G. Gyllström

Solutions graphiques d'équations différentielles du premier ordre

Stockholm : Meteorologisk-hydrografiska anstalt
1928

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SOLUTIONS GRAPHIQUES D'ÉQUATIONS DIFFÉRENTIELLES DU PREMIER ORDRE

PAR

G. GYLLSTRÖM

AVEC 8 PLANCHES

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280771

Solutions graphiques d'équations différentielles du premier ordre.

Dans la mécanique et dans la physique mathématique se présentent souvent des équations différentielles, dont le sens est inconnu, parce qu'elles ne peuvent être intégrées. L'intégration numérique, à laquelle il faut alors recourir, est très fatiguante et ne conduit que lentement au but. Des résultats plus rapides sont obtenus par la méthode graphique. Dans ce qui suit, il sera démontré comment on résout graphiquement l'équation différentielle

$$\frac{dy}{dx} = f(x, y) \qquad (1)$$

et l'équation différentielle polaire correspondante

$$\frac{rd\varphi}{dr} = f(r, \varphi) \qquad (2)$$

La solution analytique de (1) a la forme

$$F(xyC) = 0 \qquad (3)$$

ce qui correspond à un système de courbes dans le plan xy. (Voir fig. 1.) Le problème est de déduire ce système de courbes directement de l'équation différentielle (1) sans recourir à l'intégrale (3). A cet effet, on écrit d'abord à la place de (1)

$$\frac{dy}{dx} = f(x, y) = \operatorname{tg} \alpha \qquad (4)$$

où, par conséquent, α est l'angle que la tangente des courbes de la fig. 1 forme avec la direction de l'axe des x. De (4) on calcule α pour différentes valeurs de x et de y, et les directions correspondantes sont tracées, sous la forme de petits éléments linéaires dans le plan xy, après quoi on dessine les courbes intégrales sous forme de lignes de courant vers ce champ vectoriel. (Voir fig. 2.)

Un mode facile pour tracer ces éléments linéaires est le suivant, proposé par M. J. W. Sandström. On pose d'abord

$$\frac{dy}{dx} = a \qquad (5)$$

où a est une constante et, par conséquent, selon (1)

$$f(x, y) = a \qquad (6)$$

Le pendant graphique de cette équation est un système linéaire dans le plan xy, et, sur chaque ligne de ce système, la dérivée $\frac{dy}{dx}$ est, selon (1), (5) et (6) constante et égale à a. Chacune de ces lignes peut ensuite être munie d'un grand nombre de traits parallèles dont la direction correspond à $\frac{dy}{dx} = a$, après quoi les courbes intégrales sont tracées comme lignes de courant vers ce champ vectoriel. (Voir fig. 3.) Les lignes auxiliaires construites de l'équation (6) sont appelées isogones.

La propriété la plus importante des courbes intégrales est celle de ne pouvoir se couper l'une l'autre. C'est que, dans un tel point d'intersection, $\frac{dy}{dx}$ aurait deux ou plusieurs valeurs, ce qui est impossible, tant que $f(x, y)$ n'est susceptible que d'une seule valeur pour chaque point du plan xy.

Exceptionnellement, plusieurs courbes intégrales peuvent cependant se rencontrer dans un point. Ceci a lieu, lorsque l'expression $f(x, y)$ devient indéfinie et, par conséquent,

$$\frac{dy}{dx} = \frac{0}{0}. \qquad (7)$$

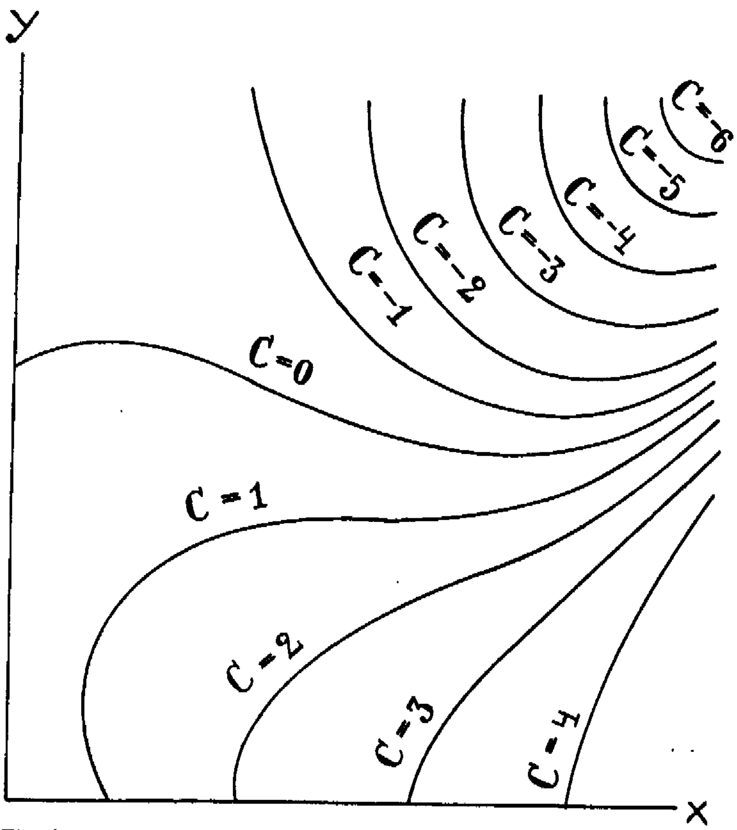


Fig. 1. La solution graphique de l'équation différentielle (1) est selon (3) un système de courbes dans le plan xy.

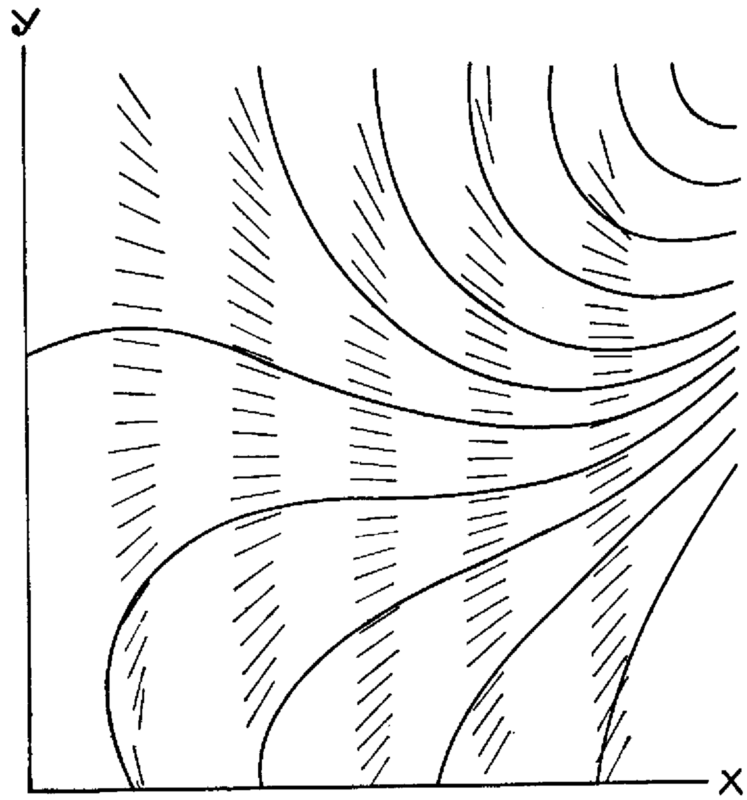


Fig. 2. Construction de la solution graphique de l'équation différentielle (1).

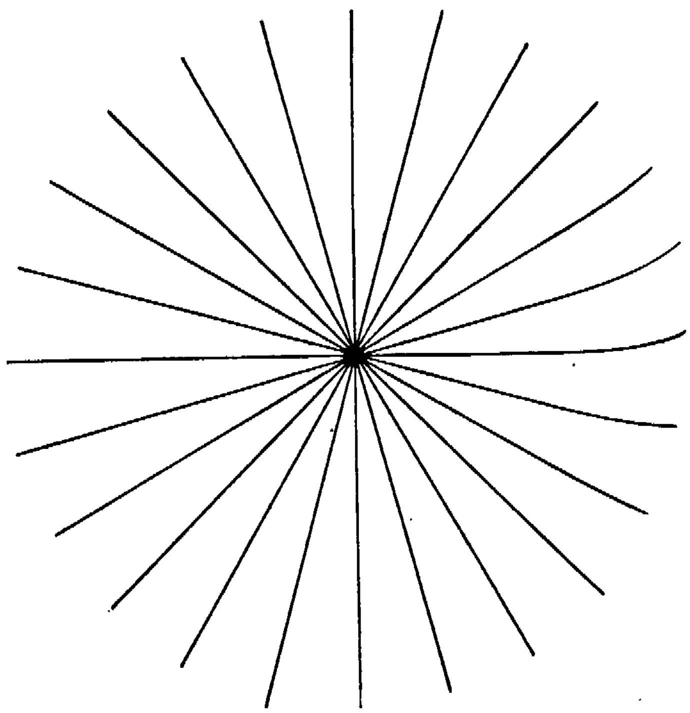


Fig. 4. Solution graphique de l'équation $\frac{dy}{dx} = \frac{y}{x}$.

