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Weak Efficiency of Foreign Exchange Rates

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WEAK EFFICIENCY OF FOREIGN EXCHANGE RATES

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Abstract

The efficient market hypothesis (EMH) concept has been fundamental in financial economics, proposing that financial markets accurately reflect all available information. However, empirical studies have raised questions about the degree of efficiency in various financial markets, including the foreign exchange market. This thesis focuses on the weak efficiency of foreign exchange rates, which implies that current exchange rates reflect past exchange rate information but not non-public or non-historical information. The study explores the presence of unit roots and cointegration relationships and utilizes Error Correction Models (ECMs) to assess the efficiency of foreign exchange rates. The empirical analysis covers 6,200 daily observations of ten exchange rates between January 1999 to March 2023. The study finds evidence of non-stationarity in the series, cointegration relationships among some exchange rates, and speed of adjustment back to equilibrium after a shock. These findings provide insights into the weak efficiency of various foreign exchange rates and include implications for market participants, policymakers, and investors.

CERCS research specialisation: P160 Statistics, operations research, programming, financial and actuarial mathematics.

Key Words: EMH, ECT, Unit roots, Cointegration, Error Correction Model.

VALUUTAKURSSIDE NÕRK EFEKTIIVSUS.

Magistritöö

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Lühikokkuvõte

Efektiivse turu hüpoteesi (EMH) kontseptsioon on olnud finantsökonoomikas põhiline, pakkudes välja, et finantsturud kajastavad täpselt kogu olemasolevat teavet. Empiirilised uuringud on aga tekitanud küsimusi erinevate finantsturgude, sealhulgas valuutaturgude efektiivsuse kohta. See magistritöö keskendub valuutaturgude nõrgale efektiivsuse uurimisele ning nõrga efektiivsuse hüpoteesist lähtudes ei tohiks valuutakursside vahel olla kointegratsiooniseoseid. Töös uuritakse valuutakursside võimalikku kointegratsiooni ning kointegratsiooni korral hinnatakse vastavaid veaparandusmudeleid (ECM). Empiiriline analüüs hõlmab 6200 kümne vahetuskursi päevaseid vaatlusi ajavahemikus jaanuar 1999 kuni märts 2023. Osutub, et enamike valuutakursside korral kointegratsiooniseoseid ei leitud ning vastavad turud olid nõrgalt efektiivsed, kuid kointegratsiooniseos oli euro ja Taani krooni vahel. Samuti uuriti, kas eksisteerib kointegratsooniseoseid, mis oleks olulised nii enne kui pärast finantskriisi 2007-2009.

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Märksõnad: Efektiivse turu hüpotees, valuutakursid, kointegratsioon, veaparandusmudel.

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Introduction

This study investigates the concept of weak efficiency in foreign exchange rates, focusing on the degree to which current exchange rates incorporate past exchange rate information while failing to reflect non-public information. The efficient market hypothesis (EMH) forms the theoretical foundation of this research, positing that financial markets efficiently integrate all available information. However, empirical evidence has cast doubts on the efficiency of various financial markets, including the foreign exchange market.

The research is structured into three main chapters. The first chapter provides the concept of the weak efficiency of foreign exchange rates and the previous studies related to the subject.

The second chapter involves the theoretical overview of time series analysis, stationary and non-stationary processes, unit roots, and cointegration. Unit root tests are employed to identify non-stationarity, and cointegration focuses on identifying long-term equilibrium relationships among multiple time series variables, which can indicate a deviation from weak efficiency in exchange rates.

The third chapter presents an empirical analysis of 6,200 daily observations of ten exchange rates from January 1999 to March 2023. Unit root tests, cointegration tests, such as the Engle-Granger two-step procedure, and Johansen Bivariate and Multivariate approaches, are utilized to investigate cointegration relationships among exchange rates. Additionally, the study employs Error Correction Models (ECMs) to estimate short-term deviations from long-term equilibrium and evaluate the speed of adjustment after a shock.

By exploring unit roots, cointegration relationships, and utilizing ECM, the findings of this research contribute to the understanding of weak efficiency in

foreign exchange markets, and valuable insights are gained into the efficiency of foreign exchange rates.

1 Weak Efficiency of Foreign Exchange Rates

Weak efficiency is a concept from the Efficient Market Hypothesis (EMH) that claims that all publicly available information is already reflected in the current market price of exchange pairs. If weak efficiency holds in the foreign exchange market, then the best forecast of one period is the last known exchange rate and no other variables that can predict future prices.

Cointegration and weak efficiency are closely related concepts that interact with the financial markets, and their relationship can be used to explain the behavior of foreign exchange rates; if the market is weak efficient, it implies the exchange rates are not cointegrated, and therefore there is no predictable long-term relationship between the exchange rates to make profitable trades in the short term.

A market is considered efficient if trading based on known private information does not result in higher profits and there is no predictable long-term relationship between exchange rates that facilitates profitable short-term transactions. Therefore, cointegration between exchange rates violates weak efficiency because of the ability to forecast.

1.1 Previous findings concerning Weak efficiency

Based on an argument presented by Granger, 1986, a market is not efficient if cointegration between pairs of exchange rates can be evaluated, as cointegration implies that at least one exchange rate is predictable using the past long-run relationship with another.

Cointegration and weak efficiency are related in the context of the foreign exchange market because cointegration implies a predictable long-term rela-

tionship between exchange pairs. This indicates that weak efficiency may not persist over the short term, as investors can use this long-term relationship to make profitable short-term trades. If, on the other hand, weak efficiency holds over the short period, this suggests that currency pairs are not cointegrated, and therefore their long-term relationship cannot be predicted.

MacDonald and Taylor, 1989 and Hakkio and Rush, 1989 research examine the efficiency of foreign exchange markets by testing cointegration analysis using Engle-Granger's two-step procedure.

The daily exchange rates data for three pairs, USD/GBP, USD/CAD, and USD/Deutsch Mark, and the period of observation is from January 1974 to December 1985. Based on the Engle-Granger two-step approach, the result of their findings suggested the rejection of the null hypothesis only on the USD/German Mark, showing evidence of cointegration between the exchange pair and indicating that the pair was not weak efficient, contrary to the other exchange pairs with no evidence of cointegration and suggest that the USD/GBP and USD/CAD were relatively more efficient in the weak form during the period of observation.

BAILLIE and BOLLERSLEV, 1989 on their cointegration analysis among daily exchange rates for seven currency pairs: the US dollar/British pound, the US dollar/Canadian dollar, the US dollar/French franc, the US dollar/German mark, the US dollar/Italian lira, the US dollar/Japanese yen, and the US dollar/Swiss franc. The data covered the period from January 1973 to December 1985. However, they employed the Johansen Multivariate approach, contrary to MacDonald and Taylor's (1989) and Hakkio and Rush's (1989) methods. Evidence of cointegration was found among the seven exchange pairs tested, suggesting that the exchange markets for these pairs were not a weak form of the efficient market during the sample period.

BAILLIE and BOLLERSLEV, 1989 concluded that their findings support the view that foreign exchange markets are not weak-form efficient, implying that past exchange rate information is insufficient to fully predict future exchange rate movements. They also suggested that technical trading rules based on past exchange rate movements were not profitable, which implies that market participants could not consistently exploit deviations from market efficiency. Using the same exchange rate pairs, Diebold, Gardeazabal, and Yilmaz, 1994 found no cointegration relationship among the exchange rate pairs of 1,245 observations, that runs through March 1980 to January 1985, after a drift (constant term) in the model.

Coleman, 1990 studied 18 daily exchange rate data between 1973 to 1989, using the Engle-Granger procedure, and found no evidence of cointegration among the currency pairs. Copeland, 1991 used the Johansen procedure to examine bivariate cointegration among major daily exchange rate pairs; US dollar, Japanese yen, British pound, Swiss franc, German mark, Canadian dollar, and Australian dollar, from 1976 to 1990. Copeland found no cointegration among the ten combinations of pairs tested and concluded that the market is efficient.

Sephton and Larsen, 1991 examined the existence of cointegration in a system of four exchange rates, namely the Canadian dollar, British pound, Japanese yen, and Swiss franc, using data from 1975 to 1986. They are inconclusive about the result based on their suggestion that cointegration among the exchange rates depends on the observation period.

Masih and Masih, 1994 explore the concept of market efficiency in the context of six European foreign exchange markets, using daily exchange rate data. They used various cointegration tests to assess the presence of a long-run relationship between exchange rates, including the Johansen and Engle-Granger

methods. Their results suggest that the six foreign exchange markets they studied were not consistently weak-form efficient during the study period and that the market inefficiency was not uniform across the markets.

Similarly, Crowder, 1994 investigated long-run relationships among the Canadian dollar, Japanese yen, and British pound, utilizing daily exchange rate data from January 1974 to July 1990. The author employed various cointegration methods, including the Johansen approach, the Engle-Granger two-step method, and vector autoregressions. The results indicate evidence of cointegration among the three currency pairs, suggesting a long-run relationship between them. However, the author also notes that short-term predictability is limited, and interpreting short-term fluctuations in exchange rates can be challenging.

Various tests have been conducted to assess cointegration and its relation to market efficiency. However, these tests have not provided conclusive and definitive results.

2 Unit Root Tests and Cointegration

2.1 Introduction to Time Series Analysis

A time series is a collection of quantitative observations ordered according to time. Time is generally considered a discrete variable. The main goal of time series analysis is to forecast but also find relationships between variables.

Time series is a set of observations x_t made sequentially over time and is considered discrete when observations are taken exclusively at specific times, typically at equal intervals.

A stochastic or random process is a family of random variables (X_t) $t \in Z$. A time series data can be seen as one realization of the random process; for every random variable X_t corresponds to only one realization x_t.

There are essential characteristics of time series; Trend and Seasonal variations.

A trend indicates a common tendency of data, either linear or non-linear. Over a long period of time, it may move upward or increase, or downward or decrease. The trend is a consistent and long-term general tendency of data movement, and the data don't need to move in the same direction to be considered a trend.

On the other hand, seasonal variations occur due to natural or artificial forces or variations; they are changes in time series that occur within less than 12 months and exhibit the same pattern of upward or downward growth over the time series' 12-month period. These variations are frequently recorded hourly, daily, weekly, quarterly, and monthly.

