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# ANALYSIS OF 1436 METEOR VELOCITIES

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# Analysis of 1436 Meteor Velocities

By Ernst Öpik.

In the year 1931 the writer devised a special apparatus for observing the velocities of meteors; the principles of this "rocking mirror" apparatus have been described briefly elsewhere (*cf.*[1] and[6]). A conically oscillating mirror, of ten oscillations per second, transforms the trail of a moving meteor into an apparently wave-shaped trajectory; from the observed number of waves, as well as from the apparent shape of the trajectory the angular velocity of the meteor can be derived. As in all kinds of visual observations of meteors in which the observer has finally to rely upon his memory, considerable accidental and systematic errors are involved in the observed velocities, too; in a statistical discussion of velocities such as given below the data must be freed, in the first place, from the influence of these errors. Only after that can the bearing of the statistical data upon cosmic problems be investigated.

The first rocking mirror apparatus of the writer's design was constructed in the work-shop of Harvard College Observatory, being destined for use in the Arizona Expedition for the Study of Meteors. This expedition, initiated by Dr. Harlow Shapley, director of Harvard College Observatory, was active in and near Flagstaff, Arizona, during the years 1931—1933. The major part of the observational funds was provided by Harvard University, namely all the expenses connected with the visual observations which formed the main portion of the programme. Supplementary telescopic observations were taken care of by Cornell University. Altogether about 26 000 visual observations of meteors were obtained during the Arizona Expedition. The writer is responsible for the entire programme of the expedition as well as for the reduction and discussion of the results which have been in progress for years at Tartu. Several papers concerning the results of the expedition have been already published (*cf.*[1, 2, 3, 4, 6]), and the present paper which deals specially with the observations of velocities does not yet conclude these investigations. In the course of the reductions and discussion, computational assistance was provided for by grants from the J. Lawrence Smith Fund of the National Academy of Sciences, Washington.

During the first two months of the activity of the Arizona Expedition the writer himself made observations with the rocking mirror apparatus; after he left, the velocity observations were continued by two other observers who worked alternately.

The present paper contains the statistical discussion of all meteor velocities observed visually during the Arizona Expedition: 611 velocities by Roger Wilson (R. W.); 546 velocities by Donald Hargrave (D. H.); 279 velocities by the writer (E. Ö.). A brief preliminary account of the first experiment, the writer's personal observations, has been already given[1] together with a description of the method. Now the entire velocity data of the expedition are summarized below.

Unpublished tables for the computation of the true radiants of extra-solar (hyperbolic) meteors were kindly put at the disposal of the writer by F. L. Whipple, of Harvard College Observatory.

1. *Reduction of Observations.* — The method of reduction is essentially the same as that described in the preliminary paper; meteors belonging, with a sufficient degree of probability, to conspicuous showers were used for the purpose of determining the systematic errors of the observers; the list of radiants in Table XXX of[3], with the mean heights given there, was adopted for the calibration of the observations of R. W. and D. H. As in[1], a parabolic velocity for all the shower meteors was assumed provisionally; the systematic corrections based on this assumption are expected to yield slightly greater velocities than the true

ones; in the final deduction of the distribution of space velocities (Section 6) a correction for the "ellipticity" of the shower meteors is introduced.

The correction factors by which the directly observed angular velocities must be multiplied are found as follows:

| Method | | $\omega_n$ | $\omega'_n$ | $\omega'$ | $\omega_1$ | $\omega_2$ |
|---|---|---|---|---|---|---|
| Correction factor, | R. W. | 0.741 | 0.528 | 0.649 | 1.46 | 1.63 |
| „ | D. H. | 0.775 | 0.775 | 0.775 | . . . | . . . |

The method designations mean: $\omega_n$, angular velocity from length of trail and duration, in the case of observed shape; $\omega'_n$ — from length of trail and number of waves, without observed shape; $\omega'$ — from length of trail, duration, and shape, when less than one whole wave was observed; $\omega_1$ — from shape (qualitative); $\omega_2$ — from ratio of height to length of wave. The shape as observed by D. H. proved to be unusable. For R. W., in the case of observed shape, the mean angular velocity was computed by assigning to $\frac{\omega_1 + \omega_2}{2}$ half-weight as compared with $\omega_n$. The number of (presumable) shower meteors was unfortunately rather small: forty-one for R. W. and thirty-two for D. H., as compared with sixty-nine for E. Ö.; therefore, the correction factors may be appreciably influenced by accidental errors. The meagre data are, nevertheless, sufficient to show for all three observers a similar run of the correction factors, although for R. W. and D. H. these differ from unity more considerably than for E. Ö.[1]. In the case of $\omega_n$, the systematic difference between E. Ö., R. W., and D. H. is in the same direction, and almost the same in amount as the systematic difference in the length of the trail (*cf.*[3], Table VII), which suggests that the systematic difference in duration may be small. The large correction factors for $\omega_1$ and $\omega_2$ point apparently to the suggested following of the meteor by the observer's eye (*cf.*[1], p. 4); the observed shape corresponds to the difference of the angular velocity of meteor and eye. After rejecting some outstanding cases, probably stray meteors (*cf.*[1], pp. 2—3), the standard deviation of $\log \omega$ for the shower meteors was found to be $\pm 0.165$ for D. H., and $\pm 0.176$ for R. W., as compared with $\pm 0.165$ for E. Ö. (*cf.*[1], p. 5). The larger figure for R. W. may be purely accidental. Allowance being made for the probable number of stray meteors retained, the observational error in log $\omega$ may be assumed to be equal to $\pm 0.106$ (*p. e.*), the same for all three observers[1]; this includes the error dispersion in the assumed mean height.

As a criterion of the rejection of some of the "standard" shower meteors the uncorrected ratio of observed to predicted angular velocity was used: the observation was rejected when this ratio exceeded 2.5 or fell below 0.4. These limits correspond to four times the logarithmic probable error. When the correction factor is introduced the limits of rejection become asymmetrical: about 1.9 and 0.3 ($\omega_n$) respectively. Nevertheless, from his own experience the writer insists upon the use of the uncorrected, directly observed ratio as a criterion of rejection. Some arguments in favour of the procedure are given below.

The correction factors as quoted above are computed without using the outstanding observations already mentioned; most of the rejected observations yield too large an angular velocity, hence a systematic influence of the rejection upon the resulting mean value of $\log \frac{\omega}{\omega_c}$ ($\omega_c$ = computed angular velocity) is exerted; therefore the rejection of eleven out of the thirty-two observations of D. H. requires special justification, whereas for R. W. (six out of forty-one) and E. Ö. (five out of sixty-nine) the effect of the rejection is of minor importance.

The probable number of stray meteors among the shower meteors of D. H., calculated according to[4], is 7.6 — smaller than the rejected number (eleven); and, nevertheless, there is evidence that the rejected ones are probably stray meteors. The angle $\Delta A$ between the direction from the radiant, and the direction of the meteor, shows the following distribution (D. H.):

| $\Delta A$ | $0' \ldots 6^0$ | $7^0 \ldots 12^0$ | $13^0 \ldots 18^0$ | All |
|---|---|---|---|---|
| Rejected m. | 3 | 0 | 8 | 11 |
| Retained m. | 10 | 5 | 6 | 21 |

There is no concentration of $\Delta A$ for the rejected meteors, contrary to what is expected for shower meteors (the observational probable error in $\Delta A$ is $\pm 8^0.0$, cf.[4], p. 7). Further, the distribution of $\omega_c$ for the rejected meteors is rather peculiar (D. H.):

| $\omega_c$, deg/sec | 0 . . . . 4.9 | 5.0 . . . . 9.9 | $\geqslant$ 10.0 | All |
|---|---|---|---|---|
| Rejected large $\omega$ | 4 | 6 | 0 | 10 |
| Rejected small $\omega$ | 0 | 0 | 1 | 1 |
| Retained | 0 | 5 | 16 | 21 |

The distribution of $\omega_c$ for the rejected and the retained meteors is opposite in character. All the rejected large $\omega$ fall near the radiant and have therefore small $\omega_c$. In the following section it is shown that the specific selection of the rocking mirror procedure is strongly unfavourable for small angular velocities; hence, near a radiant the observer is likely to omit the shower meteors, and to register stray meteors which come from farther radiants; in such a case the theoretically computed fraction of stray meteors turns out to be too small for the velocity observations. As the Perseids have their radiant within the region of observation (45$^0$ north of the zenith), they give the largest fraction of rejections. D. H. has seven Perseids, of which five are rejected on account of large $\omega$; of the ten Perseids of R. W., four are rejected because of large $\omega$, whereas his total number rejected on this account is only five.

From the above it appears that the rejected meteors represent a peculiar selection apparently of stray meteors and must not be used for standardization. At the same time, however, we must confess that on account of the small number of standard shower meteors left the velocity scale of D. H. is not well determined.

In view of the importance of the method of standardization in our problem, the complete list of the shower meteors is given in Table I; $\omega$ is the concluded angular velocity after application of the correction factors mentioned above; for the "rejected" meteors $\omega_c$ is given in parentheses. The probable number of stray meteors is: R. W., 6.5; D. H., 7.6; E. Ö., 12.0. For reasons given above these figures may be underestimated, at least for the first two observers; also, very luminous meteors with strong trains like the Leonids are not easily observed for velocity, which circumstance again favours the selection of stray meteors.

## Table I.

### List of Velocities of Shower Meteors.

| Radiant | | | | deg/sec | | | |
|---|---|---|---|---|---|---|---|
| *AR* | δ | Date | ΔA limit | ω | $\omega_c$ | $\log \frac{\omega}{\omega_c}$ | ΔA |
| | | | Observer: | E. Ö. | | | |
| 16° | +2° | Oct. 7, 31 | ≤18° | 39.6 | (11.9) | 0.522 | 0° |
| " | " | " | " | 3.7 | 11.6 | —0.498 | 13 |
| 17 | +3 | Oct. 8, 31 | ≤6 | 21.5 | 16.4 | 0.118 | 2 |
| " | " | " | " | 10.2 | 13.9 | —0.133 | 2 |
| " | " | " | " | 10.0 | 14.6 | —0.164 | 3 |
| " | " | " | " | 10.3 | 14.1 | —0.136 | 4 |
| 355 | +35 | Oct. 13, 31 | ≤18 | 12.6 | 15.2 | —0.080 | 5 |
| 32 | +5 | " | ≤12 | 7.8 | 19.2 | —0.390 | 11 |
| " | " | " | " | 20.9 | 13.8 | 0.180 | 8 |
| " | " | " | " | 14.1 | 16.8 | —0.076 | 3 |
| " | " | " | " | 15.1 | 19.9 | —0.120 | 2 |
| 93 | +14 | Oct. 21, 31 | ≤18 | 35.0 | 30.0 | 0.067 | 14 |
| " | " | " | " | 32.0 | 21.1 | 0.181 | 12 |
| " | " | " | " | 31.0 | 22.9 | 0.131 | 18 |
| " | " | " | " | 38.8 | 33.0 | 0.070 | 14 |
| " | " | " | " | 28.9 | 30.3 | —0.020 | 14 |
| " | " | " | " | 23.5 | 29.8 | —0.103 | 11 |
| " | " | " | " | 19.7 | 28.6 | —0.161 | 4 |
| " | " | " | " | 20.6 | 29.5 | —0.155 | 10 |
| 42 | +20 | Oct. 30, 31 | ≤12 | 11.4 | 16.6 | —0.162 | 6 |
| " | " | Nov. 1, 31 | " | 16.3 | 9.8 | 0.218 | 3 |
| " | " | " | " | 24.7 | 14.3 | 0.238 | 4 |
| " | " | " | " | 17.9 | 16.3 | 0.041 | 0 |
| " | " | " | " | 9.4 | 13.4 | —0.154 | 1 |
| 54 | +15 | Nov. 3, 31 | ≤18 | 5.0 | 10.6 | —0.328 | 13 |
| " | " | " | " | 11.3 | 18.9 | —0.223 | 7 |
| " | " | " | " | 15.7 | 14.6 | 0.031 | 2 |
| " | " | " | " | 9.8 | 9.3 | 0.023 | 3 |
| " | " | " | " | 10.3 | 17.6 | —0.232 | 13 |
| " | " | " | " | 11.0 | 19.0 | —0.237 | 12 |
| " | " | " | " | 8.8 | 13.2 | —0.176 | 15 |
| 64 | +25 | Nov. 4, 31 | ≤12 | 24.7 | 15.4 | 0.205 | 3 |
| " | " | " | " | 19.4 | 16.6 | 0.068 | 1 |
| " | " | " | " | 11.3 | 14.4 | —0.105 | 9 |
| 135 | +17 | " | ≤18 | 14.0 | 15.5 | —0.044 | 0 |
| 56 | +28 | Nov. 6, 31 | ≤12 | 29.0 | 14.3 | 0.308 | 0 |
| " | " | " | " | 25.5 | 14.7 | 0.239 | 6 |
| " | " | " | " | 12.0 | 12.8 | —0.028 | 5 |
| " | " | " | " | 9.7 | 12.2 | —0.099 | 10 |
| " | " | " | " | 14.1 | 12.1 | 0.068 | 12 |
| " | " | " | " | 10.3 | 18.1 | —0.244 | 3 |
| 150 | +22 | Nov. 14, 31 | ≤18 | 20.3 | 22.0 | —0.034 | 8 |
| " | " | " | " | 38.6 | 27.7 | 0.145 | 8 |
| " | " | " | " | 36.0 | 23.7 | 0.182 | 15 |
| 152 | +22 | Nov. 16, 31 | " | 60.0 | (24.8) | 0.384 | 7 |

## Table I. Continued.

| Radiant | | | | deg/sec | | log $\frac{\omega}{\omega_c}$ | $\Delta A$ |
|---|---|---|---|---|---|---|---|
| $AR$ | $\delta$ | Date | $\Delta A$ limit | $\omega$ | $\omega_c$ | | |
| 152° | + 22° | Nov. 16, 31 | ⩽ 18° | 25.5 | 10.9 | 0.369 | 1° |
| " | " | " | " | 7.9 | 12.1 | — 0.185 | 3 |
| " | " | " | " | 18.4 | 31.3 | — 0.231 | 8 |
| " | " | " | " | 30.7 | 16.3 | 0.275 | 17 |
| " | " | " | " | 30.4 | 23.5 | 0.113 | 0 |
| " | " | " | " | 19.2 | 16.7 | 0.061 | 2 |
| " | " | " | " | 38.8 | 33.0 | 0.070 | 5 |
| " | " | " | " | 12.3 | 11.2 | 0.041 | 7 |
| " | " | " | " | 18.2 | 18.2 | 0.000 | 1 |
| " | " | " | " | 25.8 | 20.9 | 0.093 | 2 |
| " | " | " | " | 21.0 | 21.0 | 0.000 | 6 |
| " | " | " | " | 29.0 | 27.6 | 0.022 | 0 |
| " | " | " | " | 23.5 | 22.3 | 0.023 | 4 |
| " | " | " | " | 20.7 | 18.7 | 0.045 | 1 |
| " | " | " | " | 11.4 | 20.8 | — 0.260 | 0 |
| " | " | " | " | 45.0 | (13.0) | 0.540 | 2 |
| 57 | + 25 | " | ⩽ 12 | 39.4 | (11.0) | 0.554 | 2 |
| " | " | " | " | 30.7 | 14.4 | 0.328 | 1 |
| " | " | " | " | 22.7 | 18.6 | 0.086 | 3 |
| " | " | " | " | 12.6 | 13.5 | — 0.029 | 12 |
| " | " | " | " | 36.8 | (10.2) | 0.558 | 0 |
| " | " | " | " | 14.7 | 11.8 | 0.096 | 2 |
| " | " | " | " | 8.6 | 12.8 | — 0.172 | 7 |
| 84 | + 17 | " | ⩽ 12 | 16.9 | 20.3 | — 0.080 | 0 |
| | | | Observer: R. W. | | | | |
| 196° | + 25° | Apr. 24, 32 | ⩽ 18° | 3.0 | (9.4) | — 0.495 | 13° |
| " | " | " | " | 11.0 | 11.7 | — 0.026 | 7 |
| " | " | " | " | 6.0 | 7.9 | — 0.119 | 15 |
| 330 | — 4 | May 4, 32 | " | 18.6 | 28.0 | — 0.176 | 9 |
| " | " | " | " | 31.2 | 37.7 | — 0.080 | 11 |
| 270 | — 35 | May 28, 32 | " | 9.5 | 12.0 | — 0.101 | 18 |
| " | " | " | " | 14.5 | 22.5 | — 0.190 | 14 |
| " | " | " | " | 10.6 | 19.0 | — 0.253 | 5 |
| " | " | " | " | 27.7 | 17.3 | 0.205 | 1 |
| " | " | " | " | 26.7 | (12.4) | 0.332 | 12 |
| 341 | — 16 | Aug. 1, 32 | " | 21.8 | 14.1 | 0.190 | 5 |
| " | " | " | " | 21.6 | 14.8 | 0.164 | 7 |
| " | " | " | " | 7.6 | 15.0 | — 0.295 | 2 |
| " | " | " | " | 10.6 | 20.1 | — 0.298 | 6 |
| " | " | " | " | 23.4 | 21.3 | 0.041 | 18 |
| 55 | + 48 | " | " | 14.8 | 16.1 | — 0.036 | 0 |
| " | " | " | " | 38.9 | (10.7) | 0.561 | 6 |
| " | " | " | " | 21.2 | 20.1 | 0.023 | 18 |
| 45 | + 57 | Aug. 11, 32 | " | 25.9 | 20.4 | 0.104 | 5 |
| " | " | " | " | 17.4 | 16.0 | 0.037 | 12 |
| " | " | " | " | 24.2 | (9.2) | 0.406 | 8 |

Table I. Continued.

| Radiant | | | | deg/sec | | log $\frac{\omega}{\omega_c}$ | $\Delta A$ |
|---|---|---|---|---|---|---|---|
| $AR$ | $\delta$ | Date | $\Delta A$ limit | $\omega$ | $\omega_c$ | | |
| 45° | +57° | Aug. 11, 32 | ⩽18° | 14.8 | 10.5 | 0.150 | 12° |
| " | " | " | " | 21.1 | 11.0 | 0.284 | 18 |
| " | " | " | " | 24.9 | (9.2) | 0.433 | 13 |
| " | " | " | " | 29.9 | (13.6) | 0.342 | 11 |
| 70 | —5 | Sept. 8, 32 | " | 18.6 | 16.4 | 0.055 | 8 |
| " | " | " | " | 14.8 | 28.5 | —0.284 | 16 |
| " | " | " | " | 17.4 | 16.9 | 0.013 | 17 |
| " | " | " | " | 28.5 | 21.5 | 0.123 | 10 |
| 150 | +25 | Nov. 12, 32 | " | 27.3 | 17.7 | 0.188 | 6 |
| " | " | Nov. 16, 32 | " | 14.2 | 11.8 | 0.081 | 13 |
| " | " | " | " | 19.7 | 17.7 | 0.046 | 11 |
| " | " | " | " | 20.1 | 19.9 | 0.004 | 10 |
| " | " | " | " | 31.2 | 20.4 | 0.185 | 11 |
| " | " | " | " | 30.6 | 22.3 | 0.138 | 5 |
| " | " | " | " | 27.5 | 25.2 | 0.039 | 0 |
| 57 | +3 | Nov. 12, 32 | " | 19.5 | 12.2 | 0.204 | 14 |
| " | " | Nov. 13, 32 | " | 15.6 | 17.7 | —0.054 | 0 |
| " | " | Nov. 17, 32 | " | 6.0 | 8.6 | —0.156 | 0 |
| 275 | +30 | Apr. 21, 33 | " | 13.1 | 8.9 | 0.168 | 3 |
| " | " | " | " | 16.4 | 14.9 | 0.042 | 15 |

Observer: D. H.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 196° | +25° | Apr. 25, 32 | ⩽18° | 19.1 | (8.8) | 0.336 | 13° |
| " | " | " | " | 11.9 | 8.4 | 0.152 | 0 |
| 224 | —16 | May 11, 32 | " | 8.9 | 11.7 | —0.118 | 1 |
| " | " | " | " | 8.5 | 9.9 | —0.066 | 14 |
| " | " | " | " | 14.0 | 10.0 | 0.146 | 9 |
| 336 | —17 | July 31, 32 | " | 9.0 | 14.9 | —0.222 | 1 |
| " | " | " | " | 11.2 | 13.2 | —0.071 | 14 |
| " | " | " | " | 11.5 | 20.4 | —0.248 | 14 |
| " | " | " | " | 28.7 | 16.6 | 0.238 | 0 |
| " | " | " | " | 11.4 | 15.2 | —0.124 | 11 |
| " | " | " | " | 9.6 | 10.9 | —0.054 | 14 |
| 45 | +57 | Aug. 10, 32 | " | 27.1 | (5.5) | 0.694 | 0 |
| " | " | " | " | 13.6 | (7.0) | 0.288 | 17 |
| " | " | " | " | 11.8 | 9.8 | 0.080 | 2 |
| " | " | " | " | 14.9 | (8.0) | 0.271 | 3 |
| " | " | " | " | 14.2 | (8.9) | 0.204 | 6 |
| " | " | " | " | 9.0 | (4.1) | 0.342 | 16 |
| " | " | " | " | 23.2 | (4.6) | 0.703 | 17 |
| 288 | +47 | Aug. 10, 32 | " | 7.7 | 6.5 | 0.073 | 15 |
| 35 | +19 | Oct. 23, 32 | " | 14.6 | 14.8 | —0.006 | 6 |
| " | " | " | " | 11.9 | 12.3 | —0.013 | 5 |
| 108 | +25 | Oct. 25, 32 | " | 30.5 | 20.2 | 0.180 | 2 |
| " | " | " | " | 16.0 | 22.3 | —0.143 | 10 |
| 230 | +47 | Jan. 2, 33 | " | 13.2 | (5.8) | 0.358 | 17 |
| " | " | " | " | 16.4 | 12.3 | 0.126 | 5 |

Table I. Continued.

| Radiant | | | | deg/sec | | $\log \frac{\omega}{\omega_c}$ | $\Delta A$ |
|---|---|---|---|---|---|---|---|
| $AR$ | $\delta$ | Date | $\Delta A$ limit | $\omega$ | $\omega_c$ | | |
| 230° | +47° | Jan. 2, 33 | ≤18° | 6.7 | (3.6) | 0.270 | 14° |
| ” | ” | ” | ” | 9.8 | 12.2 | −0.095 | 11 |
| 130 | +40 | Feb. 18, 33 | ” | 7.3 | (3.8) | 0.284 | 3 |
| ” | ” | ” | ” | 18.6 | (6.9) | 0.431 | 13 |
| ” | ” | ” | ” | 14.3 | 10.2 | 0.147 | 16 |
| 250 | +10 | May 30, 33 | ” | 3.0 | (15.7) | −0.717 | 14 |
| 286 | −20 | June 25, 33 | ” | 11.2 | 15.8 | −0.148 | 7 |

From 486 meteors with the observed height and velocity (R. W. + D. H.) the correlation represented by Table II was derived. $H_0$ is the harmonic mean height reduced to $m_z = 2.2$ and midnight[3]. The table represents the regression curve of $H_0$ upon $\omega_z = \omega \sec z$; owing to the observational error dispersion and to the spread in the projection ratio, the true correlation of the mean height and velocity must exhibit a greater range in $H_0$.

Table II.

Correlation of Height and Angular Velocity, $m_z = 2.2$.

| $\omega_z$, deg/sec | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | ≥36 |
|---|---|---|---|---|---|---|---|---|---|---|
| $H_0$, km | 81.0 | 83.0 | 84.7 | 86.3 | 88.0 | 89.7 | 91.2 | 92.4 | 92.8 | 93.0 |

These heights were corrected for zenithal magnitude (Table XXVI of[3]) and were used for the calculation of $T$, the linear velocity at right angles to the line of sight [$T = V$ in formula (3) of[1]]. In[1], the correlation is somewhat different, probably because of the small number of heights used; however, no new reduction was required, as the correction factor for E. Ö. was derived for the same mean heights and includes systematic errors in the adopted heights.

2. *Selection.* The velocity records of R. W. represent 20.0 per cent of all his observations at the rocking mirror apparatus; D. H. recorded 19.6 per cent velocities, whereas E. Ö. had 279 velocities among 418 records, or 66.7 per cent. It is true that E. Ö. did not record meteors at the margin of the field of vision, badly observed; he concentrated his attention upon the

observation of velocities in this first experiment with the rocking mirror and did not care for statistical completeness, whereas later the other two observers were instructed to record every meteor seen; it may be estimated that if he had followed the same policy the percentage of velocities in the records of E. Ö. would have been about 45—50, thus still more than twice the percentage for the other two observers. In such circumstances personal selection may become an important factor influencing the statistical results, the individual selectivity increasing with the decreasing fraction of observed velocities.

The observational selection in velocities may be considered to consist of two principal components: *the general selection*, determining the fraction of meteors seen; and the *specific selection*, determining the fraction of observed velocities among the meteors seen by the observer. Both kinds of selection must depend, in the first place, upon angular velocity and apparent magnitude; position in the field of survey, length of the trail and direction of motion may also influence the selection. The complete investigation of selection we leave for another occasion; here we trace only a few general outlines.

From the present material the general selection can be determined for the non-velocity observer who watched the same northern region at the other station: the fraction of common meteors determines the coefficient of perception (Double Count method[5]). The coefficient of perception of this observer may be set equal to

$$p = \pi(\omega) \cdot \chi(m) = \frac{n_c}{N} \quad \ldots . \ (1),$$

where $\chi(m)$ is the "magnitude function", $\pi(\omega)$ — the selection factor depending upon angular velocity. $N$ and $n_c$ are given in Table III. The figures of this table were obtained by smoothing slightly the observed figures, and by correcting the distribution for the error dispersion in magnitude; the latter is defined by[3], Table XIII. Common meteors and single observations were treated separately; Section b) of Table III represents the sum of the two distributions. Thus $\bar{m}$ of the table is the "true" magnitude, system of R. W.[3] The correction for

# Table III.

## Characteristic Data of Selection.

| $\bar{m}$ | $<0$ | 0.2 | 0.7 | 1.2 | 1.7 | 2.2 | 2.7 | 3.2 | 3.7 | 4.2 | 4.7 | $\geqslant 5.0$ | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **a) Number of common velocity meteors $(n_c)$** | | | | | | | | | | | | |
| $\omega$ deg/sec | | | | | | | | | | | | | |
| $\leqslant 4.9$ | 0.3 | 13.4 | 2.4 | 3.4 | 4.8 | 13.4 | 7.2 | 4.8 | 1.3 | 0.0 | 0.0 | 0.0 | 51.0 |
| 5.0 ... 9.9 | 1.2 | 6.1 | 0.0 | 0.0 | 12.1 | 22.1 | 41.9 | 22.2 | 1.6 | 5.6 | 0.2 | 0.0 | 113.0 |
| 10.0 ... 14.9 | 1.6 | 0.0 | 2.2 | 10.9 | 20.7 | 24.9 | 31.3 | 34.8 | 37.1 | 5.5 | 0.0 | 0.0 | 169.0 |
| 15.0 ... 19.9 | 0.0 | 2.6 | 10.8 | 15.4 | 11.1 | 11.8 | 16.2 | 27.0 | 29.0 | 4.6 | 0.5 | 0.0 | 129.0 |
| 20.0 ... 39.9 | 0.0 | 0.4 | 7.3 | 11.8 | 8.6 | 20.6 | 35.7 | 42.7 | 25.4 | 11.7 | 5.8 | 0.0 | 170.0 |
| $\geqslant 40.0$ | 0.0 | 0.0 | 0.0 | 0.3 | 3.3 | 5.1 | 5.4 | 7.7 | 3.7 | 1.9 | 2.1 | 0.5 | 30.0 |
| | **b) Number of all velocity meteors recorded during the time of the watch of the second observer $(N)$** | | | | | | | | | | | | |
| $\omega$ deg/sec | | | | | | | | | | | | | |
| $\leqslant 4.9$ | *0.3 | 13.4 | 2.4 | 4.4 | 8.1 | 20.2 | *14.0 | *8.1 | *2.1 | *0.0 | *0.0 | *0.0 | 73.0 |
| 5.0 ... 9.9 | *1.2 | 6.1 | 0.0 | 0.0 | 21.1 | 38.2 | 62.2 | 41.2 | *14.8 | *16.2 | *2.0 | *0.0 | 203.0 |
| 10.0 ... 14.9 | *1.6 | 0.0 | 2.6 | 16.3 | 29.2 | 40.6 | 54.3 | 67.1 | 53.9 | *10.4 | *0.0 | *0.0 | 276.0 |
| 15.0 ... 19.9 | *0.0 | 2.6 | 10.8 | 20.5 | 22.1 | 25.5 | 25.9 | 39.6 | 48.7 | *17.8 | *0.5 | *0.0 | 214.0 |
| 20.0 ... 39.9 | *0.4 | 0.9 | 9.7 | 19.8 | 20.8 | 37.9 | 60.6 | 73.9 | 63.8 | 40.7 | *14.5 | *0.0 | 343.0 |
| $\geqslant 40.0$ | *0.6 | 1.4 | 1.2 | 1.1 | 4.1 | 5.5 | 7.4 | 11.3 | 7.5 | 11.1 | *5.9 | *0.9 | 58.0 |
| All | *4.1 | 24.4 | 26.7 | 62.1 | 105.4 | 167.9 | *224.4 | *241.2 | *190.8 | *96.2 | *22.9 | *0.9 | 1167 |
| | **c) All Arizona records during simultaneous time of observation (common meteors counted twice)** | | | | | | | | | | | | |
| . . . | 238 | 82 | 127 | 286 | 703 | 940 | 1901 | 2574 | 4504 | 3801 | 3637 | 714 | 19507 |
| c) : b) | 60 | 3.4 | 4.7 | 4.6 | 6.7 | 5.6 | 8.5 | 10.7 | 24 | 40 | 158 | 800 | 16.8 |

error dispersion of the single observations led invariably to a marked "overcorrection" (negative numbers, *cf.* Section 5), which indicates that the adopted error is too large: the magnitude estimates of the three velocity observers are thus somewhat more accurate than the mean Arizona magnitudes are; no definite figure could be derived, but the correction for error dispersion was stopped at the first approximation (whereas normally two or three approximations are required); this procedure seemed sufficiently to balance the overestimate of the error dispersion. For the common meteors, the solution was perfectly normal with the mean error equal to $\sqrt{\frac{1}{2}}$ of the single observer as should be expected, because the magnitudes in this case depend upon all observers. The asterisks in the table refer to the "$m\,\omega$" specific selection referred to below.

