



DISSERTATIONES MATHEMATICAE UNIVERSITATIS TARTUENSIS

3



**PHENOMENOLOGICAL (CONTINUUM)
THEORY OF TURBULENCE**

by

J. Heinloo

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ABSTRACT OF THE INVESTIGATIONS PRESENTED TO OBTAIN
THE ACADEMIC DEGREE OF A DOCTOR OF MATHEMATICS

by J. Heinloo

TARTU 1992

The results of the investigations presented in the paper were obtained in the Institute of Hydrodynamics of the Siberian Branch of the Academy of Sciences of USSR (1972-1975), Institute of Thermo- and Electrodynamics of the Estonian Academy of Sciences (1975-1990) and Institute of Ecology and Marine Research of the Estonian Academy of Sciences (1990-1992)

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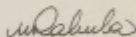
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Secretary of the Council

Dr. Phys. and Math., Professor



M. Rahula

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Jaak Heinloo was born in 1945. Graduated the Tartu University, 1972, postgraduate student 1972-1975 (in the Institute of Hydrodynamics of the Siberian Branch of the Academy of Sciences of USSR), PhD (kandidat nauk) 1982. Employed in different scientific work in the Institute of Thermo- and Electrophysics and Institute of Ecology and Marine Research of the Estonian Academy of Sciences. Since 1990 a leading scientist of Institute of Ecology and Marine Research of the Estonian Academy of Sciences. Author of 30 scientific papers, scientific advisor to 1 PhD.

INTRODUCTION

The results presented in the paper are: the doctrine of turbulence description (the turbulence metatheory) and the phenomenological (continuum) theory of turbulence as one of the node theories of the proposed turbulence metatheory.

The turbulence problem can be considered from aspects related to different scientific disciplines: mathematics, mechanics, theories of instability and chaos, synergetics, informatics, technical sciences (engineering) and others. The proposed metatheory allows to consider many of these aspects in the framework of a single doctrine. The doctrine is based on the treatment of different theories of turbulence (based on different paradigms and describing the media behaviour in the terms of fixed codes) as elements of a certain system.

The full system analysis of different turbulence descriptions is not the matter of this paper. We use the formulated principles of turbulent metatheory only with one aim in view to determine the place of phenomenological (continuum) theory of turbulence in the whole set of different theories of turbulence.

The proposed theory treats the turbulent media as a stochastic system with hierarchy of structural levels (arising from the simultaneous existence of processes with different time-space variability) in turbulent media and as a continuum with the property of rotational asymmetry (arising from the existence of some prevailing orientation of rotation to turbulent eddies in a turbulent media). The former relates the description of turbulent motions to the theory of hierarchical stochastic

systems, and the latter - to mechanics of continuums with internal rotating degrees of freedom.

In many cases equations of motion of the proposed theory can be integrated analytically. The existence of series of analytical integrals facilitate the analysis of situations under consideration. In several cases the calculated quantities are compared with the corresponding experimental data.

The presented results were obtained mostly in the period of 1972-1984 (Heinloo 1980, 1981, 1982a,b, 1984; Heinloo and Toompuu 1980, 1981, 1982; Toompuu and Heinloo 1980; Nemirovski and Heinloo 1976a,b,c, 1977a,b, 1978a,b, 1979, 1980, 1982; Makarenko, Nemirovski and Heinloo 1979). The results of the this period were generalized and summed up in the monograph "Phenomenological Mechanics of Turbulent Flows" (Heinloo,1984). Some of the results published in the monograph were accepted earlier by the Novosibirsk University as an elective subject for students of physics and mathematics (Nemirovski and Heinloo: 1980,1982).

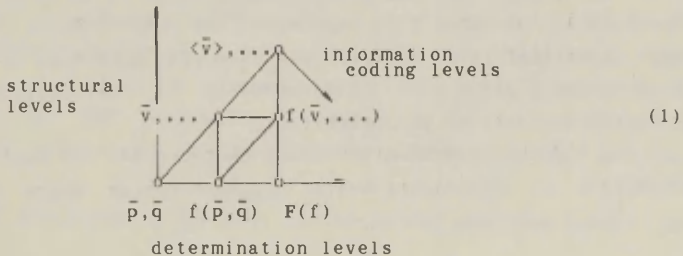
My occupational activities in the period of 1984-1991 were mostly characterized by the endeavour to put the proposed theory into practice in the marine research (Heinloo and Vösumaa 1987; Vösumaa and Heinloo 1989, 1990a)b),c); Toompuu, Heinloo, Soomere 1989). Two of the obtained results: dealing with Gibraltar salt anomaly and modelling vertical structure of the sea are shortly summarised at the end of the paper.

THE GENERAL DOCTRINE FOR DESCRIBING TURBULENT MEDIA

I. The turbulence metatheory.

The turbulent media can be considered as material systems with specific space-time structure. This structure can be presented on different structural levels corresponding to different decompositions of the media.

It follows from the essence of turbulent media as system with different structural levels that the descriptions of turbulence based on different conceptions (paradigms) of their structure and on different information coding levels can be considered as a system. (The codes - the sets of signs uniquely fixing the states of the system on the fixed levels of their description.) The elements of this "system of theories" are the node theories where every node theory is based on an independent paradigm and based on the fixed code. The connections between the two different node theories are formulated as recoding theories. The recoding theories bind the set of node theories in a system. The amount of different node theories depends on the number of structural levels under consideration and is organized according to the code network. In the case of three-level presentation of the structure of turbulent media (the canonical, the Navier-Stokes and the turbulence levels) the code network will have the form,



The codes and node theories presented in (1)

Main signs of node codes	Equations of motion
\bar{p}, \bar{q} - canonical variables	$\dot{\bar{p}} = -\frac{\partial H}{\partial \bar{q}}, \dot{\bar{q}} = \frac{\partial H}{\partial \bar{p}}$ - the Hamilton's equations
$f(\bar{p}, \bar{q})$ - probability distribution for \bar{p}, \bar{q}	$\dot{f} = [H, f]$ - the Liouville eq.
$F(f)$ - probability distribution for f	$\dot{\Phi} = i(\Theta \cdot [H, D\Phi])$ (Heinloo)
$\bar{v} = v(\bar{r}, t)$ - velocity field	$\rho(\bar{v} + \bar{v} \cdot \nabla \bar{v}) = -\nabla p + \nu \Delta \bar{v} + \bar{of}$ - the Navier-Stokes eq.
$f = (v, \dots)$ - probability distribution for v	the Hopf eq.
$\langle \bar{v} \rangle$ - averaged velocity	eq. of phenomenological (continuum) theory of turbulence (Heinloo)

(overbar denotes vector; [,] is the Poisson's brackets; $\Phi = \int F(f) \exp(i\Theta \cdot f) df$ - the characteristic functional, Θ - an arbitrary function; D is the variation derivative; a dot above the signs denotes the partial derivative $\partial/\partial t$ and between the signs a scalar product.)

According to (1), in the case of three level presentation of the structure of turbulent media, the total amount of the node theories is equal to 6. The node theories of the same determination level describe different qualities of the media with the same horizon of predictability. The lower the structural level the higher the information coding level in terms of which the description quality is expressed. The node theories of the same structural level describe the different qualities of media as different properties of an ensemble of realizations with different horizons of predictability.

The highest determination level characterizes the most stable properties of turbulence media. Any description based on the

lowest determination levels can be interpreted as the local (in time) description of these properties.

Besides the node theories, the code network (1) consists of 6 recoding theories of two types:

the recoding theories of the first type (kinetic theories)

$$\begin{array}{ccc}
 \begin{array}{c} \circ \bar{v}, \dots \\ | \\ \circ f(\bar{p}, \bar{q}) \end{array} & , & \begin{array}{c} \circ \langle \bar{v} \rangle, \dots \\ | \\ \circ f(\bar{v}, \dots) \end{array} & , & \begin{array}{c} \circ f(\bar{v}) \\ | \\ \circ F(f) \end{array}
 \end{array} \quad (2)$$

the recoding theories of the second type

$$\begin{array}{ccc}
 \begin{array}{c} \circ \text{---} \circ \\ | \quad | \\ \bar{p}, \bar{q} \quad f(\bar{p}, \bar{q}) \end{array} & , & \begin{array}{c} \circ \text{---} \circ \\ | \quad | \\ \bar{v}, \dots \quad f(\bar{v}, \dots) \end{array} & , & \begin{array}{c} \circ \text{---} \circ \\ | \quad | \\ f(\bar{p}, \bar{q}) \quad F(f) \end{array}
 \end{array} \quad (3)$$

The recoding theories of the first type bind two node theories with each other on the neighbouring structural levels. The first recoding theory in (2) is the classical kinetic theory and the second - the kinetic theory of random (stochastic) fields. The theoretical principles of the first recoding theory in (2) were found by Boltzmann and developed further by Kirkwood, Bogoljubov, Green, Rice, Olmsted, Focker, Planck and others. The first works in the field of kinetic theories of stochastic fields were written in early 60-es by Benney, Saffman, Newell, Hasselmann, Keynon and Zahharov. The most interesting results in this field in our days are connected with investigations of different wave systems (Reznik, Soomere).

It is essential to note that the determination of the probability distribution function $f=f(\bar{p}, \bar{q})$ as any probability, assumes simultaneous determination of the conditions in which the probabilities are formed. In the situation, considered in the framework of classical kinetic theory, the necessary additional information is expressed in terms of the Navier-Stokes code. The determination of $f(\bar{p}, \bar{q})$ presumes that the information expressed in terms of the Navier-Stokes code plays to some extent a permanent role relative to the statistical description of the media.

The situation in the case of the second recoding theory in

(2) establishing accordance between statistical hydromechanics and the continuum theory of turbulence, is totally similar to the first. The circumstance that the media behaviour on the higher structural levels has always some permanent quality that cannot be derived from the information obtained in its descriptions on the lowest structural levels is essential. In particular, there is no hope to obtain the full description of turbulence based only on the principles of statistical hydromechanics (Monin, Jaglom, 1967).

The main problems of recoding theories of the second type establishing accordance between two node theories on the neighbouring determination levels are connected with the questions of transitions "from order to chaos" and "from chaos to order". It is essential to note that the notions "chaos" and "order" are always connected with the fixed codes: the chaos in terms of one code may appear as order in terms of another code.

