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Assessing Robustness of Temporal Disaggregation Technique Denton in Economic Forecasting Under Anomalous Conditions

Master's Thesis

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Abstract:

This master's thesis assesses the robustness of temporal disaggregation techniques, particularly the Denton method, in handling anomalies like outliers, fat tails, and skewness in economic data. Motivated by the existing literature highlighting the importance of accurate economic forecasting and policy-making, which heavily relies on the integrity and reliability of disaggregated data, this research aims to contribute to the understanding of the performance of this technique under various conditions.

Using simulated quarterly and annual GDP data of Estonia from 2000 to 2023, the study evaluates these methods through ARIMA models that analyse growth rates and simulate future GDP scenarios. The research involves simulating 1000 GDP paths, exploring the variability and potential outcomes to understand the inherent risks in economic forecasting.

The results demonstrate that the Denton method shows accuracy and reliability, particularly in the presence of data anomalies. This is evidenced by low error metrics such as MAE, RMSE, MAPE, and SMAPE across simulations. The thesis confirms the effectiveness of the disaggregation technique under stress and highlights their value in improving economic forecasting and policy-making.

This Master's Thesis is written together yet independently. Any ideas or data taken from other authors or other sources have been fully referenced.

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Introduction

In the evolving field of economic forecasting, the accuracy and reliability of predictive models are paramount (F. Petropoulos, 2022). One of the critical challenges in this domain is effectively handling and analysing data that varies in frequency. Temporal disaggregation techniques, which transform low-frequency data into high-frequency data, play a crucial role in enhancing the granularity of economic analyses and improving decision-making processes (Sax and Steiner, 2013). Economic and financial time series often exhibit non-standard distributions and unexpected volatility, which can significantly impact the performance of disaggregation techniques (Sax and Steiner, 2013). These methods are essential not only for academic research but also for practical applications in economic policy formulation and financial market analysis. For instance, accurately disaggregating quarterly GDP data from annual figures is indispensable for timely policy interventions and market assessments (EuroStat, pub. 2020).

Previous studies have generally focused on the theoretical foundations and applications of disaggregation techniques without extensively testing their robustness against data anomalies (Latex Journal, 2021). The literature review conducted for this research highlights a significant gap in empirical evidence regarding the performance of these methods under real-world economic irregularities, such as outliers, fat tails, and skewness. This study aims to address this gap by rigorously testing how well these disaggregation methods handle such anomalies.

This thesis examines the robustness of disaggregation techniques, specifically focusing on the Denton method under conditions where data anomalies—such as outliers, fat tails, and skewness—are present. The study employs a structured experimental approach to explore the performance of temporal disaggregation techniques.The datasets used in this research are based on historical Estonian economic performance and include original annual consumption aggregates, which need to be disaggregated, simulated quarterly GDP data serving as a reference for disaggregation, and original quarterly consumption aggregates used as a benchmark for comparison.

ARIMA models are applied to generate a series of residuals, which are then used to create 1,000 possible GDP data scenarios. This process investigates how particular disaggregation techniques, such as the Denton method, perform in forecasting a range of potential economic outcomes, like quarterly consumption aggregates values.The experiments include both clean and simulated quarterly GDP datasets. The simulated datasets incorporate anomalies such as fat tails, outliers, and skewed distributions. These datasets are then subjected to the disaggregation process using the Denton method, which has demonstrated reliable results across various datasets and programming environments.

The effectiveness of each technique is evaluated based on its ability to maintain fidelity to the underlying economic trends despite the presence of data anomalies. Additionally, this thesis leverages ARIMA models to assess growth trends and predict future economic conditions, providing a foundational framework for the disaggregation process. This approach tests the method under both standard conditions and more stressed economic scenarios, simulating the impact of potential economic crises or atypical market events.

The significance of this research lies in its dual contribution: first, it provides a methodical assessment of how well the Denton method can handle anomalous conditions. Second, it offers empirical evidence of the method's efficacy, thus supporting its continued use in high-stakes economic forecasting and analysis. By conducting rigorous quantitative analysis, this thesis aims to illuminate the strengths and limitations of the Denton temporal disaggregation method, thereby guiding future research and practice in economic data analysis.

The structure of this thesis is organised as follows. The introduction provides the background, motivation, and objectives of the study, as well as the significance and contributions of the research. The literature review chapter offers a comprehensive review of existing literature on temporal disaggregation methods, focusing on the Denton method and its applications in economic forecasting, and discusses the gaps identified in previous studies and how this research aims to address them. The methodology chapter explains the research design, data collection, and analytical methods used in the study, including the mathematical formulations and models applied, such as ARIMA and the Denton method. The data and experimental setup chapter describes the datasets used, including the historical Estonian economic performance data, and the process of generating clean and simulated datasets, as well as the experimental setup for testing the robustness of the disaggregation techniques.

The results and discussion chapter presents and analyses the experimental results, evaluating the performance of the Denton method under various conditions and discussing the implications of the findings. Finally, the conclusion and future work chapter summarises the key findings, contributions to the field, and suggestions for future research directions, and reflects on the limitations of the study and potential areas for improvement. By following this structure, the thesis aims to provide a clear and comprehensive understanding of the research conducted, its findings, and its contributions to the field of economic forecasting and data analysis.

Keywords:
Temporal disaggregation; Denton method; ARIMA models; Monte-Carlo simulation; fat tails; skewness; outliers.

CERCS:
S180 Economics, econometrics, economic theory, economic systems, economic policy

1. Literature Review

1.1 Denton Disaggregation Technique and Econometric Modelling

Temporal disaggregation is a fundamental statistical technique crucial for transforming lower-frequency data into higher-frequency data, thereby enhancing the resolution and usability of economic analyses. Among the notable methodologies, the Denton method, introduced by Frederick Denton in 1971, is highly regarded for its ability to adjust high-frequency series to conform to known low-frequency totals without distorting the inherent movement of the series. This method maintains the structural characteristics of the original data while filling in data gaps. It achieves this by iteratively scaling the high-frequency series to match the low-frequency aggregate, preserving the temporal patterns present in the original data. Denton disaggregation is widely employed in various economic applications, including national accounts, price index construction, and business surveys, owing to its simplicity and effectiveness in reconciling data discrepancies across different frequencies.

In addition to these techniques, ARIMA (AutoRegressive Integrated Moving Average) models play a pivotal role in economic forecasting. Developed by Box and Jenkins in the 1970s, ARIMA models offer a versatile framework for modelling time series data, including economic indicators such as GDP growth rates, inflation rates, and unemployment rates. The ARIMA methodology encompasses three key components: autoregression (AR), differencing (I), and moving average (MA). Together, these components allow ARIMA models to capture the underlying patterns and dynamics present in the data, making them valuable tools for forecasting future values based on historical observations. ARIMA models have found widespread application in economic research, policy analysis, and decision-making, owing to their ability to accommodate various types of data and capture complex temporal dependencies.

1.2 Impact of Anomalous Data and Methodological Evaluations

Evaluating how temporal disaggregation techniques handle anomalies in data—such as outliers, fat tails, and skewness—is crucial due to their potential to distort economic forecasts and analyses. Outliers, for instance, can significantly affect the means and variances of economic data, leading to incorrect conclusions and ineffective policy decisions. The literature details various approaches to mitigating the effects of anomalies, including robust statistical techniques and adaptive filtering methods. Comparative studies aim to assess the resilience of different disaggregation methods against data distortions, providing essential insights for researchers and practitioners in choosing the right tools for their specific needs. However, few studies have delved into the subject matter of extreme kurtosis (fat tails), skewness, and outliers.

The impact of anomalies on disaggregation techniques varies depending on the method's underlying assumptions and computational algorithms. Denton disaggregation, for example, may be more robust to outliers and extreme values due to its iterative scaling approach, which focuses on preserving the overall structure of the data. Methodological evaluations play a crucial role in identifying the strengths and limitations of different disaggregation techniques under varying conditions, providing valuable guidance for researchers and practitioners seeking to apply these methods in real-world settings.

1.3 Innovations and Applications in Policy and Decision Making

The integration of newer technologies such as machine learning algorithms into traditional temporal disaggregation techniques marks a significant advancement in the field. These innovations facilitate the analysis of larger datasets and improve the accuracy of disaggregated outputs under complex scenarios. The potential of advanced methodologies to handle high-dimensional data and uncover subtle patterns in economic time series holds promise for enhancing economic forecasting and analysis. Nonetheless, the "black-box" approach of machine learning techniques may not always be relevant in economic forecasting and policy. Temporal disaggregation, based on economic models and mathematical and statistical concepts, offers a range of advantages, including the ability to explain decisions made during forecasting without excessive reliance on large amounts of data and rigorous feature extraction processes. Leveraging disaggregation techniques for transforming annual GDP data into quarterly or monthly estimates enables policymakers and analysts to react more swiftly to economic changes, enabling more timely and informed decisions, particularly crucial in rapidly evolving economic environments.

1.3.1 Previous Studies for Forecasting and Policy Formulation

Previous studies have extensively documented the application and development of temporal disaggregation techniques across various economic domains(Table 1.). Key research has influenced and shaped the methodologies used in temporal disaggregation, including Bayesian estimation for refining the accuracy of disaggregated economic data, outlier detection methodologies, ARIMA models for economic forecasting, and mixed-data sampling (MIDAS) regression models for informing policy decisions. However, while these papers have introduced various disaggregation techniques, they have yet to highlight and compare multiple possible anomalies and their effect on the disaggregation process, indicating a gap in the existing literature.

J.M. Schneider and Jurgen D. Garbrecht in 2000, focused on developing a sequence of 1-month forecast anomalies that are physically plausible and internally consistent. While Denton's method is effective in many cases, it may not be suitable for complex time series data with seasonality, irregular fluctuations, or structural breaks, as pointed out by C. Sax and P. Steiner in 2013. They highlighted limitations such as incompatibility with irregular frequencies and limited economic value in some scenarios, especially in industries like pharmaceuticals and chemicals in Switzerland.

Similarly, W. S. Wei and D. OStram (2013) addressed the problem of estimating the covariance structure for unobserved disaggregated series from the available autocovariances of an aggregate model, focusing on ARIMA disaggregation. They emphasised that the disaggregation method cannot be optimal unless the intrinsic characteristic of correlation is incorporated. However, limitations arise when dealing with data irregularities and structural breaks.

D.F. Tomasso (2003) extended methods in dynamic regression models and techniques using formulations in terms of unobserved component models/structural time series. While these methods offer flexibility in modelling complex economic time series, challenges arise with flow aggregates and time averages of stock variables, leading to significant information loss and potential obsolescence in certain situations.

Ricci L. Reber and Sarah J. Pack (2014) evaluated the performance of different mathematical methods of temporal disaggregation in estimating the output of the insurance industry. They found that while the Denton method may result in slight temporal deviations, these differences are often negligible. However, for more accurate results, especially in scenarios with high volatility, alternative methods such as the Causey-Trager method may be preferable.

