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**ANALYSIS OF THE DAILY DISTRIBUTION OF BIRTHS IN ESTONIAN
HOSPITAL AND ITS IMPLICATIONS ON THE RESOURCE PLANNING**

Master's Thesis

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Tartu 2019

Name and signature of supervisor..... Allowed for
defence on..... (date)

I have written this master's thesis independently. All viewpoints of other authors,
literary sources and data from elsewhere used for writing this paper have been
referenced.

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ABSTRACT

This study aims to study the effect of the volatility in the number of daily births on the number of beds in the Obstetric and Maternity department. This study simulates a number of mothers Length Of Stay (LOS) who transfer to the Postpartum rooms at East Tallinn and the University of Tartu hospitals when beds are full to capacity. The simulation results report 25.9% excess in the number of mothers at East Tallinn Hospital, and 3.4% excess at the University of Tartu hospital, which may put mothers life at risk at East Tallinn hospital. The simulation suggests the optimal number of beds required by limiting the percentage of mothers need to wait for the next available bed to 5%. It also suggests no more than 5% beds shortage by setting a threshold. The regression results demonstrate that relative risk at the East Tallinn high compared to the University of Tartu hospital.

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A. INTRODUCTION

The necessity of having a model understands why the inherently complex healthcare delivery system behaves like this - as a function of its medical care entity- and relationships among its components, become intrinsic. Non-linearities, iterations, and a wide range of variables that evolve dynamically over time are the characteristics of the needed model.

The way that the decision-makers evaluate allocation for healthcare delivery system resources, and improvement plans around the world are varied, but the goal is sole: efficient and effective utilisation for the available resources. Thus, the same thing applies to the Republic of Estonia regarding the adequacy of available healthcare resources and stochastic processes of mothers arrivals for delivery.

This study is vital in four ways: first, estimating the consequences of healthcare delivery system interventions. Second, optimise resources utilisation. Third, the timing of this study may consider significant. In 2018 decision-maker(s) decided to end the operations of maternity wards in southern Estonia for Põlva and Valga towns which left mothers there with no choice except The University of Tartu hospital as the closest medical care. Forth, to the best of our knowledge, this is the first study in Estonia.

It is noteworthy that many studies in the healthcare attempt to evaluate the stochastic patients flow *Cf.* available resources resource for different departments and wards such as Obstetrics, Emergency Room, et cetera, by using dynamic simulation models. Huang (1995) simulated various factors influencing bed requirement for patients as well as the random variations which follow the Poisson distribution. His paper “A Planning Model for Requirement of Emergency Beds” simulated bed occupancy via Queueing Model which Erlang in Copenhagen published his first paper in (1909), “The Theory of Probabilities and Telephone Conversations”. The Queueing theory is used widely in modelling, e.g. hospital patients in-flow we are currently studying, computer science¹ and

¹ The model of “vacation queueing system” used by Krishna and Madheswari in (2005) “*M/M/2* Queueing System with Heterogeneous Servers and Multiple Vacations”, where server vacation occurs for several

in many other fields. Harper *et al.* (2002) in the UK, also published a study “Modelling for The Planning And Management of Bed Capacities In Hospitals” where they simulated detailed model for the patients flow through wards (Cardiology, Dermatology, Elderly care, Elderly rehabilitation, Gastroenterology, General medicine, Neurology, Rehabilitation, Rheumatology, Thoracic medicine) and the number of beds in each.

Therefore, with the objective of optimising the required number of beds affected by the stochastic process of mothers in-flow, this study attempts to study the stochastic process for the daily births using System Dynamics (SD) technique where the variation of the quantitative results will be more apparent on the higher number of mothers who transfer to the Postpartum rooms. Thereby, this study analyses the patients in-flow autoregressive process (Stage 1) as well as the variation and the distribution of the births across the weekdays (Stage 2). The next stage (Stage 3) this study models the Length of Stay (LOS) which represents the consumption from the Obstetrics and Maternity department and its designated probabilities. Therefore, the required blocks for the System Dynamics (SD) simulation are specified (Stage 4). At (Stage 5), this study estimates the Incident of Ratio Rate (IRR) associated with the daily and monthly births.

The thesis contents will be developed as follows. First, a **LITERATURE REVIEW** of current research on analysis and evaluate the capacity of the planning in Obstetrics and Maternity department *vis-à-vis* stochastic patients in-flow process. It tends to identify the optimal level of resources. Then we shall illustrate the details Poisson distribution validity assumption and time-series stationarity tests, Length of Stay (OLS) and Poisson Count models, and then building the System Dynamics (SD) blocks in the section of **METHODOLOGY**. It is followed by descriptive statistics, econometric analysis, models estimation results, and the simulation results as in the section of **RESULTS**. Subsequently, the section of **CONCLUSION** shall provide an overview of the thesis and some limitations of the results.

reasons such as failure, maintenance, testing, and et cetera. Thus, server vacation takes a vacation at random time after serving clients

B. LITERATURE REVIEW

A review of the literature on healthcare resources planning exhibits many approaches to model the stochasticity of patients in-flow process, given the Obstetrics and Maternity department resources constraints. The volatility in the number of daily/weekly/annual births depends on the number of available beds, physicians, on the geographical factor(s), and et cetera. The outcome attains from healthcare utility maximisation remains inaccurately determined, because of the availability of the data, level of details the proposed model(s) deals with, the interaction with other departments, and the medical case herself (e.g. natural birth or Caesarean birth), et cetera.

Remarkably, Cochran and Bharti (2006), applied models of Discrete-Event Simulation (DES) and Queueing Network Analysis (QNA). The QNA model studied evaluated optimality of beds allocation through setting a threshold for capacity in the LDR and PR to prevent new admissions. DES model keeps receiving patients until the average level of the capacity reaches the prevention threshold rate. Their study concluded a consideration for different patient types should be taken. Therefore, applying two models of simulation was very useful to verify the simulation model outcome.

Based on the American College of Obstetrics and Gynaecology occupancy (ACOG) recommended target for the maternity unit occupancy for maternity unit at 70%, Green (2002), used the Queueing theory model to measure the average length of stay (AOLS) where patients move from Labour, Delivery, and Recovery room to Postpartum room. Intuitively, the patients' demand for bed *vis-à-vis* limited supply (entire occupation for the bed fully occupied) causes system congestion. By applying the ACOG recommended level on 148 hospitals (the study sample), it has shown significant excess in the level occupancy by 15% where overall occupancy was 60%. However, 117 of the 148 hospitals has a surplus of 15%, 27 hospitals had occupancy deficit. Her study is primarily reported occupancy in the Postpartum room while other hospitals said that most patients stayed in the same bed for both Labour, Delivery, and Recovery (LDR) room the Postpartum room where some obstetric unit combines them in one room Labour, Delivery, Recovery, and Postpartum (LDRP). This study is conservative because the resource planning optimality

of the beds' occupancy must catch a timely pattern rather than the ability to bed placement. This study also, estimated the data for the total number of beds in a hospital neglecting merged affiliated hospitals which may have a smaller size of the obstetric units. Thus, these small units reach full capacity fast.

