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UNIVERSITATIS TARTUENSIS
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MATHEMATICA, PHYSICA, MEDICA

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K. Mattieseni trükikoda o./ü.. Tartu, 1939.

Sisukord. — Contenta.

RESEARCHES ON THE PHYSICAL THEORY OF METEOR PHENOMENA

III

BASIS OF THE PHYSICAL THEORY OF METEOR PHENOMENA

BY

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TARTU 1937

Printed by K. Mattiesen, Ltd., Tartu 1937.

List of Standard Notations and Abbreviations.

A = Paper A of this series, cf. Ref. [1].
I = Paper I „ „ „ , cf. Ref. [2].
II = Paper II „ „ „ , cf. Ref. [3].
C. G. S. units used except when explicitly stated otherwise.
μ = molecular weight.
δ = density of nucleus (solid, or liquid).
ϱ = density of the atmosphere; $\varrho = \varrho_0$ for $H = 0$.
γ = density of coma.
c_1, c_2 = specific heats in solid, resp. liquid state.
f = latent heat of fusion.
h = latent heat of vaporization.
q_1 = heat up to, and including fusion, erg/gr.
q_2 = heat from fusion to vaporization.
$q = q_1 + q_2$.
P = pressure of vapours.
D = density of vapours.
ξ = intensity of vaporization (mass vaporized per cm² and sec).
p, p_n = total aërodynamic pressure, and its normal component.
p_s = average aërodynamic resistance per cm² of cross-section.
α = angle of incidence of the air molecules.
$\varkappa$, $\varkappa$ = fraction of the relative kinetic energy of the air molecules, available for heating the nucleus.
$\varkappa_0$ = value of $\varkappa$ for direct (unshielded) impact into the meteor nucleus.
d = thickness of a layer of vapours, or air, measured in units of the kinetic half-energy range.
E = heat generated by aërodynamic work and transported per second toward the nucleus.
F = total heat generated by aërodynamic work and absorbed per cm² cross-section of the nucleus.
M = air mass traversed by cm² of the cross-section of the meteor.

B = pressure in the atmosphere (barometric pressure).
g = acceleration of gravity.
z_r = zenithal angle of incidence of meteor trajectory (zenith distance of the radiant point).
H = height.
$H_{\text{ɔ}}$ = difference in height for an atmospheric density ratio $e:1$.
H_{10} = same, for a density ratio $10:1$.
L = length of visible trail.
R = radius, R_0 = initial radius of a spherical nucleus.
r = radius of the drop-fragment, detached by fissure or spraying.
Q = intensity of black body radiation.
T = absolute temperature.
k = Stefan's constant; also different coefficients of proportionality.
w = velocity of the meteor; w_0 = initial velocity.
$u_{\text{ɔ}}$ = velocity of sound.
U_e = equatorial velocity of the rotation of the nucleus.
u = proportion of air admixed to coma.
ω = angular velocity of rotation.
σ = surface tension of liquid.
ν = coefficient of viscosity of the liquid.
k_t = conductivity for heat.
η = liquefied fraction of the mass of the nucleus.
$\lambda_{\text{ɔ}}$ = kinetic half-energy range.
a, a_m = air mass accumulated in air cap, gr/cm^2.
m = apparent magnitude of meteor; m_0 = absolute magnitude; m_z = zenithal magnitude.
β = visual efficiency, or fraction of kinetic energy converted into visible radiation.
s_c = fraction of secondary atomic collisions taking place within the coma.
I = absolute luminosity (visual).

Basis of the Physical Theory of Meteor Phenomena [1]).

Abstract.

The present paper prepares a foundation for the general theory of meteor phenomena.

Probable values of the physical constants for the meteor substance are adopted. Formulae, valid for low atmospheric densities when no air cap is formed, are given for the aërodynamic pressure and work.

A first approximation theory is applied for the purpose of obtaining estimates of the physical conditions in meteors of different size, velocity, and composition. The first approximation formulae may be used for a study of observational data referring to the dependence of the mean height of meteors upon velocity and magnitude, and to estimates of the density gradient of the atmosphere; absolute estimates of the density of the atmosphere, and the calculation of the range in height for meteors cannot be made with sufficient reliability on the basis of the first approximation.

Fissure of the molten meteor as well as the spraying of the liquid during the process of fusion are found to play an important rôle in meteor phenomena; conditions of fissure and spraying are investigated, the viscosity of the liquid being one of the chief parameters. Rotation influencing spraying determines the character of the meteor display.

Conditions of the formation of an air cap, of the heat transfer within the nucleus under different conditions, and

1) Continued from [1], [2], and [3].

various other phenomena are investigated. The upper limit of size for meteor dust caught by the terrestrial atmosphere without getting vaporized is estimated.

