Size-mobility relation in the nanometer range

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1. The Millikan model

mean free path of gas molecules

the slip factor coefficients (a = 1.2, b = 0.5, c = 1)

$$K = e\frac{1+\dfrac{l}{\delta}\left[a+b\exp\left(-c\dfrac{\delta}{l}\right)\right]}{6\pi\eta\delta}$$

gas viscosity

collision distance

2. The Chapman-Enskog model

elementary charge

mass of gas molecule

mass of ion

$$K_{\text{free molecule}} = \frac{3e}{8n_\text{g}\Omega^{(1,1)}}\sqrt{\frac{\pi(1+m_\text{g}/m_i)}{2m_\text{g}kT}}$$

number concentration of molecules

first collision integral

Boltzmann constant

absolute temperature

Charged rigid spheres and (∞–4) potential: (δ = the collision distance).

$$\Omega^{(1,1)} = \Omega^{(1,1)*}\pi\delta^2,\ \ \Omega^{(1,1)*} = f(T^*),\ \ T^* = \frac{kT}{U(\delta)},\ \ U(\delta) = \begin{cases} \text{if } r<\delta \text{ then } \infty \\ \text{if } r>\delta \text{ then } \dfrac{-\alpha e^2}{8\pi\varepsilon_\circ r^4} \end{cases}$$

$$f(T^*) = \begin{cases} \text{if } T^*\le 1 \text{ then } 1.4691\times T^{*-1/2} - 0.341\times T^{*-1/4} + 0.185\times T^{*5/4} + 0.059 \\ \text{if } T^*\ge 1 \text{ then } 1 + 0.106\times T^{*-1} + 0.263\times T^{*-4/3} \end{cases}$$

Size ???

3. New model (modified Millikan equation)

$$K = f_1 f_2 e \frac{1 + \frac{l}{\delta}\left[a + b\exp\left(-c\frac{\delta}{l}\right)\right]}{6\pi\eta\delta}$$

$\delta = r_i + h_{(T_\delta)} + r_g(T_\delta),$ h – extra distance

$$f_1 = \sqrt{1 + \frac{m_g}{m_i}}, \qquad f_2 = \frac{2.25}{(a+b)\left(\Omega^{(1,1)*}_{\infty-4}(T^*) + s(r_\mathrm{i}, T_\delta) - 1\right)}$$

When $r_m \to 0$ the model approaches the free molecule regime equation with

$$\Omega^{(1,1)} = \left[\Omega^{(1,1)*}_{\infty-4} + s(r_i, T_\delta) - 1\right]\pi\delta^2$$

4. Factor $s(r_i, T_\delta)$

Assumptions:

1) The melting of internal degrees if freedom of the particle energy is described by the Einstein factor:

$$\frac{x^2 e^x}{(e^x - 1)^2} \quad \text{where} \quad x = \frac{\Delta E}{kT}$$

2) the average separation of internal energy levels is inversely proportional to the number of atoms $\Delta E = \frac{\text{const}}{r_i^3}$

It follows the model $s = 1 + (s_\infty - 1)x^2 \frac{e^x}{(e^x - 1)^2}$ where $x = \frac{273\ \text{K}}{T}\left(\frac{r_{cr}}{r_i}\right)^3.$

r_{cr} – critical radius.