Isaac O. Ajao, Femi J. Ayoola, and Joseph O. Iyaniwura (2015) conducted a study on disaggregating Nigeria's annual Gross Domestic Product (GDP) into quarterly data using various methods. They found that the Denton method performed poorly compared to other methods such as the Litterman method, which demonstrated superior results in minimising discrepancies between annual and quarterly GDP data. They recommended further exploration of methods like Cubic Spline Interpolation for improved disaggregation accuracy.

While temporal disaggregation techniques offer valuable tools for economic analysis and forecasting, their effectiveness varies depending on the nature of the data and the specific application. Denton's method remains widely used but may not always be optimal, especially in cases of data irregularities, seasonality, or structural breaks.

Table 1

Previous Studies for Forecasting Techniques

Method	Limitations	Strengths	Source
Denton Method	Sensitive to outliers and extreme values in the data.	• Minimal distortions • Accurately estimates of sub-annual series • High-frequency series closely follows the trend of the indicator series	Tomasso (2003); Reber, Pack (2014); Sax, Steiner(2023)
Chow and Lin Method	Performance may vary depending on the choice of assumptions.	• Regression-based approach • Effective handling of relationships between variables	Tomasso (2003); Munir, Riaz (2019); Wayant (2019) Essahbi, Jbir (2020);
Average and Middle Methods	Lack sophistication and may not capture complex patterns or relationships in the data.	• Suitable for quick, rough estimates • Require minimal resources and time	Schneider, Garbrecht (2000)
Method of Fragments	Can be computationally intensive, especially for high-frequency data or complex models.	• Effective with irregular events or non-uniform distributions • High degree of flexibility in handling various types of data patterns and structures	Wey(2006)
Stochastic method	intensities may still differ from observed precipitation characteristics	• Significant amount of uncertainty or variability in the data	Takhellambam (2022)
Cubic Spline Interpolation	Assumes the function being interpolated is smooth, data with fat tails can introduce extreme values that affect the	• Smooth and continuous interpolations between data points.	Fisher(1996)

	smoothness and overall fit of the spline.	● Generates twice differentiable curves	

Source: Tomasso (2003), Reber, Pack (2014), Sax, Steiner(2023), Tomasso (2003), Munir, Riaz (2019), Schneider, Garbrecht (2000), Wey(2006), Takhellambam(2022), Fisher(1996), Wayant (2019), Essahbi, Jbir (2020);

2. Methodology

2.1 Data Description

This thesis addresses the gap in the existing literature and utilises a comprehensive dataset to assess the performance of temporal disaggregation techniques under various conditions, particularly focusing on the presence of anomalies such as outliers, fat tails, and skewness. The primary data used in this study comprises economic indicators, specifically quarterly Gross Domestic Product (GDP), and quarterly and annual consumption aggregates, sourced from EuroStat. This dataset is chosen for its reliability and widespread use in economic research, providing a robust foundation for analysis (EuroStat pub. 2023).

The GDP data, central to this analysis, includes quarterly figures spanning from 2000 to 2023, covering various economic cycles, including periods of rapid growth, recession, and recovery. Quarterly GDP data for the mentioned period is used to create quarterly disaggregated estimates from the annual consumption aggregates figures. Benchmark data acts as the cornerstone of economic analysis and forecasting, as it directly influences the calibration of econometric models and the subsequent policy decisions that affect markets and societies. Accurate benchmark data ensures that the economic indicators reflect true economic performance and trends, allowing analysts to make reliable forecasts and informed decisions (Ciccarelli and Pariès 2024).

Even minor inaccuracies in reference data can lead to substantial errors in economic forecasting due to the compounded nature of economic models. For instance, incorrect reference data can misrepresent the economic conditions, leading to misguided policy decisions that may amplify economic volatility or lead to incorrect assessments of economic health. In this research, we observed how simulated anomalies within reference data significantly distorted the outputs of the Denton method, a widely used technique for temporal disaggregation (Chen 2007). By introducing synthetic anomalies into the data, we were able to study their impact on the forecasting capabilities of the Denton method. The analysis revealed that the presence of fat tails, skewness, and outliers led to forecasted values that diverged from the expected trajectory, emphasising the potential risks of unaddressed anomalies in real-world data. This underscores the necessity for economists and statisticians to scrutinise and cleanse benchmark data thoroughly to ensure the reliability of their economic models.

2.2 Growth Rate Calculation and ARIMA Modeling

Before applying the ARIMA model, the growth rates from quarterly GDP data were transformed to stabilise variance and to make the patterns in the data more linear and suitable for time series modelling(Lai and Dzombak 2020). This transformation involved converting the growth rates into their logarithmic form, a common practice in economic time series analysis to address issues of non-stationarity and heteroscedasticity. Following this, the growth rates were calculated as the difference in the logarithmic values from one quarter to the next. This step is crucial as it simplifies the underlying data structure, making it more amenable to ARIMA modelling and aiding in the detection of temporal patterns such as trends and cycles.

The ARIMA model is represented by:

$ARIMA(p,d,q)$

where:

- p is the number of lag observations included in the model (autoregressive part).
- d is the number of times that the raw observations are different (differencing part).
- q is the size of the moving average window (moving average part).

The ARIMA model equation is:

$$Y_t=\mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon t - q$$

where:

- Y_t is the actual value at time (t),
- μ is a constant,
- $\phi_1, \phi_2 \ldots \phi_p$ are the autoregressive coefficients,
- $\theta_1, \theta_2 \ldots, \theta_q$ are the moving average coefficients,
- εt is the error term at time.

The selection of an appropriate ARIMA model is crucial for effectively capturing the underlying patterns in the economic time series data. For this thesis, the ARIMA model was employed to analyse the quarterly GDP growth rates calculated from the logarithmic transformations. The model's parameters were chosen based on a combination of statistical tests for stationarity (Augmented Dickey-Fuller test) and criteria for model optimization (Akaike Information Criterion - AIC). Preliminary analyses suggested an ARIMA(1,0,0) model as optimal, incorporating one autoregressive term without differencing and no moving average term.

The fitting process involved checking the model's residuals to ensure they met the necessary assumptions of normality, homoscedasticity, and independence. Diagnostic checks followed the model estimation to validate the ARIMA model's performance. These included examining the residual plots for patterns, performing the Ljung-Box test to assess autocorrelation in residuals, and cross-validating the model's predictive accuracy (Keung Li 2004). The accuracy and reliability of disaggregated outputs were then rigorously evaluated against actual known quarterly GDP data through error metrics such as Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE).

2.3 Monte Carlo Simulation and Analysis of GDP Data

To challenge the Denton temporal disaggregation technique under various economic scenarios, the study utilised a Monte Carlo simulation approach. A total of 1000 simulated GDP data sets were generated, each reflecting different potential economic outcomes based on the ARIMA(1,0,0) model. These simulations were designed to incorporate a range of anomalies, introduced during residuals generation. To refine the analysis further, the simulated quarterly GDP data underwent a normalisation process. Each quarterly value was adjusted based on the ratio of the annual sum of the simulated quarters to the actual annual GDP. This meticulous adjustment ensured that the total annual sum of the simulated quarterly values closely matched the actual annual GDP figures.

2.4 Disaggregation of Simulated GDP Data

For each of our 1000 simulations, we carried out the disaggregation process independently using Python, ensuring that each set of quarterly data was given due consideration. The Denton function, applied to each simulation, used the quarterly GDP as reference data, disaggregating annual consumption aggregates into quarterly estimates. The resultant data frame is a repository of disaggregated series, each column a finely tuned depiction of quarterly economic activity derived from our simulations. Displaying this dataframe, we could see the individual results of the Denton method applied at scale — 1000 columns each representing a disaggregated time series, showcasing the method’s robust applicability to our simulated economic data. In effect, normalisation and disaggregation served as a bridge between the theoretical and practical aspects of our analysis. It demonstrated that our simulations, when processed through the sieve of the Denton method, could produce plausible, detailed economic trajectories that aligned with actual economic totals. This congruence bolstered confidence in the temporal disaggregation techniques, setting the stage for the subsequent validation of our methods against real-world economic data.

2.5 Evaluation of Disaggregation Quality

To evaluate the performance of the disaggregated GDP data against actual quarterly consumption data, we computed the Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), Mean Absolute Percentage Error (MAPE), and Symmetric Mean Absolute Percentage Error (SMAPE) for each of the 1000 simulations. These metrics helped us assess the accuracy of our disaggregated quarterly GDP estimates compared to available actual quarterly data.

The formulas for these metrics are as follows:

1. Mean Absolute Error (MAE):

$$MAE=\frac{1}{n}\sum_{i=1}^{n}\left|y_i-\hat{y}_i\right|$$

where:

- n is the number of observations,
- y_i is the actual value at time,
- $\hat{y}_i$ is the predicted value at time .

2. Root Mean Squared Error (RMSE):

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t-\hat{y}_t)^2}$$

3. Mean Absolute Percentage Error (MAPE):

$$MAPE = \frac{100\%}{n}\sum_{t=1}^{n}\left|\frac{y_t-\hat{y}_t}{y_t}\right|$$

4. Symmetric Mean Absolute Percentage Error (SMAPE):

$$SMAPE = \frac{100\%}{n}\sum_{t=1}^{n}\frac{|y_t-\hat{y}_t|}{(|y_t|+|\hat{y}_t|)/2}$$

These metrics helped us assess the accuracy of our disaggregated quarterly GDP estimates compared to available actual quarterly data. MAE and RMSE are absolute measures of fit, indicating the average magnitude of errors between predicted and actual values. MAPE and

SMAPE are relative measures, giving errors as a percentage of the actual values, which is often more interpretable in terms of actual impact.

2.6 Simulation of Anomalous Conditions

To simulate real-world economic behaviours, including atypical events, additional datasets incorporating anomalies were integrated. To assess the reliability of our economic forecasting under various conditions, we first investigated scenarios with the presence of fat tails in the residuals, which indicate a higher occurrence of extreme economic events than a normal distribution would predict. We employed a kurtosis analysis to measure the "tailedness" of the anomalous distribution. Our analysis with the help of ARIMA modelling of growth rates revealed that the data itself has fat tails in its distribution with given kurtosis results.

Next, to test the sensitivity of the Denton temporal disaggregation method to outliers—which can represent sudden economic shocks or data recording errors—a statistical outlier detection technique was applied. Using the Interquartile Range (IQR) method, we computed the lower and upper bounds to detect outliers in the residual data (both original and simulated). The lower bound was set at Q1 - 1.5 * IQR, and the upper bound at Q3 + 1.5 * IQR, where Q1 and Q3 are the first and third quartiles, respectively. Within our methodology, the residuals from the ARIMA model (original residuals) were first subjected to this outlier detection process, followed by outlier detection in the simulated residuals.