The conditions of radiation by atomic collisions are investigated; for most naked-eye meteors except the very brightest the degree of dilution of the coma is found to be such that secondary collisions mostly happen in the free atmosphere instead of taking place between the atoms of the vaporized meteor substance; this result is essential for quantitative estimates of the radiation from meteors.

The shielding effect of meteor vapours with respect to the flow of heat toward the nucleus is investigated; the shielding effect increases with the velocity and size of the meteor; for actual conditions the shielding effect is always only slight.

Three typical problems, for numerical treatment in a second approximation theory, are outlined (cf. Synopsis) on the basis of the scope of our estimates.

Introduction.

The interpretation of meteor observations requires as a basis a sound physical theory of meteor phenomena taking place during the flight of the meteor in our atmosphere. With such a purpose in view the present research was undertaken.

The problem is rather complicated and must be approached by consecutive approximations. The first step is an approximate theory where the radiation losses of the meteor nucleus, phenomena connected with capillarity, viscosity, and conductivity for heat, as well as rotation, are neglected; this framework theory, simple enough for direct analytical treatment, furnishes preliminary estimates of the physical conditions at different points of the meteor trajectory, estimates which are necessary for deciding such questions as whether a liquid drop breaks into pieces, or not, etc.; these estimates serve, so to speak, as traffic signals in developing the detailed numerical theory which forms the next step of our task. Some principles of our framework theory have been already published and applied elsewhere[1], and in many cases the approximate theory works well. On the other hand, such a problem as a satisfactory theory of the average light curve of a meteor can generally be solved only by numerical methods.

The physical theory of meteor phenomena may be divided into the following three major problems.

Problem 1: the theory of the flight of the meteor in the upper atmosphere without formation of an "air cap"; the meteor nucleus is bombarded by single air molecules directly, because their mean free path is large as compared with the depth of the air layer accumulated in front of the nucleus; practically all "naked eye" meteors up to very bright fireballs[1]) remain within the limits of this problem, never reaching portions of the atmosphere dense enough for the formation of an air cap; in this case the deceleration of the meteor even near the end point of its trajectory is small and may be neglected in most cases.

Problem 2: the theory of the flight of the meteor with the formation of an air cap. The latter shields the meteor from excessive heating and favours the penetration of the meteor into the atmosphere, in some cases making it possible for remnants of the nucleus to reach the ground; deceleration may be very considerable in this case; this problem refers to fire-balls of the brightness of the moon and brighter, and particularly to meteorite falls.

Problem 3: the theory of the impact of a meteorite on solid ground[2]. This problem is concerned chiefly with the formation of meteor craters, although the theory of small meteors needs some of the results of the theory of impact, namely those referring to rotation[3].

With our special purpose in view, in the present paper we shall confine ourselves to Problem 1 exclusively.

A brief review of earlier work on the theory of naked eye meteors has been given in *A* [2]). Of the former theories, Sparrow's work[4] must be regarded as the most successful. A recent contribution by the late Dr. Fisher[5] deals chiefly with Problem 2.

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1) cf. Section 3. *c* below.

2) cf. [1], pp. 5—6.

A complete theory of observable meteor phenomena can be built up only on the basis of a knowledge of the structure of the upper atmosphere. Now, this structure is yet unknown to us; we rather expect to learn something about the structure from meteor observations. Our meteor theory therefore must be adjustable to a considerable extent; it must not be bound to some particular model of the upper atmosphere. Actually we describe below a kind of standard theory, referring to an arbitrary constant logarithmic density gradient of the atmosphere; with the aid of this standard theory it may be possible, at least with a good degree of approximation, to build up a theory for an atmosphere of arbitrary structure. An iron sphere was taken as the principal basic model in our calculations, the physical properties of iron at elevated temperatures being known better than those of meteor stones. The probable difference in the behaviour of stone meteorites is sketched in general outline. As to the probable non-sphericity of actual meteors, no fundamental qualitative difference can be produced in our theory by this circumstance. However, the absolute height of meteors may be considerably influenced by their shape, a circumstance which must be kept in mind when dealing with meteor streams such as the Perseids, or Leonids[3].

Section 1.

General Physical Data and Formulae.

The data were taken mostly from Landolt-Börnstein's Tables, also from the International Critical Tables and Smithsonian Physical Tables; they are also partly guessed from general physical considerations. CGS units and absolute temperatures everywhere used.

a. Adopted numerical data for iron.

Density $\delta = 7.8$; molecular weight $\mu = 56 =$ atomic weight.

Melting point 1800^0; boiling point 3508^0; average specific heat of solid $c_1 = 6 \times 10^6$ (erg/gr. deg), of liquid $c_2 = 7.5 \times 10^6$; latent heat of fusion $f = 2.7 \times 10^9$, of vaporization $h = 6.0 \times 10^{10}$ (erg/gr).

