University of Tartu
Faculty of Science and Technology
Institute of Mathematics and Statistics

Mattias Moor

Embeddings and semilinear decompositions of automorphism groups of linear codes

Mathematics

Bachelor's Thesis (9 ECTS)

Supervisors: prof. Henk D. L. Hollmann (LTAT)
Dr. Ago-Erik Riet

Tartu 2026

LINEAARKOODIDE AUTOMORFISMIRÜHMADE SISESTUSED JA SEMILINEAARSED DEKOMPOSITSIOONID

Bakalaureusetöö

Mattias Moor

Lühikokkuvõte

Kui lineaarse koodi duaalse koodi minimaalne kaugus on suurem kui 2, on koodi permutatsiooni- ja monomiaalautomorfismide rühmad isomorfsed täieliku lineaarrühma teatud alarühmaga. Selles töös tõestame selle tulemuse otse ning näitame, et juhul kui minimaalsele kaugusele ei ole piiranguid seatud, on permutatsiooni- ja monomiaalautomorfismide rühmad isomorfsed koodi generaatormaatriksi kaudu defineeritud täieliku lineaarrühma alamrühma ja automorfismirühma alamrühma, mis fikseerib iga koodi sõna, poolotsekorrutisega.

CERCS teaduseriala: P120 Arvuteooria, korpuseteooria, algebraline geomeetria, algebra, rühmateooria.

Märksõnad: Kodeerimisteooria, automorfismirühm, lineaarne kood.

EMBEDDINGS AND SEMILINEAR DECOMPOSITIONS OF AUTOMORPHISM GROUPS OF LINEAR CODES

Bachelor's thesis

Mattias Moor

Abstract

If the minimum distance of the dual code of a linear code is greater than 2, the permutation automorphism group and monomial automorphism group

of the code are isomorphic to a subgroup of the general linear group. We reprove this ourselves by a slightly different method and proceed to show that in the general case where nothing about the minimal distance is known, the permutation or monomial automorphism group is isomorphic to a semidirect product of a subgroup of the general linear group obtained from a generator matrix of the code and the fix-group of the code, the subgroup formed by automorphisms that fix every codeword.

CERCS research specialisation: P120 Number theory, field theory, algebraic geometry, algebra, group theory.

Key Words: Coding theory, automorphism group, linear code.

Contents

Introduction

Coding theory is the field which studies the encoding and decoding of data into another form. The aim of such encodings is to give the data some desirable property, for example, protection against errors. Codes may be studied in terms of their automorphism groups. The structure of the automorphisms of a linear code are well known [4]. Each automorphism admits a triplet of a permutation, a vector of scalars and a field automorphism. The automorphisms for which the scalar vector is the all-one vector and the field automorphism is the identity automorphism are called *permutation automorphisms*. The automorphisms for which the field automorphism is the identity automorphism are called *monomial automorphisms*.

Recently, there has been interest in embedding the automorphism groups of a code into the general linear group [6]. It is known that under certain conditions, the automorphism group of a q-ary code of length n and dimension k can be embedded into the group of semilinear transformations [7]. Here we continue studying the connection between the permutation automorphism group of the code and the group of linear transformations that permute the columns of a generator matrix of the code. We will further study the same connections for monomial automorphisms.

This bachelor's thesis is divided into two main chapters. In Chapter 1, we introduce the definitions and notions necessary for the rest of the work and set the notation used going forward. We give a brief background in the coding theory used in the thesis. We define a linear permutation automorphism group and a linear monomial automorphism group for a code's generator matrix. We end the chapter with some basic results in group theory which will be used for showing the structure of the automorphism groups.

In Chapter 2, we investigate the automorphism groups of linear codes. We show that in general, the linear permutation automorphism group is not isomorphic to the permutation automorphism group. Further, we give a condition for these groups being isomorphic and define an isomorphism between them. We further show the decomposition of the permutation automorphism group in terms of the linear permutation automorphism group and the subgroup which "fixes" the code. Later in this chapter, we show similar results in the case of the monomial automorphism group instead of the permutation automorphism group. We show how these results can be proved using a different method in comparison to Chapter 2.1.

1 Preliminaries

1.1 Notation

Definition 1. We denote the set of all invertible $\mathbb{F}_q$-linear maps on a vector space V over $\mathbb{F}_q$ by $\mathrm{GL}(V, \mathbb{F}_q)$. If V is finite-dimensional, then given a fixed basis $\boldsymbol{b}_1, \ldots, \boldsymbol{b}_n$ for V over $\mathbb{F}_q$, there is a 1-1 correspondence between maps $L \in \mathrm{GL}(V, \mathbb{F}_q)$ and the invertible $n \times n$ matrices over $\mathbb{F}_q$. We denote the corresponding set of matrices by $\mathrm{GL}(n, q)$. We write $\mathcal{S}_n$ to denote the set of all permutations on the set $[n] := \{1, \ldots, n\}$; more generally, we write $\mathcal{S}_X$ to denote the set of all permutations on X.

Definition 2. Given some permutation $\pi \in \mathcal{S}_n$, we define the corresponding $n \times n$ *permutation matrix* $\boldsymbol{P}_\pi = (p_{i,j})$ as usual:

$$p_{i,j} = \begin{cases} 1 & \text{if } j = \pi(i), \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 3. *For a permutation π and a permutation matrix $\boldsymbol{P}_\pi$, $(\boldsymbol{P}_\pi)^{-1} = \boldsymbol{P}_{\pi^{-1}}$.*

Proof. Let us write $\boldsymbol{P}_{\pi^{-1}} = \left(p'_{i,j}\right)$ where

$$p'_{i,j} = \begin{cases} 1 & \text{if } j = \pi^{-1}(i), \\ 0 & \text{otherwise;} \end{cases} = \begin{cases} 1 & \text{if } i = \pi(j), \\ 0 & \text{otherwise.} \end{cases}$$

Thus, let $\boldsymbol{P}_\pi \boldsymbol{P}_{\pi^{-1}} = (pp_{i,j})$. Then

$$pp_{i,j} = \sum_{k=1}^{n} p_{i,k} p'_{k,j} = p_{i,\pi(i)} p'_{\pi(i),j} = p'_{\pi(i),j}.$$

Since a permutation is bijective, from the above we can precisely conclude that $pp_{i,j} = 1$ when $i = j$ and 0 otherwise. Therefore, since the inverse of a group element is unique, $(\boldsymbol{P}_\pi)^{-1} = \boldsymbol{P}_{\pi^{-1}}$. □

Definition 4. Given some vector $\boldsymbol{\lambda} = (\lambda_i)_{i=1}^k$ of length k, we define the corresponding $k \times k$ *diagonal* matrix $\boldsymbol{D}_{\boldsymbol{\lambda}} = (d_{i,j})$ as follows:

$$d_{i,j} = \begin{cases} \lambda_i & \text{if } i = j, \\ 0 & \text{otherwise.} \end{cases}$$

Definition 5. Given some permutation $\pi \in \mathcal{S}_k$ and vector $\boldsymbol{\lambda} = (\lambda_i)_{i=1}^k$, $\forall i \colon \lambda_i \neq 0$ of length k, we define the corresponding *monomial* matrix $\boldsymbol{M} = (m_{i,j})$ as $\boldsymbol{M} := \boldsymbol{D}_{\boldsymbol{\lambda}} \boldsymbol{P}_\pi$. We will write $\boldsymbol{M} := \boldsymbol{M}_{(\boldsymbol{\lambda},\pi)}$ to emphasize the coefficients and permutation associated with the matrix where necessary. We denote the set of all monomial matrices of size k over $\mathbb{F}_q$ as $\mathcal{M}(k, q)$.

Definition 6. We say that a set of vectors $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_n\}, \boldsymbol{a}_i \in \mathbb{F}_q^k$ are *projectively equal* if $\forall i, j \in \{1, 2, \ldots, n\}\ \exists \lambda \in \mathbb{F}_q \colon \boldsymbol{a}_i = \lambda \boldsymbol{a}_j$.

1.2 Coding theory

For further information on coding theory and for all the notions and results in this section, we refer to [8, 9, 10]. For a comprehensive reference on coding theory, see [7].

A code is defined as a collection of words of a fixed length over a finite alphabet. We will investigate a subclass of codes, called *linear codes*.

Definition 7. A *linear code* C of length n and dimension k over a finite field $\mathbb{F}_q$ is a k-dimensional subspace of $\mathbb{F}_q^n$; we will refer to such a code as an $[n, k]_q$ *code*. We call the elements of C *codewords*.

We now introduce some useful notions associated with a code.

Definition 8. The *Hamming weight* of a codeword is the number of non-zero symbols in the codeword. The *minimum weight* of a linear code C, denoted by $w(C)$, is the smallest Hamming weight of a nonzero codeword from C.

Definition 9. The *Hamming distance* between two codewords is the number of positions where their respective symbols differ. The *minimum distance* of a linear code C, denoted by $d(C)$, is the minimum distance between two distinct codewords from C. We will refer to a linear code of length n, dimension k, and minimum distance d over a finite field $\mathbb{F}$ as an $[n, k, d]_q$ *code*.

