

KONSTANTINOS PALLIKARIS

Novel exact black holes
in Einstein gravity



KONSTANTINOS PALLIKARIS

Novel exact black holes in Einstein gravity



UNIVERSITY OF TARTU
Press

This study was carried out at the University of Tartu.

The dissertation was admitted on 13.05.2026 in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Physics, and it was allowed for defence by the Council of the Institute of Physics, University of Tartu.

Supervisors: PhD Manuel Hohmann
Associate professor, Laboratory of Theoretical Physics,
Institute of Physics, University of Tartu, Estonia

PhD Emmanuel Saridakis
Research director, IAASARS,
National Observatory of Athens, Greece

Opponent: PhD Roman Konoplya
Assistant professor, RCTPA, Institute of Physics,
Silesian University in Opava, Czech Republic

Defence: 05.06.2026, University of Tartu, Estonia

The author was supported by the European Regional Development Fund via the Archimedes Foundation (Dora Pluss alategevus 2.1 välisdoktorandid LTFY, Project number 2014-2020.4.01.16-0025). Further support came from the Estonian Research Council via Team Grant PRG356 and the European Regional Development Fund via the Center of Excellence TK133. Mobility support was provided by the Archimedes Foundation through Kristjan Jaak and Dora Plus scholarships.

ISSN 1406-0647 (print)

ISBN 978-9908-57-157-7 (print)

ISSN 2806-2523 (pdf)

ISBN 978-9908-57-158-4 (pdf)

Copyright: Konstantinos Pallikaris, 2026

University of Tartu Press

<http://www.tyk.ee/>

CONTENTS

List of original publications	6
1. Introduction	9
1.1. A brief history of black holes	9
1.2. A brief history of solution-generating techniques in GR	12
1.3. Motivation	14
1.4. Aim of the thesis	16
1.5. Structure of the thesis	17
2. Quasitopological electromagnetism and black holes	18
2.1. Introduction	18
2.2. Quasitopological field theory and exact solutions	19
2.3. Black-hole thermodynamics	22
2.4. Technical summary	25
3. The electromagnetic swirling universe and novel type I spacetimes	27
3.1. Introduction	27
3.2. A detailed guide to the Ernst formalism	28
3.3. The electromagnetic swirling universe and analytic continuation	34
3.4. A Schwarzschild black hole in the EMSU	36
3.5. Mixing electric and magnetic transformations	37
3.6. Technical summary	39
4. The Kerr–Levi-Civita spacetime	41
4.1. Introduction	41
4.2. The Kerr–Levi-Civita geometry	42
4.3. Technical summary	44
5. Summary and Conclusions	45
Bibliography	47
Acknowledgements	63
Sisukokkuvõte (Summary in Estonian)	64
Attached publications	67
Quasitopological electromagnetism and black holes	70
Mixing “Magnetic” and “Electric” Ehlers–Harrison transformations: the electromagnetic swirling spacetime and novel type I backgrounds	82
A new exact rotating spacetime in vacuum: The Kerr–Levi-Civita spacetime	114
Curriculum Vitae	121
Elulookirjeldus (Curriculum Vitae in Estonian)	124

LIST OF ORIGINAL PUBLICATIONS

Publications included in the thesis

- [I] Adolfo Cisterna, Gaston Giribet, Julio Oliva, and **Pallikaris, Konstantinos**. “Quasitopological electromagnetism and black holes”. In: *Phys. Rev. D* 101 (12 June 2020), p. 124041. DOI: 10.1103/PhysRevD.101.124041. URL: <https://link.aps.org/doi/10.1103/PhysRevD.101.124041>.
- [II] José Barrientos, Adolfo Cisterna, Ivan Kolář, Keanu Müller, Marcelo Oyarzo, and Konstantinos Pallikaris. “Mixing “Magnetic” and “Electric” Ehlers–Harrison transformations: the electromagnetic swirling spacetime and novel type I backgrounds”. In: *The European Physical Journal C* 84.7 (July 2024), p. 724. ISSN: 1434-6052. DOI: 10.1140/epjc/s10052-024-13093-x. URL: <https://doi.org/10.1140/epjc/s10052-024-13093-x>.
- [III] José Barrientos, Adolfo Cisterna, Mokhtar Hassaine, Keanu Müller, and **Konstantinos Pallikaris**. “A new exact rotating spacetime in vacuum: The Kerr–Levi-Civita spacetime”. In: *Physics Letters B* 871 (2025), p. 140035. ISSN: 0370-2693. DOI: <https://doi.org/10.1016/j.physletb.2025.140035>. URL: <https://www.sciencedirect.com/science/article/pii/S0370269325007932>.

Other published work of the author

- [IV] José Barrientos, Adolfo Cisterna, Mokhtar Hassaine, and Konstantinos Pallikaris. “Electromagnetized black holes and swirling backgrounds in nonlinear electrodynamics: The ModMax case”. In: *Physics Letters B* 860 (2025), p. 139214. ISSN: 0370-2693. DOI: <https://doi.org/10.1016/j.physletb.2024.139214>. URL: <https://www.sciencedirect.com/science/article/pii/S037026932400772X>.
- [V] Damianos Iosifidis and Konstantinos Pallikaris. “Cosmology of the complete quadratic metric-affine gravity”. In: *Phys. Rev. D* 111 (2 Jan. 2025), p. 023515. DOI: 10.1103/PhysRevD.111.023515. URL: <https://link.aps.org/doi/10.1103/PhysRevD.111.023515>.
- [VI] José Barrientos, Adolfo Cisterna, and Konstantinos Pallikaris. “Plebański–Demiański à la Ehlers–Harrison: exact rotating and accelerating type I black holes”. In: *General Relativity and Gravitation* 56.9 (Sept. 2024), p. 111. ISSN: 1572-9532. DOI: 10.1007/s10714-024-03304-x. URL: <https://doi.org/10.1007/s10714-024-03304-x>.
- [VII] Adolfo Cisterna, Keanu Müller, Konstantinos Pallikaris, and Adriano Viganò. “Exact rotating wormholes via Ehlers transformations”. In: *Phys. Rev. D* 108 (2 July 2023), p. 024066. DOI: 10.1103/PhysRevD.108.024066. URL: <https://link.aps.org/doi/10.1103/PhysRevD.108.024066>.
- [VIII] Damianos Iosifidis and Konstantinos Pallikaris. “Biconnection gravity as a statistical manifold”. In: *Phys. Rev. D* 108 (4 Aug. 2023), p. 044026. DOI: 10.1103/PhysRevD.108.044026. URL: <https://link.aps.org/doi/10.1103/PhysRevD.108.044026>.

- [IX] Damianos Iosifidis and Konstantinos Pallikaris. “Describing metric-affine theories anew: alternative frameworks, examples and solutions”. In: *Journal of Cosmology and Astroparticle Physics* 2023.05 (May 2023), p. 037. DOI: 10 . 1088 / 1475 - 7516 / 2023 / 05 / 037. URL: <https://doi.org/10.1088/1475-7516/2023/05/037>.
- [X] Matteo Baggioli, Adolfo Cisterna, and Konstantinos Pallikaris. “Exploring the black hole spectrum of axionic Horndeski theory”. In: *Phys. Rev. D* 104 (10 Nov. 2021), p. 104067. DOI: 10 . 1103 / PhysRevD . 104 . 104067. URL: <https://link.aps.org/doi/10.1103/PhysRevD.104.104067>.
- [XI] Jose Pablo Figueroa and Konstantinos Pallikaris. “Quartic Horndeski, planar black holes, holographic aspects and universal bounds”. In: *Journal of High Energy Physics* 2020.9 (Sept. 2020), p. 90. ISSN: 1029-8479. DOI: 10 . 1007 / JHEP09 (2020) 090. URL: [https://doi.org/10.1007/JHEP09\(2020\)090](https://doi.org/10.1007/JHEP09(2020)090).

Author’s contribution to the publications

I, Konstantinos Pallikaris, am the main author of attached publication II. I have contributed significantly to the ideas studied therein, performed all calculations, and authored most of the manuscript after extensive discussions with my collaborators. I have made major contributions to attached publication III, where I, again, checked all the calculations and carried out a significant part of the analysis. Finally, I have contributed equally to attached publication I. In all my publications, including those not part of this thesis, I personally check all calculations, wording, and results.

LIST OF ABBREVIATIONS

Acronyms

- AdS** Anti-de Sitter. 21, 22, 25, 26
- BTZ** Bañados–Teitelboim–Zanelli. 21, 25
- CTCs** Closed Timelike Curves. 17, 36, 38, 40
- EET** Electric ET. 37, 40
- EFE** Einstein’s Field Equations. 10, 12, 14, 16, 17, 27, 41–43
- EG** Einstein gravity. 14, 16, 17, 25
- EHT** Electric HT. 37, 38, 40
- EMSU** ElectroMagnetic Swirling Universe. 34–36, 39, 43
- ET** Ehlers Transformation. 28, 39, 40
- FDAV** Frame-Dragging Angular Velocity. 17, 35, 36, 39
- GR** General Relativity. 9, 10, 13–17, 27
- HT** Harrison Transformation. 28, 39, 40
- KV** Killing Vector. 12, 13, 15–17, 27–29, 33, 35, 39, 42
- LC** Levi-Civita. 15, 16, 42, 43
- MET** Magnetic ET. 28, 34, 36, 37, 39
- MHT** Magnetic HT. 28, 34, 36, 37, 39
- NLE** Non-Linear Electrodynamics. 14, 15
- PD** Plebański–Demiański. 28
- WLP** Weyl–Lewis–Papapetrou. 29, 35, 36, 39
- ZAMO** Zero-Angular-Momentum Observer. 35, 36, 39, 43

1. INTRODUCTION

1.1. A brief history of black holes

Contrary to popular belief, the concept of a black hole dates back to studies well before the advent of Einstein's General Relativity (GR) [12] and Schwarzschild's celebrated 1916 solution [13]. It was Reverend John Michell, a close friend of Cavendish and Herschel, who speculated on the existence of "dark stars" in his 1784 letter [14]. We shall give a modernized version of his Gedankenexperiment here. Recall that the natural philosophers of the 18th century agreed that gravity obeyed Newton's law and that light was made of particles, emitted at approximately $3 \times 10^8 \text{ ms}^{-1}$. Michell assumed that the force of gravity would decelerate any particle launched from the surface of a star, and he also knew how to compute the escape velocity v_e of that particle, i.e., the initial speed at which the particle must be ejected to escape the star's gravitational pull. For a star of uniform density equal to the Sun's density, he argued that there is a critical radius, greater than 500 solar radii, for which the escape velocity equals the speed of light. Therefore, in a hypothetical scenario where the radius of our Sun would be greater than the above critical radius, the escape velocity would be greater than the speed of light, and light particles would never reach an observer far away from the star.

Twelve years later, Laplace reached the same conclusion independently. In *Exposition du système du monde* [15], he speaks of *corps obscurs*, luminous stars of uniform density equal to the Earth's density and radius approximately greater than 250 solar radii, whose rays cannot arrive at us. The idea of gigantic, invisible celestial bodies must have fascinated the British and French intelligentsia, but Michell's suggestion should appeal to us too: the irregular motion of luminous bodies could be a telltale sign of dark stars if that motion is not easily explicable on another hypothesis. This reasoning is, more or less, behind one of the many ways astronomers pinpoint black holes today. Interestingly, both Michell and Laplace opted for gigantic stars, although, ingenious as they were, they would have surely noticed that a star of solar radius and uniform density approximately 59900 times greater than the density of our planet would be an equally good "Newtonian black hole". It was unthinkable in their times that stellar matter could be that dense; we are talking densities at a whopping 30% of the density of an Earth-sized white dwarf. Ironically, Eridani 40 B appeared in Herschel's extensive *Catalogue of Double Stars* [16] one year after Michell's letter, although people did not know it was a white dwarf at the time. As Young's discovery [17] in 1801 sounded the death rattle of the corpuscular description of light, philosophers became unsure about how to combine Huygens's wave description and Newton's law of gravity. The idea of dark stars was abandoned, only to resurface more than a century later thanks to Karl Schwarzschild and his *sphère catastrophique*.

Already at the dawn of the 20th century, Schwarzschild was pondering over life in curved space. Stationed on the Russian front of World War I, he read Einstein's

newly published GR and lost no time in deriving the first exact solution to Einstein's Field Equations (EFE) that he promptly mailed to Einstein. The solution described the gravitational field in the exterior of a spherical, non-spinning star, and it was followed by an exact computation of the spacetime curvature inside the star several weeks later. The brilliant man died shortly after, but his legacy was meant to live on forever. What we today acknowledge as the event horizon of a black hole at the Schwarzschild radius, went by many names back then: Schwarzschild's singularity or sphere, magic circle, frontier, barrier, catastrophic sphere, etc. The reason was the bizarre prediction of Schwarzschild's geometry that time on the surface of a star with the circumference of the magic circle would be frozen (infinitely dilated), and light would be shifted in wavelength an infinite amount, that is, all the way out of existence. Yet no one at the time, not even Schwarzschild himself, believed that gravitational collapse could squeeze a star so much; the star's pressure would step in and save the day, they thought. Opinion setters like Einstein and Eddington were quick to dismiss such "frozen stars" as mathematical curiosities.

The majority of theoretical physicists and astrophysicists could not come to terms with the idea that black holes are the only stellar graveyard for sufficiently massive, imploding stars. Their strong resistance is reflected by the fact that it took too many decades for the scientific community to accept that black holes inhabit our Universe. Despite accumulating theoretical evidence, from Chandrasekhar's maximum mass limit for white dwarves [19] to the Tolman–Oppenheimer–Volkoff (TOV) limit for the mass of (Zwicky's) neutron stars [20, 21] and the *Continued Gravitational Contraction* of Oppenheimer and his student Snyder [22], many physicists looked the other way: black holes were probably an artifact of the high idealization of stars, perhaps there is another fate for stars more massive than approximately 2–3 Suns in their late times, or stars simply lose a sufficient amount of their mass as they age in order to die as white dwarves or neutron stars. As the world moved into the 1960s, many naysayers started becoming advocates. The calculations of Wheeler, Harrison, and Wakano [23] for cold, dead matter strengthened the case that gravity overwhelms both the electron degeneracy pressure and the nuclear force when the star is massive enough. It was also becoming increasingly implausible that stars as massive as 20–25 Suns could shed enough mass as to end up in the neutron-star graveyard. Moreover, simulations of realistic stellar implosions, carried out by Podurets [24] and May–White [25], reinforced the Oppenheimer–Snyder viewpoint that the collapse of the heaviest of stellar objects produces black holes. For relativists, the discussion unequivocally shifted from the existence of black holes to the singularity at their origin.

The year 1964 was the beginning of a brief golden age of theoretical black-hole research that, according to Thorne [26], would end in 1975. Penrose, harvesting the power of differential geometry, topology, and causality theory, gave us his beautiful 1965 singularity theorem [27] that elevated geodesic incompleteness to the necessary and sufficient condition characterizing singular spacetimes, thereby

revolutionizing our understanding of singularities. However, one has to understand that the notion of a trapped surface and the global methods introduced by Penrose felt alien to nearly all physicists at the time. Among the skeptics, Khalatnikov and Lifshitz, still convinced that the random internal deformations of a real collapsing star would prevent the formation of a singularity, kept trying to disprove Penrose. Ironically, their efforts led them to the very conclusion they hoped to avoid, the “physical” Belinski–Khalatnikov–Lifshitz singularity [28], where the temporal and spatial oscillations of tidal forces, albeit chaotic and unpredictable, become, on average, stronger and faster as one approaches the origin. That “matter does not matter” near a singularity had now emerged from traditional equation-intensive methods as well. Penrose’s work spurred other hungry minds to use the new techniques in cosmology. In a series of papers [29–32], Hawking and Geroch demonstrated that singularities are likewise inevitable in cosmological models. Hawking and Penrose, incorporating and extending their previous results, formulated a final, general singularity theorem in their joint 1970 publication *The Singularities of Gravitational Collapse and Cosmology* [33].

The golden age was an era of bold statements, none more bold than black holes going bald. The idea behind the now-famous “hairlessness” of black holes can be traced back to 1964, when Ginzburg (as in Ginzburg–Landau theory) realized that the collapse of a sufficiently massive magnetized star leads to a black hole with no magnetic field lines sticking out of it [34]. Later, in 1965 (english version in 1966), Doroshkevich, Zel’dovich, and Novikov showed that small deviations from sphericity are damped down as the star implodes [35]: nearly spherical stars must form exactly spherical black holes after the collapse has settled down. The trio quickly conjectured that, no matter the star’s shape, the black hole would always be spherical. However, Ginzburg and Zel’dovich’s team arrived at the above conclusions by idealizing implosion as a sequence of static stars, i.e., by studying “snapshots” of a collapsing star. It was Werner Israel who, in 1967, proved a part of the conjecture: the geometry of every empty, static spacetime, featuring a regular event horizon with a spherical topology, is that of Schwarzschild. He later generalized his result to the charged case [37], proving that the Reissner–Nordström (RN) solution is the only static and asymptotically flat electrovac field with a regular event horizon.¹

Meanwhile, Roy Kerr’s 1963 solution [41], initially thought to describe the spacetime outside an idealized spinning star, was already causing a sensation as the first of its kind. Its maximal analytic extension was constructed by Boyer and Lindquist [42] in 1966, while Carter studied its global structure in 1968 [43]. It became clear that the Kerr metric describes a spinning black hole! In fact, Carter [44] and Robinson [45] showed that the Kerr black hole is *unique*, i.e., the geometry

¹ The RN solution was discovered by Reissner and Nordström independently in 1916 [38] and 1918 [39], respectively, but its significance was highlighted by Brill and Graves who analytically continued the solution across the “pseudo-singularities” (event horizon and inner Cauchy horizon) [40].

of every empty, stationary, and axisymmetric spacetime, featuring a regular event horizon with spherical topology, is that of Kerr with $a^2 \leq M^2$. In 1972, Hawking proved that every stationary black hole with a spherical horizon topology is either axisymmetric or static [46], while Bose and Wang generalized Carter’s result to the (electrically) charged Kerr black hole of Newman et al. [48] in 1974 [47]. The above individual theorems comprise, in essence, the so-called “no-hair theorem”, which states that the external gravitational and electromagnetic fields of a stationary black hole in vacuum are completely determined (upon the completion of implosion) only by the *mass*, the *electric charge*, and the *angular momentum* of the star that collapsed. Everything else is converted into gravitational (or electromagnetic) radiation that carries the “hair” completely away. Equivalently, “every radiatable multipole gets radiated away completely in the late stages of collapse”—as Price had succinctly put it [49, 50].

By 1975 we knew that black holes pulsate [51], radiate [52, 53], and ultimately evaporate [53]. We also had a set of laws, the laws of black-hole mechanics [54], that turned out to be the laws of thermodynamics for black holes [53, 107, 108]. Advancements in X-ray,² radio, and optical astronomy allowed for more accurate observations, and by 1973 the evidence strongly suggested that Cygnus X-1 is a black hole [55, 56]; once mere mathematical curiosities, black holes had moved from the chalkboard to the observable Universe! After decades of debate, the existence of black holes is now widely accepted, supported by event-horizon-scale images of the supermassive black holes in M87 and at the centre of the Milky Way obtained by the Event Horizon Telescope collaboration [57, 58].

1.2. A brief history of solution-generating techniques in GR

The generation of new exact solutions of EFE from known ones emerged from the observation that the Killing-reduced field equations enjoy non-trivial symmetries. Ehlers [113] was presumably the first to discover a notable symmetry of the stationary vacuum EFE, a kind of gravitational duality transformation. Harrison [114] found another non-trivial symmetry of the stationary Einstein–Maxwell system, which mixes gravitational and electromagnetic degrees of freedom. Neugebauer and Kramer [118] showed that the Ehlers transformation is part of a non-linear realization of $SO(2,1)$, although the group itself is not mentioned in their work. In a subsequent publication [117], they reported additional solution-generating symmetry transformations in Einstein–Maxwell theory, independently rediscovering Harrison’s transformation. Later, Geroch [124] developed a general method for constructing a one-parameter family of new spacetimes from known vacuum solutions in Einstein’s theory admitting one Killing Vector (KV). He was the first to formally demonstrate that the KV reduction of EFE in vacuum yields a system of equations covariant under global $SL(2, \mathbb{R})$ transformations. Moreover,

² Recall that NASA’s *Uhuru* satellite was launched and deployed in 1970. Its mission was to study X-ray sources.

when two commuting KVs are present, he showed that the above (internal) covariance group is also the group of linear transformations acting in the 2D vector space of commuting KVs, and that it can be extended to a non-Abelian infinite-dimensional group, now called the Geroch group [125].

Kinnersley [119], using the complex-potential formulation originally introduced by Ernst [115, 116] in the case of stationary axisymmetric vacuum fields and later extended by Israel and Wilson [129] to the case of stationary electrovacuum fields,³ showed that the Einstein–Maxwell field equations possess a symmetry group isomorphic to $SU(2,1)$. Building on Geroch’s work, he introduced, in the case of stationary and axisymmetric electrovacuum fields, an infinite-dimensional group of linear transformations acting on the space of solutions, now called the Kinnersley group, which contained the Geroch group as a subgroup [120, 121]. In collaboration with Chitre, they produced a series of papers [121–123], in which they developed a formalism for constructing an infinite hierarchy of potentials. They showed that some of the non-linear symmetry transformations can be exponentiated to finite values of their parameters and, therefore, used as a means of generating new solutions. They further gave a series of transformations, which generate new stationary axisymmetric vacuum solutions from known ones, with the important property of preserving asymptotic flatness.

A complementary approach was introduced by Neugebauer using Bäcklund transformations [130]. He showed that the stationary axisymmetric vacuum field equations admit two Bäcklund transformation groups. By repeatedly using them, one can eventually obtain solutions with an increasing number of constants. For a special choice of parameters, the infinite-parameter group previously obtained by Geroch and by Kinnersley and Chitre by a rather complicated method, is contained in the free product of the two transformation groups.

A major refinement of the Kinnersley–Chitre formalism was achieved by Hauser and Ernst, who discovered a linear integral equation of the Cauchy type whose solution yields directly the generating function of the Kinnersley–Chitre hierarchy of potentials associated with the transformed spacetime metric [126]. The ingenious duo pushed things further by introducing a homogeneous Hilbert problem for studying the Kinnersley–Chitre transformations of all stationary axisymmetric vacuum spacetimes. They showed, independently of the original Kinnersley–Chitre formalism, that the Hilbert problem maps the potential of one stationary axisymmetric vacuum spacetime into that of another. In a companion work, the homogeneous Hilbert problem was generalized to handle transformations of electrovac spacetimes [128]. The neat, modern interpretation of the reduced Einstein and Einstein–Maxwell equations as field equations derived from effective Lagrangians describing non-linear σ -models coupled to 3D gravity, is probably due to Maison [131] and Mazur [132].

Taken together, these developments transformed solution generation in GR

³ See also Ref. [114].

from a collection of isolated transformation techniques into a coherent framework based on hidden symmetries, group actions, and complex-analytic structures. By the early 1980s, stationary and axisymmetric sectors of GR were firmly established as integrable systems, with solution-generating techniques providing both conceptual insight and practical tools for constructing physically relevant spacetimes.

1.3. Motivation

Exact solutions of EFE have played a central role in shaping our understanding of gravitational physics. Many of the most fundamental concepts in GR, including black holes, event horizons, and gravitational radiation, were first revealed through the study of highly symmetric exact solutions. Despite this long history, the space of exact solutions of EFE remains largely unexplored, particularly in the presence of rotation, non-trivial matter content, or non-standard asymptotic structure, although the tools for exploring it are known. Explicitly constructing new solutions is therefore essential for assessing the full predictive scope of GR and for identifying which features of black holes and other spacetimes are universal and which depend sensitively on strong underlying assumptions.

Within the strict framework of Einstein gravity (EG) and classical GR, there are two complementary and well-controlled avenues for constructing new exact solutions. The first is to modify the matter sector. Physically motivated matter models can lead to qualitatively new spacetime geometries, if the backreaction of matter is taken into account. In particular, generalized theories of electromagnetism provide a natural testing ground, as they constitute reasonable matter models allowing for richer interactions with gravity than the standard Maxwell theory. The second avenue relies on exploiting the hidden symmetries of EFE or the Einstein–Maxwell equations. The use of the solution-generating techniques described in the previous section makes it possible to map known solutions to new ones, often revealing entire families of spacetimes that would be difficult to obtain by direct integration of the field equations. It should readily be clear that such techniques are especially powerful and can be used to generate more complicated solutions from simpler seeds. The present thesis follows both of these approaches.

First, we introduce and study a promising 2-field generalization of Maxwell’s theory coupled to Lovelock [71] gravity. It is known that the consideration of Non-Linear Electrodynamics (NLE) leads to black-hole solutions with richer geometric and thermodynamic properties [59–66]. We are particularly interested in the coupling of the new model to EG in arbitrary dimensions. The matter action we propose is also understood as a generalization of quasitopological electromagnetism [73] in arbitrary spacetime dimensions. In fact, it is constructed in such a way that it reduces to the theory of Liu et al. in four dimensions (modulo field redefinitions), by which we were inspired in the first place. However, the couplings

between the Faraday tensor and the field strength of the additional form field give rise to richer configurations than those presented in the single-field model [73]. In particular, our model is valid in any number of spacetime dimensions, whereas the single-field model requires that the dimension is an even number. The black holes that we construct are reminiscent of Lovelock–Born–Infeld black holes (see [81] and references therein and thereof). They indeed exhibit rich geometric and thermodynamic properties, with phase diagrams similar to those found in everyday thermodynamic systems such as fluids (phase transitions, triple points, etc.). Interestingly, the phase structure can be quite unusual for certain parameter values (for example, disconnected phase boundaries). The equation of state for the new black holes has the form of a closed-form modification of the law of ideal gases. Another empirically sound motivation for our generalized quasitopological model is the historical association of NLE with *regular* black holes [67, 68, 97]. Quite so; one of the most significant results of this part of the work is the explicit construction of regular black holes in a strongly-coupled regime. Unlike standard black-hole solutions, which typically develop curvature singularities at some value of the radial coordinate, the curvature of regular black holes remains finite and well-defined throughout spacetime. Importantly, the regular black holes found here are *geodesically complete* spacetimes [27, 102]. These solutions therefore serve as controlled and physically consistent examples showing how the singular behaviour can be avoided within classical gravity coupled to suitably structured matter.