2.2 Stationary Process

Definition 1. The **k-th moment** of a random vector $(X_1, X_2, \ldots, X_k)$ is defined by:

$$k_{\alpha_1,\alpha_2,\cdots,\alpha_k} = \mathrm{E}(X_1^{\alpha_1}, X_2^{\alpha_2}, \cdots, X_k^{\alpha_k}),$$

where $\alpha_i, i = 1, 2, \cdots, k$ are non-negative integers and $\sum_{i=1}^{k} \alpha_i = k$.

Definition 2. Let X_t $t \in T$, be a stochastic process, then X_t is **strictly stationary** if for every positive interger k, and for every integer τ, the random vectors $(X_1, X_2, \ldots, X_k)$ and $(X_{1+\tau}, X_{2+\tau}, \ldots, X_{k+\tau})$ have the same distribution.

Definition 3. A stochastic process X_t $t \in T$ is said to be **k-th order weakly stationary** if all moments up to k of the random vectors $(X_1, X_2, \cdots, X_k)$ and $(X_{1+\tau}, X_{2+\tau}, \cdots, X_{k+\tau})$ are finite and equal, for every positive integer k and every integer τ.

Follows from definition 3, a stochastic process $X_t, t \in T$ is said to be **second-order weakly stationary** if it satisfies the following conditions:

- The mean of X_t is constant for all $t \in T$, denoted by $E[X_t] = \mu$.
- The variance of X_t is constant for all $t \in T$, denoted by $Var[X_t] = \sigma^2$.
- The autocovariance function $\gamma(t, s)$ is only a function of the time lag $|t - s|$, denoted by $\gamma(t, s) = \gamma(|t - s|)$.

Hereafter, stationarity means that the process is a second-weakly stationary process.

Some commonly used stationary models

White noise process

A white noise process is a stochastic process in which each observation is a random variable that is independently and identically distributed with a constant mean and variance; the random variable at any given time is not dependent on its past values or any other variable and is uncorrelated with all other observations in the process (i.e., no memory).

If a stochastic process ε_t is white noise process with mean value 0 and variance σ^2, then it can be presented as:

$$\varepsilon_t \sim \mathrm{WN}(0, \sigma^2).$$

If a white noise process ε_t is also assumed to be normal, then the process is strictly stationary.

Its autocovariance function *acvf* is given by:

$$\gamma(\tau) = cov(\varepsilon_t, \varepsilon_{t+\tau}) = 0; \ \tau \neq 0,$$

and the autocorrelation function *acf* is given by:

$$\rho(\tau) = \begin{cases} 1, & \text{if } \tau = 0, \\ 0, & \text{if } \tau \neq 0. \end{cases}$$

Moving-average (MA) processes

A moving average MA(q) is a time series model where each observation in the series is expressed as a linear combination of the current and past white

noise error terms. Specifically, an MA(q) process is defined as:

$$X_t = \mu + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i},$$

where ε_t is a white noise process with mean zero and variance σ^2.

$$\bar{X}_t = X_t - \mu.$$

Let L be the lag operator defined as $LX_t = X_{t-1}$, then the compact form of the process can be presented as:

$$\bar{X}_t = \theta(L)\varepsilon_t, \qquad \theta(x) = 1 - \sum_{i=1}^{q} \theta_i x^i.$$

Autoregressive (AR) processes

Suppose that ε_t is a white noise process with mean zero and variance σ^2. Then the process X_t given by

$$X_t = \mu + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + \varepsilon_t,$$

is an autoregressive process of order p, denoted by AR(p).

The compact form of AR(p) is presented as:

$$\phi(L)\bar{X}_t = \varepsilon_t, \qquad \phi(x) = 1 - \sum_{i=1}^{p} \phi_i x^i.$$

- AR(1) process is stationary if and only if $|\phi_1| < 1$.
- AR(p) process is the second order weakly stationary if and only if all

the modules of the roots x_i of the polynomial

$$\phi(x) = 1 - \sum_{i=1}^{p} \phi_i x^i,$$

are greater than one, i.e. $|x_i| > 1$, $i = 1, 2, ..., p$.

Definition. AR(p) process is said to have a unit root if at least one solution of the equation $\phi(x) = 0$ is 1.

Autoregressive moving-average (ARMA) processes

ARMA models are the combinations of AR and MA models. An ARMA(p,q) model is defined as

$$X_t = \mu + \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_p X_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + ... + \theta_q \varepsilon_{t-q}.$$

The compact form of the ARMA(p, q) is given as:

$$\phi(L)\bar{X}_t = \theta(L)\varepsilon_t.$$

The ARMA(p, q) process is the second order weakly stationary if and only if the modules of roots x_i of the polynomial $\phi(x) = 1 - \sum_{i=1}^{p} \phi_i x^i$ are greater than 1; i.e., $|x_i| > 1$, $i = 1, 2, \cdots, p$.

2.3 Non-stationary processes

Analyzing non-stationary time series requires understanding the trending behavior and the causes of the non-stationarity. Otherwise, it might generate artificial movements in the resulting time series.

Definition 4. A process is said to be **trend-stationary**, if it has a deterministic trend that can be removed through a suitable transformation, such as detrending, and the resulting time series, is stationary in the weak sense (i.e., it has a constant mean, variance, and autocovariance). In general, a trend-stationary process can be presented in the form

$$X_t = m_t + Y_t,$$

where m_t is a deterministic function that captures the long-term behavior of the process over time, and Y_t is a stationary process with a zero mean and a constant variance.

Definition 5. A stochastic process X_t is said to be **difference-stationary** if the process requires differencing to make it stationary. An example is a random walk with or without a drift term.

Consider the following processes:

- A linear trend model:

$$Y_t = a + bt + \varepsilon_t, \tag{1}$$

  where the error term ε_t is assumed to be stationary, and therefore, the process Y_t is trend-stationary.

- A random walk has the form:

$$Y_t = Y_{t-1} + \varepsilon_t. \tag{2}$$

If the error term ε_t is stationary, then by differencing, we get

$$\Delta Y_t = \varepsilon_t,$$

which means the process Y_t is considered a difference-stationary process.

- A Random walk with drift has the form:

$$Y_t = a + Y_{t-1} + \varepsilon_t, \tag{3}$$

or

$$\Delta Y_t = a + \varepsilon_t.$$

If ε_t is stationary, then the process Y_t is difference stationary.

Generally, when a nonstationary series can be transformed to be stationary by differencing once, the series is said to be **integrated of order 1**, denoted by $I(1)$. If the series needs to be differenced k times to be stationary, then the series is said to be $I(k)$.

Random walk processes are commonly used to model time series that exhibit trends or drifts, as they are not stationary, such as stock prices or exchange rates.

This means that the first differences of the series are stationary. For example, suppose we have a non-stationary time series of stock prices. In that case, we can take the first differences of the series to make it stationary, to make its behavior more predictable. Unit roots can complicate the time series data analysis, affecting the estimation of relationships between variables.

Suppose that a time series follows the following AR(1) process:

$$Y_t = \phi Y_{t-1} + \varepsilon_t, \tag{4}$$

where ε_t is a white noise error term with mean 0 and variance σ^2, and ϕ is a constant coefficient that determines the extent to which the current value of the time series depends on its past values.

If $\phi = 1$, then the time series has a unit root and is difference-stationary.

We can present equation (4) in the form

$$\Delta Y_t = \gamma Y_{t-1} + \varepsilon_t, \tag{5}$$

where $\gamma = \phi - 1$.

If $\gamma = 0$ from this equation (5), then the process Y_t has a unit root.

The hypothesis statement can be presented as follows:

$$\begin{cases} H_0 : & \gamma = 0, \ \ non - stationary, \\ H_1 : & \gamma < 0, \ \ stationary. \end{cases}$$

Unit roots are often detected using statistical tests such as the augmented Dickey-Fuller (ADF) test or the Phillips-Perron (PP) test, which test for the presence of a unit root in a time series. These tests are based on regression models that account for the existence of unit roots.

2.4 Unit Root Tests

In the case of equation (4), we assume that there is no drift and no trend; therefore, for testing unit root, the following test equations are also considered:

Equation with drift:

$$\Delta Y_t = \alpha + \gamma Y_{t-1} + \varepsilon_t, \tag{6}$$

Equation with drift and trend

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \varepsilon_t, \tag{7}$$

where ΔY_t is the first difference of the time series at time t, α is a constant term, βt is a time trend, γ is the coefficient on the lagged dependent variable Y_{t-1}, ε_t is an error term.

All the previous test equations (5), (6), and (7) are only considered with the assumption that Y_t only depend on one lag Y_{t-1}, for testing for a unit root in the case of AR(p), then we consider the Augmented Dickey-Fuller test.

The Augmented Dickey-Fuller (ADF) test is an extension of the Dickey-Fuller that takes account of the autocorrelation in the error terms.

The general form can be presented below:

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta Y_{t-i} + \varepsilon_t, \tag{8}$$

where ε_t is white noise, p is the number of lags, and the autoregressive terms included in the model is $\sum_{i=1}^{p} \phi_i \Delta Y_{t-i}$.

The ADF test statistic is based on the regression coefficient γ, given as:

$$DF_\tau = \frac{\hat{\gamma}}{SE(\hat{\gamma})},$$

where $\hat{\gamma}$ is the estimated value of γ under the null hypothesis, and $SE(\hat{\gamma})$ is the standard error of the estimated γ.

The test statistic is compared against the critical values. These critical values are not the standard t-statistics critical values but are computed by Dickey-Fuller through simulations. Another important factor is that these critical values are different for different test equations (6) - (8).

The null and alternative hypotheses are given by:

$$\begin{cases} H_0: & \gamma = 0, \ \texttt{unit root} \\ H_1: & \gamma < 0, \ \texttt{no unit root} \end{cases}$$

In addition to the critical values, Dickey-Fuller (1981) also provides F-statistics $\phi 1$, $\phi 2$, $\phi 3$ used to test the joint hypotheses in Table 1 using the formula below:

$$\phi_i = \frac{[RSS(restricted) - RSS(unrestricted)]/r}{RSS(unrestricted)/T - k},$$

RSS is the sum of squared residuals, r is the number of restrictions, T is the number of used observations, k is the number of estimated parameters, and $T - k$ is the degree of freedom.

