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VASILIKI KARANASOU

Exotic spherically symmetric objects in modified gravity

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Contents

List of publications

This thesis is based on the following three publications:

I H. Asuküla, M. Hohmann, V. Karanasou, S. Bahamonde, C. Pfeifer and J. L. Rosa, "Spherically symmetric vacuum solutions in one-parameter new general relativity and their phenomenology", Phys. Rev. D **109**, no.6, 064027 (2024)

See attached publication in Chapter 6 and [arXiv], [inSPIRE], [ETIS].

II M. Hohmann and V. Karanasou, "Symmetric teleparallel connection and spherical solutions in symmetric teleparallel gravity", Phys. Rev. D **111**, no.6, 064057 (2025)

See attached publication in Chapter 7 and [arXiv], [inSPIRE], [ETIS].

III E. F. Eiroa, G. Figueroa-Aguirre and V. Karanasou, "Stability of spherical thin-shell wormholes in scalar-tensor theories", Eur. Phys. J. Plus **141**, no.2, 134 (2026)

See attached publication in Chapter 8 and [arXiv], [inSPIRE], [ETIS].

Author's contribution

I, Vasiliki Karanasou, contributed to all the aforementioned publications and I actively participated in the scientific discussions with my co-authors, including the identification of the research problems, the development of our methodology and the analysis of the results. In particular, I verified most of the equations and results in publication I and I assisted in the writing process of some sections of this paper. I performed the majority of the calculations for the publication II and I was primarily responsible for writing the manuscript. I carried out most of the calculations for the publication III and I contributed substantially to the writing process of the paper. It should be noted that the authors' names follow alphabetical order and I am the corresponding author of publication II.

I have partly presented the results of publication I, while it was in preparation, at several conferences and schools, including the 59. Winter School of Theoretical Physics and third COST Action CA18108 Training School "Gravity – Classical, Quantum and Phenomenology which was held on 12–21 February 2023 in Poland, CosmoVerse@Lisbon held on 30 May-1 June 2023 in Portugal, the 4th Annual Conference of COST Action CA18108 held on 10-14 July 2023 in Croatia, the Metric-Affine Frameworks for Gravity held in Estonia in 2023 as well as the Space Science @ Drop Tower Seminar at ZARM, University of Bremen in October 2023. I also presented the results of both publications I and II at the Journal Club of Tartu Observatory as an invited speaker in March 2024. I presented the results of all publications at the Theoretical Physics Seminar at the Institute of Physics, University of Tartu on 31 March 2026, as well as at the Journal Club of the Laboratory of Theoretical Physics.

Chapter 1

Introduction

1.1 Motivation

The theory of General Relativity (GR), introduced by Albert Einstein in 1915 [1], offered a revolutionary way of envisaging gravity, time and space. Einstein introduced the simple and elegant idea that gravity is geometry. The existence of matter and energy curves the four-dimensional spacetime and this curvature governs the motion of objects and particles.

GR has successfully explained a wide range of phenomena on astrophysical and cosmological scales that could not be explained in the context of traditional Newtonian gravity. These phenomena include the anomalous perihelion precession of Mercury, the gravitational redshift, the Shapiro time delay, the light deflection and the gravitational lensing. Classical and modern tests and experiments have underlined the success of this theory [2–10]. Additionally, GR predicted the existence of black holes and gravitational waves long before their detection. In particular, the first gravitational wave signal was detected in 2015 by LIGO [11], while the first image of a black hole was captured by the Einstein Telescope in 2019 [12].

Despite the prominence of General Relativity, several indications suggest that this theory has limitations. Recent cosmological observations reveal tensions within the standard ΛCDM cosmological model, which is based on GR [13–15]. According to this model, the universe consists of approximately 68% dark energy, which is considered responsible for the observed accelerated expansion of the universe [16, 17], 27% dark matter and only 5% ordinary baryonic matter. However, the nature of dark matter and dark energy remains unknown. At the same time, the appearance of spacetime singularities, such as black hole singularities and the Big Bang, indicates the breakdown of the theory. Finally, GR is not compatible with the principles of quantum field theory.

Various approaches have been proposed to address these issues, including modifications of the gravitational theory itself [18–21]. Such modifications arise by introducing higher-order derivatives of the variables, higher dimensions, additional fields or new geometric quantities beyond curvature. This implies the existence of a broad range of potential candidates and while some models may resolve certain problems, they leave others unresolved. Observations have provided constraints that allow us to discard non-viable classes, yet a wide range of promising alternatives still has to be explored, since none clearly dominates the rest.

In order to better understand these theories, evaluate their advantages and assess their viability, we need to study several aspects of them, including their theoretical foundations, cosmological and astrophysical implications and their exact solutions.

1.2 Aim of the thesis

The aim of this thesis is to explore spherically symmetric objects in modified theories of gravity and investigate their properties. These objects may arise as solutions of the field equations of the gravitational theory or by the proper matching of such solutions. Solutions of the field equations describe objects such as black holes or wormholes, which are predicted by Einstein's theory, but also more exotic objects that might not have a GR counterpart. Although both black holes and wormholes are solutions of the field equations in GR, only black holes have been observed. Artificial objects such as thin-shell wormholes, thin-shell bubbles and gravastars can be constructed by properly joining solutions of the theory.

Exact solutions of the field equations provide a better understanding of the gravitational theory, its properties and its viability. In particular, the study of exact solutions in modified gravity allows us to understand how the modification affects the presence of event horizons and singularities, the asymptotic behavior of the solution, but also phenomena such as the perihelion shift, the light deflection and the Shapiro time delay. Especially in the case of solutions with a well-defined GR limit, we are able to compare directly with their GR counterparts and quantitatively determine how the modification manifests. Furthermore, potential deviations from GR could be detected in the signal of gravitational waves and provide observational indications about the existence of exotic objects.

The motivation for constructing theoretical configurations which are not direct solutions of the field equations lies mainly on two aspects. The first one arises more as a conceptual problem: What kind of spacetime constructions does the theory allow us to build? The second is a more practical one, related to the problem of exotic matter. In GR, all wormhole solutions are made from exotic matter, that is, matter with unphysical properties. This issue also emerges in several modified theories. In order to address this challenge, we construct thin-shell wormholes by properly joining two spherically symmetric solutions, such as black holes. These objects might be free of exotic matter or at least have all the exotic matter concentrated on a thin layer on the wormhole throat. A complete study of thin-shell wormholes also requires investigating their stability under radial perturbations.

In this thesis, we study exotic objects in teleparallel theories and scalar tensor gravity for which the gravitational interaction is the result of new geometric quantities or an additional scalar field, respectively. We derive spherically symmetric solutions in teleparallel gravity and we study their properties. We also construct spherically symmetric thin-shell wormholes in a scalar-tensor theory and we study their stability and the type of matter they consist of. The assumption of spherical symmetry is a good approximation of astrophysical objects such as black holes and it provides a simple but non-trivial landscape to explore.

1.3 Statements

The present thesis presents the following significant results:

1. The derivation of the field equations in one-parameter New General Relativity for spherically symmetric spacetimes using the most general tetrad for spherical symmetry.

2. Three general branches of solutions of the field equations in one-parameter New General Relativity, vacuum solutions for each branch and the study of their properties.

3. The derivation of the symmetric teleparallel connection for a stationary spherically symmetric spacetime starting from the coincident gauge.

4. The derivation of the field equations in Newer General Relativity, two families of vacuum solutions and the study of their properties.

5. The construction of thin-shell wormholes in a class of scalar-tensor gravity, the study of their stability and the type of matter on the throat.

6. The identification of stable thin-shell wormholes for a specific subclass of scalar-tensor theories.

1.4 Structure of the thesis

The present thesis is structured as follows: The first part is a review of all topics which are important for understanding the publications that form the thesis. In the second part, these publications are presented. The first part is divided into four chapters. Chapter 2 introduces the basic concepts and formalism of General Relativity and the transition to modified theories, specifically teleparallel gravity and scalar-tensor theories. In Chapter 3, the formulation of metric teleparallel gravity with a focus on New General Relativity and the key results of publication 6 are presented. In Chapter 4, symmetric teleparallel gravity is introduced, focusing on the class of Newer General Relativity and the key results of publication 7 are presented. In Chapter 5, scalar-tensor gravity is discussed, including the junction conditions in this framework and the key results of publication 8 are presented. The second part is divided into three chapters. In each chapter, one publication is presented. Chapter 6 presents the publication with the title "Spherically symmetric vacuum solutions in one-parameter new general relativity and their phenomenology". Chapter 7 presents the publication with the title "Symmetric teleparallel connection and spherical solutions in symmetric teleparallel gravity". Chapter 8 presents the publication with the title "Stability of spherical thin-shell wormholes in scalar-tensor theories". At the end, the author's CV is provided in English and Estonian.

The conventions used in this work are as follows: we use the positive signature for the metric, which is represented as $(-,+,+,+)$. Latin letters denote spatial coordinate indices, while Greek letters denote spacetime coordinate indices.

Chapter 2

From General Relativity to Modified Gravity

2.1 General Relativity

General Relativity is a gravitational theory based on Riemannian geometry. The fundamental variable is the metric tensor $g_{\mu\nu}$, a quantity used to measure distances in spacetime. From the metric tensor, we construct a non-tensorial quantity, the Levi-Civita connection, which reads

$$\mathring{\Gamma}^{\alpha}{}_{\beta\gamma} = \frac{1}{2} g^{\alpha\delta} (\partial_\beta g_{\delta\gamma} + \partial_\gamma g_{\delta\beta} - \partial_\delta g_{\beta\gamma}). \tag{2.1}$$

As we notice, it is symmetric in its lower indices. The connection is necessary for introducing differentiations in the spacetime.

In GR, the gravitational interaction is mediated by the curvature of spacetime, which is described by the Riemann tensor

$$\mathring{R}^{\alpha}{}_{\beta\gamma\delta} = \partial_\gamma \mathring{\Gamma}^{\alpha}{}_{\beta\delta} - \partial_\delta \mathring{\Gamma}^{\alpha}{}_{\beta\gamma} + \mathring{\Gamma}^{\alpha}{}_{\sigma\gamma} \mathring{\Gamma}^{\sigma}{}_{\beta\delta} - \mathring{\Gamma}^{\alpha}{}_{\sigma\delta} \mathring{\Gamma}^{\sigma}{}_{\beta\gamma}. \tag{2.2}$$

The Riemann tensor admits the following symmetries

$$\mathring{R}_{\alpha\beta\gamma\delta} = \mathring{R}_{\gamma\delta\alpha\beta}, \tag{2.3}$$

$$\mathring{R}_{\alpha\beta\gamma\delta} = -\mathring{R}_{\beta\alpha\gamma\delta} = -\mathring{R}_{\alpha\beta\delta\gamma} = \mathring{R}_{\beta\alpha\delta\gamma}, \tag{2.4}$$

$$\mathring{R}_{\alpha\beta\gamma\delta} + \mathring{R}_{\alpha\gamma\delta\beta} + \mathring{R}_{\alpha\delta\beta\gamma} = 0. \tag{2.5}$$

It also satisfies the Bianchi identity

$$\mathring{\nabla}_\sigma \mathring{R}_{\alpha\beta\gamma\delta} + \mathring{\nabla}_\delta \mathring{R}_{\alpha\beta\sigma\gamma} + \mathring{\nabla}_\gamma \mathring{R}_{\alpha\beta\delta\sigma} = 0, \tag{2.6}$$

where $\mathring{\nabla}_\alpha$ represents the covariant derivative which is related with the Levi-Civita connection and is defined as

$$\mathring{\nabla}_\mu P^{\alpha}_{\beta} = \partial_\mu P^{\alpha}_{\beta} + \mathring{\Gamma}^{\alpha}{}_{\mu\rho} P^{\rho}_{\beta} - \mathring{\Gamma}^{\rho}{}_{\mu\beta} P^{\alpha}_{\rho}, \tag{2.7}$$

for an arbitrary mixed tensor P^{α}_{β}. The definition can easily be generalized for higher rank tensors. By contracting the indices of the Riemann tensor, we obtain the second rank tensors

$$\mathring{A}_{\alpha\beta} = \mathring{R}^{\gamma}{}_{\gamma\alpha\beta}, \quad \mathring{R}_{\alpha\beta} = \mathring{R}^{\gamma}{}_{\alpha\gamma\beta}, \tag{2.8}$$

where $\mathring{R}_{\alpha\beta}$ is the Ricci tensor. One more contraction gives

$$\mathring{R} = g^{\alpha\beta}\mathring{R}_{\alpha\beta}. \tag{2.9}$$

where $\mathring{R}$ is the Ricci scalar.

In Riemannian geometry, the motion of a free particle is determined by the principle of least action. In Euclidean space, the shortest path between two points is a straight line. In a Riemannian manifold, the shortest path is typically a curved trajectory, which is described by the geodesic equations

$$\frac{d^2x^{\alpha}}{d\tau^2} + \mathring{\Gamma}^{\alpha}_{\beta\gamma}\frac{dx^{\beta}}{d\tau}\frac{dx^{\gamma}}{d\tau} = 0, \tag{2.10}$$

where τ is the proper time along the geodesics.

The action that describes GR is called Einstein-Hilbert action and it is given by

$$S_{GR} = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa}\mathring{R} + \mathcal{L}_{\mathrm{m}}\right), \tag{2.11}$$

where $\mathcal{L}_{\mathrm{m}}$ is the matter field Lagrangian. By varying the action with respect to the metric, we obtain the Einstein field equations

$$\mathring{R}_{\alpha\beta} - \frac{1}{2}\mathring{R}g_{\alpha\beta} = \kappa\Theta_{\alpha\beta}, \tag{2.12}$$

where $\Theta_{\alpha\beta}$ is the energy-momentum tensor, which determines the type of matter.