Lemma 10. *Let C be a linear code. Then $w(C) = d(C)$.*

Proof. Let $\boldsymbol{a}, \boldsymbol{b} \in C$ with $\boldsymbol{a} \neq \boldsymbol{b}$. Then $\boldsymbol{c} := \boldsymbol{a} - \boldsymbol{b} \in C$ and $\boldsymbol{c} \neq \boldsymbol{0}$, and $d(\boldsymbol{a}, \boldsymbol{b}) = w(\boldsymbol{c}) \geq w(C)$. Hence $d(C) \geq w(C)$. Conversely, if $\boldsymbol{c} \in C$ with $\boldsymbol{c} \neq \boldsymbol{0}$, then $w(\boldsymbol{c}) = d(\boldsymbol{c}, \boldsymbol{0}) \geq d(C)$, hence $w(C) \geq d(C)$. We conclude that $d(C) = w(C)$. □

Definition 11. Let $\boldsymbol{G}$ be a matrix over $\mathbb{F}_q$. The code $C(\boldsymbol{G})$ is defined as the row-space of $\boldsymbol{G}$.

Lemma 12. *If $\boldsymbol{G}$ is $k \times n$ over $\mathbb{F}_q$ with rank r, then $C(\boldsymbol{G})$ is a linear code of length n and dimension r over $\mathbb{F}_q$.*

Proof. Follows immediately from the definitions. □

Definition 13. Let C be a linear code. A full row rank matrix $\boldsymbol{G}$ (that is, with rank equal to the number of rows) for which $C = C(\boldsymbol{G})$ is called a *generator matrix* for C. Let C be an $[n, k]_q$ code. A full row rank $(n-k) \times n$ matrix $\boldsymbol{H}$ for which $\boldsymbol{H}\boldsymbol{c} = \boldsymbol{0}$ for all $\boldsymbol{c} \in C$ is called a *parity-check matrix* for C.

By combining Definition 11, Lemma 12, and Definition 13, we see that if $\boldsymbol{G}$ is a $k \times n$ matrix over $\mathbb{F}_q$ with rank k, then $k \leq n$ and $C := C(\boldsymbol{G})$ is a k-dimensional linear code of length n over $\mathbb{F}_q$. Furthermore, the rows of $\boldsymbol{G}$ form a basis for C as a vector space over $\mathbb{F}_q$. We also note that if $\boldsymbol{G}$ is a $k \times n$ matrix over $\mathbb{F}_q$ of rank k and $C = C(\boldsymbol{G})$, then an $(n-k) \times n$ full row rank matrix $\boldsymbol{H}$ over $\mathbb{F}_q$ is a parity-check matrix for C if and only if $\boldsymbol{G}\boldsymbol{H}^\top = \mathbf{O}$, where $\mathbf{O}$ denotes the all-zero matrix of the appropriate size.

Of course, a code can have different generator matrices. However, we have the following.

Lemma 14. *Let $\boldsymbol{G}$ be a generator matrix for a $[n,k]_q$ code C. Then $\boldsymbol{G}'$ is a generator matrix for the same code if and only if there exists $\boldsymbol{S} \in \mathrm{GL}(k,q)$ such that $\boldsymbol{G}' = \boldsymbol{S}\boldsymbol{G}$.*

Proof. The rows of $\boldsymbol{G}$ and the rows of $\boldsymbol{G}'$ both represent a basis for the same vector space C (the rows are linearly independent), so they are related by a linear transformation. Since this transformation is clearly surjective, and the code is finite-dimensional, it must also be injective. □

Proposition 15 ([1, page 26])**.** *Let $C = C(\boldsymbol{G})$ be a linear code. It has minimum distance d if and only if its parity check matrix $\boldsymbol{H}$ has a set of d linearly dependent columns but has no set of $d-1$ linearly dependent columns.*

Proof. Let $\boldsymbol{H} = [\boldsymbol{h}_1 \cdots \boldsymbol{h}_n]$ Assume $d(C) \geq 3$. Then, by Lemma 10, there exists $\boldsymbol{c} \in C$ such that $w(\boldsymbol{c}) \geq 3$. By Definition 13, $c_1\boldsymbol{h}_1 + \ldots + c_n\boldsymbol{h}_n = \mathbf{0}$, therefore the columns of $\boldsymbol{H}$ at the d non-zero positions of c are linearly dependent. If there were to exist a set of $d-1$ linearly dependent columns of H, say at positions $i_1, \ldots, i_{d-1}$, then there would exist coefficients $b_{i_1}, \ldots, b_{i_{d-1}}$ such that $b_{i_1}\boldsymbol{h}_{i_1} + \ldots + b_{i_{d-1}}\boldsymbol{h}_{i_{d-1}} = \mathbf{0}$. Therefore, the codeword $\boldsymbol{c}' := (c'_i)$ with

$c'_i = b_i$ when $i \in \{i_1, \ldots, i_{d-1}\}$ and $c'_i = 0$ otherwise would be in C. Since it has weight $w(\boldsymbol{c}') = d - 1$, it contradicts that $w(C) = d$.

Assume that the code's parity check matrix $\boldsymbol{H}$ has a set of d linearly dependent columns but no set of $d-1$ linearly dependent columns. Then, there must exist a codeword of weight d, but no codeword of weight $d-1$. □

Corollary 16. *Let $C = C(\boldsymbol{G})$ be a linear code. Then, $d(C) \geq 3$ if and only if any parity-check matrix $\boldsymbol{H}$ of the code has no columns which are scalar multiples of each other and no zero columns.*

Since the rowspace of a $k \times n$ matrix $\boldsymbol{G}$ of rank k is the collection of all vectors of the form $\boldsymbol{G}^\top \boldsymbol{a}$ with $\boldsymbol{a} \in \mathbb{F}_q^k$, we have that

$$C(\boldsymbol{G}) = \{\boldsymbol{G}^\top \boldsymbol{a} \mid \boldsymbol{a} \in \mathbb{F}_q^k\}. \tag{1}$$

Definition 17. Let $C = C(\boldsymbol{G})$ be the $[n,k]_q$ code with generator matrix $\boldsymbol{G}$. Let $\boldsymbol{c} \in C$, and let $\boldsymbol{a} \in \mathbb{F}_q^k$ be the (unique) vector such that $\boldsymbol{c} = \boldsymbol{G}^\top \boldsymbol{a}$. Then we will refer to $\boldsymbol{a}$ as the *information vector associated with $\boldsymbol{c}$ under $\boldsymbol{G}$*.

Next we investigate various symmetries of matrices and linear codes. Again, we begin by introducing a number of notions.

Definition 18. For a vector $\boldsymbol{c} = (c_1, \ldots, c_n) \in \mathbb{F}_q^n$ and a permutation$\pi \in \mathcal{S}_n$, we define

$$\boldsymbol{c}^\pi := (c_{\pi^{-1}(1)}, \ldots, c_{\pi^{-1}(n)}).$$

For a linear code C of length n over $\mathbb{F}_q$, we define C^π as the set of all words $\boldsymbol{c}^\pi$ for $\boldsymbol{c} \in C$.

Definition 19. Let C be a linear code. We define $\mathrm{PAut}(C)$ to be the set of all $\pi \in \mathcal{S}_n$ for which $C^\pi = C$.

Definition 20. Let $\boldsymbol{G} = [\boldsymbol{g}_1 \cdots \boldsymbol{g}_n]$ be a $k \times n$ matrix over $\mathbb{F}_q$ (so the matrix has columns $\boldsymbol{g}_1, \ldots, \boldsymbol{g}_n$), and let $\pi \in \mathcal{S}_n$. Then

$$\boldsymbol{G}^\pi := [\boldsymbol{g}_{\pi^{-1}(1)} \cdots \boldsymbol{g}_{\pi^{-1}(n)}];$$

in other words, $\boldsymbol{G}^\pi$ is the matrix that has its jth column equal to $\boldsymbol{g}_{\pi^{-1}(j)}$.

Proposition 21. *For a $k \times n$ matrix $\boldsymbol{G}$ and a permutation $\pi \in \mathcal{S}_n$, $\boldsymbol{G}^\pi = \boldsymbol{G}\boldsymbol{P}_\pi$.*

Proof. Let us fix some $j \in [n]$. Then,

$$(\boldsymbol{G}\boldsymbol{P}_\pi)_{i,\,j} = \sum_{l=1}^{n} (\boldsymbol{G})_{i,l} (\boldsymbol{P}_\pi)_{l,j} = (\boldsymbol{G})_{i,\pi^{-1}(j)} (\boldsymbol{P}_\pi)_{\pi^{-1}(j),j} = (\boldsymbol{G})_{i,\pi^{-1}(j)}$$

□

The following definition is new, and plays an important role in our work.

Definition 22. Let $\boldsymbol{G}$ be a generator matrix. We define $\mathrm{LPAut}(\boldsymbol{G})$ to be the subset of $\mathrm{GL}(k, q)$ consisting of all matrices $\boldsymbol{L} \in \mathrm{GL}(k, q)$ that permute the columns of $\boldsymbol{G}$, that is, for which there exists $\pi \in \mathcal{S}_n$ such that

$$\boldsymbol{L}\boldsymbol{G} = \boldsymbol{G}^\pi,$$

which is the case when $L\boldsymbol{g}_j = \boldsymbol{g}_{\pi^{-1}(j)}$ for all $j \in [n]$.

The reason why π^{-1} appears in these definitions instead of π can be seen from the proof of the following result.