Finally, we scrutinised the effect of skewness on the ARIMA model's residuals to simulate potential asymmetry commonly observed in economic data. To incorporate skewness into our model, we utilised a skewness transformation process on the residuals. We began by calculating the original skewness of the ARIMA model residuals to establish a baseline for comparison. In order to introduce skewness, the alpha parameter was then deliberately altered to introduce asymmetry, simulating a scenario where the residuals would be skewed to the right, indicating a longer tail on the positive side of the distribution. Using the adjusted skewness parameter, we transformed residuals, generating a new skewed dataset that mimicked the chosen level of asymmetry. This dataset was then used to observe the effects of skewness on the temporal disaggregation process, providing insights into the method's robustness under such conditions.

The anomalies were systematically introduced into the simulated GDP data using the following specifications:

- Fat Tails: To test the data in the anomalous condition we introduced fat tails into the existing one setting degrees of freedom(*df*) to 5.
- Skewness: To introduce skewness, we manipulated the data distribution to have a skewness coefficient of alpha=3. This manipulation provides a dataset that not only includes typical variations but also skewed outcomes, which are common in economic data due to shifts in policy, sudden economic shocks, or structural changes.
- Outliers: Similar to the method for fat tails, outliers were introduced using a t-distribution with 5 degrees of freedom, positioned strategically to represent unexpected shocks or errors in the data collection and reporting processes.

2.7 Cubic Spline Interpolation Method

As part of our analytical arsenal, we incorporated cubic spline interpolation to create a smooth transition between annual data points and thus generate a high-resolution quarterly economic time series. This method complements our temporal disaggregation analysis, allowing us to fill in gaps between the less frequent annual data observations with estimates that reflect the inherent continuity of economic trends. Cubic spline interpolation technique, being a widely-used method of dealing with macroeconomic data for economic forecasting, demonstrates smoothness and the precise control it affords over the curvature of the interpolated line (Paul D. Mann, 1996).

We started by organising the annual consumption aggregate data into a structured array. The consumption data was chosen for its relevance to economic growth and its representation of a key economic indicator. The cubic spline was computed by fitting third-degree polynomials between consecutive data points. The `CubicSpline` function ensures the continuity of the first and second derivatives of these polynomials across the dataset, providing a smooth curve that is differentiable throughout its domain.

Mathematically, a cubic spline interpolation fits a piecewise polynomial S_i(x) of degree three between each pair of data points (x_i, y_i). Each piece S_i(x) is defined as:

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

where a_i, b_i, c_i, and d_i are the coefficients determined for each interval [x_i, x_{i+1}]. The coefficients are computed such that:

1. Interpolation conditions: The spline passes through all data points:

$$S_i(x_i) = y_i \quad \text{and} \quad S_i(x_{i+1}) = y_{i+1}$$

2. Continuity of the first derivative: The first derivative is continuous at the interior points:

$$S_i'(x_{i+1}) = S_{i+1}'(x_{i+1})$$

3. Continuity of the second derivative: The second derivative is continuous at the interior points:

$$S_i''(x_{i+1}) = S_{i+1}''(x_{i+1})$$

4. Boundary conditions: Commonly, natural spline conditions are used where the second derivatives at the endpoints are zero:

$$S''(x_0) = 0 \quad \text{and} \quad S''(x_n) = 0$$

We established a series of quarterly points to serve as the new data points for interpolation. The cubic spline function was then applied to these points, yielding estimated values for quarters where only annual data was previously available. To visually assess the interpolation's fidelity, we plotted the original annual data against the newly interpolated quarterly estimates. This step provided an immediate, intuitive sense of how well the interpolation aligned with the annual data and highlighted the added value of more granular estimates for the analysis.

This method assumes that the economic trend between data points can be reasonably estimated by a smooth curve, which is typically valid for stable economic indicators but may not hold during volatile periods without additional adjustments. However, there are considerations to bear in mind. While cubic spline interpolation excels in smoothness, it is proven to be sensitive to extreme values(Wolberg, 1988).

3. Results

3.1 ARIMA Modelling Results

In our analysis of the log growth rate variable, we employed a SARIMAX model configured as ARIMA(1, 0, 0). This model choice was deliberate, focusing on capturing the autoregressive dynamics without incorporating differencing or moving average components. Our approach is grounded in quasi-maximum likelihood estimation theory, ensuring the robustness of our findings.

Table 2
ARIMA model

Dep. Variable:	Log Growth Rate	No. Observations:	96
Model:	ARIMA(1,0,0)	Log Likelihood:	213,210
Sample:	0	AIC:	-420.420
	-96	BIC:	-412.727
Covariance Type:	opg	HQIC:	-417.310

	Coef	std. Error	z	P> |z|	[0.025	0.975]
Const	0.019	0.004	4.612	0.0000	0.011	0.028
ar.L1	0.2630	0.064	4.082	0.0000	0.137	0.389
Sigma2	0.0007	7.98e-05	8.628	0.0000	0.001	0.001

Ljung-Box (L1) (Q):	0.8	Jarque-Bera (JB):	11.41
Prob(Q) :	0.37	Prob(JB):	0.00
Heteroskedasticity:	0.88	Skew:	-0.49
Prob(H) (two-sided):	0.73	Kurtosis:	4.37

Source: author’s calculations

We specified the SARIMAX model with an ARIMA(1, 0, 0) configuration to explore the temporal dependencies within the log growth rate variable. By focusing on the autoregressive component (AR(1)), we aimed to capture the immediate past influence on the current growth rate, avoiding the complexity of differencing (I(0)) or moving average (MA(0)) terms. The quasi-maximum likelihood estimation methodology underpins our model, providing a robust framework for parameter estimation. Our model demonstrated a high level of explanatory power, evidenced by a log likelihood of 213.210. We utilised the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) to assess the model's performance. These metrics indicated a well-balanced model that effectively captures the underlying data patterns while maintaining an appropriate level of complexity.

The coefficient estimates derived from quasi-maximum likelihood estimation proved to be both robust and consistent. Key coefficients in our model included:

- Constant term ('const'): This represents the intercept, capturing the baseline level of the log growth rate.
- Autoregressive coefficient ('ar.L1'): This reflects the influence of the previous period's growth rate on the current period, highlighting the autoregressive nature of the series.
- Residual variance ('sigma2'): This measures the dispersion of the unexplained variability in the growth rate series, providing insight into the precision of our model.

We conducted a series of diagnostic tests to validate our model assumptions:

- Jarque-Bera Test: This test for normality confirmed that our residuals were largely normally distributed, despite minor skewness. This validation supports the quasi-maximum likelihood estimation process.
- Ljung-Box Test: Our autocorrelation test yielded non-significant results, indicating no serial correlation in the residuals. This suggests that our model successfully captures the temporal structure of the data.
- Heteroskedasticity Test: The results showed no significant heteroskedasticity concerns, reinforcing the model's robustness in capturing the log growth rate dynamics.

Our findings underscore the robustness and reliability of the SARIMAX model grounded in quasi-maximum likelihood theory. Despite minor deviations from normality in the residuals, the model's performance remained resilient. Our preprocessing steps, including the generation of residuals with controlled skewness and subsequent demeaning, illustrate our meticulous approach to ensuring model adequacy and addressing potential misspecifications. The model's focus on the temporal dynamics of the log growth rate series is particularly noteworthy. By emphasising the autoregressive component, we achieved precise and accurate parameter estimates, highlighting the efficacy of our approach in capturing the underlying temporal patterns.

3.2 Residuals and Monte Carlo Simulation

After fitting our economic data to the ARIMA(1, 0, 0) model, we delved into the residual analysis to understand the model's precision. We found the average of the residuals to be exceptionally close to zero, precisely at -2.7, which indicates no bias in our model's predictions. Moreover, the standard deviation of the residuals was 0.0264, demonstrating that our predictions are not only unbiased but also clustered closely around the actual values, reflecting the model's tightness and consistency. Equipped with the parameters from our ARIMA model — a constant term of 0.0196 and an autoregressive coefficient of 0.2651 — we initiated a Monte Carlo simulation to project 1000 potential paths for our GDP. These simulations have provided us with a broad spectrum of economic scenarios, each a crucible within which the mettle of our temporal disaggregation techniques will be tested. The robustness of these methods is not merely academic; it is essential for ensuring the reliability of economic forecasting tools in the face of uncertainty and for supporting sound policy-making.

3.3 Temporal Disaggregation Results for results with original data

In this section, we delve into the results of our temporal disaggregation model applied to the baseline dataset. Initially, our model showcased commendable performance with consistently low error metrics across various measures, as previously discussed in section 4.1. However, our analysis also uncovered a notable observation regarding the residuals obtained from the ARIMA (1,0,0) model, indicating a presence of fat tails with a value of 4.37 (Figure 1.).

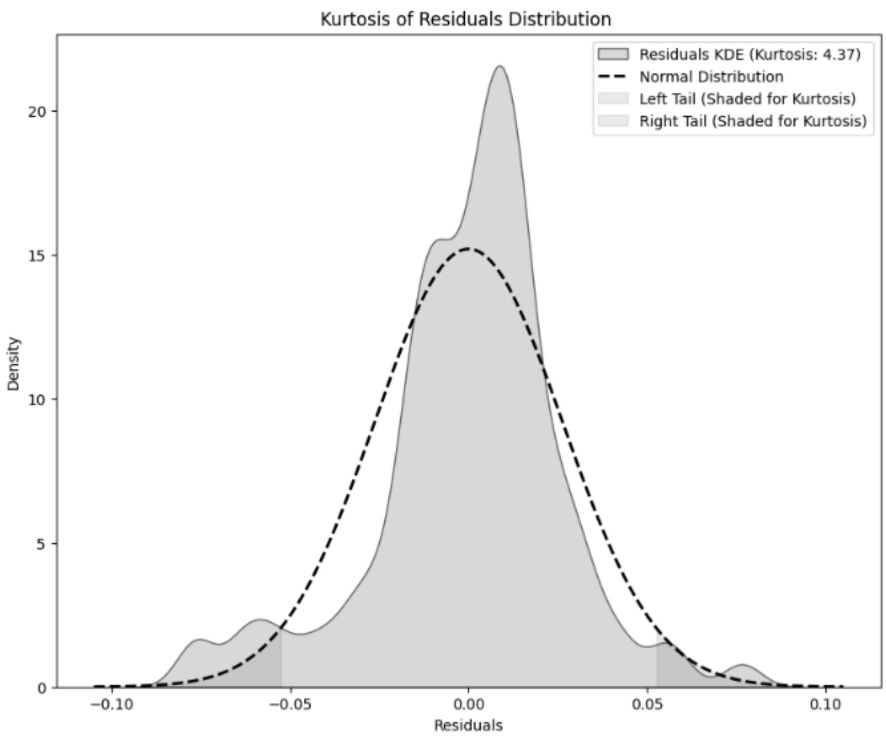

Figure 1. Distribution graph of baseline data with fat tails

Source: author’s calculations

This discovery prompted further investigation into the distribution of growth rates, revealing a kurtosis of 5.06 when examined in logarithmic form, indicating a distribution with heavier tails(Figure 2.).

