PRODUCTIVITY OF A PLAIN ELECTRIC MOBILITY CLASSIFIER

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Terminological comments

AC = Aspiration Condenser = a device in which air is continuously flowing through transversal electric field separating particles according to their electric mobilities.

EMC = Electric Mobility Classifier = a preparative instrument classifying particles according to their electric mobilities. Usually, an AC is applied. The TOF (Time Of Flight) chamber is a well known alternative.

DMA = Differential Mobility Analyzer = an analytic instrument to measure the concentrations of particles in narrow mobility intervals. Assembled of an EMC and a particle detector.

A parallelism:

Prism / Grating	Monochromator	Spectrometer
AC / TOF chamber	EMC	DMA

P_{aN} = absolute number productivity = total number of outlet particles per second,

$$P_{rV} = \text{relative volume productivity} = \frac{\text{volume of particles produced per second}}{\text{volume of the AC active zone}} = \frac{\pi d^3}{6V} P_{aN} \,.$$

Why productivity?

EMC have no essential industrial applications producing monodisperse particles today. The reason is low productivity. Tick (in mind) the correct statement:

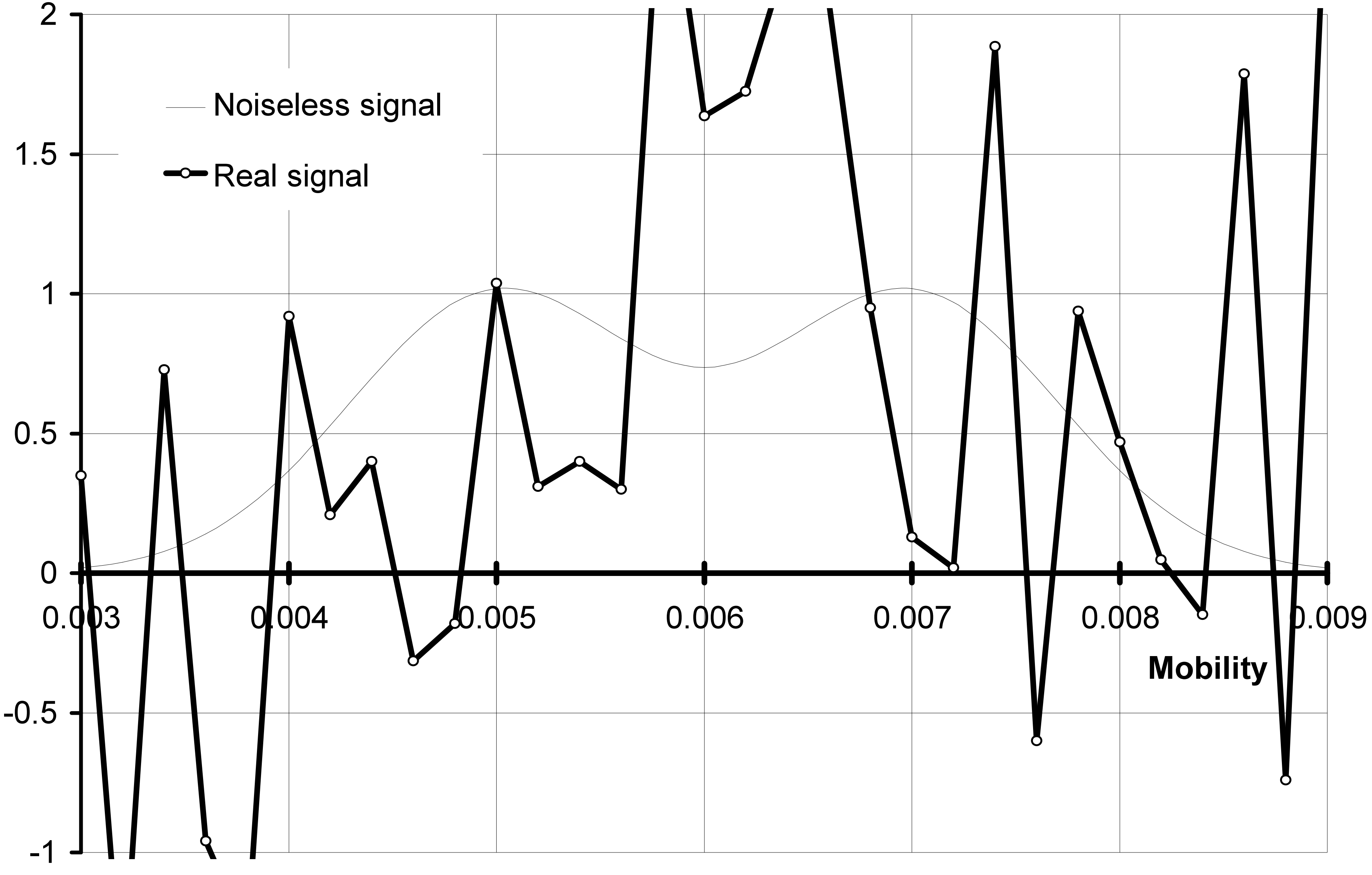
☐ the productivity of EMC will be dramatically increased as a result of a new technological invention unknown today,

