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Enhanced Design of Pure Phase Grayscale Diffractive Optical Elements by Phase-retrieval Assisted Multiplexing of Complex Functions

Shivasubramanian Gopinath,^{a*} Andrei Bleahu,^a Tauno Kahro,^a Aravind Simon John Francis Rajeswary,^a Ravi Kumar,^b Kaupo Kukli,^a Aile Tamm,^a Joseph Rosen,^{a,c} Vijayakumar Anand^{a,d}

^aInstitute of Physics, University of Tartu, W. Ostwaldi 1, 50411 Tartu, Estonia

^bDepartment of Physics, SRM University-AP, Amaravati 522502, Andhra Pradesh, India

^cSchool of Electrical and Computer Engineering, Ben-Gurion University of the Negev, Beer-Sheva 8410501, Israel

^dOptical Sciences Center, Swinburne University of Technology, Hawthorn, Melbourne, VIC 3122, Australia

*shivasubramanian.gopinath@ut.ee

ABSTRACT

Designing a pure phase multifunctional diffractive optical element (M-DOE) is a challenging task, as the regular summation of multiple pure phase functions results in a complex function. One of the widely used multiplexing methods to design a pure phase M-DOE is the random multiplexing method. In this method, different pure phase functions are multiplied to mutually exclusive binary random functions before summation. However, M-DOEs designed using the random multiplexing method are prone to scattering noise. In this study, a novel approach based on a modified Gerchberg-Saxton algorithm (GSA) has been proposed and demonstrated for the design of pure-phase multifunctional DOEs. In this approach, the complex M-DOE obtained by regular summation is used as a reference, and with suitable constraints, the amplitude component of the complex M-DOE is transported into the phase component, resulting in a pure phase MDOE. This modified algorithm is called Transport of Amplitude into Phase based on GSA (TAP-GSA). This method has been demonstrated on a well-established incoherent digital holography technique called Fresnel incoherent correlation holography (FINCH). In FINCH, it is necessary to multiplex two-phase masks, which can be achieved using random multiplexing or polarization multiplexing, resulting in reconstruction noise and low light throughput, respectively. Under low-light conditions, random multiplexing is a better choice than the polarization multiplexing method. The M-DOE designed using TAP-GSA for FINCH improved the light throughput and exhibited a higher SNR in comparison to the random multiplexing method.

Keywords: Spatial multiplexing, Gerchberg-Saxton algorithm, Fresnel incoherent correlation holography, diffractive lens, imaging, holography

1. INTRODUCTION

Imaging, holography, beam shaping, optical trapping, etc., rely heavily on multifunctional diffractive and holographic optical elements.¹⁻¹⁰ Integration of many diffractive phase masks to create multifunctionality is usually a difficult task. The modulo- 2π phase addition method is a widely used method to combine diffractive optical elements (DOEs). The resulting DOE can only generate the cumulative effect of all the functions. When a linear grating and a diffractive axicon are combined, the resulting DOE generates a shifted ring pattern in the Fourier plane.¹¹ Sometimes, such combinations may result in interesting focusing effects.^{12,3} However, the above method is not suitable for combining two diffractive functions and still retain independent behaviours. Random multiplexing allows the possibility of combining two or more diffractive functions into a multifunctional DOE (M-DOE) and obtaining independent diffractive behaviors.^{13,14} In random multiplexing, it is possible to control the optical power sharing ratio between multiple functions by controlling the number of pixels allotted for different functions. In Ref. [13], a motionless, non-scanning incoherent digital holography method known as Fresnel incoherent correlation holography (FINCH) was demonstrated using randomly multiplexed diffractive lenses. In FINCH, two diffractive lenses independently modified the light from an object point,

producing two coherent object waves to create a self-interference hologram.^{13,14} In the above cases, high background noise was observed during recording and reconstruction of holograms. At least three holograms are recorded in FINCH with phase shifts of $\theta = 0, 2\pi/3$ and $4\pi/3$ and combined to form a complex hologram that can be reconstructed without a twin image and bias. In this study, we propose a new spatial multiplexing algorithm called Transport of Amplitude into Phase using the Gerchberg-Saxton Algorithm¹⁵ (TAP-GSA) to multiplex different diffractive functions. Using amplitude and phase constraints, the proposed algorithm iteratively encodes the amplitude information into the phase information starting with the complex function produced by the summation of several diffractive phase-only functions and producing a phase-only M-DOE. The new method has been used to improve the power efficiency of FINCH in this study. The proposed method can also be used to improve interferenceless coded aperture correlation holography (I-COACH)¹⁶ using dot patterns,¹⁷ Airy beams,¹⁸ and other incoherent holography techniques.

