Chapter I

THE THEORY OF THE ASPIRATION METHOD UNDER IDEAL CONDITIONS

§1. AIR IONS

The fundamental views on the mechanism of electrical conductivity of air were established at the beginning of our century, mainly by Giese /Giese, 1882/ and J. J. Thomson /Thomson, J. J., Rutherford, 1896/. An extensive review of the earlier work may be found in the well-known monograph by Wiedemann /Wiedemann, 1885/.

The electrical conductivity of air is associated with the presence of charged particles capable of moving in an electric field. Elementary ionization events create positively charged ions and free electrons. However, the lifetime of free electrons and of monomolecular ions in air at atmospheric pressure is very short. Free electrons and monomolecular ions play an appreciable role only in very fast processes, for the investigation of which the aspiration method is not suitable for various reasons. Neutral molecules attach themselves to the initially charged particles and a bond is formed by electrical and molecular forces. The relatively stable particles formed in this way are called light air ions. Little is known about the internal structure of the light air ions /Israël, 1957b/. Segal /Segal, 1962/ tried to calculate theoretically the probability of formation of light air ions with different structures.

Light air ions are essential in the conductivity mechanism of atmospheric air.

Soon after the establishment of the ionic theory of air conductivity, the existence of larger charge carriers was discovered / Townsend, 1898; Lenard, 1900; Langevin, 1905a, 1905b/; these were called heavy air ions. Heavy air ions are formed as a result of the attachment of light air ions to aerosol particles contained in air. By their nature, heavy air ions are not ions in the usual sense, but solid or liquid charged particles suspended in air which remain stable even after losing their charge.

In atmospheric electric phenomena the heavy air ions act as spaced charge carriers.

Light and heavy air ions move with different velocities in an electric field. For the study of electric currents in air, the different nature of the charge carriers has no significance and it suffices to characterize the air ions in terms of their mobility only. It is therefore advisable to describe the light and the heavy air ions from a common point of view, using the general term a i r i o n s.

In the field of atmospheric electricity one usually uses the term a t m o s p h e r i c i o n s, which has the same significance and may be

considered as synonymous to the term a i r i o n s. However, to denote charged particles formed in laboratory or industry the term a i r i o n s is more suitable.

The mean velocity of an air ion in an electric field is proportional to the field strength $\vec{E}$:

$$\vec{v} = \underset{\cdot}{k}\vec{E}. \tag{1.1}$$

For the lightest air ions, relation (1.1) is applicable only at field strengths up to 10 kV/cm /Mitchell, Riedler, 1934; Balog, 1944/.

The mobility $\underset{\cdot}{k}$ of an air ion is by definition positive for positively charged and negative for negatively charged air ions. Usually, the absolute value of $\underset{\cdot}{k}$ is taken as the mobility of air ions. To simplify notation we shall adopt the following convention. If some quantity x has a subdot, then

$$|\underset{\cdot}{x}| = x. \tag{1.2}$$

Thus the letter k (without the subdot) stands for the mobility of air ions in the usual sense. The use of dotted letters for polar quantities is analogous to the vector notation: $|\vec{x}| = x$.

The largest particles that can be considered air ions are those for which gravity and inertia forces are still negligible. A sufficient condition for this is evidently

$$m \ll \frac{qE}{g + k\left|\frac{d\vec{E}}{dt}\right|}, \tag{1.3}$$

where m is the mass of the particle, g is the gravitational acceleration, and q is the absolute value of the particle charge.

This description of an air ion is satisfactory if the chemical nature and the mass of the charge carriers have no significance, as when considering processes in air. In many cases, however, the chemical nature and the mass of the particles cannot be neglected, but such problems are of secondary significance in our analysis.

The usual measuring methods give only the macroscopic parameters of ionized air. Charges of individual ions are undetectable. It is therefore advisable to characterize ionized air in terms of parameters which are independent of the air-ion charges.

Ionized air is characterized by the differential mobility distribution function of charge density, or by the differential mobility spectrum

$$\underset{\cdot}{\varrho}(\underset{\cdot}{k}) = \frac{d\varrho}{d\underset{\cdot}{k}}, \tag{1.4}$$

where $d\varrho$ is the charge density due to air ions with mobilities between $\underset{\cdot}{k}$ and $\underset{\cdot}{k} + d\underset{\cdot}{k}$.

The distribution function $\underset{\cdot}{\varrho}(\underset{\cdot}{k})$ is positive for positive $\underset{\cdot}{k}$ and negative for negative $\underset{\cdot}{k}$. One often uses the spectrum $\varrho(k)$, which is the absolute values of $\underset{\cdot}{\varrho}(k)$ as defined by (1.2). $\varrho(k)$ describes only the spectrum of air ions of one polarity. To describe spectra of air ions of both polarities one has to introduce the two functions $\varrho_+(k)$ and $\varrho_-(k)$. The mobility spectrum of air ions of both polarities is denoted by $\varrho_\pm(k)$. If air ions of only one polarity are considered, and the polarity is of no significance, the subscript $\pm$ may be omitted.

The charge density in the interval (k_1, k_2) is determined by the integral over the distribution function

$$\varrho(k_1, k_2) = \int_{k_1}^{k_2} \varrho(k)\,dk. \tag{1.5}$$

Similarly one finds the density of the negative charge $\varrho_- = \varrho(-\infty, 0)$, the density of the positive charge $\varrho_+ = \varrho(0, \infty)$ and the total charge density $\varrho = \varrho(-\infty, \infty)$.

The current density spectrum (or distribution function) in still air is expressed in terms of $\varrho(k)$ as follows.

$$\vec{j}(k) = \vec{v}\varrho(k) = k\varrho(k)\vec{E} = \lambda(k)\vec{E}. \tag{1.6}$$

We shall call $\lambda(k) = k\varrho(k)$ the conductivity spectrum. Integration of $\lambda(k)$ gives the conductivity in the interval (k_1, k_2)

$$\lambda(k_1, k_2) = \int_{k_1}^{k_2} \lambda(k)\,dk. \tag{1.7}$$

When taken between appropriate limits, this integral gives the negative conductivity $\lambda_- = \lambda(-\infty, 0)$, the positive conductivity $\lambda_+ = \lambda(0, \infty)$ and the conductivity $\lambda = \lambda(-\infty, \infty)$.

Neither the conductivity spectrum nor the integrated conductivity (for $k_1 \leqslant k_2$) can be negative. The interchange of the variables k_1 and k_2 reverses the sign of the charge density $\varrho(k_1, k_2)$ and of the conductivity $\lambda(k_1, k_2)$.

In the system of absolute values the charge density and the conductivity are positive if $k_1 < k_2$ and negative otherwise. Here k_1 and k_2 should represent mobilities of the same polarity. To describe the spectrum of air ions of both polarities, we should consider two complete sets of parameters: $\varrho_+(k_1, k_2)$ and $\varrho_-(k_1, k_2)$, $\lambda_+(k_1, k_2)$ and $\lambda_-(k_1, k_2)$. The conductivity spectrum also becomes double valued: $\lambda_+(k)$ and $\lambda_-(k)$. As in the case of $\varrho(k)$, the subscript + or − may be omitted sometimes, or written in the form $\pm$. Only when denoting polar charge densities $\varrho_\pm$ and polar conductivities $\lambda_\pm$, is the omission of the subscript admissible.

The use of the function $\lambda(k)$ facilitates the graphical presentation of the air-ion spectrum in large mobility intervals. The function $\varrho(k)$ is unsuitable to this end since it usually has large values in the region of small mobilities, and vice versa. Particular attention should be given to the graphical presentation of the conductivity spectrum on a logarithmic mobility scale. The area under the $\lambda(k)$ curve in this case is proportional to the charge density. This is explained by the relation

$$\int \lambda(k)\,d\left(\ln \frac{k}{k_1}\right) = \int \varrho(k)\,dk. \tag{1.8}$$

The arbitrary constant mobility k_1 in (1.8) is required in order to ensure correct dimensions.

Sometimes, the mobility distribution of air ions is characterized by the partial charge densities of hypothetical discrete groups. In the theory of measurement methods this approach is unjustified. The actual air-ion velocity distribution in any experimental arrangement is never strictly discrete (because of air-ion diffusion, to mention the most obvious factor). Even if the air-ion spectrum should turn out to be discrete, one must start with the assumption of a continuous distribution when setting up and evaluating observations, in order to prove discreteness. This is the best way to bring out the objective information. A discrete distribution can be considered as a particular case of a continuous one. To effect a mathematical transformation from a continuous to a discrete distribution, the function $\varrho(k)$ should be written as a sum

$$\varrho(k) = \sum_n \varrho_n \delta(k - k_n), \tag{1.9}$$

where ϱ_n are the specific densities of the air ions with mobilities k_n. Owing to the property of the delta function $\int f(k)\delta(k - k_n) = f(k_n)$ all integral expressions transform into sums which are characteristic of a discrete distribution.

In practice one often attempts to calculate the number density $n_\pm(k_1, k_2)$ of air ions. If the charge of all air ions equals one elementary charge, then

$$n_\pm(k_1, k_2) = \frac{\varrho_\pm(k_1, k_2)}{e}, \tag{1.10}$$

where e is the elementary charge.

Unfortunately the assumption $q = e$ holds only for light air ions. Heavy air ions and the even heavier, artificially created, charged aerosol particles may carry a larger charge. When this is the case, formula (1.10) does not give the actual number density but only some relative quantity. If the true mean charge of the air ions is unknown, then it would be more correct to speak of the charge density expressed in elementary charges per unit volume instead of the number density. For singly charged air ions the last quantity coincides with the number density of air ions.

§2. VELOCITY FIELD OF AIR IONS

Consider a laminar air flow and suppose that the motion of air ions is determined only by the fluid velocity and the action of the electric field. The velocity of the air ions is then

$$\vec{v} = \vec{u} + k\vec{E}, \tag{2.1}$$

where $\vec{u}$ is the fluid velocity. Equation (2.1) defines the flow lines of air ions with mobility k. In the case of steady flow the ion flow lines coincide with the trajectories of the air ions.

We now calculate the density distribution of the flow of air ions passing through some imaginary surface S.

Through a surface element $\vec{dS}$ there is a flux of charge given by

$$dI(k) = \varrho(k)\vec{v}\,\vec{dS} = \varrho(k)\,(d\Phi + k\,dN), \tag{2.2}$$

where $d\Phi = \vec{u}\,\vec{dS}$ is the volume rate of air flow through a surface element $\vec{dS}$ and $dN = \vec{E}\,\vec{dS}$ is the electric flux through $\vec{dS}$. The density distribution of the flow through the surface S is

$$I(k) = \iint\limits_S \varrho(k)\,(d\Phi + k\,dN) = \overline{\varrho(k)}\,(\Phi + kN), \tag{2.3}$$

where $\overline{\varrho(k)}$ is some mean value of $\varrho(k)$ on the surface S, Φ is the air flow rate and N is the electric flux through this surface.

From formula (2.3) follow two conclusions of practical importance /Tammet, 1964b/:

1. For any flow surface of air ions with mobility k we have

$$\Phi + kN = 0, \tag{2.4}$$

since $I(k)$ through the surface of the flow always equals zero.