Lemma 23. *Let $\boldsymbol{c} = (c_1, \ldots, c_n) \in C = \mathbb{F}_q^n$ and let $\pi, \sigma \in \mathcal{S}_n$. Then $(\boldsymbol{c}^\pi)^\sigma = \boldsymbol{c}^{\sigma \circ \pi}$.*

Proof. By definition of $\boldsymbol{c}^\pi$, the vector $\boldsymbol{d} := \boldsymbol{c}^\pi$ has $d_i = c_{\pi^{-1}(i)}$. Then the ith coordinate of $(\boldsymbol{c}^\pi)^\sigma = \boldsymbol{d}^\sigma$ is $d_{\sigma^{-1}(i)} = c_{\pi^{-1}(\sigma^{-1}(i))} = c_{(\sigma\circ\pi)^{-1}(i)}$. We conclude that $(\boldsymbol{c}^\pi)^\sigma = \boldsymbol{c}^{\sigma\circ\pi}$. □

Of course, this also means that $(C^\pi)^\sigma = C^{\sigma\circ\pi}$.

Corollary 24. $\mathrm{PAut}(C)$ *is a subgroup of* $\mathcal{S}_n$.

Proof. It is trivial to see that the identity permutation is in $\mathrm{PAut}(C)$. Let us fix $\pi, \sigma \in \mathrm{PAut}(C)$. We need to prove that $\pi \circ \sigma \in \mathrm{PAut}(C)$. Using the previous result,

$$C^{\pi\circ\sigma} = (C^\sigma)^\pi = C^\pi = C.$$

Additionally, we must show that $\pi^{-1} \in \mathrm{PAut}$, or equivalently $C^{\pi^{-1}} = C$. Indeed,

$$C = C^e = C^{\pi^{-1}\circ\pi} = (C^\pi)^{\pi^{-1}} = C^{\pi^{-1}}.$$

□

This justifies the word "group" in the name "Permutation automorphism group". Similarly, we have the following.

Lemma 25. *Let* $\boldsymbol{G}$ *be a generator matrix over* $\mathbb{F}_q$. *Then* $(\boldsymbol{G}^\pi)^\sigma = \boldsymbol{G}^{\sigma\circ\pi}$. *As a consequence,* $\mathrm{LPAut}(\boldsymbol{G})$ *is a subgroup of* $\mathrm{GL}(k, q)$.

Proof. Showing $(\boldsymbol{G}^\pi)^\sigma = \boldsymbol{G}^{\sigma\circ\pi}$ is much the same as in Lemma 23.

By Theorem 35, a condition for a nonempty subset H of a group G being a subgroup is that $g, h \in H \implies gh^{-1} \in H$. Let $\boldsymbol{K}, \boldsymbol{L} \in \mathrm{LPAut}$. Then

$$\boldsymbol{L}\boldsymbol{G} = \boldsymbol{G}^\pi, \quad \boldsymbol{K}\boldsymbol{G} = \boldsymbol{G}^\tau.$$

We can deduce that

$$\boldsymbol{LG} = \boldsymbol{G}^{\pi} \implies \boldsymbol{G} = \boldsymbol{L}^{-1}\boldsymbol{G}^{\pi}.$$

Using that, we find

$$\boldsymbol{GP}_{\tau} = \boldsymbol{G}^{\tau} = \boldsymbol{KG} = \boldsymbol{K}\left(\boldsymbol{L}^{-1}\boldsymbol{G}^{\pi}\right) \overset{(21)}{=} \boldsymbol{KL}^{-1}\boldsymbol{GP}_{\pi}$$

or

$$\boldsymbol{KL}^{-1}\boldsymbol{G} = \boldsymbol{GP}_{\tau}(\boldsymbol{P}_{\pi})^{-1} \overset{(3)}{=} \boldsymbol{G}^{\pi^{-1}\circ\tau}.$$

Therefore, we have shown $\mathrm{LPAut}(\boldsymbol{G})$ is a subgroup of $\mathrm{GL}(k, q)$. □

Definition 26. Let C be a linear code and $\boldsymbol{G}$ be a generator matrix for that code. We define $\mathrm{MAut}(C)$ to be the set consisting of all matrices $\boldsymbol{M} \in \mathcal{M}(n, q)$ for which $C(\boldsymbol{G}) = C(\boldsymbol{GM})$.

Lemma 27. *Let $\boldsymbol{G}, \boldsymbol{G}'$ be $k \times n$ generator matrices over $\mathbb{F}_q$. If $C(\boldsymbol{G}) = C(\boldsymbol{G}')$ and $\boldsymbol{M}$ is invertible, $C(\boldsymbol{GM}) = C(\boldsymbol{G}'\boldsymbol{M})$.*

Proof. Let us show first that $C(\boldsymbol{GM}) \subseteq C(\boldsymbol{G}'\boldsymbol{M})$. We must show $\forall \boldsymbol{c} \in C(\boldsymbol{GM}) \exists \boldsymbol{a} \in \mathbb{F}_q^k \colon \boldsymbol{c} = \boldsymbol{aG}'\boldsymbol{M}$. By Lemma 14, there exists $\boldsymbol{S}$ such that $\boldsymbol{SG} = \boldsymbol{G}'$. Therefore, we must show that $\forall \boldsymbol{c} \in C(\boldsymbol{GM}) \exists \boldsymbol{a} \in \mathbb{F}_q^k \colon \boldsymbol{c} = \boldsymbol{aSGM}$. We already know that $\forall \boldsymbol{c} \in C(\boldsymbol{GM}) \exists \boldsymbol{b} \in \mathbb{F}_q^k \colon \boldsymbol{c} = \boldsymbol{bGM}$. But since $\boldsymbol{aS} \in \mathbb{F}_q^k$, then proof is complete.

Showing $C(\boldsymbol{GM}) \subseteq C(\boldsymbol{G}'\boldsymbol{M})$ is much the same.

□

Definition 28. Let $\boldsymbol{G}$ be a generator matrix. We define $\mathrm{LMAut}(\boldsymbol{G})$ to be the subset of $\mathrm{GL}(\mathbb{F}_q^n, \mathbb{F}_q)$ consisting of all matrices $\boldsymbol{L} \in \mathrm{GL}(k, q)$ that when

multiplied by $\boldsymbol{G}$ from the right apply a monomial transformation to the columns of $\boldsymbol{G}$, that is, for which there exists $\boldsymbol{M} \in \mathrm{MAut}(C(\boldsymbol{G}))$ such that

$$\boldsymbol{L}\boldsymbol{G} = \boldsymbol{G}\boldsymbol{M},$$

which is the case when $L\boldsymbol{g}_j = \lambda_j \boldsymbol{g}_{\pi^{-1}(j)}$ for all $j \in [n]$.

Definition 29. Let C be an $[n,k]_q$ code. The *dual code* of C, denoted by $C^\perp$, is the collection of all vectors $\boldsymbol{x} \in \mathbb{F}_q^n$ such that $\boldsymbol{x} \cdot \boldsymbol{c} := x_1c_1 + \cdots + x_nc_n = 0$ for all $\boldsymbol{c} \in C$.

We will use the following simple but fundamental observations.

Lemma 30. *For all $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{F}_q^n$ and all $\pi \in \mathcal{S}_n$, we have $\boldsymbol{a}^\pi \cdot \boldsymbol{b}^\pi = \boldsymbol{a} \cdot \boldsymbol{b}$.*

Proof. The permutation represents a reordering of the summands, therefore due to commutativity of addition they are equal. □

Lemma 31. *For all generator matrices $\boldsymbol{G}$ and all $\pi \in \mathcal{S}_n$, we have $C(\boldsymbol{G})^\pi = C(\boldsymbol{G}^\pi)$.*

Proof. Let $\boldsymbol{G}$ be a $k \times n$ matrix. Then

$$C(\boldsymbol{G}) = \left\{\boldsymbol{G}^\intercal \boldsymbol{a} \mid \boldsymbol{a} \in \mathbb{F}_q^k\right\} = \left\{ \left(\sum_{i=1}^{k} \boldsymbol{G}_{i,j} a_i \right)_{j=1}^{n} \,\middle|\, \boldsymbol{a} \in \mathbb{F}_q^k \right\}.$$

By definition, any codeword $\boldsymbol{c}^\pi \in C(\boldsymbol{G})^\pi$ would be $\boldsymbol{c}^\pi = \left(\sum_{i=1}^{k} \boldsymbol{G}_{i,\pi^{-1}(j)} a_i\right)_{j=1}^{n}$. Applying the permutation instead to the columns of the matrix, the n-th column of the permutated matrix would then be the $\pi^{-1}(n)$-th column of the original matrix, therefore any codeword $\boldsymbol{c} \in C(\boldsymbol{G}^\pi)$ would also be of the form $\boldsymbol{c} = \left(\sum_{i=1}^{k} (\boldsymbol{G}^\pi)_{i,j}\, a_i\right)_{j=1}^{n} = \left(\sum_{i=1}^{k} \boldsymbol{G}_{i,\pi^{-1}(j)} a_i\right)_{j=1}^{n}$. □

These observations have a number of interesting consequences.

