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LUCY ZHENG

Cartan Khronon
– ℝeal space-time structure –

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Supervisor

Dr. Tomi S. Koivisto
Laboratory of Theoretical Physics, Institute of Physics, University of Tartu
Laboratory of High Energy and Computational Physics, National Institute of Chemical Physics and Biophysics

Opponent

Dr. William E. V. Barker
Central European Institute for Cosmology and Fundamental Physics,
Institute of Physics of the Czech Academy of Science

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Abstract

Cartan Khronon theory reformulates space-time with a symmetry-breaking clock field, the khronon, and a gauge field, incorporating internal gauge symmetry and chirality into the description of gravity. In this framework, the physical effects typically attributed to dark matter can instead be explained as effective contributions generated by gravitational dynamics. This thesis develops Cartan Khronon theory in both Euclidean and Lorentzian frameworks and studies its phenomenological aspects. A central theoretical outcome is the formulation of Cartan Khronon theory with a Spin(4) gauge structure based on a dimensionally controlled mapping between Euclidean and Lorentzian descriptions. This construction is tested with various solutions that consistently recover the corresponding general-relativistic solutions in a specific limit. Beyond this limit, the theory exhibits a broad range of phenomenological implications in cosmology and compact-object geometries. The key results include the development of cosmological perturbation theory for Cartan Khronon gravity, enabling a systematic analysis of the effective dark sector, and the pregeometric study of black holes. This thesis presents Cartan Khronon gravity as a coherent gauge-theoretic approach to space-time structure and gravitation, connecting foundational questions to concrete phenomenological consequences.

Terminology & conventions

spacetime	...	metrical continuum that combines the three dimensions of space and the one dimension of time
space-time	...	combination of the three dimensions of space and a dimension of time described with a clock field
pregeometric	...	does not require a metric tensor in formulation
khronon theory	...	Cartan Khronon theory

measurement system : $c = \hbar = 1$, $G = 1/(8\pi m_P^2)$

Levi-Civita symbol : $\epsilon_{0123} = 1$, $\epsilon_{1234} = 1$

$\mu, \nu, \rho, ...$	spacetime indices
$a, b, c, ...$	tangent space indices (regardless of the framework)
$i, j, k, ...$	spatial indices (regardless of the framework)
$A, B, C, ...$	Lorentzian indices
$I, J, K, ...$	Euclidean indices

Differential forms, which allow spacetime indices to be omitted, are denoted with bold symbols, e.g.,

$\boldsymbol{A}^{ab} = A_\mu{}^{ab}\mathrm{d}x^\mu$	:	gauge field / connection
$\boldsymbol{F}^{ab} = F_{\mu\nu}{}^{ab}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$	:	field strength / curvature
$\boldsymbol{M}^{a} = M^a{}_{\mu\nu\rho}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \wedge \mathrm{d}x^\rho$	:	effective dark matter

$$\boldsymbol{F}^{ab} = \mathbf{d}\boldsymbol{A}^{ab} + \boldsymbol{A}^a{}_c \wedge \boldsymbol{A}^{cb} \equiv (\mathbf{D}\boldsymbol{A})^{ab}$$

Contents

List of publications

This thesis is based on the following publications and further unpublished developments carried out by the author.

I **Spin(4) gauge theory of space, time, gravitation, matter and dark matter**
Tomi Koivisto, Lucy Zheng, Tom Zlosnik
Phys. Rev. D 113 (2026), p. 124069. arXiv: 2507.00968 [1]

II **Black holes in Lorentz gauge theory**
Tomi S. Koivisto, Lucy Zheng
Phys. Rev. D 111.6 (2025), p. 064008. arXiv: 2408.10100 [2]

III **Scale-invariant cosmology in de Sitter gauge theory**
Tomi S. Koivisto, Lucy Zheng
Phys. Rev. D 103.12 (2021), p. 124063. arXiv: 2101.07638 [3]

Author's contribution to the publications

In chronological order, publication III presents the results obtained through the author's computational exercise to understand the algebraic foundations of gauge theory and its cosmological implications, using de Sitter gauge theory and the minisuperspace model as an example. After preliminary work on matter coupling in de Sitter gauge theory of gravity in 2022-23, given the limited prospects for further theory development, I proposed to study phenomenological aspects of the more established four-dimensional Cartan Khronon theory, building on [4–9]. In collaboration with my supervisor, Tomi Koivisto, I obtained generalised spherically symmetric solutions in both SO(3) and SO(1,2) phases, including the charged solution presented in publication II. This was followed by the author's attempts at axisymmetric solutions, up until which computations involving differential forms were performed by hand. With the more complicated axisymmetric system, I started developing Mathematica workflows using the xTerior package to solve and analyse Cartan Khronon field equations. This facilitated the detailed computations leading to the Kerr and various cosmological solutions in both Lorentzian and Euclidean formulations presented in publication I. While looking for the correct procedure to recover the Kerr solution, I speculated that a rotating solution might require effective dark matter for recovery. Although this was not the case for Kerr black hole, it has worked for the vacuum FLRW background in the dymaxion regime instead. The pursuit of an axisymmetric solution in the Euclidean framework continued on the ferry back to Tallinn from the Nordic Cosmology meeting in Helsinki, where I found that the Wick rotation has to be performed on the spin parameter, since angular momentum is a time derivative. Together with my supervisor's idea to introduce the dimensionful parameters m_P and κ, this resulted in the development of a new Lorentzian-Euclidean mapping procedure, and consequently the construction of the Spin(4) theory. Conference and seminar talks by the author based on these publications are as listed below.

Space-time structure and phenomenology of Cartan Khronon theory
Geometric Foundations of Gravity X, University of Tartu 02/07/2026

The Spin(4) gauge theory of space, time, matter, and dark matter
hecp-theory group seminar, NICPB 29/01/2026

Cartan Khronon - the *real* space-time structure
Theoretical physics seminar, University of Tartu 18/11/2025

The modified space-time structure and cosmological aspects of the $Spin(4)$ gauge theory
hep group seminar, NORDITA 29/10/2025

Cosmology in the $Spin(4)$ gauge theory
Cosmology seminar, Helsinki Institute of Physics 15/10/2025

Chiral gauge theory of gravity and dark matter
Geometric Foundations of Gravity, University of Tartu 01/07/2025

Dark Matter candidate: Dynamics of Chiral Gravity
Cracking the Universe, Tartu & Tuorla Observatory 02/06/2025

Dark matter in gauge theories of gravity
My Favourite Dark Matter Model, University of Açores 14/04/2025

Cosmology based on gauge theories of gravity
2nd Nordic Cosmology Meeting, University of Helsinki 12/03/2025

Black Holes in Lorentz gauge theory
Metric-Affine Frameworks for Gravity, University of Tartu 18/06/2024

1. Introduction

Marked by the advent of the theories of relativity [10–12], theoretical physics has witnessed significant progress in gravitational theories over the last century where geometry started to play a central role. Together with the emergence of quantum theory overriding the deterministic framework of classical physics, the success of these transformative paradigms forms the foundation of modern theoretical physics. A slower and initially less prominent line of research pursued the construction of a more coherent, unified description of fundamental interactions based on the principle of symmetry. This framework, which only gradually came to be recognised as the standard description for three of the four fundamental interactions, is gauge theory.

Although the first indication of gauge invariance was already observed in classical electromagnetism, its formulation and establishment as a principle had to wait until much later. It was Weyl's attempt to unify gravitation and electricity [13] that shed light on the (incomplete) gauge structure in the gravitational theory and the geometric feature of electromagnetism. Though this theory was dismissed as unphysical, a decade later, after the reinterpretations of the scale invariance as a phase symmetry of the quantum wave function [14, 15] and the progress in spinor and tetrad formalism [16], Weyl returned with a refined theory of local phase invariance of spinor fields [17], establishing the modern view of the gauge principle. It required a further quarter century and several attempts [18, 19] before Yang and Mills finally constructed the non-Abelian SU(2) gauge theory [20] describing the gauge invariance of nuclear interaction, which is accepted as the basis for the Standard Model of particle physics today.

The geometric structure of the gauge theory, however, was not as appreciated as that of general relativity, partially because Yang-Mills theory was constructed as a successor of Maxwell's theory without explicit regard for its underlying geometric feature. Over two decades after the establishment of Yang-Mills theory, in 1975, Yang learned from Simons, his colleague at Stony Brook University, about the identical structure of the fibre bundle, a mathematical construction developed independently. Yang was so impressed that he drove across the US to the house of Chern, his former lecturer at the 國立西南聯合大學 (National Southwestern Associated University), and expressed his amusement as [21]

> I found it amazing that gauge fields are exactly connections on fiber bundles, which the mathematicians developed without reference to the physical world. This is both thrilling and puzzling, since you mathematicians dreamed up these concepts out of nowhere.

In the same year, Yang, together with Wu, clarified this deep relationship between the two theories in their publication [22] in which they presented a 'dictionary' [23] between gauge field and bundle terminologies. In this dictionary, the bundle term corresponding to the source in gauge theory (such as electric current) is marked as '?', indicating where physics departs from mathematics.

In fact, this is also where the difference between gravity and the other fundamental forces becomes particularly evident. In Yang-Mills theory, the source of the gauge field is a matter current associated with internal symmetry under Lie group transformations, and the gauge field is represented as a connection on a fibre bundle, a projected space distinct from the spacetime manifold. In contrast, the source current for gravity is tied to diffeomorphism invariance, symmetry under coordinate transformations, and the corresponding connection determines the geometry of the base manifold itself. Internal gauge symmetry preserves frame-independent physical quantities, such as charge, independently of the background geometry, whereas diffeomorphism invariance conserves energy and momentum, which both affect and are affected by the geometry of the spacetime manifold. This is why a gauge theoretic formulation of gravity cannot be achieved by directly replicating Yang-Mills theory, and requires a careful consideration of how to formulate gravity and its source in relation to spacetime geometry.

The tetrad formalism has played a central role in the development of gauge theoretic formulation of gravity for this reason. The establishment of this formalism was originally motivated by the integration of spin structure in gravitational theory, to enable the coupling of gravity to spinors [17]. This reformulates general relativity using the (co)frame fields of the local tangent space, substituting the metric tensor. By postulating a local base with internal symmetry based on a Lie group, tetrad formalism introduces the connection as a field independent from the metric, narrowing the gap between gravitational theory and Yang-Mills theory. The standard formulation of the tetrads in gauge gravity over the last few decades has incorporated the contribution from the translational (or transvectional [3]) part of the connection. However, as first proposed in [4], this formulation has room for optimisation.

Another key aspect of gauge theory, perhaps less commonly associated with gravitational theories, is the breaking of symmetry. The main criticism that hindered the recognition of Yang-Mills theory as the Standard Model of particle physics was the absence of mass in the theory [24], as experiments suggested that short-range interactions, such as nuclear forces, required massive particles as mediators. For the weak interaction, this was solved with the Higgs mechanism [25], which allows gauge bosons to acquire mass through spontaneous symmetry breaking while preserving the gauge invariance of the Lagrangian. While this may not appear relevant to the case of gravity, a similar mechanism has achieved the formulation of gravity and supergravity as a unified gauge theory [26] and improved our picture of cosmology by incorporating an emergent cosmological constant within the gravitational theory [27]. In fact, a stream of coincident developments of gauge structure in higher dimensional theory can be traced back to as early as the 1910-20s, where dimensional reduction via symmetry breaking induced gauge structure in 4D spacetime from diffeomorphism in higher dimensional space [28–30].

Combining tetrad formalism and symmetry breaking, the two central features of gauge theory, Cartan Khronon theory was developed. With an optimised, minimal prescription of the tetrads with a symmetry-breaking clock field, the khronon field, the choice of gauge selects the direction of time, distinguishing the temporal dimension from the other three spatial dimensions. In this theory, the spacetime metric is not necessary, and it describes geodesics only in the symmetry-broken geometric phase, making this a 'pregeometric' theory. Together with the chiral description of gravity, the theory predicts an effective dust component, which arises from gravitational dynamics. The first proposal of this framework [4] motivated this doctoral project to study gravitational theories with internal Yang-Mills type gauge symmetry, to place gravity on a more equal footing with the other fundamental forces.

An unintended theoretical implication of Cartan Khronon framework stemmed from its background-independent description of time. While modern physics assumes a Lorentzian spacetime background, the introduction of Euclidean time is essential for the mathematically well-defined formulation of path integrals in quantum field theory, statistical field theory, and black hole thermodynamics. This method is commonly regarded as a mere mathematical convenience, since all physical measurements still take place in the Lorentzian framework. However, the Euclidean framework could instead be interpreted as the foundational ground where all the physics is defined, with the Lorentzian domain serving as the observational framework imposed by our perception of time [31]. In the Cartan Khronon framework, the temporal direction is selected by the background-independent khronon field, rather than being imposed a priori through a fixed metric signature, providing a natural setting for a Euclidean description of space-time.

A major obstacle that has prevented a wider adoption of the Euclidean framework concerns the lack of a regulated mapping between the Euclidean framework and the Lorentzian description of the observational framework, as the standard mapping procedure of Wick rotation is only applicable to limited geometric structures, and a general, consistent procedure was absent [32–35]. In gravitational theory, this limitation becomes particularly evident, since the metric and the causal structures are dynamical themselves. In Cartan Khronon theory, an improved formula of dimensionally controlled mapping between Euclidean and Lorentzian frameworks is proposed to enable a consistent translation.

This thesis reports key findings established through the study and development of the Cartan Khronon framework. The central aim is to demonstrate how this gauge-theoretic formulation of gravity with a symmetry-breaking clock field lays the foundation for the Spin(4) gauge structure in gravitation, a Euclidean-Lorentzian correspondence via improved Wick rotation, and concrete cosmological and black hole solutions including the contribution of the effective dark sector. After a primer chapter on the elementary concepts of general gauge theory in Chapter 2, the rest of the thesis organises these results from the specific formalism of the Cartan Khronon framework in Chapter 3 to its phenomenological applications in Chapter 4 and 5. The main results and contributions of this thesis are as follows:

- Confirmation of preceding proposals, including the optimal tetrad prescription [4], the emergent dark matter effect [6], and the recovery of general relativity in a special limit [9], through independent derivations and analyses. — Chapter 3, 4.2.1
- Development of the Spin(4) formulation of the theory, together with an improved Wick-like formula establishing a systematic correspondence between the Euclidean and Lorentzian frameworks.
  — Chapter 3
- Formulation of FLRW perturbation theory within the Cartan Khronon framework, providing the exact properties of the dark matter effect in both general-relativistic limit and generalised scenario.
  — Chapter 4
- Derivation of explicit black hole solutions in the Cartan Khronon framework, demonstrating recovery of general relativity in a special limit and validating the Euclidean-Lorentzian mapping.
  — Chapter 5

To conclude, Chapter 6 summarises these results, guiding to a question that connects all the chapters.

2. Gauge theory

Before diving into Cartan Khronon theory, the main subject of this thesis, this chapter lays out the essential theoretical basis to understand the geometric and algebraic structures of gauge theory. The comparison with general relativity highlights the geometric analogies and key differences, outlining the motivations and methods for the gauge-theoretic approach to modifying gravitational theory.

2.1. Global & local symmetry

Symmetry, in the context of physics, refers to the invariance of physical laws under certain transformations. This is a reflection of the underlying regularities in physical phenomena, making symmetry one of the most fundamental and essential structures in the formulation of physical theories. Without symmetry, there would be no recurring patterns that govern physical systems: the very notion of physical *laws* would no longer exist. When a physical law is invariant under a transformation Λ that does not depend on the coordinates, this law exhibits global symmetry, meaning it transforms uniformly anywhere in spacetime. Meanwhile, a physical law could transform in varied ways at different positions in spacetime and still exhibits symmetry. Such symmetry with a coordinate-dependent transformation $\Lambda = \Lambda(x)$ is called local, or gauge, symmetry. [1]

A simple intuitive example can be drawn using a colour wheel (Fig. 1a). When the colour hue at every point on the wheel is increased by the same amount, the wheel is effectively rotated, but the colour hue remains in the same order and distribution. (Fig. 1b). When this effective rotation amounts to 360° or its integer multiple, the wheel remains identical to its initial configuration (Fig. 1a), exhibiting global symmetry. In contrast, if the change in hue depends on the position on the wheel, the distribution of colours can be altered (Fig. 1c), or even the overall palette may shift (Fig. 1d). However, by introducing a compensating, position-dependent adjustment at each point, one can restore the original ordering and distri-

[1] Since these transformations do not affect physical features that are observable, this is sometimes called a gauge redundancy, instead of gauge symmetry.

bution of colours. This way, the hues are "gauged" back to their original configuration with the aid of these compensations, which, in gauge theory, are known as the gauge field or gauge potential. When the colour wheel remains unchanged under such a specific coordinate-dependent transformation, it is said to possess local or gauge symmetry.

(a) Original state (b) Global (c) Local 1 (d) Local 2

Figure 1: Global and local transformations of a colour wheel

This is in fact an illustration of global and local phase transformations. The necessity of a compensating field, the gauge field, reflects the fact that a global symmetry can be promoted to a local one by replacing partial derivatives with covariant derivatives. As an explicit example, let us consider the kinetic energy of a complex scalar field ϕ under a global phase transformation $\phi \to \mathrm{e}^{i\alpha}\phi$. The scalar field ϕ can compare to the colour hue in the example of the colour wheels above, and its kinetic energy $|\partial_\mu \phi|^2$ to their layout. It is straightforward to see the kinetic energy is invariant under the global phase transformation $\Lambda = \mathrm{e}^{i\alpha}$.

$$g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi \to g^{\mu\nu}\partial_\mu(\Lambda^{-1}\phi^*)\partial_\nu(\Lambda\phi) = g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi \,. \tag{2.1}$$

With a local transformation $\Lambda(x)$, however, this invariance no longer holds, since its coordinate dependence yields an extra term.

$$\partial_\mu\phi \to \partial_\mu(\Lambda\phi) = \Lambda\partial_\mu\phi + (\partial_\mu\Lambda)\phi \,. \tag{2.2}$$

For the kinetic energy to be invariant under the local transformation, the extra term $(\partial_a\Lambda)\phi$ has to be cancelled out, and this can be achieved by introducing a gauge field/potential that transforms as

$$A_\mu \to \Lambda A_\mu \Lambda^{-1} - (\partial_\mu\Lambda)\Lambda^{-1} \,. \tag{2.3}$$

Taking this gauge field into account, the reformulated kinetic energy $|(\partial_\mu + A_\mu)\phi|^2$ is indeed gauge invariant, since

$$(\partial_\mu + A_\mu)\phi \to (\partial_\mu \Lambda)\phi + \Lambda(\partial_\mu \phi) + \Lambda A_\mu \phi - (\partial_\mu \Lambda)\phi = \Lambda(\partial_\mu + A_\mu)\phi\,, \quad (2.4)$$

then, with $\mathrm{D}_\mu \equiv \partial_\mu + A_\mu$,

$$g^{\mu\nu}\,(\mathrm{D}_\mu\phi)^*\,\mathrm{D}_\mu\phi \to g^{\mu\nu}\,\Lambda^{-1}(\mathrm{D}_\mu\phi)^*\,\Lambda\mathrm{D}_\mu\phi = |\mathrm{D}_\mu\phi|^2\,. \quad (2.5)$$

With this formulation, the analogy with the connection introduced to enable covariant parallel transport on a curved surface becomes clear. On one hand, this is not surprising considering the establishment of the gauge principle by Weyl [13, 17] was inspired by general relativity. On the other hand, it is remarkable that the construction of Yang-Mills theory [20] was achieved without geometric consideration.

2.2. Group and algebra

From the example of the phase transformation, one notices that these transformations are not arbitrary but satisfy specific conditions, such as $\Lambda^* = \Lambda^{-1}$. The set of transformations that satisfy the relevant rules of algebra is called a group. In particular, the Lie groups, the groups that provide the structure of continuous symmetry, are the transformations under which physical laws are invariant. The most intuitive example may be the rotation group. For a 3D Cartesian coordinate x^i, the rotation around the x^i-axis by the angle θ can be written as matrices R_i.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}, \quad \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}, \quad \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.6)$$

An infinitesimal θ yields generators of the 3D rotation group

$$J_1 = -i\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad J_2 = -i\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad J_3 = -i\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (2.7)$$

which satisfy the following commutation relation.

$$[J_i, J_j] = i\epsilon_{ij}{}^k J_k\,. \tag{2.8}$$

This group can be extended to the Lorentz group by including boosts and any possible combination of boosts and rotations. The boost for a frame moving along the x_1 axis at the speed v, for example, is

$$\begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{2.9}$$

where $\beta = v/c$, $\gamma = (1 - v^2/c^2)^{-1/2}$. This can be written in terms of hyperbolic functions of an angle α as

$$\begin{pmatrix} \cosh\alpha & -\sinh\alpha & 0 & 0 \\ -\sinh\alpha & \cosh\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2.10}$$

An infinitesimal ϕ yields the boost generators K_i

$$i\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad i\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad i\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \tag{2.11}$$

which satisfy the algebra

$$[J_i, J_j] = i\epsilon_{ij}{}^k J_k\,, \quad [J_i, K_j] = i\epsilon_{ij}{}^k K_k\,, \quad [K_i, K_j] = -i\epsilon_{ij}{}^k J_k\,. \tag{2.12}$$

This set of transformations preserves the spacetime interval $\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$, and hence the speed of light, making global Lorentz symmetry one of the most fundamental and essential symmetry in all the metric-based theories of physics. Such transformations Λ follow

$$\Lambda^T\eta\Lambda = \eta\,, \quad \eta = \mathrm{diag}(-1, 1, 1, 1)\,, \tag{2.13}$$

which does not precisely satisfy the orthogonality $\Lambda^T\Lambda = \mathbb{1}$, but classifies the set of transformations as an orthogonal group in a generalised sense. Together with the property $\det\Lambda = 1$ that preserves the orientation of the spacetime, the Lorentz group is also called a special orthogonal group $SO(1,3)$. For a more detailed formulation, the Lorentz transformations Λ are represented with parameters $\epsilon_{\mu\nu}$ and the generators $L_{\mu\nu}$ where $L_{0i} = K_i$ and $L_{ij} = \epsilon_{ij}{}^{k} J_k$.

$$\Lambda = \exp\left(\frac{i}{2}\epsilon^{\mu\nu} L_{\mu\nu}\right) . \tag{2.14}$$

The generators $L_{\mu\nu}$ consist of a scalar part $L^{(0)}_{\mu\nu}$ and a matrix part $\Delta_{\mu\nu}$ which differs depending on the type of the field the transformation acts on.

Table 1: Lorentz generators for scalar and vector fields

Field type	Intrinsic generator	Full generator
Scalar (spin-0) field	$\Delta^{(0)}_{\mu\nu} = 0$	$L^{(0)}_{\mu\nu} = 2x_{[\mu}\partial_{\nu]}$
Vector (spin-1) field	$(\Delta^{(1)}_{\mu\nu})^a{}_b = -2\eta_{b[\mu}\delta^a_{\nu]}$	$(L^{(1)}_{\mu\nu})^a_b = L^{(0)}_{\mu\nu}\delta^a_b + (\Delta^{(1)}_{\mu\nu})^a{}_b$
Covector (1-form)	$(\Delta^{(0,1)}_{\mu\nu})_a{}^b = 2\eta_{a[\mu}\delta^b_{\nu]}$	$(L^{(0,1)}_{\mu\nu})_a{}^b = L^{(0)}_{\mu\nu}\delta^b_a + (\Delta^{(0,1)}_{\mu\nu})_a{}^b$
	$\vdots$	

The matrix part is derived by simply computing $\left[L^{(0)}_{\mu\nu}, \boldsymbol{V}\right]$ for a relevant field, a vector $\boldsymbol{V} = V^c\partial_c$ for example.

The elements of an arbitrary Lie group satisfy certain commutation relations, and the general form of these relations is

$$[T_a, T_b] = f_{ab}{}^{c} T_c \, , \tag{2.15}$$

with the structure coefficient $f_{ij}{}^{k}$ defining the Lie algebra for relevant Lie group. Following this relation and the Jacobi identity of the Lie group elements

$$[T_a, [T_b, T_c]] + [T_b, [T_c, T_a]] + [T_c, [T_a, T_b]] = 0 \, , \tag{2.16}$$

one finds the antisymmetric nature of the structure coefficient and its Jacobi identity.

$$f_{ab}{}^{d} f_{cd}{}^{e} + f_{bc}{}^{d} f_{ad}{}^{e} + f_{ca}{}^{d} f_{bd}{}^{e} = 0 \, . \tag{2.17}$$

Isomorphic groups share the same algebraic structure, for example, ${f_{ab}}^c = {\epsilon_{ab}}^c$ for the $SO(3)$ and the $SU(2)$ groups. In light of this relation, the Lorentz generators $\{J^i, K^i\}$ can be reorganised into two sets of generators $T_i^\pm = \frac{1}{2}(J_i \pm iK_i)$ which respectively satisfy the following sets of commutation relations.

$$\left[T_i^\pm, T_j^\pm\right] = i{\epsilon_{ij}}^k T_k^\pm \,, \quad \left[T_i^\pm, T_j^\mp\right] = 0 \,. \tag{2.18}$$

Note the complexification of the generators. The generators T^+ and T^- respectively satisfy $(T^\pm)^* = T^\mp$ and $\det T^\pm = 1$, each composing a continuous group $SL(2,\mathbb{C})$. This is a manifestation of the double covering relation $\mathfrak{so}(1,3)_\mathbb{C} \simeq \mathfrak{sl}(2,\mathbb{C}) \times \mathfrak{sl}(2,\mathbb{C})$, meaning, for every complexified Lorentz generator, there exists a pair of corresponding $SL(2,\mathbb{C})$ generators. In other words, for any Lorentz transformation of a vector $x^\mu = \psi^\dagger\gamma^0\gamma^\mu\psi$, there is a corresponding $SL(2,\mathbb{C})$ transformation of the spinor ψ, and Lorentz transformations are reducible to more fundamental spinor transformations.