The solution of the system of functional equations (1), determined by the figures contained in Sections a) and b) of Table III, is obtained by successive approximations. For $\omega =$ const., $\pi(\omega) =$ const., we find the ratio of $\chi(m)$ to $\chi_0$, where $\chi_0 = \chi(m)$ for $m \leqslant 1.9$; the weighted mean values from all groups of $\omega$ are:

| | | | | | | |
|---|---|---|---|---|---|---|
| $m$ = | 2.2 | 2.7 | 3.2 | 3.7 | 4.2 | 4.7 |
| $\chi/\chi_0$ = | 0.877 | 0.939 | 0.912 | 0.780 | 0.507 | 0.700 |
| *p. e.* = | ± 0.056 | ± 0.052 | ± 0.052 | ± 0.054 | ± 0.070 | ± 0.133 |

The *p. e.* here and further is computed by assuming for equation (1) a probable error of $\frac{\pm 0.337}{\sqrt{N}}$; this is an overestimate, mostly slight: the correct formula is $\pm 0.674 \sqrt{\frac{p\,(1-p)}{N}}$.

Setting $\chi_0 = 1$, the smoothed values of the magnitude function are:

| | | | | | |
|---|---|---|---|---|---|
| true magnitude, $m$ | $\leqslant 2.9$ | 3.2 | 3.7 | 4.2 | 4.7 |
| $\chi(m)$ | 1.000 | 0.960 | 0.840 | 0.660 | 0.520 |

The selection factor depending upon velocity is then given by

$$\bar{\pi}(\omega) = \frac{\Sigma\, n_c\, \chi(m)}{\Sigma N\, [\chi(m)]^2} \pm \frac{0.337}{\sqrt{\Sigma N\, [\chi(m)]^2}} \quad . \; . \; . \; . \quad (2),$$

for a given limited range of $\omega$. An additional correction depending upon the differential parallactic displacement is, however, required: if the displacement differs from the average one, i. e. from the distance between the centres of the fields of observation of the two stations, the apparent coefficient of perception decreases on account of the fact that some meteors recorded by one observer may fall outside the field of observation of the other. With an effective field of $60^0$ diameter the correction for the Arizona observations becomes

$$\frac{\Delta p}{p} = \frac{\Delta \pi}{\pi} = + 0.0030 \, |(\bar{H} - 79)| \, . \, . \, . \quad (3);$$

$\bar{H}$ may be taken from Table II. The effect of dispersion in height does not enter into the correction except when $84 > \bar{H} > 74$, in which case $\frac{\Delta p}{p} = + 0.015 = \text{const.}$, corresponding to a cosmic *p. e.* in $H$ of $\pm 10$ km. Table IV contains the result. The mean values of $\omega$ instead of the limits of Table III are given.

Table IV.

General Selection Depending Upon $\omega$.

| | | | | | | |
|---|---|---|---|---|---|---|
| $\bar{\omega}$, deg/sec | 3.3 | 8.0 | 12.5 | 17.5 | 30.0 | 50.0 |
| $\omega_z$, „ | 6 | 11 | 18 | 25 | 40 | 67 |
| $\pi(\omega)$, uncorr. | 0.706 | 0.593 | 0.642 | 0.652 | 0.553 | 0.619 |
| Correction for parallax | +0.013 | +0.012 | +0.017 | +0.023 | +0.023 | +0.026 |
| $\pi(\omega)$, corrected | 0.719 | 0.605 | 0.659 | 0.675 | 0.576 | 0.645 |
| *p. e.* | ±0.040 | ±0.025 | ±0.022 | ±0.025 | ±0.020 | ±0.051 |
| $\pi(\omega)$, smoothed | 0.69 | 0.67 | 0.65 | 0.63 | 0.60 | 0.60 |

A decrease of the general coefficient of perception with increasing angular velocity appears to be definitely indicated; the range in $\pi(\omega)$ (smoothed) is, however, only 15 per cent. The argument here is the recorded value of $\bar{\omega}$ (corrected, of course, with the aid of the personal correction factors); on account of observational error dispersion, the range in $\pi(\omega)$ must be smaller than would have been with the true angular velocity as argument. Thus, the general selection works in favour of small velocities as anticipated by different authors (Hoffmeister, the writer); the selection is comparatively "soft", introducing but a moderate change into the distribution of velocities.

More acute may be the effect of the specific selection of the velocity observer; this is shown already by the percentages of velocity observations for different observers quoted at the beginning of this section. The specific selection may depend strongly upon $\omega$, as shown by the following figures, referring to the data of Table III (thus to the time of observation simultaneous with the other station):

Table V.

Relative Personal Selection in Angular Velocity.

| | $\omega$ | $0^0 \ldots 4^0.9$ | $5^0.0 \ldots 9^0.9$ | $10^0.0 \ldots 14^0.9$ | $15^0.0 \ldots 19^0.9$ | $20^0.0 \ldots 39^0.9$ | $\geqslant 40^0.0$ | All |
|---|---|---|---|---|---|---|---|---|
| Recorded number of velocities | E. Ö. | 2 | 34 | 56 | 38 | 86 | 32 | 248 |
| | R. W. | 53 | 77 | 91 | 88 | 142 | 15 | 466 |
| | D. H. | 18 | 92 | 129 | 88 | 115 | 11 | 453 |
| Percentage for observers | E. Ö. | 3 | 17 | 20 | 18 | 25 | 55 | 21 |
| | R. W. | 72 | 38 | 33 | 41 | 41 | 26 | 40 |
| | D. H. | 25 | 45 | 47 | 41 | 34 | 19 | 39 |

The relative proportion of velocities registered by different observers varies conspicuously with the velocity. E. Ö. has a very small number of $\omega < 5^0$, explained probably by the following of slow meteors by the observer's eye (*cf.*[1], p. 2); his proportion of high velocities is large — during the two lunations E. Ö. recorded more velocities exceeding forty deg/sec than the other observers during twenty-one lunations; one might suggest seasonal effects, but a closer examination of the material does not support this hypothesis; the difference is due to personal selection which is much more prominent than the seasonal effects are. Angular velocities less than five deg/sec little influence the concluded distribution of space velocities (*cf.* below), and the personal selection in this case is practically of no consequence. On the contrary, the frequency of medium and high angular velocities is directly reflected in the concluded distribution of the space velocities. Considering the relative completeness of the records by E. Ö., we must consider his data to be less influenced by specific selection (except $\omega \leqslant 4^0.9$); hence we conclude that the specific selection of the two other observers is unfavourable for large $\omega$; of these two, D. H. is somewhat more strongly represented at medium values of $\omega$, and more weakly at the extremes as compared with R. W. In spite of this, D. H. leads to a somewhat greater frequency of high space velocities than R. W. as shown below; the explanation is that the velocities of D. H. are frequently recorded in the lower part of the region, so that $\overline{\omega}_z$ is larger, although $\overline{\omega}$ is smaller than for R. W.

Table V, although only of relative value, gives some idea of the personal selection which may be called the specific $\omega$-selection for an average observer; the selection is unfavourable for very small, and for very large $\omega$, and more or less indifferent for medium values of $\omega$; qualitatively the coefficient of selection may be represented by a curve with a flat maximum at a certain medium value of $\omega$ declining on both sides of this maximum.

The distribution in $P$, the apical direction of the velocity meteors, reveals a specific $P$-selection which may be partly traced to the $\omega$-selection (the average $\omega$ depending upon $P$):

Table VI.

Personal Selection in Direction.

| $P$ | | 270°—359°; 0°—29° (from antapex hemisphere) | 30°—119° (upward motion) | 120°—269° (from apex hemisphere) |
|---|---|---|---|---|
| Number of vel. | E. Ö. | 65 | 11 | 203 |
| | R. W. | 142 | 15 | 454 |
| | D. H. | 175 | 38 | 333 |
| | All | 382 | 64 | 990 |
| Percentage | E. Ö. | 17 | 17 | 20 |
| | R. W. | 37 | 24 | 46 |
| | D. H. | 46 | 59 | 34 |

The $P$-selection of D. H. strongly favours the directions upward and from the antapex and is unfavourable to directions from the apex as compared with the other two observers.

Table III reveals a remarkable deficiency of slow faint meteors; as compared with Section c) of the table, the velocity meteors (all $\omega$) show a rapid decline in the relative number of faint meteors, beginning at $m = 2.7$ [*cf.* ratio c) : b)]. For separate groups of $\omega$ the decline starts later, the larger $\omega$ is: at $m = 2.7$ in the slowest group, at $m = 4.7$ in the fastest group (true magnitude limits); the figures of the table revealing a deficiency in the relative frequency of magnitudes, as compared with the non-velocity ("All Arizona") records, are marked with an asterisk. The character of the deficiency leaves no doubt that it is produced by a specific observational selection — the $m\omega$-selection. A possible change with $\omega$ of the frequency of the fainter magnitudes, if present, is entirely concealed by this effect of selection. From the experience with the rocking mirror apparatus the explanation is obvious: the resolving power for velocity increases both with the brightness and the angular speed of the meteor; the effective limiting magnitude thus changes with $\omega$ in such a manner that fainter meteors become resolved at greater angular velocities. These peculiar conditions were already noticed by the writer at an early stage of the Arizona Expedition (*cf.* [6], p. 20). The $m\omega$-selection evidently works in the opposite way to the $\omega$-selection; it increases the relative number of large $\omega$ recorded at the expense of the resolved faint meteors which at small $\omega$ would have remained unresolved.

From the unaffected parts of Table III, b), the deficient numbers affected by $m\omega$-selection may be restored (on the assumption of a distribution of $m$, the same for all $\omega$) and the effect of the selection upon the total number of velocities, recorded without regard to magnitude, may be determined. Table VII contains the result; the "restored" numbers are referred to the $m\omega$-selection of $\omega = 20^{\circ}.0 \ldots 39^{\circ}.9$.

Table VII.

$m\omega$-Selection (all observers).

| $\omega$, deg/sec | $\leqslant 4.9$ | 5.0 .. 9.9 | 10.0 ... 14.9 | 15.0 ... 19.9 | 20.0 ... 39.9 | $\geqslant 40.0$ |
|---|---|---|---|---|---|---|
| Number, recorded | 73 | 203 | 276 | 214 | 343 | 58 |
| Number, restored | 174.0 | 263.8 | 321.1 | 236.5 | 343.0 | 53.5 |
| Relative $m\omega$- selection factor (all magnitudes) | 0.420 | 0.769 | 0.859 | 0.905 | 1.000 | 1.084 |

The total selection in $\omega$, referring to the total number of velocities recorded without regard to magnitude, consists thus of the three principal components: 1) the general selection; the data of Table IV, referring to the non-velocity observer, may be assumed to define the trend of the general selection for the average velocity observer, too; 2) the $m\omega$-selection, as given by Table VII; 3) the specific $\omega$-selection, qualitatively described above in connection with Table V, quantitatively not determinable from the present material. The combined effect from all three sources runs as follows:

| $\omega$, deg/sec | $\leqslant 4.9$ | 5.0 ... 9.9 | 10.0 ... 14.9 | 15.0 ... 19.9 | 20.0 ... 39.9 | $\geqslant 40.0$ |
|---|---|---|---|---|---|---|
| Relative selection factor | $> 0.48$ | $> 0.86$ | $\gg 0.93$ | $> 0.95$ | 1.00 | $\ll 1.08$ |

The signs of inequality account for the unknown specific $\omega$-selection; evidently this selection tends to equalize the combined effect of the other two sources of selection; there may be practical equalization for the median values of $\omega$; whereas for $\omega > 40^0$ the $\omega$-selection probably outweighs the rest, and for $\omega < 5^0$, probably also for $< 10^0$ it is unable to counterbalance the deficiency. The result is that our list of velocities, uncorrected for selection, must represent well the distribution of intermediate $\omega$, revealing a deficiency of very large and of very small $\omega$. These conclusions are to some extent supported by observational evidence; the probable deficiency of large $\omega$ for R. W and D. H. — by the data of Table V; the deficiency of small $\omega$ — by the negative residuals obtained in the solution for the distribution of space velocities (Section 6) at small $T$, or at small projection ratios. The effect of the deficiency of small $\omega$ is unimportant, in any case. Besides, in the distribution of linear velocities these effects of selection are softened through the intervention of the variable zenith distance. In any case, it was decided here to use the statistical data uncorrected for selection, especially because a satisfactory determination of all the selection factors is not possible on the basis of the present data alone.

3. *First Approximation for the Distribution of Heliocentric Velocities.* Following a simplified procedure applied in preceding

papers[1,2], the heliocentric tangential velocity $T_0$ ($V_0$ in[1,2]) of each meteor was determined graphically from given $T$, $P$, and the distribution of $T_0$ was analyzed statistically. The procedure has a low statistical resolving power and details are lost by treating all directions *(P)* together; the advantage consists in being able to treat smaller groups separately and thus to obtain data referring to the seasonal effects and systematic differences between observers. As the approximate solution yields results that are not fundamentally different from the detailed analysis of velocities in separate directions, the differential results of the approximate solution for differently selected groups may be accepted with some confidence.

We are looking for individual and seasonal differences between the observers in the first place. We notice that E. Ö. observed alone during Lunations I and II, and R. W. alone

Table VIII.

Observed Distribution of Projected Heliocentric Velocities.

| $T_0$ km/sec | E. Ö. Lunations I, II (Oct.—Nov.) | | R. W. Lun. III; XIII—XV (Sep.—Dec.) | | R. W. Lun. IV—VI; XVI—XVIII (Dec.—Mar.) | | R. W. Lun. VII—IX; XIX—XXI (Mar.—June) | | R. W. Lun. X—XII; XXII, XXIII (June—Sep.) | | R. W. All |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $n$ | $\Sigma$ % | $n$ | $\Sigma$ % | $n$ | $\Sigma$ % | $n$ | $\Sigma$ % | $n$ | $\Sigma$ % | $n$ |
| 0 ... 6 | 0 | 100.0 | 1 | 100.0 | 1 | 100.0 | 0 | 100.0 | 4 | 100.0 | 6 |
| 7 ... 14 | 11 | 100.0 | 7 | 99.4 | 2 | 99.0 | 8 | 100.0 | 7 | 97.9 | 24 |
| 15 ... 17 | 6 | 96.2 | 7 | 95.6 | 3 | 97.0 | 2 | 94.2 | 9 | 94.2 | 21 |
| 18 ... 21 | 8 | 93.9 | 6 | 91.7 | 4 | 94.1 | 9 | 92.8 | 11 | 89.5 | 30 |
| 22 ... 25 | 8 | 91.0 | 13 | 88.4 | 5 | 90.1 | 11 | 86.3 | 10 | 83.7 | 39 |
| 26 ... 29 | 11 | 88.2 | 11 | 81.2 | 4 | 85.1 | 14 | 78.4 | 14 | 78.4 | 43 |
| 30 ... 35 | 33 | 84.2 | 33 | 75.1 | 16 | 81.2 | 25 | 68.3 | 32 | 71.1 | 106 |
| 36 ... 42 | 52 | 72.4 | 36 | 56.9 | 20 | 65.3 | 22 | 50.3 | 32 | 54.2 | 110 |
| 43 ... 50 | 41 | 53.8 | 23 | 37.0 | 14 | 45.5 | 17 | 34.5 | 17 | 37.4 | 71 |
| 51 ... 59 | 26 | 39.1 | 19 | 24.3 | 9 | 31.7 | 6 | 22.3 | 28 | 28.4 | 62 |
| 60 ... 71 | 23 | 29.7 | 11 | 13.8 | 12 | 22.8 | 10 | 18.0 | 14 | 13.7 | 47 |
| 72 ... 84 | 17 | 21.5 | 7 | 7.7 | 2 | 10.9 | 7 | 10.8 | 8 | 6.3 | 24 |
| 85 ... 100 | 11 | 15.4 | 1 | 3.9 | 5 | 8.9 | 4 | 5.8 | 3 | 2.1 | 13 |
| 101 ... 119 | 11 | 11.5 | 4 | 3.3 | 2 | 4.0 | 1 | 2.9 | 0 | 0.5 | 7 |
| 120 ... 143 | 11 | 7.5 | 1 | 1.1 | 1 | 2.0 | 2 | 2.2 | 0 | 0.5 | 4 |
| 144 ... 169 | 3 | 3.6 | 1 | 0.6 | 0 | 1.0 | 1 | 0.7 | 0 | 0.5 | 2 |
| 170 ... 201 | 3 | 2.5 | 0 | 0.0 | 1 | 1.0 | 0 | 0.0 | 1 | 0.5 | 2 |
| > 201 | 4 | 1.4 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 0 |
| All | 279 | ... | 181 | ... | 101 | ... | 139 | ... | 190 | ... | 611 |

Table VIII. Continued.

| $T_0$ km/sec | D. H. Lun. XIII—XV (Sep.—Dec.) | | D. H. Lun. IV—VI; XVI—XVIII (Dec.—Mar.) | | D. H. Lun. VII—IX; XIX—XXI (Mar.—June) | | D. H. Lun. X—XII; XXII—XXIII (June—Sep.) | | D. H. All |
|---|---|---|---|---|---|---|---|---|---|
| | $n$ | $\Sigma$ % | $n$ | $\Sigma$ % | $n$ | $\Sigma$ % | $n$ | $\Sigma$ % | $n$ |
| 0 ... 6 | 2 | 100.0 | 4 | 100.0 | 2 | 100.0 | 7 | 100.0 | 15 |
| 7 ... 14 | 4 | 98.3 | 6 | 96.3 | 18 | 98.8 | 14 | 95.5 | 42 |
| 15 ... 17 | 3 | 95.0 | 0 | 90.7 | 6 | 87.9 | 3 | 86.5 | 12 |
| 18 ... 21 | 2 | 92.4 | 4 | 90.7 | 11 | 84.2 | 8 | 84.5 | 25 |
| 22 ... 25 | 2 | 90.8 | 4 | 86.9 | 11 | 77.6 | 10 | 79.4 | 27 |
| 26 ... 29 | 8 | 89.1 | 8 | 83.2 | 8 | 70.9 | 10 | 72.9 | 34 |
| 30 ... 35 | 18 | 82.4 | 8 | 75.7 | 29 | 66.1 | 19 | 66.5 | 74 |
| 36 ... 42 | 19 | 67.2 | 12 | 68.2 | 16 | 48.5 | 25 | 54.2 | 72 |
| 43 ... 50 | 22 | 51.3 | 12 | 57.0 | 21 | 38.8 | 17 | 38.1 | 72 |
| 51 ... 59 | 16 | 32.8 | 12 | 45.8 | 20 | 26.1 | 13 | 27.1 | 61 |
| 60 ... 71 | 10 | 19.3 | 11 | 34.6 | 11 | 13.9 | 14 | 18.7 | 46 |
| 72 ... 84 | 7 | 10.9 | 9 | 24.3 | 3 | 7.3 | 11 | 9.7 | 30 |
| 85 ... 100 | 4 | 5.0 | 9 | 15.9 | 6 | 5.5 | 3 | 2.6 | 22 |
| 101 ... 119 | 1 | 1.7 | 6 | 7.5 | 1 | 1.8 | 0 | 0.6 | 8 |
| 120 ... 143 | 0 | 0.8 | 1 | 1.9 | 1 | 1.2 | 0 | 0.6 | 2 |
| 144 ... 169 | 0 | 0.8 | 0 | 0.9 | 0 | 0.6 | 0 | 0.6 | 0 |
| 170 ... 201 | 0 | 0.8 | 0 | 0.9 | 0 | 0.6 | 0 | 0.6 | 0 |
| > 201 | 1 | 0.8 | 1 | 0.9 | 1 | 0.6 | 1 | 0.6 | 4 |
| All | 119 | ... | 107 | ... | 165 | ... | 155 | ... | 546 |

during Lunation III; during the remaining Lunations IV—XXIII, R. W. and D. H. observed in turns, so that this period is covered more or less uniformly by both observers. This last arrangement was made against the explicit instructions of the writer after he left the expedition. Much would have been gained in the homogeneity of the observations and in the precision of the calibration (number of shower meteors) if R. W. — the ablest observer of the expedition — had been allowed to observe alone all the time at the rocking mirror apparatus.

Table VIII gives the observed distribution of $T_0$ for single observers and different seasons of the year. The shower meteors are included, a policy also followed later.

In this table $\Sigma$ % denotes the cumulative percentage of velocities from the largest to the given velocity; by the cumulative percentage it is easy to compare the different distributions; although referring to recorded velocities, the qualitative comparison is justified by the approximate equality of the error dispersion of all three observers.

The relative frequency of large velocities recorded by E. Ö. considerably exceeds the frequency for the other two observers — a circumstance which must be attributed to the $\omega$-selection discussed in the preceding section; the explanation by a seasonal difference is excluded; the Sep. — Dec. observations of R. W. and D. H. reveal the difference with respect to E. Ö. in an even more pronounced manner than their mean results do, yet the mean season closely coincides with the mean season of the observations of E. Ö. Except for the large velocities, the distribution of $T_0$ agrees, in its broad features, for all three observers; in spite of some difference in the details, there is no doubt that the observational records of all three observers belong to the same cosmic aggregate.

Seasonal effects can be studied only from the observations of R. W. and D. H. Both observers agree in assigning to the period from December to March the greater relative frequency of high velocities. To judge the seasonal variation with greater certainty we join together the observations of R. W. and D. H.:

| Season | Sep. — Dec. | Dec. — Mar. | Mar. — June | June — Sep. | All |
|---|---|---|---|---|---|
| Total number | 300 | 208 | 304 | 345 | 1157 |
| Number of $T_0 \geqslant 72$ km/sec | 27 | 37 | 27 | 27 | 118 |
| Percentage of $T_0 \geqslant 72$ km/sec | 9.0 | 17.8 | 8.9 | 7.8 | 10.2 |

Thus, according to the present observations, the proportion of high velocities is the greatest from December to March, and the smallest from June to September.

For the entire present material we assume provisionally the same law of observational error dispersion in $T_0$ as was found for E. Ö. (*cf.* [2], Table I); although the error frequency in $T_0$ depends upon the direction of motion, and although the relative frequency of directions is not identical for all observers (Table VI), the difference is not large enough to justify a special treatment in the first approximation. Further, we assume conventionally the relative frequency of projection ratios, $\frac{T_0}{W}$, according to [2], Table II, together with the special rule of use of the table. The distribution of heliocentric space velocities, resulting from the solution for error dispersion and projection ratios, is given in Table IX. In this table, $W'$ is the provisional heliocentric velocity; it requires a certain correction for the effect of the earth's attraction which cannot be taken into account individually, and a correction for the ellipticity of the orbits of shower meteors used for standardization (*cf.*[2], p. 5). The table is discontinuous with respect to $W'$, — a type of solution more easy to obtain than a continuous distribution; the statistical resolving power of the method, with the given observational error dispersion, is unable to distinguish between the two types of solution for the actually adopted intervals between the discrete values of $W'$.

Table IX.

Provisional Distribution of Heliocentric Space Velocities.

| $W'$ km/sec | E. Ö. | R. W. | D. H. | All observers |
|---|---|---|---|---|
| | $n$ | $n$ | $n$ | $n$ |
| $\leqslant 15$ | 0 | 0 | 23 | 23 |
| 18 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 2 | 2 |
| 25 | 0 | 0 | 3 | 3 |
| 30 | 0 | 0 | 3 | 3 |
| 36 | 7 | 84 | 34 | 125 |
| 42 | 76 | 173 | 64 | 313 |
| 50 | 52 | 125 | 113 | 290 |
| 60 | 47 | 102 | 164 | 313 |
| 72 | 26 | 101 | 77 | 204 |
| 85 | 16 | 25 | 61 | 102 |
| 101 | 12 | 0 | 0 | 12 |
| 120 | 15 | 0 | 0 | 15 |
| 143 | 16 | 0 | 0 | 16 |
| 170 | 10 | 0 | 0 | 10 |
| 202 | 2 | 1 | 0 | 3 |
| 240 | 0 | 0 | 2 | 2 |
| All | 279 | 611 | 546 | 1436 |

The individual differences between the observers in this table are qualitatively the same as but quantitatively more pronounced than those in Table VIII. Part of the minor details

## Table X.

Observed Distribution of Geocentric Tangential Velocities.

| $T$ km/sec | $P = 0^0 \dots 29^0$ | | | | $P = 30^0 \dots 59^0$ | | | | $P = 60^0 \dots 89^0$ | | | | $P = 90^0 \dots 119^0$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | E. Ö. | R. W. | D. H. | All | E. Ö. | R. W. | D. H. | All | E. Ö. | R. W. | D. H. | All | E. Ö. | R. W. | D. H. | All |
| 0.0 ... 7.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 7.5 ... 8.8 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8.9 ... 10.5 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 2 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 10.6 ... 12.5 | 1 | 2 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 12.6 ... 14.9 | 0 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15.0 ... 17.8 | 1 | 1 | 1 | 3 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 17.9 ... 21.1 | 1 | 0 | 2 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 3 | 5 |
| 21.2 ... 25.1 | 0 | 1 | 2 | 3 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 2 |
| 25.2 ... 29.9 | 1 | 1 | 0 | 2 | 0 | 0 | 3 | 3 | 0 | 0 | 1 | 1 | 1 | 0 | 2 | 3 |
| 30.0 ... 35.7 | 0 | 0 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 1 | 1 | 4 | 6 |
| 35.8 ... 42.3 | 0 | 0 | 2 | 2 | 0 | 0 | 2 | 2 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 2 |
| 42.4 ... 50.3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 50.4 ... 59.9 | 0 | 1 | 1 | 2 | 1 | 0 | 2 | 3 | 0 | 1 | 1 | 2 | 1 | 0 | 3 | 4 |
| 60.0 ... 71.5 | 0 | 0 | 2 | 2 | 1 | 1 | 2 | 4 | 1 | 0 | 1 | 2 | 1 | 1 | 0 | 2 |
| 71.6 ... 84.7 | 0 | 0 | 2 | 2 | 1 | 0 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 84.8 ... 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 101 ... 119 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 120 ... 143 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 144 ... 169 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 170 ... 201 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| > 201 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| All vel. | 5 | 8 | 17 | 30 | 4 | 3 | 12 | 19 | 1 | 5 | 9 | 15 | 6 | 7 | 17 | 30 |

and differences may be due to the approximate method of treatment.

4. *Observed Distribution of Velocities in Different Directions.* From the above it appears that, in spite of the individual differences explained by personal selection, the statistical data of the three observers are not radically different. Therefore, in the subsequent statistical treatment the observations of all three observers were joined together: thus they represent the observations of an average observer, with certain average coefficients of selection. The joint material is large enough to permit of a separate treatment of the different directions of motion; within each direction, however, the material is too small to allow of a similar treatment of different seasons of the year — also too heterogeneous for such a purpose considering the disproportionate role of the observations of E. Ö. in October — November.

Table X. Continued.

| $T$ km/sec | $P = 120^0 \ldots 149^0$ | | | | $P = 150^0 \ldots 179^0$ | | | | $P = 180^0 \ldots 209^0$ | | | | $P = 210^0 \ldots 239^0$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | E. Ö. | R. W. | D. H. | All | E. Ö. | R. W. | D. H. | All | E. Ö. | R. W. | D. H. | All | E. Ö. | R. W. | D. H. | All |
| 0.0 ... 7.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7.5 ... 8.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8.9 ... 10.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 10.6 ... 12.5 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 12.6 ... 14.9 | 0 | 1 | 1 | 2 | 0 | 1 | 0 | 1 | 0 | 1 | 2 | 3 | 0 | 1 | 0 | 1 |
| 15.0 ... 17.8 | 0 | 0 | 2 | 2 | 0 | 0 | 1 | 1 | 1 | 0 | 2 | 3 | 1 | 1 | 6 | 8 |
| 17.9 ... 21.1 | 1 | 5 | 0 | 6 | 1 | 1 | 3 | 5 | 0 | 3 | 4 | 7 | 2 | 5 | 1 | 8 |
| 21.2 ... 25.1 | 0 | 5 | 0 | 5 | 0 | 1 | 4 | 5 | 3 | 2 | 8 | 13 | 1 | 4 | 4 | 9 |
| 25.2 ... 29.9 | 1 | 2 | 1 | 4 | 1 | 0 | 6 | 7 | 0 | 3 | 5 | 8 | 0 | 10 | 8 | 18 |
| 30.0 ... 35.7 | 2 | 3 | 5 | 10 | 1 | 6 | 7 | 14 | 1 | 4 | 9 | 14 | 1 | 9 | 10 | 20 |
| 35.8 ... 42.3 | 0 | 2 | 4 | 6 | 4 | 11 | 6 | 21 | 5 | 7 | 11 | 23 | 1 | 13 | 7 | 21 |
| 42.4 ... 50.3 | 2 | 5 | 1 | 8 | 4 | 16 | 4 | 24 | 3 | 8 | 6 | 17 | 3 | 14 | 9 | 26 |
| 50.4 ... 59.9 | 0 | 4 | 1 | 5 | 3 | 15 | 6 | 24 | 3 | 10 | 14 | 27 | 4 | 10 | 11 | 25 |
| 60.0 ... 71.5 | 3 | 4 | 0 | 7 | 6 | 12 | 1 | 19 | 6 | 16 | 9 | 31 | 4 | 19 | 4 | 27 |
| 71.6 ... 84.7 | 0 | 4 | 2 | 6 | 4 | 8 | 3 | 15 | 2 | 15 | 9 | 26 | 7 | 12 | 8 | 27 |
| 84.8 ... 100 | 0 | 5 | 3 | 8 | 1 | 9 | 3 | 13 | 7 | 2 | 2 | 11 | 0 | 4 | 2 | 6 |
| 101 ... 119 | 2 | 0 | 1 | 3 | 0 | 4 | 2 | 6 | 6 | 3 | 1 | 10 | 0 | 3 | 2 | 5 |
| 120 ... 143 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 4 | 4 | 3 | 1 | 8 | 1 | 2 | 1 | 4 |
| 144 ... 169 | 1 | 0 | 0 | 1 | 2 | 0 | 0 | 2 | 4 | 1 | 0 | 5 | 0 | 0 | 1 | 1 |
| 170 ... 201 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 1 | 0 | 4 | 0 | 0 | 0 | 0 |
| > 201 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 2 | 5 | 0 | 3 | 8 | 1 | 1 | 1 | 3 |
| All vel. | 12 | 40 | 22 | 74 | 30 | 87 | 46 | 163 | 53 | 79 | 87 | 219 | 26 | 109 | 76 | 211 |

The distribution of *T*, the observed tangential velocities, according to observer and direction of motion, is represented by Table X.

*P* is the apical direction of motion: $P = 0^0$ means motion from antapex; $P = 180^0$ — from apex; $P = 270^0$ — toward the sun and downward (for the north polar region of observation, as in the present case). The table represents the basis of all the subsequent discussion.

A close inspection of Table X indicates that the individual differences between the observers for a given *P* are smaller than those inferred above from a comparison of the data for all directions joined: these individual differences in the apparent distribution of velocities are partly due to personal selection in direction (*cf.* Table VI and the sum-totals of Table X).

