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THE PERFORMANCE OF A MOMENTUM-BASED EQUITY PORTFOLIO ON THE EXAMPLE OF THE NASDAQ-100 (NDX)

Master's thesis

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Tartu 2021

I have written this master's thesis independently. Any ideas or data taken from other authors or other sources has been fully referenced.

Abstract[1]

The author examines the risk and reward characteristics of cross-sectional momentum, time-series momentum, and dual momentum with single equities included in the NASDAQ-100 since purchasing single stocks allows more risk-averse investors to seek higher returns. The author is unable to prove statistical significance in any J/K-strategy selected and more than 75% of the strategies are outperformed by the NASDAQ-100 itself. Cross-sectional and dual momentum both generally outperform time-series momentum while using practical benchmarks to determine the equities being purchased. The results indicate potential for a short-term time-series momentum contrarian strategy where longing losers is better than longing winners. Several economically significant strategies that outperform the NASDAQ-100 by more than five percent annually are reported.

Keywords:

G11 Investment Decisions

G11 Portfolio Choice

G23 Financial Instruments

[1] The author would like to thank supervisors Mark Kantšukov and Hakan Eratalay for their time and assistance in composing this thesis. The two supervisors offered comments on different aspects of the thesis, allowing the author to continuously improve the thesis from multiple angles. The author is grateful for the comments offered by referee Toomas Raus who proposed several ideas on how to further the thesis. The author would also like to thank Jaan Masso who provided fast-response comments with examples to all of the formatting-related questions the author had. Lastly, the author would like thank AK from *Fallible*, a financial entertainment and news *YouTube* channel. AK also is the owner of the global macro research firm called *Macro Ops* and is the original source of the idea for this thesis who pointed the author to examples of non-academic literature on the topic.

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Introduction

Participants in financial markets are always on the lookout for the next alpha-generating investment strategy. However, finding investment strategies that are capable of outperforming the indices and exchange-traded funds that are commonly utilized as benchmarks is not simple. As technology has continued to improve it has brought with it both an easier access to information and new methods to work with said information. This has enabled quantitative analysis to seek out opportunities in the market and make investment decisions across different time horizons based on the signals output by said analysis, an approach we commonly see used by today's hedge funds.

While quantitative analysis can be the foundation for thematic, value-based, fundamental, and other forms of investing, the analysis can also be built around technical analysis. However, trading strategies built around technical analysis have not received much coverage in scientific research as publication of working methods could potentially dissipate said market edge. One of the first examples of a technical trading system to receive scientific coverage was the CRISMA system by Pruitt and White (1988) which utilized cumulative volume, relative strength, and moving average components to assemble a profitable long-only investment strategy. Since Pruitt and White (1988) and subsequent follow-up papers (Pruitt & White, 1989; Pruitt et al., 1992) several authors have conducted analysis to assess the robustness of the CRISMA system, usually concluding that CRISMA merely works because of the selection of equities (Goodacre et al., 1999; Marshall et al., 2006). On the other hand, both Goodacre et al. (1999) and Marshall et al. (2006) do conclude that in some scenarios the system has outperformed markets in the past while also accounting for transaction costs. Additionally, with higher liquidity in the markets, leading to smaller bid-ask spreads, and commission-free trading it is conceivable that CRISMA is still viable before adjustments.

A form of technical analysis that has seen an exponential increase in scientific coverage over the past 30 years is momentum-based analysis. The idea behind momentum trading revolves around purchasing assets that are trending up and shorting ones that are trending down across different time horizons. Much of the research on the topic is built around the application of the "J/K-strategies" – a form of portfolio assembling where purchases are made based on the performance of the past J months and then held for K months (Jegadeesh & Titman, 1993). These strategies are commonly applied on already diversified asset classes like exchange-traded funds (see Chan et al., 2000; Tse, 2015) or work with large stock universes such as the CRSP universe (see Jegadeesh & Titman, 1993; Rouwenhorst, 1998; Marshall et al., 2017). Past research suggests that momentum trading is conducted in the equities class for strategic asset allocations (Bange & Miller, 2004) and therefore working with a smaller stock universe could lead to a systematic trading approach that will outperform a given benchmark.

Momentum comes in several forms. The initial academic coverage on the topic works with relative momentum where given a specific J/K-strategy the assets in a given universe are ranked based on their performance over the past J periods, a quantile of these assets is then purchased and held for the following K periods before liquidation (see Jegadeesh & Titman, 1993; Rouwenhorst, 1998). An alternative to relative momentum is absolute momentum where the J/K-strategies are applied, but the assets are not ranked and the purchasing decision is entirely decided on whether the asset returned more than a given benchmark over the past J periods (see Moskowitz et al., 2012; Tse, 2015),. The combination of the two where assets are ranked, a specific quantile is purchased, and the returns need to clear a specific benchmark is referred to as dual momentum and has limited scientific coverage with the existing research commonly suggesting that it is capable of outperforming both relative and absolute momentum (see Antonacci, 2017; Lim et al., 2018).

In this thesis the author works with the equities included in the NASDAQ-100 index at any given time over a 16-year period from January 2005 to December 2020. The aim of the thesis is to determine whether the application of relative momentum, absolute momentum, and dual momentum can lead to statistically significant differences in profits when comparing the returns of given strategies with the returns of the NASDAQ-100. The NASDAQ-100 is one of the three primary indices commonly cited along with the Dow Jones and the S&P500, and is the most practical for momentum-related research as it has enough coverage across different sectors while also not suffering from the inclusion of too many companies. Application of the three momentum strategies on the components of a single index is non-existent among existing academic literature and delivers the potential for individual investors to apply market strategies that are commonly applied by asset managers. The statistical testing of the momentum profits is done through the utilization of a t-testing method which overlaps with existing research on the topic.

While research has historically trended towards the application of so-called “winner minus loser” portfolios where a given percentage of the asset universe is bought long (winners) and the same percentage is sold short (losers) the research has thoroughly defused the idea that shorting is a profitable strategy (see Jegadeesh & Titman, 1993; Carhart, 1997; Rouwenhorst, 1998; Banerjee and Hung, 2013; Lim et al., 2018). As a result, the author works with just the winner portfolios, thereby also increasing the practicality of the strategy due to potential broker-dealer constraints on short selling. Additionally, the author draws distinction by working with the J/K-strategies that are on the lower end in terms of length. While assets do have a tendency to go up over longer periods of time, by rebalancing the portfolios more often it is possible to establish whether the strategies truly have a market edge.

The thesis is organized as follows. In the next section the author gives a thorough summary of the literature by covering technical analysis as a whole as well as different momentum strategies, the differentiating factors between them, and the risk-reward characteristics achieved. The subsequent sections focus on the data and the methodology before leading into the empirical results. The closing section concludes the research while both highlighting sources of weakness and offering potential points of expansion.

1. Literature review

1.1. Application of technical analysis in trading

While quantitative analysis can be the foundation for thematic, value-based, fundamental, and other forms of investing, it can also be built around technical analysis. Park and Irwin (2007) find that the number of studies surrounding technical trading has spiked since 1995 with papers relating to both stock markets and foreign exchange markets making up the majority. However, more in-depth trading strategies built around technical analysis have received little coverage in academic literature as publication of fully-developed models and methods could potentially dissipate said market edge.

One of the first examples of a technical trading system to receive coverage in academic literature is the CRISMA system by Pruitt and White (1988). It utilizes cumulative volume, relative strength, and moving average components to assemble a profitable long-only investment strategy. CRISMA uses the moving average (MA) component to determine whether the market is going up by looking at the 50-day and 200-day moving averages with the optimal result being the 50-day MA crossing the 200-day MA from below, indicating short-term market strength. Subsequently, Pruitt and White (1988) apply relative strength to compare the returns of a given asset to the market itself with the asset outperforming the market being the goal. Lastly, the strategy assumes that cumulative volume has a positive slope, referring to the fact that over a given period the stock has seen an increase in trading activity. Since Pruitt and White (1988) and subsequent follow-up papers (Pruitt & White, 1989; Pruitt et al., 1992) several authors have conducted analysis to assess the robustness of the CRISMA system, usually concluding that CRISMA merely works because of the selection of equities (Goodacre et al., 1999; Marshall et al., 2006). However, both Goodacre et al. (1999) and Marshall et al. (2006) conclude that in some scenarios the system has outperformed markets in the past while also accounting for transaction costs. Additionally, with higher liquidity in the markets today, leading to smaller bid-ask spreads and commission-free trading it is conceivable that CRISMA is still viable before adjustments.

Simpler trading rules have been proposed by other authors. Brock et al. (1992) use moving average and trading-range break[2] rules on the Dow Jones Industrial Average (DJIA), a U.S. equities index, generating buy and sell signals in the process. The results allow Brock et al. (1992) to conclude that technical analysis does assist in predicting stock prices as different variable-length moving average rules outperform any unconditional returns with the buy signals being statistically significant in most cases and sell signals being statistically significant in all cases, leading to very high t-test results for a "buy minus sell" strategy. However, Bessembinder and Chan (1998) apply the same rules as Brock et al. (1992) and find significantly smaller profits when adjusting the data for dividends with the profits decreasing even further as non-synchronous[3] trading is applied. Mills (1997) transfers the methodology of Brock et al. (1992) to the London Stock Exchange and shows that a simple moving average rule cannot outperform a buy-and-hold strategy on the most recent subsample from 1975 to 1994, suggesting that these variable-length moving averages are not the best technical indicators in a strong bull market.

The results by Brock et al. (1992) are supported by Lo et al. (2000), but in a different way. The methodology of Lo et al. (2000) ties into the most used technical patterns, commonly referred to as candlestick patterns. Candlestick patterns take the price action an asset receives during some specific period and display the highest, the lowest, the opening, and the closing price achieved during that period in the form of a rectangle. An example of a candlestick chart is presented in Figure 1. Since there are an infinite amount of periods additional rectangles will be formed, leading to the patterns themselves being formed. When Lo et al. (2000) compare the actual data of the market over the 35-year sample to the simulated data the simulation misses the mark on almost every pattern with certain patterns being present more than 14 times more in the actual data. When comparing these candlestick patterns by raw 1-day normalized returns, the top pattern among NYSE/AMEX stocks (inverse head-and-shoulders) allows for a mean daily excess return of 0.040% and the top pattern among NASDAQ stocks (rectangle top) allows for a mean daily excess return of 0.052%.

[2] The trading-range break rule in Brock et al. (1992) refers to the breaking of resistance and support levels. The resistance level is a specific value or range of values through which the stock is unable to break through to the upside. Similarly, the support level is a specific value or range of values through which the stock is unable to break through to the downside.

[3] Non-synchronous trading means purchases are made some time after the signal to purchase is given.

Figure 1. A candlestick chart of the NDX.

Notes: The chart uses a daily frequency, meaning every candle represents one trading day. The dotted line in the middle represents the closing price as of the 14^{th} of May, 2021.

Source: compiled by the author using *TradingView*

Based on existing research the best applications of technical analysis in trading extend beyond the topic of this thesis, namely into the foreign exchange markets. LeBaron (1999) looks at the foreign exchange series of both the German mark and the Japanese yen. In both weekly and daily cases the simulation estimates show mean annualized returns ranging from 7 to 10 percent with the Japanese yen having higher returns and lower volatility, including smaller standard deviations and drawdowns. However, the profitability and as a result the Sharpe ratios of these strategies heavily depend on the level of transaction costs present in the market. Neely (2002) advances the work of LeBaron (1999) by expanding the sample size to also include the Swiss franc and Australian dollar with the trading rule again showing annualized profitability, yet the results are not statistically significant for the Australian dollar. Both LeBaron (1999) and Neely (2002) also deal with intervention from central banks and show that when removing specific intervention-related observations the profitability of these daily and weekly trading rules suffers with Neely (2002) suggesting that the statistical significance of the results disappears for all currency pairings outside of the yen-dollar (JPY/USD).

1.2. Application of momentum in trading

A different approach to trading based on technical analysis first garnered attention in the 1980s after the works of De Bondt and Thaler (1985, 1987). De Bondt and Thaler find in their research that loser portfolios, i.e. portfolios that consist of stocks with negative returns over a past period, will on average outperform both the market itself and winner portfolios over 16 different three-year periods, thereby covering the entire sample period of the two authors. Another takeaway from De Bondt and Thaler (1985) ties into how most of the difference between the loser and winner portfolio is accumulated after the first year in the three-year period, suggesting that there is a specific window of up to one year where the portfolios behave similarly.

Following the research by De Bondt and Thaler (1985, 1987) there has been an exponential increase in the scientific coverage of momentum strategies with the first recognizable pieces on the topic being Jegadeesh (1990) as well as Jegadeesh and Titman (1993). The idea behind momentum and its application within both trading and asset management revolves around purchasing assets that are trending up and shorting ones that are trending down with the assets usually distributed into winner-loser portfolios where long positions are taken in the assets with strong upside momentum and short positions are taken in ones with strong downside momentum (see Jegadeesh & Titman, 1993; Carhart, 1997; Rouwenhorst, 1998; Chan et al., 2000; Tse, 2015). The reasoning behind building such portfolios stems from the assumption that the underlying assets will continue to trend in the same direction with past winners (losers) continuing to accumulate profits (losses). The portfolios themselves are frequently assembled across different time horizons and tend to follow the application of the "J/K-strategies" – a form of portfolio assembling first mentioned by Jegadeesh and Titman (1993) where asset purchases are made based on the performance of the past J months and then held for K months before being sold off.

Academic literature is dominated by the application of relative momentum (cross-sectional momentum) strategies where the aforementioned J/K-strategies are applied on a group of assets, allowing us to rank these assets prior to purchase. However, both the assets purchased and the quantity of them heavily varies. While literature does commonly list the winner portfolio, the loser portfolio, and the winner-loser portfolio, the winner (loser) portfolios themselves will usually be made up of either the top (bottom) decile or quintile of a given asset universe. Most of the research includes results on an equally-weighted portfolio where the percentage that a specific asset will make up in the winner or loser portfolio is equal. Certain parts of the literature on the topic will also analyze size-based (also referred to as value-based) portfolios where the market capitalization of the underlying asset will determine what weight we assign to the position. The opposite of relative momentum is absolute momentum (time-series momentum) where we are strictly using the past returns of an asset and given that this return clears a predefined return quota a portion of the portfolio is assigned to the asset (see Moskowitz et al., 2012; Marshall et al., 2017). Research has also started to take note of dual momentum where relative and absolute momentum are applied together (see Antonacci, 2017; Lim et al., 2018) and a recent paper by Singh et al. (2020) has brought attention to a triple momentum strategy, also termed as “macro-momentum” by the authors, where the lagged 1-month and 24-month returns of the market are compared to determine whether the approach should be long only, short only, or both.