In the second and third parts of this thesis, we use Kinnersley’s solution generating technique (mentioned in Sec. 1.2) to construct black holes with non-standard asymptotics in GR. As already mentioned in Sec. 1.2, this technique has been well-known for many decades, and it has systematically been used to generate new asymptotically flat solutions with a plethora of parameters. Nevertheless, the reduction of the Einstein–Maxwell equations along a *spacelike* KV has not been extensively explored. In particular, defining the auxiliary potentials by using the azimuthal KV in stationary axisymmetric spacetimes, leads to new solutions which are asymptotically *Levi-Civita* [106, 181] or asymptotic to a rotating generalization of the latter. Due to their unconventional asymptotic geometry, such spacetimes are usually neglected as physically irrelevant. Nevertheless, their heuristic value remains significant and should not be overlooked. Moreover, the absence of asymptotic flatness does not, by itself, render a spacetime physically irrelevant; rather, it signals that the solution does not describe an isolated system with finite total mass. Solutions obtained by using an azimuthal KV are instead naturally associated with configurations in the presence of non-compact sources or background fields. Well-known and physically meaningful examples include Melvin’s *magnetic universe* [141, 142], which represents a self-gravitating magnetic flux tube and is widely used to model strong-field backgrounds, as well as cylindrically symmetric solutions such as the *van Stockum* spacetime [164] and the Levi-Civita (LC) cylinder [69, 214] (see [70] and references therein for a

physical motivation of cylindrical systems in GR), which describe infinite or effectively long matter distributions. In all these cases, asymptotic flatness is neither expected nor appropriate. The physical relevance of such geometries is therefore local and quasi-local: provided curvature invariants remain regular and the matter content satisfies reasonable energy conditions, these solutions offer consistent local descriptions or *near-zone approximations*, which may be later matched to a meaningful “exterior” region. Such geometries, which do not themselves describe black holes, can nevertheless serve as backgrounds in which black holes may be immersed. The “magnetized” black holes constructed by Ernst [190] provide a particularly clear early example. Here, we register the electromagnetic swirling black hole attached publication II. Still, such black-hole metrics contain the continuous parameter(s) introduced by the transformation(s). While these parameters often admit a clear physical interpretation, it is natural to ask whether one can instead construct black holes with non-standard asymptotics that are characterised solely by the usual physical quantities, such as mass and angular momentum. This motivates our construction of the Kerr-LC black hole by using a discrete symmetry transformation known as *inversion*. Of course, one needs to be aware that a canonical definition of asymptotic charges cannot exist in spacetimes with LC-like asymptotics. Some of these spacetimes do not even admit an asymptotically everywhere timelike KV in the first place! Despite the absence of canonical asymptotic charges, the parameters appearing in the Kerr-LC family admit a natural physical interpretation through limiting procedures: setting the “angular-momentum” to zero yields the static Schwarzschild-LC [207] spacetime, while removing both “mass” and “angular momentum” reduces the geometry to the LC background. Accordingly, these parameters play roles analogous to mass and angular momentum in the Kerr solution.

1.4. Aim of the thesis

The aim of this thesis is to study and construct exact solutions of EFE, both in (electro)vacuum and in the presence of non-trivial matter sources, with particular emphasis on their geometric structure. The work is motivated by the fact that exact solutions remain one of the most powerful tools for understanding the non-linear dynamics of EG: 1- they provide insight into *strong-field* gravity in a geometric sense, 2- they improve our physical intuition of otherwise abstract concepts, and 3- they function as theoretical laboratories for probing analogies (black-hole mechanics-thermodynamics, gauge/gravity and fluid/gravity dualities, etc.) and testing predictions (black-hole shadows, photon rings, redshift, etc.).

The thesis is organized around two complementary research directions, each addressing a different mechanism through which EG admits novel solutions.

- The first direction concerns EG coupled to generalized electromagnetic matter, in particular a 2-field so-called quasitopological extension of classical electromagnetism. The aim here is to investigate how these modifications to the electromagnetic sector—which preserve key properties such

as gauge invariance and second-order field equations—affect the spacetime geometry. Within this framework, the thesis seeks to construct exact black-hole solutions supported by a Maxwell field and a higher-form field, and to analyse how the interaction of the latter two influences horizon geometry, thermodynamic properties, and regularity. A central goal of this part is to assess whether such theories admit physically consistent and well-behaved black holes within EG.

- The second direction focuses on vacuum and electrovac solutions generated by exploiting the hidden symmetries of EFE, made manifest in the Ernst description of GR [115, 116]. New spacetimes generated from known ones via Kinnersley’s technique using a spacelike KV, have rather “exotic” properties. For starters, they are not asymptotically flat, and it makes sense to ask what happens if we embed an asymptotically flat black hole in them. One of our goals is to investigate how the transformations affect the Petrov type, causal structure, horizon and ergoregion properties, and the removal or introduction of curvature singularities, conical defects, and Closed Timelike Curves (CTCs). The black holes we present are rotating, but the rotation here is completely different from the kind of rotation we know from Kerr black holes. We look into that. As mentioned in Sec. 1.3, there is no KV everywhere timelike at spatial infinity. This implies that energy, Frame-Dragging Angular Velocity (FNAV), ergoregions, and any other quantity measured with respect to a unique normalizable timelike KV at infinity, are now observer-dependent. Is there perhaps a new selection mechanism to pick a unique “observer at rest” somewhere at infinity among the many equally “good” candidates? There are many physically significant questions to be answered, and we answer some of them in our work.

1.5. Structure of the thesis

In the current chapter we have already sketched a brief history of black holes in Sec. 1.1, focusing on the controversy they provoked: the gradual shift of the scientific community from scepticism to acceptance, and eventually to elevating black holes to a central research object in modern theoretical and observational physics. We have also briefly reviewed the history of solution-generating techniques in Sec. 1.2 and their crucial role in exploring the space of solutions in GR. In Sec. 1.3, we have seen that, despite this progress, many questions remain open, providing the motivation for our work. The aims of this thesis were summarised in Sec. 1.4. In Chap. 2, we review the attached publication I, emphasizing its main results and the methods employed in their derivation. In Chap. 3, we give a pedagogical derivation of the Ernst equations and review Kinnersley’s solution-generating method. We present and discuss the main results of attached publication II. Finally, in Chap. 4, we review the attached publication III, focusing on the geometric properties of the Kerr–Levi-Civita black hole. We end the overview article by summarizing and concluding in Chap. 5.

2. QUASITOPOLOGICAL ELECTROMAGNETISM AND BLACK HOLES

2.1. Introduction

One can think of the Einstein–Hilbert action as the dimensional continuation of the two-dimensional Euler characteristic. Adopting this approach is useful when attempting to generalize field theories. For example, one can define higher-curvature theories of gravity in D spacetime dimensions by dimensionally continuing the $2k$ -dimensional Chern–Weil topological invariants, where $k \in \{\mathbb{Z}^+ \mid k \leq \lfloor D/2 \rfloor\}$, with the map $\lfloor \cdot \rfloor$ being the floor function. The resulting theory is known as Lovelock’s theory of gravity [71], and it is the best-understood model with higher-curvature couplings. For this reason, it has also been utilized to investigate the effects of higher-curvature terms in the framework of the AdS/CFT correspondence [72].

A similar idea has been explored in the case of Abelian gauge theories in [73]. The authors of [73] introduced the theory of quasitopological electromagnetism, an extension of the Einstein–Maxwell theory via the inclusion of new terms in the Lagrangian, terms related to topological invariants in the following manner: they are based on the $2k$ -forms

$$\mathcal{F}_{[2k]} = \bigwedge_{i=1}^k F_{[2]}, \quad (2.1)$$

where $F_{[2]} = dA_{[1]}$ is the the field strength two-form of the electromagnetic potential $A_{[1]}$. These field-strength polynomials are metric-independent and can be cast into total derivatives, much like the Pontryagin classes. In $2k$ spacetime dimensions, the integral of $\mathcal{F}_{[2k]}$ is topological.

For the forms $\mathcal{F}_{[2k]}$ to affect the dynamics of the classical theory, a metric $g = (g_{\mu\nu})$ must be introduced. Its existence allows us to define the Hodge star map and consider the D -forms $\mathcal{F}_{[2k]} \wedge \star \mathcal{F}_{[2k]}$.¹ Without further assumptions, the contribution of these top forms to the field equations is non-trivial. The theory containing such terms has been dubbed quasitopological electromagnetism for two reasons. First, the building blocks $\mathcal{F}_{[2k]}$ are topological invariants, as previously discussed. Second, the solution spectra in quasitopological electromagnetism and Maxwell’s theory coincide for static and solely electric (or magnetic) configurations, a fact no longer true when dealing with dyons.

In attached publication I, we proposed a natural generalization of quasitopological electromagnetism via the introduction of a form $B_{[p-1]}$, in addition to the Abelian gauge connection $A_{[1]}$. The field strength of $B_{[p-1]}$ is denoted by $H_p = dB_{[p-1]}$. The physical interpretation of the new field depends heavily on the context. For instance, one could identify it with fields appearing in string theory, such as the Kalb–Ramond two-form, the Ramond–Ramond forms in Type II

¹ For example, the term $-\mathcal{F}_{[2]} \wedge \star \mathcal{F}_{[2]}/2$ is the kinetic term in Maxwell’s theory.

D -form
$\mathcal{F}_{[d]} \wedge \star \mathcal{F}_{[d]}$
$\mathcal{H}_{[d]} \wedge \star \mathcal{H}_{[d]}$
$\mathcal{F}\mathcal{H}_{[d]} \wedge \star \mathcal{F}\mathcal{H}_{[d]}$
$\mathcal{F}_{[d]} \wedge \star \mathcal{H}_{[d]}$
$\mathcal{F}_{[d]} \wedge \star \mathcal{F}\mathcal{H}_{[d]}$
$\mathcal{H}_{[d]} \wedge \star \mathcal{F}\mathcal{H}_{[d]}$

Table 1. List of all D -forms one can construct using the forms $\mathcal{F}_{[2k]}$, $\mathcal{H}_{[pm]}$, $\mathcal{F}\mathcal{H}_{[2k+pm]}$, and the Hodge star map. The only restriction is that the degree d is equal to or less than the dimension of spacetime D .

theories, or the three-form featured in eleven-dimensional supergravity. An appealing field ansatz is to take $H_{[p]}$ to be proportional to the volume form on a codimension-two manifold of constant scalar curvature and spherical, flat, or hyperbolic topology. As we explicitly showed, couplings between $F_{[2]}$ and $H_{[p]}$ give rise to richer configurations than those presented in the single-field model [73]. In particular, our two-field model is valid in any number of spacetime dimensions, whereas the single-field model requires that D is an even number.

2.2. Quasitopological field theory and exact solutions

Generalizing the idea laid out in [73], we considered invariants similar to $\mathcal{F}_{[2k]}$. To be precise, let

$$\mathcal{F}_{[2k]}, \quad \mathcal{H}_{[pm]} = \bigwedge_{i=1}^m H_{[p]}, \quad \mathcal{F}\mathcal{H}_{[2k+pm]} = \mathcal{F}_{[2k]} \wedge \mathcal{H}_{[pm]} \quad (2.2)$$

be our building blocks, where $\mathcal{H}_{[pm]}$ is a pm -form with

$$m \in \{ \mathbb{Z}^+ \mid m \leq \lfloor D/p \rfloor \}$$

and $p \bmod 2 = 0$ for all $m > 1$.² Of these, we can construct all kinds of D -forms that you see in Table 2.2.

All of them would contribute to the field equations and should be included in the action. However, we are interested in configurations of the form

$$F_{[2]} \propto dx^0 \wedge dx^1, \quad H_{[D-2]} \propto \star(dx^0 \wedge dx^1), \quad (2.3)$$

where $F_{[2]}$ has solely electric components and $H_{[D-2]}$ solely “magnetic” ones. Henceforth, let $p = D - 2$ for convenience. With this in mind, it is easy to see that the only non-trivial building blocks are $\mathcal{F}_{[2]} = F_{[2]}$, $\mathcal{H}_{[n]} = H_{[n]}$, and $\mathcal{F}\mathcal{H}_{[D]} =$

² If $p \bmod 2 = 1$, then $\mathcal{H}_{[pm]} = -\mathcal{H}_{[pm]} = 0$ since the square of any odd number is an odd number itself.

$F_{[2]} \wedge H_{[n]}$. It follows that the only non-trivial D -forms contributing to the equations of motion are $F_{[2]} \wedge \star F_{[2]}$, $H_{[n]} \wedge \star H_{[n]}$, and the interaction term $\mathcal{F}\mathcal{H}_{[D]} \star \mathcal{F}\mathcal{H}_{[D]}$. For this reason, we ended up with the D -dimensional action

$$I_D[g, A_{[1]}, B_{[p-1]}] = \int \sqrt{-|g|} d^D x \left(\mathcal{L}_{\text{Lov}} - \frac{1}{4} F^2 - \frac{1}{2p!} H^2 - \alpha \mathcal{L}_{\text{int}} \right), \quad (2.4)$$

where $F^2 = F_{\mu\nu} F^{\mu\nu}$, $H^2 = H_{\mu_1 \dots \mu_n} H^{\mu_1 \dots \mu_n}$,

$$\mathcal{L}_{\text{int}} = \delta_{\nu_1 \dots \nu_D}^{\mu_1 \dots \mu_D} F_{\mu_1 \mu_2} H_{\mu_3 \dots \mu_D} F^{\nu_1 \nu_2} H^{\nu_3 \dots \nu_D} \quad (2.5)$$

is the interaction Lagrangian density, and

$$\mathcal{L}_{\text{Lov}} = \sum_{i=0}^{\lfloor D/2 \rfloor} \frac{a_i}{2^i} \delta_{\nu_1 \dots \nu_{2i}}^{\mu_1 \dots \mu_{2i}} \prod_{j=1}^i R^{\nu_{2j-1} \nu_{2j}}{}_{\mu_{2j-1} \mu_{2j}} \quad (2.6)$$

is the well-known Lanczos–Lovelock Lagrangian density.

We then made a static metric ansatz with the constant-time and unit-radius slices being codimension-2 maximally symmetric model spaces Σ_γ of constant curvature, the latter being positive when $\gamma = 1$, zero when $\gamma = 0$, and negative when $\gamma = -1$. We also made a purely electric ansatz for $F_{[2]}$, and we further assumed that $H_{[p]}$ is proportional to the volume form on Σ_γ .

First, we solved the equations of motion for the potential $A_{[1]}$. By observing that these field equations acquire the form of a generalized Gauss’ law and by integrating the canonical momentum conjugate to $A_{[1]}$ over any closed two-surface of constant time at spatial infinity, one is able to recognize the integration constant in the electric field as the electric charge density. The electric field is that of a point charge at large distances but vanishes at the radial origin. We also found that the equations of motion for $B_{[p-1]}$ are identically satisfied. By integrating $H_{[p]}$ over the aforementioned two-surfaces, one gets a topological charge in the case of a compact Σ_γ . In any case, the constant appearing in $H_{[p]}$ is recognized as the p -form flux density. By integrating the electric field, we find an electric potential, which exhibits a screened Coulomb behaviour; at large distances, it behaves as a Coulomb potential but becomes finite at short distances, a feature certainly appealing.

Next, we attacked the metric field equations. We showed that the matter stress-tensor satisfies the weak and null energy conditions. The equations of motion for the metric are of the “Wheeler” type and may, therefore, be integrated à la Wheeler [74]. A closed-form solution exists only if we truncate the Lovelock series to first order, i.e., if we consider our gravitational Lagrangian to be that of Einstein gravity with a cosmological constant in D spacetime dimensions. The square of the lapse function, $G(r)$, features a hypergeometric function, which also appears in the electric potential as the screening function. Its presence is reminiscent of what happens in the Lovelock–Born–Infeld theory (see [81] and references therein and thereof).

When $q_m \rightarrow 0$, the metric reduces to that of a Reissner–Nordström-(A)dS black hole. The same holds in the $\alpha \rightarrow 0$ limit. The lapse function can, in theory, have multiple positive roots that define the locations of Killing horizons. Since curvature invariants blow up as the radial coordinate approaches the origin, the solution describes static, charged black holes (for a certain range of values of the parameters and the coupling constant). Near the Killing horizons, the metric decomposes into a direct product of a two-dimensional Rindler space and Σ_γ . If the positive roots do not coincide, each horizon has a non-trivial temperature proportional to its surface gravity and a non-trivial entropy proportional to its area. In the case of coincident roots, the near-horizon geometry factorizes into a direct product of a two-dimensional Anti-de Sitter (AdS) space and Σ_γ . The metric is asymptotically (anti-)de Sitter, and we can appreciate that the contribution of the gauge fields to the metric decays asymptotically at least twice as fast as the mass term. Consequently, we can simply read off the Arnowitt–Deser–Misner (ADM) mass M of the black hole from the asymptotic expansion of g_{tt} (see [87, 88] for asymptotically AdS geometries).

The four-dimensional solution coincides with the one reported in [73] by construction. Let us elaborate. Consider the matter action

$$\tilde{I}_{\text{mat}}[\mathbf{g}, \tilde{A}_{[1]}] = - \int \left[\frac{1}{2} \tilde{F}_{[2]} \wedge \star \tilde{F}_{[2]} + 4\alpha \tilde{F}_{[2]} \wedge \tilde{F}_{[2]} \star (\tilde{F}_{[2]} \wedge \tilde{F}_{[2]}) \right], \quad (2.7)$$

where one makes a dyonic ansatz for $\tilde{A}_{[1]}$. It follows that, in terms of our ansätze for $F_{[2]}$ and $H_{[2]}$, $\tilde{F}_{[2]} = F_{[2]} + H_{[2]}$, and the reduced matter action becomes

$$\tilde{I}_{\text{mat}} = - \int \left[\frac{1}{2} F_{[2]} \wedge \star F_{[2]} + \frac{1}{2} H_{[2]} \wedge \star H_{[2]} + 16\alpha \mathcal{F}\mathcal{H}_{[4]} \star \mathcal{F}\mathcal{H}_{[4]} \right], \quad (2.8)$$

which is exactly the matter action we proposed in four spacetime dimensions. Requiring that the action (2.7) is stationary under variations of $\tilde{A}_{[1]}$ gives the gauge-field equations

$$d\star \tilde{F}_{[2]} = -16\alpha \tilde{F}_{[2]} \wedge d\star (\tilde{F}_{[2]} \wedge \tilde{F}_{[2]}), \quad (2.9)$$

and since $\tilde{F}_{[2]} \wedge \tilde{F}_{[2]} = 2F_{[2]} \wedge H_{[2]}$, it follows that the above equation amounts to our two independent equations for $A_{[1]}$ and $B_{[1]}$ when $p = 2$. The three-dimensional solution cannot be obtained by directly evaluating the D -dimensional solution in three spacetime dimensions, because the latter is singular there. Instead, one sets $D = 3$ at the level of the field equations and only then integrates those to get a solution that describes a deformed Bañados–Teitelboim–Zanelli (BTZ) [93] black hole. Note that the dyonic BTZ spacetime is recovered in the $\alpha \rightarrow 0$ limit.

One of the most striking results in our work is the construction of *regular* (non-singular) black-hole solutions. In conventional Einstein–Maxwell theory, curvature singularities at the origin are unavoidable for charged black holes. By contrast, the interaction between the two field strengths considered here allows for a consistent modification of the short-distance geometry. Recall that the energy

density is a monotonically decreasing function of r , falling off sufficiently fast at asymptotic infinity to provide a finite ADM mass. On the other hand, it blows up at the radial origin due to the contribution of the kinetic term for $B_{[n-1]}$. To have a finite energy density as $r \rightarrow 0$, we must be able to disregard the aforementioned kinetic term. We can do that only in the strongly coupled regime, formally accessed by taking the limit $\alpha \rightarrow \infty$ whilst keeping $q_m \sqrt{\alpha}$ finite.

We showed that if we impose a specific relation between the ADM mass, the charges, and the coupling constant,³ hyper-spherical black holes exhibit a de Sitter core; the spacetime geometry looks like the static patch of de Sitter space near the origin, with the curvature radius being a function of the charges and the coupling constant. This guarantees that the Riemannian curvature remains finite as we approach $r = 0$ and that geodesics are complete [102]. Depending on the values of the various parameters, the resulting geometries may describe regular non-extremal and extremal black holes, as well as horizonless spacetimes with a regular origin (backgrounds).

2.3. Black-hole thermodynamics

To study the thermodynamics of these novel black holes, we focused on locally asymptotically AdS solutions. Here, we shall give some tips and tricks for the calculation of the various thermodynamic quantities. We shall do so in units where $8\pi G_D = 1$.

First, to find the temperature one performs a Wick rotation $t \rightarrow -i\tau$, where we identify τ with $\tau + \beta$. Near the horizon at $r = r_+$, the Euclidean metric assumes the form

$$ds^2 \sim \rho^2 d\tilde{\tau}^2 + d\rho^2 + r_+^2 d\Sigma_\gamma^2, \quad (2.10)$$

where $\tilde{\tau}(\tau) = G'(r_+)\tau/2$ and $\rho(r) = 2\sqrt{(r-r_+)/G'(r_+)}$. The first two terms constitute the metric of a plane in polar coordinates, with Euclidean time $\tilde{\tau}$ playing the role of the angular coordinate. To avoid the conical singularity at the origin, $\tilde{\tau}$ must be 2π -periodic, which forces

$$\beta = \frac{4\pi}{G'(r_+)}. \quad (2.11)$$

In the Euclidean approach, we interpret the period of Euclidean time as the inverse of the Hawking temperature T . For the gauge field in this Euclidean space to be regular at the origin of the ρ coordinate (the horizon), it must hold that it behaves like $\sim \rho^2$ there, because

$$d\tilde{\tau} = \frac{x dy - y dx}{\rho^2} \quad (2.12)$$

³ Ultimately, this is a relation between the mass and the non-vanishing energy density of the matter fields at the origin [103].

in Cartesian coordinates $(x, y) = (\rho \cos \tilde{\tau}, \rho \sin \tilde{\tau})$. Taylor-expanding the potential about $r = r_+$, we have that

$$a(\rho) = a(r_+) + \frac{1}{2} G'(r_+) a'(r_+) \rho^2 + \mathcal{O}(\rho^4), \quad (2.13)$$

which implies that we must fix the constant $\lim_{r \rightarrow \infty} a$ such that $a(r_+) = 0$. The electric potential difference between horizon and infinity is then $\Phi_e = a(\infty) - a(r_+)$, and it is conjugate to the total charge Q_e (or the charge density q_e if Σ_γ is non-compact).

Calculating the thermodynamic potential conjugate to the topological charge Q_m (or the p -form flux density q_m if Σ_γ is non-compact) requires a bit more (trickier) work. One needs to treat $H_{[p]}$ as the independent field and force $H_{[p]} = dB_{[p-1]}$ by including a Lagrange multiplier $\tilde{B}_{[1]}$ in the quasitopological action, i.e., by adding an integral

$$\int \tilde{B}_{[1]} \wedge dH_{[p]}. \quad (2.14)$$

Variation of that new action with respect to $H_{[p]}$ gives $\tilde{H}_{[2]} = d\tilde{B}_{[1]}$, where

$$\tilde{H}_{[2]} = -\star H_{[p]} - 8\alpha(p!)^2 F_{[2]} \star \mathcal{F} \mathcal{H}_{[D]}. \quad (2.15)$$

Variation with respect to $\tilde{B}_{[1]}$ yields $dH_{[p]} = 0$, ergo $H_{[p]} = dB_{[p-1]}$. Since

$$\tilde{H}_{[2]} = \left(\frac{q_m}{r^p} - \frac{8\alpha(p!)^2 q_m^2 (a')^2}{q_m r^p} \right) dr \wedge dt, \quad (2.16)$$

consistency of the field equations for $H_{[p]}$ requires that any symmetry-compatible $\tilde{B}_{[1]}$ must be of the form $b dt$, where $b(r)$ satisfies the differential equation

$$b' = \frac{q_m}{r^p} - \frac{8\alpha(p!)^2 q_m^2 (a')^2}{q_m r^p}. \quad (2.17)$$

The solution to the above is the potential

$$b = b_\infty - \frac{q_m}{(n-1)r^{p-1}} - \frac{rq_e a'}{2pq_m} + \frac{(p-1)q_e \Phi_e}{2pq_m}. \quad (2.18)$$

Regularity of $\tilde{B}_{[1]}$ (in Euclidean space) at the horizon requires that we fix b_∞ such that $b(r_+) = 0$. Consequently, the potential difference between horizon and infinity is $\Phi_m = b(\infty) - b(r_+)$.

Finally, the Wald entropy formula [82] gives the Bekenstein–Hawking entropy density [83, 84]

$$s = 2\pi r_+^p = S/\sigma_\gamma, \quad (2.19)$$

in units where $8\pi G_D = 1$. This is expected because the gravitational sector of the action is just the Einstein–Hilbert functional. The above quantities suffice to

verify the *first law* of black-hole thermodynamics, where M is interpreted as the internal energy. Hence, the Helmholtz free energy is $F = M - TS$.