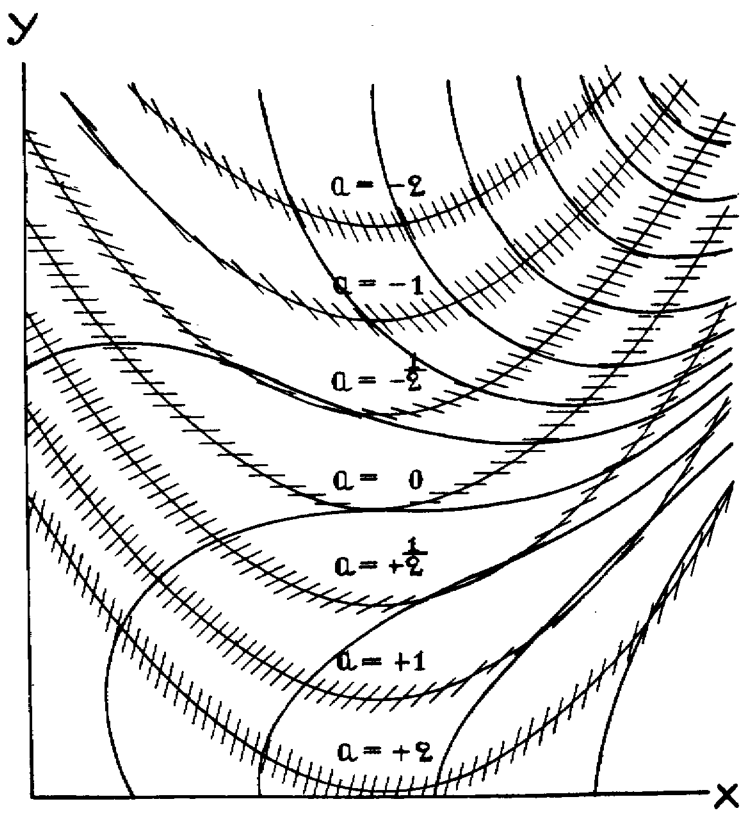


Fig. 3. Construction de la solution graphique de l'équation différentielle (1) à l'aide des isogones de M. Sandström.

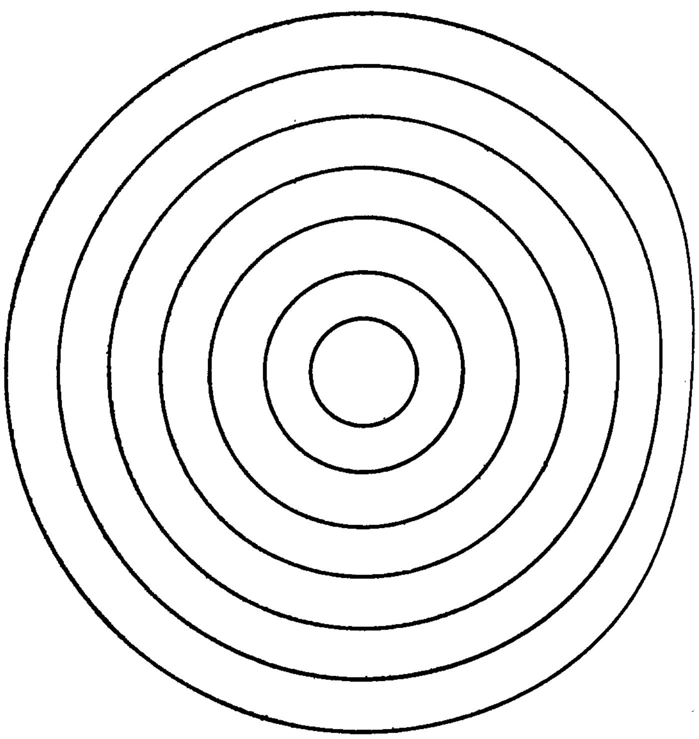


Fig. 5. Solution graphique de l'équation $\frac{dy}{dx} = -\frac{x}{y}$.

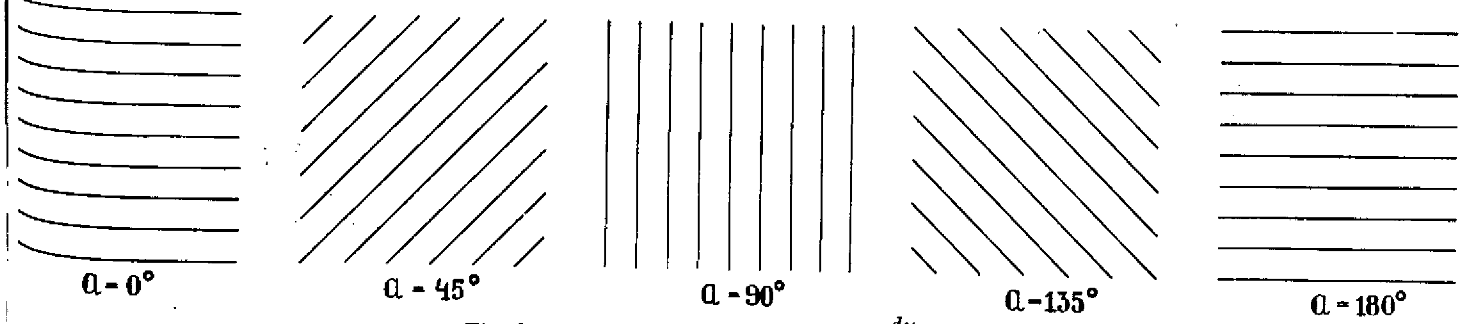


Fig. 6. Solution graphique de l'équation $\frac{dy}{dx} = \operatorname{tg} a$.

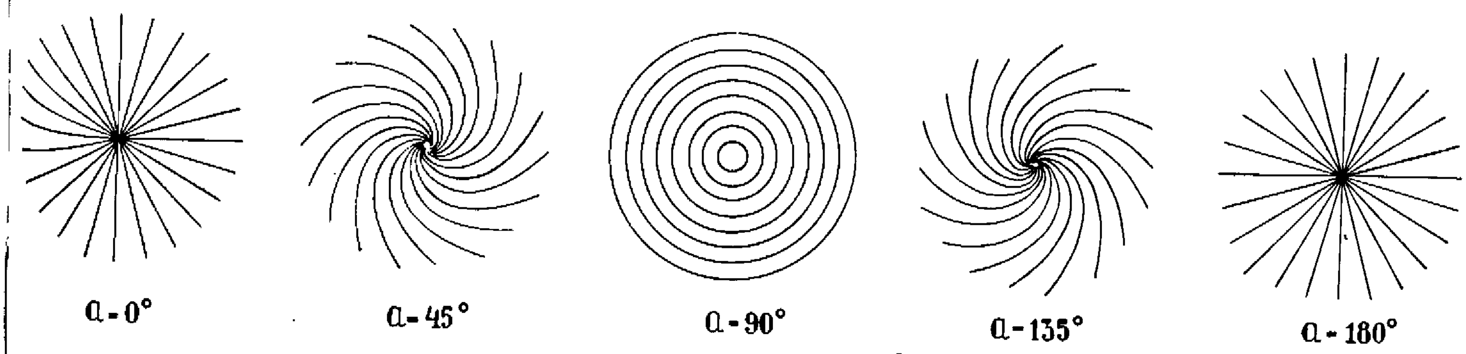


Fig. 7. Solution graphique de l'équation $\frac{r\,d\varphi}{dr} = \operatorname{tg} a$.

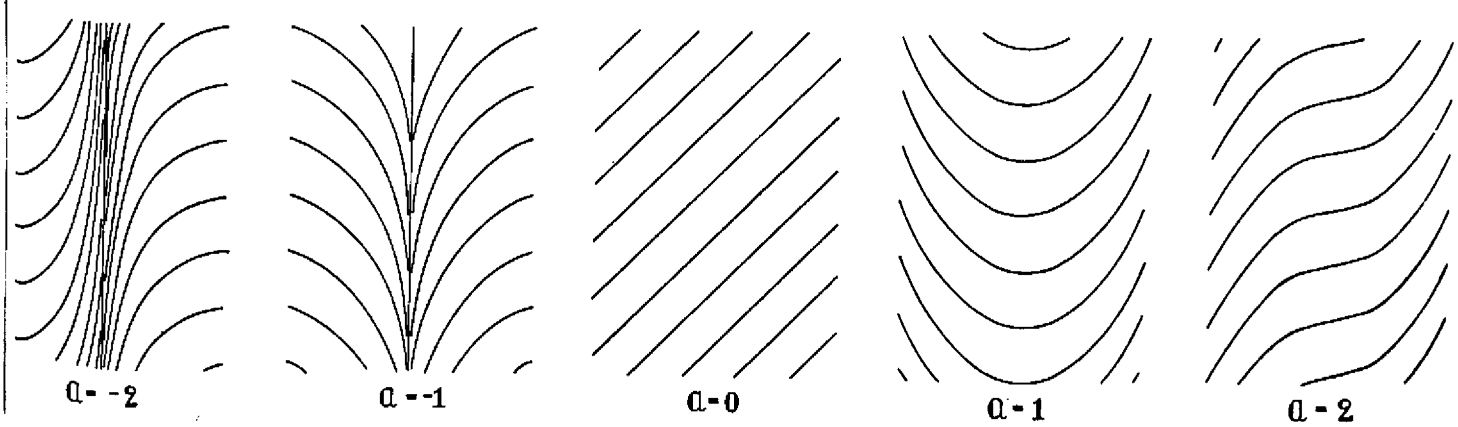


Fig. 8. Solution graphique de l'équation $\frac{dy}{dx} = x^a$.