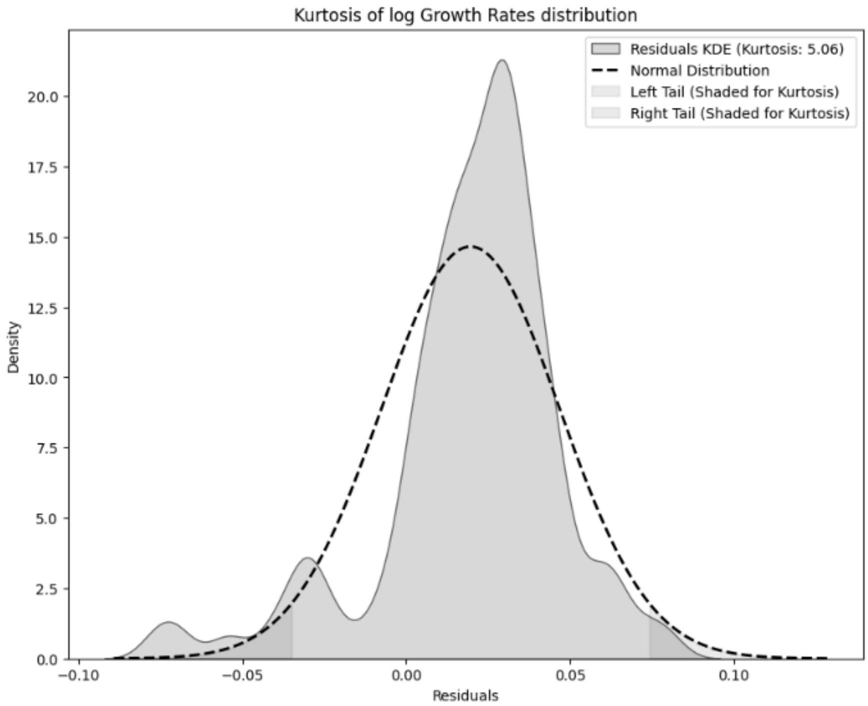

Figure 2. Growth rates distribution

Source: author's calculations

Comparing the disaggregated results to the original quarterly consumption aggregates, depicted below, our simulated data mirrors the flow of the economy as represented by the consumption figures(Figure 3.).

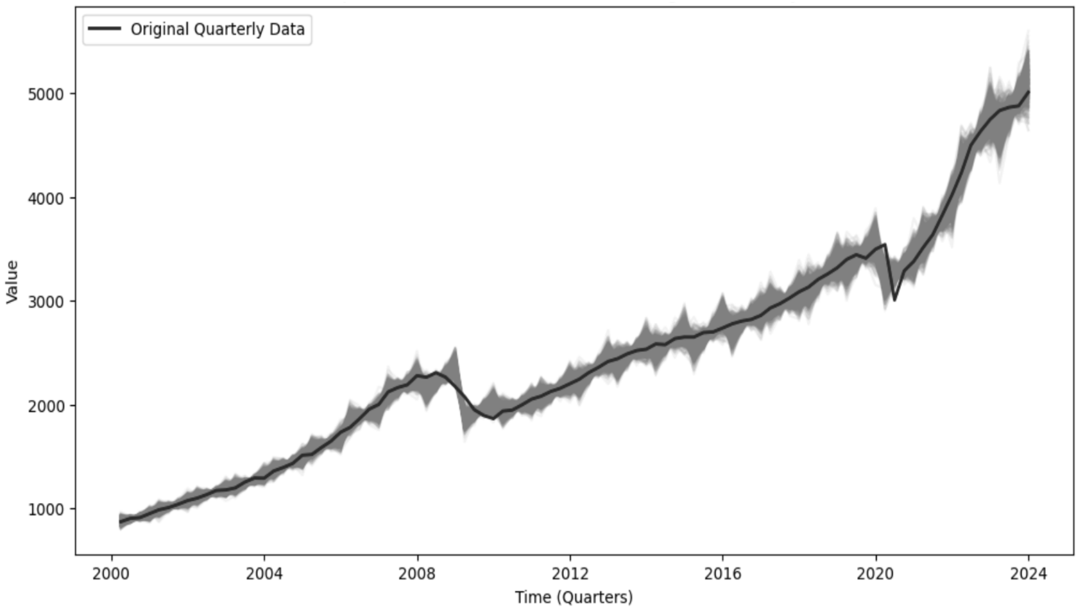

Figure 3. Comparison of 1000 disaggregation data scenarios with original quarterly data

Source: author's calculations

The convergence between our simulated data, containing fat tails in residuals and log growth rates, with the actual consumption figures affirms the methodological rigour of the approach and the reliability of the Denton method in producing economically plausible, high-frequency data from lower-frequency aggregates. However, we still cannot conclude our results entirely. Therefore, we have to calculate error metrics to find differences between our disaggregated data and the original quarterly data.

We calculate the MAE, RMSE, MAPE, and sMAPE for disaggregated series. These metrics provided us with a multifaceted view of the error landscape, revealing the magnitude and nature of the discrepancies between our model's outputs and the actual economic data. For our first column of disaggregated data, the results were as follows:

1. MAE: 64.7228, indicating that, on average, our simulated values were about 64 units away from the true values, suggesting a relatively small error given the scale of the GDP figures.
2. RMSE: 96.7012, a measure that penalises larger errors, also indicated a reasonable degree of accuracy.
3. MAPE: 0.02464, revealed that the average percentage error was about 2.4%, which in the context of economic data, is a modest deviation.
4. sMAPE: 0.012, closely mirroring the MAPE, confirmed the error's proportionality, indicating a balanced and consistent predictive performance across our simulations.

With the average percentage errors hovering around the 2.4% mark, the results corroborated the effectiveness of the Denton method in not just distributing annual values across quarters, but in doing so with a precision that closely mirrored actual economic trends. These disaggregated values not only enhance the resolution of our economic analysis but also underscore the potential of such techniques in informing policy decisions and economic forecasts.

3.4 Comparative Analysis and Robustness Assessment with further anomalies

In an effort to assess the robustness of our temporal disaggregation methods under anomalous conditions, we introduced additional controlled distortions into our GDP data simulations and observed the changes in various error metrics. These metrics serve as a barometer for the accuracy of our models under different stressors, providing critical insights into the effectiveness of our disaggregation techniques.

3.4.1 Introduction of Outliers

The original residual data revealed several outliers that corresponded to potential economic anomalies. Subsequently, the generated residuals incorporated these outliers, yielding a more varied and realistic set of economic trajectories. The visualisation of these outliers was accomplished via a scatter plot that provided a clear representation of their distribution within the IQR range(Figure 4. and 5.).