Implementing the ADF test involves testing three equations, starting by testing the general form of the equation presented in (8); which incorporates both

the trend and the drift in the test equation.

The procedure involves testing for a unit root of $H_0 : \gamma = 0$ using τ_3 statistic. If the null hypothesis is rejected, conclude that the series has no unit root or the series is trend-stationary.

Otherwise, if $H_0 : \gamma = 0$ is not rejected, test the significance of the trend term given that a unit root exists, i.e., $H_0 : \gamma = \beta = 0$ using the ϕ_3 statistic.

– If $H_0 : \gamma = \beta = 0$ is rejected, estimate the restricted model of the form:

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \sum_{i=1}^{p-1} \phi_i \Delta Y_{t-i} + \varepsilon_t,$$

and test $H_0 : \beta = 0$, using t statistic. If H_0 is rejected, conclude the process has a unit root and is difference stationary with a quadratic trend.

– If $H_0 : \gamma = \beta = 0$ is not rejected, it means there is no trend and can proceed to estimate the second test equation.

Test equation 2: ADF with drift

$$\Delta Y_t = \alpha + \gamma Y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta Y_{t-i} + \varepsilon_t. \tag{9}$$

Similar to the first, test $H_0 : \gamma = 0$ using τ_2 statistic. If H_0 is rejected, stop and conclude that the series is trend-stationary. Otherwise, if $H_0 : \gamma = 0$ is not rejected, test the null hypothesis of $H_0 : \gamma = \alpha = 0$ using ϕ_1 statistics. If H_0 is rejected in this case, estimate the restricted model of the form:

$$\Delta Y_t = \alpha + \gamma Y_{t-1} + \sum_{i=1}^{p-1} \phi_i \Delta Y_{t-i} + \varepsilon_t,$$

and test $H_0 : \alpha = 0$, using t statistic. If H_0 is rejected, conclude the series has a unit root with drift. Otherwise if $H_0 : \alpha = 0$ is not rejected, estimate

the test equation below:

Test equation 1: ADF with no drift and no trend

$$\Delta Y_t = \gamma Y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta Y_{t-i} + \varepsilon_t, \tag{10}$$

and test the null hypothesis of $H_0 : \gamma = 0$, if H_0 is rejected, it means no unit root present, otherwise, if H_0 is not rejected, conclude there is a unit root in the series with no drift and no trend.

Summary of Dickey-Fuller Test

Models	**Hypothesis**	**Test Statistic**	**Crit Values at 5% and 10%**
$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \varepsilon_t$	$\gamma = 0$ $\gamma = \beta = 0$ $\gamma = \alpha = \beta = 0$	τ_3 ϕ_3 ϕ_2	-3.45 and -4.04 6.49 and 8.73 4.88 and 6.50
$\Delta y_t = \alpha + \gamma y_{t-1} + \varepsilon_t$	$\gamma = 0$ $\gamma = \alpha = 0$	τ_2 ϕ_1	-2.89 and -3.51 4.71 and 6.70
$\Delta y_t = \gamma y_{t-1} + \varepsilon_t$	$\gamma = 0$	τ_1	-1.95 and -2.60

Table 1: Model with trend, drift, and none

ϕ_1 tests the null hypothesis of $\gamma = \alpha = 0$, ϕ_2 tests the null hypothesis of $\gamma = \alpha = \beta = 0$, and ϕ_3 tests the hypothesis of $\gamma = \beta = 0$.

The Phillips-Perron Test

The Phillips-Perron (PP) test is an alternative to the Dickey-Fuller test. It is based on a regression model that includes lagged differences in the time series and other explanatory variables that can affect the behavior of the series.

The PP test model is AR(1) which can be written as:

$$Y_t = \gamma Y_{t-1} + \varepsilon_t. \tag{11}$$

The PP test uses a modified version of the ADF test statistic that accounts for serial correlation in the residuals.

The test involves regressing the first differences of a time series against its lagged levels and testing the null hypothesis that the coefficient on the lagged level variable is zero. If the null hypothesis is rejected, it indicates that the time series is stationary and does not have a unit root.

The null and the alternative hypothesis of the PP test is stated below:

$$\begin{cases} H_0: \quad \gamma = 1, \ \texttt{has a unit root}, \\ H_1: \quad \gamma < 1, \ \texttt{has no unit root}. \end{cases}$$

The test statistic is then calculated based on the residuals of the regression and compared to critical values to determine whether the null hypothesis of a unit root can be rejected.

An advantage of the PP test over the ADF test is that it allows the errors in the regression equation to be correlated with each other, which is essential because autocorrelated errors can affect the reliability of the test. Another advantage is that it allows for various data-generating processes, such as trend-stationary, difference-stationary, and mixed processes.

2.5 Cointegration

Cointegration is a statistical concept that describes the long-term relationship between two or more non-stationary time series. When two or more time series are cointegrated, it implies that they share a common stochastic trend, even if they may exhibit short-term fluctuations or deviations from this trend.

Cointegration is a useful concept in econometrics and finance, often used to analyze the relationship between different economic variables, such as various countries' interest rates, inflation, and exchange rates. By identifying cointegrated relationships between these variables, it is possible to estimate the long-term relationship and predict the behavior of the variables over time.

The concept of cointegration is closely related to unit roots, which describe the non-stationary behavior of time series. If two or more time series have unit roots, they cannot be meaningfully compared or analyzed in a regression framework. However, if the time series are cointegrated, estimating their relationship is possible using a vector error correction model (VECM) technique.

Suppose there exists a linear combination of the non-stationary time series that is found to be stationary. It suggests that the series are not independent, and a long-term equilibrium relationship exists between them (Engle-Granger).

Consider a simple regression model:

$$Y_t = \alpha + \beta X_t + \varepsilon_t.$$

If Y_t and X_t are non-stationary and $I(1)$, and there exists a linear relationship between them, and ε_t is stationary, such that $\varepsilon_t = Y_t - \alpha - \beta X_t$. Then Y_t and X_t are cointegrated.

2.6 Spurious Regression

Time series regression models are often used in cointegration analysis, a technique for analyzing the long-term relationships between non-stationary time

series variables. Cointegration refers to a common trend between two or more time series variables with a stable, long-run relationship, even if they may exhibit short-term deviations from this relationship. Consider the general form of a regression model:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \cdots + \beta_k X_{kt} + \varepsilon_t.$$

In cointegration, time series regression models are often used to analyze the long-run relationship between two or more non-stationary variables. The basic idea behind these models is to use the concept of cointegration to transform the non-stationary variables into a stationary relationship, which can then be analyzed using standard time series regression techniques.

Spurious regression occurs when two non-stationary time series with no causal relationship are found to be statistically significant when regressed against each other. This phenomenon can occur when two or more variables have a similar trend over time or are affected by a common underlying factor, leading to incorrect inferences and conclusions.

To illustrate this, a simulated dataset of 100 observations was generated using two independent $I(1)$ processes, Y_t and X_t, following a random walk with drift. An ordinary least squares (OLS) linear regression equation was then estimated using this dataset. Since the two series are independently generated, the slope and the R^2 are expected to be zero.

The regression equation can be expressed as:

$$\begin{aligned} Y_t &= -1.2342 - 0.8946 X_t \\ (t) &= (-1.496)\ (-8.220)^{***} \\ R^2 &= 0.4081,\ \textit{Adjusted}\ R^2 = 0.4021 \end{aligned}$$

Based on the regression results, t-value = -8.220, $R^2 = 0.4081$, and adjusted $R^2 = 0.4021$, a very small p-value of 8.585e-13 indicate a strong rejection of the null hypothesis of no significant relationship between the two variables.

This outcome signifies a spurious regression, where the apparent correlation between the variables is not driven by a genuine causal relationship but rather due to random fluctuations.

To avoid spurious regression, it is recommended that the series be transformed, for example, by differencing until they no longer exhibit non-stationarity. It is also important to consider other variables influencing the relationship between the variables being studied. Furthermore, it may be helpful to use techniques such as cointegration analysis, which tests for a long-term relationship between variables rather than a short-term relationship that may be spurious.

2.7 Cointegration with only two variables

Cointegration with only two variables, also known as Bivariate Cointegration, the most common approach is using the Engle-Granger two-step method. This involves the following steps:

Determine if the two variables are integrated of the same order; the ADF test and other unit root tests can be used to test this. A false association between the variables may exist if they are not steady, which prevents cointegration.

Once the series order is determined, we can test for cointegration between the original non-stationary series using the Engle-Granger procedure. If the residuals are not stationary, we cannot conclude that cointegration exists between the two variables, and we may need to try a different approach or consider including additional variables in the analysis.

Engle-Granger Theorem states that if two series are $I(1)$, they are cointegrated if and only if a linear combination of the two series is stationary. The two-step procedure involves estimating a simple regression model for the two series and then testing the residuals for stationarity.

Let X_t and Y_t be two series integrated of order 1, i.e., $I(1)$, such that their linear combinations

$$Y_t - \beta X_t = Z_t,$$

is stationary, i.e., $I(0)$, and Z_t is the equilibrium error, then X_t and Y_t are cointegrated, suggesting both variables follow a common stochastic trend as a random walk.

Granger causality is a concept developed by economist Clive Granger to assess the causal relationship between two time series variables. It is based on the idea that if the past values of X provide additional information for predicting Y beyond what can already be predicted using Y alone, X is said to granger-cause Y.

Lemma 1. If X_t and Y_t are $I(1)$ processes and cointegrated, then X_t and $Y_{t+\tau}$ are also cointegrated for any τ. (Kirchgässner and Wolters, 2007b, pp 203-204).

Lemma 2. If X_t and Y_t are $I(1)$ processes and cointegrated, X_t is Granger-causal to Y_t, and/or Y_t is Granger causal to X_t. (Kirchgässner and Wolters, 2007b, pp 203-204).

2.8 Error Correction Model

One commonly used model for bivariate cointegration analysis is the error correction model (ECM). The ECM is a regression model that includes the differenced variables and a lagged error correction term and allows for estimating both the short-run and long-run relationships between the two time series.