☐ EMC will never reach industrial level of productivity due to the principal physical limitations.

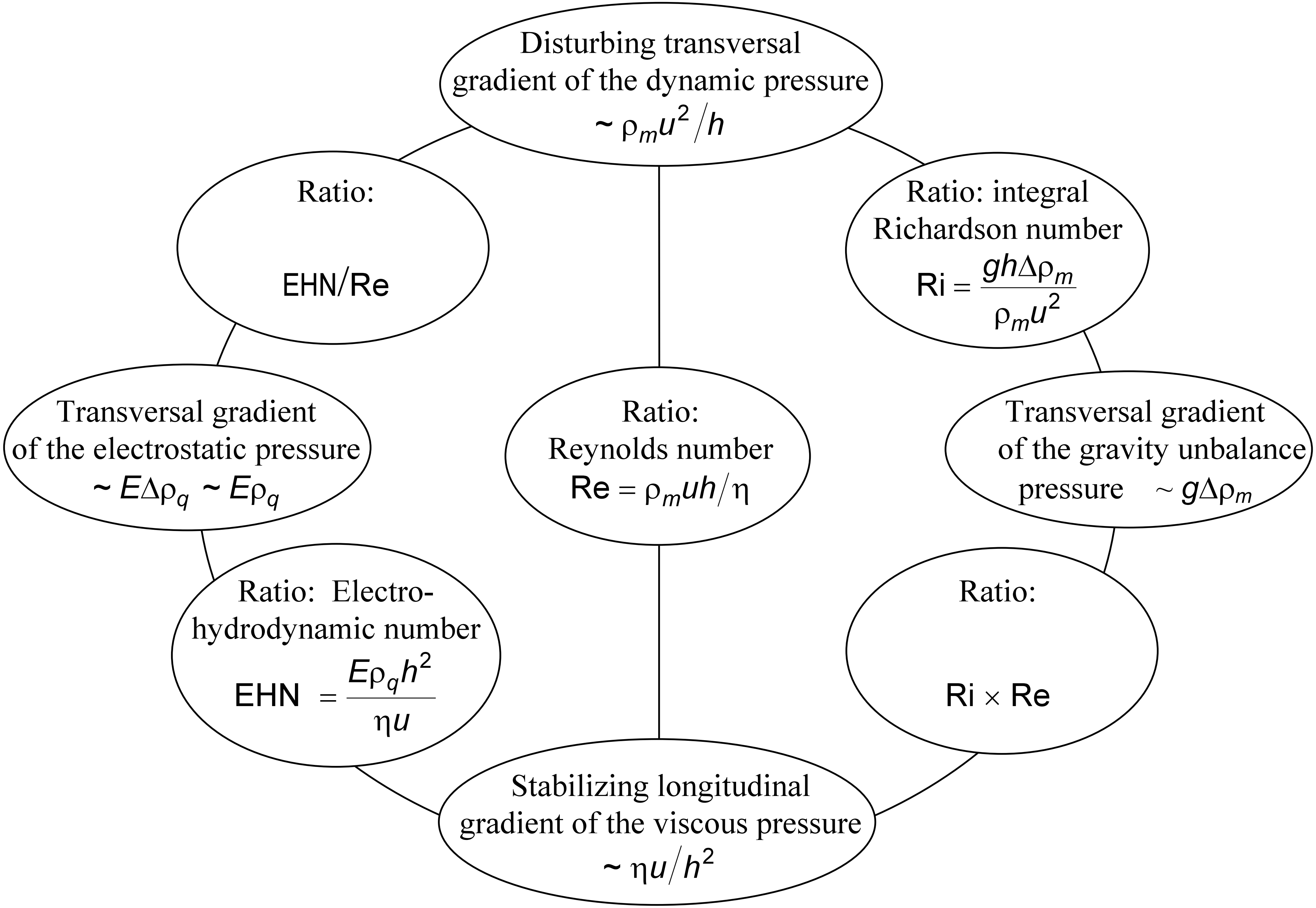
The main performance characteristic of an analytic DMA is the resolving power. Productivity as a characteristic of performance is not appreciated today.