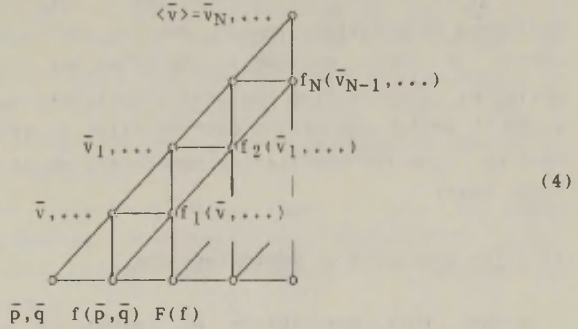
The problems of transitions "from order to chaos" are connected with the theories of instability and bifurcations. The instability problems for \bar{p}, \bar{q} code have been discussed in the classical works of Penleve, Puancare, Bruns and for the Navier-Stokes code in the works of Helmholtz, Orr, Sommerfeld, Landau, Tollmien, Schlichting et.al. The most modern problems in this field can be found in the works of Feigenbaum, Ruell and Takens.

The investigations of the problems "from chaos to order" belong to the field of synergetics. The most intricate problems in this field are associated with the "strange attractors" (Lorentz), intensively discussed in the modern mechanics.

11. Higher levels of the presentation of turbulent media.

The turbulent level in (1) includes all spectrum of space-time variability of hydrodynamic fields (defined on the Navier-Stokes level). In general, in turbulent media the hierarchy of structural levels with different space-time variability exists. It means that the turbulent level in (1) itself can be presented as a certain amount of different structural levels. In

terms of the code network it means the replacement of the code network (1) with a more general code network



where: $\bar{v}, \dots; \bar{v}_1, \dots; \langle \bar{v} \rangle = \bar{v}_N, \dots$ are the codes, fixing the states of media on different levels of their space-time variability.

It is essential to underline that the different presentations of the media corresponding to different N in (4), describe always the same situation (although, with different completeness). It follows from the above that physical, mechanical and the other properties of media have to be described in the forms, invariant from the concrete number of structural levels. The mentioned requirement for invariance plays a significant role as a requirement for formulating the concrete theories of turbulence.

The choice of a suitable number of structural levels depends on our concrete targets, our concept of the real structure (symmetry) of the media and theoretical, mathematical, experimental and etc. possibilities to solve the formulated problem.

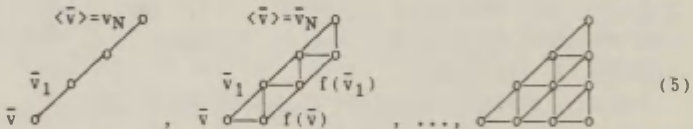
The signs of the name codes (the node codes corresponding to the lowest information coding level) can always be expressed as averaged functions defined by the name codes of the lowest structural levels. So, as any function can be presented as the corresponding Taylor's series, the signs of any name code can be expressed also through the corresponding statistical moments

defined by the signs of the name codes of the lowest structural levels. The last circumstance was used, e.g., in statistical hydromechanics as the main argument for demonstrating the priority of statistical descriptions of turbulent media to their continuum descriptions (Monin, Jaglom, 1967). Actually, the total amount of signs defined in the given way is infinite and no criteria, arising from the statistical description exists that allow to prefer any of the deduced signs to others. Such criteria follows from the qualitative specification of higher structural level only.

III. The sequences of approximations.

The full description of turbulence according to the discussed metatheory is obtained by using all codes, including the formulation of the total set of node and recoding theories. There exists a lot of reasons to consider this task too complicated to be realistic.

There exist many possibilities to build up different sequences of approximates of the metatheory. For example, in the case of a fixed number of structural levels, the sequence of approximations will have the form:



(In (5) the canonical level is omitted as usually in describing the turbulence)

The first approximation in (5) presents methatheory of turbulence as a set of separate continuum theories. Each of these theories is based on dividing the described processes into two parts of different space-time variabilities: the processes considered as "parametrized" and those assumably described by the theory. The lower the "separation" level the lower the

determination level of the theory. Assuming the scale-similarity of the turbulent flow field (Kuzmin, Patashinski 1972), the structure of equations of motion corresponding to different structural levels will have the invariant form with respect to the scales of the described processes. Belonging of the theory to concrete structural or determination level will be specified in this case by formulating concrete initial and boundary conditions for the equations of motions.

In the second case of approximations in (5), the name theories on neighbouring structural or determination levels appear to be coupled as the name theories of corresponding elementary metatheories,

$$\begin{array}{ccc}
 \bar{v}_n, \dots & \circ & \\
 & \diagdown & \\
 \bar{v}_{n-1}, \dots & \circ & \circ \quad f(\bar{v}_{n-1}, \dots)
 \end{array}
 \tag{5'}$$

The whole metatheory in the second case of approximations in (5) will be presented as the sequence of coupled N-1 elementary metatheories, corresponding to each of the elementary systems (5').

The last approximation in (5) coincides with the metatheory entirely.

There exists many reasons for the manifestation of the suggested principles of metatheory in the present paper. Some of them could be: 1) demonstration of inescapability and the specific role of continuum theories of turbulence in the whole set of theories of turbulence; 2) motivation of the ideological background of a concrete theory suggested below; 3) determination of the locality of a suggested theory in the whole set of theories of turbulence.

The presented principles of turbulence metatheory were worked out as a reaction to the prolonged ideological disorder in turbulence investigations.

THE ORIGINS OF THE PHENOMENOLOGICAL (CONTINUUM) THEORY
OF TURBULENCE

The phenomenological (continuum) theory of turbulence worked out by Heinloo (Heinloo, 1984) is based on the next two ideas. The first of them is the idea (method) of structural decomposition of stochastic systems (Heinloo, 1984). The suggested method provides simultaneous multi-scale description of turbulent media and enables to discuss problems connected with interactions between different scales of motion in turbulent media. The second idea concerns the problem of the symmetry of turbulent media and treats turbulent media as continuums with the special class of symmetry named rotational anisotropy. The rotational anisotropy can be interpreted as the essential feature of symmetry of turbulent media, arising from the orientation of the large scale eddies (Nemirovski and Heinloo, 1980; Heinloo, 1984).

I. The method of structural decomposition

The mathematical foundation for the process of increasing and reducing the number of structural levels has been worked out by Heinloo (Heinloo, 1984; pp.24-47). The suggested method is based on the interpretation of an averaging (filtration) operator discussed in (Toompuu, Heinloo, 1980). The specific feature of the suggested interpretation of an averaging process is the invariance of averaging rules with respect to the specific interpretation of an averaging operator (Heinloo, Toompuu, 1980, 1981).

According to the suggested method, any averaging (filtration) operator Q from the defined class of averaging (filtration) operators,

$$Q = \int \dots f(a) da ,$$

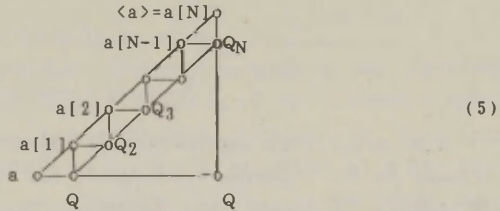
(a denotes the sign for an arbitrary name code), can be presented in the form

$$Q = Q_N Q_{N-1} \dots Q_1, \quad (4)$$

where Q_N, \dots, Q_1 are the operators

$$Q[i] = \int \dots f(a[i-1]) da[i-1],$$

operate on different structural levels,



According to (4), any $a[n] = Q_{n-1} a[n-1]$ in (5) can be represented as the sum

$$a[n] = \sum_{k=n}^{N+1} a[k]', \quad (6)$$

where $a[n]'$ is defined as

$$a[n]' = a[n-1] - a[n] \quad n=1, \dots, N+1$$

$$(a[N+1]' = a[N] = \langle a \rangle).$$

Any correlation function $ab[n]$ can be presented in the form

$$(ab)[n] = Q_n ab = \sum_{n=1}^{N+1} \langle a[n]' b[n]' \rangle \quad (7)$$

and so on.

For example, for the full kinetic energy of media defined as

$$E = \frac{1}{2} \langle v^2 \rangle$$

we have

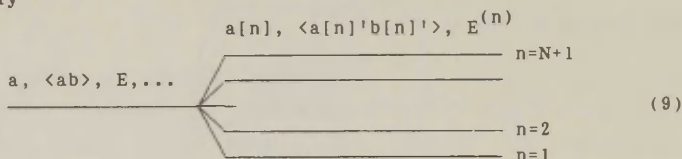
$$E = \sum_{n=1}^{N+1} E^{(n)} = \frac{1}{2} \langle v \rangle^2 + E^{\text{turb}} \quad (8)$$

where $E^{(n)} = 1/2 \langle v[n]'^2 \rangle$, $E^{(N+1)} = 1/2 \langle v \rangle^2$ and E^{turb} - the turbulent

part of energy E ($\bar{v}' = \bar{v} - \langle \bar{v} \rangle$),

$$E^{\text{turb}} = \frac{1}{2} \langle v' \rangle^2 = \sum_{n=1}^N E^{(n)} .$$

Any expression of the type (6), (7), (8) can be presented graphially



Any structural decomposition of $c = \{a, \langle ab \rangle, E, \dots\}$ is followed by the structural decomposition of the equations for c . For example, if c satisfies the balance equations

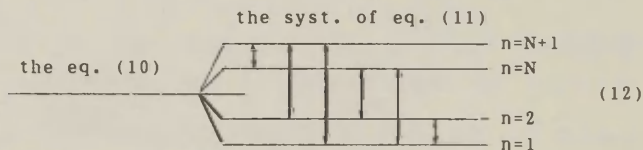
$$\frac{d}{dt} c = \nabla \cdot H + q , \quad (10)$$

according to the structural decomposition of c into the sum of $c^{(n)} = \{a[n]', \langle a[n]\'b[n]\' \rangle, E^{(n)}, \dots\}$, the equation (10) is decomposed into the system of equations for $c^{(n)}$,

$$\frac{D}{Dt} c^{(n)} = \nabla \cdot H^{(n)} + \sum_{k=n+1}^{N+1} \binom{k}{n} - \sum_{k=0}^{n-1} \binom{n}{k} + q^{(n)} , \quad (n=1, 2, \dots, N+1) . \quad (11)$$

In $d/dt = \partial/\partial t + \bar{v} \cdot \nabla$ (\bar{v} - the velocity field defined on the same level as c); $D/Dt = \partial/\partial t + \langle \bar{v} \rangle \cdot \nabla$; H and $H^{(n)}$ describes the diffusion of c and $c^{(n)}$, consequently; q and $q^{(n)}$ the sources of c and $c^{(n)}$, $\binom{k}{n}$ and $\binom{n}{k}$ describe interactions between $c^{(n)}$ and $c^{(k)}$.