The ECM can be represented as follows:

$$\Delta Y_t = \alpha_0 + \beta \Delta X_t + \alpha_1 (Y_{t-1} - \beta X_{t-1}) + \varepsilon_t. \tag{12}$$

α_0 is the intercept, $\alpha_1(Y_{t-1} - \beta X_{t-1})$ is the error correction term, and α_1 is the coefficient on the lagged error correction term.

In the bivariate case, there is an error correction representation for any cointegrating relation, and its reduced form can be presented as follows:

$$\Delta Y_t = a_0 - \gamma_y (Y_{t-1} - b\, X_{t-1}) + \sum_{j=1}^{n_x} a_{xj} \Delta X_{t-j} + \sum_{j=1}^{n_y} a_{yj} \Delta Y_{t-j} + u_{y,t}, \tag{13}$$

$$\Delta X_t = b_0 + \gamma_x (Y_{t-1} - b\, X_{t-1}) + \sum_{j=1}^{k_x} b_{xj} \Delta X_{t-j} + \sum_{j=1}^{n_y} b_{yj} \Delta Y_{t-j} + u_{x,t}, \tag{14}$$

u_x and u_y are pure random processes.

In this type of cointegration that involves two variables, we can only have two possibilities; either the two variables are cointegrated, i.e., at least one $\gamma_x, \gamma_y > 0$, which means the system has one cointegrating relation and one common stochastic trend; or the two variables are not cointegrated, i.e., $\gamma_x = \gamma_y = 0$. Then the system has two stochastic trends.

- If Y and X are cointegrated, $\gamma_x, \gamma_y \neq 0$.

- If $\gamma_y = \gamma_x = 0$, there is no longer a long-run relationship, and the two variables tested are not cointegrated and have no error correction term; the system contains two stochastic trends.

- If γ_y, $\gamma_x > 0$, then the model is stable. For example, if Y_t is greater than the equilibrium relationship, then as γ_y is negative, ΔY_t at time period t is less than usual, which means that Y_t approaches the equilibrium.

- if $\gamma_y = 0$ in (13), then it means Y does not depend on the long-term relationship, and if, in addition, the coefficients $a_{xj} = 0$, then variable Y does not depend on X.

In an error correction model (ECM), the dependent variable is regressed on its lags, the lags of the other variables in the system, and the error correction term, which is the difference from the long-run equilibrium relationship. The error correction term is the difference between the actual value of the dependent variable and its expected value based on the long-term equilibrium relationship.

When two or more non-stationary time series are cointegrated, they have a long-run equilibrium relationship that an ECM can represent. The ECM can be used to model the short-term dynamics that bring the variables back to their equilibrium relationship after a shock. In contrast, the cointegration relationship ensures that the variables eventually return to their long-run equilibrium values. This model type is often called a vector error correction model (VECM).

The VECM is an extension of the VAR model in which non-stationary variables are first differenced to make them stationary, and residuals, also known as error correction terms, are included in the model as additional variables.

The VECM captures both the short-run and long-run dynamics of the variables, making it a powerful tool for modeling complex time series data.

So, a cointegration relationship can be nested inside an ECM, and the error correction term can show the deviation from the cointegration relationship. The ECM can be used to estimate the short-term dynamics of the variables, while the cointegration relationship can be used to forecast the long-term behavior of the variables.

2.9 Cointegration with k variables

The most commonly used method is the Johansen test to test for cointegration between more than two variables. This involves each of the k variables be integrated of the same order, and this can also be tested using unit root tests such as the ADF test or PP test.

Then test for cointegration follows; once the order of integration is the same, we can test for cointegration using the Johansen test. This is done by estimating a vector autoregression (VAR) model and using likelihood ratio tests to look for cointegrating vectors.

Suppose that there exists a vector $X_t = (X_{1,t}, X_{2,t}, \cdots, X_{n,t})$, such that all its components are of the same order, say $I(1)$, and there also exists a vector $\beta = (\beta_1, \beta_2, \cdots, \beta_n)$ such that the linear combination $\beta X_t = \beta_1 X_{1,t} + \beta_2 X_{2,t} + \cdots + \beta_n X_{n,t}$ is $I(0)$.
Then β is called the cointegrating vector.

The cointegrating vectors can be estimated if cointegration is found, representing the long-term relationships between the k variables.

Consider three variables, $Y_i, i = 1, 2, 3$. Then two independent cointegrating

relations could exist

$$\begin{aligned} Y_{1,t} &= b_2 Y_{2,t} + Z_{1,t} \;\; b_2 \neq 0, \\ Y_{2,t} &= b_3 Y_{3,t} + Z_{2,t} \;\; b_3 \neq 0. \end{aligned}$$

With $k = 3$ $I(1)$ variables, $r = k - 1 = 2$ cointegrating relations, the system contains one stochastic trend, otherwise no pairwise cointegration between Y_1 and Y_2, Y_2 and Y_3, Y_1 and Y_3 would not be feasible. When there are two stochastic trends in a system of three variables, there can only be one cointegrating relation, and the corresponding vector is unique after normalization.

In the case of $I(1)$ variables, the system can contain $k-r$, $r = 1, 2, \cdots, k-1$ common stochastic trends. Otherwise, if $r = k$, then all variables are $I(0)$ processes, which contradicts the assumption of $I(1)$ for all k variables. If $r = k - 1$, we get the case of only one common stochastic trend in the system.

The Johansen Test Approach

The Johansen method is popular for testing and estimating cointegration relationships between multiple time series. The model can be represented with the following system of equations (Enders, 2014).

The VAR model of order p can be presented in this form:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \varepsilon_t, \tag{15}$$

where Y_t is n x 1 vector $(Y_{1t}, Y_{2t}, \cdots, Y_{nt})'$.

The first difference of (15), ΔY_t becomes

$$\Delta Y_t = (\phi_i - I)Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \cdots + \phi_p Y_{t-p} + \varepsilon_t. \qquad (16)$$

Add and subtract $(\phi_1 - I)Y_{t-2}$, then (16) becomes

$$\Delta Y_t = (\phi_1 - I)\Delta Y_{t-1} + (\phi_2 + \phi_1 - I)Y_{t-2} + \phi_3 Y_{t-3} + \cdots + \phi_p Y_{t-p} + \varepsilon_t.$$

if it continues like this, the following equation is arrived at:

$$\Delta Y_t = \Gamma_1 \Delta Y_{t-1} + \Gamma_2 \Delta Y_{t-2} + \cdots + \Gamma_{p-1}\Delta Y_{t-(p-1)} + \Pi Y_{t-p} + \varepsilon_t, \qquad (17)$$

where $\Pi = -(I - \sum_{i=1}^{p} \phi_i)$ and $\Gamma_i = -(I - \sum_{j=1}^{i} \phi_j)$.

Note that (17) can be presented also in the following form

$$\Delta Y_t = \Pi Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + \Gamma_2 \Delta Y_{t-2} + \cdots + \Gamma_{p-1}\Delta Y_{t-(p-1)} + \varepsilon_t, \qquad (18)$$

where $\Pi = -(I - \sum_{i=1}^{p} \phi_i)$ and $\Gamma_i = -\sum_{j=i+1}^{p} \phi_j$.

Π is a n x r matrix of cointegrating vectors, and $\Gamma_1, \cdots, \Gamma_{p-1}$ are n x n coefficient matrices that capture the lagged effects of the differenced variables, ΔY_t is a n x1 differenced vector of the dependent variable at time t, and ε_t is the vector of error term with a dimension of n x 1.

In the Johansen method, the parameters of the VAR model in (17) are estimated using the maximum likelihood method. The process involves calculating the rank of the cointegrating matrix, r, and then evaluating the matrix itself.

Determining the rank of the cointegrating matrix in the system involves

assessing the statistical significance of the eigenvalues of the matrix Π. This entails testing the null hypothesis that at most or exactly r cointegrating vectors are present. The trace and maximum eigenvalue tests, which rely on the eigenvalues of the matrix Π, are employed for this purpose.

- The trace statistic tests the null hypothesis of at most r number of cointegrating vectors against the alternate hypothesis of more than r cointegration vectors. The test statistic is stated below:

$$\lambda_{trace} = -T \sum_{i=r+1}^{p} \ln(1 - \hat{\lambda}_i), \tag{19}$$

  where $\hat{\lambda}_i$ is the eigenvalues of the matrix Π and T is the number of observations used.

  The test starts with $r = 0$ and is performed until the first time H_0 cannot be rejected. The corresponding value of r gives the cointegration rank.

- The maximum eigenvalue test, on the other hand, tests the null hypothesis of the exact number of cointegrating vectors r against the alternative of $r + 1$ cointegrating vectors. The test statistic can be as follows:

$$\lambda_{max} = -T \ln(1 - \hat{\lambda}_{r+1}). \tag{20}$$

Johansen and Juselius provide their respective critical values, and the test statistics are χ^2 asymptotically distributed with $p - r$ degrees of freedom, which depend on the significance level and the number of variables in the system.

Once the rank is determined, the cointegrating matrix is estimated using the normalized cointegrating vectors obtained from the eigenvectors of the matrix Π, which are multiplied by the elements of the matrix Π to get the estimated cointegrating matrix.

Steps involved in the estimation of Johansen Test

Suppose the variables to be tested are integrated of the same order;

- Select the appropriate lag length for the VAR model, as specified by the AIC or Schwartz Criterion (SC) of the initial non-transformed data.
- Estimate the model whether to include the drift term that includes the intercept in the cointegrating vector; such model in the case of lag $p = 2$ is:

$$\Delta Y_t = \alpha + \Gamma_1 \Delta Y_{t-1} + \Pi Y_{t-2} + \varepsilon_t. \tag{21}$$

- Determine the rank of the cointegration matrix, i.e., the maximum number of cointegrating relationships to test. Suppose we are only interested in testing the null hypothesis of no cointegration ($r = 0$) against the alternate hypothesis of at least one cointegration relationship ($r \geq 1$). In that case, the trace test statistic is sufficient to calculate the test statistic and the estimated values compared with the critical values.

The formula in (19) becomes

$$\lambda_{trace}(0) = -T[\ln(1 - \hat{\lambda}_1) + \ln(1 - \hat{\lambda}_2) + \ln(1 - \hat{\lambda}_3)].$$

Conclusions can be drawn from here with respect to the critical values provided based on the number of observations.