Formal size resolution of an DMA is monotonously increasing with a decrease in the widths of the inlet and outlet slits of the differential AC, according to the commonly accepted theoretical models. The productivity and the signal of the particle detector are decreasing with a decrease in the widths of slits. When the noise level is reached, the real resolution is lost.

Conclusion: traditional criteria of size resolution are not satisfactory because of the neglecting of the productivity as a factor of DMA performance.



Signal of a DMA in case of very low productivity


Disturbing transversal gradient of the dynamic pressure $\sim \rho_m u^2 / h$
Ratio: EHN/Re
Ratio: integral Richardson number $\mathrm{Ri} = \frac{g h \Delta\rho_m}{\rho_m u^2}$
Transversal gradient of the electrostatic pressure $\sim E\Delta\rho_q \sim E\rho_q$
Ratio: Reynolds number $\mathrm{Re} = \rho_m u h / \eta$
Transversal gradient of the gravity unbalance pressure $\sim g\Delta\rho_m$
Ratio: Electro-hydrodynamic number $\mathrm{EHN} = \frac{E\rho_q h^2}{\eta u}$
Ratio: $\mathrm{Ri} \times \mathrm{Re}$
Stabilizing longitudinal gradient of the viscous pressure $\sim \eta u / h^2$

Symbols

SI *units accepted*

d – particle diameter,
$e = 1.6\times10^{-19}$ C
h – distance between AC electrodes,
l – length of AC,
p – pressure,
q – particle charge,
t – time,
t_o – time of passage through AC,
u – average linear velocity of air in AC,
x – longitudinal co-ordinate,
y – transversal co-ordinate,

B – particle mechanical mobility,
E – electric field,
N – particle number concentration,
P_{aN} – absolute number productivity,
P_{rV} – relative volume productivity,
V – active volume of AC, $V = \Phi t_o$,
Z – particle electric mobility $Z = qB$,
$\varepsilon_o = 8.85\times10^{-12}$ F m^{-1},
η – air viscosity (≈ 18 µPa s),
ρ_m – air mass density (≈ 1.2 kg m^{-3})
ρ_q – air charge density, $\rho_q = Nq$
Φ – air flow rate.

Mobility equation $Z = \dfrac{uh}{El} = \dfrac{\eta \mathrm{Re}}{\rho_m El}$ is assumed to be satisfied.

References

Peil, I. and E. Tamm, (1984) Generation of monodisperse aerosols by the electrostatic separation method (in Russian with an English abstract). *Acta et comm. univ. Tartu.,* 669, 44–52.

Tammet, H.F, (1970) *The Aspiration Method for the Determination of Atmospheric-ion Spectra*. IPST and NSF, Jerusalem.

DIVERGENCE LIMIT

Electrostatic dispersion of charged particles along a particle trajectory in AC is described in Appendix 2. A well known simple solution is available in case of monomobile particles:

$$\frac{\partial \rho_q}{\partial t} = -\frac{Z\rho_q^2}{\varepsilon_\circ} \Rightarrow \rho_q = \frac{\rho_{qo}}{1+\frac{Z\rho_{qo}t}{\varepsilon_\circ}}$$, where ρ_{qo} is ρ_q at $t = 0$.

When increasing the particle inlet $\rho_{qo} \to \infty$, the space charge density cannot exceed the value ${}^{\lim}\rho_q = \frac{\varepsilon_\circ}{Zt}$ setting the upper limit for concentration of outlet particles possible in an AC.

As $\rho_q = Nq$ *and* $ZEt = h$, the limit is ${}^{\lim D}N = \frac{\varepsilon_\circ E}{hq}$.