Johnson (1997) modelled stochastic patients demand process for the obstetric and maternity department to identify improvements through augmenting the utilisation in the Labour, Delivery, and Recovery (LDR) room and the Postpartum room, and optimal potential expansion, in twelve models, given the resources from nurses, physicians, and other healthcare workers. Johnson adapted MedModel Software 3.0² to measure ALOS, mandatory discharge, aligned with six increase in the patient volume. Despite the high fluctuations in the bed utilisations and sizes due to variations in patient arrival and the expansions in some areas of the maternity unit (as per software recording), this study showed that quo capacity might reach its limit within a year; i.e. it is not fully utilised yet, given 75% threshold system's utilisation level.

Based on the DES model, Mahachek and Knabe (1984) used General Purpose Simulation System (GPSS)³ to model patient arrivals *vis-à-vis* available resources from Medical Practice (MD), Registered Nurse (RN), and Nurse Practitioner (NP) on the one hand, and cost/unit of MD/RN and NP/NA patient-to-cost, in the other. They found that the utilisation process should be stopped at 65% for MD and 80% for RN/NP. Patient-to-cost is higher for MD/RN compared to NP/Nurse Anaesthetist (NA). They adopted Poisson distribution to measure the variation in patient arrivals, and from the same distribution, they approximated the time needed for the epoch to occur (i.e. duration model). Consequently, staff combining clinic plan result was “no-go”, as the average volume of the patients had insufficient staff. In other words, resources do not meet patients demand for obstetric and maternity department.

² MedModel healthcare simulation software by PROMODEL Corporation 1993, to accurately record information on workload, staffing, polices, patient care process... etc. related to cost and quality, and the analyse this information statically and dynamically.

³ GPSS software is a process flow oriented simulation language originally developed by Geoffrey Gordon at IBM 1961.

Rayburn *et al.* (2012), estimated cross-sectional model based on patient arrivals and the time required to arrive at the hospital (driving time for 30 minutes and 60 minutes, estimation time base) in the one hand, and the resource of a dedicated staff of nurses, in the other. Onset symptom for the mothers to be driven to the hospital, such as vaginal bleeding or regular contraction, is an indicator for required admission to the department of obstetrics and maternity, which imply that dedicated staff should be ready to deal with the case. Given that 30-minute and 60-minute drive time are constant, the dedicated staff to serve the case capacity is subject to the volume patients and the type of the case herself. Moreover, this study deals with the hospital as a department of obstetrics and maternity represents the resource to meet pregnant mothers demand based on the geographical driving time distance. Therefore, they found that the optimal number of hospitals remains undetermined.

Study	Purpose	Resource	Data (Hospital, Period)	Finding
Cochran and Bhrati 2006 USA	Queueing Network Analysis (QNA) And DES model to balance bed unit utilisations across an obstetrics hospital by patients in-flow rate and pattern and minimise the booking of beds from up-stream units within give bed reallocation	Beds	200+ bed obstetrics Hospital in Arizona	It is necessary to consider different patient types, rather than the single patient type in the obstetric unit model, including patients with varying lengths of stay (LOS) based on their origin.
Green 2002 USA	Use queueing theory to analyse the beds' occupancy in the obstetric ward	Beds	148-New York City State Hospitals 1997	Lack of data and consistent policy(s) makes it impossible to model the consequences of bed unavailability accurately
Johnson 1997 USA	Use MedModel 3.0 to solve problems of customer satisfaction, utilisation, and patient flow.	Room/bed, Nurses, physicians and other health-care workers; Maternity unit divided into five areas: Triage, Perinatal Intensive Care Unit, Labour, Postpartum, and Antepartum	698-bed Miami Valley Hospital 1990-1997	There were large fluctuations in volume and bed utilisation in this clinical area. The existing process overextended the five regions in the maternity unit.

Mahachek <i>et al.</i>	General Purpose Simulation System (GPSS)	Nurse and Physicians	The Johns Hopkins Hospital	Should not go for combining clinics decision because the staff (costs and volumes) was insufficient for average patient volumes.
1984	patients flow, Optimal nurses and physicians	Medical Practice (MD), (Nurse Director) RN, Nurse practitioner (NP), Nurse Anaesthetist (NA)		
USA	to combine clinics			
Rayburn <i>et al.</i>	cross-sectional analysis of access to evaluate obstetric and maternity department patient arrival based on driving time to the hospital.	Nurses	2,606 hospitals	The optimal number of hospitals as a healthcare resource stills undetermined
2012				
USA				

Table 1 . Relevant Studies

C. METHODOLOGY

If we only depict the cause-and-effect relationship that determines the number of beds needed to meet the stochastic process of mothers in-flow over time, then the analysis for optimality is of a meagre. Therefore, we need to analyse this causality through models based on consistent properties and “The three golden rules of econometrics are test, test and test” (Hendry 1980).

C.1 STATISTICAL ANALYSIS AND RELEVANT TIME-SERIES

We need to analyse the process of patients in-flow, where the process defines queueing systems in the healthcare system. Standard $M/M/s$ or Erlang delay model that assumes a single queue with an unlimited waiting room that feeds into s identical servers. Thus, as the in-flow process follows the Poisson process with a constant rate – as we shall test later- and the service duration, e.g., Length Of Stay (LOS), has an exponential distribution. (These two assumptions are often called Markovian, hence the use of the two “M’s” in the notation used for the model)⁴, perfect model.

$$\text{Probability } \{N(t) = n\} = e^{-\lambda}(\lambda t)^n/n!$$

$$\mathbb{E}(N) = \mu \text{ and } \text{Var}(N) = \sigma^2 = \mu$$

Where the number of arrivals $N(t)$ during time duration t and λ is the expected number of arrivals per unit time.

We shall study the patient in-flow described as the numbers vary over a short time and the number of beds which represent the capacity of the hospital, and expressed as the quantities vary over long time variables. Patients in-flow process can be described as the Poisson process, which mathematically characterised by:

⁴ Randolph (2013), PP 368.