Momentum strategies are also the subject of trading systems within non-academic literature. The two books worthy of notation here are those by Antonacci (2014) and Clenow (2015). It must be noted that these examples of non-academic literature and the specific details used in the approaches applied by the authors have not been statistically tested, especially the elements that Clenow (2015) uses in his momentum trading system. The author of the present thesis believes that the approaches stem from the successful back-testing of certain quantitative investment strategies.

In his book, Clenow applies relative momentum within the context of a pre-defined trading system with specific market and stock criteria. However, instead of using the J/K-strategies, Clenow measures the momentum through exponential regression by finding the slope of a price series over the past 90 days which is then annualized and multiplied by the coefficient of determination. The amount of stock purchased by Clenow’s approach is determined through the value of an investor-specific risk factor and the average true range which estimates the volatility by looking at the highs and lows reached by a stock over a 20-day period. Clenow uses the top quintile approach used in momentum strategies related research and also includes a simple moving average rule to assess the general status of the market, a common approach in technical analysis that is also in the CRISMA system of Pruitt and White (1988). The notable result for the strategy is that across the duration of the sample the relative momentum strategy outperforms the S&P500 by 7 percentage points in annualized returns with a maximum drawdown that is 31 percentage points lower than that of the S&P500.

In the book of Antonacci (2014) the application of momentum is more general. Antonacci applies relative, absolute, and dual momentum on the MSCI All Country World Index (MSCI ACWI) and the S&P500 over a 40-year sample from 1974 to 2014. The conclusions drawn by Antonacci regarding the profitability of the strategies is in line with the conclusions made in a majority of academic research as relative momentum outperforms absolute momentum in most cases. However, the key takeaway from the book is that a dual momentum strategy outperforms both relative and absolute momentum across all subsamples. Given the current economic climate with extremely low or even negative interest rates the use of U.S. T-bills as a benchmark is likely to not be valid for absolute momentum. Antonacci (2014) does also highlight a handful of J/K-strategies with differing J values showing a 12 month look-back period performs the best in terms of both returns and Sharpe ratio.

Academic literature notes several advantages to using momentum strategies in capital markets over passive strategies like the buy-and-hold strategy. While the returns of these approaches either over the long-term or specific subsamples have differing views within academic literature Hurst et al. (2017) note that in the case of the biggest market drawdowns for a 60/40 portfolio a time-series momentum approach would both severely decrease drawdowns and also lead to profitable investing during the volatile periods. Similarly to the results from the books by Antonacci (2014) and Clenow (2015) the decrease in drawdowns or volatility has been showcased in academic literature for different momentum approaches (see Tse, 2015; Antonacci, 2017). Previous weak points of momentum have also started to dissipate as one-way transaction costs for equities, bonds, commodities and currencies have all seen continuous decreases dating back from 1880 to 2013 (Hurst et al., 2017).

Momentum also appears to show up in the investment strategies of institutional investors. Grinblatt et al. (1995) look at the application of momentum in the asset allocations of mutual funds and find that 76.8% of mutual funds engage in some form of momentum trading with the winning positions in a lag-free system earning quarterly returns of 1.03%. Over the sample period from the end of 1974 to the end of 1984 Grinblatt et al. (1995) suggest that mutual funds do not short losing stocks, highlighting the lack of statistical significance. Badrinath and Wahal (2002) analyze Form 13-F statements filed with the U.S. Securities and Exchange Commission (SEC) by institutional investors, concluding that momentum trading does take place and similarly to Grinblatt et al. (1995) the viable strategy is longing the winners. Bange and Miller (2004) apply a different approach to momentum by looking at how applicable the topic is to strategic asset allocation in the portfolios of institutional investors, i.e. what weights are assigned to which asset classes. The results by Bange and Miller show that higher (lower) weights in the portfolio would be assigned to the best (worst) performing assets with the results being statistically significant in the case of both equities and cash.

1.2.1. Diversified vs. non-diversified assets in momentum-based analysis

Academic literature related to different momentum strategies has some common denominators when it comes to which types of assets are being used to assemble the winner-loser portfolios. Throughout scientific coverage the strategies tend to be applied on already diversified asset classes such as exchange-traded funds (ETFs). Both ETFs themselves and the purchasing of ETFs do provide certain benefits that single equities do not, including higher liquidity, lower maximum drawdowns, lower transaction costs, and better availability among others. An additional advantage of using ETFs for momentum-related analysis ties into the data availability as the time-series of fewer assets need to be recovered and this data is also more thoroughly available.

Chan et al. (2000) work with 23 country ETFs from the Asia-Pacific region, Europe, America and Africa with a sampling period from 1980 to 1995. While Chan et al. (2000) consider only five different J/K-strategies where J and K are equal, the research by Chan et al. is one of the very few to include periods shorter than 1 month by including both a 1-week strategy and a 2-week strategy. The initial results by Chan et al. show that the profits are highest for the 1-week, 2-week and 4-week strategies while also being statistically significant and that in most cases the predictability within the equity markets is the primary cause for momentum profits, in some cases making up as much as 93% of the weekly returns.

Results that oppose Chan et al. (2000) are reported by Tse (2015) who similarly tests the viability of momentum by including ETFs at both country and sector level, among them 23 country ETFs from the same regions as Chan et al. (2000) as well as 14 U.S. sector ETFs. Tse applies 25 different J/K-strategies for both cross-sectional and time-series momentum strategies across a sample period primarily from 1997 to 2014, but is unable to prove the statistical significance of the differences in returns in most cases and cannot outperform a buy-and-hold strategy.

Alternative diversified assets have also been covered in academic literature. Carhart (1997) looks at the performance of mutual funds through several different models, including a factor-mimicking portfolio for one-year return momentum in Fama-French's four-factor model (FF4). In the equally-weighted decile portfolios based on lagged one-year returns, Carhart shows that when utilizing FF4 the momentum factor explains almost half of the 67-basis-point difference in the top and bottom deciles with approximately 88% either being described by the momentum factor or falling under exogenous factors not in the model through the intercept.

One of the first examples of the application of momentum strategies on single stocks is that of Jegadeesh and Titman (1993). The two authors show that over a sample period from

1965 to 1989 there are statistically significant profits in most cases of the 32 different J/K-strategies covered where half of the strategies form a portfolio instantly after the returns are calculated (synchronous trading) and the other half form the portfolios one week after the calculations (non-synchronous trading). Since Jegadeesh and Titman form their portfolios on the top and bottom deciles without any transaction costs, there might be an increased likelihood that the returns on these portfolios are higher than those mentioned in other articles where both the top and bottom quintiles are used, and costs are not zeroed out. When dealing with the (6, 6)-strategies where the returns are calculated based on the data of the last six months and the positions are held for six months the two authors show that in six out of seven cases the average monthly returns are highest when we are dealing with the portfolio in the top performing decile and that the best performing portfolios are those that contain either the stocks of the smallest firms in terms of market capitalization or the stocks with the highest Scholes-Williams betas. This result suggests that an approach with non-diversified assets could outperform one with diversified ones. However, the risk that the investor takes on, whether that be measured in maximum drawdown, standard deviation or some other risk metric, is bound to be far higher.

Just like in the case of ETFs, the results differ across academic literature. Rouwenhorst (1998) follows the methodology applied by Jegadeesh and Titman (1993) on a sample consisting of 2,190 firms in 12 European countries and finds statistically significant profits in every J/K-strategy at a 95% level. Novy-Marx (2012) applies Fama-MacBeth regressions on the returns of firms in a sample consisting of the stocks in the CRSP universe, a stock database hosted by the Center for Research in Security Prices, across more than 80 years and finds that the predictive power of recent returns is noticeably weaker than the predictive power of intermediate terms, suggesting that any J/K-strategies where the value of J ranges from two to six months are outclassed by the strategies where J ranges from seven to twelve months.

1.2.2. The viability of shorting as a strategy in momentum-based analysis

Literature surrounding momentum strategies tends to distribute portfolios into winner and loser portfolios by determining either the excess return or the pure return of the assets in the sample and then ranking them. Authors applying this relative momentum approach will commonly also list the risk and return statistics of a “winner minus loser” (winner-loser, WML) portfolio where a long position is taken in the best-performing quantile and a short position is taken in the worst-performing quantile. However, literature has appeared to suggest that shorting as a strategy lacks any value when applying momentum strategies. Clenow (2015) similarly suggests in his book that the short side is very difficult as professional futures trend followers make very little money shorting.

One of the reasons why shorting does not appear to be viable in the momentum strategies case ties into the profitability of the bottom-ranked portfolios. Since the loser portfolios are profitable, a short position in these portfolios would result in net losses. Jegadeesh and Titman (1993) show that for U.S. equities different J/K-strategies will always lead to profits, even when buying the loser portfolios, i.e. taking long positions. Rouwenhorst (1998) shows over a sample from 1980 to 1995 that when applying different J/K-strategies to European equities loser portfolios will yield significant profits. For the portfolios of mutual funds formed based upon a J value of 12 months, Carhart (1997) reports that even the funds with the lowest one-year returns manage a monthly excess return of 0.01% and only the bottom third of the loser portfolio will suffer negative returns. This result indicates that shorting approximately 96.7% of the stock universe of Carhart (1997) will lead to losses.

Since academic literature suggests that the bottom-ranked portfolios will quite commonly have positive returns, it is to be expected that the winner-loser portfolios will underperform the winner portfolios. In the case of global ETFs, Tse (2015) does not separate between long and short positions in the winner and loser portfolios, but the resulting winner-loser portfolios lack statistical significance and heavily fluctuate as the values of J and K change with plenty of returns even being negative. Ahn et al. (2003) show that the mean returns of a long-short strategy are most impacted by the inclusion of a short-strategy when the J values are smaller, e.g. 3 or 6 months. This means that shorting based on how the market behaves in the near-term could lead to catastrophic losses. Banerjee and Hung (2013) find that over a sample period from 1927 to 2005 the loser portfolio yields a 0.7% mean excess return per month, leading to the winner portfolio outperforming the winner-loser portfolio.

1.2.3. Different approaches to momentum-based analysis

A majority of the literature related to momentum strategies is tied to the calculation of relative strength portfolios – a form of portfolio assembling where the assets are ranked in ascending order on the performance of the past J periods at the end of each period. Antonacci (2017) similarly suggests that relative momentum is positive if an asset has appreciated more than another asset. These ranked assets are usually then assembled into deciles or quintiles, making up the portfolios on which the authors apply the J/K-strategies. However, an alternative approach would be to form portfolios on absolute momentum where the determining factor of portfolio inclusion is the past return of the underlying asset and whether that return exceeds a specific level or benchmark. Additionally, a more recent innovation has also brought attention to dual momentum where both relative and absolute momentum are applied.

Relative momentum is the foundation of the results of Jegadeesh and Titman (1993). Jegadeesh and Titman apply cross-sectional momentum in U.S. equities and find the highest monthly profits when the stocks are ordered based on a medium length period of nine to 12 months and the portfolios are held for a shorter period of three months. The monthly returns in the case of long-only winner strategies range from 1.71% to 1.92% when the portfolios are formed in synchronous fashion and from 1.79% to 1.96% when the portfolios are formed in non-synchronous fashion with a one week delay. When dealing with European equities Rouwenhorst (1998) finds that the monthly profits similarly peak when the ordering is based on a period of nine to 12 months with the portfolios held for three months. Rouwenhorst also reports similar results to Jegadeesh and Titman (1993) when it comes to which timing of portfolio formation is more profitable as the winner strategies on average return 2.12% to 2.19% when calculated immediately instead of the 2.08% to 2.09% when calculated non-synchronously with a one month delay.

The results appear to be different with exchange-traded funds as Chan et al. (2000) find that when both the ranking and the holding are two weeks then the weekly profits are at their highest at 0.48%. However, Chan et al. do not consider enough J/K-strategies to unequivocally state that a scenario where J and K are both small and equal is the best for profitability and the values of J and K selected by Chan et al. differ from most of the academic literature. Tse (2015) similarly reports that portfolios consisting of global ETFs are not statistically significant regardless of both the values of J and K selected, and the methodology applied for longing and shorting portfolio components.

Coverage of time-series momentum is not as common as that of relative momentum. Moskowitz et al. (2012) look at multiple different asset classes (commodities, currencies, equity indices, and bonds) and apply J/K-strategies on the regressions of time-series momentum strategies. While Moskowitz et al. do not directly report returns, conclusions can be drawn from the (12, 1)-strategy that they apply and then showcase through Sharpe ratios which show profitability for every single futures contract in the 58 asset sample that includes commodities, equities, bonds, and currency pairs. The results of a time-series momentum trading system are also reviewed by Marshall et al. (2017) who apply four separate J values across five CRSP quintile value-weighted size portfolios. Marshall et al. find that the mean excess returns are at their highest when we are dealing with the shortest look-back periods, meaning the smallest J values, and the portfolios consisting of the firms with the smallest market capitalization. The authors showcase mean excess returns of up to 19.5% for small cap firms when J is set to ten days, yet when dealing with the firms that have larger market capitalizations, the excess returns increase along with the J value with annual returns scaling from 2.4% to 5.2% as the J value scales from ten days to 200.

While Antonacci (2017) does not directly address the profitability of time-series momentum, he does show that trading systems which apply both relative and absolute momentum outperform those with just relative momentum in terms of both risk and return. The dual momentum strategy reported by Antonacci earns an annual return of 15.8% compared to the 13.5% of the relative momentum strategy while also decreasing the annualized standard deviation and the maximum drawdown for the 40-year sample period. The approach by Antonacci (2017) is also applied in the research of Lim et al. (2018) who apply dual momentum by initially measuring the stocks on absolute momentum and then by relative momentum. The winner (loser) portfolios established include the stocks that have both positive (negative) returns over the past 11 months and are ranked in the top (bottom) quintile of stocks based on the returns of the last 11 months. The results by Lim et al. (2018) suggest that a value-weighted dual momentum strategy outperforms a standard time-series momentum strategy with monthly profits of 1.74% compared to 0.76%.

1.2.4. The role of the J/K-strategies in momentum-based analysis

As indicated throughout the previous subchapters the J/K-strategies are the most commonly applied method of approach to momentum-related research. However, the values selected for both of these variables do vary and so do the results that go with them, whether that be in the returns themselves, the dynamics of the returns i.e. which pairings of J and K lead to which results, the statistical significance of the results, and more. The author reviews the existing scientific literature on the application of cross-sectional (relative), time-series (absolute), and dual momentum in Table 1. The ordering of the papers is based on the strategies being applied with the papers that include relative momentum either as the primary approach or as one of the approaches being listed first and the papers that cover absolute momentum as the main method or as one of the methods without the inclusion of relative momentum being listed after the relative momentum papers. No papers strictly covering dual momentum are noted in Table 1 as the existing research on the method is miniscule and commonly includes other momentum approaches with it to highlight the potential upsides of the strategy.