Table I.

Pressure (P), density (D), maximum flow (ξ) of saturated iron vapours, and black body radiation (Q).

T	1800^0	2000^0	2200^0	2400^0	2600^0	2800^0
$P \frac{\text{dynes}}{\text{cm}^2}$	40.7	321	1766	7140	23 400	64 600
$D \frac{\text{gr}}{\text{cm}^3}$	1.55×10^{-8}	1.10×10^{-7}	5.50×10^{-7}	2.04×10^{-6}	6.17×10^{-6}	1.58×10^{-5}
$\xi \frac{\text{gr}}{\text{cm}^2 \cdot \text{sec}}$	6.24×10^{-4}	4.67×10^{-3}	2.46×10^{-2}	9.50×10^{-2}	0.298	0.794
$Q \frac{\text{erg}}{\text{cm}^2 \cdot \text{sec}}$	5.99×10^8	9.12×10^8	1.34×10^9	1.88×10^9	2.60×10^9	3.50×10^9

T	3000^0	3200^0	3400^0	3600^0	3800^0
P	1.59×10^5	3.54×10^5	6.85×10^5	1.29×10^6	2.26×10^6
D	3.63×10^{-5}	7.59×10^{-5}	1.38×10^{-4}	2.46×10^{-4}	4.07×10^{-4}
ξ	1.890	4.07	7.65	14.0	23.8
Q	4.61×10^9	6.00×10^9	7.64×10^9	9.60×10^9	1.19×10^{10}

The maximum flow, ξ, is the maximum amount vaporized per cm² of surface; this quantity is computed from the adiabatic formula

$$\xi = 0.654\, u_0 D = 950 \sqrt{T} D = \frac{6.5 \times 10^{-4} P}{\sqrt{T}} \quad . \quad . \quad . \quad (1),$$

where u_0 is the velocity of sound. The amount lost by vaporization is always equal to ξ when the ratio of vapour pressure to external pressure, $\frac{P_v}{P_e}$, exceeds 2.05; for smaller ratios the vaporized amount is roughly proportional to $\frac{P_e}{P_v}\sqrt{\frac{P_v}{P_e} - 1}$, thus it varies slowly[1]). For meteors, we may assume the amount vaporized per cm² and second equal to ξ with a fair degree of approximation, when $\frac{P_v}{P_e} > 1$, and equal to zero, when $\frac{P_v}{P_e} \leqq 1$.

1) with $\frac{P_v}{P_e} = 1.1$ only, the flow still amounts to about $0.55\ \xi$.

Effective surface tension, $\sigma = 950$ (dyne/cm) (observed 950 to 997) near the temperature of fusion; at effective temperatures of vaporization of meteors σ may be smaller, about 850.

Viscosity of the liquid at vaporization temperatures small, of the order of $\nu = 0.01 \left(\frac{\text{dyne} \times \text{sec}}{\text{cm}^2}\right)$.

Conductivity for heat $k_t = 4 \times 10^6$.

b. Adopted numerical data for meteoric stone.

Density 3.4. Average atomic weight = 28 [1]). Average composition (Noddack) (including 5.5% Troïlite):

O = 39.8%; Mg = 15.1%; Si = 20.3%; Fe = 15.4%; S = 3.8%; Al = 1.5%; Ca = 1.8%; Na = 0.7%; all remaining elements = 1.6%.

The different minerals during vaporization are supposed to be dissociated into simple oxides; iron oxides, with their low energies of binding, must be completely dissociated; with the average composition by weight as given above this gives the following schematical composition of the vapours: SiO_2 = 43.5%; MgO = 25.1%; CaO = 2.5%; Fe = 15.4%; S_2 = 3.8%; O_2 = 3.8%; Al_2O_3 = 3.2%; Na_2O = 0.9%; rest 1.8%. These figures lead to an average molecular weight of the vapours $\mu = 52$.

Melting point of Olivine from 1600° to 2000°, of Troïlite 1470°; of different other minerals occurring in meteors from 1450° to 1800°. The effective melting point of a stone meteor apparently does not differ much from the melting point of iron, 1800°.

The effective boiling point (at 760 mm pressure) is determined by the boiling points of the simple constituents into which the minerals probably are decomposed during the process of vaporization, and before it: SiO_2, b. p. 2500°; MgO, b. p. 3070°; CaO, b. p. 3120°; Fe, b. p. 3508°; the effective boiling point may be assumed about 3000°, thus lower than for iron. The curve of vapour pressure probably runs flatter than for iron because of the various constituents which are volatilized step by step as the temperature rises.