However, one peculiarity persists and is even more pronounced: the excess of high velocities recorded by E. Ö..

Table X. Continued.

| T km/sec | $P = 240^0 \dots 269^0$ | | | | $P = 270^0 \dots 299^0$ | | | | $P = 300^0 \dots 329^0$ | | | | $P = 330^0 \dots 359^0$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | E. Ö. | R. W. | D. H. | All | E. Ö. | R. W. | D. H. | All | E. Ö. | R. W. | D. H. | All | E. Ö. | R. W. | D. H. | All |
| 0.0 ... 7.4 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7.5 ... 8.8 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 2 | 0 | 0 | 0 | 0 |
| 8.9 ... 10.5 | 1 | 1 | 1 | 3 | 0 | 3 | 1 | 4 | 0 | 2 | 0 | 2 | 0 | 0 | 1 | 1 |
| 10.6 ... 12.5 | 1 | 4 | 1 | 6 | 0 | 4 | 2 | 6 | 0 | 5 | 0 | 5 | 0 | 0 | 0 | 0 |
| 12.6 ... 14.9 | 0 | 4 | 1 | 5 | 1 | 6 | 5 | 12 | 0 | 1 | 3 | 4 | 0 | 3 | 0 | 3 |
| 15.0 ... 17.8 | 3 | 9 | 8 | 20 | 1 | 6 | 6 | 13 | 1 | 3 | 1 | 5 | 2 | 5 | 1 | 8 |
| 17.9 ... 21.1 | 9 | 12 | 12 | 33 | 4 | 13 | 9 | 26 | 3 | 5 | 2 | 10 | 0 | 4 | 1 | 5 |
| 21.2 ... 25.1 | 14 | 14 | 5 | 33 | 8 | 7 | 9 | 24 | 0 | 2 | 2 | 4 | 0 | 1 | 1 | 2 |
| 25.2 ... 29.9 | 11 | 20 | 15 | 46 | 4 | 6 | 13 | 23 | 1 | 2 | 8 | 11 | 0 | 2 | 4 | 6 |
| 30.0 ... 35.7 | 11 | 18 | 13 | 42 | 5 | 10 | 12 | 27 | 0 | 4 | 1 | 5 | 0 | 0 | 2 | 2 |
| 35.8 ... 42.3 | 7 | 18 | 12 | 37 | 4 | 9 | 15 | 28 | 1 | 2 | 3 | 6 | 1 | 1 | 1 | 3 |
| 42.4 ... 50.3 | 9 | 4 | 11 | 24 | 3 | 4 | 9 | 16 | 2 | 1 | 5 | 8 | 1 | 0 | 2 | 3 |
| 50.4 ... 59.9 | 5 | 11 | 13 | 29 | 3 | 4 | 5 | 12 | 2 | 0 | 2 | 4 | 1 | 0 | 2 | 3 |
| 60.0 ... 71.5 | 6 | 10 | 4 | 20 | 3 | 6 | 12 | 21 | 1 | 2 | 0 | 3 | 0 | 0 | 2 | 2 |
| 71.6 ... 84.7 | 0 | 9 | 3 | 12 | 3 | 2 | 7 | 12 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 84.8 ... 100 | 1 | 1 | 3 | 5 | 1 | 1 | 3 | 5 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 101 ... 119 | 3 | 3 | 0 | 6 | 2 | 3 | 1 | 6 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 120 ... 143 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 2 | 0 | 0 | 0 | 0 |
| 144 ... 169 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 170 ... 201 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| > 201 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| All vel. | 82 | 139 | 102 | 323 | 43 | 86 | 111 | 240 | 11 | 31 | 30 | 72 | 6 | 17 | 17 | 40 |

The excess is concentrated in the directions from the apex, $P = 150^0 \dots 209^0$, whereas for the other directions the excess is much weaker, as shown by the following table:

| | | $P = 150^0 \dots 209^0$ | | $P = 210^0 \dots 360^0 \dots 149^0$ | |
|---|---|---|---|---|---|
| $T$, km/sec | | $> 100$ | $\leqslant 100$ | $> 100$ | $\leqslant 100$ |
| E. Ö. | $n$ | 27 | 56 | 14 | 182 |
| | % | 32.5 | 67.5 | 7.1 | 92.9 |
| R. W. + + D. H. | $n$ | 22 | 277 | 21 | 837 |
| | % | 7.3 | 92.7 | 2.4 | 97.6 |

The question of the reality of these high velocities recorded by E. Ö. may be considered on this occasion. The standardization of our velocities required the rejection of a few — altogether five — high velocities (*cf.* Table I), which were considered as belonging to stray hyperbolic meteors; the rejection is based upon the assumption of a more or less normal error curve in $\log \omega$. Let us suppose that the assumption fails, and that large positive errors in $\log \omega$ are more frequent than assumed; the "rejected" shower meteors represent in this case only outstanding errors of observation. Can the observed excess of high velocities be explained by such errors? For E. Ö., $P = 150^0 \ldots 209^0$, we have:

| | All | $T > 100$ |
|---|---|---|
| Number of "shower" meteors | 27*) | 4**) |
| Percentage of high vel. | ... | 14.8 |
| Number of other meteors | 56 | 23 |
| Percentage of high vel. | ... | 41.1 |

These figures do not support the sceptical point of view; the percentage of high velocities recorded is much smaller for the "shower" meteors than for the rest; in the case of pure errors of observation we should expect equal percentages. Thus, our standardization procedure seems to be justified also in the case of the high velocities. The excess of high velocities in the observations of E. Ö. must be ascribed to the apparently more complete records, or to the smaller influence of personal selection, as compared with the other two observers. These evidently did not feel sure enough about the observation of meteors of very short duration ($0^s.1$ and less), and often preferred to record them without velocity.

5. *Correction for Observational Error Dispersion.* The correction is of primary importance in our investigation, and its theory is not widely known; we feel that it is necessary to state here briefly the mathematical basis of the procedure. Let $x$ be the true quantity, $F(x)dx$ its frequency; $y$ — the observational error, $\chi(y, x)dy$ its frequency; $\xi = x + y$ the observed quantity, $\varphi(\xi)d\xi$ the observed frequency between $\xi$ and $\xi + d\xi$. The "integral equation of diffusion" is:

$$\varphi(\xi) = \int_{-\infty}^{+\infty} F(\xi - y) \cdot \chi(y, \xi - y) \cdot dy \quad . \quad . \quad . \quad . \quad (4).$$

For tabular functions, the integral may be replaced by the numerical summation of discrete values.

*) Seven Orionids on Oct. 21 and twenty Leonids on Nov. 14 and 16.

**) Of these two are "rejected", and two are considered as errors of observation; all four are on Nov. 16, associated with the Leonids.

When $\varphi$, the observed distribution, and $\chi$, the law of errors are known, the true distribution, $F$, is determined by (4). Eddington (Monthly Notices, 73, 359, 1913) has given an analytical method of solving the equation in the case of a Gaussian $\chi = \chi(y)$, but even in this case the method is not practicable when the observational error dispersion is moderate or large. A method of successive approximations, which has been used repeatedly by the writer, is described as follows: at first we take $\varphi_1 = \int \varphi \cdot \chi \cdot dy$; the first approximation to $F$ is $F_1 = \varphi + (\varphi - \varphi_1) = 2\varphi - \varphi_1$, and $\varphi_2 = \int F_1 \cdot \chi \cdot dy$; the second approximation is $F_2 = \varphi + (F_1 - \varphi_2)$, and so forth; the numerical procedure consists actually in the direct calculation of $F_1 - \varphi_2$, instead of $\varphi_2$. From one to three approximations lead mostly to a good solution which can be checked by substituting into (4). Eventual *negative* numbers occurring in the course of the solution must be smoothed out in each approximation; the persistent occurrence of negative numbers from an initially well smoothed $\varphi(\xi)$ indicates *overcorrection*, i. e. that the observational error dispersion adopted is too large. In such a case the solution cannot be followed to the end but must be stopped at an early stage (first or second approximation), when the overcorrection becomes manifest.

The dispersions, $\Delta$ (mean square deviations from the arithmetic mean), satisfy the following equation, whatever functions $\varphi$, $F$ and $\chi$ are:

$$\Delta_x^2 = \Delta_\xi^2 - \Delta_y^2 \quad . \quad . \quad . \quad . \quad (5).$$

The above is true when $\xi$ is a "directly measured" quantity, and $\chi$ is the error function representing the distribution of $y = \xi - x$ for a given value of $x$, the true argument.

However, when the observed quantity $\eta$ is a statistical or empirical average value of $x$, corresponding to a certain observed criterion $\xi'$ (the mean absolute magnitude for given proper motion, spectroscopic absolute magnitudes, etc), $\eta = \bar{x}(\xi')$, our mathematical picture is inverted:

$$F(x) = \int_{-\infty}^{+\infty} \psi(x-y) \cdot \chi_1(y, x-y)\, dy \quad . \quad . \quad . \quad . \quad (6),$$

$$\Delta_x^2 = \Delta_\eta^2 + \Delta_y^2 \quad . \quad . \quad . \quad . \quad (7),$$

where $x = \eta + y$; $\psi(\eta)d\eta$ is the observed frequency of $\eta$, $\chi_1(y, \eta)dy$ is the apparent error function determining the distribution of $x - \eta = x - \bar{x}(\xi')$. The solution of (6) is obtained by direct integration.

Equations (5) and (7) characterize our procedure as a subtraction of the error dispersion in the first case and as an addition — in the second.

In the present case $x = \log T$ and $\chi$ is assumed to be a Gaussian depending upon the error alone, $\chi = \chi(y)$; the probable error in $\log T$ is assumed equal to $\pm 0.106$ (*cf.* Section 1). The solution was made according to equation (4), separately for each $P$-sector of table X, all observers joined; to avoid spurious maxima and minima, the observed frequencies were first carefully smoothed (graphically), within the "natural uncertainty" of a counted number $n$ equal to $\pm\sqrt{n}$ (standard devi.). A persistent overcorrection was obtained, indicating a slight overestimate in the assumed error dispersion. The explanation is as follows. The observed $\log T$ contains a direct error of observation, $\Delta_a$, and an error in the statistical mean adopted height (Table II), $\Delta_b$; the empirical error derived from the *individual* shower meteors depends equally upon both sources and is

$$\Delta_1 = \pm\sqrt{\Delta_a^2 + \Delta_b^2} = \pm 0.106 \ldots (p.\ e.) \ldots . (8).$$

According to equations (5) and (7), the true dispersion in $\log T$ is determined by

$$\Delta_x^2 = \Delta_\xi^2 - \Delta_a^2 + \Delta_b^2,$$

which gives for the effective error dispersion to be used in (4)

$$\Delta_y = \pm\sqrt{\Delta_a^2 - \Delta_b^2} \quad . \ . \ . \ . \quad (9),$$

whereas in the computations we used the empirical value $\Delta_1$. Evidently

$$\Delta_y = \pm\sqrt{\Delta_1^2 - 2\Delta_b^2} \quad . \ . \ . \ . \quad (10).$$

From [3], pp. 593—594, we estimate $\Delta_b = \pm 0.040$ (*p. e.* in $\log T$); hence $\Delta_y = \pm 0.089$ (*p. e.*): this should also be the effective dispersion to be used in the solution of (4) for the true distribution of $\log T$. As stated above, the actual solution was made nevertheless with the larger error, $\pm 0.106$; the

Table XI.

Correction for Observational Error Dispersion of the Distribution of Geocentric Tangential Velocities, All Observers.

| $T$ km/sec | $P=15^0$ obs. smooth. | $P=15^0$ corr. III | $P=45^0$ obs. sm. | $P=45^0$ corr. II | $P=75^0$ obs. sm. | $P=75^0$ corr. II | $P=105^0$ obs. sm. | $P=105^0$ corr. III | $P=135^0$ obs. sm. | $P=135^0$ corr. III | $P=165^0$ obs. sm. | $P=165^0$ corr. III |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 ... 7.4 | 0.1 | 0.0 | 0.5 | 0.5 | 0.6 | 0.0 | 0.3 | 0.2 | 0.0 | 0.0 | 0.0 | 0.0 |
| 7.5 ... 8.8 | 0.3 | 0.0 | 0.5 | 0.5 | 0.7 | 1.0 | 0.3 | 0.3 | 0.0 | 0.0 | 0.0 | 0.0 |
| 8.9 ... 10.5 | 0.9 | 0.0 | 0.5 | 0.5 | 0.8 | 1.2 | 0.4 | 0.2 | 0.4 | 0.0 | 0.0 | 0.0 |
| 10.6 ... 12.5 | 1.9 | 1.5 | 0.5 | 0.6 | 0.8 | 1.1 | 0.5 | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| 12.6 ... 14.9 | 2.6 | 3.0 | 0.5 | 0.4 | 0.6 | 0.5 | 0.9 | 0.1 | 1.6 | 0.8 | 0.5 | 0.0 |
| 15.0 ... 17.8 | 3.0 | 3.8 | 0.5 | 0.4 | 0.5 | 0.0 | 1.6 | 1.0 | 2.6 | 2.2 | 1.5 | 0.3 |
| 17.9 ... 21.1 | 3.2 | 3.7 | 0.5 | 0.2 | 0.4 | 0.0 | 2.5 | 2.6 | 3.7 | 3.3 | 3.1 | 0.2 |
| 21.2 ... 25.1 | 3.2 | 3.6 | 0.7 | 0.4 | 0.8 | 0.4 | 3.3 | 4.2 | 5.0 | 4.6 | 5.4 | 1.6 |
| 25.2 ... 29.9 | 3.1 | 3.7 | 1.1 | 1.1 | 1.3 | 1.5 | 3.8 | 5.2 | 6.4 | 6.8 | 8.5 | 3.0 |
| 30.0 ... 35.7 | 2.6 | 2.7 | 1.3 | 1.1 | 1.7 | 2.3 | 3.7 | 4.5 | 7.8 | 9.7 | 14.0 | 11.7 |
| 35.8 ... 42.3 | 2.2 | 1.9 | 1.7 | 1.4 | 1.7 | 2.1 | 3.3 | 3.4 | 8.2 | 9.9 | 21.0 | 28.2 |
| 42.4 ... 50.3 | 1.8 | 1.5 | 2.1 | 2.3 | 1.6 | 1.8 | 2.9 | 3.0 | 8.0 | 9.3 | 24.5 | 34.8 |
| 50.4 ... 59.9 | 1.5 | 1.4 | 2.7 | 3.7 | 1.5 | 2.0 | 2.4 | 2.7 | 7.2 | 7.4 | 23.5 | 31.2 |
| 60.0 ... 71.5 | 1.3 | 1.5 | 3.1 | 5.3 | 1.2 | 1.1 | 1.8 | 1.7 | 6.3 | 6.4 | 19.8 | 22.0 |
| 71.6 ... 84.7 | 1.0 | 1.2 | 2.0 | 0.6 | 0.8 | 0.0 | 1.3 | 0.9 | 5.4 | 5.8 | 15.8 | 16.0 |
| 84.8 ... 100 | 0.7 | 0.5 | 0.8 | 0.0 | 0 | 0 | 0.8 | 0.0 | 4.2 | 4.3 | 11.4 | 10.0 |
| 101 ... 119 | 0.4 | 0.0 | 0.0 | 0.0 | 0 | 0 | 0.2 | 0.0 | 3.2 | 3.5 | 7.0 | 3.5 |
| 120 ... 143 | 0.2 | 0.0 | 0.0 | 0.0 | 0 | 0 | 0.0 | 0.0 | 1.9 | 0.0 | 3.7 | 0.0 |
| 144 ... 169 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.9 | 0.0 | 1.7 | 0.0 |
| 170 ... 201 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.2 | 0.0 | 0.8 | 0.0 |
| 202 ... 239 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0 | 0.0 | 0.5 | 0.3 |
| $\geqslant 240$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0 | 0.0 | 0.3 | 0.2 |
| All vel. | 30 | 30 | 19 | 19 | 15 | 15 | 30 | 30 | 74 | 74 | 163 | 163 |

overcorrection, however, cannot be great, because of the natural limit set to the overcorrection by the forbidding of negative numbers.

Table XI contains the result. The median value of $P$ for each sector (instead of the limits quoted in Table X), the observed smoothed distribution, and the final distribution corrected for error dispersion are given, the Roman figures indicating the order of the final approximation to $F$ adopted in the solution of equation (4).

6. *The Distribution of Space Velocities.* To determine the distribution of space velocities from the data of Table XI, the frequency function of the projection ratios $\sin\lambda$ ($\lambda$ = angle between line of sight and trajectory of meteor, equal to the

Table XI. Continued.

| $T$ km/sec | $P = 195^0$ | | $P = 225^0$ | | $P = 255^0$ | | $P = 285^0$ | | $P = 315^0$ | | $P = 345^0$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | obs. smooth. | corr. IV | obs. sm. | corr. II | obs. sm. | corr. III | obs. sm. | corr. III | obs sm. | corr. III | obs. sm. | corr. II |
| 0.0 ... 7.4 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.5 | 0.2 | 0.9 | 0.0 | 0.0 | 0.0 |
| 7.5 ... 8.8 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 | 0.0 | 2.5 | 0.4 | 1.2 | 0.4 | 0.2 | 0.0 |
| 8.9 ... 10.5 | 0.3 | 0.0 | 0.4 | 0.0 | 3.0 | 0.0 | 4.0 | 2.0 | 1.9 | 0.9 | 0.6 | 0.0 |
| 10.6 ... 12.5 | 1.0 | 0.0 | 0.8 | 0.0 | 6.0 | 0.0 | 6.7 | 3.4 | 3.1 | 2.0 | 1.8 | 0.0 |
| 12.6 ... 14.9 | 2.3 | 0.0 | 1.6 | 0.0 | 11.0 | 2.8 | 10.6 | 8.3 | 4.7 | 4.4 | 4.1 | 3.8 |
| 15.0 ... 17.8 | 4.1 | 2.7 | 4.4 | 0.0 | 18.5 | 14.3 | 15.0 | 13.6 | 6.4 | 7.0 | 5.2 | 7.5 |
| 17.9 ... 21.1 | 6.3 | 7.0 | 8.5 | 6.9 | 27.3 | 27.0 | 20.4 | 22.9 | 7.8 | 9.7 | 5.3 | 7.4 |
| 21.2 ... 25.1 | 8.7 | 7.9 | 12.3 | 12.1 | 36.0 | 42.5 | 24.5 | 29.6 | 8.6 | 11.3 | 4.5 | 5.0 |
| 25.2 ... 29.9 | 11.5 | 10.4 | 16.2 | 16.5 | 42.6 | 56.9 | 26.5 | 32.5 | 8.1 | 9.3 | 3.6 | 3.1 |
| 30.0 ... 35.7 | 14.8 | 12.9 | 20.0 | 21.3 | 43.6 | 58.5 | 26.1 | 30.1 | 7.2 | 7.6 | 3.0 | 2.5 |
| 35.8 ... 42.3 | 18.0 | 13.8 | 22.8 | 24.3 | 38.6 | 45.8 | 24.0 | 25.8 | 6.1 | 5.8 | 2.6 | 2.2 |
| 42.4 ... 50.3 | 22.0 | 20.4 | 25.5 | 28.3 | 30.4 | 27.9 | 21.2 | 21.2 | 5.1 | 5.1 | 2.3 | 2.3 |
| 50.4 ... 59.9 | 27.0 | 36.8 | 27.3 | 33.7 | 23.5 | 20.1 | 18.3 | 18.8 | 4.0 | 3.9 | 2.0 | 2.1 |
| 60.0 ... 71.5 | 28.4 | 44.5 | 26.5 | 36.3 | 17.5 | 15.9 | 15.2 | 16.9 | 3.0 | 3.0 | 1.6 | 1.7 |
| 71.6 ... 84.7 | 23.8 | 33.3 | 20.2 | 24.3 | 12.0 | 10.5 | 11.2 | 11.8 | 1.9 | 1.5 | 1.2 | 1.1 |
| 84.8 ... 100 | 15.8 | 9.1 | 11.5 | 5.0 | 7.3 | 0.8 | 7.3 | 2.5 | 1.0 | 0.0 | 0.9 | 0.9 |
| 101 ... 119 | 10.7 | 3.1 | 5.4 | 0.0 | 3.7 | 0.0 | 4.0 | 0.0 | 0.5 | 0.0 | 0.6 | 0.4 |
| 120 ... 143 | 7.5 | 1.8 | 3.0 | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 | 0.3 | 0.1 | 0.4 | 0.0 |
| 144 ... 169 | 5.8 | 4.7 | 1.6 | 0.0 | 0.0 | 0.0 | 0 | 0 | 0.2 | 0.0 | 0.1 | 0.0 |
| 170 ... 201 | 4.6 | 5.9 | 1.2 | 0.3 | 0.0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 202 ... 239 | 3.6 | 4.7 | 1.0 | 1.5 | 0.0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\geqslant 240$ | 2.8 | 0.0 | 0.8 | 0.5 | 0.0 | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 |
| All vel. | 219 | 219 | 211 | 211 | 323 | 323 | 240 | 240 | 72 | 72 | 40 | 40 |

angular distance of the centre of the meteor trail from its radiant) is required, separately for each $P$-sector; our former primitive way of treatment (*cf.*[2], Table II, and Section 3 of the present investigation), good for exploratory purposes, must be replaced by a more correct procedure.

Simplifications, nevertheless, are introduced; we assume the whole area of observation to be replaced by a point at its centre, exactly $45^0$ north of the zenith ($\delta = +80^0$); within each of the $P$-sectors we assume a homogeneous distribution of $P$, $\frac{dn}{dP} = \text{const.}$, of a density corresponding to the median value of $P$. The consequences of these simplifications, although not negligible, are smaller than the neglect of observational selection, and are roughly estimated in a subsequent section.

We start from the considerations exposed on pp. 568—569 of[3]; if $B$ is the density of radiants at vertical incidence per unit of solid angle (square radian), we have $f(a, \lambda) = B \cdot (\cos z_i)^k$ (*cf.*[3]); $B$ may be variable. With $a = A$, and $B = 1$, the auxiliary Tables XII and XIII are calculated, from the exact formulae:

$$\frac{n_\lambda}{c} = \sin^2 \lambda - (\lambda - \sin \lambda \cos \lambda) \cos A \quad . \quad . \quad . \quad . \quad (11), \text{ for } k = 1,$$

and

$$\frac{n_\lambda}{c} = \frac{\sqrt{2}}{3} \sin^2 A \, (1 - \cos^3 \lambda) + \sqrt{2} \cos^2 A \, (1 - \cos \lambda) -$$

$$- \frac{2\sqrt{2}}{3} \cos A \sin^3 \lambda \quad . \quad . \quad . \quad . \quad (12), \text{ for } k = 2;$$

here

$c = \frac{\sqrt{2}}{4} dA$; $n_\lambda$ is the integral number of radiants from $\lambda = 0$ to $\lambda$, within the limits of direction $A$ and $A + dA$. In[3] we mentioned a preliminary value of $k = 1.34$. For our purposes it is more convenient to represent the law intermediate between $k = 1$ and $k = 2$ by linear interpolation:

$$\cos^k z_i \, (=) \, \beta \cos z_i + (1 - \beta) \cos^2 z_i \quad . \quad . \quad . \quad . \quad (13);$$

for $k = 1$, $\beta = 1$; $k = 2$, $\beta = 0$.

For the entire Arizona expedition the observed meteor streaming near the meridian (the least influenced by the apical effect) was as follows:

| | Meteors moving up | | | Meteors moving down | | | Ratio down : up |
|---|---|---|---|---|---|---|---|
| | $A$ | number | $\bar{\delta}$ | $A$ | number | $\bar{\delta}$ | |
| North of zenith | 0° ... 29° | 143 | +76° | 150° ... 179° | 3266 | +14° | 20.0:1 |
| | 330 ... 359 | 141 | | 180 ... 209 | 2419 | | |
| South of zenith | 150 ... 179 | 138 | −34° | 0 ... 29 | 1935 | +56° | 13.4:1 |
| | 180 ... 209 | 108 | | 330 ... 359 | 1369 | | |

Here $\bar{\delta}$ is the mean declination for the given sector, estimated on the basis of Tables XII and XIII with $\beta = 0.5$. The inequality in the ratios for North and South is apparently an effect of declination, which we choose to represent by an additional factor $b^{\cos^2 \delta}$ in (13). The theoretical ratio down: up for the chosen limits of $A$ is $\frac{3.16}{0.236} = 13.4$ for $k = 1$ (Table XII), and $\frac{2.50}{0.090} = 27.8$

for $k = 2$ (Table XIII). With these, and the observed ratios, equation (13) yields the following solution:

$$\beta = 0.54 \pm 0.045; \quad b = 1.374.$$

Table XII.

Stream Intensity, $\frac{n_\lambda}{c}$ (from $\lambda = 0^0$ to $\lambda = \lambda$), for $k = 1$, $B = 1$, at $z = 45^0$ North *).

| $\lambda$ | A | | | | | | |
|---|---|---|---|---|---|---|---|
| | $0^0$ (up) | $30^0$ 330 | $60^0$ 300 | $90^0$ 270 | $120^0$ 240 | $150^0$ 210 | $180^0$ (down) |
| $0^0$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0.027 | 0.027 | 0.028 | 0.030 | 0.032 | 0.033 | 0.033 |
| 20 | 0.090 | 0.094 | 0.104 | 0.117 | 0.130 | 0.140 | 0.144 |
| 30 | 0.159 | 0.171 | 0.204 | 0.250 | 0.296 | 0.329 | 0.341 |
| 40 | 0.207 | 0.235 | 0.310 | 0.413 | 0.516 | 0.591 | 0.619 |
| 45 | 0.215 | ... | ... | ... | ... | ... | ... |
| 49.1 | — | 0.258 | ... | ... | ... | ... | ... |
| 50 | — | — | 0.396 | 0.587 | 0.778 | 0.917 | 0.968 |
| 60 | — | — | 0.443 | 0.750 | 1.057 | 1.282 | 1.364 |
| 63.4 | — | — | 0.446 | ... | ... | ... | ... |
| 70 | — | — | — | 0.883 | 1.334 | 1.663 | 1.784 |
| 80 | — | — | — | 0.970 | 1.582 | 2.031 | 2.195 |
| 90 | — | — | — | 1.000 | 1.786 | 2.360 | 2.571 |
| 100 | — | — | — | — | 1.928 | 2.629 | 2.886 |
| 110 | — | — | — | — | 2.005 | 2.826 | 3.126 |
| 116.6 | — | — | — | — | 2.016 | ... | ... |
| 120 | — | — | — | — | — | 2.940 | 3.278 |
| 130 | — | — | — | — | — | 2.978 | 3.348 |
| 130.9 | — | — | — | — | — | 2.980 | ... |
| $135^0$ | — | — | — | — | — | — | 3.356 |

With a sufficient degree of approximation we assume $\beta = 0.5$; actually the sum of Tables XII and XIII was used below, which corresponds to $\frac{2n_\lambda}{c}$.

A moderate but definite increase of the radiant density toward the equator (which may be identified as the mean position of the ecliptic for the whole year) is indicated by $b$; the density at the equator is 37 per cent greater than at the pole; if this is an effect of the solar meteors only, their true degree of concentration toward the ecliptic must be considerably greater, because the solar meteors represent only a fraction of all the meteors.

*) At $z = 45^0$ South, take $A \pm 180^0$ for $A$.

Table XIII.

Stream Intensity, $\frac{n_\lambda}{c}$ (from $\lambda = 0^0$ to $\lambda = \lambda$), for $k = 2$, $B = 1$, at $z = 45^0$ North.

| $\lambda$ | A: $0^0$ (up) | $30^0$ 330 | $60^0$ 300 | $90^0$ 270 | $120^0$ 240 | $150^0$ 210 | $180^0$ (down) |
|---|---|---|---|---|---|---|---|
| $0^0$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0.016 | 0.017 | 0.019 | 0.021 | 0.023 | 0.025 | 0.026 |
| 20 | 0.047 | 0.051 | 0.062 | 0.080 | 0.100 | 0.117 | 0.123 |
| 30 | 0.071 | 0.081 | 0.112 | 0.165 | 0.230 | 0.285 | 0.307 |
| 40 | 0.080 | 0.096 | 0.152 | 0.260 | 0.402 | 0.530 | 0.582 |
| 45 | 0.080 | ... | ... | ... | ... | ... | ... |
| 49.1 | — | 0.099 | ... | ... | ... | ... | ... |
| 50 | — | — | 0.175 | 0.346 | 0.597 | 0.830 | 0.928 |
| 60 | — | — | 0.182 | 0.412 | 0.792 | 1.161 | 1.319 |
| 63.4 | — | — | 0.182 | ... | ... | ... | ... |
| 70 | — | — | — | 0.452 | 0.963 | 1.487 | 1.712 |
| 80 | — | — | — | 0.469 | 1.094 | 1.772 | 2.068 |
| 90 | — | — | — | 0.471 | 1.179 | 1.993 | 2.357 |
| 100 | — | — | — | — | 1.222 | 2.142 | 2.560 |
| 110 | — | — | — | — | 1.235 | 2.223 | 2.697 |
| 116.6 | — | — | — | — | 1.235 | ... | ... |
| 120 | — | — | — | — | — | 2.251 | 2.733 |
| 130 | — | — | — | — | — | 2.256 | 2.746 |
| 130.9 | — | — | — | — | — | 2.257 | ... |
| $135^0$ | — | — | — | — | — | — | 2.748 |

The apical effect produces an increase of the apparent density of the radiants toward the apex; without inquiring into the true nature of this effect we try to represent the observed distribution by a simple formula of interpolation. A range of the apical effect much greater than the one observed has been computed by the writer upon certain assumptions[7]; even this extreme case could be represented satisfactorily by the simple expression

$$B(\alpha_1) \sim a^{\cos \alpha_1} \times \text{const.} \quad . \quad . \quad . \quad . \quad (14),$$

as shown by the following figures:

| $\alpha_1$ | $0^0.0$ | $11^0.7$ | $23^0.4$ | $35^0.5$ | $48^0.4$ | $61^0.5$ |
|---|---|---|---|---|---|---|
| $B(\alpha_1)$, exact (*cf.*[7]) | 13.6 | 12.8 | 11.0 | 7.1 | 4.4 | 2.3 |
| $B(\alpha_1)$, from (14) ($a = 50$) | 15.0 | 14.0 | 10.7 | 7.2 | 4.0 | 1.9 |

| $\alpha_1$ | $76^0.5$ | $94^0.4$ | $125^0.6$ | $149^0.2$ | $180^0.0$ |
|---|---|---|---|---|---|
| $B(\alpha_1)$, exact (*cf.*[7]) | 0.80 | 0.21 | 0.026 | 0.009 | 0.006 |
| $B(\alpha_1)$, from (14) ($a = 50$) | 0.75 | 0.22 | 0.031 | 0.011 | 0.006 |

($\alpha_1$ is the distance of the apparent radiant from the apex).

