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A GALVANOMETRICALLY REGISTERING VERTICAL SEISMOGRAPH WITH TEMPERATURE COMPENSATION

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Introduction.

The Vertical Seismograph described in the following pages is not merely an experimental apparatus, but has been constructed for several seismic stations by the firm of H. Masing, Tartu (Estonia). The model in its latest form is a most reliable and elegant piece of work and will be found useful in every seismic station. In connection with two other horizontal seismographs of recent construction[1]) it is intended to complete the equipment of a first-class seismic station.

Every seismic expert is well aware of the importance which attaches to the vertical component in seismology. Without a reliable apparatus for recording this component, a seismic station can only furnish scanty materials for seismometry. Records, however, which are obtained by means of a properly constructed vertical seismograph can be utilized for calculations and determinations of the most varied character.

Very simple types of seismographs will suffice where the object is merely the recording of time intervals. But even then the registration of the vertical component is attended with much difficulty if the apparatus is very sensitive, because large disturbances are caused by variations of temperature.

At the present time we are no longer satisfied with the mere determination of the time element. Seismic workers are interested in absolute values of the dislocation components of the earth's surface, which are not obtainable with apparatus of the ordinary kind.

In this respect the galvanometric method of registration possesses certain advantages, because it ensures a more exact

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[1]) J. Wilip: On New Precision-Seismographs. Acta et Commentationes Universitatis Tartuensis (Dorpatensis). A. X. 7. 1926.

and constant maintenance of the true zero-position than the other methods.

With particular regard to the Galitzin-Seismograph for the vertical component, it must be mentioned that this apparatus was constructed without temperature compensation, which necessitated a careful maintenance of a constant temperature in the pendulum room.

This object, however, is never perfectly attainable, as a spiral-spring apparatus with long proper vibration periods ($13^s.5$) shows a distinct reaction already to a difference of 0.0001^0 C.

In a system with Ewing's suspension[1]) the periods above the true zero-position are shorter, and below it longer, than the desired period for which the apparatus has been adjusted. Hence the migration of the zero-point through the altered proper period of the pendulum would be a source of error. Moreover, the centre of gravity of the oscillating system must remain permanently in the chosen horizontal plane. Should this condition not be fulfilled, the forces of inertia would bring about an effect of the horizontal dislocations of the earth's surface. The vertical seismograph in this case also acts at right angles to the axis of rotation, like a seismograph for the horizontal component.

Hence we see that under all circumstances care must be taken that the true rest position be permanently maintained.

These considerations made it necessary to look for means whereby such sources of error could be avoided.

Description of the New Vertical Seismograph.

A diagrammatic sketch of the new vertical seismograph is given in fig. 1. A substantial bed-plate serves as support for a stand built up of 2 trapeze-shaped frames, in the interior of which the oscillating system is contained. The latter consists of a double metal frame *B*, which comprises the following parts: 1) the Principal Weight *G*, which is displaceable upwards and downwards (in the actual model this weight has been divided into 4 parts), 2) the Registering Coils *J*, 3) the Damping-plate *D*,

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[1]) According to Dr. R. Ehlert this method of suspension dates from the year 1891. cf: Gerlands Beiträge zur Geophysik, Bd. III. S. 426. 1898.

and 4) 2 bi-metallic flat spiral springs *T*, arranged symmetrically on both sides and provided with regulating weights.

The horizontal axis of rotation is at *A* and is formed by plate-springs. The Ewing-suspension is here formed by a couple

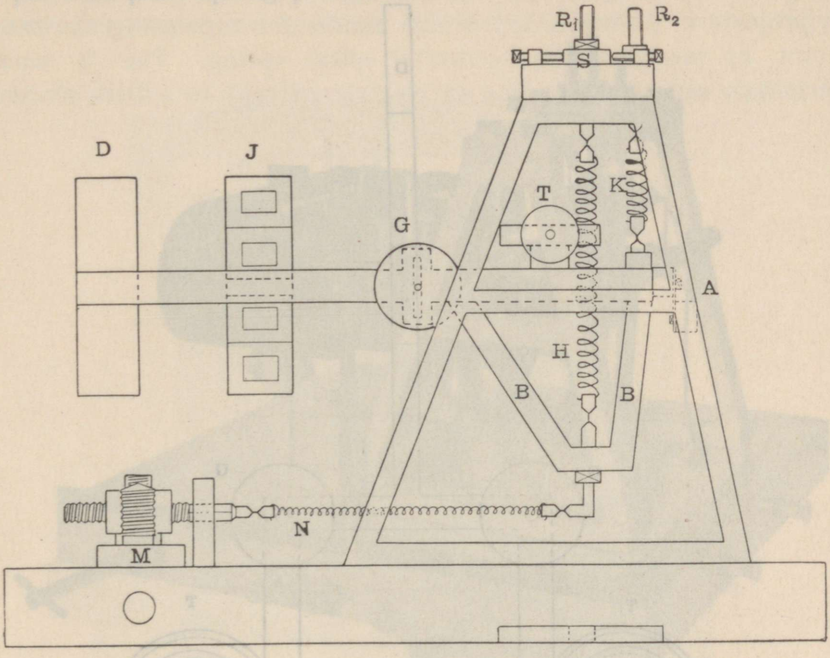

Fig. 1.

of spiral springs, identical in every respect, whose lower points of application are situated below the plane containing the centre of gravity of the oscillating system.

In order to secure a constant proper period in the upper and lower parts of the plane of the centre of gravity, there is another short spiral spring *K* having its lower point of application above the plane of the centre of gravity[1]).

The upper points of application are situated below the screw-nuts R_1 and R_2, by means of which they may be raised or lowered. The screw-nuts of the main spirals *R* can be shifted to right or left by means of the slides *S*.

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[1]) J. Wilip: Zur Theorie und Konstruktion von Vertikalseismographen. Gerlands Beiträge zur Geophysik. Bd. XIX. H. 4. S. 387. 1928.

In addition to these spirals which carry the oscillating system, there is another very weak horizontal spiral spring N, which has its point of application at the lower end of the oscil-

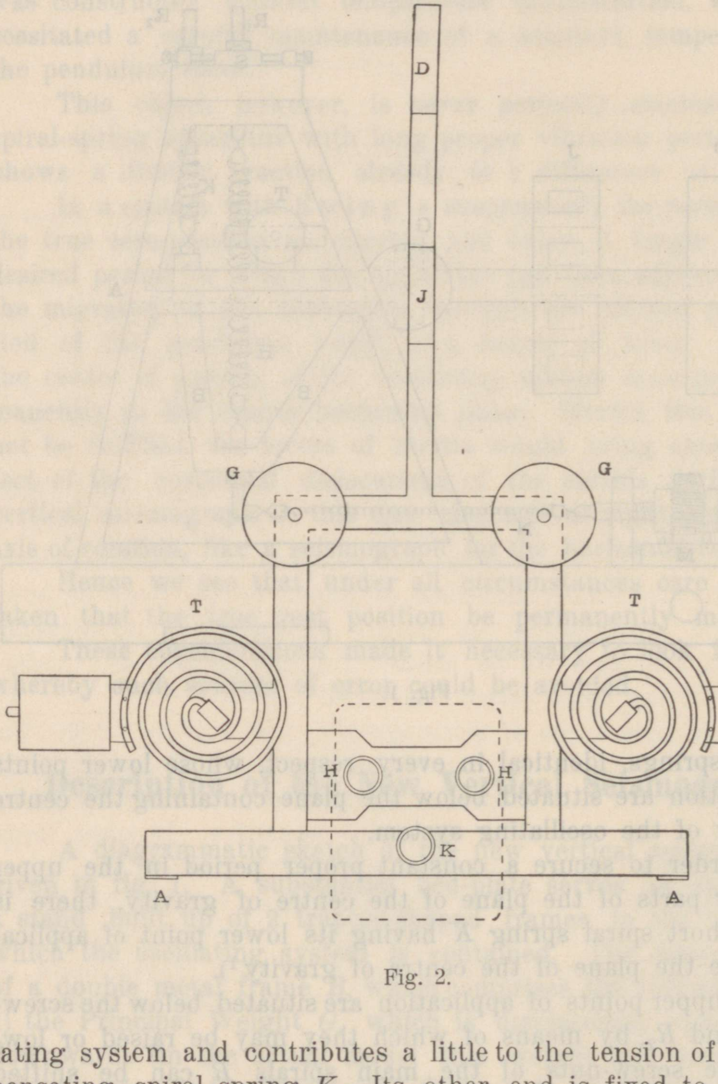

Fig. 2.

lating system and contributes a little to the tension of the compensating spiral spring K. Its other end is fixed to a screw-contrivance M, by means of which it may be tightened or slackened. It serves for the regulation of the zero-point, which is effected from the outside by means of a key.

All the points of application of the spirals are formed by delicate plate-springs, which cause a very slight and constant amount of friction. Including the two plate-springs of the horizontal axis of rotation A, there are in the apparatus altogether 5 pairs of plate-spring joints.

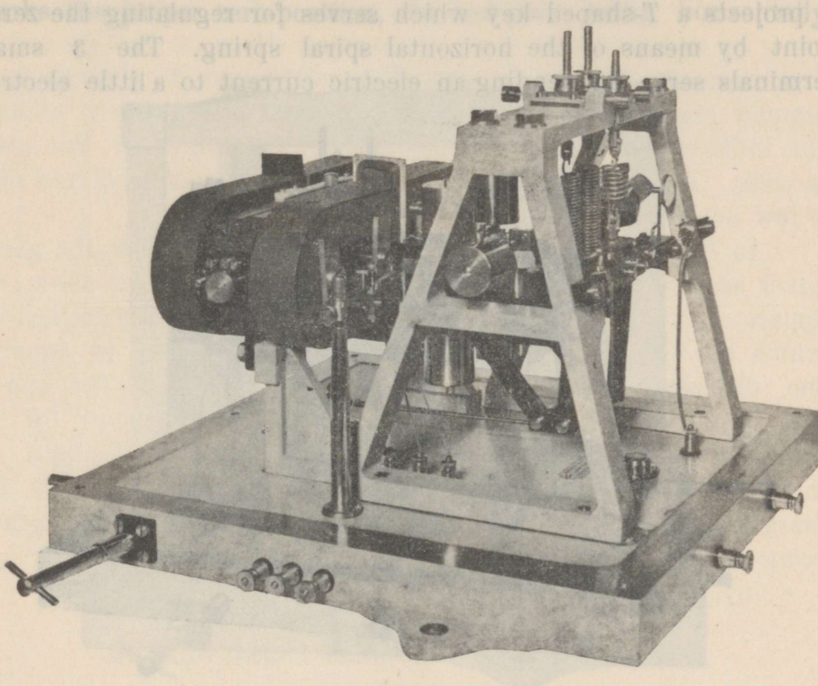

Fig. 3.

The oscillating system is roughly sketched in fig. 2 as seen from above. It includes a weight of about 6 kg, and is of a fairly complicated construction. At the corners of the rectangle are placed the movable principal weights. At the sides there are to be seen flat spiral contrivances carrying weights. Each of these consists of a bi-metallic strip of invar and brass, 4 mm thick and 13 mm wide, the brass being on the outside. With rise of temperature the spiral bends and approaches the weight to the axis of rotation, so that both the point of oscillation and the centre of gravity are shifted towards the axis of rotation. If the invar were placed on the outside, the flat spirals would have to be inverted, and the same result would be obtained.

At the back of the frame the vertically placed spiral springs are indicated. The dotted rectangle marks the place of a radiation reflector, which encloses the spiral springs.