2.2 Fundamental concepts in gravitational physics

As we discussed, the main topic of the present thesis is the study of spherically symmetric objects in modified gravity. These objects may arise as solutions of the field equations or we construct them by using spherical solutions of the theory. Spherical solutions of the field equations can be described by the line element of the form

$$ds^2 = -A(t,r)^2dt^2 + B(t,r)^2dr^2 + r^2(d\theta^2 + r^2\sin^2\theta d\phi^2). \tag{2.13}$$

If we also impose stationarity, which implies no time dependence, then we have $A(t,r) = A(r)$ and $B(t,r) = B(r)$. In GR, for $A(r) = 1/B(r) = 1 - 2m/r$, we obtain the Schwarzschild black hole, where m is the black hole mass. A simple generalization of the Schwarzschild solution is the Reissner-Nordström, which describes a charged black hole with $A(r) = 1/B(r) = 1 - \frac{2m}{r} - \frac{Q^2}{r^2}$. In modified theories of gravity, we might find identical solutions of the field equations, modified solutions with a well-defined GR limit or even exotic solutions that do not have a GR counterpart. Before we start discussing the main work and results of this thesis, we introduce several concepts that are useful in gravitational physics and especially for the study of spherically symmetric objects.

2.2.1 Asymptotic flatness

A spacetime is asymptotically flat when the curvature vanishes at large distances and consequently, it approaches the Minkowski flat spacetime. This implies the conditions

$$\lim_{r\to\infty} A(r) = 1, \ \lim_{r\to\infty} B(r) = 1 \,. \tag{2.14}$$

2.2.2 Komar mass

The Komar mass describes the mass of an object as computed by an observer at infinity [22, 23]. It is a quantity defined only for stationary spacetimes. We introduce a timelike Killing vector field $\xi^\mu = \{1, 0, 0, 0\}$ and a spacelike hypersurface Σ_t at a constant slice t from the event horizon to spatial infinity. The Komar mass is expressed as

$$\mathcal{M} = -\frac{1}{8\pi} \int_{S_t} \mathring{\nabla}^\mu \xi^\nu dS_{\mu\nu} \,, \tag{2.15}$$

where S_t is the 2-boundary of Σ_t and $dS_{\mu\nu}$ is the surface element of S_t. The Komar mass for any spherically symmetric spacetime takes the form

$$\mathcal{M} = \frac{1}{8\pi} \lim_{r\to\infty} \int_0^\pi \int_0^{2\pi} \frac{r^2 g_{tt}'}{\sqrt{g_{tt} g_{rr}}} \sin\theta \, d\phi \, d\theta. \tag{2.16}$$

2.2.3 Gravitational redshift

The gravitational redshift describes how the frequency of a light signal changes, as measured by a stationary observer, when the light travels outward towards infinity. It can be calculated by

$$\omega_\infty g_{tt\infty} = \omega g_{tt} \,, \tag{2.17}$$

where the frequencies ω and ω_∞ of a photon are measured by the stationary observer at some radius r and at infinity respectively.

2.2.4 Singularities

We are able to identify curvature singularities by calculating some scalar quantity of the theory such as the Kretschmann invariant given by

$$\mathring{K} = \mathring{R}_{\alpha\beta\gamma\delta}\mathring{R}^{\alpha\beta\gamma\delta}. \tag{2.18}$$

The singularities correspond to values of the radial coordinate for which $\mathring{K}$ becomes infinite.

2.2.5 Horizons

In gravitational physics, we encounter the concept of horizons. The event horizon is a null hypersurface around a massive object, such as a black hole, beyond which nothing can escape to infinity, not even light. A Killing horizon is a surface that admits a Killing vector, that is, a vector that represents a spacetime symmetry and becomes null. In regions containing trapped surfaces, that is, closed, two-dimensional surfaces S for which the expansion ϑ (more details about ϑ can be found in (2.2.6)) of all future-directed null geodesic congruences orthogonal to S is negative, the boundary of this region defines the apparent horizon.

The event horizon, the Killing horizon and the apparent horizon of static spacetimes coincide [23]. In the context of this thesis, we study static spacetimes and thus, in order to determine if an object admits horizons, it is enough to check if there is a Killing vector that becomes null.

2.2.6 Geodesic motion

We would also like to study how the motion of a test particle is affected by the presence of a gravitating object. The spherical symmetry allows us to work at the equatorial plane $\theta = \pi/2$. The energy E and the angular momentum L of the particle are given respectively by

$$E = A^2\dot{t}, \quad L = r^2\dot{\phi}, \tag{2.19}$$

and they are constants of the motion. The dot represents differentiation with respect to the proper time τ or the affine parameter λ for massive particles or photons respectively.

For studying the radial motion of particles for which $L = 0$, we consider a beam of worldlines. In order to check the convergence or divergence of the worldlines, we compute the expansion scalar

$$\vartheta = \mathring{\nabla}_\alpha u^\alpha, \tag{2.20}$$

where u^α is the tangent vector field to the worldlines. The worldlines converge when the expansion scalar is negative and diverge when it is positive. Additionally, we explore how this behavior evolves with time by investigating the Raychaudhuri equations given by

$$\frac{d\vartheta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - \mathring{R}_{\alpha\beta}u^\alpha u^\beta, \tag{2.21}$$

$$\frac{d\vartheta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - \mathring{R}_{\alpha\beta}u^\alpha u^\beta, \tag{2.22}$$

for massive and massless particles respectively. We note that $\sigma_{\alpha\beta}$ is the shear tensor, $\omega_{\alpha\beta}$ is the rotation tensor and $h_{\alpha\beta}$ is the transverse metric [23].

Starting from the line element, we obtain the following equation

$$\frac{1}{2}\dot{r}^2 + V_{\text{eff}}(r) = 0\,, \tag{2.23}$$

where the effective potential $V_{\text{eff}}(r)$ has the form

$$V_{\text{eff}}(r) = -\frac{1}{2B^2}\left(s + \frac{E^2}{A^2} - \frac{L^2}{r^2}\right)\,. \tag{2.24}$$

We set $s = -1$ for massive particles and $s = 0$ for photons. For a particle that follows a bound orbit around an object, there should be two radii $r_{1,2}$ for which

$$\dot{r} = 0, \quad V_{\text{eff}}(r) = 0, \tag{2.25}$$

and for $r_1 < r < r_2$

$$V_{\text{eff}}(r) < 0. \tag{2.26}$$

In the special case of circular orbits for which $r_1 = r_2$, we should have the additional conditions

$$\ddot{r} = 0, \quad V'_{\text{eff}}(r) = 0. \tag{2.27}$$

The orbits are stable for $V''_{\text{eff}}(r_{1,2}) > 0$.

2.2.7 Phenomena: Light deflection, Perihelion shift & Shapiro time delay

An other topic that is interesting to study is the phenomena of light deflection, the perihelion shift and the Shapiro time delay, which are related with the orbit of a test particle when it passes near a gravitating object. These phenomena are linked to classical observables and consequently, observations could reveal potential deviations from GR or indications about the existence of exotic objects predicted in modified theories. We are able to derive these observables from the different parameterizations of the orbits.

The perihelion shift $\Delta\phi_{\text{peri}}$ is the deviation of the particle's orbit from a perfect ellipse around the object. We use the orbits parametrized as curves $r(\phi)$ or $\phi(r)$ and we define this deviation as follows

$$\Delta\phi_{\text{peri}} = 2\left|\int_{R_-}^{R_+} \frac{d\phi}{dr} dr\right| - 2\pi, \tag{2.28}$$

where R_- is the perihelion and R_+ the aphelion.

The light deflection $\Delta\phi_{\text{light}}$ is the angle about which a light ray bends while passing by the gravitating object. Following [24], the deflection angle $\Delta\phi_{\text{light}}$ can be expressed as

$$\Delta\phi_{\text{light}} = \int_{R_S}^{r_c} \frac{d\phi}{dr} dr + \int_{r_c}^{R_R} \frac{d\phi}{dr} dr + \Psi_R - \Psi_S, \tag{2.29}$$

where $r = R_S$ is the location of the source of the light ray with angle $\Psi_S(R_S) = L/E\sqrt{f(R_S)}/R_S$ with respect to the radial direction and $r = R_R$ is the location of a receiver with an angle $\Psi_R(R_R) = L/E\sqrt{f(R_R)}/R_R$ with respect to the radial direction (Figure 2.1).

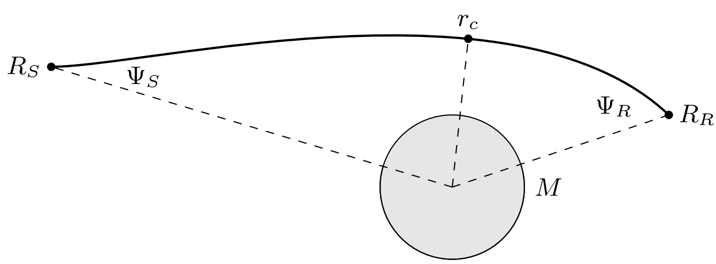

Figure 2.1: The phenomenon of light deflection

The Shapiro time delay Δt_{Shap} is the additional time that a light signal takes to propagate between two points r_1 and r_2 due to the presence of the gravitating object, compared to the absence of this object. It is defined as

$$\Delta t_{\text{Shap}} = \int_{r_1}^{r_2} \frac{dt}{dr} dr - \left(\int_{r_1}^{r_2} \frac{dt}{dr} dr\right)_{\text{Minkowski}}. \tag{2.30}$$

2.2.8 Traversable wormholes, energy conditions and exotic matter

Wormholes arise as solutions of the field equations in gravity theories, as black holes. A wormhole geometry can be described by a line element of the form

$$ds^2 = -e^{2\varphi(r)}dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1/2} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \,, \tag{2.31}$$

where $\varphi(r)$ is the redshift function and $b(r)$ the shape function. We assume $r > 0$. From the form of the metric, we realize that wormhole solutions do not admit horizons since $g_{tt} \neq 0$ for any function $\varphi(r)$. Additionally, the flaring-out condition, expressed as

$$b(r) < r_0 < r, \quad b(r_0) = r_0, \quad b'(r_0) \leq 1 \,, \tag{2.32}$$

should be satisfied. We notice that r_0 is the minimum radius which corresponds to the radius of the wormhole throat [25]. The flaring out condition implies that there are two asymptotically flat regions that are connected on the throat.

An other important aspect of wormhole physics is the type of matter from which wormholes are made. GR predicts that these objects require exotic matter, that is, matter which exhibits physically unreasonable properties [25–27]. In order to understand how exotic matter emerges, we have to introduce the energy conditions. These conditions are some simple, physically motivated constraints imposed on the stress-energy tensor which determines the type of matter. We classify them as weak, null, strong and dominant energy conditions and they can be expressed, respectively, as follows

$$\Theta_{\alpha\beta} u^\alpha u^\alpha \geq 0, \tag{2.33}$$

$$\Theta_{\alpha\beta} k^\alpha k^\alpha \geq 0, \tag{2.34}$$

$$(\Theta_{\alpha\beta} - \tfrac{1}{2}\Theta g_{\alpha\beta}) u^\alpha u^\beta \geq 0, \tag{2.35}$$

$$-\Theta^\alpha_{\ \beta} u^\beta \text{ is a future-directed, timelike or null, vector field,} \tag{2.36}$$

where u^α is a timelike vector while k^α is a null vector. The weak energy condition implies that the energy density of any matter distribution, as measured by an observer, should not be a negative quantity and a similar statement holds for the null energy condition. Considering the field equations, the strong energy condition is actually a statement for the Ricci tensor $R_{\alpha\beta} u^\alpha u^\beta \geq 0$. The dominant energy condition implies that matter should follow timelike or null worldlines.

The energy conditions can also be expressed in terms of the energy density and the pressures if we express the stress-energy tensor as follows

$$\Theta^{\alpha\beta} = \rho \hat{e}_0^\alpha \hat{e}_0^\beta + p_1 \hat{e}_1^\alpha \hat{e}_1^\beta + p_2 \hat{e}_2^\alpha \hat{e}_2^\beta + p_3 \hat{e}_3^\alpha \hat{e}_3^\beta, \tag{2.37}$$

where the vectors $\hat{e}_{\mu}^{\alpha}$ form an orthonormal basis. Thus, the weak, null, strong and dominant energy conditions take the following form, respectively

$$\rho \geq 0, \quad \rho + p_i \geq 0, \tag{2.38}$$

$$\rho + p_i \geq 0, \tag{2.39}$$

$$\rho + \textstyle\sum_i p \geq 0, \quad \rho + p_i \geq 0, \tag{2.40}$$

$$\rho \geq 0, \quad \rho \geq |p_i|. \tag{2.41}$$

We notice that the null energy condition is the most important in the sense that its violation implies the violation of all the other conditions.

2.2.9 Junction conditions and thin-shell wormholes

In gravitational physics, it is common that we might have two regions V^+ and V^- in the four-dimensional spacetime, which are separated by a hypersurface Σ and we would like to find proper conditions for their smooth match on their boundary. This also implies that these conditions should allow the union of the metrics to be a valid solution of the field equations.

A hypersurface is a three-dimensional manifold embedded in the four-dimensional spacetime and can be expressed either with restrictions on the coordinates $\Phi(x^\alpha) = 0$ or by parametric equations $x^\alpha = x^\alpha(y^a)$, where x^α represent the coordinates of the spacetime and y^a are the intrinsic coordinates of the hypersurface. For a spacelike or timelike hypersurface we can define the unit normal as

$$n_\alpha = \frac{\epsilon \Phi_{,\alpha}}{|g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}|^{1/2}}, \tag{2.42}$$

pointing from V^- to V^+. We have $n^\alpha n_\alpha = \epsilon$ with $\epsilon = 1$ or -1 for a timelike or spacelike hypersurface respectively. The tangent vector to the hypersurface is given by

$$e_a^\alpha = \frac{\partial x^\alpha}{\partial y^a}. \tag{2.43}$$

We define the induced metric or first fundamental form of the hypersurface as

$$h_{ab} = g_{\alpha\beta} e_a^\alpha e_b^\beta, \tag{2.44}$$

which describes the intrinsic aspects of the hypersurface. We also define the extrinsic curvature or second fundamental form of the hypersurface as

$$K_{ab} = \mathring{\nabla}_\beta n_\alpha e_a^\alpha e_b^\beta, \tag{2.45}$$

which describes how the hypersurface is embedded in the four-dimensional spacetime.