We can also test the null hypothesis of one cointegration relationship ($r \leq 1$) against the alternative hypothesis of at least two cointegration relationships ($r \geq 2$), then the trace statistic becomes

$$\lambda_{trace}(1) = -T[\ln(1 - \hat{\lambda}_2) + \ln(1 - \hat{\lambda}_3)].$$

Further analysis of the resulting cointegrating vectors to understand the long-term relationships and the speed of adjustment coefficients through the error-correction model.

3 Testing of Weak Efficiency of Foreign Exchange Rates

3.1 Overview of the Dataset used

The present study utilized a dataset of 6200 daily observations of exchange rates of 10 currencies per US dollar. The currency pairs included the Euros (EUR), British pounds (GBP), Japanese yen (JPY), Swedish krona (SEK), Czech koruna (CZK), Danish krone (DKK), Hungarian forint (HUF), Polish złoty (PLN), Swiss franc (CHF), and Norwegian krone (NOK). The observation period spanned from January 1999 to March 2023, and the data was sourced from the Statistical Data Warehouse of the European Central Bank.

As evidenced by prior literature, exchange rate data are typically non-stationary and integrated of the same order, or sufficiently $I(1)$. Therefore, conducting a unit root test to ascertain which of the series is non-stationary before undertaking any cointegration analysis is imperative.

The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests were used to test for the presence of unit roots in ten exchange rates. The analysis was conducted on the exchange rates at levels and the first difference. The results from both tests indicate that the null hypothesis of unit roots present in levels cannot be rejected at a significance level of 0.05. These findings suggest that the observed exchange rate series are non-stationary.

Consequently, the series underwent differencing once, followed by a unit root test to evaluate the presence of unit roots. The results revealed that the null hypothesis of unit roots could be rejected, confirming that all series are considered integrated of order one, or $I(1)$.

Furthermore, it is essential to note that a unit root test incorporating trend and drift terms in the model was conducted. The ADF test was utilized, and the results revealed that the null hypothesis of no trend and no drift could not be rejected at the 0.05 significance level. This suggests that the series has no trend or intercept term, that is, $\alpha = 0$ and $\beta = 0$.

The series is visually inspected for stationarity using graph plots.

Upon examination of the output, it was found that all ten exchange rate data series under consideration are not looking stationary in levels, as appended below.

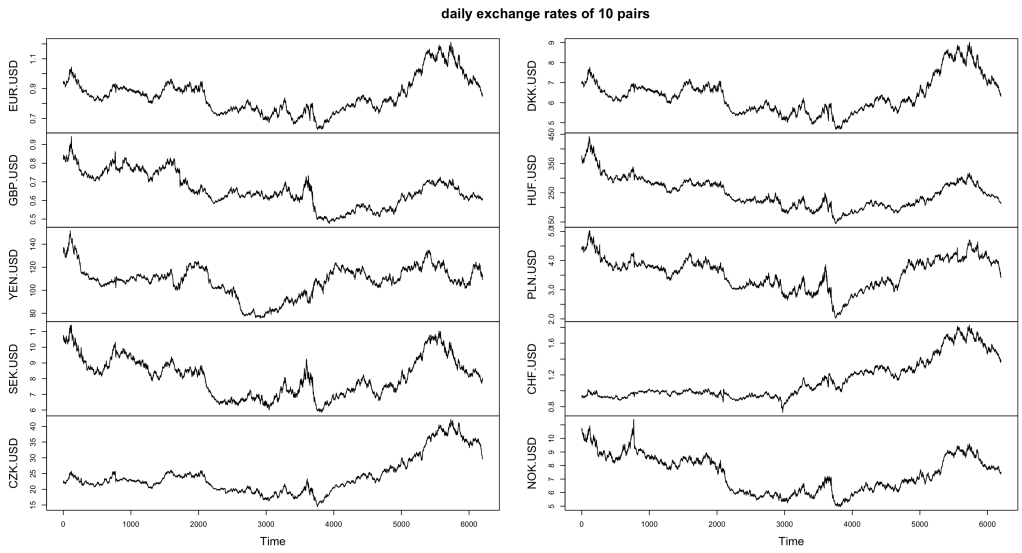

Figure 1: Non-stationary series at levels.

The exchange rates considered in this analysis are categorized into three groups:

- EUR and the Europa Countries that are members of the European Union, i.e., EUR, SEK, CZK, DKK, HUF, and PLN.

- Europa countries no-members of the EU, i.e., NOK and CHF.
- Other countries considered, i.e., GBP and YEN.

The exchange rates (Y_t) are standardized and rescaled so it can be plot together and visualize using the formula below:

$$Std.\ (Y_t) = \frac{Y_t - min(Y_t)}{max(Y_t) - min(Y_t)}$$

Their respective graph plots are also presented below:

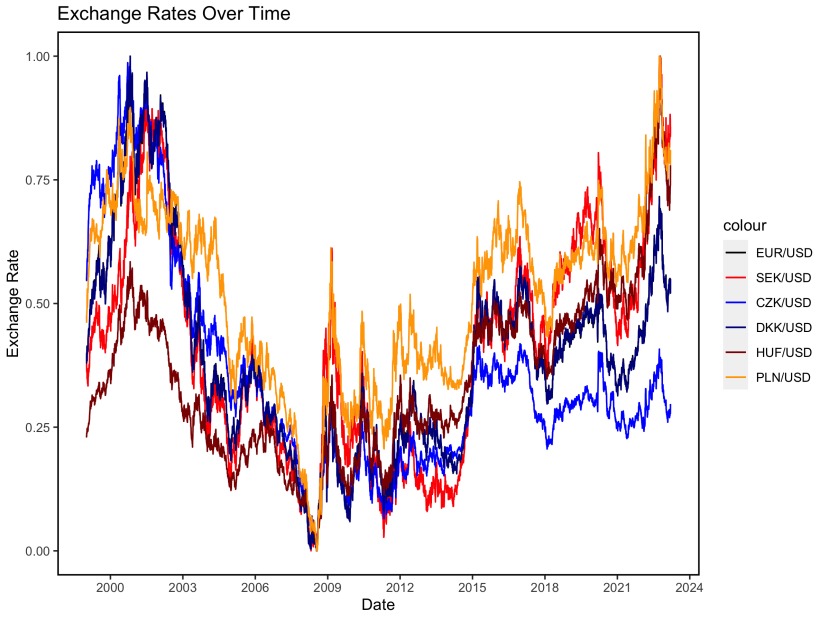

Figure 2: EUR and currencies of Europa countries members of EU.

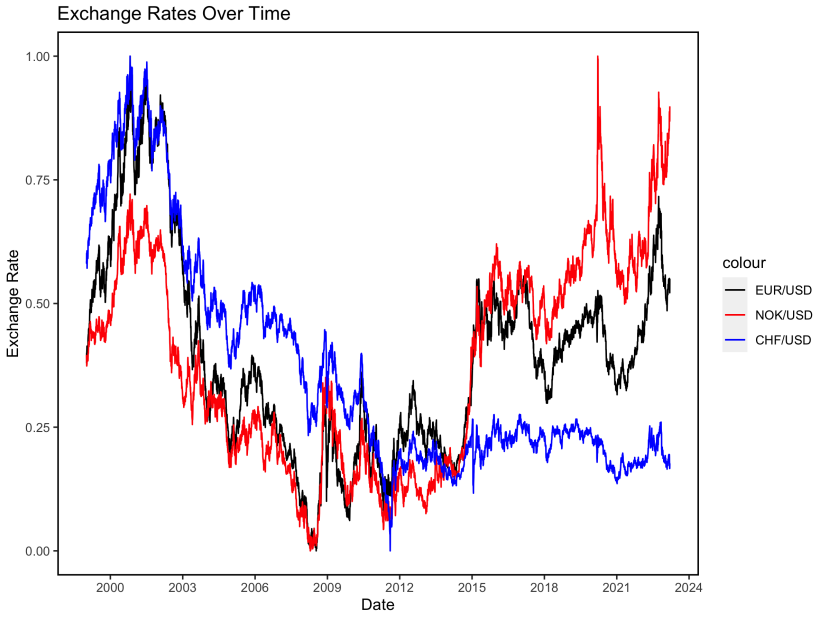

Figure 3: EUR and currencies of Europa countries non-members of EU.

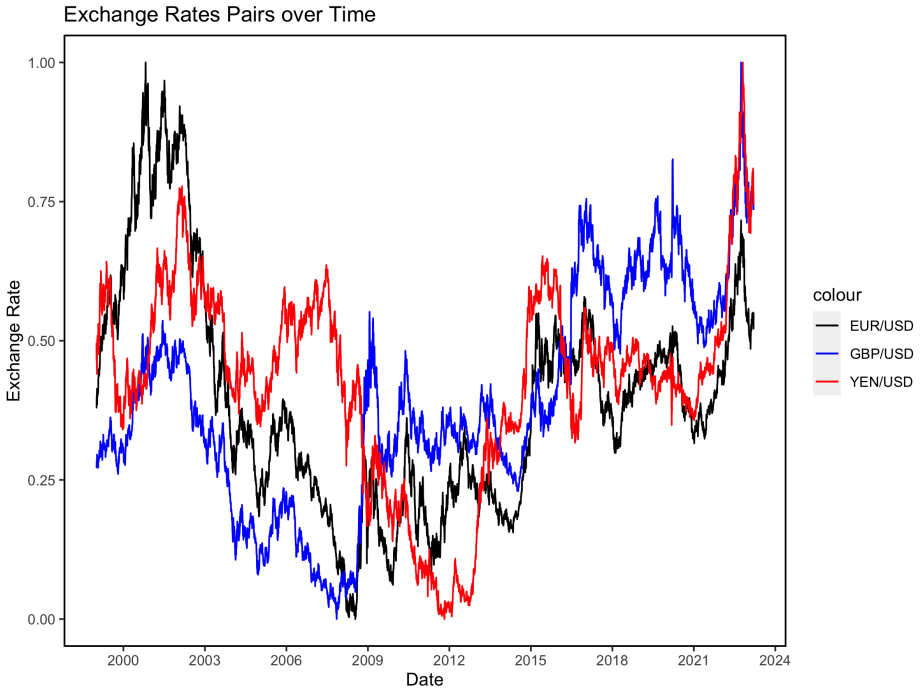

Figure 4: EUR and currencies of other countries considered.