2. METHODS

The representation of the TAP-GSA algorithm is shown in Figure 1. The algorithm consists of two planes, namely, the mask plane and the sensor plane connected by Fresnel propagation. There are two steps in the algorithm. As seen at the top of Figure 1, the first step involves adding up two or more pure phase functions to create a complex function that we call the ideal function at the mask plane. The ideal complex function at the mask plane is propagated to the sensor plane to obtain the ideal complex function at the sensor plane. In the next step, the TAP-GSA starts at the mask plane, but the magnitude of the ideal complex function is replaced by a uniform valued matrix, and using the Fresnel propagator, the complex amplitude at the sensor plane is estimated. At the sensor plane, the amplitude constraint of the ideal complex function of the sensor plane is applied. The phase calculated at the sensor plane is combined with the ideal phase, as shown in Figure 1. This process is iterated until the algorithm converges to a non-changing phase-only matrix at the mask plane. The ratio between the number of pixels replaced in the phase matrix to the total number of pixels at the sensor plane is defined as the degrees of freedom (DOFs).

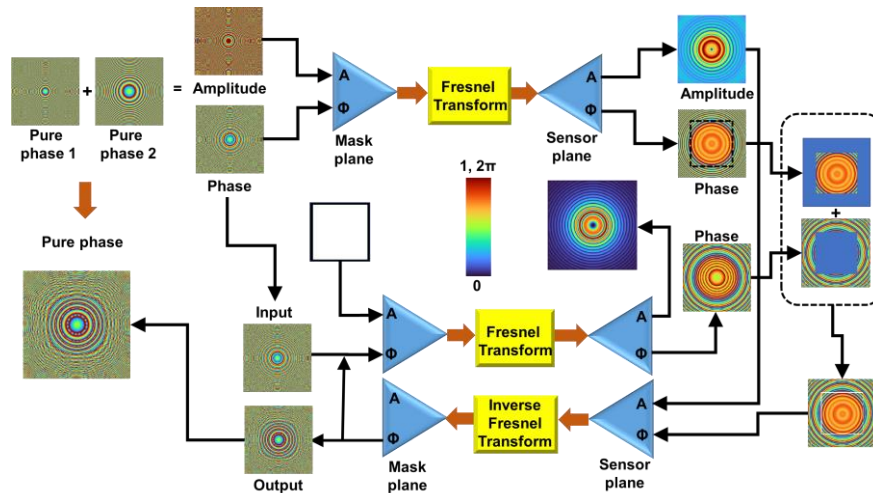


Figure 1. Representation of the TAP-GSA Algorithm.

The optical configuration of the optimal FINCH is shown in Figure 2.¹⁹ In FINCH, light from an object point is modulated by a refractive lens L with a focal length f_1 and divided into two beams by a spatial light modulator (SLM) displaying multiplexed diffractive lenses. A polarizer P_1 was used to polarize light along the active axis of the SLM. The diffractive lenses displayed on the SLM split the object wave into two and modulated them differently, which interfered with the sensor. FINCH, in the inline configuration, requires a minimum of three camera recordings with phase shifts ($\theta = 0, 2\pi/3$ and $4\pi/3$), which are superposed to obtain a complex hologram. The complex hologram is numerically back propagated to reconstruct the object's image.²⁰

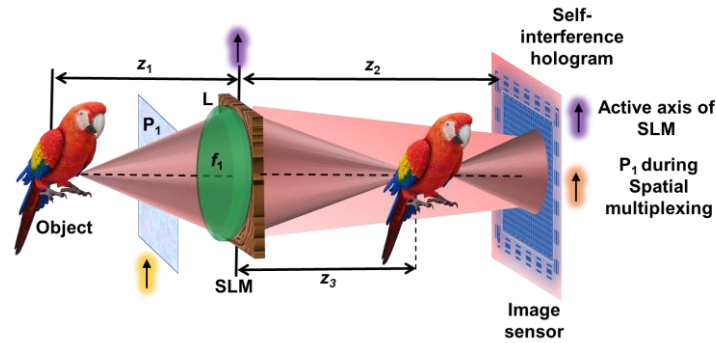


Figure 2. Optical configuration of FINCH with reduced path difference.