2. For a homogeneous distribution of air ions over some surface the density distribution of the flow through this surface is expressed by

$$I(k) = (\Phi + kN)\,\varrho(k). \tag{2.5}$$

Consider now the behavior of the spectrum $\varrho(k)$ in the velocity field of the air ions /Cagniard, 1943, 1944; Tammet, 1960/. Further, let us assume that the air is incompressible and neglect the recombination and generation of air ions as well as processes in which the air-ion velocity varies. On the basis of these assumptions and the law of charge conservation, the law of conservation of the charge density spectrum follows, and is given by

$$\frac{\partial \varrho(k)}{\partial t} = -\operatorname{div}\vec{j}(k). \tag{2.6}$$

Since $\vec{j}(k) = \vec{v}\,\varrho(k)$, expression (2.6) becomes

$$\frac{\partial \varrho(k)}{\partial t} = -\varrho(k)\operatorname{div}\vec{v} - \vec{v}\operatorname{grad}\varrho(k). \tag{2.7}$$

The variation of $\varrho(k)$ in a point moving along the trajectory of an air ion with velocity $\vec{v}$ is characterized by the total differential $d\varrho(k)$. This is connected with the partial derivative via the expression

$$\frac{d\varrho(k)}{dt} = \frac{\partial \varrho(k)}{\partial t} + \vec{v}\operatorname{grad}\varrho(k), \tag{2.8}$$

from which we obtain

$$\frac{d\varrho(k)}{dt} = -\varrho(k)\operatorname{div}\vec{v}. \tag{2.9}$$

Since $\operatorname{div}\vec{u} = 0$ and $\operatorname{div}\vec{E} = 4\pi\varrho$, we obtain

$$\frac{d\varrho(k)}{dt} = -4\pi\varrho\lambda(k). \tag{2.10}$$

This expression describes the phenomenon known as electrostatic dissipation /Townsend, 1898; Wolodkewitsch, 1933a, 1933b; Forster 1959; Dunskii, Kitaev, 1960; Whitby, McFarland, 1961; Kitaev, 1962/. Electrostatic dissipation is due to the mutual repulsion or attraction of air ions.

In many cases one may neglect the electric field of the space charge and assume that $\operatorname{div}\vec{E} = 0$. Then

$$\frac{d\varrho(k)}{dt} = 0. \tag{2.11}$$

This formula is a supplement to Liouville's well-known theorem on the motion of air ions. Formula (2.11) leads to a conclusion which is of great importance in the theory of the aspiration method: if one neglects generation, recombination, variation of mobility, diffusion and electrostatic interaction of air ions, then, in a laminar incompressible air flow $\varrho(k)$ is constant along the trajectory of an air ion of corresponding mobility.

A similar result first appeared in a work of Becker /Becker, 1910/. The same result was obtained and applied in a little-known work /Cagniard, 1943/ on the theory of an aspiration counter for the case of a discrete air-ion spectrum. For charged aerosols a similar result was obtained by Levin /Levin, 1957, 1959/.

When studying the motion of air ions it is sometimes advantageous to utilize the methods of the theory of similarity. To ensure similarity between the flow lines of air ions in two separate systems, then in addition to the condition of hydrodynamic and electrostatic similarity, we also require agreement between the values given by the special criterion

$$K_I = \frac{kE}{u} = \frac{kU}{ux}. \tag{2.12}$$

In this expression E denotes the characteristic field strength, u the characteristic flow velocity, U the characteristic voltage, x the characteristic dimension and k the mobility of the air ions under consideration.

§3. ARRANGEMENT OF AN ASPIRATION COUNTER AND THE PRINCIPLE OF MEASUREMENT OF THE AIR-ION SPECTRUM

The aspiration counter comprises a measuring capacitor and arrangements for drawing air through the capacitor, for supplying voltage to the capacitor and for measuring the currents generated. One usually measures only the current through one capacitor plate, called the collector plate.

For the case of steady-state operation of the counter and a stationary spectrum $\varrho(k)$, the current through the plates is determined by conduction of the air ions.

The current flowing through the capacitor is either grounded directly or otherwise via the voltage supply or current meter. The total current flowing through the measuring capacitor may differ from zero if the mean charge densities of the air entering and leaving the capacitor are not equal to one another.

In most cases when using the aspiration counter one may assume that the motion of air ions in the capacitor is independent of the presence of other air ions in the air sample. Then the density distribution of the current $I(k)$ through the collector plate is proportional to the corresponding value $\varrho(k)$. This can be expressed in the form

$$I(k)=G\varrho(k). \tag{3.1}$$

The quantity G is here a constant depending on the structural and operational parameters of the measuring capacitor.

In what follows we shall assume that G does not depend on $\varrho(k)$ except in special cases where the interaction of air ions in the capacitor is allowed.

The current through the collector plate generated by the incoming air ions is expressed by the integral

$$I=\int_{-\infty}^{+\infty} G\varrho(k)dk. \tag{3.2}$$

This current can be directly measured, thus enabling the experimental determination of the dependence of the current I on the operational parameters of the counter. The function $I(\psi)$, where ψ is an arbitrarily chosen variable operational parameter, is called the characteristic of the aspiration counter. Usually a volt-ampere characteristic is used, whereby ψ denotes the voltage across the capacitor plates. Each type of counter design is characterized by a particular form of the function $G(\psi, k)$, which is the Green's function for the air-ion current through the collector plate. Expression (3.2) can be regarded as an integral equation with respect to $\varrho(k)$:

$$I(\psi)=\int_{-\infty}^{+\infty} G(\psi, k)\varrho(k)dk;$$

if for one polarity $G = 0$, then (3.3)

$$I(\psi)=\int_{0}^{\infty} G(\psi, k)\varrho(k)dk.$$

The physical significance of the function $G(\psi, k)$ is explained as follows. Suppose that the air sample contains air ions of mobility k_1 only. In this case $\varrho(k)=|\varrho\delta(k-k_1)$ and equation (3.3) yields $G(\psi, k_1)=I(\psi)/\varrho$. Consequently, $G(\psi, k)$ may be regarded as the counter characteristic (reduced to unit charge density) in the presence of air ions of mobility k only.

Considering counters of a given type we have to find the actual form of the function G and the method for solving equation (3.3). The solution of the integral equation (3.3) in the general case is possible only with the aid of numerical analysis, which is rather laborious.

The solution is simplified only when certain design requirements of the capacitor are fulfilled, ensuring a special form of the kernel of equation G.

These requirements, common for all aspiration counters, are listed below:

1. The measuring capacitor should possess axial symmetry.
2. There should be two openings in the outer plate of the capacitor. These openings are called the entrance and exit opening, respectively, depending on the flow direction.

The surface of the opening is defined as the imaginary surface covering the opening. The surface of the entrance opening should be such that $E = 0$ on it and the flow velocity $\vec{u}$ is directed into the measuring capacitor.

3. In the internal capacitor plates there should be no openings through which part of the air can flow.
4. All the requirements stated in the preceding paragraph should be satisfied inside the capacitor. Outside the capacitor $\varrho(k)$ should be homogeneous.

Instead of an axially symmetric capacitor, the capacitor may have the form of a sector. One often uses a parallel-plate capacitor which is a limiting case of an axially symmetric capacitor of infinite radius. In a parallel-plate capacitor a homogeneous distribution of velocities of the air flow in the longitudinal section must be ensured.

In the general theory of aspiration counters we assume the above mentioned requirements are satisfied. The effect of deviations from these requirements on the measuring results is dealt with in the second chapter.

§4. INTEGRAL COUNTER

The measuring capacitor of an integral counter comprises two plates. For the collector plate one usually chooses the inner plate /McClelland, Kennedy, 1912; Nolan, J. J., Nolan, P. J., 1930; Weger, 1953a; Siksna, 1961a/. The collector plate is connected to the current meter. A voltage U is applied across the collector and repulsing plates. The air sample is drawn into the capacitor at a volume flow rate Φ.

The integral counter is the most widely used version of the aspiration counter due to its simple design and high sensitivity. One reason for the wide use of integral counters is the simplicity of measuring integral quantities. Apart from measuring devices for ion concentration and conductivity, the integral method is applied in aerosol detectors /Sekiyama, 1959; Rich, 1959; Hasenclever, Siegmann, 1960; Siksna, 1961b/.

To determine the G-function of the integral counter, let us consider the behavior of air ions with a given mobility k in the measuring capacitor having an inner collector plate (Figure 4.1). When $Uk > 0$, the air ions are repelled by the inner plate and $G = 0$. When $Uk < 0$, the air ions will settle on the plate. The flow lines for air ions terminating on the inner plate enclose part of the space between the capacitor plates. This spatial portion is limited by the flow surface which we shall call the boundary surface. The boundary surface comes into contact with the rear edge of the inner plate.

When the mobility k is sufficiently small the boundary surface does not intersect the outer plate, as shown in Figure 4.1. The electric flux through the boundary surface is then $N = 4\pi CU$, where C is the active capacitance of the measuring capacitor. The active capacitance is the capacitance of the measuring capacitor proper and differs from the capacitance of the insulated counter system, which includes the electrometer capacitance and the parasitic capacitance of the connections. Methods for determining the active capacitance are treated in §33. According to (2.4) the air-flow rate through the boundary surface equals

$$\Phi' = 4\pi CUk. \tag{4.1}$$

The flow of air ions which enter the capacitor through that portion of the entrance surface bounded by the boundary surface is $\Phi'\varrho$ (2.5). Since all the air ions passing through this region settle on the inner plate, $I = \Phi'\varrho$ and $G = \Phi' = 4\pi CUk$. This result was obtained for certain restricted initial conditions as early as 1903 /Riecke, 1903/. For less restricted initial conditions the problem is treated in studies carried out by Swann /Swann, 1914a, b, d/.

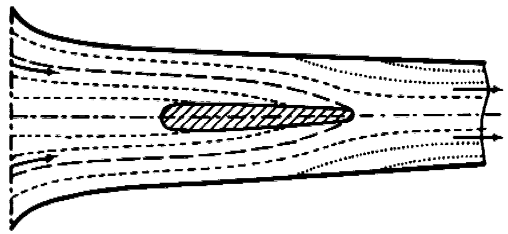

FIGURE 4.1. Flow lines in the measuring capacitor:

- - - - - - flow lines; — — — — boundary surface; · · · · · flow lines on which $\varrho(k) = 0$; — · — · — · — entrance surface.

It should be noted that in the derivation of the expression

$$I = 4\pi CUk\varrho \tag{4.2}$$

no assumptions were made on the symmetry of the measuring capacitor. Therefore expression (4.2) is also valid in the case of an asymmetric system of any geometrical configuration. Such an approach was first outlined for somewhat different conditions in the work /Kohlrausch, 1906/.

More mobile ions with a mobility k_0 will have a boundary surface that intersects the entrance surface at the outer plate. In this case $\Phi' = \Phi$.

The corresponding mobility k is called the limiting mobility. From formula (4.1) we obtain for $\Phi' = \Phi$

$$\left.\begin{array}{l} k_0 = \dfrac{\Phi}{4\pi CU}, \\ \text{and considering the polarity} \\ k_0 = -\dfrac{\Phi}{4\pi CU} \end{array}\right\} \tag{4.3}$$

when $k_0 \leqslant k$, all air ions passing through the entrance surface settle and $G = \Phi$.

On the basis of the above results we may write for the air ions attracted to the collector plate

$$G = \begin{cases} 4\pi CUk & \text{for} \quad k \leqslant k_0 \\ \Phi & \text{for} \quad k_0 \leqslant k. \end{cases} \tag{4.4}$$

Similar considerations for a capacitor with an outer collector plate lead to results expressed by formulas (4.3) and (4.4).

From the above it follows that the basic formulas describing an integral counter are valid independently of the geometry of the measuring capacitor and the flow velocity distribution only when the capacitor is axially symmetric. The latter applies only to axially symmetric steady flows. Air turbulence in the capacitor is inadmissible in the general case.