- i. Additive property: Sum of n independent Poisson processes with parameter $\lambda, \lambda_1 + \lambda_2 + \dots + \lambda_n$
- ii. Decomposition property: $\{M(t), t \geq 0\}$ and $\{M_1(t), t \geq 0\}$ are independent i.e. randomness in Poisson process yields a Poisson process.
- iii. Interarrival times: the interval between two successive realisations of Poisson processes with λ is independent and identically distributed (iid).
- iv. Memoryless/Markovian property of Exponential Distribution: Assume X is exponentially distributed with mean $\frac{1}{\lambda}$; then
 $\Pr\{X \geq x + y \mid X \geq x\} = \Pr\{X \geq y\}$ is independent of x ,
for LHS equals

$$\begin{aligned} \frac{\Pr\{X \geq x + y \text{ and } X \geq x\}}{\Pr\{X \geq x\}} &= \frac{\Pr\{X \geq x + y\}}{\Pr\{X \geq x\}} \\ &= \frac{e^{-\lambda(x+y)}}{e^{-\lambda x}} \\ &= e^{-\lambda y} = \Pr\{X \geq y\}. \end{aligned}$$

- v. Randomness/stochastic property
Given $\{N(t), t \geq 0\}$ occurred by epoch T , then the interval $\gamma \in [0, T]$ i.e.
 $\Pr\{t < \gamma < t + dt \mid N(T) = 1\} = \frac{dt}{T}, 0 < t < T$ ⁵

Moreover, these properties show the stationarity in the process where the realisation (γ values at a given point of time) of the process implies a patient arrival is independent of another patient arrival, and occurs once at a time, and is not a function of time, therefore:

$$\begin{aligned} \mathbb{E}(\gamma_t) &= \mu \neq f(t) \\ \text{VAR}(\gamma_t) &= \sigma^2 \neq f(t) \\ \text{COVAR}(\gamma_t, \gamma_{t+1}) &= f(\eta) \\ &\neq f(t) \end{aligned}$$

Besides, the randomness in the value of a particular realisation does not mean just any value; it means that before we see any the individual realisations, we have no certainty what value the process is going to take. In other words, γ is not a deterministic process.

⁵ Medhi (2003), pp 28.

Therefore, the daily number of births is the deterministic independent variable in our model, which it is the factor that affects the average number of the non-negative dependent variable; i.e. beds.

Poisson distribution $\chi^2 = \sum \left(\frac{X_i - \mu_i}{\sigma_{x_i}} \right)^2$, test for the daily birth would be the cornerstone.

It is crucial to know whether the daily births, i.e. observed frequency differs from a postulated theoretical one.

To verify the stationarity in the process, Augmented Dickey-Fuller (ADF) and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests, shall be used, where:

Augmented Dickey-Fuller (ADF) test: the basic idea of ADF-test (1979) is testing whether we have a non-stationary time series, or we have a unit root, i.e. $\rho = 1$.

$$Y_t = \mu + \rho Y_{t-1} + e_t; \quad e_t \sim NID(0, \sigma^2) \quad \text{where } Y_t \text{ and } Y_{t-1} \text{ are non - startionary}$$

$$H_0: \rho = 1 \text{ against } H_a: \rho < 1$$

$$\Delta X_t = \mu + (\rho - 1)X_{t-1} + e_t$$

therefore under H_0 , X_{t-1} vanishes

Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test: the basic idea of KPSS test (1991) is testing of the null hypothesis of stationarity against the alternative of a unit root. KPSS null hypothesis is trend stationary.

$$y_t = \xi_t + r_{t-1} + \epsilon_t;$$

$$r_t = r_{t-1} + u_t; \quad u_t \sim iid(0, \sigma_u^2)$$

$$H_0: \sigma_u^2 = 0$$

Contradiction in the results we check the first-order differencing, then run ADF and KPSS tests again:

$$y_t = y_t - y_{t-1}$$

C.2 LENGTH OF STAY (LOS)

The idea of modelling the Length of Stay (LOS) is to estimate the length of stay for a mother who admits for delivery until she discharges ('discharge' includes deceased and transferred to another hospital), complete follow-up.

Modelling the Length of Stay (LOS) is based on the sampling patterns for collecting; thereby, there are three models⁶:

- The Retrospective sample is collected by identifying a series of mothers at a specific time and ascertaining the number of the completed time each mother has been institutionalised.
- Partial follow-up sampling involves identifying a series of mothers at a specific time and observing each mother's entire stay from admission to discharge.
- Interval sampling consists of identifying many mothers at a specific time and following these and any newly admitted mothers over a specified period. At the end of this interval, some mothers will have been discharged, and others will still be institutionalised.

This study shall model the Retrospective sampling pattern. Retrospective means that the theoretical density function $f(x)$ reflects that the mother actual stays higher than or equals to Average Length of Stay (ALOS), $(P(X \geq x))$. To fulfil the model unbiasedness estimation requirement; the probability distribution of Length of Stay (LOS) must be discrete, and its probability distribution is the same for each cohort admitted to the obstetrics and maternity department.

⁶ Selvin, Steve (1977) PP 324

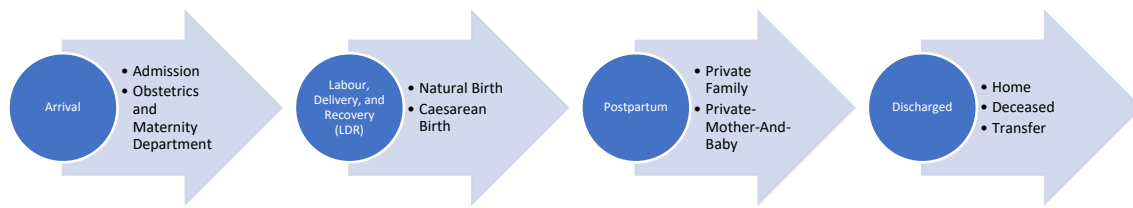


Figure 8. Retrospective Length of Stay (LOS) Model

Assuming normal birth process, i.e. without risk of medical complication; the length of stay for a mother who arrives for delivery can be divided into four stages as Figure 8. Mother starts from the Arrival, where she admits into the Obstetrics and Maternity department, thereby she gives birth whether Natural birth or Caesarean birth at the Labour, Delivery, and Recovery (LDR) room. Subsequently, she transfers to the one of Postpartum room (Private Family or Private-Mother-And-Baby). At the final stage mother will be discharged, where she goes home.

C.3 SYSTEM DYNAMICS SIMULATION (SD)

Dynamic Simulation replicates the real-world system for ‘what-if?’ and thereby refers to the explanation for the complexity in the hospital system behaviour over time.