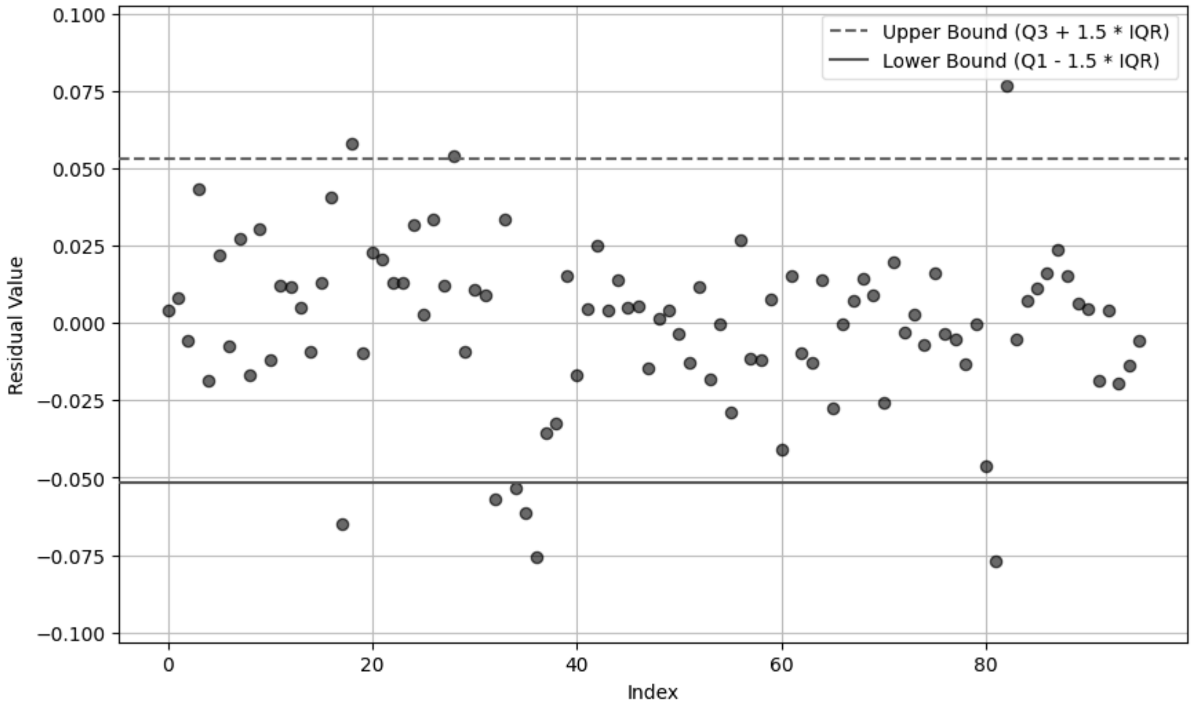

Figure 4. Outliers in residuals of original residuals

Source: author’s calculations

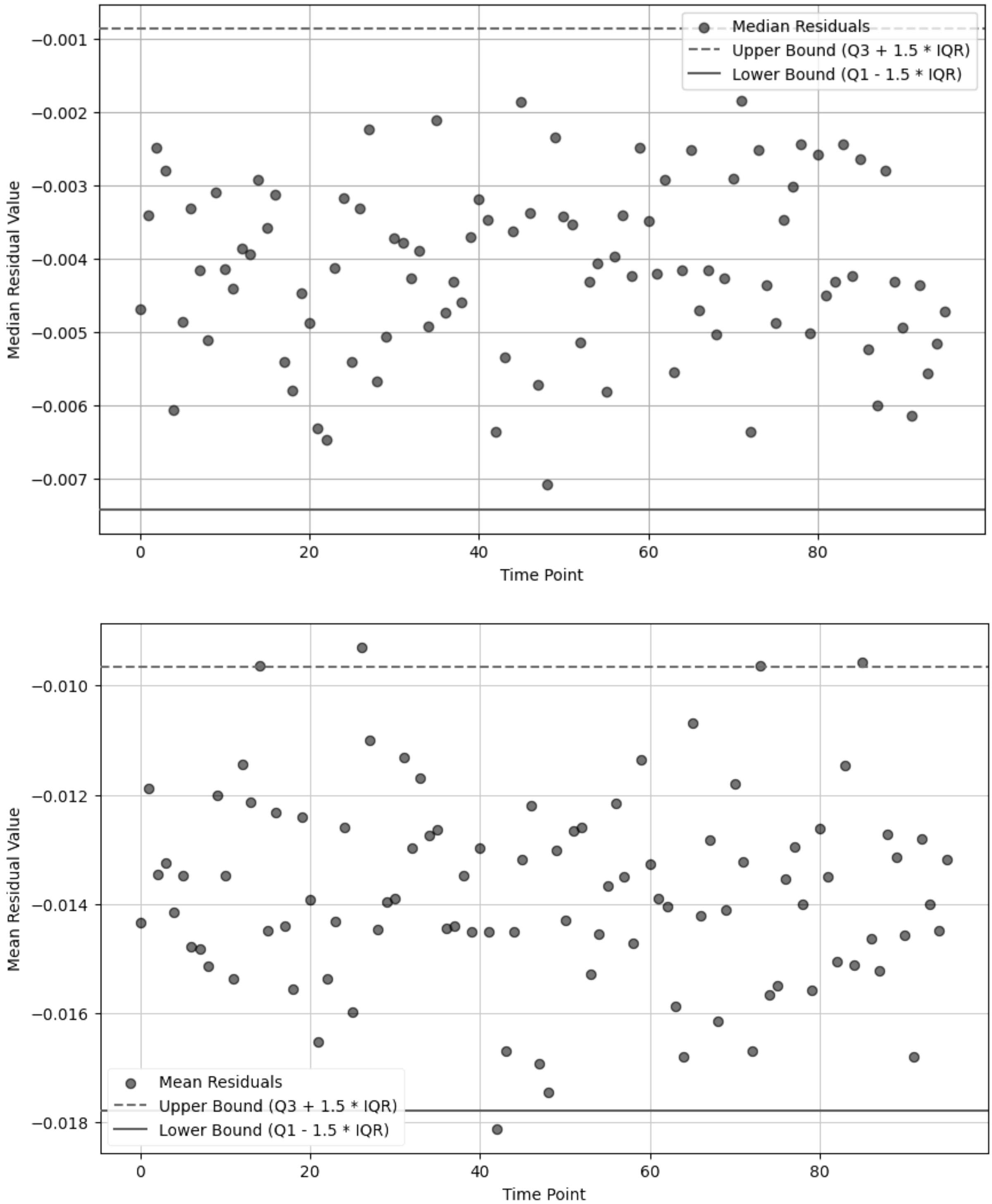

Figure 5. Median and mean of outliers from simulated residuals

Source: author's calculations

Notably, the plot showed the potential influence of these extreme values on the consistency and accuracy of the economic forecasts. The computed bounds (lower: -0.0515 and upper:0.0531) offered a quantitative criterion for the identification of outliers, and the resultant scatter plot served as a visual confirmation of their presence. The outliers emphasised the necessity for robust forecasting methods that can accommodate such extremes, validating the necessity of the Denton method in our approach. The simulation with outliers, meant to represent sudden, atypical changes in the economy, resulted in a marked increase in error metrics, indicating a larger divergence from the actual data:

MAE: 96.8431 (an increase of 49.59% from the baseline)

RMSE: 144.415 (an increase of 49.31% from the baseline)

MAPE: 0.03955(a 60.44% increase from the baseline)

sMAPE: 0.0199(34.16%% increase from the baseline)

The significant increase in all error metrics in the presence of outliers suggests that our model is most sensitive to sudden, large deviations from the norm. This underscores the need for robust detection and mitigation strategies in economic modelling to handle potential crises or unexpected economic events.

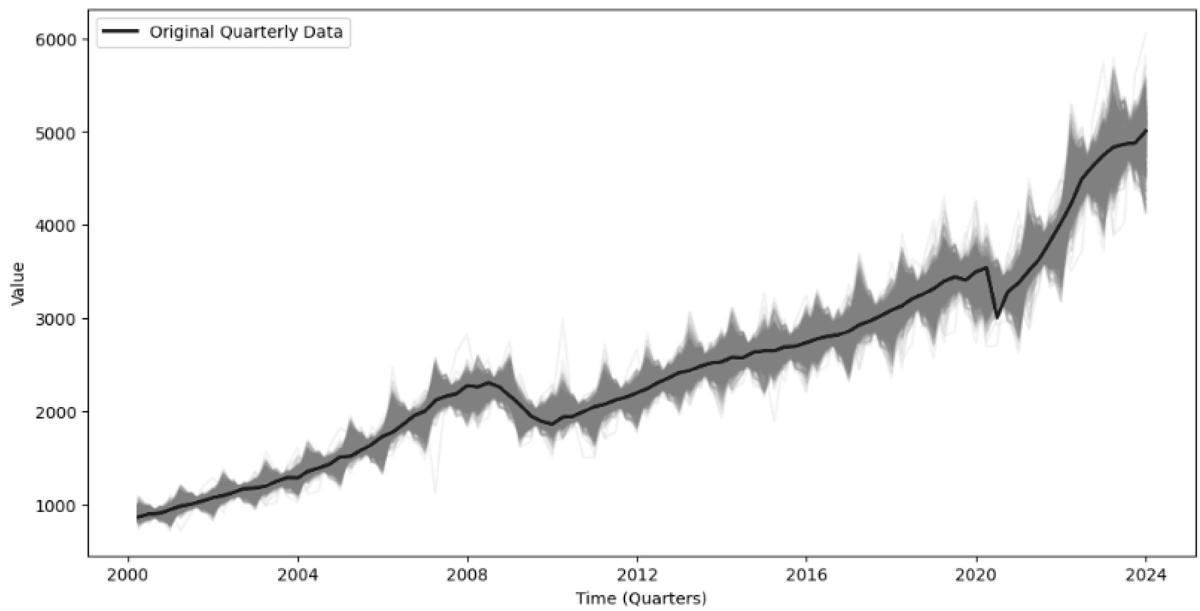

Figure 6. Comparison of 1000 disaggregation data scenarios with original quarterly data

Source: author's calculations

3.4.2 Assessment with Skewness

The exploration into the skewness of residuals yielded findings that provide nuanced insights into the model's performance under skewed conditions.The original residuals exhibited a leftward skewness of -0.499, indicating a distribution with a tail extending towards the left, or lower values. This asymmetry was further illustrated by the Kernel

Density Estimate plot, which depicted a noticeable deviation from the symmetrical bell curve expected under normal distribution assumptions(Figure 7.).

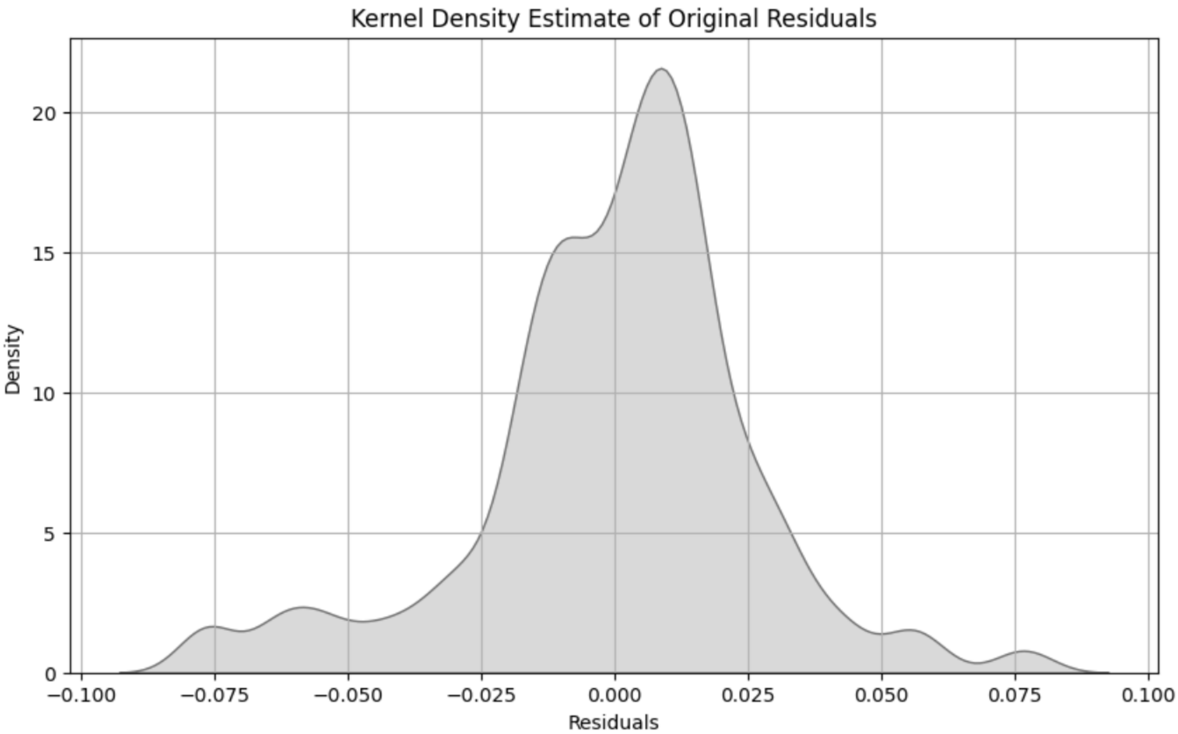

Figure 7. Original Residual Density
Source: author’s calculations

When skewness was artificially introduced, the skewness metric for the generated residuals averaged 0.637, signalling a rightward shift. This implies that the simulation introduced a bias towards higher values, as the residuals' distribution displayed a tail leaning towards the right. The histogram below of the skewness for generated residuals captured a clear shift towards positive skewness, suggesting an increased frequency of higher-than-average residuals compared to the original model(Figure 8.).