The question concerning the permissible capacitor geometry and flow velocity distribution has been a matter of controversy for a long time, although the above conclusion could be reached from known results /Swann, 1914b; Cagniard, 1943, 1944; Levin, 1959/. The reason for misunderstandings lay in that the basic formulas for the integral counter were derived from the calculation of air-ion trajectories, which involved serious difficulties due to the occurrence of radial components of the flow velocity. To simplify the calculations certain assumptions on the capacitor geometry and distribution of the air-flow velocities were made. Corresponding requirements were set up with regard to counter design. Such superfluous requirements may lead to appreciable limitations when designing counters. Consider as an example Becker's well-known counter /Becker, 1909/, whereby special measures were taken to ensure a homogeneous electric field at the capacitor entrance. This, however, only complicated the design without producing any advantages /Scholz, 1931b/. Recent works propose a uniform distribution of air-flow velocities /Misaki, 1960; Paltridge, 1965/ or the absence of radial components /Hoegl, 1963b/. When choosing the capacitor dimensions, large deviations from a cylindrical geometry are considered to be inadmissible /Komarov, Seredkin, 1960/.

It should be pointed out that the calculation of air-ion trajectories is complicated and at times requires computer-programmed numerical calculations /Kraemer, Johnstone, 1955/. The solution of many basically simple problems when reduced to formulas describing the integral counter appears to be complicated when calculating trajectories /Shimizu, 1956, 1957, 1960/.

§ 5. SOLUTION OF THE EQUATION OF THE INTEGRAL COUNTER

Apart from depending on the mobility, the G-function of the integral counter depends on the three parameters C, U and Φ. When determining the characteristics of a counter one of the parameters is varied while the remaining ones are kept constant. Usually the voltage is varied and, less frequently, the air-flow rate /Nolan, P. J., Kenny, 1952/. The reason is that variation of the air-flow rate may have an adverse effect on the $\varrho(k)$ of the air sample.

To solve equation (3.3) it should be transformed by applying a suitable operator into a form in which the desired quantities can be eliminated by algebraic operations. In the case of the integral counter this can be achieved by a single and double differentiation with respect to ψ and the operator h_ψ

$$h_\psi = 1 - \psi \frac{\partial}{\partial \psi}. \tag{5.1}$$

The operator h_ψ is often used for the evaluation of measurements, since a convenient method — "the tangent method" — exists for determining $h_\psi I$ /Israël 1931, 1957b; Gerasimova, 1939; Imyanitov, 1957/ (see Figure 5.1).

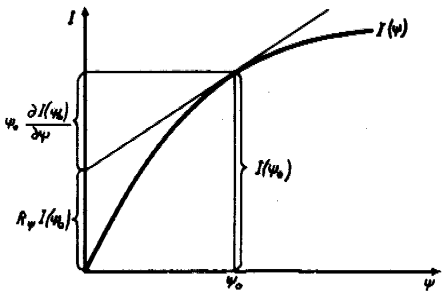

FIGURE 5.1. Determination of $h_\psi I$ by the tangent method.

In the integrand of equation (3.3) the differential operator or h_ψ acts only upon the function G. Below we list the transformations of $G(\psi, k)$ which will be used later:

$$\frac{\partial G}{\partial U} = \begin{cases} 4\pi C k & \text{for} \quad k < k_0 \\ 0 & \text{for} \quad k_0 < k, \end{cases} \tag{5.2}$$

$$\frac{\partial G}{\partial \Phi} = \begin{cases} 0 & \text{for} \quad k < k_0 \\ 1 & \text{for} \quad k_0 < k, \end{cases} \tag{5.3}$$

$$h_U G = \begin{cases} 0 & \text{for} \quad k < k_0 \\ \Phi & \text{for} \quad k_0 < k, \end{cases} \tag{5.4}$$

$$h_\Phi G = \begin{cases} 4\pi C U k & \text{for} \quad k < k_0 \\ 0 & \text{for} \quad k_0 < k, \end{cases} \tag{5.5}$$

$$\frac{\partial^2 G}{\partial U^2} = -\frac{\Phi^2}{4\pi C U^3} \delta(k - k_0), \tag{5.6}$$

$$\frac{\partial^2 G}{\partial \Phi^2} = -\frac{1}{4\pi C U} \delta(k - k_0). \tag{5.7}$$

Using expressions (5.2)—(5.7) we can readily derive formulas for calculating different parameters of ionized air in accordance with the well-known function $I(U)$ or $I(\Phi)$. The results are

$$\varrho(k_0) = -\frac{4\pi C U^3}{\Phi^2} \frac{\partial^2 I}{\partial U^2}, \tag{5.8}$$

$$\varrho(k_0) = -4\pi C U \frac{\partial^2 I}{\partial \Phi^2}, \tag{5.9}$$

$$\varrho(k_1, k_2) = \frac{h_U I(U_1)}{\Phi_1} - \frac{h_U I(U_2)}{\Phi_2}, \tag{5.10}$$

$$\varrho(k_1, k_2) = \frac{\partial I(\Phi_1)}{\partial \Phi} - \frac{\partial I(\Phi_2)}{\partial \Phi}, \tag{5.11}$$

$$\lambda(k_1, k_2) = \frac{1}{4\pi C}\left[\frac{\partial I(U_2)}{\partial U} - \frac{\partial I(U_1)}{\partial U}\right], \tag{5.12}$$

$$\lambda(k_1, k_2) = \frac{1}{4\pi C}\left[\frac{h_\Phi I(\Phi_2)}{U_2} - \frac{h_\Phi I(\Phi_1)}{U_1}\right]. \tag{5.13}$$

The use of the δ-function for the derivation of formulas (5.8) and (5.9) is not obligatory. Instead, the integral in equation (3.3) may be written as the sum of two integrals corresponding to the ranges $k \leqslant k_0$ and $k_0 \leqslant k$, respectively. Differentiating with respect to the limits of the integrals, we obtain the same results /Langevin, 1905a; Israël, 1957b/.

It is possible to derive a more general and concise notation for the solutions of the integral-counter equation /Siksna, 1950; Israël, 1957b/. In this case we take for the starting function

$$\mathrm{P}(k_0) = \frac{I(k_0)}{\Phi}. \tag{5.14}$$

The quantity $\mathrm{P}(k_0)$ has the dimension of charge density and will be called the conventional charge density for the limiting mobility k_0. In simplified calculations the value of P, expressed in elementary charges per unit volume, is often represented as the concentration of air ions. Thus the calculated concentration is actually a conventional quantity, since it does not correspond to any definite mobility interval. In practice, however, one often encounters conditions in which the actual character of the mobility spectrum enables us to assume

$$\varrho(k_0, \infty) \approx \mathrm{P}(k_0). \tag{5.15}$$

This expression is used very often to calculate the concentration of light air ions.

Starting with the function $P(k_0)$, we transform equation (3.3) into

$$P(k_0)=\int_0^\infty \frac{G(k_0,k)}{\Phi}\,\varrho(k)\,dk. \tag{5.16}$$

Since

$$\frac{G(k_0,k)}{\Phi}=\begin{cases} k/k_0 & \text{for} \quad k\leqslant k_0 \\ 1 & \text{for} \quad k_0\leqslant k, \end{cases} \tag{5.17}$$

equation (5.16) has only one independent operation parameter. For convenience, we take as the variable parameter the reciprocal limiting mobility $\omega=1/k_0$.

In similar fashion we obtain the formulas

$$\varrho(k_0)=-\omega^3\frac{\partial^2 P}{\partial\omega^2}, \tag{5.18}$$

$$\varrho(k_1,k_2)=h_\omega P(k_1)-h_\omega P(k_2), \tag{5.19}$$

$$\lambda(k_1,k_2)=\frac{\partial P(k_2)}{\partial\omega}-\frac{\partial P(k_1)}{\partial\omega}. \tag{5.20}$$

In the method of varying the air-flow rate, formulas (5.18)—(5.20) are inconvenient. Here it is better to start with the function

$$\Lambda(k_0)=\frac{I(k_0)}{4\pi CU}, \tag{5.21}$$

which has the dimension of conductivity and will be called the conventional conductivity at the limiting mobility k_0. The kernel of the integral equation is

$$\frac{G}{4\pi CU}=\begin{cases} k & \text{for} \quad k\leqslant k_0 \\ k_0 & \text{for} \quad k_0\leqslant k. \end{cases} \tag{5.22}$$

Calculations similar to those carried out above yield

$$\varrho(k_0)=-\frac{\partial^2\Lambda}{\partial k_0^2}, \tag{5.23}$$

$$\varrho(k_1,k_2)=\frac{\partial\Lambda(k_1)}{\partial k_0}-\frac{\partial\Lambda(k_2)}{\partial k_0}, \tag{5.24}$$

$$\lambda(k_1,k_2)=h_{k_0}\Lambda(k_2)-h_{k_0}\Lambda(k_1). \tag{5.25}$$

The integral counter is often used for measuring the polar charge density $\varrho_\pm$ and the polar conductivity $\lambda_\pm$. This is possible only under specific conditions. To measure $\varrho_\pm$ it is necessary that $\varrho(k)\equiv 0$ in the range $k<k_0$.

For the range in which $\varrho(k)\equiv 0$ an arbitrary function may be taken for G without affecting the results. Setting G equal to Φ everywhere, we obtain

$$\varrho_\pm=P_\pm=\frac{I_\pm}{\Phi}. \tag{5.26}$$

$I_\pm$ denotes current at the respective polarity.

To verify formula (5.26) k_0 must be chosen sufficiently small in order that $\varrho(k)\equiv 0$ for $k<k_0$. The condition is satisfied if upon increasing the voltage, the current I remains unaltered and upon decreasing the air-flow rate I decreases proportionally.

To measure the polar conductivity k_0 must be chosen sufficiently large in order to satisfy the condition $\varrho(k)\equiv 0$ in the range $k_0<k$. Then for all mobilities one may take $G=4\pi CUk$, which yields

$$\lambda_\pm=\Lambda_\pm=\frac{I_\pm}{4\pi CU}. \tag{5.27}$$

For the experimental verification of this formula it is necessary that the current I remains constant when increasing the air-flow rate and is proportional to the voltage when decreasing the voltage.

§6. VARIOUS INTEGRAL COUNTER TYPES

Some variants of the aspiration method possess integral characteristics. The most widespread is the method of the precondenser ascribed to Mache /Mache, 1903/, but actually proposed by McClelland /McClelland, 1898/.

The arrangement of the counter with a precondenser is shown in Figure 6.1.

When plotting the characteristics of the counter the voltage of the precondenser is varied and the current $I_2=I_2(U_1)$ through the collector plate is measured. The voltage of the main capacitor remains unaltered.

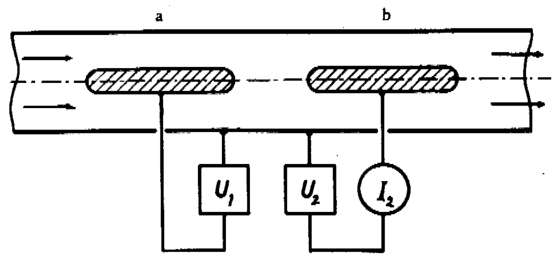

FIGURE 6.1. Counter with a precondenser:

a—precondenser; b—main capacitor; U—voltage supply; I— ammeter.