After taking the first difference of the time series data and plotting the resulting series, a stationary pattern with constant mean and variance is observed. This suggests that the series have achieved stationarity, a desirable property in the cointegration analysis, as appended below.

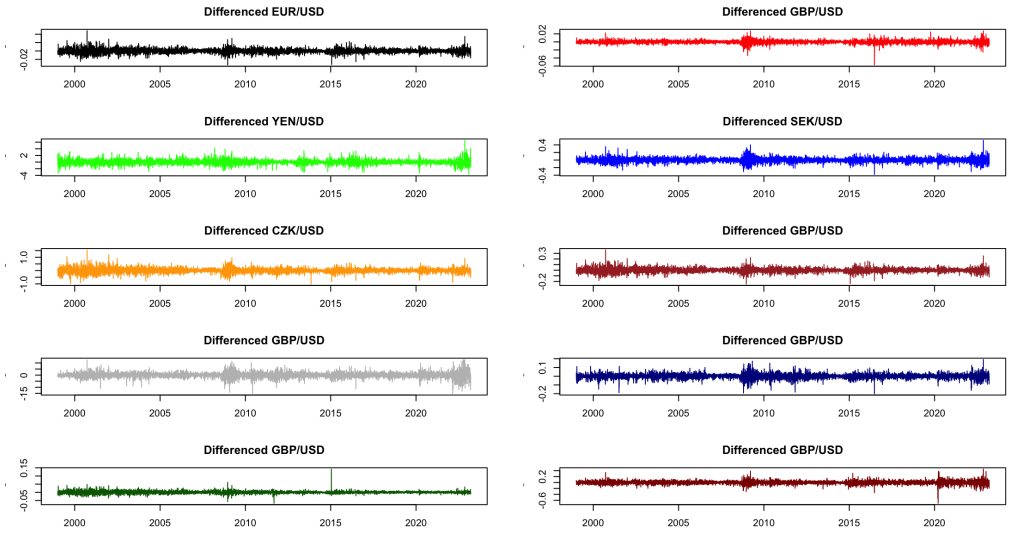

Figure 5: All series stationary on the first difference

3.2 Unit root Test results

The test result for unit roots was conducted using URDF and PP Tests considering the three models; with trend and drift, with drift only, and without drift and trend.

The result table is presented below:

Currency	**PP Test** p**-values**	Y_t	**URDF Test statistic**	**Eqtn. type**	Y_t
EUR/USD	0.694	Not $I(0)$	0.0136	none	Not $I(0)$
SEK/USD	0.751	Not $I(0)$	0.3534	none	Not $I(0)$
CZK/USD	0.710	Not $I(0)$	-0.7938	none	Not $I(0)$
DKK/USD	0.740	Not $I(0)$	0.0130	none	Not $I(0)$
HUF/USD	0.587	Not $I(0)$	0.7498	none	Not $I(0)$
PLN/USD	0.694	Not $I(0)$	0.1673	none	Not $I(0)$
NOK/USD	0.815	Not $I(0)$	0.5883	none	Not $I(0)$
CHF/USD	0.717	Not $I(0)$	-0.9835	none	Not $I(0)$
GBP/USD	0.581	Not $I(0)$	0.4749	none	Not $I(0)$
YEN/USD	0.771	Not $I(0)$	0.2072	none	Not $I(0)$

Table 2: Unit root test results for the initial time series Y_t

- The lag length selection for the PP Test was based on the formula implemented in the pp.test() function in R, which determined a long lag length as follows:

$$lags = 12 * \left(\frac{\text{length}(x)}{100}\right)^{0.25}.$$

- The short lag length was also considered, and found that the choice of

lag length did not significantly impact the test results.

The results indicate that the p-values are higher than the 0.05 significance level for all the tested series, indicating that the null hypothesis of unit roots cannot be rejected.

- The lag selection for the PP Test is based on the Akaike Information Criteria (AIC). In the URDF test output, the p-values correspond to the F-statistic of the significance of the test regression. However, comparing the critical values and test statistics is more reliable when testing for unit roots with trend, drift, or none variant, especially with a large sample size. It is noted that all the test statistics fall within the critical region, leading to the failure to reject the null hypothesis of unit roots.

- In addition to the PP and URDF Tests, the ADF Test was also conducted to confirm the results. The adf.test() function in R was used for this purpose, and the results were consistent with those of the previous tests. This indicates no evidence of the presence of trend, drift, or both in the series and that the series contains a unit root. This suggests that all the series tested were non-stationary.

Unit root tests results for the differenced series

Using PP and URDF Test, all the p-values are ≤ 0.01; based on these results, no evidence of unit roots indicates that they are stationary on the first difference, i.e., ΔY_t is I(0) and Y_t is $I(1)$.

This finding is further confirmed using the adf.test() function. Therefore, it can be concluded that the time series data analyzed in this study are integrated of order one ($I(1)$).

3.3 Johansen Cointegration Test results

The Johansen Trace test for pairwise cointegration was carried out using the initial data, the drift variant was applied, and the test statistic and the critical values are presented in the tables below.

Johansen Pairwise Cointegration Table

Pairs	SEK	CZK	DKK	HUF	PLN	NOK	CHF	GBP	YEN
EUR	5.11	11.66	20.28**	4.91	9.88	5.70	4.66	4.88	8.11
SEK	-	6.85	5.09	8.30	11.02	5.11	12.13	6.89	9.88
CZK	-	-	11.49	6.96	6.67	7.89	4.40	5.25	7.14
DKK	-	-	-	4.89	9.79	5.71	4.60	4.89	8.14
HUF	-	-	-	-	7.70	6.59	15.40	12.12	10.38
PLN	-	-	-	-	-	6.12	13.17	8.20	11.65
CHF	-	-	-	-	-	-	5.83	6.58	4.54
NOK	-	-	-	-	-	-	-	7.48	7.59
GBP	-	-	-	-	-	-	-	-	6.36
YEN	-	-	-	-	-	-	-	-	-

Table 3: Johansen Trace Test Statistic table for cointegration

Critical values	**10%**	**5%**	**1%**
	17.85	19.96	24.60

Hypotheses:

H_0: There is no cointegration relationship.

H_1: There is a cointegration relationship.

The resulting output found a cointegration relationship between EUR and DKK at 5%

** The initial time series Y_t was transformed, and a cointegration test with the "none" variant was carried on the transformed series $\hat{Y}_t$; the results were identical (result in appendix).

The Johansen multivariate cointegration test was carried out among EU member countries;

The drift variant is used for the initial data, and to double-check the results, the none variant is used on transformed data.

For the initial time series data, the results show that there is no cointegration relationship among members of the European Union. While for the transformed data, only the trace statistic detects cointegration at the 10% significance level, while the max eigen statistic could not find any cointegration relationship (result in appendix).

3.4 Engle-Granger Cointegration Test results

Suppose two times series Y_{1t} and Y_{2t} are integrated of the same order, in the case of the series tested, $I(1)$. Granger two-step method involves checking the order of the residuals using a unit root test; if the residuals ε_{1t} and ε_{2t}

are $I(0)$, then cointegration relationship exist between the two series Y_{1t} and Y_{2t}.

Of all the ten currency pairs tested, two pairs were found to have a cointegration relationship, and the table is presented below.

	EUR-DKK		**HUF-NOK**	
	statistics	**p-values**	**statistic**	**p-values**
ADF Test	-3.529	0.0271	-3.081	0.090
PP Test	-30.661	0.00855	-22.418	0.03681
Johansen's Trace Test	-21.031	0.04131	-15.666	0.19337

Table 4: Engle-Granger cointegration result

From the output table, all the p-values for EUR – DKK is < 0.05 significance level; the null hypothesis of no cointegration can be rejected, and conclude that EUR and DKK are cointegrated, since the residuals do not contain unit roots, i.e. $(I(0))$. Again, this supports the cointegration relationship found using the Johansen approach in Table 3.

The pair HUF and NOK seem cointegrated, as detected by PP Test for unit roots 0.036 at a 5% significance level; ADF Test shows a mild cointegration at 0.09, while Johansen's Trace Test shows no cointegration between the two series.

3.5 Sub-periods Cointegration results

This crisis is due to the Global Financial Crisis that started in the second quarter of 2007 but escalated in 2008; it originated from the collapse of the subprime mortgage market in the United States and led to declines in the stock market, the bankruptcy of major financial institutions, high unemployment rates, and a real estate market crash.

The crisis had implications for the EU economy due to some countries exposure to the collapse of inflated housing markets and high levels of private and public debt, leading to recessions and rising unemployment rates.

However, governments and central banks intervened and implemented measures such as bailouts, interest rate reductions, and regulatory reforms to stabilize the financial system, and the market eventually recovered in 2010.

A structural change test was also conducted using statistical tests to find the structural breakpoints in the series, but they have no common changepoints. Hence, the dataset is divided into two periods, with the split date being 01.01.2010, to observe the effect of the Global financial crisis on the exchange rates market.

Period 1 is the pre-crisis and during the crisis period, i.e., from 01.01.1999 to 01.01.2010, and period 2 is after the crisis period, i.e., 01.02.2010 to 15.03.2023.

For period 1:

- Using the Engle-Granger two-step method, the result shows a cointegration relationship between the pairs HUF-NOK; also, CZK-CHF are cointegrated at a 5% significance level.

- Using the Johansen pairwise cointegration, HUF-NOK are found to be cointegrated at a 5% significant level.

For period 2:

- Cointegration relationship was found between SEK-NOK at 0.05 significance level, using the Engle-Granger 2-step method.
- Under the Johansen pairwise cointegration, evidence of cointegration was found in pairs EUR-CHF and DKK-CHF at a 5% significance level. In contrast, GBP-CHF, SEK-NOK, CZK-CHF, and PLN-CHF are found to be cointegrated at a 10% significant level.

3.6 Error-Correction Model Presentation

The error correction model is presented for the cointegrated series in the whole period, for which, at least in one equation, the error correction term is significant; in other cointegrated pairs, the error correction term is not significant.

For HUF-NOK cointegration:

The lag selection was based on the AIC and Schwartz Criterion (SC).