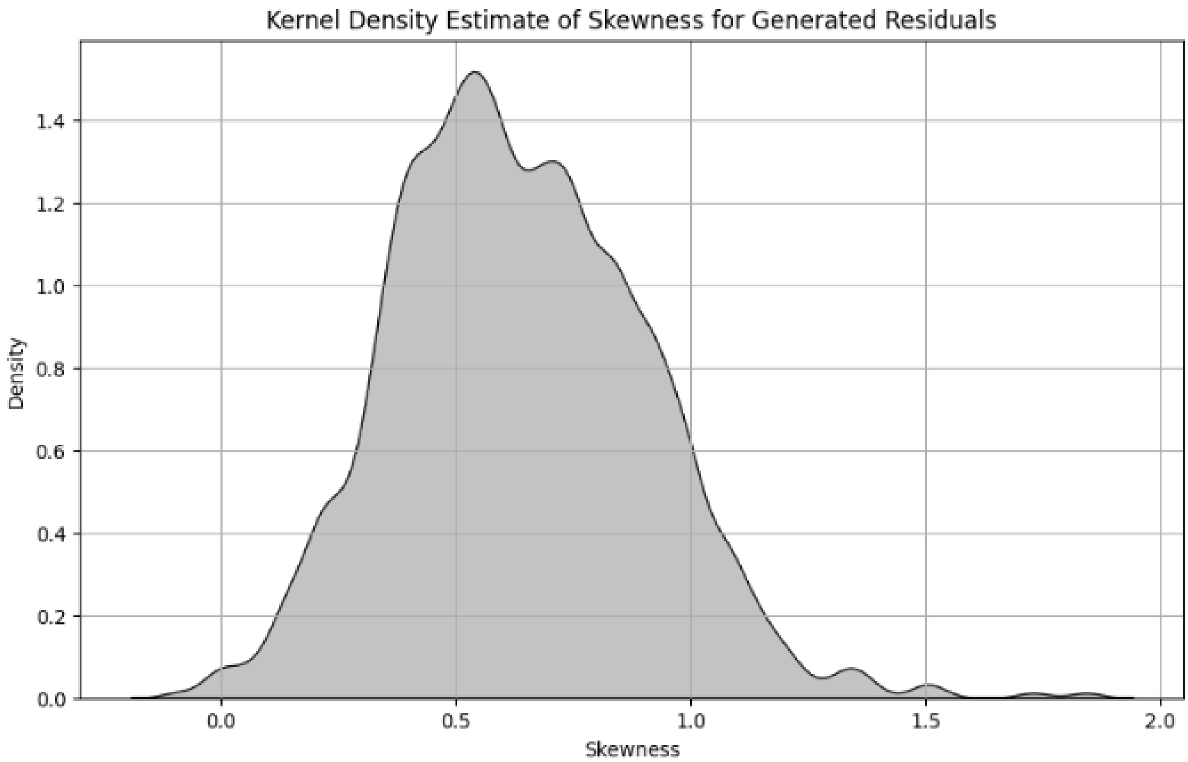

Figure 8. Generated residual density

Source: author's calculations

Kernel Density Estimate plots for the original residuals showed a distribution slightly leaning left, while the generated residuals displayed a pronounced skew to the right. Introducing skewness to the data to represent lopsided economic growth or contraction led to moderate changes in the error metrics(Figure 9.):

MAE: 93.5 (an increase of 45% from the baseline)

RMSE: 132.5 (an increase of 26% from the baseline)

MAPE: 0.03635 (a 35% increase from the baseline)

sMAPE: 0.01822 (a 18% increase from the baseline)

The impact of skewness, though noticeable, was not as significant as outliers but was more pronounced than fat tails. This informs us that asymmetric growth patterns can be accounted for by our model to a reasonable extent, but caution should be exercised in heavily skewed environments.

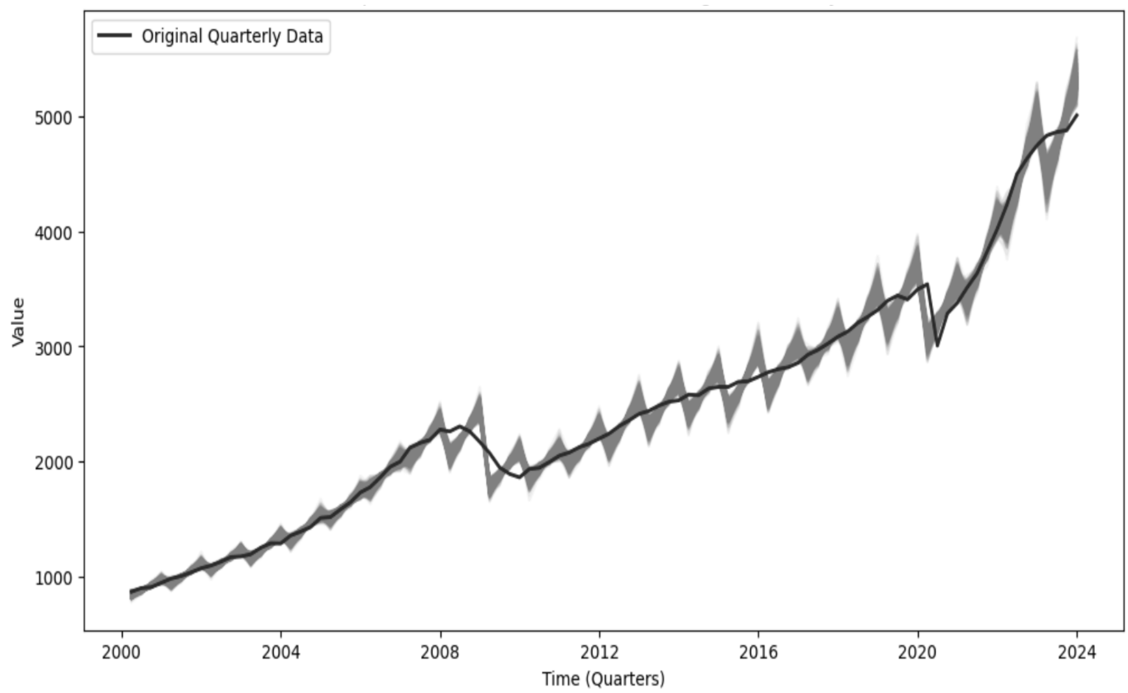

Figure 9. Comparison of 1000 disaggregation data scenarios with original quarterly data

Source: author's calculations

3.4.3 Assessment with Fat Tails

The exploration into the presence of fat tails within the ARIMA model residuals provided detailed insights into the impact of extreme values on the temporal disaggregation process. Upon integration, the fat tails significantly altered the distribution characteristics of the residuals. The standard kurtosis of the residuals increased to 4.37(Figure 10.), indicating a leptokurtic distribution with a propensity for extreme outlier occurrences. Subsequently, the excess kurtosis rose to 4.52(Figure 11.), further accentuating the presence of fat tails and highlighting a more pronounced deviation from a normal distribution.

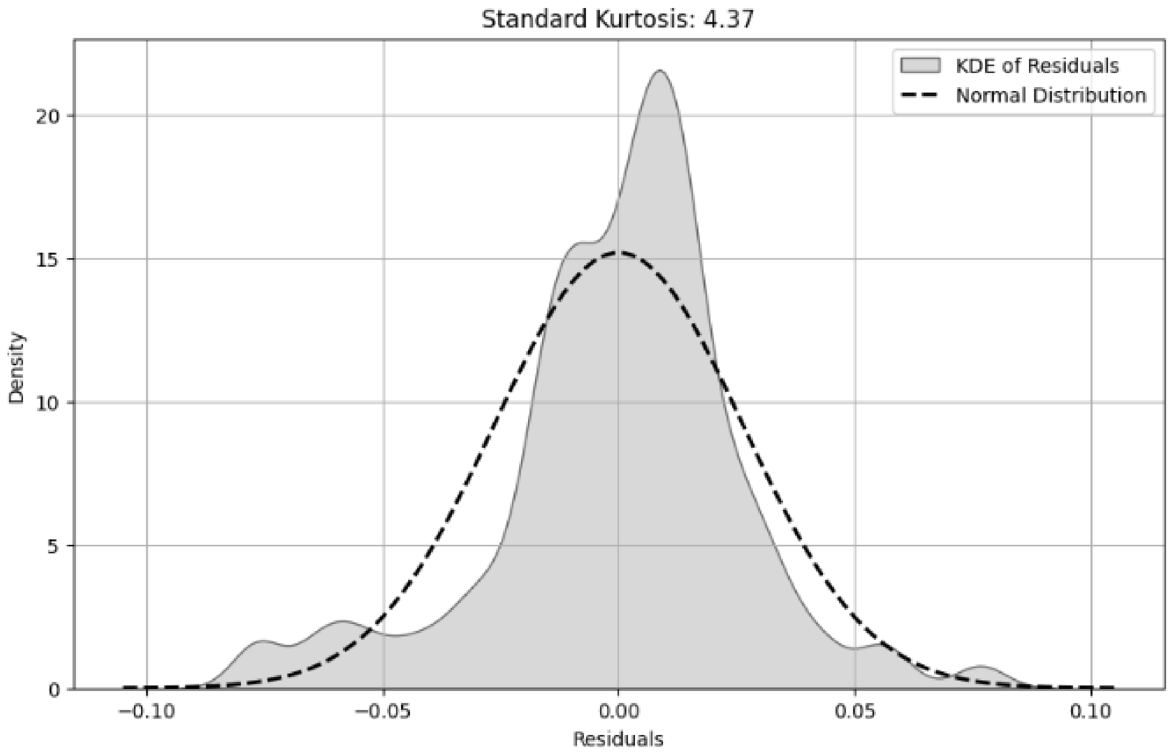

Figure 10. Standard Kurtosis distribution

Source: author's calculations

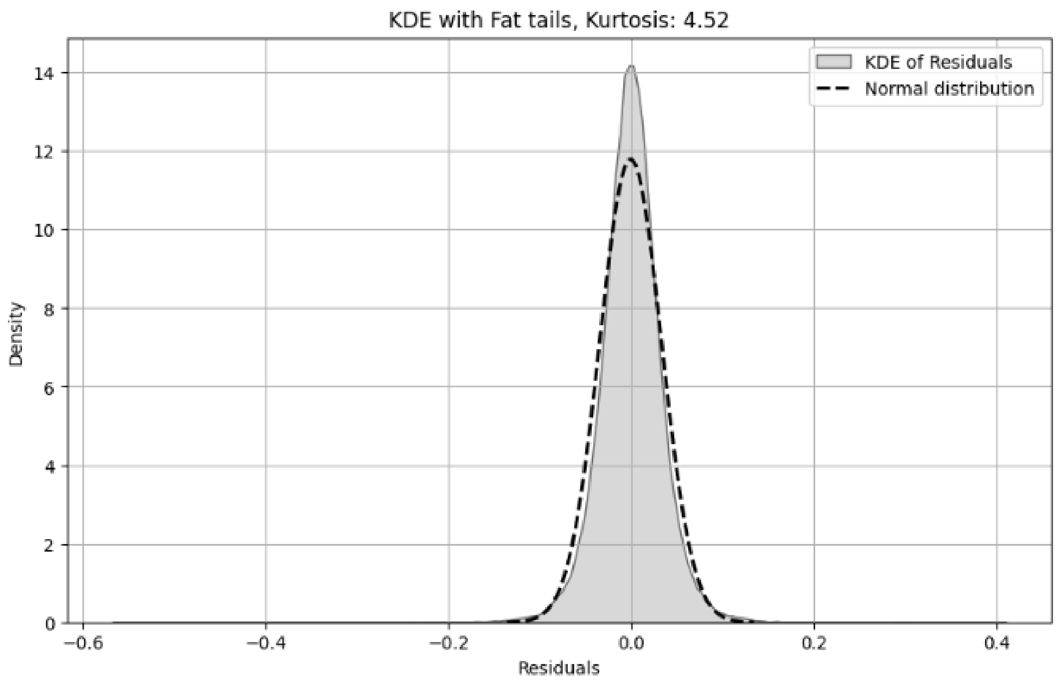

Figure 11. Fat tailed distribution

Source: author’s calculations

The introduction of fat tails led to noticeable increases in error metrics, indicating a greater degree of error and variability in the disaggregation process. Specifically, the Mean Absolute Error (MAE) increased by approximately 7.2%, from 62.39 to 72.97, and the Root Mean Square Error (RMSE) escalated by approximately 4.9%, from 92.61 to 106.67. Similarly, the Mean Absolute Percentage Error (MAPE) and the Symmetric Mean Absolute Percentage Error (sMAPE) registered increases of approximately 7.7%, from 0.0261 to 0.029, and approximately 7.7%, from 0.012 to 0.014, respectively. Despite the observed increases in error metrics, it is essential to note that the Denton method maintained its robustness in handling fat tails. While the introduction of fat tails led to variability in the disaggregation process, the Denton method's performance remained satisfactory, indicating its resilience in capturing underlying trends even in the presence of extreme values(Figure 12.).