Table 1

Summary of the results of previous studies surrounding the application of relative, absolute, and dual momentum.

Author(s)	Sample period and stock universe	Momentum approaches applied with additional details	Values of J and K selected with total strategies	Profitability	Statistical significance of results
Jegadeesh and Titman, 1993	Sample period: from 1965 to 1989 Universe: NYSE and AMEX stocks from the CRSP universe[4]	Relative momentum • decile approach • W/L/WML[5] • synchronous vs. non-synchronous[6] • equally-weighted	J and K values of three, six, nine, and 12 months. 32 total strategies - 16 with synchronous and 16 with non-synchronous trading.	Best profitability for WML: (12, 3) Most profitable: long only (12, 3) Profitability increases as the J value increases, but profitability decreases as the K value increases.	Methodology: standard t-statistics Results: Statistical significance at a 99% level for all long-only strategies. Significance for both the loser and the WML portfolios vary at a 95% level, but tend towards statistical significance as K increases.
Rouwenhorst, 1998	Sample period: from 1978 to 1995 Universe: 2,190 firms from 12 European countries	Relative momentum • decile approach • W/L/WML • synchronous vs. non-synchronous • equally-weighted	J and K values of three, six, nine, and 12 months. 32 total strategies - 16 with synchronous and 16 with non-synchronous trading.	Best profitability for WML: (12, 3) for synchronous, (9, 3) for non-synchronous. Most profitable: long only (12, 3) for synchronous, and long only (9, 3) for non-synchronous. Profitability increases as the J value increases, but profitability decreases as the K value increases.	Methodology: standard t-statistics Results: Statistical significance only reported for WML portfolios. Statistical significance at a 95% level supported across all WML portfolios with most cases also supporting it at a 99% level.

[4] The CRSP universe refers to the stock data universe hosted by the Center for Research in Security Prices in affiliation with the University of Chicago Booth. As of the 8th of May, 2021, the universe includes data on stocks that are listed on NYSE, NYSE American, NASDAQ, and NYSE Arca. Read more at http://www.crsp.org/.

[5] Abbreviations for the winner (W), the loser (L), and the winner minus loser (WML) portfolios. The winner portfolio includes the top performers over a given J period that are bought long, the loser portfolio consists of the bottom performers over a given J that are shorted, and the winner minus loser portfolio is a simultaneous long-short approach.

[6] Synchronous trading refers to the act of purchasing instantly after the tracking period (J period) concludes. For non-synchronous trading there is a period between the tracking period and the purchasing period, usually 1 week or 1 month.

Table 1 cont.

Author(s)	Sample period and stock universe	Momentum approaches applied with additional details	Values of J and K selected with total strategies	Profitability and volatility	Statistical significance of results
Chan et al., 2000	Sample period: from January 1980 to June 1995 Universe: Equity market indices of 23 countries, one index per country	Relative momentum • WML with weights based on deviation from cross-sectional average[7]	J and K values of one week, two weeks, four weeks, 12 weeks, and 26 weeks. Total of five (5) strategies.	Most profitable: the (0.5, 0.5) strategy where a two week holding period generates a 0.48% weekly return. Profitability appears to decline as K is increased as the longest K periods have the smallest returns.	Methodology: z-statistics that are corrected for heteroscedasticity and autocorrelation based on the Newey-West adjustment (HAC). Results: Four strategies out of five are statistically significant at a 95% level.
Ahn et al., 2003	Sample period: from 1963 to 1997 Universe: NYSE and AMEX firms	Relative momentum • decile approach • W/L/WML • equally-weighted	J and K values of three, six, nine, and 12 months. Total of 16 strategies.	Best profitability for WML: (12, 3) Most profitable: long only (12, 3) Profitability increases as the J value increases, but profitability decreases as the K value increases.	No statistical testing of the results of the relative momentum strategy as it is not the primary subject of the paper.
Carhart, 1997	Sample period: from January 1962 to December 1993 Universe: 1,892 diversified equity funds (mutual funds)	Relative momentum • decile approach • W/L/WML, in-between portfolios[8] • equally-weighted • active rebalancing[9]	J and K values of one year for a total of one strategy. Paper mainly focused on CAPM and FF4[10].	Winner portfolio is both the most profitable and the most volatile (std. dev.). WML portfolio has the same return at half the volatility. Returns decline from winner to loser, std. dev. lowest in the middle.	No statistical testing of the results of the relative momentum strategy as it is not the primary subject of the paper.

[7] Chan et al. (2000) take the returns of all 23 indices and find their average. If the return of a given index is higher (lower) than the average it is assigned to the winner (loser) portfolio. Weights are assigned based on the difference between the return of a given index and the average of the returns of all indices.
[8] The phrase 'in-between portfolios' refers to the portfolios that are between the winner portfolio and the loser portfolio in terms of returns over a given J period.
[9] Constant purchasing and selling of assets in all portfolios to guarantee that the weights of positions remain true to whichever strategy the author is applying.
[10] The abbreviation 'FF4' refers to the Fama-French four-factor model, an extension of the three-factor model (FF3) that includes a momentum proxy.

Table 1 cont.

Author(s)	Sample period and stock universe	Momentum approaches applied with additional details	Values of J and K selected with total strategies	Profitability and volatility	Statistical significance of results
Banerjee and Hung, 2013	Sample period: from January 1926 to December 2005 Universe: NYSE, NASDAQ, and AMEX stocks from the CRSP universe	Relative momentum • decile approach • W/L/WML • equally-weighted	J and K values of six months for a total of one strategy. Paper mainly focused on the comparison of risk-reward metrics between momentum and NDS[11].	Both the winner and the loser portfolio are profitable, leading to the winner portfolio outperforming the WML portfolio. Returns for the whole sample period are lower than those reported in Jegadeesh and Titman (1993).	Methodology: standard t-statistics Results: Both the winner and the WML portfolios are statistically significant at a 99% level in all sub-samples outside of the Great Depression.
Novy-Marx, 2012	Sample period: January 1926 to December 2010 Universe: all of the stocks in the CRSP universe	Relative momentum • decile approach • WML • equally-weighted and value-weighted are both used	No direct application of the J/K strategies. 15 strategies with K set to one month are included as the paper seeks to determine the optimal length of J. CAPM, FF3 and FF4 are also applied.	The (1, 1) strategy loses money, but the returns increase as J goes up. Returns reach their peak at a J of 12 months. Intermediate horizon (seven to 12 months) outperforms recent horizon (two to six months). The equally-weighted portfolios generally generate lower returns at a lower level of volatility, leading to similar Sharpe ratios.	Methodology: standard t-statistics Results: Relative momentum as a strategy brings with it returns that are statistically significant at a 99% level regardless of whether J is a more recent period (two to six months) or an intermediate one (seven to 12 months).

[11] 'NDS' is the abbreviation for 'naive diversification strategy', a passive investment strategy described by Banerjee and Hung (2013) where given a universe with N stocks each stock is allocated an equal weight of $(1/N)$ with rebalancing done every period.

Table 1 cont.

Author(s)	Sample period and stock universe	Momentum approaches applied with additional details	Values of J and K selected with total strategies	Profitability and volatility	Statistical significance of results
Tse, 2015	Sample period varies. Most assets are from January 1997 to December 2014, for U.S. sector ETFs the earliest is from January 1999. Universe: 23 country ETFs and 14 U.S. sector ETFs	Relative momentum • custom approach[12] • WML • equal weights and proportional weights[13] are both used Absolute momentum • the excess returns are calculated through the 1-month T-bill rate	For the relative momentum approach J and K values of one, three, six, nine, and 12 months are used. For the absolute momentum approach the same values are used, but only the five strategies where J equals K are used.	For country ETFs the relative momentum approach is weakly profitable in approx. 60% of the cases with (3, 1) performing best. For US sector ETFs the (6, 6) ranks as the top relative momentum strategy. For absolute momentum the pooled returns across different ETFs peak in the case of the (3, 3) strategy.	Methodology: t-statistics that are calculated with HAC consistent errors. Results: No relative momentum strategy, regardless of weight approach, reaches even a statistical significance of 90%. Some examples of statistical significance at a 95% level are present for absolute momentum.
Antonacci, 2017	Sample period: from 1974 to 2011 Universe: different assets are covered, including equity indices, real estate, commodities and more	Relative momentum • active rebalancing Absolute momentum • 1-month T-bill rate as benchmark Dual momentum	The paper is mainly focused on the (12, 1) strategy, but (3, 1); (6, 1); and (9, 1) are also reported. The approaches are applied within asset classes[14].	Dual momentum outperforms relative momentum in an equities universe, but the strategies are virtually tied for other universes in terms of returns. Decreasing the J period from 12 to three, six, or nine will reduce the returns in an equities universe.	No statistical testing of the results of the relative momentum and dual momentum strategies are reported.

[12] Instead of applying the decile or quintile approach commonly used in literature Tse (2015) buys long (sells short) four country ETFs and two U.S. sector ETFs which make up the winner (loser) portfolio, approximately corresponding to a quintile approach.
[13] Proportional weights in Tse (2015) refers to an asset being assigned a weight based on the degree to which a given ETF outperforms the equally-weighted mean of all ETFs.
[14] The approach of Antonacci (2017) is to select a couple assets per asset class and to use momentum in order to see if it is possible to outperform the assets in that class.

Table 1 cont.

Author(s)	Sample period and stock universe	Momentum approaches applied with additional details	Values of J and K selected with total strategies	Profitability and volatility	Statistical significance of results
Singh et al., 2020	Sample period: from 2005 to 2020 Universe: all of the stocks listed on the Bombay Stock Exchange	Relative momentum • decile approach • W/L/WML • non-synchronous • equally-weighted Absolute momentum • selection based on the returns[15] Dual momentum Triple momentum[16]	Fixed J value of 12 months with varying K values of one, three, six, nine, and 12 months. Five (5) strategies per momentum approach. Paper also reports the results from CAPM and FF3.	For relative and dual momentum the top strategies for the best raw returns are (12, 3) and for absolute momentum it is (12, 1). The Sharpe ratios favor the smaller K values of one, three and six months. Triple momentum outperforms the other approaches in several metrics. Profitability is generally declining as K increases.	Methodology: t-statistics that are calculated with HAC consistent errors. Results: Relative, absolute, and dual momentum achieve a 95% statistical significance regardless of the K period. Statistical significance of 99% and more for all cases where K is either one or three months.
Moskowitz et al., 2012	Sample period: from Jan 1965 to December 2009 Universe: futures prices for equity indices, cross-currency pairs, commodities, and government bonds.	Absolute momentum • the excess returns are calculated through the 1-month T-bill rate	J and K values of one, three, six, nine, 12, 24, 36 and 48 months. Total of 64 strategies observed. Paper mainly focused on regressions.	The profitability of the strategies is not directly reported. The authors do report regression results where the returns of absolute momentum are regressed on the FF4 factors and the returns of the MSCI World Index. The variables are ineffective as the intercept remains stat. significant at a >99.9% level and R-squared ranges from 14 to 34%.	Methodology: standard t-statistics on the independent variables of a regression where absolute momentum returns depend on the FF4 factors and the returns of global equities, bonds, and commodities. Results: Statistical significance declines as J and K increase. Most small J, small K (up to 12 months) strategies are statistically significant at a 99% level. The largest values of J and K are mostly statistically insignificant, even at a 90% level.

[15] The absolute momentum approach does not have a specific benchmark beyond an asset needing to have positive returns.

[16] Triple momentum is a concept first mentioned in Singh et al. (2020) where the authors include a macro-momentum factor that allows for a more dynamic approach to the market by determining whether just the winner portfolio, just the loser portfolio, or the WML portfolio should be invested in. The decision is made by comparing the lagged 1-month and 24-month returns with the comparison between the two and their values (i.e. whether the returns are positive or negative) being the purchasing criteria.

Table 1 cont.

Author(s)	Sample period and stock universe	Momentum approaches applied with additional details	Values of J and K selected with total strategies	Profitability and volatility	Statistical significance of results
Hurst et al., 2017	Sample period varies, ranging from January 1880 to December 2013. Universe: total of 67 markets across four major asset classes (currency pairs, , commodities, bonds, equity indices)	Absolute momentum • equally-weighted • WML • selection based on the returns	The author covers just one strategy where they are applying a J period of one, three, and 12 months at the same time, weighting them equally and then holding them for a K period of one month.	Since there is just one strategy the takeaways are not of much value. The authors do show that allocating a weight of 20% to an absolute momentum strategy improves the characteristics for both risk and reward in the case of a 60/40 portfolio. Absolute momentum is able to profit in eight out of the ten worst drawdowns for a 60/40 portfolio[17].	No statistical testing of the results of the absolute momentum strategy are reported.
Lim et al., 2018	Sample period varies. CRSP stocks start from 1926, four different starts are noted for Europe[18], periods end in 2017 Universe: NYSE, NASDAQ, and AMEX stocks from the CRSP universe; equities from 12 European countries	Absolute momentum • equally-weighted, value-weighted (based on market capitalization), volatility-weighted (based on std. dev) • non-synchronous • W/L/WML Dual momentum	The authors report on a (12, 1) strategy where the J period is set to 11 months and the 12th month is an intermediate period for non-synchronous means. Risk-adjusted returns are also regressed on CAPM, FF3 and FF5.	The equally-weighted approach for absolute momentum has the highest returns for all of the W/L/WML portfolios. Dual momentum outperforms the best absolute momentum approach in returns, but does so while taking on more volatility with higher standard deviations.	Methodology: standard t-statistics Results: For both absolute and dual momentum the winner portfolios achieve the highest levels of statistical significance (>99.9%) with the statistical significance of WML portfolios fluctuating between 95% and 99%.

Source: compiled by the author

[17] The ten worst drawdowns for a 60/40 portfolio over the entire sample period of the authors includes several wars, flash crashes, and economic crises. See Hurst et al. (2017), page 4, exhibit 3 for more details. By drawdown the authors are referring to the decrease in the price of an asset or portfolio from its peak to its trough over a given period.
[18] Nine of the twelve countries start in 1975; the remaining three countries (UK, Sweden, and Spain) start in 1956, 1984, and 1988, respectively.