Although not included in the original publication, we mention here that the thermodynamical framework can be extended by treating the coupling constant and the cosmological constant as thermodynamic variables. This is the so-called extended phase space [88–90]. The quantity conjugate to α is given by

$$\mathcal{A} = -\frac{4(p!)^2}{\beta} \int \mathcal{FH}_{[D]} \star \mathcal{FH}_{[D]} = -4(p!)^2 q_m^2 \sigma_\gamma \int_{r_+}^{\infty} \frac{(a')^2}{r^p} dr, \quad (2.20)$$

which evaluates to

$$\mathcal{A} = \frac{q_e}{4p\alpha} [r_+ a'(r_+) - (p-1)\Phi_e] \sigma_\gamma. \quad (2.21)$$

If we also define the thermodynamic pressure $P = -\Lambda$, its conjugate quantity is known as the thermodynamic volume V . It is the coefficient of δP in the on-shell variation of the renormalized Euclidean action, namely

$$V = -\sigma_\gamma \lim_{\varepsilon \rightarrow \infty} \left(\int_{r_+}^{\varepsilon} dr \sqrt{|g|} - \int_0^{\varepsilon} dr \sqrt{|g|} \right) = \frac{r_+^{p+1}}{p+1} \sigma_\gamma. \quad (2.22)$$

Thus, allowing both Λ and α to vary, an extended first law is obtained by taking the on-shell variation of the renormalized Euclidean action, with the mass interpreted as enthalpy rather than internal energy. It reads

$$\delta M = T \delta S + \Phi_e \delta Q_e + \Phi_m \delta Q_m + V \delta P + \mathcal{A} \delta \alpha. \quad (2.23)$$

Note that, in the grand canonical ensemble (fixed potentials), the renormalized Euclidean action evaluates to $\beta \Omega$, where Ω is the grand potential

$$\Omega = \mathcal{G} - \Phi_e Q_e - \Phi_m Q_m, \quad (2.24)$$

with

$$\mathcal{G} = M - TS \quad (2.25)$$

being the Gibbs free energy. On the other hand, in the canonical ensemble (fixed charges), the renormalized Euclidean action evaluates to $\beta \mathcal{G}$. In that case, additional surface terms are necessary to ensure a well-posed variational problem.

Recall that $G(r_+) = 0$ can be solved for M as a function of S , P , Q_e , Q_m , and α , if one replaces r_+ by S via Eq. (2.19). Under a change $r \mapsto \lambda r$, invariance of G suggests that

$$P \mapsto \frac{P}{\lambda^2}, \quad (M, Q_e, Q_m) \mapsto \lambda^{p-1} (M, Q_e, Q_m), \quad \alpha \mapsto \lambda^2 \alpha. \quad (2.26)$$

Indeed, since $S \mapsto \lambda^p S$, it holds that

$$M(S', P', Q_e', Q_m', \alpha') = \lambda^{p-1} M(S, P, Q_e, Q_m, \alpha), \quad (2.27)$$

where, in this equation only, we use primes to denote the rescaled variables. Therefore, M is a homogeneous function of degree $p - 1$, and Euler's theorem implies the *Smarr relation*

$$M = \frac{p}{p-1}TS + \frac{2}{p-1}(\mathcal{A}\alpha - VP) + \Phi_e Q_e + \Phi_m Q_m, \quad (2.28)$$

which could not have been obtained, had Λ and α been held fixed, using scaling arguments.

As is well-known, in EG in AdS, for a given temperature above a certain threshold, there exist two-black hole solutions, a small black hole and a large black hole, and there is a minimum temperature below which no black hole exists. Here, the presence of the two gauge fields and their contact term modifies this picture: in a given temperature range, there exist up to five configurations competing to dominate the ensemble by minimizing the free energy. First-order phase transitions between small, intermediate, and large black holes can occur. For completeness we mention that in the extended phase space, it is possible to show that the Hawking temperature can be interpreted as an equation of state $P(T, V)$. Within this interpretation, black holes with a hyper-spherical Σ_γ ($\gamma = 1$) behave as non-ideal gases, with the gravitational constant playing the role of the “inter-particle attraction”, the entropy proportional to the number of microscopic degrees of freedom of the “black-hole fluid”, and with V/S proportional to the “volume per particle”. This approach to the thermodynamics results in a quite interesting phase structure, featuring triple points, re-entrant phase transitions, and even a peculiar case with disconnected phase boundaries. However, it is out of the scope of this thesis.

2.4. Technical summary

We have extended the quasitopological electromagnetic Lagrangian introduced in [73] by considering a higher-degree fundamental form $B_{[p-1]}$. This field, independent of the potential $A_{[1]}$, interacts with the field strength of the latter via its field strength $H_{[p]}$. This allows us to construct new dyonic black holes in $D \neq 4$ dimensions (even and odd), also recovering the four-dimensional solution presented in [73]. A salient property of these spacetimes is the existence of a screened Coulomb potential. Asymptotically, the new geometries behave like (A)dS–Reissner–Nordström black holes up to a certain order, since the contribution of the interaction term is suppressed by one additional inverse power of r relative to the standard electromagnetic contribution. However, the short-distance geometry is quite distinct. The three-dimensional case is special, because the matter action depends on the gauge potential and an axion field, with their interaction resulting in deformed BTZ black holes. Most importantly, we construct new regular black holes and backgrounds in *arbitrary* spacetime dimensions. The new black holes have interesting thermodynamic properties. First-order phase

transitions between small, intermediate, and large black holes are possible, and at a given temperature there can be up to five black-hole branches competing to dominate the canonical ensemble. Small black holes can have arbitrarily low temperature. They are the ones that minimize the free energy at low temperatures in the canonical ensemble, where thermal AdS is excluded.

3. THE ELECTROMAGNETIC SWIRLING UNIVERSE AND NOVEL TYPE I SPACETIMES

3.1. Introduction

Since the advent of Einstein's GR, exact solutions to EFE have played a crucial part in modern Physics [106]. They have significantly aided us in understanding many classical and semi-classical properties of gravity, thus becoming integral to our hitherto comprehension of diverse phenomena occurring at astrophysical and cosmological scales. Furthermore, exact solutions have set the stage for the theoretical exploration of many groundbreaking concepts, including, but not limited to, black hole thermodynamics [83, 107–109], the information paradox [110], and holography [111].

Nevertheless, obtaining sufficiently general classes of exact solutions in GR has always been a difficult task due to the highly non-linear character of EFE. Nowadays, the structure of vacuum and electrovacuum fields possessing a KV is well understood, and a large class of transformations generating new stationary and axially symmetric solutions from a given one is known [119–128, 130]. Ehlers [113] was presumably the first to discover a notable symmetry of the stationary vacuum EFE, a kind of gravitational duality transformation. Harrison [114] found another non-trivial symmetry of the stationary Einstein–Maxwell system, which mixes gravitational and electromagnetic degrees of freedom. Neugebauer and Kramer [118] showed that the Ehlers transformation is part of a non-linear realization of $SO(2,1)$, although the group itself is not mentioned in their work. In a subsequent publication [117], they reported additional solution generating symmetry transformations in Einstein–Maxwell theory, independently re-discovering Harrison's transformation. Later, Geroch [124] developed a general method for constructing a one-parameter family of new spacetimes from known vacuum solutions in Einstein's theory admitting one KV. He was the first to formally demonstrate that the Killing-vector reduction of EFE in vacuum yields a system of equations covariant under global $SL(2, \mathbb{R})$ transformations. Moreover, when two commuting KVs are present, he showed that the above (internal) covariance group is also the group of linear transformations acting in the 2D vector space of commuting KVs, and that it can be extended to a non-Abelian infinite-dimensional group, now called the Geroch group [125].

Kinnersley [119], using the complex-potential formulation originally introduced by Ernst [115, 116] in the case of stationary axisymmetric vacuum fields and later extended by Israel and Wilson [129] to the case of stationary electrovacuum fields,¹ showed that the Einstein–Maxwell field equations possess a symmetry group isomorphic to $SU(2,1)$. Building on Geroch's work, he introduced, in the case of stationary and axisymmetric electrovacuum fields, an infinite-dimensional

¹ See also Ref. [114].

group of linear transformations acting on the space of solutions, now called the Kinnersley group, which contained the Geroch group as a subgroup [120, 121]. The neat, modern interpretation of the reduced Einstein and Einstein–Maxwell equations as field equations derived from effective Lagrangians describing non-linear σ -models coupled to 3D gravity, is probably due to Maison [131] and Mazur [132].

Recently, there has been a renewed interest in exploring new stationary axisymmetric Einstein–Maxwell fields, generated by applying the solution-generating mechanism of Kinnersley [119] to known vacuum or electrovacuum seeds. In particular, the significant effects of Ehlers and Harrison transformations applied to accelerating spacetimes have been discussed extensively [6, 133, 135, 136]. Using Ehlers or Harrison transformations, or a combination thereof, one may map known algebraically special accelerating solutions to new algebraically general solutions. For example, some of us have successfully constructed a complete hierarchy of type I spacetimes [6], obtained via this generating technique, thereby generalizing the well-known Plebański–Demiański (PD) family, i.e., the most general family of type D Einstein–Maxwell fields.

In light of these recent developments, we wish to shed light on novel spacetimes one can generate from Minkowski spacetime by composing Ehlers and Harrison transformations. To further elaborate on our agenda, it is best if we adhere to the following terminology. We shall refer to an Ehlers Transformation (ET) as *electric* (resp., *magnetic*) when it acts on the Ernst potentials defined using the timelike KV ∂_t (resp., the spacelike KV ∂_ϕ). The same terminology applies mutatis mutandis to a Harrison Transformation (HT). To our knowledge, only the combination of electric transformations has been studied in the literature thus far. In this work, we fill the gaps, and we do so by first combining a Magnetic ET (MET) with a Magnetic HT (MHT). Afterwards, we take all possible combinations of electric and magnetic transformations.

3.2. A detailed guide to the Ernst formalism

Here, we shall present a detailed derivation of the Ernst equations. The action for Einstein–Maxwell theory (after a redefinition of the Maxwell field) is

$$S[g, A_{[1]}] = \frac{1}{2\kappa} \int \sqrt{-|g|} d^4x \left(R - F^2 \right). \quad (3.1)$$

Varying it with respect to the metric and the gauge field, we obtain the field equations

$$G_{\mu\nu} = 2 \left(F_\mu{}^\lambda F_{\nu\lambda} - \frac{1}{4} g_{\mu\nu} F^2 \right), \quad (3.2a)$$

$$0 = \partial_\nu \left(\sqrt{-|g|} F^{\nu\mu} \right), \quad (3.2b)$$

respectively. Since the RHS of Eq. (3.2a) is traceless, the metric field equations imply that $R = 0$ and thus admit the simple Ricci form

$$R_{\mu\nu} = 2 \left(F_{\mu}{}^{\lambda} F_{\nu\lambda} - \frac{1}{4} g_{\mu\nu} F^2 \right). \quad (3.3)$$

Let us now consider a chart $x = \{t, \rho, \phi, z\}$, with ϕ identified with $\phi + 2\pi$, and let us next make the Weyl–Lewis–Papapetrou (WLP) ansatz

$$ds^2 = -f(dt - \omega d\phi)^2 + \frac{1}{f} [e^{2\gamma}(d\rho^2 + dz^2) + \rho^2 d\phi^2], \quad (3.4)$$

where f , ω , and γ depend on ρ and z . Without further assumptions, the motion group is locally generated by the two commuting KVs ∂_t and ∂_ϕ . We will also make the gauge field ansatz $A_{[1]} = A_t(\rho, z)dt + A_\phi(\rho, z)d\phi$, the Lie derivative of which with respect to the two KVs vanishes. The non-trivial components of the field-strength tensor are

$$F_{t\rho} = -A'_t, \quad F_{tz} = -\dot{A}_t, \quad F_{\rho\phi} = A'_\phi, \quad F_{\phi z} = -\dot{A}_\phi, \quad (3.5)$$

where f' denotes the derivative of a function $f(\rho, z)$ with respect to ρ , while $\dot{f}$ denotes the derivative of that function with respect to z . Since the metric and the field-strength tensor do not depend on t and ϕ , it follows that the equations of motion for the gauge field read

$$(\sqrt{-|g|}F^{\rho\mu})' = -(\sqrt{-|g|}F^{z\mu}). \quad (3.6)$$

We shall define the four-vectors

$$u = \sqrt{-|g|} \frac{F^{\rho\mu}}{\rho} \partial_\mu, \quad v = \sqrt{-|g|} \frac{F^{z\mu}}{\rho} \partial_\mu, \quad (3.7)$$

whose explicit expressions are

$$u = \frac{1}{\rho^2} \left(\frac{\omega^2 f^2 - \rho^2}{f} A'_t + \omega f A'_\phi \right) \partial_t + \frac{f}{\rho^2} (A'_\phi + \omega A'_t) \partial_\phi, \quad (3.8a)$$

$$v = \frac{1}{\rho^2} \left(\frac{\omega^2 f^2 - \rho^2}{f} \dot{A}_t + \omega f \dot{A}_\phi \right) \partial_t + \frac{f}{\rho^2} (\dot{A}_\phi + \omega \dot{A}_t) \partial_\phi. \quad (3.8b)$$

It is straightforward to find that

$$\frac{1}{\rho} \partial_\nu (\sqrt{-|g|} F^{\nu\mu}) = \frac{1}{\rho} (\rho u^\mu)' + v^\mu. \quad (3.9)$$

Given an arbitrary three-vector $\vec{X} = X^i \vec{e}_i$,² whose components depend only on the radius and height of the cylinder, namely ρ and z , respectively, its divergence has the form

$$\vec{\nabla} \cdot \vec{X} = \frac{1}{\rho} (\rho X^1)' + \dot{X}^2, \quad (3.10)$$

² The vectors $\{\vec{e}_i\} = \{\vec{\rho}, \vec{z}, \vec{\phi}\}$ are the unit vectors in the cylindrical coordinate system.

where ∇ is the so-called del operator in cylindrical coordinates. Therefore, we may define four three-vectors $\vec{V}^\mu = u^\mu \vec{\rho} + v^\mu \vec{z}$ (only $\vec{V}^t$ and $\vec{V}^\phi$ are non-trivial) and cast the Maxwell equations into the neatly compact form

$$\vec{\nabla} \cdot \vec{V}^\mu = 0. \quad (3.11)$$

Let us now introduce the tensor $\mathcal{E}_{\mu\nu} = (2/\rho) \sqrt{-|g|} (R_{\mu\nu} - 2T_{\mu\nu})$, where $T_{\mu\nu}$ is the parenthesis on the RHS of Eq. (3.3). The latter tensor vanishes when the field equations hold. We shall first present the equation $\mathcal{E}_{tt} = 0$, whose explicit form is

$$\nabla^2 f - \frac{|\vec{\nabla} f|^2}{f} + \frac{f^3 |\vec{\nabla} \omega|^2}{\rho^2} - 2 \left(\frac{f^2 |\vec{\nabla} A_\phi + \omega \vec{\nabla} A_t|^2}{\rho^2} + |\vec{\nabla} A_t|^2 \right) = 0. \quad (3.12)$$

The other equation, $\mathcal{E}_{t\phi} = 0$, reads

$$\begin{aligned} -\nabla^2(\omega f) + \frac{\omega |\vec{\nabla} f|^2}{f} - \frac{\omega f^3 |\vec{\nabla} \omega|^2}{\rho^2} + \frac{2f\omega'}{\rho} - 4\vec{\nabla} A_t \cdot \vec{\nabla} A_\phi \\ + 2\omega \left(\frac{f^2 |\vec{\nabla} A_\phi + \omega \vec{\nabla} A_t|^2}{\rho^2} - |\vec{\nabla} A_t|^2 \right) = 0. \end{aligned} \quad (3.13)$$

Multiplying the former with ω and adding the result to the latter, we get the simpler equation

$$\frac{f\omega'}{\rho} - \vec{\nabla} \omega \cdot \vec{\nabla} f - \frac{f\nabla^2 \omega}{2} = 2\vec{\nabla} A_t \cdot (\vec{\nabla} A_\phi + \omega \vec{\nabla} A_t). \quad (3.14)$$

Both sides of the above equation can be written—modulo the Maxwell equations—as divergences, so Eq. (3.14) is equivalent to

$$\vec{\nabla} \cdot \left(\frac{f^2}{\rho^2} \vec{\nabla} \omega + 4A_t \vec{V}^\phi \right) = 0. \quad (3.15)$$

Equations (3.11), (3.12), and (3.15) suffice to determine A_t , A_ϕ , f , and ω . Furthermore, equations

$$\rho(R_{\rho z} - 2T_{\rho z}) = 0, \quad \frac{1}{2}\rho[R_{\rho\rho} - R_{zz} - 2(T_{\rho\rho} - T_{zz})] = 0 \quad (3.16)$$

provide us with $\dot{\gamma}$ and γ' , respectively, them being explicit functions of ρ and z that we can integrate to get γ . Finally, note that $\mathcal{E}_{\phi\phi} = 0$ is not an independent equation; it holds identically if $\mathcal{E}_{tt} = 0 = \mathcal{E}_{t\phi}$ holds.

At this stage, recall that if h is an arbitrary function of ρ and z , then

$$\vec{\nabla} \cdot \left(\frac{1}{\rho} \vec{\phi} \times \vec{\nabla} h \right) = 0. \quad (3.17)$$

Consequently, if there is a divergence-free three-vector $\vec{X}$ with components $X^i(\rho, z)$, there exists a function $h(\rho, z)$ such that

$$\vec{X} = \frac{1}{\rho} \vec{\phi} \times \vec{\nabla} h. \quad (3.18)$$

That function is called a *twist scalar potential*.

Revisiting the Maxwell equations (3.11), we observe that

$$\vec{V}^t = \omega \vec{V}^\phi - \frac{1}{f} \vec{\nabla} A_t. \quad (3.19)$$

One of the Maxwell equations reads $\vec{\nabla} \cdot \vec{V}^\phi = 0$ and we may thus introduce a twist scalar potential $\tilde{A}_\phi(\rho, z)$ such that

$$\vec{V}^\phi = \frac{1}{\rho} \vec{\phi} \times \vec{\nabla} \tilde{A}_\phi. \quad (3.20)$$

Since

$$\vec{\phi} \times (\vec{\phi} \times \vec{X}) = -\vec{X} \quad (3.21)$$

for any vector $\vec{X} = X^1 \vec{\rho} + X^2 \vec{z}$, we may cross both sides of Eq. (3.20) with $\vec{\phi}$ from the left to obtain the vector equation

$$\frac{1}{f} \vec{\nabla} \tilde{A}_\phi + \frac{\omega}{\rho} \vec{\phi} \times \vec{\nabla} A_t + \frac{1}{\rho} \vec{\phi} \times \vec{\nabla} A_\phi = \mathbf{0}, \quad (3.22)$$

where we also considered the definition of $\vec{V}^\phi$. Taking the divergence of both sides of the above equation, we finally find

$$\vec{\nabla} \cdot \left(\frac{1}{f} \vec{\nabla} \tilde{A}_\phi + \frac{\omega}{\rho} \vec{\phi} \times \vec{\nabla} A_t \right) = 0, \quad (3.23)$$

an equation replacing the Maxwell equation $\vec{\nabla} \cdot \vec{V}^\phi = 0$. We used the property (3.17) to get the above expression.

The other Maxwell equation, $\vec{\nabla} \cdot \vec{V}^t = 0$, can be written as

$$\vec{\nabla} \cdot \left(\frac{\omega}{\rho} \vec{\phi} \times \vec{\nabla} \tilde{A}_\phi - \frac{1}{f} \vec{\nabla} A_t \right) = 0. \quad (3.24)$$

Multiplying Eq. (3.23) by i and subtracting the above equation from the result, we get the complex equation

$$\vec{\nabla} \cdot \left(\frac{1}{f} \vec{\nabla} \Phi + \frac{i\omega}{\rho} \vec{\phi} \times \vec{\nabla} \Phi \right) = 0, \quad (3.25)$$

which is equivalent to the Maxwell equations $\vec{\nabla} \cdot \vec{V}^\mu = 0$. Here, we have introduced the complex potential $\Phi = A_t + i\tilde{A}_\phi$.

We shall now focus on the metric field equations, particularly Eq. (3.15). Observing that

$$\Im(\bar{\Phi}\vec{\nabla}\Phi) = (A_t\vec{\nabla}\tilde{A}_\phi - \tilde{A}_\phi\vec{\nabla}A_t), \quad (3.26)$$

where $\bar{\Phi}$ denotes the complex conjugate of Φ , and since

$$\vec{\nabla} \cdot \left(\frac{2}{\rho} \vec{\phi} \times \tilde{A}_\phi \vec{\nabla} A_t \right) = -\vec{\nabla} \cdot \left[\frac{1}{\rho} \vec{\phi} \times \Im(\bar{\Phi}\vec{\nabla}\Phi) \right], \quad (3.27)$$

it is straightforward to cast Eq. (3.15) into

$$\vec{\nabla} \cdot \left[\frac{f^2}{\rho^2} \vec{\nabla} \omega + \frac{2}{\rho} \vec{\phi} \times \Im(\bar{\Phi}\vec{\nabla}\Phi) \right] = 0. \quad (3.28)$$

Next, we introduce another twist scalar potential, namely $\chi(\rho, z)$, such that

$$\frac{f^2}{\rho} \vec{\nabla} \omega + 2\vec{\phi} \times \Im(\bar{\Phi}\vec{\nabla}\Phi) = (-)^{\tilde{\sigma}} \vec{\phi} \times \vec{\nabla} \chi, \quad (3.29)$$

where $\tilde{\sigma} = 0, 1$ encodes the sign freedom in the definition. Crossing both sides of the above equation with $\vec{\phi}$ from the left, dividing them by f^2 , and taking their divergence, we find that

$$\vec{\nabla} \cdot \vec{W} = 0, \quad \vec{W} := \frac{1}{f^2} [(-)^{\tilde{\sigma}} \vec{\nabla} \chi - 2\Im(\bar{\Phi}\vec{\nabla}\Phi)]. \quad (3.30)$$

The above is equivalent to Eq. (3.15).

Moving on, we recall that

$$\begin{aligned} |\vec{\nabla}\phi|^2 &= \frac{1}{\rho^2} |\vec{\nabla}\tilde{A}_\phi|^2, \\ \vec{\nabla}\Phi \cdot \vec{\nabla}\bar{\Phi} &= |\vec{\nabla}A_t|^2 + |\vec{\nabla}\tilde{A}_\phi|^2, \\ \vec{\nabla}\omega &= \rho\vec{\phi} \times \vec{W}. \end{aligned} \quad (3.31)$$

Using the above together with the identity

$$|\vec{\phi} \times \vec{X}|^2 = |\vec{X}|^2, \quad (3.32)$$

which holds for an arbitrary vector $\vec{X} = X^1\vec{\rho} + X^2\vec{z}$, we may quickly cast Eq. (3.12) into

$$\nabla^2 f - \frac{1}{f} |\vec{\nabla} f|^2 + f^3 W^2 - 2|\vec{\nabla}\Phi|^2 = 0. \quad (3.33)$$

Multiplying $\vec{\nabla} \cdot \vec{W}$ by if^2 and subtracting the result from the LHS of Eq. (3.33), we get the complex equation

$$\nabla^2 f - \frac{1}{f} |\vec{\nabla} f|^2 + f^3 W^2 - 2|\vec{\nabla}\Phi|^2 - if^2 \vec{\nabla} \cdot \vec{W} = 0, \quad (3.34)$$

which is equivalent to the system of eqs. (3.12) and (3.15).