Fig. 3 gives a photographic representation of the seismograph. The bed-plate is 43.3 cm wide and 45.3 cm long. From it projects a *T*-shaped key which serves for regulating the zero-point by means of the horizontal spiral spring. The 3 small terminals serve for sending an electric current to a little electro-

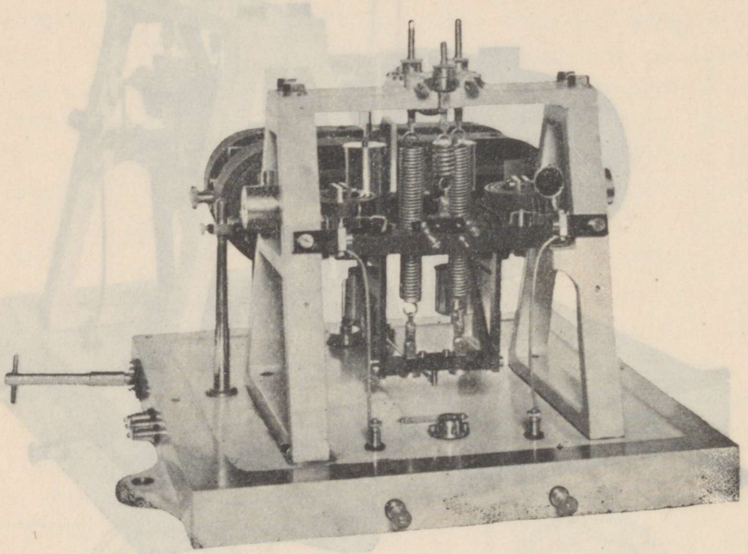

Fig. 4.

magnet, which is fixed to the rod in front and has a double action, enabling excursions of the rod to be produced in an upward and downward direction by commuting the current. This is of importance in the determination of the constants and in the symmetrical adjustment of the proper period. The two terminals on the right are there for connecting the coils with the galvanometer. To the left of the bed-plate there are two small feet, which together with a third foot attachable to the upper part of the stand, serve for ascertaining the reduced pendulum length, the whole being tilted through an angle of 90^0 and the system being allowed to oscillate freely.

The induction and damping magnets are attached to the bed-plate by a special frame, which enables the magnetic poles to be approached to each other, or moved apart. The exact amount of displacement is indicated by scales provided with deci-verniers. Between the poles of the damping magnets a wire index is fixed. The damping-plate bears a little scale, the middle graduation-line of which by coinciding with the wire index, marks the proper zero-position, the bed-plate being horizontally

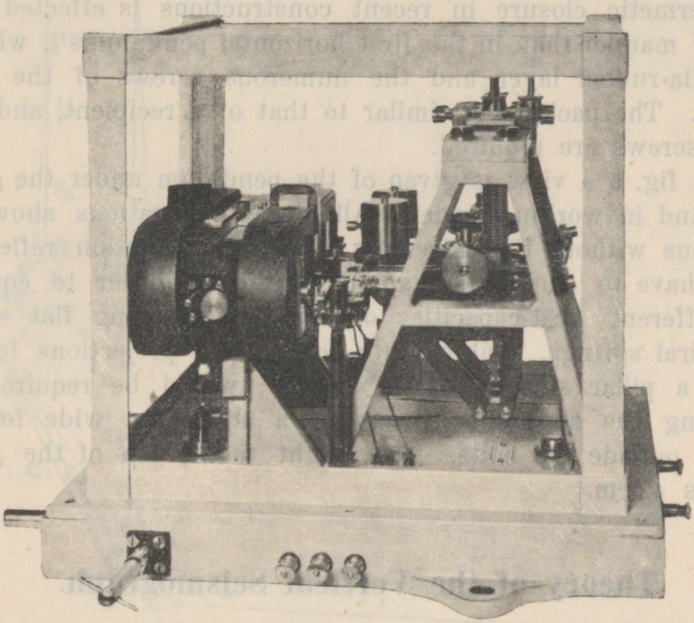

Fig. 5.

adjusted with the help of a spirit-level. The compensating flat spirals are also provided with a graduation enabling the desired amount of compensation to be effected.

On the upper part of the frame three screws are to be seen marking the upper points of fixation of the spiral springs. All of these are provided with graduated nuts. The two in front pass through slides, the displacement of which is also ascertainable from graduations. To the bed-plate a spirit-level is attached, which is intended to assist in the approximate horizontal adjustment of the bed-plate.

Fig. 4 is a back view, which allows some details to be more clearly discerned. The outer rim of the bed-plate shows three projections with perforations (only one of them is visible in figs. 3 and 4). These serve for fixing the bed-plate to the bolts embedded in the masonry pillar.

The upper surface of the rim of the bed-plate is polished, so that a glass-cover resting on it can enclose the apparatus hermetically. The pendulum works in perfect isolation from the outer air and must not show any suction-effect in windy weather. The hermetic closure in recent constructions is effected in a simpler manner than in the first horizontal pendulums[1]), without the india-rubber layer and the numerous screws of the older models. The packing is similar to that of a recipient, and only a few screws are required.

In fig. 5 a view is given of the pendulum under the glass-cover and in working order. All these illustrations show the apparatus without the extremely important radiation reflectors, which have to enclose the spiral springs in order to equalise the different heat-capacities of the compensating flat spirals and spiral springs. Taking into account the projections for the bolts, a pillar surface of $60 \times 60\ \mathrm{cm}^2$ would be required for mounting the apparatus, reserving a strip 5 cm wide for the margin outside the bolts. The height to the top of the glass-cover is 42 cm.

Theory of the Vertical Seismograph.

The theory of the vertical seismograph with double-spring suspension has been discussed by the author in a previous publication[2]). In the following the most important points that are necessary for judging the proper working of the apparatus will be recapitulated with the addition of some supplementary remarks.

In fig. 6 let the axis of rotation of the pendulum be at O, and the points of application of the spiral springs at A, A_1, A_2, and B, B_1, B_2.

As was explained in the communication referred to above, all the spiral springs may be made to act horizontally. In the case here given it is the zero-point spiral which is arranged horizontally.

1) J. Wilip. loc. cit.
2) J. Wilip. loc. cit.

Let the centre of gravity of the mass M of the pendulum be at a distance r from the axis of rotation, which is assumed to be deflected upwards through the angle Θ. In this case 4 moments of force are acting, viz., that of gravity $\mathfrak{M}_1$, that of

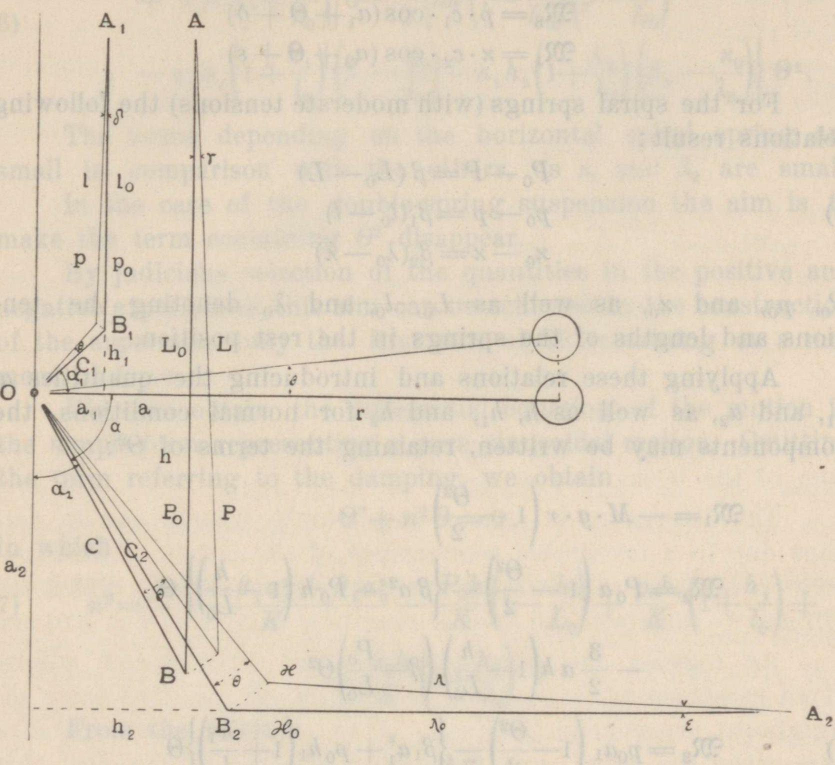

Fig. 6.

the pair of long springs $\mathfrak{M}_2$, that of the short spring $\mathfrak{M}_3$, and that of the small zero correction $\mathfrak{M}_4$.

According to the fundamental law of mechanics we have:

$$K\Theta'' = \mathfrak{M}_1 + \mathfrak{M}_2 + \mathfrak{M}_3 + \mathfrak{M}_4 \tag{1}$$

K being the moment of inertia of the system and Θ'' the angular acceleration of the deflection Θ.

If we denote the arms of the lower points of application of the spring tensions P, p and $\varkappa$ by c, c_1, and c_2, the angles between these and the horizontal, respectively vertical, plane, by α, α_1, and α_2, the angles between the new direction of the spirals and their rest position by γ, δ, and ε, then the 4 moments of force may be written as follows:

$$
2)\qquad
\begin{aligned}
\mathfrak{M}_1 &= -M\cdot g\cdot r\cdot\cos\Theta\\
\mathfrak{M}_2 &= P\cdot c\cdot\cos(a-\Theta+\gamma)\\
\mathfrak{M}_3 &= p\cdot c_1\cdot\cos(a_1+\Theta+\delta)\\
\mathfrak{M}_4 &= \varkappa\cdot c_2\cdot\cos(a_2+\Theta+\varepsilon)
\end{aligned}
$$

For the spiral springs (with moderate tensions) the following relations result:

$$
3)\qquad
\begin{aligned}
P_0-P &= \beta(L_0-L)\\
p_0-p &= \beta_1(l_0-l)\\
\varkappa_0-\varkappa &= \beta_2(\lambda_0-\lambda)
\end{aligned}
$$

P_0, p_0, and $\varkappa_0$, as well as L_0, l_0, and λ_0 denoting the tensions and lengths of the springs in the rest position.

Applying these relations and introducing the quantities a, a_1, and a_2, as well as h, h_1, and h_2 for normal conditions, the components may be written, retaining the terms of Θ^2:

$$
4)\qquad
\begin{aligned}
\mathfrak{M}_1 &= -M\cdot g\cdot r\left(1-\frac{\Theta^2}{2}\right)\\
\mathfrak{M}_2 &= P_0 a\left(1-\frac{\Theta^2}{2}\right)-\left\{\beta a^2-P_0 h\left(1-\frac{h}{L_0}\right)\right\}\Theta\\
&\quad -\frac{3}{2}a h\left(1-\frac{h}{L_0}\right)\left(\beta-\frac{P_0}{L_0}\right)\Theta^2\\
\mathfrak{M}_3 &= p_0 a_1\left(1-\frac{\Theta^2}{2}\right)-\left\{\beta_1 a_1^2+p_0 h_1\left(1+\frac{h_1}{l_0}\right)\right\}\Theta\\
&\quad +\frac{3}{2}a_1 h_1\left(1+\frac{h_1}{l_0}\right)\left(\beta_1-\frac{p_0}{l_0}\right)\Theta^2\\
\mathfrak{M}_4 &= \varkappa_0 a_2\left(1-\frac{\Theta^2}{2}\right)-\left\{\beta_2 a_2^2+\varkappa_0 h_2\left(1+\frac{h_2}{\lambda_0}\right)\right\}\Theta\\
&\quad +\frac{3}{2}a_2 h_2\left(1+\frac{h_2}{\lambda_0}\right)\left(\beta_2-\frac{\varkappa_0}{\lambda_0}\right)\Theta^2
\end{aligned}
$$

In the rest position the moment of gravity balances the tensions of all the springs.

Hence we have

$$
5)\qquad M\cdot g\cdot r = P_0 a + p_0 a_1 + \varkappa_0 a_2
$$

For the sum of all the moments of force we obtain

$$\begin{aligned}\mathfrak{M} = -\Big\{&\beta a^2 + \beta_1 a_1^2 + \beta_2 a_2^2 - P_0 h\left(1-\frac{h}{L_0}\right) + p_0 h_1\left(1+\frac{h_1}{l_0}\right) \\ &+ \varkappa_0 h_2\left(1+\frac{h_2}{\lambda_0}\right)\Big\}\Theta - \frac{3}{2}\Big\{ah\left(1-\frac{h}{L_0}\right)\left(\beta-\frac{P_0}{L_0}\right) \\ &- a_1 h_1\left(1+\frac{h_1}{l_0}\right)\left(\beta_1-\frac{p_0}{l_0}\right) - a_2 h_2\left(1+\frac{h_2}{\lambda_0}\right)\left(\beta_2-\frac{\varkappa_0}{\lambda_0}\right)\Big\}\Theta^2. \end{aligned} \tag{6}$$

The terms depending on the horizontal spiral spring are small in comparison with the others, as $\varkappa_0$ and β_2 are small.