Assuming that a quantity F is defined on both sides of the hypersurface Σ, we define the jump of F across Σ as

$$[F] = F^{+} - F^{-}, \tag{2.46}$$

where $F^{\pm}$ is the quantity in the area $V^{\pm}$.

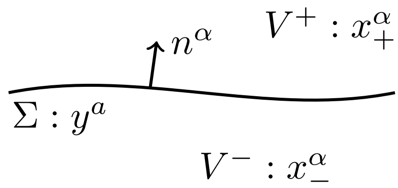

Figure 2.2: Two regions V^{+} and V^{-} described by the coordinates x_{+}^{a} and x_{-}^{a} respectively separated by a hypersurface Σ with intrinsic coordinates y^{a}. n^{α} is the unit normal on Σ.

A standard method for deriving the junction conditions is to express the field equations in a distributional form. During this process, singular terms arise which we have to either eliminate or interpret physically. The junction conditions in GR are given by the continuity of the induced metric and the extrinsic curvature across the hypersurface [28], [29]

$$[h_{ab}] = [K_{ab}] = 0. \tag{2.47}$$

When the second condition fails, a thin layer of matter emerges on Σ with stress-energy tensor

$$S_{ab} = -\frac{\epsilon}{\kappa^2}([K_{ab}] - [K]h_{ab}). \tag{2.48}$$

The singular term here describes a thin distribution of matter on the hypersurface that we call thin-shell with stress-energy tensor $S_{\alpha\beta}$.

Junction conditions have several applications, such as in astrophysics for ensuring the proper match of the exterior and the interior of a star, in cosmology for the construction of brane world models, but also in wormhole physics. As we mentioned previously, all wormhole solutions in GR (but also in some modified theories) require of exotic matter. By applying the thin-shell formalism, we construct new objects, which we call thin-shell wormholes, in an attempt to completely eliminate or reduce the need for exotic matter or concentrate it on a thin layer, which is located on the wormhole throat. These objects are not solutions of the field equations, but we construct them by joining two solutions [30–33]. In this context, assuming two spherically symmetric spacetimes which usually describe black hole solutions, we remove a region with $r < a$ from each geometry where

a will be the radius of the shell and we apply a "cut-and-paste" procedure. We connect these two areas by matching their boundaries on the surface

$$\Sigma = r - a = 0. \tag{2.49}$$

The new spacetime describes a wormhole with the wormhole throat on Σ and two asymptotically flat regions connected on the throat with radius a.

2.3 Teleparallel gravity

In this subsection, we discuss a broad class of modified theories, which arise by introducing new geometric contributions along with the curvature of the spacetime. By assuming a general affine connection $\Gamma^{\mu}_{\ \nu\rho}$, we can construct more general theories, the so-called metric affine theories. Using the affine connection, we define three quantities which describe different geometric properties of spacetime. These quantities are the curvature

$$R^{\mu}_{\ \nu\rho\sigma} = \partial_{\rho}\Gamma^{\mu}_{\ \nu\sigma} - \partial_{\sigma}\Gamma^{\mu}_{\ \nu\rho} + \Gamma^{\mu}_{\ \tau\rho}\Gamma^{\tau}_{\ \nu\sigma} - \Gamma^{\mu}_{\ \tau\sigma}\Gamma^{\tau}_{\ \nu\rho}, \tag{2.50}$$

the torsion

$$T^{\mu}_{\ \nu\rho} = \Gamma^{\mu}_{\ \rho\nu} - \Gamma^{\mu}_{\ \nu\rho} \tag{2.51}$$

and the non-metricity

$$Q_{\mu\nu\rho} = \nabla_{\mu}g_{\nu\rho} = \partial_{\mu}g_{\nu\rho} - \Gamma^{\sigma}_{\ \nu\mu}g_{\sigma\rho} - \Gamma^{\sigma}_{\ \rho\mu}g_{\nu\sigma}. \tag{2.52}$$

While the curvature constructed from the Levi-Civita connection $\mathring{\Gamma}^{\mu}_{\ \nu\rho}$ is the only quantity that accounts for the gravitational effects in GR, in metric-affine theories, gravity can be the result of all three aforementioned geometric quantities. We are able to comprehend their different effect when we parallel transport a vector in such geometries, as it is explained in Figure 2.3.

There is an interesting subclass of metric affine theories, which is called teleparallel gravity for which the curvature vanishes identically. When non-metricity vanishes as well, the theory is called metric teleparallel gravity and gravity is the result of torsion only. Alternatively, when torsion vanishes along with the curvature, the theory is called symmetric teleparallel and gravity is the result of non-metricity only. The following actions describe the simplest metric and symmetric teleparallel theories respectively

$$S_{TEGR} = \int d^4x\sqrt{-g}\ \left(-\frac{1}{2\kappa}T + \mathcal{L}_m\right), \tag{2.53}$$

$$S_{STEGR} = \int d^4x\sqrt{-g}\ \left(-\frac{1}{2\kappa}Q + \mathcal{L}_m\right). \tag{2.54}$$

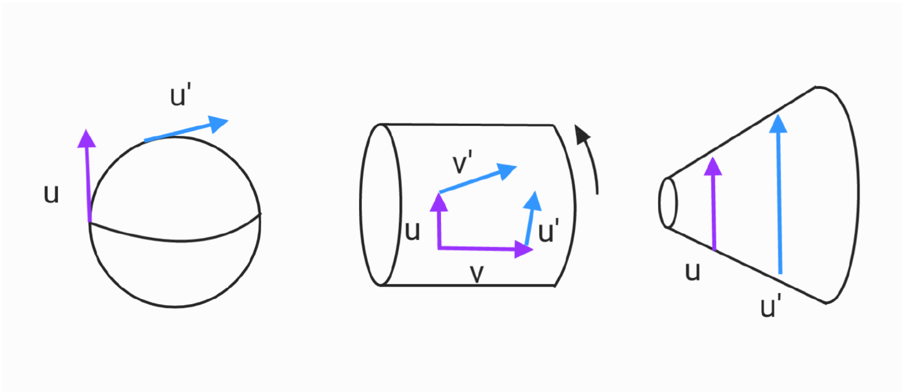

Figure 2.3: The effect of curvature, torsion and non-metricity on the parallel transport of a vector. In the case of a spacetime with curvature only, when we parallel transport a vector u, its direction changes. In a spacetime with torsion, when we parallel transport two vectors u and v, we realize that the parallelogram formed by these vectors does not close. In a spacetime with non-metricity, when we parallel transport a vector u, we notice that its magnitude changes.

It is worth highlighting the similarity to the Einstein-Hilbert action. The Ricci scalar is replaced by the torsion scalar or the non-metricity scalar respectively. The theories are equivalent to GR up to a boundary term and for this reason, they are called Teleparallel Equivalent of General Relativity (TEGR) and Symmetric Teleparallel Equivalent of General Relativity (STEGR). This also implies that the field equations are identical to the ones from GR. These three theories form the so-called Geometric Trinity of Gravity [34].

More general teleparallel theories can be constructed by substituting the torsion scalar T or the non-metricity scalar Q with a function $f(T)$ or $f(Q)$ respectively, which correspond to $f(T)$ or $f(Q)$ theories [35]. This resembles the construction of $f(R)$ gravity, but we emphasize that these theories are not equivalent. Scalar-tensor extensions involving torsion or non-metricity have also been explored [36–39]. In this thesis, we will focus on quadratic metric and symmetric teleparallel theories which are constructed by decomposing the torsion scalar or non-metricity scalar into its irreducible components [35, 40], as we shall see in the following sections.

2.4 Scalar-tensor gravity

In this subsection, we discuss a different class of theories which is constructed by adding new fields. Scalar-tensor theories introduce a tensor and a scalar field as the mediators of gravity. In this thesis, we focus on a curvature-based subclass of scalar-tensor gravity where the metric tensor $g_{\mu\nu}$ and a scalar field ϕ account for the gravitational interaction.

A long debate in the context of scalar-tensor gravity is the choice of the working frame. There are two frames, the Einstein frame and the Jordan frame, which is considered the physical frame [41, 42]. In the Jordan frame, the scalar field is non-minimally coupled to gravity through the Ricci scalar while the matter sector does not depend on the scalar field. In this framework, particles follow the geodesic equations and this is the reason why this frame is called physical. By applying a proper conformal transformation and a redefinition of the scalar field, we obtain the Einstein frame where the scalar field is non-minimally coupled to matter. The main question that arises is whether these two frames are equivalent. If they are not equivalent, then which frame should we use? There is no definite answer to that. In some problems, the one frame might be well behaved while in others not.

In this thesis, we work only in the Jordan frame, but for better understanding, we briefly discuss both frames in the context of Brans-Dicke theory as an example. The action of the theory in the Jordan frame is given by

$$S_J = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\,\left[\phi R - \frac{\omega}{\phi}\,g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\right] + \int d^4x\,\mathcal{L}_m[g_{\mu\nu},\psi], \tag{2.55}$$

where ψ represents matter fields. After applying the conformal transformation

$$\tilde{g}_{\mu\nu} = \phi\, g_{\mu\nu} \tag{2.56}$$

and redefining the scalar field as follows

$$\phi \to \sqrt{\frac{2\omega+3}{16\pi}}\phi, \tag{2.57}$$

we obtain the action in the Einstein frame

$$S_E = \int d^4x\,\sqrt{-\tilde{g}}\,\left[\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2}\tilde{g}^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\right] + \int d^4x\,\mathcal{L}_m[\tilde{g}_{\mu\nu},\psi]. \tag{2.58}$$

For second-order derivatives of the metric and the scalar field, scalar-tensor theories can be represented by the general class of Horndeski theories described by the following action [43]

$$S_H = \int d^4x\sqrt{-g}(\mathcal{L}+\mathcal{L}_m), \tag{2.59}$$

where

$$\begin{aligned}\mathcal{L} \;=\;& G_2(\phi,X) + G_3(\phi,X)\Box\phi + G_4(\phi,X)\,R + \\ & G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right] + G_5(\phi,X)\,G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ & -\frac{1}{6}G_{5,X}\big((\Box\phi)^3 - 3\,\Box\phi\,(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) \\ & +2\,(\nabla_\mu\nabla_\nu\phi)(\nabla^\nu\nabla^\rho\phi)(\nabla_\rho\nabla^\mu\phi)\big),\end{aligned} \tag{2.60}$$

with $X = -\frac{1}{2}\nabla^{\mu}\phi\nabla_{\mu}\phi$. It is easy to check that Horndeski gravity is a generalization of other well-established gravitational theories. If we set

$$G_2 = G_3 = G_5 = 0,\ G_4 = 1/2\kappa, \tag{2.61}$$

we retrieve General Relativity. For

$$G_2 = \omega X/\phi - V(\phi),\ G_3 = G_5 = 0,\ G_4 = \phi/2, \tag{2.62}$$

Horndeski gravity reduces to Brans-Dicke theory. Finally, for the following form of the G functions

$$G_2 = -\frac{1}{2}(R\partial_R f - f)),\ G_3 = G_5 = 0,\ G_4 = \partial_R f/2, \tag{2.63}$$

we retrieve $f(R)$ gravity. ∂_R denotes derivatives with respect to the Ricci scalar.

Chapter 3

Spherically symmetric solutions in New General Relativity

In this chapter, we discuss metric teleparallel gravity, the class of teleparallel theories for which gravity is the result of the torsion of spacetime while the curvature and the non-metricity vanish identically. In subsection 3.1, we introduce the main concepts and the formulation commonly employed in the context of metric teleparallelism. In 3.2, we focus on New General Relativity (New GR), a simple generalization of TEGR, which is constructed by decomposing the torsion scalar into its irreducible components. Finally, in 3.3, we present the results of the attached publication 6. We summarize the methods used and the main steps we followed to solve the field equations and find spherically symmetric solutions in New GR.

3.1 Geometrical aspects

In contrast to GR, metric teleparallel gravity is formulated in terms of tetrads rather than the metric tensor. In particular, the fundamental variables in this class of theories are the tetrad e^a with inverse e_a and the spin connection ${\omega^a}_{b\mu}$ [44–47]. They are expressed, respectively, as

$$e^a = {e^a}_\mu dx^\mu, \quad e_a = {e_a}^\mu \partial_\mu, \quad {\omega^a}_{b\mu} = {\Lambda^a}_c \partial_\mu {\Lambda_b}^c, \tag{3.1}$$

where Λ^a_b is a local Lorentz transformation. The tetrad and its inverse satisfy the following conditions

$${e^a}_\mu {e_b}^\mu = \delta^a_b, \quad {e^a}_\mu {e_a}^\nu = \delta^\nu_\mu, \tag{3.2}$$

while the metric and its inverse are defined by

$$g_{\mu\nu} = \eta_{ab} {e^a}_\mu {e^b}_\nu, \quad g^{\mu\nu} = \eta^{ab} {e_a}^\mu {e_b}^\nu, \tag{3.3}$$

where η_{ab} is the Minkowski metric.

The connection takes the following form

$$\Gamma^{\rho}{}_{\mu\nu} = e_a{}^{\rho}(\partial_\nu e^a{}_\mu + \omega^a{}_{b\nu} e^b{}_\mu). \tag{3.4}$$

We can relate the metric teleparallel connection $\Gamma^{\mu}{}_{\nu\rho}$ with the Levi-Civita connection $\mathring{\Gamma}^{\mu}{}_{\nu\rho}$ through the following decomposition

$$\Gamma^{\mu}{}_{\nu\rho} = \mathring{\Gamma}^{\mu}{}_{\nu\rho} + K^{\mu}{}_{\nu\rho}\,, \tag{3.5}$$

where $K^{\mu}{}_{\nu\rho}$ is the contortion tensor

$$K^{\mu}{}_{\nu\rho} = \frac{1}{2}(T_{\nu}{}^{\mu}{}_{\rho} - T_{\rho}{}^{\mu}{}_{\nu} - T^{\mu}{}_{\nu\rho})\,. \tag{3.6}$$

We recall here that in metric teleparallel gravity, the curvature and the non-metricity vanish identically $R^{\rho}{}_{\sigma\mu\nu} = 0, Q_{\alpha\beta\gamma} = 0$, while the torsion tensor $T^{\rho}{}_{\mu\nu}$ accounts for the gravitational interaction.