To incorporate spinor representations in Lorentz algebra, it requires the assistance of Clifford algebra $\mathrm{Cl}_{1,3}$. The generators of the group are Pauli matrices σ^a which corresponds to Hamilton's quaternions $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$.

$$\sigma_1 = -i\boldsymbol{i} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \,, \quad \sigma_2 = -i\boldsymbol{j} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \,, \quad \sigma_3 = -i\boldsymbol{k} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \,. \tag{2.19}$$

The identity is often included as the 0th element $\sigma^0 = \mathbb{1}$. These generators satisfy

$$[\sigma_i, \sigma_j] = 2i{\epsilon_{ij}}^k \sigma_k \,, \tag{2.20}$$

corresponding to complexified Lorentz algebra (2.18). The spatial rotation of a two-component spinor by an angle θ about an axis $\boldsymbol{n}$ is formulated in terms of these generators as $\psi \to \mathrm{e}^{\frac{i}{2}\theta}\boldsymbol{\sigma}\cdot\boldsymbol{n}\psi$, describing the peculiar nature of spinorial rotation. A rotation by $\theta = 2\pi$ flips the sign of a spinor, and it requires a $\theta = 4\pi$ phase change to bring the spinor back to its original state. This double-valued behaviour is the reflection of the fact that $SU(2)$ is a double cover of $SO(3)$, and this characterises the intrinsic property of spin-1/2 particles.

For the description of the Lorentz transformation of the four-component Dirac spinors, it requires gamma matrices

$$\gamma^0 = i\sigma^1 \otimes \sigma^0 \,, \quad \gamma^i = \sigma^2 \otimes \sigma^i \,, \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \sigma^3 \otimes \sigma^0 \,, \tag{2.21}$$

as the fundamental representations. The intrinsic Lorentz generators for the Dirac spinor are constructed from these $SL(2,\mathbb{C})$ generators as

$$S_{\mu\nu} = -\frac{i}{2}\gamma_{[\mu}\gamma_{\nu]} = -\frac{i}{4}\left[\gamma_\mu, \gamma_\nu\right] \,, \tag{2.22}$$

introducing additional generators to the Lorentz group that are more fundamental and irreducible. Under the transformation $U = \exp\left(\frac{i}{2}\epsilon^{\mu\nu}S_{\mu\nu}\right)$, the vector bilinear $x^\mu = \bar{\psi}\gamma^\mu\psi$ transforms as

$$\bar{\psi}\gamma^\mu\psi \to \bar{\psi}U^{-1}\gamma^\mu U\psi = \Lambda^\mu{}_\nu\bar{\psi}\gamma^\nu\psi \,, \tag{2.23}$$

where $\bar{\psi} \equiv \psi^\dagger\gamma^0$ is Dirac adjoint and $\Lambda^\mu{}_\nu = \delta^\mu_\nu + \epsilon^\mu{}_\nu$ is Lorentz transformation, demonstrating the nature of x^a as a Lorentz vector. These additional generators introduced by Clifford algebra complete the Table 1 as follows.

Table 2: Lorentz generators for spinor fields

Field type	Intrinsic generator	Full generator
	$\vdots$	
Weyl spinors $\psi_\pm = P_\pm\psi$	$P_\pm S_{\mu\nu} P_\pm \,, \quad P_\pm = \frac{1}{2}\left(1 \pm \gamma_5\right)$	$P_\pm\left(L^{(0)}_{\mu\nu}\mathbb{1} + S_{\mu\nu}\right)P_\pm$
Dirac spinor $\psi = \psi_+ + \psi_-$	$S_{\mu\nu} = -\frac{i}{4}\left[\gamma_\mu, \gamma_\nu\right]$	$L^{(1/2)}_{\mu\nu} = L^{(0)}_{\mu\nu}\mathbb{1} + S_{\mu\nu}$

2.3. Dynamics

The Standard Model of particle physics formulates the electromagnetic and nuclear forces based on the gauge symmetry of their dynamics under the relevant Lie group transformations. With the new field introduced to implement local symmetry, there arise interactions that are absent in theories that consider only global symmetry.

2.3.1. The source

Let us consider a general Lagrangian $L(Q^a, \partial_\mu Q^a)$ to describe the dynamics of a physical field Q^a. Under an infinitesimal global transformation ${\Lambda^a}_b$, the field Q^a transforms as

$$Q^a \to Q^a + \delta Q^a \,, \quad \delta Q^a = \epsilon^c {\Lambda_c}^a{}_b Q^b \,, \tag{2.24}$$

then, the variation of the Lagrangian is

$$\delta L = \frac{\partial L}{\partial Q^a}\delta Q^a + \frac{\partial L}{\partial {Q^a}_{,\mu}}\delta {Q^a}_{,\mu} = \left[\frac{\partial L}{\partial Q^a} - \frac{\partial}{\partial x^\mu}\left(\frac{\partial L}{\partial Q^a_{,\mu}}\right)\right]\delta Q^a + \frac{\partial}{\partial x^\mu}\left(\frac{\partial L}{\partial Q^a_{,\mu}}\delta Q^a\right) \,. \tag{2.25}$$

The variational principle identifies the Euler-Lagrange equation as the field equation, and the conservation of the Noether current ${J_N}^\mu = \frac{\partial L}{\partial Q^a_{,\mu}}\delta Q^a$.

As discussed in the previous section, for a gauge-invariant action, it requires the gauge-covariant derivative to be introduced. In addition, the dynamics of the gauge field itself plays into how a gauge-invariant action must be formulated: the gauge field cannot enter the Lagrangian independently, but only through the covariant derivative or the field strength, which take the following form.

$$F^a{}_{\mu\nu} = 2\partial_{[\mu} A^a{}_{\nu]} + f^a{}_{bc} A^b{}_{[\mu} A^c{}_{\nu]} \,. \tag{2.26}$$

Here, the structure constant acts as a coupling constant for the self interaction of the gauge field. With an Abelian group, such as $U(1)$ group of phase transformations, due to its commutative nature, the structure constant is null and the gauge field does not couple to itself.

To provide an explicit example of a gauge invariant action that includes the self interaction of gauge field, let us consider the Yang-Mills action.

$$I_{YM} = -\frac{1}{4g^2}\int \mathrm{d}^4x F^a{}_{\mu\nu}F_a{}^{\mu\nu} = -\frac{1}{2g^2}\int \boldsymbol{F}^a \wedge \star \boldsymbol{F}_a\,. \tag{2.27}$$

The variation of the action

$$\delta I_{YM} = -\frac{1}{2g^2}F^a{}_{\mu\nu}\mathrm{D}_\mu\left(\delta A_\nu{}^a\right) = \frac{1}{2g^2}\left[-\partial_\mu\left(F_a{}^{\mu\nu}\delta A_\nu{}^a\right) + \left(\mathrm{D}_\mu F_a{}^{\mu\nu}\right)\delta A_\nu{}^a\right]\,, \tag{2.28}$$

when coupled to a matter source current

$$\delta I_{\mathrm{matter}} = J_{\mathrm{m}}{}^{\nu a}\delta A_{\nu a}\,, \tag{2.29}$$

identifies the equation of motion $\mathrm{D}_\mu F^{a\mu\nu} = J_{\mathrm{m}}{}^{\nu a}$, and covariant derivative of this relation demonstrates the gauge-covariant conservation of the matter current $\mathrm{D}_\mu J_{\mathrm{m}}{}^\mu = 0$. By considering an infinitesimal global transformation α^a, the variation of the gauge field becomes $\delta A_\nu{}^a = f^a{}_{bc}A_\nu{}^b\alpha^c$, and the boundary term of (2.28)

$$-\frac{1}{2g^2}\partial_\mu\left(F_{a\mu\nu}\delta A_\nu{}^a\right) = -\frac{1}{2g^2}\partial_\mu\left(F_{a\mu\nu}f^a{}_{bc}A_\nu{}^b\right)\alpha^c = 0 \tag{2.30}$$

identifies the conserved Noether current

$$J_{\mathrm{N}}{}^\mu{}_a = -\frac{1}{2g^2}f_{abc}A^b{}_\nu F^{c\mu\nu}\,. \tag{2.31}$$

Each matter field interacting with gauge field carries a charge corresponding to Noether current.

$$Q^a = \int dx^3 J_{\mathrm{N}}{}^{0a}\,. \tag{2.32}$$

For the Standard Model of particle physics, the gauge group is often described as $SU(3)\times SU(2)\times U(1)$ with the $SU(3)$ gauge symmetry for the strong nuclear force, the $SU(2)$ for unified framework for the electroweak theory, and the $SU(2)\times U(1)$ for the electromagnetism. The elements of these groups each satisfy $\Lambda\Lambda^\dagger = 1$, with different ranks and representations, respectively corresponding to different Noether currents and charges.

Table 3: Gauge symmetry, associated fields and sources in the Standard Model and gauge-theoretic picture of Cartan Khronon theory

Gauge group	Generator	Gauge field / excitation	Source / charge
$SU(3)_{\mathrm{C}}$	Gell-Mann matrices	$\boldsymbol{G}^A$ (gluons)	colour charge
$SU(2)_{\mathrm{L}}$	Pauli matrices	$\boldsymbol{W}^i$ (weak gauge bosons)	weak isospin
$U(1)_{\mathrm{Y}}$	phase transformation	$\boldsymbol{B}$ (hypercharge gauge field)	hypercharge
$U(1)_{\mathrm{EM}}$	phase transformation	$\boldsymbol{A}_{\mathrm{EM}}$ (photon)	electric charge
$SU(2) \times SU(2)$	gamma matrices	$\boldsymbol{A}^{ab}$ (spin connection)	angular momentum

2.3.2. Spontaneous symmetry breaking

The physical mechanism of the fundamental interactions is not as straightforward as the elegant mathematical structure of the gauge theory. A gauge-invariant action faces a significant issue with the absence of a mass term for gauge bosons, as it would violate the gauge symmetry.

$$m^2 A_\mu A^\mu \to m^2 \left(\Lambda A_\mu A^\mu \Lambda^{-1} - \Lambda_{,\mu} A^\mu \Lambda^{-1} - \Lambda A^\mu \Lambda^{-1} \Lambda_{,\mu} \Lambda^{-1} + \Lambda_{,\mu} \Lambda^{-1} \Lambda^{,\mu} \Lambda^{-1} \right) .$$

This implies that a gauge-invariant action can only describe the dynamics of massless particles, in spite of experimental evidence indicating that the short-range forces within atomic nuclei must be mediated by massive particles.

In the Standard Model, this problem for the weak gauge bosons is circumvented by the Higgs mechanism. Instead of the kinetic term of the gauge field, a scalar field ϕ, now known as the Higgs field, is introduced as the potential.

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \frac{\mu^2}{v^2} \left(\phi^\dagger \phi \right)^2 . \tag{2.33}$$

It is straightforward to confirm that the introduction of this term does not perturb the gauge invariance of the action.

$$\phi^\dagger \phi \to (\Lambda \phi)^\dagger \Lambda \phi = \phi^\dagger \Lambda^\dagger \Lambda \phi = \phi^\dagger \phi . \tag{2.34}$$

When the Higgs field rolls down the "Mexican hat" potential well (2.33), it acquires a non-zero vacuum expectation value at its minima $\phi^\dagger \phi = v^2/2$. With a certain gauge choice, the corresponding direction is selected, e.g.,

$\langle \phi \rangle = \frac{1}{\sqrt{2}} (0,\, v)^T$, breaking the electroweak $SU(2) \times U(1)$ gauge symmetry into the electromagnetic $U(1)_{EM}$ symmetry only for the Higgs vacuum. The gauge bosons associated with the broken symmetry, the W and Z bosons, acquire mass via their couplings to the Higgs field, while photons remain massless as they follow the unbroken $U(1)_{EM}$ symmetry. Thus, the particles that couple to the Higgs boson acquire mass, while the gauge invariance of the underlying Lagrangian is preserved.

2.4. Gauge structure in gravitation

In a simplified view, Yang-Mills theory and general relativity can both be interpreted as frameworks that introduced covariant derivatives to advance the state of physics at their respective times. Both theories consider the local symmetry of the physical systems containing covariant structure. In Yang-Mills theory, the gauge field is introduced as the connection, while general relativity introduces the Levi-Civita connection. The physical fields in each theory, namely the field strength and the curvature, are constructed from the respective connections. The covariant derivatives in each theory satisfy the Bianchi identity, each as a consequence of the Jacobi identity of the gauge field (2.16) and the Levi-Civita connection.

$$[\mathrm{D}_\mu, [\mathrm{D}_\nu, \mathrm{D}_\rho]] + [\mathrm{D}_\nu, [\mathrm{D}_\rho, \mathrm{D}_\mu]] + [\mathrm{D}_\rho, [\mathrm{D}_\mu, \mathrm{D}_\nu]] = 3!\, \mathrm{D}_{[\mu} F_{\nu\rho]} = 0\,. \quad (2.35)$$

The crucial distinction, however, lies in how the connection is constructed in each theory, reflecting the difference in the transformations under which each physical system remains invariant. Let us recall the example of the colour wheel. In this case, the colour wheel is invariant under a transformation of its physical feature, whereas the physical system of general relativity is invariant under coordinate transformations. The former is a Yang-Mills type symmetry where the transformation acts on an abstract space, while the latter is the diffeomorphism invariance, where the transformation takes place directly on the spacetime manifold. The gauge-field connection lives on the fibre bundle, a projection lifted from the base object, unaffected by the geometry of the base object, whereas the Levi-Civita connection lives on the base object – the spacetime manifold. The gauge field in Yang-Mills theory is a fundamental field that does not require another field in its construction, while the Levi-Civita connection depends on the spacetime metric. This distinction manifests in how the gauge field and the Levi-Civita connection transform.

2.4.1. Tetrad formalism

Since Weyl's second attempt at unification [17], tetrad formalism has been the standard approach for incorporating a gauge field into gravitational theory. This formalism employs the orthonormal frame basis ${\rm e}_\mu{}^a$ of the tangent space at each point in spacetime, bridging the background spacetime and the tangent space as

$$g_{\mu\nu} = {\rm e}^a{}_\mu {\rm e}^b{}_\nu \eta_{ab} \,. \tag{2.36}$$

This provides the framework for defining spinor fields on a curved spacetime, since they transform under the local Lorentz transformations (2.22), not under coordinate transformations, and the spacetime metric alone does not provide the algebraic basis for spinor representations. Due to this implementation of spin structure, the connection that supports covariant parallel transport in this orthonormal frame is called the spin connection. The spin connection $\omega_\mu = \omega_\mu{}^{ab} L_{ab}$ transforms in the same manner as (2.3), functioning as a gauge field, and relates to the affine connection as

$$\omega_\mu{}^{ab} = {\rm e}_\nu{}^a \left(\partial_\mu {\rm e}^{\nu b} + \Gamma^\nu{}_{\rho\mu} {\rm e}^{\rho b} \right) \,. \tag{2.37}$$

While the Levi-Civita connection ensures that distances and angles measured on the spacetime manifold are preserved along parallel transport, the spin connection preserves distances and angles in the local orthonormal frame. This difference allows the latter framework to accommodate non-vanishing torsion, a field sourced by spin that represents twisting of the tangent space along parallel transport.

$$T^a{}_{\mu\nu} \equiv 2{\rm D}_{[\mu} {\rm e}^a{}_{\nu]} = 2{\rm e}^a{}_\rho \Gamma^\rho{}_{[\mu\nu]} \,. \tag{2.38}$$

In general relativity, the symmetry between the indices μ, ν makes it a torsion-free theory. The presence of torsion can, in principle, lead to physical effects such as modifications to spinning particle motion and altered interactions in high spin density regions. Geometric extension and modification of general relativity involving torsion include Einstein Cartan gravity [36], teleparallel gravity [37], and metric affine gravity [38].

The gauge field ω_μ now allows the formulation of the curvature on equal footing with the field strength in Yang-Mills theory.

$$\boldsymbol{F}^{ab} = F^{ab}{}_{\mu\nu} \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu = \mathrm{D}_{[\mu}\omega_{\nu]ab} = \mathbf{d}\boldsymbol{\omega}^{ab} + \boldsymbol{\omega}^a{}_c \wedge \boldsymbol{\omega}^{cb} \,, \tag{2.39}$$

Then, the Yang-Mills type gauge-invariant gravitational action follows as

$$I = \int \epsilon_{abcd} \boldsymbol{F}^{ab} \wedge \boldsymbol{F}^{cd} \,. \tag{2.40}$$

This corresponds to the quadratic Gauss-Bonnet term in general relativity, which reduces to a surface term and does not contribute to the local dynamics in 4D spacetime. The first order gauge-invariant action corresponding to the Einstein-Hilbert action is known as the Palatini action.

$$I = \int \mathrm{d}^4 x \; \mathrm{e} \; \mathrm{e}^a{}_\mu \mathrm{e}^b{}_\nu F_{ab}{}^{\mu\nu} = \int \epsilon_{abcd} \mathbf{e}^a \wedge \mathbf{e}^b \wedge \boldsymbol{F}^{cd} \,. \tag{2.41}$$

The rest of this thesis enjoys the brevity offered by differential forms. This formalism allows coordinate-independent formulations with reduced indices, and naturally encodes important geometric properties, such as antisymmetry. These advantages make them particularly well-suited for the discussions that follow. A systematic list of mathematical definitions can be found in [39] and a comprehensive explanation for the technical details can be found in [40].

2.4.2. Which Lie group?

For the description of the tetrad formalism in Section 3.2, we employed the Lorentz generator to construct the gauge field for convenience, but the same procedure to construct a Yang-Mills type action can be applied to different Lie groups by adopting relevant generators. In fact, historical attempts at gauge-theoretic formulations of gravity have involved the following Lie groups, where the Lorentz group is a subgroup to all the other four groups, and the additional invariances imposed by these groups are listed below.

Table 4: Options for internal gauge symmetry of gravitation

Gauge group	Transformations	Geometry
Lorentz [41]	rotation and boost *	R, T
Poincaré [42]	* + translation	R, T
Weyl [43, 44]	* + scale transformation	R, Q
$GL(4)$ [45]	* + scale and shear	R, T, Q
(anti) de Sitter [26, 27]	* + transvection	R, T

Geometrically speaking, Lorentz, Poincaré, and (a)dS gauge theories introduce torsion (indicated as T in Table 4) in addition to curvature (R), coinciding with the geometric modification provided by Einstein Cartan gravity. Similarly, GL(4) gauge gravity introduces torsion and non-metricity, and together with the translational symmetry $T(4)$, corresponds to the metric affine gravity.

A specific formulation of tetrads has become the standard over the course of these developments in the gauge-theoretic formulation of gravity. This involves two fundamental fields: the non-Lorentz part of the gauge field $\boldsymbol{\theta}$ and the symmetry-breaking field ξ.

$$\mathbf{e}^a = \boldsymbol{\theta}^a + \mathbf{D}\xi^a \,. \tag{2.42}$$

A similar approach to the Higgs mechanism in Section 2.3.2 can be incorporated into a gauge theory of gravity via ξ to ensure the resulting symmetry to be Lorentz symmetry: the foundational spacetime symmetry. This mechanism has been implemented in various gauge theories of gravity, and the (anti) de Sitter gauge theory has been particularly successful, being the first gravitational theory to induce cosmological constant without adding it by hand. To showcase this example, we can consider a dS gauge invariant action.

$$I = \frac{1}{4!}\int \epsilon_{ABCDE}\boldsymbol{F}^{AB} \wedge \boldsymbol{F}^{CD}\xi^E \,. \tag{2.43}$$

Here, the capital indices run through $A = 0, ..., 4$, and we denote the 4D indices with lowercase letters $a = 0, ..., 3$. The dS symmetry is broken into

Lorentz symmetry by $\xi^A = \ell\delta^A_4$. Since the cosmological constant is the intrinsic curvature of the spacetime, this relates to the dS scale length as $\ell = \sqrt{\xi^A\xi_A}$ as $\Lambda = 3\ell^{-2}$. Redefining the additional gauge field component $\boldsymbol{A}^{4a}$ as $\boldsymbol{A}^a \equiv \ell^{-1}\boldsymbol{A}^{4a}$ clarifies the additional terms acquired by dS covariant field strength.

$$\boldsymbol{F}^{ab} = (\mathbf{D}\boldsymbol{A})^{ab} - \ell^{-2}\boldsymbol{A}^a \wedge \boldsymbol{A}^b\,, \tag{2.44a}$$

$$\boldsymbol{F}^{a} = (\mathbf{D}\boldsymbol{A})^{a} - \mathbf{d}\log\ell \wedge \boldsymbol{A}^a\,. \tag{2.44b}$$

With the coframe fields defined as $\mathbf{e}^a = \boldsymbol{A}^a + \mathbf{D}\xi^a$ and the Lorentz gauge covariant field strength $\boldsymbol{R}^{ab} = (\mathbf{D}\boldsymbol{A})^{ab}$, the symmetry broken Lorentz gauge invariant action reproduces the gravitational action with a Gauss-Bonnet term and the Palatini action, and incorporates the cosmological constant as a kinematic effect.

$$\int \epsilon_{abcd}\,\ell\left(\boldsymbol{R}^{ab} \wedge \boldsymbol{R}^{cd} - \frac{2}{3}\Lambda\,\mathbf{e}^a \wedge \mathbf{e}^b \wedge \boldsymbol{R}^{cd} + \frac{\Lambda^2}{9}\,\mathbf{e}^a \wedge \mathbf{e}^b \wedge \mathbf{e}^c \wedge \mathbf{e}^d\right)\,. \tag{2.45}$$

The non-vanishing torsion $\boldsymbol{T}^a = \mathbf{D}\mathbf{e}^a$ indicates the theory prescribes Cartan geometry, and as we will see, the inclusion of cosmological constant term in the action is in fact not unique to the (a)dS gauge theory, but a feature shared with other Cartan geometric theories, such as Lorentz and Poincaré gauge theory.

3. Cartan Khronon

Building on the advantages of tetrad formalism and the symmetry-breaking mechanism in gravitational theory, an optimised formulation of tetrad with a symmetry-breaking field was first proposed in [4], with potential implications for dark matter effect. The original theory chooses the Lorentz group, the most basic symmetry for spacetime, as the gauge symmetry. Then a natural question one may ask is: what is the resulting symmetry after the symmetry breaking? As it will turn out, the symmetry-breaking field in this theory is a clock field, hence dubbed the "khronon", which breaks Lorentz symmetry into spatial symmetry of the $SO(3)$ group.

Traditionally, the asymmetry between space and time has been modelled by assigning different metric signatures for temporal and spatial dimensions, with a Minkowski manifold as the spacetime background. This description of spacetime necessitates the imaginary treatment of time and contributes to the lack of time evolution in quantum theories, often referred to as the problem of time [46, 47]. The newly introduced formulation with a fundamental, background-independent clock field no longer requires the fixed signature split of $(-,+,+,+)$, redefining how spacetime structure is formulated. This prescription offers a framework for a real-valued description of space-time, entailing rich theoretical and phenomenological implications.

3.1. Theory framework

This theory develops from a minimal number of fundamental fields: the gauge field $\boldsymbol{A}^{ab}$ and the symmetry-breaking field ϕ^a. An economical formulation of the tetrad is introduced as

$$\mathbf{e}^a = \mathbf{D}\phi^a \,. \tag{3.1}$$

With the gauge choice $\phi^a = \delta^a{}_0\phi$, a more explicit expression of the tetrads in the symmetry broken phase

$$\mathbf{e}^0 = \mathbf{d}\phi \,, \quad \mathbf{e}^i = \phi \boldsymbol{A}^i{}_0 \,, \tag{3.2}$$

reveals the role of the khronon field as a clock field, and how the space-time asymmetry emerges from symmetry breaking. The spacetime metric is not required as an independent fundamental field, but is instead defined from these two fundamental fields only in the symmetry-broken phase, making this a pregeometric theory.

$$g_{\mu\nu}\mathbf{d}x^{\mu} \otimes \mathbf{d}x^{\nu} = \eta_{00}\mathbf{d}\phi^2 + \eta_{ij}\phi^2 \boldsymbol{A}^i{}_0 \otimes \boldsymbol{A}^j{}_0 \,. \tag{3.3}$$

In this theory, only the electric components of the connection enter the corresponding metric.

As discussed in the previous chapter for the general gauge theory, a gauge invariant action only permits a specific structure (2.26) for the gauge field to enter the action. With the minimal fundamental fields of this theory, we have 0-form ϕ^a, 1-form $\mathbf{e}^a = \mathbf{D}\phi^a$ and 2-form $\boldsymbol{F}^{ab} = (\mathbf{D}\boldsymbol{A})^{ab}$ as the building blocks for a four dimensional gauge invariant action. When the invariance under the generic $\mathrm{O}(m,n)$ $(m+n=4)$ gauge transformations and the global shift $\phi^a \to \phi^a + \xi^a$ transformations (where $\mathbf{D}\xi^a = 0$) are imposed, possible components of the action reduce to a handful of terms below. Classified by the order of the khronon field ϕ^a,

$$I_{(0)} = g_G \int \epsilon_{abcd}\boldsymbol{F}^{ab} \wedge \boldsymbol{F}^{cd} + g_P \int \boldsymbol{F}_{ab} \wedge \boldsymbol{F}^{ab} \,, \tag{3.4a}$$

$$\begin{aligned} I_{(2)} = &\int \epsilon_{abcd}\mathbf{e}^a \wedge \mathbf{e}^b \wedge \left(g_+ {}^+\boldsymbol{F}^{cd} + g_- {}^-\boldsymbol{F}^{cd}\right) \\ &+ \tilde{g} \int \left(\boldsymbol{F}_{ab} \wedge \boldsymbol{F}^a{}_c \phi^b \phi^c - \boldsymbol{F}_{ab} \wedge \mathbf{e}^a \wedge \mathbf{e}^b\right) \,, \end{aligned} \tag{3.4b}$$

$$I_{(4)} = \frac{\lambda}{4!} \int \epsilon_{abcd}\mathbf{e}^a \wedge \mathbf{e}^b \wedge \mathbf{e}^c \wedge \mathbf{e}^d \,. \tag{3.4c}$$

This is the generic form of the gauge-invariant action for an arbitrary subgroup of $O(m,n)$, which includes the Lorentz $SO(1,3)$, Euclidean $SO(4)$, and their double covers. The quadratic terms of $I_{(0)}$ reduce to boundary terms, each corresponding to the Gauss-Bonnet and Pontryagin terms, and the quadratic sector $I_{(2)}$ contains the Palatini and Holst combinations. In this framework, however, a weighted (anti) self-dual decomposition is introduced, as will be explained in more detail later in Section 3.1.3. As demonstrated in (2.45) for the dS gauge-invariant action with symmetry-breaking mechanism, the $I_{(4)}$ term is associated with the cosmological constant.

This makes g_+, g_-, λ to be the effective coupling constants that determine the dynamics of the theory at classical level. Some key theories and phases that stem from the choice of gauge group and these constants are as schematised below.