Corollary 32. *Let C be an $[n,k]_q$ code, with dual code $C^\perp$, and let $\pi \in \mathrm{PAut}(C)$. Then $(C^\perp)^\pi = (C^\pi)^\perp$.*

Proof. By Lemma 30, we have $\boldsymbol{x} \in C^\perp$ if and only if $\boldsymbol{x}^\pi \in (C^\pi)^\perp$. Since $\boldsymbol{x} \in C^\perp$ if and only if $\boldsymbol{x}^\pi \in (C^\perp)^\pi$, the claim follows. □

Corollary 33. *For every linear code C, we have $\mathrm{PAut}(C^\perp) = \mathrm{PAut}(C)$.*

Proof. By Corollary 32, if $\pi \in \mathrm{PAut}(C)$, then $C^\perp = (C^\pi)^\perp = (C^\perp)^\pi$, hence $\pi \in \mathrm{PAut}(C^\perp)$; we conclude that $\mathrm{PAut}(C) \subseteq \mathrm{PAut}(C^\perp)$. Since $(C^\perp)^\perp = C$, we can obtain the reverse inclusion by interchanging C and $C^\perp$ in the above argument. □

Lemma 34. *If $\boldsymbol{G}$ is a generator matrix and $L_1\boldsymbol{G} = L_2\boldsymbol{G}$ with $L_1, L_2 \in \mathrm{GL}(k,q)$, then $L_1 = L_2$.*

Proof. Since we can cancel full row rank matrices from the right, $L_1\boldsymbol{G} = L_2\boldsymbol{G} \implies L_1 = L_2$. □

1.3 Short exact sequences

We will briefly introduce the necessary definitions for our work. We begin by introducing some group-theoretic results.

We will describe a simple condition for identifying subgroups.

Theorem 35 ([11, Theorem 2.2.]). *A subset S of a group G is a subgroup if and only if $1 \in S$ and $s, t \in S$ imply $st^{-1} \in S$.*

The following is the classic "First isomorphism theorem".

Theorem 36 ([11, Theorem 2.24.]). *Let $f\colon G \to H$ be a homomorphism with kernel K. Then K is a normal subgroup of G and $G/K \cong \operatorname{Im} f$.*

Let us be given some sequence of groups G_n and homomorphisms $f_n\colon G_n \to G_{n+1}$. This sequence can be either finite or infinite. We can write this down as such:

$$\ldots \xrightarrow{f_{n-1}} G_n \xrightarrow{f_n} G_{n+1} \xrightarrow{f_{n+1}} \ldots$$

Definition 37. We call a sequence *exact* at point n if $\operatorname{Ker} f_n = \operatorname{Im} f_{n-1}$.

Definition 38. We call a (finite) sequence

$$G_0 \xrightarrow{f_1} G_1 \xrightarrow{f_2} G_2 \xrightarrow{f_3} \ldots \xrightarrow{f_n} G_n$$

exact if it is exact at every i, $1 \leq i \leq n-1$.

Definition 39. We call an exact sequence of the form

$$1 \to A \xrightarrow{f} B \xrightarrow{g} C \to 1$$

a *short exact sequence*. Here, 1 denotes the trivial one-element group.

We note that the homomorphisms to and from the 1-element group are always the same, and therefore we leave them unmarked.

Definition 40. A short exact sequence $1 \to A \xrightarrow{f} B \xrightarrow{g} C \to 1$ is called *split* if there exists a homomorphism $h\colon C \to B$ such that $g \circ h = \mathrm{id}_C$.

Split exact sequences are often discussed in the context of abelian groups, in which case a different definition for splitting may sometimes be used. For abelian groups, this definition is equivalent to $B \cong A \oplus C$.

Lemma 41. *For a short exact sequence, $C \cong B/\operatorname{Ker} g = B/\operatorname{Im} f$.*

Proof. We know that $C = \operatorname{Im} g$. Since $\operatorname{Ker} f = 0$, we know that f is injective. Since it is a homomorphism, we know that $A \cong \operatorname{Im} f$. By Theorem 36, we can conclude that $B/\operatorname{Ker} g \cong C$. □

For further information on group theory and short exact sequences, see [11] or [3, Section 5.5, Section 10.5].

We will look at a concept called the *semidirect product* which extends the direct product of two groups and relates closely to short exact sequences. For further information on semidirect products, see, e.g., [12, 2].

Definition 42. Let N and H be groups and $\varphi\colon H \to \operatorname{Aut}(N)$. Then we can construct a new group $N \rtimes_\varphi H$ where the underlying set is $N \times H$ and the group action is

$$(n_1, h_1) \cdot (n_2, h_2) = (n_1 \varphi_{h_1}(n_2), h_1 h_2).$$

This new group is called the *semidirect product* of N and H.

Lemma 43 ([12])**.** *Let G be a group, $N \trianglelefteq G$, and $H \leq G$. Let $i\colon H \to G$ be the embedding given by $i(h) := h$ and $\pi\colon G \to G/N$ be the projection given by $\pi(g) := g + N$. Then the following are equal:*

1. *There exists some $\varphi\colon H \to \mathrm{Aut}(N)$. Consequently, $G = N \rtimes_\varphi H$.*
2. *$G = NH$ and $N \cap H = \{1\}$.*
3. *Every $g \in G$ has a unique decomposition, i.e. there exist unique $n \in N, h \in H$ such that $g = nh$.*
4. *The function $\pi \circ i$ is an isomorphism.*
5. *There exists a split exact sequence*

$$1 \to N \to G \to H \to 1.$$

2 Results

2.1 Connections between PAut and LPAut

We will first show a certain connection between elements of $\mathrm{LPAut}(\boldsymbol{G})$ and $\mathrm{PAut}(C(\boldsymbol{G}))$.

Lemma 44. *Let $\boldsymbol{G}$ be a $k \times n$ matrix of rank k over $\mathbb{F}_q$, $\pi \in \mathcal{S}_n$. Then,*

$$\pi \in \mathrm{PAut}(C(\boldsymbol{G})) \iff \exists \boldsymbol{L} \in \mathrm{LPAut}(\boldsymbol{G}) \colon \boldsymbol{L}\boldsymbol{G} = \boldsymbol{G}^{\pi}.$$

Proof. Let us first assume $\pi \in \mathrm{PAut}(C(\boldsymbol{G}))$. Then,

$$C(\boldsymbol{G}) \overset{(19)}{=} C(\boldsymbol{G})^{\pi} \overset{(31)}{=} C(\boldsymbol{G}^{\pi}).$$

Therefore $\boldsymbol{G}$ and $\boldsymbol{G}^{\pi}$ are both generator matrices for C. By Lemma 14, $\exists \boldsymbol{L} \in \mathrm{GL}(k, q)$ such that $\boldsymbol{L}\boldsymbol{G} = \boldsymbol{G}^{\pi}$. By Definition 22, $\boldsymbol{L} \in \mathrm{LPAut}(\boldsymbol{G})$.

Let us now assume $\exists \boldsymbol{L} \in \mathrm{LPAut}(\boldsymbol{G}) \colon \boldsymbol{L}\boldsymbol{G} = \boldsymbol{G}^{\pi}$. By Lemma 14, $C(\boldsymbol{G}) = C(\boldsymbol{G}^{\pi})$. By Lemma 31, $C(\boldsymbol{G})^{\pi} = C(\boldsymbol{G}^{\pi})$, therefore $\pi \in \mathrm{PAut}(C(\boldsymbol{G}))$. □

Lemma 45. *If $\boldsymbol{L} \in \mathrm{LPAut}$ and $\boldsymbol{L}\boldsymbol{G} = \boldsymbol{G}^{\pi}$, then $\boldsymbol{L}^{-1}\boldsymbol{G} = \boldsymbol{G}^{\pi^{-1}}$.*

Proof.

$$\boldsymbol{L}\boldsymbol{G} = \boldsymbol{G}\boldsymbol{P}_{\pi} \implies \boldsymbol{G} = \boldsymbol{L}^{-1}\boldsymbol{G}\boldsymbol{P}_{\pi} \implies \boldsymbol{G}\boldsymbol{P}_{\pi}^{-1} = \boldsymbol{L}^{-1}\boldsymbol{G}$$

□

We note that a similar result to Lemma 44 been published before in [9], although it is not stated in the context of a group of linear matrices. As the

proof given there is quite cursory, we felt it appropriate to include our own version.

The result in [9] we refer to is the following.

Lemma 46 ([9, Section 7, Lemma 12])**.** *The permutation of coordinate places represented by the $n \times n$ matrix A is in* $\operatorname{Aut} \mathscr{C}$ *if and only if*

$$KM = MA$$

for some invertible $k \times k$ matrix K.

By Proposition 21, we can see that this follows immediately from Lemma 44.

It turns out that for certain types of generator matrices, these groups are isomorphic.

Theorem 47. *Let $\boldsymbol{G}$ be a full row rank $k \times n$ matrix over $\mathbb{F}_q$, and let $C = C(\boldsymbol{G})$ be the corresponding linear code. Assume the columns of $\boldsymbol{G}$ are distinct.*[1] *Then* $\operatorname{LPAut}(\boldsymbol{G}) \cong \operatorname{PAut}(C)$*, with isomorphism $\phi : \operatorname{LPAut}(\boldsymbol{G}) \mapsto \operatorname{PAut}(C)$ defined by $\phi(\boldsymbol{L}) = \pi$ precisely when $\boldsymbol{LG} = \boldsymbol{G}^{\pi}$.*

Proof. Let us first show the function ϕ is well-defined. First, we know that for each $\boldsymbol{L} \in \operatorname{LPAut}(\boldsymbol{G})$ there exists a corresponding permutation by the definition of LPAut. Let us show this permutation is unique. Let us fix $\pi, \sigma \in \operatorname{PAut}(C(\boldsymbol{G}))$ for which $\boldsymbol{LG} = \boldsymbol{G}^{\pi} = \boldsymbol{G}^{\sigma}$. If the permutation were to differ at some index i, that is $\pi(i) \neq \sigma(i)$, then there would be a corresponding j s.t. $\pi^{-1}(j) \neq \sigma^{-1}(j)$. Since the columns are unique, this would imply that $\boldsymbol{g}_{\pi^{-1}(j)} \neq \boldsymbol{g}_{\sigma^{-1}(j)}$. We know from $\boldsymbol{G}^{\pi} = \boldsymbol{G}^{\sigma}$ that $\boldsymbol{g}_{\pi^{-1}(i)} = \boldsymbol{g}_{\sigma^{-1}(i)}$. Thus, the permutation cannot differ for any $i \in \{1, \ldots, n\}$, therefore $\pi = \sigma$.