The selection of J and K values noted in Table 1 shows that existing literature rarely considers values beyond the commonly applied ones. A significant share of the articles covering relative momentum look at just the strategies where J and K are either three, six, nine or 12 months. Another common approach in literature is to fix J (K) to then seek out the value of K (J) that brings with it the best risk-reward characteristics. This is mainly due to the fact that research has repeatedly suggested that a (12, 1) strategy where the look-back period is 12 months and the holding period of the assets is one month is best suited for the application of relative momentum (see Jegadeesh & Titman, 1993; Rouwenhorst, 1998; Ahn et al., 2003). Antonacci (2017) similarly focuses on the (12, 1) strategy, but does also report three others strategies where J is either three, six, or nine months. Furthermore, papers that are comparing momentum to different strategies will usually stick to just one J/K-strategy (see Banerjee & Hung, 2013) and papers examining the market anomaly that is momentum tend to also look at just one J/K-strategy (see Carhart, 1997).

Specific trends can be noted across different J/K-strategies in most of the literature. A majority of the literature in Table 1 that is focused on non-diversified assets supports the claim that the best profitability is reached either when J is set to 12 months or when it is relatively close to 12 months with an intermediate horizon of seven to 12 months. Values beyond 12 months are seldom covered, but the general understanding is that as soon as the tracking period exceeds 12 months the statistical significance of the results starts to decrease and the profitability plummets. For the values of K the literature is less clear since K is more commonly fixed in the existing research. Several papers (see Jegadeesh & Titman, 1993; Rouwenhorst, 1998; Chan et al., 2000; Ahn et al., 2003) showcase profitability declining as the holding period extends, suggesting that a shorter holding period is superior. Asset selection does not appear to reject this idea either as peak profitability in existing literature tends to be reached when the value of K is small, most commonly either one or three months.

2. Empirical analysis on the example of the NASDAQ-100 index

2.1. Data

Since there is no freely accessible method of acquiring the historical composition of the NASDAQ-100 (NDX) index over the sample selected, the author has manually assembled the data set used in this thesis. A list including all of the stocks within the NDX as of the 31st of December, 2020 is used as the initiation moment and then the index is reverse-engineered for the authors' sample period starting from the 31st of December, 2020 and ending on the 1st of January, 2005. The author uses two primary sources for cross-validation of the data: the website

ETFdb.com[19] and the NASDAQ Investor Relations (NIR) database. Additional sources are sought out in cases where the two sources do not validate one another or when one of the two is missing values. These additional sources most commonly are traditional financial media companies such as Bloomberg, CNBC, TheStreet, and others.

The resulting overview of companies being added and removed is accurate for a wide majority of the sample period, but some inaccuracies are still present. The main issue with the primary sources selected is that there is a period from January 2006 to June 2007 where cross-validation through the primary sources is not possible as the NIR database has no announcements regarding any changes taking place in the NDX for said period. Additional sources are unable to fill in the gaps, meaning for the aforementioned period, the NDX removes 14 tickers from the index while only bringing in just four. Another issue for the sample period concerns 2012 and 2013 where a missing cross-validation leads to the author making an exclusion by keeping a stock in the dataset for an additional six months. Figure 2 demonstrates how outside of 2006 and 2007, there is a tendency for stocks to get replaced by other stocks at a 1:1 ratio. Furthermore, later differences can largely be attributed to the simultaneous removal or addition of a company that has multiple types of shares, e.g. class A and class C shares.

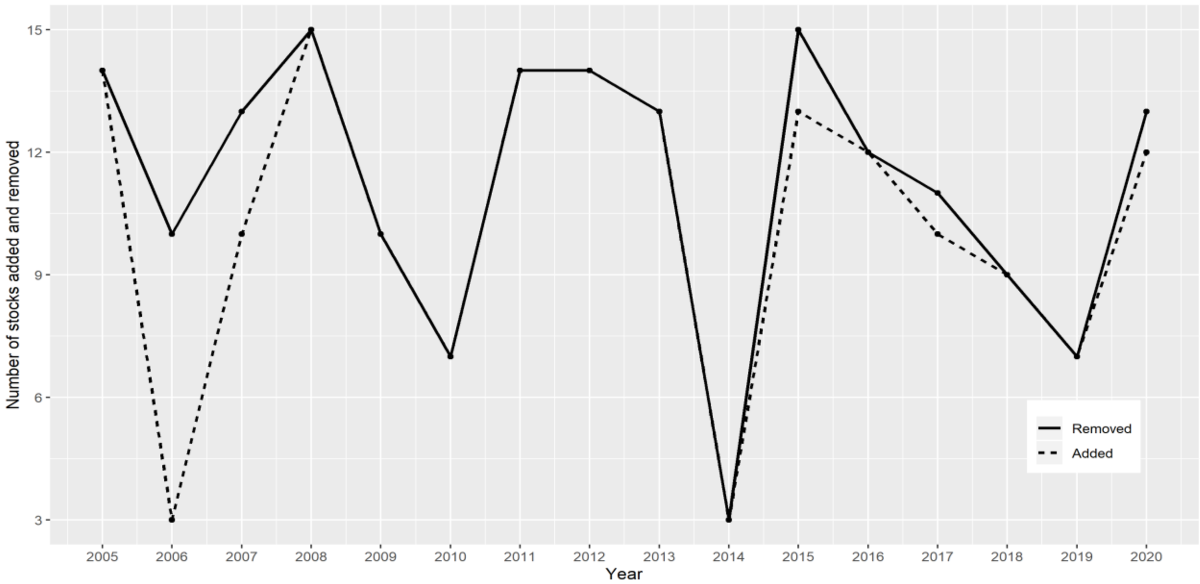

Figure 2. Changes in the composition of the NDX by year.
Notes: Changes made to the index are summed by year and then marked as either added (dashed line) or removed (solid line).
Source: compiled by the author

[19] ETFdb lists all of the stocks kicked out of the NASDAQ-100 starting from 1995 all the way up until 2013 here.

Some smaller issues also arise in cases where based on the publications in the NIR database stocks are removed, but not instantly replaced. In those cases, the author is simply opting to apply an instant remove-and-replace strategy as the maximum delay mentioned is seven days, meaning the adjustment is unlikely to have an impact. Another issue worth considering is the removal and addition of a company within the same year of which there is only one case in the sample period which the author is opting to ignore. Since the data is built from the current day backwards, the stock is being excluded from the index for just 13 calendar days, meaning it is unlikely to have an impact.

The final result of the cross-validating process is a list of 243 unique tickers after checking for repeat values. The acquisition of the data for these 243 unique tickers comes primarily through the R package *Quantmod* which is able to gather the full *Yahoo Finance* data for 166 companies. An additional two companies are missing values for one trading day which is resolved by averaging the values of the trading days before and after. Since the two primary sources include tickers that are not present in the market at the end of the sample period, the author is able to recover a further eight companies by replacing the previous ticker symbols of the companies with their current ones as of the 31st of December, 2020. Another 18 companies are added through the historical data available on the website *Investing.com*, for a total data set of 194 companies or approximately 80% of the unique tickers. From the 49 missing companies *Quantmod* recovers data for several of them, but all of these companies only have partial data with most of them missing years' worth of observations. These companies are excluded from the data set. The remaining companies that *Quantmod* is unable to track down, primarily the targets of mergers and acquisition transactions, either have a significant amount of observations missing or have no freely accessible data based on the research of the author, regardless of whether the transactions took place towards the start or the end of the sample period.

The data set assembled by the author brings significant limitations to the conclusions of this thesis. The primary limitation of the research will be survivorship bias, meaning it is impossible to determine whether the purchases made when following the momentum strategies covered in this thesis would have actually been the ones performed at the time. This is due to the fact that at any given time the historic composition of the index is not 100% accurate. Figure 3 demonstrates how application of the momentum strategies covered in this thesis are impacted by the unavailability of data as the author has just 82 of the 113 companies included in the index (approximately 73%) at the start of the sample period. However, the more time goes on and the closer we get to the current day, the smaller the survivorship bias becomes. The coverage first crosses 90%, meaning the author has data for 90% of the companies in the index at a given time, in November of 2015 and remains above the threshold for the remainder of the sample period. Additionally, outside of two trading days, coverage does not drop below 80% once after the 4th of June, 2007. This is likely due to how the author constructed the data set where a stock is removed on its final day, but a new stock is introduced on the following day. The author also draws attention to the fact that more than a quarter of the stocks excluded come in multiples as the author simultaneously excludes shares of different classes. This means the survivorship bias most likely is smaller than Figure 3 suggests, given that there are no additional unique tickers or few of them.

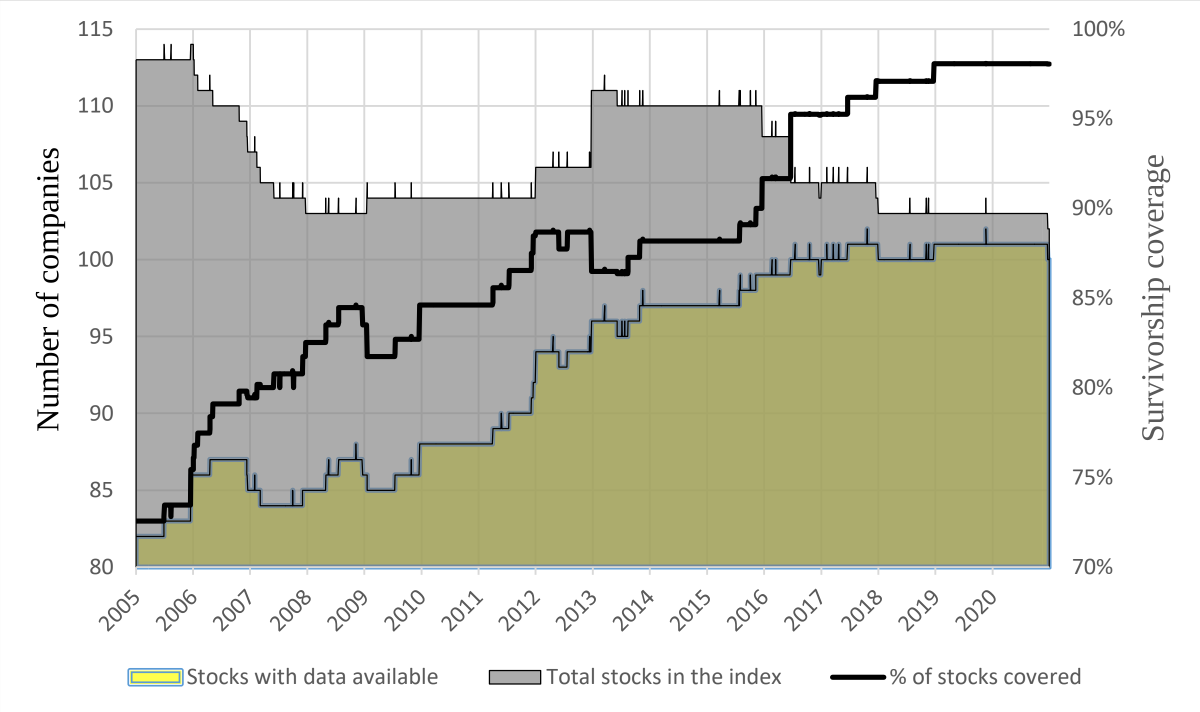

Figure 3. The potential impact of survivorship bias over time.

Notes: Stocks with data available (yellow) and total stocks in the index (grey) are indicated by the left-side Y-axis while the percentage of stocks covered (black) is indicated by the right-side Y-axis.

Source: compiled by the author

The process of manually assembling the data for a single equities momentum approach is a lengthy one. Gathering the data on which stocks were in the index during the 16-year sample period and when (if) they entered or exited alone took the author approximately 150 hours with the verification of sources taking the most time. This is due to the fact that financial news platforms seldom publish one stock being replaced with another one, leaving the author reliant on the two primary data sources and their correctness. This makes the application of momentum on the level of an individual investor impractical for even medium-sized stock universes such as the NASDAQ-100. Furthermore, since the R package *Quantmod* is able to recover the necessary data for approximately just two-thirds of the stock universe, an additional 20 to 30 hours are spent seeking out alternative sources of data beyond *Yahoo Finance*. With most of these sources either being locked behind paywalls or suffering from the same partial data issues as certain stocks recovered by *Quantmod* the ability to assemble a dataset that is free of survivorship bias without outright purchasing said data is basically impossible. In addition, the time spent on gathering the aforementioned data does not include any potential time spent on data correction which could tack on further hours due to improper formatting, currency differences, adjustments for splits and dividends, and more.

Other issues that may arise also need discussion. The method that the author uses to gather data can be referred to as a "first entry to last exit" approach. The method entails acquiring the data for a specific company starting from the date when they were first included in the index until the date when they were last removed. For companies present in the index at the end of the sample period the 31st of December, 2020 is used as the value for "last exit". In the cases where a company is first added, removed and then eventually added back the author utilizes the periods during which the company is outside the index to calculate the look-back returns, but periods before the first entry are not used to calculate these J period values as this would introduce further data and survivorship-related issues. The author also draws attention to the fact that the historic data from the website *Investing.com* does not include an adjusted close value for the 18 companies for which data is gathered through the website. In these cases the author instead utilizes the closing values. In some unique cases, application of momentum is not possible. These cases are all related to the rebalancing of the NDX at the end of 2020 with some stocks being included in the index for just six trading days. For a few companies the J/K-strategies may not fulfil their primary goal if the sum of J and K is 6 months as not all companies remain in the index for extended periods of time. The author covers these more unique cases tied to the availability of data in the next subchapter.

Since two of the three momentum strategies covered in the literature review require a benchmark, the author also utilizes the U.S. 10-year Treasury bond (T-bill) for which data is gathered for the same sample period through *Yahoo Finance*. The data provided by *Yahoo Finance* is missing data for 38 trading days with two days not present and 36 days having no values. For all of these observations interpolation is used since the day-to-day change of the 10-year tends to be a few basis points. Additionally, the author uses the returns of the NASDAQ-100 itself for statistical testing with data also gathered through *Yahoo Finance* and applies it as the benchmark if the U.S. 10-year does not lead to a successful filtering of equities.

2.2. Methodology

The author is applying relative, absolute, and dual momentum on single equities in the NASDAQ-100. All of the approaches utilize the daily adjusted closing prices of the stocks in the index at a given time outside of the companies that did not have this data available for which the closing prices are used instead as described in Subchapter 2.1. The way the author applies these three momentum strategies is similar to that of existing literature as the author utilizes different J/K-strategies to help determine the most successful strategy. However, the examined values of J ("tracking period") and K ("holding period") selected are smaller than those covered by other authors in an attempt to determine whether the momentum strategies can outperform given benchmarks by limiting downside in bear markets. The 25 combinations examined by the author include J and K values of 1 week, 2 weeks, 1 month, 2 months and 3 months which in trading day terms are equivalent to five (5), ten (10), 20, 40 and 65 trading days. While one month is typically 21 trading days, by reducing the amount of trading days per month merely by redefining the word 'month', it allows for a constant common denominator of five (5) which reduces the run time of the strategies from approximately 350 hours to just 45. To avoid notation-related issues stemming from these values of J and K, the author makes an adjustment to the notation used in existing literature. For example, in the case where J is 1 week and K is 2 weeks, the strategy is denoted by (0.25, 0.5). In the cases where either J or K is in months the whole numbers 1, 2 and 3 are used.