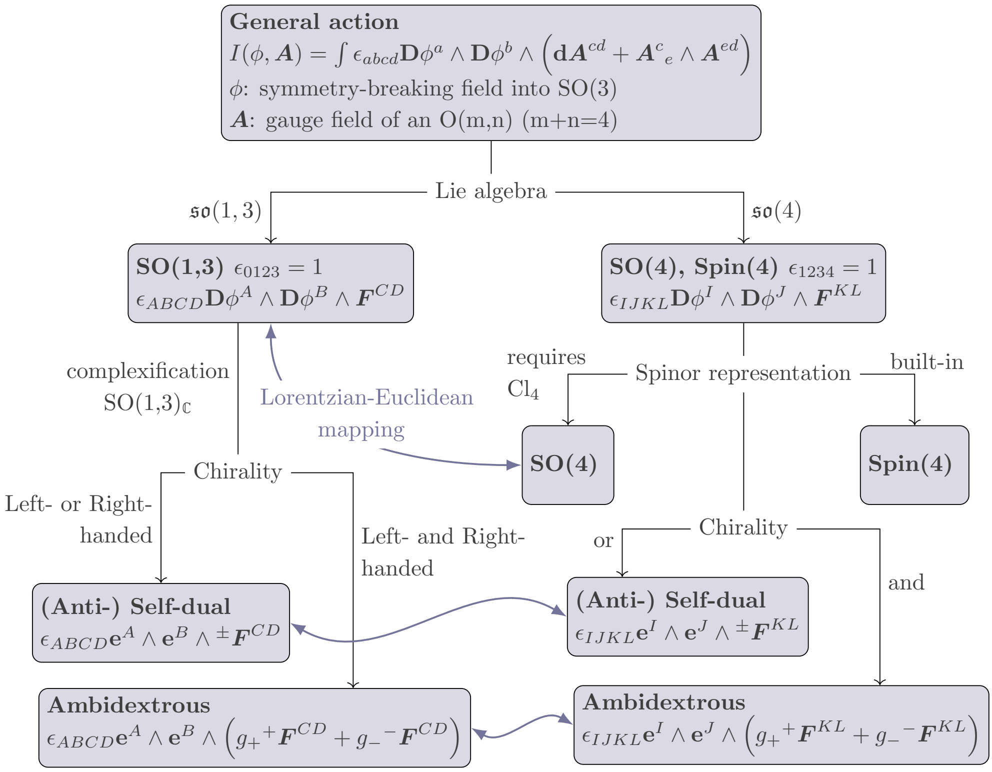

Figure 2: Theory scheme

3.1.1. Group and algebra

As outlined in Section 2.2, the choice of symmetry group determines the algebraic structure of the theory. Due to its tie to the Minkowski metric (2.13), the Lorentz group $SO(1,3)$ is the most fundamental symmetry for all the theories formulated on the metrical spacetime manifold. After complexification, the Lorentz group exhibits an interesting correspondence with the electroweak symmetry $SU(2)$, as demonstrated through the decomposition of the Lorentz generators into two sets that satisfy respective commutation relations (2.18). The chiral formulation of spinors is a notable manifestation of this split into two subgroups.

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} , \quad \psi_\pm = \frac{1}{2}\left(1 \pm \gamma^5\right)\psi \,. \tag{3.5}$$

The full Dirac spinor ψ is the wave function of the Dirac equation, and the chiral parts $\psi_\pm$ describe spin-$\frac{1}{2}$ fermions, where the $\pm$ indicates the right- and left-handedness of the spin orientation with respect to the fermion's motion.

However, such correspondence between different groups is not unique to the Lorentz group. Most fundamentally, $SU(2)$ and $SO(3)$ share the same algebra

$$\mathfrak{su}(2) = \mathfrak{so}(3) \,, \tag{3.6a}$$

and $SU(2)$ is a double cover of $SO(3)$, i.e., there is a 2-to-1 map from $SU(2)$ to $SO(3)$. In a similar manner, for $SU(2) \times SU(2)$ and $SO(4)$,

$$\mathfrak{su}(2) \oplus \mathfrak{su}(2) = \mathfrak{so}(4) \,. \tag{3.6b}$$

and $SU(2) \times SU(2)$ is a double cover of $SO(4)$. Constructed as the double covering of respective $SO(n)$ ($n > 2$), spin groups $Spin(n)$ represent spinorial rotations and play a significant role in the description of fermions. This reveals the relation

$$Spin(4) = SU(2) \times SU(2) \,. \tag{3.7}$$

The two $SU(2)$ groups each correspond to the left- and right-handed spinor components in the Euclidean framework. The difference between $Spin(n)$ and $SO(n)$ lies in their topology: the former is simply connected whilst the latter is not. Intuitively, a simply connected space has no holes, meaning any closed loop can be shrunk down to a single point.

In the Cartan Khronon framework, since the background-independent clock field assists the description of space-time asymmetry, it is unnecessary to assume a Minkowski background, and a Euclidean background can be employed. The gauge symmetry for the gravitational action is, then, not limited to $SO(1,3)$, but $SO(4)$ and $Spin(4)$ also become viable candidates. In terms of algebra, the difference between these groups comes down to the choice between $\mathfrak{so}(1,3)$ and $\mathfrak{so}(4)$. A subtle, yet pivotal difference between these algebras is found in the definition of the Levi-Civita symbol. We employ the following index convention for $\mathfrak{so}(1,3)$,

$$\epsilon_{ABCD} = \begin{cases} 1 & \text{if } (A,B,C,D) \text{ is an even permutation of (0,1,2,3)}\,, \\ -1 & \text{if } (A,B,C,D) \text{ is an odd permutation of (0,1,2,3)}\,, \\ 0 & \text{otherwise}\,, \end{cases} \tag{3.8a}$$

and for $\mathfrak{so}(4)$,

$$\epsilon_{IJKL} = \begin{cases} 1 & \text{if } (I,J,K,L) \text{ is an even permutation of (1,2,3,4)}\,, \\ -1 & \text{if } (I,J,K,L) \text{ is an odd permutation of (1,2,3,4)}\,, \\ 0 & \text{otherwise}\,, \end{cases} \tag{3.8b}$$

where we treat the index 4 as the temporal index in the Euclidean setting.

Between $Spin(4)$ and $SO(4)$ groups, while both groups yield the same gravitational action based on their shared algebra $\mathfrak{so}(4)$, the conceptual difference lies in what each fundamental representation describes. An $SO(4)$ transformations rotate vectors, while a $Spin(4)$ transformation 'rotates' spinors. As demonstrated for the example of Lorentz group in Section 2.2, in an $SO(1,3)$ or $SO(4)$ frameworks where spinorial rotation is not intrinsic, it requires Clifford algebra to describe spinor transformations, whereas in a $Spin(4)$ gauge theory, the spinor representations are the fundamental representations of the group, hence the description of fermions is inherent, and the $SO(4)$ generators can be constructed from the spinor representations.

3.1.2. Euclidean-Lorentzian correspondence

Having established the algebraic difference between the groups, a bridge between the $\mathfrak{so}(1,3)$ and $\mathfrak{so}(4)$ frameworks is required. Even when the entire derivation process of a physical system can be carried out in the Euclidean framework with no adversity, once the result is obtained, a Euclidean description of a time-dependent phenomenological solution requires a translation into the Lorentzian framework where the physical measurements take place. For example, with the different formulations of time between the two frameworks, a wave equation in one framework becomes an equation of exponential growth in the other.

A simple replacement of time $t \to it_E$, known as the Wick rotation, is the standard procedure to introduce Euclidean time in quantum theories and black hole thermodynamics. In quantum theories, the probability amplitude of transition from an initial state $|g_1, \phi_1, S_1\rangle$ to a final state $|g_2, \phi_2, S_2\rangle$ is formulated using path-integral

$$\langle g_1, \phi_1, S_1 | g_2, \phi_2, S_2 \rangle = \int \mathrm{D}[g, \phi] \exp(iI[g, \phi]) \,, \tag{3.9}$$

to measure the phase contribution $\exp(iI[g, \phi])$ over all the possible field configurations $\mathrm{D}[g, \phi]$. With a real action I, the path-integral fails to converge, and Euclidean time is introduced for mathematical convenience. This approach has been proved to be particularly useful for the description of black hole thermodynamics, as the partition function Z can be formulated as a periodic function in Euclidean time

$$Z = \mathrm{Tr}\{\exp(-\beta H)\} = \int \mathrm{D}[g, \phi] \exp(-I_E[g, \phi]) \,, \tag{3.10}$$

and this allows the identification of Hawking temperature $T = 1/\beta$ and intrinsic entropy $S = \beta \frac{\partial I_E}{\partial \beta} - I_E$ for various black hole solutions, such as Schwarzschild and Kerr-Newman, as well as de Sitter space [31].

This procedure is an extrapolation of Green functions and path integrals in Minkowski spacetime into Euclidean space via analytic continuation, treating the time coordinate as a complex variable [48]. However, this procedure is not applicable to arbitrary equation, e.g., requires a spacetime metric to be purely electric to have a Euclidean correspondence [32]. In addition, a simple index rotation of the gamma matrices with $\gamma^4 = -i\gamma^0$ fails to establish a viable Euclidean description of spinors, causing the loss of Euclidean symmetry, hermiticity $\psi_\pm^\dagger = \psi_\mp$, and the violation of the reality condition [49]. In the attempt to maintain these qualities in both frameworks, several approaches have been considered. The Osterwalder–Schrader reflection positivity [50, 51] provides a method to reconstruct Lorentz symmetry and restore hermiticity of spinors at the cost of non-hermitian action, and the Schwinger/Zumino approach [52, 53] introduces a consistent rotation matrix for spinors to restore Euclidean symmetry and hermiticity of both spinors and action. However, neither restores reality for Majorana spinors, i.e., their components become complex in the Euclidean regime. In addition, often referred to as the doubling problem, the former approach requires the spinor and its conjugate to be treated as independent Grassmann integration variables, and the latter requires the number of spinor components to be doubled [54].

In the Cartan Khronon framework, we propose a simple yet improved procedure that may evade these problems. With the khronon field serving as a clock field independent of the background, this allows a formulation of coordinate-independent bridge between the two frames, instead of a background-dependent coordinate trick. When constructing tetrads from the khronon field, the physical dimensions of these fields must be handled with a care. If the khronon were introduced as a dimensionless field, this causes the tetrads (3.2) to acquire dimensions of energy, even though frame fields are generally dimensionless. Considering the role of the khronon as a clock field, a more natural prescription would be to introduce dimensionful parameters m_P and κ in respective framework, and establish the mapping procedure as below.

$$(m_P, t) \xleftrightarrow[\text{correspondence}]{\text{Lorentzian-Euclidean}} (\kappa, \tau) \tag{3.11}$$

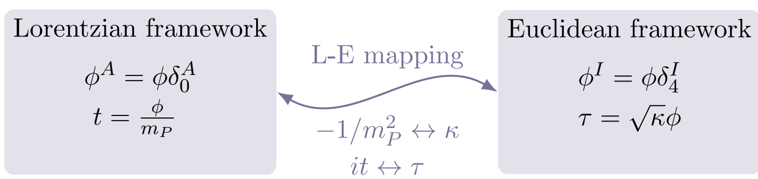

Figure 3: Treatment of time in and between $\mathfrak{so}(1,3)$ and $\mathfrak{so}(4)$ frameworks

In this formulation, the Wick rotation acts not only on time but also on two dimensionful parameters m_P and κ, as it should be. Time and these newly introduced parameters are all real-valued in their own 'home' frameworks, and m_P in the Lorentzian framework is identified as the Planck mass from the Friedmann equation, as will be discussed later in Chapter 4. This means that the correspondence between the Lorentzian and Euclidean descriptions is not just a formal substitution of coordinates: it is a dimensionally controlled map between two physically meaningful setups.

The bridge between the spinor representations in each framework is then constructed using a unitary rotation $U = \exp\left(\gamma^4\gamma^5\frac{\theta}{2}\right)$ of the spinor basis. With this rotation, the gamma matrices can be mapped between the two frameworks without violating each group symmetry [49]. The rotation by $\theta = \pi/2$ allows the consistent formulation of the gamma matrices in Euclidean framework

$$\gamma_E^i = \gamma_L^i\,, \quad \gamma_E^4 = \gamma_L^5\,, \quad \gamma_E^5 = \gamma_E^1\gamma_E^2\gamma_E^3\gamma_E^4 = -\gamma^4\,, \tag{3.12}$$

which satisfy the Euclidean Clifford algebra and the corresponding hermiticity relations.

$$\left\{\gamma_E^I, \gamma_E^J\right\} = 2\delta^{IJ}\mathbb{1}, \qquad (\gamma_E^I)^\dagger = -\gamma_E^5\gamma_E^I\gamma_E^5 = \gamma_E^I, \qquad (\gamma_E^5)^2 = \mathbb{1}. \tag{3.13a}$$

In particular, the Euclidean temporal matrix reflects the spatial gamma matrices under conjugation.

$$\gamma_E^4\gamma_E^i\gamma_E^4 = -\gamma_E^i, \qquad \gamma_E^4\gamma_E^4\gamma_E^4 = \gamma_E^4. \tag{3.13b}$$

This new system can be understood as a reinterpretation of γ^4 and γ^5. Then Euclidean Weyl spinors are

$$\psi_{E\pm} = \frac{1}{2}\left(1 \mp \gamma_E^5\right)\psi_E = \frac{1}{2}\left(1 \pm \gamma^4\right)\psi_E\,, \tag{3.14}$$

and the Dirac adjoint in Euclidean description is $\bar{\psi} = \psi_E^\dagger\gamma_E^5 = -\psi_E^\dagger\gamma^4$. In a theory where the gamma matrices are available as the generators, a vector field can be written in terms of spinors, and the khronon field is no exception.

$$\phi^I = \bar{\psi}\gamma_E^I\psi \tag{3.15}$$

This shows that the khronon is not an external add-on, but can be represented internally within the spinorial structure of the theory. It can be checked that the khronon field in the symmetry broken phase $\phi^4 = \bar{\psi}\gamma_E^4\psi$ is invariant under the parity transformation

$$\gamma_E^i \to -\gamma_E^i\,, \quad \gamma_E^4 \to \gamma_E^4\,, \quad \gamma_E^5 \to -\gamma_E^5\,, \quad \psi \to \gamma_E^5\psi\,. \tag{3.16}$$

3.1.3. Chirality in gravitation

With the $Spin(4)$ structure or the spin structure supplied by Clifford algebra, it is possible to implement chiral structure not only in spinors but also in gravitation. The chiral description of gravity has been studied in several contexts, most notably in loop quantum gravity [55–57]. Although its physical interpretation remains somewhat ambiguous, it induces observable implications in phenomenology, as we will see across the following chapters. The chiral projection of gravity is formulated through its (anti) self-dual decomposition, which arises from the following structure of the Lie algebra we prescribe.

$$\mathfrak{so}(1,3)_{\mathbb{C}} \simeq \mathfrak{sl}(2,\mathbb{C}) \oplus \overline{\mathfrak{sl}(2,\mathbb{C})}\,, \quad \mathfrak{so}(4) = \mathfrak{su}(2) \oplus \mathfrak{su}(2)\,. \tag{3.17}$$

This decomposition requires the Hodge star operator $\star$ which represents self-duality for the 2-forms.

$$\star X_{ab} = \frac{1}{2}\epsilon_{ab}{}^{cd}X_{cd}\,, \quad \star\star X_{ab} = \frac{1}{4}\epsilon_{abcd}\epsilon^{cdef}X_{ef} = \sigma\delta^c_{[a}\delta^d_{b]}X_{cd}\,. \tag{3.18}$$

The parameter σ is introduced for a general formulation and differs depending on the algebra, such that $\sigma = 1$ for $\mathfrak{so}(4)$ and $\sigma = -1$ for $\mathfrak{so}(1,3)$. This operator plays a role analogous to that of the γ^5 matrix in the Weyl spinor decomposition (3.5). The corresponding chiral projectors are then defined so that the (anti) self-dual parts are separated consistently in both Euclidean and Lorentzian settings.

$${}^{\pm}P_{ab}{}^{cd} = \frac{1}{2}\left(1 \pm \frac{1}{\sqrt{\sigma}}\star\right)_{ab}{}^{cd} = \frac{1}{2}\left(\delta^c_{[a}\delta^d_{b]} \pm \frac{1}{2\sqrt{\sigma}}\epsilon_{ab}{}^{cd}\right)\,. \tag{3.19}$$

The contrast of the complexification in $\mathfrak{so}(1,3)$ and the preserved reality in $\mathfrak{so}(4)$ is most evident here. It is convenient to note that the chiral projections ${}^{\pm}X_{ab} \equiv {}^{\pm}P_{ab}{}^{cd}X_{cd}$ satisfy the following relation.

$$\star{}^{\pm}X_{ab} = \pm\sqrt{\sigma}\,{}^{\pm}X_{ab}\,. \tag{3.20}$$

With a $\mathfrak{so}(4)$ or $\mathfrak{so}(1,3)$ generator S_{ab}, the (anti) self-dual decomposition can be formulated in an algebra-independent way, providing an explicit decomposition with respect to the generators.

$$
\begin{aligned}
X &= \tfrac{1}{2}X^{ab}S_{ab} = \tfrac{1}{2}\left({}^{+}X^{ab}\,{}^{+}S_{ab} + {}^{-}X^{ab}\;{}^{-}S_{ab}\right) \\
&= {}^{+}X^{ti}\,{}^{+}S_{ti} + \tfrac{1}{2}{}^{+}X^{ij}\,{}^{+}S_{ij} + {}^{-}X^{ti}\,{}^{-}S_{ti} + \tfrac{1}{2}{}^{-}X^{ij}\,{}^{-}S_{ij} \\
&= \left({}^{+}X^{tk} + \tfrac{\sqrt{\sigma}}{2}{}^{+}X^{ij}\epsilon_{ij}{}^{tk}\right)\,{}^{+}S_{tk} + \left({}^{-}X^{tk} - \tfrac{\sqrt{\sigma}}{2}{}^{-}X^{ij}\epsilon_{ij}{}^{tk}\right)\,{}^{-}S_{tk} \\
&= {}^{++}X^{tk}\;{}^{+}S_{tk} + {}^{--}X^{tk}\,{}^{-}S_{tk} = 2^{+}X^{tk}\;{}^{+}S_{tk} + 2^{-}X^{tk}\,{}^{-}S_{tk} \\
&\equiv {}^{+}X^{i}\;{}^{+}S_{ti} + {}^{-}X^{i}\,{}^{-}S_{ti}\,.
\end{aligned}
\tag{3.21}
$$

The corresponding (anti) self-dual connection ${}^{\pm}\boldsymbol{A}$, and the (anti) self-dual curvature ${}^{\pm}\boldsymbol{F}$ are then straightforwardly defined as

$$
{}^{\pm}\boldsymbol{F}^{ab} = \mathbf{d}^{\pm}\boldsymbol{A}^{ab} + {}^{\pm}\boldsymbol{A}^{a}{}_{c} \wedge {}^{\pm}\boldsymbol{A}^{cb}\,. \tag{3.22}
$$

The (anti) self-duality imposes the following relation between the time-related 'electric' components and the purely spatial 'magnetic' components of the spin connection and curvature.

$$
{}^{\pm}\boldsymbol{A}^{i} \equiv 2\,{}^{\pm}\boldsymbol{A}^{ti} = \pm\frac{1}{\sqrt{\sigma}}\epsilon^{ti}{}_{jk}\boldsymbol{A}^{jk}\,, \tag{3.23a}
$$

$$
{}^{\pm}\boldsymbol{F}^{i} \equiv 2^{\pm}\boldsymbol{F}^{ti} = \pm\frac{1}{\sqrt{\sigma}}\epsilon^{ti}{}_{jk}\boldsymbol{F}^{jk}\,. \tag{3.23b}
$$

Meanwhile, being rank-1 2-forms, torsion, along with the tetrads, cannot be decomposed into (anti) self-dual parts in the same manner. Instead, these fields can be written in terms of (anti) self-dual pieces of the connection and curvature.

$$
\mathbf{e}^{i} = \phi\left({}^{+}\boldsymbol{A}^{i}{}_{t} + {}^{-}\boldsymbol{A}^{i}{}_{t}\right)\,, \tag{3.24a}
$$

$$
\boldsymbol{T}^{i} = \mathbf{D}\mathbf{e}^{i} = \phi\left({}^{+}\boldsymbol{F}^{i}{}_{t} + {}^{-}\boldsymbol{F}^{i}{}_{t}\right)\,. \tag{3.24b}
$$

This shows that chirality is now assigned to the spin connection, instead of a physical particle. This decomposition tells which components of these geometric fields couple to right- and left-handed fermions through the covariant derivative.

$$
\mathbf{D}\psi = \left(\mathbf{d} + {}^{+}\boldsymbol{A}\right)\psi_{+} + \left(\mathbf{d} + {}^{-}\boldsymbol{A}\right)\psi_{-}\,. \tag{3.25}
$$

How respective chiral components would manifest as observable geometric features of gravity remains an open question.

3.2. Dynamics

Having established the theoretical framework, we can now discuss the dynamics of each model. Since the (anti) self-dual case is only a special limit, we proceed with the general case with weighted handedness. Since the surface integrals and topological terms do not affect the classical dynamics, the simplest gravitational action we can consider without losing generality reads

$$I = \frac{1}{2}\int \epsilon_{abcd}\mathbf{e}^a \wedge \mathbf{e}^b \wedge \left(g_+{}^+\boldsymbol{F}^{cd} + g_-{}^-\boldsymbol{F}^{cd}\right) \\ = \sqrt{\sigma}\int \mathbf{D}\phi^a \wedge \mathbf{D}\phi^b \wedge \left(g_+{}^+(\mathbf{D}\boldsymbol{A})_{ab} - g_-{}^-(\mathbf{D}\boldsymbol{A})_{ab}\right) , \tag{3.26}$$

where $(\mathbf{D}\boldsymbol{A})_{ab} \equiv \mathbf{d}\boldsymbol{A}_{ab} + \boldsymbol{A}_a{}^c \wedge \boldsymbol{A}_{cb}$ and the hidden σ-dependence of the (anti) self-dual 2-forms determines whether the action is real or complex.

3.2.1. Canonical source currents

In general relativity and many other gravitational theories, the energy-momentum tensor, as the source of gravity, is defined as the variation of matter action with respect to the metric.

$$\delta I_{matter} = \frac{1}{2}\int \sqrt{-g}\delta g^{\mu\nu}T_{\mu\nu} . \tag{3.27}$$

This metrical energy-momentum tensor is symmetric by construction. However, in the present pregeometric and gauge-theoretic formulation, it is possible to establish canonical source currents in line with the Noether currents. Through the variation of the matter action with respect to each fundamental field, energy-momentum current $\boldsymbol{t}^a$ associated with translational symmetry and angular momentum current $\boldsymbol{O}^{ab}$ associated with the relevant Lie group symmetry are introduced.

$$\delta I_{matter} = \int \left(\delta\mathbf{D}\phi_a \wedge \boldsymbol{t}^a + \delta\boldsymbol{A}_{ab} \wedge \boldsymbol{O}^{ab}\right) . \tag{3.28}$$

The source current $\boldsymbol{O}^{ab}$ corresponds to what is also known as the hypermomentum tensor. [58] The canonical energy momentum current $\boldsymbol{t}^a$ can be written in terms of its tensor components $t_{ab} = t_{\mu\nu}\partial_a{}^\mu\partial_b{}^\nu$ as $\boldsymbol{t}^a = t^a{}_b \star \mathbf{e}^b$. Let us define these current components to be dimensionless across

Lorentzian formulation t_{AB} and Euclidean formulation t_{IJ}.

$$\boldsymbol{t}^A = \frac{i}{m_P} t^A{}_B \star \mathbf{e}^B \,, \quad \boldsymbol{t}^I = \sqrt{\kappa} t^I{}_J \star \mathbf{e}^J \,, \tag{3.29}$$

This allows the matter sector to be described in a way that matches the dimensions of the geometric variables used in the theory.