There are three conventional methods in dynamic simulation modelling; each of them has its aspects in term of the type of problems, the focus, and the handling of time:

- System Dynamics (SD) deals with the Strategic and Operational kind of problems. It concerns with Group/Aggregate (Cohort), and it handles Continuous time.
- Discrete-Event Simulation (DES) deals with the Operational and Tactical type of problems. It concerns with Individual (Entity), and it handles Discrete time.
- Agent-Based Modelling/Simulation (ABM/S) deals with Strategic, Operational, and Tactical type of problems. It concerns with Individual (Agent), it handles discrete time.

Congestion in the hospital system depends on two factors: Unscheduled in-flow stochastic process for mothers and the number of beds at the Postpartum rooms.

Therefore, we shall concern with the aggregate resolution for a continuous time provided by the System Dynamics (SD) model. The System Dynamics technique does not study the reason behind variability in individual daily birth interaction with the system; it is a *cohort*. Its technique is more straightforward and less detailed. In other words, this technique identifies the implications of fluctuation and unpredictable demands for Obstetrics and Maternity department admissions on the hospital bed capacity.

The System Dynamics (SD) technique has the primary building blocks *feedback loops*, *stocks* and *flows*. The *stock* is to keep track of the levels of the daily number of births, while the *flows* are for the rate of change of this number. *Feedback loops* describe the circular interaction between the demand by the patients for labour and delivery, and the supply represented by the number of beds in the Labour, Delivery, and Recovery (LDR) and Postpartum rooms.

C.4 POISSON COUNT MODEL

Poisson count data belongs to general linear models' (GLM) family; recalling the Poisson distribution:

$$P(Y = y) = \frac{e^{-\mu} \mu^y}{y!},$$

where, y is a certain number for the Poisson model, and μ is the intensity or rate parameter:

$$\mu = \exp(x_i' \beta)$$

In words, creating the Poisson model can be by the parametrisation. This model has a property:

$$\mathbb{E}(y|x) = \text{var}(y|x) = \mu$$

In words, there is an equality of mean and variance. This restrictive property often fails to hold in practice.

D. DATA AND RESULTS

This section shall empirically model, simulate, and test public data for East Tallinn and the University of Tartu hospitals during (2017-2018).

D.1 DISTRIBUTION AND TIME-SERIES

Figures 1 and 2 show the time-series of daily birth for mothers who admitted for delivery in the East Tallinn and the University of Tartu hospitals. East Tallinn Hospital reports $\mu = 11.62$ of the daily number of births exceeds available rooms = 9, comparing to the $\mu = 6.67$ in the University of Tartu where available rooms are 8.

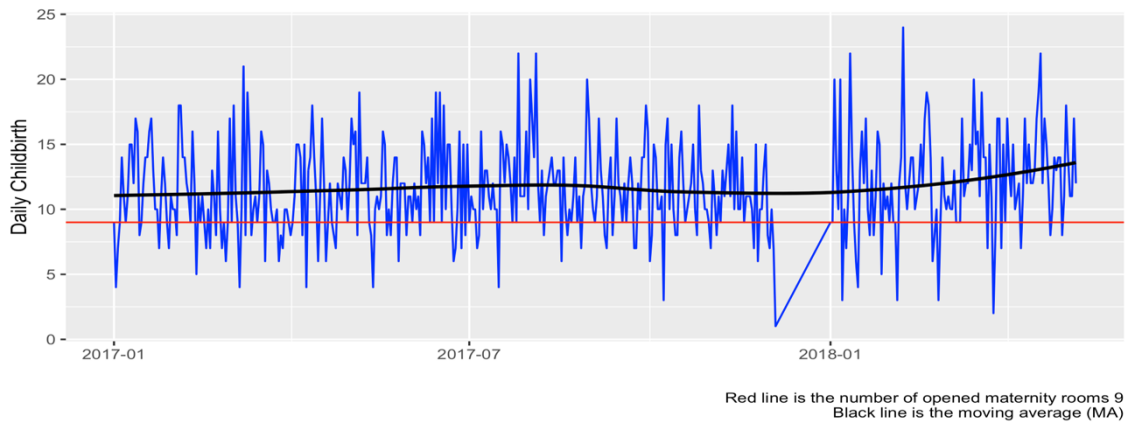


Figure 1. Distribution of the number of daily births and moving average, ET Hospital (01.01.2017-06.05.2018).

Source: Author's calculations

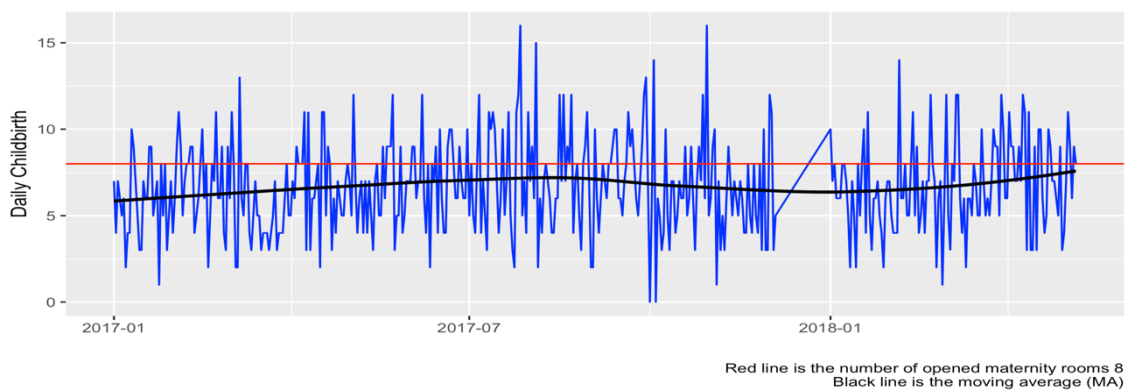
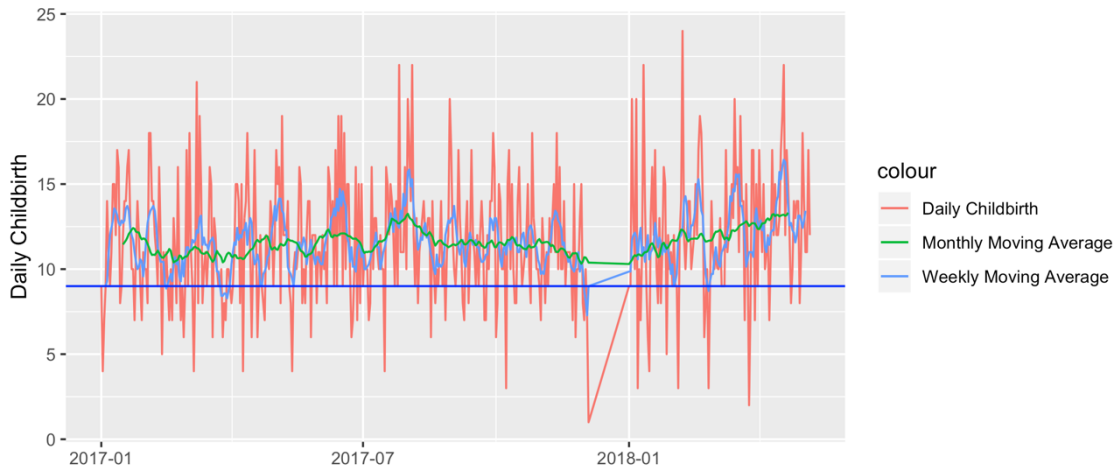


Figure 2. Distribution of the number of daily births and moving average, UT Hospital (01.01.2017-06.05.2018).