[1] Using Corollary 16, we may replace this with the stronger condition $d\left(C^{\perp}\right) \geq 3$.

Let us show the function is a homomorphism. Let us fix matrices $\boldsymbol{K}, \boldsymbol{L} \in \mathrm{LPAut}(\boldsymbol{G})$. Then, there exists $\sigma, \pi \in S_n$ such that $\boldsymbol{LG} = \boldsymbol{G}^{\pi}$, $\boldsymbol{KG} = \boldsymbol{G}^{\sigma}$. Then, $\exists \tau \in \mathcal{S}_n \colon \phi(\boldsymbol{LK}) = \tau$. We know that $\boldsymbol{G}^{\tau} = (\boldsymbol{LK})\boldsymbol{G}$. On the other hand, $\phi(\boldsymbol{L}) \cdot \phi(\boldsymbol{K}) = \pi \circ \sigma$. Thus we need to show that $\tau = \pi \circ \sigma$. Since all of $\boldsymbol{G}$'s columns are distinct, this is equivalent to showing $\boldsymbol{G}^{\tau} = \boldsymbol{G}^{\pi \circ \sigma} \overset{(25)}{=} (\boldsymbol{G}^{\sigma})^{\pi}$ i.e. $(\boldsymbol{LK})\boldsymbol{G} = \boldsymbol{L}(\boldsymbol{KG})$. By associativity of matrix multiplication, this is true.

We will now show the function is surjective. Let us fix some $\pi \in \mathrm{PAut}(C)$. Then, by Lemma 44, we see that $\exists \boldsymbol{L} \in \mathrm{LPAut}(C(\boldsymbol{G})) \colon \boldsymbol{LG} = \boldsymbol{G}^{\pi}$.

We now show the function is injective. Assume that $\phi(\boldsymbol{L}) = \phi(\boldsymbol{K})$, therefore $\boldsymbol{LG} = \boldsymbol{G}^{\pi} = \boldsymbol{KG}$. By Lemma 34, $\boldsymbol{L} = \boldsymbol{K}$. □

We note that a similar result for the full automorphism group has been published before, in [7]. It is the following:

Theorem 48 ([7, Section 7, Theorem 7.1.])**.** *Let $\mathcal{C}$ be an $[n, k, d]$ code over $\mathbb{F}_q$, with $d \geq 3$ and $r \times n$ parity check matrix H, where $r = n - k$.*

1. *If $(M; \gamma) \in \mathrm{Aut}(\mathcal{C})$, where $M \in \mathcal{M}_n(\mathbb{F}_q)$ and $\gamma \in \mathrm{Gal}(\mathbb{F}_q)$, then there is a unique $N \in \mathrm{GL}_T(q)$ such that $M(H^T\gamma^{-1}) = H^T N$.*

2. *The map $\Theta \colon \mathrm{Aut}(\mathcal{C}) \to \Gamma L_T(q)$ given by $(M; \gamma)\Theta = (N; \gamma)$ is an isomorphism with image all $(N; \gamma)$ such that $H^T N$ is $H^T\gamma^{-1}$ with its rows permuted and rescaled.*

The restriction for the generator matrix to have no multiple columns seems somewhat arbitrary. It turns out that for matrices with duplicate columns, LPAut is not directly isomorphic to PAut. We will give a description of the general case.

Theorem 49. *Let $\boldsymbol{G}$ be a full row rank $k \times n$ matrix over $\mathbb{F}_q$, and let $C = C(\boldsymbol{G})$ be the corresponding linear code. Let* $\mathrm{PFix}(\boldsymbol{G}) := \{\pi \in S_n \colon \boldsymbol{G}^\pi = \boldsymbol{G}\}$. *Then there exists a short exact sequence*

$$1 \to \mathrm{PFix}(\boldsymbol{G}) \xrightarrow{\varphi} \mathrm{PAut}(C) \xrightarrow{\Phi} \mathrm{LPAut}(\boldsymbol{G}) \to 1.$$

Proof. The function $\varphi \colon \mathrm{PFix}(\boldsymbol{G}) \to \mathrm{PAut}(C)$ is just the trivial insertion.

Similarly to the previous proof, let us define a function $\Phi : \mathrm{PAut}(C) \mapsto \mathrm{LPAut}(\boldsymbol{G})$ defined by $\Phi(\pi) = \boldsymbol{L}$ for which $\boldsymbol{LG} = \boldsymbol{G}^\pi$. By Lemma 34 (uniqueness), Lemma 14 (existence) we can conclude that this is well defined.

It remains to be proven that $\operatorname{Im}\varphi = \ker\Phi$, $\ker\varphi = \{1\}$ and $\operatorname{Im}\Phi = \mathrm{LPAut}(\boldsymbol{G})$. It is clear that $\ker\varphi = \{1\}$. Let us show that Φ is surjective. Fixing some $\boldsymbol{L} \in \mathrm{LPAut}(\boldsymbol{G})$, by definition of LPAut, $\exists\pi \colon \boldsymbol{LG} = \boldsymbol{G}^\pi$, by Lemma 44 $\pi \in \mathrm{PAut}(C)$, therefore $\operatorname{Im}\Phi = \mathrm{LPAut}(\boldsymbol{G})$.

Let us denote the $n \times n$ identity matrix as I_n. Since $\operatorname{Im}\varphi = \mathrm{PFix}(\boldsymbol{G})$, we must show that $\ker\Phi = \mathrm{PFix}(\boldsymbol{G})$. Indeed,

$$\pi \in \ker\Phi \iff \Phi(\pi) = I_k \iff I_k\boldsymbol{G} = \boldsymbol{G}^\pi \iff \boldsymbol{G} = \boldsymbol{G}^\pi \iff \pi \in \mathrm{PFix}(\boldsymbol{G}).$$

□

Corollary 50. *As an immediate consequence,* $\mathrm{LPAut}(\boldsymbol{G}) \cong \mathrm{PAut}(C)/\mathrm{PFix}(\boldsymbol{G})$.

Proof. Follows directly using Lemma 41. □

Another immediate consequence is that this gives us a new way to prove Theorem 47: when there are no duplicate columns, it is easy to see that $\mathrm{PFix}(\boldsymbol{G})$ is the one-element group. Therefore, $\mathrm{LPAut}(\boldsymbol{G}) \cong \mathrm{PAut}(C)/\{e\} \cong \mathrm{PAut}(C)$.

We finally come to the main result in this section, showing we can express PAut as a semidirect product in any case.

Theorem 51. $\mathrm{PAut}(C) \cong \mathrm{PFix}(\boldsymbol{G}) \rtimes_\varphi \mathrm{LPAut}(\boldsymbol{G})$, *where* $\varphi\colon \mathrm{LPAut}(\boldsymbol{G}) \to \mathrm{Aut}\left(\mathrm{PFix}(\boldsymbol{G})\right)$.

Proof. We recall the short exact sequence defined in Theorem 49. By Lemma 43, we need only to show that this sequence splits. Therefore we need to show that there exists a homomorphism $\Phi'\colon \mathrm{LPAut}(\boldsymbol{G}) \to \mathrm{PAut}(C)$ such that $\Phi(\Phi'(\boldsymbol{L})) = \boldsymbol{L}$ for all $\boldsymbol{L} \in \mathrm{LPAut}(\boldsymbol{G})$.

Let us define the function $\Phi'\colon \mathrm{LPAut}(\boldsymbol{G}) \to \mathrm{PAut}(C)$. For each column $\boldsymbol{g}$, let us denote the corresponding set of indices at which that column occurs in the generator matrix as $I_{\boldsymbol{g}} := \{a_{\boldsymbol{g},1}, \ldots, a_{\boldsymbol{g},t}\}, t < [n]$. Since we know that the multiplication of $\boldsymbol{L}$ by $\boldsymbol{G}$ induces a permutation of the columns of $\boldsymbol{G}$, this action must map these columns to some set of positions $I_{\boldsymbol{Lg}} := \{a_{\boldsymbol{Lg},1}, \ldots, a_{\boldsymbol{Lg},t}\}$. Since the action of $\boldsymbol{L}$ constitutes a bijection between these columns, it must preserve the multiplicities of the columns. Therefore, $a_{\boldsymbol{g},i} \mapsto a_{\boldsymbol{Lg},i}$ is a bijection on the blocks $I_{\boldsymbol{g}}$. Let us fix some permutation π that satisifies the equation $\boldsymbol{LG} = \boldsymbol{G}^\pi$. We note that any permutation in the coset $\pi\,\mathrm{PFix}(\boldsymbol{G})$ also satisfies that equation. Therefore, let $\Phi'(\boldsymbol{L})$ be the permutation from the coset $\pi\,\mathrm{PFix}(\boldsymbol{G})$ which has the property that for any $a_{\boldsymbol{g},i}$, π maps that to $a_{\boldsymbol{Lg},i}$.