In the case of the double-spring suspension the aim is to make the term containing Θ^2 disappear.

By judicious selection of the quantities in the positive and negative expressions, this aim can be achieved in the construction of the apparatus, only the first part of $\mathfrak{M}$ remaining as a moment of force.

We then obtain the differential equation of the motion in the simple form representing a pure sinusoidal motion. Omitting the term referring to the damping, we obtain

$$\Theta'' + n^2\Theta = 0,$$

in which

$$n^2 = \frac{\beta a^2 + \beta_1 a_1^2 + \beta_2 a_2^2}{K} - \frac{P_0 h}{K}\left(1-\frac{h}{L_0}\right) + \frac{p_0 h_1}{K}\left(1+\frac{h_1}{l_0}\right) + \frac{\varkappa_0 h_2}{K}\left(1+\frac{h_2}{\lambda_0}\right). \tag{7}$$

From the relation

$$T = \frac{2\pi}{n}$$

the proper period of the seismograph can be deduced.

The confirmation of this theory shall now be given by measurements obtained from a vertical seismograph. These will demonstrate the exactness of the theory and reveal the disturbing influence which the horizontal zero-point correction may exercise on the general moment of force.

The most important factor is here the coefficient of Θ^2, which is expressible as follows:

$$ah\left(1-\frac{h}{L_0}\right)\left(\beta-\frac{P_0}{L_0}\right) - a_1 h_1\left(1+\frac{h_1}{l_0}\right)\left(\beta_1-\frac{p_0}{l_0}\right) - a_2 h_2\left(1+\frac{h_2}{\lambda_0}\right)\left(\beta_2-\frac{\varkappa_0}{\lambda_0}\right)$$

This coefficient has to become infinitesimal.

The measurements carried out by the aid of a cathetometer, ordinary compasses and beam-compasses, had yielded the following values:

$a = 4.23$ cm	$a_1 = 1.21$ cm	$a_2 = 11.19$ cm
$h = 7.591$ cm	$h_1 = 3.110$ cm	$h_2 = 4.50$ cm
$L_0 = 17.595$ cm	$l_0 = 7.693$ cm	$\lambda_0 = 15.50$ cm
$\beta = 1963 \cdot g$ Dyne/cm	$\beta_1 = 4673 \cdot g$ Dyne/cm	$\beta_2 = 8.7 \cdot g$ Dyne/cm
$P_0 = 11740 \cdot g$ Dyne	$p_0 = 2042 \cdot g$ Dyne	$\varkappa_0 = 34.35 \cdot g$ Dyne

Herefrom it follows that

$$+ah\left(1-\frac{h}{L_0}\right)\left(\beta-\frac{P_0}{L^0}\right)=+23660 \cdot g$$

$$-a_1 h_1\left(1+\frac{h_1}{l_0}\right)\left(\beta_1-\frac{p_0}{l_0}\right)=-23290 \cdot g$$

$$-a_2 h_2\left(1+\frac{h_2}{\lambda_0}\right)\left(\beta_2-\frac{\varkappa_0}{\lambda_0}\right)=-\quad 421 \cdot g$$

This example demonstrates convincingly that with Θ^2 the sum of the terms may vanish.

The divergence is indeed surprisingly minute and is perhaps due to a favourable concurrence of the errors of measurement committed in obtaining the quantities a and a_1, which are difficult to ascertain, the limit of exactness being above $\pm$ 0.01 cm.

As regards the other quantities, the cathetometer admits of an exactness of $\pm$ 0.002 cm in reading off, so that quite insignificant errors arise herefrom. Most probably in the instance here given the result was favourably affected by the fact that the spiral springs were suspended in an absolutely vertical direction, and no corrections on account of their inclinations had to be considered.

An objection might here be raised to the effect that the zero-correction during adjustment may alter the proper period.

When looking at the value of β_2, it is seen that the tension of the zero-correction spring is very weak. Moreover, the screw for adjustment has a very short pitch. 30 complete turns of the key alter the length of λ_0 only by 0.07 cm, which would correspond to an alteration of tension of about $0.6 \cdot g$ Dyne.

If the apparatus has been properly adjusted in the station it may happen occasionally that only a couple of turns are required for finding the zero-point. For this reason no apprehension need be entertained that the zero correction will disturb the proper period.

The centre of gravity of the oscillating system was ascertained by means of supports, finding the position where equilibrium prevailed. It was discovered at a distance of 8.75 cm from the axis of rotation.

In the rest position we obtain the following equation for equilibrium:

$$Mgr = P_0 a + p_0 a_1 + \varkappa_0 a_2$$

In accordance with the values ascertained we ought to find

$$6020 \cdot 8.75 \cdot g = 11740 \cdot 4.23 \cdot g + 2042 \cdot 1.21 \cdot g + 34.35 \cdot 11.19 \cdot g$$

This condition yielded on the one side $52675 \cdot g$, and on the other, as the sum of the terms, $52515 \cdot g$. The divergence is to be attributed to errors of measurement for r, a, a_1, and a_2, so that this agreement may also be considered as sufficiently close.

With a seismograph of this kind, however, the condition that the term referring to Θ^2 should vanish, is not all that is required. It must also be demanded that at the same time the proper period T should become sufficiently long, say $11^s.0$—$12^s.5$. Hence $n^2 = \left(\frac{2\pi}{T}\right)^2$ must be made to vary only within certain definite quantitative limits. To realize this purpose, either all quantities must be selected with the utmost exactness in the construction of the apparatus, in order to obtain the desired period, which is practically impossible, or some of the values must be rendered variable. We have at our disposal the quantities a, h, a_1, h_1, P_0, and p_0, if the zero-correction is to be excluded here. The variations of the quantities L_0 and l_0 correspond to those of P_0 and p_0.

In the model here described chiefly a, h, P_0 and p_0 are affected by the adjustment, the compensation spiral retaining the values a_1 and h_1. The alteration of a and h is not effected directly by a horizontal and vertical displacement, but use is made of the circumstance that an inclination of the axes of the spiral springs has the same effect as an alteration of the distances a and h, which in practice is much more convenient.

Let us assume the upper point of application of the main spiral springs to be shifted from the perpendicular position nearer to the plane of the axis of rotation, so that the new direction forms an angle ϑ with the perpendicular. Then the moment of force becomes

$$8)\qquad (\mathfrak{M}_2) = P_1 \cdot c \cdot \cos\{(a+\vartheta) - \Theta + \gamma_1\}$$

In case the shifting is effected in the opposite direction through the angle ϑ, we have the equation:

$$9)\qquad (\mathfrak{M}_2)_1 = P_1 \cdot c \cdot \cos\{(a-\vartheta) - \Theta + \gamma_2\}$$

These expressions are completely analogous to those valid in the normal condition, the only difference being that in the former case the angle a is increased by the amount ϑ, and in the latter case reduced by the same quantity. This is identical with saying that in the former case a has been reduced and h increased, the reverse having been done in the latter case.

The inclination of the main spirals is effected by the adjustable slides, and the alteration of the normal tensions P_0 and p_0 by the graduated nuts. Hereby L_0 and P_0 are somewhat increased in the rest position. Now let the displacement of the slides amount to b cm, this being small compared with L_0.

Then for small values of ϑ we may put

$$10)\qquad \vartheta = \frac{b}{L_0}$$

In an inclined position the length of the spirals is

$$11)\qquad L_1 = \frac{L_0}{\cos\vartheta} = L_0\left(1 + \frac{\vartheta^2}{2}\right)$$

Or the difference

$$12)\qquad \Delta L_0 = \frac{b^2}{2\,L_0}$$

Accordingly the difference of the tensions becomes

$$13)\qquad \Delta P_0 = \frac{\beta \cdot b^2}{2\,L_0}$$

In our example we have

$$\beta = 1963 \cdot g \,\text{Dyne}, L_0 = \text{about } 17.6 \text{ cm}$$

and the maximum shifting of the slides $b = 0.5$ cm.

For the maximum differences we should obtain

$$\Delta L_0 = 0.007 \text{ cm}$$

and

$$\Delta P_0 = 14 \cdot g \text{ Dyne}.$$

Let us next consider to what shifting of the spirals, in a position parallel to their own towards the axis of rotation and downwards, our inclination would correspond.

In this combined parallel shifting and lowering we should have to form, in place of a and h, expressions for a_3 and h_3, differing from the preceding ones, *viz.*:

14) $$a_3 = c \cdot \cos(\alpha + \vartheta) = a \cdot \cos\vartheta - h \cdot \sin\vartheta$$

and

15) $$h_3 = c \cdot \sin(\alpha + \vartheta) = h \cdot \cos\vartheta + a \cdot \sin\vartheta,$$

or for smaller values of ϑ

16) $$a_3 = a\left(1 - \frac{\vartheta^2}{2}\right) - h\vartheta$$

and

17) $$h_3 = h\left(1 - \frac{\vartheta^2}{2}\right) + a\vartheta.$$

Substituting according to the equation $\vartheta = \frac{b}{L_0}$ we obtain

18) $$a_3 = a - \frac{b}{L_0}\left(h + \frac{ab}{2L_0}\right) = a - \Delta a$$

and

19) $$h_3 = h + \frac{b}{L_0}\left(a - \frac{hb}{2L_0}\right) = h + \Delta h.$$

Making use of the values in our example and selecting the greatest possible shifting $b = 0.5$ cm, we find:

$$\Delta a = 0.217 \text{ cm}$$

$$\Delta h = 0.117 \text{ cm}.$$

This is to say that in the maximum case with the given model an inclination of the main spirals would correspond to a shifting of the spirals towards the axis of rotation by roughly 2 mm, and to a lowering of their lower points of application by roughly 1 mm.

The product

$$+ah\left(1-\frac{h}{L_0}\right)\left(\beta-\frac{P_0}{L_0}\right)$$

would yield $22520 \cdot g$, *i.e.* a somewhat smaller amount than the two negative terms in the equation of Θ^2 taken together.

The absolute amount of the moment of force would thus decrease in an upwards excursion and increase in a downward excursion, which implies a longer half-period above and a shorter one below.

By shifting the slide in the opposite direction, the product just referred to is given a greater value than the two negative terms taken together. Thereby the asymmetry of the period is made to occur in the opposite direction.

The example sufficiently illustrates the idea of the shifting of the upper point of suspension of the spirals from the perpendicular position for the purpose of altering the values which are of importance for the proper period of the pendulum.

A few remarks may be added here.

In our discussion we have attributed special importance to the perpendicular and horizontal positions of the spiral springs only for the reason that the apparatus are constructed in this manner.

Strictly speaking the spiral springs may be oriented in any desired direction, as this is done practically on a small scale when we give a slanting position to the one point of application.

The tensions required for the maintenance of equilibrium remain unaltered as long as the angle between the spring axes and the arms c, c_1, and c_2 remains the same.

From this point of view it is a matter of indifference which point of application is chosen in the construction of the apparatus for the spirals A, A_1, and A_2 (and also B, B_1, and B_2). This choice may in certain cases be determined by considerations of practical utility.

The Temperature Compensation.

The most important part of every spiral-spring seismograph is a special contrivance for automatically maintaining the true rest position of the apparatus during changes of temperature. If a contrivance of this kind is properly arranged, even a somewhat asymmetrically oscillating seismograph may furnish trustworthy records with the galvanometric method of registration.

A person not versed in thermic questions of this kind will naturally make very stringent demands in regard to such a compensation arrangement and will occasionally point out errors which in his opinion are quite inadmissible.