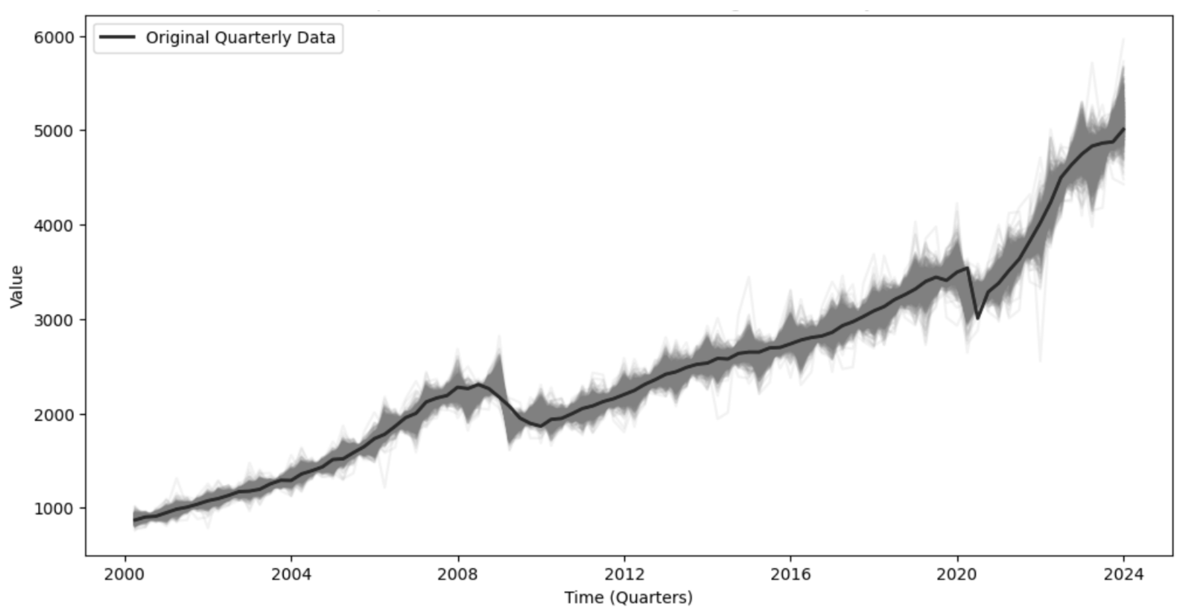

Figure 12. Comparison of 1000 disaggregation data scenarios with original quarterly data

Source: author’s calculations

3.5 Cubic Spline Interpolation Analysis

As we applied cubic spline interpolation to annual data, this statistical technique allowed us to construct a series of intermediate points that gave us a smooth and continuous representation of the economic trajectory at a quarterly frequency, bridging the gap between the annual data points. Upon reviewing the interpolated data, we observed a coherent progression of estimated values that followed the general trend of the annual data, as evidenced by the plotted visualisation. The continuity and smoothness characteristic of cubic spline interpolation were evident, providing a refined perspective of the underlying economic patterns.

The figure displayed a close alignment of the interpolated quarterly values with the actual annual data, underscoring the reliability of the interpolation in capturing the core trend of the data (Figure 13.). Notably, the spline curve managed to replicate the key movements and shifts observed in the yearly data.

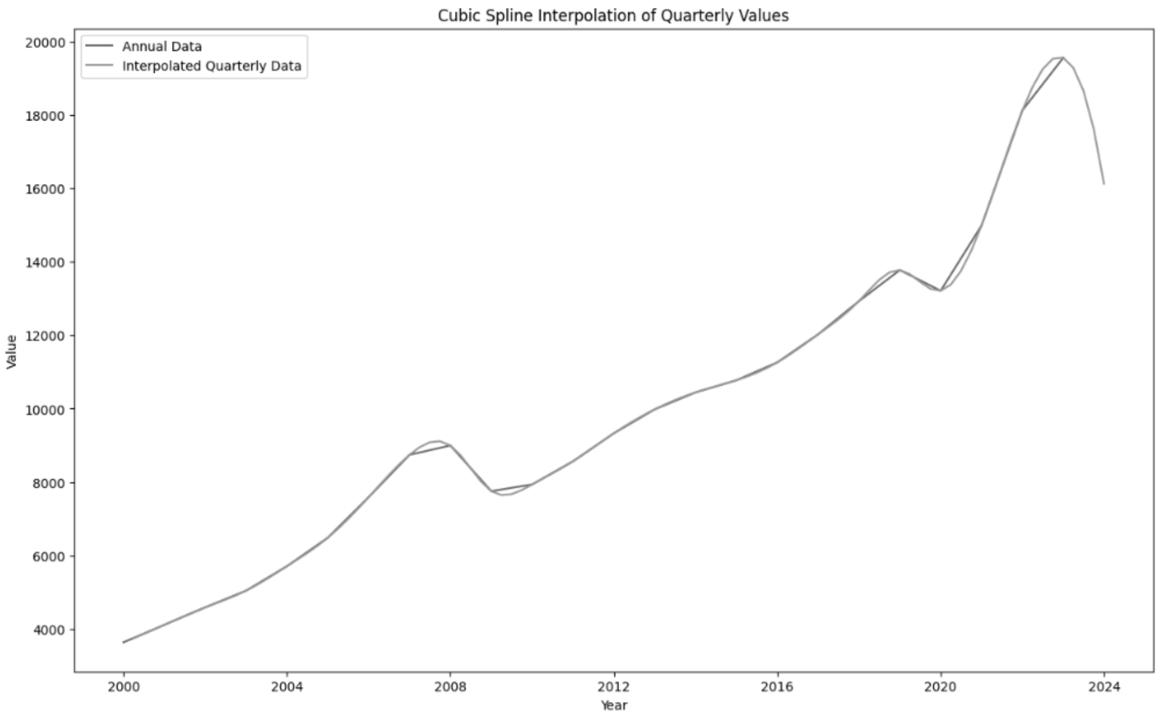

Figure 13. Cubic Spline interpolation of quarterly values

Source: author's calculations

The Cubic Spline interpolation method, while useful for smoothing and generating high-resolution data from low-resolution data, shows limitations with data containing anomalies, like fat tails and outliers(Wolberg, 1988). Not to mention, this method creates quarterly running totals, unlike standalone quarterly values. In contrast, the Denton method, specifically designed for temporal disaggregation to ensure that aggregated totals of the disaggregated data match the original totals, exhibits far superior resistance to extreme values, like fat tails. The low values in these metrics indicate that the Denton method preserves the integrity and structure of the original data more faithfully than cubic spline interpolation.

Although cubic spline interpolation is widely used by economists, our quantitative, as well as qualitative evidence suggests that Denton disaggregation is better suited for working with anomalous data, especially when quarterly forecasting values are needed.

3.6 Comparative Insights

The comparison of error metrics(Table 3.) across different types of anomalies yielded several key insights:

- Outliers Impact: The significant increase in all error metrics in the presence of outliers suggests that our model is most sensitive to sudden, large deviations from the norm. This underscores the need for robust detection and mitigation strategies in economic modelling to handle potential crises or unexpected economic events.
- Resilience to Fat Tails: The relatively small increase in errors when dealing with fat tails indicates a degree of resilience within the model. This suggests that while extreme events can affect forecast accuracy, our disaggregation methods can still provide a reliable economic outlook, albeit with some limitations.
- Moderation in Skewness: The impact of skewness, though noticeable, was not as significant as outliers but was more pronounced than fat tails. This informs us that asymmetric growth patterns can be accounted for by our model to a reasonable extent, but caution should be exercised in heavily skewed environments.

In conclusion, our analysis demonstrated that while the introduction of anomalies can significantly influence the performance of temporal disaggregation methods, these techniques—particularly the Denton method—maintain a level of robustness that can be trusted in a wide array of scenarios, reinforcing their utility in economic analysis and forecasting.