Source: Author's calculations

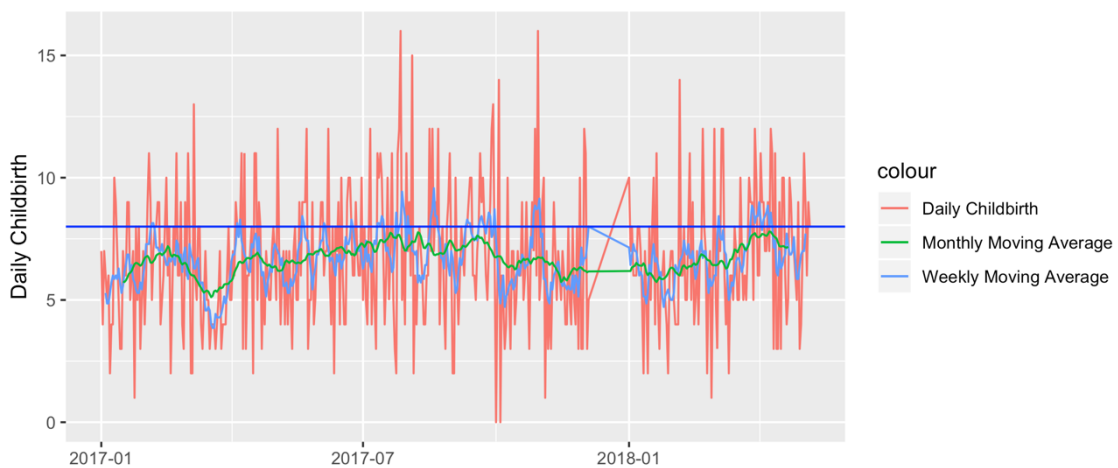
Seasonal distribution gives closer picture as Figures 3 and 4 show that the daily, weekly, and monthly (MA) in the East Tallinn hospital is higher available rooms, while they are lower than available rooms.



Red line is the number of opened maternity rooms 9

Figure 3. Seasonal distribution of the number of daily births and moving average, ET Hospital (01.01.2017-06.05.2018).

Source: Author's calculations



Red line is the number of opened maternity rooms 8

Figure 4. Seasonal distribution of the number of daily births and moving average, UT Hospital (01.01.2017-06.05.2018).

Source: Author's calculations

Figure 5 shows that the distribution and the variation in the number daily birth in both hospitals are not similar. East Tallinn hospital reports $\sigma^2 = 13.30$, while the University of Tartu reports lower variance $\sigma^2 = 7.32$.

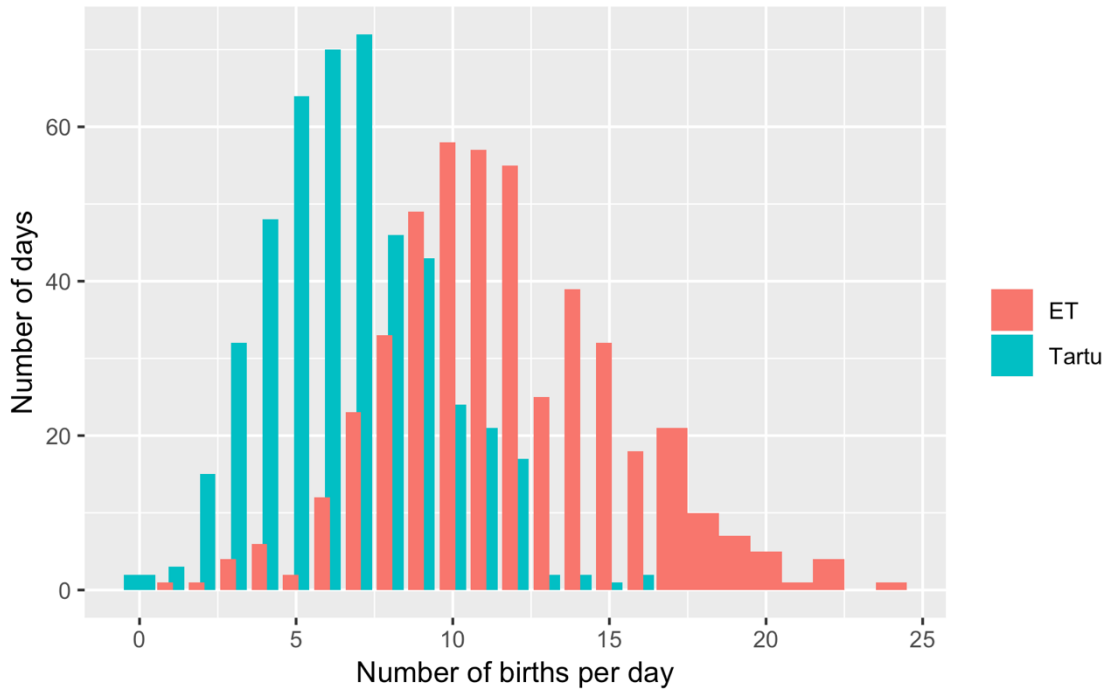


Figure 5. Distribution of the number of daily births, UT Hospital (01.01.2017-06.05.2018).