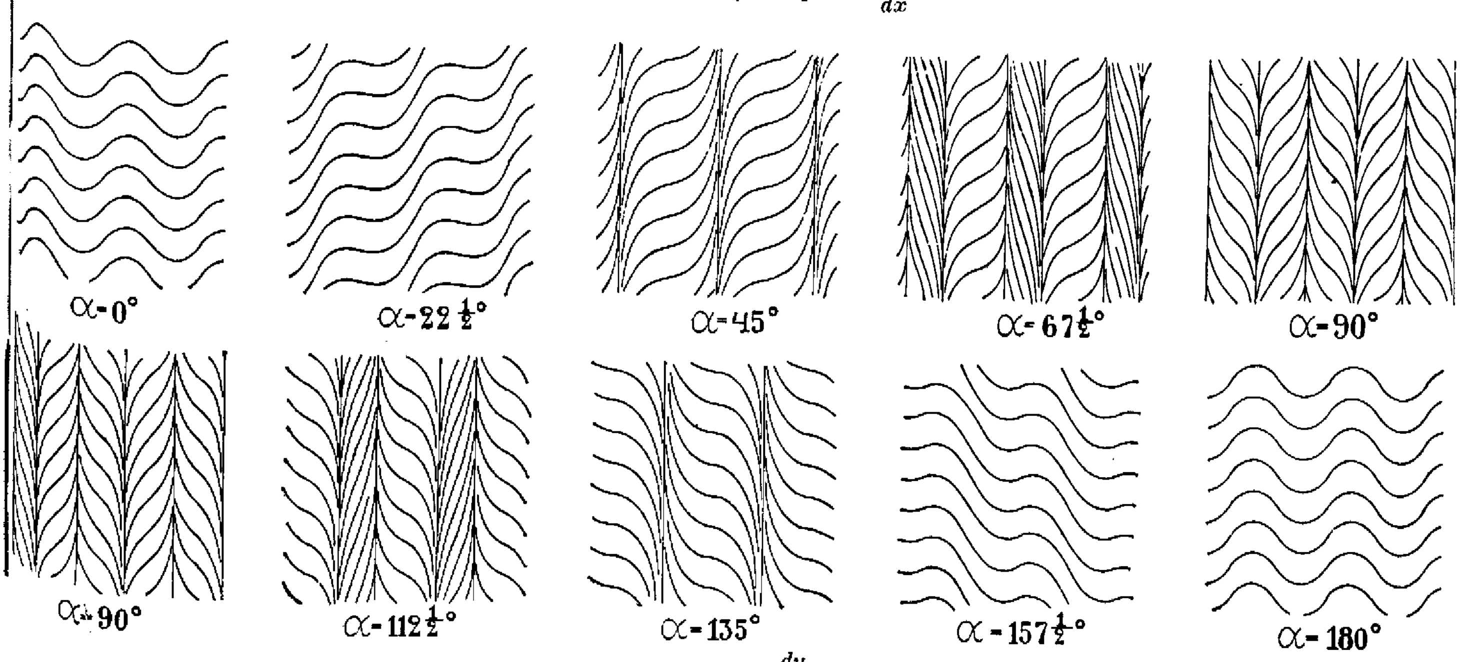


Fig. 9. Solution graphique de l'équation $\frac{dy}{dx} = \operatorname{tg}(\operatorname{arctg} \sin x + a.)$

Les points singuliers contribuent beaucoup à l'orientation et à l'aspect de tout le champ des lignes de courant. Ainsi, p. ex., dans l'équation différentielle

$$\frac{dy}{dx} = \frac{y}{x} \tag{8}$$

l'expression $\frac{dy}{dx}$ devient indéfinie à $x = 0$, $y = 0$. La fig. 4 reproduit la solution graphique de cette équation, laquelle consiste en droites rayonnant en tous sens de l'origine.

L'équation différentielle, dont les solutions graphiques coupent les solutions de (1) à angle droit est de la forme

$$\frac{dy}{dx} = -\frac{1}{f(xy)} \tag{9}$$

et, si, au lieu de (8) on écrit

$$\frac{dy}{dx} = -\frac{x}{y} \tag{10}$$

c'est là l'équation des cercles concentriques ayant l'origine pour centre. (Voir fig. 5.) De (10), il résulte que, dans ce cas encore, l'origine est un point singulier.

Si, dans (1), on introduit un paramètre a,

$$\frac{dy}{dx} = f(x, y, a) \tag{11}$$

on obtient des solutions différentes pour les différentes valeurs de a. L'équation de cette espèce la plus simple est

$$\frac{dy}{dx} = f(a) \tag{12}$$

à laquelle correspondent des lignes droites parallèles (voir fig. 6) et

$$\frac{r\,d\varphi}{dr} = f(a) \tag{13}$$

qui donne des spirales logarithmiques. (Voir fig. 7.) Le cas spécial suivant de (11) est

$$\frac{dy}{dx} = f(x, a) \tag{14}$$

dont les solutions sont continuellement répétées en sens vertical, comme il ressort de la fig. 8, qui contient les courbes intégrales de

$$\frac{dy}{dx} = x^a \tag{15}$$

Un intérêt spécial est présenté par la solution graphique qui dans chaque point coupe la solution de l'équation différentielle (1) sous un angle constant α. L'équation en est

$$\frac{dy}{dx} = \operatorname{tg}(\operatorname{arc\,tg} f(x, y) + \alpha) \tag{16}$$

La fig. 9 contient la solution graphique de

$$\frac{dy}{dx} = \operatorname{tg}(\operatorname{arc\,tg} \sin x + \alpha) \tag{17}$$

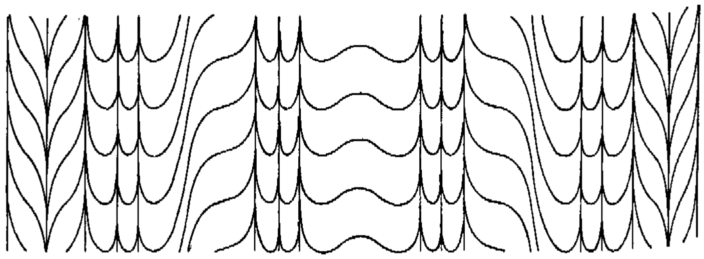


Fig. 10. Solution graphique de l'équation $\frac{dy}{dx} = \operatorname{tg} 2{,}75\,(\operatorname{arctg} \operatorname{tg}^2 x - x)$.

Les différentes formes que les solutions de (14) peuvent prendre ne sont pas nombreuses. Toute équation un peu compliquée de la forme

$$\frac{dy}{dx} = f(x) \tag{18}$$

les contient toutes. (Voir fig. 10).

La solution graphique de l'équation différentielle

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right) \tag{19}$$

ou

$$\frac{r\,d\varphi}{dr} = f(\varphi) \tag{20}$$

a un point singulier dans l'origine et la solution graphique en reste inchangée et identique, en cas d'agrandissement ou de réduction de l'image.

La dérivée $\frac{dy}{dx}$ est une tangente trigonométrique, ce qui la rend plus facilement infinie que $f(x, y)$. Sous ce rapport, l'on obtient une meilleure équivalence en introduisant une tangente aussi du côté droit de (1). Si, au lieu de (1), on écrit

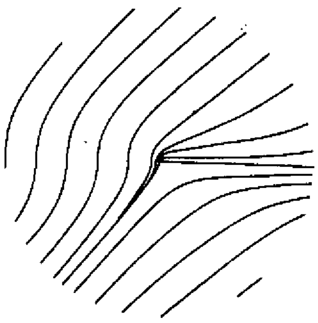
$$\frac{dy}{dx} = f(\operatorname{tg} x, \operatorname{tg} y) \tag{21}$$

les variables du membre droit deviennent infinies aussi souvent et aussi facilement que $\frac{dy}{dx}$, et les images qui en résultent deviennent plus diverses et plus variées.

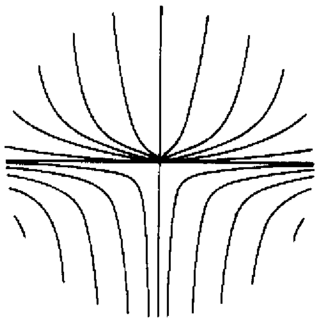
Les pl. I—V montrent différents points singuliers et les pl. VI—VIII des solutions graphiques de l'équation différentielle (21).

La facilité extraordinaire avec laquelle les solutions graphiques des équations différentielles sont construites selon la méthode décrite ci-dessus, et les résultats aussi instructifs qu'intéressants qu'elle produit, devraient encourager tous ceux qui s'occupent de physique et de mécanique à l'appliquer à la cinématique de l'air et de l'eau, spécialement dans la météorologie et dans l'océanographie, mais aussi aux champs magnétiques et électriques, etc. Enfin la clarté de la méthode et son adhésion directe à la définition de la dérivée $\frac{dy}{dx}$ la rendraient éminemment utile, lorsqu'il s'agit d'introduire et d'appliquer le calcul infinitésimal dans les écoles techniques et dans les lycées.

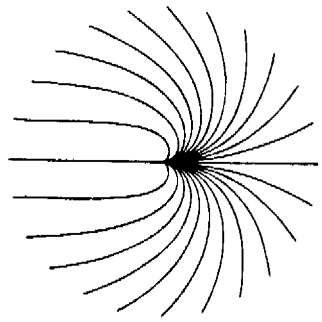
$\frac{dy}{dx} = \operatorname{tg} 0.5 \operatorname{arctg} \frac{y^2}{x}$



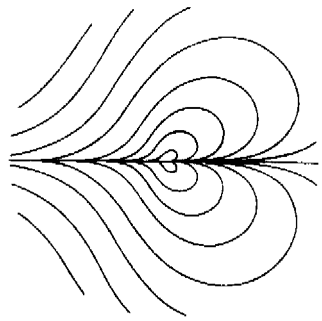
$\frac{dy}{dx} = \frac{y^2}{x}$



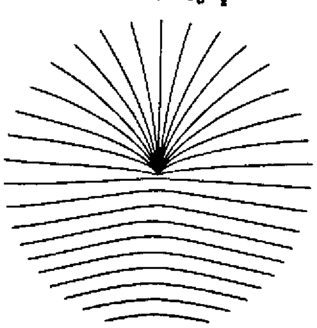
$\frac{r\,d\varphi}{dr} = \sin\varphi$



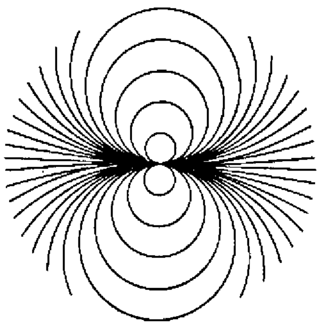
$\frac{r\,d\varphi}{dr} = \frac{3\sin\varphi - \sin^3\varphi}{1 - 3\sin^2\varphi}$



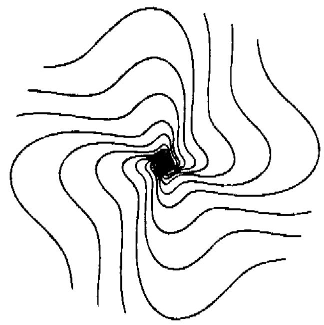
$\frac{r\,d\varphi}{dr} = \frac{\operatorname{tg}^2\frac{\varphi}{2} - \operatorname{tg}\frac{\varphi}{2}}{1 + \operatorname{tg}^3\frac{\varphi}{2}}$