The total air flow Φ through an AC is divided into an outlet flow $\delta_{\text{out}}\Phi$ carrying the separated particles and a waste flow $(1-\delta_{\text{out}})\Phi$. Thus, the absolute number productivity of an EMC cannot exceed the limit

$${}^{\lim D}P_{aN} = \delta_{\text{out}}\Phi\frac{\varepsilon_\circ E}{hq} .$$

When calculating the relative volume productivity, the ratio $\frac{\Phi}{V}$ can be replaced with $\frac{EBq}{h}$.

The result is $$^{\lim D}P_{rV} = \delta_{\text{out}} \frac{\pi\varepsilon_\circ}{6} Bd^3 \frac{E^2}{h^2}.$$

The parameters of the AC are presented by a single control parameter E/h in the equation above. A conclusion: to keep the bound of the productivity high, the field E should be increased and the distance h should be decreased as much as possible. h is limited to a technological minimum $h_{\min}$. E is limited or to a technological maximum $E_{\max1} \approx 10$ kV/cm (considering local breakdowns) or to a value set by the mobility equation $E_{\max2} = \frac{\eta \mathrm{Re}_{\max}}{\rho_m Z l_{\min}}$, where $\mathrm{Re}_{\max}$ is the maximum Reynolds number and $l_{\min}$ is the technological minimum length of the AC. Thus, the absolute limit of the relative volume productivity is

$$^{\lim D}P_{rV} = \delta_{\text{out}} \frac{\pi\varepsilon_\circ}{6} Bd^3 \frac{E_{\text{opt}}^{\;2}}{h_{\min}^{\;2}}$$

where $E_{\text{opt}} = \min\,(E_{\max1}\,,\,E_{\max2})$.

Question: when $E_{\text{opt}} = E_{\max1}$? Answer: if $E_{\max1} < E_{\max2}$ or $Z < \frac{\eta \mathrm{Re}_{\max}}{\rho_m l_{\min} E_{\max 1}}$.

Example: if $\mathrm{Re}_{\max} = 1000$, $E_{\max} = 10$ kV/cm and $l_{\min} = 15$ mm, the critical mobility is

0.01 $cm^2V^{-1}s^{-1}$. Conclusion: both alternatives of the field boundary are important in the practice.

TURBULENCE LIMIT

Transversal electrostatic forces applied to the air flow in an AC can destroy the laminar flow and initiate turbulence (electric wind or electrostatic convection) as shown by Peil and Tamm (1984). Result: aerosol will be mixed and size resolution of the EMC will be lost when the electrostatic forces exceed a critical limit.

Reynolds number (a well known criterion) $\mathrm{Re} = \dfrac{\text{a measure of disturbing inertial forces}}{\text{a measure of stabilizing viscous forces}}$.

ElectroHydrodynamic Number $\mathrm{EHN} = \dfrac{\text{a measure of disturbing electrostatic forces}}{\text{a measure of stabilizing viscous forces}}$.

The measure of electrostatic forces is $\sim \dfrac{\mathrm{d}p}{\mathrm{d}y} \sim \rho_q E$, the measure of viscous forces is $\sim \dfrac{\mathrm{d}p}{\mathrm{d}x} \sim \dfrac{\eta u}{h^2}$.

Thus, $\mathrm{EHN} = \dfrac{\rho_q E h^2}{\eta u}$.

When $\mathrm{EHN} \ll \mathrm{EHN}_{\mathrm{cr}}$ the critical value of Re can be measured: $\mathrm{Re}_{\mathrm{cr}} \approx 1000$.
When $\mathrm{Re} \ll \mathrm{Re}_{\mathrm{cr}}$ the critical value of EHN can be measured: $\mathrm{EHN}_{\mathrm{cr}} \approx 2$.

The last very rough estimate is based on exclusive examples published by Peil and Tamm (1984).

A simplest approximate composite condition of laminar flow can be written:

$$\frac{\mathrm{Re}}{\mathrm{Re}_{cr}} + \frac{\mathrm{EHN}}{\mathrm{EHN}_{cr}} < 1 \qquad \Rightarrow \qquad \mathrm{EHN} < \mathrm{EHN}_{\max} = \left(1 - \frac{\mathrm{Re}}{\mathrm{Re}_{cr}}\right)\mathrm{EHN}_{cr}\ .$$

The particle number concentration $N = \rho_q / q$ is limited to $\frac{\eta u}{Eh^2 q}\mathrm{EHN}_{\max}$. Thus, the absolute number productivity of an EMC with laminar air flow cannot exceed the limit

$$^{\lim T}P_{aN} = \delta_{\text{out}}\Phi \frac{\eta u}{qEh^2}\mathrm{EHN}_{\max}\,.$$

It follows,

$$^{\lim T}P_{rV} = \delta_{\text{out}} \frac{\pi\eta^2}{6\rho_m} Bd^3 \frac{\mathrm{Re}_{\max} \times \mathrm{EHN}_{\max}}{h_{\min}{}^4}\,.$$

This boundary does not depend on E and q at all.