But in dealing with this question we must consider that in a compensation arrangement two things are of importance. Firstly, the thermic expansions and the alterations of the elastic forces which are due to fluctuations of temperature must be compensated for, these effects manifesting themselves in a lengthening of the spirals and a reduction of the length of the pendulum. Secondly, however, the temperature gradient must at the same time be taken into account.

Thus it is a rather complicated technical task to compensate for the heat capacities and conductivities of the various parts of the apparatus in such a manner that the same result is obtained with all gradients of temperature.

Notwithstanding all difficulties, the problem of devising an efficient compensation arrangement for the ordinary basement temperatures has been successfully solved in the present apparatus.

This arrangement is so efficient that an observer may remain for hours in the pendulum room working at other seismographs, which sometimes causes the temperature to rise more than 1^0, without noticing any striking against the lower stop-screw or any dangerous amount of erratic motion, and all this without any special protection against heat except the glass cover.

This allows the conclusion to be drawn that a vertical seismograph of the kind described here would keep sufficiently quiet already at ordinary room temperature, and indeed, this has occasionally been shown to be true in the workshop.

The importance of this question renders it necessary to devote a little more attention to the compensation arrangement.

Before proceeding to examine a temperature compensation of this kind, one must provisionally mount the apparatus in the proper manner.

It was mentioned in the description of the seismograph that in front of the damping magnet there is fixed a wire index, which must have exactly the same distance from the bed-plate as the axis of rotation. If the bed-plate is oriented horizontally by the aid of spirit-levels, the plane of the centre of gravity, in the case of the suspended system, must pass through the axis of rotation and the wire index. Only in this position does the vertical seismograph not react to horizontal displacements. In order to fulfil or test this condition, the oscillating system is clamped between two stop-screws, after which the magnets are unscrewed and the spiral springs removed. Next the long third foot is attached and the whole tilted through an angle of 90^0. By means of a plumb-line the bed-plate is then placed vertically and the stop-screws loosened, so that the whole system can oscillate freely. One of the long spiral springs is tied to the frame near its usual place so that it can take part in the oscillation.

Now the little mirror attached to the apparatus is temporarily removed and by its help the zero-mark of the little scale on the damping-plate is made to coincide with the wire index. Adjustment is effected by shifting the four principal weights along their axes.

This having been done, the apparatus may be mounted in the station and the further adjustment carried out on the spot.

After the bed-plate has been laid horizontally on the pillar by the aid of the spirit-level and firmly screwed on to the bolts, all the spiral springs are replaced and the radiation reflectors placed in position. Usually a telescope with a vertical scale is set up at a distance of 5 m from the mirror and the pendulum clamped between the two stop-screws at such an altitude that the zero-mark on the damping-plate exactly coincides with the wire index. To this position the zero-line of the telescope scale must be adjusted. Now the two stop-screws are screwed back until the telescope makes an excursion of 100 mm both when adjusted to the pendulum resting on the lower screw and striking against the upper screw. Within this range a precise adjustment is obtainable.

At first, for the sake of convenience, the proper period should be about 10^s. For this purpose the nuts of the upper spiral springs should be used. The rule is: If the compensation spiral is slackened and the main spirals tightened, the period will be lengthened. In the reverse case it will be shortened.

This period having been obtained on the zero-mark, the values above and below it may be tested. In doing so, it must be borne in mind that the pendulum occasionally creeps up, thereby impeding the determination of the period. In this case one must carefully follow with the zero-correction, until a constant position is attained. Only when this has been achieved is it worth while determining the period. While doing this, it is advisable to screw back the induction magnets as far as possible and then to observe the periods by the aid of the coupled galvanometer, selecting chiefly small amplitudes not exceeding $\pm$ 5 mm according to the vertical pendulum scale. (The damping magnets must, of course, be kept at a distance.) If it appears that the periods above and below do not agree with the rest-position period, the two slides above must be shifted.

The rule is here: If the slides are shifted back, the period decreases in the lower half and increases in the upper; if they are shifted forward, the reverse is the result. Afterward the desired long period must again be obtained by alteration of the tensions.

This procedure is repeated alternately until a good symmetry has been obtained. Only after this has been achieved does one proceed to the adjustment and testing of the temperature compensation.

If the weights attached to the bi-metallic flat spirals are wrongly placed, the impression may be produced that these apparatus have no fixed position of rest. The slightest change of temperature will cause them to touch either below or above.

But one hardly believes one's eyes when once the correct position of the weights has been discovered. Then the ordinary gradients of temperature in the basement are of no importance, and the apparatus themselves regulate their zero-point without any other protection against heat than the hermetically closing glass cover.

If in the bi-metallic flat spirals the brass forms the outer side, the spiral curves more strongly with higher temperatures, approaches the regulating weight nearer to the axis of rotation and thereby counteracts the alteration of the rest position produced by the extension of the spiral springs. Whether the compensation has been properly adjusted is easily ascertainable if the temperature in the pendulum room can be slowly altered by $0^{\circ}.5$—$1^{\circ}.0$ C in the course of 24 hours.

It is best to begin the examination in the morning, entering the basement at intervals of an hour, and each time using the key to correct the zero-point, if this should prove necessary. While doing so, attention must be paid that the pendulum be never allowed to reach the zero-mark, but that scope be given for creeping up, as the pendulum, owing to an elastic after-effect, gradually follows up by itself.

If it appears that the pendulum invariably deviates from the rest position to one side, one must, in correcting, go a little beyond the zero-point in the opposite direction and wait for the result.

If the compensation is approximately correct, the rest position will usually be adjusted by nightfall. Should the temperature change during the night, one sees the next morning to which side the regulating weights must be shifted. In case of slight deviations it is advisable to wait another 24 hours before correcting the position of the regulating weights, which may then no longer be necessary.

Having slackened the lower screws of the flat spirals by means of a pin, so that they can be turned a little, the screws of the regulating weights attached to the flat spirals are slackened and the weights shifted to another division of the scale and again screwed firmly on. Thereupon the lower screws of the flat spirals are tightened, taking care to adjust their position relative to their axes in such a manner that the pendulum just touches the upper stop-screw and that there is no need to alter the tensions of the springs.

If this is done, the proper period originally adjusted for is afterwards retained during oscillation.

As by this manipulation the flat spirals have become warmed they have curled up and on cooling begin to uncurl. For this reason the recommendation was given above to allow the pendulum to touch the upper stop-screw from the very beginning, as it will have a tendency to sink. When correcting, one first of all goes once more so far that contact takes place above, and then corrects more carefully. By nightfall one will usually have succeeded in finding the proper rest position.

Supposing that a sufficient constancy of rest position has been found for the proper period of 10^s, and that the pendulum has not deviated more than ± 10 mm during three or four days, the longer period corresponding with the galvanometer is adjusted for, and the exactness and permanence of the adjustment tested. Any further correction of the position of the regulating weights is performed in the same manner as described above.

The first models of the apparatus showed an elastic after-effect, found in all spiral-spring instruments, owing to which the pendulum, in spite of the temperature compensation, always tends downwards and rests against the lower stop-screw. The firm of H. Masing has recently invented a method of tempering by the application of which this elastic after-effect can be rendered extremely small.

In the following, some examples may be adduced to demonstrate with what exactness the zero-point is maintained during considerable alterations of temperature.

In one experiment the compensating weights of the flat spirals were fixed at division 15 of the scale. The result was that at the temperature gradient occurring in the basement the pendulum sank about 10—15 mm according to the telescope scale during a fall of temperature of about $0^\circ.2$—$0^\circ.5$ C. This meant that the compensation was too strong. The weights were shifted to division 18, which was followed by a very efficient compensation, as may be seen from the following observations.

Table I.

Date	Hour	Temperature	Zero-point
Nov. 21, 1929	$19^1/_2{}^h$	$12^0.00$ C	— 8.0 mm
„ 22, „	$8^1/_2$	11.73	— 5.1
	19	11.61	— 7.7
„ 23, „	$8^1/_2$	11.40	— 4.2
	18	11.35	— 3.0
„ 24, „	$9^1/_2$	11.13	+ 1.2
	19	11.07	+ 1.8
„ 25, „	9	10.93	+ 3.8
	19	10.85	+ 4.7
„ 26, „	$8^1/_2$	10.71	+ 6.8
	12	10.70	+ 7.2
	16	10.90	+ 2.7
	18	10.80	+ 1.1
„ 27, „	$8^1/_2$	10.90	—11.8
	13	10.45	+ 4.8
	$16^1/_2$	10.65	— 3.3
	19	10.61	— 4.8
„ 28, „	$8^1/_2$	10.50	— 6.3
	19	10.40	— 6.5
„ 29, „	$8^1/_2$	10.33	— 8.3
	19	10.00	+ 6.8
„ 30, „	$8^1/_2$	10.20	—10.0

During this whole period the key serving for correction was never touched.

With such sudden changes of temperature as were produced artificially on the 26th, 27th and 29th, a certain amount of irregularity arises, which cannot be avoided at such gradients with the given of degree compensation, unless the influence were retarded by further protection against heat.

The fact that at very small gradients the pendulum does not return again to the division of the scale corresponding to the former temperature, seems to indicate that a slight elastic after-effect is interfering. This will only diminish in course of time, as can be observed in other spiral-spring apparatus.

The alterations of the zero-point in the table show that at rather unfavourable basement temperatures, like those given above, the deviations are very insignificant.

Thus the conclusion may well be drawn that the principles of construction applied here will also yield favourable results with vertical seismographs for mechanical and optical registration.

To one circumstance attention may be directed. The regulating weights of 450 g each produced a proper compensation when placed at division 18, *i. e.* at a considerable distance from the extreme end of the flat-spiral. A smaller weight at the extreme end would therefore suffice for effecting compensation.

Thus it was shown by experiments that for this purpose two single weights of 417.5 g each, one on each side, at division 2, were sufficient. In order now to facilitate a proper selection of the weights, it is advisable to place the flat spiral-spring system as much as possible into the plane of the centre of gravity. If this is done there is no need, when selecting a new pair of weights, again to adjust the centre of gravity by the troublesome tilting of the apparatus and disconnecting of the spiral springs.

In future this constructional improvement will be taken into consideration.

At the same time these apparatus will be improved in another respect.

In future models the bi-metallic flat spiral springs of invar and brass are to be replaced by a bi-metal consisting of invar and alloys of nickel-steel, which is already obtainable commercially and which in its elastic properties is in no way inferior to steel.

Another example may here be-given for a somewhat weaker compensation effected by the above-mentioned smaller weight of 417.5 g placed at division 5.

Here again the key was never touched during the whole series of observations. The proper period amounted to $11^s.5$.

Table II.

Date	Hour	Temperature	Zero-point
Dec. 11, 1929	15^h	$11^0.88$ C	+ 0.2 mm
	20	11.80	— 2.3
„ 12, „	$8^1/_2$	11.58	— 5.0
	17	11.49	—10.8
„ 13, „	$8^1/_2$	11.02	+ 1.3
	$19^1/_2$	10.98	+10.5
„ 14, „	$8^1/_2$	11.05	+ 9.8

At 12^h two horizontal pendulums were brought in through a window of the basement which was very close to the pillar with the vertical seismograph, which caused the temperature near the vertical seismograph for a time to fall to $8^0.0$ C. This is the explanation of the great change in the position of the zero-point next recorded.

Date	Hour	Temperature	Zero-point
Dec. 14, 1929	$12^1/_2{}^h$	$10^0.95$ C	+24.0 mm
	19	10.94	+ 3.8
„ 15, „	10	10.89	— 5.5
	18	10.61	— 1.6
„ 16, „	$8^1/_2$	10.36	+ 2.3
	16	10.60	4.5
	$18^1/_2$	10.30	+ 1.4
„ 17, „	9	10.12	+10.0

3 hours' work had been done in the pendulum room with 4—5 electric lamps burning.