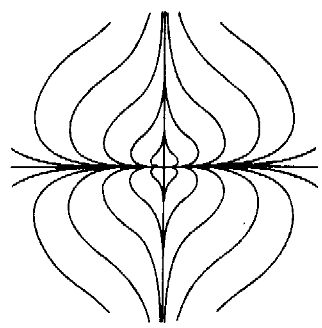
$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$



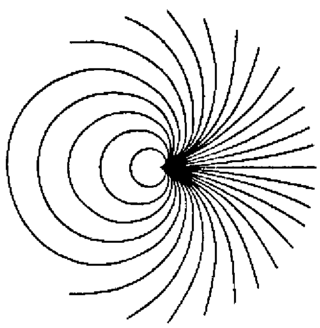
$\frac{r\,d\varphi}{dr} = \operatorname{tg}(\operatorname{arctg} \sin 4\varphi + 67.5)$



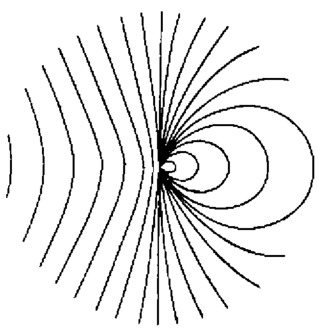
$\frac{r\,d\varphi}{dr} = \frac{2\sin 2\varphi}{1 - \sin^2 2\varphi}$



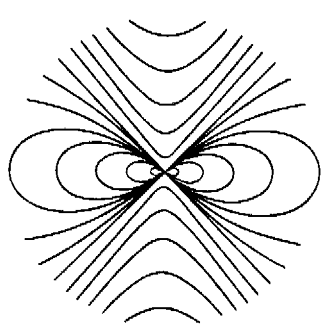
$\frac{dy}{dx} = \operatorname{tg} 1.5 \operatorname{arctg} \frac{y}{x}$



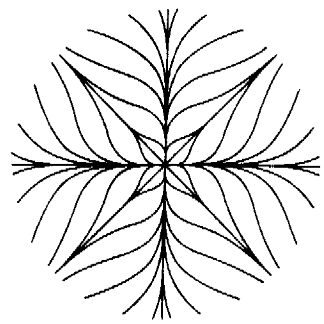
$\frac{r\,d\varphi}{dr} = -\frac{\cos^2\varphi}{\sin\varphi}$



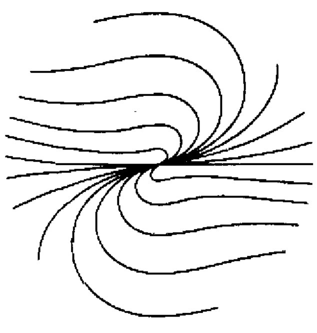
$\frac{r\,d\varphi}{dr} = \sin 2\varphi - \frac{1}{\sin 2\varphi}$



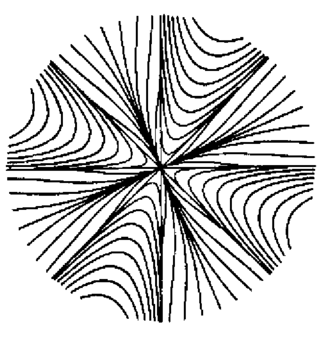
$\frac{r\,d\varphi}{dr} = \sin 4\varphi$



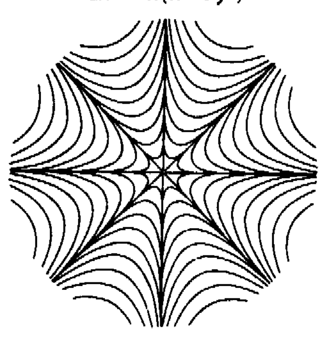
$\frac{r\,d\varphi}{dr} = \frac{2\sin^2\varphi}{1 - \sin^4\varphi}$



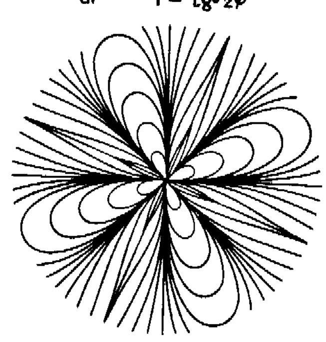
$\frac{r\,d\varphi}{dr} = \frac{\operatorname{tg}^2 2\varphi - \operatorname{tg} 2\varphi}{1 + \operatorname{tg}^3 2\varphi}$



$\frac{dy}{dx} = \frac{y(3x^2 - y^2)}{x(x^2 - 3y^2)}$



$\frac{r\,d\varphi}{dr} = \frac{\operatorname{tg}^2 2\varphi + \operatorname{tg} 2\varphi}{1 - \operatorname{tg}^3 2\varphi}$



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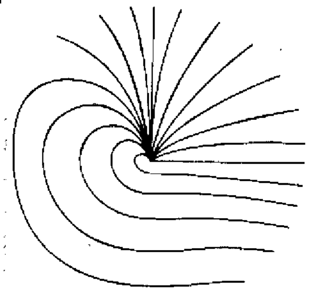
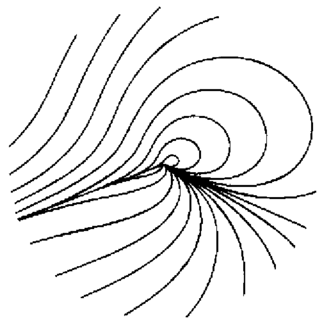
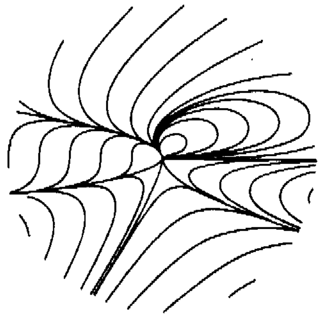
$$\frac{rd\varphi}{dr} = -\frac{1-\cos\varphi}{\sin\varphi} \cdot \frac{1-\cos\varphi-\sin\varphi}{1-\cos\varphi+\sin\varphi}$$



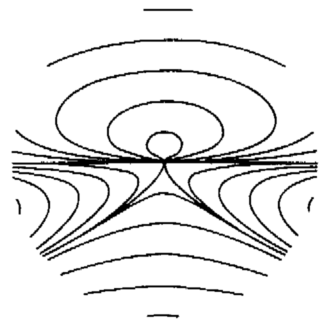
$$\frac{rd\varphi}{dr} = \frac{1+2\sin\varphi-\sin^2\varphi}{1-2\sin\varphi-\sin^2\varphi}$$



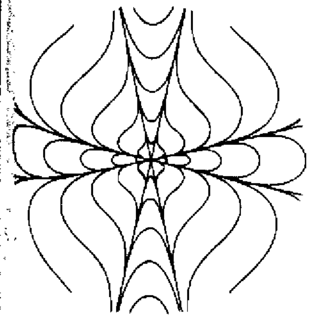
$$\frac{dy}{dx} = \operatorname{tg} 2.5 \operatorname{arctg}\frac{y^2}{x}$$



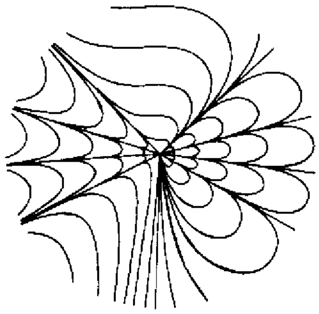
$$\frac{dy}{dx} = \frac{2xy^2}{x^2-y^4}$$



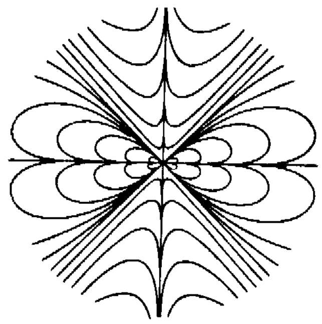
$$\frac{rd\varphi}{dr} = -\frac{1-6\sin^2 2\varphi+\sin^4 2\varphi}{4\sin^2 2\varphi(1-\sin^2 2\varphi)}$$



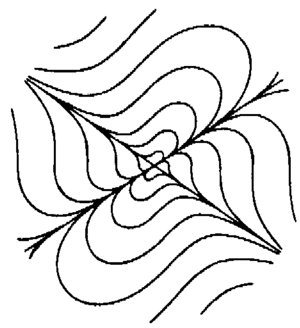
$$\frac{rd\varphi}{dr} = \operatorname{tg}(9 \operatorname{arctg} \sin\varphi + 45°)$$



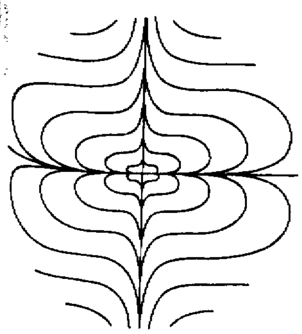
$$\frac{rd\varphi}{dr} = \frac{4\sin^2 2\varphi(1-\sin^2 2\varphi)}{1-6\sin^2 2\varphi+\sin^4 2\varphi}$$



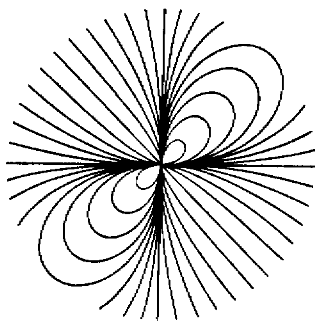
$$\frac{rd\varphi}{dr} = -\operatorname{ctg} 5 \operatorname{arctg} \sin^2\varphi$$



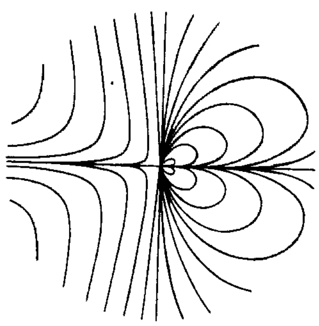
$$\frac{rd\varphi}{dr} = \frac{3\sin 2\varphi-\sin^3 2\varphi}{1-3\sin^2 2\varphi}$$