Source: Author's calculations

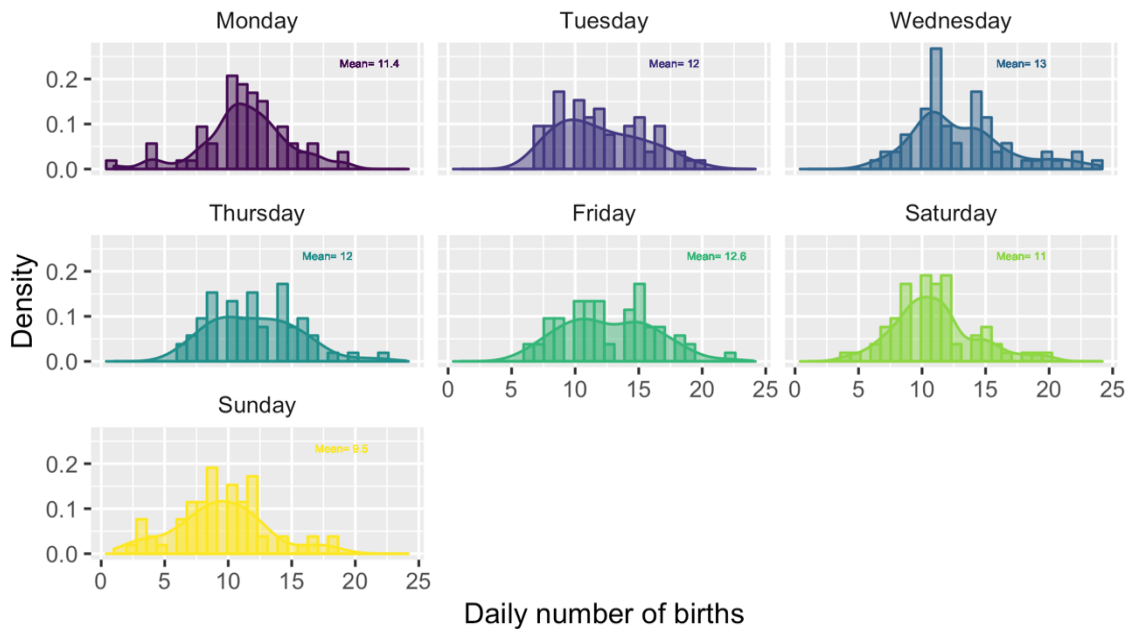


Figure 6. Distribution of the number of daily births with mean, UT Hospital (01.01.2017-06.05.2018).

Source: Author's calculations

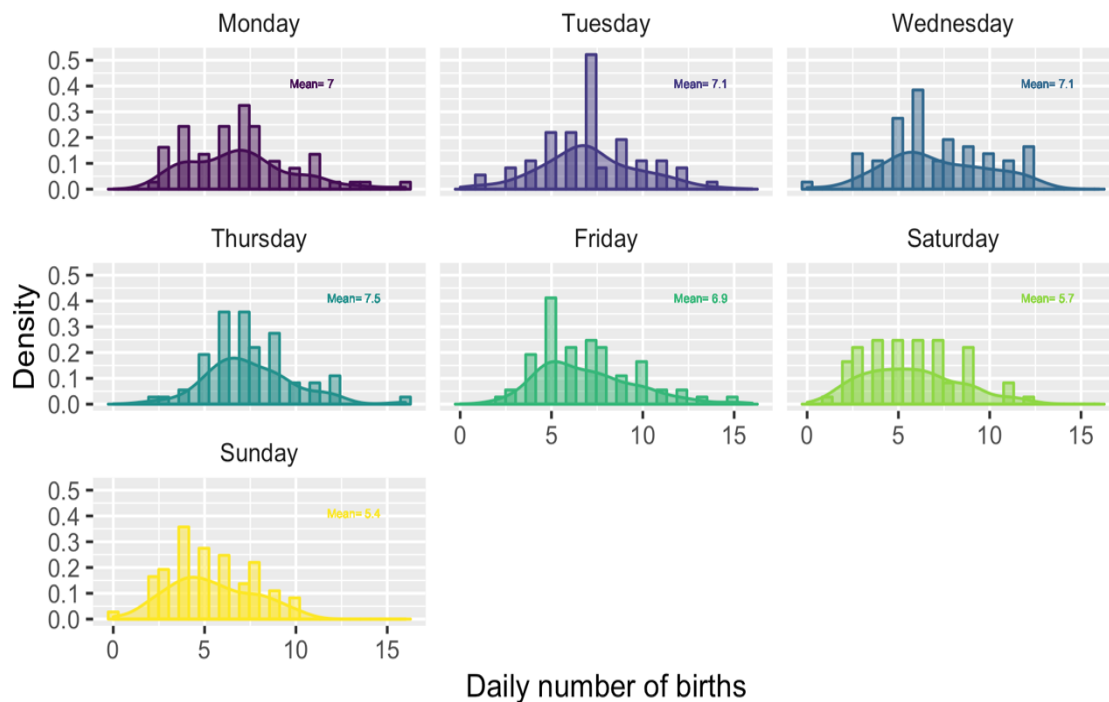


Figure 7. Distribution of the number of daily births with mean, UT Hospital (01.01.2017-06.05.2018).

Source: Author's calculations

Figures 6 and 7, show the variation in the daily number of births across the weekdays at both hospitals. East Tallinn hospital shows $\mu = 13$ on Wednesdays is higher than any other day of the week especially on Friday $\mu = 9.5$. While, the variation across the weekdays at the University of Tartu hospital shows Tuesdays have the highest $\mu = 7.5$ Cf. any other days of the week especially, on Fridays $\mu = 5.4$ too.

We noticed that the variation during the mid-week births is high compared to other days. This result helps the decision-maker to plan resource to meet highly non-elective birth during the mid-week. Furthermore, Poisson distribution describes the stochastic variation in the number of daily births very well.

It is noteworthy that this study is inspired by Gam et al. (2013) analysed the daily birth distribution. His descriptive retrospective study in Denmark for the period between 2000 and 2009. He similarly found that the mid-week variation is high, and the Poisson is adequate for describing daily births.

Table 2. shows that the distribution of the daily birth at both hospitals follows Poisson distribution as we fail to reject H_0 : True value equals event rate at statistical significance level $\alpha = 0.05$. ADF test results confirm stationarity in the process for the both hospital where we reject the H_0 : we have a unit root at significance level $\alpha = 0.05$.

KPSS test results for the East Tallinn hospital confirms trend stationarity where we fail to reject the H_0 : we have a trend stationary at statistical significance level $\alpha = 0.05$ but we reject H_0 for the University of Tartu process at statistical significance level $\alpha = 0.05$. Therefore, we run the tests again for the first-order difference for the University of Tartu process whereby KPSS test confirms trend stationary as we fail to reject the H_0 : we have a trend stationary at statistical significance level $\alpha = 0.05$.

	Exact Poisson χ^2	ADF	KPSS	First-Order Difference
East Tallinn	11.629 P-Value = 1	-6.760 P-Value = 0.01	0.079 P-value = 0.1	
University of Tartu	6.762 P-Value = 1	6.455 P-Value = 0.01	1.55 P-value = 0.04	ADF: -13.163, P-value = 0.01 KPSS: 0.007, P-value = 0.1

Table 2. Tests Results for Poisson distribution and Stationarity

Source: Author's calculations

D.2 POISSON COUNT MODEL

Decision-maker(s) may also interest in the distributional consequences of the decision, and by estimating the effect of the change based on the average count. Poisson count model provides Incident Rate Ratio (IRR) or relative risk estimation as Table

Table 3. shows the Poisson count model estimation results for the hospitals. The Goodness of fit χ^2 test results report that residual deviance is close the residual degree of freedom, which means the models are fit well.