When decreasing the value of E, the length of the AC should be simultaneously increased.
A question: is there a danger, that the boundary above cannot be reached due to the technological restrictions to the length of the AC? The answer is not: the length can be kept below 1 m in any realistic situation.

WHAT A LIMIT IS LIMITING?

$$\frac{{}^{\lim T}P_{rV}}{{}^{\lim D}P_{rV}} = \frac{\eta^2}{\rho_m \varepsilon_\circ} \frac{\mathrm{Re}_{\mathrm{max}}\,\mathrm{EHN}_{\mathrm{max}}}{E_{\mathrm{opt}}^2 h_{\mathrm{min}}^2} = \left(\frac{h_{cr}}{h_{\mathrm{min}}}\right)^2, \text{ where } \; h_{cr} = \frac{\eta}{E_{\mathrm{opt}}} \sqrt{\frac{\mathrm{Re}_{\mathrm{max}}\,\mathrm{EHN}_{\mathrm{max}}}{\rho_m \varepsilon_\circ}} .$$

If $h_{\mathrm{cr}} < h_{\mathrm{min}}$ then the turbulence is limiting the EMC productivity, otherwise the divergence.

Variant $E_{\mathrm{opt}} = E_{\mathrm{max1}} \; \left(Z < \frac{\eta \mathrm{Re}_{\mathrm{max}}}{\rho_m l_{\mathrm{min}} E_{\mathrm{max\,1}}}\right)$:

$E_{\mathrm{max1}} = 10$ kV/cm & $\mathrm{Re}_{\mathrm{max}}\mathrm{EHN}_{\mathrm{max}} = 1000 \quad \Rightarrow \quad h_{\mathrm{cr}} \approx 0.18$ mm.

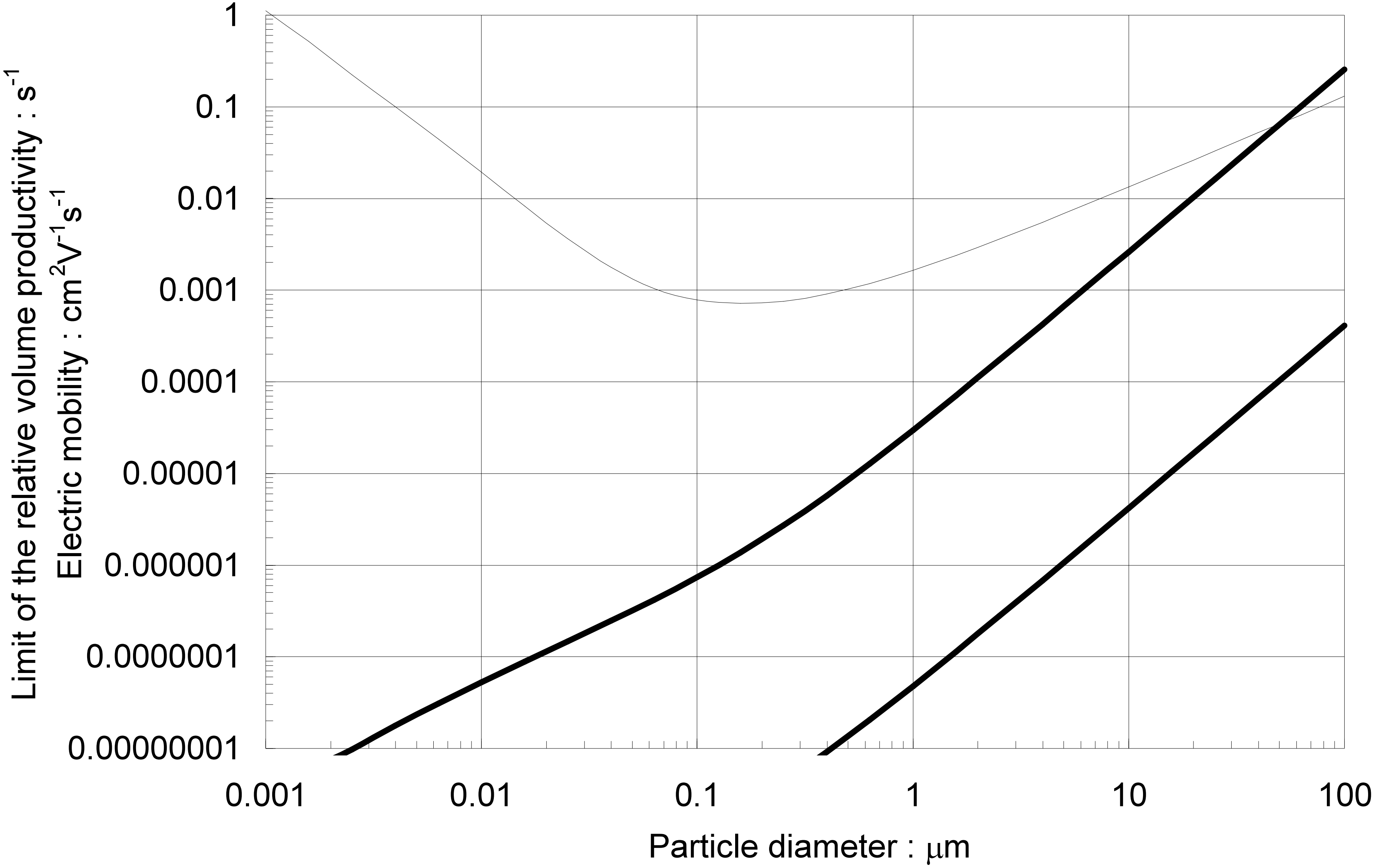
Conclusion: electrostatic turbulence is the limiting phenomenon in case of a low mobility.