Lag length of 2, as chosen by AIC

$$\begin{array}{lccccccc} \Delta HUF_t = & 0.0095 & - & 0.0011.ECT & + & 0.0412\ \Delta HUF_{t-1} & - & 1.8599\ \Delta NOK_{t-1} \\ & (0.0352) & & (0.0014) & & (0.0172)^{*} & & (0.6492)^{**} \end{array}$$

$$- \underset{(0.0172)^*}{0.0359\ \Delta HUF_{t-2}} + \underset{(0.6497)}{0.9353\ \Delta NOK_{t-2}}$$

$$\Delta NOK_t = \underset{(0.0009)^\cdot}{0.0018} + \underset{(0.00004)^*}{0.00008.ECT} + \underset{(0.0005)^*}{0.0010\ \Delta HUF_{t-1}} - \underset{(0.0171)^*}{0.0416\ \Delta NOK_{t-1}}$$

$$- \underset{(0.0005)}{0.0006\ \Delta HUF_{t-2}} + \underset{(0.0172)}{0.0258\ \Delta NOK_{t-2}}$$

Where $ECT = HUF_{t-1} - 35.6016\ NOK_{t-1}$.

Lag length of 1, as chosen by SC

$$\Delta HUF_t = \underset{(0.0340)}{0.0089} - \underset{(0.0014)}{0.0013.ECT} + \underset{(0.0172)^*}{0.0408\ \Delta HUF_{t-1}} - \underset{(0.6492)^{**}}{1.8794\ \Delta NOK_{t-1}}$$

$$\Delta NOK_t = \underset{(0.0009)^\cdot}{0.0016} + \underset{(0.00004)^*}{0.00008.ECT} + \underset{(0.0005)^*}{0.0010\ \Delta HUF_{t-1}} - \underset{(0.0171)^*}{0.0419\ \Delta NOK_{t-1}}$$

Where $ECT = HUF_{t-1} - 35.3994\ NOK_{t-1}$.

The error correction term coefficient of -0.0013 is insignificant, indicating that no adjustment occurs, and the short-term deviation might not be corrected back to equilibrium as the ECT is close to zero. This means predicting future NOK values better using the long-term relationship is possible. Still, in the case of HUF, only the NOK short-term relationship can be used, as they have a one-sided relationship.

Conclusions

The findings of this study reveal important insights into the weak efficiency of foreign exchange rates. Firstly, the unit root tests showed that the exchange rates exhibit non-stationary behavior, the first difference in the series ensured stationarity which makes all the exchange rates considered integrated of the same order, i.e., $I(1)$, which is a necessary condition for cointegration.

A pairwise cointegration was tested among the series using the Engle-Granger and Johansen Pairwise Method, and found a cointegration relationship between EUR – DKK and HUF – NOK in the data for the entire period, indicating a long-run relationship, and deviations from this equilibrium are corrected over time.

An error correction model was presented to analyze the dynamics of the cointegrated variables, which captures the short-term adjustments toward the long-term equilibrium, indicating the existence of a corrective mechanism that brings the exchange rates back to their long-term equilibrium. The error correction term is only significant in the HUF – NOK relationship, a small ECT coefficient closer to zero means that the adjustment of the short-term deviation back to its equilibrium after a shock is weak and slow, or most likely not.

Subsequently, the dataset was divided into two sub-periods to examine cointegration separately and identify any changes within each period; this is to ascertain the effect of the Global Financial Crisis of 2008 on the cointegration of exchange rates; for the first period, which is before and during the crisis, dated from 1999 – 2010, the Engle-Granger pairwise cointegration method found two distinct cointegration relationships between HUF-NOK and CZK-CHF, while the Johansen pairwise cointegration using the Trace statistic

found only a cointegration in HUF – NOK at 5% significance level.

In the period after the crisis, i.e., 2010 – 2023, a cointegration was found between SEK – NOK using the Engle-Granger two-step method, while the Johansen Pairwise cointegration found six cointegration between EUR – CHF, DKK – CHF at a 5% significant level, and GBP – CHF, SEK – NOK, EUR – CHF, PLN – CHF at a 10% significance level, which has subjected the cointegrating pairs for a Granger-causality test, to explore the presence of causal relationships between the variables. The results suggest no causal relationships among the variables considered, there is no directional influence or predictability between the variables under investigation during the second sub-period. In other words, changes in one variable do not lead to predictable changes in the other variable, and vice versa.

Furthermore, a Johansen Multivariate cointegration test was carried out among EUR and five EU member countries; i.e., EUR, SEK, CZK, DKK, HUF, PLN, and the result indicates that at 10% significance, there is a weak cointegration relationship using trace statistic, no cointegration in case the maximum eigenvalue statistic. These results are inconsistent, and there is a weak or no cointegration relationship between EU member countries, which implies the EU market is almost efficient in the weak form.

In summary, these findings imply that the foreign exchange market is not fully efficient and may not adhere to the weak-form efficiency hypothesis. Several cointegration relationships were tested, and found only one, implying no long-term cointegration, and the results depend on the periods considered.

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Appendix 1. Unit root results

PP Test results for EUR (variable d1)

```
> pp.test(d1, lag.short = FALSE)
Type 1: no drift no trend
 lag  Z_rho p.value
  33 0.0068   0.694
-----
 Type 2: with drift no trend
 lag Z_rho p.value
  33 -5.72   0.422
-----
 Type 3: with drift and trend
 lag Z_rho p.value
  33 -5.71   0.755
---------------
Note: p-value = 0.01 means p.value <= 0.01
       lag        Z_rho   p.value
type 1  33  0.006796415 0.6936975
type 2  33 -5.717759180 0.4217430
type 3  33 -5.705519385 0.7550173
```

URDF results for EUR (variable d1)

```
> print(ur.df(d1, type = "trend", selectlags = "AIC"))
> print(ur.df(d1, type = "drift", selectlags = "AIC"))
> print(ur.df(d1, type = "none", selectlags = "AIC"))
```

```
    Test results for EUR_trend :
                tau3      phi2     phi3
statistic -1.594619 0.9114926 1.337909
      1pct  5pct 10pct
tau3 -3.96 -3.41 -3.12
phi2  6.09  4.68  4.03
phi3  8.27  6.25  5.34

Test results for EUR_drift :
                tau2     phi1
statistic -1.635806 1.367265
      1pct  5pct 10pct
tau2 -3.43 -2.86 -2.57
phi1  6.43  4.59  3.78

Test results for EUR_none :
                tau1
statistic 0.01363515
      1pct  5pct 10pct
tau1 -2.58 -1.95 -1.62
```

PP Test on the differenced EUR (variable d1)

```
> diff_d1 = diff(d1, differences = 1)
> summary(pp.test(diff_d1, lag.short = FALSE))

Type 1: no drift no trend
 lag Z_rho p.value
```

```
  11 -6132    0.01
-----
 Type 2: with drift no trend
 lag Z_rho p.value
  11 -6132    0.01
-----
 Type 3: with drift and trend
 lag Z_rho p.value
  11 -6132    0.01
---------------
Note: p-value = 0.01 means p.value <= 0.01
```

Appendix 2. Engle-Granger Test Result

1. Testing pair EUR and DKK (variable d1 and d6)

```
> summary(egcm(d1, d6, include.const = TRUE))
Y[i] =   7.4338 X[i] +   0.0119 + R[i], R[i] =   0.9955 R[i-1] + eps[i],
eps ~ N(0,  0.0010^2)
        (0.0011)        (0.0010)                (0.0013)

R[6200] = -0.0016 (t = -0.163)

WARNING: The series seem cointegrated but the residuals are not AR(1).

Unit Root Tests of Residuals
                                    Statistic    p-value
  Augmented Dickey-Fuller (ADF)        -3.529    0.02968
  Phillips-Perron (PP)                -30.661    0.00878
  Johansen's Trace Test (JOT)         -21.031    0.04131

  EUR and DKK are cointegrated.
```

Note: If the residuals do not follow an AR(1) process, it implies that the cointegrating relationship may not be well-specified, and the EGCM results may be unreliable.

2. Testing pair HUF and NOK (variable d7 and d10)

```
> summary(egcm(d7, d10, include.const = TRUE))
Y[i] =   0.0250 X[i] +   1.2030 + R[i],
```

```
R[i] =   0.9965 R[i-1] + eps[i], eps ~ N(0,  0.0459^2)
        (0.0001)        (0.0414)        (0.0011)
R[6200] = 0.1393 (t = 0.259)

Unit Root Tests of Residuals
                                    Statistic    p-value
  Augmented Dickey-Fuller (ADF)        -3.081    0.09035
  Phillips-Perron (PP)                -22.418    0.03681
  Johansen's Trace Test (JOT)         -15.666    0.19337
  HUF and NOK are cointegrated.
```

EGCM for series that are not cointegrated (EUR-GBP)

```
> summary(egcm(d1, d2, include.const = TRUE))
Y[i] =   0.7621 X[i] +   0.0000 + R[i],
R[i] =   0.9998 R[i-1] + eps[i], eps ~ N(0,  0.0035^2)
        (0.0013)        (0.0000)        (0.0005)
R[6200] = 0.1046 (t = 1.150)

WARNING: X and Y do not appear to be cointegrated.

Unit Root Tests of Residuals
                                       Statistic    p-value
  Augmented Dickey Fuller (ADF)           -1.188    0.85256
  Phillips-Perron (PP)                    -3.828    0.82056
  Johansen's Trace Test (JOT)             -5.011    0.98097
EUR and GBP are not cointegrated.
```

Appendix 3. Johansen Cointegration Result

Johansen Pairwise Cointegration Table (None Variant)

Pairs	SEK	CZK	DKK	HUF	PLN	NOK	CHF	GBP	YEN
EUR	4.59	9.43	20.23**	3.32	9.66	2.90	3.96	4.35	7.94
SEK	-	4.34	4.56	7.36	10.67	2.64	11.59	6.38	9.45
CZK	-	-	9.26	2.28	4.87	7.31	1.96	3.50	6.61
DKK	-	-	-	3.30	9.57	2.90	3.89	4.36	7.97
HUF	-	-	-	-	6.49	3.16	14.48	11.17	9.42
PLN	-	-	-	-	-	4.57	12.64	7.72	11.35
CHF	-	-	-	-	-	-	3.14	4.54	3.44
NOK	-	-	-	-	-	-	-	6.86	7.00
GBP	-	-	-	-	-	-	-	-	5.80
YEN	-	-	-	-	-	-	-	-	-