Let us check Φ' is a homomorphism. Let us fix some $\boldsymbol{L}_1, \boldsymbol{L}_2$, and some $a_{\boldsymbol{g},i}$

$$\begin{aligned}
\left(\Phi'(\boldsymbol{L}_1)\Phi'(\boldsymbol{L}_2)\right)(a_{\boldsymbol{g},i}) &= \Phi'(\boldsymbol{L}_1)\left(\Phi'(\boldsymbol{L}_2)(a_{\boldsymbol{g},i})\right) \\
&= \Phi'(\boldsymbol{L}_1)(a_{\boldsymbol{L}_2\boldsymbol{g},i}) \\
&= a_{\boldsymbol{L}_1\boldsymbol{L}_2\boldsymbol{g},i} \\
&= \Phi'(\boldsymbol{L}_1\boldsymbol{L}_2)(a_{\boldsymbol{g},i})
\end{aligned}$$

Therefore, Φ' is a homomorphism.

Let us fix some random $\boldsymbol{L} \in \mathrm{LPAut}$. We must show that $\Phi(\Phi'(\boldsymbol{L})) = \boldsymbol{L}$. By definition of Φ', we know that $\Phi'(\boldsymbol{L}) = \pi$ for some $\pi \in \mathrm{PAut}(C)$ for which $\boldsymbol{LG} = \boldsymbol{G}^\pi$. Then, by definition of Φ, we know that $\Phi(\pi) = \boldsymbol{L}'$ for some $\boldsymbol{L}'$ for which $\boldsymbol{L}'\boldsymbol{G} = \boldsymbol{G}^\pi$. Then,

$$\boldsymbol{LG} = \boldsymbol{G}^\pi = \boldsymbol{L}'\boldsymbol{G} \implies \boldsymbol{LG} = \boldsymbol{L}'\boldsymbol{G} \overset{(34)}{\implies} \boldsymbol{L} = \boldsymbol{L}'.$$

□

Proposition 52. *Let $\boldsymbol{G}$ be a generator matrix with columns $\boldsymbol{g}_1, \boldsymbol{g}_2, \ldots, \boldsymbol{g}_n$. Let us denote the unique column vectors of $\boldsymbol{G}$ as $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_t$ with respective multiplicities $m_1, m_2, \ldots, m_t$. Then,* $\mathrm{PFix}(\boldsymbol{G}) \cong \prod_{i=1}^{t} S_{m_i}$.

Proof. Let us partition the set of $\boldsymbol{G}$-s column indices into sets of indices of equal columns $E_1, E_2, \ldots, E_t$ where $E_i := \left\{ j \in \boldsymbol{G} \mid \boldsymbol{v}_i = \boldsymbol{g}_j \right\}$. Note that a permutation $\pi \in \mathrm{PFix}(\boldsymbol{G})$ must necessarily also fix every partition, i.e. $\pi(E_i) = E_i$. Having $S(E_i)$ denote the symmetric group $\mathcal{S}_{m_i}$ acting on the set E_i, we have therefore shown that $\mathrm{PFix}(\boldsymbol{G}) \subseteq S(E_1) \times S(E_2) \times \ldots \times S(E_t)$.

Let us now fix some permutation $\sigma \in\subseteq S(E_1) \times S(E_2) \times \ldots \times S(E_t)$. Note that $\sigma \in \mathrm{PFix}(\boldsymbol{G})$ since $\boldsymbol{G}\boldsymbol{P}_\sigma = \boldsymbol{G}$ because σ permutes only columns that are equal with each other. Therefore,

$$\mathrm{PFix}(\boldsymbol{G}) = S(E_1) \times S(E_2) \times \ldots \times S(E_t) \cong \prod_{i=1}^{t} S_{m_i}.$$

□

2.2 Connections between MAut and LMAut

In this chapter, we will aim to generalize the results in the previous chapter to $\mathrm{MAut}(C)$. Explicitly, we will prove that $\mathrm{MAut}(C)$ can be expressed as a semidirect product. The core ideas behind many of the proofs here are the same as in the previous section, however there exist distinct complications regarding how mappings between the linear and regular automorphism groups can be defined.

Let $\boldsymbol{G} := [\boldsymbol{g}_1, \boldsymbol{g}_2, \ldots, \boldsymbol{g}_n]$ be a generator matrix. Let us define an equivalence relation $i \sim j \iff \boldsymbol{g}_i \neq \boldsymbol{0} \wedge \boldsymbol{g}_i = \lambda \boldsymbol{g}_j$ for some non-zero scalar $\lambda \in \mathbb{F}_q$. Let $E_1, \ldots, E_i$ denote the equivalence classes. Then, E_i is the set of indices of columns which are projectively equal to $\boldsymbol{g}_i$. Let $Z := [i \in [n] \mid \boldsymbol{g}_i = \boldsymbol{0}]$. Note that these two cases cover all the columns of the generator matrix. Let $\boldsymbol{M} \in \mathrm{MAut}(C(\boldsymbol{G}))$ be some monomial transformation. We can see that for matrices which have multiple zero or projectively equal columns this transformation is not unique, i.e. there exists a transformation $\boldsymbol{M}_2,\ \boldsymbol{M} \neq \boldsymbol{M}_2$ such that $\boldsymbol{G}\boldsymbol{M} = \boldsymbol{G}\boldsymbol{M}_2$.

Example. Let the matrices be over $\mathbb{F}_3$. Let us define

$$\boldsymbol{G} := \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \boldsymbol{M} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \boldsymbol{M}_2 := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

A trivial calculation shows us that indeed, $\boldsymbol{G}\boldsymbol{M} = \boldsymbol{G}\boldsymbol{M}_2$.

For every set of projectively equal column indices, let us order it by the natural order on integers so that we may write $E_i = \left\{a_{E_i,1}, \ldots, a_{E_i,|E_i|}\right\}$ and choose some arbitrary vector $\boldsymbol{v}_{E_i}$ amongst the column vectors such that for

every $t \in \{1, \ldots, |E_i|\}$ $\exists \kappa_{E_i,t} \in \mathbb{F}_q \colon \boldsymbol{g}_{a_{E_i,t}} = \kappa_{E_i,t} \boldsymbol{v}_{E_i}$. We call $\boldsymbol{v}_{E_i}$ the *canonical representation vector of a set of projectively equal columns*. Let us order Z by the natural order of integers so we may write $Z = \left\{a_{Z,1}, \ldots, a_{Z,|Z|}\right\}$.

Definition 53. We define the *canonical monomial transformation* μ_{canon} corresponding to some monomial transformation μ as follows:

Since a monomial transformation maps projectively equal columns to projectively equal columns, let us define $\mu(E_i) := \left[j \in [n] \mid \exists \lambda \in \mathbb{F}_q \colon \mu(\boldsymbol{g}_i) = \lambda \mu(\boldsymbol{g}_j)\right]$. Note that this means that for the canonical representation vector, $\exists \eta_{E_i} \colon \mu(\boldsymbol{v}_{E_i}) = \eta_{E_i} \boldsymbol{v}_{\mu(E_i)}$. Let us define $\mu(Z) := [i \in [n] \mid \mu(\boldsymbol{g}_i) = \boldsymbol{0}]$. Then, let us define the *canonical permutation* π_{canon} as

$$\pi_{\text{canon}}(i) := \begin{cases} a_{\mu(E_i),t} & \text{if } \boldsymbol{g}_i \neq \boldsymbol{0} \text{ and therefore } \exists t \colon i = a_{E_i,t} \\ a_{\mu(Z),t} & \text{if } \boldsymbol{g}_i = \boldsymbol{0} \text{ and therefore } \exists t \colon i = a_{Z,t}. \end{cases}$$

Let us now define the *canonical scalar vector* $\boldsymbol{\lambda}_{\text{canon}} = (\lambda_i)_{i=1}^n$ as

$$\lambda_i := \begin{cases} \frac{\kappa_{E_i,t} \eta_{E_i}}{\kappa_{\mu(E_i),t}} & \text{if } \boldsymbol{g}_i \neq \boldsymbol{0} \text{ and therefore } \exists t \colon i = a_{E_i,t} \\ 1 & \text{if } \boldsymbol{g}_i = \boldsymbol{0}. \end{cases}$$

Then, $\mu_{\text{canon}} := (\pi_{\text{canon}}, \boldsymbol{\lambda}_{\text{canon}})$. The corresponding *canonical monomial matrix* is denoted as $\boldsymbol{M}_{\text{canon}}$.

Lemma 54. *The composition of two canonical monomial transformations is also a canonical monomial transformation.*

Proof. Let us fix some canonical monomial transformations $\mu_1, \mu_2 \in \text{MAut}(C)$, $\mu_1 = (\mu_1)_{\text{canon}},\ \mu_2 = (\mu_2)_{\text{canon}}$. Let $(\pi, \boldsymbol{\lambda}) = \mu := \mu_1 \circ \mu_2$.

We must show that $\mu_1 \circ \mu_2 = \mu_{\text{canon}} = (\pi_{\text{canon}}, \boldsymbol{\lambda}_{\text{canon}})$.

First, $\mu_1(\mu_2(\boldsymbol{v}_{E_i})) = \mu_1(\eta^{(2)}_{E_i}\boldsymbol{v}_{\mu_2(E_i)}) = \eta^{(1)}_{\mu_2(E_i)}\eta^{(2)}_{E_i}\boldsymbol{v}_{\mu_1(\mu_2(E_i))}$. Therefore, $\eta_{E_i} = \eta^{(1)}_{\mu_2(E_i)}\eta^{(2)}_{E_i}$.