It is worth mentioning that the tetrad and the spin connection are uniquely determined only up to a local Lorentz transformation. Under this transformation, the metric remains invariant, while the connection remains invariant only if the condition $\partial_\mu \Lambda^a_{\ b} = 0$ holds. The tetrad and the spin connection transform respectively as follows

$$e'^a{}_\mu = \Lambda^a{}_b e^b{}_\mu, \tag{3.7}$$

$$\omega'^a{}_{b\mu} = \Lambda^a{}_c (\Lambda^{-1})^d{}_b \omega^c{}_{d\mu} + \Lambda^a{}_c \partial_\mu (\Lambda^{-1})^c{}_b. \tag{3.8}$$

It is also important to discuss the Weitzenböck gauge. Taking into account the properties of flatness and metric compatibility of the connection, it is always equivalent to work in a Lorentz frame for which the spin connection vanishes

$$\omega^a{}_{b\mu} = 0. \tag{3.9}$$

This is the so-called Weitzenböck gauge. In this gauge, the metric teleparallel connection reduces to

$$\Gamma^{\rho}{}_{\mu\nu} = e_a{}^{\rho} \partial_\nu e^a{}_\mu. \tag{3.10}$$

For the rest of our discussion, we adopt the Weitzenböck gauge and thus, the tetrad is the only fundamental dynamical variable in this framework.

Before we close this subsection, it is necessary to introduce some useful quantities. We express the torsion scalar as follows

$$T = \frac{1}{4}T_1 + \frac{1}{2}T_2 - T_3, \tag{3.11}$$

where we define

$$T_1 = T^{\mu\nu\rho}T_{\mu\nu\rho}, \quad T_2 = T^{\mu\nu\rho}T_{\rho\nu\mu}, \quad T_3 = T^{\mu}{}_{\mu\rho}T_{\nu}{}^{\nu\rho}. \tag{3.12}$$

The torsion scalar can be related with the Ricci scalar of the Levi-Civita connection in the following way

$$\mathring{R} = -T + B\,, \quad B = 2\mathring{\nabla}_\lambda T^\lambda\,. \tag{3.13}$$

When we study metric teleparallel gravity, it is convenient to introduce the axial, vector and tensor torsion, respectively

$$a_\mu = \tfrac{1}{6}\epsilon_{\mu\nu\sigma\rho}T^{\nu\sigma\rho}\,, \quad v_\mu = T^\sigma{}_{\sigma\mu}\,, \tag{3.14}$$

$$t_{\sigma\mu\nu} = \tfrac{1}{2}\left(T_{\sigma\mu\nu} + T_{\mu\sigma\nu}\right) + \tfrac{1}{6}\left(g_{\nu\sigma}v_\mu + g_{\nu\mu}v_\sigma\right) - \tfrac{1}{3}g_{\sigma\mu}v_\nu\,, \tag{3.15}$$

The tensor torsion satisfies the following symmetries

$$t_{\alpha\mu\nu} = t_{\mu\alpha\nu}, \quad t_{\alpha\mu\nu} + t_{\nu\alpha\mu} + t_{\mu\nu\alpha} = 0, \quad t^{\alpha\mu}{}_\alpha = t_\alpha{}^{\alpha\mu} = t^{\mu\alpha}{}_\alpha = 0\,. \tag{3.16}$$

The torsion can be expressed in terms of the new quantities as follows

$$T_{\mu\nu\rho} = \frac{1}{3}(g_{\mu\nu}v_\rho - g_{\mu\rho}v_\nu) + \epsilon_{\mu\nu\rho\sigma}a^\sigma + \frac{2}{3}(t_{\mu\nu\rho} - t_{\mu\rho\nu})\,. \tag{3.17}$$

We are then able to construct the following independent parity-even scalars

$$T_{\text{axi}} = a_\mu a^\mu = \frac{1}{18}\left(2T_{\sigma\mu\nu}T^{\mu\sigma\nu} - T_{\sigma\mu\nu}T^{\sigma\mu\nu}\right)\,, \tag{3.18a}$$

$$T_{\text{vec}} = v_\mu v^\mu = T^\sigma{}_{\sigma\mu}T_\rho{}^{\rho\mu}\,, \tag{3.18b}$$

$$T_{\text{ten}} = t_{\sigma\mu\nu}t^{\sigma\mu\nu} = \frac{1}{2}\left(T_{\sigma\mu\nu}T^{\sigma\mu\nu} + T_{\sigma\mu\nu}T^{\mu\sigma\nu}\right) - \frac{1}{2}T^\sigma{}_{\sigma\mu}T_\rho{}^{\rho\mu}\,. \tag{3.18c}$$

3.2 New General Relativity

After having introduced the metric teleparallel formulation, we are able to focus on a specific metric teleparallel theory, the so-called New General Relativity (New GR), which is a simple generalization of TEGR [40]. The theory is constructed by decomposing the torsion scalar into its irreducible components that we previously introduced

$$T = c_{\text{axi}}T_{\text{axi}} + c_{\text{ten}}T_{\text{ten}} + c_{\text{vec}}T_{\text{vec}}, \tag{3.19}$$

where $c_{\text{ten}}, c_{\text{vec}}$ and c_{axi} are the parameters of the theory. Different values of the parameters correspond to different subclasses of New GR. For the following values

$$c_{\text{ten}} = \frac{2}{3}, \quad c_{\text{vec}} = -\frac{2}{3}, \quad c_{\text{axi}} = \frac{3}{2}, \tag{3.20}$$

the theory reduces to TEGR. Thus, it is equivalent to GR up to a boundary term B.

Alternatively, we can write

$$T = c_1 T_1 + c_2 T_2 + c_3 T_3, \tag{3.21}$$

and we retrieve GR for the parameter values

$$c_1 = \frac{1}{4}, \quad c_2 = -\frac{1}{2}, \quad c_3 = -1. \tag{3.22}$$

The relation between the different parameter sets is given by

$$c_1 = \frac{1}{2}\, c_{\text{ten}} - \frac{1}{18}\, c_{\text{axi}}, \quad c_2 = \frac{1}{2}\, c_{\text{ten}} + \frac{1}{9}\, c_{\text{axi}}, \quad c_3 = c_{\text{vec}} - \frac{1}{2}\, c_{\text{ten}}. \tag{3.23}$$

Previous studies, such as the post-Newtonian analysis [48], have obtained constraints on these parameters. In particular, it has been shown that the only non-trivial parameters are β and γ whose value is equal to 1 in GR. Their deviation from GR can be expressed as follows

$$\beta - 1 = -\frac{\delta}{2}, \quad \gamma - 1 = -2\delta, \tag{3.24}$$

where δ in New GR is given by

$$\delta = \frac{2c_1 + c_2 + c_3}{2(2c_1 + c_2 + 2c_3)}. \tag{3.25}$$

Thus, for obtaining $\beta = \gamma = 1$, we should have $2c_1 + c_2 + c_3 = 0$.

Additionally, it has been proved that in order to avoid ghost modes [49], the coefficients c_{ten} and c_{vec} should satisfy the following condition

$$c_{\text{ten}} + c_{\text{vec}} = 0\,. \tag{3.26}$$

Since multiplication of the lagrangian with a constant factor does not affect the equations, we can set the parameters c_{ten} and c_{vec} equal to their TEGR values and introduce a new parameter ϵ that will be the only one determining the new theory as follows

$$c_{\text{ten}} = \frac{2}{3},\; c_{\text{vec}} = -\frac{2}{3},\; c_{\text{axi}} = \frac{3}{2} + \epsilon. \tag{3.27}$$

Since there is only one free parameter, we call this theory 1-parameter New GR.

The action of this theory can be written in the form of TEGR action plus a correction coming from the axial part

$$S_{\text{NGR}} = -\frac{1}{2\kappa^2} \int d^4x\; e\; (T + \epsilon\, T_{\text{axi}} - L_m)\,. \tag{3.28}$$

where $e = \det(e^a{}_\mu)$. By varying the action with respect to the tetrad, we obtain the field equations, which again can be expressed as a GR part plus teleparallel terms in the following way

$$\kappa^2\Theta_{\mu\nu} = \mathring{G}_{\mu\nu} + \epsilon\left(\frac{1}{2}a^\rho a_{(\rho}g_{\mu\nu)} - \frac{4}{9}\epsilon_{\nu\alpha\beta\gamma}a^\alpha t_\mu{}^{\beta\gamma} - \frac{2}{9}\epsilon_{\mu\nu\rho\sigma}a^\rho u^\sigma - \frac{1}{3}\epsilon_{\mu\nu\rho\sigma}\nabla^\rho a^\sigma\right). \tag{3.29}$$

We also note that $\mathring{G}_{\mu\nu}$ is the Einstein tensor constructed from the Levi-Civita connection and $\Theta_{\mu\nu}$ is the energy-momentum tensor.

The most general spherically symmetric tetrad in the Weitzenböck gauge has been derived [50]

$$e^0 = C_1\,dt + C_2\,dr\,, \tag{3.30a}$$

$$e^1 = C_3\sin\theta\cos\phi\,dt + C_4\sin\theta\cos\phi\,dr + (C_5\cos\theta\cos\phi - C_6\sin\phi)\,d\theta - \sin\theta(C_5\sin\phi + C_6\cos\theta\cos\phi)\,d\phi\,, \tag{3.30b}$$

$$e^2 = C_3\sin\theta\sin\phi\,dt + C_4\sin\theta\sin\phi\,dr + (C_5\cos\theta\sin\phi + C_6\cos\phi)\,d\theta + \sin\theta(C_5\cos\phi - C_6\cos\theta\sin\phi)\,d\phi\,, \tag{3.30c}$$

$$e^3 = C_3\cos\theta\,dt + C_4\cos\theta\,dr - C_5\sin\theta\,d\theta + C_6\sin^2\theta\,d\phi, \tag{3.30d}$$

where $C_i = C_i(t,r)$ are unknown functions of the radial and time coordinate. The line element takes the following form

$$ds^2 = \left(C_3^2 - C_1^2\right)dt^2 + 2(C_3C_4 - C_1C_2)\,dt\,dr + \left(C_4^2 - C_2^2\right)dr^2 + \left(C_5^2 + C_6^2\right)d\Omega^2\,, \tag{3.31}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric on the two-sphere.

3.3 Key results

In the attached publication 6, we present three general branches of solutions of the field equations in 1-parameter New GR for spherically symmetric spacetimes, we derive solutions for the vacuum case and we study the properties of the solution of the second branch, which describes a spacetime beyond Schwarzschild.

3.3.1 Solving the antisymmetric equations

Before we explain the steps for solving the field equations, we introduce the following parametrization for the tetrad to simplify our calculations

$$C_1(t,r) = A(t,r)\cosh\beta(t,r)\,, \qquad C_3(t,r) = A(t,r)\sinh\beta(t,r)\,, \tag{3.32a}$$

$$C_4(t,r) = B(t,r)\cosh\beta(t,r)\,, \qquad C_2(t,r) = B(t,r)\sinh\beta(t,r)\,, \tag{3.32b}$$

$$C_5(t,r) = R(t,r)\cos\alpha(t,r)\,, \qquad C_6(t,r) = R(t,r)\sin\alpha(t,r)\,. \tag{3.32c}$$

Consequently, the line element can be written as

$$ds^2 = -A(t,r)^2 dt^2 + B(t,r)^2 dr^2 + R(t,r)^2 d\Omega^2 \,. \tag{3.33}$$

Due to the additional coordinate freedom, we can set $R(t,r) = r$. We emphasize that the metric has two degrees of freedom because of the functions $A(t,r)$ and $B(t,r)$ while the tetrad has four degrees of freedom because of the functions $A(t,r)$, $B(t,r)$, $\alpha(t,r)$ and $\beta(t,r)$.

The first step for solving the field equations is to take advantage of the symmetry of the energy-momentum tensor which implies the vanishing antisymmetric components of the field equations. This requires us to solve the following system

$$E_{[tr]} \propto \epsilon \sin\alpha \left(B\alpha_{,t}\cosh\beta - A\alpha_{,r}\sinh\beta\right) = 0 \,, \tag{3.34}$$

$$\begin{aligned} E_{[\theta\phi]} \propto \epsilon \Big[& rA^2B\Big\{ rA_{,r}\alpha_{,r} - 2B\sin\alpha\Big(\left(B\beta_{,t} - A_{,r}\right)\cosh\beta + B_{,t}\sinh\beta\Big)\Big\} \\ & - r^2AB^2\left(B_{,t}\alpha_{,t} + B\alpha_{,tt}\right) + r^2A_{,t}B^3\alpha_{,t} + A^3\Big\{ r\Big(\left(2B - rB_{,r}\right)\alpha_{,r} + rB\alpha_{,rr}\Big) \\ & + 2B^2\left(r\beta_{,r}\sinh\beta + \cosh\beta\right)\sin\alpha - 2B^3\sin(2\alpha)\Big\}\Big] = 0 \,, \end{aligned} \tag{3.35}$$

where commas represent derivatives with respect to t or r. We identify three branches of solutions.

1. Branch 1: $\sin\alpha = 0$ which implies $\alpha = k\pi$ ($k \in Z$).
2. Branch 2: $\sin\alpha \neq 0$ with $\alpha = \alpha_0$ (constant).
3. Branch 3: $\sin\alpha \neq 0$ with $\alpha = \alpha(r,t)$ variable and $B\alpha_{,t}\cosh\beta - A\alpha_{,r}\sinh\beta = 0$.