The actual range in $B(\alpha_1)$ is much smaller than in the above example, and an interpolation formula must thus work better. From the relative frequency of velocities in $P = 165^0 + 195^0$, as compared with $P = 15^0 + 345^0$ (*cf.* sum totals in Table XI), and with the value of $b$ as found above, we finally set up the following preliminary working formula for the apparent density of radiants (vertical incidence):

$$B' = a^{\cos \alpha_1} \cdot b^{\cos^2 \delta_1} \quad . \quad . \quad . \quad . \quad (15),$$

with $a = 2.77$, $b = 1.374$.

Our auxiliary tables above are arranged according to the angle $A$, whereas our statistics refer to the apical direction $P$; for the centre of the area of observation, the mean of the whole year, these angles are connected by the equation

$$A = P - 90^0 + t \quad . \quad . \quad . \quad . \quad (16),$$

where $t$ is the mean local time. Thus, for a given $P$-sector, different $A$-sectors contribute during the night.

The distribution of observational time and of the difference $A - P$ for the Northern region, all Arizona records, is as follows:

| $t$ | $19^h.0$ | $20^h.0$ | $21^h.0$ | $22^h.0$ | $23^h.0$ | $24^h.0$ |
|---|---|---|---|---|---|---|
| $A - P$ | $195^0$ | $210^0$ | $225^0$ | $240^0$ | $255^0$ | $270^0$ |
| Hours of observation | 90 | 222 | 316 | 352 | 296 | 263 |
| Per cent | 3.3 | 8.2 | 11.6 | 12.9 | 10.9 | 9.7 |

| $t$ | $1^h.0$ | $2^h.0$ | $3^h.0$ | $4^h.0$ | $5^h.0$ | All |
|---|---|---|---|---|---|---|
| $A - P$ | $285^0$ | $300^0$ | $315^0$ | $330^0$ | $345^0$ | ... |
| Hours of observation | 314 | 327 | 299 | 178 | 66 | 2723 |
| Per cent | 11.5 | 12.0 | 11.0 | 6.5 | 2.4 | 100.0 |

The theoretical distribution of the stream intensity for a given sector $P$ is computed from Tables XII or XIII by assuming weights proportional to the time of observation as quoted above. Table XIV contains the concluded weights of the $A$-distributions as they contribute to a given $P$-distribution (linear interpolation for intermediate values of $A$ being used).

Table XIV.

Relative Weights of the $A$-Distributions Contributing to the $P$-Distribution, North.

| $A$, $360^0 - A$ | $P$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $15^0$ | $45^0$ | $75^0$ | $105^0$ | $135^0$ | $165^0$ | $195^0$ | $225^0$ | $255^0$ | $285^0$ | $315^0$ | $345^0$ |
| $0^0$ | 0.056 | 0.203 | 0.223 | 0.222 | 0.222 | 0.074 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $30^0$ | 0.203 | 0.279 | 0.425 | 0.445 | 0.296 | 0.222 | 0.074 | 0.000 | 0.000 | 0.000 | 0.000 | 0.056 |
| $60^0$ | 0.223 | 0.222 | 0.278 | 0.277 | 0.223 | 0.222 | 0.222 | 0.074 | 0.000 | 0.000 | 0.056 | 0.203 |
| $90^0$ | 0.222 | 0.222 | 0.074 | 0.056 | 0.203 | 0.223 | 0.222 | 0.222 | 0.074 | 0.056 | 0.203 | 0.223 |
| $120^0$ | 0.222 | 0.074 | 0.000 | 0.000 | 0.056 | 0.203 | 0.223 | 0.222 | 0.278 | 0.277 | 0.223 | 0.222 |
| $150^0$ | 0.074 | 0.000 | 0.000 | 0.000 | 0.000 | 0.056 | 0.203 | 0.279 | 0.425 | 0.445 | 0.296 | 0.222 |
| $180^0$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.056 | 0.203 | 0.223 | 0.222 | 0.222 | 0.074 |
| Sum | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

For the calculation of $B'$ [formula (15)], the mean for the whole year, we assume $\delta_{\text{apex}} = 0^0$ and, with sufficient precision (centre of the area of observation near the north pole), for a point $(\delta_1, P)$,

$$\cos a_1 = -\cos\delta_1 \cos P \quad . \quad . \quad . \quad . \quad (17).$$

Our original coordinates are $\lambda$, $P$; these given, the declination slightly varies with $A$; counting with the weights of Table XIV, the average declination for given $(\lambda, P)$ may be found; from that, $B'$, according to (15) and (17), is computed. With that, the final theoretical distribution as given in Table XV is computed. The table gives for the limits $\lambda_1$ and $\lambda_2$, and for $\Delta P = 1$ (one radian), corresponding to a solid angle of $S = \cos\lambda_1 - \cos\lambda_2$:

1) the intensity of streaming (radiation) for $B' = 1$ (uniform distribution of directions),

$$N_z = 4\sqrt{2}\int_{\lambda_1}^{\lambda_2} (\cos z_i + \cos^2 z_i) \sin\lambda \, d\lambda;$$

$N_z$ is found by a summation of Tables XII and XIII, and by taking the corresponding differences between $\lambda_1$ and $\lambda_2$;

2) the adopted (provisional) radiant density, $B'$, computed according to (15);

3) the concluded intensity of radiation corresponding to $B'$,

$$N_\lambda = B' N_z.$$

If the true mean density of apparent radiants is $\bar{B}$, the observable probable number of meteors originating from a sector $P$ to $P + \Delta P$, and from a distance $\lambda_1$ to $\lambda_2$, is

$$n_\lambda = \frac{\Delta P}{4\sqrt{2}} \bar{B} N_z \quad . \quad . \quad . \quad . \quad (18),$$

$\Delta P$ being given in radians.

For our adopted width of the sector, $30^0$, the formula becomes

$$n_\lambda = \frac{\bar{B} N_z}{10.80} \quad . \quad . \quad . \quad . \quad (18').$$

With the aid of this formula, the relative figures of Table XV may be converted into absolute ones for the specific value of $\Delta P = 30^0$.

At the foot of Table XV the coordinates of the effective point, or centre of radiation, are given; $\lambda_{\text{eff.}}$ is the value of $\lambda$ that halves $\Sigma N_\lambda$; $\bar{\delta}_1$, $\bar{\alpha}_1$, and $\bar{z}_i$ refer to the point $(P, \lambda_{\text{eff.}})$.

The degree of approximation reached by Table XV may be judged from a comparison of the observed distribution of the velocities according to $P$ with the theoretical distribution which is determined by the sum of $N_\lambda$:

| $P$ | $15^0$ | $45^0$ | $75^0$ | $105^0$ | $135^0$ | $165^0$ |
|---|---|---|---|---|---|---|
| Observed $n$ | 30 | 19 | 15 | 30 | 74 | 163 |
| Computed $n \sim \Sigma N_\lambda$ | 37 | 24 | 18 | 21 | 48 | 132 |
| Ratio obs.: comp. | 0.81 | 0.79 | 0.83 | 1.43 | 1.54 | 1.23 |

| $P$ | $195^0$ | $225^0$ | $255^0$ | $285^0$ | $315^0$ | $345^0$ | All |
|---|---|---|---|---|---|---|---|
| Observed $n$ | 219 | 211 | 323 | 240 | 72 | 40 | 1436 |
| Computed $n \sim \Sigma N_\lambda$ | 254 | 312 | 262 | 170 | 99 | 59 | 1436 |
| Ratio obs.: comp. | 0.86 | 0.68 | 1.23 | 1.41 | 0.73 | 0.68 | . . . |

The difference of observation and computation reveals a doubtlessly systematic trend with $P$ of such a character that no improvement in the adopted $\cos z$ and $\delta$-effect can help; perhaps a value of $a = 3.20$ in the $\alpha_1$-effect [*cf.* formula (15)] might do a little better without removing the major systematic fluctuations. These are not larger than the order of personal selection (*cf.* Section 2). In any case, the representation must be considered satisfactory in view of the different simplifying

# Table XV.

## Stream Intensity, Calculated.

(Northern Region, Arizona).

| $\lambda$ | $P=15^0$ | | | $P=45^0$ | | | $P=75^0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $N_z$ | $\bar{B}'$ | $N_\lambda$ | $N_z$ | $\bar{B}'$ | $N_\lambda$ | $N_z$ | $\bar{B}'$ | $N_\lambda$ |
| $0^0$–10 | 0.048 | 0.951 | 0.046 | 0.046 | 1.000 | 0.046 | 0.046 | 0.993 | 0.046 |
| 10–20 | 0.140 | 0.818 | 0.114 | 0.120 | 0.893 | 0.107 | 0.108 | 0.973 | 0.105 |
| 20–30 | 0.201 | 0.718 | 0.144 | 0.152 | 0.813 | 0.123 | 0.123 | 0.953 | 0.117 |
| 30–40 | 0.232 | 0.646 | 0.149 | 0.152 | 0.757 | 0.115 | 0.104 | 0.948 | 0.099 |
| 40–50 | 0.236 | 0.582 | 0.140 | 0.124 | 0.720 | 0.089 | 0.065 | 0.959 | 0.062 |
| 50–60 | 0.219 | 0.561 | 0.123 | 0.098 | 0.698 | 0.068 | 0.031 | 0.982 | 0.030 |
| 60–70 | 0.192 | 0.538 | 0.103 | 0.072 | 0.678 | 0.049 | 0.014 | 1.007 | 0.014 |
| 70–80 | 0.154 | 0.519 | 0.080 | 0.051 | 0.676 | 0.034 | 0.007 | 1.030 | 0.007 |
| 80–90 | 0.115 | 0.516 | 0.059 | 0.029 | 0.673 | 0.020 | 0.003 | 1.052 | 0.003 |
| 90–100 | 0.071 | 0.513 | 0.036 | 0.013 | 0.668 | 0.009 | 0 | . . . | 0 |
| 100–110 | 0.041 | 0.520 | 0.021 | 0.006 | 0.673 | 0.004 | 0 | . . . | 0 |
| 110–120 | 0.011 | 0.528 | 0.006 | 0.002 | 0.675 | 0.001 | 0 | . . . | 0 |
| 120–130 | 0.004 | 0.550 | 0.002 | 0.000 | . . . | 0.000 | 0 | . . . | 0 |
| 130–135 | 0.001 | 0.562 | 0.000 | 0.000 | . . . | 0.000 | 0 | . . . | 0 |
| Sum | 1.665 | . . . | 1.023 | 0.865 | . . . | 0.665 | 0.501 | . . . | 0.483 |
| | $\lambda_{\text{eff.}}=44^0.2$; $\bar{\delta}_1=+48^0$; | | | $\lambda_{\text{eff.}}=34^0.9$; $\bar{\delta}_1=+60^0$; | | | $\lambda_{\text{eff.}}=27^0.8$; $\bar{\delta}_1=+69^0$; | | |
| | $\bar{\alpha}_1=130^0$; $\bar{z}_i=67^0.8$ | | | $\bar{\alpha}_1=111^0$; $\bar{z}_i=72^0.9$ | | | $\bar{\alpha}_1=95^0$; $\bar{z}_i=72^0.1$ | | |

assumptions made. For our purposes we do not need a better first approximation theory; our aim is to get a reliable distribution of $\lambda$, or $\sin\lambda$ inside a given $P$-sector; the error in this internal distribution must be smaller by at least an order of magnitude than the error in the absolute frequency for different $P$-sectors. A difference in the distribution depending

Table XV. Continued.

| $\lambda$ | $P=105^0$ | | | $P=135^0$ | | | $P=165^0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $N_z$ | $\bar{B}'$ | $N_\lambda$ | $N_z$ | $\bar{B}'$ | $N_\lambda$ | $N_z$ | $\bar{B}'$ | $N_\lambda$ |
| $0^0$ | 0.046 | 1.007 | 0.046 | 0.046 | 1.000 | 0.046 | 0.048 | 1.052 | 0.050 |
| 10 | 0.106 | 1.062 | 0.113 | 0.118 | 1.140 | 0.134 | 0.136 | 1.268 | 0.172 |
| 20 | 0.121 | 1.122 | 0.136 | 0.144 | 1.324 | 0.191 | 0.192 | 1.538 | 0.295 |
| 30 | 0.102 | 1.219 | 0.124 | 0.144 | 1.545 | 0.223 | 0.220 | 1.871 | 0.411 |
| 40 | 0.060 | 1.330 | 0.080 | 0.113 | 1.799 | 0.203 | 0.216 | 2.27 | 0.490 |
| 50 | 0.026 | 1.445 | 0.038 | 0.084 | 2.09 | 0.175 | 0.197 | 2.67 | 0.526 |
| 60 | 0.011 | 1.574 | 0.017 | 0.062 | 2.38 | 0.147 | 0.169 | 3.07 | 0.519 |
| 70 | 0.006 | 1.679 | 0.010 | 0.041 | 2.59 | 0.106 | 0.138 | 3.43 | 0.474 |
| 80 | 0.002 | 1.746 | 0.003 | 0.024 | 2.73 | 0.065 | 0.096 | 3.62 | 0.347 |
| 90 | 0 | . . . | 0 | 0.010 | 2.82 | 0.028 | 0.062 | 3.68 | 0.228 |
| 100 | 0 | . . . | 0 | 0.004 | 2.73 | 0.011 | 0.033 | 3.52 | 0.116 |
| 110 | 0 | . . . | 0 | 0.001 | 2.59 | 0.003 | 0.010 | 3.23 | 0.032 |
| 120 | 0 | . . . | 0 | 0.000 | . . . | 0.000 | 0.002 | 2.84 | 0.006 |
| 130 | 0 | . . . | 0 | 0.000 | . . . | 0.000 | 0.001 | 2.43 | 0.002 |
| 135 | | | | | | | | | |
| Sum | 0.480 | . . . | 0.567 | 0.791 | . . . | 1.332 | 1.520 | . . . | 3.668 |
| | $\lambda_{\text{eff.}}=29^0.2$; $\bar{\delta}_1=+68^0$; | | | $\lambda_{\text{eff.}}=43^0.5$; $\bar{\delta}_1=+53^0$; | | | $\lambda_{\text{eff.}}=57^0.9$; $\bar{\delta}_1=+34^0$; | | |
| | $\bar{\alpha}_1=84^0$; $\bar{z}_i=73^0.5$ | | | $\bar{\alpha}_1=65^0$; $\bar{z}_i=80^0.3$ | | | $\bar{\alpha}_1=37^0$; $\bar{z}_i=77^0.2$ | | |

upon velocity may be expected; but even this can produce only a small change in our results: as shown by Table XVI, the two very different assumptions regarding $B'$, — $B'$ = const., and $B'$ as given by (15) — lead to laws of projection ratios which still are very similar to each other; yet the first law, $B'$ = const., does not represent the observed frequency of $P$ at all (for $B'$ = const., the frequency should be proportional to $\Sigma N_z$ of Table XV, with practical equality of the apex and antapex

Table XV. Continued.

| $\lambda$ | $P=195^0$ | | | $P=225^0$ | | | $P=255^0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $N_z$ | $\bar{B}'$ | $N_\lambda$ | $N_z$ | $\bar{B}'$ | $N_\lambda$ | $N_z$ | $\bar{B}'$ | $N_\lambda$ |
| $0^0$ | | | | | | | | | |
| | 0.051 | 1.135 | 0.058 | 0.054 | 1.140 | 0.062 | 0.057 | 1.067 | 0.061 |
| 10 | | | | | | | | | |
| | 0.159 | 1.365 | 0.217 | 0.179 | 1.324 | 0.237 | 0.190 | 1.159 | 0.220 |
| 20 | | | | | | | | | |
| | 0.250 | 1.667 | 0.417 | 0.301 | 1.545 | 0.465 | 0.335 | 1.262 | 0.423 |
| 30 | | | | | | | | | |
| | 0.335 | 2.023 | 0.676 | 0.409 | 1.799 | 0.734 | 0.467 | 1.368 | 0.637 |
| 40 | | | | | | | | | |
| | 0.331 | 2.43 | 0.804 | 0.483 | 2.089 | 1.006 | 0.566 | 1.493 | 0.846 |
| 50 | | | | | | | | | |
| | 0.353 | 2.84 | 1.000 | 0.513 | 2.38 | 1.220 | 0.618 | 1.614 | 0.996 |
| 60 | | | | | | | | | |
| | 0.329 | 3.23 | 1.060 | 0.503 | 2.59 | 1.300 | 0.621 | 1.714 | 1.064 |
| 70 | | | | | | | | | |
| | 0.281 | 3.52 | 0.987 | 0.442 | 2.73 | 1.204 | 0.559 | 1.778 | 0.991 |
| 80 | | | | | | | | | |
| | 0.223 | 3.68 | 0.820 | 0.360 | 2.82 | 1.015 | 0.465 | 1.791 | 0.833 |
| 90 | | | | | | | | | |
| | 0.154 | 3.62 | 0.557 | 0.262 | 2.73 | 0.716 | 0.343 | 1.746 | 0.599 |
| 100 | | | | | | | | | |
| | 0.097 | 3.43 | 0.333 | 0.172 | 2.59 | 0.445 | 0.226 | 1.679 | 0.380 |
| 110 | | | | | | | | | |
| | 0.041 | 3.07 | 0.126 | 0.076 | 2.38 | 0.181 | 0.103 | 1.574 | 0.162 |
| 120 | | | | | | | | | |
| | 0.013 | 2.67 | 0.035 | 0.034 | 2.09 | 0.071 | 0.034 | 1.445 | 0.049 |
| 130 | | | | | | | | | |
| | 0.003 | 2.46 | 0.007 | 0.003 | 1.94 | 0.006 | 0.003 | 1.387 | 0.004 |
| 135 | | | | | | | | | |
| Sum | 2.620 | . . . | 7.097 | 3.791 | . . . | 8.662 | 4.587 | . . . | 7.265 |
| | $\lambda_{\text{eff.}}=63^0.5$; $\bar{\delta}_1=+24^0$; $\bar{\alpha}_1=28^0$; $\bar{z}_i=61^0.4$ | | | $\lambda_{\text{eff.}}=64^0.7$; $\bar{\delta}_1=+20^0$; $\bar{\alpha}_1=48^0$; $\bar{z}_i=48^0.3$ | | | $\lambda_{\text{eff.}}=64^0.2$; $\bar{\delta}_1=+19^0$; $\bar{\alpha}_1=76^0$; $\bar{z}_i=22^0.8$ | | |

directions). From these considerations it appears that no further approximation is required, and that case *(b)* of Table XVI may finally be assumed to represent the distribution of projection ratios for the present material.

The distribution of space velocities is in principle determined by the same equation (4), by which the error dispersion was accounted for: set for $\chi$ the frequency function of projection ratios (Table XVI, case *b*), for $\varphi$ the frequency of tangential

Table XV. Continued.

| $\lambda$ | $P = 285^0$ | | | $P = 315^0$ | | | $P = 345^0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $N_z$ | $\bar{B}'$ | $N_\lambda$ | $N_z$ | $\bar{B}'$ | $N_\lambda$ | $N_z$ | $\bar{B}'$ | $N_\lambda$ |
| $0^0$–10 | 0.057 | 0.964 | 0.055 | 0.055 | 0.893 | 0.049 | 0.053 | 0.889 | 0.047 |
| 10–20 | 0.191 | 0.946 | 0.180 | 0.180 | 0.813 | 0.146 | 0.161 | 0.774 | 0.124 |
| 20–30 | 0.336 | 0.948 | 0.318 | 0.310 | 0.757 | 0.235 | 0.257 | 0.682 | 0.175 |
| 30–40 | 0.472 | 0.968 | 0.457 | 0.420 | 0.719 | 0.301 | 0.332 | 0.622 | 0.206 |
| 40–50 | 0.573 | 0.995 | 0.570 | 0.500 | 0.698 | 0.349 | 0.374 | 0.575 | 0.215 |
| 50–60 | 0.624 | 1.014 | 0.633 | 0.533 | 0.678 | 0.361 | 0.378 | 0.550 | 0.208 |
| 60–70 | 0.630 | 1.033 | 0.651 | 0.527 | 0.676 | 0.355 | 0.355 | 0.528 | 0.187 |
| 70–80 | 0.574 | 1.047 | 0.601 | 0.468 | 0.673 | 0.314 | 0.309 | 0.520 | 0.160 |
| 80–90 | 0.472 | 1.054 | 0.497 | 0.383 | 0.668 | 0.255 | 0.244 | 0.513 | 0.125 |
| 90–100 | 0.349 | 1.052 | 0.367 | 0.278 | 0.673 | 0.187 | 0.170 | 0.516 | 0.088 |
| 100–110 | 0.233 | 1.030 | 0.240 | 0.183 | 0.676 | 0.123 | 0.108 | 0.519 | 0.056 |
| 110–120 | 0.103 | 1.007 | 0.104 | 0.087 | 0.678 | 0.059 | 0.046 | 0.538 | 0.025 |
| 120–130 | 0.038 | 0.982 | 0.037 | 0.029 | 0.698 | 0.020 | 0.017 | 0.561 | 0.010 |
| 130–135 | 0.003 | 0.970 | 0.003 | 0.003 | 0.708 | 0.002 | 0.002 | 0.578 | 0.001 |
| Sum | 4.655 | . . . | 4.713 | 3.956 | . . . | 2.756 | 2.806 | . . . | 1.627 |
| | $\lambda_{\text{eff.}} = 62^0.2$; $\bar{\delta}_1 = +21^0$; $\bar{\alpha}_1 = 104^0$; $\bar{z}_i = 20^0.8$ | | | $\lambda_{\text{eff.}} = 58^0.3$; $\bar{\delta}_1 = +27^0$; $\bar{\alpha}_1 = 129^0$; $\bar{z}_i = 38^0.7$ | | | $\lambda_{\text{eff.}} = 52^0.3$; $\bar{\delta}_1 = +36^0$; $\bar{\alpha}_1 = 141^0$; $\bar{z}_i = 54^0.7$ | | |

velocities, corrected for error dispersion (Table XI), and for $F$ — the frequency function of space velocities sought. The solution, if represented by discrete values of the space velocity, is less complicated than in the case of the error dispersion: successive approximations are not required. The numerical solution proceeds in the following way: the discrete values of the space velocity, $U$, are spaced by the same ratio $\sqrt[4]{2} : 1$ as were $T$ (in Table XI) and $\sin \lambda$ (in Table XVI) spaced; let the highest value of $T$

## Table XVI.

### Relative Frequency of Projection Ratios (Sin $\lambda$).

Case $a$ : $B$ = const.; case $b$ : $B'$ defined by formula (15). Case $b$ is adopted in the solution for space velocities.

| Sin $\lambda$ | $P = 15^0$ | | $P = 45^0$ | | $P = 75^0$ | | $P = 105^0$ | |
|---|---|---|---|---|---|---|---|---|
| | $a$ | $b$ | $a$ | $b$ | $a$ | $b$ | $a$ | $b$ |
| 1.000 – 0.842 | 0.384 | 0.328 | 0.228 | 0.202 | 0.064 | 0.066 | 0.052 | 0.071 |
| 0.842 – 0.707 | 0.173 | 0.160 | 0.157 | 0.144 | 0.110 | 0.110 | 0.104 | 0.120 |
| 0.707 – 0.594 | 0.117 | 0.117 | 0.131 | 0.123 | 0.135 | 0.132 | 0.133 | 0.143 |
| 0.594 – 0.500 | 0.093 | 0.097 | 0.116 | 0.116 | 0.137 | 0.137 | 0.142 | 0.146 |
| 0.500 – 0.421 | 0.066 | 0.077 | 0.090 | 0.102 | 0.128 | 0.132 | 0.131 | 0.132 |
| 0.421 – 0.354 | 0.049 | 0.055 | 0.073 | 0.072 | 0.102 | 0.096 | 0.104 | 0.094 |
| 0.354 – 0.297 | 0.037 | 0.046 | 0.060 | 0.063 | 0.082 | 0.087 | 0.085 | 0.079 |
| 0.297 – 0.250 | 0.023 | 0.033 | 0.042 | 0.048 | 0.064 | 0.064 | 0.065 | 0.060 |
| 0.250 – 0.210 | 0.016 | 0.023 | 0.031 | 0.033 | 0.044 | 0.046 | 0.046 | 0.042 |
| 0.210 – 0.177 | 0.012 | 0.017 | 0.021 | 0.026 | 0.038 | 0.033 | 0.038 | 0.030 |
| 0.177 – 0.148 | 0.009 | 0.014 | 0.015 | 0.021 | 0.028 | 0.027 | 0.029 | 0.023 |
| 0.148 – 0.125 | 0.006 | 0.010 | 0.010 | 0.015 | 0.020 | 0.021 | 0.021 | 0.018 |
| 0.125 – 0.105 | 0.005 | 0.007 | 0.007 | 0.011 | 0.014 | 0.014 | 0.015 | 0.012 |
| 0.105 – 0.000 | 0.010 | 0.016 | 0.019 | 0.024 | 0.034 | 0.035 | 0.035 | 0.030 |
| Sum | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Mean Sin $\lambda$ | 0.699 | 0.654 | 0.596 | 0.570 | 0.479 | 0.478 | 0.468 | 0.494 |

be $T_1$, and let the number between $T_1$ and $T_2 = \frac{T_1}{\sqrt[4]{2}}$ be $n_1$ (Table XI, corr.); let the frequency of projection ratios from 1.000 to 0.842 be $\chi_0$ (Table XVI, b); the frequency of space velocity $U_1 = T_1$ is $N_1 = \frac{n_1}{\chi_0}$; this number is then multiplied consecutively by the fractions $\chi$ given under $b$ in Table XVI,

Table XVI. Continued.

| Sin λ | P = 135° | | P = 165° | | P = 195° | | P = 225° | |
|---|---|---|---|---|---|---|---|---|
| | *a* | *b* | *a* | *b* | *a* | *b* | *a* | *b* |
| 1.000 | | | | | | | | |
| | 0.206 | 0.303 | 0.367 | 0.504 | 0.465 | 0.583 | 0.516 | 0.598 |
| 0.842 | | | | | | | | |
| | 0.151 | 0.175 | 0.170 | 0.176 | 0.170 | 0.167 | 0.173 | 0.170 |
| 0.707 | | | | | | | | |
| | 0.133 | 0.131 | 0.119 | 0.104 | 0.106 | 0.088 | 0.100 | 0.087 |
| 0.594 | | | | | | | | |
| | 0.121 | 0.112 | 0.097 | 0.075 | 0.085 | 0.064 | 0.072 | 0.056 |
| 0.500 | | | | | | | | |
| | 0.094 | 0.079 | 0.069 | 0.044 | 0.054 | 0.032 | 0.044 | 0.030 |
| 0.421 | | | | | | | | |
| | 0.076 | 0.056 | 0.051 | 0.032 | 0.035 | 0.023 | 0.030 | 0.021 |
| 0.354 | | | | | | | | |
| | 0.057 | 0.041 | 0.040 | 0.020 | 0.025 | 0.014 | 0.020 | 0.012 |
| 0.297 | | | | | | | | |
| | 0.043 | 0.031 | 0.025 | 0.014 | 0.017 | 0.009 | 0.014 | 0.008 |
| 0.250 | | | | | | | | |
| | 0.033 | 0.021 | 0.017 | 0.010 | 0.013 | 0.006 | 0.010 | 0.006 |
| 0.210 | | | | | | | | |
| | 0.026 | 0.015 | 0.012 | 0.007 | 0.010 | 0.005 | 0.007 | 0.004 |
| 0.177 | | | | | | | | |
| | 0.018 | 0.011 | 0.010 | 0.004 | 0.006 | 0.003 | 0.004 | 0.003 |
| 0.148 | | | | | | | | |
| | 0.012 | 0.008 | 0.007 | 0.003 | 0.004 | 0.002 | 0.003 | 0.002 |
| 0.125 | | | | | | | | |
| | 0.009 | 0.005 | 0.005 | 0.002 | 0.003 | 0.001 | 0.002 | 0.001 |
| 0.105 | | | | | | | | |
| | 0.021 | 0.012 | 0.011 | 0.005 | 0.007 | 0.003 | 0.005 | 0.002 |
| 0.000 | | | | | | | | |
| Sum | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Mean Sin λ | 0.583 | 0.655 | 0.689 | 0.769 | 0.746 | 0.809 | 0.775 | 0.819 |

and the products subtracted from the corresponding $T$-frequencies; a new distribution of $T$ is obtained, where $T_2$ is the highest value, with $n_2$ between $T_2$ and $T_3$; the frequency of space velocity $U_2 = T_2$ is evidently $N_2 = \frac{n_2}{\chi_0}$, and so forth.

The distribution of the space velocities determined by the above method is given in column $n$ of Table XVII. The other data given in Table XVII, are: $U$, the discrete values of the space velocity corrected for the "ellipticity" of the standard shower meteors (*cf.* below); $V = \sqrt{U^2 - 125}$, the geocentric

Table XVI. Continued.

| Sin λ | $P = 255^0$ | | $P = 285^0$ | | $P = 315^0$ | | $P = 345^0$ | |
|---|---|---|---|---|---|---|---|---|
| | *a* | *b* | *a* | *b* | *a* | *b* | *a* | *b* |
| 1.000 | | | | | | | | |
| | 0.541 | 0.590 | 0.543 | 0.557 | 0.523 | 0.503 | 0.474 | 0.427 |
| 0.842 | | | | | | | | |
| | 0.169 | 0.167 | 0.169 | 0.168 | 0.170 | 0.168 | 0.173 | 0.167 |
| 0.707 | | | | | | | | |
| | 0.096 | 0.087 | 0.095 | 0.093 | 0.099 | 0.100 | 0.106 | 0.109 |
| 0.594 | | | | | | | | |
| | 0.068 | 0.058 | 0.068 | 0.065 | 0.071 | 0.073 | 0.079 | 0.085 |
| 0.500 | | | | | | | | |
| | 0.040 | 0.032 | 0.040 | 0.037 | 0.043 | 0.047 | 0.052 | 0.059 |
| 0.421 | | | | | | | | |
| | 0.028 | 0.023 | 0.028 | 0.026 | 0.030 | 0.033 | 0.034 | 0.042 |
| 0.354 | | | | | | | | |
| | 0.019 | 0.014 | 0.018 | 0.016 | 0.019 | 0.022 | 0.023 | 0.031 |
| 0.297 | | | | | | | | |
| | 0.012 | 0.009 | 0.012 | 0.011 | 0.014 | 0.016 | 0.017 | 0.023 |
| 0.250 | | | | | | | | |
| | 0.008 | 0.006 | 0.008 | 0.008 | 0.010 | 0.011 | 0.013 | 0.016 |
| 0.210 | | | | | | | | |
| | 0.006 | 0.005 | 0.006 | 0.006 | 0.007 | 0.008 | 0.009 | 0.012 |
| 0.177 | | | | | | | | |
| | 0.004 | 0.003 | 0.004 | 0.004 | 0.004 | 0.005 | 0.006 | 0.009 |
| 0.148 | | | | | | | | |
| | 0.003 | 0.002 | 0.003 | 0.003 | 0.003 | 0.004 | 0.004 | 0.006 |
| 0.125 | | | | | | | | |
| | 0.002 | 0.001 | 0.002 | 0.002 | 0.002 | 0.003 | 0.003 | 0.004 |
| 0.105 | | | | | | | | |
| | 0.004 | 0.003 | 0.004 | 0.004 | 0.005 | 0.007 | 0.007 | 0.010 |
| 0.000 | | | | | | | | |
| Sum | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Mean Sin λ | 0.787 | 0.814 | 0.788 | 0.794 | 0.777 | 0.764 | 0.753 | 0.716 |

velocity corrected for the attraction of the earth; $\bar{a}_1$, $\bar{\delta}_1$ — the apparent distance from the apex, and the declination of the centre of radiation of the $P$-sector, corrected for zenithal attraction (the uncorrected coordinates of the centre are given at the foot of Table XV); $W$, the mean heliocentric velocity corresponding to the centre of radiation; $a$, $\delta$ — the true, or heliocentric distance from the apex, and the declination of the centre of radiation; $b_z'$, $b_z''$ — the correction factors of stream intensity depending upon the gravitational action of the earth; $\frac{Wb_a}{V}$, the correction factor of stream intensity depending upon the orbital velocity of the earth. The exact meaning of the three last mentioned quantities is defined in the next section following.