The reason why the expected run time of the program is so long stems from the way that the strategies themselves are built. The author offers a simplified flow chart of how the strategies are built in Figure 4 and also provides a generalized code outtake from the program in Appendix A. Additionally, the author notes that a flow chart like this is used to calculate five or ten different strategies per recursive cycle. These strategies are ones where the J period is fixed, but the K period is dynamic. For example, a loop would look at the (1, 0.25), (1, 0.5), (1, 1), (1, 2), and (1, 3) strategies for both the decile and the quintile approach. It works in the same way for absolute, and dual momentum, but since there are no decile or quintile approaches for absolute momentum the amount of strategies per loop is five instead of ten.

The author is utilizing an approach at times referred to in the literature as "overlapping J's". When applying the three strategies while dealing with longer K periods the strategies run into a bias issue stemming from the timing of the entry. When no overlapping is applied and portfolios are both sold and purchased only as the previous K period concludes and the next one begins then during the time when assets are being held there are other winner portfolios that are being skipped over. This is due to the fact that look-back periods are constantly ending.

There are certain strategies that do not need this feature, namely the strategies where K is set to the smallest value possible which in the case of the author is one week. This is the case only due to the fact that the author is applying a common denominator of five across the lengths of J and K, resulting in look-back periods concluding on a weekly basis, not a daily one.

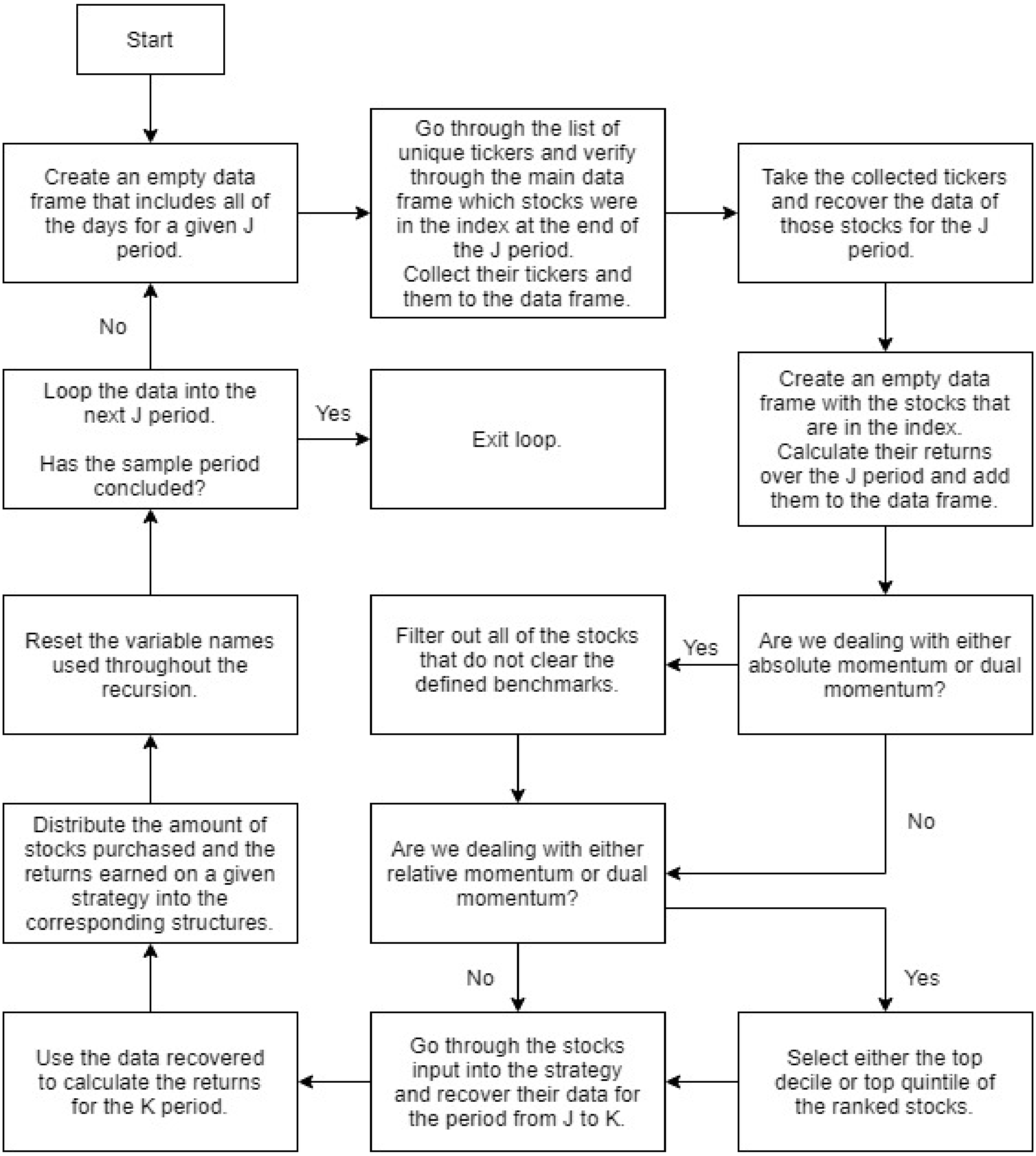

Figure 4. Flow chart to describe the structure of the code of the author in software *R*.

Notes: The main data frame mentioned in Step 2 of the flow chart is a data set assembled by the author that includes all of the 243 unique tickers and the 4,026 trading days for the sample period. The data frame is used to verify whether a stock is in the index on a given day.

Source: compiled by the author

For the data to be comparable the author also adjusts the data set described in Subchapter 2.1. Since the maximum value for J is 65 days the author needs to eliminate any portfolios purchased prior to the 66th day in the data set. This is due to the fact that the winner portfolios purchased prior to the 66th day are making purchasing decisions on data that does not cover the entire length of the J period for certain other strategies. The data for days one through 65 is still used to rank the stocks for a given J period, but only if it leads to the first purchase landing on the 66th date and not before that. This correction is also needed for the backend of the sample period since the purchasing of winner portfolios where K is three months cannot be done beyond the 3961st day (4026 minus 65). Therefore the data set on which the purchasing of stocks is applied ranges from the 66th day to the 3961st day instead of from the first (1st) to the 4026th.

The form of portfolio assembling applied by the author will look at just the winner portfolio as the loser portfolio and by virtue the winner-loser portfolio are both outperformed by the winner portfolio in a majority of the available momentum-related research. The returns of the stocks in the index at the end of each J period are calculated using discrete returns, i.e.

(1) $$r_t = (P_t / P_{t-1}) - 1$$

where r_t is the return of a stock from time $t-1$ to time t;

P_t is the price of a stock at time t; and

P_{t-1} is the price of a stock at time $t-1$.

Different winner portfolios are considered by working with both the top decile (10%) and the top quintile (20%) of stocks at any given time for both relative momentum and dual momentum. Applying either of these approaches to absolute momentum is obsolete since it would lead to the purchasing of the same stocks as the corresponding relative momentum strategy as long as enough stocks clear a given benchmark. After ranking the stocks on their discrete returns depending on the strategy being used either the top 10% or 20% will be selected from them (relative momentum and dual momentum), the stocks unable to outperform a given benchmark are dropped (absolute momentum and dual momentum), or both (dual momentum). The remaining stocks that are purchased and held for K periods have their returns calculated using continuous returns, i.e.

(2) $$r_t = \log(P_t / P_{t-1})$$

The author is working with equally-weighted portfolios, meaning each stock will have the same weight in the portfolio as of the formation date. If the stock universe available for a given J period at time t has n stocks and after eliminating some segment of those stocks that do not fulfill the criteria of a given strategy, leaving us with m stocks then the returns of the J period winner portfolio being held for the entirety of K for a given strategy can be described by

(3) $$r_P = \sum_i^m w_i r_i$$

where r_P is the return of the portfolio;

w_i is the weight of stock i;

r_i is the continuous return of stock i over the K period; and

$m \leq n$.

It is important to note that the NASDAQ itself is not an equally-weighted index, meaning the exposure that a given strategy has to certain stocks will always be either excessive or insufficient, potentially leading to both lower returns and higher volatility.

The performance of the three strategies is assessed through both risk and reward characteristics. For profitability the author reports the mean annualized returns of the strategies along with the corresponding T-statistics. For the absolute momentum approach the utilized benchmark is the U.S. 10-year Treasury bond that has been standardized to weekly form. It is commonly applied as the risk-free rate in the capital asset pricing model and therefore is a reasonable metric to use. In practice, the application of the 10-year as a benchmark eliminates any chance of the strategy investing in an asset that had either negative returns or no returns for a given period. However, as noted in the literature review the expected result is that the 10-year is simply not a high enough benchmark and therefore the author also considers the annualized rate of return of the NASDAQ-100 through the sample period of 11.19% as the benchmark for both the absolute momentum and the dual momentum strategies. For the risk characteristics the author reports the mean annual standard deviation. As an additional risk-reward metric the author reports the Sharpe ratios of the strategies.

For statistical testing the author is applying t-statistics calculated with Newey-West heteroscedasticity and autocorrelation (HAC) consistent standard errors. The t-test is one-tailed since the goal is to determine whether the momentum strategies can successfully outperform the NASDAQ-100. The null hypothesis is that a given strategy has the same returns as the NDX whereas the alternative hypothesis is that the strategy has greater returns than the NDX. The levels of significance that the author is testing are 90%, 95%, and 99% with 1.282, 1.645, and 2.326 being the corresponding critical values. The reason why this methodology with HAC errors is popular in existing literature stems from the fact that stock market data has autocorrelation present, meaning stocks will usually move in the same direction based on some exogenous factors like central bank policy towards interest rates, inflation, unemployment, and others. The author finds the HAC consistent errors by running a linear regression on the data in the following form:

$$Y_S - r_{NDX} = \beta_0 + e \quad (4)$$

where Y_S is the vector of returns of a given strategy S;

r_{NDX} is the annualized rate of return of the NASDAQ-100 over the sample period of the author at 11.19%;

β_0 is a constant; and

e is the error term.

The standard error for the linear regression is recovered using the *coeftest* command from the R package *sandwich*. This is the reason why the author is utilizing linear regressions as *coeftest* allows for the quick gathering of the standard errors and by virtue the t-statistics. The t-statistics are calculated in the following form:

$$t_S = (\overline{r_S} - r_{NDX})/se_S \quad (5)$$

where t_S is the t-statistic for the returns of a given strategy S;

$\overline{r_S}$ is the mean annualized return of a given strategy S;

r_{NDX} is the annualized return of the NASDAQ-100 at 11.19%; and

se_S is the standard error found through the application of the regression in equation (4) for a given strategy S.

Since the annualized return of the data set that the author has does not equal the annualized return of the NASDAQ-100 it would be impractical to apply it as the population mean when calculating the t-statistics. The reason why stems from the fact that the stock universe of the author does not fully overlap with that of the NDX at any point in the sample period. Therefore the returns of the stock universe covered in this thesis lack practical value as it is not a purchasable asset while exchange-traded funds tracking the NASDAQ-100 such as the QQQ are purchasable.

2.3. Results

As noted in Subchapters 2.1 and 2.2 the author applies three separate momentum strategies on a total of 194 companies. Some general descriptive statistics for the data available are noted in Table 2. Out of the 194 companies 38 of them remain in the index throughout the sample period for a total of 4,026 trading days and the six companies that were added in the annual rebalancing of the NDX at the end of 2020 feature for just six trading days.

A key differentiator outside of the risk and reward characteristics is the amount of stocks purchased by each strategy. While modern broker-dealers do have significantly smaller transaction fees with the prevalence of the payment for order flow (PFOF) system the results of the author are not adjusted for any transaction costs. Since any momentum strategy will start to lose some of its edge when a set amount of money is to be distributed into smaller segments it remains a possibility that the accumulation of so-called “minimum costs” where investors are charged at least some flat-rate on every transaction could lead to lower capital gains or furthering losses.

Table 2

Descriptive statistics for the stocks that have data available.

	N	Min	Mean	Median	Max	Mode	Skewness	Kurtosis
Stocks in the index	194	82	93	95	102	101	1.43	-0.19
Trading days in the index	194	6	1930	1966	4026	4026	0.25	1.69
Days until removal	129	144	1464	1216	4020	243	0.60	2.19
Days until re-addition	35	112	763	506	3020	251	1.63	5.26

Notes: The mean and median are rounded down to a full integer. Both days until removal and days until re-addition count the same stock multiple times if it has been removed or re-added multiple times. Days until re-addition does not count the initial adding of a stock.
Source: compiled by the author

The author highlights the average amount of stocks bought in all three momentum approaches in Table 3. For relative momentum the averages mainly describe the amount of stocks in the index at a given time, but due to the methodology rounding values down it is slightly lower than 10% or 20% of the 93 value reported in Table 2. The results for absolute momentum and dual momentum also trend higher as the look-back period increases. For the absolute momentum case it is apparent that the reason why the amount of stocks purchased continues to increase as the J period increases stems from the added likelihood of stocks clearing the standardized U.S. 10-year benchmark. Since the mean amount of stocks being purchased in the absolute momentum strategies make up around 50 percent of the companies in the NDX it is to be expected that dual momentum cannot significantly reduce the top decile or top quintile in terms of size. Due to this the values reported in Table 3 for dual momentum are calculated with the NASDAQ-100 as the benchmark. However, when comparing the average amount of stocks purchased the difference between relative momentum and dual momentum remains minimal. The average decile approach purchases 0.18 fewer stocks and the average quintile approach purchases 0.91 fewer stocks. When standardizing the 11.19% that the NDX earns annually over the sample period to a weekly rate of return the strategies with J set to one week need stocks to outperform a mere 0.22%. Therefore it is reasonable to claim that a practical benchmark may not be the most optimal for both absolute momentum and dual momentum strategies, at least when working with smaller values of both J and K.

Table 3

The mean amounts of stocks purchased per J period across different strategies.