We emphasize that only the first branch has been found and analyzed for the full parameter New GR in [51] and this branch corresponds to GR.

3.3.2 Vacuum Solutions

Having solved the antisymmetric components of the field equations and now setting the energy-momentum tensor equal to zero, we are able to find vacuum spherically symmetric solutions by solving the symmetric components of the field equations.

Branch 1

For the first branch, we obtain

$$A(r)^2 = \left(1 - \frac{2M}{r}\right), \quad B(r)^2 = \left(1 - \frac{2M}{r}\right)^{-1}, \tag{3.36}$$

where M is a constant that represents the mass of the object. We notice that the metric functions correspond to the Schwarzschild metric. This result is expected since, as we mentioned, this branch reduces to GR. It is, however, important to emphasize that while there is a unique spherically symmetric solution in terms of the metric, the tetrad is not unique, as it depends on the free function $\beta(t,r)$.

Branch 2

For the second branch, we find

$$A(r)^2 = \Big(1 - \frac{2M}{r}\Big)\,, \quad B(r)^2 = h(\alpha_0,\epsilon)\Big(1 - \frac{2M}{r}\Big)^{-1}\,, \tag{3.37}$$

where we have introduced the parameter

$$h(\alpha_0,\epsilon) = h = \frac{1}{1 - \frac{4}{9}\epsilon\left[\cos\left(2\alpha_0\right) - 1\right]}\,. \tag{3.38}$$

The metric is static and corresponds to a Schwarzschild-like solution. We also manage to obtain an analytical solution for the function β which determines the form of the tetrad, for the specific case $\beta(t,r) = \beta(r)$. The solution is given by

$$\beta(r) = \text{arccosh}\Big[\frac{2B(r)\cos\alpha_0}{r}\left(M^2\beta_0 + r\right)\Big]\,. \tag{3.39}$$

Branch 3

Considering that the third branch is more complicated, we adopt the assumption $B = 1/A$ that simplifies the system of equations. We obtain the following expressions for the metric functions

$$A(t,r)^2 = \Big(1 - \frac{2M}{r}\Big)\,, \quad B(r)^2 = \Big(1 - \frac{2M}{r}\Big)^{-1}\,. \tag{3.40}$$

This means that there is a unique Schwarzschild solution for the metric. We also have to determine the form of the functions $\alpha(t,r)$ and $\beta(t,r)$ of the tetrad. We find an analytical solution for the specific case $\beta(t,r) = \beta(r)$, given by

$$\beta(r) = \text{arcsinh}\Big(\tfrac{c_1 r^{3/2}}{\sqrt{r-2M}}\Big)\,, \quad \alpha(t,r) = 2\ \text{arccot}\big(e^{2c_1 t + \alpha_2(r)}\big)\,, \tag{3.41}$$

$$\alpha_2(r) = \textstyle\int dr\, \frac{2\sqrt{c_1^2 r^2 + 1 - \frac{2M}{r}}}{r-2M}\,. \tag{3.42}$$

We note that a static metric does not imply a static tetrad.

3.3.3 Properties of the Schwarzschild-like solution

In the final part of publication 6, we explore the properties of the beyond-Schwarzschild solution, which was found for the second branch. In particular, we investigate geometric properties, we compute classical observables, the Komar mass, we identify singularities and we discuss the photon sphere.

Geometric properties

Preserving the Lorentzian signature of the metric requires $h\left(\alpha,\epsilon\right) > 0$ and consequently

$$\epsilon > \frac{9}{4}\left[\cos\left(2\alpha\right)-1\right]^{-1} = \delta_{min} < 0. \tag{3.43}$$

The form of the metric implies the existence of an event horizon at $r = 2M$. Considering the conditions (2.14), we also conclude that the solution is asymptotically non-flat.

Regarding the geodesic motion of particles near this object, we find that the effective potential (2.24) takes a form that differs only by a factor $h(\epsilon,\alpha)^{-1}$ from the Schwarzschild counterpart

$$V_{\text{eff}} = h(\epsilon,\alpha)^{-1} V_{\text{effSchw}}. \tag{3.44}$$

We find similar expressions for the orbits of particles around this solution

$$\dot{r} = \frac{1}{\sqrt{h(\alpha,\epsilon)}}\dot{r}_{\text{Schw}}, \quad \frac{d\phi}{dr} = \sqrt{h(\alpha,\epsilon)}\frac{d\phi}{dr}_{\text{Schw}}, \quad \frac{dt}{dr} = \sqrt{h(\alpha,\epsilon)}\frac{dt}{dr}_{\text{Schw}}. \tag{3.45}$$

Additionally, the circular orbits, determined by $\dot{r} = \ddot{r} = 0$, coincide with those in Schwarzschild spacetime.

Classical observables

We compute the classical observables using (2.28), (2.29) and (2.30) and we find that we can also express them in terms of their Schwarzschild counterparts as follows

$$\Delta\phi_{\text{peri}} = \sqrt{h(\alpha,\epsilon)}\,\Delta\phi_{\text{periSchw}} + (\sqrt{h(\alpha,\epsilon)}-1)2\pi\,, \tag{3.46}$$

$$\Delta\phi_{\text{light}} = \sqrt{h(\alpha,\epsilon)}\,\Delta\phi_{\text{lightSchw}} + (1-\sqrt{h(\alpha,\epsilon)})(\Psi_{R\text{Schw}} - \Psi_{S\text{Schw}})\,, \tag{3.47}$$

$$\Delta t_{\text{Shap}} = \sqrt{h(\alpha,\epsilon)}\,\Delta t_{\text{ShapSchw}} + (\sqrt{h(\alpha,\epsilon)}-1)\left(\int_{r_1}^{r_2}\frac{dt}{dr}_{\text{Schw}}dr\right)_{M=0}. \tag{3.48}$$

Komar mass and singularities

We compute the Komar mass from (2.16) and we obtain

$$\mathcal{M} = M\sqrt{1+\frac{8}{9}\epsilon\sin^2(\alpha_0)}\,, \tag{3.49}$$

which differs from the Schwarzschild mass by a teleparallel correction. By computing the Kretschmann invariant (2.18), we conclude that there is a spacetime singularity at $r = 0$, similarly with the Schwarzschild case.

Photon sphere and shadow

Finally, we investigate the photon sphere and the black hole shadow by employing a Mathematica-based ray-tracing code for obtaining the images and intensity profiles of the solutions [52–60] and we compare the results with the ones from the analysis of the geodesics.

As previously mentioned, the geodesic equation can be written in the following way

$$\phi'(r) = \sqrt{h}\phi'_{\text{Schw}} = \pm\frac{b}{r^2}\frac{\sqrt{-g_{tt}g_{rr}}}{\sqrt{1+g_{tt}\frac{b^2}{r^2}}}, \tag{3.50}$$

where $b \equiv L/E$ is the impact parameter. It is convenient to introduce a new parameter $\bar{\delta}$ expressed as

$$h(\alpha,\epsilon) = \frac{1}{1-\bar{\delta}}. \tag{3.51}$$

We investigate the propagation of photons for different values of the parameters and we find that for $\bar{\delta} < 0$, there is an attractive effect in the photons approaching the object, while for $\bar{\delta} > 0$ we identify a repulsive effect.

The accretion disk in the equatorial plane follows the Gralla-Lupsasca-Marrone (GLM) intensity profile [61], given by

$$I_e(r;\gamma,\beta,\sigma) = \frac{\exp\left\{-\frac{1}{2}\left[\gamma+\text{arcsinh}\left(\frac{r-\beta}{\sigma}\right)\right]^2\right\}}{\sqrt{(r-\beta)^2+\sigma^2}}, \tag{3.52}$$

in the reference frame of the emitter, where γ, β, and σ are free parameters. The following expression

$$I_o(r) = g_{tt}^2 I_e(r)\,, \tag{3.53}$$

describes the observed intensity profile.

In the attached publication 6, we study the observed intensity profiles for some interesting models and we conclude that for $\bar{\delta} > 0$, there is an increase light-ring contribution to the observed profiles and images and vice versa for $\bar{\delta} < 0$. This also affects the size of the black hole shadow, which increases in the first case and decreases in the second. Thus, these results are in agreement with the theoretical conclusions.

Chapter 4

Symmetric teleparallel connection and spherically symmetric solutions in Newer General Relativity

In this chapter, we discuss symmetric teleparallel gravity, the class of teleparallel theories in which gravity arises from non-metricity while the curvature and the torsion of spacetime vanish identically. In subsection 4.1, we introduce the main geometric quantities and concepts of this framework. In 4.2, we discuss the coincident gauge, a convenient system of coordinates in the context of symmetric teleparallel gravity. In 4.3, we focus on Newer General Relativity (Newer GR), a simple generalization of STEGR, which is constructed by decomposing the non-metricity scalar into its irreducible components. Finally, in 4.4, we present the main results of the attached publication 7. We summarize the derivation of the most general spherically symmetric and stationary connection in the context of symmetric teleparallelism and present two vacuum spherical solutions along with their properties.

4.1 Geometrical aspects

In symmetric teleparallel theories of gravity, the metric $g_{\mu\nu}$ and an affine connection $\Gamma^{\rho}{}_{\mu\nu}$ are the variables that determine the dynamics. The curvature and the torsion vanish identically $R^{\rho}{}_{\sigma\mu\nu} = 0, T^{\rho}{}_{\mu\nu} = 0$ while gravity is the result of the non-metricity tensor $Q_{\alpha\beta\gamma} = \nabla_{\alpha} g_{\beta\gamma}$. We notice that this tensor is symmetric in its last two indices. Thus, we can introduce two independent traces expressed as

$$Q_{\alpha} = Q_{\alpha\nu}{}^{\nu}, \quad \tilde{Q}_{\alpha} = Q^{\nu}{}_{\nu\alpha} \, . \tag{4.1}$$

The following decomposition relates the symmetric teleparallel connection $\Gamma^{\mu}_{\ \nu\rho}$ with the Levi-Civita connection $\mathring{\Gamma}^{\mu}_{\ \nu\rho}$

$$\Gamma^{\mu}_{\ \nu\rho} = \mathring{\Gamma}^{\mu}_{\ \nu\rho} + L^{\mu}_{\ \nu\rho}\,, \tag{4.2}$$

where $L^{\mu}_{\ \nu\rho}$ is the disformation tensor

$$L^{\mu}_{\ \nu\rho} = \frac{1}{2} g^{\mu\lambda}(Q_{\lambda\nu\rho} - Q_{\nu\lambda\rho} - Q_{\rho\lambda\nu})\,. \tag{4.3}$$

We also provide the expression for the non-metricity scalar

$$Q = -\frac{1}{4} Q_{\alpha\mu\nu}Q^{\alpha\mu\nu} + \frac{1}{2} Q_{\alpha\mu\nu}Q^{\alpha\nu\mu} + \frac{1}{4} Q_{\alpha}Q^{\alpha} + \frac{1}{2} Q_{\alpha}\tilde{Q}^{\alpha}\,. \tag{4.4}$$

The non-metricity scalar can be related with the Ricci scalar of the Levi-Civita connection in the following way

$$\mathring{R} = -Q + B\,, \quad B = \mathring{\nabla}_{\lambda}(Q^{\lambda} - \tilde{Q}^{\lambda})\,. \tag{4.5}$$

4.2 Coincident gauge

Considering the vanishing curvature and torsion in the symmetric teleparallel theories, we realize that we can always find a coordinate system, the so-called coincident gauge, for which the connection vanishes $\tilde{\Gamma}^{\mu}_{\ \nu\rho} = 0$ [35, 62, 63]. The transformation of the connection from one coordinate system $\tilde{x}^{\mu}$ to an other x^{μ} can be expressed as

$$\tilde{\Gamma}^{\mu}_{\ \nu\rho} = \frac{\partial \tilde{x}^{\mu}}{\partial x^{\mu}} \frac{\partial x^{\rho}}{\partial \tilde{x}^{\rho}} \frac{\partial x^{\nu}}{\partial \tilde{x}^{\nu}} \Gamma^{\mu}_{\ \nu\rho} - \frac{\partial x^{\rho}}{\partial \tilde{x}^{\rho}} \frac{\partial x^{\nu}}{\partial \tilde{x}^{\nu}} \frac{\partial^2 \tilde{x}^{\mu}}{\partial x^{\nu} \partial x^{\rho}}\,. \tag{4.6}$$

If $\tilde{x}$ represents the coordinates in the coincident gauge and $x^{\mu} = (t, r, \theta, \phi)$, then the components of the connection in spherical coordinates take the following form

$$\Gamma^{\mu}_{\ \nu\rho} = \frac{\partial x^{\mu}}{\partial \tilde{x}^{\sigma}} \frac{\partial^2 \tilde{x}^{\sigma}}{\partial x^{\nu} \partial x^{\rho}}\,. \tag{4.7}$$

By using this gauge, it is much easier to compute the components of the connection in any other coordinate system.

4.3 Newer General Relativity

A simple generalization of STEGR is Newer General Relativity (Newer GR) where the non-metricity scalar is decomposed into the five possible quadratic scalars as follows

$$Q = c_1 Q^{\mu\nu\rho}Q_{\mu\nu\rho} + c_2 Q^{\mu\nu\rho}Q_{\rho\mu\nu} + c_3 Q^{\rho\mu}_{\ \ \ \mu}Q_{\rho\nu}^{\ \ \ \nu} + c_4 Q^{\mu}_{\ \mu\rho}Q_{\nu}^{\ \nu\rho} + c_5 Q^{\mu}_{\ \mu\rho}Q^{\rho\nu}_{\ \ \ \nu}\,. \tag{4.8}$$

$c_1, ..., c_5$ are the parameters of the theory and different values of the parameters correspond to different theories of gravity. For the following values

$$c_1 = -\frac{1}{4}, \quad c_2 = \frac{1}{2}, \quad c_3 = \frac{1}{4}, \quad c_4 = 0, \quad c_5 = -\frac{1}{2}, \tag{4.9}$$

the theory reduces to STEGR and thus, it is equivalent to GR up to a boundary term. We emphasize that Newer GR and other modifications of STEGR are not equivalent to GR.