The limiting mobilities are determined as follows:

$$k_1 = -\frac{\Phi}{4\pi C U_1}, \tag{6.1}$$

$$k_2 = -\frac{\Phi}{4\pi C U_2}. \tag{6.2}$$

If $k/k_2 \leqslant 0$, then

$$G_2 = 0.$$

If $0 \leqslant k/k_2 \leqslant 1$, then

$$G_2 = \begin{cases} 0 & \text{for} \quad k/k_1 \leqslant -k/k_2 \\ -4\pi(C_1U_1 + C_2U_2)k & \text{for} \quad -k/k_2 \leqslant k/k_1 \leqslant 0 \\ -4\pi C_2 U_2 k & \text{for} \quad 0 \leqslant k/k_1 \leqslant 1 - k/k_2 \\ \Phi + 4\pi C_1 U_1 k & \text{for} \quad 1 - k/k_2 \leqslant k/k_1 \leqslant 1 \\ 0 & \text{for} \quad 1 \leqslant k/k_1 \end{cases} \tag{6.3}$$

If $1 \leqslant k/k_2$ then

$$G_2 = \begin{cases} 0 & \text{for} \quad k/k_1 \leqslant -1 \\ \Phi - 4\pi C_1 U_1 k & \text{for} \quad -1 \leqslant k/k_1 \leqslant 0 \\ \Phi + 4\pi C_1 U_1 k & \text{for} \quad 0 \leqslant k/k_1 \leqslant 1 \\ 0 & \text{for} \quad 1 \leqslant k/k_1 \end{cases}$$

The counter with a precondenser is used, as a rule, for the study of air ions with mobilities $1 \leqslant k/k_2$. In the range $0 \leqslant k/k_1$ the function G_2 of the counter with a precondenser equals the difference of the fixed parameter Φ and the function G of the standard integral counter. The value of G_2 for $1 \leqslant k/k_2$ does not depend on the polarity of the voltage U_1. This is shown in Figure 6.2, which gives a plot of the function G for a counter with a precondenser and for a standard integral counter.

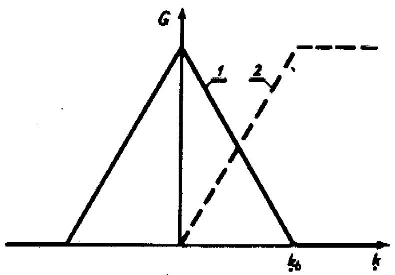

FIGURE 6.2. The G-function:

1—for a counter with a precondenser; 2—for a standard integral counter.

If in the range $0 \leqslant k/k_2 \leqslant 1$ the condition $\varrho(k) \equiv 0$ is satisfied, then all calculations for a counter with a precondenser differ from those for a standard counter only in the sign before the derivative and the operator h_ψ.

In the opposite case the situation is somewhat more complicated. When the polarities of the voltage across the preliminary capacitor and the main capacitor are the same, partial conductivities of the type $\lambda(0, k)$ cannot be measured. It is thus recommended to plot the characteristics for opposite voltage polarities. In this case the partial conductivities are connected with the derivatives $\partial I_2/\partial U_1$ as in the case of the standard integral counter.

Some of the problems connected with the precondenser counter are treated in detail in the works /Kohlrausch, 1914; Israël, 1931; Polovko, Nichkevich, 1937; Nolan, P. J., O'Connor, 1955/.

The method of the condensation nucleus counter is closely related to the methods of the precondenser /Nolan, P. J., Deignan, 1948/. Since in the former the nonelectric quantity (concentration of condensation nuclei) is recorded, the method of the condensation nucleus counter with a precondenser will not be discussed in detail here.

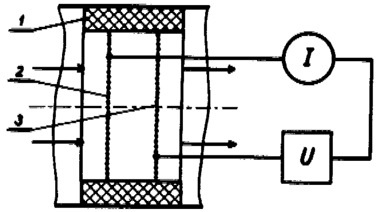

FIGURE 6.3. Counter in which air is blown through grids:

1—insulator; 2—collector grid; 3—repulsing grid.

We shall now briefly describe the method of blowing air through grids which may be used as an integral or differential method. This method was proposed in the work /Zeleny, 1898b/.

The method of blowing air through grids is not widespread /Kähler, 1903; Aselmann, 1906/, although it certainly is of interest because of the possibility of decreasing the dimensions of the measuring capacitor. The application of this method is apparently restricted to large limiting mobilities, for which it is advisable to consider a large air-flow rate with a relatively small active capacitance of the measuring capacitor.

A schematic diagram of the measuring capacitor with grids is shown in Figure 6.3.

The limiting mobility can be calculated from formula (4.3). However, more explicit is the expression

$$k_0 = \frac{u}{E}, \tag{6.4}$$

where u is the flow velocity, and E the electric field strength between the grids. The function G is given by (4.4).

The method of blowing air through grids deviates from the requirements mentioned in §3. These requirements are replaced by the condition of a uniform electric field and air flow and the condition of total screening of the electric field by the collector grid. In the case of a wide-mesh grid the last condition is not satisfied /Loeb, 1923; Kaden, 1950, Izergin, 1958/, which should be taken into account when designing the counter. At the same time attention should be given to the adsorption of air ions, which is of particular significance in the above described method.

§ 7. DIFFERENTIAL COUNTERS OF THE FIRST ORDER

There are two types of differential counters of the first order: one has a divided electrode and the other employs a divided air flow. The method of the divided electrode was proposed by Zeleny /Zeleny 1901/. The theory of the air-ion spectrum was developed later /Blackwood, 1920; Hogg, 1939; Misaki, 1950; Tammet, 1960; Whipple, 1960; Hoppel, Kraakevik, 1965/. This method is the most widely used version of the aspiration method.

The counter with a divided electrode differs from the integral counter in that one electrode of the measuring capacitor is divided into two mutually insulated parts. During aspiration the second part serves as a collector plate, while the first part, called the forward electrode, is maintained at the same potential as the collector plate. The repulsing plate is standard. The design of such a capacitor is schematically shown in Figure 7.1. Here the inner plate is divided. As in the integral counter with an outer collector plate one can divide the outer plate, the rear part of which then becomes the collector plate. The inner part is not divided and is standard. The differential counter of the first order with a divided electrode is similar in design to an integral counter with a precondenser, but differs in its operation and the ratio of the plate capacitances.

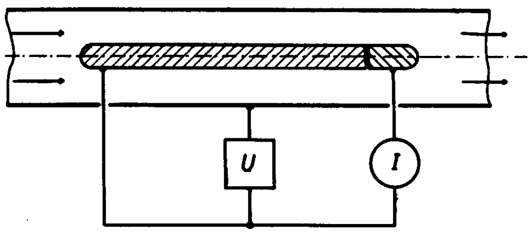

FIGURE 7.1. Differential counter of the first order with a divided electrode.

Let us now define the function G for a counter with a divided electrode. The sum of the currents through the forward and collector plates equals the current in the corresponding integral counter, the active capacitance of which is C_1+C_2, where C_1 is the mutual capacitance (the absolute value of the coefficient of electrostatic induction) between the forward plate and the main plate, and C_2 is the mutual capacitance between the collector plate and the main plate.

The current through the forward plate equals the current in the integral counter with an active capacitance C_1. The current through the collector plate equals the difference of the total current and the current through the forward plate. Consequently, the G-function for the differential counter with a divided electrode corresponds to the difference of the G-function of the integral counter with an active capacitance C_1+C_2 and the G-function of an integral counter with an active capacitance C_1. When describing differential counters it suffices to consider air ions of one polarity only, as in §§ 4—5. Denoting the corresponding limiting mobilities by k_a and k_b, respectively, we obtain

$$k_a=\frac{\Phi}{4\pi(C_1+C_2)U}, \tag{7.1}$$

$$k_b=\frac{\Phi}{4\pi C_1 U}, \tag{7.2}$$

$$G=\begin{cases} 4\pi C_2 Uk & \text{for} \quad k\leqslant k_a \\ \Phi-4\pi C_1 Uk & \text{for} \quad k_a\leqslant k\leqslant k_b \\ 0 & \text{for} \quad k_b\leqslant k. \end{cases} \tag{7.3}$$

The behavior of the G-function is shown in Figure 7.2.

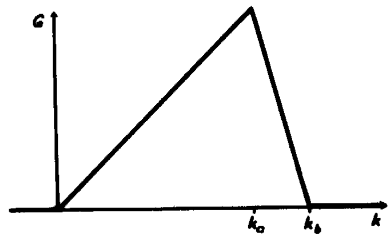

FIGURE 7.2. The G-function of a differential counter with a divided electrode.

The G-function (7.3) has the following properties:

$$h_U G=\begin{cases} 0 & \text{for} \quad k<k_a \\ \Phi & \text{for} \quad k_a<k<k_b \\ 0 & \text{for} \quad k_b<k. \end{cases} \tag{7.4}$$

$$\frac{\partial G}{\partial\Phi}=\begin{cases} 0 & \text{for} \quad k<k_a \\ 1 & \text{for} \quad k_a<k<k_b \\ 0 & \text{for} \quad k_b<k. \end{cases} \tag{7.5}$$

These properties enable the use of the differential counter of the first order with a divided electrode for measuring the partial charge density. It can be readily shown that

$$\varrho(k_a, k_b)=\frac{h_U I}{\Phi}, \tag{7.6}$$

$$\varrho(k_a, k_b) = \frac{\partial I}{\partial \Phi}, \tag{7.7}$$

where I is the absolute value of the current through the collector plate.

If the interval (k_a, k_b) is sufficiently narrow, the spectrum can be approximately defined. Using the theorem of the mean integral value, we obtain

$$\varrho(\bar{k}) = \frac{4\pi C_1(C_1 + C_2)U}{C_2\Phi^2} h_U I, \tag{7.8}$$

$$\varrho(\bar{k}) = \frac{4\pi C_1(C_1 + C_2)U}{C_2\Phi} \frac{\partial I}{\partial \Phi}. \tag{7.9}$$

In this interval the mobility $\bar{k}$ has the mean value

$$\bar{k} = \frac{(C_1 + C_2/2)\Phi}{4\pi C_1(C_1 + C_2)U} \tag{7.10}$$

and the relative half-width

$$\delta_k = \frac{C_2}{2C_1 + C_2}. \tag{7.11}$$

The smaller the ratio C_2/C_1, the smaller is the quantity δ_k and, consequently, the better the resolution of the method.

When the ratio C_2/C_1 is small, the partial conductivities can also be calculated. This follows immediately from the form of the G-function and we can write

$$\lambda(\bar{k}_1, \bar{k}_2) = \frac{1}{4\pi C_2}\left(\frac{I_2}{U_2} - \frac{I_1}{U_1}\right). \tag{7.12}$$

The mobilities $\bar{k}_1$ and $\bar{k}_2$ are defined by formulas (7.10) and (7.11) for the voltages U_1 and U_2, respectively.

The method of Misaki /Misaki, 1950/, not treated here, differs from that described above and is an independent version of the differential method with a divided electrode.

We shall now consider the differential counter of the first order in which the air flow is divided. This method is not widespread /Nolan, J.J., 1919; Nolan, J.J., Harris, 1922/. The quantitative theory is given in the work /Tammet, 1960/.

The measuring capacitor of this counter has two plates and differs from the capacitor of the integral counter only in that the capacitor inlet is divided into two parts (a circular inner and outer opening, respectively) by a coaxial tube (Figure 7.3). If the inner plate is the collector then the air sample is drawn into the capacitor at a volume flow rate Φ_2 through the outer opening. If the outer plate is the collector, the air is drawn through the inner opening. An additional flow of specially deionized air with a flow rate Φ_1 is drawn in through another opening. If the additional air flow Φ_1 were not deionized then the G-function would equal the G-function of a standard

integral counter with an air-flow rate $\Phi_1 + \Phi_2$. Assuming that the air flow Φ_2 is deionized, the G-function equals the G-function of an integral counter with an air-flow rate Φ_1. The G-function for a differential counter with a divided air flow corresponds to the difference of the G-functions for the above stated cases. Consequently,

$$G = \begin{cases} 0 & \text{for} \quad k \leqslant k_a \\ 4\pi CUk - \Phi_1 & \text{for} \quad k_a \leqslant k \leqslant k_b \\ \Phi_2 & \text{for} \quad k_b \leqslant k, \end{cases} \tag{7.13}$$

where

$$k_a = \frac{\Phi_1}{4\pi CU} \tag{7.14}$$

$$k_b = \frac{\Phi_1 + \Phi_2}{4\pi CU}. \tag{7.15}$$

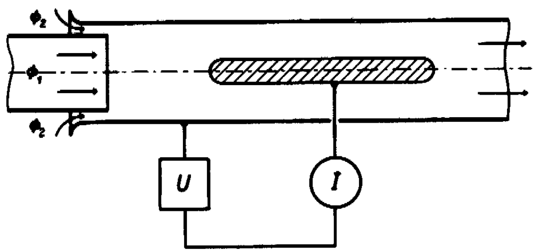

FIGURE 7.3. Differential counter of the first order with a divided air flow.