With the fundamental spinor representations, it is natural to consider fermions as the matter source. In the $Spin(4)$ framework, we can formulate the Euclidean Dirac Lagrangian in Hermitian form, by rotating the Lorentzian Dirac Lagrangian $i\psi^\dagger\gamma^0 \left(\not{D} + m\right)\psi$, using the unitary matrix $U = \exp\!\left(\gamma^4\gamma^5\frac{\theta}{2}\right)$ introduced in Section 3.1.2.

$$i\psi^\dagger U\gamma^0 \left(\not{D} + m\right) U\psi = \psi^\dagger\gamma^4 U^{-1} \left(\not{D} + m\right) U\psi = \bar{\psi}_E \left(\not{D}_E + m\right)\psi_E \,. \tag{3.30}$$

In the pregeometric formulation, the covariant derivative of spinor fields is defined with the matrix representation of the generator $S_{ab} = \frac{1}{2}\gamma_{[a}\gamma_{b]}$.

$$\mathbf{D}\psi = \mathbf{d}\psi + \frac{1}{2}\boldsymbol{A}^{ab} S_{ab}\psi \,, \quad \mathbf{D}\bar{\psi} = \mathbf{d}\bar{\psi} - \frac{1}{2}\boldsymbol{A}^{ab}\bar{\psi} S_{ab} \,. \tag{3.31}$$

Then a generalised action for fermions can be built from following Hermitian 1-form.

$$\frac{1}{2}\left[\left(\bar{\psi}\gamma^a\mathbf{D}\psi\right) + \left(\bar{\psi}\gamma^a\mathbf{D}\psi\right)^\dagger\right] = \frac{1}{2}\left(\bar{\psi}\gamma^a\mathbf{D}\psi - \mathbf{D}\bar{\psi}\gamma^a\psi\right) \tag{3.32}$$

From the full action including mass term

$$I = \int \left[\frac{1}{2}\star\mathbf{e}^a \wedge \left(\bar{\psi}\gamma_a\mathbf{D}\psi - \mathbf{D}\bar{\psi}\gamma_a\psi\right) + \star m\bar{\psi}\psi\right] \,, \tag{3.33}$$

the energy and spin currents are derived as

$$\boldsymbol{t}^a_\psi = -\frac{1}{2}\star\left(\mathbf{e}^a \wedge \mathbf{e}^b\right) \wedge \left(\bar{\psi}\gamma_b\mathbf{D}\psi - \mathbf{D}\bar{\psi}\gamma_b\psi\right) + \star\mathbf{e}^a m\bar{\psi}\psi \,, \tag{3.34a}$$

$$\boldsymbol{O}^{ab}_\psi = -\frac{1}{4}\star\mathbf{e}^c\bar{\psi}\left(\gamma_c o^{ab} + o^{ab}\gamma_c\right)\psi \,, \tag{3.34b}$$

where a more explicit expression $t^a{}_b$ shows it is not necessarily symmetric.

$$t^{ab}_\psi \star \mathbb{1} = -\frac{1}{2}\star\left(\mathbf{e}^a \wedge \mathbf{e}^c\right) \wedge \epsilon^{\mu\nu}\mathrm{e}^b{}_\mu \left(\bar{\psi}\gamma_c\mathrm{D}_\nu\psi - \mathrm{D}_\nu\bar{\psi}\gamma_c\psi\right) + \star\delta^{ab} m\bar{\psi}\psi \,. \tag{3.35}$$

3.2.2. Equations of motion

With all the ingredients on the table, we can now consider the gravitational field equations. The variation for respective fundamental field yields

$$\frac{\delta I}{\delta\phi_a} = \mathbf{D}\left[2\left(g_+{}^+\boldsymbol{F}^a{}_b - g_-{}^-\boldsymbol{F}^a{}_b\right)\wedge\mathbf{D}\phi^b - \boldsymbol{t}^a - \lambda\star\mathbf{D}\phi^a\right], \tag{3.36a}$$

$$\begin{aligned}\frac{\delta I}{\delta\boldsymbol{A}_{ab}} &= \mathbf{D}\left[g_+{}^+\left(\mathbf{D}\phi^{[a}\wedge\mathbf{D}\phi^{b]}\right) - g_-{}^-\left(\mathbf{D}\phi^{[a}\wedge\mathbf{D}\phi^{b]}\right)\right] \\ &\quad + 2\mathbf{D}\phi^c\wedge\left(g_+{}^+\boldsymbol{F}^{[a}{}_c - g_-{}^-\boldsymbol{F}^{[a}{}_c\right)\phi^{b]} + \phi^{[a}\boldsymbol{t}^{b]} - \boldsymbol{O}^{ab} - \lambda\phi^{[a}\star\mathbf{D}\phi^{b]},\end{aligned} \tag{3.36b}$$

and the action principle tells the equations of motion. What makes this theory distinctive is the conserved 3-form $\boldsymbol{M}^a$ defined through the khronon field equation as $\mathbf{D}\boldsymbol{M}^a \equiv (3.36\text{a}) = 0$.[1] The same structure can be found in (3.36b) and this simplifies the equation significantly.

$$2\left(g_+{}^+\boldsymbol{F}^a{}_b - g_-{}^-\boldsymbol{F}^a{}_b\right)\wedge\mathbf{D}\phi^b = \boldsymbol{t}^a + \lambda\star\mathbf{D}\phi^a + \boldsymbol{M}^a\,, \quad \mathbf{D}\boldsymbol{M}^a = 0\,, \tag{3.37a}$$

$$g_+\mathbf{D}^+\left(\mathbf{D}\phi^{[a}\wedge\mathbf{D}\phi^{b]}\right) - g_-\mathbf{D}^-\left(\mathbf{D}\phi^{[a}\wedge\mathbf{D}\phi^{b]}\right) = \phi^{[a}\boldsymbol{M}^{b]} + \boldsymbol{O}^{ab}\,. \tag{3.37b}$$

As we break the symmetry between time and space by gauge-fixing the khronon field to align with the time coordinate $\phi^a = \phi\delta^a_t$, we enter the SO(3) phase. In this phase, the khronon field equation becomes

$$\left(g_+{}^+\boldsymbol{F}_i - g_-{}^-\boldsymbol{F}_i\right)\wedge\mathbf{e}^i = \boldsymbol{t}^t + \lambda\star\mathbf{e}^t + \boldsymbol{M}^t\,, \quad \mathbf{D}\boldsymbol{M}^t = 0\,, \tag{3.38a}$$

$$2\left(g_+{}^+\boldsymbol{F}^i{}_j - g_-{}^-\boldsymbol{F}^i{}_j\right)\wedge\mathbf{e}^j = \boldsymbol{t}^i + \lambda\star\mathbf{e}^i + \boldsymbol{M}^i\,, \quad \mathbf{D}\boldsymbol{M}^i = 0\,. \tag{3.38b}$$

Noting $\boldsymbol{T}^a = \mathbf{D}\mathbf{e}^a$ and hence $\boldsymbol{T}^t = \mathbf{D}\mathbf{D}\phi^t = \boldsymbol{F}^t{}_i\phi^i = 0$, the connection field equation becomes

$$(g_+ - g_-)\,\boldsymbol{T}^i\wedge\mathbf{e}^t + \sqrt{\sigma}\,(g_+ + g_-)\,\epsilon^i{}_{jk}\boldsymbol{T}^j\wedge\mathbf{e}^k = -\phi\boldsymbol{M}^i - \boldsymbol{O}^{ti}\,, \tag{3.39a}$$

$$2\,(g_+ - g_-)\,\boldsymbol{T}^{[i}\wedge\mathbf{e}^{j]} + \sqrt{\sigma}\,(g_+ + g_-)\,\epsilon^i{}_{jk}\boldsymbol{T}^k\wedge\mathbf{e}^t = \boldsymbol{O}^{ij}\,. \tag{3.39b}$$

Without explicitly solving these equations, we already see the interesting nature of this 3-form $\boldsymbol{M}^a$. Firstly, by definition, it satisfies the continuity equation, hence it is conserved. When gravity is (anti) self-dual $g_+g_- = 0$

[1] If one insists on tensor formulation, corresponding formulation of $\boldsymbol{M}^a$ can be included as the Lagrange multiplier , and finding components correspondence requires the inclusion of covariant boundary term.

and the spin current is negligible $\boldsymbol{O} = 0$, the connection equation of motion reduces to

$$\boldsymbol{T}^i \wedge \mathbf{e}^t + \sqrt{\sigma}\epsilon^i{}_{jk}\boldsymbol{T}^j \wedge \mathbf{e}^k = -\phi \boldsymbol{M}^i \,, \tag{3.40a}$$

$$2\boldsymbol{T}^{[i} \wedge \mathbf{e}^{j]} + \sqrt{\sigma}\epsilon^{ij}{}_k \boldsymbol{T}^k \wedge \mathbf{e}^t = 0 \,. \tag{3.40b}$$

Multiplying one of the equations by $\sqrt{\sigma}$ and the Levi-Civita tensor, this system reveals the vanishing spatial components $\boldsymbol{M}^i = 0$. Interpreting $\boldsymbol{M}$ as matter, this would correspond to the pressureless nature. Finally, applying the vanishing spatial components to the conservation equations,

$$\mathbf{D}\boldsymbol{M}^t = \mathbf{d}\boldsymbol{M}^t = 0 \,, \tag{3.41a}$$

$$\mathbf{D}\boldsymbol{M}^i = \boldsymbol{A}^i{}_t \wedge \boldsymbol{M}^t = \phi^{-1}\mathbf{e}^i \wedge \boldsymbol{M}^t = 0 \,, \tag{3.41b}$$

we find $\boldsymbol{M}^t$ has to satisfy

$$\boldsymbol{M}^t = \hat{\rho} \star \mathbf{e}^t = \hat{\rho} v \mathbf{d}x^1 \wedge \mathbf{d}x^2 \wedge \mathbf{d}x^3 \,, \quad \partial_t(\hat{\rho} v) = 0 \,, \tag{3.42}$$

where $\hat{\rho}$ is the effective energy density of the 3-form $\boldsymbol{M}$ and v is the unit volume of the space it spans. This suggests that its effective energy dilutes according to the spatial expansion, and concentrates as space contracts, as desired for a dark matter candidate. In this scenario, the effective energy of this component is determined by the initial condition.

In the case of ambidextrous gravity $g_+ g_- \neq 0$ where both chiral sectors contribute, and the spin current does not vanish $\boldsymbol{O} \neq 0$, while $\boldsymbol{M}$ remains conserved, some of the above properties are modified. In this case, The following rearranged connection equations of motion make the left- and right-handed sectors explicit

$$g_+ \left(\boldsymbol{T}^i \wedge \mathbf{e}^t + \epsilon^i{}_{jk}\boldsymbol{T}^j \wedge \mathbf{e}^k \right) = -\frac{1}{2}\phi \boldsymbol{M}^i - {}^+\boldsymbol{O}^i \,, \tag{3.43a}$$

$$g_- \left(\boldsymbol{T}^i \wedge \mathbf{e}^t - \epsilon^i{}_{jk}\boldsymbol{T}^j \wedge \mathbf{e}^k \right) = \frac{1}{2}\phi \boldsymbol{M}^i - {}^-\boldsymbol{O}^i \,, \tag{3.43b}$$

which becomes particularly useful for phenomenological analysis, as will be demonstrated in the subsequent chapters.

So far, we have seen that Cartan Khronon theory introduces a component $\boldsymbol{M}$ that imitates the behaviours of matter with specific properties, together with the spin current $\boldsymbol{O}$ as an additional feature beyond general relativity. Furthermore, its background-independent formulation of time provides a framework wherein the gravitational and material dynamics can be described in both Euclidean and Lorentzian settings.

4. Cosmology

Despite being the weakest among the fundamental forces, when it comes to the large-scale structure, gravity is the dominant force that governs physical phenomena, making cosmology a perfect testing ground for gravitational theories. The widely accepted standard model of cosmology today is built on general relativity, but largely relies on manually incorporated elements to account for observations that cannot be explained with general relativity. A data source responsible for the vast majority of these predictions for the early universe is the Cosmic Microwave Background (CMB), the residual electromagnetic radiation from the time of photon decoupling. Since these data provide an accumulated imprint from earlier times, this makes it possible to make deductions and place constraints on theories to explain how the universe's structure was formed.

Table 5: Timeline for the standard model of cosmology [59–62]

$t = 0$		(Singularity)	
		Planck Epoch	$\sim 10^{-33}$ cm
$\sim 10^{-43}$ sec	$\sim 10^{19}$ GeV	From QG to GR	
		Inflationary era	
$\sim 10^{-35}$ sec	$\sim 10^{16}$ GeV	Inflaton decays into hot plasma	
		Radiation-Dominated Era	
$\sim 10^{4}$ yr	~ 1 eV	Start of structure formation	$a \sim 3.3 \times 10^{-4}$
		Matter-Dominated Era	
$\sim 3.8 \times 10^{5}$ yr	~ 0.4 eV	• Photon decoupling produces CMB	$a \sim 9 \times 10^{-4}$
$\sim 9.8 \times 10^{9}$ yr	$\sim 10^{-3}$ eV	Freezing of structure formation	$a \sim 0.73$
		Dark Energy-Dominated Era	
$\sim 10^{10}$ yr	$\sim 10^{-4}$ eV	• Present	$a = 1$

Cartan Khronon theory provides a framework in which the manually incorporated factors in the standard model can instead be described with effective contribution of gravitational dynamics itself, thereby offering a self-contained, more complete picture of the universe compared to general relativity. The configuration of gravitational chirality (g_+, g_-), introduced in (3.26), alters consequent phenomenology significantly. The initial state of the universe may be determined with the chirally symmetric phase of gravity, and later transitions into the (anti) self-dual phase which recovers the standard ΛCDM model.

Each section of this chapter starts with a concise review of the standard model, then explores cosmological implications provided by Cartan Khronon theory, demonstrating its more comprehensive portrayal of the universe.

4.1. Cosmological background

The standard model of cosmology employs the conjectures that, on sufficiently large scales, space and the density distribution are uniform and there is no preferred direction. Together with the assumption that the matter content can be approximated to be a perfect fluid, this shapes the large-scale structure of the simplified model of the universe. The standard model requires additional energy and mass inferred from observational data, that cannot be explained with the matter content we know of. These are collectively known as the dark sector of the universe.

4.1.1. Homogeneous & isotropic universe

The generic homogeneous and isotropic spacetime metric, established independently by Friedmann [63], Lemaître [64], Robertson [65], and Walker [66], after whom named the FLRW metric, reads

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left(\frac{1}{1 - kr^2} \mathrm{d}r^2 + r^2 \mathrm{d}\theta^2 + r^2 \sin^2\theta \mathrm{d}\phi^2 \right) , \tag{4.1}$$

where k is the parameter for spatial curvature (open<0, flat$=0$, closed>0), and the scale factor $a(t)$ measures the change in distance between two points in space, normalised so that $a(t_0) = 1$ at the present time. Applying this configuration to Einstein's field equations yields the Friedmann equations, which relate the expansion of the universe to the energy density ρ and pressure p of its matter content.

$$\left(\frac{a'}{a}\right)^2 + \frac{k}{a^2} = \frac{1}{3} m_P^{-2} \rho \,, \tag{4.2a}$$

$$2\frac{a''}{a} + \left(\frac{a'}{a}\right)^2 + \frac{k}{a^2} = -m_P^{-2} p \,, \tag{4.2b}$$

Following from this system, conservation law for the matter content is derived as

$$\rho' + 3\frac{a'}{a}(\rho + p) = 0 \,. \tag{4.3}$$

In the simplest case where the universe is spatially flat $k = 0$, i.e., Euclidean, and the source matter is assumed to be a perfect fluid $p = w\rho$, the Friedmann equations are solved for the scale factor as

$$a \sim t^{\frac{2}{3(1+w)}} \,. \tag{4.4}$$

For ultra-relativistic particles $w = 1/3$ that dominated the early universe, this predicts decelerating expansion $a \sim t^{1/2}$, and the equation (4.3) reveals the relation $\rho_r \sim a^{-4}$. Similarly, for a matter dominated era where the pressure is negligible $w = 0$, one finds the corresponding expansion rate.

Radiation dominated	$a \sim t^{1/2}$	$\rho_r \sim a^{-4}$
Matter dominated	$a \sim t^{2/3}$	$\rho_m \sim a^{-3}$

A more realistic cosmological picture with mixed composition can be introduced by hand.

$$\rho = \rho_r + \rho_m + \dots \,. \tag{4.5}$$

Introducing Hubble parameter $H = a'/a$ and density parameters $\Omega_i = \rho_i/\rho_c$, the expansion of the universe can be formulated as

$$\frac{H^2}{H_0^2} = \Omega_r a^{-4} + \Omega_m a^{-3} + \dots \,, \tag{4.6}$$

where $\sum_i \Omega_i = 1$. This model, however, is not the full picture of the universe, as it fails to explain several observed phenomena. Firstly, if the current universe was matter-dominated, this model would predict a decelerating expansion at $\sim t^{2/3}$. In contrast, observations of supernova distances across a range of redshifts carried out in the late 1990s revealed that the expansion of our universe is in fact accelerating [67]. This points to an energy source in vacuum space that would overcompensate the pulling force of gravity and drive the accelerated expansion of space. Secondly, the rotation speed of a spiral galaxy is theoretically expected to decrease towards its outer regions. Yet, successive measurements reveal that the rotation speed does not decline as sharply as predicted, implying the existence of undetected mass required to sustain this nearly constant rotational velocity [68]. Due to the characteristic curved profiles in velocity-radius plots, e.g., [69], this phenomena is commonly known as the galaxy rotation curve.

4.1.2. ΛCDM model

The vacuum energy responsible for the accelerated expansion of the universe can be incorporated into the Friedmann equations by hand as a constant Λ.

$$\left(\frac{a'}{a}\right)^2 + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{1}{3} m_P^{-2} \rho \,, \tag{4.7a}$$

$$2\frac{a''}{a} + \left(\frac{a'}{a}\right)^2 + \frac{k}{a^2} - \Lambda = -m_P^{-2} p \,. \tag{4.7b}$$

This constant is known as the cosmological constant, which first appears in Einstein's early consideration of cosmology based on general relativity [70]. It was later abandoned after Hubble's work [71] which explained the expansion of the universe without the cosmological constant. Even though there were sporadic revivals of the cosmological constant after Einstein's time, it was not widely accepted until multiple observational evidences indicated the accelerating expansion of the universe. The vacuum energy responsible for this effect came to be known as dark energy [72]. In a dark energy dominated universe, the universe's scale would grow exponentially, with its density unaffected by the scale factor.

$$\text{Dark Energy dominated} \quad a \sim \exp\left(\sqrt{\Lambda/3}t\right) \quad \rho_\Lambda \sim a^0$$

The observational data consistently suggest that dark energy accounts for roughly $\Omega_\Lambda \sim 0.7$ of the total mass-energy content of the universe, leaving

$\Omega_m \sim 0.3$ for matter. However, the ordinary (baryonic) matter we know of only accounts for $\Omega_m \sim 0.04$ [59, 73].

The missing mass to explain the galaxy rotation curve has been sought after since 1930s, and this invisible matter was named dark matter after its non-luminous nature. Unlike dark energy whose sole role is to accelerate the expansion of the universe, dark matter is considered to be also responsible for other large scale and galactic effects, such as structure formation in the early universe and gravitational lensing of distant galaxies. [74] In order to be able to explain these phenomena, dark matter has to be a non-relativistic perfect fluid that interacts only gravitationally. While other possibilities, such as hot or warm dark matter, have also been considered, the standard model of cosmology adopts Cold Dark Matter, since its pressureless nature explains observations ranging from CMB anisotropies to galaxy clusters. A matter source with vanishing pressure is sometimes referred to as ideal dust, and its difference from ordinary matters (baryons and photons) is more evident in its perturbation growth. Since it does not interact with radiation, cold dark matter does not feel the high pressure in the radiation dominated era that prevents the density perturbation of ordinary matter. Consequently, while baryons and photons undergo acoustic oscillations, cold dark matter starts to cluster. This makes cold dark matter responsible for the formation of gravitational potential during this era, attracting other matters and forming early structures of the universe [61].

In Cartan Khronon theory, this dark matter effect arises from gravitational dynamics. As we have seen briefly in Section 3.2.2, this dynamical effect $\boldsymbol{M}$ behaves like conserved matter, with its effective density and pressure satisfying continuity equations. In the (anti) self-dual sector of the theory, the effective pressure vanishes in vacuum and its total mass in a co-moving volume remains constant, demonstrating its nature as an effective candidate for ideal dust, even before explicitly solving the field equations. To investigate deeper into the cosmological background offered by Cartan Khronon theory, we consider the Spin(4) gauge-invariant self-dual action for standard cosmology

$$I = \int \mathbf{D}\phi^I \wedge \mathbf{D}\phi^J \wedge {}^+\boldsymbol{F}_{IJ} - \int \star\lambda - \int \mathbf{D}\phi^I \wedge \boldsymbol{t}_I \,. \tag{4.8}$$

For Euclidean description, this thesis employs dimensional parameter κ, time τ, and dotted time derivative $\dot{}$, in contrast to m_P, t and prime $'$ in the Lorentzian framework.

The homogeneous & isotropic configuration for the gauge field is

$$\boldsymbol{A}^{4i} = A(\tau)\mathbf{d}x^i \,, \quad \boldsymbol{A}^{ij} = B(\tau)\epsilon^{ij}{}_k\mathbf{d}x^k \,, \tag{4.9a}$$

and for energy-momentum current is

$$\boldsymbol{t}^4 = -\sqrt{\kappa}\rho \star \mathbf{d}\tau \,, \quad \boldsymbol{t}^i = \sqrt{\kappa}p \star \mathbf{e}^i \,, \tag{4.9b}$$

where factors $\sqrt{\kappa}$ are introduced for correct dimension. Together with the cosmological constant

$$\Lambda = \kappa\lambda = -m_P^{-2}\lambda \,, \tag{4.10}$$

and conserved 3-form

$$\boldsymbol{M}^4 = -\sqrt{\kappa}\hat{\rho} \star \mathbf{d}\tau \,, \quad \boldsymbol{M}^i = 0 \,, \tag{4.11}$$

these lead to the Friedmann equations with Euclidean time.

$$3\left(\frac{\dot{a}}{a}\right)^2 = \kappa\left(\rho + \hat{\rho}\right) + \Lambda \,, \tag{4.12a}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\kappa p + \Lambda \,. \tag{4.12b}$$

After the dimensionally controlled mapping, we obtain the Lorentzian Friedmann equations for the ΛCDM model.

$$3H^2 = m_P^{-2}\left(\rho + \hat{\rho}\right) + \Lambda \,, \tag{4.12c}$$

$$2H' + 3H^2 = -m_P^{-2}p + \Lambda \,. \tag{4.12d}$$

4.1.3. Λβ DM model

The self-dual action (4.8) can be generalised by introducing weighted chirality of gravitation.

$$I = \int \mathbf{D}\phi^I \wedge \mathbf{D}\phi^J \wedge \left(g_+{}^+\boldsymbol{F}_{IJ} - g_-{}^-\boldsymbol{F}_{IJ}\right) - \int \star\lambda - \int \mathbf{D}\phi^I \wedge \boldsymbol{t}_I \,. \tag{4.13}$$

In the $g_+g_- = 0$ limit where gravity is either fully right- or left-handed, we recover the phenomenology of general relativity with effective cold dark matter [1, 2, 8, 9]. For $g_+g_- \neq 0$ where both chiral sectors of gravity contributes to the dynamics, the conventional procedure leads to a trivial solution $\dot{A} = B = 0$, and does not recover the FLRW background. However, this is resolved by including the effective dark matter $\boldsymbol{M}$ in the vacuum background. Since $\boldsymbol{M}$ is not a substantial matter but a dynamical effect that imitates the behaviour of matter, the inclusion of $\boldsymbol{M}$ does not alter the vacuum nature of the background. For convenience, we introduce parameters of $g_\pm$

$$\alpha \equiv \frac{(g_+ - g_-)^2}{g_+ + g_-}, \quad \beta \equiv -\frac{4g_+g_-}{g_+ + g_-}, \quad \gamma \equiv \frac{g_+ + g_-}{g_+ - g_-} = \pm\sqrt{1 - \beta/\alpha}\,. \tag{4.14}$$

where $\alpha = \gamma = 1$, $\beta = 0$ recovers the limit of purely left- or right-handed gravity and the ΛCDM model as described in the previous subsection. The special cases $g_+ = -g_-$, $g_+ = g_-$ are separately studied as potentially significant limits. This generalised ambidextrous gravitational action (4.13) is comparable to that of loop quantum gravity in which the Barbero-Immirzi parameter $\gamma = \sqrt{1 - \beta/\alpha}$ represents the size of a discrete quantum area. A range of physical implications, including black hole entropy, has been investigated, however, the potential significance of this parameter remains to be fully understood. In current context, the parameter γ determines the relative weight of the Palatini and the additional term of the Holst action.

$$\int \epsilon_{abcd}\mathbf{e}^a\wedge\mathbf{e}^b\wedge\Big(g_+{}^+\boldsymbol{F}^{cd} + g_-{}^-\boldsymbol{F}^{cd}\Big) = \frac{g_+ + g_-}{4}\int \mathbf{e}^a\wedge\mathbf{e}^b\wedge\Big(\frac{2}{\gamma}\boldsymbol{F}_{ab} + \star\boldsymbol{F}_{ab}\Big)\,. \tag{4.15}$$

The system of connection equations of motion (3.40) determines the effective pressure of $\boldsymbol{M}$ as

$$\hat{p} = -2\beta\frac{1 - Ht}{t^2}\,, \tag{4.16}$$

and the continuity equation of $\boldsymbol{M}$ reveals this to be the source of its effective density.

$$\hat{\rho}' + 3H\hat{\rho} = -3\frac{\hat{p}}{t}\,. \tag{4.17}$$

The khronon equations of motion (3.38) then lead to the Friedmann equations with additional β-terms.

$$3\alpha H^2 - 3\beta t^{-2} = m_P^{-2}\left(\rho + \hat{\rho}\right) + \Lambda\,, \tag{4.18a}$$
$$\alpha\left(3H^2 + 2H'\right) - \beta t^{-2} = -m_P^{-2} p + \Lambda\,. \tag{4.18b}$$

It is straightforward to verify that ordinary matter indeed satisfies its standard conservation law. In a vacuum without ordinary matter and dark energy, the evolution of the scale factor and the effective dark matter density is determined by the chiral parameters g_+, g_-.

$$a(t) \sim t^{\frac{1}{3}\left(1 \pm \sqrt{1 + 3\frac{\beta}{\alpha}}\right)}\,, \quad \hat{\rho}(t) = \frac{2m_P^2}{3t^2}\left(\alpha - 3\beta \pm \sqrt{\alpha^2 + 3\alpha\beta}\right)\,. \tag{4.19}$$

The self-dual phase $\alpha = 1$, $\beta = 0$ recovers the Minkowski vacuum and effective ideal dust, as the dust effect dilutes, and the inclusion of the dust effect in the vacuum background does not affect the scale factor. With a slight deviation from general relativity $\beta = \epsilon \ll 1$, the effective dark matter acquires a nonzero effective pressure and no longer behaves exactly as ideal dust, though, as we will see, this does not disqualify $\boldsymbol{M}$ as a dark matter candidate.

$$\begin{array}{lll} g_\pm = 1,\ g_\mp = 0 & a \sim t^{\frac{2}{3}} & \hat{\rho} = \frac{4}{3} m_P^2 t^{-2} \\ g_\pm = 1,\ g_\mp = \epsilon \ll 1 & a \sim t^{\frac{2}{3} - \frac{\epsilon}{2}} & \hat{\rho} \simeq \left(\frac{4}{3} + \epsilon\right) m_P^2 t^{-2} \end{array}$$

In the chirally symmetric phase $\alpha = 0$, $\beta = -2g$, a vacuum solution does not exist. In the presence of matter, the system instead leads to a peculiar relation $\rho + \hat{\rho} = -3p$.

$$g_+ = g_- \quad a \sim \phi^{2/[3(1+w)]} \quad \hat{\rho} = -(1 + 3w)\rho$$

A physically more meaningful scenario is introduced with a seemingly bizarre configuration $g_+ = -g_-$. In this special phase, we find the khronon field takes the role of scale factor, and recovers the expansion rate $H \sim t^{-1}$ of the radiation dominated universe.

$$g_+ = -g_- \quad a \sim \phi \quad \hat{\rho} = -\frac{\rho}{1 - 3w} + \frac{m_0}{\phi^4} \quad \rho \sim \phi^{-3(1+w)}$$

The effective energy of $\boldsymbol{M}$ in this phase can be a candidate for dark radiation, the excessive radiation in the early universe, predicted from observational data [75, 76]. These physical significances make the $g_+ = -g_-$ phase a well-motivated candidate for the chirally symmetric phase, as can be supported by the following reorganisation.

$$\begin{aligned}
&\frac{1}{2}\int \epsilon_{IJKL}\mathbf{e}^I \wedge \mathbf{e}^J \wedge \left(g_+{}^+\boldsymbol{F}^{KL} + g_-{}^-\boldsymbol{F}^{KL}\right) \\
&\qquad = \int \mathbf{e}^I \wedge \mathbf{e}^J \wedge \left(g_+{}^+\boldsymbol{F}_{IJ} - g_-{}^-\boldsymbol{F}_{IJ}\right) . \qquad (4.20)
\end{aligned}$$

4.1.4. With the spin current

For the study of the effective dark matter, we have so far focused on the simple case where the spin current is absent. However, extending the cosmological analysis by including this additional contribution reveals its important role in Cartan Khronon theory. The full action including the effect of spin current reads

$$\begin{aligned}
I = &\int \mathbf{D}\phi^I \wedge \mathbf{D}\phi^J \wedge \left(g_+{}^+\boldsymbol{F}_{IJ} - g_-{}^-\boldsymbol{F}_{IJ}\right) \qquad (4.21) \\
&- \int \star\lambda - \int \mathbf{D}\phi^I \wedge \boldsymbol{t}_I - \int \boldsymbol{A}^{IJ} \wedge \boldsymbol{O}_{IJ} ,
\end{aligned}$$

and the homogeneous, isotropic configuration for the spin current can be set up as

$$^{\pm}\boldsymbol{O}^i = \sqrt{\kappa}\,{}^{\pm}O \star \mathbf{e}^i , \qquad (4.22)$$

to fit the rearranged formulation of the connection equations of motion (3.43). Let us introduce the following parameters for convenience.

$$\Omega \equiv \frac{\kappa}{2\gamma}\left({}^+O + {}^-O\right) , \quad \Sigma = \frac{\kappa^{3/2}}{\alpha\gamma^2\phi}\left(g_+{}^-O - g_-{}^+O\right) . \qquad (4.23)$$

The solution to the connection equations of motion elucidate an interesting relation between the spin current and the effective pressure of the dark matter candidate $\boldsymbol{M}$:

$$\hat{p} = \frac{\Sigma}{\kappa^{3/2}} + \frac{2\beta}{\kappa\tau}\left(H - \tau^{-1}\right) . \qquad (4.24)$$

This relation is particularly significant when gravity is purely left- or right-handed, as this is the identification of a direct coupling between left- or right-handed spin current ${}^{\pm}O$ and the effective pressure of $\boldsymbol{M}$. The continuity equation $\mathbf{D}\boldsymbol{M}^a = 0$ substantiates this role of the spin current.

$$\dot{\hat{\rho}} + 3H\left(\hat{\rho} + \frac{2\beta}{\kappa\tau^2}\right) = \frac{6\beta}{\kappa\tau^3} - \frac{\Omega}{\kappa\tau^2}\,. \tag{4.25}$$

The khronon equations yield the Friedmann equation with the new effect of spin current as

$$3\alpha H^2 + 3H\Omega + \frac{3}{4}\frac{\Omega^2}{\alpha} = \kappa\left(\rho + \hat{\rho}\right) + 3\beta\tau^{-2}\,, \tag{4.26a}$$

$$3\alpha H^2 + 2\alpha\dot{H} + 2H\Omega + \frac{\Omega^2}{4\alpha} + \dot{\Omega} + \frac{\Sigma}{\sqrt{\kappa}} = -\kappa p + \beta\tau^{-2}\,, \tag{4.26b}$$

and solving this for ordinary matter, one finds its continuity equation sourced by its own spin current and the expansion of universe.

$$\dot{\rho} + 3H\left(\rho + p\right) = \frac{3\Omega}{\kappa}\left(H^2 + \dot{H} + \frac{1}{\tau^2} + \frac{H\Omega + \dot{\Omega}}{2\alpha}\right) - 3H\frac{\Sigma}{\kappa^{3/2}}\,. \tag{4.27}$$

When we employ the fermions (3.34) to be the source, a more explicit formulation of source currents is available. A more detailed study of how spin currents affect the cosmological background is currently under investigation, and this thesis only teases the presence of such an unseen contribution that arises in Cartan Khronon theory. The dark matter effect is now coupled to the material spin, and this introduces effective pressure for $\boldsymbol{M}$ at the background level. This leads to potential implications in phenomenology, such as altered mechanism of inflation and structure formation.