Interactions between $c^{(n)}$ ($n=1, \dots, N+1$) described by the system of equations (11) can be represented on the graph (9) in the following form,



The symbols \uparrow in (12) correspond to $(\frac{n}{k})$ in (11)

The graphical form of representation of the systems of equations of the type (11) makes the structure of the system (11) and, in particular, the invariance of the system (11) with respect to the number of structural levels under consideration more evident.

II. The symmetry of the turbulent media.

The main problem in describing turbulence on the continuum level description is the type of symmetry of the turbulent media.

According to Richardson (Richardson, 1922), the turbulence can be treated as a motion of hierarchy of eddies with different scales. Kolmogorov (Kolmogorov, 1941) noted that the large scale eddies deriving their energy directly from the mean flow are oriented by it. The circumstance that the existence of some prevailing orientation of eddies is characteristic only to large scale turbulence was used by Kolmogorov as the motivation of proposed by him theory of small scale turbulence regarded as isotropic and homogeneous (Kolmogorov, 1941).

Due to the absence of suitable mathematical quantity describing the orientation of large scale turbulence, the ideas of Richardson and Kolmogorov fell into oblivion when building up concrete theories aimed to the description of large scale turbulence.

From the mechanical point of view it is clear that the existence of any prevailing orientation in a turbulent media includes the continuum theories of turbulence in the class of mechanics of oriented media. The main principles of the mechanics of oriented media were established by E. and F. Cosserat (Cosserat E., Cosserat F. 1909). These principles were reformulated in the of modern continuum mechanics by Toupin and Truesdell (Truesdell, Toupin, 1960 and Toupin, 1962) and intensively discussed in 60-es in the theory of elasticity (Eringen, Muki Sternberg, Grioiy) and in hydrodynamics (Eringen, Ericksen, Leslie, Stokes, Arriman, Cakeman, Condiff, Dahler, Aero, Kuvshinski and others). For a detailed concept of the

mentioned field of investigations, we recommend to see the paper (Arriman et. al, 1973).

The idea that turbulent media could be regarded as oriented media provoked Eringen (Eringen, 1966; Eringen and Chang, 1970) to suggest that the principles worked out in the mechanics of oriented fluids could be applied in describing turbulence. Later, Nikolaevski (Nikolaevski, 1969 and 1972) made a concrete attempt to relate the description of turbulence to the mechanics of oriented media. Due to the relationship between rotating degrees of freedom and the size of so called differential macro-volumes, the attempt of Nikolaevski was not successful.

It became evident in the middle of 70-ies (Nemirovsky and Heinloo, 1978) that the determination of the internal rotational degrees of freedom had to be directly associated with the Richardson-Kolmogorov's conception of turbulence. This possibility seems to be rather intricate as it provides the possibility to overcome the existing contradiction between the ideological backgrounds of the descriptions of small-scale turbulence (based on Richardson-Kolmogorov conception) and the large scale turbulence (based on other conceptions).

The named contradiction was surpassed after Heinloo had defined (Nemirovski and Heinloo, 1976) the specific measure of rotational-anisotropy of turbulent media. The proposed measure is defined as

$$\bar{\Omega} = \langle \bar{\Omega}^* \rangle, \quad (13)$$

where

$$\bar{\Omega}^* = \frac{\bar{v} \cdot \bar{x} \times \bar{R}}{R^2}. \quad (14)$$

(\bar{R} is radius of the momentary curve of the flow field of \bar{v} ; $\langle \rangle$ denotes averaging; \times denotes the vector product.)

It is easy to conclude that the condition $\bar{\Omega} \neq 0$ denotes the existence of preferred orientation in turbulent media and $\bar{\Omega}=0$ the absence of that orientation. In the first case the turbulent media is called rotationally anisotropic and in the second rotationally isotropic.

It should be noted that: 1) condition $\bar{\Omega} \neq 0$ arises from the

correlation between kinematic and geometrical characteristics of the structure of a flow field; 2) interpretation of $\bar{\Omega}$ differs from that of the purely local characteristics of a flow field; 3) $\bar{\Omega}$ has been interpreted mechanically as the mean angular velocity associated with the rotational motion of turbulent eddies; 4) $\bar{\Omega}$ defined in every point of a flow field forms the continuum and 5) $\bar{\Omega}$ is defined independently from the mean velocity $\langle \bar{v} \rangle$ and differs in general from $\bar{\omega}$

$$\bar{\omega} = \frac{1}{2} \nabla \times \langle \bar{v} \rangle.$$

It is essential to note that the proposed definition of $\bar{\Omega}$ does not demand the definition of the eddies, the latter being a very complicated problem.

According to the laws in mechanics of continuums with internal rotational degrees of freedom (Eringen, 1966; Ariman et al., 1973), the full description of the motions of rotationally anisotropic turbulent media have to be based on independent laws of balance of a momentum and moment of momentum:

$$\begin{aligned} \rho \frac{D}{Dt} \langle \bar{v} \rangle &= \nabla \cdot \bar{P}^+ + \frac{1}{2} \nabla \times \bar{p} + \rho \bar{f}, \\ \rho \frac{D}{Dt} \bar{M} &= \nabla \cdot \bar{Q} - \bar{p} + \rho \bar{m} \end{aligned} \quad (15)$$

where the moment of momentum \bar{M} is defined accordingly to the definitions of $\bar{\Omega}$ as

$$\bar{M} = \langle \bar{M}^* \rangle \quad (16)$$

where

$$\bar{M}^* = \bar{v}' \times \bar{R} = R^2 \bar{\Omega}^* \quad (17)$$

The other quantities in (15), not mentioned above, are: \bar{P}^+ is symmetric part of the stress tensor; \bar{p} - dual vector of the antisymmetric part of the stress tensor; \bar{Q} - moment stress tensor; ρ - mean density of the environment; \bar{f} and \bar{m} densities

(per unit mass) of mass forces and mass moments.

After defining the effective moment of inertia J

$$J\bar{\Omega} = \langle R^2 \bar{\Omega}^* \rangle, \quad (18)$$

the expression for \bar{M} will take the form

$$\bar{M} = J\bar{\Omega}. \quad (19)$$

Let's note that in the case of $\bar{\Omega} \neq 0$ the turbulent energy $E^{\text{turb}} = 1/2 \langle v'^2 \rangle$ is divided into the sum

$$E = \frac{J}{2} \bar{\Omega}^2 + E^{(0)} \quad (20)$$

where

$$E^{(0)} = \frac{1}{2} \langle \bar{M}^* \cdot \bar{\Omega}^* \rangle.$$

The first term on the right hand side of (20) can be interpreted as energy of large scale (oriented) eddies and the second as energy of relatively small scale (not oriented) eddies.

Due to the structural decomposition of turbulent media into N structural levels, the system of equations (15) is transformed into the system (Heinloo, 1984; pp.68-72)

$$\begin{aligned} \rho \frac{D}{Dt} \langle \bar{v} \rangle &= \nabla \cdot P^* + \frac{1}{2} \sum_{n=1}^N \nabla \cdot x \bar{p}^{(n)} + \rho \bar{f}, \\ \rho \frac{D}{Dt} \bar{M}^{(n)} &= \nabla \cdot Q^{(n)} - \bar{p}^{(n)} + \sum_{k=n+1}^N \binom{k}{n} - \sum_{k=1}^{k=n-1} \binom{n}{k} + \rho \bar{m}^{(n)} \end{aligned} \quad (21)$$

($n=1, 2, \dots, N$) where $\bar{M}^{(n)}$ is defined as

$$\bar{M}^{(n)} = \langle \bar{v}[n] \cdot x \bar{R}[n] \rangle = J^{(n)} \bar{\Omega}^{(n)}. \quad (22)$$

The terms $\binom{k}{n}$ and $\binom{n}{k}$ in (21) describe interactions between $\bar{M}^{(k)}$ and $\bar{M}^{(n)}$. The meaning of other quantities in (24) is analogous to the analogous terms in (17).

Let's note that according to (7), (19) and (22),

$$J\bar{\Omega} = \sum_{n=1}^N J^{(n)}\bar{\Omega}^{(n)}. \quad (23)$$

The total turbulence energy corresponding to the situation described by the system of equations (21), is decomposed into the sum

$$E = \sum_{n=1}^N \sum_{p=n}^{N+1} E^{(np)} \quad (24)$$

where

$$E^{(np)} = \frac{1}{2} \langle \bar{M}^{(n)} [p]' \cdot \bar{\Omega}^{(n)} [p]' \rangle$$

($\bar{\Omega}^{(n)} = \langle \bar{M}^{(n)} / R[n]' \rangle$) describes the sublevels of energies $E^{(n)}$ with different ranges of regularity.