J	Relative momentum - decile	Relative momentum - quintile	Absolute momentum	Dual momentum - decile	Dual momentum - quintile
0.25	8.87	18.28	49.46	8.68	17.01
0.5	8.87	18.29	50.65	8.64	17.25
1	8.88	18.30	52.04	8.69	17.46
2	8.89	18.32	53.39	8.75	17.62
3	8.90	18.34	54.59	8.74	17.65

Notes: The words 'decile' and 'quintile' indicate the strategies are purchasing either the top 10% or 20% of stocks in a given stock universe. The absolute momentum values reported use the U.S. 10-year T-bill as the benchmark and the dual momentum values reported use the annualized return of the NASDAQ-100 as the benchmark when selecting the winner portfolios. Source: compiled by the author

The author reports the results of the application of 25 different J/K-strategies in the case of a relative momentum strategy in Appendix B. Similar trends can be spotted regardless of whether the strategy is to purchase the top decile or quintile. There is a general tendency for the values to increase from both left to right and top to bottom, meaning the short-term strategies with constant exiting and re-entry are outclassed by ones that more resemble buy-and-hold. In the case of statistical significance the results reported in Appendix B are entirely insignificant, even at a 90% level. The main culprit for this lack of statistical significance is the return of the NASDAQ-100 which returned 11.19% annually over the sample period. Due to this high benchmark statistical significance at a 90% level or higher would require a strategy to outperform the index by at least 5% on a relatively small standard error or by as much as 15% on a larger standard error. With the peak return over 50 different strategies being 16.87% for the (3, 1) strategy in the top decile approach the benchmark is outperformed by 5.68%, but with a reported standard error of 6.92% the strategy would have to return closer to 20.07% for statistical significance at the 90% level, suggesting that the top strategy needs its excess return to increase by more than 50% to achieve statistical significance at a 90% level.

The results of Appendix B also include some logical takeaways that could have been assumed prior. The top quintile approach is likely to bring with it superior diversification when compared to the top decile approach, meaning the followers of these strategies would most likely be sacrificing some returns for less risk. This is indicated by several factors. For example, the average J/K-strategy for the decile approach returns 10.41% compared to the 8.95% of the quintile approach. Furthermore, in 24 out of the 25 cases the decile approach has higher returns than the quintile approach with the (1, 0.25) strategy being the outlier. The same trends can also be spotted over both the J periods and the K periods. When comparing the corresponding K periods the decile approaches outperform the quintile approaches on average by 1.32% to 1.75%. The range is even larger across the corresponding J periods with the improvement in returns ranging from 0.31% to 3.13%.

The returns stemming from the application of the absolute momentum strategy are presented in Appendix C. Similarly to the relative momentum approaches the results are not able to clear statistical significance. Unlike the relative momentum results reported in Appendix B there are no absolute momentum strategies that are capable of outperforming the index in absolute terms. Outside of a few outliers longer K periods continue to outperform shorter ones with the average returns increasing from 7.66% to 9.51% as K increases, but for the J periods the results do not have this linear uptrend and have more variance. When comparing the corresponding J/K-strategies across groups the returns reported by the absolute momentum approach are outperformed by the decile approach of the relative momentum strategy in 20 out of 25 cases, but absolute momentum beats the quintile approach in 13 out of 25 cases. However, across the strategies with J periods of either 2 or 3 months, both of the relative momentum approaches outperform absolute momentum in all ten cases. Appendix B and Appendix C both showcase that these strategies are the ones closest to reaching statistical significance as these tend to be the strategies capable of outperforming the NASDAQ-100.

The downfall of the absolute momentum strategy might be related to the amount of shares bought as described in Table 3. However, when testing the absolute momentum strategy with the NASDAQ-100 return set as the benchmark, the mean return over the 25 J/K strategies declines by 0.01% even as the average amount of stocks bought across all strategies declines from 52.0 in the case of the U.S. 10-year to 48.1 in the case of the NDX, indicating that the benchmarks themselves are most likely the cause of concern. This continuous purchasing of large portfolios will lead to more exposure in different sectors and given the correlations of returns between sectors the absolute momentum approach is likely to be negatively impacted by the amount of companies in the winner portfolio. Potential transaction costs would weigh on the returns even more, meaning the absolute momentum approach appears impractical when calculated with both a low-returning risk-free benchmark, and a practical benchmark.

The results of the dual momentum approaches are reported in Appendix D with the author separating between decile and quintile approaches just like in the case of relative momentum. Since both Appendix C and Table 3 indicate that the 10-year T-bill is not a viable benchmark the author is applying the 11.19% returned by the NDX over the 16-year sample period as the benchmark for these strategies. The returns are lackluster when compared to relative momentum with the average dual momentum strategy decreasing returns. In the case of the decile approach this decrease is equal to 0.25% and for the quintile approach it equals 0.24% annually. Similarly to relative and absolute momentum the author observes no cases of statistical significance across the 50 different strategies. As first indicated in Table 3 the change in the amount of shares purchased after clearing the returns of the benchmark did not drastically change, meaning the expected impact on the returns is minimal. The author does observe outlier cases, primarily among the cases where J is set to two months. In the ten strategies that have such a J period value the dual momentum strategies are outperformed by the relative momentum ones on average by 0.99% and 1.64% respectively for the decile and quintile approaches. However, excluding these strategies does not tilt the scales in favor of the dual momentum strategies in a significant way.

A potential source of weakness of the momentum strategies described in Appendices B, C, and D is the selection of values for J and K. Existing literature is significantly more focused on longer periods, typically not including periods below 1 month and usually going as high as 12 months. Additionally, the author's decision to work with such small incremental steps from value to value could be adding limited utility if any since the high returns of the NDX mean higher returns need to be achieved in any strategy to reach statistical significance. With the higher values of both J and K outclassing the lower values it leaves space for analysis beyond the values selected by the author.

The author believes that both the absolute and dual momentum strategies are unviable without a benchmark that lacks practical value. In the absolute momentum case the 10-year Treasury bond allows for the purchasing of approximately half of the index at any given time. Given the fact that the author is not checking for transaction costs the strategy does not seem fitting even if the strategies could compete with the NASDAQ-100 in pure returns. In total when all of the strategies are observed the index itself is outperforming most strategies. Across the absolute momentum strategies not a single one outperforms the NDX regardless of the benchmark. For relative momentum 16 strategies out of 50 and for dual momentum 14 strategies out of 50 outperform the NASDAQ-100 with the decile approach making up 18 of the 30 total strategies. For both the decile and the quintile approach in the case of dual momentum the author utilizes the annualized returns of the NDX over the selected sample period as the benchmark. When standardizing the rate of return to the corresponding J value the benchmark is too low to filter out any significant portion of the stock universe, ranging from 0.22% for the strategies with J set to one week and up to 2.80% for the strategies with J set to three months. The author believes that in order to see any real changes in terms of filtering out more equities the benchmark needs to be at least twice as high with an annualized return of around 22.5%. While setting the benchmark at such a value would help to eliminate the worst performers that are currently being included in all of the winner portfolios the benchmark does lack practical value since equity markets return 6-8 percent in the long-term, suggesting that the viability of trading systems that are based around momentum could come into question.

The volatility of the strategies is compared in Appendix E through the annualized standard deviations of the returns. The mean results indicate that absolute momentum grants the least exposure to risk with an average standard deviation of 20.42% over the 25 J/K-strategies. For both relative momentum and dual momentum the top decile approaches bring with them additional risk when compared to the quintile approach. The means for the top decile approaches are 24.20% and 24.94% for relative and dual momentum respectively compared to the corresponding 21.48% and 22.44% of the quintile approaches. Additionally, the mean standard deviations are higher for the dual momentum strategies than they are for the relative momentum ones. This result indicates that increasing the weights in the top stocks of the winner portfolios will lead to higher risks as exposure gets compressed into fewer stocks. The results in Appendix E do not show much variance when comparing over the five unique K values with the means over all strategies settling between 22.38% and 23.18%.

To compare the risk-adjusted returns the author highlights the Sharpe ratios of the momentum strategies in Appendix F. A trend across the results is that the ratios are generally low as a Sharpe ratio of 0.9 to 1.1 is the preferred landing zone for risk-reward trade-offs. The values of the ratios highlight the same strategies as the returns in Appendices B, C, and D with the highest Sharpe ratios almost always belonging to the strategies that have a tracking period of three months and a holding period of one, two, or three months. The top Sharpe ratios reported in Appendix F belong to the (1, 3) and the (2, 3) strategies for the decile approaches of both relative momentum and dual momentum with values ranging from 0.60 to 0.64. Similarly, when comparing over different tracking and holding periods the best strategies based on mean Sharpe ratio are those where the length of the J period is set to three months and the length of the K period is set to three months. Only in the case of absolute momentum does a two month look-back period outperform the three month look-back period.

Since the results reported in this thesis for the strategies with small J and K values are very low it raises a question about whether a contrarian strategy would outperform the strategies themselves. While in the cases of the top decile and quintile approaches for the relative momentum and dual momentum strategies contrarianism would lead to purchasing more than 80% of the stock universe the share of stocks purchased would closer to 50% for absolute momentum. Furthermore, absolute momentum with either of the benchmarks utilized in this thesis has zero total strategies capable of outperforming the NASDAQ-100. The author reports the annualized returns of a contrarian absolute momentum strategy in Table 4. The results do indicate that a short-term contrarian strategy for absolute momentum can outperform the NDX. Unlike the standard absolute momentum strategies the returns now decline as both J and K increase which is to be expected as a weighted average of both strategies should lead to similar returns. The reason why the returns will not be equal to those of the index stems from both the NASDAQ-100 not being an equally-weighted index and the data set of the author being partial. Additionally, an equally-weighted approach has a lot of variance in the amount of stocks purchased for absolute momentum with the two different extremes being a few stocks having high weights and a lot of stocks having low weights.

Table 4

The annualized returns of different contrarian absolute momentum strategies.

J	K = 0.25	0.5	1	2	3
0.25	13.44%	10.70%	9.55%	9.53%	9.04%
0.5	11.71%	11.51%	10.50%	9.68%	8.88%
1	9.90%	10.10%	10.15%	9.07%	8.63%
2	11.31%	11.14%	9.24%	8.25%	7.63%
3	6.47%	7.31%	7.17%	6.85%	6.41%

Notes: The returns reported in this table are formed by purchasing stocks that did not outperform the U.S. 10-year T-bill over a given J period, and holding them for a given K period before selling.
Source: compiled by the author

While no cases of statistical significance are reported in this thesis the results do still have economic significance. For relative and dual momentum the author reports returns that outperform the NDX by more than five percent annually. Compounded over a 16-year sample this would indicate that these strategies more than double the return of the NASDAQ-100. While the validity of the benchmarks used in this thesis is there the benchmarks themselves do not offer value for the absolute and dual momentum strategies. The author believes that on a single equities level with short look-back periods it is unlikely that a practical benchmark can lead to great returns for the absolute momentum and dual momentum strategies. Additionally, the author provides a basis for the idea that short-term momentum contrarianism where some segment of the losing equities is purchased long can lead to significant profits beyond those of the standard momentum strategies. While the existing scientific literature has noted that longing loser portfolios is profitable no cases of loser portfolios outperforming the winner portfolios can be noted over the literature reviewed by the author. This result could be valuable to institutional investors, primarily quant strategists looking for edges in the market.

Conclusion

While scientific coverage of momentum-related investment strategies has continually increased since the cornerstone papers by both De Bondt and Thaler (1985, 1987) as well as Jegadeesh and Titman (1993) it still remains a well-kept secret in the world of investing. The author seeks to determine whether specific momentum strategies are capable of outperforming the NASDAQ-100 when a defined subset of the equities in the index are bought based on given criteria. A total of 194 unique companies are analyzed and three different momentum approaches are applied. While individual equities bring with them higher volatility when compared to diversified assets such as exchange-traded funds the approach of the author on a smaller sample size could assist in the potential creation of trading systems among both individual and institutional investors.

The data set of the author uses recent periods from January 2005 until December 2020, thereby including several bull and bear markets. Regardless of whether the author is applying a relative momentum approach, an absolute momentum approach, or a dual momentum approach there are no examples of statistical significance across all of the strategies. Cases where the strategies outperform the NASDAQ-100 index can be observed, but they account for less than 25% of all strategies. The profitability of the results is in-line with existing literature as relative momentum outperforms absolute momentum. While the existing research on dual momentum is limited the author is unable to disprove the idea that dual momentum is superior to relative momentum as the results between the two momentum approaches are similar given a practical benchmark.

While academic literature on the topic of momentum-based investing does go back and forth in terms of achieving statistically significant results the author believes there are several reasons as to why the strategies not only cannot achieve statistical significance, but also are unable to outperform the NASDAQ-100 in most cases. The data set that the author is working with in this thesis is lacking in several categories and this could be forcing the strategies into equities that it normally would not consider. In the case of absolute momentum the author has a hard time making a case for its individual application. When working with individual stocks there are bound to be some assets going up and this will make most realistic benchmarks such as the U.S. 10-year or the NDX insufficient regardless of the selection of J and K values as the returns of the benchmarks get converted into yields over the corresponding J values. This idea is also supported by the contrarian absolute momentum strategy outperforming the standard one. Furthermore, increasing the benchmark to a rate that is multiple times higher than the return of the stock market as a whole in the long-term seems impractical, thereby supporting the idea that additional qualifiers are needed when designing a trading system around any of the strategies.

The author has several propositions for future research on the topic of momentum-based analysis and how the results reported in this thesis can be extended. Momentum strategies could become the basis of real trading strategies given a specific market niche can be discovered. More recent literature has already started to take notice of dual momentum and has continued to extend the terms on which assets get purchased. For the sample of the author the strategies that are the closest to statistical significance could serve as the basis of a system with additional rules being applied on top. Alternatively, research could focus on extending the results on a selection of J and K values that are more common across existing literature either on the same data, on the full historic data of the NDX, or on the S&P500. The practicality of the approach itself must remain an essential component to such strategies as the increase in passive investing and the inability to outperform buy-and-hold investing continues to cause troubles for institutional investors. By making sure that the approach is practical it could encourage more active investing in the market without the need to have a great understanding for the market.