Let us now expand Eq. (3.25) to obtain

$$-\frac{1}{f^2}\vec{\nabla}f \cdot \vec{\nabla}\Phi + \frac{1}{f}\nabla^2\Phi + \frac{i}{\rho}\vec{\nabla}\omega \cdot (\vec{\phi} \times \vec{\nabla}\Phi) = 0. \quad (3.35)$$

Using the last equation in eqs. (3.31), we can cast the above into

$$-\vec{\nabla}f \cdot \vec{\nabla}\Phi + f\nabla^2\Phi + if^2\vec{W} \cdot \vec{\nabla}\Phi = 0. \quad (3.36)$$

Proceeding with the educated introduction of the complex gravitational potential

$$\mathcal{E} = f - \Phi\bar{\Phi} - i(-)^{\tilde{\sigma}}\chi, \quad (3.37)$$

we can express Eq. (3.36) as

$$(\Re\mathcal{E} + \Phi\bar{\Phi})\nabla^2\Phi = (\vec{\nabla}\mathcal{E} + 2\bar{\Phi}\vec{\nabla}\Phi) \cdot \vec{\nabla}\Phi. \quad (3.38)$$

Interestingly, modulo the above, the complex gravitational equation (3.34) also assumes a similar form, namely

$$(\Re\mathcal{E} + |\Phi|^2)\nabla^2\mathcal{E} = (\vec{\nabla}\mathcal{E} + 2\bar{\Phi}\vec{\nabla}\Phi) \cdot \vec{\nabla}\mathcal{E}. \quad (3.39)$$

Equations (3.16), which we solve by quadrature to get γ , can also be expressed in terms of the two complex potentials $\mathcal{E}$ and Φ . We will not bother writing them down since we will determine γ differently. The calculations in the magnetic case are quite similar to those in the electric case. One starts by making the ansatz

$$ds^2 = f(d\phi - \omega dt)^2 + \frac{1}{f}[e^{2\gamma}(d\rho^2 + dz^2) - \rho^2 dt^2], \quad (3.40)$$

where the functions f , ω , and γ are related to those in the ansatz (3.4) by redefinitions, although we retain the same notation. The metric ansatz (3.40) is suitable for reduction of the field equations along the orbits of the azimuthal KV ∂_ϕ . Then, it is possible to show that the field equations one needs to satisfy are again Eqs. (3.39) and (3.38), provided we define the Ernst potentials as

$$\mathcal{E} = -f - \Phi\bar{\Phi} - i\chi, \quad \Phi = A_\phi + i\tilde{A}_t, \quad (3.41)$$

where χ and $\tilde{A}_t$ are the twist scalars, which satisfy

$$\vec{\phi} \times \vec{\nabla}\chi = -\frac{f^2}{\rho}\vec{\nabla}\omega + 2\vec{\phi} \times \Im(\bar{\Phi}\vec{\nabla}\Phi), \quad (3.42a)$$

$$\vec{\phi} \times \vec{\nabla}\tilde{A}_t = \frac{f}{\rho}(\vec{\nabla}A_t + \omega\vec{\nabla}A_\phi). \quad (3.42b)$$

Equations (3.39) and (3.38) are invariant under symmetry transformations in potential space whose domain and target are the auxiliary Ernst potentials associated with Einstein–Maxwell fields. Such transformations leave γ invariant. Their

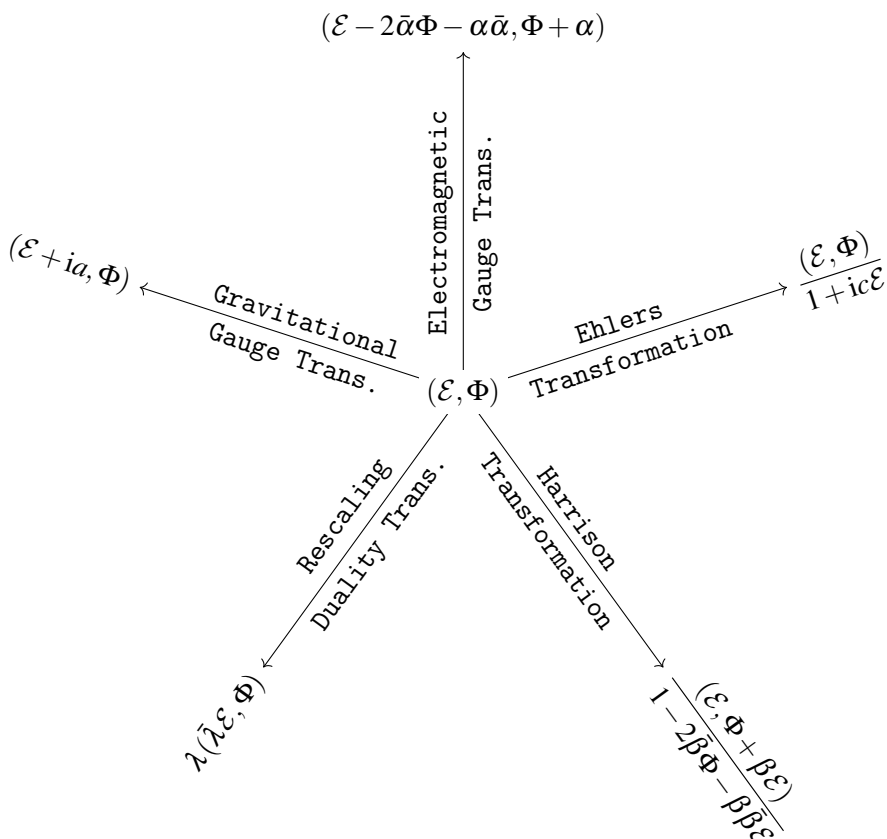


Figure 1. Available symmetry transformations of the auxiliary potentials $\mathcal{E}$ and Φ . Greek parameters are complex, while Latin ones are real.

finite forms and their names are shown in Fig. 1. Electromagnetic and gravitational gauge transformations are gauge transformations in potential space; they do not map known solutions to new ones. The rescaling/duality transformation amounts to a global rescaling of f and χ , together with a mixing of the real and imaginary parts of the electromagnetic Ernst potential. When $\lambda = e^{ib}$, it leaves the metric invariant and corresponds to a pure electromagnetic duality rotation. The Ehlers and Harrison transformations are the ones we extensively use in attached publication II.

3.3. The electromagnetic swirling universe and analytic continuation

First, we combined a MET with a MHT, generating the ElectroMagnetic Swirling Universe (EMSU) from Minkowski spacetime. The (electro)magnetic (Bonnor–Melvin) universe is the limit of the EMSU as the Ehlers parameter approaches zero, while the swirling spacetime is the limiting value of the new metric at van-

ishing Harrison parameter. In coordinates adapted to ∂_t , we determined the ergoregions in the new spacetime. We found that they extend to infinity, which implies that there are regions at spatial infinity where the KV ∂_t is not timelike. In fact, there is no linear combination of the KVs which is timelike everywhere at infinity, and the concept of “observers at rest” is inherently ambiguous; even the most general combination of KVs defines a time flow in some parts of the asymptotic region, and we showed that there is no selection mechanism that singles out a preferred KV within this general family. As a consequence, energy, ergoregions, and FDAV now become *observer-dependent*. Of course, this is true also in a swirling spacetime. For convenience, we chose to define the ergoregions and the FDAV with respect to ∂_t , keeping in mind that this is just a choice. In particular, we emphasized that the values of the FDAV—which coincides with the angular velocity of a Zero-Angular-Momentum Observer (ZAMO) whose four-velocity is aligned with ∂_t —are meaningless; only the velocity differences are useful, since we can use them to probe for counter-rotating spacetime regions.

Next, we proved that the spacetime is free of horizons and curvature singularities. We showed that the axis is regular. Moreover, we verified the absence of a Misner string by observing that the gravitomagnetic potential associated with ∂_t vanishes on both components of the axis as $\mathcal{O}(\rho^2)$. The asymptotic behaviour of the metric depends on how fast ρ and z grow relative to each other. More or less, it is swirling-like. We also showed that the new background is free of closed timelike curves. There are electric and magnetic fields present in the EMSU. Both depend on ρ , with the field lines parallel to the symmetry axis, in the vicinity of which they acquire a constant profile. The fields decay asymptotically. Interestingly, the electric field is non-trivial even if the initial external electric field is switched off, since the initial external magnetic field sources it in that case. Similarly, the initial external electric field sources the magnetic field in the absence of an initial external magnetic field. The new geometry has Petrov type D, and we proved, in great detail, that it belongs to Kundt’s class.

Continuing, we recalled two well-known facts: 1- a planar Reissner–Nordström (RN) spacetime can be analytically continued to Melvin’s magnetic universe [156], and 2- a planar Taub–NUT spacetime can be analytically continued to the swirling solution [144]. These analytic continuations involve a double Wick rotation, coordinate transformations and parameter redefinitions. Similarly, one may reasonably expect to be able to analytically continue a planar RN–NUT spacetime to the EMSU, and we proved this exactly. That motivated us to use the cosmological extension of the RN–NUT spacetime to construct a generalization of the EMSU that includes a non-vanishing cosmological constant Λ . In general, when we consider a cosmological constant, the solution-generating technique of Kinnersley ceases to apply, because the symmetries of the reduced field equations, on which the method relies, are broken. Equations that were homogeneous in the absence of Λ become inhomogeneous in its presence. In particular, the WLP metric itself is not suitable for stationary axisymmetric fields in the presence of a cosmological

constant,³ ergo the machinery used so far is not applicable. Extending stationary and axisymmetric solutions of the Ernst equations to account for the presence of a cosmological constant is a highly non-trivial task. Indeed, we were able to analytically continue the cosmological extension of the planar RN–NUT spacetime to that of the EMSU, obtaining another Petrov-type D spacetime, which, again, belongs to Kundt’s class. We argued that the cosmological constant must be negative to avoid CTCs at spatial infinity. String-like defects are present in the extended spacetime, and one may remove them by imposing a particular relation between the transformation parameters. Limits to the cosmological extensions of the Bonnor–Melvin and swirling solutions are discussed.

3.4. A Schwarzschild black hole in the EMSU

The EMSU has a rather exotic asymptotic behaviour, and it is natural to ask what happens when we “throw” an asymptotically flat black hole in it. The Schwarzschild-EMS black hole is obtained by acting with a composition of a MET and a MHT on the Ernst potentials associated with the Schwarzschild spacetime. The new spacetime describes the exterior of a black hole with its horizon located at $r = 2M$. The curvature singularity is at the origin of the radial coordinate. The embedding of the horizon surface can be visualized as a 2-sphere of area $16\pi M^2$, which is deformed when the transformation parameters are switched on. The horizon circumference at the equator shrinks as those parameters grow large, which leads to a squeezing effect.

The new black hole is stationary; ZAMOs in the exterior region rotate due to the swirling property of the background. It also is an electrovac spacetime; there is a Maxwell field present, different from that in the EMSU. The metric is asymptotically swirling-like, and the two spacetime regions separated by the equator are counter-rotating. The F_{DAV} grows infinitely large as $|z| \rightarrow \infty$, while it vanishes at the equator and at $r = 2M$. The ergoregions prove to be more or less insensitive to introducing a mass. Their behaviour at asymptotic infinity is the same as that in the case of the EMSU, and nothing is worth reporting close to the horizon. We also concluded that the new black-hole spacetime has Petrov type I, which becomes D on the axis, while the solution is algebraically special on the horizon surface. When both transformation parameters vanish, we recover the Schwarzschild black hole. When only the Harrison parameter is switched off, we obtain the swirling black hole (Petrov type is I) [144], which describes a Schwarzschild black hole embedded in a swirling spacetime. Finally, when only the Ehlers parameter is switched off, we recover the static Petrov-type I Schwarzschild–Melvin spacetime [161], which describes a Schwarzschild black hole embedded in an electromagnetic universe.

³ See [157] for a generalized metric which, given a specific harmonic condition, can be reduced to the WLP metric.

3.5. Mixing electric and magnetic transformations

Next, we scrutinized another route. Instead of composing Ehlers and Harrison transformations of the same “kind”, i.e., electric or magnetic, we explored their *mixture*. In general, there are eight possible mixed compositions

$$U^\alpha_\beta U^\gamma_\delta, \quad (3.43)$$

where $\beta \neq \delta$ ($\alpha, \beta, \gamma, \delta = 1, 2$) and

$$U = \begin{pmatrix} E_e & E_m \\ H_e & H_m \end{pmatrix}. \quad (3.44)$$

The symbols E_e , E_m , H_e , and H_m denote the operator forms of an Electric ET (EET), MET, Electric HT (EHT), and MHT, respectively. If the seed spacetime is Minkowski, the number of available compositions reduces to four, namely

$$U^\alpha_1 U^\beta_2. \quad (3.45)$$

That happens because

$$U^\alpha_2 U^\beta_1 \sim U^\alpha_2, \quad (3.46)$$

where $\sim$ denotes a rough equivalence relation here, in that electric transformations of the seed potentials associated with Minkowski result in a metric also Minkowski modulo coordinate transformations.

Our goal was to register all novel spacetimes obtained via mixed compositions in the case of a Minkowski seed. Are these new Petrov-type I geometries free of curvature singularities, topological defects, and other sorts of pathologies? Since we know that $U^1_\alpha U^2_\alpha = U^2_\alpha U^1_\alpha$, we could directly operate on the seed potentials with $E_e[c]H_e[\sigma]E_m[j]H_m[\beta]$, where j, c are real parameters, $\beta = (B + iE)/2$, and $\sigma = (q_e + iq_m)/2$. The above operation would be tantamount to casting the EMSU metric in the form (3.4) and acting on the associated electric potentials with $E_e[c]H_e[\sigma]$. We could then consider all possible limits, in which we have compositions of two transformation operators, one magnetic and one electric. However, that would not be a wise strategy in terms of computational complexity. Hence, we generated each case separately.

We collect the novel spacetimes in Fig. 2 for the reader’s convenience. There are four inequivalent spacetimes one can obtain from Minkowski by operating on its associated seed potentials with one electric and one magnetic transformation, Ehlers or Harrison:

- A four-parameter family containing the parameters E and B of the EMU and two additional parameters q_e and q_m entering via an EHT.
- A three-parameter family containing the parameters E and B of the EMU and one additional parameter c entering via an EET.
- A two-parameter family containing the parameter j of the swirling universe and one additional parameter c entering via an EET.

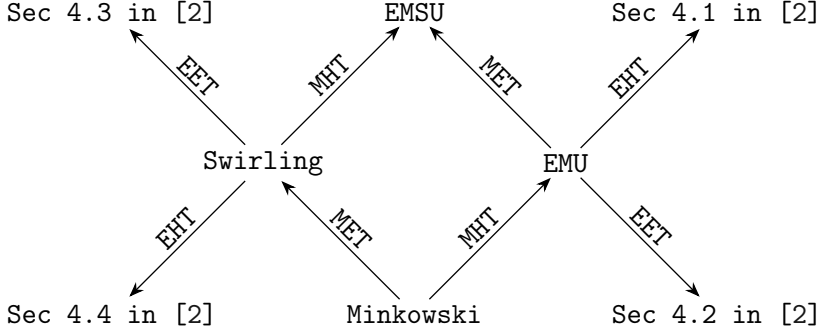


Figure 2. Inequivalent spacetimes obtained from Minkowski via ETs or HTs up to two sequential transformations.

- A three-parameter family containing the parameter j of the swirling universe and two additional parameters q_e and q_m entering via an EHT.

As expected, the new spacetimes are complicated modifications of the electromagnetic universe or the swirling solution. However, they are neither “NUTty” nor charged extensions of the latter. To be precise, we have not presented any evidence to support the association of c with a NUT parameter or that of q_e, q_m with monopolic charges. The “physical” meaning (if any) of c, q_e , and q_m remains a puzzling problem.

Besides the spacetime of Sec. 4.3 in attached publication II, which describes a naked singularity, all other spacetimes are curvature-regular under some parameter conditions. They are also free of topological defects, and their Petrov type is I. However, there is a common problem in all of them, i.e. the appearance of *non-chronal* regions, regions where we can draw CTCs. The fact that we gave a “physical” reason, namely that the soaring rotation is what tips the light cones over, is not a relief. One should primarily check if a freely falling observer can violate causality, i.e., if such curves are *geodesic*, though even non-geodesic CTCs can be problematic if traversable under other sensible conditions. The whole topic is delicate anyway. Protection mechanisms [162, 165] forbidding causality violation have yet to acquire the form of formal theorems, and it also has been conjectured that causality-violating curves exist only in the classical theory (and would disappear in the quantum version). Nevertheless, those pathologies exist at the classical level, and the very fact that they beset honest solutions to the Einstein-Maxwell field equations (or even to pure general relativity), namely that they do not arise due to some artificial distribution of matter or some weird glueing of spacetimes, actually makes them interesting in our opinion, though from a very different perspective.

3.6. Technical summary

We focused on obtaining the complete list of stationary and axisymmetric spacetimes, generated from a Minkowski spacetime by operating on the associated seed potentials with a composition of ETs and HTs. We operationally defined “electric” and “magnetic” transformations as transformations of potentials associated with the “electric” and “magnetic” forms of the WLP metric, Eqs. (3.4) and (3.40), respectively.

Our first goal was to investigate the new spacetime obtained via the composition of a MET with a MHT. As expected, the resulting Petrov-type D spacetime, the EMSU, inherits key properties from the swirling [134, 143, 144] and the Bonnor–Melvin [141, 142] solutions, two well-studied spacetimes which we recover in appropriate limits. We emphasized that there is: 1- no KV timelike everywhere at infinity and 2- many KVs timelike “somewhere” at infinity. Observers at rest with respect to these KVs are equally valid; there is no criterion that singles out one member of this class over another. Different observers will “map” the ergoregions differently and, in general, measure different values of the FDAV. However, they will agree that the ergoregion extends to spatial infinity, that the FDAV grows infinitely large when $|z| \mapsto \infty$, and that regions above and below an observer-dependent value of z counter-rotate. The EMSU is free of singularities and string-like defects. The asymptotic behaviour of the spacetime metric depends on the growth rate of the ratio ρ/z . When this ratio satisfies a certain inequality, the metric is asymptotically swirling; however, the spacetime itself is not, since the Faraday tensor does not vanish in that limit. Finally, we gave a detailed proof that the EMSU belongs to the Kundt family. Next, we established a precise relation between a planar Reissner–Nordström–NUT black hole and the EMSU by analytically continuing the former via a double Wick rotation, followed by suitable coordinate reparametrizations and parameter redefinitions. The cosmological extension of this black hole is known, and we exploited the above correspondence to reverse-engineer the form of the EMSU in the presence of a cosmological constant. A fine tuning of the transformation parameters was necessary to avoid string-like defects.

Another highlight of attached publication II is the usefulness of the EMSU as a “background” spacetime for black holes. Indeed, we “embedded” a Schwarzschild black hole into this background and analysed how the former interacts with the latter. The new spacetime is a rotating electrovac solution describing the exterior region of a black hole. The rotation, however, is not of the Kerr type; in the frame adapted to the generator of the Killing horizon, the black-hole angular velocity—equivalently, the angular velocity of a ZAMO at the horizon—vanishes. The ZAMOs nevertheless rotate in the exterior region, and their angular velocity increases without bound as they move farther away from the black hole. In this sense, the compact object itself is *effectively static*, while the exterior region rotates as a consequence of the swirling property of the background geometry. Al-

though the horizon is located at $r = 2M$ and its surface area is that of a 2-sphere of that radius, the surface itself is deformed in the presence of the transformation parameters. In an isometric embedding into Euclidean space, the horizon appears as an axisymmetrically distorted surface, reminiscent of a sphere compressed along its equator.

Finally, we constructed all spacetimes obtainable from Minkowski space by composing electric and magnetic ETs and HTs. Three of these were found to be conditionally free of curvature singularities and string-like defects. All of these contain regions where CTCs are possible. One might naively expect the resulting geometries to represent extensions of the Melvin or swirling solutions, for instance, that an EET would generate a NUT charge, or that monopole-type charges would arise through an EHT. This expectation is not borne out. The physical interpretation, if any, of the electric-transformation parameters in these spacetimes remains unclear and warrants further investigation.

4. THE KERR–LEVI-CIVITA SPACETIME

4.1. Introduction

To date, only two exact solutions with well-defined static limits are known to describe the exterior gravitational field of a spinning mass in vacuum. The first is the Kerr metric [176], which remains the most astrophysically relevant solution to EFE, underpinning much of our theoretical understanding of rotating black holes. The second is its less familiar generalization, the Tomimatsu–Sato metric [211]. Unlike the Kerr metric, the Tomimatsu–Sato solution does not reduce to the Schwarzschild geometry in the limit of vanishing angular momentum, but to a class of Weyl metrics known as Zipoy–Voorhees spacetimes [212, 213].

The circular, stationary, and axisymmetric Einstein and Einstein–Maxwell field equations become remarkably integrable in the complex-potential formalism of Ernst [185, 186]. That formalism reveals a hidden set of Lie point symmetries which, under appropriate conditions, serves as a powerful tool for generating sophisticated solutions [112, 187–189]. We already discussed these symmetries in attached publication II and in Sec. 3.2 of the present thesis. One notices that every symmetry transformation introduces a continuous parameter into the new solution. Our aim was to generate a new rotating vacuum solution without introducing any additional parameters beyond those already present in the seed spacetime, namely the mass m and the angular momentum per unit mass a . To accomplish this, one must therefore resort to a discrete symmetry of the reduced EFE. The symmetry transformation we employed is known as *inversion*.

Denoting the inversion operator by I , its action is defined by

$$I : \mathcal{E} \mapsto \frac{1}{\mathcal{E}}. \quad (4.1)$$

The operator can be expressed as the composition

$$D_{i/b} E_{1/b} G_b^{(g)} \quad (4.2)$$

in the limit $b \rightarrow \infty$, where

$$D_\alpha : (\mathcal{E}, \Phi) \mapsto \alpha(\alpha^* \mathcal{E}, \Phi), \quad (4.3a)$$

$$E_j : (\mathcal{E}, \Phi) \mapsto (\mathcal{E}, \Phi)(1 + ij\mathcal{E})^{-1}, \quad (4.3b)$$

$$G_b^{(g)} : (\mathcal{E}, \Phi) \mapsto (\mathcal{E} + ib, \Phi), \quad (4.3c)$$

are the rescaling/duality, Ehlers, and gravitational gauge transformations, respectively.¹ It can also be written as

$$G_j^{(g)} D_{ij} E_j G_{1/j}^{(g)} \quad (4.4)$$

¹ The parameters j and b are real, while the parameter α is complex.

at the level of symmetry operators, which proves that no limit whatsoever is necessary in the process. Do also note that (4.4) and $D_{ij}E_jG_{1/j}^{(g)}$ produce the same solution, since the last gravitational gauge transformation cannot affect the form of the physical metric. Despite the presence of an Ehlers transformation in the decomposition (4.4), inversion does not generate stationary spacetimes from static ones. Consequently, to obtain a rotating solution we shall start from a rotating seed. However, inverting the “electric” potentials (see Sec. 3.2) associated with the Kerr metric does nothing up to diffeomorphisms. In fact, an electric inversion does not generate new solutions from known solutions in the Plebański–Demiański class [180]. Hence, it is important to consider magnetic transformations, i.e., to invert potentials defined using the azimuthal KV ∂_ϕ .

4.2. The Kerr–Levi-Civita geometry

We considered the Kerr spacetime as our seed, and we applied the inversion machinery to the magnetic potentials associated with it. Initially, we obtained a new spacetime with a conical singularity; to remove it we rescaled the azimuthal coordinate and re-glued. The new spacetime is asymptotic to a rotating generalization of the LC metric [181],² and we address it as the Kerr–LC spacetime. The rr component of the new metric has poles at the inner and outer horizon radii of the Kerr black hole. Unlike Kerr, however, the Kerr–LC geometry is curvature-regular everywhere; there is no ring singularity. This Petrov-type I solution of the EFE describes either a regular black hole or a regular horizon-free geometry, depending on the value of $(a/m)^2$. Moreover, recall that the equatorial circumference of the outer horizon at $r = r_+$ of a Kerr black hole is independent of the angular momentum. By contrast, for a Kerr–LC black hole it does depend on it: for fixed mass, increasing a leads to a larger equatorial circumference of the outer horizon. As a increases (within the range for which the embedding exists), the outer horizon evolves from a surface with an equatorial constriction to a prolate, nearly cylindrical geometry, and ultimately to an oblate shape.

Next, we encounter the same issue discussed in attached publication II and in Sec. 3.3. In spacetimes with an exotic asymptotic behaviour such as that of the Kerr–LC spacetime, the notion of “rest” is ambiguous: no Killing vector remains timelike everywhere at infinity, and the members

$$\partial_{t^{(\alpha)}} = \partial_t + \alpha \partial_{\hat{\phi}} \quad (4.5)$$

of a one-parameter family of KVs all define admissible time flows in certain parts of the asymptotic region. Passing to coordinates adapted to $\partial_{t^{(\alpha)}}$ therefore corresponds to choosing a rotating frame. In such frames, the ZAMO angular velocity shifts according to

$$\Omega^{(\alpha)} = \Omega^{(0)} - \alpha, \quad (4.6)$$

² This is valid when $\rho, z \rightarrow \infty$ with ρ/z finite.

and becomes constant at the horizon. This constant, denoted $\Omega^{(\alpha)}|_H$, may be interpreted as the *relative* angular velocity of the black hole in the chosen frame. Importantly, differences such as

$$\Delta\Omega = \Omega^{(\alpha)} - \Omega^{(\alpha)}|_H \quad (4.7)$$

are independent of α and therefore frame-invariant. By construction, the outer horizon is a Killing horizon generated by

$$\partial_{t^{(\alpha)}} + \Omega^{(\alpha)}|_H \partial_{\hat{\phi}}. \quad (4.8)$$

We noted that the asymptotic geometry is insensitive to the choice of frame; ZAMOs defined with respect to $\partial_{t^{(\alpha)}}$ experience the same dragging far away. This dragging, however, is not of the type produced by an isolated, massive, rotating compact object dragging spacetime in its vicinity. Rather, it resembles the rotation characteristic of swirling black holes. In essence, it is closer to the dragging encountered in swirling black holes. In fact, asymptotic frame-dragging is common to all solutions with rotating asymptotic forms. Note, however, that the frame-dragging in swirling black holes exists even if we remove the mass source, be it static or non-static, due to the intrinsic rotation of the background. On the other hand, dragging in the Kerr–LC spacetime is solely due to the angular momentum of the Kerr seed. The price we pay for removing the conical singularity is that we can no longer remove the source or its angular momentum. For the sake of the argument, however, let us consider the spacetime with a conical defect and take the limits $m \rightarrow 0$ and $J \rightarrow 0$ while keeping a finite. The resulting spacetime is the LC spacetime. If we only switch off the angular momentum of the seed, we recover the Schwarzschild–LC spacetime [205–207]. In both cases, we end up with *static* solutions to EFE.

Because the ergo-geometry depends on the choice of rotating frame, the conclusions drawn in different frames can differ markedly and may even appear contradictory. In one frame, for instance, a static observer can “hover” arbitrarily close to the black hole, whereas this is impossible in others. In a different frame, there exists a radius at which no observer can remain at rest at any altitude. Finally, there is yet another frame in which the ergoregions closely resemble those of the EMSU attached publication II. Nevertheless, there is a universal conclusion to be drawn, namely that ergoregions extend to infinity. Interestingly, the Kerr–LC metric does not admit a Kerr–Schild representation: the overdetermined system that arises in attempting such a construction has no solution, in contrast to the Kerr case and to the static Schwarzschild–LC spacetime. Finally, we explicitly showed that the Kerr-swirling and Kerr–LC metrics correspond to a more general metric in, simply put, different potential-space gravitational *gauges*; there is no physical-space invariance that maps one into the other.

4.3. Technical summary

The attached publication III has been devoted to the construction of a novel rotating vacuum solution of Einstein’s field equations, the Kerr–Levi-Civita spacetime. The new geometry is a rotating generalization of the recently studied Schwarzschild–Levi-Civita black hole [205–207]. The solution was generated from the Kerr metric by exploiting a discrete symmetry of the field equations in the complex-potential formalism of Ernst, known as *inversion*. The Kerr–LC spacetime has a remarkable amount of good properties that make it attractive: it is free of curvature singularities, conical defects, spinning strings, and closed timelike curves. When $m^2 > a^2$, it describes a rotating black hole with the same horizon radii as the Kerr black hole, but with the above nice features, which also hold in the extremal and super-spinning regimes. It appears that the interplay between the rotation of the Kerr seed and that of a generalized Levi-Civita asymptotic geometry regularizes the ring singularity of the Kerr black hole. Here, we achieve the curvature regularization without introducing additional parameters, recovering the Schwarzschild–LC spacetime in the static limit. We claimed that in all spacetimes with a similar asymptotic geometry it is impossible to define a Killing vector that is asymptotically timelike everywhere. Since the ergo-geometry depends on the choice of frame and no preferred frame exists, we cannot really distinguish between the qualitatively different ergoregion profiles. The frame-independent conclusions are that the ergoregion extends to infinity in some directions and that ZAMOs experience a more intense dragging as they move farther from the source.