Table 5: Johansen cointegration table - none variant

Johansen Pairwise Cointegration Result

EUR and DKK with drift

```
d1 = EUR, d6 = DKK
```

```
> summary(ca.jo(cbind(d1,d6), type = "trace", K = 3, ecdet = "const"))
Trace Test results:
Eigenvalues (lambda):
[1] 2.829353e-03 4.396814e-04 1.488015e-20

Values of test statistic and critical values of test:
          test 10pct  5pct  1pct
r <= 1 |  2.73  7.52  9.24 12.97
r = 0  | 20.28 17.85 19.96 24.60
```

EUR - DKK without drift

```
d1 = EUR, d6 = DKK
t1 = d1/mean(d1), t6 = d6/mean(d6)
> summary(ca.jo(cbind(t1,t6), type = "trace", K = 3, ecdet = "none"))
Trace Test results
Eigenvalues (lambda):
[1] 0.0028290844 0.0004304929

Values of test statistic and critical values of test:
          test 10pct  5pct  1pct
r <= 1 |  2.67  6.50  8.18 11.65
r = 0  | 20.23 15.66 17.95 23.52
```

Some output of series that are not cointegrated

Example:

EUR - GBP with drift

```
> d1 = EUR, d2 = GBP
```

```
> summary(ca.ja(cbind(d1,d2), type = "trace", K = 3, ecdet = "none"))
Trace Test results
Eigenvalues (lambda):
[1] 4.357841e-04 3.520054e-04 5.421011e-20

Values of test statistic and critical values of test:
         test 10pct  5pct  1pct
r <= 1 | 2.18  7.52  9.24 12.97
r = 0  | 4.88 17.85 19.96 24.60
```

EUR - GBP without drift

```
t1 = d1/mean(d1), t2 = d2/mean(d2)
> summary(ca.ja(cbind(d1,d2), type = "trace", K = 3, ecdet = "none"))
Trace Test results
Eigenvalues (lambda):
[1] 0.0004310383 0.0002710562

Values of test statistic and critical values of test:
         test 10pct  5pct  1pct
r <= 1 | 1.68  6.50  8.18 11.65
r = 0  | 4.35 15.66 17.95 23.52
```

Johansen Multivariate Cointegration Result

1. All exchange rates, with drift

```
> summary((ex_series, type = "trace", K = 3, ecdet = "const"))
Test type: trace statistic
```

```
Eigenvalues (lambda):
 [1]  1.111117e-02  7.742474e-03  5.899868e-03  4.097229e-03
 [5]  3.367344e-03  2.710244e-03  1.718779e-03  8.481675e-04
 [9] 8.112820e-04  6.360551e-04  -1.351486e-17

Values of test statistic and critical values of test:
           test  10pct   5pct   1pct
r <= 9 |   3.94   7.52   9.24  12.97
r <= 8 |   8.97  17.85  19.96  24.60
r <= 7 |  14.23  32.00  34.91  41.07
r <= 6 |  24.89  49.65  53.12  60.16
r <= 5 |  41.71  71.86  76.07  84.45
r <= 4 |  62.61  97.18 102.14 111.01
r <= 3 |  88.05 126.58 131.70 143.09
r <= 2 | 124.72 159.48 165.58 177.20
r <= 1 | 172.89 196.37 202.92 215.74
r = 0  | 242.13 236.54 244.15 257.68

> summary((ex_series, type = "eigen", K = 3, ecdet = "const"))
Test type: max eigenvalue statistic (lambda max)
Eigenvalues (lambda):
 [1]  1.111117e-02  7.742474e-03  5.899868e-03  4.097229e-03
 [5]  3.367344e-03  2.710244e-03  1.718779e-03  8.481675e-04
 [9]  8.112820e-04  6.360551e-04 -1.351486e-17

Values of test statistic and critical values of test:
          test 10pct  5pct  1pct
```

```
r <= 9 |  3.94  7.52  9.24 12.97
r <= 8 |  5.03 13.75 15.67 20.20
r <= 7 |  5.26 19.77 22.00 26.81
r <= 6 | 10.66 25.56 28.14 33.24
r <= 5 | 16.82 31.66 34.40 39.79
r <= 4 | 20.90 37.45 40.30 46.82
r <= 3 | 25.44 43.25 46.45 51.91
r <= 2 | 36.67 48.91 52.00 57.95
r <= 1 | 48.17 54.35 57.42 63.71
r = 0  | 69.24 60.25 63.57 69.94
```

2. All exchange rates, without drift

```
> summary((t_series, type = "trace", K = 3, ecdet = "none"))
Test type: trace statistic
Eigenvalues (lambda):
 [1] 1.105745e-02 7.741918e-03 5.549523e-03 4.012317e-03
 [5] 3.348745e-03 2.648403e-03 1.695762e-03 8.307917e-04
 [9] 6.399515e-04 2.134543e-05

Values of teststatistic and critical values of test:
           test  10pct   5pct   1pct
r <= 9 |   0.13   6.50   8.18  11.65
r <= 8 |   4.10  15.66  17.95  23.52
r <= 7 |   9.25  28.71  31.52  37.22
r <= 6 |  19.77  45.23  48.28  55.43
r <= 5 |  36.20  66.49  70.60  78.87
r <= 4 |  56.99  85.18  90.39 104.20
```

```
r <= 3 |  81.90 118.99 124.25 136.06
r <= 2 | 116.39 151.38 157.11 168.92
r <= 1 | 164.55 186.54 192.84 204.79
r = 0  | 233.46 226.34 232.49 246.27
```

```
> summary((ex_series, type = "eigen", K = 3, ecdet = "const"))
Test type: maximal eigenvalue statistic (lambda max)
Eigenvalues (lambda):
 [1] 1.105745e-02 7.741918e-03 5.549523e-03 4.012317e-03
 [5] 3.348745e-03 2.648403e-03 1.695762e-03 8.307917e-04
 [9] 6.399515e-04 2.134543e-05

Values of teststatistic and critical values of test:
          test 10pct  5pct  1pct
r <= 9 |  0.13  6.50  8.18 11.65
r <= 8 |  3.97 12.91 14.90 19.19
r <= 7 |  5.15 18.90 21.07 25.75
r <= 6 | 10.52 24.78 27.14 32.14
r <= 5 | 16.43 30.84 33.32 38.78
r <= 4 | 20.79 36.25 39.43 44.59
r <= 3 | 24.91 42.06 44.91 51.30
r <= 2 | 34.49 48.43 51.07 57.07
r <= 1 | 48.16 54.01 57.00 63.37
r = 0  | 68.90 59.00 62.42 68.61
```

Johansen cointegration result for EU member countries

All EU members exchange rates - With drift

```
d1 = EUR, d4 = SEK, d5 = CZK, d6 = DKK, d7 = HUF, d8 = PLN
eu_series = cbind(d1, d4, d5, d6, d7, d8)
> summary(ca.jo(eu_series, type = "trace", K = 2, ecdet = "const"))
Test type: trace statistic
Eigenvalues (lambda):
[1]  5.332061e-03  4.041673e-03  2.482193e-03  1.997975e-03
[5]  9.468824e-04  4.771050e-04 -1.319543e-18

Values of test statistic and critical values of test:
          test 10pct   5pct   1pct
r <= 5 |  2.96  7.52   9.24  12.97
r <= 4 |  8.82 17.85  19.96  24.60
r <= 3 | 21.21 32.00  34.91  41.07
r <= 2 | 36.60 49.65  53.12  60.16
r <= 1 | 61.68 71.86  76.07  84.45
r = 0  | 94.79 97.18 102.14 111.01

> summary(ca.jo(eu_series, type = "eigen", K = 2, ecdet = "const"))
Test type: max eigenvalue statistic (lambda max)
Eigenvalues (lambda):
[1]  5.332061e-03  4.041673e-03  2.482193e-03  1.997975e-03
[5]  9.468824e-04  4.771050e-04  -1.319543e-18

Values of test statistic and critical values of test:
```

```
          test 10pct  5pct  1pct
r <= 5 |  2.96  7.52  9.24 12.97
r <= 4 |  5.87 13.75 15.67 20.20
r <= 3 | 12.39 19.77 22.00 26.81
r <= 2 | 15.39 25.56 28.14 33.24
r <= 1 | 25.08 31.66 34.40 39.79
r = 0  | 33.11 37.45 40.30 46.82
```

All EU members exchange rates - without drift

```
t_eu_series = cbind(t1, t4, t5, t6, t7, t8)
> summary(ca.jo(t_eu_series, type = "trace", K = 2, ecdet = "none"))
Test type: trace statistic
Eigenvalues (lambda):
[1] 5.267623e-03 4.041505e-03 2.376170e-03 1.997566e-03
[5] 6.197854e-04 1.081973e-05

Values of test statistic and critical values of test:
          test 10pct  5pct   1pct
r <= 5 |  0.07  6.50  8.18  11.65
r <= 4 |  3.91 15.66 17.95  23.52
r <= 3 | 16.29 28.71 31.52  37.22
r <= 2 | 31.02 45.23 48.28  55.43
r <= 1 | 56.10 66.49 70.60  78.87
r = 0  | 88.81 85.18 90.39 104.20

> summary(ca.jo(t_eu_series, type = "eigen", K = 2, ecdet = "none"))
Test type: max eigenvalue statistic (lambda max)
```

```
Eigenvalues (lambda):
[1] 5.267623e-03 4.041505e-03 2.376170e-03 1.997566e-03
[5] 6.197854e-04 1.081973e-05

Values of test statistic and critical values of test:
          test 10pct  5pct  1pct
r <= 5 |  0.07  6.50  8.18 11.65
r <= 4 |  3.84 12.91 14.90 19.19
r <= 3 | 12.38 18.90 21.07 25.75
r <= 2 | 14.73 24.78 27.14 32.14
r <= 1 | 25.08 30.84 33.32 38.78
r = 0  | 32.71 36.25 39.43 44.59
```

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