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APPENDIX A

The generalized code of the R program written by the author in the case of the relative momentum strategy (0.25, 0.25)

```
# The program is looped over the entire data universe outside of the first and the last 65
days due to the overlapping J's concept.
for (j in seq(1+65, 4027-65, 5)){
# Create an empty data frame that has the days and some empty columns where we will place the
adjusted closing prices.
  Stocks_In_The_Index <- c()
  RelMomentum1wk <- data.frame(matrix(NA, nrow=6, ncol=120))
  colnames(RelMomentum1wk)[1]<- "Day"
  RelMomentum1wk$Day <- as.Date(RelMomentum1wk$Day)
  RelMomentum1wk$Day[1:6] <- c(main_df_fin[j:(j+5),1])
  for (m in 2:120){
    RelMomentum1wk[,m] <- as.numeric(RelMomentum1wk[,m])
  }
# For every stock that was present in the index from 01.01.2005-31.12.2020
  for (i in 2:244){
# Exclude all the stocks for which the author has no data
    if (!(colnames(FirstEntry_To_LastExit)[i] %in% Stocks_With_No_Data)){
# Find the stocks that are in the index after 1 week from the current date
      if (main_df_fin[j+5,i]=="Yes"){
        Stocks_In_The_Index <- c(Stocks_In_The_Index, colnames(FirstEntry_To_LastExit)[i])
      }
# Once we get to the last stock, we replace the column names of the empty data frame that was
created at the start of the loop.
      if (i==244){
        colnames(RelMomentum1wk)[2:(length(Stocks_In_The_Index)+1)] <- Stocks_In_The_Index
# Since the empty data frame was a 6x120 matrix, we remove the columns where there is no
stock name.
        if ((substr(colnames(RelMomentum1wk)[1+length(Stocks_In_The_Index)], 2, 2) %in% c(1,
2, 3, 4, 5, 6, 7, 8, 9, 0))==FALSE
            & (substr(colnames(RelMomentum1wk)[2+length(Stocks_In_The_Index)], 2, 2) %in%
c(1, 2, 3, 4, 5, 6, 7, 8, 9, 0))==TRUE){
          RelMomentum1wk <- RelMomentum1wk[,1:(length(Stocks_In_The_Index)+1)]
        }
# Now we have all of the stocks that were in the index at the end of the J period.
# For each stock in the index:
        for (k in 2:length(colnames(RelMomentum1wk))){
# We check how many days' worth of data that stock has
          duration <- dim(eval(parse(text=paste0(colnames(RelMomentum1wk)[k]))))[1]
          DatesOfStock <- eval(parse(text=paste0(colnames(RelMomentum1wk)[k], "$Index")))
# If the value is not 4,026 then the stock is not always in the index.
          if (!(length(DatesOfStock)==4026)){
            for (p in 1:duration){
# We track down the day where the date at the end of the J period overlaps with that of the
stock we are currently looking at.
              if (main_df_fin$Day[j]==DatesOfStock[p]){
# We assign the adjusted closing prices of those days to the previously empty data frame.
                StockInRel <- paste0("RelMomentum1wk$",colnames(RelMomentum1wk)[k])
                PasteValue <- paste0(colnames(RelMomentum1wk)[k], "$",
                                     colnames(RelMomentum1wk)[k], ".Adjusted[p:(p+5)]")
                eval(parse(text=paste0(StockInRel, " <- c(", PasteValue, ")")))
                rm(StockInRel, PasteValue)
              }
            }
          }
# If the value is 4,026 then the stock was in the index the whole time and there is no need
to track down the specific days.
          if (length(DatesOfStock)==4026){
            StockInRel2 <- paste0("RelMomentum1wk$",colnames(RelMomentum1wk)[k])
            PasteValue2 <- paste0(colnames(RelMomentum1wk)[k], "$",
                                  colnames(RelMomentum1wk)[k], ".Adjusted[j:(j+5)]")
            eval(parse(text=paste0(StockInRel2, " <- c(", PasteValue2, ")")))
            rm(StockInRel2, PasteValue2)
          }
        }
# The author now has the stocks that were in the index for the J period and the daily
adjusted closing prices of those stocks.
        if (k==length(colnames(RelMomentum1wk))){
# Create a new data frame with 2 columns - one for tickers, one for returns.
          Returns_1week <- data.frame(matrix(NA, nrow=(dim(RelMomentum1wk)[2]-1), ncol=2))
          colnames(Returns_1week)<-c("Stock", "Return")
          Returns_1week$Stock <- as.character(Returns_1week$Stock)
          Returns_1week$Return <- as.numeric(Returns_1week$Return)
          Returns_1week$Stock[1:(dim(RelMomentum1wk)[2]-1)] <-
colnames(RelMomentum1wk)[2:dim(RelMomentum1wk)[2]]
          for (i in 1:(length(colnames(RelMomentum1wk))-1)){
            if (Returns_1week$Stock[i]==colnames(RelMomentum1wk)[i+1]){
```

```
            Returns_1week$Return[i] <- (RelMomentum1wk[6,i+1]/RelMomentum1wk[1,i+1])
          }
        }
# The author now has the returns over the J period.
# We arrange the stocks and then select the top deciles and quintiles.
        Returns_1week <- Returns_1week %>% arrange(desc(Return))
        WinnerPortfolio_decile <- Returns_1week[1:floor(0.1*dim(Returns_1week)[1]), 1:2]
        WinnerPortfolio_quintile <- Returns_1week[1:floor(0.2*dim(Returns_1week)[1]), 1:2]
# Now we have the top decile and top quintile that will be used to build the portfolios.
# For J = 1 week, K = 1 week the returns are calculated from J to J+5.
# We first look at the top decile approach.
        for (i in 1:floor(0.1*dim(Returns_1week)[1])){
# Again we look at whether the stock was in the universe the whole time or not.
          dimens <- dim(eval(parse(text=paste0(WinnerPortfolio_decile$Stock[i]))))[1]
# If the stock was in the index for the whole time:
          if (dimens == 4026){
# Then we can recover it's data from the end of J until the end of K without adjusting
looping values.
            numerator <- eval(parse(text=paste0(WinnerPortfolio_decile$Stock[i], "$",
                                                WinnerPortfolio_decile$Stock[i],
".Adjusted[",j+5+5,"]")))
            denominator <- eval(parse(text=paste0(WinnerPortfolio_decile$Stock[i], "$",
                                                  WinnerPortfolio_decile$Stock[i],
".Adjusted[",j+5,"]")))
# If the stock is removed from the index before the end of the K period.
            if (is.na(numerator)){
# We count for how many days it is missing data.
              values2count <- is.na(eval(parse(text=paste0(WinnerPortfolio_decile$Stock[i],
"$",

WinnerPortfolio_decile$Stock[i], ".Adjusted[",j,":",j+5+5,"]"))))
              missing <- sum(values2count==TRUE)
# And we adjust the end of the K period to coincide with the end of our data for that stock.
              if (missing > 0){
                if (missing < 6){
                  numerator <- eval(parse(text=paste0(WinnerPortfolio_decile$Stock[i], "$",

WinnerPortfolio_decile$Stock[i], ".Adjusted[",j+5+5-missing,"]")))
                  rm(values2count)
                }
              }
            }
# We calculate the log returns
            WinnerPortfolio_decile$Return[i] <- log(numerator/denominator)
            rm(numerator, denominator)
          }
# If the stock was NOT in the index for the whole time:
          if (!(dimens==4026)){
# We again need to match up the date and the data for these stocks.
            for (r in 1:dimens){
              CurrentStockDate <- eval(parse(text=paste0(WinnerPortfolio_decile$Stock[i],
"$Index[", r, "]")))
              if (main_df_fin$Day[j+5]==CurrentStockDate){
                numerator <- eval(parse(text=paste0(WinnerPortfolio_decile$Stock[i], "$",
                                                    WinnerPortfolio_decile$Stock[i],
".Adjusted[",r+5,"]")))
                denominator <- eval(parse(text=paste0(WinnerPortfolio_decile$Stock[i], "$",
                                                      WinnerPortfolio_decile$Stock[i],
".Adjusted[",r,"]")))
# We again make this correction if the stock gets removed earlier.
                if (is.na(numerator)){
                  values2count <-
is.na(eval(parse(text=paste0(WinnerPortfolio_decile$Stock[i], "$",

WinnerPortfolio_decile$Stock[i], ".Adjusted[",r,":",r+5,"]"))))
                  missing <- sum(values2count==TRUE)
                  if (missing > 0){
                    if (missing < 6){
                      numerator <- eval(parse(text=paste0(WinnerPortfolio_decile$Stock[i],
"$",

WinnerPortfolio_decile$Stock[i], ".Adjusted[",r+5-missing,"]")))
                      rm(values2count)
                    }
                  }
                }
# And again, calculate the log returns.
                WinnerPortfolio_decile$Return[i] <- log(numerator/denominator)
                rm(numerator, denominator)
              }
            }
          }
        }
# We run this exact same methodology now for the top quintile approach.
        for (i in 1:floor(0.2*dim(Returns_1week)[1])){
            dimens <- dim(eval(parse(text=paste0(WinnerPortfolio_quintile$Stock[i]))))[1]
            if (dimens == 4026){
```

```
                numerator <- eval(parse(text=paste0(WinnerPortfolio_quintile$Stock[i], "$",
                                                    WinnerPortfolio_quintile$Stock[i],
".Adjusted[",j+5+5,"]")))
                denominator <- eval(parse(text=paste0(WinnerPortfolio_quintile$Stock[i], "$",
                                                      WinnerPortfolio_quintile$Stock[i],
".Adjusted[",j+5,"]")))
                if (is.na(numerator)){
                  values2count <-
is.na(eval(parse(text=paste0(WinnerPortfolio_quintile$Stock[i], "$",

WinnerPortfolio_quintile$Stock[i], ".Adjusted[",j,":",j+5+5,"]"))))
                  missing <- sum(values2count==TRUE)
                  if (missing > 0){
                    if (missing < 6){
                      numerator <- eval(parse(text=paste0(WinnerPortfolio_quintile$Stock[i],
"$",

WinnerPortfolio_quintile$Stock[i], ".Adjusted[",j+5+5-missing,"]")))
                      rm(values2count)
                          }
                        }
                      }
                WinnerPortfolio_quintile$Return[i] <- log(numerator/denominator)
                rm(numerator, denominator)
                      }
              if (!(dimens==4026)){
                for (r in 1:dimens){
                  CurrentStockDate <-
eval(parse(text=paste0(WinnerPortfolio_quintile$Stock[i], "$Index[", r, "]")))
                  if (main_df_fin$Day[j+5]==CurrentStockDate){
                    numerator <- eval(parse(text=paste0(WinnerPortfolio_quintile$Stock[i],
"$",
                                                        WinnerPortfolio_quintile$Stock[i],
".Adjusted[",r+5,"]")))
                    denominator <- eval(parse(text=paste0(WinnerPortfolio_quintile$Stock[i],
"$",
                                                          WinnerPortfolio_quintile$Stock[i],
".Adjusted[",r,"]")))
                    if (is.na(numerator)){
                      values2count <-
is.na(eval(parse(text=paste0(WinnerPortfolio_quintile$Stock[i], "$",

WinnerPortfolio_quintile$Stock[i], ".Adjusted[",r,":",r+5,"]"))))
                      missing <- sum(values2count==TRUE)
                      if (missing > 0){
                        if (missing < 6){
                          numerator <-
eval(parse(text=paste0(WinnerPortfolio_quintile$Stock[i], "$",

WinnerPortfolio_quintile$Stock[i], ".Adjusted[",r+5-missing,"]")))
                          rm(values2count)
                        }
                      }
                    }
                    WinnerPortfolio_quintile$Return[i] <- log(numerator/denominator)
                    rm(numerator, denominator)
                  }
                }
              }
          }
# We take note of our results.
# First we mark down how many stocks were purchased.
# Decile approach:
          TrackRel_J1w_K1w_d <- c(TrackRel_J1w_K1w_d, floor(0.1*dim(Returns_1week)[1]))
# Quintile approach:
          TrackRel_J1w_K1w_q <- c(TrackRel_J1w_K1w_q, floor(0.2*dim(Returns_1week)[1]))
# And more importantly, we take note of the returns earned by these portfolios.
          J1wk_K1wk_decile <- c(J1wk_K1wk_decile,
(sum(WinnerPortfolio_decile$Return[1:floor(0.1*dim(Returns_1week)[1])]))/floor(0.1*dim(Return
s_1week)[1]))
          J1wk_K1wk_quintile <- c(J1wk_K1wk_quintile,
(sum(WinnerPortfolio_quintile$Return[1:floor(0.2*dim(Returns_1week)[1])]))/floor(0.2*dim(Retu
rns_1week)[1]))
          rm(WinnerPortfolio_decile, WinnerPortfolio_quintile)
# At this point, the author would change to the J = 1 week, K = 2 weeks approach.
# Only a few changes are made, primarily around which data is recovered.
# For example: Instead of taking the data from J+5 to J+10, we would instead be interested in
the data from J+5 to J+15.
# Since the author overwrites the data frames 'WinnerPortfolio_decile' and
'WinnerPortfolio_quintile' these are read in again.
        }
      }
    }
  }
}
```

APPENDIX B

Returns of the relative momentum strategies

Equal weights of equities, top decile (10%)					
J	K = 0.25	0.5	1	2	3
0.25	5.80%	6.56%	7.75%	8.80%	9.33%
	(-0.84)	(-0.65)	(-0.56)	(-0.45)	(-0.40)
0.5	7.75%	8.18%	8.11%	8.97%	9.95%
	(-0.56)	(-0.44)	(-0.49)	(-0.38)	(-0.23)
1	6.16%	7.64%	7.30%	10.01%	11.32%
	(-0.86)	(-0.53)	(-0.57)	(-0.19)	(0.02)
2	9.75%	11.00%	12.03%	12.87%	13.69%
	(-0.24)	(-0.03)	(0.12)	(0.25)	(0.41)
3	13.80%	15.16%	16.87%	16.67%	14.89%
	(0.42)	(0.55)	(0.82)	(0.83)	(0.57)

Equal weights of equities, top quintile (20%)					
J	K = 0.25	0.5	1	2	3
0.25	4.83%	6.09%	7.27%	8.37%	9.05%
	(-1.13)	(-0.79)	(-0.65)	(-0.52)	(-0.43)
0.5	4.30%	5.27%	6.46%	8.00%	8.63%
	(-1.26)	(-0.92)	(-0.76)	(-0.53)	(-0.44)
1	7.22%	7.39%	7.02%	9.25%	10.00%
	(-0.72)	(-0.60)	(-0.63)	(-0.30)	(-0.19)
2	8.50%	10.61%	10.52%	11.24%	11.97%
	(-0.51)	(-0.09)	(-0.11)	(0.01)	(0.12)
3	11.79%	11.94%	12.04%	13.12%	12.86%
	(0.11)	(0.12)	(0.13)	(0.30)	(0.26)

Notes: This table reports the mean annualized returns of different relative momentum strategies. The strategies assume the formation of a winner portfolio, either consisting of the top 10% ("top decile") or the top 20% ("top quintile") of equities, based on their performance over the past J weeks (months). These portfolios are then bought and held for K weeks (months). The corresponding J and K values are either towards the left of or above each column with the values signifying either weeks or months as described in Subchapter 2.2. The rates of return per strategy are annualized using the same amount of trading days as described in Subchapter 2.2 with a year having 252 trading days. The values in the parenthesis denote the t-statistics calculated through the usage of Newey-West heteroscedasticity and autocorrelation (HAC) consistent errors. The test for the null hypothesis when calculating the t-statistics is that the mean annualized return of a given strategy is equal to the annualized return of the NASDAQ-100 over the 16-year sample period. One asterisk (*) denotes statistical significance at a 90% level with p-values below 0.1, two asterisks (**) at a 95% level with p-values below 0.05, and three asterisks (***) at a 99% level with p-values below 0.01.