The factor $\frac{Wb_\alpha}{V}$ refers to the mean conditions for the whole year, when $\bar{\delta}_1 = \bar{\beta}_1$ and $\bar{\delta} = \bar{\beta}$ is the mean latitude (ecliptical); the computations remain valid for individual cases, when $\beta_1$ instead of $\bar{\delta}_1$ is taken (or $\beta$ instead of $\bar{\delta}$).

When dealing with the direct results of observation, we were contented by a relative scale of velocities based upon the assumption of parabolic velocities of the shower meteors; there is no doubt that the solar shower meteors actually move in elliptical orbits; hence our values of $T$ (also $T_0$) are slightly overestimated. At present the correction cannot be exactly determined; therefore it is better to leave the observed values uncorrected as they stand. However, for the subsequent discussion, in the first place for Table XVII, a tentative correction factor, 0.986, is used; the factor is the mean for the four showers of our list (Table I) for which some more or less vague information is available:

| Shower | Lyrids | May Aquarids | Perseids | Leonids |
|---|---|---|---|---|
| Ratio of velocity to parabolic velocity | 0.995: | 0.986 | 0.989 | 0.974 |

There seem to be fair reasons for believing that *persistent* showers are likely to move along strongly elongated ellipses, whereas elliptical showers of a short period of revolution are short-lived and are soon dissolved into a "sporadic" aggregate; therefore we hardly expect our scale of velocities to be much influenced by short period streams in Table I, and may consider the factor 0.986 as close to the true value.

7. *Transformation of Stream Intensities.* We define the stream intensity from a given direction in the sky as the number per unit of solid angle of meteor radiants *) observable at normal incidence per unit of time and area, under prescribed conditions of selection. True group radiants, which give infinite

*) Individual radiants.

# Table XVII.

## Distribution of Space Velocities and Reduction Factors for Heliocentric Intensity of Radiation.

| $U$ km/sec | $V$ km/sec | $n$ | $\bar{\alpha}_1$ | $\bar{\delta}_1$ $(\bar{\beta}_1)$ | $W$ km/sec | $\bar{\alpha}$ | $\bar{\delta}$ $(\beta)$ | $b'_z$ | $b''_z$ | $\frac{Wb_\alpha}{V}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $P = 15^0$ | | | | | | |
| 281 | 281 | 0 | $130^0$ + | $48^0$ | 301 | $134^0$ + | $44^0$ | 1.00 | 1.00 | 1.23 |
| 235 | 235 | 0 | | „ | 255 | 135 | 43 | „ | „ | 1.28 |
| 198 | 198 | 0 | | „ | 218 | 136 | 42 | „ | „ | 1.34 |
| 166 | 166 | 0 | | „ | 187 | 137 | 41 | „ | „ | 1.41 |
| 141 | 141 | 0 | | „ | 162 | 138 | 40 | „ | „ | 1.50 |
| 117 | 117 | 0 | | „ | 138 | 140 | 39 | „ | „ | 1.50 |
| 99 | 98.5 | 1.5 | | „ | 120 | 141 | 38 | „ | 1.00 | 1.62 |
| 83.5 | 82.7 | 3.0 | | „ | 104 | 143 | 36 | „ | 0.99 | 1.78 |
| 70.5 | 69.6 | 2.4 | | „ | 91.7 | 144 | 34 | 1.00 | 0.99 | 1.97 |
| 59.0 | 58.0 | 1.5 | 130 + | 48 | 80.5 | 146 | 32 | 0.99 | 0.99 | 2.21 |
| 49.6 | 48.3 | 1.8 | 130 | 47 | 71.3 | 149 | 30 | 0.98 | 0.98 | 2.56 |
| 41.7 | 40.2 | 2.4 | 131 | 47 | 63.9 | 152 | 27 | 0.97 | 0.97 | 3.05 |
| 35.2 | 33.4 | 3.7 | 131 | 47 | 57.5 | 154 | 25 | 0.96 | 0.96 | 3.75 |
| 29.5 | 27.3 | 5.4 | 132 | 46 | 52.2 | 157 | 22 | 0.95 | 0.95 | 4.70 |
| 24.7 | 22.0 | 3.3 | 132 | 45 | 47.4 | 160 | 19 | 0.93 | 0.92 | 6.32 |
| 20.8 | 17.5 | 2.6 | 134 | 43 | 43.8 | 163 | 16 | 0.90 | 0.89 | 8.84 |
| 17.5 | 13.5 | 1.9 | 135 | 41 | 40.5 | 164 | 13 | 0.86 | 0.83 | 13.6 |
| 14.7 | 9.5 | 0.5 | 138 | 37 | 37.4 | 170 | 9 | 0.80 | 0.74 | 22.4 |
| 12.3 | 5.1 | 0.0 | $144^0$ + | $28^0$ | 34.3 | $175^0$ + | $4^0$ | 0.70 | 0.59 | 50.8 |
| | | | | | | | | 0.61 | 0.29 | 270 |
| Sum . . . . | | 30 | | | | | | | | |
| | | | | $P = 45^0$ | | | | | | |
| 281 | | 0 | $111^0$ + | $60^0$ | 293 | $116^0$ + | $56^0$ | 1.00 | 1.00 | 1.13 |
| 235 | | 0 | | „ | 247 | 117 | 55 | „ | „ | 1.15 |
| 198 | | 0 | | „ | 210 | 119 | 54 | „ | „ | 1.19 |
| 166 | | 0 | | „ | 179 | 120 | 54 | „ | „ | 1.23 |
| 141 | | 0 | | „ | 154 | 121 | 53 | „ | „ | 1.28 |
| 117 | | 0 | | „ | 131 | 123 | 51 | „ | 1.00 | 1.36 |
| 99 | | 0 | | „ | 113 | 125 | 49 | „ | 0.99 | 1.45 |
| 83.5 | *cf.* $P = 15^0$ | 3.0 | 111 + | 60 | 97.3 | 128 | 47 | 1.00 | 0.99 | 1.57 |
| 70.5 | | 16.0 | 111 | 59 | 84.9 | 130 | 45 | 0.99 | 0.99 | 1.71 |
| 59.0 | | 0 | 111 | 59 | 74.0 | 133 | 42 | 0.98 | 0.98 | 1.92 |
| 49.6 | | 0 | 111 | 59 | 65.1 | 136 | 39 | 0.97 | 0.97 | 2.20 |
| 41.7 | | 0 | 112 | 58 | 58.4 | 140 | 36 | 0.96 | 0.96 | 2.69 |
| 35.2 | | 0 | 112 | 58 | 52.4 | 144 | 33 | 0.95 | 0.95 | 3.27 |
| 29.5 | | 0 | 112 | 57 | 47.4 | 148 | 29 | 0.93 | 0.92 | 4.25 |
| 24.7 | | 0 | 113 | 56 | 43.4 | 152 | 25 | 0.90 | 0.89 | 5.24 |
| 20.8 | | 0 | 114 | 54 | 40.2 | 157 | 20 | 0.86 | 0.83 | 9.00 |
| 17.5 | | 0 | 116 | 51 | 37.7 | 161 | 16 | 0.80 | 0.74 | 15.4 |
| 14.7 | | 0 | 119 | 44 | 35.4 | 167 | 11 | 0.71 | 0.58 | 34.5 |
| 12.3 | | 0 | $125^0$ + | $33^0$ | 33.1 | $173^0$ + | $5^0$ | 0.64 | 0.27 | 192 |
| Sum . . . . | | 19 | | | | | | | | |

Table XVII. Continued.

(For $V$, *cf.* $P = 15^0$).

| $U$ km/sec | $n$ | $\alpha_1$ | $\bar{\delta}_1$ | $W$ km/sec | $\bar{\alpha}$ | $\bar{\delta}$ | $b'_z$ | $b''_z$ | $\frac{Wb_\alpha}{V}$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $P = 75^0$ | | | | | |
| 281 | 0 | $95^0+$ | $69^0$ | 285 | $101^0+$ | $67^0$ | 1.00 | 1.00 | 1.04 |
| 235 | 0 | „ | | 240 | 102 | 66 | „ | „ | 1.05 |
| 198 | 0 | „ | | 203 | 103 | 66 | „ | „ | 1.06 |
| 166 | 0 | „ | | 171 | 105 | 65 | „ | „ | 1.08 |
| 141 | 0 | „ | | 147 | 107 | 64 | „ | „ | 1.11 |
| 117 | 0 | „ | | 123 | 109 | 62 | „ | 1.00 | 1.14 |
| 99 | 0 | „ | | 106 | 111 | 60 | „ | 0.99 | 1.17 |
| 83.5 | 0 | 95 + | 69 | 90.6 | 114 | 58 | 1.00 | 0.99 | 1.24 |
| 70.5 | 15.0 | 95 | 68 | 78.3 | 117 | 55 | 0.99 | 0.99 | 1.31 |
| 59.0 | 0 | 95 | 68 | 67.7 | 121 | 53 | 0.98 | 0.98 | 1.41 |
| 49.6 | 0 | 95 | 68 | 59.1 | 125 | 49 | 0.97 | 0.97 | 1.58 |
| 41.7 | 0 | 95 | 67 | 52.3 | 130 | 45 | 0.96 | 0.96 | 1.79 |
| 35.2 | 0 | 95 | 67 | 46.8 | 134 | 41 | 0.95 | 0.95 | 2.11 |
| 29.5 | 0 | 96 | 66 | 42.5 | 140 | 36 | 0.93 | 0.92 | 2.70 |
| 24.7 | 0 | 96 | 64 | 38.8 | 146 | 31 | 0.90 | 0.89 | 3.57 |
| 20.8 | 0 | 96 | 62 | 36.1 | 151 | 26 | 0.86 | 0.83 | 5.00 |
| 17.5 | 0 | 97 | 58 | 34.2 | 157 | 20 | 0.80 | 0.74 | 8.11 |
| 14.7 | 0 | 99 | 50 | 32.6 | 165 | 13 | 0.71 | 0.58 | 16.6 |
| 12.3 | 0 | $102^0+$ | $38^0$ | 31.3 | $171^0+$ | $6^0$ | 0.64 | 0.28 | 87.1 |
| Sum | 15 | | | | | | | | |
| | | | | $P = 105^0$ | | | | | |
| 281 | 0 | $84^0+$ | $68^0$ | 280 | $90^0+$ | $68^0$ | 1.00 | 1.00 | 0.981 |
| 235 | 0 | „ | | 234 | 91 | 68 | „ | „ | 0.978 |
| 198 | 0 | „ | | 197 | 93 | 68 | „ | „ | 0.980 |
| 166 | 0 | „ | | 166 | 94 | 68 | „ | „ | 0.976 |
| 141 | 0 | „ | | 141 | 96 | 68 | „ | „ | 0.981 |
| 117 | 0 | „ | | 118 | 99 | 67 | „ | 1.00 | 1.00 |
| 99 | 0 | „ | | 100 | 101 | 65 | „ | 0.99 | 1.00 |
| 83.5 | 12.3 | 84 + | 68 | 85.2 | 104 | 64 | 1.00 | 0.99 | 1.02 |
| 70.5 | 2.4 | 84 | 67 | 73.0 | 108 | 61 | 0.99 | 0.99 | 1.05 |
| 59.0 | 7.2 | 84 | 67 | 62.6 | 112 | 59 | 0.98 | 0.98 | 1.10 |
| 49.6 | 0.0 | 84 | 67 | 54.2 | 117 | 55 | 0.97 | 0.97 | 1.17 |
| 41.7 | 0.0 | 84 | 66 | 47.7 | 122 | 51 | 0.96 | 0.96 | 1.29 |
| 35.2 | 8.1 | 84 | 66 | 42.6 | 128 | 45 | 0.95 | 0.95 | 1.47 |
| 29.5 | 0 | 83 | 65 | 37.9 | 134 | 41 | 0.93 | 0.92 | 1.69 |
| 24.7 | 0 | 83 | 63 | 34.8 | 142 | 34 | 0.90 | 0.89 | 2.14 |
| 20.8 | 0 | 83 | 61 | 32.7 | 148 | 28 | 0.86 | 0.83 | 2.78 |
| 17.5 | 0 | 82 | 57 | 31.0 | 154 | 22 | 0.80 | 0.74 | 3.54 |
| 14.7 | 0 | 80 | 49 | 29.7 | 162 | 14 | 0.71 | 0.58 | 4.78 |
| 12.3 | 0 | $78^0+$ | $37^0$ | 29.1 | $170^0+$ | $6^0$ | 0.64 | 0.27 | 11.2 |
| Sum | 30 | | | | | | | | |

## Table XVII. Continued.

(For $V$, *cf.* $P = 15^0$).

| $U$ km/sec | $n$ | $\bar{\alpha}_1$ $\bar{\delta}_1$ | $W$ km/sec | $\bar{\alpha}$ $\bar{\delta}$ | $b'_z$ | $b''_z$ | $\frac{Wb_\alpha}{V}$ |
|---|---|---|---|---|---|---|---|
| | | | $P = 135^0$ | | | | |
| 281 | 0 | $65^0 + 53^0$ | 270 | $71^0 + 56^0$ | 1.00 | 1.00 | 0.880 |
| 235 | 0 | „ | 224 | 72 57 | „ | „ | 0.860 |
| 198 | 0 | „ | 187 | 73 58 | „ | „ | 0.840 |
| 166 | 0 | „ | 156 | 75 59 | „ | „ | 0.814 |
| 141 | 0 | „ | 131 | 77 60 | „ | „ | 0.786 |
| 117 | 11.4 | „ | 108 | 80 61 | „ | 1.00 | 0.760 |
| 99 | 7.5 | „ | 89.9 | 82 62 | „ | 0.99 | 0.722 |
| 83.5 | 9.6 | 65 + 53 | 75.0 | 86 62 | 1.00 | 0.99 | 0.698 |
| 70.5 | 7.6 | 65 52 | 63.0 | 90 61 | 0.99 | 0.99 | 0.670 |
| 59.0 | 9.2 | 65 52 | 52.7 | 96 60 | 0.98 | 0.98 | 0.649 |
| 49.6 | 13.0 | 65 52 | 44.7 | 102 58 | 0.97 | 0.97 | 0.630 |
| 41.7 | 9.8 | 64 51 | 38.1 | 108 55 | 0.96 | 0.96 | 0.600 |
| 35.2 | 5.9 | 64 50 | 33.7 | 117 49 | 0.95 | 0.94 | 0.622 |
| 29.5 | 0 | 64 50 | 30.3 | 126 44 | 0.93 | 0.92 | 0.644 |
| 24.7 | 0 | 63 48 | 27.9 | 135 36 | 0.90 | 0.89 | 0.604 |
| 20.8 | 0 | 62 46 | 26.5 | 144 28 | 0.86 | 0.82 | 0.416 |
| 17.5 | 0 | 61 43 | 26.1 | 153 21 | 0.81 | 0.73 | 0.594 |
| 14.7 | 0 | 57 35 | 25.9 | 161 12 | 0.74 | 0.56 | 5.56 |
| 12.3 | 0 | $52^0 + 22^0$ | 26.8 | $172^0 + 4^0$ | 0.69 | 0.25 | 76.0 |
| Sum | 74 | | | | | | |
| | | | $P = 165^0$ | | | | |
| 281 | 0.6 | $37^0 + 34^0$ | 258 | $41^0 + 38^0$ | 1.00 | 1.00 | 0.770 |
| 235 | 0.0 | „ | 212 | 42 38 | „ | „ | 0.730 |
| 198 | 0.0 | „ | 175 | 43 39 | „ | „ | 0.691 |
| 166 | 0.0 | „ | 144 | 44 40 | „ | „ | 0.640 |
| 141 | 0.0 | „ | 119 | 46 42 | „ | „ | 0.590 |
| 117 | 6.8 | „ | 94.9 | 48 44 | „ | 1.00 | 0.526 |
| 99 | 17.5 | „ | 76.7 | 51 46 | „ | 0.99 | 0.460 |
| 83.5 | 23.8 | 37 + 34 | 61.6 | 54 49 | 1.00 | 0.99 | 0.397 |
| 70.5 | 29.2 | 37 33 | 49.1 | 59 51 | 0.99 | 0.99 | 0.331 |
| 59.0 | 38.4 | 37 33 | 38.6 | 65 55 | 0.98 | 0.98 | 0.261 |
| 49.6 | 33.0 | 37 33 | 30.3 | 73 60 | 0.97 | 0.97 | 0.200 |
| 41.7 | 13.7 | 36 32 | 23.8 | 83 63 | 0.96 | 0.96 | 0.139 |
| 35.2 | 0 | 36 32 | 20.0 | 97 62 | 0.95 | 0.94 | 0.0971 |
| 29.5 | 0 | 35 31 | 17.4 | 116 54 | 0.93 | 0.92 | 0.0452 |
| 24.7 | 0 | 34 30 | 16.9 | 134 41 | 0.90 | 0.89 | 0.0660 |
| 20.8 | 0 | 34 28 | 18.1 | 147 27 | 0.86 | 0.83 | 0.457 |
| 17.5 | 0 | 32 25 | 19.7 | 159 17 | 0.81 | 0.74 | 1.88 |
| 14.7 | 0 | 29 20 | 22.0 | 168 8 | 0.73 | 0.57 | 9.40 |
| 12.3 | 0 | $23^0 + 10^0$ | 24.9 | $176^0 + 2^0$ | 0.67 | 0.26 | 112 |
| Sum | 163 | | | | | | |

Table XVII. Continued.

(For $V$, *cf.* $P = 15^0$).

| $U$ km/sec | $n$ | $\bar{\alpha}_1$ | $\bar{\delta}_1$ | $W$ km/sec | $\bar{\alpha}$ | $\bar{\delta}$ | $b'_z$ | $b''_z$ | $\frac{Wb_\alpha}{V}$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $P = 195^0$ | | | | | |
| 281 | 0.0 | $28^0+$ | $24^0$ | 255 | $31^0+$ | $27^0$ | 1.00 | 1.00 | 0.749 |
| 235 | 8.1 | „ | | 209 | 32 | 27 | „ | „ | 0.704 |
| 198 | 7.7 | „ | | 172 | 33 | 28 | „ | „ | 0.658 |
| 166 | 4.4 | „ | | 141 | 34 | 29 | „ | „ | 0.603 |
| 141 | 0.0 | „ | | 116 | 35 | 30 | „ | „ | 0.546 |
| 117 | 3.3 | „ | | 91.9 | 37 | 31 | „ | 1.00 | 0.478 |
| 99 | 13.2 | „ | | 73.6 | 39 | 33 | „ | 0.99 | 0.407 |
| 83.5 | 51.6 | „ | | 58.2 | 42 | 35 | 1.00 | 0.99 | 0.341 |
| 70.5 | 57.9 | 28 + | 24 | 45.6 | 46 | 38 | 0.99 | 0.99 | 0.267 |
| 59.0 | 35.4 | 27 | 23 | 34.2 | 50 | 42 | 0.98 | 0.98 | 0.187 |
| 49.6 | 9.0 | 27 | 23 | 25.6 | 59 | 48 | 0.97 | 0.97 | 0.126 |
| 41.7 | 5.2 | 27 | 23 | 19.1 | 72 | 56 | 0.96 | 0.96 | 0.0770 |
| 35.2 | 9.0 | 26 | 23 | 14.9 | 87 | 61 | 0.95 | 0.95 | 0.0362 |
| 29.5 | 6.7 | 25 | 22 | 12.5 | 113 | 55 | 0.93 | 0.92 | 0.0112 |
| 24.7 | 4.2 | 24 | 21 | 13.2 | 137 | 37 | 0.90 | 0.89 | 0.0862 |
| 20.8 | 3.3 | 23 | 20 | 15.2 | 153 | 23 | 0.86 | 0.83 | 0.442 |
| 17.5 | 0.0 | 20 | 18 | 17.7 | 165 | 13 | 0.80 | 0.75 | 1.88 |
| 14.7 | 0.0 | 15 | 15 | 20.8 | 173 | 7 | 0.70 | 0.60 | 9.70 |
| 12.3 | 0.0 | $8^0+$ | $8^0$ | 24.4 | $178^0+$ | $2^0$ | 0.60 | 0.30 | 121 |
| Sum | 219 | | | | | | | | |
| | | | | $P = 225^0$ | | | | | |
| 281 | 0.8 | $48^0+$ | $20^0$ | 262 | $53^0+$ | $22^0$ | 1.00 | 1.00 | 0.809 |
| 235 | 1.5 | „ | | 216 | 54 | 22 | „ | „ | 0.776 |
| 198 | 0.0 | „ | | 180 | 55 | 22 | „ | „ | 0.741 |
| 166 | 0.0 | „ | | 148 | 57 | 23 | „ | „ | 0.699 |
| 141 | 0.0 | „ | | 123 | 58 | 23 | „ | „ | 0.656 |
| 117 | 0.0 | „ | | 99.8 | 61 | 24 | „ | 1.00 | 0.604 |
| 99 | 8.4 | „ | | 81.9 | 64 | 24 | 1.00 | 0.99 | 0.550 |
| 83.5 | 37.8 | „ | | 66.9 | 67 | 25 | 0.99 | 0.99 | 0.495 |
| 70.5 | 47.8 | „ | | 54.6 | 72 | 26 | 0.98 | 0.99 | 0.438 |
| 59.0 | 35.3 | 48 + | 20 | 44.2 | 78 | 27 | 0.98 | 0.98 | 0.380 |
| 49.6 | 25.1 | 47 | 19 | 35.5 | 85 | 26 | 0.97 | 0.97 | 0.313 |
| 41.7 | 20.1 | 47 | 19 | 29.5 | 95 | 26 | 0.96 | 0.96 | 0.268 |
| 35.2 | 16.7 | 47 | 19 | 25.5 | 106 | 25 | 0.95 | 0.95 | 0.230 |
| 29.5 | 10.5 | 46 | 19 | 22.5 | 119 | 23 | 0.93 | 0.92 | 0.166 |
| 24.7 | 5.9 | 46 | 18 | 21.5 | 132 | 18 | 0.89 | 0.89 | 0.0592 |
| 20.8 | 1.1 | 45 | 17 | 21.3 | 145 | 14 | 0.86 | 0.84 | 0.315 |
| 17.5 | 0.0 | 43 | 15 | 22.0 | 156 | 9 | 0.79 | 0.75 | 1.57 |
| 14.7 | 0.0 | 39 | 12 | 23.2 | 165 | 5 | 0.69 | 0.61 | 8.50 |
| 12.3 | 0.0 | $33^0+$ | $6^0$ | 25.4 | $174^0+$ | $1^0$ | 0.57 | 0.32 | 105 |
| Sum | 211 | | | | | | | | |

## Table XVII. Continued.

(For $V$, *cf.* $P = 15^0$).

| $U$ km/sec | $n$ | $\bar{\alpha}_1$ | $\bar{\delta}_1$ | $W$ km/sec | $\alpha$ | $\bar{\delta}$ | $b'_z$ | $b''_z$ | $\frac{Wb_\alpha}{V}$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $P = 255^0$ | | | | | |
| 281 | 0 | $76^0 +$ | $19^0$ | 275 | $82^0 +$ | $19^0$ | 1.00 | 1.00 | 0.935 |
| 235 | 0 | „ | | 230 | 83 | 19 | „ | „ | 0.926 |
| 198 | 0 | „ | | 193 | 85 | 20 | „ | „ | 0.918 |
| 166 | 0 | „ | | 162 | 86 | 20 | „ | „ | 0.908 |
| 141 | 0 | „ | | 137 | 88 | 20 | „ | „ | 0.896 |
| 117 | 0 | „ | | 113 | 91 | 20 | „ | 1.00 | 0.887 |
| 99 | 1.4 | „ | | 95.7 | 94 | 20 | 1.00 | 0.99 | 0.875 |
| 83.5 | 17.5 | „ | | 80.8 | 97 | 19 | 0.99 | 0.99 | 0.874 |
| 70.5 | 21.6 | „ | | 68.7 | 101 | 19 | 0.98 | 0.99 | 0.876 |
| 59.0 | 25.1 | „ | | 58.3 | 106 | 19 | 0.98 | 0.98 | 0.891 |
| 49.6 | 34.4 | „ | | 50.1 | 111 | 18 | 0.97 | 0.97 | 0.916 |
| 41.7 | 59.1 | „ | | 43.8 | 117 | 17 | 0.96 | 0.96 | 0.970 |
| 35.2 | 68.7 | „ | | 39.0 | 124 | 16 | 0.95 | 0.95 | 1.07 |
| 29.5 | 55.0 | „ | | 35.1 | 131 | 15 | 0.93 | 0.93 | 1.21 |
| 24.7 | 28.9 | 76 + | 19 | 32.4 | 139 | 13 | 0.90 | 0.89 | 1.46 |
| 20.8 | 10.7 | 75 | 18 | 30.4 | 146 | 10 | 0.85 | 0.84 | 1.67 |
| 17.5 | 0.6 | 75 | 18 | 29.4 | 154 | 8 | 0.79 | 0.76 | 2.13 |
| 14.7 | 0.0 | 74 | 17 | 28.7 | 162 | 6 | 0.66 | 0.62 | 1.65 |
| 12.3 | 0.0 | $72^0 +$ | $16^0$ | 28.5 | 170 + | $3^0$ | 0.54 | 0.34 | 30.4 |
| Sum | 323 | | | | | | | | |

| $U$ km/sec | $n$ | $\bar{\alpha}_1$ | $\bar{\delta}_1$ | $W$ km/sec | $\alpha$ | $\bar{\delta}$ | $b'_z$ | $b''_z$ | $\frac{Wb_\alpha}{V}$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $P = 285^0$ | | | | | |
| 281 | 0 | $104^0 +$ | $21^0$ | 290 | $110^0 +$ | $20^0$ | 1.00 | 1.00 | 1.09 |
| 235 | 0 | „ | | 244 | 111 | 20 | „ | „ | 1.11 |
| 198 | 0 | „ | | 207 | 112 | 20 | „ | „ | 1.14 |
| 166 | 0 | „ | | 176 | 113 | 20 | „ | „ | 1.17 |
| 141 | 0 | „ | | 151 | 115 | 20 | „ | „ | 1.21 |
| 117 | 0 | „ | | 128 | 117 | 19 | „ | 1.00 | 1.26 |
| 99 | 4.5 | „ | | 110 | 119 | 19 | 1.00 | 0.99 | 1.32 |
| 83.5 | 19.8 | „ | | 94.4 | 122 | 18 | 0.99 | 0.99 | 1.42 |
| 70.5 | 23.7 | „ | | 82.0 | 125 | 18 | 0.98 | 0.99 | 1.54 |
| 59.0 | 22.6 | „ | | 71.3 | 128 | 17 | 0.98 | 0.98 | 1.70 |
| 49.6 | 24.6 | „ | | 62.6 | 132 | 16 | 0.97 | 0.97 | 1.94 |
| 41.7 | 30.6 | $104^0 +$ | 21 | 55.5 | 135 | 15 | 0.96 | 0.96 | 2.23 |
| 35.2 | 34.4 | 104 | 20 | 49.8 | 140 | 14 | 0.95 | 0.95 | 2.73 |
| 29.5 | 35.1 | 104 | 20 | 45.0 | 144 | 12 | 0.93 | 0.93 | 3.43 |
| 24.7 | 26.9 | 104 | 20 | 41.1 | 149 | 11 | 0.90 | 0.89 | 4.65 |
| 20.8 | 14.1 | 105 | 19 | 38.3 | 154 | 9 | 0.85 | 0.84 | 6.91 |
| 17.5 | 3.3 | 105 | 18 | 35.8 | 158 | 7 | 0.79 | 0.76 | 10.8 |
| 14.7 | 0.4 | 106 | 17 | 33.7 | 164 | 5 | 0.66 | 0.62 | 23.2 |
| 12.3 | 0.0 | $108^0 +$ | $14^0$ | 31.8 | $171^0 +$ | $2^0$ | 0.54 | 0.34 | 111 |
| Sum | 240 | | | | | | | | |

## Table XVII. Continued.

(For $V$, *cf.* $P = 15^0$).

| $U$ km/sec | $n$ | $\bar{\alpha}_1$ | $\bar{\delta}_1$ | $W$ km/sec | $\bar{\alpha}$ | $\bar{\delta}$ | $b'_z$ | $b''_z$ | $\frac{Wb_\alpha}{V}$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $P = 315^0$ | | | | | |
| 281 | 0 | $129^0$ | $+27^0$ | 301 | $133^0$ | $+25^0$ | 1.00 | 1.00 | 1.22 |
| 235 | 0 | „ | | 255 | 134 | 25 | „ | „ | 1.26 |
| 198 | 0 | „ | | 218 | 135 | 24 | „ | „ | 1.33 |
| 166 | 0 | „ | | 186 | 136 | 24 | „ | „ | 1.40 |
| 141 | 0.1 | „ | | 161 | 137 | 23 | „ | „ | 1.49 |
| 117 | 0.0 | „ | | 138 | 139 | 23 | „ | 1.00 | 1.61 |
| 99 | 0.0 | „ | | 120 | 140 | 22 | 1.00 | 0.99 | 1.75 |
| 83.5 | 3.0 | „ | | 104 | 142 | 21 | 0.99 | 0.99 | 1.94 |
| 70.5 | 5.0 | „ | | 91.3 | 144 | 20 | 0.98 | 0.99 | 2.19 |
| 59.0 | 5.6 | 129 | + 27 | 80.2 | 146 | 19 | 0.98 | 0.98 | 2.52 |
| 49.6 | 6.8 | 130 | 26 | 71.2 | 149 | 17 | 0.97 | 0.97 | 3.04 |
| 41.7 | 6.9 | 130 | 26 | 63.6 | 151 | 16 | 0.96 | 0.96 | 3.69 |
| 35.2 | 9.7 | 130 | 26 | 57.3 | 154 | 15 | 0.95 | 0.95 | 4.48 |
| 29.5 | 11.2 | 131 | 26 | 52.0 | 157 | 13 | 0.93 | 0.93 | 6.24 |
| 24.7 | 13.0 | 131 | 26 | 47.2 | 160 | 12 | 0.89 | 0.89 | 8.73 |
| 20.8 | 7.7 | 131 | 25 | 43.3 | 162 | 10 | 0.86 | 0.84 | 12.9 |
| 17.5 | 2.7 | 133 | 24 | 40.2 | 166 | 8 | 0.79 | 0.75 | 22.3 |
| 14.7 | 0.3 | 136 | 22 | 37.2 | 170 | 6 | 0.68 | 0.61 | 49.9 |
| 12.3 | 0.0 | $143^0$ | $+18^0$ | 34.2 | $175^0$ | $+3^0$ | 0.56 | 0.33 | 266 |
| Sum | 72 | | | | | | | | |
| | | | | $P = 345^0$ | | | | | |
| 281 | 0 | $141^0$ | $+36^0$ | 305 | $144^0$ | $+33^0$ | 1.00 | 1.00 | 1.28 |
| 235 | 0 | „ | | 259 | 145 | 32 | „ | „ | 1.33 |
| 198 | 0 | „ | | 222 | 146 | 32 | „ | „ | 1.40 |
| 166 | 0 | „ | | 190 | 147 | 31 | „ | „ | 1.49 |
| 141 | 0 | „ | | 165 | 147 | 30 | „ | „ | 1.60 |
| 117 | 0.9 | „ | | 141 | 148 | 29 | „ | 1.00 | 1.75 |
| 99 | 1.6 | „ | | 123 | 150 | 28 | 1.00 | 0.99 | 1.93 |
| 83.5 | 1.6 | „ | | 107 | 151 | 27 | 0.99 | 0.99 | 2.15 |
| 70.5 | 2.6 | 141 | + 36 | 94.7 | 152 | 26 | 0.98 | 0.99 | 2.46 |
| 59.0 | 3.0 | 142 | 35 | 83.6 | 155 | 23 | 0.98 | 0.98 | 2.92 |
| 49.6 | 2.9 | 142 | 35 | 74.1 | 156 | 22 | 0.97 | 0.97 | 3.48 |
| 41.7 | 2.3 | 142 | 35 | 66.3 | 158 | 20 | 0.96 | 0.96 | 4.29 |
| 35.2 | 2.3 | 142 | 35 | 59.7 | 160 | 19 | 0.95 | 0.95 | 5.44 |
| 29.5 | 3.5 | 142 | 35 | 54.0 | 162 | 17 | 0.93 | 0.92 | 7.30 |
| 24.7 | 6.3 | 143 | 34 | 49.2 | 164 | 15 | 0.89 | 0.89 | 10.4 |
| 20.8 | 8.3 | 145 | 33 | 45.3 | 167 | 12 | 0.86 | 0.84 | 16.0 |
| 17.5 | 4.7 | 147 | 32 | 41.8 | 170 | 10 | 0.79 | 0.75 | 27.5 |
| 14.7 | 0.0 | 150 | 29 | 38.3 | 173 | 7 | 0.69 | 0.60 | 60.8 |
| 12.3 | 0.0 | $157^0$ | $+23^0$ | 34.8 | $177^0$ | $+3^0$ | 0.58 | 0.31 | 312 |
| Sum | 40 | | | | | | | | |

stream intensity at the radiant and zero intensity outside, we assume to be more or less uniformly distributed over the sector. For relative data unit of time and area are unimportant; for the present material the horizontal area at the average height $H_c$ of the meteors covered by the velocity observer, and the total time covered by the observations are taken as units. Also, no correction for general or special selection is introduced, nor is the influence of velocity upon the apparent luminosity of the meteor (which in its turn influences the observable number) taken into account; the conditions of selection thus are: the specific selection of the velocity observers and the effective minimum mass visible corresponding to $U$, the actual velocity at the entrance into the terrestrial atmosphere.