Date	Hour	Temperature	Zero-point
Dec. 17, 1929	13^h	$10^0.80$ C	—1.0 mm
	15	10.49	—3.7
Another two hours' work had been done.			
„ 17, „	17	10.81	—4.2
	$19^1/_2$	10.45	—2.8
„ 18, „	$8^1/_2$	10.26	+9.3
	$19^3/_4$	10.15	+15.0
„ 19, „	$9^1/_2$	10.51	+5.7
	13	10.60	+1.0
	17	10.32	+5.8
„ 20, „	$8^3/_4$	10.40	—0.2
	12	10.70	—3.0
	$15^1/_2$	10.28	—1.5
„ 21, „	9	10.07	+3.0
	18	9.98	—1.0

This table clearly shows the importance of the compensation contrivance.

Not enough importance can be attached to the circumstance that one can quite safely control the constants of all the apparatus in the station, without running the risk of the vertical seismograph coming in contact with the lower stop-screw. After the termination of the work the zero-point quickly readjusts itself.

As in the example given the full compensation had not yet been attained, the compensation weights were augmented by 5 g each, after which adjustment was again made to the desired period of $12^s.0$ and the zero-mark. The following series of observations demonstrates that the compensation has improved in comparison with the preceding example.

Table III.

Date	Hour	Temperature	Zero-point
Dec. 28, 1929	$9^1/_4{}^h$	$10^0.09$ C	— 8.0 mm
	20	9.81	— 3.0
„ 29, „	$9^3/_4$	9.57	+ 0.8
	18	9.40	+ 1.3
„ 30, „	$9^1/_2$	9.20	— 6.8
	18	9.14	— 6.1
„ 31, „	$9^1/_2$	9.00	— 7.7
	19	8.91	—10.5
Jan. 1, 1930	10	8.45	— 1.3
	18	8.37	— 0.8
„ 2, „	$9^3/_4$	8.19	— 1.0
	18	8.15	— 1.5
„ 3, „	$9^1/_2$	7.91	+ 3.7
	18	7.90	+ 2.0
„ 4, „	$9^1/_2$	7.82	— 6.3
	18	7.82	— 8.0

In some places irregular jumps of the zero-point occur, which appear inadmissible according to the gradient of temperature.

As in all these experiments the glass cover was put on without effecting hermetical closure, the causes of these jumps must be sought in variations of the atmospheric pressure. Particularly striking is the fall of the zero-point between Dec. 29, 18^h, and Dec. 30, $9^1/_2{}^h$. On the first of these dates the barometer showed 754.1 mm, and on the second 743.6 mm.

Let us calculate for our oscillating system the approximate buoyancy of the air. The total mass amounted to 6000 g. As the principal weights consist of lead enclosed in brass, the remaining parts being chiefly made of brass, copper and invar, we may assume a mean density of 10 and obtain 600 cm^3 as the volume of air displaced.

Assuming that the barometer at 10^0 C fell from 760 to 740 mm, 1 cm^3 of air in these cases will have a weight of 0.0012470 and 0.0012142 g. Hence in the first case the buoyancy amounts to 0.7482, and in the second to 0.7285 g.

With our system, therefore, the buoyancy varies by 1 mg for every millimeter change of atmospheric pressure.

As a matter of fact an excess weight of 20 mg placed at the distance of the centre of gravity from the axis of rotation caused the pendulum to be deflected 11.7 mm, the distance of the scale being 5 m from the mirror. This example shows clearly to what a remarkable extent the atmospheric pressure influences the rest position of this type of spiral-spring apparatus with large gradients of pressure. If the temperature compensation is imperfect and the glass cover does not close hermetically, and if these two effects act in the same direction, serious disturbances of the zero-point may result.

In the construction of the apparatus this influence might also be avoided by introducing into the oscillating system aneroid capsules which automatically displace additional weights of the required magnitude in response to alterations of the atmospheric pressure.

This circumstance shows clearly how important it is for the vertical seismograph to be withdrawn from the variations of atmospheric pressure by means of a hermetically closing cover.

With this construction a similar vertical seismograph may even be used for the purely optical registration method employing a maximum magnification of 1500—2000, without any apprehension that disturbances might be caused by the lines overlapping or wandering beyond the scale.

As the tables given above show, a good approximate compensation is obtainable with a weight of 450.0 g at division 18, and 423.5 g at division 5, on each flat spring. From these values we can derive the magnitude of the weight to be employed for every other division of the scale in order to effect a similar degree of compensation as in the cases observed. A survey is given by the following table.

Table IV.

Division of the graduation scale	Mass of the single weight
0	412 g
5	423
10	433
15	444
20	454
25	465

The initial experiments had indeed shown that the original weight of 404.5 g placed at division 0, i. e. at the very end of the flat spiral, did not allow a proper compensation to be effected.

The finished apparatus must be provided with such large compensation weights that the regulation need not be effected by alteration of the masses, but may be performed by shifting the weights on to another division of the scale.

It is probable that in general a still stronger compensation will have to be selected, as the compensations occurring in the examples given would be adequate only for falling temperatures and are also intended for larger gradients.

It is advisable to adapt the compensation to the gradient occurring in the basement, so that the elastic after-effect which sometimes appears in the apparatus would also be compensated. This after-effect, as was mentioned above, has lately been very much reduced and appears only in rare cases with long proper periods.

Mention may still be made of the possibility that with greater intervals of temperature an influence on the proper period may result, because so many alterations take place in the pendulum through thermic influences. This question can be solved only by means of experiments.

During the adjustment of the long proper period of the pendulum it may sometimes happen that after some days the period changes automatically. This seems to be due to the order in which the upper adjusting screws have been used for the variation of the tensions.

Hence times must be chosen for the experiment mentioned when the pendulum has already been hanging quietly for a number of days.

In the present case, after completing the tests recorded in table III, it was found that for $t = 7^0.80$ C on an average

$$T = 11^s.64.$$

In a special experiment the temperature was raised to $t = 14^0.20$ C by heating the room for two days, during which period the pendulum was kept at the zero-point by means of a support so as to prevent the necessity of larger corrections by means of the zero-point spiral-spring later on. It was then found that on an average

$$T = 12^s.54.$$

Thus for a difference of temperature of $6^0.4$C we would in fact have an alteration of the proper period amounting to about $0^s.90$, which makes $0^s.14$ for one degree.

This dependence of the proper period on the temperature is an inherent property of all spring-seismographs of similar construction and must be taken into consideration in every determination of constants in the station.

The influence shows itself in an alteration of the interval of time, t_0, and of the constant of damping μ. 1)

At the normal basement temperature the divergence in the course of a year is not very perceptible.

Nevertheless it may sometimes happen that a control of the constants would be too often needed, which would unnecessarily interrupt the work of the station.

But this difficulty may be very well avoided by once for all ascertaining, for the seismograph in question, the dependence of the proper period on the temperature, and of the alteration of the damping constant μ on the proper period, and then from the basement temperature deducing the proper period and μ, choosing as a starting-point the 3 values of the last determination of constants.

An example of the dependence of the damping constant on the proper period may be given here.

For the required t_0 $(T_1 = 11^s.60)$ the magnetic poles were so adjusted that μ was equal to zero. Hereafter the tensions of the spiral springs were altered and t_0 and μ determined each time. The values ascertained are listed in the following table, which was obtained from 5 series of determinations, namely for $T = 9^s.64$, $10^s.98$, $11^s.60$, $11^s.87$ and $13^s.62$, each group containing more than 10 observations.

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1) As proposed by O. Somville the symbol μ^2, originally introduced by B. Galitzin for the damping constant, has here been replaced by the simpler μ.

Table V.

T	μ	T	μ
$9^s.40$	0.530	$11^s.40$	0.048
9.60	0.481	11.60	0.000
9.80	0.434	11.80	−0.047
10.00	0.385	12.00	−0.098
10.20	0.337	12.20	−0.144
10.40	0.288	12.40	−0.194
10.60	0.242	12.60	−0.240
10.80	0.193	12.80	−0.290
11.00	0.145	13.00	−0.339
11.20	0.096		

Thus we see that these fluctuations of the proper period under the influence of the changes of temperature may be taken into account without a frequent determination of the constants.

A test control undertaken from time to time will show in the beginning how far such a determination of the constants can be depended upon.

In addition to this the verification of the constants is to be recommended for the reason that changes of temperature alter the resistance of the circuit $R_\alpha + r$, and that this influence upon the damping constant μ and the galvanometric transmission factor k must be taken into consideration. This influence, which is comparatively slight, may also be easily ascertained by special experiments.

Practically insignificant is the influence of the altered reduced pendulum-length caused by the fluctuations of temperature.

At two different temperatures l was determined and the following results obtained:

$$t = 6^0.90\ \mathrm{C} \qquad l = 14.87\ \mathrm{cm}$$

and

$$t = 13^0.10\ \mathrm{C} \qquad l = 14.82\ \mathrm{cm}.$$

Thus the alteration amounts only to 0.05 cm for a difference of temperature of $6^0.2$ C, a value which involves no perceptible errors in the calculation of seismic dislocations.

As all the above-mentioned experiments did not show complete compensation, the two weights at division 5 were augmented by 5 g each.

With this small additional weight the apparatus proved to be already over-compensated, as the pendulum slowly sank with falling temperature and rose with rising temperature.

But at the same time it had become quite insensitive to any sudden changes of temperature.

Whereas before on opening the window a considerable deflection resulted, the zero-point now remained unaltered when 2 horizontal seismographs were taken out by a window in the neighbourhood of the apparatus.

The following table demonstrates how the apparatus reacted when a rise of temperature was artificially produced.

Table VI.

Date	Hour	Temperature	Zero-point
Jan. 11, 1930	18^h	$10^0.80$ C	+ 5.1 mm
„ 12, „	10	10.54	− 0.5
The stove was gently heated for half an hour.			
	12	10.88	+ 4.8
	18	11.30	+ 3.7
„ 13, „	$8^1/_2$	11.08	− 5.7
Heated for half an hour.			
	12	11.75	+ 5.8
	19	12.02	+ 3.0
„ 14, „	$8^1/_2$	11.60	− 6.3
Heated for half an hour.			
	12	12.30	+ 6.0
	18	12.52	+ 3.7
„ 15, „	$8^1/_2$	12.05	− 9.0
Heated strongly for an hour.			
	12	13.43	+22.3
	18	14.00	+15.0
„ 16, „	$8^1/_2$	13.30	− 1.5

The proper period finally proved to be on an average

$$T = 11^s.83.$$

The result of this table is really amazing and satisfactory at the same time. It is seen that by a slightly smaller weight one could effect a still more exact compensation. To reach the true limit the values in table IV would have to be augmented by about 3—4 g. Nevertheless no objection can be raised to the latter compensation, as it will resist quite unusual fluctuations of temperature.

It is interesting to contrast the working of this method of construction with the former[1]), in which the spiral springs were on the outside and the temperature compensation in the middle. There the spiral springs are affected by the influence of heat before the compensation, while here the compensation, which is on the outside, is affected a very little earlier. The deviations from the true rest position are very small and are practically without significance.

All the experiments described above were undertaken with detached magnets, so that the apparatus possessed but a weak proper damping.

In all similar experiments the pendulum should, if possible, not be allowed to reach the extreme position where it strikes against the stop-screws, because time would thus be wasted, as the tendency to creep up will be stronger.

Later on, when the apparatus is already registering, the telescope with the scale may be omitted in the control of the zero-point, the wire-index alone being used and the position being examined by means of a magnifying glass. If the zero-point has travelled about the fourth part of a division, correction is effected by turning the key about 2 whole revolutions back. Then there is usually enough scope given for the creeping-up tendency.

The more precise adjustment of the period-symmetry is effected together with the determination of the galvanometric transmission factor and the other constants of the apparatus.

The conclusions which can be drawn from the observations given above are of far-reaching importance.

Formerly it was customary to select for a seismic station a basement with as small an annual amplitude of temperature as possible. This condition has now been rendered altogether superfluous. Any room that is protected from the concussions due to traffic can be used as a station for seismic observations.

As regards temperature, a gentle regular heating is just what is required in order that the fluctuations of temperature

1) J. Wilip. Ueber Temperaturkompensation bei Vertikalseismographen. Sitzungsberichte der Naturforschergesellschaft bei der Universität Tartu, XXXV (3, 4). p. 152. 1929.

may not perceptibly influence the proper period of the vertical seismograph.