4.2. Cosmological perturbation

Moving beyond the simplified homogeneous and isotropic background, this section considers the evolution of spacetime structure by introducing perturbation fields. In cosmological perturbation theory, these fluctuations are classified into three distinct modes of tensor, vector, and scalar, each of which evolves independently at linear order. While vector perturbations, which represent rotational velocities within cosmic fluids, are generally trivial in standard models due to their rapid decay in an expanding universe, tensor perturbations describe the propagation of gravitational waves, which provide the theoretical foundation of recently confirmed phenomena, and scalar perturbations are essential for modelling structure formation, as they govern the evolution of density fluctuations responsible for the formation of galaxies and stars. Within the Cartan Khronon framework, this analysis is further refined by introducing additional pseudo-tensorial and scalar fluctuations to account for the unique behaviours of the dark sector.

4.2.1. Gravitational waves

In the standard cosmological perturbation theory [61, 77], the spatial fluctuation of the cosmic background is represented with a symmetric, traceless and divergence-free tensor h_{ij}.

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2 \left(\delta_{ij} + h_{ij}\right) \mathrm{d}x^i \mathrm{d}x^j \, . \tag{4.28}$$

The traceless condition $h^i{}_i = 0$ removes the components that behave like scalar, and divergence-free/transverse condition $h_i{}^j{}_{,j} = 0$ removes vector-like behaviour, separating the tensor mode evolution. Solving Einstein's field equations for this metric, one obtains a wave equation for h_{ij}, predicting gravitational waves.

$$h''_{ij} + 3Hh'_{ij} - a^{-2}\nabla^2 h_{ij} = 0 \, . \tag{4.29}$$

In Cartan Khronon theory, we take into account additional pseudo-tensors $\tilde{h}$ for the gauge field and m for effective dark matter, allowing a more detailed analysis of the perturbations.

$$\boldsymbol{A}^{4i} = A \left(\delta^i_j + h^i{}_j\right) \mathbf{d}x^j \, , \quad \boldsymbol{A}^{ij} = B \left(\epsilon^{ij}{}_k + \epsilon^{ijl} \tilde{h}_{lk}\right) \mathbf{d}x^k \, , \tag{4.30}$$

$$\boldsymbol{M}^4 = -\sqrt{\kappa}\hat{\rho} \star \mathbf{e}^0 \, , \quad \boldsymbol{M}^i = \left(\sqrt{\kappa}\hat{p}\delta^i_j + \kappa^{-3/2} m^i{}_j\right) \star \mathbf{e}^j \, , \tag{4.31}$$

Inserting this setup into the connection equations (3.40a), we obtain the relation between the pseudo-tensorial perturbation $\tilde{h}_{ij}$ and the rest of the tensorial perturbation fields.

$$\tilde{h}_{ij} = h_{ij} + \frac{1}{B} \left(\gamma a \dot{h}_{ij} + \epsilon_{(i}{}^{kl} h_{j)k,l} - a\alpha\gamma\kappa^{-1}\tau m_{ij}\right) \, . \tag{4.32}$$

The other connection equations (3.40b) further reveal the effective dark matter perturbation m_{ij} to be dependent on the metric perturbation h_{ij}.

$$m_{ij} = \frac{\beta\gamma\kappa\dot{h}_{ij}}{\tau} \,. \tag{4.33}$$

As a result, the khronon equations (3.38) yield

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + \frac{\gamma^2}{a^2}\nabla^2 h_{ij} = 0 \,, \tag{4.34}$$

in Euclidean framework, which is translated into the gravitational wave equations of the metric perturbation h_{ij} in Lorentzian framework:

$$h''_{ij} + 3Hh'_{ij} - \frac{\gamma^2}{a^2}\nabla^2 h_{ij} = 0 \,. \tag{4.35}$$

This equation has the same form with the gravitational wave equation derived in general relativity, but with a modified propagation speed controlled by the chirality parameter $\gamma \equiv \frac{g_+ + g_-}{g_+ - g_-}$. When the speed of light is constant and remains to be the absolute speed limit, this identification places a constraint $\gamma^2 \leq 1$, which limits the possible configurations to $(g_+ = 0\,,\ g_- \neq 0)$, $(g_+ > 0\,,\ g_- \leq 0)$, and $(g_+ < 0\,,\ g_- \geq 0)$, ruling out the $g_+ = g_-$ configuration and supporting $g_+ = -g_-$ to be the chirally symmetric configuration as discussed in Section 4.1.3.

In the $\gamma = 1$ limit, the speed of gravity is equal to that of light. Thus, Cartan-khronon theory reproduces the standard gravitational-wave equation of general relativity when gravity is purely (anti) self-dual $\alpha = 1$, $\beta = 0$. The multi-messenger data from the neutron star merger GW170817 [78] show the difference between the speed of gravitational and electromagnetic waves in vacuum is within the range of

$$-3 \times 10^{-15} c < v_{\mathrm{GW}} - v_{\mathrm{EM}} < 7 \times 10^{-16} c \,. \tag{4.36}$$

This data indicates that, at present time, the left(right)-handed component of the gravitation has to be much smaller than the right(left)-handed component, to the order of $|g_\mp / g_\pm| \lesssim 10^{-15}$.

4.2.2. Structure formation

In the perturbation theory for general relativity, the scalar perturbations in the metric consist of the fluctuations in the lapse function $\bar{A}$, the scalar part of the shift vector $\bar{B}$, and the spatial components $\bar{D}$, $\bar{E}$

$$\mathrm{d}s^2 = -\left(1+2\bar{A}\right)\mathrm{d}t^2 + 2a\bar{B}_{,i}\mathrm{d}t\mathrm{d}x^i + a^2\left[\left(1-\bar{D}-\tfrac{1}{3}\bar{E}_{,kk}\right)\delta_{ij} + \bar{E}_{,ij}\right]\mathrm{d}x^i\mathrm{d}x^j\,. \tag{4.37}$$

Including perturbations in the metrical energy-momentum tensor

$$T^0{}_0 = -\rho - \delta\rho\,, \quad T^i{}_j = (p+\delta p)\,\delta^i_j + p\Pi_,{}^i{}_j\,, \tag{4.38a}$$

$$T^0{}_i = (\rho+p)\left(v_{,i}+\bar{B}_{,i}\right)\,, \quad T^i{}_0 = -(\rho+p)\,v_,{}^i\,, \tag{4.38b}$$

we can derive evolution equations for the fluctuations in matter density $\delta\rho$ and pressure δp, velocity potential v, and anisotropic pressure Π.

$$\delta\rho' = -3H\,(\delta\rho+\delta p) + (\rho+p)\left(3\bar{D}' - \frac{1}{a}\nabla\cdot\vec{v}\right)\,, \tag{4.39a}$$

$$\begin{aligned}\left[(\rho+p)\left(v-\bar{B}\right)\right]' = &-(\rho+p)'\left(v-\bar{B}\right) - 4H\,(\rho+p)\left(v-\bar{B}\right) \\ &+\frac{1}{a}\left[\delta p + \tfrac{2}{3}p\nabla^2\Pi + (\rho+p)\,\bar{A}\right]\,.\end{aligned} \tag{4.39b}$$

In order to identify physical fluctuations and remove the false fluctuations generated by coordinate choice, one can study gauge-invariant quantities, called Bardeen potentials [79].

$$\bar{\Phi} \equiv \bar{A} + \left[a\left(\bar{B}-a\bar{E}'\right)\right]'\,, \quad \bar{\Psi} \equiv D + \tfrac{1}{3}\nabla^2\bar{E} - aH\left(\bar{B}-a\bar{E}'\right)\,. \tag{4.40}$$

Adopting the conformal Newtonian gauge, i.e., choosing the coordinate where $\bar{B}=\bar{E}=0$, the metric fluctuations reduce to diagonal components.

$$\mathrm{d}s^2 = -\left(1+2\bar{\Phi}\right)\mathrm{d}t^2 + a^2\left(1-2\bar{\Psi}\right)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\,. \tag{4.41}$$

In this gauge, the perturbed Einstein's field equations reduce to constraint equations for relative energy-density perturbation $\delta^N \equiv \delta\rho^N/\rho$, velocity, equation of state w, and anisotropic pressure,

$$3a^2H^2\left[\delta^N + 3aH\,(1+w)\,v^N\right] = \nabla^2\bar{\Psi}\,, \quad 3a^2H^2w\Pi = \bar{\Psi}-\bar{\Phi}\,, \tag{4.42a}$$

and evolution equations

$$\tfrac{3}{2}aH^2\,(1+w)\,v^N = \bar{\Psi}' + H\bar{\Phi}\,, \tag{4.42b}$$

$$\tfrac{3}{2}H^2\delta p^N = \rho\left[\bar{\Psi}'' + H(\bar{\Phi}' + 2\bar{\Psi}') + (H^2 + 2H')\bar{\Phi} + \tfrac{1}{3a^2}\nabla^2\left(\bar{\Phi} - \bar{\Psi}\right)\right]. \tag{4.42c}$$

The lapse potential $\bar{\Phi}$ governs for the acceleration of matter, and the curvature potential $\bar{\Psi}$ measures the depth of gravitational wells. These are generalisations of the gravitational potential in the Newtonian limit. The difference between the two Bardeen potentials is sourced by the anisotropic stress. Hence, for a perfect fluid, the two become identical. Derived from field equations, comoving density perturbation for arbitrary matter content i follows the second order equation

$$\begin{aligned}\delta_i'' + \left(1 - 6w_i + 3c_s^2\right)H\delta_i' - \tfrac{3}{2}\left[\Omega_i + (10 - 2\Omega_i)\,w_i - 6c_s^2 - 3\Omega w_i^2\right]\delta_i H^2 \\ = \frac{c_s^2}{a^2}\nabla^2\left(\delta_i - \frac{\delta\rho_i}{\rho_i} + \frac{1}{c_s^2}\frac{\delta p_i}{\rho_i}\right),\end{aligned} \tag{4.43}$$

where $c_s^2 \equiv p_i'/\rho_i'$ is called the speed of sound, representing the evolution of pressure which acts against gravitational pulling force. For cold dark matter, both the pressure and sound speed vanish, which is precisely what allows its clustering effect. In the radiation-dominated early universe, overdensities deepen the potential wells to draw in more matter, forming structures such as stars and galaxies. In a matter-dominated universe, these potentials stabilise to provide the source for stable growth of the structure.

In Cartan Khronon theory, the same procedure to examine the perturbations is applied to an increased number of scalar fields. The general perturbative analysis must then take into account fluctuations in the khronon field φ, the connection components c, r, s, ψ, $\tilde{c}, \tilde{r}, \tilde{s}, \tilde{\psi}$, the effective dark matter density $\delta\hat{\rho}$ and pressure $\delta\hat{p}$, along with the lapse and shift components $n, m, \chi, \tilde{m}$,

$$\phi = \tau/\sqrt{\kappa} + \varphi \quad\Rightarrow\quad \mathbf{e}^4 = \left(1 + \sqrt{\kappa}\dot{\varphi}\right)\mathbf{d}\tau + \sqrt{\kappa}\varphi_{,i}\mathbf{d}x^i\,, \tag{4.44a}$$

$$\boldsymbol{A}^4{}_i = A\dot{c}_{,i}\mathbf{d}\phi + A\left((1-\psi)\,\delta_{ij} + \Delta^i_j r + \epsilon_{ij}{}^k\tilde{s}_{,k}\right)\mathbf{d}x^j\,, \tag{4.44b}$$

$$\boldsymbol{A}^{ij} = B\epsilon^{ijk}\dot{\tilde{c}}_{,k}\mathbf{d}\phi + B\left(\epsilon^{ij}{}_k\left(1 - \tilde{\psi}\right) + \epsilon^{ijl}\Delta_{lk}\tilde{r} + 2s_{,[i}\delta_{j]k}\right)\mathbf{d}x^k\,, \tag{4.44c}$$

$$\boldsymbol{M}^4 = -\sqrt{\kappa}\,(\hat{\rho} + \delta\hat{\rho}) \star \mathbf{e}^4 - \chi_{,i} \star \mathbf{e}^i \,, \tag{4.44d}$$

$$\boldsymbol{M}_i = n_{,i} \star \mathbf{e}^0 + \left(\sqrt{\kappa}\,(\hat{p} + \delta\hat{p})\,\delta_{ij} + \Delta_{ij} m + \epsilon_{ij}{}^k \tilde{m}_{,k}\right) \star \mathbf{e}^j \,, \tag{4.44e}$$

and those in the matter source $\delta\rho$, δp, u, Π,

$$\boldsymbol{t}^4 = -\sqrt{\kappa}\,(\rho + \delta\rho) \star \mathbf{e}^4 + (\rho + p)\left(\sqrt{\kappa} u_{,i} + a^{-1}\kappa\varphi\right)_{,i} \star \mathbf{e}^i \,, \tag{4.44f}$$

$$\boldsymbol{t}^i = (\rho + p)\left(\sqrt{\kappa} u + a^{-1}\kappa\varphi\right)^{,i} \star \mathbf{e}^4 + \sqrt{\kappa}\left[(p + \delta p)\,\delta^i_j + \Delta^i{}_j \Pi\right] \star \mathbf{e}^j \,. \tag{4.44g}$$

By constructing corresponding metric tensor from Cartan Khronon fundamental fields using the relation (3.3), one finds that the current configuration corresponds to the standard case (4.37) as

$$\bar{A} : \frac{\partial\varphi}{\partial\phi} \,, \quad \bar{B} : \sqrt{\kappa}\frac{\varphi}{a} + a\dot{c} \,, \quad \bar{D} : \psi - \frac{\varphi}{\phi} \,, \quad \bar{E} : r \,, \tag{4.45}$$

and the energy momentum current is in fact equivalent to the standard case (4.38a), only the linear perturbations included in the background $\star\mathbf{e}^I$ cause the current to appear differently.

The first half of the connection equations (3.40a) identifies the relation between perturbation fields in the magnetic part of the connection and the rest of the scalar fields.

$$\dot{\tilde{c}} = \dot{c} - \frac{\tilde{s}}{B} - \frac{\sqrt{\kappa}\tau}{2\alpha\gamma^3 B}\,(n + 2\gamma\tilde{m}) \,, \quad \tilde{r} = r - \frac{\tilde{s}}{B} + \gamma\frac{\tau A}{B}\left(\dot{c} - \dot{r} + \frac{\sqrt{\kappa}\tau}{\alpha\gamma^2} m\right) \,, \tag{4.46a}$$

$$s = -\tilde{s} + \frac{1}{B}\left(\frac{\sqrt{\kappa}\dot{A}}{A}\phi + \psi + \frac{1}{3}\nabla^2 r - \frac{\sqrt{\kappa}\tau^2 A}{2\alpha\gamma^2} n\right) \,, \tag{4.46b}$$

$$\begin{aligned}\tilde{\psi} = {} & \frac{\sqrt{\kappa}\tau^2 A}{4\alpha\gamma B}\left(\sqrt{\kappa}\delta\hat{p} + \frac{2}{3}\nabla^2 m + 2\beta\frac{\dot{A}}{\tau^2 A}\varphi\right) - \frac{1}{2}\left(\sqrt{\kappa}\frac{\varphi}{\phi} - \psi - \frac{1}{3}\nabla^2 r\right) \\ & - \frac{\gamma\tau A}{2B}\left(\sqrt{\kappa}\frac{\dot{A}}{A}\dot{\varphi} + \dot{\psi} + \frac{1}{3}\nabla^2 \dot{r}\right) \,, \end{aligned} \tag{4.46c}$$

The second half of the connection equations (3.40b) then determines the fluctuations for some components of the effective dark matter.

$$n = 0 \,, \quad m = \frac{\beta}{\sqrt{\kappa}\tau}\,(\dot{c} + \dot{r}) \,, \quad \tilde{m} = 0 \,, \tag{4.47a}$$

$$\delta\hat{p} = -2\frac{\beta}{\kappa\tau}\left(\frac{\sqrt{\kappa}\dot{A}}{\tau A}\varphi + \sqrt{\kappa}\frac{\dot{A}}{A}\dot{\varphi} + \dot{\psi} + \frac{1}{3}\nabla^2 \dot{c}\right) \,. \tag{4.47b}$$

The Euler equation $\mathbf{D}\boldsymbol{M}_i = 0$ further identifies

$$\chi = \frac{2\beta}{\sqrt{\kappa}\tau A}\left(\sqrt{\kappa}\left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right)\varphi + \sqrt{\kappa}\frac{\dot{A}}{A}\dot{\varphi} + \dot{\psi} + \frac{1}{3}\nabla^2\dot{r}\right) . \tag{4.48}$$

At this point, the independent perturbation fields reduce to the fluctuations in khronon field φ, the connection components c, r, ψ, $\tilde{s}$, and the effective dark matter density $\delta\hat{\rho}$. Finally, applying these results, the background solution for A, B, and Wick rotation, the khronon equations (3.38) in Lorentz framework result in

$$\begin{aligned}\delta\rho + \delta\hat{\rho} = & -6\alpha m_P^2 H\left(H\Phi' + \Psi' + \frac{1}{3}\nabla^2 c'\right) + 2\alpha\frac{m_P^2}{a^2}\nabla^2\left(H\Phi + \Psi + \frac{1}{3}\nabla^2 r\right)\\ & - 2\beta\frac{m_P^2}{a^2}\nabla^2\left(H\Phi + \Psi + \frac{1}{3}\nabla^2 r\right) + 6\beta\frac{m_P^2}{t^3}\Phi\,,\end{aligned} \tag{4.49a}$$

$$(\rho + p)\, v_{,i} = -\frac{\Phi_{,i}}{a}\hat{\rho} + 2\alpha\frac{m_P^2}{a}\left(H\Phi'_{,i} + \Psi'_{,i} + \frac{1}{3}\nabla^2 r'_{,i}\right) - 2\beta\frac{m_P^2}{at^2}\Phi_{,i}\,, \tag{4.49b}$$

$$\begin{aligned}\delta p = & 2\alpha m_P^2\left[\left(3H^2 + 2H'\right)\Phi' + H\left(\Phi'' + 3\Psi' + \nabla^2 c'\right) + \left(\Psi'' + \frac{1}{3}\nabla^2 c''\right)\right]\\ & - \frac{2}{3}\left(\alpha - \beta\right)\frac{m_P^2}{a^2}\nabla^2\left(H\Phi + \Psi + \frac{1}{3}\nabla^2 r\right) - 2\beta\frac{m_P^2}{t^3}\Phi\,,\end{aligned} \tag{4.49c}$$

$$\begin{aligned}\Pi_{,i,j} = & \left(\alpha - \beta\right)\frac{m_P^2}{a^2}\left(H\Phi_{,i,j} + \Psi_{,i,j} + \frac{1}{3}\nabla^2 r_{,i,j}\right)\\ & - 3\alpha m_P^2 H\left(c'_{,i,j} - r'_{,i,j}\right) - \alpha m_P^2\left(c''_{,i,j} - r''_{,i,j}\right)\,,\end{aligned} \tag{4.49d}$$

where $\Phi \equiv \frac{\varphi}{m_P}$ and $\Psi \equiv \psi - \frac{\varphi}{\phi}$ were introduced for brevity. The direct correspondence between these parameters and the Bardeen potentials in the standard case can be identified as follows.

$$\bar{\Phi} : \sqrt{\kappa}\dot{\Phi} = m_P^{-1}\Phi'\,, \quad \bar{\Psi} : \Psi\,. \tag{4.50}$$

In the $\alpha = 1$, $\beta = 0$ limit, the obtained khronon equation system (4.49) corresponds to the standard case (4.38a), confirming the recovery of general relativity once again, and the perturbation in the effective dark matter density $\delta\hat{\rho}$ becomes the only additional field with physical meaning.

To further ensure consistency, the continuity equation obtained from the system of field equations agrees with the one directly derived from $\mathbf{D}\boldsymbol{M}_4 = 0$ in the conformal Newtonian gauge.

$$\begin{aligned}\delta\hat{\rho}' + 3H\delta\hat{\rho} = & \ \Phi'\hat{\rho}' + 3\left(H\Phi' + \Psi' + \frac{1}{3}\frac{\nabla^2}{a^2}\Phi\right)\hat{\rho} \\ & +6\beta\frac{m_P^2}{t^2}\left[\left(\frac{2H}{t} - \frac{3}{t^2}\right)\Phi + H\Phi' + \Psi'\right] \\ & -2\beta\frac{m_P^2}{a^2}\nabla^2\left[H'\Phi + H\Phi' - \Phi t^{-2} + \Psi'\right] \\ \approx & \ \frac{\nabla^2}{a^2}\left[-2\beta m_P^2\left(H\Phi' + \Psi'\right) + \left(\hat{\rho} - 2\beta m_P^2 H' + \frac{2\beta m_P^2}{t^2}\right)\Phi\right],\end{aligned} \tag{4.51}$$

This reveals the clustering nature of $\boldsymbol{M}$ regardless for any value of β. The approximation in the bottom line shows the quasi-static limit $\nabla^2 f \gg H^2 f$ where $f' \sim Hf$, $f'' \sim H^2 f$, and also takes into account that gravitational potentials are negligible compared to density perturbations $\delta\rho, \delta\hat{\rho} \gg \rho\Phi', \rho\Psi$. Now the second order differential equation of the density perturbation, with the absence of $\nabla^2\hat{\delta}$ term, shows the vanishing sound speed of $\boldsymbol{M}$.

$$\begin{aligned}&\hat{\delta}'' + \left[2H - \frac{2\beta m_P^2}{t^3}\left(\frac{6-6Ht}{\hat{\rho}} + \frac{t^2}{\hat{\rho}t^2 + 2\beta m_P^2}\right)\right]\hat{\delta}' \\ &= \frac{\hat{\rho}}{2\alpha m_P^2}\hat{\delta} + 2\beta\frac{m_P^2\hat{\rho}t^2\left[3\alpha Ht(5Ht-7) + 9\alpha - 5\beta\right]}{\alpha\hat{\rho}t^4\left(2\beta m_P^2 + \hat{\rho}t^2\right)}\hat{\delta} \\ &\quad +2\beta\frac{6\beta m_P^4\left[\alpha Ht(5Ht-8) + 4\alpha - \beta\right] - \hat{\rho}^2 t^4}{\alpha\hat{\rho}t^4\left(2\beta m_P^2 + \hat{\rho}t^2\right)}\hat{\delta}\,.\end{aligned} \tag{4.52}$$

In the $\beta = 0$ limit, this reduces to the standard evolution equation for cold dark matter, and even for $\beta \neq 0$, this does not alter the vanishing of sound speed, supporting the interpretation of the effective dark matter as a clustering medium.

4.3. Cosmological timeline and outlook

We have demonstrated the standard model of cosmology assumes a manually introduced dark sector, whereas Cartan Khronon theory provides a self-contained framework in which explicit solution of the dark matter effect can be derived. So far, we have found

- The special limits of (anti) self-dual $\beta = 0$ and symmetric gravity $g_+ = \pm g_-$ lead to different phases of cosmological background and effective dark matter behaviour.
- The parameter γ represents the gravitational wave speed, which places a constraint $\gamma^2 \leq 1$, and observational data further imply another constraint $|\beta| \lesssim 10^{-15}$ at the present time.
- When the spin current is neglected, the exact $\beta = 0$ limit of the theory recovers general relativity and effective ideal dust.
- For $\beta \neq 0$, the effective dark matter is no longer pressureless, though its sound speed still vanishes locally.