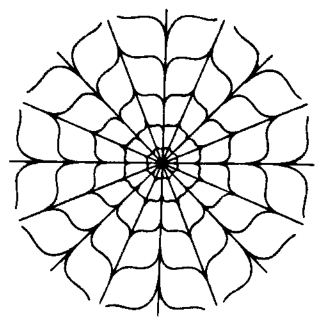
$$\frac{rd\varphi}{dr} = \frac{\operatorname{tg}^2\varphi+\operatorname{tg}\varphi}{1-\operatorname{tg}^3\varphi}$$



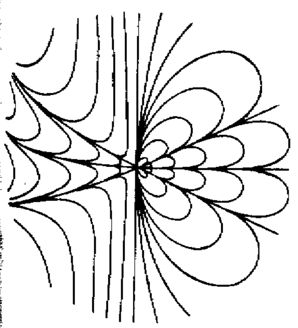
$$\frac{rd\varphi}{dr} = \operatorname{tg} 4 \operatorname{arctg} \sin\varphi$$



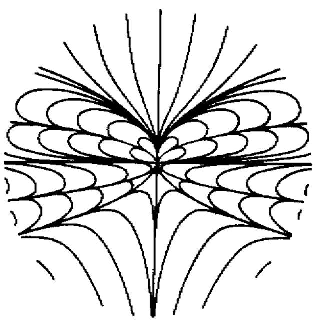
$$\frac{rd\varphi}{dr} = \frac{2\sin 8\varphi}{1-\sin^2 8\varphi}$$



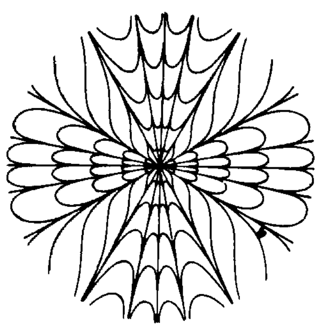
$$\frac{rd\varphi}{dr} = \operatorname{tg} 8 \operatorname{arctg} \sin\varphi$$



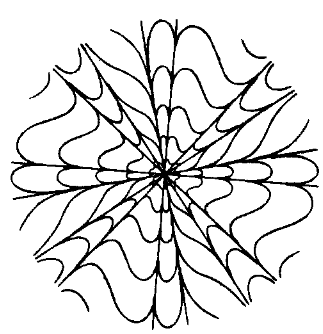
$$\frac{dy}{dx} = \operatorname{tg} 5 \operatorname{arctg}\frac{y^2}{x}$$



$$\frac{rd\varphi}{dr} = \operatorname{tg} 9 \operatorname{arctg} \sin 2\varphi$$



$$\frac{rd\varphi}{dr} = \operatorname{tg}(4 \operatorname{arctg} \sin 4\varphi + 45°)$$



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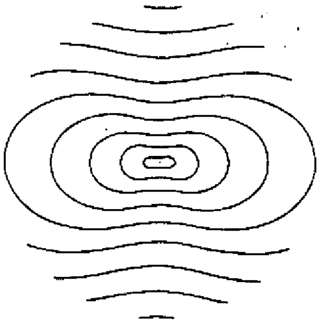
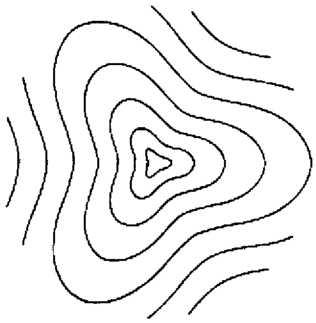
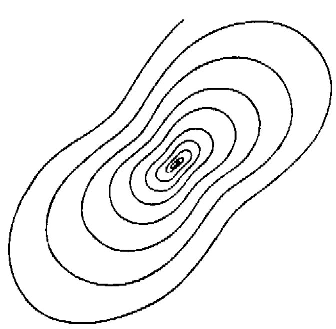
$\frac{r\,d\varphi}{dr} = -\frac{1}{\sin 2\varphi}$



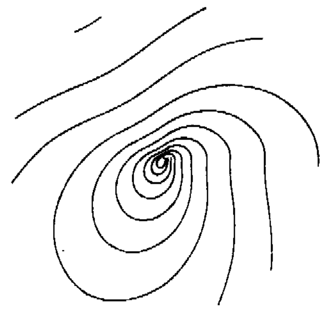
$\frac{r\,d\varphi}{dr} = -\frac{1}{\sin 3\varphi}$



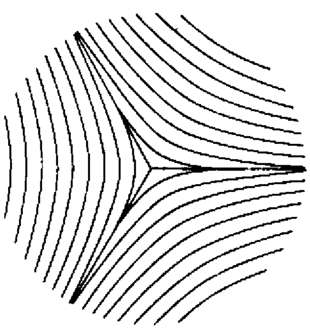
$\frac{r\,d\varphi}{dr} = \frac{1+2\sin^2\varphi-\sin^4\varphi}{1-2\sin^2\varphi-\sin^4\varphi}$



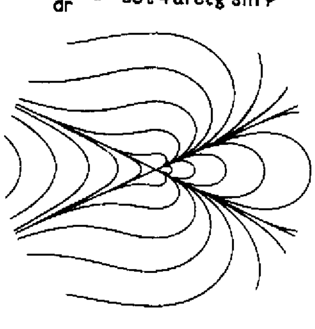
$\frac{r\,d\varphi}{dr} = -\frac{1}{\sin 1{,}5\varphi}$



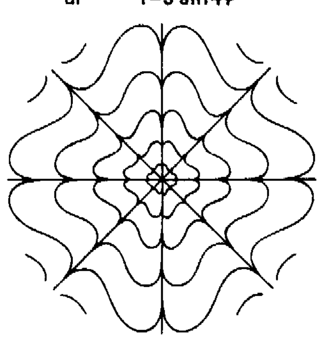
$\frac{dy}{dx} = -\operatorname{tg} 0{,}8 \operatorname{arctg}\frac{y}{x}$



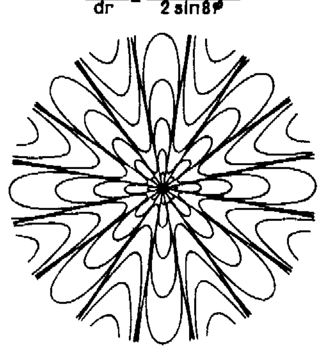
$\frac{r\,d\varphi}{dr} = -\cot 4 \operatorname{arctg} \sin\varphi$



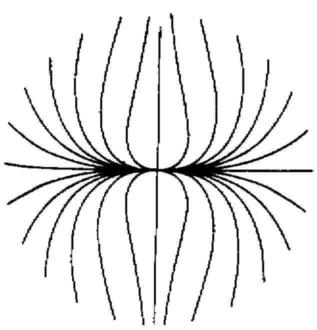
$\frac{r\,d\varphi}{dr} = \frac{3\sin 4\varphi - \sin^3 4\varphi}{1-3\sin 4\varphi}$



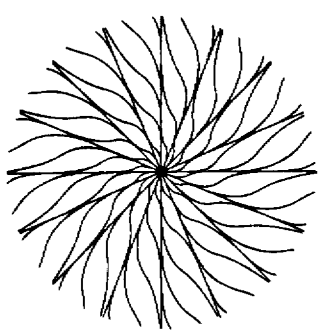
$\frac{r\,d\varphi}{dr} = \frac{\sin^2 8\varphi - 1}{2\sin 8\varphi}$



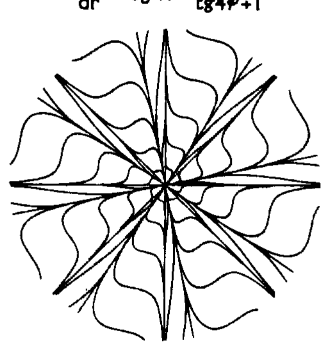
$\frac{r\,d\varphi}{dr} = \sin 2\varphi$



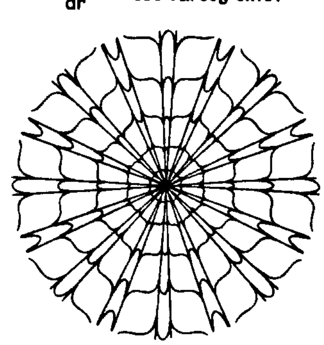
$\frac{r\,d\varphi}{dr} = \sin^2 8\varphi$



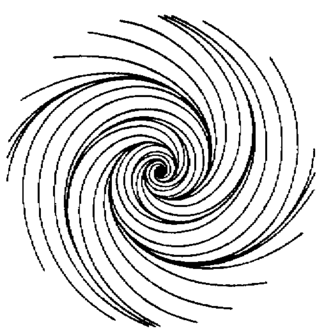
$\frac{r\,d\varphi}{dr} = \operatorname{tg} 4\varphi \cdot \frac{\operatorname{tg} 4\varphi - 1}{\operatorname{tg} 4\varphi + 1}$



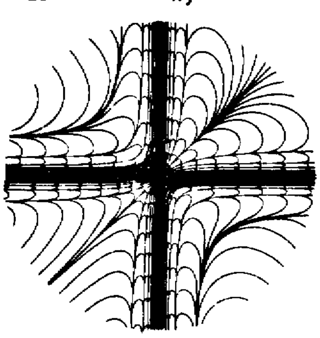
$\frac{r\,d\varphi}{dr} = -\cot 4 \operatorname{arctg} \sin 8\varphi$



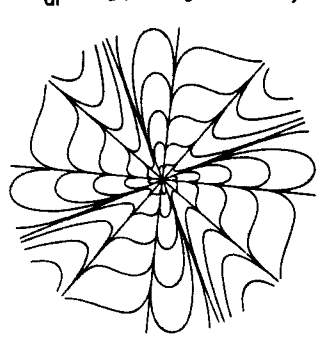
$\frac{du}{dt} = \frac{(k-l)\operatorname{tg} u}{1+k\operatorname{tg}^2 u}; \quad \frac{u = lr - k\varphi}{t = klr + \varphi}$



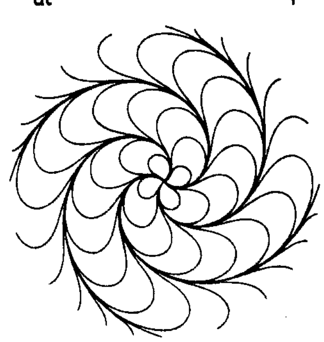
$\frac{du}{dt} = \operatorname{tg} ku; \; u = \frac{y-x}{xy}, \; t = x^3 + y^3$