Lately F. J. Scrase[1]) has investigated the thermic and elastic properties of spirals springs made of elinvar (chrome--nickel-steel) with regard to utilizing them for the suspension of the oscillating system. If this material through tempering could be made to acquire those remarkable properties which in the case here given the firm of H. Masing has produced in steel, the compensation weights on the flat spiral springs might be made considerably smaller.

Final Adjustment of the Apparatus and Determination of the Constants.

When the temperature compensation is at last working well the determination of the constants may be taken in hand.

In order to ascertain the reduced pendulum-length, the system has again to be removed from the pillar, the spirals being detached, the whole tilted through an angle of 90^0 and placed exactly in a perpendicular position, etc., as was done in the beginning.

Next the system is allowed to oscillate and 100 periods at a time are counted by means of a stop-watch, which marks tenths, or better still, hundredths of a second.

From the pendulum formula the reduced pendulum-length l is then calculated. This amounted to 14.00—15.00 cm with the latest models.

This length varies but little at the expense of the temperature compensation, as has been ascertained by an experiment, so that it involves no errors in the calculation of the registrations.

Hereupon the bed-plate is again mounted in its proper position. Exactly in the manner described above, the spiral springs are replaced, the radiation reflectors screwed on, the stop-screws and the scale together with the telescope properly placed in position, the system made to oscillate, and the period of the galvanometer belonging to the apparatus again brought to the proper value.

1) Journal of Scientific Instruments. Vol. VI. Nr. 12. p. 385. 1929.

The damping magnets are screwed on, the poles adjusted for ca 8—10 mm, and the poles of the induction magnets are approached so closely to the surface of the coils that a visiting card may pass through without much friction.* Now the pendulum may be adjusted to zero by the aid of the telescope and the search for the proper limit of aperiodicity may be begun.

Using a triple lead, the wire-ends of which are differently marked, the 3 terminals are connected with the 3 corresponding screws on the contact-key belonging to the apparatus. To the two terminals which are left free, a battery of 4—6 Volts is connected, and then a beginning can be made with the deflection of the pendulum, the commutator being used as desired. When doing so, the outer limit of resistance must already have been properly supplemented. It is advisable to adjust the distances of the deflecting pins from the pendulum in such a manner, that in the galvanometer the first deflection should not be greater than 150—160 mm with the usual distance of 1 m of the scale from the mirror. The large deflections are not used at all with this method of registration.

If the proportion of the deflections of the galvanometer does not diverge too much from 2.294 with an upward or downward deflection of the pendulum, the pendulum is left at rest for some hours. Then a series of deflections downwards and another series upwards are carried out separately, all values being determined, *viz.* the two galvanometer deflections m_1 and m_2, the deflection of the pendulum m, and the time interval t_0 from the beginning of the movement of the galvanometer coil to its passage through the zero-point. The expected t_0 is calculated in advance from the galvanometer period.

An example may be given here, where the determination was carried out with a vertical seismograph, the slide being at division 4.5 mm. In the present case the previously calculated t_0 amounted to $5^s.53$.

I) The deflection being from below upwards:

$a = 2.243 \quad m_1/m = 17.81 \quad m_2/m = 7.94 \quad t_0 = 5^s.44$

II) The deflection being from above downwards:

$a = 2.275 \quad m_1/m = 19.59 \quad m_2/m = 8.61 \quad t_0 = 5^s.45$

The divergence of the two a, as well as that of m_1/m show at once that period-symmetry has not yet been attained. The value of t_0 varies less.

Therefore the slides had to be shifted further back, the division selected being 4.25 mm. In this position the magnets were left in their places and the proper period was regulated according to t_0, which is quickly done, by selecting the proper tensions of the spiral springs and every time gradually determining t_0.

After a few hours the determination was carried out. For I and II the results were respectively:

I) $a = 2.336 \pm 0.018$; $m_1/m = 19.18 \pm 0.71$;
$m_2/m = 8.21 \pm 0.33$; $t_0 = 5^s.57 \pm 0.06$

II) $a = 2.356 \pm 0.024$; $m_1/m = 19.41 \pm 0.55$;
$m_2/m = 8.24 \pm 0.14$; $t_0 = 5^s.58 \pm 0.05$

It is seen that the divergences are already quite insignificant.

In order to decide whether the asymmetry is greater above or below the zero-position, similar determinations were carried out in wrong rest positions, once at +50 mm deflection from the true rest position and again at —50 mm. For each position the observations were taken a considerable time after the adjustment to the division.

The deflections of 50 mm always refer to a distance of 5 m from the scale to the mirror.

The following results were obtained:

at +50 mm

I) $a = 2.389 \pm 0.020$; $m_1/m = 19.35 \pm 0.31$;
$m_2/m = 8.10 \pm 0.17$; $t_0 = 5^s.56 \pm 0.05$

II) $a = 2.385 \pm 0.008$; $m_1/m = 19.46 \pm 0.59$;
$m_2/m = 8.16 \pm 0.27$; $t_0 = 5^s.61 \pm 0.07$

at —50 mm

I) $a = 2.461 \pm 0.035$; $m_1/m = 19.86 \pm 0.39$;
$m_2/m = 8.07 \pm 0.24$; $t_0 = 5^s.66 \pm 0.05$

II) $a = 2.556 \pm 0.014$; $m_1/m = 18.76 \pm 0.22$;
$m_2/m = 7.34 \pm 0.13$; $t_0 = 5^s.74 \pm 0.04$

From these series of observations it is seen that even at slide-division 4.25 mm complete symmetry had apparently not been attained, as the period below the zero-point proved to be longer. Hence it may be expected that slide-division 4.1 mm would produce a still better agreement.

An arithmetic mean, however, for the true zero-position would already be satisfactory, as attention must be paid in any case that the pendulum should always work close to the zero-position.

In summer, when the microseismic movements are very slight, the mean error in determinations of this kind is considerably smaller than in the examples given above.

Great caution must, however, be applied in the proper valuation of the above result.

The observations have yielded the curious result that the damping has increased not only below, but also above, though in a smaller degree. This fact might be thought to cast an unfavourable light on the exact working of these apparatus.

It is clear that the cause of this phenomenon must be sought partly in the thermic influence on the proper period.

We shall here give a few readings of thermometers, one of which, t_a, was outside the glass cover, and a second, t_i, inside the same, having its bulb in contact wiht the heat reflector, while the temperature in the room was being altered by heating and cooling.

The stove was heated when the thermometer outside showed 10°.00 C, and after closing it readings were taken for a certain length of time. In the following table these temperatures are given, together with their differences.

Table VII.

Time	t_a	t_i	$t_a - t_i$
Jan. 26, 1930 $9^1/_2{}^h$	10°.00 C	—	—
11	10.61	10°.15 C	0°.46 C
12	11.08	10.27	0.81
13	11.42	10.53	0.89
15	11.83	10.96	0.87
16	11.97	11.12	0.85
17	12.07	11.27	0.80
18	12.13	11.40	0.73
20	12.22	11.56	0.66
„ 27, „ 9	12.02	11.60	0.42
19	11.77	11.43	0.34
„ 28, „ 9	11.29	11.05	0.24
19	11.01	10.76	0.25
„ 29, „ 9	9.68	9.77	—0.09
20	9.53	9.56	—0.03
„ 30, „ 9	8.73	8.84	—0.11
18	8.48	8.60	—0.12
„ 31, „ 9	7.67	7.80	—0.13

From the temperatures observed on Jan. 26th, it is seen how slowly the temperature rises inside, compared with greater gradients outside. This shows that the glass cover alone forms a good protection against heat, whereby the influence of sudden changes of temperature with a good compensation is reduced to a minimum.

Somewhat surprising is the condition on Jan. 27th, at 19^h, when no fall of temperature would be expected inside, the thermometer, however, showing less than in the morning.

This may be explained by the great heat capacity of the bed-plate and the pillar beneath it, the temperature of which could rise only a little during the preceding warm period.

Hence a radiation of heat from the inside downwards has played a part.

The last observations record a difference in the opposite direction. Even with very slow cooling down, the thermometer inside showed a somewhat higher temperature.

The deflection experiments described above were first of all carried out for the middle division of the scale; then, after some hours, followed those for the division + 50, and last of all, again after a considerable space of time, those for — 50.

After every series of observations the air temperature in the pendulum room rose temporarily $0^0.5$—$1^0.0$ C, decreasing again after the interruption of the observations. Such a heat-effect is, however, very slowly transmitted to all parts of the seismograph and causes a longer proper period, which again brings about a stronger damping.

For this reason one is justified in considering the adjustment for period-symmetry as much better than may be inferred directly from the figures.

It is true that a certain amount of asymmetry results from the values for a and m_1/m in the first series of observations, but in the last case this is made to appear excessively large by the influence of heat.

On Free Proper Vibrations of the Seismograph and Secondary Phenomena.

All registering apparatus in which elastic forces are utilized show an elastic after-effect which manifests itself with special distinctness in sudden changes. With the present vertical seismographs it is chiefly the zero-correction in which a troublesome after-effect appears when the key is turned a great number of times in the same direction.

For this reason it is advisable never to adjust the pendulum directly to the zero-point, but to allow a number of divisions for the creeping-up effect.

For the same reason observations should only be taken some hours after an important correction, otherwise the results may easily prove deceptive, as the pendulum tends more to one side than to the other.

The alteration of the zero-point appearing after every use of the correcting key is of different magnitude with different models.

It seems that weaker springs produce a much smaller after-effect than stronger ones. In the present instance the horizontal spiral spring might be 3 times weaker. Then the only difference would be that in regulating the key would have to be given three times the number of revolutions.

However, this existence of an after-effect does not imply any considerable interference with the correct working of the apparatus, as may be ascertained without difficulty by unscrewing all the magnets and allowing the system to oscillate with its own damping.

At this point observations of oscillations may be given having the same period-symmetry, the slides being at division 4.25 mm, at which the determinations given above were carried out.

First of all the logarithmic decrement was ascertained by means of the telescope and scale, the distance from the mirror being again 5 m as in the former observations.

In the following table the results of the observations made for the calculation of the logarithmic decrement are listed, $m_s + m_{s+1}$ representing the sum of deflections to right and left.

Table VIII.

$m_s + m_{s+1}$	$m_s + m_{s+1}$	$m_s + m_{s+1}$	$m_s + m_{s+1}$
109.0 mm	36.7 mm	12.6 mm	4.5 mm
92.4	31.7	10.8	3.9
79.5	27.5	9.4	3.3
67.7	23.5	8.3	2.9
58.3	20.2	7.2	2.5
50.1	17.3	6.1	
42.7	14.7	5.2	

The observations may be divided into 2 groups, 1) the larger sums of amplitudes, which never occur in the definition of k, and 2) small sums of amplitudes from the range within which the deflections are selected for the determination of the constants.

The mean logarithmic decrements

$$\Lambda = \log(m_s + m_{s+1}) - \log(m_{s+1} + m_{s+2})$$

resulted for the interval 109.0—23.5 mm

$$\Lambda = 0.0666$$

and that of the small amplitudes between 20.2—2.5 mm

$$\Lambda = 0.0648.$$

Firstly, the proper damping is very small and amounts to very little more than in the turning-coil galvanometer employed with open outer circuit, where Λ amounted to 0.05. Secondly, this damping is remarkably regular, because the logarithmic decrement is very little smaller for small deflections than for large.

Besides, this slight alteration need not be real and may perhaps be explained by the fact that the traffic of the town causing small tremors becomes more evident in small amplitudes by reducing the friction.

This feature is a great advantage of these apparatus and shows that the proper damping, in spite of the 5 pairs of plate-spring joints, is in no way inferior, as regards regularity, to an electromagnetic damping.

This property suggests already that the dependence of the proper period on the amplitude will likewise not be of much significance.

From several series of observations the dependence of the proper period on the amplitude has been ascertained by graphical methods and the average result listed in the following table.

Table IX.