This provides a theoretical model that explains a large proportion of the late time cosmology since the matter-dominated era. However, its potential to provide further insights into earlier universe cosmology has not been fully explored.

A number of different scenarios are under consideration. If the universe starts from a point $\phi^a = \vec{0}$, the elapse of time spontaneously breaks the symmetry between space and time $\phi^a = (\phi, 0, 0, 0)$. A toy model with this beginning called "khronogenesis" has been considered in the $\beta = 0$, $w = const.$ background [8]. This model predicts a non-singular beginning in the presence of something material, and the universe starts with an inflationary era $a \sim \phi^{2/3(1+w)}$. This may be combined with the chirally symmetric configuration $g_+ = -g_-$ discussed in Section 4.1.3. In this phase, the background includes an $\boldsymbol{M}$ effect that imitates the behaviour of radiation, potentially providing an initial condition for the origin of the matter content.

The transition mechanism from the chirally symmetric phase into the (anti) self-dual configuration of gravity $g_+ \sim 1$, $|g_-| \lesssim 10^{-15}$ at present time is currently under investigation. Simply promoting the parameters $g_\pm$ to dynamical variables induces a significant drawback of altering the

Table 6: Timeline for Cartan Khronon model of cosmology

$t = 0$		Symmetric Phase $\phi^a = \vec{0}$, $g_+ = -g_-$	
		Inflationary era — Phase transition	
?		• Phase transition towards $g_+ \sim 1$, $g_- \sim 0$	
		Radiation-Dominated Era	
~ 1 sec	$\sim 10^{10}$ eV	• Big Bang Nucleosynthesis	
$\sim 10^4$ yr	~ 1 eV	Start of structure formation	$a \sim 3.3 \times 10^{-4}$
		Matter-Dominated Era	
$\sim 3.8 \times 10^5$ yr	~ 0.4 eV	• Photon decoupling produces CMB	$a \sim 9 \times 10^{-4}$
$\sim 9.8 \times 10^9$ yr	$\sim 10^{-3}$ eV	Freezing of structure formation	$a \sim 0.73$
		Dark Energy-Dominated Era	
$\sim 10^{10}$ yr	$\sim 10^{-4}$ eV	• Present	$a = 1$

khronon field equations, spoiling the signature of this theory – the effective dark matter. This confines $g_\pm$ to remain independent of the khronon field, at least at the time of Big Bang Nucleosynthesis (BBN) and thereafter, where the phenomenology is established by observations and the effect of dark matter is not negligible. Another possible route is an alternative inflationary mechanism, as the alignment of khronon field with the scale factor in the $g_+ = -g_-$ phase is highly compatible with models such as [80].

The inclusion of spin current opens the door to various alternative scenarios. In the context of Einstein Cartan gravity, the inflation and more general cosmology with the effect of spin has been studied in [81–83]. Each study considers a different source, from anisotropic spinning fluid with high density of spin-1/2 particles to fermion condensate, yet all describe a spin-induced accelerated expansion that does not require the vacuum energy or scalar field in standard inflation model. In addition, each model predicts a bounce, a repulsive centrifugal effect that avoids the Big Bang singularity. The khronon theory predicts yet newer effect of spin current as hinted in Section 4.1.4. This is expected to provide an altered description of phenomenology in inflationary and radiation-dominated eras. While spin effect is not expected to influence the gravitational waves, it is curious to see whether it would feed the vector mode to yield a propagating effect, and how it would affect the structure formation.

In summary, several plausible scenarios of the early universe cosmology can be considered within the Cartan Khronon framework. A non-singular origin could be realised from space-time symmetry breaking, a spin-induced bounce, or a combination of both, leading to a smooth onset of cosmic inflation without appealing to a separate quantum theory of gravity. Ongoing studies of spin current effects and mechanisms of chiral symmetry breaking may reveal further phenomenological consequences in inflationary and radiation dominated eras, potentially with further observable signatures. Overall, Cartan Khronon theory offers a self-contained analytical description of dark matter effects and provides a systematic framework for studying the evolution of the universe, with structure formation in the matter-dominated era naturally emerging from gravitational dynamics.

5. Black holes

The origin of the concept of the black hole goes a long way back to the 18th century [84], when it was predicted that a sufficiently massive and compact star could induce gravitational attraction that is so strong that even light cannot escape. Not long after general relativity was introduced, a theoretical solution, known as the Schwarzschild solution today, provided the theoretical foundation for this idea. This dense region in spacetime enclosed by an event horizon holds theoretical significance as a testing ground across the entire spectrum of spacetime curvature.

However, a black hole, as a physical object in reality, may differ from what one would imagine based on its name or what some theoretical solutions predict it to be. Contrary to the image implied by the word “hole,” a black hole is not a physical cavity in spacetime, but a substantial region with extreme mass density. It is possible to travel around a black hole and observe its opposite side. Just like any other star, black holes are not static but rotate in reality, and other stars can form a stable orbit around a black hole outside its horizon. Recent observations confirm that the boundary of a black hole is, in fact, brighter than the background region, due to the radiation from a gas disk formed by heated matter about to cross the event horizon.

This chapter demonstrates that the standard black hole solutions of general relativity are reproduced within the Cartan Khronon framework, confirming the recovery of general relativity as a special limit of the theory. The proposed mapping procedure for Euclidean-Lorentzian correspondence remains effective in the non-trivial context of the Kerr solution. Several unconventional spacetime configurations enabled by this framework are considered in addition.

5.1. Spherically symmetric spacetime

The simplest theoretical solution of a black hole is given as a spherically symmetric vacuum solution to gravitational field equations [85].

$$\mathrm{d}s^2 = -\left(1 - \frac{r_s}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{r_s}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\,. \tag{5.1}$$

This metric describes a spherical region with radius $r = r_s$, from which nothing can escape. The region itself is electromagnetically neutral with net zero charge, and the geometry outside $r > r_s$ also applies to a vacuum region outside a star. The Schwarzschild radius r_s is related to the mass m_s inside this region as $r_s = 2Gm_s = \frac{m_s}{4\pi m_P^2}$. While Schwarzschild metric provides a description in the standard spherical coordinates, an improved description was found in a different coordinate system found by Lemaître [86].

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \frac{r_s}{r(\tau,\rho)}\mathrm{d}\rho^2 + r(\tau,\rho)^2\mathrm{d}\theta^2 + r(\tau,\rho)^2\sin^2\theta\mathrm{d}\phi^2\,. \tag{5.2}$$

The Lemaître coordinate system $\{\tau, \rho\}$ is a compound of Schwarzschild temporal and radial coordinates $\{t, r\}$

$$\mathrm{d}\tau = \mathrm{d}t + \sqrt{r_s/r}\,(1 - r_s/r)^{-1}\,\mathrm{d}r\,, \tag{5.3a}$$

$$\mathrm{d}\rho = \mathrm{d}t + \sqrt{r/r_s}\,(1 - r_s/r)^{-1}\,\mathrm{d}r\,, \tag{5.3b}$$

which removes the coordinate singularity at the horizon, providing a smooth, continuous horizon-crossing.

In Cartan Khronon theory where the metric is not fundamental, we consider spherically symmetric configuration for tetrads and connection to obtain the vacuum black hole solution. Due to the temporal nature the khronon field induces, it is natural to employ a synchronous configuration. A general function $F(r)$ can include spherical solutions that are not necessarily in vacuum.

$$\mathbf{e}^0 = \mathbf{d}\tau, \quad \mathbf{e}^1 = F(r)\mathbf{d}\rho, \quad \mathbf{e}^2 = r\mathbf{d}\theta, \quad \mathbf{e}^3 = r\sin\theta\mathbf{d}\varphi, \tag{5.4}$$

The corresponding electric components of the connection are

$$\boldsymbol{\omega}^1{}_0 = \frac{F(\tau,\rho)}{\tau}\mathbf{d}\rho\,, \quad \boldsymbol{\omega}^2{}_0 = \frac{r(\tau,\rho)}{\tau}\mathbf{d}\theta\,, \quad \boldsymbol{\omega}^3{}_0 = \frac{r(\tau,\rho)\sin\theta}{\tau}\mathbf{d}\varphi\,, \tag{5.5}$$

and the remaining components should follow the spherically symmetric configuration [87].

$$\boldsymbol{\omega}^1{}_2 \;=\; A\mathbf{d}\theta - iB\sin\theta\mathbf{d}\varphi\,, \tag{5.6a}$$

$$\boldsymbol{\omega}^1{}_3 = +iB\mathbf{d}\theta + A\sin\theta\mathbf{d}\varphi\,, \tag{5.6b}$$

$$\boldsymbol{\omega}^2{}_3 = -iC\mathbf{d}\tau - iD\mathbf{d}\rho - \cos\theta\mathbf{d}\varphi\,. \tag{5.6c}$$

Solving the connection equations of motion (3.40), these parameters are identified to be

$$A = -\frac{r'}{F}\,, \quad B = \frac{r}{\tau} - \dot{r}\,, \quad C = 0\,, \quad D = \dot{F} - \frac{F}{\tau}\,, \tag{5.7}$$

where $\dot{} = \frac{\partial}{\partial\tau}$, $' = \frac{\partial}{\partial\rho}$ and the khronon equations of motion (3.38) find the general solution

$$F(r) = \pm\sqrt{\frac{r_s}{r} + A^2 - 1}\,, \tag{5.8}$$

where the integration constant is absorbed into r_s, and the parameter A is constant. The Lemaître solution is recovered in the $A = 1$ limit, showing the theory reproduces the standard black-hole exterior of general relativity.

5.1.1. Singularity and causal structure

While the exterior properties of a black hole are increasingly being elucidated with observations, what happens inside a black hole is not observable by nature. This is where the theoretical prediction comes into play. For a particle of mass m moving in the equatorial plane $\theta = \pi/2$, its trajectory into a Schwarzschild black hole follows

$$u^0 = \frac{e}{1 - r_s/r}\,, \quad u^3 = \frac{L}{mr^2}\,, \quad u^1 = \sqrt{e^2 - \left(1 - \frac{r_s}{r}\right)\left(1 + \frac{L^2}{m^2r^2}\right)}\,, \tag{5.9}$$

due to the conserved energy $E = em$ and angular momentum L [88]. The specific energy $e = E/m = 1/\sqrt{1 - v_\infty^2}$ classifies the trajectory based on the initial velocity of the test particle at infinite distance.

$e > 1$	hyperbolic "hail" geodesics with finite $v_\infty > 0$
$e = 1$	parabolic "rain" geodesics with $v_\infty = 0$
$e < 1$	elliptic "drip" geodesics with $v_\infty < 0$ (starts from finite r)
$e \leq 0$	white hole

For a trajectory without angular motion $L = 0$, a comparison with the spherically symmetric solution in Cartan Khronon theory reveals that the additional parameter A coincides with this specific energy e. Despite being one of the connection components, this parameter A turns out to carry a physical meaning. A study to find the reason for this identification may entertain interesting properties of black hole geometry in Cartan Khronon description.

As the test particle passes through the event horizon $r = r_s$, the Schwarzschild lapse vanishes, and the explicit solution for the test particle orbit with $L = 0$ and $e = 1$,

$$t = t_0 \pm r_s \left(\frac{2}{3} \left(\frac{r}{r_\mathrm{s}} \right)^{\frac{3}{2}} + 2\sqrt{\frac{r}{r_\mathrm{s}}} + \ln \frac{\left| \sqrt{\frac{r}{r_\mathrm{s}}} - 1 \right|}{\sqrt{\frac{r}{r_\mathrm{s}}} + 1} \right) , \tag{5.10}$$

shows the Schwarzschild time is not well-defined at the event horizon $r = r_s$. However, as briefly discussed, this is a coordinate-dependent false singularity, and switching to proper-time coordinates of Lemaître or Painlevé-Gullstrand descriptions allows a smooth passage through the event horizon. The explicit orbit in terms of proper time

$$\tau = \tau_0 \pm \frac{2}{3} r_s \left(\frac{r}{r_\mathrm{s}} \right)^{\frac{3}{2}} , \tag{5.11a}$$

or equivalently,

$$r(\tau) = \left[\frac{9}{4} r_s \left(\tau - \tau_0 \right)^2 \right]^{1/3} , \tag{5.11b}$$

demonstrates the test particle itself would not observe any significant discontinuity at the event horizon. Nevertheless, what distinguishes a black hole from other astrophysical compact objects is exactly this event horizon, which forms a one-way boundary instead of a physical surface. At the horizon, the radial coordinate of an infalling object becomes timelike, so the infalling object cannot return to the exterior or remain on a stable orbit. As the object experiences the descent towards the centre of the black hole, the theoretical trajectory reaches its limitation at the very centre $r = 0$ of the black hole, where the curvature diverges to infinity, forming a singularity.

5.1.2. Orthoradial spacetime

The description of the event horizon in Cartan Khronon theory is non-singular, following the continuous formulation of the Lemaître coordinates. In the $SO(3)$ phase after the symmetry breaking, the khronon field acts as a canonical clock field, corresponding to the time coordinate in Lemaître coordinate, measuring the proper time for observer. The Cartan Khronon framework, however, can accommodate the unconventional $SO(1,2)$ phase in which the radial coordinate becomes timelike, corresponding to the black hole interior. In this phase, the "radion" $\phi^a = \delta^a_1 \phi(r)$ replaces the role previously played by the khronon. In the absence of black hole $m_s = 0$, radion is linearly related to the radial coordinate. In the presence of a black hole, the radion transitions smoothly from real values outside the horizon to imaginary values inside, with the field vanishing at the horizon.

$$\phi(r) = \pm\sqrt{r\,(r - r_s)} \pm \frac{r_s}{2} \log\left[\frac{4r}{r_s}\left(1 + \sqrt{1 - \frac{r_s}{r}}\right) - 2\right] + \phi_0\,. \tag{5.12}$$

This real-to-imaginary transition of the radion field offers a natural framework for the imaginary shielding, a method to reinterpret the central singularity that has been considered in a different theory [89] where this transition was implemented by gluing the Lorentzian and Euclidean metrics.

With the corresponding setup for the gauge field

$$\boldsymbol{\omega}^0{}_1 = \frac{f}{\phi}\mathbf{d}t\,, \quad \boldsymbol{\omega}^1{}_2 = -\frac{r}{\phi}\mathbf{d}\theta\,, \quad \boldsymbol{\omega}^1{}_3 = -\frac{r\sin\theta}{\phi}\mathbf{d}\varphi\,, \tag{5.13a}$$

$$\boldsymbol{\omega}^0{}_2 = A(r)\mathbf{d}\theta - iB(r)\sin\theta\mathbf{d}\varphi\,, \quad \boldsymbol{\omega}^0{}_3 = iB(r)\mathbf{d}\theta + A(r)\sin\theta\mathbf{d}\varphi\,,$$
$$\boldsymbol{\omega}^2{}_3 = iC(r)\mathbf{d}t + iD(r)\mathbf{d}r - \cos\theta\mathbf{d}\varphi\,, \tag{5.13b}$$

the spherically symmetric solution in radion phase is obtained with the following lapse function.

$$f = \alpha\left[1 + \sqrt{1 - \frac{r_s}{r}} \log\left(\sqrt{\frac{r}{r_s}} - \sqrt{\frac{r}{r_s} - 1}\right)\right] + \beta\sqrt{1 - \frac{r_s}{r}}\,. \tag{5.14}$$

This lapse function recovers the Schwarzschild solution for the integration constants $\alpha = 0$ and $\beta \neq 0$, whereas the $\alpha \neq 0$ and $\beta = 0$ case is not asymptotically flat. For the modified Schwarzschild solution with $\alpha \neq 0$ and $\beta \neq 0$, only either of the exterior or the interior is real-valued, and an additional singularity appears outside the event horizon, which coincides with the caustic singularity in mimetic gravity [90]. This scenario is ruled out by recent observations of black hole shadows [91], making the $\alpha \neq 0$, $\beta \neq 0$ solution purely theoretical.

5.1.3. Electromagnetised spacetime

An electrically charged black hole is usually described with the solution to the Einstein-Maxwell equation. The static solution is known as the Reissner-Nordström metric [92, 93]

$$\mathrm{d}s^2 = -\left(1 - \frac{r_s}{r} + \frac{r_q^2}{r^2}\right)\mathrm{d}t^2 + \left(1 - \frac{r_s}{r} + \frac{r_q^2}{r^2}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\,, \tag{5.15}$$

which describes the spacetime around a charged analogue of the Schwarzschild solution with mass $m_s = 4\pi m_P^2 r_s$ and charge $q = \sqrt{4\pi\epsilon_0 r_q^2}$. This can be recovered in Cartan Khronon theory with a similar procedure as the Schwarzschild metric.

However, with a pregeometric formulation of electromagnetism, as established in [94], the source electromagnetic fields can be embedded into the gravitational frame, allowing an unconventional formulation of an electromagnetised metric. As discussed in more detail in one of the attached publications [2], in order to recover the known phenomenology of electromagnetism in such a first-order formalism, it seems inevitable to formulate the electromagnetic field strength $\boldsymbol{F}$ and the gauge potential $\boldsymbol{H}^{ab} = \boldsymbol{h}^a \wedge \mathbf{e}^b$ independently, with an additional coframe field $\boldsymbol{h}^a$. The Yang-Mills type action

$$I = \int \boldsymbol{H}^{ab} \wedge \left(\star\boldsymbol{H}_{ab} - \eta_{ab}\boldsymbol{F}\right)\,, \tag{5.16}$$

yields field equations

$$\mathbf{d}\boldsymbol{H} = 0\,, \tag{5.17a}$$

$$2\star\boldsymbol{H}_{ab} \wedge \mathbf{e}^b - \boldsymbol{F} \wedge \mathbf{e}_a = 0\,, \tag{5.17b}$$

and find the spherically symmetric configuration for the hyperframe to be

$$\boldsymbol{h}^0 = -\frac{q_b}{2r^2}\mathbf{e}^1 \,, \quad \boldsymbol{h}^1 = -\frac{q_b}{2r^2}\mathbf{e}^0 \,, \quad \boldsymbol{h}^2 = -\frac{q_e}{2r^2}\mathbf{e}^3 \,, \quad \boldsymbol{h}^3 = \frac{q_e}{2r^2}\mathbf{e}^2 \,, \quad (5.18)$$

from which an electromagnetised (hyper)metric is constructed as

$$\mathbf{d}\tilde{s}^2 = \boldsymbol{h}^a \otimes \boldsymbol{h}_a = \frac{q_b^2}{4r^4}\left(-f^{-2}\mathbf{d}r^2 + f^2\mathbf{d}t^2\right) + \frac{q_e^2}{4r^2}\left(\mathbf{d}\theta^2 + \sin^2\theta\mathbf{d}\varphi^2\right) \,. \quad (5.19)$$

This is a reformulation of the static charged black hole, with electromagnetism embedded in the Lorentz gauge, offering a unified framework for gravitational and electromagnetic interactions, though this makes the geometric interpretation of the metric unclear. More insights towards unification in line with this approach can be found in [95].

5.1.4. Symmetric configuration in $\beta \neq 0$ phase

As discussed in the previous Chapter 4, observational evidence suggests gravitation in Cartan Khronon theory has to be mostly (anti) self-dual $\beta \sim 0$ at present time, leaving slight room $\beta \lesssim 10^{-15}$ for ambidextrous description. In the early universe, however, gravitation was expected to be chirally symmetric $g_+ = -g_-$, that is, $\beta \neq 0$. We have seen in Section 4.1.3 that, in the $\beta \neq 0$ phase, it is essential to include the dark matter effect to recover the FLRW cosmological background. With the spherically symmetric configuration (5.4) and

$$\boldsymbol{M}^1 = -\frac{\sqrt{\kappa}\hat{p}_r}{2}\star\mathbf{e}^1 - \frac{\hat{v}}{2\kappa^{\frac{3}{2}}}\star\mathbf{e}^4 \,, \quad (5.20a)$$

$$\boldsymbol{M}^2 = -\frac{\sqrt{\kappa}\hat{p}_\theta}{2}\star\mathbf{e}^2 + \hat{\pi}\star\mathbf{e}^3 \,, \quad (5.20b)$$

$$\boldsymbol{M}^3 = \frac{\hat{\pi}}{\kappa^{\frac{3}{2}}}\star\mathbf{e}^2 + \frac{\sqrt{\kappa}\hat{p}_\theta}{2}\star\mathbf{e}^3 \,, \quad (5.20c)$$

$$\boldsymbol{M}^4 = \frac{\hat{w}}{2\kappa^{\frac{3}{2}}}\star\mathbf{e}^1 + \frac{\sqrt{\kappa}\hat{\rho}}{2}\star\mathbf{e}^4 \,, \quad (5.20d)$$

accompanied by the continuity equations for $\boldsymbol{M}^a$, field equations result in the following system involving the function $F(r)$, the chirality parameter β, and the effective density $\hat{\rho}$.

$$(g_+ + g_-)\left(1 - A^2\right) + \alpha\left(F + 2F_{,r}r\right)F + 3\beta\frac{r^2}{\tau^2} = -r^2\kappa\hat{\rho}\,, \tag{5.21a}$$

$$(g_+ + g_-)\left(1 - A^2\right) + \alpha\left(F^2 + 2F_{,r}Fr\right) = -\beta\frac{r^2}{\tau^2}\,, \tag{5.21b}$$

$$\alpha\left[F_{,rr}Fr + (F_{,r})^2 r + 2F_{,r}F\right] = -\beta\frac{r}{\tau^2}\,, \tag{5.21c}$$

In contrast to the general relativistic limit, no valid spherically symmetric solution has not been found in the presence of the $\boldsymbol{M}$ effect. The attempt to recover an axisymmetric solution with a similar procedure has also been unsuccessful, indicating there may be no straightforward black hole solution in the $\beta \neq 0$ phase.

Observations indicate the existence of accreting black holes at $z \simeq 10.6$ [96], corresponding to a few hundred million years after the Big Bang, during the matter-dominated era. At present, there is no evidence of a black hole from an earlier time. If the absence of a black hole solution in $\beta \neq 0$ phase is confirmed, this could constrain the theory by requiring the transition from the symmetric phase to the $\beta = 0$ phase to occur before $t \sim 10^8$ years, and the speed of gravity to be exactly the speed of light. This would also imply the effective dark matter $\boldsymbol{M}$ acted as ideal dust during the structure formation. The former constraint is in agreement with the cosmic timeline, which places the transition before the Big Bang Nucleosynthesis.

However, finding an exact analytical solution for a black hole in the presence of matter faces difficulties even in the context of general relativity. Some attempts to construct analytical black hole metrics surrounded by dark matter have tested pasting an isolated black hole spacetime onto a background to match the matter distribution [97], while others have adopted a numerical approach [98] or a mathematically convenient halo profile [99]: no straightforward analytical solution has been found in the presence of matter. For this reason, the possible absence of a black hole solution is not regarded as a critical limitation of the $\beta \neq 0$ phase, and the strictness of the second constraint, $\beta = 0$ at present time, remains open to discussion.

5.2. Axisymmetric spacetime

The theoretical model of a black hole closest to the observed black holes is obtained as the axisymmetric solution of gravitational field equations. Taking into account the angular momentum $J = m_s a$, the Kerr metric describes a rotating spacetime [100]

$$\mathrm{d}s^2 = -\left(1 - \frac{r_s r}{\Sigma}\right)\mathrm{d}t^2 - \frac{2r_s ar}{\Sigma}\sin^2\theta \mathrm{d}t\mathrm{d}\varphi + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma \mathrm{d}\theta^2 + \frac{A}{\Sigma}\sin^2\theta \mathrm{d}\varphi^2\,, \tag{5.22}$$

where $r_s = m_s/(4\pi m_P^2) \equiv 2m$, $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta = r^2 + a^2 - r_s r$, and $A = \left(r^2 + a^2\right)\Sigma + r_s ra^2\sin^2\theta$. The spin parameter a must satisfy $0 \leq a \leq m$ for this geometry to be a valid black hole solution with an event horizon.

Unlike the static black holes discussed in Section 5.1.1, the Kerr metric predicts two horizons, $r_\pm = m \pm \sqrt{m^2 - a^2})$, where the outer horizon r_+ is a coordinate singularity, while the inner horizon r_- may develop into a physical singularity. In addition, there is a ring-shaped singularity located at $r = 0$, $\theta = \pi/2$. The close vicinity of the event horizon is predicted to have a dragging effect due to the rotation where observers cannot remain at rest. In this region between the outer event horizon $r_+ = m + \sqrt{m^2 - a^2}$ and the ergosphere $r = m + \sqrt{m^2 - a^2\cos^2\theta}$, the Killing vector becomes spacelike, allowing trajectories with negative energy relative to infinite distance. This results in the Penrose process, the effective extraction of energy past the event horizon [101].

The stable circular orbit of a particle outside a Kerr black hole would be

$$r^2 - 6mr \pm 8a\sqrt{mr} - 3a^2 = 0\,, \tag{5.23}$$

where the sign $\pm$ depends on whether the particle orbit is prograde, meaning it revolves in the same direction as the black hole rotation, or retrograde, revolving against the black hole rotation. For a prograde orbit, as the spin parameter increases, the innermost stable circular orbit moves inwards, reaching the event horizon for a maximally spinning black hole where $a = m$. Whereas, for a retrograde orbit, as the spin parameter increases, the innermost stable circular orbit moves outwards, extending beyond the ergosphere up to $r = 9m$. Strongly lensed emission from plasma associated with photon capture in the vicinity of event horizon is expected to produce a bright emission ring surrounding the central shadow. Although the size and asymmetry of the shadow depend on the mass, spin, inclination and plasma model, the flattening effect of the mass quadrupole moment and the compressing effect of frame dragging are predicted to cancel each other, keeping the Kerr central shadow close to circular [102–104].

The Event Horizon Telescope images of M87* [105, 106] reveal an asymmetric bright emission ring enclosing a central brightness depression, consistent with the shadow expected from a Kerr solution in general relativity. The observed ring asymmetry can be accounted for by strong gravitational lensing and relativistic beaming in the emitting plasma, while the measured circularity constrains the deviations from the Kerr exterior geometry. Related Event Horizon Telescope observations of Sgr A* [107, 108] provide further strong-field tests of stationary black-hole metrics, and the black hole spin is estimated to be near theoretical maximum, corresponding to the case where the prograde innermost stable circular orbit lies very close to the event horizon. These observations probe the exterior region of Kerr geometry, while the interior structure including the Cauchy horizon the ring singularity remains to be unknown. This makes the recovery of the Kerr exterior in the Lorentzian description a necessary consistency test for the Cartan Khronon construction, while leaving room for alternative interior descriptions.