$\frac{r\,d\varphi}{dr} = \operatorname{tg}(3 \operatorname{arctg} \sin 4\varphi + 45^\circ)$



$\frac{du}{dt} = \operatorname{tg} ku; \; u = \varphi - r, \; t = \varphi - \frac{1}{r}$



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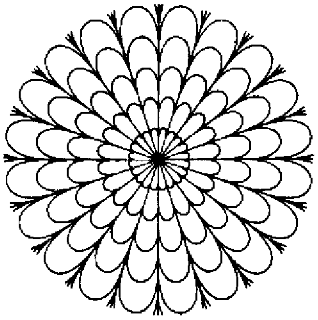
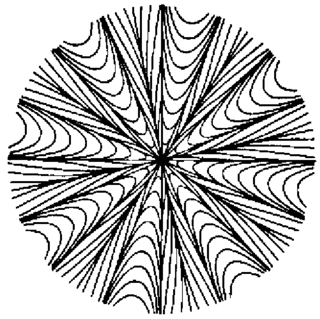
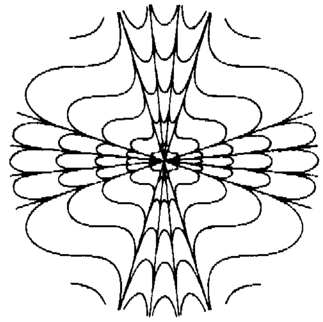
$\frac{dy}{dx} = \mathrm{tg}\,11\,\mathrm{arctg}\frac{y}{x}$



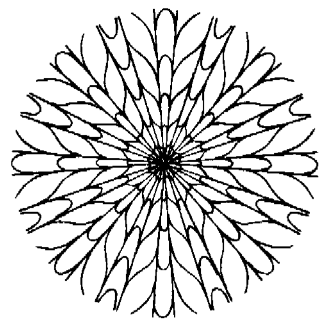
$\frac{rd\varphi}{dr} = \frac{\mathrm{tg}^2 4\varphi - \mathrm{tg}\,4\varphi}{1+\mathrm{tg}^3 4\varphi}$



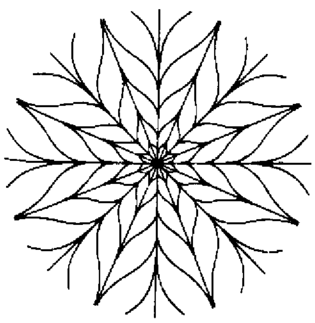
$\frac{rd\varphi}{dr} = -\cot 9\,\mathrm{arctg}\,\sin 2\varphi$



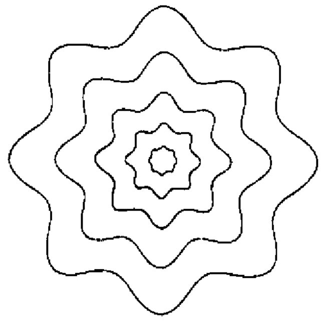
$\frac{rd\varphi}{dr} = -\cot 3\,\mathrm{arctg}\,\sin 8\varphi$



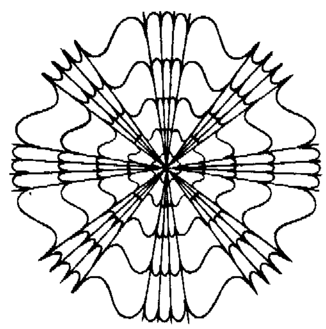
$\frac{rd\varphi}{dr} = \sin 8\varphi$



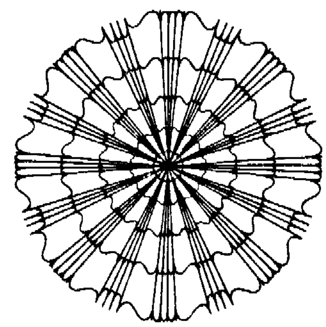
$\frac{rd\varphi}{dr} = -\frac{1}{\sin 8\varphi}$



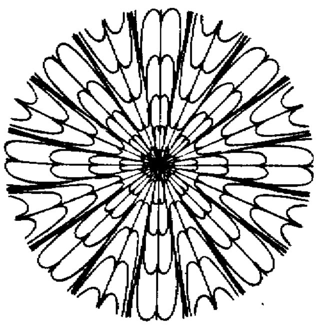
$\frac{rd\varphi}{dr} = -\cot 9\,\mathrm{arctg}\,\sin 4\varphi$



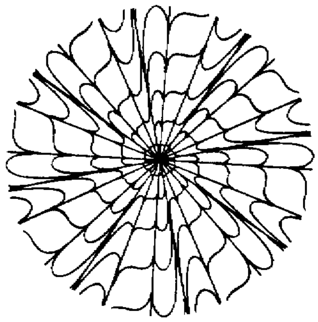
$\frac{rd\varphi}{dr} = -\cot 9\,\mathrm{arctg}\,\sin 8\varphi$



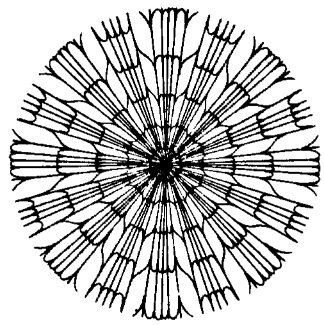
$\frac{rd\varphi}{dr} = \mathrm{tg}\,4\,\mathrm{arctg}\,\sin 8\varphi$



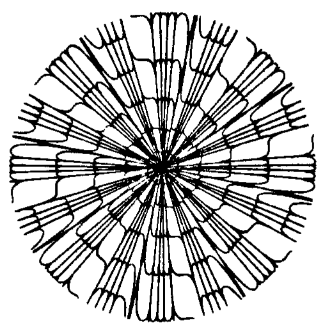
$\frac{rd\varphi}{dr} = \mathrm{tg}(3\,\mathrm{arctg}\,\sin 8\varphi + 45^\circ)$



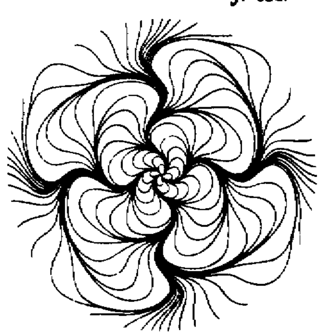
$\frac{rd\varphi}{dr} = \mathrm{tg}\,9\,\mathrm{arctg}\,\sin 8\varphi$



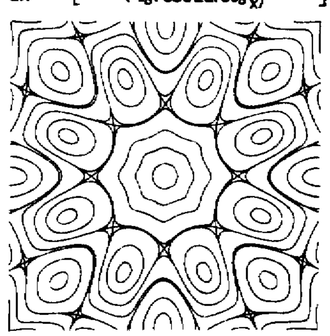
$\frac{rd\varphi}{dr} = \mathrm{tg}(9\,\mathrm{arctg}\,\sin 8\varphi + 45^\circ)$



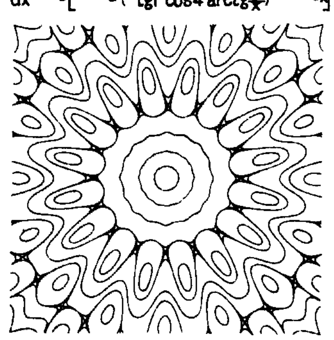
$\frac{du}{dt} = \mathrm{tg}\,2u;\quad u = \sin r - \varphi;\ t = \int\frac{dr}{r^2\cos r} + \varphi$



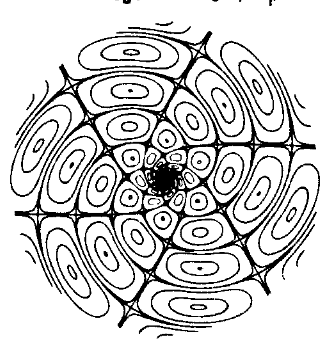
$\frac{dy}{dx} = \mathrm{tg}\left[\mathrm{arctg}\left(\frac{\mathrm{tg}\,r\sin 2\,\mathrm{arctg}\frac{y}{x}}{\mathrm{tg}\,r\cos 2\,\mathrm{arctg}\frac{y}{x}}\right) - \mathrm{arctg}\frac{y}{x}\right]$



$\frac{dy}{dx} = \mathrm{tg}\left[\mathrm{arctg}\left(-\frac{\mathrm{tg}\,r\sin 4\,\mathrm{arctg}\frac{y}{x}}{\mathrm{tg}\,r\cos 4\,\mathrm{arctg}\frac{y}{x}}\right) - 3\,\mathrm{arctg}\frac{y}{6x}\right]$



$\frac{du}{dt} = \frac{\mathrm{tg}\,u}{\mathrm{tg}\,t}$ $\quad u = \varphi - r$, $t = \varphi - \frac{1}{r}$



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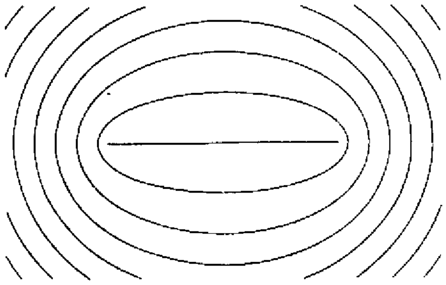
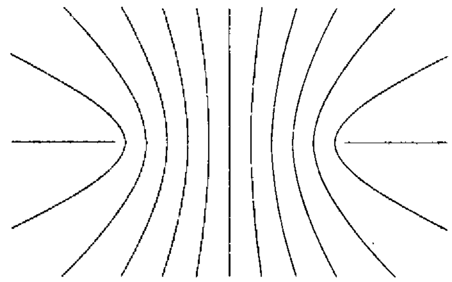
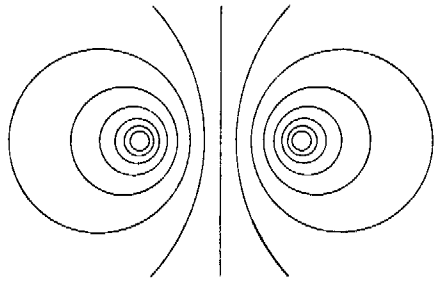
$$\frac{dy}{dx} = -\frac{x\sqrt{(x-a)^2+y^2}+(x-a)\sqrt{x^2+y^2}}{y\left(\sqrt{(x-a)^2+y^2}+\sqrt{x^2+y^2}\right)}$$