Variant $E_{\mathrm{opt}} = E_{\mathrm{max2}} \; \left(Z > \frac{\eta \mathrm{Re}_{\mathrm{max}}}{\rho_m l_{\mathrm{min}} E_{\mathrm{max\,1}}}\right)$:

$$h_{cr} = Z l_{\mathrm{min}} \sqrt{\frac{\mathrm{EHN}_{\mathrm{max}} \rho_m}{\mathrm{Re}_{\mathrm{max}}\, \varepsilon_\circ}} , \qquad Z_{\mathrm{cr}} = Z_{(h_{\mathrm{cr}} = h_{\mathrm{min}})} = \frac{h_{\mathrm{min}}}{l_{\mathrm{min}}} \sqrt{\frac{\mathrm{Re}_{\mathrm{max}}\, \varepsilon_\circ}{\mathrm{EHN}_{\mathrm{max}} \rho_m}} .$$

Conclusion: if $Z < Z_{\mathrm{cr}}$ then the turbulence is the limiting phenomenon.
if $Z > Z_{\mathrm{cr}}$ then the divergence is the limiting phenomenon.

Estimate of Z_{cr} at $\mathrm{Re}_{\mathrm{max}} = 500$ & $\mathrm{EHN}_{\mathrm{max}} = 1$ is $Z_{\mathrm{cr}} = (h_{\mathrm{min}}/l_{\mathrm{min}}) \times 0.43\ \mathrm{cm}^2\mathrm{V}^{-1}\mathrm{s}^{-1}$. Realistic value of Z_{cr} is $0.1 \ldots 0.3\ \mathrm{cm}^2\mathrm{V}^{-1}\mathrm{s}^{-1}$. $Z > Z_{\mathrm{cr}}$ is possible only when classifying extra big particles charged in a strong electric field, or extra fine nanometer particles.



Black curves: Limit of the relative volume productivity ${}^{\lim T}P_{rV}$, $\delta_{\mathrm{out}}\mathrm{Re}_{\max}\mathrm{EHN}_{\max} = 500$, lower curve at $h = 10$ mm, and upper curve at $h = 2$ mm.

Red curve: Electric mobility at $n_{\mathrm{o}}t = 10^6\ \mathrm{cm}^{-3}\mathrm{s}$ and $E_{\mathrm{Pauthenier}} = 10$ kV/cm.