The wide range of these theories can be reduced by considering the constraints that arise when the post-Newtonian limit is satisfied. It has been proved that there are two sets of conditions for which the Parametrized Post-Newtonian parameters coincide with those of General Relativity [64], given by

$$c_1 = -\frac{1}{4}, \quad c_2 = \frac{1}{2}\left(\frac{1}{2} + V\right), \quad c_3 = \frac{1}{4}, \quad c_4 = \frac{1}{2}\left(\frac{1}{2} - V\right), \quad c_5 = -\frac{1}{2}, \tag{4.10}$$

$$c_1 = -\frac{1}{4}, \quad c_2 = \frac{1}{2}, \quad c_3 = \frac{1}{4} + V, \quad c_4 = 0, \quad c_5 = -\frac{1}{2}, \tag{4.11}$$

where V is a real number. We shall call the theories that correspond to these sets of parameters Type 1 and Type 2 Newer GR respectively. We realize that in both cases there is only one free parameter V. It is also important to mention that the second set corresponds to a ghost-free theory, which is constructed based on a symmetry under transverse diffeomorphisms [65].

In order to derive the field equations, we have to vary the action of the theory with respect to the metric [62]

$$\begin{aligned}
\kappa^2 E_{\mu\nu} =& -2\mathring{\nabla}_\rho \Big[c_1 Q^\rho{}_{\mu\nu} + c_2 Q_{(\mu\nu)}{}^\rho + c_3 Q^{\rho\sigma}{}_\sigma g_{\mu\nu} + c_4 Q^\sigma{}_{\sigma(\mu}\delta^\rho_{\nu)} \\
&+ \frac{1}{2}c_5\left(Q_\sigma{}^{\sigma\rho} g_{\mu\nu} + \delta^\rho_{(\mu} Q_{\nu)\sigma}{}^\sigma\right)\Big] + \frac{1}{2}(c_1 Q^{\rho\sigma\tau}Q_{\rho\sigma\tau} + c_2 Q^{\rho\sigma\tau}Q_{\tau\rho\sigma} \\
&+ c_3 Q^{\tau\rho}{}_\rho Q_{\tau\sigma}{}^\sigma + c_4 Q^\rho{}_{\rho\tau} Q_\sigma{}^{\sigma\tau} + c_5 Q^\rho{}_{\rho\tau} Q^{\tau\sigma}{}_\sigma) g_{\mu\nu} \\
&- c_3 Q_{\mu\rho}{}^\rho Q_{\nu\sigma}{}^\sigma + c_1\left(2Q^{\rho\sigma}{}_\mu Q_{\sigma\rho\nu} - Q_\mu{}^{\rho\sigma} Q_{\nu\rho\sigma} - 2Q^{\rho\sigma}{}_{(\mu} Q_{\nu)\rho\sigma}\right) \\
&+ c_2\left(Q^{\rho\sigma}{}_\mu Q_{\sigma\rho\nu} - Q_\mu{}^{\rho\sigma} Q_{\nu\rho\sigma} - Q^{\rho\sigma}{}_{(\mu} Q_{\nu)\rho\sigma}\right) \\
&+ c_4\left[Q_\rho{}^{\rho\sigma}(Q_{\sigma\mu\nu} - 2Q_{(\mu\nu)\sigma}) + Q^\rho{}_{\rho\mu} Q^\sigma{}_{\sigma\nu} - Q^\rho{}_{\rho(\mu} Q_{\nu)\sigma}{}^\sigma\right] \\
&+ \frac{1}{2}c_5\left[Q^{\rho\sigma}{}_\sigma(Q_{\rho\mu\nu} - 2Q_{(\mu\nu)\rho}) - Q_{\mu\rho}{}^\rho Q_{\nu\sigma}{}^\sigma\right].
\end{aligned} \tag{4.12}$$

In this case, the variation with respect to the symmetric teleparallel connection gives only a redundant equation. However, we emphasize that in any symmetric teleparallel theory these variations should always maintain the flatness and torsion-free properties [66, 67].

4.4 Key results

In the attached publication 7, we present the derivation of the most general affine connection for stationary spacetimes with spherical symmetry, we then focus on Newer GR and we identify two families of vacuum solutions in Type 1 and Type 2 theories and finally, we study their properties. In this subsection, we present the main results and steps.

4.4.1 Symmetric teleparallel connection

In order to derive the components of the most general symmetric teleparallel connection for spherically symmetric and stationary spacetimes, we start from the coincident gauge (4.2). Using the ansatz

$$\tilde{x}^0 = \tilde{t}(t,r), \quad \tilde{x}^1 = \tilde{r}(t,r)\sin\theta\cos\phi, \quad \tilde{x}^2 = \tilde{r}(t,r)\sin\theta\sin\phi, \quad \tilde{x}^3 = \tilde{r}(t,r)\cos\theta\,, \tag{4.13}$$

where $\tilde{t}, \tilde{r}$ are some arbitrary functions, we express (4.7) in terms of these functions. We determine the general form of the functions $\tilde{r}(t,r)$ and $\tilde{t}(t,r)$ for the stationary case by integrating directly the components of the connection and we obtain

$$\tilde{r}(t,r) = e^{\int f_1(r)\,dr}\, e^{lt}\,, \tag{4.14}$$

$$\tilde{t}(t,r) = e^{\int f_2(r)dr} e^{mt}\,, \tag{4.15}$$

where $f_1(r) = \Gamma^{\theta}{}_{r\theta}$, $f_2(r) = \left(\frac{\Gamma^r{}_{tr}}{l\,\Gamma^r{}_{\theta\theta}} + f_1(r)\right)$ and l, m real numbers. We then express the components of the connection in terms of the functions f_1, f_2 and the parameters l, m as follows

$$\begin{aligned}
\Gamma^t{}_{tt} &= \frac{m^2 f_1 - l^2 f_2}{m f_1 - l f_2}, \quad \Gamma^t{}_{tr} = \frac{(l-m) f_1 f_2}{l f_2 - m f_1}, \quad \Gamma^t{}_{rr} = \frac{f_1^2 f_2 + f_2 f_1' - f_1(f_2^2 + f_2')}{l f_2 - m f_1}, \\
\Gamma^r{}_{tt} &= \frac{lm(m-l)}{l f_2 - m f_1}, \quad \Gamma^r{}_{tr} = \frac{lm(f_2 - f_1)}{l f_2 - m f_1}, \quad \Gamma^r{}_{rr} = \frac{l(f_2^2 + f_2') - m(f_1^2 + f_1')}{l f_2 - m f_1}, \\
\Gamma^t{}_{\theta\theta} &= \frac{f_2}{m f_1 - l f_2}, \quad \Gamma^r{}_{\theta\theta} = \frac{m}{l f_2 - m f_1}, \quad \Gamma^t{}_{\phi\phi} = \Gamma^t{}_{\theta\theta}\sin^2\theta, \quad \Gamma^r{}_{\phi\phi} = \Gamma^r{}_{\theta\theta}\sin^2\theta, \\
\Gamma^\theta{}_{t\theta} &= \Gamma^\phi{}_{t\phi}, \quad \Gamma^\theta{}_{r\theta} = \Gamma^\phi{}_{r\phi}, \quad \Gamma^\theta{}_{\phi\phi} = -\cos\theta\sin\theta, \quad \Gamma^\phi{}_{\theta\phi} = \cot\theta\,,
\end{aligned} \tag{4.16}$$

where we consider $m f_1 - l f_2 \neq 0$ for a well-defined coincident gauge and connection.

It is important to mention the work [68] that presents similar results. In this paper, they derive two sets for the components of the connection and we notice that the second set coincides with (4.16) for $c \to l$ and $k \to m - l$. We emphasize that in our work, we computed the components of the connection starting from the coincident gauge with direct integration. Also, our parameterization represents both sets and thus, there is no need to use any ambiguous limits to relate them. Additionally, we note that our approach is different from the one in [63]. In this work, they use the two sets of the connection coefficients derived in [68] and they present different parameterizations of the coincident gauge transformation, while our parameterization covers both of them.

4.4.2 Two families of vacuum solutions

For the next part of the publication, we focus on Newer GR, we derive the field equations in Type 1 and Type 2 theory and we obtain two families of solutions. The first solution is a solution of Type 1 theory and it is described by the following metric functions

$$A(r) = 1, \quad B(r) = \sqrt{r f_1(r)(2 - r f_1(r))}\,, \tag{4.17}$$

with $V = 0,\ f_2(r) = 0$ and $l = 0$. For reasons that are explained in the following subsections, we use the function

$$f_1(r) = \frac{1}{r-k}, \tag{4.18}$$

where k is a constant. The second solution which is common for both Type 1 and Type 2 Newer GR has the following metric functions

$$A(r) = qr \quad \text{and} \quad B(r) = B = \text{constant}\,, \tag{4.19}$$

where q is a constant. For Type 1 theories, we have

$$m = 0, \quad f_1(r) = 0, \quad f_2(r) = k/r\,, \tag{4.20}$$

while for Type 2

$$l = 0, \quad f_1(r) = 1/r, \quad f_2(r) = 1/r\,. \tag{4.21}$$

These solutions do not have counterparts in GR.

4.4.3 Properties of the solutions

Finally, we investigate the properties of these solutions thoroughly. We study in particular the following.

Possibility of traversable wormhole

A wormhole geometry should have a metric of the form (2.31) and it should satisfy the flaring out condition (2.32). After investigating these expressions, we conclude that the flaring-out condition is violated for both solutions and thus, they cannot describe traversable wormholes.

Asymptotic flatness

Considering the conditions (2.14) for asymptotic flatness, we realize that the second solution is not asymptotically flat, while for the first solution, this property depends on the choice of the function $f_1(r)$. We selected the function (4.18) in such a way that asymptotic flatness is satisfied.

Komar mass

We compute the Komar mass of the solutions using the expression (2.16). It is easy to find that for the first solution the Komar mass vanishes, while for the second solution it becomes infinite.

Gravitational redshift

Using the expression (2.17), we find that for the first solution there is no gravitational redshift $\omega_\infty = \omega$, while for the second solution it becomes infinite since the frequency at infinity vanishes $\omega_\infty = 0$.

Singularities

By calculating the Kretschmann invariant (2.18), we find that for each solution there is one spacetime singularity. The only exception is the special case of Minkowski spacetime for the first solution.

Horizons

The vector $t^a = \partial x^a/\partial t$ is a Killing vector for both metrics. However, the quantity $g_{\alpha\beta}t^\alpha t^\beta$ does not change sign and thus, the solutions do not admit any horizons.

Geodesic motion

Regarding the radial motion of test particles near these objects, we compute the expansion of a beam of timelike or null worldlines (2.20) and its evolution (2.21), (2.22). We find that the expansion is positive for outward motion and negative for inward motion and the geodesics converge as time passes for both solutions. These results are in line with GR, the focusing theorem and the attractive behavior of gravity.

Regarding bound orbits, we compute the effective potential for each solution (2.24) and we investigate the conditions (2.25). We conclude that there are no bound orbits for any of the solutions.

Light deflection

In principle, we can use (2.29) to compute the deflection angle. For the first solution, this calculation is cumbersome without providing any meaningful result. For this reason, we focus on the behavior only at large distances. By considering a Taylor expansion around an infinitesimally small quantity ϵ, we conclude that the light deflection at large distances vanishes, which agrees with the case of a flat spacetime. For the second solution, we realize that there is no turning point. Consequently, there is no deflection and the particle falls into the singularity regardless of its initial motion.

Causal structure

In order to comprehend the causal structure of the solutions, we construct the Penrose diagrams. The method for constructing the diagrams involves the following steps. First, we bring the metric into the form

$$ds^2 = -F^2 dt^2 + F^{-2} d\rho^2 + r^2(\rho) d\Omega^2 \,, \tag{4.22}$$

where $F = F(r(\rho))$ by finding a proper coordinate transformation $r = r(\rho)$ and then we solve the equation

$$B \; dr = \frac{1}{F} \; d\rho \,. \tag{4.23}$$

We compute the tortoise coordinate ρ^*

$$\rho^* = \int_0^\rho \frac{d\rho'}{F^2} \,, \tag{4.24}$$

and we use the null coordinates

$$u = t - \rho^*, \quad U = t + \rho^* \,. \tag{4.25}$$

To ensure a final range of the coordinates, we introduce a new set of coordinates

$$\tilde{u} = \tanh u, \quad \tilde{U} = \tanh U \,. \tag{4.26}$$

Finally, for the Penrose diagrams, we use the coordinates

$$T = \frac{\tilde{U} + \tilde{u}}{2}, \quad L = \frac{\tilde{U} - \tilde{u}}{2} \,. \tag{4.27}$$

and thus, the metric can be written as

$$ds^2 = -F^2 \frac{dL^2 - dT^2}{(1 - (T - L)^2)(1 - (T + L)^2)} \,. \tag{4.28}$$

In the attached publication, we present the Penrose diagrams and the conclusions for each case.

Chapter 5

Thin-shell wormholes in scalar-tensor gravity

In this chapter, we focus on scalar-tensor gravity, a type of theory for which gravity is the result of the curvature of spacetime along with a scalar field ϕ. In subsection 5.1, we introduce the specific class of scalar-tensor theories in which we are interested. In 5.2, we discuss the junction conditions in the framework of scalar-tensor gravity, which we apply to construct thin-shell wormholes. Finally, in 5.3, we present the results of the attached publication 8. We summarize the construction of thin-shell wormholes, the study of their stability and the type of matter on the throat and we present and analyze stable wormhole structures in a specific subclass of scalar-tensor theories.