In a counter with a divided air flow one usually applies only the method of varying the voltage. Calculations similar to those for the method of the divided electrode yield

$$\lambda(k_a, k_b) = \frac{1}{4\pi C}\frac{\partial I}{\partial U}. \tag{7.16}$$

The spectrum $\lambda(k)$ is calculated from

$$\lambda(\bar{k}) = \frac{U}{\Phi_2}\frac{\partial I}{\partial U}, \tag{7.17}$$

where $\bar{k}$ lies in the interval of mean mobility

$$\bar{k} = \frac{\Phi_1 + \Phi_2/2}{4\pi CU} \tag{7.18}$$

and relative half-width

$$\delta_k = \frac{\Phi_2}{2\Phi_1 + \Phi_2}. \tag{7.19}$$

The partial charge density is calculated from the two current values $I_1(U_1)$ and $I_2(U_2)$:

$$\varrho(\tilde{k}_1, \tilde{k}_2) = \frac{I_1 - I_2}{\Phi_2}, \tag{7.20}$$

where $\tilde{k}_1$ and $\tilde{k}_2$ are determined by (7.18) and (7.19) for the voltage U_1 and U_2, respectively.

The differential version of the method in which air is blown through grids /Zeleny, 1898; Altberg, 1912; Griffiths, Awberry, 1929/ may be considered as a differential counter of the first order. The measuring capacitor of such a counter is similar to that of the integral grid counter (Figure 6.3). The difference is that in the differential method the air drawn through the net is deionized and the air ions are generated directly in the space between the grids. To generate air ions, radiation from a radioactive source or some other ionizing agency is used. This, of course, limits the application of the differential method employing grids to the laboratory.

In the theory of the differential counter with grids one should start, instead of with the function $\varrho(k)$, with the spectrum of the flow of air ions regenerated in the space between the grids. The current through the collector grid is

$$I = \int_{k_0}^{\infty} I(k)\, dk. \tag{7.21}$$

The limiting mobility k_0 is given, as in the case of the integral method, by (6.5) or (4.3). Expression (7.21) is similar to the equation of the counter having a G-function of the form

$$G = \begin{cases} 0 & \text{for} \quad k < k_0 \\ 1 & \text{for} \quad k_0 < k. \end{cases} \tag{7.22}$$

The solution of equation (7.21) is

$$I(k_0) = \frac{U^2}{ud} \frac{\partial I}{\partial U}, \tag{7.23}$$

where U is the voltage between the collector grid and the rear grid and d is the distance between the two grids.

When comparing the differential methods of the first order with the integral method, we note that in the differential method $\varrho(k)$ is calculated in terms of the first derivative of the current instead of the second derivative as in the integral method. A single differentiation is actually carried out in the experimental arrangement since either the measuring capacitor or the air flow is divided. When studying the air-ion spectrum, the differential method of the first order has appreciable advantages over the integral method. Differential counters also possess certain advantages when measuring partial charge densities. To determine the partial charge density in a given mobility interval by means of the integral method, four values of the current $I(U)$ should be known (see §29). The differential counter of the first order requires the knowledge of only two $I(U)$-values. When measuring $\varrho(\tilde{k}_1, \tilde{k}_2)$, it is advisable to use the divided-electrode method with a relatively large C_2/C_1 ratio. When measuring $\varrho(\tilde{k}, \infty)$, the divided air-flow counter is recommended, which requires only one $I(U)$-value.

§8. DIFFERENTIAL COUNTER OF THE SECOND ORDER

The differential counter of the second order is the most perfected version of the aspiration method for the study of air-ion spectra. This method was first discussed in the works /Erikson 1921, 1922, 1924, 1929; Zeleny, 1929; Val'ta, 1929/. The quantitative theory was given by the author /Tammet, 1960/, who used a differential form of the aspiration-counter equation, which is not treated in the present book.

The measuring capacitor of a differential counter of the second order is similar to that of the differential counter of the first order with a divided electrode. However, the air is drawn in as in the case of a divided air-flow counter. The G-function for the differential counter of the second order corresponds to the difference between the G-function of the first-order differential counter with a divided electrode and air-flow rate $\Phi_1 + \Phi_2$ and the G-function for the same counter with an air-flow rate Φ_1. We denote the limiting mobilities as follows:

$$k_{aa} = \frac{\Phi_1}{4\pi(C_1 + C_2)U}, \tag{8.1}$$

$$k_{ab} = \frac{\Phi_1 + \Phi_2}{4\pi(C_1 + C_2)U}, \tag{8.2}$$

$$k_{ba} = \frac{\Phi_1}{4\pi C_1 U}, \tag{8.3}$$

$$k_{bb} = \frac{\Phi_1 + \Phi_2}{4\pi C_1 U}. \tag{8.4}$$

In the case $\Phi_2/\Phi_1 \leqslant C_2/C_1$, we have $k_{ab} \leqslant k_{ba}$, and

$$G = \begin{cases} 0 & \text{for} \quad k \leqslant k_{aa} \\ 4\pi(C_1 + C_2)Uk - \Phi_1 & \text{for} \quad k_{aa} \leqslant k \leqslant k_{ab} \\ \Phi_2 & \text{for} \quad k_{ab} \leqslant k \leqslant k_{ba} \\ \Phi_1 + \Phi_2 - 4\pi C_1 U k & \text{for} \quad k_{ba} \leqslant k \leqslant k_{bb} \\ 0 & \text{for} \quad k_{bb} \leqslant k \end{cases} \tag{8.5}$$

In the case $C_2/C_1 \leqslant \Phi_2/\Phi_1$, we have $k_{ba} \leqslant k_{ab}$, and

$$G = \begin{cases} 0 & \text{for} \quad k \leqslant k_{aa} \\ 4\pi(C_1 + C_2)Uk - \Phi_1 & \text{for} \quad k_{aa} \leqslant k \leqslant k_{ba} \\ 4\pi C_2 U k & \text{for} \quad k_{ba} \leqslant k \leqslant k_{ab} \\ \Phi_1 + \Phi_2 - 4\pi C_1 U k & \text{for} \quad k_{ab} \leqslant k \leqslant k_{bb} \\ 0 & \text{for} \quad k_{bb} \leqslant k \end{cases} \tag{8.6}$$

To determine the spectrum the theorem of the mean integral value is applied to the counter equation (3.3). Since in both expressions (8.5) and (8.6) integration of the G-function yields the same result, namely

$$\int_0^{\infty} G\, dk = \frac{C_2\Phi_2(\Phi_1 + \Phi_2/2)}{4\pi C_1(C_1 + C_2)U}, \tag{8.7}$$

we obtain

$$\varrho(\tilde{k}) = \frac{4\pi C_1(C_1+C_2)U}{C_2\Phi_2(\Phi_1+\Phi_2/2)}\, I. \tag{8.8}$$

$\tilde{k}$ lies in the interval defined by the mean mobility

$$\bar{k} = \frac{(C_1+C_2)(\Phi_1+\Phi_2)-C_1\Phi_1}{8\pi C_1(C_1+C_2)U} \tag{8.9}$$

and the relative half-width

$$\delta_k = \frac{(C_1+C_2)(\Phi_1+\Phi_2)-C_1\Phi_1}{(C_1+C_2)(\Phi_1+\Phi_2)+C_1\Phi_1}. \tag{8.10}$$

Introducing the notation

$$\gamma_C = \frac{2C_1+C_2}{C_2}, \tag{8.11}$$

$$\gamma_\Phi = \frac{2\Phi_1+\Phi_2}{\Phi_2}, \tag{8.12}$$

the above expressions can be written more clearly in the form

$$\varrho(\tilde{k}) = \frac{\gamma_C+\gamma_\Phi}{2(2\Phi_1+\Phi_2)\bar{k}\delta_k}\, I = \frac{1+\gamma_C\gamma_\Phi}{2(2\Phi_1+\Phi_2)\bar{k}}\, I, \tag{8.13}$$

$$\bar{k} = \frac{\Phi_1}{4\pi(C_1+C_2)U(1+\delta_k)}, \tag{8.14}$$

$$\delta_k = \frac{\gamma_C+\gamma_\Phi}{1+\gamma_C\gamma_\Phi}. \tag{8.15}$$

From expression (8.13) it follows that for certain values of the mean limiting mobility and mean air-flow rate $\Phi_1+\Phi_2/2$ the current I depends on the counter parameters only via the product $\gamma_C\gamma_\Phi$. It is readily seen that when the quantity $\gamma_C\gamma_\Phi$ is not varied, the parameter δ_k has a minimum if $\gamma_C=\gamma_\Phi$. Consequently, the condition of best resolution for constant current is

$$\frac{\Phi_2}{\Phi_1} = \frac{C_2}{C_1}. \tag{8.16}$$

This expression was derived in a less strict way in the work /Tammet, 1960/.

The condition (8.16) is close to that of optimum operation, but it does not exactly coincide with the latter. The reason is that the counter sensitivity, apart from depending on the ratio $I/\varrho(k)$, depends also on the capacitance C_2. For optimum operation conditions, $C_2/C_1 < \Phi_2/\Phi_1$.

Besides the above application of the differential counter of the second order, the possibility of measuring the partial charge density or partial conductivity is of interest. This is borne out by the occurrence of the

segment $G=\Phi_2$ or $G=4\pi C_2Uk$ in the mobility interval between k_{ab} and k_{ba}. Choosing the counter parameters in such a way that $\Phi_2/\Phi_1 \ll C_2/C_1$ or $C_2/C_1 \ll \Phi_2/\Phi_1$, one can obtain the G-function in the form shown in Figure 8.1. We shall divide the integral in equation (3.3) into terms corresponding to the linear intervals of the G-function. Most important is the integral in the interval (k_{ab}, k_{ba}), which directly gives the partial charge density or the partial conductivity. To reduce the two remaining integrals to the form of partial charge density or partial conductivity, we employ the theorem according to which in the interval (a, b) there always exists a ξ, such that

$$\int_a^b f(t)\,dt = \int_a^\xi g(t)\,dt \tag{8.17}$$

only if in the whole interval the condition

$$0 \leqslant \frac{f(t)}{g(t)} \leqslant 1 \tag{8.18}$$

is satisfied.

In the case $\Phi_2/\Phi_1 < C_2/C_1$ we obtain

$$\varrho(\tilde{k}_1, \tilde{k}_2) = \frac{I}{\Phi_2}, \tag{8.19}$$

where $\tilde{k}_1$ and $\tilde{k}_2$ lie in intervals with mean mobilities

$$\bar{k}_1 = \frac{\Phi_1+\Phi_2/2}{4\pi(C_1+C_2)U}, \tag{8.20}$$

$$\bar{k}_2 = \frac{\Phi_1+\Phi_2/2}{4\pi C_1 U}. \tag{8.21}$$

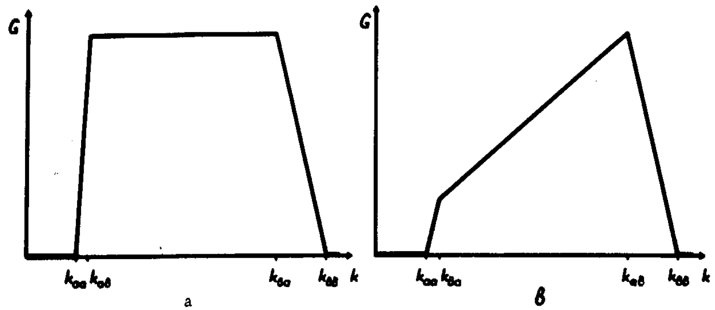

FIGURE 8.1. G-function of the second-order differential counter: a—when $\Phi_2/\Phi_1 \ll C_2/C_1$; b—when $C_2/C_1 \ll \Phi_2/\Phi_1$.