$m_s + m_{s+1}$	T	$m_s + m_{s+1}$	T
100.0 mm	$12^s.18$	30.0 mm	$12^s.01$
90.0	12.16	25.0	12.00
80.0	12.13	20.0	11.98
70.0	12.11	15.0	11.97
60.0	12.08	10.0	11.96
50.0	12.06	5.0	11.94
40.0	12.03		

As is seen from these values, the proper period does indeed diminish with the amplitude, but the divergences are very small in the range of practical application (5.0—10.0 mm), being practically without significance. Besides, in the case mentioned complete symmetry had not yet been attained. As some experiments have shown, the proper period in an unsymmetrical position depends to a considerable extent on the amplitude.

These considerations lead one to believe that a multiple-spring seismograph, in spite of its complicated nature, when properly treated will be in no way inferior to a precision instrument.

Here a phenomenon must not be left unmentioned which may sometimes arise at the expense of the compensating flat spiral springs, especially in cases of resonance.

The bi-metallic flat spiral springs represent elastic spiral objects carrying weights, and therefore they may execute vibrations of a very short proper period. In fact the traffic of the town in certain cases produces an alteration of the zero-point at the pendulum itself which is caused by these vibrations. These tremors of short period are strongest when the centre of gravity does not fall into the plane of the rest position and the period-symmetry is imperfect.

It is a striking fact that an upward deflection of the pendulum is produced, in such a manner as if by these tremors the flat spiral springs were curling up.

To find a direct explanation for this phenomenon is rendered more difficult by the fact that the oscillating motion is complicated. While the flat spiral springs are oscillating in their short proper period, the weight in vibrating is shifted to and fro along the tangent to the spiral. Thereby the distance of the weight from the centre is shortened during the curling up and lengthened during the uncurling.

In the first phase the centrifugal forces are smaller on account of the shorter distance from the centre, in the second they are greater. It is difficult to conceive that these variable centrifugal forces should produce resultants with the elastic forces in such a way that the moments of force are smaller in the more curled condition than in the less curled. The compensation-weights, however, are not fixed directly to the curved surfaces, but to a projecting piece. Therefore these weights, when set trembling, may bend the end of the flat spiral more towards the centre than in the phase of curling towards the outside. The result is that the period of oscillation becomes asymmetric, whereby at every complete period the weight is shifted farther towards the axis of rotation than it is shifted back at the half-period in the opposite direction. The effect is the summation of a great number of half-periods and thus an upward deflection is produced.

This phenomenon might be more convincingly demonstrated by mathematical analysis, which, however, would lead too far.

In addition to this way of oscillating, the weights may also oscillate vertically to the plane of the flat spiral, and this is conceivable especially with vertical shocks, such as are caused by the vehicular traffic in the streets. In vibrating thus the weight will not describe a straight path, but every time when leaving the plane of the flat spiral it will curl up the spiral a little. Hence it follows that also in this case the impulses of short period give rise to an excess weight which must deflect the pendulum upwards.

There is still a third cause which has here to be taken into account, and which follows from the principle of energy.

Every time that the flat spiral springs begin to oscillate, the short periodical oscillations are very strongly damped. This fact causes the energy of oscillation to be consumed by internal mutual friction of the molecules. This energy must be transformed into heat and raise the temperature of the flat spiral springs. As a result this heat effect must cause a slight curling up of the flat spiral springs. How great this effect may be and whether it is at all ascertainable, can hardly be demonstrated by a calorimetric experiment.

Quite apart from these causes each of the above-named kinds of motion taken separately may, even on the assumption of symmetrical oscillations, at the same time produce a deviation due to the resistance of the air, as the flat compensation spirals with their weights may move like the wings of a bird. For the present it is difficult to make definite statements about this assumption.

When the compensating flat spiral springs begin to tremble, they suddenly produce an upward deflection of the pendulum, which is recorded by the very sensitive galvanometer.

For the determination of weak local earthquakes this circumstance would sometimes be very desirable, if the intention is not to measure absolute values, but only to record the moments of occurrence. This would be equivalent to saying that galvanometric registration by means of such a vertical seismograph is much more sensitive than all others, and perfectly registers violent shocks.

It may also be mentioned here that by some critics the galvanometric method is stated to fail in cases of sudden shocks, as is indeed shown by the theory.

According to my experience this reproach is generally not justified in practice. Violent shocks that strike the earth's crust from below every time produce a vibration with a period of about $1^s.4$, and for this the sensitiveness is still quite sufficient. This I have found to be true especially in the earthquakes from the direction of Bokhara.[1] I believe that this is not only true for Pulkovo, but may be regarded as a general rule.

But here the trembling of the flat spiral springs with violent shocks would add a disturbing element for distant earthquakes, which though helpful in registering very precisely the moments of arrival, would injuriously influence the amplitudes.

As my experiments have shown, these secondary movements can be made to disappear by placing a layer of cotton or cloth between the windings of the flat spiral springs at their ends. This prevents the appearance of oscillation and the compensation of temperature does not suffer in the least thereby.

This influence may be also very easily suppressed by constructional devices. All that is needful is to construct one of the flat spiral springs in such a manner that in the bi-metallic band the invar metal is placed towards the outside. Then the spring has only to be fastened at the opposite side and it will regulate the temperature compensation as well as the other. Then during the oscillations there will be an equally great excess-weight working in the opposite direction as in the other flat spiral spring, whereby every disturbance of the zero-point will be prevented. If any deviation should still remain, this may be accounted for by the conversion of vibratory energy into heat.

After this it may well be affirmed that these last secondary effects can be eliminated in a very simple manner, should this be desired.

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1) J. Wilip, Ueber ein in Pulkovo registriertes künstliches Erdbeben. C. R. de l' Ac. Imp. des Sc. de St.-Pétersb. T. VI. 2. p. 173—184. 1914.

On the Sensitiveness of the Seismograph.

In all seismic registrations it is desirable to have a great efficiency especially for the vertical component. If the sensitiveness of a vertical seismograph is very great, this apparatus will still register sharply the preliminary phase of the most distant earthquakes, while at the same time the horizontal components do not show any trace of a movement, and one would believe to be in the region of a seismic shadow. But for a sensitive vertical seismograph no seismic shadows exist.

If the station is situated near the sea, on which in certain seasons violent cyclones occur, little advantage is derived from an exaggerated sensitiveness of the apparatus in the period of cyclones, as the microseismic pulsations blur the registrations. Nevertheless a high sensitiveness is very useful in the epoch when cyclones are absent. With the galvanometric registration method the sensitiveness may be easily reduced by removing the poles of the induction magnets farther from each other. It is always advantageous on every seismic station to make arrangements for using at least 2 degrees of sensitiveness, one of which has to be taken as high as possible for the calm period. In this way one is enabled to collect a very rich body of observations in the period of calm.

For this reason care must be taken that a vertical seismograph should allow a very high sensitiveness to be produced.

Let us first of all consider what sensitiveness may be attained in the present case without having recourse to long optical levers in connection with the galvanometer mirror.

For the galvanometric registration method the magnification i. e. the relation of the maximum excursion of the point of light on the drum y_m to the maximum ground displacement x_m, is expressed by the following equation:

$$\mathfrak{V} = \frac{y_m}{x_m} = \frac{k \cdot A \cdot T \cdot u}{\pi l (1 + u^2)^2} \tag{20}$$

for the case where the periods of the pendulum and of the galvanometer are equal and the limit of aperiodicity is exactly adjusted in the pendulum. Here $u = \frac{T_p}{T}$ indicates the relation of the period of the earth-wave to that of the apparatus.

With the vertical seismograph in question, k, T and l have been ascertained, while for the distance of the galvanometer mirror from the registration drum $A = 125$ cm may be taken. For the remaining values the following quantities were found: the galvanometric transmission factor $k = 155$, the proper period of the galvanometer $T_1 = T = 11^s.57$ and the reduced pendulum length $l = 14.82$ cm.

On the basis of these values the following table has been compiled:

Table X.

T_p	$\mathfrak{B}$	T_p	$\mathfrak{B}$
$0^s.1$	43	$6^s.0$	1551
0.5	206	7.0	1561
1.0	408	8.0	1524
1.5	601	9.0	1454
2.0	785	10.0	1364
3.0	1099	15.0	869
4.0	1329	20.0	524
5.0	1477	30.0	209

This table shows that the present vertical seismograph for a vibration period of only $0^s.5$ equals in efficiency all the most commonly used seismographs with mechanical and purely optical registration of 200 fold magnification in regard to the indication of near earthquakes. With distant earthquakes, however, the secondary waves of the first preliminary phase possess a period of $1^s.0$—$2^s.0$, for which reason the sensitiveness here is about 2—4 times as great.

Here it may be pointed out that owing to the nature of the case these motions of short period representing superficial waves of distortion at the point of registration come out very strongly in the horizontal components, where they regularly indicate the arrival of the longitudinal waves at a sufficiently early moment. They yield an angle of emergence which differs totally from that of the longitudinal waves, about 15^0—20^0, as I have often noticed in my seismographic practice at Pulkovo.

The criticism of a delayed emergence of the longitudinal waves in the galvanometric method applies only to the antiquated Galitzin pendulum with a proper period of ab. 25^s, as has recently been shown by B. Gutenberg[1]) with sufficient clearness by means of registration experiments.

From this table it may be seen that the magnification amounts to considerably over 1000 for a wide range of periods, just for those wave-periods which so often appear in the first preliminary phase. On the other hand, these periods characterise also the strongest microseismic movements, which take their origin from the waves of the sea.

At the time of strong microseismic movements, the above-mentioned sensitiveness would be of little advantage, as from the tangle of lines it is scarcely possible to ascertain any definite values.

In the absence of stronger microseismic movements, however, a much higher sensitiveness would be very desirable.

By selecting a slightly thicker wire and by augmenting the number of windings in the coils, the galvanometric transmission factor may easily be still further increased, without any great effect on the regular form of the coils. Thus with a new instrument k was made considerably larger, being equal to 190.

In order to increase this magnification still further, the positions of the damping plate and the coils would simply have to be exchanged. To obtain a damping which is required for reading the limit of aperiodicity, a further approaching of the magnetic poles to the plate would be sufficient, as the four induction-coils are filled with a damping metal and exercise a considerable damping-effect at the longer lever arm. By such an arrangement the magnification in the above table would be augmented 1.425 times, so that the maximum augmentation would amount to considerably over 2000.

In the calm season registration would be effected with the greatest possible sensitiveness, while for the period of strong microseismic movements the magnetic poles of the induction coils would be screwed further apart, the damping magnets being also approached in the same proportion to the damping-plate.

In addition to this possibility there are some others, which however are a little more complicated.

The seismic station of the observatory at Irkutsk would

1) B. Gutenberg: Gerlands Beiträge zur Geophysik. Bd. 25. H. 1. p. 74—80. 1930.

have to use a galvanometric transmission factor of about 300 for all the three components all the year round.

It is to be expected that this station under the direction of such an experienced scientist as the director of the observatory of Irkutsk, W. B. Shostakovich, is becoming the central seismic station for the whole of Russia, because no other station in the Soviet-Union, and perhaps in the whole world, is placed under such favourable conditions with regard to microseismic movements as that of Irkutsk.

The vertical seismograph with vertical temperature compensation springs has been sent to Copenhagen, Scoresby-Sund, Saint Louis, New York, Cincinnati, Santa Clara and Berkeley.

This new pendulum type with horizontally arranged flat spiral springs has already been furnished to the seismic stations of La Paz, Manila, Potsdam, Stuttgart, Uccle, Buffalo, New-Zealand, Ksara and Estonia, where recently a first-class station has been established.

Summary.

A description is given here of a vertical seismograph with temperature compensation, which until now has only been constructed in Estonia.

The theory of this apparatus is deduced and proved by means of measurements, the conformity being found to be sufficiently great.

Directions are given for the adjustment of the apparatus and especially of the temperature compensation.

The temperature compensation proves to be very correct with proper treatment, and the vertical seismograph will stand prolonged working in the pendulum room without any protection against heat.

Methods are discussed for a more precise adjustment of the period symmetry.