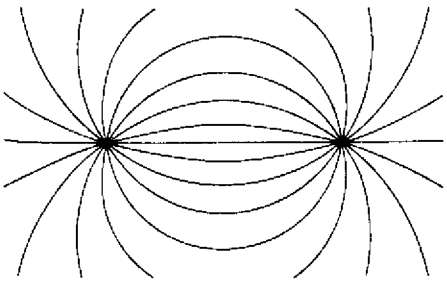
$$\frac{dy}{dx} = \frac{(x-a)\sqrt{x^2+y^2}-x\sqrt{(x-a)^2+y^2}}{y\left(\sqrt{(x-a)^2+y^2}-\sqrt{x^2+y^2}\right)}$$



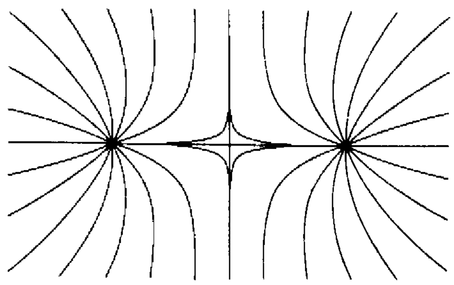
$$\frac{dy}{dx} = \frac{x[(x-a)^2-x^2]+a(x^2+y^2)}{y(2ax-a^2)}$$



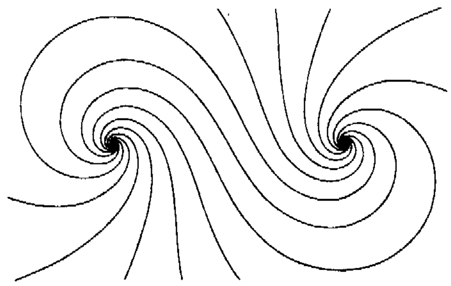
$$\frac{dy}{dx} = \frac{y[(x-a)^2-x^2]}{x[(x-a)^2-x^2]+a(x^2+y^2)}$$



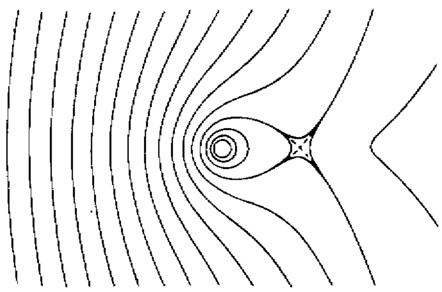
$$\frac{dy}{dx} = \frac{y[(x-a)^2+x^2+2y^2]}{x[(x-a)^2+x^2+2y^2]-a(x^2+y^2)}$$



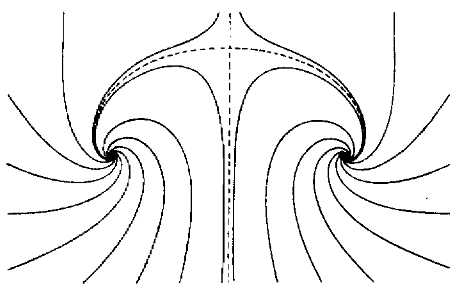
$$\frac{dy}{dx} = \frac{(y\,\mathrm{tg}\,30-x+a)(x^2+y^2)+(x-y\,\mathrm{tg}\,30^\circ)[(x-a)^2+y^2]}{[(x-a)\,\mathrm{tg}\,30+y](x^2+y^2)-(y+x\,\mathrm{tg}\,30^\circ)[(x-a)^2+y^2]}$$



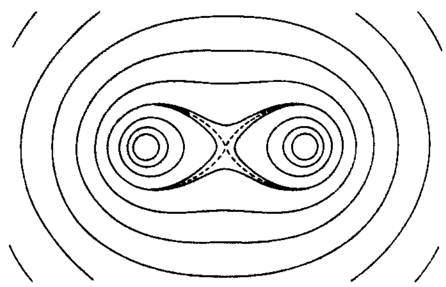
$$\frac{dy}{dx} = -\frac{x^2-x+y^2}{y}$$



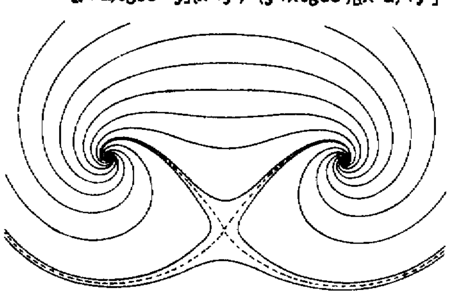
$$\frac{dy}{dx} = \frac{(x-a+y)(x^2+y^2)-(x-y)[(x-a)^2+y^2]}{(x-a-y)(x^2+y^2)+(x+y)[(x-a)^2+y^2]}$$



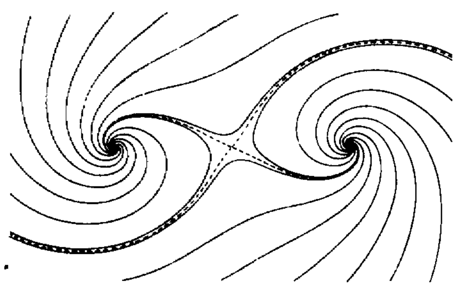
$$\frac{dy}{dx} = -\frac{x[(x-a)^2+y^2]+(x-a)(x^2+y^2)}{y[(x-a)^2+x^2+2y^2]}$$



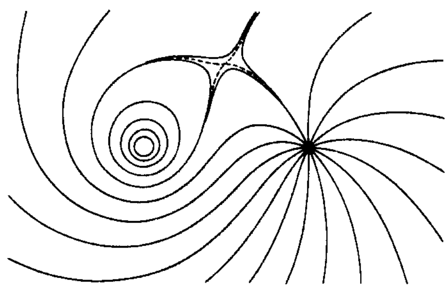
$$\frac{dy}{dx} = \frac{(y\,\mathrm{tg}\,30^\circ+x-a)(x^2+y^2)+(x-y\,\mathrm{tg}\,30^\circ)[(x-a)^2+y^2]}{[(x-a)\,\mathrm{tg}\,30^\circ-y](x^2+y^2)-(y+x\,\mathrm{tg}\,30^\circ)[(x-a)^2+y^2]}$$



$$\frac{dy}{dx} = \frac{(y\,\mathrm{tg}\,30^\circ-x+a)(x^2+y^2)-(x-y\,\mathrm{tg}\,30^\circ)[(x-a)^2+y^2]}{[(x-a)\,\mathrm{tg}\,30^\circ+y](x^2+y^2)+(y+x\,\mathrm{tg}\,30^\circ)[(x-a)^2+y^2]}$$



$$\frac{dy}{dx} = \frac{x[(x-a)^2+y^2]-y(x^2+y^2)}{(x-a)(x^2+y^2)-y[(x-a)^2+y^2]}$$



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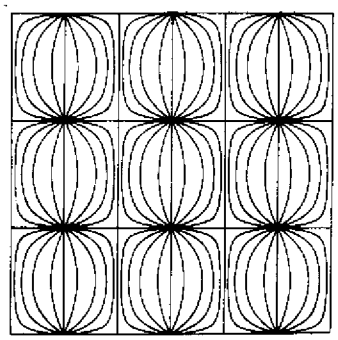
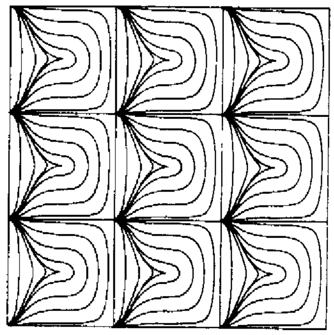
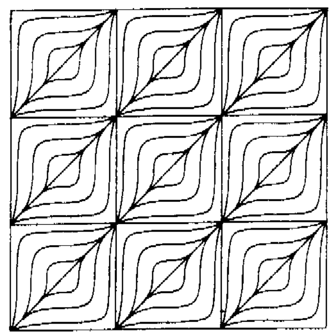
$$\frac{dy}{dx} = \frac{\mathrm{tg}\,x}{\sin 2x}$$



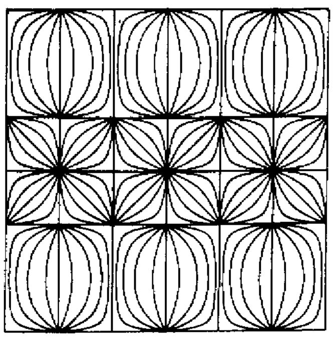
$$\frac{dy}{dx} = \frac{\mathrm{tg}\,y}{\mathrm{tg}^2 x}$$



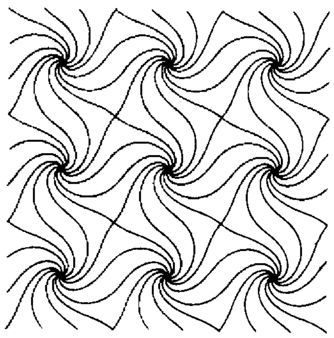
$$\frac{dy}{dx} = \frac{\mathrm{tg}^2 y}{\mathrm{tg}^2 x}$$



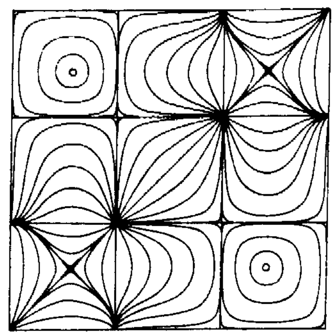
$$\frac{dy}{dx} = \frac{\mathrm{tg}^2 y - \mathrm{tg}\,y}{\sin 4x\,(1+\mathrm{tg}^2 y)}$$