The relative half-width of the $(\tilde{k}_1, \tilde{k}_2)$ interval is expressed by (7.19).

In the case $C_2/C_1 < \Phi_2/\Phi_1$, we obtain

$$\lambda(\tilde{k}_1, \tilde{k}_2) = \frac{I}{4\pi C_2 U}, \tag{8.22}$$

where $\bar{k}_1$ and $\bar{k}_2$ lie in intervals, the mean mobilities of which are given by

$$\bar{k}_1 = \frac{(C_1 + C_2/2)\Phi_1}{4\pi C_1(C_1 + C_2)U}, \tag{8.23}$$

$$\bar{k}_2 = \frac{(C_1 + C_2/2)(\Phi_1 + \Phi_2)}{4\pi C_1(C_1 + C_2)U}. \tag{8.24}$$

The relative half-width of the interval is given by formula (7.11).

The advantage of this new method for measuring $\varrho(k_1, k_2)$ and $\lambda(k_1, k_2)$ over other variants of the aspiration method is that it suffices to measure directly only one value of the current through the collector plate in order to obtain one value of the desired parameter. It should be mentioned that in the integral method the measuring of four and in the first-order differential method the measuring of two current values is required to obtain the same result.

§9. COUNTERS WITH SEVERAL COLLECTOR PLATES

In the following we consider counters in which the current through several plates is measured. Counters with reversible plates, for example, that are described in the work /Ortner, El Nadi, 1955/ do not belong to this group. There are many possible designs of a measuring capacitor with several collector plates. A general treatment of the problem is very involved and beyond the scope of this book. We shall therefore confine ourselves to some practical examples.

The simplest counter with two collector plates is an integral counter measuring the current through both plates. It enables the simultaneous recording of air ions of both polarities, but does not present any new possibilities of measurement.

Consider the behavior of air ions in the measuring capacitor under more general conditions. Suppose the capacitor consists of many plates to which different voltages are applied, and consider the expression for the total current through some group of plates, which we shall call the collector group. This expression is similar to that for the current in an integral counter. However, two conditions must be satisfied.

1. Air ions of one polarity only settle on the collector group and if the conductivity is sufficiently large, all air ions of this polarity will settle.

2. The boundary surface of the air ions that are being collected (the current of which is unsaturated) should separate the collector group from all the other plates, so that the electric flux through the boundary surface will correspond to the sum of the charges on the plates of the collector group.

These conditions will be called the normal conditions.

The limiting mobility of the collector group is given by

$$k_0 = -\frac{\Phi}{4\pi \sum_{n=1}^{m} Q_n}, \tag{9.1}$$

where Q_n corresponds to the charges on the plates of the collector-group plates. This is defined by the sums $\sum_l C_{n,l} U_{n,l}$, where $C_{n,l}$ and $U_{n,l}$ are respectively the mutual capacitance and the voltage between the plates with indices n and l.

The question whether the normal conditions are satisfied is complex and can be answered only in special cases.

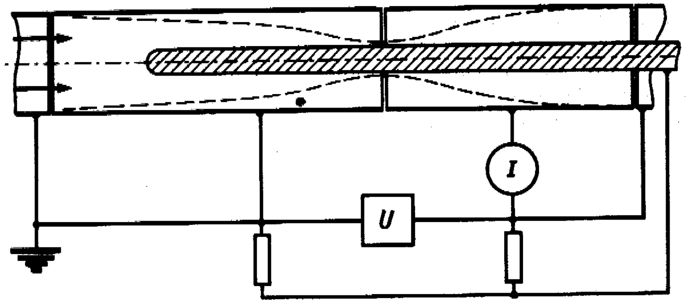

FIGURE 9.1. Example of a measuring capacitor in which the normal conditions are not satisfied.

------ boundary surface for a certain mobility.

An example of a measuring capacitor in which the normal conditions are not satisfied is shown in Figure 9.1. The normal conditions are fully satisfied in the trivial case in which the potentials of all plates are equal to one another. This occurs in the differential counter with an electrode divided into several collecting plates /Israël, 1931/.

The current through the first plate of this counter is calculated in the same way as in the case of an integral counter, and the current through the remaining plates as in the case of a first-order differential counter with a divided electrode. Suppose the current is measured through the plate with index $l \neq 1$ and the plate with index $m > l$ in a capacitor designed such that $C_m = C_l$. The currents I_l and I_m are expressed by the integral (3.3) and G by equations (7.1)–(7.3), where $C_2 = C_l = C_m$ and $C_1 = \sum_{n=1}^{l-1} C_n$ for G_l and $C_1 = \sum_{n=1}^{m-1} C_n$ for G_m. The difference $I_l - I_m$ is also expressed by the integral (3.3), where $G = G_l - G_m$. The expression

$$G = \begin{cases} 0 & \text{for} \quad k \leqslant k_{ma} \\ 4\pi \sum_{n=1}^{m} C_n U k - \Phi & \text{for} \quad k_{ma} \leqslant k \leqslant k_{mb} \\ 4\pi C_l U k & \text{for} \quad k_{mb} \leqslant k \leqslant k_{la} \\ \Phi - 4\pi \sum_{n=1}^{l+1} C_n U k & \text{for} \quad k_{la} \leqslant k \leqslant k_{lb} \\ 0 & \text{for} \quad k_{lb} \leqslant k, \end{cases} \tag{9.2}$$

where k_{ma}, k_{mb}, k_{la} and k_{lb} are the limiting mobilities corresponding to the capacitances $\sum_{n=1}^{m} C_n$, $\sum_{n=1}^{m-1} C_n$, $\sum_{n=1}^{l} C_n$ and $\sum_{n=1}^{l-1} C_n$. The G-function (9.2) has the same form as the G-function of the second-order differential counter (9.6). We can therefore apply the same method of calculation, which yields

$$\lambda(\bar{k}_1, \bar{k}_2) = \frac{I_l - I_m}{4\pi C_l U}, \tag{9.3}$$

where $\bar{k}_1$ and $\bar{k}_2$ lie in the intervals with mean velocities

$$\bar{k}_1 = \frac{\Phi\left(\sum\limits_{n=1}^{m-1} C_n + C_l/2\right)}{4\pi U \sum\limits_{n=1}^{m-1} C_n \sum\limits_{n=1}^{m} C_n}, \tag{9.4}$$

$$\bar{k}_2 = \frac{\Phi\left(\sum\limits_{n=1}^{l-1} C_n + C_l/2\right)}{4\pi U \sum\limits_{n=1}^{l-1} C_n \sum\limits_{n=1}^{l} C_n}. \tag{9.5}$$

The relative half-widths of the intervals are respectively

$$\delta_{k_1} = \frac{C_l}{2\sum\limits_{n=1}^{m-1} C_n + C_l}, \tag{9.6}$$

$$\delta_{k_2} = \frac{C_l}{2\sum\limits_{n=1}^{l-1} C_n + C_l}. \tag{9.7}$$

When studying the spectrum $\varrho(k)$ it is advisable to measure the currents through adjacent plates, the capacities of which are equal to one another. Assuming $m = l+1$, we can arrive at the results obtained above. Similarly, as in the case of the second-order differential counter, we obtain

$$\varrho(\bar{k}) = \frac{4\pi U \sum\limits_{n=1}^{l-1} C_n \sum\limits_{n=1}^{l} C_n \sum\limits_{n=1}^{l+1} C_n}{\Phi^2 C_l^2}\,(I_l - I_{l+1}). \tag{9.8}$$

$\bar{k}$ lies in the interval with mean mobility

$$\bar{k} = \frac{\Phi \sum\limits_{n=1}^{l} C_n}{4\pi U \sum\limits_{n=1}^{l-1} C_n \sum\limits_{n=1}^{l+1} C_n} \approx \frac{\Phi}{4\pi U \sum\limits_{n=1}^{l} C_n} \tag{9.9}$$

and relative half-width

$$\delta_k = \frac{C_l}{\sum\limits_{n=1}^{l} C_n}. \tag{9.10}$$

In formulas (9.3) and (9.8) the difference between the currents through both plates appears.

Therefore, in the case of the above-described counter, it is useful to employ one differential electrometer instead of two electrometers /Imyanitov, 1952; Zachek, 1964/. This enables us to obtain directly a count which is proportional to the desired quantity.

An interesting new counter version with two collector plates was recently proposed /Imyanitov, 1963; Imyanitov, Pavlyuchenkov, 1964; Schmeer, 1966/ for integral measurements.

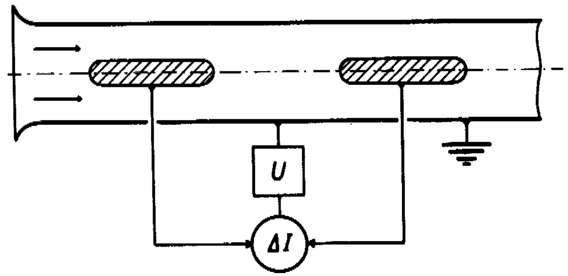

FIGURE 9.2. The counter of Imyanitov. ΔI is the differential electrometer.

A schematic diagram of the counter is given in Figure 9.2. The measuring capacitor consists of two collector plates with equal capacitances and equal voltages U. The difference $\Delta I = I_1 - I_2$ of the currents through the first and second collector plate is measured with the aid of the differential electrometer. It is readily shown that for ΔI the function G_Δ has the form

$$G_\Delta = \begin{cases} 0 & \text{for} \quad k \leqslant k_0/2 \\ (2k/k_0 - 1)\Phi & \text{for} \quad k_0/2 \leqslant k \leqslant k_0 \\ \Phi & \text{for} \quad k_0 \leqslant k \end{cases} \tag{9.11}$$

where k_0 is calculated from the active capacitance C of one inner plate. The trend of the function G_Δ is shown in Figure 9.3.

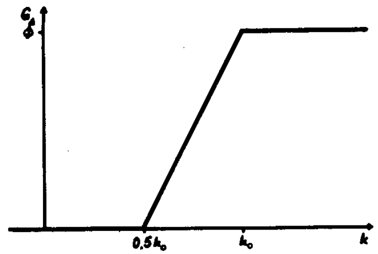

FIGURE 9.3. The G-function for the counter of Imyanitov.

Starting with expression (9.11) we can readily derive a formula useful for practical calculations:

$$\varrho[(0.75 \pm 0.25)k_0, \infty] = \frac{\Delta I}{\Phi}. \qquad (9.12)$$

To obtain one value of $\varrho(\bar{k}, \infty)$, one count will suffice.

The differential counter of Misaki /Misaki, 1950/ may also be considered as a counter with several plates.

Let us finally consider a problem indirectly related to the present section. It appears that the normal conditions are fulfilled for the integral counter shown in Figure 9.4. The special feature of this counter is that the active capacitance of the measuring capacitor is accurately given by the formula for the capacitance of an ideal cylindrical capacitor, in spite of the fact that the electric field is of a rather complex form. Charges induced by the voltage between the forward plate and the collector plate mutually compensate one another. The normal conditions are satisfied in all capacitor designs in which some symmetrical component is connected in front of the collector plate to the outer plate. This enables the use of an annular entrance opening which is of interest in counters with a divided air flow as well as in certain specific designs of the integral counter /Reinet, 1958, 1959a/.