The pendulum shows a very regular oscillation with a weak proper damping, the period depending only very little on the amplitude.

The principles of construction advocated by the author have proved very successful and allow a general application of

them in the construction of vertical seismographs of different sensitiveness and methods of registration to be expected.

With a loosely fitting cover the influence of variable atmospheric pressure is observed. The seismograph acts like a baroscope.

The dependence of the proper period on the temperature is ascertained and a method is recommended for easily avoiding possible errors.

The constants are determined and the sensitiveness is examined, which gives a maximum magnification of about 1500 times and may be increased at wish.

A secondary phenomenon is observed in the apparatus and precautionary measures are indicated for removing the same.

The difference of temperature outside and inside the glass cover is examined.

Henceforth cellar rooms with small annual amplitude of temperature are found to be no longer necessary for the erection of seismographs.

Any quiet room with provision for heating can be used as a room for a seismic station.

Appendix.

In conclusion the principal formulæ shall be given which may be conveniently applied in the galvanometric registration method. They have been computed chiefly after the elaborations of B. Galitzin[1]) and O. Somville[2]).

In the determination of the constants of the galvanometer and the limit of resistance of the outer circuit for complete aperiodicity, a series of deflections to the left and right is observed, which are corrected on the sheet, and values are formed for the logarithmic decrement according to

(1) $$\Lambda = \log(m_s + m_{s+1}) - \log(m_{s+1} + m_{s+2})$$

wherein m denotes deflections which have already been corrected.

The proper period of the galvanometer without damping, when T'_1 is measured, is as follows:

(2) $$T_1 = \frac{T'_1}{\sqrt{1 + 0.5372\,\Lambda_0^2}}$$

and the galvanometer constant

(3) $$c_0 = \varepsilon_0 = 4.6052\,\frac{\Lambda_0}{T'_1}.$$

The logarithmic decrement is ascertained with different outer resistances (300, 250, 200, 150, 100 Ω) and ε_s is calculated every time according to

(4) $$\varepsilon_s = \frac{4.6052}{T_1} \cdot \frac{\Lambda_s}{\sqrt{1 + 0.5372\,\Lambda_s^2}}.$$

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1) Fürst B. Galitzin, Vorlesungen über Seismometrie, deutsch bearbeitet von O. Hecker. Leipzig u. Berlin. B. G. Teubner. 1914.

2) O. Somville, Constantes des sismographes Galitzin. Annales de l'Observatoire Royal de Belgique. 3e série, t. I. 1922.

From a series of R_s and ε_s the values for the second galvanometric constant c are obtained according to

(5) $$c = (\varepsilon_s - c_0)(R_s + r)$$

where r denotes the inner resistance of the galvanometer itself. For the limit of aperiodicity we get

(6) $$\varepsilon_s = n_1 = \frac{2\pi}{T_1}.$$

Therefore the limit of resistance follows from

(7) $$R_a = \frac{c}{n_1 - c_0} - r.$$

In the station $R_a + r$ must be constant at any temperature, therefore the magnitude of the supplementary resistance, which is added to the resistance of the coils through the connections, etc., is so chosen that it would correspond to the average of the annual temperature. In the calculation we apply the equation

(8) $$R_t = R_o(1 + 0.004\,t),$$

where 0.004 denotes the coefficient of temperature of copper wire.

If the reduced pendulum length l is determined directly from the period of oscillation of the pendulum tilted through an angle of 90°, then

(9) $$l = \frac{T^2 \cdot g}{4\pi^2}.$$

For all horizontal pendulums it is more convenient to use the method of varying the inclination of the axis of rotation, by allowing the pendulum to oscillate, for the periods T_0 and T, at two different inclinations of the bed-plate. This inclination Δi is measured by means of a mirror affixed to the bed-plate. Then

(10) $$l = \frac{\Delta i}{n^2 - n_0^2} \cdot g.$$

where $\Delta i = \frac{m}{2D}$ is the angle of inclination measured by the mirror method, $n_0 = \frac{2\pi}{T_0}$ and $n = \frac{2\pi}{T}$. g is the acceleration of gravity at the place of observation.

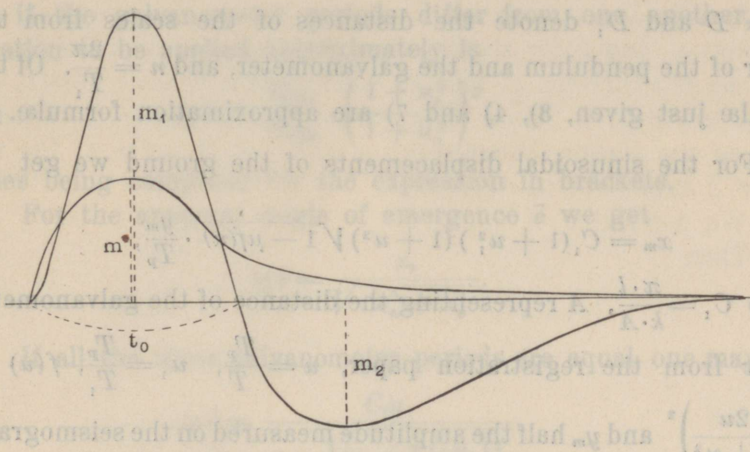

Fig. 7.

In Fig. 7, m denotes the deflections of the pendulum at the limit of aperiodicity, m_1 and m_2 the corresponding deflections of the galvanometer, and t_0 the time from the beginning of the galvanometer movement till the passage through the rest position.

For the galvanometric registration method the following formulæ apply:

$$1)\ t_0 = \frac{3\,T_1}{2\pi}$$

$$2)\ a = \frac{m_1 - \Delta m_1}{m_2 - \Delta m_2}$$

$$3)\ \mu = \frac{2{,}294 - a}{0{,}795}$$

(11) $$4)\ \xi = \tfrac{2}{3} n_1 t_0 - 2 + \tfrac{1}{10}\mu$$

$$5)\ \xi = \frac{T - T_1}{T_1} = \frac{n_1 - n}{n}$$

$$6)\ T = T_1(1 + \xi)$$

$$7)\ k = n_1 \cdot \frac{D}{D_1} \cdot \frac{m_1 - \Delta m_1}{m}(2.817 + 0.018\mu)$$

$$= n_1 \cdot \frac{D}{D_1} \cdot \frac{m_2 - \Delta m_2}{m}(6.461 - 2.196\mu)$$

where D and D_1 denote the distances of the scales from the mirror of the pendulum and the galvanometer, and $n = \frac{2\pi}{T_1}$. Of the formulæ just given, 3), 4) and 7) are approximation formulæ.

For the sinusoidal displacements of the ground we get

$$x_m = C_1(1 + u_1^2)(1 + u^2)\sqrt{1 - \mu f(u)} \cdot \frac{y_m}{T_p}, \tag{12}$$

where $C_1 = \frac{\pi \cdot l}{k \cdot A}$, A representing the distance of the galvanometer mirror from the registration paper, $u = \frac{T_p}{T}$, $u_1 = \frac{T_p}{T_1}$, $f(u) = \left(\frac{2u}{1+u^2}\right)^2$ and y_m half the amplitude measured on the seismogram.

Tables are compiled for the expression

$$\frac{C_1(1 + u_1^2)(1 + u^2)\sqrt{1 - \mu f(u)}}{T_p},$$

which are corrected after every determination of the constants, should this prove to be necessary.

The moments of maxima require a correction for delay by the pendulum τ and the galvanometer τ_1.

Thus we get

$$\tau = T_p\left[\frac{1}{2\pi} arctg \left\{\frac{2u\sqrt{1-\mu}}{u_2 - 1}\right\}\right] \tag{13}$$

and

$$\tau_1 = T_p\left[\frac{1}{2\pi} arctg \left\{\frac{2u_1}{u_1^2 - 1}\right\} - \frac{1}{4}\right]. \tag{14}$$

For these equations tables are constructed.

The azimuth is obtained from

$$tg\,\alpha = \frac{x_e}{x_n} = \frac{C_e(1 + u_{1e}^2)(1 + u_e^2)\sqrt{1 - \mu_e f(u_e)}}{C_n(1 + u_{1n}^2)(1 + u_n^2)\sqrt{1 - \mu_n f(u_n)}}. \tag{15}$$

If no considerable deviations occur in the periods we may put

$$tg\,\alpha = \frac{C_e y_e}{C_n y_n}. \tag{16}$$

But if the galvanometer periods differ from one another, the equation to be applied approximately is

$$(17) \qquad tg\, a = \frac{C_e y_e}{C_n y_n} \cdot \left(\frac{1+u_e^2}{1+u_n^2}\right)^2,$$

tables being computed for the expression in brackets.

For the apparent angle of emergence $\bar{e}$ we get

$$(18) \qquad tg\, \bar{e} = \frac{x_z}{\sqrt{x_n^2 + x_e^2}}.$$

If all the three galvanometer periods are equal, one may put

$$(19) \qquad tg\, \bar{e} = \frac{C_z y_z}{C_n y_n \sqrt{1 + \left(\frac{C_e y_e}{C_n y_n}\right)^2}}.$$

If this is not the case, the following approximation will suffice:

$$(20) \qquad tg\, \bar{e} = \frac{C_z y_z}{C_n y_n}\left(\frac{1+u_z^2}{1+u_n^2}\right)^2 \cdot \frac{1}{\sqrt{1 + \left[\frac{C_e y_e}{C_n y_n}\left(\frac{1+u_e^2}{1+u_n^2}\right)^2\right]^2}}.$$

For the squares in brackets containing u, tables are computed.

In all these formulæ the indices n, e and z denote the *N-S-*, *E-W-* and vertical components.

For the determination of the geographical co-ordinates of the epicentre a spherical triangle has to be solved.

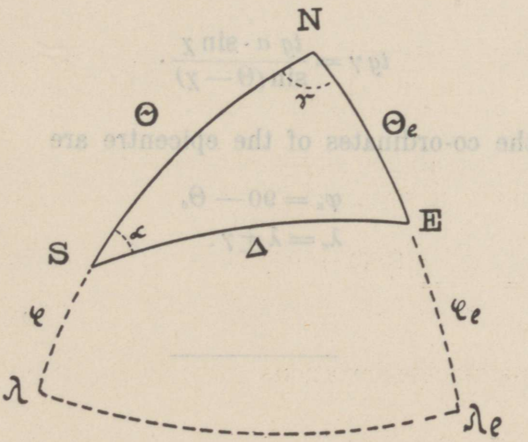

Fig. 8.

In Fig. 8, let S be the station with the geographical co-ordinates φ and λ for the northern hemisphere, E the epicentre with the co-ordinates φ_e and λ_e, which are to be ascertained, and N the north-pole. Thus as known values we have the co-ordinates of the station, the azimuth a and the distance of the epicentre Δ expressed in degrees.

We determine Θ_e and γ.

Then

$$\cos\Theta_e = \cos\Theta \cdot \cos\Delta + \sin\Theta \cdot \sin\Delta \cdot \cos a,$$

$$\sin\gamma = \sin a \cdot \frac{\sin\Delta}{\sin\Theta_e}$$

(21) and

$$\cos\gamma = \frac{\cos\Delta - \cos\Theta \cdot \cos\Theta_e}{\sin\Theta \cdot \sin\Theta_e}$$

or

$$tg\,\gamma = \frac{\sin a \cdot \sin\Theta \cdot \sin\Delta}{\cos\Delta - \cos\Theta \cdot \cos\Theta_e}.$$

If in order to simplify the calculation we put

$$tg\,\chi = \cos a \cdot tg\,\Delta \tag{22}$$

we may write

$$\cos\Theta_e = \frac{\cos\Delta \cdot \cos(\Theta - \chi)}{\cos\chi} \tag{23}$$

and

$$tg\,\gamma = \frac{tg\,a \cdot \sin\chi}{\sin(\Theta - \chi)}. \tag{24}$$

Then the co-ordinates of the epicentre are

$$\begin{aligned} \varphi_e &= 90 - \Theta_e \\ \lambda_e &= \lambda \pm \gamma. \end{aligned} \tag{25}$$