Source: compiled by the author

APPENDIX C

Returns of the absolute momentum strategies

J	K = 0.25	0.5	1	2	3
0.25	4.84%	7.37%	7.42%	8.45%	9.22%
	(-1.18)	(-0.63)	(-0.63)	(-0.48)	(-0.39)
0.5	9.68%	8.40%	8.02%	8.85%	9.38%
	(-0.30)	(-0.47)	(-0.52)	(-0.40)	(-0.33)
1	8.65%	7.55%	7.75%	8.54%	9.31%
	(-0.48)	(-0.58)	(-0.52)	(-0.42)	(-0.35)
2	4.98%	7.68%	8.41%	9.05%	9.89%
	(-0.97)	(-0.63)	(-0.53)	(-0.41)	(-0.24)
3	10.16%	9.03%	9.88%	9.39%	9.74%
	(-0.16)	(-0.35)	(-0.22)	(-0.30)	(-0.26)

Notes: This table reports the mean annualized returns of different absolute momentum strategies. The strategies assume the formation of a winner portfolio based on the performance of the equities in the NASDAQ-100 over the past J weeks (months). The inclusion of stocks in the winner portfolio is contingent upon the return of the stock being higher than the standardized return of the benchmark (which is the U.S. 10-year Treasury bond) over J weeks (months). The portfolios are then bought and held for K weeks (months). The corresponding J and K values are either towards the left of or above each column with the values signifying either weeks or months as described in Subchapter 2.2. The rates of return per strategy are annualized using the same amount of trading days as described in Subchapter 2.2 with a year having 252 trading days. The values in the parenthesis denote the t-statistics calculated through the usage of Newey-West heteroscedasticity and autocorrelation (HAC) consistent errors. The test for the null hypothesis when calculating the t-statistics is that the mean annualized return of a given strategy is equal to the mean annualized return of the NASDAQ-100 over the 16-year sample period. One asterisk (*) denotes statistical significance at a 90% level with p-values below 0.1, two asterisks (**) at a 95% level with p-values below 0.05, and three asterisks (***) at a 99% level with p-values below 0.01.
Source: compiled by the author

APPENDIX D

Returns of the dual momentum strategies

Equal weights of equities, top decile (10%)

J	K = 0.25	0.5	1	2	3
0.25	4.21%	6.51%	7.35%	8.79%	9.28%
	(-1.05)	(-0.66)	(-0.61)	(-0.45)	(-0.41)
0.5	9.49%	8.83%	8.28%	9.04%	9.94%
	(-0.28)	(-0.34)	(-0.45)	(-0.37)	(-0.24)
1	6.88%	7.37%	7.15%	9.89%	11.31%
	(-0.72)	(-0.55)	(-0.58)	(-0.20)	(0.02)
2	6.69%	10.19%	11.34%	12.55%	13.63%
	(-0.63)	(-0.15)	(0.02)	(0.22)	(0.41)
3	13.54%	14.50%	16.70%	16.21%	14.44%
	(0.36)	(0.45)	(0.82)	(0.76)	(0.52)

Equal weights of equities, top quintile (20%)

J	K = 0.25	0.5	1	2	3
0.25	3.46%	6.51%	7.17%	8.48%	9.19%
	(-1.31)	(-0.73)	(-0.68)	(-0.49)	(-0.41)
0.5	6.92%	6.30%	7.02%	8.48%	8.99%
	(-0.78)	(-0.76)	(-0.67)	(-0.48)	(-0.41)
1	8.20%	6.98%	6.58%	8.96%	10.05%
	(-0.53)	(-0.63)	(-0.67)	(-0.34)	(-0.20)
2	4.18%	8.55%	9.52%	10.56%	11.81%
	(-1.04)	(-0.44)	(-0.30)	(-0.11)	(0.10)
3	11.42%	11.23%	12.28%	12.60%	12.34%
	(0.04)	(0.00)	(0.18)	(0.22)	(0.19)

Notes: This table reports the mean annualized returns of different dual momentum strategies. The strategies assume the formation of a winner portfolio, either consisting of the top 10% ("top decile") or the top 20% ("top quintile") of equities, based on their performance over the past J weeks (months). The inclusion of stocks in the winner portfolio is contingent upon the return of the stock being higher than the standardized return of the benchmark (which is the annual return of the NASDAQ-100 over the 16-year sample period of the author) over J weeks (months). The portfolios are then bought and held for K weeks (months). The corresponding J and K values are either towards the left of or above each column with the values signifying either weeks or months as described in Subchapter 2.2. The rates of return per strategy are annualized using the same amount of trading days as described in Subchapter 2.2 with a year having 252 trading days. The values in the parenthesis denote the t-statistics calculated through the usage of Newey-West heteroscedasticity and autocorrelation (HAC) consistent errors. The test for the null hypothesis when calculating the t-statistics is that the mean annualized return of a given strategy is equal to the mean annualized return of the NASDAQ-100 over the 16-year sample period. One asterisk (*) denotes statistical significance at a 90% level with p-values below 0.1, two asterisks (**) at a 95% level with p-values below 0.05, and three asterisks (***) at a 99% level with p-values below 0.01.

Source: compiled by the author

APPENDIX E

Standard deviations of the returns of different J/K-strategies across the three momentum approaches applied

Strategy	J	K = 0.25	0.5	1	2	3
Relative momentum: top decile	0.25	24.21%	25.30%	24.21%	25.37%	25.49%
	0.5	23.71%	24.12%	23.55%	24.68%	25.43%
	1	23.29%	24.06%	23.28%	23.90%	24.41%
	2	24.08%	23.73%	23.85%	24.52%	24.20%
	3	24.55%	24.45%	23.50%	23.31%	23.83%
Relative momentum: top quintile	0.25	21.88%	22.20%	21.65%	21.92%	22.06%
	0.5	21.42%	21.63%	21.30%	21.67%	22.06%
	1	21.26%	21.62%	21.05%	21.05%	21.78%
	2	21.39%	21.19%	20.88%	21.39%	21.44%
	3	21.70%	21.59%	21.21%	20.68%	20.91%
Absolute momentum	0.25	20.87%	20.21%	19.64%	19.69%	20.03%
	0.5	19.67%	20.01%	19.77%	19.71%	19.70%
	1	19.99%	21.23%	20.21%	19.62%	19.51%
	2	26.11%	21.86%	21.93%	19.92%	19.30%
	3	20.94%	20.97%	20.03%	19.69%	19.80%
Dual momentum: top quintile	0.25	24.72%	25.47%	24.34%	25.40%	25.56%
	0.5	23.82%	24.59%	23.93%	24.91%	25.63%
	1	23.58%	25.04%	23.73%	23.93%	24.51%
	2	29.49%	25.58%	26.02%	25.10%	24.43%
	3	25.82%	25.51%	24.13%	23.84%	24.37%
Dual momentum: top quintile	0.25	22.73%	22.41%	21.86%	22.38%	22.66%
	0.5	21.54%	22.11%	21.62%	21.88%	22.42%
	1	21.69%	23.01%	21.86%	21.42%	21.97%
	2	27.66%	23.52%	23.52%	22.26%	21.73%
	3	23.29%	23.00%	21.81%	21.20%	21.57%

Notes: This table reports the annualized standard deviations of the returns of different J/K-strategies for relative, absolute, and dual momentum. The values are found by taking all of the returns of a given strategy, finding the standard deviation of said strategy, and then multiplying the standard deviations by the square root of the ratio of 252 and K where K describes the amount of trading days for which the winner portfolio is held as described in Subchapter 2.2.

Source: compiled by the author

APPENDIX F

Sharpe ratios of different J/K-strategies across the three momentum approaches applied

Strategy	J	K = 0.25	0.5	1	2	3
Relative momentum: top decile	0.25	0.16	0.18	0.24	0.27	0.29
	0.5	0.25	0.26	0.26	0.28	0.32
	1	0.18	0.24	0.23	0.34	0.38
	2	0.32	0.38	0.42	0.45	0.49
	3	0.48	0.54	0.64	0.63	0.54
Relative momentum: top quintile	0.25	0.13	0.19	0.25	0.29	0.32
	0.5	0.11	0.15	0.21	0.28	0.30
	1	0.25	0.25	0.24	0.35	0.37
	2	0.31	0.41	0.41	0.44	0.47
	3	0.45	0.46	0.48	0.54	0.52
Absolute momentum	0.25	0.14	0.27	0.28	0.33	0.36
	0.5	0.39	0.32	0.31	0.35	0.38
	1	0.34	0.26	0.29	0.34	0.38
	2	0.12	0.26	0.30	0.36	0.41
	3	0.39	0.34	0.40	0.38	0.39
Dual momentum: top quintile	0.25	0.09	0.18	0.22	0.27	0.29
	0.5	0.32	0.28	0.27	0.29	0.31
	1	0.21	0.22	0.22	0.33	0.38
	2	0.16	0.32	0.36	0.42	0.48
	3	0.45	0.49	0.61	0.60	0.51
Dual momentum: top quintile	0.25	0.07	0.20	0.24	0.29	0.32
	0.5	0.23	0.20	0.24	0.30	0.31
	1	0.29	0.22	0.21	0.33	0.37
	2	0.08	0.28	0.32	0.39	0.45
	3	0.41	0.40	0.47	0.50	0.48

Notes: This table reports the Sharpe ratios of different J/K-strategies for relative, absolute, and dual momentum. The values are found by taking the annualized returns of a given strategy at a specific moment, subtracting the risk-free rate of the U.S. 10-year Treasury bond at that moment, and then dividing the difference with the standard deviation of the returns that the strategy had across the whole sample period.

Source: compiled by the author

Kokkuvõte

MOMENTUMIL PÕHINEVA AKTSIAPORTFELLI TULEMUSLIKKUS NASDAQ-100 (NDX) INDEKSI NÄITEL

Antud lõputöös uurib autor kolme erinevat momentum analüüsil põhinevat kauplemisstrateegiat. Need strateegiad on ristandmetel põhinev momentum, kus varade tootluseid võrreldakse teiste varade omadega; aegridadel põhinev momentum, kus varade tootluseid võrreldakse kindla võrdlusalusega; ja eelneva kahe kombineerimisel saadud duaal momentum. Ostude ja müükide tegemisel rakendatakse teemaalases kirjanduses populaarseid J/K-strateegiaid, kus eelneva J perioodi baasil määratakse, milliseid varasid ostetakse, ja sellele järgneval K perioodil hoitakse antud varasid kuni nad K perioodi lõpus realiseeritakse. Strateegiaid võrreldakse nii volatiilsuse kui ka tulususe suhtarvude põhjal.

Autori valitud andmestik katab aktsiaid, mis kuuluvad NASDAQ-100 kooslusesse perioodil 1. jaanuar 2005 kuni 30. detsember 2020. Üksikaktsiate põhine analüüs on teemaalase kirjanduse raames vähemuses - mittehajutatud varad nagu börsil kaubeldavad fondid moodustavad enamuse kirjanduses, kuna analüüsiks täielike andmete saamine on praktiliselt võimatu. Ka autor peab lõpliku andmestiku puhul välja tooma mitmeid puudujääke, mis on tingitud ajalooliste andmete kättesaamatusest. Siia hulka kuulub näiteks ellujäämise kallutatus, kus autor ei saa garanteerida, et strateegiad ostavad õigeid aktsiaid, kuna andmestik pole täielik. Strateegiate jaoks vajalikud võrdlusalused on USA 10-aastane võlakiri ja indeksi NASDAQ-100 aastane tootlus valimiperioodi raames.

Teemaalasest kirjandusest tulevad välja kindlad trendid, mida autor kasutab ära oma metoodikas. Peamiselt rakendatakse kirjanduses ristandmetel põhinevat momentumi ning võrreldakse tulemusi erinevate J/K-strateegiate raames. Kirjanduses domineerivad sarnased J/K-strateegiad, kus J ja K väärtused on kõrged – parimaid tulemusi kiputakse teadustama, kui J periood on üks aasta ja K periood on üks kuu. Lisaks ei ole kirjanduse põhjal lühikeseks müümine tulus ettevõtmine, kuna ka J perioodide halvimad varad teenivad tulu K perioodi raames. Autor rakendab oma metoodikas väiksemaid väärtuseid J ja K puhul, seekaudu keskendudes strateegiate lühiperioodi potentsiaalile. Metoodikas vaadeldakse ainult võitja portfelle, s.t. ristandmetel põhineva ja duaal momentumi puhul ostetakse J perioodi raames 10% või 20% suurima tootlusega aktsiatest ning aegridadel põhineva ja duaal momentumi puhul ostetakse varasid ainult siis, kui J perioodi raames teenitud tootlus ületab kindla võrdlusaluse.

Erinevate strateegiate raames ei teadusta autor ühtegi statistiliselt olulist tulemust. Strateegiate tootlused kipuvad kasvama koos J ja K väärtustega, s.t. mida pikema perioodi peale on teatud aktsia tootlus piisavalt kõrge, seda kõrgemat tootlust ta sellele järgneva perioodi raames toodab. Sellest hoolimata on NASDAQ-100 indeksil parem aastane tootlus valimiperioodi raames kui 75% strateegiatest. Parima tootlusega strateegiad suudavad iga-aastaselt toota vähemalt viis protsenti rohkem kui NASDAQ-100, mis tähendaks, et 16-aastase valimiperioodi raames suudavad need strateegiad teenida enam kui kaks korda rohkem antud indeksist. Sharpe'i suhtarvu baasil ei ole strateegiate riskile kohandatud tootlused silmapaistvad, kuna maksimaalne saavutatud väärtus kõigi strateegiate raames on 0.64. Autor teadustab aegridadel põhineva momentumi puhul ka vastandstrateegiat, kus ostetakse aktsiaid, mis ei ületa võrdlusalust. Sarnase strateegia puhul on võimalik tihedama kauplemise puhul saavutada NASDAQ-100 indeksist kõrgemaid aastaseid tootlusi.

Võtmesõnad:

G11 Investeerimisotsused

G11 Portfelli valik

G23 Finantsinstrumendid

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Sten Mirski
24/05/2021