The total system of the equations of energy balance, describing energy scatterings in the media, corresponding to the expansion (24), has the form (Heinloo, 1984; pp. 76-82):

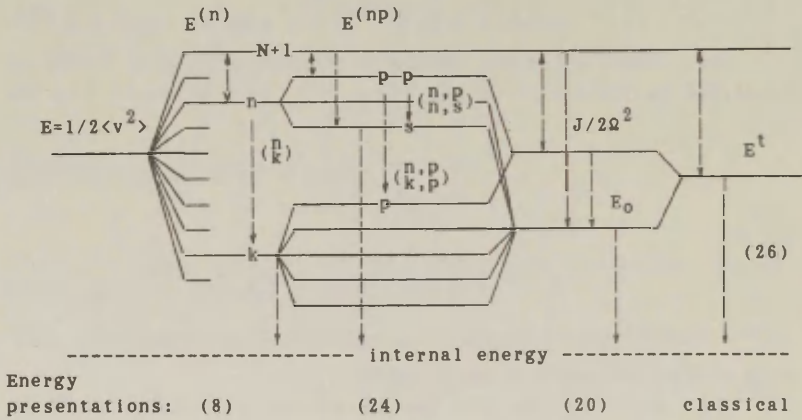
$$\varrho \frac{D}{Dt} E^{(N+1)} = \nabla \cdot \bar{h}^{(N+1)} - \sum_{n=0}^N \sum_{p=n}^{N+1} \langle \bar{M}^{(n+1,p)} \rangle, \quad (25)$$

$$\varrho \frac{D}{Dt} E_{np} = \nabla \cdot \bar{h}^{(np)} + \sum_{k=n+1}^{N+1} \langle \bar{M}^{(k,p)} \rangle - \sum_{k=0}^{n-1} \langle \bar{M}^{(k,p)} \rangle + \sum_{s=p+1}^{N+1} \langle \bar{M}^{(n,s)} \rangle - \sum_{s=n}^{p-1} \langle \bar{M}^{(n,s)} \rangle$$

$$\varrho \frac{D}{Dt} E^{(0)} = \nabla \cdot \bar{h}^{(0)} + \sum_{n=1}^{N+1} \sum_{p=n}^{N+1} \langle \bar{M}^{(n,p)} \rangle.$$

In (25): $E^{(N+1)}$ - the energy of mean flow, $E^{(0)}$ - the internal energy, $\bar{h}^{(N+1)}$, $\bar{h}^{(np)}$, $\bar{h}^{(0)}$ describes the diffusion of energies $E^{(N+1)}$, $E^{(np)}$, $E^{(0)}$, the quantities in (25) under sum signs describe energy scatterings between the corresponding energetic levels. The expressions of different quantities in (25) have quite voluminous forms and can't be reproduced here.

The types of energy interactions described on different levels of energy presentation are shown in the next graph,



Higher organisation level of motions corresponds to higher positions of energetic levels in (26). Let's note that interactions between the energetic sub-levels (n, p) with different n occur only between these sublevels that correspond to the same range of order (to the same p).

The detailed presentation of energy exchange processes in the media is essential, in particular, in motivating the specific constitutive equations for closing the system of equations (21) (or (15)) (Heinloo, 1984, p. 90).

THE PRESENTATION OF DIFFERENT QUANTITIES IN THE BALANCE EQUATIONS (15) and (21)

The expressions for P^+ , Q , \bar{p} and \bar{m} in equations (15), deduced by (Heinloo, 1984, pp.64-66) using the Navier-Stokes equation can be presented in following form:

$$\begin{aligned}
 P^+ &= \{ \langle \sigma_{ij} \rangle - \rho \langle v'_i v'_j \rangle \}, \\
 Q &= \{ e_{kis} \langle P_{ij}^* R_s' \rangle - \rho \langle v_j' M_k' \rangle \}, \\
 \bar{p} &= - \frac{1}{2} \rho e_{kij} \langle v'_i v'_j \rangle,
 \end{aligned}
 \tag{27}$$

where

$$\bar{m} = \bar{m}_0 + \bar{m}_1 + \bar{m}_2 + \bar{m}_3 + \bar{m}_4 + \bar{m}_5$$

$$\bar{m}_0 = \langle \bar{f}' x \bar{R}' \rangle,$$

$$\bar{m}_1 = \langle \bar{v}' x \frac{d}{dt} \bar{R} \rangle,$$

$$\bar{m}_2 = \frac{J}{2} (\nabla \langle \bar{v} \rangle) \cdot \bar{\Omega}, \quad (28)$$

$$\bar{m}_3 = \frac{1}{\rho} \{-e_{kis} \langle \sigma_{ij}' R_{s,j}' \rangle\},$$

$$\bar{m}_4 = \{-e_{kis} \langle v_i' v_j' \rangle \langle R_s \rangle_{,j}\},$$

$$\bar{m}_5 = \{\langle v' ({}_j R_s) \rangle \langle v \rangle_{,j}\}.$$

(In (27) and (28): σ denotes the molecular stress tensor:

$$\sigma = \{-p \delta_{ij} + \mu v_{(i,j)}\}$$

(p is thermodynamic pressure, δ_{ij} the Kronecker's symbol, μ coefficients of molecular viscosity, $v_{(i,j)}$ symmetric part of a gradient of the velocity field);

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\bar{v} + \bar{v}') \cdot \nabla$$

and $P_{ij}^* = -\rho v_i' v_j'$.

In writing the expressions (27), (28) we confined with the case of constant density.

Let's note that the first of the equations in (15) is the Reynold's equation, but according to the equation of moment of momentum in (15), the turbulent part of the stress tensor has to be considered asymmetrical.

An assumption of symmetry of the turbulent stress tensor in classical theories of turbulence is based on the interpretation of $-\rho \langle v_i' v_j' \rangle$ as a correlation functions between different components of the pulsating part of velocity field. It is easy to see that such interpretation of $-\rho \langle v_i' v_j' \rangle$ is not adequate in its physical meaning as a component of the stress tensor. The latter assumes the differentiation of $-\rho \langle v_i' v_j' \rangle$ and

$-\rho \langle v'_j v'_i \rangle$ that describe the stress situation on different planes of the differential volume. When the internal moment of momentum is defined, $-\rho \langle v'_i v'_j \rangle$ and $-\rho \langle v'_j v'_i \rangle$ do not compensate each other and the turbulent stress tensor appears to be asymmetric (Heinloo, 1984, pp. 66-68). It is substantial to note that the answer to the question about the symmetry of turbulent stress tensor depends always on the concrete interpretation of the moment of momentum. It gives relative content to the statement about the symmetry of stress tensor.

The expressions for the terms in (23), (24) have the following forms (Heinloo, 1984, pp.68-72):

$$P^* = \sum_{n=0}^N P^{+(n)},$$

$$Q^{(n)} = -\rho \sum_{k=1}^n \langle \bar{v}[k] \bar{M}^{(n)}[k] \rangle - \sum_{k=n+1}^N [k, n] + \sum_{k=0}^{n-1} [n, k] \quad (29)$$

$$\begin{aligned} \langle \bar{k} \rangle &= \{-\rho \langle e_{k i s} (v_i[k] v_j[k]) \rangle [k] R_s[n]', j \rangle + \\ &+ \rho \frac{J^{(n)}}{2} v_{i, j} \Omega_k^{(n)} + \rho \langle e_{k i s} v_j [k] R_s \rangle [k] v_i [n]', j \rangle\}, \\ \bar{m}^n &= \bar{m}_0^{(n)} + \bar{m}_1^{(n)} + \bar{m}_2^{(n)} + \bar{m}_3^{(n)} + \bar{m}_4^{(n)} + \bar{m}_5^{(n)} \end{aligned}$$

where:

$$P^{+(n)} = \{-\rho \langle v_i [n] v_j [n] \rangle\},$$

$$[n, k] = -\{\rho e_{k i s} (v_i [k] v_j [k]) \rangle [k] R_s [n]'\},$$

$$\bar{m}_0^{(n)} = \langle \bar{v} [n] \bar{x} \bar{R} [n] \rangle,$$

$$\bar{m}_1^{(n)} = \langle \bar{v} [n] \cdot x \left(\frac{\delta}{\delta t} + \bar{v} [n-1] \cdot \nabla \right) \bar{R} [n] \rangle,$$

$$\bar{m}_2^{(n)} = \frac{J^{(n)}}{2} \langle \nabla \langle \bar{v} \rangle \rangle \cdot \bar{\Omega}^{(n)}, \quad (30)$$

$$\bar{m}_3^{(n)} = \left\{ -\frac{1}{\rho} e_{k i s} \sigma_{i j} R_s [n] \right\},$$

$$\bar{m}_4^{(n)} = \{-e_{k i s} \langle v_j [n] v_j [n] \rangle \langle R_s \rangle, j\},$$

$$\bar{m}_5^{(n)} = \{e_{kis} \langle v_{(j[n]R_s)[n]'} v_{i,j} \rangle\}$$

It is easy to conclude that

$$\sum_{n=1}^N Q^{(n)} = Q, \quad \sum_{n=1}^N \left(\sum_{k=n+1}^N \binom{k}{n} - \sum_{k=1}^{n-1} \binom{n}{k} \right) = 0, \quad \sum_{n=1}^N \bar{m}_r^{(n)} = \bar{m}_r \quad (31)$$

(r=0, ..., 5)

CLOSURE OF THE EQUATIONS OF BALANCE

All the above said is valid for every turbulent media, but the deduced equations of balance do not determine the situation uniquely. The necessary additional information has to be based on certain supplementary concept of the processes in the turbulent media.

The necessary additional information can be divided into three classes: restrictions to the structure of the media, restrictions to the energy processes resulting from the cascade nature of energy scatterings in the media and constitutive equations.

a) The restrictions to the structure of the turbulent media.

$$\begin{aligned} \langle \bar{R} \rangle &= 0 & (\langle \bar{R}[n]' \rangle &= 0), \\ \langle v_{(j'R_s)} \rangle &= 0 & (\langle v_{(j[n]R_j)[n]'} \rangle &= 0), \\ \langle \sigma_{ij'R_s,j} \rangle &= 0 & (\langle \sigma_{ij[n]R_s,j}[n]' \rangle &= 0). \end{aligned} \quad (32)$$

The kinematic meaning of first restriction in (8) needs no special explanation. It is shown (Heinloo, 1984, p.94) that the second restriction in (32) means the absence of correlation between $|\bar{M}^*$ and the direction of \bar{v}' . The third restriction means (Heinloo, 1984, p.94) the distinguished role of molecular dissipation in describing large scale turbulence (associated with the $\bar{\Omega}$ field).

The expression for $\bar{m}^{(1)}$ can be presented in the form

(Heinloo, 1984, p.95)

$$\bar{m}^{(1)} = \langle \bar{v}' \times \bar{v}^* \rangle + \frac{J}{2} (\nabla \langle \bar{v} \rangle) \cdot \bar{\Omega} \quad (33)$$

where \bar{v}^* is velocity of the displacement of the center of curvature of \bar{v}' stream line. Using (33) for the full expression of mass moment \bar{m} we shall have

$$\bar{m} = J(\nabla \langle \bar{v} \rangle) \cdot \bar{\Omega} + \langle \bar{v}' \times \bar{v}^* \rangle + \langle \bar{r}' \times \bar{R}' \rangle \quad (34)$$

Let's note that the term $\langle \bar{v}' \times \bar{v}^* \rangle$ in (34) can be interpreted as the moment associated with the loss of mean orientation in the environment due to splitting of eddies during their motion (Heinloo, 1984, p. 95).