Introducing oblate spheroidal coordinates $\cos\theta = z/r$, $\tan\phi = y/x$, the Kerr metric can be reorganised into a tetrad-friendly form [109].

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \left(\frac{\Sigma}{\sqrt{r^2+a^2}}\mathrm{d}r + \frac{\sqrt{r_s r}}{\Sigma}\left(\mathrm{d}t - a\sin^2\theta\mathrm{d}\phi\right)\right)^2 \qquad (5.24)$$
$$+\Sigma^2\mathrm{d}\theta^2 + \left(r^2+a^2\right)\sin^2\theta\mathrm{d}\phi^2\,.$$

Since a rotating spacetime configuration is a non-trivial test for whether the mapping procedure Section 3.1.2 is indeed well-defined, let us introduce the tetrads corresponding to the above metric, but with unspecified functions f, σ, α, β in Euclidean setting.

$$\mathbf{e}^1 = \beta\mathbf{d}\phi + \frac{\sigma}{f}\mathbf{d}r - \alpha\sin^2\theta\mathbf{d}\varphi\,, \quad \mathbf{e}^2 = \sigma\mathbf{d}\theta\,, \quad \mathbf{e}^3 = f\sin\theta\mathbf{d}\varphi\,, \quad \mathbf{e}^4 = \mathbf{d}\phi\,. \qquad (5.25)$$

Then electric part of the connection is determined following the relation $\mathbf{e}^i = -\boldsymbol{A}^i\phi$. Solving the connection equations of motion (3.40) for the most generic configuration of the magnetic components of the connection, one finds the solution

$$^{\pm}\boldsymbol{A}^1 = -\left(\pm\frac{\beta}{\phi} + W\right)\mathbf{d}\phi - \left(\pm\frac{\sigma}{\phi f} + X\right)\mathbf{d}r - Y\mathbf{d}\theta - \left(Z \mp \frac{\alpha}{\phi}\sin^2\theta\right)\mathbf{d}\varphi\,, \tag{5.26a}$$

$$^{\pm}\boldsymbol{A}^2 = S\mathbf{d}\phi + T\mathbf{d}r + U\mathbf{d}\theta + \left(V \mp \frac{\sigma}{\phi}\right)\mathbf{d}\varphi\,, \tag{5.26b}$$

$$^{\pm}\boldsymbol{A}^3 = -O\mathbf{d}\phi - P\mathbf{d}r - Q\mathbf{d}\theta - \left(R \pm \frac{f}{\phi}\sin\theta\right)\mathbf{d}\varphi\,, \tag{5.26c}$$

where

$$O = \frac{\beta'\sigma + \beta\sigma'}{2\sigma^2} + \frac{\alpha'\beta^2 - \alpha\beta\beta'}{2\sigma}\sin\theta\,, \quad P = \frac{\sigma'}{f\sigma} + \frac{\alpha'\beta - \alpha\beta'}{2f}\sin\theta\,,$$

$$Q = -\frac{f\sigma'}{\sigma}\,, \quad R = -\frac{\alpha}{2\sigma}\left(\alpha'\beta - \alpha\beta'\right)\sin^3\theta - \frac{\alpha'\sigma + \alpha\sigma'}{2\sigma^2}\sin^2\theta - \frac{1}{\sigma}\left(\alpha\cos\theta + \beta f f'\right)\sin\theta - \frac{f}{\phi}\sin\theta\,, \tag{5.26d}$$

$$S = \frac{\alpha'\beta + \alpha\beta'}{2\sigma}\sin\theta + \frac{\beta\beta'\sigma - \beta^2\sigma'}{2\sigma^2}\,, \quad T = \frac{\alpha'}{f}\sin\theta + \frac{\beta'\sigma - \beta\sigma'}{2f\sigma}\,,$$

$$U = \frac{\alpha'\sigma - \alpha\sigma'}{2f\sigma}\sin\theta + \frac{\alpha}{f}\cos\theta + \frac{f\beta\sigma'}{\sigma} + \frac{\sigma}{\phi}\,, \tag{5.26e}$$

$$V = -\frac{\alpha\alpha'}{\sigma}\sin^3\theta - \frac{\alpha\beta'\sigma - \alpha\beta\sigma'}{2\sigma^2}\sin^2\theta - \frac{ff'}{\sigma^2}\sin\theta\,,$$

$$W = \frac{\alpha'\sigma - \alpha\sigma'}{2f\sigma^2}\beta\sin\theta + \frac{\alpha}{f\sigma}\beta\cos\theta - \frac{f\beta\beta'}{\sigma} - \frac{\beta}{\phi}\,, \tag{5.26f}$$

$$X = \frac{\alpha'\sigma - \alpha\sigma'}{2f^2\sigma}\sin\theta + \frac{\alpha}{f^2}\cos\theta - \beta' - \frac{\sigma}{\phi f}\,, \quad Y = \frac{\alpha\sigma'}{\sigma}\sin\theta - \frac{\beta'\sigma - \beta\sigma'}{2\sigma}\,,$$

$$Z = -\frac{\alpha\alpha'\sigma - \alpha^2\sigma'}{2f\sigma^2}\sin^3\theta - \frac{\alpha^2}{f\sigma}\sin^2\theta\cos\theta + \frac{\phi f\alpha'\beta - \phi f\alpha\beta' - 2\alpha\sigma}{2\phi\sigma}\sin^2\theta - \frac{f}{\sigma}\cos\theta\,.$$

Together with the khronon equations of motion (3.38), we identify the unspecified functions introduced for the tetrad configuration:

$$f = \pm\sqrt{r^2 - a_E^2}\,, \quad \sigma = \pm\sqrt{r^2 - a_E^2\cos^2\theta}\,, \quad \alpha = \pm a_E\beta\,, \quad \beta = \pm\sqrt{\frac{\kappa m_s r}{4\pi\sigma^2}}\,. \tag{5.27}$$

Since the angular momentum is a time-derivative, the framework-dependence of the parameter a is marked with its subscript, and the translation into the Lorentzian framework involves $a_E \to -ia$. The mapped Lorentzian solution recovers the Doran metric (5.24), confirming both the consistency of the (anti) self-dual sector of Cartan Khronon theory with general relativity and the reliability of the newly introduced mapping procedure (3.11) between Euclidean and Lorentzian frameworks.

The rotating black hole solution in the geometric phase of Cartan Khronon theory, i.e., the Doran metric, predicts no physical distinction from the standard Kerr black hole. However, in the same manner as how the Lemaître-Painlevé-Gullstrand metric regularises the coordinate singularity in the Schwarzschild solution, the Doran metric also employs the proper time of free-falling observers, yielding a smooth description of the motion across the event horizon. Although the Doran form is regular at the outer horizon, the Cauchy horizon of Kerr metric persists at r_- where infalling matter is expected to accumulate, and this influx undergoes an unbounded relative blue-shift, driving the effective internal mass parameter and curvature to diverge. This is known as the mass-inflation instability [104, 110].

In the Euclidean framework of Cartan Khronon theory, however, a real solution requires $r > a_E$. While certain components of tetrads and connection diverge at $r = a_E$, the scalar curvature $\boldsymbol{F}^{IJ} \wedge \boldsymbol{F}_{IJ}$ vanishes, indicating this boundary is not a curvature singularity. When $a_E < m$, as required for a valid Kerr solution with event horizon, the Cauchy horizon falls into the region $r < a_E$: the Euclidean axisymmetric domain terminates before the Lorentzian inner-horizon region is reached. Since $r_+ > a_E$ holds for any a_E, the boundary $r = a_E$ lies between the event horizon r_+ and the inner horizon r_-. This points towards the establishment of a shielding method that does not require the glueing of two metrics [89, 111], and its regularity and extendibility will be studied.

6. Conclusion

Over a century since the establishment of the theories of relativity, it has been the standard practice to formulate classical aspects of physics in Lorentzian spacetime. There are, however, notable exceptions in which a Euclidean spacetime provides a more suitable framework, in quantum physics and black hole thermodynamics. In fact, Euclidean description of physics entails real-valued formulation of physical observables, in contrast to the complexified formulation in Lorentzian description. This incomplete Lorentzian picture of physics with extended numbers has led some to ponder on the idea whether a Euclidean background might in fact represent the fundamental structure of the physical world, and a Lorentzian background only provides a framework for observation.

With time formulated as a background independent field, the Cartan Khronon framework provides a foundation for unconventional space-time structures, accommodating both Lorentzian and Euclidean backgrounds. An improved procedure of mapping between these frameworks, akin to Wick rotation, incorporates dimensionful parameters and rotation of the gamma matrix, providing a consistent description of time. This comprehensive framework incorporates a Spin(4) gauge structure, offering a real-valued chiral formulation of the gravitational fields on par with the fundamental fields of the theory, Weyl spinors. Due to the internal gauge structure of this theory, gravity is sourced by canonical currents: the energy-momentum current associated with the translational symmetry and the spin current associated with the Spin(4) or relevant Lie group symmetry.

The successful derivation of various phenomenological solutions and recovery of the general relativistic solutions in their specific limits provide consistency verifications of the Cartan Khronon gravity and the proposed dimensional mapping procedure between Euclidean and Lorentzian descriptions. In particular, general-relativistic solutions are recovered in the (anti) self-dual limit $\beta = 0$ where the dynamical effect of the gravity $\boldsymbol{M}$ behaves as effective ideal dust, and the propagation speed of the gravitational waves coincides with the speed of light. The inclusion of the spin current reveals its coupling to the effective dark matter, allowing the $\boldsymbol{M}$-effect to be included at the background level in this sector.

A more general configuration of gravitational chirality (g_+, g_-) beyond this limit predicts altered phenomenology. In this phase $\beta \neq 0$, new homogeneous solutions have been identified where the dynamical effect of $\boldsymbol{M}$ no longer behaves as ideal dust. However, its sound speed remains negligible at the relevant scales in structure formation, keeping $\boldsymbol{M}$ a plausible candidate for effective dark matter. The recovery of FLRW solution in this phase requires the inclusion of the effective dark matter in the background. This makes the derivation of a spherically symmetric or an axisymmetric geometry less straightforward.

A constraint can be imposed on the chiral parameter γ, whose absolute value coincides with the gravitational wave speed. Given the speed of light is constant and not surpassed by the speed of gravity at any phase of cosmology, it has to satisfy $\gamma^2 \leq 1$. This excludes $g_+ = g_-$ from viable configurations, and $g_+ = -g_-$ provides a feasible chirally symmetric phase of gravity instead. In this specific phase, the khronon field plays the role of scale factor and dynamical effect of $\boldsymbol{M}$ imitates the behaviour of radiation, potentially providing an effective source of dark radiation. Comparing the gravitational wave speed $|\gamma|$ predicted by the theory with neutron-star merger data, we find $|g_\mp/g_\pm| \lesssim 10^{-15}$ for present time: gravity is mostly (anti) self-dual today, only allowing a deviation of $|\beta| \lesssim 10^{-15}$.

Table 7: Various phases of Cartan Khronon framework ($\boldsymbol{O} = 0$)

$(g_+ ,\ g_-)$	α	β	$\gamma = v_{\text{GW}}$	$\boldsymbol{M}$	cosmology	BH
$(1,0)$, $(0,1)$	1	0	1	ideal dust	ΛCDM	GR
$-g_\mp/g_\pm \lesssim 10^{-15}$	$\lesssim 1$	$\gtrsim 0$	$\lesssim 1$	$\hat{p} \neq 0$, $c_S = 0$	$\Lambda\beta$DM	?
$g_+ = -g_-$	∞	∞	0	dark radiation	$a \sim \phi$	?
$g_+ = g_- = g$	0	$-2g$	∞	$\hat{\rho} = -\rho - 3p$	$a \sim \phi^{2/[3(1+w)]}$	?

The scalar sector of the perturbative framework makes it possible to assess the behaviour of $\boldsymbol{M}$ in structure formation as the clustering mediator, allowing the Cartan Khronon theory to provide analytical solution for dark matter effect and advance our picture of the universe, particularly in the matter dominated and the current dark energy dominated eras. Possible scenarios for earlier time before BBN are under investigation. Future studies on the spin effect and the chiral symmetry breaking mechanism should elucidate more phenomenological consequences in the inflationary and radiation-dominated eras. These results suggest that the theory captures the standard late-time cosmological evolution while also leaving open plausible alternatives for the early universe.

Black hole and other spherically symmetric geometries have been studied in the general-relativistic limit of Cartan Khronon framework. An alternative description with a radial symmetry-breaking field offers an exotic causal structure, yielding a caustic singularity comparable to that of the compact object in mimetic gravity. Integrated formulation of electromagnetised metric tensor is demonstrated by incorporating first order pregeometric formalism for electromagnetism. No stationary black hole solution in the presence of $\boldsymbol{M}$-effect has been found with the configurations considered here. This leaves the study open whether a more general, possibly dynamical, configuration is required in the $\beta \neq 0$ sector.

An axisymmetric Euclidean spacetime geometry was established, whose mapping into the Lorentzian description recovers the rotating Kerr geometry. This, along with the cosmological perturbation theory across Euclidean and Lorentzian descriptions, provides a particularly robust validation to support the proposed dimensional mapping procedure. The real Euclidean solution terminates before reaching the space-time domain that corresponds to the Lorentzian Cauchy horizon. This suggests a new interpretation of the interior structure, potentially shielding both the inner horizon and the central singularity, though a nonsingular completion requires further analysis.

Altogether, these results collectively demonstrate that Cartan Khronon theory does not only offer an alternative formulation of gravitation, but provides a concrete framework where the emergence of time, compact-object geometry, and dark-sector phenomenology become different aspects of the same gauge-theoretic dynamics. The theory connects the foundational structure of space-time with observable cosmological and black hole phenomena, offering a coherent starting point for exploring extreme regimes of the gravitational physics, the early universe, black hole interiors, and the dynamical origin of the dark sector within a single theory of space-time.

Through improved descriptions of phenomenology, this thesis asks a fundamental question:

> Is Lorentzian spacetime – where we *observe* physics – the optimal framework to *define* physics?

Sisukokkuvõte

Cartan kroonon – Reaalne aeg-ruumi struktuur –

Üldrelatiivsusteoorias kirjeldatakse gravitatsiooni aegruumi geomeetriana, milles aja- ja ruumisuundade erinevus sisaldub Lorentzi struktuuris. Käesolev väitekiri käsitleb teistsugust võimalust: ajasuund ei ole teoorias algusest peale ette antud, vaid selle valib dünaamiline kellaväli ehk kroonon. Cartani kroononi gravitatsiooniteoorias on fundamentaalseteks muutujateks sisemine seostus $\boldsymbol{A}^{ab}$ ja kroononiväli ϕ^a, millest defineeritakse kaasraam $\mathbf{e}^a = \mathbf{D}\phi^a$. Sellises kirjelduses ei pea meetrika olema teooria lähteobjekt, vaid aegruumi geomeetria tekib sümmeetriat rikkunud faasis. Väitekirja eesmärk on arendada seda kalibratsiooniteoreetilist gravitatsioonikäsitlust ning uurida selle konkreetseid tagajärgi kosmoloogias ja mustade aukude füüsikas.

Töö alguses antakse ülevaade kalibratsioonisümmeetriast, spiinoresitustest ja gravitatsiooni esimest järku formulatsioonist. See annab aluse Cartani kroononi teooria dünaamika käsitlemiseks. Teoorias seostuvad gravitatsioonivälja allikad loomulikult kanooniliste vooludega: energia-impulsi vooluga ning sisemise pöördsümmeetriaga seotud spinni vooluga. kroononvälja kaudu saadud kaasraam võimaldab kirjeldada gravitatsiooni sisemise sümmeetria alusel ilma sõltumatut meetrikat või tetraadi algselt sisse toomata.

Väitekirja keskne teoreetiline tulemus on Cartani kroononi teooria eukleidilise Spin(4)-formulatsiooni väljatöötamine. Tavapärases Lorentzi kirjelduses eristab aega ruumist meetrika signatuur, samas kui eukleidilises Spin(4)-kirjelduses saab aja suund kujuneda kroononivälja valitud suunana. Spin(4)-struktuur on oluline ka seetõttu, et see võimaldab käsitleda kiraalseid spiinoresitusi teooria sisemise sümmeetria osana. Töö käigus formuleeritakse dimensionaalselt kontrollitud vastavus eukleidilise ja Lorentzi kirjelduse vahel. Selle vastavuse ülesanne on siduda eukleidilises raamistikus tuletatud lahendid Lorentzi geomeetriaga, mille kaudu kirjeldatakse füüsikalisi vaatlusi. Vastavust ei käsitleta üksnes formaalse konstruktsioonina, vaid seda kontrollitakse konkreetsete kosmoloogiliste ja musta augu lahendite abil.

Kosmoloogilises osas rakendatakse Cartani kroononi teooriat homogeensele ja isotroopsele universumile. Töö käigus leitakse uusi taustlahendeid, mis sõltuvad gravitatsiooniteooria kiraalse sektori valikust. Teatud piiris taastuvad üldrelatiivsusteooriast tuttavad kosmoloogilised lahendid, kuid teooria sisaldab ka gravitatsioonidünaamikast tekkivat efektiivset komponenti. Sobivas sektoris võib see komponent käituda tolmutaolise ainena ning anda panuse, mis meenutab külma tumeaine rolli kosmiliste struktuuride kujunemisel.

Taustlahendite uurimise kõrval arendatakse väitekirjas Cartani kroononi gravitatsiooni kosmoloogiliste häirituste teooriat. Tensorhäirituste analüüs võimaldab uurida gravitatsioonilainete levikut ja selle sõltuvust teooria kiraalsetest parameetritest. Skalaarses sektoris käsitletakse tihedushäirituste arengut ning efektiivse komponendi mõju struktuuritekkele. Analüüsitud spinita skalaarses sektoris ja väikestel skaaladel saadud tulemus näitab, et efektiivsel komponendil võib olla kaduv helikiirus ning seetõttu külma tumeainega sarnane klasterdumiskäitumine. See ei tõesta veel täielikku alternatiivi tumeainele, kuid annab konkreetse aluse teooria edasiseks fenomenoloogiliseks kontrollimiseks.

Musta augu lahendite osas uuritakse kõigepealt sfääriliselt sümmeetrilisi konfiguratsioone. Tuletatakse lahendid eri sümmeetriasektorites ning käsitletakse ka elektromagnetvälja sisaldavat juhtu. Need lahendid võimaldavad kontrollida Cartani kroononi teooria seost üldrelatiivsusteooriast tuntud staatiliste musta augu geomeetriatega. Lisaks uuritakse, millistel tingimustel võib efektiivne gravitatsiooniline komponent musta augu konfiguratsioonides esineda. Valitud statsionaarsete ansatsite piires ei leita üldises sektoris mittetriviaalse efektiivse komponendiga lahendit; see jätab avatuks küsimuse, kas selliste konfiguratsioonide kirjeldamiseks on vaja üldisemaid või dünaamilisi lahendeid.

Töö üks olulisemaid konkreetseid kontrolle on pöörleva musta augu lahendi konstrueerimine. Eukleidilises formulatsioonis saadud aksiaalsümmeetriline lahend seotakse dimensionaalselt kontrollitud kaardistuse abil Lorentzi kirjeldusega ning asjakohases piiris taastub Kerri geomeetria Dorani koordinaatides. Kuna Kerri lahend on astrofüüsikaliste pöörlevate mustade aukude standardne kirjeldus, annab selle taastumine tugeva testi nii Cartani kroononi teooriale kui ka eukleidilise ja Lorentzi raamistiku vahelisele vastavusele. Eukleidiline lahend katab seejuures teistsuguse reaalse koordinaatpiirkonna kui Lorentzi Kerri siselahend. See tähelepanek pakub uue lähtekoha musta augu sisegeomeetria uurimiseks, kuid regulaarse siselahendi või singulaarsuse kõrvaldamise tõestamine nõuab edasist analüüsi.

Kokkuvõttes arendab väitekiri Cartani kroononi gravitatsiooniteooria sidusaks Spin(4)-põhiseks raamistikuks, milles aja suunda käsitletakse dünaamilise struktuurina. Töö näitab, et eukleidilise ja Lorentzi kirjelduse vahelist vastavust saab rakendada konkreetsetele kosmoloogilistele ja musta augu lahenditele ning et teoorial on uuritavas sektoris uued, füüsikaliselt huvipakkuvad tagajärjed. Väitekiri loob seega aluse edasiseks uurimistööks kahes gravitatsioonifüüsika äärmuslikus valdkonnas: varase universumi dünaamikas ja mustade aukude sisegeomeetrias.

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Bibliography

[1] Tomi Koivisto, Lucy Zheng, and Tom Zlosnik. "$Spin(4)$ gauge theory of space, time, gravitation, matter, and dark matter". *Phys. Rev. D* 113.12 (2026), page 124069. DOI: 10.1103/xx3x-s5wj. arXiv: 2507.00968 [gr-qc].

[2] Tomi S. Koivisto and Lucy Zheng. "Black holes in Lorentz gauge theory". *Phys. Rev. D* 111.6 (2025), page 064008. DOI: 10.1103/PhysRevD.111.064008. arXiv: 2408.10100 [gr-qc].

[3] Tomi S. Koivisto and Luxi Zheng. "Scale-invariant cosmology in de Sitter gauge theory". *Phys. Rev. D* 103.12 (2021), page 124063. DOI: 10.1103/PhysRevD.103.124063. arXiv: 2101.07638 [gr-qc].

[4] Tom Złośnik, Federico Urban, Luca Marzola, and Tomi Koivisto. "Spacetime and dark matter from spontaneous breaking of Lorentz symmetry". *Class. Quant. Grav.* 35.23 (2018), page 235003. DOI: 10.1088/1361-6382/aaea96. arXiv: 1807.01100 [gr-qc].

[5] Tomi Koivisto, Manuel Hohmann, and Tom Złośnik. "The General Linear Cartan Khronon". *Universe* 5.6 (2019), page 168. DOI: 10.3390/universe5070168. arXiv: 1905.02967 [gr-qc].

[6] Priidik Gallagher and Tomi Koivisto. "The Λ and the CDM as Integration Constants". *Symmetry* 13.11 (2021), page 2076. DOI: 10.3390/sym13112076. arXiv: 2103.05435 [gr-qc].

[7] Tomi S. Koivisto and Tom Zlosnik. "Paths to gravitation via the gauging of parametrized field theories". *Phys. Rev. D* 107.12 (2023), page 124013. DOI: 10.1103/PhysRevD.107.124013. arXiv: 2212.04562 [gr-qc].

[8] Tomi Koivisto. "Cosmology in the Lorentz gauge theory". *Int. J. Geom. Meth. Mod. Phys.* 20.Supp01 (2023), page 2450040. DOI: 10.1142/S0219887824500403. arXiv: 2306.00963 [gr-qc].

[9] Mehraveh Nikjoo and Tom Zlosnik. "Hamiltonian formulation of gravity as a spontaneously-broken gauge theory of the Lorentz group". *Class. Quant. Grav.* 41.4 (2024), page 045005. DOI: 10.1088/1361-6382/ad1c84. arXiv: 2308.01108 [gr-qc].

[10] Albert Einstein. "On the electrodynamics of moving bodies". *Annalen Phys.* 17 (1905), pages 891–921. DOI: 10.1002/andp.200590006.

[11] Hermann Minkowski. “Raum und Zeit”. *Jahresbericht der Deutschen Mathematiker-Vereinigung* 18 (1909). Address delivered at the 80th Assembly of German Natural Scientists and Physicians, Cologne, September 21, 1908, pages 75–88.

[12] Albert Einstein. “Zur Allgemeinen Relativitätstheorie”. *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* 1915 (1915). [Addendum: Sitzungsber.Preuss.Akad.Wiss.Berlin (Math.Phys.) 1915, 799–801 (1915)], pages 778–786.

[13] H. Weyl. “Gravitation and electricity”. *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* 1918 (1918), page 465.

[14] V. Fock. “On the invariant form of the wave equation and the equations of motion for a charged point mass. (In German and English)”. *Z. Phys.* 39 (1926). Edited by J. C. Taylor, pages 226–232. DOI: 10.1007/BF01321989.

[15] F. London. “Quantum mechanical interpretation of the Weyl theory. (In German and English)”. *Z. Phys.* 42 (1927). Edited by J. C. Taylor, pages 375–389. DOI: 10.1007/BF01397316.

[16] E. Cartan. “Sur les variétés à connexion affine et la théorie de la relativité généralisée. (première partie)”. *Annales Sci. Ecole Norm. Sup.* 40 (1923), pages 325–412.

[17] H. Weyl. “Electron and Gravitation. 1. (In German)”. *Z. Phys.* 56 (1929), pages 330–352. DOI: 10.1007/BF01339504.

[18] O. Klein. “On the Theory of Charged Fields”. *Surveys High Energ. Phys.* 5 (1986), pages 269–285. DOI: 10.1080/01422418608228775.

[19] Julian S. Schwinger. “The Theory of quantized fields. 2.” *Phys. Rev.* 91 (1953). Edited by K. A. Milton, pages 713–728. DOI: 10.1103/PhysRev.91.713.

[20] Chen-Ning Yang and Robert L. Mills. “Conservation of Isotopic Spin and Isotopic Gauge Invariance”. *Phys. Rev.* 96 (1954). Edited by Jong-Ping Hsu and D. Fine, pages 191–195. DOI: 10.1103/PhysRev.96.191.

[21] Mu-ming Poo and Alexander Wu Chao. “Conversation with Chen-Ning Yang: reminiscence and reflection”. *National Science Review* 7.1 (Aug. 2019), pages 233–236. ISSN: 2095-5138. DOI: 10.1093/nsr/

nwz113. eprint: https://academic.oup.com/nsr/article-pdf/7/1/233/38881258/nwz113.pdf.

[22] Tai Tsun Wu and Chen Ning Yang. "Concept of Nonintegrable Phase Factors and Global Formulation of Gauge Fields". *Phys. Rev. D* 12 (1975). Edited by Jong-Ping Hsu and D. Fine, pages 3845–3857. DOI: 10.1103/PhysRevD.12.3845.

[23] I. M. Singer. "SOME PROBLEMS IN THE QUANTIZATION OF GAUGE THEORIES AND STRING THEORIES". *Proc. Symp. Pure Math.* 48 (1988). Edited by R. O. Wells, pages 199–216.

[24] L. O'Raifeartaigh. *The dawning of gauge theory.* Princeton, NJ, USA: Princeton Univ. Press, 1997. ISBN: 978-0-691-02977-1.