3. SIMULATION RESULTS

A test object featuring the letters "CIPHR" was used in the simulation. The simulated holograms ($\theta = 0, 2\pi/3$ and $4\pi/3$), the phase and magnitude of the complex hologram, and the reconstructed image using Fresnel back propagation for random multiplexing and spatial multiplexing using TAP-GSA with a DOF of 10% are shown in Figure 3. The proposed methodology using spatial multiplexing with TAP-GSA has, as seen from the reconstruction results, less reconstruction noise than random multiplexing. Since FINCH has a resolution that is 1.5 times higher than that of the direct imaging system with the same numerical aperture, both cases have superior resolution to direct imaging, as shown in Figure 3.

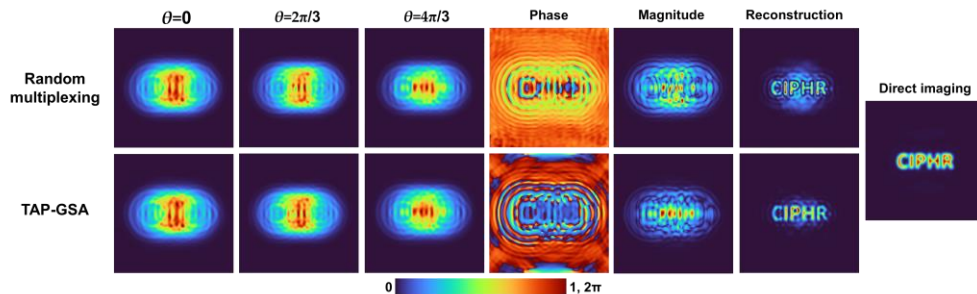


Figure 3. Simulation results of random multiplexing and the TAP-GSA method of holograms with phase shifts $\theta = 0, 2\pi/3$ and $4\pi/3$, the phase and magnitude of the complex hologram and the reconstruction results by Fresnel back propagation. The simulation result of direct imaging is also shown.

4. EXPERIMENTAL RESULTS

An experimental setup was built based on Figure 2. A high-power LED (Thorlabs, 170 mW, $\lambda = 650$ nm and $\Delta\lambda = 20$ nm) was used to illuminate a USAF object (Group – 5, Element 1) number 5 and gratings with a line width of $15.63 \mu\text{m}$. The light from the object was collimated using a refractive biconvex lens with a focal length of 5 cm. The collimated light was polarized along the active axis of the SLM (Thorlabs Exulus HD2, 1920×1200 pixels, pixel size = $8 \mu\text{m}$). On the SLM, M-DOEs for focal lengths of 13 and 28 cm were displayed, and the holograms were recorded using an image sensor (Zelux CS165MU/M 1.6 MP monochrome CMOS camera, 1440×1080 pixels with pixel size $\sim 3.5 \mu\text{m}$). The experiment was carried out for both random multiplexing and spatial multiplexing with TAP-GSA. The distance between the refractive lens and SLM is ~ 18 cm. The distance between the SLM and image sensor was 17.8 cm. The diffractive lenses were combined into M-DOEs using random multiplexing and spatial multiplexing with TAP-GSA. The phase masks synthesized by phase-shifting the diffractive lens with a focal length of 13 cm for $\theta = 0, 2\pi/3$ and $4\pi/3$ for random multiplexing and spatial multiplexing with TAP-GSA are shown in Figures 4(a)-4(c) and Figures 4(d)-4(f), respectively.