The expressions for $\bar{m}^{(n)}$ in the equations (21) will have similar to (34) forms

$$\bar{m}^{(n)} = J^{(n)} (\nabla \langle \bar{v} \rangle) \cdot \bar{\Omega}^{(n)} + \langle \bar{v}'^{(n)} \times \bar{v}^{*(n)} \rangle + \langle \bar{r}'^{(n)} \times \bar{R}'^{(n)} \rangle, \quad (35)$$

b) The restrictions to energy processes.

There are three different type of unreversible processes in the turbulent media (Heinloo, 1984, pp.90-91):

I. molecular dissipation converting energy into heat on different energetic levels;

II. operation of the symmetric part of stress tensors, $P^{+(n)}$ converting the energy of mean flow into energies $E^{(nn)}$,

$$- (N_{n, n}^{n+1, n+1}) = - P^{+(n)} \cdot \nabla \langle \bar{v} \rangle \leq 0 ;$$

III. cascade process of eddies generation, converting the energies $E^{(np)}$ into energies $E^{(ns)}$ with $s=n, \dots, p-1$. In terms of the equations of energy balance (25) the just said means that the part of works $(\frac{n, p}{n, s})$ do not include the works of the mass moments, is always non-positive,

$$- (\frac{n, p}{n, s}) \leq 0,$$

The position I follows from the Navier-Stokes equations. The positions II, III can be regarded as additional postulates that determine the direction of energetic processes in the turbulent media.

c) The constitutive equations.

The theory worked out in (Heinloo, 1984, pp.89-96) is based on the following constitutive equations:

$$\begin{aligned}
 P^+(n) &= \{v^{(n)} \langle v \rangle_{(1,j)}\}, \\
 \bar{p}^{(n)} &= 2 \gamma^{(n)} (2\bar{\Omega}^{(n)} - \nabla x \langle \bar{v} \rangle), \\
 -\rho \langle \bar{v}[k] \cdot \bar{M}^{(n)}[k] \rangle &= \left\{ -\frac{\theta_1^{(n)} + \theta_2^{(n)}}{3} \Omega_{k,k}^{(n)} \delta_{ij} + \theta_1^{(n)} \Omega_{i,j}^{(n)} + \theta_2^{(n)} \Omega_{j,i}^{(n)} \right\}, \\
 [k,n] &= \{ \chi_1^{(k,n)} (\Omega_k^{(k)} - \Omega_k^{(n)})_{,k} + \chi_2^{(k,n)} (\Omega_j^{(k)} - \Omega_j^{(n)})_{,i} + \\
 &\quad + \chi_3^{(k,n)} (\Omega_1^{(k)} - \Omega_1^{(n)})_{,i}, \\
 \left(\frac{k}{n} \right) &= \varphi^{(k,n)} (\bar{\Omega}^{(k)} - \bar{\Omega}^{(n)}),
 \end{aligned} \tag{36}$$

$$\rho \langle \bar{v}[n] \cdot x \bar{v}^*[n] \rangle = -4\mathfrak{E}^{(n)} \bar{\Omega}^{(n)} + \rho \frac{J^{(n)}}{2} (\nabla \langle \bar{v} \rangle) \cdot \bar{\Omega}^{(n)}$$

where $\mu^{(n)}$; $\theta_1^{(n)}$, $\theta_2^{(n)}$, $\chi_1^{(k,n)}$, $\chi_2^{(k,n)}$, $\chi_3^{(k,n)}$; $\gamma^{(n)}$; $\mathfrak{E}^{(n)}$; $\varphi^{(k,n)} \geq 0$ are the coefficients describing the diffusion of

momentum and moment of momentum, the friction due to relative rotation of eddies the trend to the irregularity of the orientation of eddies due to their interaction and the interaction

between different $\bar{\Omega}^{(n)}$ ($n=1, \dots, N$); $\chi_r^{(k,n)} = \chi_r^{(n,k)}$, $\varphi^{(k,n)} = \varphi^{(n,k)}$,

According to (30), (31) and (36) the expressions for $P^{(s)}$, Q , \bar{p} and $\rho \bar{m}^{(1)}$ will have the forms:

$$P^+ = \{v \langle v \rangle_{(i,j)}\},$$

$$\bar{p} = 2\gamma(2\bar{\Omega} - \nabla \times \langle \bar{v} \rangle), \quad (37)$$

$$Q = \left\{ -\frac{\theta_1 + \theta_2}{3} \Omega_k, k^{\delta} \delta_{ij} + \theta_1 \Omega_{i,j} + \theta_2 \Omega_{j,i} \right\},$$

$$\rho_m^{-1} = -4\bar{\alpha}\bar{\Omega} + \rho \frac{J}{2} (\nabla \times \langle \bar{v} \rangle) \cdot \bar{\Omega}$$

where

$$\nu = \sum_{n=1}^N \nu^{(n)},$$

and τ , α and θ_r ($r=1,2$) are defined according to the relationships

$$\gamma \bar{\Omega} = \sum_{n=1}^N \gamma^{(n)} \bar{\Omega}^{(n)}, \quad \alpha \bar{\Omega} = \sum_{n=1}^N \alpha^{(n)} \bar{\Omega}^{(n)}, \quad \theta_r \bar{\Omega} = \sum_{n=1}^N \theta_r^{(n)} \bar{\Omega}^{(n)}, \quad (38)$$

Taking into account (23) and (38), it is easy to conclude that the constitutive equations (36) are in accordance with the principle of structural invariance.

Let's note that the quantities

$$\tilde{\gamma} = \gamma^{(n)} / J^{(n)}, \quad \tilde{\alpha} = \alpha^{(n)} / J^{(n)} \quad \text{and} \quad \tilde{\theta} = \theta_r^{(n)} / J^{(n)}$$

do not depend on the scales of the described processes determined by $J^{(n)}$.

THE EQUATIONS OF MOTION

Replacing the constitutive equations from (37) into the equations of balance (21), we get the following system of equations of motion for rotationally anisotropic turbulent flows (for the case of $N=1$; henceforth \bar{v} denotes the averaged velocity),

$$\rho \frac{D}{Dt} \bar{v} = -\nabla p + (\nu_{mol} + \nu + \gamma) \Delta \bar{v} + 2\gamma \nabla \times \bar{\Omega} + \rho \bar{f}, \quad (39)$$

$$\rho J \frac{D}{Dt} \bar{\Omega} = \theta_1 \Delta \bar{\Omega} + \frac{2\theta_2 - \theta_1}{3} \nabla \nabla \cdot \bar{\Omega} - 2\gamma (2\bar{\Omega} - \nabla \nabla \cdot \bar{\Omega}) - 4\alpha \bar{\Omega} +$$

$$+ \rho J (\nabla \bar{v}) \cdot \bar{\Omega} + \rho \langle \bar{f}' x \bar{R}' \rangle.$$

The expression for $\langle \bar{f}' x \bar{R}' \rangle$ depends on the specific nature of the mass forces \bar{f} .

Let's note (Nemirovski, Heinloo, 1980):

I. If $J \rightarrow 0$ ($R \rightarrow 0$), $\theta_1, \theta_2, \gamma, \alpha \rightarrow 0$ and the equation of the moment of momentum in (39) is reduced to the condition of symmetry of the stress tensor and the equation of momentum to the Boussinesque approximation of the Reynold's equation

$$\rho \frac{D}{Dt} \bar{v} = -\nabla \bar{p} + (\nu_{m01} + \nu) \Delta \bar{v} + \rho \bar{f}. \quad (40)$$

II. If $J \neq 0$, $\langle \bar{f}' x \bar{R}' \rangle = 0$, $\alpha = 0$, from the condition $\bar{\Omega} = \bar{\omega}$ in every point of the flow field for some initial time moment $t = t_0$, it follows that $\bar{\Omega}$ remains equal to $\bar{\omega}$ for all $t > t_0$. The equation of the moment of momentum in this case is reduced to the Gromeko-Lemba equation and the equation of momentum to the equation (40) (with the relationship between viscosity coefficients $\theta_1 = J(\nu_{m01} + \nu)$). The stress tensor in this case, as in the previous case, appears to be symmetric.

III. If the velocity field is potential and the mass moment is absent ($\langle \bar{f}' x \bar{R}' \rangle = 0$) from the condition $\bar{\Omega} = 0$ in every point of the flow field for some initial time moment $t = t_0$, it follows that $\bar{\Omega}$ remains equal to 0 for all $t > t_0$. The equation of momentum in this case is reduced to the equation for perfect fluids and there arises no question about the symmetry of the stress tensor.

The positions I, II and III include three different ways to ascertain the correspondence of equations of the suggested theory to these of classical theories.

For of $N > 1$, the system of equations (39) will take the form (Heinloo, 1984, pp.97-98):

$$\rho \frac{D}{Dt} \bar{v} = -\sqrt{P} + (\nu_{mol} + \nu + \sum_{n=1}^N \tau^{(n)}) \Delta \bar{v} + 2 \sum_{n=1}^N \tau^{(n)} \nabla \times \bar{\omega}^{(n)} + \rho \bar{f}, \quad (41)$$

$$\rho J^{(n)} \frac{D}{Dt} \bar{\omega}^{(n)} = \sum_{n=1}^N (a^{(k,n)} \Delta \bar{\omega}^{(k)} + b^{(k,n)} \nabla \nabla \cdot \bar{\omega}^{(k)} - c^{(k,n)} \bar{\omega}^{(k)}) +$$

$$+ 2 \tau^{(n)} \nabla \times \bar{v} + \rho J^{(n)} (\nabla \bar{v}) \cdot \bar{\omega}^{(n)} + \rho \langle \bar{f} [n] \cdot x \bar{R} [n] \rangle$$

where $n=1, 2, \dots, N$: $a^{(k,n)}$, $b^{(k,n)}$, $c^{(k,n)}$ are the matrices, defined by the coefficients $\tau^{(n)}$, $\alpha^{(n)}$, $\theta_1^{(n)}$, $\theta_2^{(n)}$, $\chi_1^{(k,n)}$, $\chi_2^{(k,n)}$, $\varphi^{(k,n)}$ in the following way:

for $k=n$

$$a^{(n,n)} = \theta_3^{(n)} + \sum_{k=1}^N \chi_3^{(k,n)}, \quad b^{(n,n)} = \theta_1^{(n)} + \theta_2^{(n)} + \sum_{k=1}^N (\chi_1^{(k,n)} + \chi_2^{(k,n)})$$

$$c^{(n,n)} = 4(\tau^{(n)} + \alpha^{(n)}) + \sum_{k=1}^N \varphi^{(k,n)},$$

for $k \neq n$

$$a^{(k,n)} = -\chi_3^{(k,n)}, \quad b^{(k,n)} = -\chi_1^{(k,n)} - \chi_2^{(k,n)}, \quad c^{(k,n)} = -\varphi^{(k,n)}.$$

The special cases of the equations of motion (41) corresponding to the turbulent boundary layer and to the creeping flow have been derived in (Heinloo, 1984, pp.100-109 and 111-114).