5. SUMMARY AND CONCLUSIONS

Black holes are among the most extreme and fascinating objects in the Universe. They push our understanding of gravity to its limit. Because of this, these compact astrophysical objects are also powerful theoretical laboratories for better understanding gravity, matter, and geometry. This doctoral thesis dives into new black holes in Einstein gravity in arbitrary spacetime dimensions and Einstein's four-dimensional theory of general relativity, focusing on how new types of matter and well-known solution-generating techniques can give rise to previously unknown spacetime geometries.

Exact solutions play a special role in gravitational physics. While many real-world systems must be studied numerically or approximately, exact solutions allow researchers to explore the geometry of spacetime (and other properties) in a mathematically controlled way. Historically, such solutions have led to major breakthroughs, including the discovery of black holes themselves. The work presented in this thesis continues this tradition by constructing and analysing new exact black-hole solutions that deepen our understanding of how gravity behaves under extreme conditions.

The first major part of the thesis focuses on Einstein gravity coupled to a generalization of classical electromagnetism. In standard physics, electromagnetism is described by Maxwell's theory. However, from a theoretical perspective, it is natural to ask whether electromagnetism can be extended in a systematic way, much like gravity has been extended beyond Einstein's original theory using geometric and topological ideas. Inspired by this analogy, the thesis builds on and extends quasitopological electromagnetism by introducing a new two-field matter model, the building blocks of which are topological invariants. When this generalized electromagnetic model is minimally coupled to gravity, it gives rise to new families of black hole solutions in various spacetime dimensions. These black holes exhibit rich geometric structure and non-trivial thermodynamic behaviour. In particular, their thermodynamics reveals phase transitions that closely resemble those of everyday systems such as fluids, providing a striking link between gravitational physics and statistical mechanics. A key result of this part of the work is the existence of regular black holes. In standard black-hole solutions, spacetime typically becomes singular at the origin of the coordinate system, meaning that physical quantities such as curvature become infinite and the theory breaks down. In contrast, the regular black holes found here avoid such singularities while remaining fully consistent solutions of classical field equations. These solutions are important because they offer controlled examples where gravity remains well-behaved even under extreme conditions, providing valuable insight into how singularities might be resolved or avoided in a complete theory of gravity.

The second major part of the thesis explores new spacetime geometries generated using powerful solution-generating techniques developed in Einstein's (and Einstein–Maxwell theory). A new vortex-like background is constructed, the elec-

tromagnetic swirling universe, and a delicate technique is used to build its cosmological extension. The new spacetime possesses unusual symmetry properties, asymptotic geometry, and electromagnetic fields. A black hole, immersed in it, differs fundamentally from the usual electrovacuum fields. Both the black hole and observers in its exterior are forced to rotate due to this new type of inherent spacetime rotation, while the black hole also interacts with the external electromagnetic fields.

Another highlight of the thesis is the construction of a new exact rotating spacetime in vacuum, referred to as the Kerr–Levi-Civita spacetime, using solution-generating methods. Rotating solutions of Einstein’s equations are of exceptional importance, since all realistic astrophysical black holes are expected to rotate. Despite this, surprisingly few exact rotating solutions are known, and most discussions rely almost exclusively on the Kerr black hole. The Kerr–Levi-Civita spacetime represents a genuinely new member of this rare class. It differs fundamentally from Kerr, while still solving Einstein’s vacuum equations exactly. Unlike Kerr, which describes an isolated rotating mass in otherwise empty space, the Kerr–LC solution inherits features from the the Levi-Civita spacetime and its rotating generalization, leading to qualitatively new properties of the geometry. Most importantly, the new solution involves only the Kerr parameters, but in such a way that the rotation no longer resembles that of a Kerr black hole. For certain parameter ranges, it describes a regular black hole that rotates in a rather swirling-like fashion.

Taken together, the results of this thesis demonstrate that exact solutions remain an indispensable tool in gravitational physics. By introducing new matter models, or exploiting hidden symmetries, our work reveals previously unexplored black-hole geometries, which deepen our theoretical understanding of gravity.

BIBLIOGRAPHY

- [1] Adolfo Cisterna, Gaston Giribet, Julio Oliva, and **Pallikaris, Konstantinos**. “Quasitopological electromagnetism and black holes”. In: *Phys. Rev. D* 101 (12 June 2020), p. 124041. DOI: 10.1103/PhysRevD.101.124041. URL: <https://link.aps.org/doi/10.1103/PhysRevD.101.124041>.
- [2] José Barrientos, Adolfo Cisterna, Ivan Kolář, Keanu Müller, Marcelo Oyarzo, and Konstantinos Pallikaris. “Mixing “Magnetic” and “Electric” Ehlers–Harrison transformations: the electromagnetic swirling spacetime and novel type I backgrounds”. In: *The European Physical Journal C* 84.7 (July 2024), p. 724. ISSN: 1434-6052. DOI: 10.1140/epjc/s10052-024-13093-x. URL: <https://doi.org/10.1140/epjc/s10052-024-13093-x>.
- [3] José Barrientos, Adolfo Cisterna, Mokhtar Hassaine, Keanu Müller, and **Konstantinos Pallikaris**. “A new exact rotating spacetime in vacuum: The Kerr–Levi-Civita spacetime”. In: *Physics Letters B* 871 (2025), p. 140035. ISSN: 0370-2693. DOI: <https://doi.org/10.1016/j.physletb.2025.140035>. URL: <https://www.sciencedirect.com/science/article/pii/S0370269325007932>.
- [4] José Barrientos, Adolfo Cisterna, Mokhtar Hassaine, and Konstantinos Pallikaris. “Electromagnetized black holes and swirling backgrounds in nonlinear electrodynamics: The ModMax case”. In: *Physics Letters B* 860 (2025), p. 139214. ISSN: 0370-2693. DOI: <https://doi.org/10.1016/j.physletb.2024.139214>. URL: <https://www.sciencedirect.com/science/article/pii/S037026932400772X>.
- [5] Damianos Iosifidis and Konstantinos Pallikaris. “Cosmology of the complete quadratic metric-affine gravity”. In: *Phys. Rev. D* 111 (2 Jan. 2025), p. 023515. DOI: 10.1103/PhysRevD.111.023515. URL: <https://link.aps.org/doi/10.1103/PhysRevD.111.023515>.
- [6] José Barrientos, Adolfo Cisterna, and Konstantinos Pallikaris. “Plebański–Demiański à la Ehlers–Harrison: exact rotating and accelerating type I black holes”. In: *General Relativity and Gravitation* 56.9 (Sept. 2024), p. 111. ISSN: 1572-9532. DOI: 10.1007/s10714-024-03304-x. URL: <https://doi.org/10.1007/s10714-024-03304-x>.
- [7] Adolfo Cisterna, Keanu Müller, Konstantinos Pallikaris, and Adriano Viganò. “Exact rotating wormholes via Ehlers transformations”. In: *Phys. Rev. D* 108 (2 July 2023), p. 024066. DOI: 10.1103/PhysRevD.108.024066. URL: <https://link.aps.org/doi/10.1103/PhysRevD.108.024066>.
- [8] Damianos Iosifidis and Konstantinos Pallikaris. “Biconnection gravity as a statistical manifold”. In: *Phys. Rev. D* 108 (4 Aug. 2023), p. 044026. DOI: 10.1103/PhysRevD.108.044026. URL: <https://link.aps.org/doi/10.1103/PhysRevD.108.044026>.
- [9] Damianos Iosifidis and Konstantinos Pallikaris. “Describing metric-affine theories anew: alternative frameworks, examples and solutions”. In: *Journal of Cosmology and Astroparticle Physics* 2023.05 (May 2023), p. 037. DOI: 10.1088/1475-7516/2023/05/037. URL: <https://doi.org/10.1088/1475-7516/2023/05/037>.
- [10] Matteo Baggioli, Adolfo Cisterna, and Konstantinos Pallikaris. “Exploring the black hole spectrum of axionic Horndeski theory”. In: *Phys. Rev. D* 104 (10 Nov.

- 2021), p. 104067. DOI: 10.1103/PhysRevD.104.104067. URL: <https://link.aps.org/doi/10.1103/PhysRevD.104.104067>.
- [11] Jose Pablo Figueroa and Konstantinos Pallikaris. “Quartic Horndeski, planar black holes, holographic aspects and universal bounds”. In: *Journal of High Energy Physics* 2020.9 (Sept. 2020), p. 90. ISSN: 1029-8479. DOI: 10.1007/JHEP09(2020)090. URL: [https://doi.org/10.1007/JHEP09\(2020\)090](https://doi.org/10.1007/JHEP09(2020)090).
 - [12] Albert Einstein. “Zur Allgemeinen Relativitätstheorie”. In: *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* 1915 (1915). [Addendum: *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* 1915, 799–801 (1915)], pp. 778–786.
 - [13] Karl Schwarzschild. “On the gravitational field of a mass point according to Einstein’s theory”. In: *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* 1916 (1916), pp. 189–196. arXiv: physics/9905030.
 - [14] John Michell. “On the Means of Discovering the Distance, Magnitude, &c. of the Fixed Stars, in Consequence of the Diminution of the Velocity of Their Light, in Case Such a Diminution Should be Found to Take Place in any of Them, and Such Other Data Should be Procured from Observations, as Would be Farther Necessary for That Purpose. By the Rev. John Michell, B. D. F. R. S. In a Letter to Henry Cavendish, Esq. F. R. S. and A. S.” In: *Philosophical Transactions of the Royal Society of London* 74 (1784), pp. 35–57. ISSN: 02610523. URL: <http://www.jstor.org/stable/106576> (visited on 06/18/2025).
 - [15] Pierre Simon Laplace. “Beweis des Satzes, dass die anziehende Kraft bey einem Weltkörper so groß seyn könne, dass das Licht davon nicht ausströmen kann”. In: *Allgemeine Geographische Ephemeriden* 4.1 (July 1799), pp. 1–6.
 - [16] William Herschel. “Catalogue of Double Stars. By William Herschel, Esq. F. R. S.” In: *Philosophical Transactions of the Royal Society of London* 75 (1785), pp. 40–126. ISSN: 02610523. URL: <http://www.jstor.org/stable/106749> (visited on 06/22/2025).
 - [17] Thomas Young. “The Bakerian Lecture. On the theory of light and colours”. In: *Philosophical Transactions of the Royal Society of London* 92 (1802), pp. 12–48. DOI: 10.1098/rstl.1802.0004.
 - [18] S. Chandrasekhar and. “XLVIII. The density of white dwarf stars”. In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 11.70 (1931), pp. 592–596. DOI: 10.1080/14786443109461710. eprint: <https://doi.org/10.1080/14786443109461710>. URL: <https://doi.org/10.1080/14786443109461710>.
 - [19] S. Chandrasekhar and. “XLVIII. The density of white dwarf stars”. In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 11.70 (1931), pp. 592–596. DOI: 10.1080/14786443109461710. URL: <https://doi.org/10.1080/14786443109461710>.
 - [20] Richard C. Tolman. “Static Solutions of Einstein’s Field Equations for Spheres of Fluid”. In: *Phys. Rev.* 55 (4 Feb. 1939), pp. 364–373. DOI: 10.1103/PhysRev.55.364. URL: <https://link.aps.org/doi/10.1103/PhysRev.55.364>.
 - [21] J. R. Oppenheimer and G. M. Volkoff. “On Massive Neutron Cores”. In: *Phys. Rev.* 55 (4 Feb. 1939), pp. 374–381. DOI: 10.1103/PhysRev.55.374. URL: <https://link.aps.org/doi/10.1103/PhysRev.55.374>.
 - [22] J. R. Oppenheimer and H. Snyder. “On Continued Gravitational Contraction”. In: *Phys. Rev.* 56 (5 Sept. 1939), pp. 455–459. DOI: 10.1103/PhysRev.56.455. URL: <https://link.aps.org/doi/10.1103/PhysRev.56.455>.

- [23] B Kent Harrison, Masami Wakano, and John A Wheeler. “Matter-energy at high density: end point of thermonuclear evolution”. In: *La structure et évolution de l’univers* 124 (1958).
- [24] M. A. Podurets. “The collapse of a star with allowance for counterpressure”. In: *Doklady Akademii Nauk SSSR* 154.2 (1964). Translated title, pp. 300–301. URL: <http://mi.mathnet.ru/dan29032>.
- [25] Michael M. May and Richard H. White. “Hydrodynamic Calculations of General-Relativistic Collapse”. In: *Phys. Rev.* 141 (4 Jan. 1966), pp. 1232–1241. DOI: 10.1103/PhysRev.141.1232. URL: <https://link.aps.org/doi/10.1103/PhysRev.141.1232>.
- [26] Kip S. Thorne. *Black Holes and Time Warps: Einsteins Outrageous Legacy*. WW Norton, 1995.
- [27] Roger Penrose. “Gravitational Collapse and Space-Time Singularities”. In: *Phys. Rev. Lett.* 14 (3 Jan. 1965), pp. 57–59. DOI: 10.1103/PhysRevLett.14.57. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.14.57>.
- [28] V. A. Belinski, E. M. Lifshitz, and I. M. Khalatnikov. “Oscillatory approach to the singular point in relativistic cosmology”. In: *Usp. Fiz. Nauk* 102.11 (1970), pp. 463–500. DOI: 10.3367/UFNr.0102.197011d.0463. URL: <https://ufn.ru/ru/articles/1970/11/d/>.
- [29] Stephen W. Hawking. “The occurrence of singularities in cosmology”. In: *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 294.1439 (1966), pp. 511–521. DOI: 10.1098/rspa.1966.0221.
- [30] Stephen W. Hawking. “The Occurrence of Singularities in Cosmology. II”. In: *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 295.1443 (1966), pp. 490–493. DOI: 10.1098/rspa.1966.0255.
- [31] Stephen Hawking. *Singularities and the geometry of space-time*. 1966. DOI: 10.1140/epjh/e2014-50013-6.
- [32] R. P. Geroch. “Singularities in Closed Universes”. In: *Phys. Rev. Lett.* 17 (8 Aug. 1966), pp. 445–447. DOI: 10.1103/PhysRevLett.17.445. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.17.445>.
- [33] Stephen W. Hawking and Roger Penrose. “The Singularities of Gravitational Collapse and Cosmology”. In: *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 314.1519 (1970), pp. 529–548. DOI: 10.1098/rspa.1970.0021.
- [34] V. L. Ginzburg. “The Magnetic Fields of Collapsing Masses and the Nature of Superstars”. In: *Soviet Physics Doklady* 9 (Nov. 1964), p. 329.
- [35] A. G. Doroshkevich, Ya. B. Zel’dovich, and I. D. Novikov. “Gravitational Collapse of Nonsymmetric and Rotating Masses”. In: *Soviet Physics JETP* 22.1 (1966). <http://www.jetp.ac.ru/cgi-bin/e/index/e/22/1/p122?a=list>, pp. 122–130.
- [36] Werner Israel. “Event Horizons in Static Vacuum Space-Times”. In: *Phys. Rev.* 164 (5 Dec. 1967), pp. 1776–1779. DOI: 10.1103/PhysRev.164.1776. URL: <https://link.aps.org/doi/10.1103/PhysRev.164.1776>.
- [37] Werner Israel. “Event horizons in static electrovac space-times”. In: *Communications in Mathematical Physics* 8.3 (Sept. 1968), pp. 245–260. ISSN: 1432-0916. DOI: 10.1007/BF01645859. URL: <https://doi.org/10.1007/BF01645859>.

- [38] H. Reissner. “Über die Eigengravitation des elektrischen Feldes nach der Einsteinschen Theorie”. In: *Annalen der Physik* 355.9 (1916), pp. 106–120. DOI: <https://doi.org/10.1002/andp.19163550905>. eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1002/andp.19163550905>. URL: <https://onlinelibrary.wiley.com/doi/abs/10.1002/andp.19163550905>.
- [39] G. Nordström. “On the Energy of the Gravitation field in Einstein’s Theory”. In: *Koninklijke Nederlandse Akademie van Wetenschappen Proceedings Series B Physical Sciences* 20 (Jan. 1918), pp. 1238–1245.
- [40] John C. Graves and Dieter R. Brill. “Oscillatory Character of Reissner-Nordström Metric for an Ideal Charged Wormhole”. In: *Phys. Rev.* 120 (4 Nov. 1960), pp. 1507–1513. DOI: 10.1103/PhysRev.120.1507. URL: <https://link.aps.org/doi/10.1103/PhysRev.120.1507>.
- [41] Roy P. Kerr. “Gravitational Field of a Spinning Mass as an Example of Algebraically Special Metrics”. In: *Phys. Rev. Lett.* 11 (5 Sept. 1963), pp. 237–238. DOI: 10.1103/PhysRevLett.11.237. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.11.237>.
- [42] Robert H. Boyer and Richard W. Lindquist. “Maximal Analytic Extension of the Kerr Metric”. In: *Journal of Mathematical Physics* 8.2 (Feb. 1967), pp. 265–281. ISSN: 0022-2488. DOI: 10.1063/1.1705193. eprint: <https://pubs.aip.org/aip/jmp/article-pdf/8/2/265/19040647/265\1\online.pdf>. URL: <https://doi.org/10.1063/1.1705193>.
- [43] Brandon Carter. “Global Structure of the Kerr Family of Gravitational Fields”. In: *Phys. Rev.* 174 (5 Oct. 1968), pp. 1559–1571. DOI: 10.1103/PhysRev.174.1559. URL: <https://link.aps.org/doi/10.1103/PhysRev.174.1559>.
- [44] B. Carter. “Axisymmetric Black Hole Has Only Two Degrees of Freedom”. In: *Phys. Rev. Lett.* 26 (6 Feb. 1971), pp. 331–333. DOI: 10.1103/PhysRevLett.26.331. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.26.331>.
- [45] D. C. Robinson. “Uniqueness of the Kerr Black Hole”. In: *Phys. Rev. Lett.* 34 (14 Apr. 1975), pp. 905–906. DOI: 10.1103/PhysRevLett.34.905. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.34.905>.
- [46] S. W. Hawking. “Black holes in general relativity”. In: *Communications in Mathematical Physics* 25.2 (June 1972), pp. 152–166. ISSN: 1432-0916. DOI: 10.1007/BF01877517. URL: <https://doi.org/10.1007/BF01877517>.
- [47] S. K. Bose and M. Y. Wang. “Stationary, axially symmetric perturbations of charged Kerr black holes”. In: *Phys. Rev. D* 10 (6 Sept. 1974), pp. 1675–1677. DOI: 10.1103/PhysRevD.10.1675. URL: <https://link.aps.org/doi/10.1103/PhysRevD.10.1675>.
- [48] E. T. Newman, E. Couch, K. Chinnapared, A. Exton, A. Prakash, and R. Torrence. “Metric of a Rotating, Charged Mass”. In: *Journal of Mathematical Physics* 6.6 (June 1965), pp. 918–919. ISSN: 0022-2488. DOI: 10.1063/1.1704351. eprint: <https://pubs.aip.org/aip/jmp/article-pdf/6/6/918/19113862/918\1\online.pdf>. URL: <https://doi.org/10.1063/1.1704351>.
- [49] Richard H. Price. “Nonspherical Perturbations of Relativistic Gravitational Collapse. I. Scalar and Gravitational Perturbations”. In: *Phys. Rev. D* 5 (10 May 1972), pp. 2419–2438. DOI: 10.1103/PhysRevD.5.2419. URL: <https://link.aps.org/doi/10.1103/PhysRevD.5.2419>.

- [50] Richard H. Price. “Nonspherical Perturbations of Relativistic Gravitational Collapse. II. Integer-Spin, Zero-Rest-Mass Fields”. In: *Phys. Rev. D* 5 (10 May 1972), pp. 2439–2454. DOI: 10.1103/PhysRevD.5.2439. URL: <https://link.aps.org/doi/10.1103/PhysRevD.5.2439>.
- [51] William H. Press. “Long Wave Trains of Gravitational Waves from a Vibrating Black Hole”. In: *Astrophysical Journal, Letters* 170 (Dec. 1971), p. L105. DOI: 10.1086/180849.
- [52] Ya. B. Zel’Dovich. “Generation of Waves by a Rotating Body”. In: *Soviet Journal of Experimental and Theoretical Physics Letters* 14 (Aug. 1971), p. 180.
- [53] S. W. HAWKING. “Black hole explosions?” In: *Nature* 248.5443 (Mar. 1974), pp. 30–31. ISSN: 1476-4687. DOI: 10.1038/248030a0. URL: <https://doi.org/10.1038/248030a0>.
- [54] J. M. Bardeen, B. Carter, and S. W. Hawking. “The four laws of black hole mechanics”. In: *Communications in Mathematical Physics* 31.2 (June 1973), pp. 161–170. ISSN: 1432-0916. DOI: 10.1007/BF01645742. URL: <https://doi.org/10.1007/BF01645742>.
- [55] H. L. Shipman. “The Implausible History of Triple Star Models for Cygnus X-1: Evidence for a Black Hole”. In: *Astrophysics Letters* 16 (Feb. 1975), p. 9.
- [56] R. E. Rothschild, E. A. Boldt, S. S. Holt, and P. J. Serlemitsos. “Millisecond Temporal Structure in Cygnus X-1”. In: *Astrophysical Journal, Letters* 189 (Apr. 1974), p. L13. DOI: 10.1086/181452.
- [57] The Event Horizon Telescope Collaboration et al. “First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole”. In: *The Astrophysical Journal Letters* 875.1 (Apr. 2019), p. L1. DOI: 10.3847/2041-8213/ab0ec7. URL: <https://doi.org/10.3847/2041-8213/ab0ec7>.
- [58] Event Horizon Telescope Collaboration et al. “First Sagittarius A* Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole in the Center of the Milky Way”. In: *The Astrophysical Journal Letters* 930.2 (May 2022), p. L12. DOI: 10.3847/2041-8213/ac6674. URL: <https://doi.org/10.3847/2041-8213/ac6674>.
- [59] Tanay Kr. Dey. “Born-Infeld black holes in the presence of a cosmological constant”. In: *Phys. Lett. B* 595 (2004), pp. 484–490. DOI: 10.1016/j.physletb.2004.06.047. arXiv: hep-th/0406169.
- [60] S. H. Hendi and M. H. Vahidinia. “Extended phase space thermodynamics and $P - V$ criticality of black holes with a nonlinear source”. In: *Phys. Rev. D* 88 (8 Oct. 2013), p. 084045. DOI: 10.1103/PhysRevD.88.084045. URL: <https://link.aps.org/doi/10.1103/PhysRevD.88.084045>.
- [61] Rong-Gen Cai, Da-Wei Pang, and Anzhong Wang. “Born-Infeld black holes in (A)dS spaces”. In: *Phys. Rev. D* 70 (12 Dec. 2004), p. 124034. DOI: 10.1103/PhysRevD.70.124034. URL: <https://link.aps.org/doi/10.1103/PhysRevD.70.124034>.
- [62] Sharmila Gunasekaran, Robert B. Mann, and David Kubiznak. “Extended phase space thermodynamics for charged and rotating black holes and Born-Infeld vacuum polarization”. In: *JHEP* 11 (2012), p. 110. DOI: 10.1007/JHEP11(2012)110. arXiv: 1208.6251 [hep-th].
- [63] Rabin Banerjee and Dibakar Roychowdhury. “Critical phenomena in Born-Infeld AdS black holes”. In: *Phys. Rev. D* 85 (2012), p. 044040. DOI: 10.1103/PhysRevD.85.044040. arXiv: 1111.0147 [gr-qc].

- [64] De-Cheng Zou, Shao-Jun Zhang, and Bin Wang. “Critical behavior of Born-Infeld AdS black holes in the extended phase space thermodynamics”. In: *Phys. Rev. D* 89 (4 Feb. 2014), p. 044002. DOI: 10.1103/PhysRevD.89.044002. URL: <https://link.aps.org/doi/10.1103/PhysRevD.89.044002>.
- [65] Igor Bandos, Kurt Lechner, Dmitri Sorokin, and Paul K. Townsend. “A non-linear duality-invariant conformal extension of Maxwell’s equations”. In: *Phys. Rev. D* 102 (2020), p. 121703. DOI: 10.1103/PhysRevD.102.121703. arXiv: 2007.09092 [hep-th].
- [66] Alvaro Ballon Bordo, David Kubizňák, and Tales Rick Perche. “Taub-NUT solutions in conformal electrodynamics”. In: *Phys. Lett. B* 817 (2021), p. 136312. DOI: 10.1016/j.physletb.2021.136312. arXiv: 2011.13398 [hep-th].
- [67] Kirill A. Bronnikov. “Regular magnetic black holes and monopoles from nonlinear electrodynamics”. In: *Phys. Rev. D* 63 (2001), p. 044005. DOI: 10.1103/PhysRevD.63.044005. arXiv: gr-qc/0006014.
- [68] Leonardo Balart and Elias C. Vagenas. “Regular black holes with a nonlinear electrodynamics source”. In: *Phys. Rev. D* 90 (12 Dec. 2014), p. 124045. DOI: 10.1103/PhysRevD.90.124045. URL: <https://link.aps.org/doi/10.1103/PhysRevD.90.124045>.
- [69] M. F. A. da Silva, L. Herrera, F. M. Paiva, and N. O. Santos. “The Levi-Civita space-time”. In: *Journal of Mathematical Physics* 36.7 (July 1995), pp. 3625–3631. ISSN: 0022-2488. DOI: 10.1063/1.530986. eprint: https://pubs.aip.org/aip/jmp/article-pdf/36/7/3625/19124375/3625_1_online.pdf. URL: <https://doi.org/10.1063/1.530986>.
- [70] Kirill A Bronnikov, N O Santos, and Anzhong Wang. “Cylindrical systems in general relativity”. In: *Classical and Quantum Gravity* 37.11 (May 2020), p. 113002. DOI: 10.1088/1361-6382/ab7bba. URL: <https://doi.org/10.1088/1361-6382/ab7bba>.
- [71] David Lovelock. “The Einstein Tensor and Its Generalizations”. In: *Journal of Mathematical Physics* 12.3 (Mar. 1971), pp. 498–501. ISSN: 0022-2488. DOI: 10.1063/1.1665613.
- [72] Mauro Brigante, Hong Liu, Robert C. Myers, Stephen Shenker, and Sho Yaida. “Viscosity bound violation in higher derivative gravity”. In: *Phys. Rev. D* 77 (12 June 2008), p. 126006. DOI: 10.1103/PhysRevD.77.126006.
- [73] Hai-Shan Liu, Zhan-Feng Mai, Yue-Zhou Li, and H. Lü. “Quasi-topological electromagnetism: Dark energy, dyonic black holes, stable photon spheres and hidden electromagnetic duality”. In: *Science China Physics, Mechanics & Astronomy* 63.4 (Dec. 2019), p. 240411. ISSN: 1869-1927. DOI: 10.1007/s11433-019-1446-1.
- [74] James T. Wheeler. “Symmetric solutions to the maximally Gauss-Bonnet extended Einstein equations”. In: *Nuclear Physics B* 273.3 (1986), pp. 732–748. ISSN: 0550-3213. DOI: [https://doi.org/10.1016/0550-3213\(86\)90388-3](https://doi.org/10.1016/0550-3213(86)90388-3).
- [75] Julio Oliva and Sourya Ray. “A new cubic theory of gravity in five dimensions: black hole, Birkhoff’s theorem and C-function”. In: *Classical and Quantum Gravity* 27.22 (Oct. 2010), p. 225002. DOI: 10.1088/0264-9381/27/22/225002.
- [76] Robert C. Myers and Brandon Robinson. “Black holes in quasi-topological gravity”. In: *Journal of High Energy Physics* 2010.8 (Aug. 2010), p. 67. ISSN: 1029-8479. DOI: 10.1007/JHEP08(2010)067.