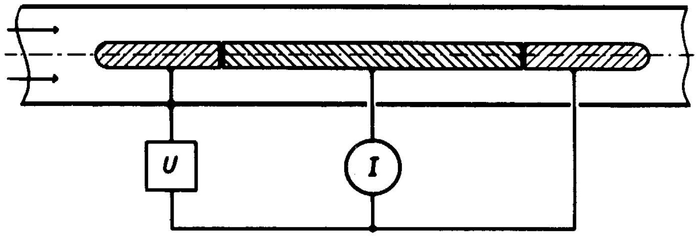

FIGURE 9.4. Measuring capacitor, the active capacitance of which can be accurately calculated from the geometrical length of the collector plate.

§10. COUNTERS WITH INTERPLATE AIR MIXING

Counters are known in which provision is made for mixing the air flow between the two inner plates of the measuring capacitor. This presents new measuring possibilities, as proposed in the works /Israël, 1949; Israël, Schultz, 1933; Gerasimova, 1941b/.

Interplate mixing of the air flow is effected by an arrangement in which the flow is first rendered turbulent and then laminar again. The mixing is facilitated by dividing the current into two branches /Israël, 1931/. Complete mixing is obtained when two measuring capacitors having plates are connected in series and separated by a grid-like screen.

Mixing of the air is unavoidably accompanied by a loss of air ions due to increased adsorption. Distortions may also arise in the case of incomplete mixing. These phenomena, which are neglected in the present chapter,

considerably reduce the practical importance of methods in which interplate air mixing is employed.

Consider a measuring capacitor with two inner plates, between which the air flow is mixed. The density spectrum of the flow of air ions passing through the first part of the capacitor is defined as the difference of the density spectrum of the incoming flow and the density spectrum of the flow of settling air ions.

$$I'(k) = (\Phi - G_a - G_b)\varrho(k), \qquad (10.1)$$

where G_a and G_b are respectively the G-functions for the inner and outer plate of the first part of the capacitor. After mixing the spectrum is given by

$$\varrho'(k) = \frac{I'(k)}{\Phi} = \left(1 - \frac{G_a + G_b}{\Phi}\right)\varrho(k). \qquad (10.2)$$

The second part of the measuring capacitor corresponds to a standard integral counter. The function G_2 for the current through the second inner plate is calculated by multiplying the G-function of a standard integral counter by the multiplicative factor of formula (10.2). Denoting the limiting mobility of the first part of the capacitor by k_1 and the limiting mobility of the second part by k_2, we obtain for the condition $k_1 \leqslant k_2$

$$G_2 = \begin{cases} \Phi \frac{k}{k_2}\left(1 - \frac{k}{k_1}\right) & \text{for } k \leqslant k_1 \\ 0 & \text{for } k_1 \leqslant k. \end{cases} \qquad (10.3)$$

For the condition $k_2 \leqslant k_1$

$$G_2 = \begin{cases} \Phi \frac{k}{k_2}\left(1 - \frac{k}{k_1}\right) & \text{for } k \leqslant k_2 \\ \Phi\left(1 - \frac{k}{k_1}\right) & \text{for } k_2 \leqslant k \leqslant k_1 \\ 0 & \text{for } k_1 \leqslant k. \end{cases} \qquad (10.4)$$

In the latter case we have

$$h_{U_2} G_2 = \begin{cases} 0 & \text{for } k < k_2 \\ \Phi\left(1 - \frac{k}{k_1}\right) & \text{for } k_2 < k \leqslant k_1 \\ 0 & \text{for } k_1 \leqslant k. \end{cases} \qquad (10.5)$$

Applying the theorem of the mean integral value we can derive

$$\varrho(\tilde{k}) = \frac{2k_1}{\Phi(k_1 - k_2)^2} h_{U_2} I_2, \qquad (10.6)$$

where $\tilde{k}$ lies in the interval with mean mobility

$$\bar{k} = \frac{k_1 + k_2}{2} \qquad (10.7)$$

and relative half-width

$$\delta_k = \frac{k_1 - k_2}{k_1 + k_2}. \qquad (10.8)$$

It should be pointed out that in the described method the condition $k_2 < k_1$ must be satisfied.

When the air flow is divided into two branches (after it has passed through the first part of the measuring capacitor), the latter are led through two respective counters with different limiting mobilities.

This enables us to determine $\varrho(k)$ from two simultaneous measurements. A detailed description of this method may be found in the works /Israël, 1931; Israël, Shultz, 1933; Israël, 1957b/.

A comparison of the above method with the usual differential methods does not reveal any advantages. On the contrary, the method of interplate air mixing is undoubtedly less accurate and less sensitive, which can be easily seen from formulas (10.6) and (7.8).

Therefore, the above method can hardly be expected to be of any practical use.

Interplate air mixing can also be effected in a standard integral counter with a precondenser. If the second capacitor operates under saturation conditions, no deviation from the usual operating conditions will occur. In the opposite case certain complications arise. A detailed treatment of this problem may be found in the works /Israël, 1931, 1957b/. Compared with the usual method of the precondenser, the method of interplate air mixing offers no advantages.

Gerasimova /Gerasimova, 1941b/ proposed a method which on first sight seemed very promising. Here the partial charge density $\varrho(0, \bar{k})$ of heavy and super-heavy ions is measured, whereby an integral counter is employed in conjunction with a precondenser and interplate air mixing. The measuring capacitors can have a relatively high limiting mobility and a simple design.

The method of Gerasimova requires that the condition $k_1 \leqslant k_2$ be satisfied. The current through the inner plate is expressed by equation (3.3), where the G-function has the form given by (10.3). Differentiation of this equation yields

$$\frac{\partial I_2}{\partial U_1} = -\frac{\Phi}{k_1 k_2 U_1}\int_0^{k_1} k^2 \varrho(k)\,dk, \tag{10.9}$$

$$h_{U_1} I_2 = \frac{\Phi}{k_2}\int_0^{k_1} k\,\varrho(k)\,dk. \tag{10.10}$$

Let us now introduce the mean square mobility of the air ions and write the integrals in the form

$$\int_0^{k_1} k^2\varrho(k)\,dk = \overline{k^2}(0,\ k_1)\,\varrho(0,\ k_1), \tag{10.11}$$

$$\int_0^{k_1} k\,\varrho(k)\,dk = \bar{k}(0,\ k_1)\,\varrho(0,\ k_1). \tag{10.12}$$

Forming the ratio $(h_{U_1} I_2)^2/\frac{\partial I_2}{\partial U_1}$, we obtain

$$\varrho(0,\ k_1) = \frac{C_1}{\Phi\, C_2 U_2}\,\frac{\overline{k^2}(0,\ k_1)}{[\bar{k}(0,\ k_1)]^2}\,\frac{(h_{U_1} I_2)^2}{\partial I_2/\partial U_1}. \tag{10.13}$$

The expression $\overline{k^2}(0,\ k_1)/[\bar{k}(0,\ k_1)]^2$ cannot be determined from the measurement results, which is a serious drawback of the method. Formula (10.13) can be applied only in the case when the parameter $\overline{k^2}(0,\ \bar{k}_1)/[\bar{k}(0,\ k_1)]^2$ can be estimated on the basis of previously known results. Gerasimova /Gerasimova, 1941b/ assumed $\overline{k^2}(0, \bar{k}_1) = [\bar{k}(0, k)]^2$, which leads to erroneous conclusions.

§11. MEASUREMENT OF THE SPACE CHARGE DENSITY

Many methods based on various principles are available for measuring the charge density. Those not related to aspiration methods are the Thomson method /Thomson, W., 1882, Daunderer, 1909/, the three-collector method /Daunderer, 1907/ and the Imyanitov method (related to the Thomson method) /Imyanitov, 1951; In'kov, 1958, 1965; Kitaev, 1962/, and finally the Mühleisen-Holl method /Mühleisen, Holl, 1952/ and the indirect method of calculating the charge density from the spatial distribution of the electric field and the conductivity /Hansen, 1935; Mecklenburg, Lautner, 1940/. A survey of the various methods can be found in the book /Israël, 1961/.

The aspiration device for measuring the charge density should satisfy the condition $G = \Phi$ for any mobility. Since this is impossible, we must be satisfied with the approximate fulfillment of this condition in the practical range of mobilities. In the integral counter the condition $G = \Phi$ can be fulfilled for mobilities of one polarity only. The limiting mobility should be less than the mobility of heaviest ions still contributing to the charge density. When measuring charge densities, the polar densities are recorded separately and then added. The accuracy of this method is not very high and the relative error often exceeds the relative errors of the initial measurements. Nevertheless, this method is sometimes applied in practice /Gockel, 1917; Reinet, 1958/. The simultaneous counting of air ions of both polarities can be effected by means of devices with a filter, through which the air is drawn. Here the current which flows through the filter to the ground is measured.

The operation of a fiber filter is based on air-ion adsorption. The application of the method was first described in the work /Zeleny, 1898a/. This method is considered to be one of the best methods for measuring the charge density /Mühleisen, 1957a/. The measuring filter of the device is filled with cotton wool /Becker, 1910; Obolensky, 1925/, with glass wool /Zeleny, 1898a; McClelland, 1898/, with metal shavings /Aselmann, 1906; Brown, 1930/, or with metallic grids /Krasnogorskaya, Seredkin, 1964/.

The theory of the fiber-filter method /Fuks, Stechnika, 1962/ has been little studied. To verify the effectiveness of the filter the air flow is drawn through two filters arranged in series. The filter is considered effective if the current through the second filter is sufficiently small compared to the current through the first filter. The filter in question can be connected in series with the standard integral counter /Paltridge, 1967/. The charge density is given by an elementary formula

$$\varrho = \frac{I}{\Phi}. \tag{11.1}$$

The disadvantage of fiber filters lies in their relatively large resistance to the air flow. Accordingly, the electrostatic filter suggested by Gunn /Gunn, 1953/ is more efficient. The Gunn method is a combination of the integral-counter method and the filter method. This device is schematically shown in Figure 11.1. The G-function for such a filter has the form shown in Figure 11.2. For a sufficiently small limiting mobility the approximation $G = \Phi$ is permissible and does not depend on the mobility.

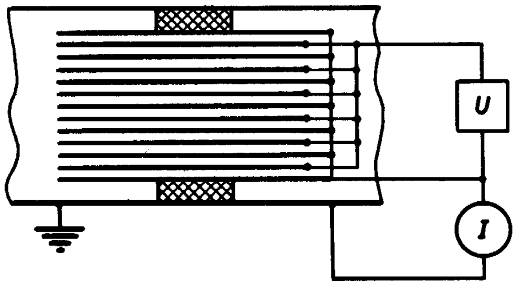

FIGURE 11.1. Gunn filter.

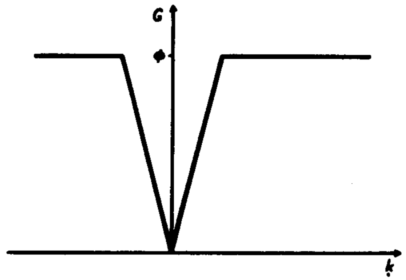

FIGURE 11.2. The G-function for the Gunn filter.

The Gunn filter usually consists of plane-parallel plates. The maximum allowable voltage between the plates should be chosen on the basis of reliability considerations. Since breakdown of the capacitor occurs at a certain value of the electric field strength E, the latter should be chosen before designing the filter. The formula for the limiting mobility (4.3) now becomes

$$k_0 = \frac{\Phi d}{EV}, \tag{11.2}$$

where d is the distance between the plates, and V is the filter volume.

§12. MODULATED COUNTERS

In the usual aspiration counters one measures a fairly weak direct current through the collector plate. Amplification and recording of such a weak direct current constitutes a difficult technical problem. For the measurement of the very weak direct current in more recent instruments it is preferable to transform the incoming signal into an alternating one, since amplification of an alternating current is much simpler.

