.}\,b\,\text{Sin.}\,B = \text{Sin.}\,N\,\text{Cos.}\,E$

$\text{Sin.}\,a\,\text{Sin.}\,A = \text{Cos.}\,N$

$\text{Sin.}\,c\,\text{Sin.}\,C = \text{Sin.}\,N\,\text{Sin.}\,E$

Ex quibus transmutationibus, solutione instituta, prodeunt valores:

$$\left.\begin{array}{l} \text{Sin.}\,d.\ \text{Cos.}\,c \\ +\,\text{Cos.}\,d\,\text{Sin.}\,as.\ \text{Cos.}\,b \\ +\,\text{Cos.}\,d\,\text{Cos.}\,as.\ \text{Cos.}\,a \end{array}\right| \ \left.\begin{array}{l} y = \text{Sin.}\,d.\ \ \text{Cos.}\,E\,\text{Sin.}\,(T - N) \\ \quad\ \ \text{Cos.}\,d\,\text{Sin.}\,as.\ \ \text{Sin.}\,E\,\text{Sin.}\,(T + N) \\ \quad - \text{Cos.}\,d\,\text{Cos.}\,as.\ \ \text{Sin.}\,E\,\text{Cos.}\,E\,\text{Sin.}\,N\,\text{Sin.}\,T \end{array}\right\}R$$

Similiter

$$\left.\begin{array}{l} -\,\text{Sin.}\,d.\ \text{Cos.}\,c \\ -\,\text{Cos.}\,d\,\text{Sin.}\,as\,\text{Cos.}\,b \\ -\,\text{Cos.}\,d\,\text{Sin.}\,as\,\text{Cos.}\,a \end{array}\right| \ \left.\begin{array}{l} x = \text{Sin.}\,d\,\text{Cos.}\,E\,\text{Cos.}\,(T + N).\,\frac{\text{Sin.}\,(n - I)}{\text{Sin.}\,n} \\ \quad + \text{Cos.}\,d\,\text{Sin.}\,as.\ \text{Sin.}\,E\,\text{Cos.}\,(T + N)\,\frac{\text{Sin.}\,(m - I)}{\text{Sin.}\,m} \\ \quad - \text{Cos.}\,d\,\text{Cos.}\,as\,\text{Sin.}\,T\,\text{Sin.}\,I \end{array}\right\}R$$

Positis scilicet

$$\text{Cotg.}\,m = \frac{\text{Cos.}\,T.\,\text{Cos.}\,E}{\text{Cos.}\,(T + N)}$$

$$\text{Cotg.}\,n = \frac{\text{Cos.}\,T.\,\text{Tg}\,E}{\text{Cos.}\,(T + N)}$$

, Formulis hisce ita adaptatis, ut quantitates quæ in calculo variantur, separatæ appareant.

Innotescente D autem adsunt æquationes

$p = x\,\text{Sin.}\,a\,\text{Sin.}\,(A) + y\,\text{Sin.}\,a\,\text{Cos.}\,(A)$

$q = x\,\text{Sin.}\,b\,\text{Sin.}\,(B) + y\,\text{Sin.}\,b\,\text{Cos.}\,(B)$; etc.

Ex quibus ipsæ x et y facillime determinantur, Scilicet

$y.\ \text{Cos.}\,c = q\,\text{Cos.}\,N - p\,\text{Sin.}\,N\,\text{Cos.}\,E$

$x\ \ \text{Cos.}\,c = p\,\text{Sin.}\,b\,\text{Cos.}\,B - q\,\text{Sin.}\,a\,\text{Cos.}\,A.$

Problema 4. Datis p, q, t tribus óbservationibus, determínare angulum inter radios vectõres interceptum

Solutio. $\text{Cos.}\,n = \frac{P_1}{rr_2}$; æquando $P_1 = pp_2 + qq_2 + tt_2$

Quantitas P_1 formam pro se fert

$$1 + o\,D + (o)\,D_2 + [o]\,D_1\,D_2$$

o, ex observationibus eruta; D ope calculi (approximantis) obtinuisti: vel ex r tibi innotescet ope æquationis

$$D - \frac{r\,\text{Sin.}\,a\,\text{Sin.}\,[(A) + u] - R\,\text{Cos.}\,I.}{\text{Cos.}\,d\,\text{Cos.}\,as.};$$ vel ex hypothesi sumis.

Denique ex problemate 3 determinari potest, inventis I, N atque X, Y: vel tandem ex triangulorum serie determinabis n, Olbers. l. c. p. 79. not.

Problema 5. Datis $p, q, t, p_2, q_2, t_2 \ldots$ ex calculo, atque observationibus, determinare orbitæ elementa.

Solutio. $r^2 = p^2 + q^2 + t^2$, definit radios vectores.

Aequatione $\frac{r^2 (q2 \, \text{Sin. } E - t2 \, \text{Cos. } E)}{q \, \text{Sin. } E - t \, \text{Cos. } E} = P1 + \sqrt{rr^2 - P1^2} . \text{Cotg. } u$

invenis angulum radii vectoris et rectæ per nodos transeunte tempore observationis primæ.

quibus inventis habebis inclinationem orbitæ;

$$\text{Sin. } I = \frac{q \, \text{Sin. } E - t \, \text{Cos. } E}{r \, \text{Sin. } u}$$

atque longitudinem Nodi

$$\text{Tin. I. Sin. } N = \frac{tq2 - qt2}{(p2 \, q - q2 \, p) \, \text{Cos. } E + (tp2 - t2p) \, \text{Sin. } E}$$

Ex natura parabolæ determinabis anomaliam U

$$\text{Tang.} \frac{U}{2} = \text{Cotg.} \frac{n2}{2} - \frac{\sqrt{r}}{r3} . \frac{1}{\text{Sin.} \frac{n2}{2}}$$

atque distantiam Perihelii Q

$$Q = r \, \overline{\text{Cos. } \tfrac{1}{2} U^2}$$

Pro ellipsi habebis æquationes tres; quarum prima

$$2r\left(\frac{(e)}{a} \text{Cos. } U + 1\right) - (p) = 0$$ (p) parameter, (e) eccentricitas.

Ex quibus ope formularum quas du Sejour l. c. amplius exponit, elementa orbitæ determinabis.

Problema 6. Calculare rectascensionum atque declinationum seriem, sive ephemeriden.

Solutio. Ex theoria virium centralium innotescunt methodis notissimis, (de quibus hic agere non licet) x, y; a, A, b B innotescent ex elementis; proinde ex æquationibus problematis secundi d, as, D determinatur.

§. 28.

Hæc fere sunt precipua problemata, quibus calculus nititur trajetoriarum. Solutionis cujusvis præstantia non solum calculi compendio, sed et rigore, ordine atque toto illius tenore æstimari debet. Volvuntur problemata alia adhuc subsidiaria ut ita dicam aut præparatoria, ad dirigendas hypotheses &c. apta, ad determinationem maximorum, aut minimorum, ad investigationem perihelii, ad selectum observationum atque temporum juvantia ... Supersunt denique quæ de casibus traduntur quibus æquationes rigorem calculi respuunt, et quæ sunt his similia, gravia atque magni momenti, de quibus sectione altera dicendi locus aderit.

Corrigenda.

Excidit

Aeq. 17. $t2S \left[rr2 \, r3^{\frac{3}{2}} + m^2 t . \, r3^{\frac{3}{2}} \right] = tS2 \left[rr2 \, r3^{\frac{3}{2}} + m^2 t2 . \, r^{\frac{3}{2}} \right]$

quæ est Euleri. Theorie der Planeten, übersetzt v. Paccassi, §. 25, p. 13.