- [77] M. H. Dehghani, A. Bazrafshan, R. B. Mann, M. R. Mehdizadeh, M. Ghanaatian, and M. H. Vahidinia. “Black holes in (quartic) quasitopological gravity”. In: *Phys. Rev. D* 85 (10 May 2012), p. 104009. DOI: 10.1103/PhysRevD.85.104009.
- [78] Adolfo Cisterna, Luis Guajardo, Mokhtar Hassaine, and Julio Oliva. “Quintic quasi-topological gravity”. In: *Journal of High Energy Physics* 2017.4 (Apr. 2017), p. 66. ISSN: 1029-8479. DOI: 10.1007/JHEP04(2017)066.
- [79] Pablo Bueno, Pablo A Cano, and Robie A Hennigar. “(Generalized) quasi-topological gravities at all orders”. In: *Classical and Quantum Gravity* 37.1 (Nov. 2019), p. 015002. DOI: 10.1088/1361-6382/ab5410.
- [80] M. Abramowitz and I.A. Stegun. *Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables*. Applied mathematics series. Dover Publications, 1965.
- [81] Matías Aiello, Rafael Ferraro, and Gastón Giribet. “Exact solutions of Lovelock-Born-Infeld black holes”. In: *Phys. Rev. D* 70 (10 Nov. 2004), p. 104014. DOI: 10.1103/PhysRevD.70.104014.
- [82] Robert M. Wald. “Black hole entropy is the Noether charge”. In: *Phys. Rev. D* 48 (8 Oct. 1993), R3427–R3431. DOI: 10.1103/PhysRevD.48.R3427.
- [83] S. W. Hawking. “Particle creation by black holes”. In: *Communications in Mathematical Physics* 43.3 (Aug. 1975), pp. 199–220. ISSN: 1432-0916. DOI: 10.1007/BF02345020.
- [84] Jacob D. Bekenstein. “Extraction of Energy and Charge from a Black Hole”. In: *Phys. Rev. D* 7 (4 Feb. 1973), pp. 949–953. DOI: 10.1103/PhysRevD.7.949.
- [85] Ming-Da Li, Hui-Min Wang, and Shao-Wen Wei. “Triple points and novel phase transitions of dyonic AdS black holes with quasitopological electromagnetism”. In: *Phys. Rev. D* 105 (10 May 2022), p. 104048. DOI: 10.1103/PhysRevD.105.104048. URL: <https://link.aps.org/doi/10.1103/PhysRevD.105.104048>.
- [86] M. Cvetič, G. W. Gibbons, H. Lü, and C. N. Pope. “Killing horizons: Negative temperatures and entropy super-additivity”. In: *Phys. Rev. D* 98 (10 Nov. 2018), p. 106015. DOI: 10.1103/PhysRevD.98.106015. URL: <https://link.aps.org/doi/10.1103/PhysRevD.98.106015>.
- [87] L.F. Abbott and S. Deser. “Stability of gravity with a cosmological constant”. In: *Nuclear Physics B* 195.1 (1982), pp. 76–96. ISSN: 0550-3213. DOI: [https://doi.org/10.1016/0550-3213\(82\)90049-9](https://doi.org/10.1016/0550-3213(82)90049-9).
- [88] David Kastor, Sourya Ray, and Jennie Traschen. “Enthalpy and the mechanics of AdS black holes”. In: *Classical and Quantum Gravity* 26.19 (Sept. 2009), p. 195011. DOI: 10.1088/0264-9381/26/19/195011.
- [89] David Kubizňák and Robert B. Mann. “Black hole chemistry”. In: *Canadian Journal of Physics* 93.9 (2015), pp. 999–1002. DOI: 10.1139/cjp-2014-0465. eprint: <https://doi.org/10.1139/cjp-2014-0465>.
- [90] Brian P Dolan. “Pressure and volume in the first law of black hole thermodynamics”. In: *Classical and Quantum Gravity* 28.23 (Nov. 2011), p. 235017. DOI: 10.1088/0264-9381/28/23/235017.
- [91] Nigel D Goldenfeld. *Lectures On Phase Transitions And The Renormalization Group*. English (US). Frontiers in physics. Publisher Copyright: © 1992 Taylor & Francis Group LLC. Westview Press, 1992. DOI: 10.1201/9780429493492.

- [92] Andrew Chamblin, Roberto Emparan, Clifford V. Johnson, and Robert C. Myers. “Charged AdS black holes and catastrophic holography”. In: *Phys. Rev. D* 60 (1999), p. 064018. DOI: 10.1103/PhysRevD.60.064018. arXiv: hep-th/9902170.
- [93] Máximo Bañados, Claudio Teitelboim, and Jorge Zanelli. “Black hole in three-dimensional spacetime”. In: *Phys. Rev. Lett.* 69 (13 Sept. 1992), pp. 1849–1851. DOI: 10.1103/PhysRevLett.69.1849.
- [94] David Kubizňák and Robert B. Mann. “P - V criticality of charged AdS black holes”. In: *Journal of High Energy Physics* 2012, 33 (July 2012), p. 33. DOI: 10.1007/JHEP07(2012)033. arXiv: 1205.0559 [hep-th].
- [95] David Kubizňák and Robert B. Mann. “Black hole chemistry I”. In: *Canadian Journal of Physics* 93.9 (Sept. 2015), pp. 999–1002. DOI: 10.1139/cjp-2014-0465. arXiv: 1404.2126 [gr-qc].
- [96] James Bardeen. “Non-singular general relativistic gravitational collapse”. In: *Proceedings of the 5th International Conference on Gravitation and the Theory of Relativity*. Sept. 1968, p. 87.
- [97] Eloy Ayón-Beato and Alberto García. “Regular Black Hole in General Relativity Coupled to Nonlinear Electrodynamics”. In: *Phys. Rev. Lett.* 80 (23 June 1998), pp. 5056–5059. DOI: 10.1103/PhysRevLett.80.5056.
- [98] Eloy Ayen-Beato and Alberto Garcia. “Non-Singular Charged Black Hole Solution for Non-Linear Source”. In: *General Relativity and Gravitation* 31 (May 1999), p. 629. DOI: 10.1023/A:1026640911319.
- [99] Eloy Ayón-Beato and Alberto García. “New regular black hole solution from nonlinear electrodynamics”. In: *Physics Letters B* 464.1 (1999), pp. 25–29. ISSN: 0370-2693. DOI: [https://doi.org/10.1016/S0370-2693\(99\)01038-2](https://doi.org/10.1016/S0370-2693(99)01038-2).
- [100] Eugeny Babichev, Christos Charmousis, Adolfo Cisterna, and Mokhtar Hassaine. “Regular black holes via the Kerr-Schild construction in DHOST theories”. In: *Journal of Cosmology and Astroparticle Physics* 2020.06 (June 2020), p. 049. DOI: 10.1088/1475-7516/2020/06/049.
- [101] Valeri P. Frolov. “Notes on nonsingular models of black holes”. In: *Phys. Rev. D* 94 (10 Nov. 2016), p. 104056. DOI: 10.1103/PhysRevD.94.104056.
- [102] A. D. Sakharov. “The Initial Stage of an Expanding Universe and the Appearance of a Nonuniform Distribution of Matter”. In: *Soviet Journal of Experimental and Theoretical Physics* 22 (Jan. 1966), p. 241.
- [103] Euro Spallucci and Anais Smailagic. “Regular black holes from semi-classical down to Planckian size”. In: *International Journal of Modern Physics D* 26.07 (2017), p. 1730013. DOI: 10.1142/S0218271817300130.
- [104] Xing-Hui Feng and H. Lü. “Higher-derivative gravity with non-minimally coupled Maxwell field”. In: *European Physical Journal C* 76.4, 178 (Apr. 2016), p. 178. DOI: 10.1140/epjc/s10052-016-4007-y.
- [105] Adolfo Cisterna, Sebastián Fuenzalida, and Julio Oliva. “Lovelock black p -branes with fluxes”. In: *Phys. Rev. D* 101 (6 Mar. 2020), p. 064055. DOI: 10.1103/PhysRevD.101.064055.
- [106] Hans Stephani, Dietrich Kramer, Malcolm MacCallum, Cornelius Hoenselaers, and Eduard Herlt. *Exact Solutions of Einstein’s Field Equations*. 2nd ed. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2003. DOI: <https://doi.org/10.1017/CB09780511535185>.

- [107] J. D. Bekenstein. “Black holes and the second law”. In: *Lettere al Nuovo Cimento* (1971-1985) 4.15 (Aug. 1972), pp. 737–740. ISSN: 1827-613X. DOI: 10.1007/BF02757029. URL: <https://doi.org/10.1007/BF02757029>.
- [108] Jacob D. Bekenstein. “Black Holes and Entropy”. In: *Phys. Rev. D* 7 (8 Apr. 1973), pp. 2333–2346. DOI: 10.1103/PhysRevD.7.2333. URL: <https://link.aps.org/doi/10.1103/PhysRevD.7.2333>.
- [109] S. W. HAWKING. “Black hole explosions?” In: *Nature* 248.5443 (Mar. 1974), pp. 30–31. ISSN: 1476-4687. DOI: 10.1038/248030a0. URL: <https://doi.org/10.1038/248030a0>.
- [110] Steven B. Giddings. “The black hole information paradox”. In: *arXiv e-prints*, hep-th/9508151 (Aug. 1995), hep-th/9508151. DOI: 10.48550/arXiv.hep-th/9508151. arXiv: hep-th/9508151 [hep-th].
- [111] Leonard Susskind. “The world as a hologram”. In: *Journal of Mathematical Physics* 36.11 (Nov. 1995), pp. 6377–6396. ISSN: 0022-2488. DOI: 10.1063/1.531249. eprint: https://pubs.aip.org/aip/jmp/article-pdf/36/11/6377/19188778/6377_1_online.pdf. URL: <https://doi.org/10.1063/1.531249>.
- [112] J. Ehlers. “Transformations of static exterior solutions of Einstein’s gravitational field equations into different solutions by means of conformal mapping”. In: *Colloq. Int. CNRS* 91 (1962). Ed. by M. A. Lichnerowicz and M. A. Tonnelat, pp. 275–284.
- [113] Jurgen Ehlers. “Konstruktionen und Charakterisierung von Lösungen der Einsteinschen Gravitationsfeldgleichungen”. Other thesis. 1957.
- [114] B Kent Harrison. “New Solutions of the Einstein-Maxwell Equations from Old”. In: *Journal of Mathematical Physics* 9.11 (1968), pp. 1744–1752.
- [115] Frederick J. Ernst. “New Formulation of the Axially Symmetric Gravitational Field Problem”. In: *Phys. Rev.* 167 (5 Mar. 1968), pp. 1175–1178. DOI: 10.1103/PhysRev.167.1175. URL: <https://link.aps.org/doi/10.1103/PhysRev.167.1175>.
- [116] Frederick J. Ernst. “New Formulation of the Axially Symmetric Gravitational Field Problem. II”. In: *Phys. Rev.* 168 (5 Apr. 1968), pp. 1415–1417. DOI: 10.1103/PhysRev.168.1415. URL: <https://link.aps.org/doi/10.1103/PhysRev.168.1415>.
- [117] G. Neugebauer and D. Kramer. “Eine Methode zur Konstruktion stationärer EINSTEIN-MAXWELL-Felder”. In: *Annalen der Physik* 479.1-2 (1969), pp. 62–71. DOI: <https://doi.org/10.1002/andp.19694790108>. eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1002/andp.19694790108>. URL: <https://onlinelibrary.wiley.com/doi/abs/10.1002/andp.19694790108>.
- [118] D. Kramer and G. Neugebauer. “Zu axialsymmetrischen stationären Lösungen der Einsteinschen Feldgleichungen für das Vakuum”. In: *Communications in Mathematical Physics* 10.2 (June 1968), pp. 132–139. ISSN: 1432-0916. DOI: 10.1007/BF01654237. URL: <https://doi.org/10.1007/BF01654237>.
- [119] William Kinnersley. “Generation of stationary Einstein–Maxwell fields”. In: *Journal of Mathematical Physics* 14.5 (May 1973), pp. 651–653. ISSN: 0022-2488. DOI: 10.1063/1.1666373. eprint: https://pubs.aip.org/aip/jmp/article-pdf/14/5/651/19171512/651_1_online.pdf. URL: <https://doi.org/10.1063/1.1666373>.

- [120] William Kinnersley. “Symmetries of the stationary Einstein–Maxwell field equations. I”. In: *Journal of Mathematical Physics* 18.8 (Aug. 1977), pp. 1529–1537. ISSN: 0022-2488. DOI: 10.1063/1.523458. eprint: https://pubs.aip.org/aip/jmp/article-pdf/18/8/1529/19033786/1529_1_online.pdf. URL: <https://doi.org/10.1063/1.523458>.
- [121] William Kinnersley and D. M. Chitre. “Symmetries of the stationary Einstein–Maxwell field equations. II”. In: *Journal of Mathematical Physics* 18.8 (Aug. 1977), pp. 1538–1542. ISSN: 0022-2488. DOI: 10.1063/1.523459. eprint: https://pubs.aip.org/aip/jmp/article-pdf/18/8/1538/19033920/1538_1_online.pdf. URL: <https://doi.org/10.1063/1.523459>.
- [122] William Kinnersley and D. M. Chitre. “Symmetries of the stationary Einstein–Maxwell field equations. III”. In: *Journal of Mathematical Physics* 19.9 (Sept. 1978), pp. 1926–1931. ISSN: 0022-2488. DOI: 10.1063/1.523912. eprint: https://pubs.aip.org/aip/jmp/article-pdf/19/9/1926/19170653/1926_1_online.pdf. URL: <https://doi.org/10.1063/1.523912>.
- [123] William Kinnersley and D. M. Chitre. “Symmetries of the stationary Einstein–Maxwell equations. IV. Transformations which preserve asymptotic flatness”. In: *Journal of Mathematical Physics* 19.10 (Oct. 1978), pp. 2037–2042. ISSN: 0022-2488. DOI: 10.1063/1.523580. eprint: https://pubs.aip.org/aip/jmp/article-pdf/19/10/2037/19078583/2037_1_online.pdf. URL: <https://doi.org/10.1063/1.523580>.
- [124] Robert Geroch. “A Method for Generating Solutions of Einstein’s Equations”. In: *Journal of Mathematical Physics* 12.6 (June 1971), pp. 918–924. ISSN: 0022-2488. DOI: 10.1063/1.1665681. eprint: https://pubs.aip.org/aip/jmp/article-pdf/12/6/918/19296427/918_1_online.pdf. URL: <https://doi.org/10.1063/1.1665681>.
- [125] Robert Geroch. “A Method for Generating New Solutions of Einstein’s Equation. II”. In: *Journal of Mathematical Physics* 13.3 (Mar. 1972), pp. 394–404. ISSN: 0022-2488. DOI: 10.1063/1.1665990. eprint: https://pubs.aip.org/aip/jmp/article-pdf/13/3/394/19124918/394_1_online.pdf. URL: <https://doi.org/10.1063/1.1665990>.
- [126] Isidore Hauser and Frederick J. Ernst. “Integral equation method for effecting Kinnersley–Chitre transformations”. In: *Phys. Rev. D* 20 (2 July 1979), pp. 362–369. DOI: 10.1103/PhysRevD.20.362. URL: <https://link.aps.org/doi/10.1103/PhysRevD.20.362>.
- [127] Isidore Hauser and Frederick J. Ernst. “A homogeneous Hilbert problem for the Kinnersley–Chitre transformations”. In: *Journal of Mathematical Physics* 21.5 (May 1980), pp. 1126–1140. ISSN: 0022-2488. DOI: 10.1063/1.524536. eprint: https://pubs.aip.org/aip/jmp/article-pdf/21/5/1126/19093137/1126_1_online.pdf. URL: <https://doi.org/10.1063/1.524536>.
- [128] Isidore Hauser and Frederick J. Ernst. “A homogeneous Hilbert problem for the Kinnersley–Chitre transformations of electrovac space-times”. In: *Journal of Mathematical Physics* 21.6 (June 1980), pp. 1418–1422. ISSN: 0022-2488. DOI: 10.1063/1.524567. eprint: https://pubs.aip.org/aip/jmp/article-pdf/21/6/1418/19126413/1418_1_online.pdf. URL: <https://doi.org/10.1063/1.524567>.
- [129] Werner Israel and Gordon A. Wilson. “A Class of Stationary Electromagnetic Vacuum Fields”. In: *Journal of Mathematical Physics* 13.6 (June 1972), pp. 865–

867. ISSN: 0022-2488. DOI: 10.1063/1.1666066. eprint: https://pubs.aip.org/aip/jmp/article-pdf/13/6/865/19006105/865_1_online.pdf. URL: <https://doi.org/10.1063/1.1666066>.
- [130] G Neugebauer. “Backlund transformations of axially symmetric stationary gravitational fields”. In: *Journal of Physics A: Mathematical and General* 12.4 (Apr. 1979), p. L67. DOI: 10.1088/0305-4470/12/4/001. URL: <https://doi.org/10.1088/0305-4470/12/4/001>.
 - [131] D. Maison. “Ehlers-Harrison-type transformations for Jordan’s extended theory of gravitation”. In: *General Relativity and Gravitation* 10.8 (June 1979), pp. 717–723. ISSN: 1572-9532. DOI: 10.1007/BF00756907. URL: <https://doi.org/10.1007/BF00756907>.
 - [132] Pawel Mazur. “A relationship between the electrovacuum Ernst equations and nonlinear sigma model”. In: *Acta Physica Polonica* B14 (Apr. 1983), pp. 219–234.
 - [133] José Barrientos and Adolfo Cisterna. “Ehlers transformations as a tool for constructing accelerating NUT black holes”. In: *Phys. Rev. D* 108 (2 July 2023), p. 024059. DOI: 10.1103/PhysRevD.108.024059. URL: <https://link.aps.org/doi/10.1103/PhysRevD.108.024059>.
 - [134] I. G. Contopoulos, F. P. Esposito, K. Kleidis, D. B. Papadopoulos, and L. Witten. “Generating solutions to the Einstein field equations”. In: *International Journal of Modern Physics D* 25.02 (2016), p. 1650022. DOI: 10.1142/S021827181650022X. eprint: <https://doi.org/10.1142/S021827181650022X>. URL: <https://doi.org/10.1142/S021827181650022X>.
 - [135] Marco Astorino and Giovanni Boldi. “Plebanski-Demianski goes NUTs (to remove the Misner string)”. In: *Journal of High Energy Physics* 2023.8 (Aug. 2023), p. 85. ISSN: 1029-8479. DOI: 10.1007/JHEP08(2023)085. URL: [https://doi.org/10.1007/JHEP08\(2023\)085](https://doi.org/10.1007/JHEP08(2023)085).
 - [136] Marco Astorino. “Accelerating and charged type I black holes”. In: *Phys. Rev. D* 108 (12 Dec. 2023), p. 124025. DOI: 10.1103/PhysRevD.108.124025. URL: <https://link.aps.org/doi/10.1103/PhysRevD.108.124025>.
 - [137] E. T. Newman and L. A. Tamburino. “New Approach to Einstein’s Empty Space Field Equations”. In: *Journal of Mathematical Physics* 2.5 (Sept. 1961), pp. 667–674. ISSN: 0022-2488. DOI: 10.1063/1.1703754. eprint: https://pubs.aip.org/aip/jmp/article-pdf/2/5/667/19279640/667_1_online.pdf. URL: <https://doi.org/10.1063/1.1703754>.
 - [138] I. Robinson and A. Trautman. “Some spherical gravitational waves in general relativity”. In: *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 265 (1323 1962), pp. 463–473. DOI: 10.1098/rspa.1962.0036.
 - [139] L. Witten. *Gravitation: An Introduction to Current Research*. Wiley, 1962.
 - [140] Rudolf Bach. “Neue Lösungen der Einsteinschen Gravitationsgleichungen”. In: *Mathematische Zeitschrift* 13.1 (Dec. 1922), pp. 119–133. ISSN: 1432-1823. DOI: 10.1007/BF01485283. URL: <https://doi.org/10.1007/BF01485283>.
 - [141] W B Bonnor. “Static Magnetic Fields in General Relativity”. In: *Proceedings of the Physical Society. Section A* 67.3 (Mar. 1954), p. 225. DOI: 10.1088/0370-1298/67/3/305. URL: <https://dx.doi.org/10.1088/0370-1298/67/3/305>.

- [142] M. A. Melvin and J. S. Wallingford. “Orbits in a Magnetic Universe”. In: *Journal of Mathematical Physics* 7.2 (Feb. 1966), pp. 333–340. ISSN: 0022-2488. DOI: 10.1063/1.1704937. eprint: <https://pubs.aip.org/aip/jmp/article-pdf/7/2/333/19272606/333\1\online.pdf>. URL: <https://doi.org/10.1063/1.1704937>.
- [143] G W Gibbons, A H Muftaba, and C N Pope. “Ergoregions in magnetized black hole spacetimes”. In: *Classical and Quantum Gravity* 30.12 (May 2013), p. 125008. DOI: 10.1088/0264-9381/30/12/125008. URL: <https://dx.doi.org/10.1088/0264-9381/30/12/125008>.
- [144] Marco Astorino, Riccardo Martelli, and Adriano Viganò. “Black holes in a swirling universe”. In: *Phys. Rev. D* 106 (6 Sept. 2022), p. 064014. DOI: 10.1103/PhysRevD.106.064014. URL: <https://link.aps.org/doi/10.1103/PhysRevD.106.064014>.
- [145] A. Z. Petrov. “The Classification of Spaces Defining Gravitational Fields”. In: *General Relativity and Gravitation* 32.8 (Aug. 2000), pp. 1665–1685. ISSN: 1572-9532. DOI: 10.1023/A:1001910908054. URL: <https://doi.org/10.1023/A:1001910908054>.
- [146] R. A. d’Inverno and R. A. Russell-Clark. “Classification of the Harrison Metrics”. In: *Journal of Mathematical Physics* 12.7 (July 1971), pp. 1258–1263. ISSN: 0022-2488. DOI: 10.1063/1.1665729. URL: <https://doi.org/10.1063/1.1665729>.
- [147] Jerry B. Griffiths and Jiří Podolský. *Exact Space-Times in Einstein’s General Relativity*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2009.
- [148] A. H. Taub. “Empty Space-Times Admitting a Three Parameter Group of Motions”. In: *Annals of Mathematics* 53.3 (1951), pp. 472–490. ISSN: 0003486X, 19398980. URL: <http://www.jstor.org/stable/1969567> (visited on 01/27/2025).
- [149] William Kinnersley. “Type D Vacuum Metrics”. In: *Journal of Mathematical Physics* 10.7 (July 1969), pp. 1195–1203. ISSN: 0022-2488. DOI: 10.1063/1.1664958. URL: <https://doi.org/10.1063/1.1664958>.
- [150] J. Patera and P. Winternitz. “Subalgebras of real three- and four-dimensional Lie algebras”. In: *Journal of Mathematical Physics* 18.7 (July 1977), pp. 1449–1455. ISSN: 0022-2488. DOI: 10.1063/1.523441. URL: <https://doi.org/10.1063/1.523441>.
- [151] Rogério Capobianco, Betti Hartmann, and Jutta Kunz. “Geodesic motion in a swirling universe: The complete set of solutions”. In: *Phys. Rev. D* 109 (6 Mar. 2024), p. 064042. DOI: 10.1103/PhysRevD.109.064042. URL: <https://link.aps.org/doi/10.1103/PhysRevD.109.064042>.
- [152] Matilde Illy. “Accelerated Reissner-Nordstrom black hole in a swirling, magnetic universe”. Dec. 2023, arXiv:2312.14995, arXiv:2312.14995. DOI: 10.48550/arXiv.2312.14995. arXiv: 2312.14995 [gr-qc].
- [153] Valeri P. Frolov and Igor D. Novikov. *Black Hole Physics*. Springer Netherlands, 1998. DOI: 10.1007/978-94-011-5139-9. URL: <http://dx.doi.org/10.1007/978-94-011-5139-9>.
- [154] Wolfgang Kundt. “The plane-fronted gravitational waves”. In: *Zeitschrift für Physik* 163.1 (Feb. 1961), pp. 77–86. ISSN: 0044-3328. DOI: 10.1007/BF01328918. URL: <https://doi.org/10.1007/BF01328918>.