The possibility of transforming the output signal of the measuring capacitor by introducing certain changes in the counter design was proposed quite recently /Junod, Sänger, Thams, 1962/. Apart from the outlined advantage, this method opens the way to some other new possibilities of major importance. In modulated counters it appears possible to suppress various distorting effects which often cause serious trouble in usual counters. This problem will be discussed in the following chapters.

The main advantage of the modulated counters lies in the possibility of differentiating by means of modulation techniques. This was the main reason underlying the development of modulated measuring capacitors /Junod, Sänger, Thams, 1962/.

Modulated counters record the amplitude of the alternating component of the current through the collector plate.

The modulation frequency is very low (of the order of several Hz or lower) so that the use of amplifiers is required. For simplicity, we assume that the modulation period is sufficiently large compared to the time constant of the current generated by the air ions flowing toward the collector plate of the capacitor. This enables us to forego allowance for transient processes which considerably complicate the calculation procedure.

The amplitude of the alternating current through the collector plate is given by

$$I_{\sim} = \frac{1}{2}(I_{max} - I_{min}). \tag{12.1}$$

$I_{\sim}$ can be correlated with the spectrum $\varrho(k)$ via an integral transformation similar to formula (3.3). The kernel of the integral will be denoted by $G_{\sim}$.

Let us briefly consider the specific possibilities of counters with modulated measuring and modulated precondenser. For simplicity, we assume the modulating signal is rectangular everywhere. Rectangular modulation is in most cases the most appropriate.

1. Modulation by the means of the gas flow rate. Suppose the measuring capacitor is similar to the measuring capacitor of a usual integral counter. Denote the gas flow rate during the first half-period by Φ_1, and during the second half-period by Φ_2. Assume further that $\Phi_1 < \Phi_2$ and denote the respective limiting mobilities by k_1 and k_2. The function $G_{\sim}$ is then given by

$$2G_{\sim} = \begin{cases} 0 & \text{for} \quad k \leqslant k_1 \\ 4\pi CUk - \Phi_1 & \text{for} \quad k_1 \leqslant k \leqslant k_2 \\ \Phi_2 - \Phi_1 & \text{for} \quad k_2 \leqslant k. \end{cases} \tag{12.2}$$

This expression is similar to (7.13), which represents the G-function of a first-order differential counter with a divided air current. Analysis of formula (12.2) leads correspondingly to results obtained in § 7 and expressed by formulas (7.16) and (7.20).

It may be noted that for the case $\Phi_2 = 2\Phi_1$ the function G_- assumes the same form as the function G_- of the Imyanitov counter (9.11), so that the described counter possesses the same properties as this counter.

For the case $\Phi_1 = 0$ the function G_- corresponds to the function G of an ordinary integral counter.

Two antiphase-modulated measuring capacitors may be fed by one fan. The gas flow rate through the fan will then be constant.

A disadvantage of gas flow rate modulation is the tendency of pulsating flows to become turbulent and the necessity of mechanical commutator arrangements in gas flow rate modulation.

2. Modulation of the measuring capacitor voltage. Let us denote the voltage in successive half-periods by U_1 and U_2, respectively, and assume $U_1 < U_2$. In accordance with formula (4.3) $k_2 < k_1$. The function G_- then becomes

$$2G_- = \begin{cases} 4\pi C(U_2 - U_1)k & \text{for} \quad k \leqslant k_2 \\ \Phi - 4\pi C U_1 k & \text{for} \quad k_2 \leqslant k \leqslant k_1 \\ 0 & \text{for} \quad k_1 \leqslant k. \end{cases} \tag{12.3}$$

When $U_1 = 0$, expression (12.3) corresponds to the G-function of a usual integral counter. Junod, Sänger and Thams /1962/ developed a method of double differentiation by means of a complex modulating voltage. A typical modulating voltage ensuring double differentiation is shown in Figure 12.1. Here the difference between the amplitudes of the variable high-frequency component is recorded for different half-periods of the low-frequency component.

The double differentiation method is very sensitive to fluctuations in the relative charge density.

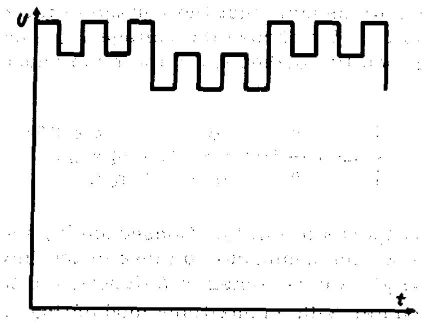

FIGURE 12.1. Modulating voltage signal ensuring double differentiation.

A serious disadvantage of this method is the necessity to accurately compensate the induced current which exceeds the measured current by several orders of magnitude. The compensation of the induced current is effected by a bridge circuit /Junod, Sänger, Thams, 1962/, which will be described in §31.

3. Counter with a modulated precondenser. The measuring capacitor is schematically shown in Figure 12.2. The intermediate shorted screening capacitor is not necessary in principle, and merely serves to decrease the total length and to prevent a widening of the air flow after it has passed through the precondenser. Widening of the flow may cause turbulence. The auxiliary *RC* circuit serves to separate the alternating and direct current.

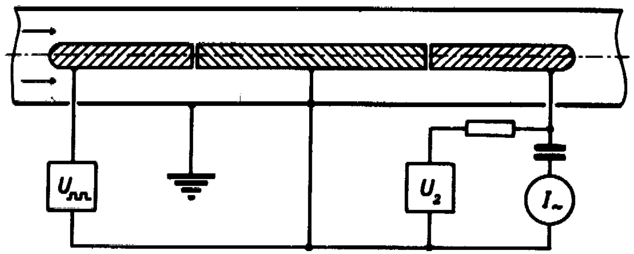

FIGURE 12.2. Counter with a modulated precondenser.

This counter has none of the shortcomings connected with the aerodynamic instability of the pulsating flow and induced current.

Counters employing a modulated precondenser offer many additional possibilities for work under various operating conditions. The function G_- is calculated as the difference of two expressions of the type (6.3). Since the general form of this function is of current interest, we shall consider only one example. Suppose the active capacitances of the preliminary and main capacitors are C_1 and C_2, respectively, and the voltage of the preliminary capacitor is 0 in the first half-period and U_1 in the second half-period. Further, we assume that the voltage of the main capacitor is constant and equals U_2. We choose the voltage U_1, such that $U_1C_1 = U_2C_2$. Denoting the limiting mobility of the ions in the main capacitor by k_0, we obtain

$$2G_- = \begin{cases} 0 & \text{for} \quad k \leqslant 0{,}5k_0 \\ (2k/k_0 - 1)\Phi & \text{for} \quad 0.5k_0 \leqslant k \leqslant k_0 \\ \Phi & \text{for} \quad k_0 \leqslant k. \end{cases} \tag{12.4}$$

This expression corresponds to (9.11). Consequently, the counter under consideration has properties analogous to those of the Imyanitov counter.

Choosing $U_1C_1 = -U_2C_2$, we can obtain a G-function of the type (4.4).

4. Differential counter with an auxiliary modulating capacitor. According to Figure 12.3 the auxiliary capacitor is located at the center of the first inner plate of the measuring capacitor. The auxiliary capacitor can be placed directly in front of the differential measuring capacitor but this would lead to complications because of the deviation from normal conditions and because of the appearance of an edge effect at the boundary between the auxiliary to the measuring capacitor.

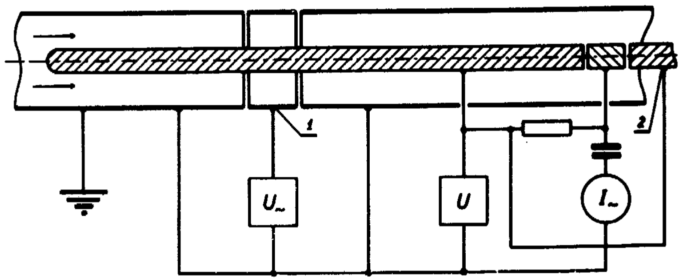

FIGURE 12.3. Differential counter with an auxiliary modulated capacitor:

1—modulated plate; 2—equipotential elongation of the inner plate intended to improve the aerodynamic properties of the measuring capacitor.

Suppose the mutual capacitance between the first inner and the entire outer plate (including the modulated plate) is C_1, the capacitance between the first inner and the modulated plate is $C_{\sim}$, the active capacitance of the second inner plate is C_2. Denoting the constant voltage by U and the amplitude of the alternating modulating voltage by $U_{\sim}$, we obtain the four limiting mobilities:

$$k_{a+} = \frac{\Phi}{4\pi[(C_1 + C_2)U + C_{\sim}U_{\sim}]}, \qquad (12.4)^*$$

$$k_{a-} = \frac{\Phi}{4\pi[(C_1 + C_2)U - C_{\sim}U_{\sim}]}, \qquad (12.5)$$

$$k_{b+} = \frac{\Phi}{4\pi(C_1U + C_{\sim}U_{\sim})}, \qquad (12.6)$$

$$k_{b-} = \frac{\Phi}{4\pi(C_1U - C_{\sim}U_{\sim})}. \qquad (12.7)$$

When $k_{a-} \leqslant k_{b+}$, the function $G_{\sim}$ is expressed by

$$2G_{\sim} = \begin{cases} 0 & \text{for } k \leqslant k_{a+} \\ 4\pi[(C_1 + C_2)U + C_{\sim}U_{\sim}]k - \Phi & \text{for } k_{a+} \leqslant k \leqslant k_{a-} \\ 8\pi C_{\sim}U_{\sim}k & \text{for } k_{a-} \leqslant k \leqslant k_{b+} \\ \Phi - 4\pi(C_1U - C_{\sim}U_{\sim})k & \text{for } k_{b+} \leqslant k \leqslant k_{b-} \\ 0 & \text{for } k_{b-} \leqslant k. \end{cases} \qquad (12.8)$$

In the case of strong modulation $k_{b+} \leqslant k_{a-}$, so that

$$2G_{\sim} = \begin{cases} 0 & \text{for } k \leqslant k_{a+} \\ 4\pi[(C_1 + C_2)U + C_{\sim}U_{\sim}]k - \Phi & \text{for } k_{a+} \leqslant k \leqslant k_{b+} \\ 4\pi C_2Uk & \text{for } k_{b+} \leqslant k \leqslant k_{a-} \\ \Phi - 4\pi(C_1U - C_{\sim}U_{\sim})k & \text{for } k_{a-} \leqslant k \leqslant k_{b-} \\ 0 & \text{for } k_{b-} \leqslant k. \end{cases} \qquad (12.9)$$

* [Both this and the previous formula are denoted as (12.4) in the Russian text.]

The two last expressions are similar to formula (8.6), which describes the second-order differential counter under conditions whereby $C_2/C_1 \leqslant \Phi_2/\Phi_1$. Without further calculations it may be stated that, in principle, the above method offers the same possibilities as the second-order differential method for the conditions stated above.

When measuring the spectrum, it is advisable to choose the ratio $U_{\sim}/U$ equal or slightly larger than the ratio $C_2/2C_{\sim}$.

The voltage of the modulated plate need not necessarily have only an alternating component, as the alternating component can itself be modulated by a lower frequency component.

In the above-described method modulation by differentiation replaces the technically rather inconvenient separation of the air flow.

The possibilities of the method of modulating the precondenser are not confined to the examples considered above. Since the method is relatively new, it is difficult to indicate the practical significance of the different variants. The general theoretical calculations pertaining to the different counter variants are standard, and we shall forego any further treatment of additional variants of the method.

The theory of modulation counters must be further refined because of the need to calculate or estimate the transient processes. This problem will not be considered in the present book.