THE EQUATIONS OF TRANSPORT

In the case of rotationally anisotropic turbulence, the turbulent part of the vector of diffusion of any substance, c ,

$$\bar{H}^{\text{turb}} = -\langle \bar{v}' c' \rangle,$$

in the equation of balance

$$\frac{D}{Dt} c = k^{\text{mol}} \Delta c + \nabla \cdot \bar{H}^{\text{turb}}$$

where k^{mol} is the coefficient of molecular transport of c , can be presented as the sum (Heinloo, 1984, p.193)

$$\bar{H}^{turb} = \langle \bar{\Omega}'x(c'\bar{R})' \rangle + \bar{\Omega}x\langle c'\bar{R} \rangle, \quad (42)$$

where the first and second terms describe the transport of c resulting from the action of small scale (rotationally isotropic) and large scale (rotationally anisotropic) "parts" of turbulence. So far small scale turbulence can be considered isotropic for the first term in (42), we suggest that

$$\langle \bar{\Omega}'x(c'\bar{R})' \rangle = k_0 \nabla c \quad (43)$$

where k_0 denotes the diffusion coefficient for small scale turbulence.

According to (Heinloo 1984, p.194), $\langle c'\bar{R} \rangle$ can be expressed as

$$\langle c'\bar{R} \rangle = -k_1 \nabla c - k_2 \bar{\Omega}x \nabla c. \quad (44)$$

(k_1 and k_2 - the coefficients describing different properties of transport processes with large scale turbulence.)

Using (43) and (44), the expression for \bar{H}^{turb} , (42) will take the form

$$\bar{H}^{turb} = K \nabla c \quad (45)$$

$$K = k_1 + k_1(\Omega^2 I - \bar{\Omega}\bar{\Omega}) + k_2 E \cdot \bar{\Omega} \quad (46)$$

is the tensor of the turbulent transport of c (I is the unit tensor, E the Levi-Civitta tensor).

Let's note that due to rotational asymmetry of turbulence the tensor K is asymmetric. Using the identity

$$\nabla \cdot (K^{(as)} \cdot \nabla c) = -\bar{s} \cdot \nabla c$$

where $K^{(as)}$ is the antisymmetric part of the tensor K ,

$$\bar{s} = \nabla x \bar{k} = -k_2 \nabla x \bar{\Omega},$$

and \bar{K} is the dual vector of the tensor $K^{(as)}$, it is easy to note that the effect of the antisymmetric part of the tensor of turbulent transport is analogous to that of the transport with the mean velocity of incompressible fluid.

Using (45) and (46) the equation for c will have the form

$$\frac{\partial}{\partial t} c + (\bar{v} - k_2 \nabla \times \bar{\Omega}) \cdot \nabla c = \nabla \cdot [(k_{m01} + k)I + k_1(\Omega^2 I - \bar{\Omega}\bar{\Omega})] \cdot \nabla c. \quad (47)$$

The effect of the substance transport describing the antisymmetric part of the tensor K was used by Toompuu, Heinloo and Soomere as one of the possible mechanism for forming the Gibraltar Salt Anomaly (Toompuu, Heinloo, Soomere, 1989).

It is important to note that the usual assumption of the symmetry of the tensor K in (45) has not been argued and it comes out from the absence of the suitable quantity (axial vector) which can be assigned to the antisymmetric part of the tensor K in classical turbulence descriptions.

The described situation is generalized for the case of $N > 1$ in (Heinloo, 1984; pp.192-202)

In many cases, besides considering the transfer of any substance on the level of mean concentration, it is essential to consider other statistical moments of the concentration field. The most important role besides the mean concentration is played by the dispersion $q = \langle c'^2 \rangle$ there. The problems resulting from describing the transformation of dispersion in rotationally anisotropic turbulent environments with the hierarchy of structural levels have been discussed in detail in (Heinloo, 1984; pp 204-208).

Some principles of the suggested description of dispersion transformation were used by A.Toompuu in his Ph.D. (kand.nauk.) dissertation (scientific co-adviser J.Heinloo) as theoretical background for his integro-differential model of dispersion transformation of scalar substances (Toompuu, 1981).

THE TURBULENT MEDIA UNDER THE DIFFERENT FIELDS OF MASS FORCES

Let's discuss three specific cases of mass forces \bar{f} : the gravity force (I), the Lorentz force (in the case of electrical conductivity of the media under the influence of homogeneous magnetic field) (II) and the inertial force in rotating systems (III).

I. The case of gravity forces with the constant density of the media is trivial. The situation is the most interesting when the density is not strongly constant. Limiting with the Boussinesque approximation

$$\frac{|\langle \rho^* - \bar{\rho} \rangle|}{\bar{\rho}} \ll 1,$$

where ρ^* is the actual density of media, it can be shown that all the terms in equations of motion (39)((41)) will preserve their form, except the terms of $\rho \langle \bar{f}' x \bar{R} \rangle$ replaced by $\rho^* \bar{g}$ and $\langle \rho^* \bar{g} x \bar{R}' \rangle$, i.e.

$$\rho \bar{f} = \rho^* \bar{g}, \quad \rho \bar{m} = \langle \rho^* \bar{g} x \bar{R}' \rangle.$$

Using the expression (44) (with $c=0$), we will have for $\rho \bar{m}$ (Heinloo, 1984, pp.59-61)

$$\rho \bar{m} = -k_2 \bar{g} x \nabla \rho^* - k_1 [(\bar{g} \cdot \nabla \rho^*) - \bar{g} \nabla \rho^*] \cdot \bar{\omega}. \quad (48)$$

Let's note that due to (48) the equations of motion (39) appear to be coupled with the equation of transport for ρ^* (or for c_1, c_2, \dots then $\rho^* = \rho^*(c_1, c_2, \dots)$). The described situation is typical for turbulent processes in the stratified environments and, in particular, in describing the vertical exchange processes in the sea (Heinloo, Vösumaa: 1987; Vösumaa, Heinloo, 1989).

The described situation is generalized to the case of $N > 1$ in (Heinloo, 1984; pp. 208-209). The energy balance of the flows in stratified environments is discussed in (Heinloo, 1984:pp.82-88 and pp.214-216)

II. The terms of $\rho\bar{f}$ and $\rho\bar{m}$ for media with electrical conductivity under the influence of homogeneous magnetic field can be presented in the forms (Heinloo, 1984, pp.160-162):

$$\rho\bar{f} = \frac{\sigma}{1+\alpha^2 B^2} \{ \bar{E}x\bar{B} + (\bar{v}x\bar{B})x\bar{B} + \alpha[\bar{B}x(\bar{E}+\bar{v}x\bar{B})] \},$$

$$\rho\bar{m}^{(n)} = - \frac{(1-\beta)\sigma J}{2(1+\alpha^2 B^2)} [(\bar{B}\bar{B}+B^2 I)\bar{\Omega}^{(n)} + \alpha B^2 \bar{B}x\bar{\Omega}^{(n)}]. \quad (49)$$

where \bar{E} is the strength of the electric field, \bar{B} induction of magnetic field α the Holl's parameter, σ electrical conductivity, β a parameter characterizing the characteristics of pulsations of an electric field; expressions (49) are confined with the condition $\langle \bar{B} \rangle = \bar{B}$.

The expressions (49) couple the equation of motion (39) with Maxwell equations (in magneto-hydrodynamical approximation):

$$\frac{\partial B}{\partial t} = -\nabla \cdot x\bar{E}, \quad \nabla \cdot x\bar{B} = \mu_0 \bar{j}, \quad \nabla \cdot \bar{E} = \nabla \cdot \bar{B} = 0 \quad (50)$$

where μ_0 is the magnetic permeability and \bar{j} intensity of the electric current,

$$\bar{j} = \frac{\sigma}{1+\alpha^2 B^2} [\bar{E} + (\bar{v}x\bar{B}) + \alpha(\bar{E} + \bar{v}x\bar{B})].$$

The balance energy of magnetohydrodynamic turbulent flows is discussed in (Heinloo, 1984, pp.163-166). Let's note here only that the total effect of magnetic field under turbulent media always diminishes the turbulent energy.

The system of equations (39), (49) was used for describing and analyzing different cases of turbulent flows of media with electric conductivity under different orientations of the induction of magnetic field with respect to the characteristics of the mean flow (Heinloo, 1984, pp. 166-191). The equations for a boundary layer in the magnetic field are deduced in (Heinloo,

1984, pp.188-191)

III. The expressions for inertial forces and moments in the case of turbulent flows in rotating systems follow from the equations (39) after the replacements:

$$\frac{D}{Dt} \bar{v} \longrightarrow \frac{D}{Dt} \bar{v} + \bar{\omega}_0 \times \bar{v}, \quad \frac{D}{Dt} \bar{\Omega} \longrightarrow \frac{D}{Dt} \bar{\Omega} + \bar{\omega}_0 \times \bar{\Omega}$$

and

$$\bar{v} \longrightarrow \bar{v} + \bar{\omega}_0 \times \bar{r}, \quad \bar{\Omega} \longrightarrow \bar{\Omega} + \bar{\omega}_0,$$

where the quantities in the inertial system of coordinates are on the left side of "→" and the same quantities in the rotating system of coordinates on the right side of "→"; $\bar{\omega}_0$ - the angular velocity of the system rotation.

After the named replacements the expressions for the inertial force and moment will have the forms:

$$\begin{aligned} \rho \bar{f} &= 2\sigma \bar{v} \times \bar{\omega}_0 + \rho \bar{\omega}_0 \times (\bar{\omega}_0 \times \bar{r}), \\ \rho \bar{m} &= [4(\tau - \alpha)I - 2\rho J E \cdot \bar{\Omega} + \rho J \nabla \bar{v}] \cdot \bar{\omega}_0. \end{aligned} \quad (51)$$

SOME INTEGRALS OF THE EQUATIONS OF MOTION

Despite of the voluminous character of the equations of motion these equations became linear and can be integrated analytically in many cases. Let's sum up some of the integrals, deduced in (Heinloo, 1984).