- [155] W. Kundt. “Exact solutions of the field equations: twist-free pure radiation fields”. In: *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 270 (1342 1962), pp. 328–334. DOI: 10.1098/rspa.1962.0224.
- [156] G W Gibbons and C A R Herdeiro. “The Melvin universe in Born-Infeld theory and other theories of nonlinear electrodynamics”. In: *Classical and Quantum Gravity* 18.9 (May 2001), p. 1677. DOI: 10.1088/0264-9381/18/9/305. URL: <https://dx.doi.org/10.1088/0264-9381/18/9/305>.
- [157] Christos Charmousis, David Langlois, Danièle Steer, and Robin Zegers. “Rotating spacetimes with a cosmological constant”. In: *Journal of High Energy Physics* 2007.02 (Feb. 2007), p. 064. DOI: 10.1088/1126-6708/2007/02/064. URL: <https://dx.doi.org/10.1088/1126-6708/2007/02/064>.
- [158] Martin Žofka. “Bonnor-Melvin universe with a cosmological constant”. In: *Phys. Rev. D* 99 (4 Feb. 2019), p. 044058. DOI: 10.1103/PhysRevD.99.044058. URL: <https://link.aps.org/doi/10.1103/PhysRevD.99.044058>.
- [159] Marco Astorino. “Charging axisymmetric space-times with cosmological constant”. In: *Journal of High Energy Physics* 2012.6 (June 2012), p. 86. ISSN: 1029-8479. DOI: 10.1007/JHEP06(2012)086. URL: [https://doi.org/10.1007/JHEP06\(2012\)086](https://doi.org/10.1007/JHEP06(2012)086).
- [160] V Pravda and O B Zaslavskii. “Curvature tensors on distorted Killing horizons and their algebraic classification”. In: *Classical and Quantum Gravity* 22.23 (Nov. 2005), p. 5053. DOI: 10.1088/0264-9381/22/23/009. URL: <https://dx.doi.org/10.1088/0264-9381/22/23/009>.
- [161] Frederick J. Ernst. “Black holes in a magnetic universe”. In: *Journal of Mathematical Physics* 17.1 (Jan. 1976), pp. 54–56. ISSN: 0022-2488. DOI: 10.1063/1.522781. eprint: https://pubs.aip.org/aip/jmp/article-pdf/17/1/54/19046555/54_1_online.pdf. URL: <https://doi.org/10.1063/1.522781>.
- [162] S. W. Hawking. “Chronology protection conjecture”. In: *Phys. Rev. D* 46 (2 July 1992), pp. 603–611. DOI: 10.1103/PhysRevD.46.603. URL: <https://link.aps.org/doi/10.1103/PhysRevD.46.603>.
- [163] W B Bonnor. “The rigidly rotating relativistic dust cylinder”. In: *Journal of Physics A: Mathematical and General* 13.6 (June 1980), p. 2121. DOI: 10.1088/0305-4470/13/6/033. URL: <https://dx.doi.org/10.1088/0305-4470/13/6/033>.
- [164] W. J. van Stockum. “IX.—The Gravitational Field of a Distribution of Particles Rotating about an Axis of Symmetry”. In: *Proceedings of the Royal Society of Edinburgh* 57 (1938), pp. 135–154. DOI: 10.1017/S0370164600013699.
- [165] John Friedman, Michael S. Morris, Igor D. Novikov, Fernando Echeverria, Gunnar Klinkhammer, Kip S. Thorne, and Ulvi Yurtsever. “Cauchy problem in space-times with closed timelike curves”. In: *Phys. Rev. D* 42 (6 Sept. 1990), pp. 1915–1930. DOI: 10.1103/PhysRevD.42.1915. URL: <https://link.aps.org/doi/10.1103/PhysRevD.42.1915>.
- [166] Jiri Bicak. “Selected solutions of Einstein’s field equations: Their role in general relativity and astrophysics”. In: *Lect. Notes Phys.* 540 (2000). Ed. by Bernd G. Schmidt, pp. 1–126. arXiv: [gr-qc/0004016](https://arxiv.org/abs/gr-qc/0004016).
- [167] Jiri Bicak. “Exact solutions and their interpretation”. In: *16th International Conference on General Relativity and Gravitation (GR16)*. July 2001, pp. 337–357. DOI: 10.1142/9789812776556_0017. arXiv: [gr-qc/0201011](https://arxiv.org/abs/gr-qc/0201011).

- [168] J. D. Bekenstein. “Black holes and the second law”. In: *Lett. Nuovo Cim.* 4 (1972), pp. 737–740. DOI: 10.1007/BF02757029.
- [169] Jacob D. Bekenstein. “Black holes and entropy”. In: *Phys. Rev. D* 7 (1973), pp. 2333–2346. DOI: 10.1103/PhysRevD.7.2333.
- [170] S. W. Hawking. “Black hole explosions”. In: *Nature* 248 (1974), pp. 30–31. DOI: 10.1038/248030a0.
- [171] S. W. Hawking. “Particle Creation by Black Holes”. In: *Commun. Math. Phys.* 43 (1975). Ed. by G. W. Gibbons and S. W. Hawking. [Erratum: *Commun. Math. Phys.* 46, 206 (1976)], pp. 199–220. DOI: 10.1007/BF02345020.
- [172] S. L. Shapiro and S. A. Teukolsky. *Black holes, white dwarfs, and neutron stars: The physics of compact objects*. 1983. DOI: 10.1002/9783527617661.
- [173] Weinberg, S. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*. 1972. DOI: 10.1002/9783527617661.
- [174] J. Polchinski. *String theory. Vol. 1: An introduction to the bosonic string*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Dec. 2007. DOI: 10.1017/CB09780511816079.
- [175] J. Polchinski. *String theory. Vol. 2: Superstring theory and beyond*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Dec. 2007. DOI: 10.1017/CB09780511618123.
- [176] Roy P. Kerr. “Gravitational field of a spinning mass as an example of algebraically special metrics”. In: *Phys. Rev. Lett.* 11 (1963), pp. 237–238. DOI: 10.1103/PhysRevLett.11.237.
- [177] B. P. Abbott et al. “GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral”. In: *Phys. Rev. Lett.* 119.16 (2017), p. 161101. DOI: 10.1103/PhysRevLett.119.161101. arXiv: 1710.05832 [gr-qc].
- [178] B. P. Abbott et al. “Gravitational Waves and Gamma-rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A”. In: *Astrophys. J. Lett.* 848.2 (2017), p. L13. DOI: 10.3847/2041-8213/aa920c. arXiv: 1710.05834 [astro-ph.HE].
- [179] Hans Stephani, D. Kramer, Malcolm A. H. MacCallum, Cornelius Hoenselaers, and Eduard Herlt. *Exact solutions of Einstein’s field equations*. Cambridge Monographs on Mathematical Physics. Cambridge: Cambridge Univ. Press, 2003. DOI: 10.1017/CB09780511535185.
- [180] J. F. Plebanski and M. Demianski. “Rotating, charged, and uniformly accelerating mass in general relativity”. In: *Annals Phys.* 98 (1976), pp. 98–127. DOI: 10.1016/0003-4916(76)90240-2.
- [181] Jerry B. Griffiths and Jiri Podolsky. *Exact Space-Times in Einstein’s General Relativity*. Cambridge Monographs on Mathematical Physics. Cambridge: Cambridge University Press, 2009. DOI: 10.1017/CB09780511635397.
- [182] P. K. Townsend. “The eleven-dimensional supermembrane revisited”. In: *Phys. Lett. B* 350 (1995), pp. 184–187. DOI: 10.1201/9781482268737-25. arXiv: hep-th/9501068.
- [183] Roberto Emparan, Gary T. Horowitz, and Robert C. Myers. “Exact description of black holes on branes”. In: *JHEP* 01 (2000), p. 007. DOI: 10.1088/1126-6708/2000/01/007. arXiv: hep-th/9911043.
- [184] S. W. Hawking and Simon F. Ross. “Pair production of black holes on cosmic strings”. In: *Phys. Rev. Lett.* 75 (1995), pp. 3382–3385. DOI: 10.1103/PhysRevLett.75.3382. arXiv: gr-qc/9506020.

- [185] Frederick J. Ernst. “New formulation of the axially symmetric gravitational field problem”. In: *Phys. Rev.* 167 (1968), pp. 1175–1179. DOI: 10.1103/PhysRev.167.1175.
- [186] Frederick J. Ernst. “New Formulation of the Axially Symmetric Gravitational Field Problem. II”. In: *Phys. Rev.* 168 (1968), pp. 1415–1417. DOI: 10.1103/PhysRev.168.1415.
- [187] Robert P. Geroch. “A Method for generating solutions of Einstein’s equations”. In: *J. Math. Phys.* 12 (1971), pp. 918–924. DOI: 10.1063/1.1665681.
- [188] W Kinnersley. “Generation of stationary Einstein-Maxwell fields”. In: *J.Math.Phys.* 14 (1973), p. 651. DOI: 10.1063/1.1666373.
- [189] BK Harrison. “New solutions of the Einstein-Maxwell equations from old”. In: *J.Math.Phys.* 9 (1968), p. 1744. DOI: 10.1063/1.1664508.
- [190] Frederick J. Ernst. “Black holes in a magnetic universe”. In: *J. Math. Phys.* 17.1 (1976), pp. 54–56. DOI: 10.1063/1.522781.
- [191] Frederick J. Ernst and Walter J. Wild. “Kerr black holes in a magnetic universe”. In: *J. Math. Phys.* 17.2 (1976), p. 182. DOI: 10.1063/1.522875.
- [192] D. V. Galtsov and V. I. Petukhov. “Black Hole in an External Magnetic Field”. In: *Zh. Eksp. Teor. Fiz.* 74 (1978), pp. 801–818.
- [193] V. I. Dokuchaev. “A black hole in a magnetic universe”. In: *Sov. Phys. JETP* 65 (1987), pp. 1079–1986.
- [194] Adriano Viganò. “Black Holes and Solution Generating Techniques”. PhD thesis. Milan U., 2022. arXiv: 2211.00436 [gr-qc].
- [195] Andrea Di Pinto. “Charged and Rotating Black Holes in a Melvin-swirling Universe”. PhD thesis. Milan U., 2024. arXiv: 2407.11270 [gr-qc].
- [196] I. G. Contopoulos, F. P. Esposito, K. Kleidis, D. B. Papadopoulos, and L. Witten. “Generating Solutions to the Einstein Field Equations”. In: *Int. J. Mod. Phys. D* 25.02 (2015), p. 1650022. DOI: 10.1142/S021827181650022X. arXiv: 1501.03968 [gr-qc].
- [197] Andrea Di Pinto, Silke Klemm, and Adriano Viganò. “Kerr-Newman black hole in a Melvin-swirling universe”. In: *JHEP* 06 (2025), p. 150. DOI: 10.1007/JHEP06(2025)150. arXiv: 2503.07780 [gr-qc].
- [198] Hans A. Buchdahl. “Reciprocal static solutions of the equations of the gravitational field”. In: *Austral. J. Phys.* 9 (1956), pp. 13–18.
- [199] Matilde Illy. “Accelerated Reissner-Nordstrom black hole in a swirling, magnetic universe”. Other thesis. Dec. 2023. arXiv: 2312.14995 [gr-qc].
- [200] B. Carter. “Republication of: Black hole equilibrium states Part II. General theory of stationary black hole states”. In: *Gen.Rel.Grav.* 42 (2010), p. 653.
- [201] Ericourgoulhon. “An Introduction to the theory of rotating relativistic stars”. In: *CompStar 2010: School and Workshop on Computational Tools for Compact Star Astrophysics*. Mar. 2010. arXiv: 1003.5015 [gr-qc].
- [202] Brandon Carter. “Killing horizons and orthogonally transitive groups in space-time”. In: *J. Math. Phys.* 10 (1969), pp. 70–81. DOI: 10.1063/1.1664763.
- [203] Hans Stephani. *Differential Equations: Their Solution Using Symmetries*. 1990. DOI: 10.1017/CB09780511599941.
- [204] Riccardo Martelli. “THE ACTION OF THE AXISYMMETRIC AND STATIONARY SYMMETRY GROUP OF GENERAL RELATIVITY ON A STATIC BLACK HOLE”. Tesi di Laurea. Milan U., 2021.

- [205] M. M. Akbar and M. A. H. MacCallum. “Static Axisymmetric Einstein Equations in Vacuum: Symmetry, New Solutions and Ricci Solitons”. In: *Phys. Rev. D* 92.6 (2015), p. 063017. DOI: 10.1103/PhysRevD.92.063017. arXiv: 1508.05196 [gr-qc].
- [206] S. Habib Mazharimousavi. “Schwarzschild-Levi-Civita black hole”. In: *Phys. Lett. B* 861 (2025), p. 139234. DOI: 10.1016/j.physletb.2024.139234. arXiv: 2403.02365 [gr-qc].
- [207] José Barrientos, Adolfo Cisterna, Mokhtar Hassaine, and Julio Oliva. “Revisiting Buchdahl transformations: new static and rotating black holes in vacuum, double copy, and hairy extensions”. In: *Eur. Phys. J. C* 84.10 (2024), p. 1011. DOI: 10.1140/epjc/s10052-024-13383-4. arXiv: 2404.12194 [gr-qc].
- [208] B. Carter. “Axisymmetric Black Hole Has Only Two Degrees of Freedom”. In: *Phys. Rev. Lett.* 26 (1971), pp. 331–333. DOI: 10.1103/PhysRevLett.26.331.
- [209] Werner Israel. “Event horizons in static electrovac space-times”. In: *Commun. Math. Phys.* 8 (1968), pp. 245–260. DOI: 10.1007/BF01645859.
- [210] Robert M. Wald. “Final states of gravitational collapse”. In: *Phys. Rev. Lett.* 26 (1971), pp. 1653–1655. DOI: 10.1103/PhysRevLett.26.1653.
- [211] Akira Tomimatsu and Humitaka Sato. “New Exact Solution for the Gravitational Field of a Spinning Mass”. In: *Phys. Rev. Lett.* 29 (1972), pp. 1344–1345. DOI: 10.1103/PhysRevLett.29.1344.
- [212] David M. Zipoy. “Topology of Some Spheroidal Metrics”. In: *J. Math. Phys.* 7.6 (1966), p. 1137. DOI: 10.1063/1.1705005.
- [213] B. H. Voorhees. “Static axially symmetric gravitational fields”. In: *Phys. Rev. D* 2 (1970), pp. 2119–2122. DOI: 10.1103/PhysRevD.2.2119.
- [214] T. Levi-Civita. “ds² einsteiniani in campi newtoniani. IX: L’analogo del potenziale logaritmico”. In: *Rend. Accad. Lincei* 28 (1919), 101–109. §§7.3, 10.2.
- [215] Eloy Ayón-Beato, Mokhtar Hassaine, and Daniel Higueta-Borja. “Role of symmetries in the Kerr-Schild derivation of the Kerr black hole”. In: *Phys. Rev. D* 94.6 (2016). [Addendum: *Phys.Rev.D* 96, 049902 (2017)], p. 064073. DOI: 10.1103/PhysRevD.94.064073. arXiv: 1512.06870 [hep-th].
- [216] Marco Astorino. “Removal of conical singularities from rotating C-metrics and dual CFT entropy”. In: *JHEP* 10 (2022), p. 074. DOI: 10.1007/JHEP10(2022)074. arXiv: 2207.14305 [gr-qc].

ACKNOWLEDGEMENTS

I am especially thankful to my partner Ioanna for her patience, understanding, and unwavering support throughout this long and challenging journey. Her encouragement during difficult moments and her belief in me made an essential difference, and without her this work would not have been possible. I would also like to express my sincere gratitude to my supervisor, Prof. Manuel Hohmann, for his guidance, patience, and continued support throughout this work. He has always been there for me, from answering the most naive questions to providing the most valuable suggestions. I am grateful to my collaborators and colleagues for many valuable discussions, especially Adolfo Cisterna, José Barrientos, and Damianos Iosifidis, with whom I had continuous and inspiring exchanges on the topics we worked on together. I am also grateful to the members of the Theoretical Physics Laboratory at the University of Tartu for providing a stimulating and supportive research environment, especially Laur Järv and Tomi Koivisto. Finally, I acknowledge the financial support provided by the European Regional Development Fund via the Archimedes Foundation (Dora Pluss alategevus 2.1 välisdoktorandid LTFY, Project number 2014-2020.4.01.16-0025). Part of the support also came from the Estonian Research Council via Team Grant PRG356 and the European Regional Development Fund via the Center of Excellence TK133. Additional financial support came from Archimedes Foundation through Kristjan Jaak and Dora Plus scholarships.

SISUKOKKUVÕTE

Uued analüütilised musta augu lahendid Einsteini gravitatsiooniteoorias

Mustad augud on ühed Universumi kõige äärmuslikumad ja paeluvamad objektid. Need tähistavad punkti, kus meie arusaam gravitatsioonist on viidud selle piirini. Seetõttu on need kompaktsed astrofüüsikalised objektid ühtlasi võimsad teoreetilised laborid gravitatsiooni, aine ja geomeetria mõistmiseks. Käesolev doktoritöö uurib musti auke Einsteini võrrandite täpsete lahendite abil, keskendudes sellele, kuidas uued aine tüübid ja tuntud lahendite genereerimise meetodid võivad viia seni tundmatute aegruumi geomeetriateni. Analüütilised lahendid mängivad gravitatsioonifüüsikas erilist rolli. Kui paljusid reaalse maailma süsteeme tuleb uurida arvutuslikult või ligikaudsete meetoditega, siis analüütilised lahendid võimaldavad uurida aegruumi geomeetriat (ja muid omadusi) matemaatiliselt kontrollitud viisil. Ajalooliselt on just sellised lahendid viinud oluliste läbimurreteni, sealhulgas mustade aukude endi avastamiseni. Käesolevas töös jätkatakse seda traditsiooni, konstrueerides ja analüüsides uusi analüütilisi mustade aukude lahendeid, mis süvendavad meie arusaama gravitatsiooni käitumisest äärmuslikes tingimustes. Doktoritöö esimene põhiosa keskendub Einsteini gravitatsiooniteooriale koos klassikalise elektromagnetismi üldistusega. Tavapärases füüsikas kirjeldab elektromagnetismi Maxwelli teooria. Teoreetilisest vaatepunktist on aga loomulik küsida, kas elektromagnetismi saab süstemaatiliselt laiendada, sarnaselt sellele, kuidas gravitatsiooniteooriat on laiendatud Einsteini algsest teooriast edasi geomeetriliste ja topoloogiliste ideede abil. Sellest analoogiast inspireerituna tugineb käesolev töö kvasi-topoloogilisele elektromagnetismile ning laiendab seda, tutvustades uut kaheväljalist aine mudelit, mille ehitusplokid on topoloogilised invariantid. Kui see üldistatud elektromagnetiline mudel on gravitatsiooniga minimaalselt seotud, tekivad uued mustade aukude lahendite perekonnad erinevates aegruumi dimensioonides. Need mustad augud omavad rikkalikku geomeetrilist struktuuri ja mittetriviaalset termodünaamilist käitumist. Eelkõige ilmnevad nende termodünaamikas faasisiirded, mis meenutavad tugevalt igapäevaseid süsteeme, nagu vedelikud, luues silmatorkava seose gravitatsioonifüüsika ja statistilise mehaanika vahel. Selle töö üheks keskseks tulemuseks on regulaarsete mustade aukude olemasolu. Tavapärastes mustade aukude lahendites muutub aegruum tavaliselt keskpunktis singulaarseks, mis tähendab, et füüsikalised suurused, nagu kõverus, muutuvad lõpmatuks ning teooria lakkab kehtimast. Vastupidiselt sellele väldivad käesolevas töös leitud regulaarsed mustad augud selliseid singulaarsusi, olles samal ajal täielikult kooskõlalised klassikaliste väljateooriate lahendid. Need lahendid on olulised, kuna pakuvad kontrollitud näiteid olukordadest, kus gravitatsioon püsib hästi määratletuna ka äärmuslikes tingimustes, ning annavad väärtuslikku teavet selle kohta, kuidas singulaarset käitumist võiks täielikus gravitatsiooniteoorias lahendada või vältida. Doktoritöö teine põhiosa uurib uusi aegruu-

mi geomeetriaid, mis on saadud võimsate lahendite genereerimise meetodite abil, mis on välja töötatud Einsteini ja Einsteini–Maxwelli teoorias. Konstrueeritakse uus keerisetaoline taustaegruum, elektromagnetiline keerlev aegruum, ning selle kosmoloogiline laiendus saadakse delikaatse meetodi abil. Uus aegruum omab ebatavalisi sümmeetriaomadusi, asümptootilist geomeetriat ja elektromagnetvälju. Sellesse paigutatud must auk erineb põhimõtteliselt tavapärastest elektrovaakuumlahenditest. Nii must auk kui ka vaatlejad selle välispiirkonnas on sunnitud pöörama selle uue aegruumi sisemise pöörmise tõttu, samal ajal kui must auk interakteerub väliste elektromagnetväljadega. Teine töö fookus on uue analüütilise pööleva vaakumlahendi konstrueerimine Einsteini võrranditele, mida nimetatakse Kerr–Levi-Civita aegruumiks, kasutades lahendite genereerimise meetodeid. Pöörlavad lahendid Einsteini võrranditele on erakordselt olulised, kuna kõik realistlikud astrofüüsikalised mustad augud eeldatavasti pöörlavad. Sellest hoolimata on analüütilised pöörlavad lahendid teada üllatavalt vähe ning enamik käsitlusi tugineb peaaegu eranditult Kerri mustale augule. Kerr–Levi-Civita aegruum kujutab endast selle haruldase klassi tõeliselt uut liiget. See erineb Kerri lahendist fundamentaalselt, lahendades samal ajal analüütiliselt Einsteini vaakumvõrrandid. Erinevalt Kerrist, mis kirjeldab isoleeritud pöörlavat massi muidu tühjas ruumis, sisaldab Kerr–LC lahend täiendavat geomeetrilist struktuuri, mis on pärit Levi-Civita aegruumist ja selle pöörlavast üldistusest, ning mis viib kvalitatiivselt uute geomeetriliste omadusteni. Kõige olulisem on see, et uus lahend sõltub ainult Kerri parameetritest, kuid viisil, mille korral pöörlamine ei sarnane enam Kerri musta augu pöörlamisega. Teatud parameetrite vahemikes kirjeldab see regulaarset musta auku, mis pöörlab oma olemuselt keerisetaolisel viisil. Kokkuvõttes näitavad käesoleva doktoritöö tulemused, et analüütilised lahendid on gravitatsioonifüüsikas jätkuvalt asendamatud tööriistad. Uute aine mudelite sisseviimise või varjatud sümmeetriate ärakasutamise kaudu paljastab töö seni uurimata mustade aukude geomeetriaid, mis süvendavad meie teoreetilist arusaama gravitatsioonist.

ATTACHED PUBLICATIONS

CURRICULUM VITAE

Personal data

Name: Konstantinos Pallikaris
Date of birth: 25.09.1988
Phone: 00306985710122
ORCID: 0000-0003-2403-892X
E-mail: konstantinos.pallikaris@gmail.com

Education

2018–Present University of Tartu, Laboratory of Theoretical Physics, Institute of Physics, PhD
2018 National Technical University of Athens, School of Applied Mathematics and Physics, Integrated Master (4+1 years)

Research work

Fields of research

ETIS RESEARCH FIELD: 4. Natural Sciences and Engineering; 4.10. Physics and Technical Physics; CERCS RESEARCH FIELD: P190 Mathematical and general theoretical physics, classical mechanics, quantum mechanics, relativity, gravitation, statistical physics, thermodynamics; SPECIFICATION: Black holes

Completed projects

- PRG356 "Gauge gravity: unification, extensions and phenomenology" (01.01.2019–31.12.2023)
Principal Investigator: Tomi Sebastian Koivisto
University of Tartu, Faculty of Science and Technology, Institute of Physics (coordinator)
Financier: Estonian Research Council; Financing: 1 148 375 EUR.
- SLTFY16133T (TK133) "The Dark Side of the Universe (TK133)" (01.03.2016–01.03.2023)
Principal Investigator: Manuel Hohmann
University of Tartu, Faculty of Science and Technology, Institute of Physics (partner)
Financier: Archimedes Foundation; Financing: 778 050 EUR

Publications

2025

Iosifidis, Damianos; Pallikaris, Konstantinos (2025). Cosmology of the complete quadratic metric affine gravity. *Physical Review D*, 111 (2), 30.

DOI: 10.1103/PhysRevD.111.023515.

Barrientos, Jose; Cisterna, Adolfo; Hassaine, Mokhtar; Pallikaris, Konstantinos (2025). Electromagnetized black holes and swirling backgrounds in nonlinear electrodynamics: The ModMax case. *Physics Letters B*, 860, ARTN 139214.

DOI: 10.1016/j.physletb.2024.139214.

Barrientos, Jose; Cisterna, Adolfo; Hassaine, Mokhtar; Muller, Keanu; Pallikaris, Konstantinos (2025). A new exact rotating spacetime in vacuum: The Kerr-Levi-Civita spacetime. *Physics Letters B*, 871, ARTN 140035.

DOI: 10.1016/j.physletb.2025.140035.

2024

Barrientos, Jose; Cisterna, Adolfo; Pallikaris, Konstantinos (2024). Plebański-Demiański à la Ehlers Harrison: exact rotating and accelerating type I black holes. *General Relativity and Gravitation*, 56 (9), ARTN 111.

DOI: 10.1007/s10714-024-03304-x.