The changes in the spherical coordinates of the radiant, due to relative motion (aberration) or gravitational action we are concerned with, are *polar:* the radiants, under the action of a given cause, are all displaced along great circles converging in a certain pole, and the amount of displacement depends solely upon the distance $\Theta$ of the radiant from the pole. In such a case the ratio of the original to the changed stream intensities is evidently

$$\frac{B_1}{B_2} = \frac{\sin\Theta_2}{\sin\Theta_1}\left|\frac{d\Theta_2}{d\Theta_1}\right| \cdot \frac{\varrho_1 V_1}{\varrho_2 V_2} = b_\Theta \frac{\varrho_1 V_1}{\varrho_2 V_2} \quad . \quad . \quad . \quad . \quad (19);$$

$$\text{for gravitational action } \frac{\varrho_1 V_1}{\varrho_2 V_2} = \frac{dS_2}{dS_1}.$$

Here $\varrho$ is the space density, $dS$ — the infinitesimal cross-section of a parallel beam, $V$ — the stream velocity; for a single radiant the intensity of streaming depends solely upon $\varrho V$, whereas $b_\Theta$ takes into account the effect of the crowding of radiants toward the pole.

For the terrestrial field of gravitation — the zenithal attraction — we have:

$$\frac{B_e}{B_i} = b_z = \frac{\sin z_i \, dz_i}{\sin z_e \, dz_e} \cdot \frac{dS_i}{dS_e} = b'_z \cdot b''_z \quad . \quad . \quad . \quad . \quad (20).$$

Here the subindex $i$ refers to the perturbed, $e$ — to the unperturbed directions.

The values of $b'_z = \frac{\sin z_i dz_i}{\sin z_e dz_e}$ in Table XVII were computed numerically, with the help of Nielsen's tables[8]. $b''_z$ may be determined in the following way. Let the elements of the hyperbolic orbit be: $e = -\sec\gamma$, where $\gamma$ is the true anomaly of the asymtote of approach, $90^0 < \gamma < 180^0$; $\xi = -p \operatorname{ctg}\gamma$, the distance of the asymtote from the focus (minor axis of hyperbola). Let $U$ be the orbital velocity, $V$ — the velocity at $r = \infty$, $c$ — the circular velocity at $r$; the polar equation of the hyperbola is $r = \frac{\xi \sin\gamma}{\cos\varphi - \cos\gamma}$, where $\varphi$ is the true anomaly counted in the direction opposite to the motion. $z_i$ is the angle between the radius vector and the tangent direction toward the radiant; $z_e = \gamma - \varphi$. Take an element of the cross-section of the beam (in the normal plane) $dS = dx\, dy$, $dx$ in the orbital plane, $dy$ at right angles to it; we have $\frac{dS_i}{dS_e} = \frac{dx_i}{dx_e} \cdot \frac{dy_i}{dy_e}$. Now, $\frac{dx_i}{dx_e} = \left(\frac{\partial r}{\partial \xi} + \frac{\partial r}{\partial \gamma}\frac{\partial \gamma}{\partial \xi} + \frac{\partial r}{\partial \varphi}\frac{\partial \varphi}{\partial \gamma} \cdot \frac{\partial \gamma}{\partial \xi}\right) \sin z_i$. $V = \text{const.}$ and $\frac{dy_i}{dy_e} = \frac{\sin z_e \sin\gamma}{\cos\varphi}$; further, $\operatorname{tg}\gamma = -\frac{\xi}{r}\frac{V^2}{c^2}$, whence $\frac{\partial\gamma}{\partial\xi} = \frac{\sin\gamma\cos\gamma}{\xi}$; for the same radius vector $z_e = \text{const.}$, whence $\frac{\partial\varphi}{\partial\gamma} = 1$. After appropriate substitution we obtain:

$$b''_z = \frac{\sin^2\gamma \sin z_e \sin z_i \left[2(\cos\varphi - \cos\gamma) - \sin\gamma \sin z_e\right]}{(\cos\varphi - \cos\gamma)^3}.$$

At $z_e = 180^0$ (not at $z_e = 0^0$), $\varphi = \gamma - 180^0$, $b''_z = 0$: the stream intensity is infinite when the radius vector is directed toward the anti-radiant; such a case of the influence of zenithal attraction cannot be observed because the earth intercepts the meteors ($z_i > 90^0$); in the case of the action of the solar gravitational field upon a hyperbolic meteor stream, the effect comes fully into play: when the apparent position of the sun happens to be near the cosmic (true) radiant, the stream intensity may become very great. In the limiting case of coincidence, a shower converging from all directions (all inclinations) toward the radius vector of the anti-radiant is formed; the probability of the coincidence is, however, very small (the radiant should coincide with the position of the sun's centre within 0.″1).

The above formula is conveniently transformed for computation:

$$\left.\begin{aligned} b_z'' &= \frac{V}{U}\,(1 - N^2 \cos^4 \gamma) \quad . \quad . \quad . \quad . \quad . \\ \operatorname{tg} \gamma &= -\frac{UV}{c^2} \sin z_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ N &= \frac{V^2 + c^2 - UV \cos z_i}{c^2} \quad . \quad . \quad . \quad . \quad . \\ U^2 &= V^2 + 2c^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \end{aligned}\right\} \quad (20\,\mathrm{a}).$$

For the gravitational field of the earth, $c^2 = 62.6$ (km$^2$/sec$^2$). It may be added that the same formulae and Nielsen's tables may be used for the transition from heliocentric to extra-solar stream intensities, if for $z$ the angular distance $\zeta$ from the anti-solar point is taken, and all velocities changed in the ratio 3.769 : 1.

For the transition from unperturbed geocentric to heliocentric stream intensities we have ($\varrho_1 = \varrho_2$, $V_1 = W$, $V_2 = V$):

$$\frac{\sin \alpha_1}{\sin \alpha} = \frac{W}{V}; \quad \frac{\sin(\alpha - \alpha_1)}{\sin \alpha} = \frac{c}{V}; \quad \frac{\sin(\alpha - \alpha_1)}{\sin \alpha_1} = \frac{c}{W} \quad . \quad . \quad . \quad (21);$$

here $c$ is the mean orbital velocity of the earth.

$$W^2 = V^2 + c^2 - 2Vc \cos \alpha_1; \quad V^2 = W^2 + c^2 + 2Wc \cos \alpha \quad . \; . \; . \quad (22);$$

with the aid of these formulae we find:

$$b_\alpha = \left| \frac{d(\cos \alpha_1)}{d(\cos \alpha)} \right| = \frac{W}{V} \left| \left(1 - \frac{c^2}{V^2} - \frac{cW \cos \alpha}{V^2}\right) \right| \quad . \quad . \quad . \quad . \quad (23).$$

At $\frac{W}{c} = -\cos \alpha$, $b_\alpha = 0$; when $b_\alpha$ passes over zero, a mean absolute value over the tabular interval must be specially computed; this can happen, evidently, only when the heliocentric velocity is smaller than the orbital velocity of the earth. In most cases, however, it is safe to calculate $b_\alpha$ for the middle of the tabular interval. Formulae (23) and (19) yield

$$\frac{B_1}{B_2} = \frac{W}{V}\, b_\alpha = \left| \left(\frac{W}{V}\right)^2 \left[1 - \frac{1 + \frac{W}{c} \cos \alpha}{\left(\frac{V}{c}\right)^2}\right] \right| \quad . \quad . \quad . \quad . \quad (24).$$

The values in the corresponding column of Table XVII were computed with the aid of this formula.

The transformation of ecliptical latitude is:

$$\sin \beta = \sin \beta_1 \cdot \frac{\sin \alpha}{\sin \alpha_1} \quad . \quad . \quad . \quad . \quad (25);$$

by setting $\bar{\beta} = \bar{\delta}$, and $\bar{\beta}_1 = \bar{\delta}_1$ (mean of the year), the transformation of geocentric coordinates into heliocentric ones in Table XVII is accomplished.

From our observations it is possible to derive only the mean stream intensity per $P$-sector; the probable distribution within the sector was assumed to be defined by $N_\lambda$ of Table XV, but the present material does not provide a means to prove that the distribution is such as adopted. According to formulae (18′), (20) and (24), the heliocentric stream intensity at ($\bar{\alpha}$, $\bar{\delta}$), for given $W$, is:

$$B_0 = \frac{10.8\,n}{\Sigma N_z} \cdot b_z \cdot \frac{W}{V} b_\alpha = E n \quad . \quad . \quad . \quad . \quad (26);$$

here $\Sigma N_z$ is the sum of $N_z$ in Table XV, and $n$ is the frequency of $W$ in Table XVII.

8. *Distribution of Heliocentric Velocities and Heliocentric Stream Intensities.* In Table XVII the heliocentric velocity is spaced by odd intervals unsuited for the comparison of relative frequencies. Therefore the data were redistributed according to uniform logarithmic limits of $W$; the result is given in Table XVIII.

The table is arranged according to $P$ — the observed (apparent) direction of motion; it gives for different limits of the heliocentric velocity, $W$: $\bar{\alpha}$, the distance from the apex, and $\bar{\delta}$, the declination of the true (heliocentric) centre of radiation; $n$, the concluded number of observed meteors; $\sigma$, the effective heliocentric area of radiation (*cf.* below); $B_0 = \frac{n}{\sigma}$, the heliocentric stream intensity per unit of solid angle; $\bar{U}$, the mean "velocity of impact" upon which the luminosity of a given mass must depend.

In Table XVII each discrete value of $W$ was assumed to cover a continuous interval with the limits half-way between the adjacent discrete values; for the given interval $\Delta W$ the heliocentric stream intensity is given by (26); assuming for given narrow limits of $W$ proportionality of $n$, $B_0$, and $\Delta W$,

## Table XVIII.

### Distribution of Heliocentric Velocities, and Heliocentric Stream Intensities.

| $W$ km/sec limits | | $P$ 135° | 165° | 195° | 225° |
|---|---|---|---|---|---|
| | $\bar{\alpha}$ | 163° | | | |
| | $\bar{\delta}$ | + 11° | | | |
| 29.8 ... | $n$ | 0.0 | — | — | — |
| ... 25.1 | $\sigma$ | 0.0041 | | | |
| | $B_0$ | < 120 | | | |
| | $\bar{U}$ | 14 | | | |
| | $\bar{\alpha}$ | | 169° | 174° | 152° |
| | $\bar{\delta}$ | | + 8° | + 6° | + 11° |
| 25.1 ... | $n$ | — | 0.0 | 0.0 | 0.8 |
| ... 21.1 | $\sigma$ | | 0.0231 | 0.029 | 0.248 |
| | $B_0$ | | < 22 | < 17 | 3.2 |
| | $\bar{U}$ | | 14 | 13 | 16 |
| | $\bar{\alpha}$ | | 151° | 167° | |
| | $\bar{\delta}$ | | + 24° | + 12° | |
| 21.1 ... | $n$ | — | 0.0 | 0.0 | — |
| ... 17.7 | $\sigma$ | | 0.226 | 0.128 | |
| | $B_0$ | | < 2.2 | < 4 | |
| | $\bar{U}$ | | 20 | 16 | |
| | $\bar{\alpha}$ | | 135° | 155° | |
| | $\bar{\delta}$ | | + 40° | + 21° | |
| 17.7 ... | $n$ | — | 0.0 | 2.2 | — |
| ... 14.9 | $\sigma$ | | 0.604 | 0.512 | |
| | $B_0$ | | < 0.8 | 4.3 | |
| | $\bar{U}$ | | 24 | 20 | |

| $W$ km/sec limits | | $P$ 135° | 165° | 195° | 225° |
|---|---|---|---|---|---|
| | $\bar{\alpha}$ | | | 123° | |
| | $\bar{\delta}$ | | | + 48° | |
| 14.9 ... | $n$ | — | — | 6.6 | — |
| ... 12.5 | $\sigma$ | | | 5.42 | |
| | $B_0$ | | | 1.22 | |
| | $\bar{U}$ | | | 26 | |
| | $\bar{\alpha}$ | | | 107° | |
| | $\bar{\delta}$ | | | + 56° | |
| 12.5 ... | $n$ | — | — | 8.7 | — |
| ... 14.9 | $\sigma$ | | | 16.3 | |
| | $B_0$ | | | 0.53 | |
| | $\bar{U}$ | | | 32 | |
| | $\bar{\alpha}$ | | 121° | 85° | |
| | $\bar{\delta}$ | | + 50° | + 60° | |
| 14.9 ... | $n$ | — | 0.0 | 6.4 | — |
| ... 17.7 | $\sigma$ | | 0.945 | 6.43 | |
| | $B_0$ | | < 0.5 | 1.00 | |
| | $\bar{U}$ | | 28 | 37 | |
| | $\bar{\alpha}$ | | 106° | 72° | |
| | $\bar{\delta}$ | | + 58° | + 56° | |
| 17.7 ... | $n$ | — | 0.0 | 3.2 | — |
| ... 21.1 | $\sigma$ | | 2.23 | 3.42 | |
| | $B_0$ | | < 0.22 | 0.9 | |
| | $\bar{U}$ | | 34 | 42 | |

Table XVIII. Continued.

| W km/sec limits | | P | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 15° | 45° | 75° | 105° | 135° | 165° | 195° | 225° | 255° | 285° | 315° | 345° |
| | $\bar{\alpha}$ | | | | | | 86° | 65° | 122° | | | | |
| | $\bar{\delta}$ | | | | | | + 63° | + 51° | + 21° | | | | |
| 21.1 ... | $n$ | — | — | — | — | — | 8.6 | 4.6 | 21.9 | — | — | — | — |
| ... 25.1 | $\sigma$ | | | | | | 1.22 | 2.48 | 2.89 | | | | |
| | $B_0$ | | | | | | 7.1 | 1.9 | 7.6 | | | | |
| | $\bar{U}$ | | | | | | 40 | 46 | 27 | | | | |
| | $\bar{\alpha}$ | | | | 166° | 138° | 78° | 59° | 101° | 158° | | | |
| | $\bar{\delta}$ | | | | + 10° | + 33° | + 62° | + 48° | + 25° | + 7° | | | |
| 25.1 ... | $n$ | — | — | — | 0.0 | 0.0 | 17.6 | 5.6 | 20.8 | 0.5 | — | — | — |
| ... 29.8 | $\sigma$ | | | | 0.006 | 0.142 | 0.889 | 2.03 | 1.57 | 0.115 | | | |
| | $B_0$ | | | | < 80 | < 4 | 19.9 | 2.8 | 13.3 | 4.4 | | | |
| | $\bar{U}$ | | | | 13 | 22 | 46 | 52 | 38 | 16 | | | |
| | $\bar{\alpha}$ | 175° | 168° | 159° | 148° | 121° | 72° | 50° | 90° | 139° | 163° | 175° | 177° |
| | $\bar{\delta}$ | + 4° | + 9° | + 18° | + 28° | + 47° | + 59° | + 42° | + 26° | + 13° | + 5° | + 3° | + 3° |
| 29.8 ... | $n$ | 0.0 | 0.0 | 0.0 | 0.0 | 5.1 | 24.6 | 19.6 | 20.8 | 67.2 | 1.3 | 0.0 | 0.0 |
| ... 35.4 | $\sigma$ | 0.0016 | 0.0019 | 0.0052 | 0.0226 | 0.132 | 0.714 | 1.36 | 1.30 | 0.377 | 0.0289 | 0.0030 | 0.0017 |
| | $B_0$ | < 300 | < 260 | < 100 | < 22 | 39 | 34.5 | 14.4 | 16.0 | 179 | 45 | < 170 | < 300 |
| | $\bar{U}$ | 12 | 13 | 16 | 20 | 33 | 52 | 57 | 46 | 24 | 14 | 12 | 12 |
| | $\bar{\alpha}$ | 166° | 163° | 146° | 133° | 108° | 65° | 49° | 83° | 125° | 152° | 167° | 172° |
| | $\bar{\delta}$ | + 11° | + 18° | + 31° | + 42° | + 55° | + 55° | + 41° | + 26° | + 16° | + 10° | + 8° | + 8° |
| 35.4 ... | $n$ | 2.3 | 0.0 | 0.0 | 3.1 | 11.8 | 27.4 | 26.5 | 23.6 | 103.6 | 36.1 | 3.7 | 2.7 |
| ... 42.1 | $\sigma$ | 0.0088 | 0.0100 | 0.0159 | 0.0310 | 0.131 | 0.561 | 1.21 | 1.12 | 0.435 | 0.0915 | 0.0224 | 0.0107 |
| | $B_0$ | 260 | < 50 | < 31 | 100 | 90 | 48.9 | 21.9 | 21.2 | 238 | 396 | 165 | 250 |
| | $\bar{U}$ | 16 | 19 | 24 | 30 | 42 | 59 | 64 | 54 | 34 | 22 | 17 | 16 |

Table XVIII. Continued.

| $W$ km/sec limits | | $P$ 15° | 45° | 75° | 105° | 135° | 165° | 195° | 225° | 255° | 285° | 315° | 345° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\bar{\alpha}$ | 161° | 149° | 136° | 124° | 101° | 61° | 46° | 78° | 115° | 143° | 160° | 166° |
| | $\bar{\delta}$ | +18° | +28° | +40° | +49° | +58° | +52° | +38° | +27° | +17° | +13° | +12° | +13° |
| 42.1 ... | $n$ | 6.3 | 0.0 | 0.0 | 5.0 | 13.2 | 22.8 | 38.6 | 29.8 | 66.5 | 60.3 | 21.1 | 14.5 |
| ... 50.1 | $\sigma$ | 0.0195 | 0.0210 | 0.0231 | 0.0361 | 0.122 | 0.461 | 0.924 | 0.954 | 0.471 | 0.152 | 0.0481 | 0.0246 |
| | $B_0$ | 320 | $< 24$ | $< 22$ | 140 | 108 | 49.7 | 41.9 | 31.2 | 141 | 397 | 440 | 590 |
| | $\bar{U}$ | 23 | 28 | 34 | 40 | 51 | 68 | 71 | 62 | 44 | 31 | 24 | 22 |
| | $\bar{\alpha}$ | 155° | 142° | 128° | 117° | 95° | 57° | 43° | 72° | 108° | 136° | 155° | 161° |
| | $\bar{\delta}$ | +24° | +34° | +47° | +55° | +60° | +50° | +36° | +26° | +18° | +15° | +14° | +18° |
| 50.1 ... | $n$ | 8.1 | 0.0 | 0.0 | 0.9 | 9.0 | 20.5 | 37.1 | 39.8 | 34.2 | 49.0 | 18.5 | 6.6 |
| ... 59.6 | $\sigma$ | 0.0324 | 0.0292 | 0.0293 | 0.0405 | 0.116 | 0.401 | 0.758 | 0.829 | 0.495 | 0.203 | 0.0791 | 0.0440 |
| | $B_0$ | 250 | $< 17$ | $< 17$ | 22 | 78 | 51.3 | 49.1 | 48.0 | 69.1 | 242 | 235 | 150 |
| | $\bar{U}$ | 32 | 35 | 44 | 50 | 62 | 76 | 80 | 70 | 55 | 42 | 32 | 30 |
| | $\bar{\alpha}$ | 151° | 136° | 122° | 111° | 89° | 54° | 41° | 68° | 103° | 130° | 151° | 158° |
| | $\bar{\delta}$ | +28° | +39° | +52° | +60° | +61° | +49° | +34° | +25° | +19° | +16° | +16° | +20° |
| 59.6 ... | $n$ | 3.8 | 0.0 | 0.0 | 7.0 | 7.8 | 18.4 | 27.1 | 33.0 | 24.6 | 31.9 | 11.1 | 3.8 |
| ... 70.9 | $\sigma$ | 0.0460 | 0.0378 | 0.0333 | 0.0426 | 0.110 | 0.351 | 0.667 | 0.738 | 0.499 | 0.247 | 0.112 | 0.0636 |
| | $B_0$ | 83 | $< 13$ | $< 15$ | 160 | 71 | 52.4 | 40.8 | 44.7 | 49.3 | 129 | 99 | 60 |
| | $\bar{U}$ | 43 | 50 | 56 | 62 | 73 | 87 | 90 | 82 | 66 | 53 | 44 | 40 |
| | $\bar{\alpha}$ | 147° | 132° | 118° | 106° | 85° | 51° | 39° | 65° | 98° | 126° | 147° | 156° |
| | $\bar{\delta}$ | +31° | +43° | +55° | +62° | +62° | +46° | +33° | +25° | +19° | +18° | +18° | +22° |
| 70.9 ... | $n$ | 2.3 | 6.7 | 15.0 | 6.4 | 9.1 | 14.1 | 9.5 | 14.8 | 19.9 | 29.2 | 8.7 | 4.3 |
| ... 84.3 | $\sigma$ | 0.0593 | 0.0453 | 0.0357 | 0.0435 | 0.105 | 0.310 | 0.593 | 0.668 | 0.498 | 0.280 | 0.141 | 0.0853 |
| | $B_0$ | 39 | 150 | 420 | 150 | 87 | 46 | 16.0 | 22.2 | 40.0 | 104 | 62 | 51 |
| | $\bar{U}$ | 56 | 63 | 69 | 76 | 86 | 100 | 102 | 94 | 80 | 66 | 56 | 53 |

## Table XVIII. Continued

| W km/sec limits | | P 15° | 45° | 75° | 105° | 135° | 165° | 195° | 225° | 255° | 285° | 315° | 345° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\bar{\alpha}$ | 144° | 129° | 114° | 103° | 82° | 48° | 37° | 62° | 95° | 123° | 144° | 153° |
| | $\bar{\delta}$ | +34° | +46° | +58° | +64° | +62° | +44° | +31° | +24° | +20° | +18° | +20° | +25° |
| 84.3 ... | $n$ | 3.1 | 11.2 | 0.0 | 7.6 | 7.3 | 6.2 | 2.5 | 3.4 | 6.1 | 24.9 | 6.3 | 3.9 |
| ... 100.2 | $\sigma$ | 0.0715 | 0.0501 | 0.0380 | 0.0444 | 0.102 | 0.272 | 0.505 | 0.609 | 0.492 | 0.306 | 0.172 | 0.104 |
| | $B_0$ | 44 | 224 | <13 | 170 | 72 | 23 | 5.0 | 5.6 | 12.5 | 82 | 37 | 38 |
| | $\bar{U}$ | 71 | 78 | 84 | 91 | 101 | 114 | 117 | 109 | 96 | 81 | 72 | 68 |
| | $\bar{\alpha}$ | 142° | 126° | 111° | 100° | 80° | 47° | 35° | 60° | 92° | 119° | 141° | 151° |
| | $\bar{\delta}$ | +37° | +49° | +60° | +66° | +61° | +43° | +30° | +24° | +20° | +19° | +21° | +27° |
| 100.2 ... | $n$ | 3.2 | 1.1 | 0.0 | 0.0 | 10.5 | 2.2 | 0.6 | 0.0 | 0.4 | 7.3 | 2.5 | 2.1 |
| ... 119.2 | $\sigma$ | 0.0825 | 0.0548 | 0.0400 | 0.0447 | 0.0962 | 0.250 | 0.456 | 0.564 | 0.483 | 0.326 | 0.198 | 0.126 |
| | $B_0$ | 39 | 20 | <12 | <11 | 109 | 9 | 1.3 | <1.0 | 0.8 | 22 | 13 | 17 |
| | $\bar{U}$ | 88 | 96 | 103 | 108 | 119 | 132 | 134 | 127 | 114 | 98 | 88 | 86 |
| | $\bar{\alpha}$ | 140° | 123° | 108° | 97° | 77° | 45° | 34° | 58° | 89° | 117° | 139° | 149° |
| | $\bar{\delta}$ | +39° | +51° | +63° | +67° | +60° | +41° | +29° | +23° | +20° | +19° | +22° | +28° |
| 119.2 ... | $n$ | 0.9 | 0.0 | 0.0 | 0.0 | 0.2 | 0.0 | 2.1 | 0.0 | 0.0 | 0.0 | 0.0 | 1.6 |
| ... 141.7 | $\sigma$ | 0.0923 | 0.0584 | 0.0410 | 0.0451 | 0.0936 | 0.230 | 0.418 | 0.529 | 0.476 | 0.343 | 0.220 | 0.142 |
| | $B_0$ | 10 | <9 | <12 | <11 | 2 | < 2 | 5.0 | < 1.0 | <1.0 | < 1.5 | < 2 | 11 |
| | $\bar{U}$ | 110 | 116 | 124 | 130 | 140 | 152 | 156 | 148 | 135 | 120 | 110 | 106 |
| | $\bar{\alpha}$ | 138° | 121° | 106° | 95° | 75° | 44° | 34° | 57° | 87° | 115° | 137° | 148° |
| | $\bar{\delta}$ | +40° | +53° | +64° | +68° | +59° | +40° | +29° | +23° | +20° | +20° | +23° | +30° |
| 141.7 ... | $n$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 5.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.5 |
| ... 168.5 | $\sigma$ | 0.100 | 0.0632 | 0.0421 | 0.0455 | 0.0906 | 0.214 | 0.379 | 0.499 | 0.469 | 0.360 | 0.240 | 0.157 |
| | $B_0$ | <5 | <8 | <12 | <11 | <6 | <2 | 13 | < 1.0 | < 1.0 | < 1.5 | 0.4 | 3 |
| | $\bar{U}$ | 134 | 142 | 150 | 155 | 166 | 176 | 180 | 173 | 160 | 145 | 134 | 131 |
| | $\bar{\alpha}$ | 136° | 119° | 103° | 92° | 72° | 41° | 33° | 54° | 84° | 112° | 135° | 146° |
| | $\bar{\delta}$ | +42° | +54° | +66° | +68° | +57° | +38° | +28° | +22° | +20° | +20° | +24° | +32° |
| > 168.5 | $n$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.6 | 13.1 | 2.3 | 0.0 | 0.0 | 0.0 | 0.0 |
| | $\sigma$ | 0.116 | 0.068 | 0.0439 | 0.0455 | 0.0862 | 0.183 | 0.351 | 0.446 | 0.461 | 0.383 | 0.275 | 0.184 |
| | $B_0$ | < 4 | <7 | <11 | <11 | <6 | 3 | 37 | 5.1 | < 1.0 | < 1.5 | < 2 | <3 |
| | $\bar{U}$ | 200 | 210 | 220 | 220 | 230 | 284 | 226 | 252 | 224 | 230 | 200 | 200 |

the component of stream intensity reduced to the interval $W_2 - W_1 = \Delta_0$, based on the particular values of $n'$ and $E$, is

$$B_0' = \frac{n'E \cdot \Delta_0}{\Delta W} = \frac{n'}{\sigma'},$$

where $\sigma' = \frac{\Delta W}{E\Delta_0}$ is the effective heliocentric area of radiation, and $E$ is defined by (26); the weight of $B_0'$ is evidently equal to $\sigma'$; hence, from several values of $B_0'$ entering into the given interval $\Delta_0$, the weighted mean results:

$$B_0 = \frac{n}{\sigma} \quad . \quad . \quad . \quad . \quad (26'),$$

where $n = \Sigma n'$, and

$$\sigma = \frac{1}{\Delta_0} \Sigma \frac{\Delta W}{E} \quad . \quad . \quad . \quad . \quad (27).$$

The component values $n'$ and $\Delta W$ were taken so that the interval of $W$ fell inside $\Delta_0$ (for which purpose an entry in Table XVII often had to be divided between two entries of Table XVIII); this policy led to a sum of $n$ in Table XVIII equal to the total observed.