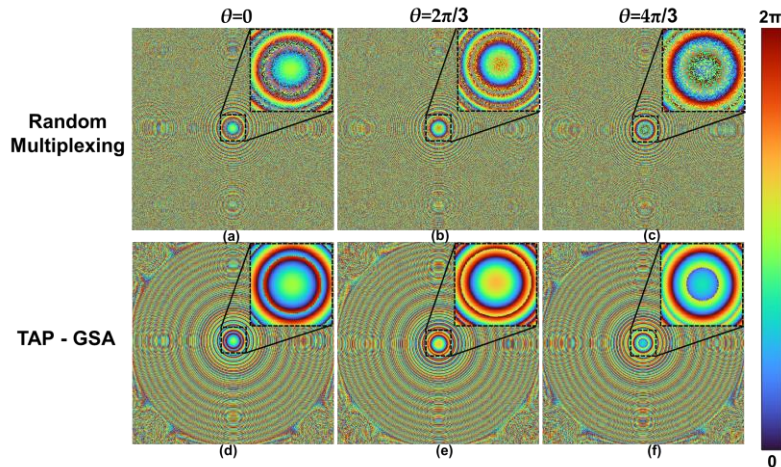


Figure 4. Phase masks for optimal FINCH for the random multiplexing method with phase shifts $\theta =$ (a) 0, (b) $2\pi/3$ and (c) $4\pi/3$ and for spatial multiplexing with TAP-GSA with phase shifts $\theta =$ (d) 0, (e) $2\pi/3$ and (f) $4\pi/3$.

A USAF object was mounted, and the phase masks for both methods shown in Figure 4 were displayed one after another on the SLM and FINCH holograms for the corresponding phase masks were recorded. Figures 5(a)-5(c) show phase-shifted holograms for the random multiplexing method for $\theta = 0, 2\pi/3$ and $4\pi/3$, respectively. Figures 5(d)-5(f) show the phase and magnitude of the complex hologram and the reconstruction result by Fresnel back propagation, respectively. Similarly, Figures 5(g)-5(l) show the phase-shifted holograms for $\theta = 0, 2\pi/3$ and $4\pi/3$, phase and magnitude of the complex hologram and reconstruction by Fresnel back propagation for the TAP-GSA method. The average background noise (ABN) was measured as 10.8×10^{-3} and 3.51×10^{-3} for random multiplexing and spatial multiplexing with TAP-GSA, respectively. The ABN for direct imaging was 0.53×10^{-3} . For the same input optical power, the exposure times of random multiplexing and spatial multiplexing with TAP-GSA were 71 ms and 42 ms, respectively, to achieve the same dynamic range in the image sensor.

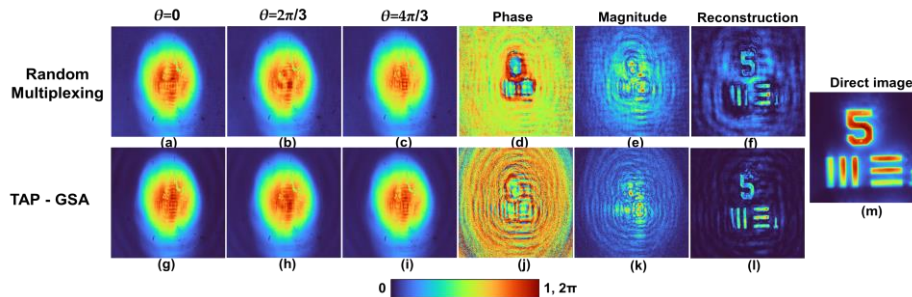


Figure 5. Holograms of the USAF object recorded for optimal FINCH for random multiplexing and TAP-GSA methods. Holograms of the random multiplexing method with phase shifts $\theta =$ (a) 0, (b) $2\pi/3$ and (c) $4\pi/3$. (d) Phase and (e) magnitude of the complex hologram and (f) reconstructed image of the USAF object. Holograms of a USAF object by the TAP-GSA method with phase shifts $\theta =$ (g) 0, (h) $2\pi/3$ and (i) $4\pi/3$, (j) phase, (k) magnitude, and (l) reconstructed image of the USAF object. (m) Direct image of USAF object.

5. SUMMARY AND CONCLUSION

A novel spatial multiplexing method based on GSA called TAP-GSA has been developed for multiplexing phase functions into a pure phase function. The method has been demonstrated on a well-established incoherent digital holography technique FINCH and found to improve the optical power throughput by ~ 1.7 times compared to the random multiplexing method. The ABN of the new method was found to be nearly 3 times lower than that of the random

multiplexing method. We believe that the developed method will be useful for improving I-COACH and other multiplane imaging applications.²¹

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