Barrientos, Jose; Cisterna, Adolfo; Kolar, Ivan; Mueller, Keanu; Oyarzo, Marcelo; Pallikaris, Konstantinos (2024). Mixing "Magnetic" and "Electric" Ehlers-Harrison transformations: the electromagnetic swirling spacetime and novel type I backgrounds. *The European Physical Journal C*, 84 (7), ARTN 724.

DOI: 10.1140/epjc/s10052-024-13093-x.

2023

Iosifidis, Damianos; Pallikaris, Konstantinos (2023). Biconnection gravity as a statistical manifold. *Physical Review D*, 108 (4), ARTN 044026.

DOI: 10.1103/PhysRevD.108.044026.

Iosifidis, Damianos; Pallikaris, Konstantinos (2023). Describing metric-affine theories anew: alternative frameworks, examples and solutions. *Journal of Cosmology and Astroparticle Physics*, 2023 (5), ARTN 037.

DOI: 10.1088/1475-7516/2023/05/037.

Cisterna, Adolfo; Mueller, Keanu; Pallikaris, Konstantinos; Vigano, Adriano (2023).

Exact rotating wormholes via Ehlers transformations. *Physical Review D*, 108 (2), ARTN 024066.

DOI: 10.1103/PhysRevD.108.024066.

2021

Baggioli, Matteo; Cisterna, Adolfo; Pallikaris, Konstantinos (2021). Exploring the black hole spectrum of axionic Horndeski theory. *Physical Review D*, 104 (10), ARTN 104067.

DOI: 10.1103/PhysRevD.104.104067.

2020

Cisterna, Adolfo; Giribet, Gaston; Oliva, Julio; Pallikaris, Konstantinos (2020). Quasitopological electromagnetism and black holes. *Physical Review D*, 101 (12), ARTN 124041.

DOI: 10.1103/PhysRevD.101.124041.

Pablo Figueroa, Jose; Pallikaris, Konstantinos (2020). Quartic Horndeski, planar black holes, holographic aspects and universal bounds. *Journal of High Energy Physics*, 9, ARTN 090.

DOI: 10.1007/JHEP09(2020)090.

ELULOOKIRJELDUS

Isikuandmed

Nimi: Konstantinos Pallikaris
Sünniaeg: 25.09.1988
Telefon: 00306985710122
ORCID: 0000-0003-2403-892X
E-post: konstantinos.pallikaris@gmail.com

Haridus

2018–Present Tartu Ülikool, Teoreetilise füüsika labor, Füüsika instituut, PhD
2018 National Technical University of Athens, School of Applied Mathematics and Physics, Integrated Master (4+1 years)

Teadustöö

Teadustöö põhisuunad

ETIS VALDKOND: 4. Loodusteadused ja tehnika; 4.10. Füüsika; CERCS VALDKOND: P190 Matemaatiline ja üldine teoreetiline füüsika, klassikaline mehaanika, kvantmehaanika, relatiivsus, gravitatsioon, statistiline füüsika, termodünaamika

Lõppenud projektid

- PRG356 "Gravitatsiooni kalibratsiooniteooria: ühendamine, laiendused ja fenomenoloogia"
(01.01.2019–31.12.2023)
Vastutav täitja: Tomi Sebastian Koivisto
Tartu Ülikool, Loodus- ja täppisteaduste valdkond, füüsika instituut (koordinaator)
Finantseerija: Sihtasutus Eesti Teadusagentuur; Eraldatud summa: 1 148 375 EUR.
- SLTFY16133T (TK133) "Tume Universum (TK133)"
(01.03.2016–01.03.2023)
Vastutav täitja: Manuel Hohmann
Tartu Ülikool, Loodus- ja täppisteaduste valdkond, füüsika instituut (partner)
Finantseerija: Sihtasutus Archimedes; Eraldatud summa: 778 050 EUR.

Publikatsioonid

2025

Iosifidis, Damianos; Pallikaris, Konstantinos (2025). Cosmology of the complete quadratic metric affine gravity. *Physical Review D*, 111 (2), 30.

DOI: 10.1103/PhysRevD.111.023515.

Barrientos, Jose; Cisterna, Adolfo; Hassaine, Mokhtar; Pallikaris, Konstantinos (2025). Electromagnetized black holes and swirling backgrounds in nonlinear electrodynamics: The ModMax case. *Physics Letters B*, 860, ARTN 139214.

DOI: 10.1016/j.physletb.2024.139214.

Barrientos, Jose; Cisterna, Adolfo; Hassaine, Mokhtar; Muller, Keanu; Pallikaris, Konstantinos (2025). A new exact rotating spacetime in vacuum: The Kerr-Levi-Civita spacetime. *Physics Letters B*, 871, ARTN 140035.

DOI: 10.1016/j.physletb.2025.140035.

2024

Barrientos, Jose; Cisterna, Adolfo; Pallikaris, Konstantinos (2024). Plebański-Demiański à la Ehlers Harrison: exact rotating and accelerating type I black holes. *General Relativity and Gravitation*, 56 (9), ARTN 111.

DOI: 10.1007/s10714-024-03304-x.

Barrientos, Jose; Cisterna, Adolfo; Kolar, Ivan; Mueller, Keanu; Oyarzo, Marcelo; Pallikaris, Konstantinos (2024). Mixing "Magnetic" and "Electric" Ehlers-Harrison transformations: the electromagnetic swirling spacetime and novel type I backgrounds. *The European Physical Journal C*, 84 (7), ARTN 724.

DOI: 10.1140/epjc/s10052-024-13093-x.

2023

Iosifidis, Damianos; Pallikaris, Konstantinos (2023). Biconnection gravity as a statistical manifold. *Physical Review D*, 108 (4), ARTN 044026.

DOI: 10.1103/PhysRevD.108.044026.

Iosifidis, Damianos; Pallikaris, Konstantinos (2023). Describing metric-affine theories anew: alternative frameworks, examples and solutions. *Journal of Cosmology and Astroparticle Physics*, 2023 (5), ARTN 037.

DOI: 10.1088/1475-7516/2023/05/037.

Cisterna, Adolfo; Mueller, Keanu; Pallikaris, Konstantinos; Viganò, Adriano (2023).

Exact rotating wormholes via Ehlers transformations. *Physical Review D*, 108 (2), ARTN 024066.

DOI: 10.1103/PhysRevD.108.024066.

2021

Baggioli, Matteo; Cisterna, Adolfo; Pallikaris, Konstantinos (2021). Exploring the black hole spectrum of axionic Horndeski theory. *Physical Review D*, 104 (10), ARTN 104067.

DOI: 10.1103/PhysRevD.104.104067.

2020

Cisterna, Adolfo; Giribet, Gaston; Oliva, Julio; Pallikaris, Konstantinos (2020). Quasitopological electromagnetism and black holes. *Physical Review D*, 101 (12), ARTN 124041.

DOI: 10.1103/PhysRevD.101.124041.

Pablo Figueroa, Jose; Pallikaris, Konstantinos (2020). Quartic Horndeski, planar black holes, holographic aspects and universal bounds. *Journal of High Energy Physics*, 9, ARTN 090.

DOI: 10.1007/JHEP09(2020)090.

DISSERTATIONES PHYSICAE UNIVERSITATIS TARTUENSIS

1. **Andrus Ausmees.** XUV-induced electron emission and electron-phonon interaction in alkali halides. Tartu, 1991.
2. **Heiki Sõnajalg.** Shaping and recalling of light pulses by optical elements based on spectral hole burning. Tartu, 1991.
3. **Sergei Savihhin.** Ultrafast dynamics of F-centers and bound excitons from picosecond spectroscopy data. Tartu, 1991.
4. **Ergo Nõmmiste.** Leelishalogeniidide röntgenelektronemissioon kiiritamisel footonitega energiaga 70–140 eV. Tartu, 1991.
5. **Margus Rätsep.** Spectral gratings and their relaxation in some low-temperature impurity-doped glasses and crystals. Tartu, 1991.
6. **Tõnu Pullerits.** Primary energy transfer in photosynthesis. Model calculations. Tartu, 1991.
7. **Olev Saks.** Attoampri diapsoonis volude mõõtmise füüsikalised alused. Tartu, 1991.
8. **Andres Virro.** AlGaAsSb/GaSb heterostructure injection lasers. Tartu, 1991.
9. **Hans Korge.** Investigation of negative point discharge in pure nitrogen at atmospheric pressure. Tartu, 1992.
10. **Jüri Maksimov.** Nonlinear generation of laser VUV radiation for high-resolution spectroscopy. Tartu, 1992.
11. **Mark Aizengendler.** Photostimulated transformation of aggregate defects and spectral hole burning in a neutron-irradiated sapphire. Tartu, 1992.
12. **Hele Siimon.** Atomic layer molecular beam epitaxy of A^2B^6 compounds described on the basis of kinetic equations model. Tartu, 1992.
13. **Tõnu Reinot.** The kinetics of polariton luminescence, energy transfer and relaxation in anthracene. Tartu, 1992.
14. **Toomas Rõõm.** Paramagnetic H^{2-} and F^+ centers in CaO crystals: spectra, relaxation and recombination luminescence. Tallinn, 1993.
15. **Erko Jalviste.** Laser spectroscopy of some jet-cooled organic molecules. Tartu, 1993.
16. **Alvo Aabloo.** Studies of crystalline celluloses using potential energy calculations. Tartu, 1994.
17. **Peeter Paris.** Initiation of corona pulses. Tartu, 1994.
18. **Павел Рубин.** Локальные дефектные состояния в CuO_2 плоскостях высокотемпературных сверхпроводников. Тарту, 1994.
19. **Olavi Ollikainen.** Applications of persistent spectral hole burning in ultrafast optical neural networks, time-resolved spectroscopy and holographic interferometry. Tartu, 1996.
20. **Ülo Mets.** Methodological aspects of fluorescence correlation spectroscopy. Tartu, 1996.
21. **Mikhail Danilkin.** Interaction of intrinsic and impurity defects in CaS:Eu luminophors. Tartu, 1997.

22. **Ирина Кудрявцева.** Создание и стабилизация дефектов в кристаллах KBr, KCl, RbCl при облучении ВУФ-радиацией. Тарту, 1997.
23. **Andres Osvet.** Photochromic properties of radiation-induced defects in diamond. Tartu, 1998.
24. **Jüri Örd.** Classical and quantum aspects of geodesic multiplication. Tartu, 1998.
25. **Priit Sarv.** High resolution solid-state NMR studies of zeolites. Tartu, 1998.
26. **Сергей Долгов.** Электронные возбуждения и дефектообразование в некоторых оксидах металлов. Тарту, 1998.
27. **Kaupo Kukli.** Atomic layer deposition of artificially structured dielectric materials. Tartu, 1999.
28. **Ivo Heinmaa.** Nuclear resonance studies of local structure in $\text{RBa}_2\text{Cu}_3\text{O}_{6+x}$ compounds. Tartu, 1999.
29. **Aleksander Shelkan.** Hole states in CuO_2 planes of high temperature superconducting materials. Tartu, 1999.
30. **Dmitri Nevedrov.** Nonlinear effects in quantum lattices. Tartu, 1999.
31. **Rein Ruus.** Collapse of 3d (4f) orbitals in 2p (3d) excited configurations and its effect on the x-ray and electron spectra. Tartu, 1999.
32. **Valter Zazubovich.** Local relaxation in incommensurate and glassy solids studied by Spectral Hole Burning. Tartu, 1999.
33. **Indrek Reimand.** Picosecond dynamics of optical excitations in GaAs and other excitonic systems. Tartu, 2000.
34. **Vladimir Babin.** Spectroscopy of exciton states in some halide macro- and nanocrystals. Tartu, 2001.
35. **Toomas Plank.** Positive corona at combined DC and AC voltage. Tartu, 2001.
36. **Kristjan Leiger.** Pressure-induced effects in inhomogeneous spectra of doped solids. Tartu, 2002.
37. **Helle Kaasik.** Nonperturbative theory of multiphonon vibrational relaxation and nonradiative transitions. Tartu, 2002.
38. **Tõnu Laas.** Propagation of waves in curved spacetimes. Tartu, 2002.
39. **Rünno Lõhmus.** Application of novel hybrid methods in SPM studies of nanostructural materials. Tartu, 2002.
40. **Kaido Reivelt.** Optical implementation of propagation-invariant pulsed free-space wave fields. Tartu, 2003.
41. **Heiki Kasemägi.** The effect of nanoparticle additives on lithium-ion mobility in a polymer electrolyte. Tartu, 2003.
42. **Villu Repän.** Low current mode of negative corona. Tartu, 2004.
43. **Алексей Котлов.** Оксианионные диэлектрические кристаллы: зонная структура и электронные возбуждения. Тарту, 2004.
44. **Jaak Talts.** Continuous non-invasive blood pressure measurement: comparative and methodological studies of the differential servo-oscillometric method. Tartu, 2004.
45. **Margus Saal.** Studies of pre-big bang and braneworld cosmology. Tartu, 2004.

46. **Eduard Gerškevičs.** Dose to bone marrow and leukaemia risk in external beam radiotherapy of prostate cancer. Tartu, 2005.
47. **Sergey Shchemelyov.** Sum-frequency generation and multiphoton ionization in xenon under excitation by conical laser beams. Tartu, 2006.
48. **Valter Kiisk.** Optical investigation of metal-oxide thin films. Tartu, 2006.
49. **Jaan Aarik.** Atomic layer deposition of titanium, zirconium and hafnium dioxides: growth mechanisms and properties of thin films. Tartu, 2007.
50. **Astrid Rekker.** Colored-noise-controlled anomalous transport and phase transitions in complex systems. Tartu, 2007.
51. **Andres Punning.** Electromechanical characterization of ionic polymer-metal composite sensing actuators. Tartu, 2007.
52. **Indrek Jõgi.** Conduction mechanisms in thin atomic layer deposited films containing TiO₂. Tartu, 2007.
53. **Aleksei Krasnikov.** Luminescence and defects creation processes in lead tungstate crystals. Tartu, 2007.
54. **Küllike Rägo.** Superconducting properties of MgB₂ in a scenario with intra- and interband pairing channels. Tartu, 2008.
55. **Els Heinsalu.** Normal and anomalously slow diffusion under external fields. Tartu, 2008.
56. **Kuno Kooser.** Soft x-ray induced radiative and nonradiative core-hole decay processes in thin films and solids. Tartu, 2008.
57. **Vadim Boltrushko.** Theory of vibronic transitions with strong nonlinear vibronic interaction in solids. Tartu, 2008.
58. **Andi Hektor.** Neutrino Physics beyond the Standard Model. Tartu, 2008.
59. **Raavo Josepson.** Photoinduced field-assisted electron emission into gases. Tartu, 2008.
60. **Martti Pärs.** Study of spontaneous and photoinduced processes in molecular solids using high-resolution optical spectroscopy. Tartu, 2008.
61. **Kristjan Kannike.** Implications of neutrino masses. Tartu, 2008.
62. **Vigen Issahhanjan.** Hole and interstitial centres in radiation-resistant MgO single crystals. Tartu, 2008.
63. **Veera Krasnenko.** Computational modeling of fluorescent proteins. Tartu, 2008.
64. **Mait Müntel.** Detection of doubly charged higgs boson in the CMS detector. Tartu, 2008.
65. **Kalle Kepler.** Optimisation of patient doses and image quality in diagnostic radiology. Tartu, 2009.
66. **Jüri Raud.** Study of negative glow and positive column regions of capillary HF discharge. Tartu, 2009.
67. **Sven Lange.** Spectroscopic and phase-stabilisation properties of pure and rare-earth ions activated ZrO₂ and HfO₂. Tartu, 2010.
68. **Aarne Kasikov.** Optical characterization of inhomogeneous thin films. Tartu, 2010.
69. **Heli Valtna-Lukner.** Superluminally propagating localized optical pulses. Tartu, 2010.

70. **Artjom Vargunin.** Stochastic and deterministic features of ordering in the systems with a phase transition. Tartu, 2010.
71. **Hannes Liivat.** Probing new physics in e^+e^- annihilations into heavy particles via spin orientation effects. Tartu, 2010.
72. **Tanel Mullari.** On the second order relativistic deviation equation and its applications. Tartu, 2010.
73. **Aleksandr Lissovski.** Pulsed high-pressure discharge in argon: spectroscopic diagnostics, modeling and development. Tartu, 2010.
74. **Aile Tamm.** Atomic layer deposition of high-permittivity insulators from cyclopentadienyl-based precursors. Tartu, 2010.
75. **Janek Uin.** Electrical separation for generating standard aerosols in a wide particle size range. Tartu, 2011.
76. **Svetlana Ganina.** Hajusandmetega ülesanded kui üks võimalus füüsikaõppe efektiivsuse tõstmiseks. Tartu, 2011
77. **Joel Kuusk.** Measurement of top-of-canopy spectral reflectance of forests for developing vegetation radiative transfer models. Tartu, 2011.
78. **Raul Rammula.** Atomic layer deposition of HfO_2 – nucleation, growth and structure development of thin films. Tartu, 2011.
79. **Сергей Наконечный.** Исследование электронно-дырочных и интерстициал-вакансионных процессов в монокристаллах MgO и LiF методами термоактивационной спектроскопии. Тарту, 2011.
80. **Niina Voropajeva.** Elementary excitations near the boundary of a strongly correlated crystal. Tartu, 2011.
81. **Martin Timusk.** Development and characterization of hybrid electro-optical materials. Tartu, 2012, 106 p.
82. **Merle Lust.** Assessment of dose components to Estonian population. Tartu, 2012, 84 p.
83. **Karl Kruusamäe.** Deformation-dependent electrode impedance of ionic electromechanically active polymers. Tartu, 2012, 128 p.
84. **Liis Rebane.** Measurement of the $W \rightarrow \tau\nu$ cross section and a search for a doubly charged Higgs boson decaying to τ -leptons with the CMS detector. Tartu, 2012, 156 p.
85. **Jevgeni Šablonin.** Processes of structural defect creation in pure and doped MgO and NaCl single crystals under condition of low or super high density of electronic excitations. Tartu, 2013, 145 p.
86. **Riho Vendt.** Combined method for establishment and dissemination of the international temperature scale. Tartu, 2013, 108 p.
87. **Peeter Piksarv.** Spatiotemporal characterization of diffractive and non-diffractive light pulses. Tartu, 2013, 156 p.
88. **Anna Šugai.** Creation of structural defects under superhigh-dense irradiation of wide-gap metal oxides. Tartu, 2013, 108 p.
89. **Ivar Kuusik.** Soft X-ray spectroscopy of insulators. Tartu, 2013, 113 p.
90. **Viktor Vabson.** Measurement uncertainty in Estonian Standard Laboratory for Mass. Tartu, 2013, 134 p.

91. **Kaupo Voormansik.** X-band synthetic aperture radar applications for environmental monitoring. Tartu, 2014, 117 p.
92. **Deivid Pugal.** hp-FEM model of IPMC deformation. Tartu, 2014, 143 p.
93. **Siim Pikker.** Modification in the emission and spectral shape of photo-stable fluorophores by nanometallic structures. Tartu, 2014, 98 p.
94. **Mihkel Pajusalu.** Localized Photosynthetic Excitons. Tartu, 2014, 183 p.
95. **Taavi Vaikjärv.** Consideration of non-adiabaticity of the Pseudo-Jahn-Teller effect: contribution of phonons. Tartu, 2014, 129 p.
96. **Martin Vilbaste.** Uncertainty sources and analysis methods in realizing SI units of air humidity in Estonia. Tartu, 2014, 111 p.
97. **Mihkel Rähn.** Experimental nanophotonics: single-photon sources- and nanofiber-related studies. Tartu, 2015, 107 p.
98. **Raul Laasner.** Excited state dynamics under high excitation densities in tungstates. Tartu, 2015, 125 p.
99. **Andris Slavinskis.** EST Cube-1 attitude determination. Tartu, 2015, 104 p.
100. **Karlis Zalite.** Radar Remote Sensing for Monitoring Forest Floods and Agricultural Grasslands. Tartu, 2016, 124 p.
101. **Kaarel Piip.** Development of LIBS for *in-situ* study of ITER relevant materials. Tartu, 2016, 93 p.
102. **Kadri Isakar.** ²¹⁰Pb in Estonian air: long term study of activity concentrations and origin of radioactive lead. Tartu, 2016, 107 p.
103. **Artur Tamm.** High entropy alloys: study of structural properties and irradiation response. Tartu, 2016, 115 p.
104. **Rasmus Talviste.** Atmospheric-pressure He plasma jet: effect of dielectric tube diameter. Tartu, 2016, 107 p.
105. **Andres Tiko.** Measurement of single top quark properties with the CMS detector. Tartu, 2016, 161 p.
106. **Aire Olesk.** Hemiboreal Forest Mapping with Interferometric Synthetic Aperture Radar. Tartu, 2016, 121 p.
107. **Fred Valk.** Nitrogen emission spectrum as a measure of electric field strength in low-temperature gas discharges. Tartu, 2016, 149 p.
108. **Manoop Chenchiliyan.** Nano-structural Constraints for the Picosecond Excitation Energy Migration and Trapping in Photosynthetic Membranes of Bacteria. Tartu, 2016, 115p.
109. **Lauri Kaldamäe.** Fermion mass and spin polarisation effects in top quark pair production and the decay of the higgs boson. Tartu, 2017, 104 p.
110. **Marek Oja.** Investigation of nano-size α - and transition alumina by means of VUV and cathodoluminescence spectroscopy. Tartu, 2017, 89 p.
111. **Viktoriia Levushkina.** Energy transfer processes in the solid solutions of complex oxides. Tartu, 2017, 101 p.
112. **Mikk Antsov.** Tribomechanical properties of individual 1D nanostructures: experimental measurements supported by finite element method simulations. Tartu, 2017, 101 p.
113. **Hardi Veermäe.** Dark matter with long range vector-mediated interactions. Tartu, 2017, 137 p.

114. **Aris Auzans.** Development of computational model for nuclear energy systems analysis: natural resources optimisation and radiological impact minimization. Tartu, 2018, 138 p.
115. **Aleksandr Gurev.** Coherent fluctuating nephelometry application in laboratory practice. Tartu, 2018, 150 p.
116. **Ardi Loot.** Enhanced spontaneous parametric downconversion in plasmonic and dielectric structures. Tartu, 2018, 164 p.
117. **Andreas Valdmann.** Generation and characterization of accelerating light pulses. Tartu, 2019, 85 p.
118. **Mikk Vahtrus.** Structure-dependent mechanical properties of individual one-dimensional metal-oxide nanostructures. Tartu, 2019, 110 p.
119. **Ott Vilson.** Transformation properties and invariants in scalar-tensor theories of gravity. Tartu, 2019, 183 p.
120. **Indrek Sünter.** Design and characterisation of subsystems and software for ESTCube-1 nanosatellite. Tartu, 2019, 195 p.
121. **Marko Eltermann.** Analysis of samarium doped TiO₂ optical and multi-response oxygen sensing capabilities. Tartu, 2019, 113 p.
122. **Kalev Erme.** The effect of catalysts in plasma oxidation of nitrogen oxides. Tartu, 2019, 114 p.
123. **Sergey Koshkarev.** A phenomenological feasibility study of the possible impact of the intrinsic heavy quark (charm) mechanism on the production of doubly heavy mesons and baryons. Tartu, 2020, 134 p.
124. **Kristi Uudeberg.** Optical Water Type Guided Approach to Estimate Water Quality in Inland and Coastal Waters. Tartu, 2020, 222 p.
125. **Daniel Blixt.** Hamiltonian analysis of covariant teleparallel theories of gravity. Tartu, 2021, 142 p.
126. **Ulbossyn Ualikhanova.** Gravity theories based on torsion: theoretical and observational constraints. Tartu, 2021, 154 p.
127. **Iaroslav Iakubivskiy.** Nanospacecraft for Technology Demonstration and Science Missions. Tartu, 2021, 177 p.
128. **Heido Trofimov.** Polluted clouds at air pollution hot spots help to better understand anthropogenic impacts on Earth's climate. Tartu, 2022, 96 p.
129. **Ott Rebane.** *In situ* non-contact sensing of microbiological contamination by fluorescence spectroscopy. Tartu, 2022, 157 p.
130. **Juhan Saaring.** Ultrafast Relaxation Processes in Ternary Hexafluorides Studied under Synchrotron Radiation Excitation. Tartu, 2022, 106 p.
131. **Ahmet Ilker Topuz.** Quantitative and qualitative investigations for muon scattering tomography via GEANT4 simulations: A computational study. Tartu, 2023, 163 p.
132. **Nico Benincasa.** Phase transitions and gravitational waves in models of dark matter. Tartu, 2023, 206 p.
133. **Kaja Pae.** Electron-phonon interactions in local degenerate electronic states in solids. Tartu, 2024, 201 p.
134. **Kristjan Määrsepp.** Phenomenological implications of Standard Model extensions. Tartu, 2024, 136 p.

135. **Ye Wang.** Investigating the properties of metal surfaces under high electric fields based on ab initio calculations. Tartu, 2024, 107 p.
136. **Laxmipriya Pati.** The effects of non-Riemannian connection in teleparallel gravity. Tartu, 2025, 172 p.
137. **Débora Aguiar Gomes.** Theoretical and astrophysical aspects of extended general relativity. Tartu, 2025, 169 p.
138. **Mina Hajizadeh Omaslanolya.** Structure and dynamics of photoactive proteins studied by (in situ-) neutron scattering methods. Tartu, 2025, 150 p.
139. **Aditya Savio Paul.** Advancing the study of small solar system bodies through multi-agent mapping and characterization. Tartu, 2025, 244 p.
140. **Sanu Bifal Maji.** Synthesis and luminescence investigation of nanoparticles doped with Pr^{3+} ions in selected fluoride and phosphate hosts. Tartu, 2025, 114 p.
141. **Ernest Michael Priidik Gallagher.** On the internal gauge theory analogy to the Cartan Khronon theory of gravity. Tartu, 2026, 221 p.
142. **Maria Naeem.** First order electroweak radiative corrections to the decay of the polarised W boson. Tartu, 2026, 236 p.