The effective area, according to (27), may exceed $2\pi$ under certain circumstances due to the effect of zenithal attraction ($b_z$), and especially to the factor $\frac{W}{V}$ of stream velocity in $E$. Without these influences we should have, for given limits of $W$ and $P$:

$$\Sigma_P \sigma = \int_0^{\frac{\pi}{2}} 2\pi \sin z \cdot \frac{1}{2} (\cos z + \cos^2 z)\, dz = \frac{5}{6} \pi = 2.62$$

(the sum over all $P$ sectors).

For small $W$ ($< 29.8$), the computed values of $\sigma$ are too large on account of the approximate method of computation: whereas the assumed value of $E$ corresponds to the centre of radiation, the average $E$ over the whole sector is larger; the error is serious only for the very small velocities ($W < 17.7$, especially $W = 12.5 \ldots 14.9$), and no correction is introduced.

In Table XVIII data for $W < 29.8$ occur twice; it was not advisable to join these together as they represent different heliocentric directions. A dash in the table indicates that the corresponding limits of $W$ cannot occur at given $P$. In the case of $n = 0$, an upper limit to $B_0$ is given, assuming $n < 0.5$; of course, this modest upper limit can be exceeded in reality. We notice on this occasion that our values of $n$ are based on smoothed observational data for which the natural uncertainty is less than $\pm \sqrt{n}$; on the other hand, our methods of treatment (correction for error dispersion and solution for frequency of projection ratios) involve systematic errors which may be large at small values of $n$.

The general frequency of heliocentric velocities among the observed data is represented by Table XIX; for elliptical meteors the corresponding values of $a$, the semi-major axis of the orbit, are given. It appears that quite a number of small values of $a < 1$ are represented (of which only those with large orbital eccentricities are accessible to observation): these may be identified as "Zodiacal Light" meteors.

Table XIX.

Distribution of Heliocentric Velocities, All Directions.

| $W$ km/sec | $a$ | $n$ | % | $W$ km/sec | $a$ | $n$ | % | $W$ km/sec | $n$ | % | $W$ km/sec | $n$ | % |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12.5 | 0.55 | | | 29.8 | 1.00 | | | 70.9 | | | 168.5 | | |
| | | 15.3 | 1.1 | | | 138.6 | 9.6 | | 140.0 | 9.8 | | 7.0 | 0.5 |
| 14.9 | 0.57 | | | 35.4 | 1.71 | | | 84.3 | | | 200.5 | | |
| | | 8.6 | 0.6 | | | 240.8 | 16.8 | | 82.5 | 5.7 | | 7.6 | 0.5 |
| 17.7 | 0.61 | | | 42.1 | ∞ | | | 100.2 | | | 238.3 | | |
| | | 3.2 | 0.2 | | | 278.1 | 19.4 | | 29.9 | 2.1 | | 1.4 | 0.1 |
| 21.1 | 0.67 | | | 50.1 | ... | | | 119.2 | | | 283.3 | | |
| | | 35.9 | 2.5 | | | 223.7 | 15.6 | | 4.8 | 0.3 | | | |
| 25.1 | 0.77 | | | 59.6 | ... | | | 141.7 | | | All | 1436 | 100.0 |
| | | 44.5 | 3.1 | | | 168.5 | 11.7 | | 5.6 | 0.4 | | | |
| 29.8 | 1.00 | | | 70.9 | ... | | | 168.5 | | | | | |

The figures of Table XIX are in general agreement with the first approximation as found in Table IX, all observers; the agreement may be regarded as an adequate check of the methods of the first approximation. On the other hand, Table XIX shows much more detail, owing without doubt to the separate treatment of the different directions.

The total number of shower meteors used for standardization (Table I) is 142, or 59 per cent of the concluded number for $W = 35.4 \ldots 42.1$; the average fraction of shower meteors in the Arizona records is 21.1 per cent [4], thus slightly more than the percentage of $W = 35.4 \ldots 42.1$ in Table XIX.

Table XX gives the broad outline of the distribution of heliocentric velocities in different directions. The figures indicate a highly variable composition of meteor streaming according to apparent direction; there are few solar meteors from the antapex directions ($P = 315^0 - 15^0$), and few moving upwards ($P = 45^0 - 105^0$); for apex directions ($P = 165^0 - 195^0$), and downward directions ($P = 225^0 - 285^0$) solar meteors are numerous. These figures evidently reflect the effect of the earth's orbital motion *plus* a concentration of the radiants of solar meteors toward the ecliptic.

The total number of solar meteors in Table XX is 486.9, or 33.8 per cent; of moderate hyperbolic meteors — 670.3, or 46.6 per cent; of high hyperbolic meteors — 278.8, or 19.6 per cent.

Table XX.

Frequency of Solar and Hyperbolic Meteors.

| | $P=15^0$ | | $P=45^0$ | | $P=75^0$ | | $P=105^0$ | | $P=135^0$ | | $P=165^0$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $n$ | % | $n$ | % | $n$ | % | $n$ | % | $n$ | % | $n$ | % |
| Solar ($W < 42.1$) . . | 2.3 | 7.7 | 0 | 0 | 0 | 0 | 3.1 | 10.3 | 16.9 | 22.8 | 78.2 | 48.0 |
| Moderate hyperbolic ($W = 42.1 \ldots 70.9$) . | 18.2 | 60.7 | 0 | 0 | 0 | 0 | 12.9 | 43.0 | 30.0 | 40.5 | 61.7 | 37.8 |
| High hyperbolic ($W > 70.9$) . . . . | 9.5 | 31.7 | 19.0 | 100 | 15.0 | 100 | 14.0 | 46.7 | 27.1 | 36.6 | 23.1 | 14.2 |
| Sum . . . . | 30 | 100 | 19 | 100 | 15 | 100 | 30 | 100 | 74 | 100 | 163 | 100 |

| | $P=195^0$ | | $P=225^0$ | | $P=255^0$ | | $P=285^0$ | | $P=315^0$ | | $P=345^0$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $n$ | % | $n$ | % | $n$ | % | $n$ | % | $n$ | % | $n$ | % |
| Solar ($W < 42.1$) . . | 83.4 | 38.1 | 87.9 | 41.7 | 171.3 | 53.0 | 37.4 | 15.6 | 3.7 | 5.1 | 2.7 | 6.8 |
| Moderate hyperbolic ($W = 42.1 \ldots 70.9$) | 102.8 | 46.9 | 102.6 | 48.6 | 125.3 | 38.8 | 141.2 | 58.8 | 50.7 | 70.4 | 24.9 | 62.2 |
| High hyperbolic ($W > 70.9$) . . . | 32.8 | 15.0 | 20.5 | 9.7 | 26.4 | 8.2 | 61.4 | 25.6 | 17.6 | 24.4 | 12.4 | 31.0 |
| Sum . . . . | 219 | 100 | 211 | 100 | 323 | 100 | 240 | 100 | 72 | 100 | 40 | 100 |

Among the observed solar meteors orbits of direct motion are predominant ($\bar{a} > 90^0$ for direct motion, $\bar{a} < 90^0$ for retrograde motion); the same, however, is true also for the hyperbolic meteors, although in a minor degree, as shown by Table XXI.

Table XXI.

Frequency of Direct and Retrograde Orbits.

| | Limits of $W$, km/sec | | | | | | | | | | All Solar $W < 42.1$ | All Hyperbolic $W > 42.1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\leqslant 21.1$ | 21.1 ... 29.8 | 29.8 ... 35.4 | 35.4 ... 42.1 | 42.1 ... 50.1 | 50.1 ... 59.6 | 59.6 ... 70.9 | 70.9 ... 84.3 | 84.3 ... 100.2 | $> 100.2$ | | |
| a. Direct | (14.4) | 47.4 | 91.9 | 168.9 | 189.2 | 123.0 | 82.5 | 89.7 | 61.7 | 19.5 | 322.6 | 565.6 |
| b. Retrograde | (12.7) | 33.0 | 46.7 | 71.9 | 88.9 | 100.7 | 86.0 | 50.3 | 20.8 | 36.8 | 164.3 | 383.5 |
| Ratio a : b | (1.13) | 1.44 | 1.97 | 2.35 | 2.13 | 1.22 | 0.96 | 1.78 | 2.97 | 0.53 | 1.96 | 1.47 |

In constructing the table, at the transition of $\bar{a}$ over the critical value $90^0$ the discrete values of $n$ of Table XVIII were redistributed by assuming a continuous and uniform distribution over $a$ in the particular interval.

Although the data of Table XXI are only approximate, as referring to effective centres of radiation instead of the true individual radiants, they show very clearly the effect of the preference for direct, or antapex directions in the heliocentric velocities; a still greater effect of the same kind is revealed by the general trend of the variation of $B_0$ with $\bar{a}$ in Table XVIII. The result is the more surprising because the influence of the relative velocity $(\bar{U})$ upon the luminosity has not been taken into account; if the luminosity increases with $\bar{U}$, the correction for this effect — to obtain figures referring to the same inferior limit of meteor masses — must add to the contrast in $B_0$. The possible physical and cosmical causes of this phenomenon are considered in Section 12. Here we try to evaluate the maximum influence of observational selection and of computational simplification upon $B_0$. Indeed, a number of factors tend to increase our values of $B_0$ from antapex directions as compared with the apex directions:

a) the general selection of velocities favours small velocities, thus antapex directions; according to Table IV, the preference ratio for the antapex is $r < 1.2$;

b) the specific $\omega$-selection (Table V), together with the $P$-selection (Table VI), $r < 1.5$;

c) the $m\omega$-selection (Table VII), $r < 1.0$;

d) the circumstance that $\sigma$ is referred to the centre of radiation, instead of the average for the sector, $r \leqslant 1.15$;

e) the finite extent of the area of observation, instead of the fictitious point at $z = 45^0$, adds more to the antapex values of $N_z$ and $N_\lambda$ (Table XV), $r \leqslant 1.15$;

f) the excentric position of the centre of attention of the velocity observer, displaced on the average $6^0$ east of the pole, $r \leqslant 1.3$.

The combined effect from all sources is the *maximum* preference ratio for the antapex as compared with the apex direction:

$$r \leqslant 1{,}2 \cdot 1{,}5 \cdot 1{,}0 \cdot 1{,}15 \cdot 1{,}15 \cdot 1{,}3 = 3{,}1.$$

This value, although strongly exaggerated, is nevertheless unable in most cases (except in the case of very high velocities) to remove the excess in $B_0$ of antapex over apex. There is no doubt left that, when observational selection and computational effects are taken into account, the general character of the dependence of $B_0$ upon $\bar{a}$ will not differ much from the dependence represented by Table XVIII.

Another kind of computational effect is produced by the smoothing which preceded the correction for error dispersion; the influence of smoothing makes the concluded frequencies of two adjacent classes of velocity depend, to some extent, upon each other; this, apparently, is the reason why the transition from solar to hyperbolic velocities is not marked by a discontinuity in the frequencies; with the resolving power of the present method and material it is impossible to avoid the influence of smoothing: the numbers in separate directions are too small. Further, if our assumed mean velocity of the shower meteors, 0.986 of the parabolic, is too large, all our velocities have to be decreased, and the group $W = 42.1 \ldots 50.1$ may partly include solar meteors; however, a considerable systematic error of this kind does not seem to be probable.

9. *Search for Preferential Directions.* We are not here concerned with group radiants which are derived from the observations of one or two nights; the number of velocities recorded for each shower is too small to allow of drawing conclusions regarding the mean velocity. We may look for more general preferential directions, active for longer intervals of time; these should be detectable through seasonal variation in the relative frequency from different apical directions.

The material is too small to allow of a complete treatment of separate seasons with the derivation of the distribution of heliocentric velocities in each case. Instead of that we apply a substitute method; we count the observed uncorrected velocities within limits of $T$ (observed) chosen in such a manner that the total number for given $P$ within each velocity class is equal to the concluded number of heliocentric velocities as quoted in Table XX; three classes of velocity are thus considered. Although our counts refer now to uncorrected velocities, it is obvious that our substitute "high hyperbolic" class must reflect the behaviour of the true "high hyperbolic" class chiefly, and so forth; on account of error dispersion real seasonal fluctuations must appear weakened in our substitute tables.

The effective limits of velocity are given in Table XXII.

Table XXII.

Adopted Effective Limits of $T$.

| | $P$ | | | | | |
|---|---|---|---|---|---|---|
| | 15° | 45° | 75° | 105° | 135° | 165° |
| Solar . . . . . . . | $\leqslant 11.3$ | — | — | $\leqslant 17.8$ | $\leqslant 26.1$ | $\leqslant 50.4$ |
| Moderate hyperbolic | 11.4 . . . . . . 35.0 | — | — | 17.9 . . . . . . 32.9 | 26.2 . . . . . . 55.9 | 50.5 . . . . . . 89.3 |
| High hyperbolic . . | $\geqslant 35.1$ | All | All | $\geqslant 33.0$ | $\geqslant 56.0$ | $\geqslant 89.4$ |

Table XXII. Continued.

| | P | | | | | |
|---|---|---|---|---|---|---|
| | 195° | 225° | 255° | 285° | 315° | 345° |
| Solar . . . . . . . | $\leqslant 47.7$ | $\leqslant 42.5$ | $\leqslant 33.3$ | $\leqslant 17.4$ | $\leqslant 10.2$ | $\leqslant 13.9$ |
| Moderate hyperbolic | 47.8 . . . . . . 104 | 42.6 . . . . . . 83.9 | 33.4 . . . . . . 70.1 | 17.5 . . . . . . 47.7 | 10.3 . . . . . . 42.7 | 14.0 . . . . . . 37.0 |
| High hyperbolic . . | $\geqslant 105$ | $\geqslant 84.0$ | $\geqslant 70.2$ | $\geqslant 47.8$ | $\geqslant 42.8$ | $\geqslant 37.1$ |

Six groups of lunations were considered:

Table XXIII.

Groups of Lunations.

| Group | A | B | C |
|---|---|---|---|
| Lunations | I. II. XIII. XIV | III. IV. XV. XVI | V. VI. XVII. XVIII |
| Mean date | Oct. 28 | Dec. 17 | Feb. 14 |
| Extreme dates | Sep. 20 — Nov. 19 | Nov. 12 — Jan. 16 | Jan. 15 — Mar. 17 |
| Total number of velocities | 399 | 238 | 150 |
| $f$ | 0.600 | 1.005 | 1.595 |

| Group | D | E | F |
|---|---|---|---|
| Lunations | VII. VIII. XIX. XX | IX. X. XXI. XXII | XI. XII. XXIII |
| Mean date | Apr. 14 | June 12 | Aug. 9 |
| Extreme dates | Mar. 14 — May 13 | May 12 — July 10 | July 16 — Sep. 10 |
| Total number of velocities | 222 | 188 | 239 |
| $f$ | 1.078 | 1.273 | 1.001 |

The mean number observed per group is $\frac{1436}{6} = 239.3$; in the above table the factor $f$ is the ratio of this mean to the observed number of the particular group. Fluctuations in the observed totals depend partly upon real seasonal variation, partly upon unequal observing conditions; to obtain data free from the latter source of uncertainty, but fit for a direct comparison of the relative frequency in $P$, we calculate reduced numbers for each seasonal group:

$$n_0 = fn \quad . \quad . \quad . \quad . \quad (28).$$

## Table XXIV.

### Seasonal Variation of $n_0$.

| Group of Lunations | P 15° | 45° | 75° | 105° | 135° | 165° | 195° | 225° | 255° | 285° | 315° | 345° | All P |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | a) Solar | | | | | | |
| A | 0.2 | 0.0 | 0.0 | 0.1 | 0.7 | 10.8 | 9.6 | 6.7 | *33.4 | 2.3 | 0.0 | 0.0 | 63.8 |
| B | 0.9 | 0.0 | 0.0 | 0.0 | *8.0 | 10.0 | 11.4 | 15.2 | 31.8 | 8.7 | *1.8 | *1.6 | 89.4 |
| C | *1.6 | 0.0 | 0.0 | 0.0 | 3.5 | 9.6 | 12.8 | 8.1 | 19.6 | 1.6 | 1.4 | 0.8 | 59.0 |
| D | 0.0 | 0.0 | 0.0 | *3.3 | 3.2 | 14.1 | *22.0 | 15.3 | 26.4 | 8.1 | 1.1 | 0.6 | *94.1 |
| E | 0.0 | 0.0 | 0.0 | 0.0 | 1.5 | 14.0 | 13.1 | 15.4 | *33.1 | *17.2 | 0.0 | 0.0 | *94.3 |
| F | 0.0 | 0.0 | 0.0 | 0.0 | 1.3 | *20.1 | 17.4 | *30.3 | 21.2 | 2.8 | 0.0 | 0.0 | *93.1 |
| Mean, $\bar{n}_0$ | 0.4 | 0.0 | 0.0 | 0.6 | 3.0 | 13.1 | 14.4 | 15.2 | 27.6 | 6.8 | 0.7 | 0.5 | 82.3 |
| Weight of max. | (0.8) | — | — | 13 | 50 | 6.7 | 6.5 | 4700 | 10 | 500 | (0.9) | (1.0) | 1000 |
| | | | | | | | b) Moderate hyperbolic | | | | | | |
| A | 3.4 | 0.0 | 0.0 | 2.6 | 4.4 | 11.8 | *20.0 | 14.7 | *29.1 | 24.7 | 7.9 | 3.1 | *121.7 |
| B | 4.0 | 0.0 | 0.0 | *4.0 | 5.6 | 11.7 | 15.3 | 16.8 | 18.1 | 20.1 | 7.2 | 5.6 | 108.4 |
| C | *4.5 | 0.0 | 0.0 | 2.4 | 1.3 | 5.1 | 16.0 | 17.0 | 15.1 | 18.2 | 10.2 | *10.3 | 100.1 |
| D | 2.0 | 0.0 | 0.0 | 1.1 | *6.0 | 8.9 | 8.4 | 13.6 | 18.4 | *35.1 | 8.6 | 1.5 | 103.6 |
| E | 1.3 | 0.0 | 0.0 | 1.3 | 5.5 | 7.6 | *20.0 | 17.3 | 19.0 | 25.3 | *11.4 | 1.3 | 110.0 |
| F | 2.9 | 0.0 | 0.0 | 1.0 | *6.3 | *13.1 | *20.8 | *24.5 | 17.3 | 16.2 | 7.0 | 5.2 | 114.3 |
| Mean, $\bar{n}_0$ | 3.0 | 0.0 | 0.0 | 2.1 | 4.8 | 9.7 | 16.8 | 17.3 | 19.5 | 23.3 | 8.7 | 4.5 | 109.7 |
| Weight of max. | (0.6) | — | — | (1.3) | 2.0 | (1.2) | 30 | 4.7 | 80 | 20 | (0.8) | 8 | 13 |
| | | | | | | | c) High hyperbolic | | | | | | |
| A | 1.2 | 3.0 | 0.6 | 1.5 | 4.5 | 4.5 | *14.2 | 2.0 | 4.7 | 10.7 | 4.1 | *2.9 | 53.9 |
| B | 2.1 | 4.0 | 3.0 | 3.0 | *6.5 | 3.4 | 3.5 | 3.2 | 3.4 | 5.4 | 1.0 | *2.8 | 41.3 |
| C | *3.5 | *4.8 | *9.6 | *5.6 | *6.4 | *7.6 | 3.2 | *8.3 | 5.1 | *15.3 | *9.1 | 1.6 | *80.1 |
| D | 1.2 | 3.2 | 3.2 | 0.0 | 5.8 | 1.8 | 2.0 | 2.4 | *6.9 | 9.7 | 3.3 | 2.1 | 41.6 |
| E | 1.3 | 2.5 | 2.5 | 1.3 | 1.9 | 1.3 | 1.3 | 6.7 | 2.7 | 10.9 | 1.3 | 1.3 | 35.0 |
| F | 1.1 | 2.0 | 0.0 | 4.0 | 2.4 | 4.8 | 0.8 | 1.2 | 3.5 | 11.0 | 0.0 | 0.8 | 31.6 |
| Mean, $\bar{n}_0$ | 1.7 | 3.2 | 3.2 | 2.6 | 4.6 | 3.9 | 4.2 | 4.0 | 4.4 | 10.5 | 3.1 | 1.9 | 47.2 |
| Weight of max. | (0.9) | (0.6) | 30 | (1.7) | 2.2 | 2.0 | $10^8$ | 3.0 | (1.2) | (1.3) | 20 | 3.2 | 4000 |

The result is given in Table XXIV. We notice that the values of $n$ upon which this table is based are unsmoothed values *) and, therefore, their natural uncertainty is comparable

*) Fractional values of $n$ are produced by interpolation in applying the odd limits of Table XXII to counts according to the standard limits of $T$.

to $\pm \sqrt{n}$. Allowing for the procedure of normalization, the effective Gaussian unit for positive deviations in $n_0 - \bar{n}_0$ is $\sqrt{\frac{238}{239} \cdot \frac{5}{6} f} \, (\sqrt{2\bar{n}_0} + \frac{1}{2})$ (*cf.* [9]). The "weight" of the maximum is computed by a procedure explained in [9], pp. 22—23; the weight is equal to the reciprocal mathematical expectation of an accidental maximum, or maxima (marked in the table with asterisks) to exceed the observed ones; as the maxima for given $P$ are chosen out of six cases, the mathematical expectation is $6p$, $p$ being the probability; the weight is $\frac{1}{6p}$.

Large weights indicate that the observed maxima are probably not accidental, — they may indicate either real preferential streaming, or peculiar personal selection; the latter explanation seems, at the first glance, to claim some probability in the case of the seasonal group C of the "high hyperbolic", where maxima occur in almost all directions and where the relative total is high. Into group C of 1932 falls the beginning of velocity observations by D. H. while R. W. was still near his beginning; if at the beginning they recorded a greater percentage of high velocities than later, the peculiarity may be explained. In such a case the observations made a year later, in 1933, should show a small percentage of high velocities. The observational records of R. W. and D. H., however, do not support such a view (*cf.* Table VIII):

| | | C, 1932 | C, 1933 | A, B, D, E, F 1931—1933 |
|---|---|---|---|---|
| $T_0 > 100$ | number | 8 | 3 | 18 |
| | % | 9 | 5 | 1.8 |
| $T_0 > 71$ | number | 21 | 10 | 87 |
| | % | 24 | 16 | 8.6 |
| $T_0 > 50$ | number | 40 | 27 | 267 |
| | % | 45 | 44 | 26.5 |
| $T_0 \leqslant 50$ | number | 49 | 34 | 740 |
| | % | 55 | 56 | 73.5 |

Thus, although the frequency of very large velocities is somewhat reduced in C, 1933, the relative frequency still considerably exceeds the figures for the rest of the year. It appears, therefore, that the excess of high velocities in group C cannot be explained by personal selection.

Real streaming can be advocated as a cause of the excess influencing all directions, if the centre, or centres of radiation lie inside the region of observation, the north polar region in the present case. Let us examine group C of the "high hyperbolic" from this standpoint. Although the final list of radiants in [4] does not contain a single northern radiant for the period under question, the negative evidence is not convincing; the radiants selected in [4] are those which are of conspicuous activity during 1—2 nights, whereas radiants of low intensity and long duration could not find their way into the final list. Indeed, "cosmic",

Table XXV.

Radiants with $\delta \geqslant 50^0$ for Group C of Lunations.

| *AR* | $\delta$ + | Date of Chart |
|---|---|---|
| *a.* Active on 18 charts = = 56% | | |
| 90° | 85° | Feb. 4, 32 |
| 160 | 88 | Feb. 13, 32 |
| 250 | 85 | Mar. 2, 32 |
| 180° | 84° | Feb. 3, 33 |
| *b.* Active on 20 charts = = 62% | | |
| 95° | 69° | Jan. 28, 32 |
| 117 | 71 | Jan. 31, 32 |
| 90 | 68 | Feb. 2, 32 |
| 71 | 69 | Feb. 26, 32 |
| 90 | 73 | Feb. 16, 33 |
| 100° | 70° | Feb. 27, 33 |
| *c.* Active on 27 charts = = 84% | | |
| 144° | 52° | Feb. 27, 32 |
| 128 | 61 | Mar. 4, 32 |
| 140 | 64 | Mar. 6, 32 |
| 138 | 57 | Jan. 30, 33 |
| 152 | 53 | Feb. 14, 33 |
| 150 | 60 | Feb. 18, 33 |
| 140 | 50 | Feb. 19, 33 |
| 126° | 62° | Feb. 20, 33 |

| *AR* | $\delta$ + | Date of Chart |
|---|---|---|
| *d.* Active on 26 charts = = 81% | | |
| 168° | 61° | Feb. 27, 32 |
| 170 | 60 | Mar. 2, 32 |
| 180 | 63 | Mar. 14, 32 |
| 180 | 60 | Feb. 20, 33 |
| 176 | 65 | Feb. 26, 33 |
| 166° | 62° | Feb. 27, 33 |
| *e.* Active on 25 charts = = 78% | | |
| 218° | 60° | Feb. 2, 32 |
| 210 | 57 | Feb. 4, 32 |
| 204 | 61 | Mar. 1, 32 |
| 212 | 50 | Mar. 2, 32 |
| 214 | 50 | Mar. 10, 32 |
| 218 | 55 | Feb. 21, 33 |
| 205 | 50 | Feb. 22, 33 |
| 213° | 60° | Mar. 4, 33 |
| *f.* Active on 17 charts = = 53% | | |
| 247° | 70° | Mar. 1, 32 |
| 217 | 70 | Jan. 26, 33 |
| 228 | 74 | Jan. 30, 33 |
| 201 | 74 | Feb. 18, 33 |
| 212 | 69 | Feb. 22, 33 |
| 233° | 65° | Mar. 2, 33 |

| *AR* | $\delta$ + | Date of Chart |
|---|---|---|
| *g.* Active on 22 charts = = 69% | | |
| 234° | 57° | Feb. 27, 32 |
| 250 | 52 | Mar. 4, 32 |
| 240 | 50 | Mar. 14, 32 |
| 250 | 53 | Feb. 2, 33 |
| 228° | 52° | Feb. 3, 33 |
| Single radiants | | |
| 45° | 72° | Jan. 28, 32 |
| 288 | 69 | Feb. 10, 32 |
| 202 | 64 | Feb. 13, 32 |
| 270 | 69 | Mar. 2, 32 |
| 186 | 50 | Mar. 4, 32 |
| 116 | 50 | Mar. 4, 32 |
| 273 | 59 | Mar. 6, 32 |
| 96 | 51 | Mar. 6, 32 |
| 154 | 75 | Mar. 14, 32 |
| 166° | 73° | Mar. 4, 33 |

or extra-solar streaming of high velocity should lead to such persistent radiants; these may be detected by the method of "coincidence" if applied over a longer interval of time (*cf.*[4], p. 31). The "working list of 2000 radiants" (*cf.*[4], p. 5) was consulted for this purpose. For the period under question (Lunations V, VI, XVII, XVIII) there were thirty-two "combined charts" (*cf.*[4], p. 4), on which fifty-three radiants north of declination $+49^0$ were identified; the complete list of these radiants is given in Table XXV. The radiants revealed conspicuous clustering into seven distinct groups *(a—g)*, according to which the table is arranged. The number and percentage of northern "combined charts", on which radiation from the given radiant may have been active (at least four apparent members of the radiant per chart), is also given.

The radiants of this table are based upon the observations of all observers, a total of 2562 records on northern and southern charts; the activity of the radiants on the northern charts is controlled from about one-half of the total number of records. The velocity records for group C are only 150 in number, — too few to advise an independent determination of the radiants. The velocity meteors were referred to the radiants *a—g* on the basis of the apparent direction of motion. It then appeared that the meteors assigned to the radiants, except *f*, showed a higher mean velocity than the "stray" meteors; *f* was also feebly represented among the velocity records (only five probable members), and was excluded from the discussion. The frequency of heliocentric tangential velocities was found as follows:

| Observed $T_0$ km/sec | $<50$ | | 50—71 | | $>71$ | | All | |
|---|---|---|---|---|---|---|---|---|
| | n | % | n | % | n | % | n | % |
| Radiants *a-e* and *g* | 47 | 50 | 25 | 27 | 22 | 23 | 94 | 100 |
| "Stray" meteors | 36 | 64 | 11 | 20 | 9 | 16 | 56 | 100 |

An attempt was made to determine the space velocity for each radiant, from the correlation $T = U \sin \lambda$. For this purpose $\log T$ was plotted against $\log \sin \lambda$. The few common meteors, with their directly determined radiants (although not very accurate), were also used to separate apparent members of the radiants from "stray" meteors. As the number of "stray" meteors in our present case must be considerable, the determination of $U$ is very uncertain; nevertheless, radiants $g$ and $c$ gave satisfactory correlations. The results are as follows (mean date = Feb. 14):

| Radiant | *a* | *b* | *c* | *d* | *e* | *g* |
|---|---|---|---|---|---|---|
| Apparent centre of radiation: $AR$, $\delta$ | 180°; +87° | 93°; +69° | 140°; +57° | 174°; +63° | 210°; +54° | 238°; +53° |
| $n$ | 14 | 12 | 19 | 17 | 18 | 24 |
| $\alpha_1$ | 104° | 128° | 107° | 96° | 74° | 72° |
| $U$ km/sec | 150 | 72 | 89 | 94 | 130 | 170 |
| $W$ „ | 160 | 93 | 102 | 102 | 125 | 162 |
| $W_\infty$ „ | 154 | 83 | 93 | 92 | 117 | 158 |
| Extra-solar radiant: $AR_\infty$, $\delta_\infty$ ($W_\infty = 101$) | 24°; +65° | 73°; +60° | 109°; +56° | 135°; +71° | 203°; +67° | 240°; +74° |

The "extra-solar" radiant, or the supposed direction in interstellar space (asymptotic direction), at an assumed constant velocity $W_\infty = 101$ for all radiants, is computed with the aid of manuscript tables constructed at Harvard by F. L. Whipple. The total number of members, $\Sigma n = 104$, exceeds the true number of individuals (94), because a few meteors had to be assigned to two radiants at the same time.