By definition, $\pi_2(a_{E_i,t}) = a_{\mu_2(E_i),t}$. Then, $\pi_1(\pi_2(a_{E_i,t})) = \pi_1(a_{\mu_2(E_i),t}) = a_{\mu_1(\mu_2(E_i)),t} = a_{(\mu_1\circ\mu_2)(E_i),t} = \pi(a_{E_i}, t)$. For the zero columns, the same argument follows. Therefore, $\pi = \pi_{\text{canon}}$.

The scalar for zero columns is trivially also 1. For non zero columns, the scalar of the composition would be $\lambda^1_{\pi_2(a_{E_i,t})}\lambda^2_{a_{E_i,t}}$. By definition,

$$\lambda^2_{a_{E_i,t}} = \frac{\kappa_{E_i,t}\eta^{(2)}_{E_i}}{\kappa_{\mu_2(E_i),t}}, \quad \lambda^1_{\pi_2(a_{E_i,t})} = \lambda^1_{a_{\mu_2(E_i),t}} = \frac{\kappa_{\mu_2(E_i),t}\eta^{(1)}_{\mu_2(E_i)}}{\kappa_{\mu_1(\mu_2(E_i)),t}}.$$

Therefore

$$\lambda^1_{\pi_2(a_{E_i,t})}\lambda^2_{a_{E_i,t}} = \frac{\kappa_{E_i,t}\eta^{(2)}_{E_i}}{\kappa_{\mu_2(E_i),t}}\cdot\frac{\kappa_{\mu_2(E_i),t}\eta^{(1)}_{\mu_2(E_i)}}{\kappa_{\mu_1(\mu_2(E_i)),t}} = \frac{\kappa_{E_i,t}\eta^{(2)}_{E_i}\eta^{(1)}_{\mu_2(E_i)}}{\kappa_{\mu_1(\mu_2(E_i)),t}} = \frac{\kappa_{E_i,t}\eta_{E_i}}{\kappa_{\mu_1(\mu_2(E_i)),t}} = \lambda_{a_{E_i,t}}.$$

Hence $\boldsymbol{\lambda} = \boldsymbol{\lambda}_{\text{canon}}$. □

Definition 55. We call the group of monomial matrices that leaves a generator matrix $\boldsymbol{G}$ unchanged

$$\text{Fix}(\boldsymbol{G}) := \{\boldsymbol{M} \in \text{MAut}(C) \mid \boldsymbol{G}\boldsymbol{M} = \boldsymbol{G}\}$$

Proposition 56. *Let $\boldsymbol{G}, \boldsymbol{G}'$ be two generator matrices for the same code C. Then,* $\text{Fix}(\boldsymbol{G}) = \text{Fix}(\boldsymbol{G}')$.

Proof. By Lemma 14, $\exists \boldsymbol{S} \colon \boldsymbol{G} = \boldsymbol{S}\boldsymbol{G}'$. Then,

$$\begin{aligned}
\boldsymbol{M} \in \mathrm{Fix}(\boldsymbol{G}) &\iff \boldsymbol{G}\boldsymbol{M} = \boldsymbol{G} \\
&\iff \boldsymbol{S}\boldsymbol{G}'\boldsymbol{M} = \boldsymbol{S}\boldsymbol{G}' \\
&\iff \boldsymbol{S}\left(\boldsymbol{G}'\boldsymbol{M} - \boldsymbol{G}'\right) = 0 \\
&\iff \boldsymbol{G}'\boldsymbol{M} - \boldsymbol{G}' = 0 \\
&\iff \boldsymbol{G}'\boldsymbol{M} = \boldsymbol{G}' \\
&\iff \boldsymbol{M} \in \mathrm{Fix}(\boldsymbol{G}').
\end{aligned}$$

□

Often we do not speak of a code in terms of a particular generator matrix. Therefore it is useful to think about fixing the code itself instead of fixing a generator matrix of the code.

Definition 57. Let $\mathrm{Fix}(C)$ denote the group of monomial matrices that fixes every codeword $\boldsymbol{c}$ in a linear code C.

$$\mathrm{Fix}(C) := \{\boldsymbol{M} \in \mathrm{MAut}(C) \mid \forall \boldsymbol{c} \in C \colon \boldsymbol{c}\boldsymbol{M} = \boldsymbol{c}\}$$

The following shows that these notions of fixing either the code or the generator matrix are equal.

Proposition 58. *Let C be a linear code and $\boldsymbol{G}$ be a $k \times n$ generator matrix such that $C = C(\boldsymbol{G})$. Then,* $\mathrm{Fix}(\boldsymbol{G}) = \mathrm{Fix}(C)$.

Proof. Let us first show $\mathrm{Fix}(\boldsymbol{G}) \subseteq \mathrm{Fix}(C)$. Fixing some $\boldsymbol{M} \in \mathrm{Fix}(\boldsymbol{G})$, for all $\boldsymbol{c} \in C$, $\exists \boldsymbol{a} \colon \boldsymbol{a}\boldsymbol{G} = \boldsymbol{c}$. Then, $\boldsymbol{c}\boldsymbol{M} = \boldsymbol{a}\boldsymbol{G}\boldsymbol{M} = \boldsymbol{a}\boldsymbol{G} = \boldsymbol{c}$.

Let us now show $\mathrm{Fix}(C) \subseteq \mathrm{Fix}(\boldsymbol{G})$. Fixing some $\boldsymbol{M} \in \mathrm{Fix}(C)$, let $\boldsymbol{r}_i, i \in [k]$ be a row of the generator matrix. Then, the matrices $\boldsymbol{G}$ and $\boldsymbol{GM}$ are equal if $\forall i \in [k]\colon \boldsymbol{r}_i = \boldsymbol{r}_i\boldsymbol{M}$. Since the rows are also codewords, this is true. □

We will now look at two mappings between $\mathrm{LMAut}(\boldsymbol{G})$ and $\mathrm{MAut}(C)$.

Definition 59. Let us define a function $\Phi\colon \mathrm{MAut}(C) \to \mathrm{LMAut}(\boldsymbol{G})$ where $\Phi(\boldsymbol{M}) = \boldsymbol{L}$ precisely when $\boldsymbol{LG} = \boldsymbol{GM}$.

By Lemmas 14 and 34 we can conclude that this function is well defined.

Proposition 60. *The function Φ as given in Definition 59 is a homomorphism.*

Proof. Let us fix $\boldsymbol{M}_1, \boldsymbol{M}_2 \in \mathrm{MAut}(C)$. Let $\Phi(\boldsymbol{M}_1 \cdot \boldsymbol{M}_2) = \boldsymbol{L}_3$.[2] Then, $\boldsymbol{L}_3\boldsymbol{G} = \boldsymbol{G}(\boldsymbol{M}_1 \cdot \boldsymbol{M}_2) = (\boldsymbol{GM}_2)\boldsymbol{M}_1 = \boldsymbol{L}_2\boldsymbol{GM}_1 = \boldsymbol{L}_2\boldsymbol{L}_1\boldsymbol{G}$. Therefore, by Lemma 34, $\boldsymbol{L}_3 = \boldsymbol{L}_2\boldsymbol{L}_1$. □

Corollary 61. $\mathrm{Fix}(\boldsymbol{G})$ *is a normal subgroup of* $\mathrm{Aut}(C)$.

Proof. $\boldsymbol{L} \in \operatorname{Ker}\Phi$ if and only if $\boldsymbol{GM} = \boldsymbol{G}$, hence $\operatorname{Ker}\Phi = \mathrm{Fix}(\boldsymbol{G})$. Therefore by Theorem 36, $\mathrm{Fix}(\boldsymbol{G})$ is a normal subgroup of $\mathrm{MAut}(C)$. □

Lemma 62. *There exists a subgroup $H \leq \mathrm{MAut}(C)$ such that $H \cong \mathrm{LMAut}(\boldsymbol{G})$.*

Proof. Let us consider a mapping $\Phi'\colon \mathrm{LMAut}(\boldsymbol{G}) \to \mathrm{MAut}(C)$. By Definition 28, there exists some $\boldsymbol{M}$ such that $\boldsymbol{LG} = \boldsymbol{GM}$. Let us define $\Phi'(\boldsymbol{L}) := \boldsymbol{M}_{\mathrm{canon}}$. Let us show Φ' is a homomorphism. Let us fix some $\boldsymbol{L}_1, \boldsymbol{L}_2 \in \mathrm{LMAut}(\boldsymbol{G})$. Let $\Phi'(\boldsymbol{L}_1) = \boldsymbol{M}_1, \Phi'(\boldsymbol{L}_2) = \boldsymbol{M}_2, \Phi'(\boldsymbol{L}_1\boldsymbol{L}_2) = \boldsymbol{M}_3$. We must then show that $\boldsymbol{M}_3 = \boldsymbol{M}_1\boldsymbol{M}_2$.

[2] Note that $\cdot$ represents the group operation, which is $\boldsymbol{M}_1 \cdot \boldsymbol{M}_2 = \boldsymbol{M}_2\boldsymbol{M}_1$ where $\boldsymbol{M}_2\boldsymbol{M}_1$ is the product obtained by matrix multiplication.