The restrictive property of the Poisson count model for the hospitals hold. Thereby, we fail to reject $H_0: \mathbb{E}(y|x) = var(y|x) = \mu$, i.e.; there is no over-dispersion $var(y) > \mathbb{E}(y)$. The ratio of residual deviance confirms our failure to reject the null hypothesis and the adequacy for Poisson count model assumption.

According to the estimated Incident Rate Ratio (IRR) results for the East Tallinn hospital, they report that on Sundays there will be 81.6% less births compared to Mondays -the weekdays base- which is statistically significant at $\alpha = 0.01$. While The estimated coefficients for the University of Tartu hospital report that on Sundays there is 22.9% less birth compared to Mondays, which is statistically significant at $\alpha = 0.01$. Seasonal IRR results report for the East Tallinn hospital report that on Aprils, there will be 7.4% less births compared to Januarys -the months base- which is not statistically significant at $\alpha = 0.05$. While The estimated coefficients for the University of Tartu hospital report that on Aprils there is 18.2% less birth compared to Januarys, which is statistically significant at $\alpha = 0.05$

Dependent Variable:				
	East Tallinn		University of Tartu	
	Beta	IRR	Beta	IRR
Tuesday	0.046	1.047	0.024	1.024
	-0.051	-0.053	-0.065	-0.067
Wednesday	0.131***	1.139***	0.025	1.026
	-0.05	-0.057	-0.066	-0.067
Thursday	0.052	1.053	0.084	1.087
	-0.051	-0.053	-0.065	-0.07
Friday	0.105**	1.111**	-0.011	0.989
	-0.05	-0.056	-0.066	-0.065
Saturday	-0.037	0.964	-0.207***	0.813***
	-0.052	-0.05	-0.069	-0.056
Sunday	-0.184***	0.832***	-0.260***	0.771***
	-0.054	-0.045	-0.07	-0.054
February	-0.007	0.993	0.119	1.127
	-0.055	-0.054	-0.073	-0.083
March	0.011	1.011	0.043	1.044
	-0.053	-0.054	-0.073	-0.076
April	0.071	1.074	0.167**	1.182**
	-0.053	-0.057	-0.071	-0.084
May	0.041	1.042	0.133	1.142
	-0.061	-0.063	-0.082	-0.093
June	0.059	1.06	0.186**	1.205**
	-0.065	-0.068	-0.086	-0.103
July	0.03	1.03	0.227***	1.255***
	-0.065	-0.067	-0.084	-0.106
August	0.078	1.081	0.196**	1.216**
	-0.063	-0.069	-0.085	-0.103
September	0.019	1.02	0.237***	1.267***
	-0.065	-0.067	-0.085	-0.107
October	-0.007	0.993	0.075	1.078
	-0.065	-0.065	-0.088	-0.095
November	-0.022	0.978	-0.048	0.953
	-0.066	-0.065	-0.093	-0.088
December	-0.557***	0.573***	0.336*	1.400*
	-0.204	-0.117	-0.188	-0.263
Constant	2.413***	11.166***	1.824***	6.198***
	-0.05	-0.559	-0.067	-0.414
Observations	464	464	464	464
Log Likelihood	-1,225.612	-1,225.61	-1,082.08	-1,082.08
Akaike Inf. Crit.	2,487.225	2,487.225	2,200.16	2,200.16
Goodness of fit	0.138	0.138	0.271	0.271
Over-dispersion	0.38	0.38	0.70	0.70
Residual Deviance	1.076	1.76	1.030	1.030
Note:		*p<0.	1; **p<0.05;	***p<0.01

Table 3. Estimated Poisson count model.

Source: Author's calculations

D.3 LENGTH OF STAY (LOS) AND SYSTEM DYNAMICS SIMULATION (SD)

The Obstetrics and Maternity department at East Tallinn and the University of Tartu hospitals has two types of Postpartum rooms at different prices. Where East Tallinn hospital has thirty-seven beds, the University of Tartu has thirty beds.

According to the National Institute for Health Development in Estonia⁷, the Average Length of Stay (ALOS) at the Obstetrics and Maternity department is 2.8 days. We assume *no* medical complications during mothers Length of Stay (LOS) at the Postpartum room, and two categories of births: non-elective *Natural and elective Caesarean (C-Section)*. Thus, we can calculate the probability of daily natural births as 2, 3, or 4 days with a probability of 50%, 40%, and 10%, respectively. The probability of daily Caesarean birth is 10%, 20%, and 70%, respectively⁸.

Therefore, the simulated number of mothers who transfers to the Postpartum when beds are full to capacity as we see in Figure 9. The results show that East Tallinn hospital simulated mothers excess the number of beds by 25.9, while the University of Tartu excess was 3.4.

Furthermore, this simulation suggests that an optimal number of beds meets at least 95% of mothers demand for beds. Calculating 95% upper-quartile from simulated mother distribution yields, 43.65 assumes that only 5% of mothers will need to wait for the next available beds. Put it differently, by calculating the 5% lower-quartile 13, decision-maker may set a threshold for beds shortage up-to 5% and then no more 13 mothers need to wait.

⁷ Tervise Arenu Insituut, Table: RV301, 2017. $ALOS = \frac{\text{Number of in bed days in observed period}}{\text{Number of discharged in observed period}}$