5.1 Scalar-tensor theory

In the present thesis, we focus on a class of scalar-tensor theories with a non-minimally coupled scalar field ϕ described by the following action in the Jordan frame

$$S = \int d^4x \sqrt{-g} \left(f(\phi)R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - U(\phi) \right) + \int d^4x \sqrt{-g}\mathcal{L}_m, \tag{5.1}$$

where $U(\phi)$ has the role of a self-interaction potential and $f(\phi)$ is an arbitrary function of the scalar field. Comparing with the Horndeski action (2.60), we identify

$$G_2 = -U(\phi), \quad G_3 = -\frac{1}{2}, \quad G_4 = f(\phi), \quad G_5 = G_6 = 0. \tag{5.2}$$

By varying the action with respect to the metric and the scalar field, we obtain the following field equations, respectively

$$2f(\phi)G_{\mu\nu} + g_{\mu\nu}\left(\frac{1}{2}(\nabla\phi)^2 + U(\phi)\right) - \nabla_\mu\phi\nabla_\nu\phi - 2\nabla_\mu\nabla_\nu f(\phi) + 2g_{\mu\nu}\Box f(\phi) = \Theta_{\mu\nu}, \tag{5.3}$$

$$\Box\phi + f'(\phi)R - U'(\phi) = 0, \tag{5.4}$$

where the prime denotes derivative with respect to ϕ. $\Theta_{\mu\nu}$ is the energy-momentum tensor related with the matter field Lagrangian $\mathcal{L}_m$.

5.2 Junction conditions in scalar-tensor theories

In subsection 2.2.9, we discussed the junction conditions for the smooth join of two regions of spacetime on their boundary and the thin-shell formalism in the context of GR. We also mentioned that the junction conditions can be derived by introducing distributional expressions and identifying singular terms. This method can be employed for the derivation of the junction conditions in modified theories as well. In modified gravity, the extra degrees of freedom introduce additional conditions for the proper matching of the two regions. In particular, the junction conditions in scalar-tensor theories are given by [69]

$$[g_{\mu\nu}] = 0, \tag{5.5}$$

$$[\phi] = 0, \tag{5.6}$$

$$-2\epsilon f(\phi)([K_{\mu\nu}] - h_{\mu\nu}[K]) + 2\Omega f'(\phi)h_{\mu\nu} = S_{\mu\nu}, \tag{5.7}$$

$$\Omega = 2\epsilon f'(\phi)[K], \tag{5.8}$$

where Ω is defined as

$$\Omega = \epsilon[n^\mu\partial_\mu\phi]. \tag{5.9}$$

As we mentioned in 2.2.9, ϵ takes the values 1 or -1 for timelike or spacelike hypersurfaces respectively. In this work, we assume a timelike hypersurface. The first two conditions, which imply the continuity of the metric and the scalar field across the hypersurface, ensure the smooth join of the two regions without the appearance of matter on their boundary. The third condition implies the existence of a thin layer of matter due to the non-vanishing stress-energy tensor $S_{\mu\nu}$ on the hypersurface. The last condition is relevant only when a thin-shell exists and it can be considered to describe an additional contribution to the stress-energy tensor $S_{\mu\nu}$. It is easy to notice that the first condition is the same as in GR while the second and the forth are directly related to the presence of the scalar field. The third condition reduces to its GR counterpart for $f(\phi) = 1$. It is also worth noting that the special case $\Omega = 0$ further implies $[K] = 0$ and the third condition reduces to an expression that resembles its counterpart from $f(R)$ gravity [70].

5.3 Key results

In the attached publication 8, we present the construction of spherical thin-shell wormholes in the class of scalar-tensor theories discussed in 5.1 by properly matching two spherically symmetric solutions on their boundary. We study their stability under radial perturbations and the type of matter on the shell. We also apply our analysis to a particular example using a hairy charged black hole solution for the wormhole construction. In this subsection, we present the main results and methods of this work.

5.3.1 Construction of thin-shell wormholes

We have already discussed the construction of a thin-shell in the context of GR by employing the thin-shell formalism in 2.2.9. We now apply this method in the framework of the scalar-tensor theory described by the action (5.1). Assuming two spherically symmetric spacetimes described by the line elements of the following form

$$ds^2 = -A(r)^2_{1,2}dt^2 + A(r)^2_{1,2}dr^2 + r^2 d\theta^2 + r^2 \sin\theta^2 d\phi^2, \tag{5.10}$$

and a hypersurface

$$\Sigma = r - a, \tag{5.11}$$

the junction condition (5.8) can be written in the following way

$$4f'(\phi)\left(\frac{\sqrt{A_1(a)+\dot{a}^2}+\sqrt{A_2(a)+\dot{a}^2}}{a}+\frac{A_1'(a)+2\ddot{a}}{4\sqrt{A_1(a)+\dot{a}^2}}+\frac{A_2'(a)+2\ddot{a}}{4\sqrt{A_2(a)+\dot{a}^2}}\right)-\Omega = 0. \tag{5.12}$$

It is convenient to work in the orthonormal basis for which the energy-momentum tensor for a perfect fluid reads $S_{\hat{\imath}\hat{\jmath}} = \mathrm{diag}(\sigma, p_{\hat{\theta}}, p_{\hat{\phi}})$, where σ is the surface energy density and $p = p_{\hat{\theta}} = p_{\hat{\phi}}$ the transverse pressure. The junction condition can be written in terms of σ and p as well

$$\sigma = -2\Omega f'(\phi) - \frac{4f(\phi)}{a}(\sqrt{A_1(a)+\dot{a}^2}+\sqrt{A_2(a)+\dot{a}^2}), \tag{5.13}$$

$$p = 2\Omega f'(\phi)+f(\phi)\left(\frac{2(\sqrt{A_1(a)+\dot{a}^2}+\sqrt{A_2(a)+\dot{a}^2})}{a}\right)+$$

$$+f(\phi)\left(\frac{A_1'(a)+2\ddot{a}}{\sqrt{A_1(a)+\dot{a}^2}}+\frac{A_2'(a)+2\ddot{a}}{\sqrt{A_2(a)+\dot{a}^2}}\right). \tag{5.14}$$

5.3.2 Stability

When we construct such objects it is important to study if they would describe stable configurations. For determining the stability of static thin-shell wormholes under radial perturbations, it is standard to express $\dot{a}^2$ in the form of a master equation

$$\dot{a}^2 = -V(a), \tag{5.15}$$

where $V(a)$ describes a potential. Stable solutions satisfy the condition $V''(a_0) > 0$. In GR and other gravity theories, it is usually easy to bring the dynamics into the form of (5.15). However, in our case, it appears quite challenging to derive such an expression in a generalized way.

We notice that our equations can be simplified for thin-shell wormholes symmetric across the throat for which $A_1(r) = A_2(r) = A(r)$, $\phi_1(r) = \phi_2(r) = \phi(r)$ and consequently $\phi_1'(r) = \phi_2'(r) = \phi'(r)$. An additional interesting assumption that simplifies the expressions is the condition $\phi'(a) = 0$, which implies $\Omega = [n^\mu\partial_\mu\phi] = 0$ and consequently $[K] = 0$. This assumption leads to equations similar to their $f(R)$ gravity counterparts [70], as we previously mentioned. This similarity emphasizes the close relation between $f(R)$ and scalar-tensor gravity.

After applying these assumptions, the junction condition (5.12) simplifies to

$$4f'(\phi)\left(A(a)+\dot{a}^2\right)+af'(\phi)\left(A'(a)+2\ddot{a}\right)=0, \tag{5.16}$$

which for the static configurations reads

$$4A(a_0)+a_0A'(a_0)=0, \tag{5.17}$$

considering $f'(\phi)\neq 0$. The potential then takes the form

$$V(a) = A(a) - \frac{a_0^4}{a^4}A(a_0). \tag{5.18}$$

We Taylor expand $V(a)$ around a_0 as follows

$$V(a) = V(a_0)+V'(a_0)(a-a_0)+\frac{V''(a_0)}{2}(a-a_0)^2+\mathcal{O}(a-a_0)^3. \tag{5.19}$$

It is easy to check that $V(a_0) = V'(a_0) = 0$. The stability is then determined by the sign of $V''(a_0)$, the dominant term of the expansion, which has the form

$$V''(a_0) = A''(a_0) - 20\frac{A(a_0)}{a_0^2} \geq 0. \tag{5.20}$$

5.3.3 Type of matter

As we have previously discussed, the main motivation for constructing thin-shell wormholes is to completely remove the need for exotic matter or at least to constrain all the exotic matter on the throat. After imposing the aforementioned assumptions, we find that the energy density (5.13) and pressure (5.14) of the perfect fluid when computed on the shell reduce respectively to

$$\sigma_0 = -\frac{8f(\phi_0)}{a_0}\sqrt{A(a_0)}, \tag{5.21}$$

$$p_0 = -\frac{4f(\phi_0)}{a_0}\sqrt{A(a_0)}, \tag{5.22}$$

where $\phi_0 = \phi(a_0)$. We realize that the equation of state is fixed and has the form $p_0 = \sigma_0/2$. This is a consequence of the condition $[K] = 0$, which is not present in GR. For determining the type of matter on the throat, we have to study the energy conditions (2.38) and especially the condition $\sigma_0 \geq 0$. Taking into account (5.21), the only case that would allow a positive surface energy density is for $f(\phi) < 0$. However, this would imply a negative effective gravitational constant $G_{eff} \propto 1/f(\phi_0) < 0$ and the existence of ghost modes [71]. Thus, the presence of exotic matter on the throat is unavoidable for any choice of the parameters.

5.3.4 Application

In order to apply our analysis and identify stable wormholes solutions, we focus on the Einstein-Maxwell theory coupled to a conformally invariant scalar field [72] with

$$f(\phi) = \frac{1}{16\pi G} - \frac{\phi^2}{12}, \quad U(\phi) = 0, \quad \mathcal{L}_m = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}, \tag{5.23}$$

where $F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ is the electromagnetic tensor and $\mathcal{A}_\mu$ the vector potential.

For the construction of thin-shell wormholes, we use the static hairy Reissner-Nordström black hole solution of the theory described by the line element [72]

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{Q^2+s}{r^2}\right)dt^2 + \left(1 - \frac{2m}{r} + \frac{Q^2+s}{r^2}\right)^{-1} dr^2 + d\Omega^2, \tag{5.24}$$

along with the conformally coupled constant scalar field given by

$$\phi = \pm\sqrt{\frac{6}{8\pi}}\sqrt{\frac{s}{s+Q^2}}, \tag{5.25}$$

where m is the black hole mass, Q is the electric charge and s the scalar hair. We notice that for $s > 0$, Q can take any value, while for $s < 0$, we should have $Q^2 < -s$. For $s = 0$, we retrieve the Reissner-Nordström solution of GR.

The horizons are expressed as

$$r_{\pm} = m \pm \sqrt{m^2 - Q^2 - s}, \tag{5.26}$$

where the plus sign represents the event horizon r_h and the minus sign a Cauchy horizon. There is a critical value of the charge given by

$$Q_c = \sqrt{m^2 - s}. \tag{5.27}$$

For $Q < Q_c$, there are two horizons, for $Q = Q_c$ there is one horizon and no horizons exist for $Q > Q_c$.

For our analysis, we take into account the junction condition (5.17), the stability condition (5.20), the condition for having a real scalar field $Q^2 < -s$ when $s < 0$ and conditions for having physically allowed areas such as $r > 0$. We obtain physically relevant configurations for $s > 0$. In particular, for $0 < s/m^2 < 1$, we find a pair of solutions with large values of Q/m, one stable and the other unstable, while for $1 \leq s < 9/8$ the stable and unstable configurations correspond to smaller values of Q/m. As we should expect from our analysis, the energy conditions are not satisfied on the throat. We notice, though, that in contrast to standard exotic wormhole solutions, the exotic matter in this case is all concentrated on the throat since the energy conditions are satisfied at the bulk.

Future Directions

In this dissertation, we explored a wide range of gravitating objects from black holes and wormholes to completely exotic solutions. Despite the fact that we did not explore the most general solutions, we realized that we already encountered deviations from GR as well as a rich new ground to examine. It would still be intriguing to attempt to identify more general solutions that could potentially provide a rich phenomenology.

With respect to the first publication, it would be interesting to explore whether we could derive more general solutions in terms of the metric for the third branch by dropping the assumption $A = 1/B$ for the metric functions. More general solutions could potentially describe black holes, wormholes or even more exotic objects. For both the second and third branches, we could also investigate more general solutions in terms of the tetrad since it provides extra degrees of freedom by maintaining time dependence.

With respect to the second publication, it would also be challenging to explore more general solutions that might describe black holes or wormholes and examine how they differ from the standard GR objects. In that case, we could also explore black hole shadows and rings. We remind that we have been able to solve the field equations for a very specific form of the functions and values of the parameters. Generally, solving high-order differential equations is admittedly challenging. Thus, numerical approaches could be adopted in that case.

A natural way of further extending the previous work is by introducing perturbations around the background solutions we have derived. This provides the base framework for studying quasinormal modes, a spectrum of frequencies that arise from the last stage of the gravitational wave signal and is directly related to the properties of the sources of the waves. The importance of this study is that the existence of exotic objects could be directly confirmed by their observational signatures.

Regarding the third publication, we could drop the assumption of symmetry and the equality of the derivative of the scalar field across the wormhole throat in order to explore more general solutions. This could potentially provide more general stable wormhole solutions and the possibility of completely eliminating the need for exotic matter on the wormhole throat, an important challenge in wormhole physics.

It would also be interesting to study thin-shell wormholes in the less explored case of metric teleparallel gravity.

All previous studies can, of course, be extended to more general spacetimes where the spherical symmetry is dropped. For example, it would be interesting to study more realistic solutions, such as rotational objects and their properties. At the same time, we could investigate exotic objects in other modified theories of gravity.

The present thesis explores only a small part of the vast range of aspects that need to be explored in order to complement and enhance our understanding of modified theories and their viability. An other direction that would be interesting to follow is cosmological aspects. In particular, investigating the cosmological dynamics of the theories studied in this thesis would provide a better understanding of the role of dark energy in this context and the potential cosmological scenarios. Additionally, it would also be useful to compare the modified cosmological models with observational data and draw conclusions about the benefits of these models in comparison to the ΛCDM, the need for a cosmological constant and the possibility of addressing cosmological tensions.