1) Homogeneous $\bar{\Omega}$ field for $\bar{v}=0$ (Heinloo, 1984, pp.115-116):

$$\bar{\Omega}^{(n)} = \sum_{m=1}^N \bar{C}_m^{(n)} \exp(\lambda_m t), \quad (53)$$

where $\bar{C}_m^{(n)}$ are the constant vectors and λ_m are determined from the equation

$$|c^{(k,n)} + \lambda \rho J^{(n)} \delta_{kn}| = 0.$$

2) Stationary $\bar{\Omega}$ field for $\bar{\Omega} = (0, \bar{\Omega}^{(n)}(\bar{x}), 0)$ and $\bar{v}=0$ (Heinloo,

1984, pp.117-118).

The integrals of equations of motion, corresponding to the situation under consideration, expressed in the form analogous to (53) where λ_m are determined from the equation

$$|a^{(k,n)}_{\lambda_m} - c^{(k,n)}| = 0.$$

3) The creeping flows:

$$\bar{v} = \bar{v}^0 - \frac{2}{\mu + \tau} \sum_{k=1}^N \sum_{n=1}^N \gamma^{(n)} \alpha^{(k,n)} \bar{\Phi}_1^{(k)}, \quad (54)$$

$$\bar{Q}^{(n)} = d^{(n)} \mu_e \nabla \nabla x \bar{v}^0 - \sum_{k=1}^N \alpha^{(k,n)} \nabla \nabla x \bar{\Phi}_1^{(k)} + \nabla \nabla \psi^{(n)}$$

where \bar{v}^0 and $\bar{\Phi}_1$ are defined from the equations

$$-\nabla \nabla p + \mu_{ef} \Delta \bar{v}^0 + \rho \bar{f} = 0$$

and

$$\sum_{k=1}^N \alpha^{(k,n)} \nabla \nabla x \nabla \nabla x \bar{\Phi}_1^{(k)} + \bar{\Phi}_1^{(n)} = 0$$

$$d^{(n)} = \sum_{k=1}^N [c_*^{(k,n)}]^{-1} \frac{2 \delta^{(k)}}{\mu + \tau}, \quad \alpha^{(k,n)} = [c_*^{(k,n)}]^{-1} a^{(m,n)}$$

$$\mu_{ef} = (\mu + \tau) [1 + 2 \sum_{n=1}^N \delta^{(n)} d^{(n)}], \quad c_*^{(k,n)} = c^{(k,n)} - 4 \frac{\delta^{(n)} a^{(k)}(k)}{\mu + \tau}$$

$\psi^{(n)}$ - an arbitrary function satisfying the equation $\Delta \psi^{(n)} = \nabla \nabla \cdot \bar{Q}^{(n)}$

$$\delta = \sum_{n=1}^N \delta^{(n)}.$$

4) The Poiseuille and Couette flows in tubes, channels and between rotating cylinders (Heinloo, 1984, pp.118-134).

The integrals for the Poiseuille and Couette flows in tubes, channels and between rotating cylinders are derived from (54).

The calculated velocity profiles were compared with the series of experimental data of Nikuradse (Nikuradse, 1936),

Compte-Bellot (Compte-Bellot, 1968) and Zmeikov and Ustremenko (Zmeikov and Ustremenko, 1964). As an example, in Fig.1. the data of Compte-Bellot (the channel flow; $Re=1,14 \cdot 10^5$) is compared with calculated profiles for $N=1$ and $N=2$.

The energy balance of channel flow is discussed in (Heinloo, 1984, pp.139-141).

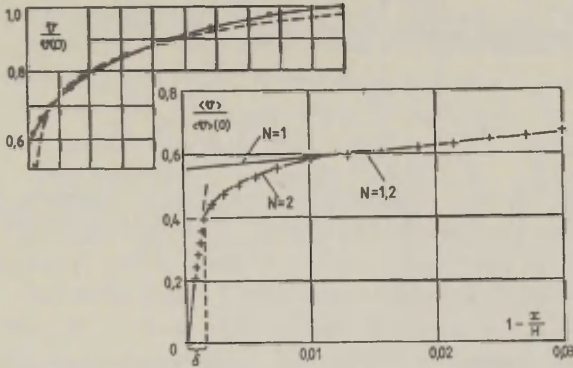


Fig.1. The Poiseuille channel flow: presented theory (—), classical theory (- -) and experiment (·, +)

5) The flows in tubes and channels under pulsating pressure field

$$\frac{\partial p}{\partial z} = p_0 + \operatorname{Re}[p_1 \exp(-i \nu t)] \quad (55)$$

(Heinloo, 1984, pp.142-148).

According to the deduced integral of equations of motion, corresponding to the situation (55), the velocity profiles for tube flows will have the form

$$v = v_0(r) + \operatorname{Re}[v_1(r) \exp(-i \nu t)]$$

where $v_0(r)$ is the solution of the stationary Poiseuille flow with $\partial p / \partial z = p_0$.

$$v_1(r) = -\frac{i p_1}{\theta} + C_1 J_0(\lambda r) + C_2 J_0(\lambda^* r)$$

λ and λ^* are defined from the equation

$$\lambda^4 - \lambda^2 \left[\frac{1}{1^2} - i \nu \theta \left(\frac{1}{\mu + \gamma} + \frac{J}{\theta} \right) \right] - \frac{i \nu \theta}{\nu_{ef}} \left(\frac{1}{1^2} \frac{\mu_{ef}}{\mu + \gamma} - \frac{i \nu \theta J}{\theta} \right) = 0$$

(J_0 - the modified Bessel function of zero order;

$$\nu_{ef} = \nu + \frac{\varepsilon}{\varepsilon + \gamma} \quad 1^2 = \frac{\theta_1(\mu + \gamma)}{4 \nu_{ef}(\gamma + \varepsilon)}$$

Despite of the simplicity of the deduced velocity profile, it reproduces exactly the behavior of real velocity profiles. As an example, in the Fig.2. results of the realized comparison of calculated profiles with experimental data (Bukreev, Sahhin, 1976), corresponding to $\nu = 10\pi$ rad./sek. and $\nu = 20\pi$ rad/sek for the time moments $t_n = n\pi/4$ ($n=1, \dots, 8$) are given.

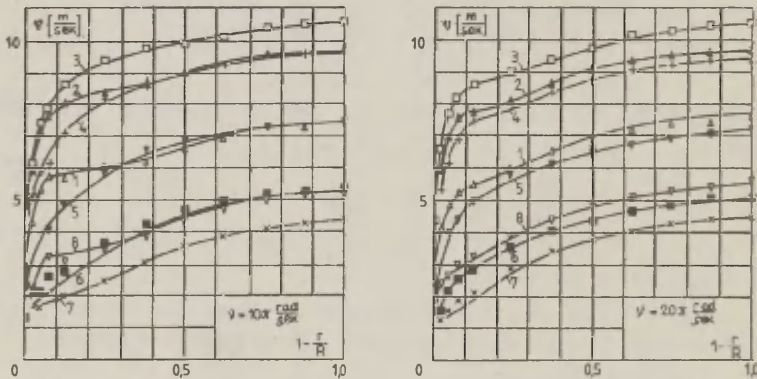


Fig.2. The calculated profiles compared with the experimental data of Bukreev and Sahhin.

6) Arbitrary one-dimensional flows in tubes, channels and between rotating cylinders (Heinloo, 1984, pp.148-155).

The integrals of the equations of motion corresponding to the arbitrary time-depending boundary conditions and/or pressure gradients are deduced in terms of the corresponding Fourier series.

7) The steady and unsteady MGD turbulent flows in channels, tubes and between rotating cylinders (Heinloo, 1984, pp. 166-188).

All the integrals mentioned above are also deduced for the case of MGD turbulent flows under the homogeneous electric and magnetic fields for three different orientations of magnetic field with respect to mean flow. In Fig.3. the calculated velocity profiles are compared with the corresponding experimental data (Kovner and Levin, 1964) for five different Hartmann numbers, Ha , in the case of perpendicular magnetic field.

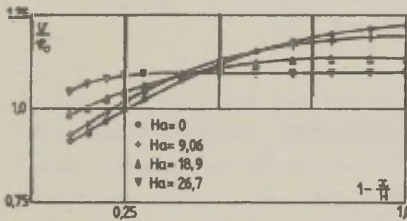


Fig.3. The calculated velocity profiles (—) in the case of perpendicular magnetic field compared with experimental data.

It is shown that for longitudinal and transversal magnetic fields, the velocity profiles of Poiseuille flows for sufficiently great Hartmann number differ from the corresponding profiles in the case of laminar flows only by viscosity: if for the latter case the viscosity is molecular, it will be equal to $\mu + \tau$ for the first case. It means that despite of the "laminarization" of velocity profiles, the motion itself remains turbulent. This conclusion is in total agreement with the experimental studies published in (Branover and Tsinober, 1970; Levin and Tsinenko, 1966).

SOME OCEANOLOGICAL APPLICATIONS OF THE PROPOSED THEORY

The oceanological applications of the proposed theory will be demonstrated on two examples. The first example treats the horizontal exchange processes with large scale eddies and, the second vertical exchange processes with small scale (but oriented) eddies.

1) Let's discuss the situation in the first example that occurs in the Atlantic Ocean due to the injection of the Mediterranean salty water into the Ocean (Toompuu, Heinloo, Soomere, 1989).

It is essential to assume that due to the strong vertical stratification, the motions determining the described processes are quasi-two-dimensional. It means that the $\bar{\Omega}$ field is oriented longitudinal to the radius r . In this case it follows from the equations of motion (in rotating system of coordinates) that

$$\bar{\Omega} = -a\omega_0 \sin \bar{n} \quad (56)$$

where $a = \frac{\partial \varphi}{\partial \varphi} + \gamma$ and \bar{n} - the normal to the sea surface.