Since monomial transformations are linear maps and linear maps are entirely determined by their values on basis vectors,

We know that $\boldsymbol{L}_1\boldsymbol{G} = \boldsymbol{G}\boldsymbol{M}_1$, $\boldsymbol{L}_2\boldsymbol{G} = \boldsymbol{G}\boldsymbol{M}_2$, and $\boldsymbol{L}_1\boldsymbol{L}_2\boldsymbol{G} = \boldsymbol{G}\boldsymbol{M}_3$. Then,

$$\boldsymbol{L}_1\boldsymbol{L}_2\boldsymbol{G} = \boldsymbol{L}_1(\boldsymbol{G}\boldsymbol{M}_2) = \boldsymbol{G}\boldsymbol{M}_1\boldsymbol{M}_2.$$

By Lemma 54, $\boldsymbol{M}_3 = \boldsymbol{M}_1\boldsymbol{M}_2$. Let us show that Φ' is injective. Taking some $\boldsymbol{L}_1, \boldsymbol{L}_2 \in \mathrm{LMAut}(\boldsymbol{G})$ such that $\Phi'(\boldsymbol{L}_1) = \boldsymbol{M}_1, \Phi'(\boldsymbol{L}_2) = \boldsymbol{M}_2$ and $\Phi'(\boldsymbol{L}_1) = \Phi'(\boldsymbol{L}_2)$, by Lemma 34 we can infer

$$\boldsymbol{L}_1\boldsymbol{G} = \boldsymbol{G}\boldsymbol{M}_1 = \boldsymbol{G}\boldsymbol{M}_2 = \boldsymbol{L}_2\boldsymbol{G} \implies \boldsymbol{L}_1 = \boldsymbol{L}_2.$$

Therefore $\mathrm{Im}(\Phi')$ is a subgroup of $\mathrm{MAut}(C)$ and $\mathrm{Im}(\Phi') \cong \mathrm{LMAut}(\boldsymbol{G})$ by Theorem 36. □

Theorem 63. $\mathrm{MAut}(C) = \mathrm{Fix}(\boldsymbol{G}) \rtimes \mathrm{Im}(\Phi')$.

Proof. We have already shown that $\mathrm{Fix}(\boldsymbol{G})$ is a normal subgroup of $\mathrm{MAut}(C)$ by Corollary 61 and that $\mathrm{Im}(\Phi')$ is a subgroup of $\mathrm{MAut}(C)$ by Lemma 62.

Let us show that $\mathrm{MAut}(C) = \mathrm{Fix}(\boldsymbol{G})\,\mathrm{Im}(\Phi')$. Let us fix some $\boldsymbol{M} \in \mathrm{MAut}(C)$. Then, let $\boldsymbol{L} := \Phi(\boldsymbol{M}) \in \mathrm{LMAut}(\boldsymbol{G})$. Let us define $\boldsymbol{K} := \boldsymbol{M}\Phi'(\boldsymbol{L})^{-1}$. Then, $\boldsymbol{K} \in \mathrm{Fix}(\boldsymbol{G}) = \mathrm{Ker}\,\Phi$ because $\Phi(\boldsymbol{K}) = \Phi(\boldsymbol{M})\Phi\left(\Phi'(\boldsymbol{L})^{-1}\right) = \boldsymbol{L}\boldsymbol{L}^{-1} = \boldsymbol{I}$. Note that $\boldsymbol{M} = \boldsymbol{K}\Phi'(\boldsymbol{L})$. Since $\boldsymbol{K} \in \mathrm{Ker}\,\Phi$ and $\Phi'(\boldsymbol{L}) \in \mathrm{Im}(\Phi')$, we can conclude $\mathrm{MAut}(C) = \mathrm{Fix}(\boldsymbol{G})\,\mathrm{Im}\,\Phi'$.

Let us show that the intersection of $\mathrm{Fix}(\boldsymbol{G})$ and $\mathrm{Im}\,\Phi'$ is trivial. Let us fix some element $\Phi'(\boldsymbol{L}) \in \mathrm{Im}\,\Phi'$. If $\Phi'(\boldsymbol{L}) \in \mathrm{Fix}(\boldsymbol{G}) = \mathrm{Ker}\,\Phi$, then $\boldsymbol{I} = \Phi(\Phi'(\boldsymbol{L})) = \boldsymbol{L}$. Then, it is trivial to see that $\Phi'(\boldsymbol{I}) = \boldsymbol{M}_{\mathrm{id}}$ by definition.

By Lemma 43, $\mathrm{MAut}(C) \cong \mathrm{Fix}(\boldsymbol{G}) \rtimes \mathrm{Im}(\Phi')$. □

Conclusions

In this thesis, we first define the automorphism group of a matrix over a finite field. We then proceed to study the connection between the permutation or monomial automorphism group of a linear code and the automorphism group of a generator matrix of the code. We show that the matrix automorphism group is isomorphic to the permutation or monomial automorphism group if the dual code has minimum distance greater than or equal to three. Further, in the general case, the permutation or monomial automorphism group of the code is a semidirect product of the linear permutation or monomial automorphism group of the code's generator matrix and the subgroup of the permutation or monomial automorphism group that fixes every codeword in the code. Furthermore, in the permutation group case, we show how to connect these groups by constructing a short exact sequence involving these groups. In the monomial case, we instead show the semidirect product structure directly instead of constructing a short exact sequence. We choose to present different approaches for the two cases, but fundamentally both methods could be used for either case.

In coding theory, the permutation automorphism group is a subgroup of the monomial automorphism group which consists of the monomial automorphisms where all the scalars are 1. The monomial automorphism group itself is the subgroup of the *full* automorphism group. In the full automorphism group, we consider "field automorphisms" applied to each position of the codeword in addition to the monomial automorphisms. Therefore, the monomial automorphism group is the subgroup of the full automorphism group where the field automorphism is the identity automorphism. Recently, in [5], we have also shown that an analogous version of our result exists for the full

automorphism group.

References

[1] *Coding Theory: Linear Codes and Distances*. 2021. URL: https://feog.github.io/4-coding.pdf (visited on 02/12/2026).

[2] Keith Conrad. *Splitting of Short Exact Sequences for Groups*. https://kconrad.math.uconn.edu/blurbs/grouptheory/splittinggp.pdf. Accessed: 2025-11-25. 2009.

[3] David S. Dummit and Richard M. Foote. *Abstract Algebra*. 3rd ed. Hoboken, NJ: Wiley, 2004.

[4] Thomas Feulner. *The automorphism groups of linear codes and canonical representatives of their semilinear isometry classes*. 2009. DOI: 10.3934/amc.2009.3.363. URL: https://www.aimsciences.org/article/id/471d510d-205a-4e91-a9be-2374e1d93e31.

[5] Henk D. L. Hollmann, Mattias Moor, and Ago-Erik Riet. *The semilinear automorphism group of a matrix over a finite field*. Submitted to ISITA 2026.

[6] Henk D.L. Hollmann. "Non-standard linear recurring sequence subgroups in finite fields and automorphisms of cyclic codes I". In: *Finite Fields and Their Applications* 86 (2023), p. 102146. ISSN: 1071-5797. DOI: https://doi.org/10.1016/j.ffa.2022.102146. URL: https://www.sciencedirect.com/science/article/pii/S1071579722001551.

[7] W. C. Huffman. "Codes and groups". In: *Handbook of Coding Theory*. Ed. by V. S. Pless, W. C. Huffman, and R. A. Brualdi. Vol. II. Amsterdam: Elsevier, 1998, pp. 1345–1440.

[8] W. Cary Huffman and Vera Pless. *Fundamentals of Error-Correcting Codes*. Cambridge University Press, 2003.

[9] F.J. MacWilliams and N.J.A. Sloane. *The Theory of Error-Correcting Codes*. Vol. 16. North-Holland Mathematical Library. Elsevier, 1977, pp. 188–215. DOI: https://doi.org/10.1016/S0924-6509(08)70532-4. URL: https://www.sciencedirect.com/science/article/pii/S0924650908705324.

[10] Ron Roth. *Introduction to Coding Theory*. Cambridge University Press, 2006.

[11] Joseph J. Rotman. "Abelian Groups". In: *An Introduction to the Theory of Groups*. Fourth. New York, NY: Springer New York, 1995, pp. 307–342. ISBN: 978-1-4612-4176-8. DOI: 10.1007/978-1-4612-4176-8_10. URL: https://doi.org/10.1007/978-1-4612-4176-8_10.

[12] Wikipedia contributors. *Semidirect product*. https://en.wikipedia.org/wiki/Semidirect_product. Accessed June 11, 2026. 2026.

Lihtlitsents lõputöö reprodutseerimiseks ja üldsusele kättesaadavaks tegemiseks

Mina, Mattias Moor,

1. annan Tartu Ülikoolile tasuta loa (lihtlitsentsi) minu loodud teose "Lineaarkoodide automorfismirühmade sisestused ja semilineaarsed dekompositsioonid", mille juhendaja(d) on Ago-Erik Riet ja Hendrik Dirk Lodewijk Hollmann, reprodutseerimiseks eesmärgiga seda säilitada, sealhulgas lisada digitaalarhiivi DSpace kuni autoriõiguse kehtivuse lõppemiseni.

2. Annan Tartu Ülikoolile loa teha punktis 1 nimetatud teos üldsusele kättesaadavaks Tartu Ülikooli veebikeskkonna, sealhulgas digitaalarhiivi DSpace kaudu Creative Commonsi litsentsiga CC BY NC ND 3.0, mis lubab autorile viidates teost reprodutseerida, levitada ja üldsusele suunata ning keelab luua tuletatud teost ja kasutada teost ärieesmärgil, kuni autoriõiguse kehtivuse lõppemiseni.

3. Olen teadlik, et punktides 1 ja 2 nimetatud õigused jäävad alles ka autorile.

4. Kinnitan, et lihtlitsentsi andmisega ei riku ma teiste isikute intellektuaalomandi ega isikuandmete kaitse õigusaktidest tulenevaid õigusi.

Mattias Moor
11.06.2026