The most pronounced maximum in Table XXIV occurs in $P = 195^0$, seasonal group A of the "high hyperbolic"; with a weight of $10^8$ it is doubtlessly real and due to the observations of E. Ö. in October, 1931, whereas the November observations of E. Ö. do not add to the effect. For the October observations of E. Ö. altogether two "cosmic" radiants were indicated:

| Radiant | *i* | *k* |
|---|---|---|
| Period | Oct. 8—21, 1931 | Oct. 6—21, 1931 |
| Mean date | Oct. 14 | Oct. 11 |
| Mean radiant: $AR$; $\delta$ | 115°; +2° | 19°; +4° |
| Number of members | 22 | 15 |
| $\alpha_1$ | 20° | 71° |
| $U$ km/sec | 100 | 63 |
| $W$ „ | 73 | 60 |
| $W_\infty$ „ | 59 | 43 |
| Extra-solar radiant: $AR_\infty$; $\delta_\infty$ | 129°; −8° | 347°; −11° |

The data for radiant *i* are fair and account for most of the excess of high velocities in $P \sim 195^0$ (*cf.* Section 4). In addition to that, there was an excess of high velocities from about $AR = 81^0$, $\delta = +31^0$ (Oct. 1931) which is near the expected position of the radiants of meteors belonging to the Ursa Major stream (ephemeris by F. L. Whipple, unpublished); however, no solution for velocity could be obtained because of the disturbing effect of the Orionids.

Radiant *k* is less certain, being strongly disturbed by a conspicuous solar radiant near the same position (*cf.* Table I); it corresponds to the maximum in $P = 285^0$ (Table XXIV), but the concluded velocity is comparatively low.

The data for the "moderate hyperbolic" meteors (Table XXIV. b) indicate some real streaming, as can be judged from the weights ($P = 195^0$, $255^0$, $285^0$) which are, however, smaller than in the two other sections of the table. The maxima are spread over all seasons of the year. Owing to the moderate velocity the separation of distinct radiants appears to be difficult (radiant *k* mentioned above is, nevertheless, an example). For each *P* we tried to estimate graphically the dates of maximum and minimum activity; assuming the position of the mean radiant to coincide with the effective centre of radiation (Table XVII), the "cosmic" radiants were determined for $W_\infty = 30$ km/sec as given below:

| $P$ | | $15^0$ | $105^0$ | $135^0$ | $165^0$ | $195^0$ | $225^0$ | $255^0$ | $285^0_a$ | $285^0_b$ | $315^0$ | $345^0$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Relative Maxima of Activity in *P*** | | | | | | | | | | | | |
| Date | | May 1 | Dec. 15 | July 20 | Sep. 20 | Sep. 15 | Aug. 20 | Oct. 20 | Apr. 20 | — | May 1 | Feb. 15 |
| Extra-solar radiant | $AR_\infty$ | $84^0$ | $294^0$ | $115^0$ | $155^0$ | $82^0$ | $358^0$ | $359^0$ | $141^0$ | — | $123^0$ | $35^0$ |
| | $\delta_\infty$ | $+32^0$ | $+16^0$ | $-19^0$ | $+17^0$ | $+26^0$ | $+24^0$ | $+5^0$ | $+32^0$ | — | $+25^0$ | $+20^0$ |
| **Relative Minima of Activity in *P*** | | | | | | | | | | | | |
| Date | | Nov. 1 | July 15 | Feb. 15 | Mar. 15 | Apr. 1 | Feb. 20 | Apr. 15 | Aug. 15 | Feb. 15 | Nov. 1 | May 20 |
| Extra-solar radiant | $AR_\infty$ | $271^0$ | $109^0$ | $337^0$ | $338^0$ | $300^0$ | $0^0$ | $164^0$ | $260^0$ | $85^0$ | $310^0$ | $122^0$ |
| | $\delta_\infty$ | $+7^0$ | $+17^0$ | $+27^0$ | $+32^0$ | $+48^0$ | $+45^0$ | $+43^0$ | $-20^0$ | $+26^0$ | $0^0$ | $+30^0$ |

The maxima and minima in the table occur promiscuously and no general preferential direction is indicated; of course, the failure may be explained by the incorrectness of our assumptions regarding the mean velocity and centre of radiation: the case of the "high hyperbolic" of C-lunations discussed above is an example of how strong preferential directions may be concealed leading to a more or less uniform apparent intensity in all directions. Further, by counting the effective limits of observed *T*, the variation in the frequency of the true velocities may be concealed by observational error dispersion. The general statistics of directions, where observations north and south of the zenith form the basis for the determination of the true distribution of radiants over the celestial sphere, may help to decide upon the question. The present evidence points to a comparatively uniform distribution of directions in space without a noticeable effect of the solar motion. In this latter point at least the results agree with those of Hoffmeister (*cf.* [10], p. 54).

The "solar" meteors, according to Table XXIV. a), reveal conspicuous streaming easily explained by the alternate activity of different showers.

10. *Velocity and Luminosity.* The dependence of the observed fraction of high velocities upon apparent magnitude, noticed in [2], may be partly due to the $m\omega$-selection; its effect, however, must be smaller than would follow from Table III: the table refers to *true* magnitudes, whereas the correlation in practice can be studied only on the basis of *recorded* magnitudes for which the

Table XXVI.

Velocity and Zenithal Magnitude.

| $m_z$ (recorded) | | $\geqslant 3.5$ | 3.2 | 2.7 | 2.2 | 1.45 | 0.45 | −0.55 | −1.55 | $\leqslant -2.0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| High hyperbolic | n | 23 | 25 | 34 | 42 | 77 | 51 | 15 | 3 | 2 |
| | % | 21 | 21 | 20 | 18 | 20 | 20 | 15 | 12 | 7 |
| Moderate hyperbolic | n | 59 | 54 | 72 | 130 | 160 | 93 | 51 | 13 | 15 |
| | % | 54 | 44 | 42 | 55 | 42 | 37 | 50 | 52 | 56 |
| Solar | n | 28 | 43 | 66 | 64 | 148 | 111 | 36 | 9 | 10 |
| | % | 25 | 35 | 38 | 27 | 38 | 43 | 35 | 36 | 37 |

$m\omega$-selection is softened by error dispersion. For zenithal magnitudes the $m\omega$-effect should be still less pronounced.

Using practically the same classification of velocities as defined by Table XXII, the distribution of velocities according to zenithal magnitude was found as given in Table XXVI.

The table refers to uncorrected distributions of $T$ and $m_z$, and has only a qualitative significance.

We find that for $m_z > 0$ the distribution does not reveal a systematic change with magnitude; for $m_z < 0$, a noticeable decline in the frequency of the "high hyperbolic" starts. These qualitative data are substantiated by an analysis of the distribution of $T_0$ according to the "first approximation" method (*cf.* Section 3):

### Table XXVII.

Distribution of Heliocentric Velocities for High and Low Luminosity.

| $W$ km/sec | | $\leqslant 36$ | 42 | 50 | 60 | 72 | 85 | 101 | 120 | 143 | 170 | $\geqslant 202$ | All |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $m_z < 0$ | n | 4 | 35 | 62 | 44 | 3 | 0 | 0 | 0 | 0 | 0 | 6 | 154 |
| ($\bar{m}_z = -0.6$ true) | % | 2.6 | 22.7 | 40.3 | 28.6 | 1.9 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3.9 | 100.0 |
| $m_z > 0$ | n | 152 | 278 | 228 | 269 | 201 | 102 | 12 | 15 | 16 | 9 | 0 | 1282 |
| ($\bar{m}_z = 2.5$ true) | % | 11.8 | 21.7 | 17.8 | 21.0 | 15.7 | 8.0 | 0.9 | 1.2 | 1.2 | 0.7 | 0.0 | 100.0 |

The bright meteors show a distribution of the heliocentric velocities which definitely differs from the rest; conspicuous is the absence of high hyperbolic velocities, $W = 72$ to 170, among bright meteors, whereas the group $W \geqslant 202$ is, on the contrary, represented by bright meteors only. The small number of $W \leqslant 36$ among the bright meteors may partly be due to the $m\omega$-selection: as shown by the data given at the bottom of Table III (ratio c : b), the recorded number of velocity meteors brighter than apparent magnitude zero decreases suddenly to about one-fifteenth of the number expected from the general distribution of magnitudes; this is doubtlessly the effect of specific selection, explained perhaps by the eye following the bright meteors, or by a decrease in the resolving power due to strong coma and train.

11. *Velocity and Height.* Table XXVIII contains the mean logarithm of velocity for different directions $P$, together with

the mean reduced height, $H_0$ (*cf.* [3]), of the centre of the trail for the northern region only.

## Table XXVIII.

Mean Logarithmic Velocity and Height: Northern Heights Only.

| $P$ | 15° | 45° | 75° | 105° | 135° | 165° | 195° | 225° | 255° | 285° | 315° | 345° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{\log U}$ | 1.581 | 1.860 | 1.848 | 1.778 | 1.817 | 1.818 | 1.855 | 1.762 | 1.602 | 1.615 | 1.546 | 1.530 |
| *p. e.* | ±.040 | ±.052 | ±.058 | ±.040 | ±.026 | ±.016 | ±.015 | ±.015 | ±.012 | ±.014 | ±.026 | ±.034 |
| $U$ km/sec | 38.1 | 72.4 | 70.5 | 60.0 | 65.6 | 65.8 | 71.6 | 57.8 | 40.0 | 41.2 | 35.2 | 33.9 |
| $\bar{\alpha}_1$ | 130° | 111° | 95° | 84° | 65° | 37° | 28° | 48° | 76° | 104° | 129° | 141° |
| $H_0$ km | 80.2 | 87.3 | 79.6 | 90.5 | 93.1 | 97.5 | 97.6 | 90.1 | 86.4 | 84.9 | 82.7 | 80.2 |
| *p. e.* | ± 2.3 | ± 2.9 | ± 3.0 | ± 3.1 | ± 1.8 | ± 1.2 | ± 1.1 | ± 0.9 | ± 0.6 | ± 0.7 | ± 1.2 | ± 1.8 |
| $H_0'$ | 79.8 | 87.3 | 79.6 | 90.0 | 92.0 | 95.1 | 95.7 | 88.0 | 83.8 | 84.1 | 82.4 | 79.9 |

The probable errors in the table refer to the combined observational and cosmic dispersion. The correlation of $H_0$ and $\log U$, although quite definite, is somewhat contradictory; for a smooth curve led through three normal points the sum of the squares of the deviations *), $\Sigma \Delta^2$, is 2.25 times larger than expected from the above quoted probable errors. The deviations seem to be related to the fraction $f_s$ of solar meteors observed in the given direction (*cf.* Table XX). Observational evidence [3] points to a systematically greater height of shower meteors as compared with the rest for equal velocity (influence of composition and shape). Assuming tentatively 5 km as the difference in height, the figures at the bottom of Table XXVIII were computed from

$$H_0' = H_0 - 5.0\, f_s.$$

These heights, supposed to refer to a zero fraction of solar meteors, yield a much better correlation, $\Sigma \Delta^2$ equalling only 1.16

*) The ordinates and the abscissae are both subject to considerable errors; therefore the deviations considered are measured at right angles to the line of correlation. The expected probable error in this direction is $\bar{\varepsilon} = \pm \sqrt{\varepsilon_1^2 \cos^2\gamma + \varepsilon_2^2 \sin^2\gamma}$, where $\gamma$ is the inclination of the line, $\varepsilon_1$ and $\varepsilon_2$ — the component probable errors; the expected value of $\Sigma \Delta^2$ is assumed to be $\frac{9}{12} \frac{\Sigma \bar{\varepsilon}^2}{0.674^2}$, corresponding to three normal points and twelve observations.

times the expected value, or one-half of the uncorrected $\Sigma \Delta^2$. From this standpoint even the outstanding case of $P = 75^0$ falls within the possible limits of error dispersion.

Besides, the correlation between $H_0$ and $\log U$ may have been impaired by specific observational selection, because the two sets of data do not refer to exactly the same objects. The total number of northern heights upon which Table XXVIII is based exceeds by about 30 per cent the total number of velocities; but only 620, or 43 per cent of the velocity meteors have observed heights. The overlapping of the two sets of data is especially bad for meteors moving upward: at $P = 45^0$, there are four heights among nineteen velocities; at $P = 75^0$, one height in fifteen velocities; at the same time, the total number of heights is twenty-four in $P = 45^0$, and fourteen in $P = 75^0$. In such circumstances the discordant data in these directions are not surprising.

To get the clearest view of the significance of the data contained in Table XXVIII, we join them (none rejected) into two normal points (weight = number of velocities), using $H_0$ (not $H_0'$, as this is a problematic quantity):

| | $\overline{H_0}$ | $\overline{\log U}$ |
|---|---|---|
| a: regression line of $H_0$ upon $\log U$ | | |
| | $84.9 \pm 0.4$ | $1.596 \pm 0.008$ |
| | $93.9 \pm 0.6$ | $1.813 \pm 0.008$ |
| b: regression line of $\log U$ upon $H_0$ | $84.8 \pm 0.4$ | $1.610 \pm 0.008$ |
| | $94.5 \pm 0.6$ | $1.811 \pm 0.008$ |
| c: true correlation . . . . . . . . | $84.8 \pm 0.4$ | $1.603 \pm 0.008$ |
| | $94.2 \pm 0.6$ | $1.812 \pm 0.008$ |

This gives $H_0 = (44.9 \pm 4.1) \log U + \text{const.}$ . . . (29).

The above data depend upon only one-half of the Arizona heights, the advantage consisting in equal principles of selection ($P$ of northern region) for $U$ and $H_0$.

A less direct way of constructing the correlation which makes possible the use of all the Arizona heights consists in grouping both kinds of data according to $\alpha_1$. For the heights the data are given in [3], Table XIX; the mean values of $\alpha_1$ were slightly corrected as quoted below, in agreement with the law of the intensity of streaming adopted in the present paper (Section 6), and the mean heights were reduced to exactly the same values of $a_1$ as found for the $\log U$ groups with the aid of differential corrections from [3], Table XXI.

Corrections of $\bar{a}_1$ of [3], Table XIX:

| | | | | |
|---|---|---|---|---|
| $\bar{a}_1$ cited | $41^0$ | $78^0$ | $102^0$ | $134^0$ |
| $\bar{a}_1$ corrected | $41^0$ | $79^0$ | $101^0$ | $128^0$ |

Correlation of $\log U$ and $H_0$, all Arizona heights:

| | | | | |
|---|---|---|---|---|
| $\bar{a}_1$ limits | $120^0$—$180^0$ | $90^0 - 119^0$ | $60^0 - 89^0$ | $0^0 - 59^0$ |
| $\bar{a}_1$ mean | $133^0$ | $104^0$ | $75^0$ | $38^0$ |
| $\overline{\log U}$ | $1.549 \pm .018$ | $1.649 \pm .013$ | $1.650 \pm .011$ | $1.812 \pm .009$ |
| $H_0$ (reduced to $\bar{a}_1$ mean) | $78.4 \pm 0.7$ | $82.5 \pm 0.5$ | $87.7 \pm 0.4$ | $96.0 \pm 0.5$ |

The mean correlation coincides with the straight line joining the two extreme points:

$$H_0 = (66.9 \pm 4.6) \log U + \text{const.} \quad . \quad . \quad . \quad . \quad . \quad (30).$$

The difference between the coefficients in (29) and (30) equals 3.7 times the probable error and is not likely to be accidental (especially because the data in the two cases are not quite independent); the different way of selection has evidently influenced the result. In such a case (29) is to be preferred as the more direct one in spite of (30) showing a smaller relative error.

12. *The Heliocentric Stream Intensity.* The peculiar behaviour of $B_0$ in Table XVIII is already noticed in Section 8; the values of $B_0$ which are not corrected for the influence of velocity upon luminosity increase systematically toward the antapex, in spite of decreasing $U$. Observational selection, although acting in the same direction, is unable to alter these results considerably. It appears that at present we are unable to give a satisfactory interpretation of the phenomenon; a discussion of the possible causes may nevertheless be useful in future research.

Let the velocity dependence of luminosity be defined by

$$I \sim U^s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31);$$

according to this the limiting mass observable changes also and the observable stream intensity changes in the ratio

$$B \sim f(I) \sim U^i \quad . \quad . \quad . \quad . \quad (32).$$

When the frequency function of meteor masses and the law of the dependence of luminosity upon mass are given (direct proportionality must be nearly fulfilled) the exponent $i$ is in a definite manner determined by $s$ (effective values of the exponents are to be used, of course). Generally $i$ and $s$ must have the same sign; further, for a monotonous increment of meteor frequency with decreasing mass (luminosity), such as indicated by actual observations of meteors, approximate equality of $i$ and $s$ must hold ( $\pm$ 30 per cent).

Assuming the view hitherto upheld by the writer of $s \sim 3$, $i \sim 3$, and allowing for some observational selection, the data of Table XXI if referred to the same lower limit of meteor mass indicate for the solar meteors a ratio of direct to retrograde motions of about 10 : 1; for the hyperbolic meteors, the estimated ratio is about 4 : 1. Whereas for the solar meteors the preference for direct motion may be explained by the well known properties of the solar system, for the hyperbolic meteors the preference is much more difficult to explain. The results refer to the mean of the whole year; a "cosmic" radiant, active from the antapex direction during one season, will turn to the apex direction in another season, which may lead to compensation even in the case of strong preferential directions. It is true that on account of the combined effect of the displacement due to the orbital motion of the earth, of the inclination of the ecliptic toward the equator, and of the gravitational action of the sun a hyperbolic radiant of not too high a velocity may be low, or below the horizon when near the apex, passing high above the horizon when near the antapex; such, for example, would be the case of radiant $k$, Section 9; in view of the dependence of the incident stream intensity upon $\cos z_i$, an effect in the required direction may result. The "Taurus stream" of v. Niessl-Hoffmeister[10] should lead to a similar effect. Nevertheless, although the influence of preferential directions is not excluded, from the data of Table XXIV. b) it appears that the streaming tendency of the "moderate hyperbolic" meteors alone can hardly account for the entire preference factor of 4:1.

There exists another general systematic difference between the apex and antapex directions: on account of the displacement of the geocentric radiant toward the apex, the heliocentric declinations of the centres of radiation from antapex directions ($P = 255^0 - 345^0$ and $15^0$) are smaller than the declinations from the apex directions ($P = 135^0 - 225^0$, *cf.* Table XVIII); a *P*-sector in an antapex direction reaches farther south than in the apex direction, as far as heliocentric motion is concerned. A general increase of the heliocentric stream intensity toward south may in such a case account, at least partly, for the observed increase of $B_0$ toward the antapex. The high horary meteor rates recorded by McIntosh in New Zealand[16] lend some support to the hypothesis, as they indicate indeed a greater meteor frequency in the southern hemisphere.

Another cause to be considered is a possible deviation of the exponent in (32) from the suggested value $i = 3$. In fact, $i = 0$ would do well; this is the assumption upon which Hoffmeister actually bases his investigations in meteor statistics. A slightly negative value of $i$ would do even better. Independently of the law of meteor radiation a value of $i$ near zero would result in case meteors around the effective limiting magnitude of the visual observer are absent; the Leonid shower seems to represent such a case. However, for the sporadic meteors such a peculiar distribution of luminosities is contradicted by observations (*cf.* also Table XXVI).

The physical law of the variation of meteor luminosity with velocity remains to be considered.

A small exponent in (31) would result in case the continuous radiation of the meteor nucleus is considerable: the temperature of vapourization (considerably higher for iron than for stone) does not rise much with the velocity, and the duration decreases, so that a nearly constant total amount of continuous radiation emerges from the nucleus; a very low efficiency of the atomic radiation would be necessary in such a case — the observed strong emission spectra of meteors do not support such a possibility.

The physical theory of radiation produced by atomic collisions[11] may next be examined; the correctness of the theory of the radiation of an average atom, developed by the writer, is confirmed by recent experimental data[12, 13] referring to the light emitted by $K_{II}$ collisions with He. The relative change in the intensity of emission observed by Bumann agrees well enough with the figures of Table XIII in[11], — the table which is considered by the writer to correspond to the degree of dilution in the coma of observable meteors[14]; the absolute values of the "heat factors" cannot, of course, be warranted. In spite of the agreement, the applicability of the above mentioned table to equation (31) may be questioned. The table refers to the radiation of an average atom of unchanged mean properties — thus of an average unchanged state of ionization; practically unchanged are also the $K_{II}$ atoms in the above mentioned experiments because the high ionization potential of He prevents $K_{II}$ from becoming neutral, whereas the second ionization of K cannot set in efficiently, as the corresponding ionization potential of $K_{II}$ exceeds the value for He. In the terrestrial atmosphere, however, the moving meteor atoms may easily become singly ionized because the ionization potentials of Fe, Mg, Ca, Si are smaller than the corresponding energies of the air molecules; at high velocities second ionization may set in.

Now, at least in the case of Fe we know that the neutral atom is able to emit much more visible light than the ionized atom does; the analysis of meteor spectra by Millman[17] indicates that most of the meteor emission is due to neutral iron. If the average meteor atom loses in visual efficiency with the increasing state of ionization, a mechanism reducing the effective value of $s$ in (31) presents itself, because the higher the velocity, the higher is the average state of the ionization of the moving atom, and the shorter the duration of the neutral state. The question is to be considered theoretically for particular atoms. In the meanwhile methods for the direct determination of $s$ may be looked for.

In the correlation of height with velocity and magnitude we take into account a minor factor, disregarded in our preliminary treatment in [3]: the change of apparent magnitude with height. The zenithal magnitude $m_z$ defines the apparent luminosity at a distance equal to $H$; for equal $m_z$, the absolute luminosity, thus also the mass is proportional to $H^2$. Assuming for the atmospheric density $\varrho \sim 10^{-\frac{H}{a}}$, the correlation of height and velocity, originally referred to $m_z =$ const., together with (31) yields, according to the first approximation theory of [14]:

$$a = a_u = \frac{H_1 - H_2}{\left(2 + \frac{s}{3}\right) \log \frac{U_1}{U_2} - \frac{2}{3} \log \frac{H_1}{H_2}} \quad . \ . \ . \ . \ . \ . \ (33).$$

The term $-\frac{2}{3} \log \frac{H_1}{H_2}$ accounts for the dependence of mass (and radius) upon height, at $m_z =$ const.

Similarly, from the correlation of the height of the endpoint with $m_z$, at constant velocity, we obtain:

$$a = a_m = \frac{H_1 - H_2}{\frac{2}{15}(m_1 - m_2) - \frac{2}{3} \log \frac{H_1}{H_2}} \quad . \ . \ . \ . \ . \ . \ (34).$$

Substituting into (33) the value of $a$ found from (34), the exponent $s$ may be determined.

For the determination of $a_m$ we take (*cf.* [3], p. 580) $\frac{dH}{dm} = 3.32$ km/mag; the correlation is determined from an effective difference $H_1 - H_2 = 16 + \Delta$ km, $H_1 = 91$ km, $H_2 = 75$ km, $\frac{2}{15}(m_1 - m_2) = 0.653$, $m_1 = +3$, $m_2 \sim -2$ (*cf.* [3], Table XXVa); the difference in height contains a small correction $\Delta$ to take into account the systematic change of velocity with $m_z$; from Table XXVII we estimate the systematic difference in velocity between $m_z = -2$ and $+3$ at $\Delta \log U = \Delta \log W = -0.022$; to equal velocity a smaller difference of $H_1 - H_2$ corresponds; according to the mean of (29) and (30) we find $\Delta = -0.022 \times 56 = -1.2$ km; (34) gives then

$$a_m = \frac{14.8}{0.653 - 0.055} = 24.8 \pm 3.0 \text{ km}.$$

The value of $a_m$ is exactly the same as preliminarily found in [3], but the probable error is assumed to be equal to almost the double of the value following from the dispersion in $H$ alone, and is supposed to take also into account the uncertainty in the scale of magnitudes.

Setting in (33) $a_u = a_m$ as found above, and taking for $H$ and $\log V$ the extreme values for the normal points of the corresponding correlation as found in Section 11, the following values of the exponent $s$ result:

from the "direct" correlation (29), $s = -0.1 \pm 0.7$ *(p. e.)*;
from the "indirect" correlation (30), $s = +2.8 \pm 1.1$ *(p. e.)*.

The two conflicting values accidentally nearly represent the two extreme cases considered in connection with meteor statistics ($s = 0$ and $s = 3$).

Another method of estimating $s$ is based on accurate photographic data, comprising real paths, velocities and magnitudes, such as recent data by Whipple[15]. The "first approximation theory"[14] requires for the radius

$$R \propto e^{-\frac{H}{a}} U^2 \sec z_i \quad . \; . \; . \; . \; . \quad (35),$$

when $H$ is the height of a characteristic point, e. g. the centre of the trail. The reduction of luminosity to unit radius is given by

$$m_0 = m + 7.5 \log R \quad . \; . \; . \; . \quad (36).$$

Whipple's data[15] lead to the following figures, with $\bar{a} = 20$ km:

| Meteor No. | 642 | 651 | 660 | 663 "hyp." | 663 "solar" | 670 | 689 |
|---|---|---|---|---|---|---|---|
| $U$ km/sec | 30.6 | 36.6 | 13.3 | 79.7 | (68.4) | 23.9 | 61.2 |
| $R$ (arbitrary units) | 1.14 | 1.09 | 5.06 | 1.20 | (8.3) | 3.50 | 1.79 |
| $m$ = absolute mag. | −2.3 | −2.6 | −2.2 | −4.0 | −4.0 | −3.1 | −4.1 |
| $m_0$ = same, reduced to $R = 1.0$ | −1.9 | −2.3 | +3.0 | −3.4 | (+2.9) | +0.9 | −2.2 |

For meteor No. 663 two solutions are given by Whipple: one, with a hyperbolic velocity following from the direct (although uncertain) identification of details on the two trails; the other, with a lower velocity, assumed to be more probable on the basis of certain theoretical considerations referring to the physical theory of meteor phenomena. Our view is that the theory of the upper atmosphere is at present too much subject to a kind of "uncertainty principle" to form a sure basis of judgment upon direct observational data. The observed deceleration is influenced (diminished) by the reaction of meteor vapours and cannot be used for absolute estimates. The safest ground is presented by differential data based upon the density gradient of the atmosphere, instead of the absolute density, such as the above table; $m_0$ there expresses in stellar magnitudes the visual efficiency of radiation per unit mass. All the values of $m_1$ fall upon a straight-line correlation with $\log U$, with a mean deviation of only $\pm$ 0.8 mag, except the "solar" case of 663 which falls by as much as 6.2 magnitudes below the line: there is a weighty argument in favour of the directly found "hyperbolic" solution to be the right one. Moreover, the discrepancy in $m_0$ for No. 663 "solar" cannot be removed by any plausible value of $\bar{a}$. Rejecting the "solar" hypothesis of No. 663, the correlation of $m_0$ and $\log U$, for different values of $\bar{a}$, gave the following values of $s$ in (31):

| | | | |
|---|---|---|---|
| $\bar{a} =$ | 15 km | 20 km | 25 km |
| $s =$ | 5.6 | 3.4 | 1.2 |

The determination is not very certain, as it depends largely upon the adopted value of $\bar{a}$; it is perhaps significant that for $\bar{a} = a_m = 25$, $s = 1.2$ is nearly equal to the mean of the two values found above from the correlation of velocity with height. Until a better determination is available, we think that $s = 1.3$ is the most probable value to be assigned to the exponent on the basis of the feeble direct observational evidence available. With such a low value of $s$, and perhaps a slightly smaller value of $i \sim 1$ in (32), the difficulty of interpreting $B_0$ in Table XVIII is much reduced; all the residual effect, besides observational selection, may apparently be accounted for by some streaming among the "moderate hyperbolic" meteors and by a preference for direct motion among the "solar" meteors.

Tartu,
February 21, 1939.

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*Abbreviations:* H. C. = Circular of Harvard College Observatory
H. A. = Annals of Harvard College Observatory

## Notation.

| | |
|---|---|
| $\omega$ | angular velocity of the meteor |
| $\omega_c$ | predicted angular velocity of a shower meteor |
| $\omega_z$ | zenithal angular velocity |
| $\Delta A$ | angle between direction of meteor, and direction from radiant |
| $AR$ | right ascension |
| $\delta$, $\delta_1$ | declination } the subindex $_1$ refers specially to geocentric coordinates |
| $\alpha$, $\alpha_1$ | angular distance from the orbital apex of the earth } the subindex $_1$ refers specially to geocentric coordinates |
| $\beta$ | latitude (ecliptical) |
| $z$ | zenith distance |
| $z_i$ | zenith angle of incidence (zenith distance of the radiant) |
| $m$ | magnitude |
| $m_z$ | zenithal magnitude |
| $T$ | linear tangential geocentric velocity, observed |
| $T_0$ | heliocentric tangential velocity |
| $U$ | space velocity at the boundary of the atmosphere |
| $V$ | geocentric space velocity corrected for the earth's attraction |
| $W$ | heliocentric space velocity |
| $W_\infty$ | velocity outside the solar system |
| $P$ | apical direction of meteor, counted at the centre of the trail counter-clockwise from the direction toward the orbital apex of the earth ($P = 0^0$ . . . meteor moving from antapex toward apex) |
| $\lambda$ | angular distance of the centre of the trail from the radiant |
| $A$ | direction of meteor with respect to horizontal coordinates, counted counter-clockwise; $A = 0^0$ denotes motion vertically up in the northern region |
| $B$, $B'$ | geocentric density of radiants, or stream intensity per square radian for normal incidence |
| $B_0$ | heliocentric stream intensity per square radian for normal incidence |
| $k$ | exponent in $\cos^k z_i$, the law of incident stream intensity |
| $t$ | mean local time |
| $n$ | number of meteors |
| $b_z = b'_z b''_z$ | ratio of extra-terrestrial stream intensity to the stream intensity influenced by the terrestrial field of gravitation |

| | |
|---|---|
| $\frac{Wb_{\alpha}}{V}$ | ratio of heliocentric to extra-terrestrial geocentric stream intensity |
| $E$ | total transition factor to heliocentric stream intensity, $B_0 = En$; personal selection and influence of velocity upon luminosity not included |
| $\sigma$ | effective heliocentric solid angle of radiation (streaming) |
| $H_0$, $H_c$ | height |
| $\zeta$ | angular distance of the heliocentric radiant from the antisolar point |
| $c$ | mean orbital velocity of the earth; also generally velocity in a circular orbit |
| $a$, $a_u$, $a_m$ | difference in height for which the density of the atmosphere changes in the ratio 10 : 1 |
| $a$ | semi-major axis of the orbit |
| $s$ | exponent of the velocity dependence of luminosity, $I \propto U^s$ |
| $i$ | exponent of the velocity dependence of stream intensity, $f(I) \propto U^i$ |