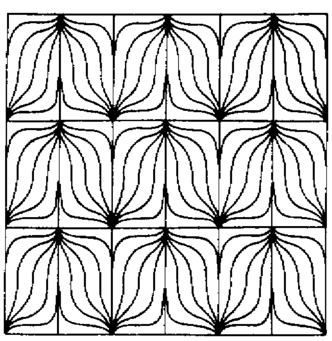
$$\frac{dy}{dx} = \frac{\mathrm{tg}\,x + \mathrm{tg}\,y}{\mathrm{tg}\,x - \mathrm{tg}\,y}$$



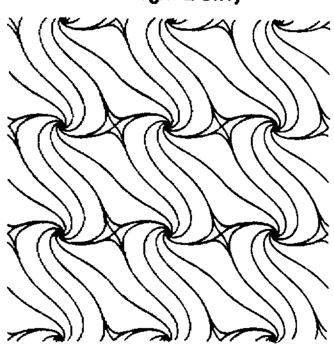
$$\frac{dy}{dx} = \frac{\mathrm{tg}\,5\,\mathrm{arctg}\,\sin^2 y}{\mathrm{tg}\,5\,\mathrm{arctg}\,\sin^2 x}$$



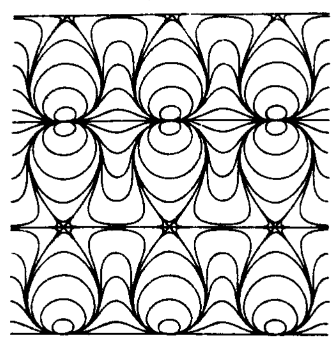
$$\frac{dy}{dx} = \frac{\mathrm{tg}^1 y}{\sin 2x}$$



$$\frac{dy}{dx} = \frac{\mathrm{tg}\,x + 2\sin y}{\mathrm{tg}\,x - 2\sin y}$$



$$\frac{dy}{dx} = \frac{4\,\mathrm{tg}\,x\,\sin y}{\mathrm{tg}^2 x - 4\sin^2 y}$$



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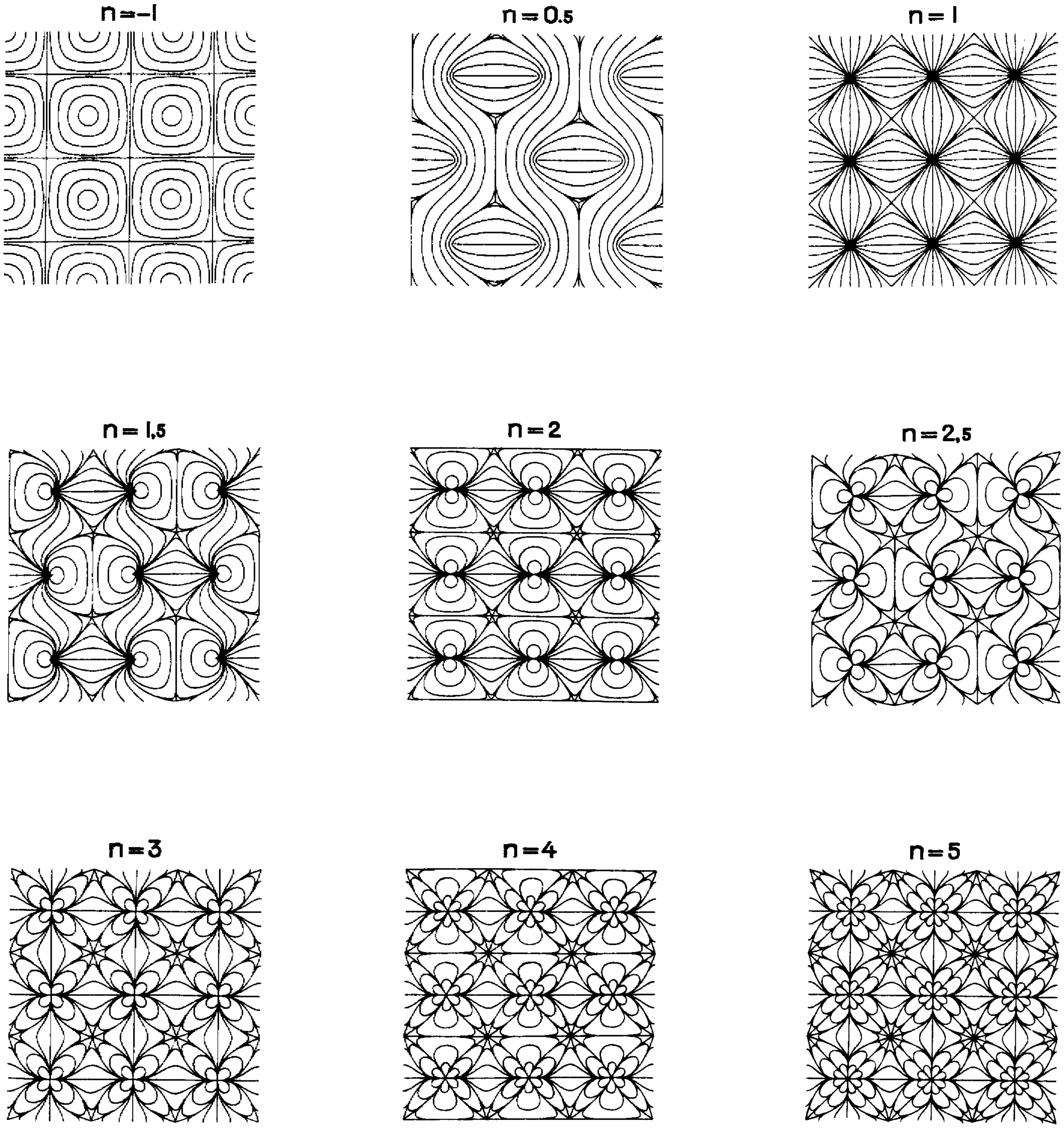
$$\frac{dy}{dx} = \operatorname{tg} n \operatorname{arctg} \frac{\operatorname{tg} y}{\operatorname{tg} x}$$



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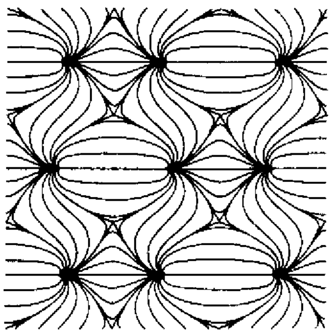
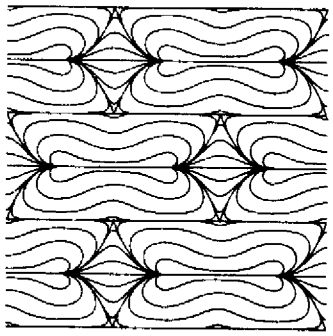
$$\frac{dy}{dx} = \operatorname{tg}\left[n \operatorname{arctg} \sin \operatorname{arctg}\frac{\operatorname{tg} y}{\operatorname{tg} x} + \operatorname{arctg}\frac{\operatorname{tg} y}{\operatorname{tg} x}\right]$$

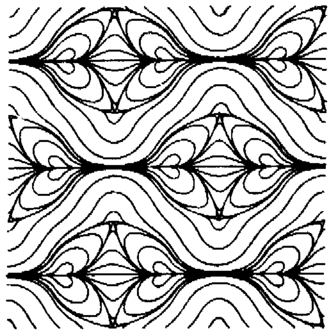
n = 1



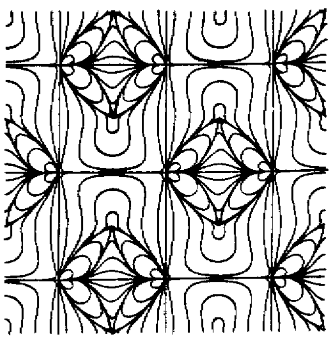
n = 2



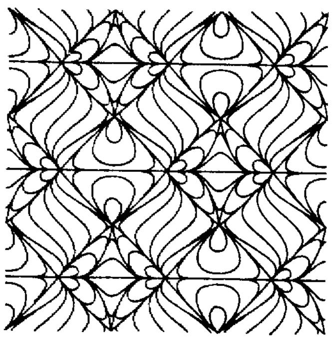
n = 3



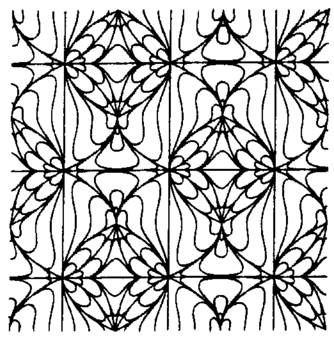
n = 4



n = 5

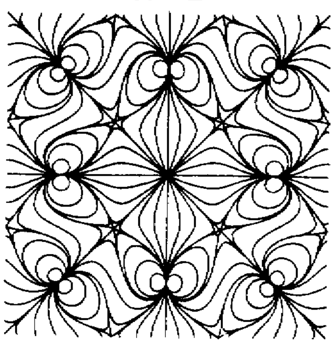


n = 8

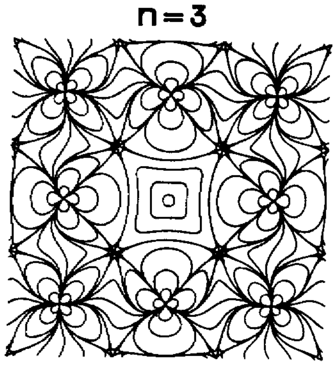


$$\frac{dy}{dx} = \operatorname{tg}\left[n \operatorname{arctg}\left(\frac{y \operatorname{tg} y + x \operatorname{tg} x}{y \operatorname{tg} x - x \operatorname{tg} y} + \operatorname{arctg}\frac{y}{x}\right)\right]$$

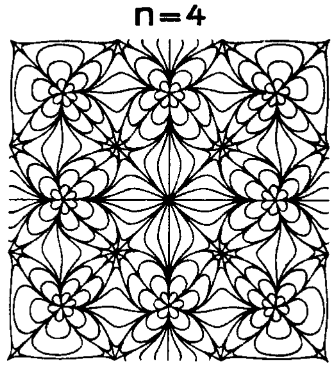
n = 2



n = 3



n = 4



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