Summary

In the present doctoral thesis, we study exotic spherically symmetric objects in modified theories of gravity. The motivation for exploring theories beyond General Relativity arises from theoretical challenges of the theory, discrepancies in cosmological observations and the requirement for a quantum theory of gravity. In order to gain a deeper understanding of the gravitational interaction, it is essential to explore several aspects of these theories, including exact solutions of the field equations and their properties. Such solutions might describe objects like black holes or wormholes, but also more exotic objects that do not necessarily have a general relativistic limit. A notable issue in General Relativity is that wormhole solutions require exotic matter, a form of matter with unphysical properties. This problem can be partially mitigated by constructing artificial spherical solutions in which all exotic matter is confined to a thin-shell.

In publication 6, we find three general branches of solutions in New General Relativity, a torsion-based theory, by solving the antisymmetric field equations, using the most general spherically symmetric tetrad. The first branch is equivalent to General Relativity. With respect to vacuum solutions, the second branch gives a Schwarzschild-like solution, while we solve the third branch only for a special case, which corresponds to a Schwarzschild solution. We explore the properties of the second solution, which has a well-defined general relativistic limit.

In publication 7, we first compute the components of the most general symmetric teleparallel connection with spherical symmetry starting from the coincident gauge in the framework of non-metricity-based theories. We then find two exotic vacuum spherical solutions in Newer General Relativity and we study their properties. These solutions do not have a counterpart in General Relativity.

In publication 8, we construct spherical wormholes, the so-called thin-shell wormholes, by properly joining two spherical solutions of the field equations of a scalar-tensor theory. We study their stability and the type of matter on the wormhole throat. We find stable wormhole structures in a subclass of theories for which all exotic matter is concentrated on the thin-shell on the throat.

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Kokkuvõte (in Estonian)

Eksootilised sfääriliselt sümmeetrilised objektid modifitseeritud gravitatsioonis

Käesolevas doktoritöös uuritakse eksootilisi sfääriliselt sümmeetrilisi objekte modifitseeritud gravitatsiooniteooriates. Üldrelatiivsusteooriast edasi arenenud teooriate uurimise ajendiks on uued väljakutsed, kosmoloogiliste vaatlustega seotud lahknevused ning huvi gravitatsiooni kvantteooria vastu. Gravitatsioonilisest vastasmõjust sügavama arusaamise saavutamiseks on oluline käsitleda nende teooriate erinevaid aspekte, sealhulgas välja võrrandite täpseid lahendeid ja nende omaduste uurimist. Sellised lahendid võivad kirjeldada objekte nagu mustad augud või ussiaugud, aga ka eksootilisemaid objekte, millel ei pruugi olla üldrelativistlikut piiri. Tähelepanuväärne probleem Üldrelatiivsusteoorias on ussiaugu lahendid, mis nõuavad eksootilist ainet ehk ainevormi, millel on ebafüüsikalised omadused. Seda probleemi saab osaliselt käsitleda, konstrueerides kunstlikke sfäärilisi lahendeid, kus kogu eksootiline aine on piiratud õhukese kestaga.

Esimeses artiklis tutvustatakse kolme üldist lahendite haru Uues üldrelatiivsusteoorias, mille käigus lahendatakse antisümmeetrilised väljavõrrandid kasutades kõige üldisemat sfääriliselt sümmeetrilist tetraadi. Esimene haru on ekvivalentne üldrelatiivsusteooriaga. Vaakumlahenduste puhul annab teine haru Schwarzschildi-laadse lahendi, samas kui kolmandat haru lahendame ainult erijuhtumi puhul, mis vastab Schwarzschildi lahendile. Uuritakse teise haru lahendite omadusi, millel on hästi määratletud üldrelativistlik piir.

Teises artiklis arvutatakse esmalt sfäärilise sümmeetria korral kõige üldisema sümmeetrilise teleparalleelse seostuse komponendid, lähtudes kokkulangevast kalibratsioonist mittemetrilisusel põhinevate teooriate raamistikus. Seejärel leitakse Uues üldrelatiivsusteoorias kaks eksootilist sfäärilist vaakumlahendit ja uuritakse nende omadusi. Nendel lahenditel puudub vaste üldrelatiivsusteoorias.

Kolmandas artiklis konstrueeritakse sfäärilisi ussiauke, nn õhukese kestaga ussiauke, ühendades sobival viisil kaks sfäärilist skalaar-tensori teooria väljavõrrandi lahendit. Uuritakse nende stabiilsust ning ussiaugu kurgu piirkonnas esineva aine tüüpi. Artiklis on leitud stabiilseid ussiaugu struktuure teooriate alamklassis, kus kogu eksootiline aine on koondunud kurgu piirkonna õhukesele kestale.

Attached publications

Curriculum Vitae
(in English)

Personal information:

Name	Vasiliki Karanasou
Date of birth	16.12.1995
Address	Laboratory of Theoretical Physics, Institute of Physics W. Ostwaldi 1, 50411, Tartu, Estonia
E-mail	vasiliki.karanasou@ut.ee

Education:

2022 – 2026	PhD in Theoretical Physics University of Tartu, Estonia
2018 – 2020	MSc in Theoretical Physics Uppsala University, Sweden
2013 – 2018	BSc in Physics Aristotle University of Thessaloniki, Greece
2011 – 2013	High School Diploma 1st General Lyceum of Levadia, Greece

Publications:

1. H. Asuküla, M. Hohmann, V. Karanasou, S. Bahamonde, C. Pfeifer and J. L. Rosa, *"Spherically symmetric vacuum solutions in one-parameter new general relativity and their phenomenology"*, Phys. Rev. D **109**, no.6, 064027 (2024), [arXiv], [inSPIRE], [ETIS]

2. M. Hohmann and V. Karanasou, *"Symmetric teleparallel connection and spherical solutions in symmetric teleparallel gravity"*, Phys. Rev. D **111**, no.6, 064057 (2025), [arXiv], [inSPIRE], [ETIS]

3. E. F. Eiroa, G. Figueroa-Aguirre and V. Karanasou, *"Stability of spherical thin-shell wormholes in scalar-tensor theories"*, Eur. Phys. J. Plus **141**, no.2, 134 (2026), [arXiv], [inSPIRE], [ETIS]

Awards and stipends:

Estonian Doctoral School	Mobility grant approved for attending PASCOS 2026, the 31st International Symposium on Particles, Strings, and Cosmology (UK, 2026)
Erasmus+	Stipend for a research traineeship at the Instituto de Astronomía y Física del Espacio (CONICET-UBA) (Argentina, 2025)
COST Action CA21136	Approved funding for attending the CosmoVerseSchool @ Sofia (Bulgaria, 2026)
	Grant for visiting Dr. C. Pfeifer at ZARM under the STSM programme (Germany, 2023)
COST Action CA18108	Funding for participating in the CA18108 Fourth Annual Conference (Croatia 2023)
	Funding for participating in 59. Winter School of Theoretical Physics and third COST Action CA18108 Training School "Gravity – Classical, Quantum and Phenomenology" (Poland, 2023)
ACIF 2022	PhD Fellowship by the Spanish Government - Declined (2022)

Invited talks:

▷ Friday Cosmology Seminars @Szczecin, Poland (online, 2026)
"Dynamical systems analysis in Newer GR cosmology"

▷ Journal Club, Tartu Observatory, Estonia (2024)
"Spherically symmetric solutions in New and Newer GR"

Teaching experience:

Lecturer for the course "Selected Topics in the Theories of Gravity" (LTFY.00.010), University of Tartu (2026)

Lecture 1 – Quasinormal Modes: From GR to teleparallel gravity
Lecture 2 – Junction conditions in GR and modified gravity

Supervision experience:

▷ Supervision of the undergraduate student Merili Knoll as part of the course "Loodusteadusliku meetodi seminar" (LTFY.01.012), University of Tartu (2025)

▷ Supervision of the student Alicia Osadči for the bachelor project *"Gravitational behavior of timelike worldlines"* within the Talendid Tartusse program, University of Tartu (2024-2025)

▷ Supervision of the undergraduate student Rainis Puusepp as part of the course "Loodusteadusliku meetodi seminar" (LTFY.01.012), University of Tartu (2024)

Public outreach:

Author of the popularized article "Εναλλακτικές θεωρίες βαρύτητας" [Alternative theories of gravity] for the Greek electronic magazine UNI-MAG (2025)

Translation of teaching material in astrophysics and cosmology for the COST Action CA21136 in the Greek language (2025)

Participation in the video prepared for the Women in Science Week 2024: "Women in physics - Laboratory of Theoretical Physics of the University of Tartu" (2024)

Participation in the "Training Camp 2023: The Art of Giving a Popular Science Talk", where I gave the popular talk *"Wormholes: Reality or Science Fiction?"*

Peer review and other activities:

Peer reviewer	European Physical Journal C International Journal of Geometric Methods in Modern Physics
BSc thesis reviewer	Marie Femke Jaarma, University of Tartu (2024)
Member of COST Actions	CA21136, CA18108

Curriculum Vitae

(eesti keeles)

Isiklik informatsioon:

Nimi	Vasiliki Karanasou
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Hariduskäik:

2022 – 2026	Filosoofiadoktor (PhD) teoreetilises füüsikas Tartu Ülikool, Eesti
2018 – 2020	Magistrikraad (MSc) teoreetilises füüsikas Uppsala Ülikool, Rootsi
2013 – 2018	Bakalaureusekraad (BSc) füüsikas Aristotle Aristotelese Ülikool Thessalonikis, Kreeka
2011 – 2013	Keskkooli lõputunnistus Levadia 1. üldgümnaasium, Kreeka

Publikatsioonid:

1. H. Asuküla, M. Hohmann, V. Karanasou, S. Bahamonde, C. Pfeifer and J. L. Rosa, *"Spherically symmetric vacuum solutions in one-parameter new general relativity and their phenomenology"*, Phys. Rev. D **109**, no.6, 064027 (2024), [arXiv], [inSPIRE], [ETIS]

2. M. Hohmann and V. Karanasou, *"Symmetric teleparallel connection and spherical solutions in symmetric teleparallel gravity"*, Phys. Rev. D **111**, no.6, 064057 (2025), [arXiv], [inSPIRE], [ETIS]

3. E. F. Eiroa, G. Figueroa-Aguirre and V. Karanasou, *"Stability of spherical thin-shell wormholes in scalar-tensor theories"*, Eur. Phys. J. Plus **141**, no.2, 134 (2026), [arXiv], [inSPIRE], [ETIS]

Uurimistoetused ja stipendiumid:

Eesti doktorikool	Mobiilsusstipendium heaks kiidetud osalemiseks PASCOS 2026-l, 31. rahvusvahelisel sümpoosionil osakestest, kiududest ja kosmoloogiast (Suurbritannia, 2026)
Erasmus+	Stipendium teadusuuringute praktika läbimiseks Instituto de Astronomía y Física del Espacio (CONICET-UBA) juures (Argentina, 2025)
COST Action CA21136	Heaks kiidetud rahastus osalemiseks CosmoVerseSchool @ Sofia (Bulgaaria, 2026)
	Stipendium külastuseks Dr C. Pfeiferi juures ZARM-is STSM programmi raames (Saksamaa, 2023)
COST Action CA18108	Rahastamine osalemiseks CA18108 Fourth Annual Conference (Horvaatia 2023)
	Rahastamine osalemiseks 59. Winter School of Theoretical Physics and third COST Action CA18108 Training School "Gravity – Classical, Quantum and Phenomenology" (Poola, 2023)
ACIF 2022	Filosoofiadoktori stipendium Hispaania valitsuselt – Keeldutud (2022)

Kutsutud ettekanded:

▷ Friday Cosmology Seminars @Szczecin, Poola (online, 2026)
"Dynamical systems analysis in Newer GR cosmology"

▷ Tartu Observatooriumi Teadusklubi, Eesti (2024)
"Spherically symmetric solutions in New and Newer GR"

Õppetöö kogemus:

Kursuse lektor "Valitud peatükke gravitatsiooniteooriast" (LTFY.00.010), Tartu Ülikool, (2026)

Loeng 1 – Quasinormal Modes: From GR to teleparallel gravity
Loeng 2 – Junction conditions in GR and modified gravity

Juhendamiskogemus:

▷ Bakalaureuseüliõpilase Merili Knolli juhendamine kursuse "Loodusteadusliku meetodi seminar" (LTFY.01.012) raames, Tartu Ülikool (2025)

▷ Bakalaureuseüliõpilase Alicia Osadči juhendamine bakalaureusetöö *"Gravitational behavior of timelike worldlines"* raames Talendid Tartusse program, Tartu Ülikool (2024–2025)

▷ Bakalaureuseüliõpilase Rainis Puusepa juhendamine kursuse "Loodusteadusliku meetodi seminar" (LTFY.01.012) raames, Tartu Ülikool (2024)

Avalik teavitustöö:

Populaarteadusliku artikli "Εναλλακτικές θεωρίες βαρύτητας" [Gravitatsiooni alternatiivsed teooriad] autor Kreeka elektroonilisele ajakirjale UNI-MAG (2025)

Õppevahendite tõlkimine astronoomia ja kosmoloogia valdkonnas Kreeka keelde COST Action CA21136 jaoks (2025)

Osalemine videos, mis valmis Naiste Teaduspäeva 2024 raames: " Naised teaduses - Tartu Ülikooli teoreetilise füüsika labor" (2024)

Osalemine koolituslaagris "Training Camp 2023: The Art of Giving a Popular Science Talk", kus pidasin populaarteadusliku ettekande *"Wormholes: Reality or Science Fiction?"*

Võrdlev hindamine ja muud tegevused:

Vastastik hindaja	European Physical Journal C
	International Journal of Geometric Methods in Modern Physics
Bakalaureusetöö retsensent	Marie Femke Jaarma, Tartu Ülikool (2024)
COST Actioni liige	CA21136, CA18108

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