Assuming the absence of the mean flow, the connection (56) determines the whole kinematical situation in the region under consideration. Taking into account that $\bar{\Omega}$ in the transport equation (46) is determined in the absolute system of coordinates and using (55) we will have (in the spherical system of coordinates)

$$-A \frac{\partial S}{\partial \varphi} - B \sin 2\vartheta \frac{\partial S}{\partial \vartheta} = (k_{mol} + k + B \sin^2 \vartheta) \Delta S \quad (57)$$

(φ and ϑ geographical longitude and latitude; Δ the two-dimensional Laplacian defined on the surface φ, ϑ ; $A = k^{(2)}(1-a)\omega_0$, $B = k^{(1)}(1-a)^2 \omega_0^2$; S salinity deviations from arbitrary homogeneous salinity field).

The equation (57) has been solved numerically for the part of Atlantic Ocean of $-70^\circ \leq \varphi \leq -8^\circ, 12^\circ \leq \vartheta \leq 60^\circ$. The calculated

salinity distribution has been compared with the experimental map of Griffiths and Hopfinger (Griffiths and Hopfinger, 1984) in fig.4. The calculated distribution of salinity deviations from arbitrary homogeneous salinity field, corresponds to the case $k_{mol} + k \ll B \sin^2 \vartheta$, $A=1$, $B=0,03$. The boundary conditions are taken at the western, northern and southern boundaries $S=0$, and at the eastern boundary $\partial c / \partial \psi = 0$, except $\vartheta \neq 36^\circ$ (the latitude of the Strait of Gibraltar), where $\partial c / \partial \psi = 1$.

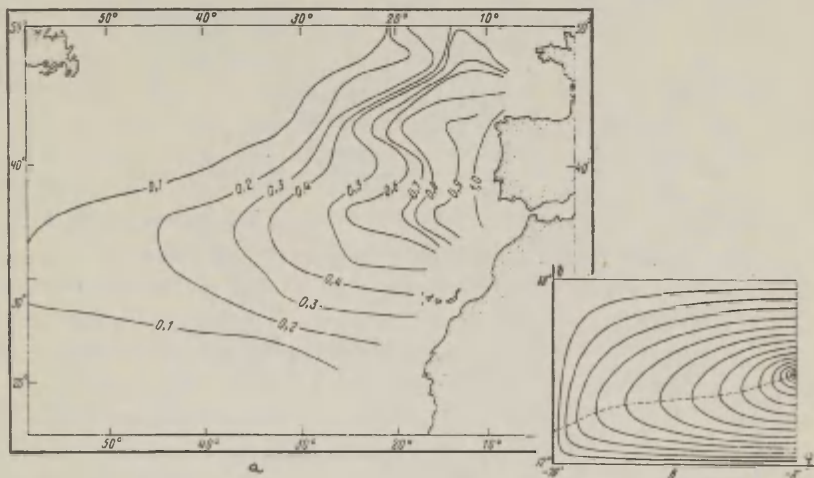


Fig.4. The Gibraltar Salt Anomaly: field situation (a) and numerical experiment (b).

Let's note the good qualitative accordance at least with respect to the next two effects: the tongue-shaped form of the salinity distribution and the displacement of the maximum of salinity towards the equator. It is important to emphasize that the occurrence of the named effects follows only from the existence of preferential orientation of the eddies in the GSA region, due to Earth rotation.

2) In the second example the one-dimensional model of vertical structure of the sea (Heinloo, Võsumaa, 1984; Võsumaa,

Heinloo, 1987) is treated. The model is based on the assumption of horizontal homogeneity of all hydrophysical fields and linear state equation for the sea water. The system of equations of motion and transport in this case will take the form:

$$\begin{aligned}
 \rho \frac{\partial v}{\partial t} &= (\nu + \gamma) \frac{\partial^2 v}{\partial z^2} + 2\gamma \nabla \bar{x} \bar{\omega} + 2\rho \bar{v} x \bar{\omega}_0 \\
 \rho J \frac{\partial \bar{\omega}}{\partial t} &= \theta \frac{\partial^2 \bar{\omega}}{\partial z^2} - 4(\gamma + \kappa) \bar{\omega} + 2\gamma \nabla \bar{x} \bar{v} + 2\rho J \bar{\omega} x \bar{\omega}_0 + \\
 &\quad + k_2 g \left[-\alpha \frac{\partial T}{\partial z} + \beta \frac{\partial S}{\partial z} \right] \bar{\omega} \\
 \frac{\partial}{\partial t} E^{(0)} &= - \frac{\partial^2 E^{(0)}}{\partial z^2} - \left\{ \frac{1}{t_0} + k_0 g \left[-\alpha \frac{\partial T}{\partial z} + \beta \frac{\partial S}{\partial z} \right] \right\} E^{(0)} + \\
 &\quad + 2\nu \left| \frac{\partial \bar{v}}{\partial z} \right|^2 + 4\gamma \left| \frac{1}{2} \nabla \bar{x} \bar{v} - \bar{\omega} \right|^2 + \theta \left| \frac{\partial \bar{\omega}}{\partial z} \right|^2 + 4\gamma \bar{\omega}^2 \\
 \frac{\partial T}{\partial t} &= \frac{\partial}{\partial z} \left[(k_T + k_0 E^{(0)} + k_2 \bar{\omega}^2) \frac{\partial T}{\partial z} \right], \\
 \frac{\partial S}{\partial t} &= \frac{\partial}{\partial z} \left[(k_S + k_0 E^{(0)} + k_2 \bar{\omega}^2) \frac{\partial S}{\partial z} \right]
 \end{aligned} \tag{58}$$

($\bar{v} = (v_\varphi, v_D, 0)$, $\bar{\omega} = (\omega_\varphi, \omega_D, 0)$; k_T and k_S - the coefficients of molecular diffusion for T and S; t_0 - the characteristic time of molecular dissipation of energy $E^{(0)}$; α and β - the coefficients of thermal expansion and saline contraction; z - the vertical coordinate, directed downward)

In the case of the absence of stratification, the solution of the equations of motion in (58) can be expressed in the analytical form (Heinloo, 1984, pp.229-232). The deduced solution generalizes the corresponding classical Ekman's solution.

In the case of environment stratification, the system of equations (58) was investigated numerically for the next two cases;

a) when $k_T, k_S \ll k_0 E^{(0)} + k_2 \Omega^2$ and b) when Ω and v are equal to zero.

The first case was investigated numerically for different (steady and unsteady) boundary conditions on the sea surface. The Ph.D. dissertation has been prepared by Ü. Võsumaa based on the results of the realized investigations (for an academic degree of Ph.D. in geophysics; scientific adviser J.Heinloo). Let's list only some results of the realized investigations, more interesting from the point of view of mechanics.

1) Due to the decrease of turbulence intensity with the increase of z , the vertical profiles of T and S take a typical form, consisting from mixed (quasi-homogeneous) layer, "jump" layer and a nondisturbed or periodically disturbed layer.

2) The thickness of the mixed layer varies greatly. The maximum thickness of the mixed layer is achieved at the end of the convection and it depends on the duration of the convection.

3) The halocline deepness is formed during convection and it is always situated deeper than thermocline.

4) Any "jump" layer hinder to the penetration of turbulence from the mixed layer into the layer under the jump" layer, but does not hinder the penetration of the momentum.

5) Because of the last effect a series of second stage "jump" layers may be formed under the main "jump" layer.

6) So, as the mean flow interacts with the large scale turbulence and the latter interacts with the stratification, the mean flow appears to be coupled with stratification of the environment. Due to the mentioned interaction, the velocity field appears to be modulated, for example, by the diurnal changes of vertical heat fluxes through the sea surface.

The second case was investigated in the typical for double diffusion and salt fingers situations. It is shown that due to difference of the coefficients of molecular diffusion for T and S , the released potential energy transforms into the turbulent energy by means of the series of turbulent "explosions". As a result of these "explosions" the vertical profiles of T and S take the typical stairs-like form.

THE CONCLUSIONS

1. The principles of the turbulence metatheory have been formulated and the concrete theory of turbulence is built up.

2. The suggested principles of the turbulence metatheory are based on the system treatment of a number of different turbulence theories as elements of a certain system.

3. The proposed theory belongs to the class of continuum theories of turbulence and are based on the understanding of the turbulent media as hierarchic media of structural levels with the rotational asymmetry of kinematical structure of a turbulent flow field.

4. Due to the rotational asymmetry of a turbulent flow field, the equations of motion are based on the independent laws of balance of momentum and moment of momentum.

5. The correspondence of the equations of motion to the equations of classical theories has been discussed.

6. The results of the applications of the proposed theory to concrete steady and unsteady turbulent flows, including MGD turbulent flows have been presented.

7. Some recent results of the applications of the proposed theory to oceanological problems have been shortly summed up.

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Resümee

Töös vaadeldakse kahte küsimuste ringi. Esimene neist käsitleb erinevatel paradigmatel põhinevate turbulentsi kirjeldamise teooriate omavahelise seostatuse küsimusi, ning, teine - turbulentsete liikumiste ja turbulentsetes keskkondades toimuvate protsesside kontinuaalse kirjeldamise probleeme.

Esimese probleemi käsitus põhineb süsteemanalüüsi põhimõtetel. Esitatud käsitus võimaldab süstematiseerida erinevatel ettekujutlustel põhinevate turbulentsi teooriate hulka ning, konkreetset, määratleda turbulentsete liikumiste kontinuaalsete teooriate koht ja roll üldises turbulentsiõpetuse kontekstis.

Töö põhiosas käsitletakse autori poolt aastatel 1972-1984 arendatud turbulentsete liikumiste kontinuaalse teooria ülesehituse küsimusi. Esitatud materjal põhineb autori poolt avaldatud monograafial "Turbulentsete voolamiste fenomenoloogiline mehaanika" (Tallinn, "Valgus", 1984).

Põhiline erinevus esitatud teooria ja vastavate klassikaliste käsitluste vahel seisneb erinevas keskkonna sümmeetria omaduste käsitluses. Defineeritud turbulentsete keskkondade pöördeline mitteisotroopsus ning mainitud omaduse seostamine turbulentsete liikumiste kiirusvälja keeriselise struktuuriga võimaldab peale konkreetse teooria ülesehituse kõrvaldada ka oluline ja pikka aega kestnud ideoloogiline vastuolu väiksemastaabilise ja suuremastaabilise turbulentsi kirjeldamise põhimõtete vahel.

Töö viimases osas antakse ülevaade esitatud teooria rakendustest mõningate konkreetsete turbulentsete liikumiste ja turbulentsetes keskkondades toimuvate protsesside kirjeldamisel.

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