⁸ $P(X = k) = p^k \cdot (1 - p)$

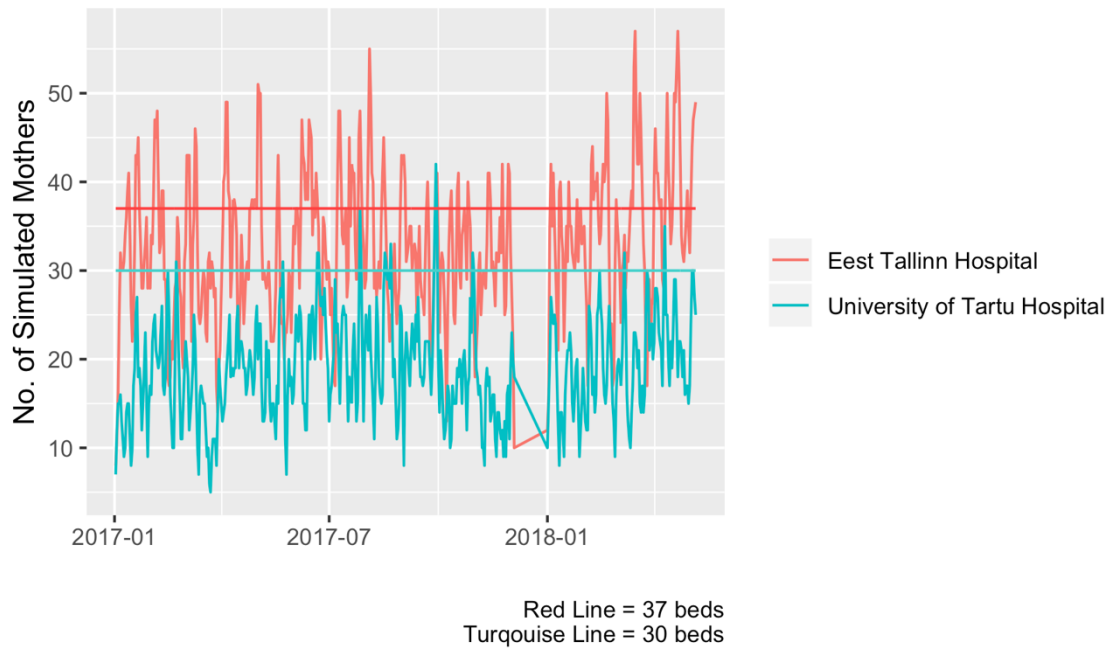


Figure 9. Distribution of the simulated number of mothers who transfer to the Postpartum room from Labour, Delivery, and Recovery (LDR) room.

Source: Public Data

These results also imply that - in particular - elective Caesarean birth can be scheduled on days which the department has lower beds occupancy in the Postpartum rooms. Furthermore, Figure 10. shows that East Tallinn hospital on Sundays the median = 29, minimum number (there is no outlier) = 10, quartile 1 = 25, quartile 3 = 34.5, and thus interquartile = 9.5. On Fridays the median = 34, the maximum number (there is no outlier) = 53, quartile 1 = 29, quartile 3 = 39, and thus interquartile = 9. Sundays are the optimal days for delivery and Fridays are the optimal days for discharge.

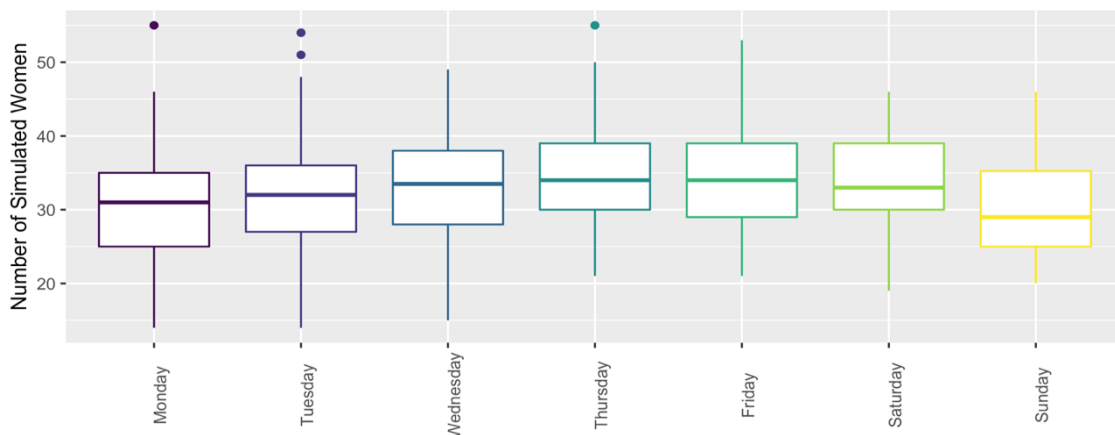


Figure 10. Distribution of the simulated number of mothers who transfer to the Postpartum room from Labour, Delivery, and Recovery (LDR) room at East Tallinn hospital.

Source: Author's calculations

Figure 10. shows that the University of Tartu hospital on Mondays the median = 16, minimum number (there is no outlier) = 10, quartile 1 = 13, quartile 3 = 19.5, and thus interquartile = 6.5. On Fridays the median = 20, the maximum number (there is no outlier) = 34, quartile 1 = 16.25, quartile 3 = 25, and thus interquartile = 8.5. Mondays are the optimal days for delivery and Fridays are the optimal days for discharge.

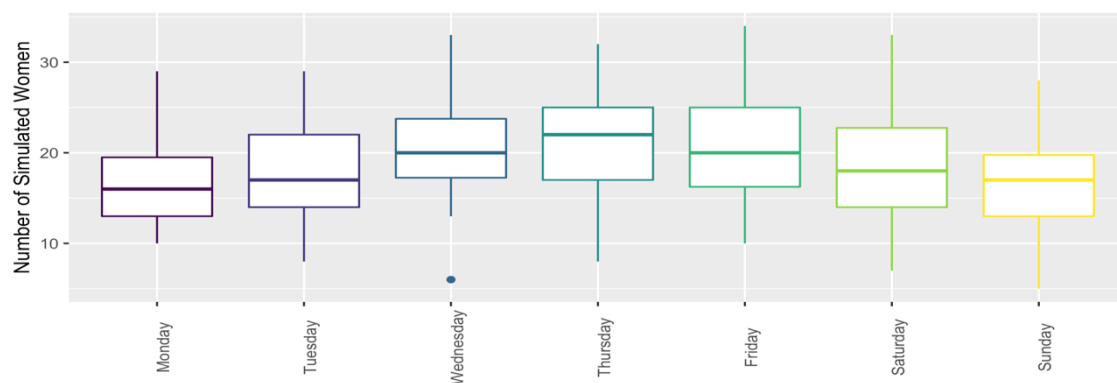


Figure 11. Distribution of the simulated number of mothers who transfer to the Postpartum room from Labour, Delivery, and Recovery (LDR) room at the University of Tartu hospital.

Source: Author's calculations

E. CONCLUSION

It is difficult to arrive at any conclusions concerning the optimal number of beds required to meet mothers stochastic in-flow process. The in-flow lacks partly for the upper limit; it increases as an effect of a more efficient system. The complexity in this industry can never be optimised but handled, prioritised, and balanced

Despite the limitation of data whereby births registered by date, not on time; this study captures the variation in daily births across weekdays, the daily number of births *vis-à-vis* available number of beds, and the relative risk that corresponds to the variation in the daily births.

Poisson distribution well describes the number of daily births where the variation is high during the mid-weeks. The System Dynamics simulation technique may provoke the decision-maker to set a threshold where the hospital admission system stops when beds occupancy is 5% more to full to capacity; or set a threshold for the beds running short to 5%. Decision-maker may consider budget allocation towards increasing the Obstetrics and Maternity department beds capacity.

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