Table 3

Comparison of Error Metrics

Condition/Error Metrics	Mean Average Error (MAE)	Root Mean Square Error(RMSE)	Mean Average Percentage Error(MAPE)	Symmetric Mean Average Percentage Error (sMAPE)

Baseline	63.93	93.52	0.025	0.012
Outliers	97.59	141.33	0.039	0.019
Skewness	93.59	132.49	0.036	0.018
Fat tails	72.97	106.67	0.029	0.014

Source: author's calculations

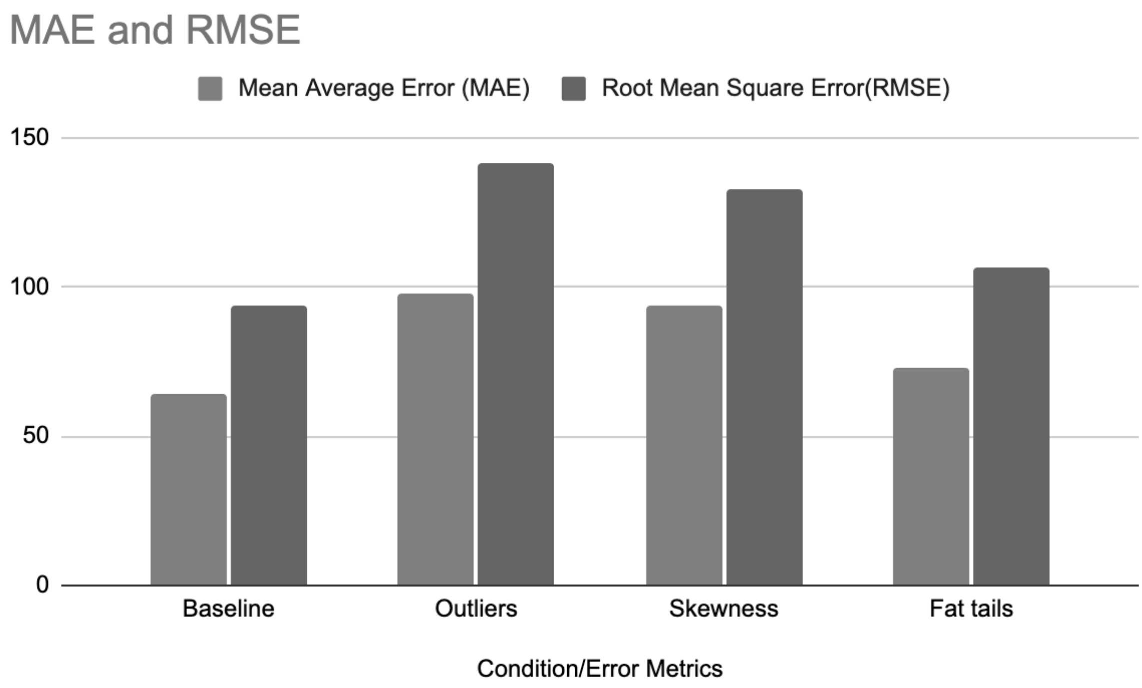

Figure 14. MAE and RMSE Comparison

Source: author's calculations

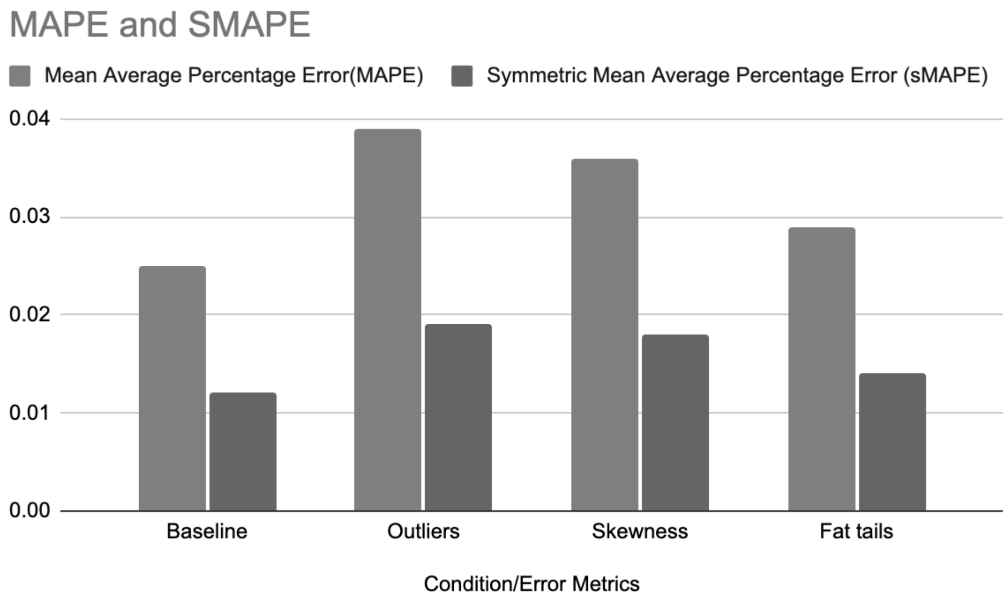

Figure 15. MAPE vs SMAPE Comparison

Source: author's calculations

Conclusion

This thesis rigorously explored temporal disaggregation methods, focusing specifically on assessing the performance of the Denton method under conditions challenging economic datasets with introduced anomalies. Through meticulous simulations incorporating outliers, fat tails, and skewness into quarterly and annual GDP data for Estonia, the aim was to replicate the unpredictable nature of economic data and evaluate the resilience of the Denton method in such scenarios.

The comparison of error metrics across different types of anomalies yielded several key insights. Notably, the Denton method consistently showcased robustness, demonstrating lower error metrics across Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE), and Symmetric Mean Absolute Percentage Error (sMAPE). This is particularly evident in the presence of outliers, where the Denton method showcased a comparatively moderate increase in errors, indicating its resilience in handling sudden, large deviations from the norm. Regarding fat tails, while there was an increase in errors, the Denton method exhibited remarkable performance. Despite the introduction of fat tails, the Denton method maintained relatively low error rates compared to the baseline. This indicates that even in scenarios of extreme events, the Denton method can still provide a reliable economic outlook, emphasising its robustness in handling unpredictable data distributions.

The impact of skewness, though noticeable, still displayed better results compared to outliers. This suggests that while asymmetric growth patterns can influence forecast accuracy, the Denton method can still produce reliable results in such conditions. However, the extreme levels of each anomalous condition can also change it. This means we would face a problem in a scenario where the outliers and skewness perform significantly high.

Additionally, while cubic spline interpolation may be suited for applications requiring data smoothing, our work underscores that it does not perform as well when precise and accurate high-frequency economic data generation is required from lower-frequency totals. In contrast, the Denton method proves to be more adept at ensuring the integrity of disaggregated economic data, making it an indispensable tool for economic forecasting and policy-making. This is especially crucial in scenarios where economic decisions hinge on the accuracy of high-frequency data derived from annual figures.

In conclusion, our analysis demonstrates the Denton method's robustness in the face of anomalies, reaffirming its utility in economic analysis and forecasting. Its consistent performance across different types of anomalies signifies its reliability in providing stable forecasts, crucial for informing economic policy effectively even under challenging conditions. This research contributes to the advancement of economic analysis by emphasising the critical need for accurate temporal disaggregation methods, particularly in environments marked by complexity and unpredictability.

Strengths: The study's methodological framework, characterised by stringent evaluation methods and sophisticated simulations, ensures the reliability and validity of the insights drawn. Leveraging ARIMA models and Monte Carlo simulations, the study provides a nuanced understanding of temporal disaggregation methods' performance, offering valuable insights for applied economic forecasting.

Limitations: While the study acknowledges its reliance on synthetic anomalies and the limited scope of examined disaggregation techniques, future research could address these limitations by incorporating real-world economic data and exploring additional methods. Moreover, integrating machine learning models could further enhance the robustness and accuracy of temporal disaggregation methods under varied economic conditions, extending their application beyond traditional economic analysis.

By addressing these recommendations, future studies can continue to advance the reliability of economic data disaggregation, thereby supporting more informed policy-making and economic analysis.

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Appendix A
Github repository and research papers table

- Github repository with code: https://github.com/kostiantynvoskovtsov/masters-thesis
- Research papers and interactive error metric comparison tables: https://docs.google.com/spreadsheets/d/1LaGiawJsxFG7vGhlcKSkeMn2HLmSjxYZ6ry9O_rupXU/edit?pli=1#gid=1532041382

Appendix B
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21/05/2024

Kokkuvõte

Hinnates Dentoni ajalisest lagundamise tehnika robustsust majandusprognoosides anomaalsetes tingimustes

See lõputöö uuris põhjalikult ajalisel jaotamisel põhinevaid meetodeid, keskendudes eriti Dentoni meetodi toimivuse hindamisele keerukate majandusandmekogumite tingimustes, kus on sisse viidud anomaaliaid. Hoolikate simulatsioonide kaudu, mis sisaldasid Eesti kvartali- ja aasta põhistes SKP andmetes kõrvalekaldeid, rasvaseid sabasid ja kalduvust, püüti jäljendada majandusandmete ettearvamatust ning hinnata Dentoni meetodi vastupidavust sellistes olukordades.

Erinevate anomaaliate puhul kasutatud veemõõdikute võrdlus tõi esile mitu olulist tähelepanekut. Eriti paistis silma, et Dentoni meetod näitas järjepidevalt vastupidavust, demonstreerides madalamaid veemõõdikuid nagu keskmine absoluutne viga (MAE), ruutkeskmine viga (RMSE), keskmine absoluutne protsentuaalne viga (MAPE) ja sümmeetriline keskmine absoluutne protsentuaalne viga (sMAPE). See oli eriti ilmne kõrvalekallete olemasolu korral, kus Dentoni meetodi vead suurenesid mõõdukalt, näidates selle meetodi võimet toime tulla ootamatute ja suurte kõrvalekalletega normist. Rasvasabade puhul suurenesid küll vead, kuid Dentoni meetod näitas märkimisväärset jõudlust. Vaatamata rasvasabadele säilitas Dentoni meetod suhteliselt madalad veamäärad võrreldes algväärtusega.

See näitab, et isegi äärmuslike sündmuste korral suudab Dentoni meetod pakkuda usaldusväärset majanduslikku ülevaadet, rõhutades selle vastupidavust ettearvamatute andmejaotuste käsitlemisel. Kuigi kalduvuse mõju oli märgatav, näitas Dentoni meetod siiski paremaid tulemusi võrreldes kõrvalekalletega. See viitab sellele, et kuigi asümmeetrilised kasvumustrid võivad mõjutada prognooside täpsust, suudab Dentoni meetod sellistes tingimustes ikkagi usaldusväärseid tulemusi toota. Kuid iga anomaalia äärmuslikud tasemed võivad samuti seda muuta. See tähendab, et probleem võib tekkida olukorras, kus kõrvalekalded ja kalduvus on märkimisväärselt kõrged. Lisaks, kuigi kuupsplaini interpolatsioon võib sobida andmete silumiseks, rõhutab meie analüüs, et see ei toimi nii hästi, kui on vaja täpset ja täpset kõrgsagedusliku majandusandmete genereerimist madalsageduslik est andmetest. Seevastu Dentoni meetod osutub sobivamaks tagama jaotatud majandusandmete terviklikkust, muutes selle hädavajalikuks tööriistaks majandusprognooside ja poliitika kujundamise jaoks. See on eriti oluline olukordades, kus majandusotsused sõltuvad täpsete kõrgsageduslike andmete saamisest aasta põhistest arvudest. Kokkuvõttes näitab meie analüüs, et Dentoni meetod on anomaaliate korral vastupidav, kinnitades selle kasulikkust majandusanalüüsis ja prognoosimisel.

Selle järjepidev jõudlus erinevate anomaaliate korral tõendab selle usaldusväärsust stabiilsete prognooside pakkumisel, mis on majanduspoliitika tõhusaks teavitamiseks kriitilise tähtsusega isegi keerulistes tingimustes. See uurimus aitab kaasa majandusanalüüsi edendamisele, rõhutades täpsete ajaliste jaotamise meetodi kriitilist vajadust, eriti keerukuse ja ettearvamatuse tingimustes.