[25] Peter W. Higgs. "Broken Symmetries and the Masses of Gauge Bosons". *Phys. Rev. Lett.* 13 (1964). Edited by J. C. Taylor, pages 508–509. DOI: 10.1103/PhysRevLett.13.508.

[26] S. W. MacDowell and F. Mansouri. "Unified Geometric Theory of Gravity and Supergravity". *Phys. Rev. Lett.* 38 (1977). [Erratum: Phys.Rev.Lett. 38, 1376 (1977)], page 739. DOI: 10.1103/PhysRevLett.38.739.

[27] K. S. Stelle and Peter C. West. "Spontaneously Broken De Sitter Symmetry and the Gravitational Holonomy Group". *Phys. Rev. D* 21 (1980), page 1466. DOI: 10.1103/PhysRevD.21.1466.

[28] Gunnar Nordstrom. "On the possibility of unifying the electromagnetic and the gravitational fields". *Phys. Z.* 15 (1914), pages 504–506. arXiv: physics/0702221.

[29] Th. Kaluza. "Zum Unitätsproblem der Physik". *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* 1921 (1921), pages 966–972. DOI: 10.1142/S0218271818700017. arXiv: 1803.08616 [physics.hist-ph].

[30] Oskar Klein. "Quantum Theory and Five-Dimensional Theory of Relativity. (In German and English)". *Z. Phys.* 37 (1926). Edited by J. C. Taylor, pages 895–906. DOI: 10.1007/BF01397481.

[31] G. W. Gibbons and S. W. Hawking, editors. *Euclidean quantum gravity.* 1994.

[32] Christer Helleland and Sigbjorn Hervik. "A Wick-rotatable metric is purely electric". *J. Geom. Phys.* 123 (2018), pages 424–429. DOI: 10.1016/j.geomphys.2017.09.015. arXiv: 1504.01244 [math-ph].

[33] Alessio Baldazzi, Roberto Percacci, and Vedran Skrinjar. "Wicked metrics". *Class. Quant. Grav.* 36.10 (2019), page 105008. DOI: 10.1088/1361-6382/ab187d. arXiv: 1811.03369 [gr-qc].

[34] Maxim Kontsevich and Graeme Segal. "Wick Rotation and the Positivity of Energy in Quantum Field Theory". *Quart. J. Math. Oxford Ser.* 72.1-2 (2021), pages 673–699. DOI: 10.1093/qmath/haab027. arXiv: 2105.10161 [hep-th].

[35] Matt Visser. "How to Wick rotate generic curved spacetime" (Feb. 2017). arXiv: 1702.05572 [gr-qc].

[36] Hans F. Westman and T. G. Zlosnik. "An introduction to the physics of Cartan gravity". *Annals Phys.* 361 (2015), pages 330–376. DOI: 10.1016/j.aop.2015.06.013. arXiv: 1411.1679 [gr-qc].

[37] Sebastian Bahamonde et al. "Teleparallel gravity: from theory to cosmology". *Rept. Prog. Phys.* 86.2 (2023), page 026901. DOI: 10.1088/1361-6633/ac9cef. arXiv: 2106.13793 [gr-qc].

[38] Friedrich W. Hehl, J. Dermott McCrea, Eckehard W. Mielke, and Yuval Ne'eman. "Metric affine gauge theory of gravity: Field equations, Noether identities, world spinors, and breaking of dilation invariance". *Phys. Rept.* 258 (1995), pages 1–171. DOI: 10.1016/0370-1573(94)00111-F. arXiv: gr-qc/9402012.

[39] Alejandro Jiménez Cano. "Metric-affine Gauge theories of gravity. Foundations and new insights". PhD thesis. Granada U., Theor. Phys. Astrophys., 2021. arXiv: 2201.12847 [gr-qc].

[40] Kirill Krasnov. *Formulations of General Relativity.* Cambridge Monographs on Mathematical Physics. Cambridge University Press, Nov. 2020. ISBN: 978-1-108-67465-2, 978-1-108-48164-9. DOI: 10.1017/9781108674652.

[41] Ryoyu Utiyama. "Invariant theoretical interpretation of interaction". *Phys. Rev.* 101 (1956). Edited by Jong-Ping Hsu and D. Fine, pages 1597–1607. DOI: 10.1103/PhysRev.101.1597.

[42] T. W. B. Kibble. "Lorentz invariance and the gravitational field". *J. Math. Phys.* 2 (1961). Edited by Jong-Ping Hsu and D. Fine, pages 212–221. DOI: 10.1063/1.1703702.

[43] John M. Charap and W. Tait. “A GAUGE THEORY OF THE WEYL GROUP”. *Proc. Roy. Soc. Lond. A* 340 (1974), pages 249–262. DOI: 10.1098/rspa.1974.0151.

[44] A. Bregman. “Weyl transformations and poincare gauge invariance”. *Prog. Theor. Phys.* 49 (1973), pages 667–692. DOI: 10.1143/PTP.49.667.

[45] Chen-Ning Yang. “Integral Formalism for Gauge Fields”. *Phys. Rev. Lett.* 33 (1974). Edited by Jong-Ping Hsu and D. Fine, pages 445–447. DOI: 10.1103/PhysRevLett.33.445.

[46] Edward Anderson. “Problem of Time in Quantum Gravity”. *Annalen Phys.* 524 (2012), pages 757–786. DOI: 10.1002/andp.201200147. arXiv: 1206.2403 [gr-qc].

[47] Lucía Menéndez-Pidal de Cristina. “The problem of time in quantum cosmology”. PhD thesis. Nottingham U., University of Nottingham, Nottingham U., Nov. 2022. arXiv: 2211.09173 [gr-qc].

[48] Peter Woit. “Spacetime is Right-handed” (Nov. 2023). arXiv: 2311.00608 [hep-th].

[49] Peter van Nieuwenhuizen and Andrew Waldron. “On Euclidean spinors and Wick rotations”. *Phys. Lett. B* 389 (1996), pages 29–36. DOI: 10.1016/S0370-2693(96)01251-8. arXiv: hep-th/9608174.

[50] Konrad Osterwalder and Robert Schrader. “AXIOMS FOR EUCLIDEAN GREEN’S FUNCTIONS”. *Commun. Math. Phys.* 31 (1973), pages 83–112. DOI: 10.1007/BF01645738.

[51] Konrad Osterwalder and Robert Schrader. “Axioms for Euclidean Green’s Functions. 2.” *Commun. Math. Phys.* 42 (1975), page 281. DOI: 10.1007/BF01608978.

[52] Julian Schwinger. “Euclidean Quantum Electrodynamics”. *Phys. Rev.* 115 (1959), pages 721–731. DOI: 10.1103/PhysRev.115.721.

[53] B. Zumino. “Euclidean Supersymmetry and the Many-Instanton Problem”. *Phys. Lett. B* 69 (1977). Edited by Mikhail A. Shifman, page 369. DOI: 10.1016/0370-2693(77)90568-8.

[54] C. Wetterich. “Spinors in euclidean field theory, complex structures and discrete symmetries”. *Nucl. Phys. B* 852 (2011), pages 174–234. DOI: 10.1016/j.nuclphysb.2011.06.013. arXiv: 1002.3556 [hep-th].

[55] A. Ashtekar. “New Variables for Classical and Quantum Gravity”. *Phys. Rev. Lett.* 57 (1986), pages 2244–2247. DOI: 10.1103/PhysRevLett.57.2244.

[56] A. Ashtekar. “A Note on Helicity and Selfduality”. *J. Math. Phys.* 27 (1986), pages 824–827. DOI: 10.1063/1.527187.

[57] J. Fernando Barbero G. “Real Ashtekar variables for Lorentzian signature space times”. *Phys. Rev. D* 51 (1995), pages 5507–5510. DOI: 10.1103/PhysRevD.51.5507. arXiv: gr-qc/9410014.

[58] Friedrich W. Hehl, G. David Kerlick, and Paul von der Heyde. “On Hypermomentum in General Relativity I. The Notion of Hypermomentum”. *Zeitschrift für Naturforschung A* 31.2 (1976), pages 111–114. DOI: doi:10.1515/zna-1976-0201.

[59] E. J. Copeland. “Inflation: In the early universe and today”. *Lect. Notes Phys.* 646 (2004). Edited by Nora Breton, Jorge L. Cervantes-Cota, and Marcelo Salgado, pages 53–107. DOI: 10.1007/978-3-540-40918-2_3.

[60] Viatcheslav Mukhanov. *Physical Foundations of Cosmology.* Cambridge University Press, 2005.

[61] Hannu Kurki-Suonio. *Cosmological Perturbation Theory I.* 2024. URL: https://www.mv.helsinki.fi/home/hkurkisu/cpt/CosPer.pdf.

[62] Antonio Riotto. “Inflation and the theory of cosmological perturbations”. *ICTP Lect. Notes Ser.* 14 (2003). Edited by G. Dvali, Abdel Perez-Lorenzana, G. Senjanovic, G. Thompson, and F. Vissani, pages 317–413. arXiv: hep-ph/0210162.

[63] A. Friedmann. “On the Possibility of a world with constant negative curvature of space”. *Z. Phys.* 21 (1924), pages 326–332. DOI: 10.1007/BF01328280.

[64] Georges Lemaitre. “A Homogeneous Universe of Constant Mass and Growing Radius Accounting for the Radial Velocity of Extragalactic Nebulae”. *Annales Soc. Sci. Bruxelles A* 47 (1927), pages 49–59. DOI: 10.1007/s10714-013-1548-3.

[65] H. P. Robertson. “Kinematics and World-Structure”. *Astrophys. J.* 82 (1935), pages 284–301. DOI: 10.1086/143681.

[66] A. G. Walker. "On Milne's Theory of World-Structure". *Proc. Lond. Math. Soc. s* 2-42.1 (1937), pages 90–127. DOI: 10.1112/plms/s2-42.1.90.

[67] S. Perlmutter et al. "Cosmology from Type Ia supernovae". *Bull. Am. Astron. Soc.* 29 (1997), page 1351. arXiv: astro-ph/9812473.

[68] Yoshiaki Sofue and Vera Rubin. "Rotation curves of spiral galaxies". *Ann. Rev. Astron. Astrophys.* 39 (2001), pages 137–174. DOI: 10.1146/annurev.astro.39.1.137. arXiv: astro-ph/0010594.

[69] K. G. Begeman, A. H. Broeils, and R. H. Sanders. "Extended rotation curves of spiral galaxies: Dark haloes and modified dynamics". *Mon. Not. Roy. Astron. Soc.* 249 (1991), page 523. DOI: 10.1093/mnras/249.3.523.

[70] Albert Einstein. "Cosmological Considerations in the General Theory of Relativity". *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* 1917 (1917), pages 142–152.

[71] Edwin Hubble. "A relation between distance and radial velocity among extra-galactic nebulae". *Proc. Nat. Acad. Sci.* 15 (1929), pages 168–173. DOI: 10.1073/pnas.15.3.168.

[72] Michael S. Turner. "Dark matter and dark energy in the universe". *ASP Conf. Ser.* 165 (1999). Edited by Brad K. Gibson, Rim S. Axelrod, and Mary E. Putman, page 431. arXiv: astro-ph/9811454.

[73] Varun Sahni. "Dark matter and dark energy". *Lect. Notes Phys.* 653 (2004). Edited by E. Papantonopoulos, pages 141–180. DOI: 10.1007/b99562. arXiv: astro-ph/0403324.

[74] Gianfranco Bertone, Dan Hooper, and Joseph Silk. "Particle dark matter: Evidence, candidates and constraints". *Phys. Rept.* 405 (2005), pages 279–390. DOI: 10.1016/j.physrep.2004.08.031. arXiv: hep-ph/0404175.

[75] Willy Fischler and Joel Meyers. "Dark Radiation Emerging After Big Bang Nucleosynthesis?" *Phys. Rev. D* 83 (2011), page 063520. DOI: 10.1103/PhysRevD.83.063520. arXiv: 1011.3501 [astro-ph.CO].

[76] Maria Archidiacono, Erminia Calabrese, and Alessandro Melchiorri. "The Case for Dark Radiation". *Phys. Rev. D* 84 (2011), page 123008. DOI: 10.1103/PhysRevD.84.123008. arXiv: 1109.2767 [astro-ph.CO].

[77] Robert H. Brandenberger. “Lectures on the theory of cosmological perturbations”. *Lect. Notes Phys.* 646 (2004). Edited by Nora Breton, Jorge L. Cervantes-Cota, and Marcelo Salgado, pages 127–167. DOI: 10.1007/978-3-540-40918-2_5. arXiv: hep-th/0306071.

[78] LIGO Scientific Virgo Fermi-GBM INTEGRAL collaboration. “Gravitational Waves and Gamma-rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A”. *Astrophys. J. Lett.* 848.2 (2017), page L13. DOI: 10.3847/2041-8213/aa920c. arXiv: 1710.05834 [astro-ph.HE].

[79] James M. Bardeen. “Gauge Invariant Cosmological Perturbations”. *Phys. Rev. D* 22 (1980), pages 1882–1905. DOI: 10.1103/PhysRevD.22.1882.

[80] Xingang Chen, Mohammad Hossein Namjoo, and Yi Wang. “Models of the Primordial Standard Clock”. *JCAP* 02 (2015), page 027. DOI: 10.1088/1475-7516/2015/02/027. arXiv: 1411.2349 [astro-ph.CO].

[81] M. Gasperini. “Spin Dominated Inflation in the Einstein-cartan Theory”. *Phys. Rev. Lett.* 56 (1986), pages 2873–2876. DOI: 10.1103/PhysRevLett.56.2873.

[82] João Magueijo, T. G. Zlosnik, and T. W. B. Kibble. “Cosmology with a spin”. *Phys. Rev. D* 87.6 (2013), page 063504. DOI: 10.1103/PhysRevD.87.063504. arXiv: 1212.0585 [astro-ph.CO].

[83] Giorgi Tukhashvili and Paul J. Steinhardt. “Cosmological Bounces Induced by a Fermion Condensate”. *Phys. Rev. Lett.* 131.9 (2023), page 091001. DOI: 10.1103/PhysRevLett.131.091001. arXiv: 2307.16098 [gr-qc].

[84] John Michell. “On the Means of Discovering the Distance, Magnitude, &c. of the Fixed Stars, in Consequence of the Diminution of the Velocity of Their Light, in Case Such a Diminution Should be Found to Take Place in any of Them, and Such Other Data Should be Procured from Observations, as Would be Farther Necessary for That Purpose.” *Phil. Trans. Roy. Soc. Lond.* 74 (1784), pages 35–57. DOI: 10.1098/rstl.1784.0008.

[85] Karl Schwarzschild. “On the gravitational field of a mass point according to Einstein’s theory”. *Sitzungsber. Preuss. Akad. Wiss.*

Berlin (Math. Phys.) 1916 (1916), pages 189–196. arXiv: physics/9905030.

[86] G. Lemaitre. "The expanding universe". *Annales Soc. Sci. Bruxelles A* 53 (1933), pages 51–85. DOI: 10.1023/A:1018855621348.

[87] Manuel Hohmann. "Metric-affine Geometries With Spherical Symmetry". *Symmetry* 12.3 (2020), page 453. DOI: 10.3390/sym12030453. arXiv: 1912.12906 [math-ph].

[88] Edwin Taylor, John Archibald Wheeler, and Edmund Bertschinger. *Exploring Black Holes: Introduction to General Relativity Second Edition.* 2nd edition. 2017.

[89] G. W. Gibbons and S. W. Hawking. "Cosmological Event Horizons, Thermodynamics, and Particle Creation". *Phys. Rev. D* 15 (1977), pages 2738–2751. DOI: 10.1103/PhysRevD.15.2738.

[90] Mohammad Ali Gorji, Alireza Allahyari, Mohsen Khodadi, and Hassan Firouzjahi. "Mimetic black holes". *Phys. Rev. D* 101.12 (2020), page 124060. DOI: 10.1103/PhysRevD.101.124060. arXiv: 1912.04636 [gr-qc].

[91] Mohsen Khodadi, Sunny Vagnozzi, and Javad T. Firouzjaee. "Event Horizon Telescope observations exclude compact objects in baseline mimetic gravity". *Sci. Rep.* 14.1 (2024), page 26932. DOI: 10.1038/s41598-024-78264-y. arXiv: 2408.03241 [gr-qc].

[92] H. Reissner. "Über die Eigengravitation des elektrischen Feldes nach der Einsteinschen Theorie". *Annalen Phys.* 355.9 (1916), pages 106–120. DOI: 10.1002/andp.19163550905.

[93] Gunnar Nordstrom. "On the Energy of the Gravitation field in Einstein's Theory". *Kon. Ned. Akad. Wetensch. Proc.* 20.2 (1918), pages 1238–1245.

[94] Priidik Gallagher, Tomi Koivisto, and Luca Marzola. "Pregeometric first order Yang-Mills theory". *Phys. Rev. D* 105.12 (2022), page 125010. DOI: 10.1103/PhysRevD.105.125010. arXiv: 2202.05657 [hep-th].

[95] Ernest Michael Priidik Gallagher. "On the internal gauge theory analogy to the Cartan Khronon theory of gravity". PhD thesis. University of Tartu, 2026. URL: https://dspace.ut.ee/server/api/core/bitstreams/e7068909-c960-4a14-8bf4-fcc8788dee22/content.

[96] Roberto Maiolino et al. “A small and vigorous black hole in the early Universe”. *Nature* 627.8002 (2024). [Erratum: Nature 630, E2 (2024)], pages 59–63. DOI: 10.1038/s41586-024-07494-x. arXiv: 2305.12492 [astro-ph.GA].

[97] Zhaoyi Xu, Xian Hou, Xiaobo Gong, and Jiancheng Wang. “Black Hole Space-time In Dark Matter Halo”. *JCAP* 09 (2018), page 038. DOI: 10.1088/1475-7516/2018/09/038. arXiv: 1803.00767 [gr-qc].

[98] R. A. Konoplya and A. Zhidenko. “Solutions of the Einstein Equations for a Black Hole Surrounded by a Galactic Halo”. *Astrophys. J.* 933.2 (2022), page 166. DOI: 10.3847/1538-4357/ac76bc. arXiv: 2202.02205 [gr-qc].

[99] Vitor Cardoso, Kyriakos Destounis, Francisco Duque, Rodrigo Panosso Macedo, and Andrea Maselli. “Black holes in galaxies: Environmental impact on gravitational-wave generation and propagation”. *Phys. Rev. D* 105.6 (2022), page L061501. DOI: 10.1103/PhysRevD.105.L061501. arXiv: 2109.00005 [gr-qc].

[100] Roy P. Kerr. “Gravitational field of a spinning mass as an example of algebraically special metrics”. *Phys. Rev. Lett.* 11 (1963), pages 237–238. DOI: 10.1103/PhysRevLett.11.237.

[101] R. Penrose and R. M. Floyd. “Extraction of rotational energy from a black hole”. *Nature* 229 (1971), pages 177–179. DOI: 10.1038/physci229177a0.

[102] Matt Visser. “The Kerr spacetime: A Brief introduction”. *Kerr Fest: Black Holes in Astrophysics, General Relativity and Quantum Gravity.* June 2007. arXiv: 0706.0622 [gr-qc].

[103] Christian Heinicke and Friedrich W. Hehl. “Schwarzschild and Kerr Solutions of Einstein’s Field Equation – an introduction”. *Int. J. Mod. Phys. D* 24.02 (2014), page 1530006. DOI: 10.1142/S0218271815300062. arXiv: 1503.02172 [gr-qc].

[104] Saul A. Teukolsky. “The Kerr Metric”. *Class. Quant. Grav.* 32.12 (2015), page 124006. DOI: 10.1088/0264-9381/32/12/124006. arXiv: 1410.2130 [gr-qc].

[105] Event Horizon Telescope collaboration. “First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole”.

Astrophys. J. Lett. 875 (2019), page L1. DOI: 10.3847/2041-8213/ab0ec7. arXiv: 1906.11238 [astro-ph.GA].

[106] Event Horizon Telescope collaboration. "First M87 Event Horizon Telescope Results. VI. The Shadow and Mass of the Central Black Hole". *Astrophys. J. Lett.* 875.1 (2019), page L6. DOI: 10.3847/2041-8213/ab1141. arXiv: 1906.11243 [astro-ph.GA].

[107] Event Horizon Telescope collaboration. "First Sagittarius A* Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole in the Center of the Milky Way". *Astrophys. J. Lett.* 930.2 (2022), page L12. DOI: 10.3847/2041-8213/ac6674. arXiv: 2311.08680 [astro-ph.HE].

[108] Event Horizon Telescope collaboration. "First Sagittarius A* Event Horizon Telescope Results. VI. Testing the Black Hole Metric". *Astrophys. J. Lett.* 930.2 (2022), page L17. DOI: 10.3847/2041-8213/ac6756. arXiv: 2311.09484 [astro-ph.HE].

[109] Chris Doran. "A New form of the Kerr solution". *Phys. Rev. D* 61 (2000), page 067503. DOI: 10.1103/PhysRevD.61.067503. arXiv: gr-qc/9910099.

[110] Eric Poisson and Werner Israel. "Inner-horizon instability and mass inflation in black holes". *Physical Review Letters* 63 (1989), pages 1663–1666. DOI: 10.1103/PhysRevLett.63.1663.

[111] Salvatore Capozziello, Silvia De Bianchi, and Emmanuele Battista. "Avoiding singularities in Lorentzian-Euclidean black holes: The role of atemporality". *Phys. Rev. D* 109.10 (2024), page 104060. DOI: 10.1103/PhysRevD.109.104060. arXiv: 2404.17267 [gr-qc].

[112] David Tong. *Lectures on the Standard Model.* 2019. URL: https://www.damtp.cam.ac.uk/user/tong/sm/standardmodel.pdf.

[113] David Tong. *Lectures on Gauge Theory.* 2018. URL: https://www.damtp.cam.ac.uk/user/tong/gaugetheory.html.

[114] Derek K. Wise. "MacDowell-Mansouri gravity and Cartan geometry". *Class. Quant. Grav.* 27 (2010), page 155010. DOI: 10.1088/0264-9381/27/15/155010. arXiv: gr-qc/0611154.

[115] Hendrik Antoon Lorentz. "Electromagnetic phenomena in a system moving with any velocity smaller than that of light". *Proceedings of the Royal Netherlands Academy of Arts and Sciences* 6 (1904),

pages 809–831. URL: https://dwc.knaw.nl/DL/publications/PU00014148.pdf.

[116] Henri Poincaré. "Sur la dynamique de l'électron". *Comptes rendus hebdomadaires des séances de l'Académie des sciences* 140 (June 1905), pages 1504–1508.

[117] Emmy Noether. "Invariant Variation Problems". *Gott. Nachr.* 1918 (1918), pages 235–257. DOI: 10.1080/00411457108231446. arXiv: physics/0503066.

[118] M. Kasuya. "On the Gauge Theory in the Einstein-Cartan-Weyl Space-Time". *Nuovo Cim. B* 28 (1975), pages 127–137. DOI: 10.1007/BF02722810.

[119] E. Schrödinger. "Quantisierung als Eigenwertproblem". *Annalen Phys.* 384.4 (1926), pages 361–376. DOI: 10.1002/andp.19263840404.

[120] Chen-Ning Yang and R. L. Mills. "Isotopic spin conservation and a generalized gauge invariance" (1954).

[121] Ronald Shaw. "The problem of particle types and other contributions to the theory of elementary particles". PhD thesis. Cambridge U., 1955.

[122] T. Y. Cao. *Conceptual developments of 20th century field theories.* Cambridge University Press, Oct. 2019. ISBN: 978-0-521-63420-5, 978-0-511-88190-9, 978-1-108-47607-2, 978-1-108-75374-6, 978-1-108-56692-6. DOI: 10.1017/9781108566926.

[123] M. Leclerc. "Canonical and gravitational stress-energy tensors". *Int. J. Mod. Phys. D* 15 (2006), pages 959–990. DOI: 10.1142/S0218271806008693. arXiv: gr-qc/0510044.

[124] R. P. Kerr. "Do Black Holes have Singularities?" (Nov. 2023). arXiv: 2312.00841 [gr-qc].

[125] Kirill Krasnov. "Field redefinitions and Plebanski formalism for GR". *Class. Quant. Grav.* 35.14 (2018), page 147001. DOI: 10.1088/1361-6382/aac844. arXiv: 1708.07694 [gr-qc].

Attached publications

Curriculum Vitae

Personal data

Name:	Lucy Zheng 郑 露西 [1]
Email:	zhenglucy@icloud.com

Physics education

2022–2026 2019–2021	Doctoral study in Physics,	University of Tartu
2017–2018	MSc in Gravity, Particles, and Fields,	University of Nottingham
2013–2017	BSc in Physics,	Tokyo University of Science
2009–2012	Ochanomizu University Senior High School	
1997–1998	University of Science and Technology Beijing kindergarten	

Relevant Experience

2025	Seminar & research visit,	NORDITA (Nordic Institute for Theoretical Physics)
	Invited seminar visit,	University of Helsinki
2024–	Peer review,	CQG, Physica Scripta (IOP), PRD (APS)
2021	Maths contents writer,	freelance
2017	Museum staff,	Space Museum TeNQ
2013	Museum attendant,	Miraikan (National Museum of Emerging Science and Innovation)

[1] I stopped using Pinyin spelling to avoid confusions and mispronunciations. Please refer to me as Lucy in alphabetical context and 露西, the literal Chinese equivalent of Lucy, in Hanzi (Kanji).

Elulookirjeldus

Isikuandmed

Nimi:	Lucy Zheng 郑 露西 [2]
E-post:	luxi.zheng@ut.ee

Füüsika haridus

2022–2026 2019–2021	Doktorantuur füüsikas,	Tartu Ülikool
2017–2018	MSc gravitatsiooni, osakeste ja väljade alal,	University of Nottingham
2013–2017	BSc füüsikas,	東京理科大学
2009–2012	Gümnaasium,	お茶の水女子大学附属高校
1997–1998	Lasteaed,	北京科技大学附属幼儿园

Kogemus

2025	Seminari- ja uurimisvisiit,	NORDITA (Põhjamaade Teoreetilise Füüsika Instituut)
	Kutsutud seminarivisiit,	Helsingin yliopisto
2024–	Ajakirja retsensioon,	CQG, Physica Scripta (IOP), PRD (APS)
2021	Matemaatika sisu kirjutaja,	vabakutseline
2017	Muuseumi töötaja,	Kosmosemuuseum TeNQ
2013	Muuseumi teenindaja,	科学未来館 (Riiklik Uue Teaduse ja Innovatsiooni Muuseum)

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