Average specific heat of solid $c_1 = 1.1 \times 10^7$; of liquid $c_2 = 1.3 \times 10^7$; for the latent heat of fusion no data are available;

1) The average is computed weighting by relative proportion of weight, and not by relative number of atoms.

assuming it equal to the heat of fusion of Li_2SiO_3, we get as a guess $f = 3.4 \times 10^9$ (erg/gr). The latent heat of vaporization can be estimated from Trouton's law with more certainty; applying this law separately to the constituents SiO_2, MgO, etc., and adding about $200 \frac{\text{cal}}{\text{gr}}$ for the heat of dissociation of silicates (a quantity estimated from the known thermochemical data of a few silicates), we get $h = 5.6 \times 10^{10}$ (with an uncertainty of perhaps $\pm$ 10 per cent).

For surface tension $\sigma = 400 \left(\frac{\text{dyne}}{\text{cm}}\right)$ may be a fair guess at fusion, and $\sigma = 360$ at temperatures of volatilization.

The viscosity of molten silicates is known to be very large; in Landolt-Börnstein we find for artificial Diopside (group of Pyroxenes, common in meteorites) at $T = 1553^0$, $\nu = 106$ and at $T = 1573^0$, $\nu = 33$; these values refer to temperatures close to the melting point; unfortunately we are unable to make any reliable estimate of the viscosity at the temperatures of volatilization, about 2500^0; it is not probable that the viscosity continues decreasing as fast as it does near the melting point; we may set the following probable limits for the effective viscosity of the liquid at volatilization: $10 > \nu > 0.1$. Conductivity for heat $k_t = 2 \times 10^5$.

c. *Radiation from the nucleus.*

The total loss by radiation from the nucleus (solid or liquid) is given by Stefan's formula

$$Q = kT^4, \text{ where } k = 5.7 \times 10^{-5} \frac{\text{erg.}}{\text{cm}^2.\text{sec}} \quad . \quad . \quad . \quad . \quad (2).$$

Values of Q are given in Table I.

We notice that the radiation from the nucleus is of little significance in producing observable radiation[1]); nevertheless, radiation losses must be considered in a detailed meteor theory because of their influence upon the balance of heat of the nucleus.

d. *Aërodynamic resistance and coefficient of accomodation.*

In the absence of an air cap, i. e. of hydrodynamic stream-lines, each molecule of the air hits the nucleus of the meteor directly and rebounds independently of other air molecules; the

1) cf. [1] and earlier authors.

theory of air resistance in such a case leads invariably to the square law of velocity, whatever the velocity is.

Let p denote the total aërodynamic pressure, p_n its component normal to the surface (thus p_n is the radial component for a spherical meteor), p_s the average resistance[1]) per cm² cross section of a spherical body, ϱ the density of the atmosphere, w the velocity, α the angle of incidence, and $\varkappa$ the fraction of relative kinetic energy converted into heat available for the heating of the meteor nucleus. $\varkappa$ is practically very close to the so called "coefficient of accomodation". Three typical cases may be considered.

1) Perfectly elastic impacts, when the particles rebound according to the laws of ideal reflection; evidently $\varkappa = 0$; in this case the pressure is normal to the surface,

$$p = p_n = 2\varrho\, w^2 \cos^2\alpha \quad \ldots \ldots \quad (3),$$

and

$$p_s = \varrho\, w^2 \quad \ldots \ldots \quad (4);$$

the heating effect upon the nucleus is zero, $\bar{\varkappa} = 0$.

2) Inelastic impacts of the first type, when the impinging particles lose their normal component, but preserve their tangential component of velocity; thus $\varkappa = \cos^2\alpha$. The pressure is again normal to the surface,

$$p = p_n = \varrho\, w^2 \cos^2\alpha \quad \ldots \ldots \quad (5),$$

and

$$p_s = \frac{1}{2}\,\varrho\, w^2 \quad \ldots \ldots \quad (6).$$

3) Inelastic impacts of the second type, when the impinging particles lose all their relative velocity and are thus completely carried away by the meteor; $\varkappa = 1$. The resultant total pressure is in the direction of motion; we have

$$p_n = \varrho\, w^2 \cos^2\alpha \quad \ldots \ldots \quad (5');$$

$$p_s = \varrho\, w^2 \quad \ldots \ldots \quad (4').$$

The total heat absorbed by the nucleus is equal to the relative kinetic energy of the penetrated air mass. The total work of aërodynamic resistance per cm² and second is $\varrho\, w^3$, but the heating effect is only one-half of that[2]), or

$$E' = \frac{1}{2}\,\varrho\, w^3 \quad \ldots \ldots \quad (7).$$

1) In the direction of motion.

2) The other half of the aërodynamic work is spent in accelerating the air to the velocity of the meteor.

In the case of velocities of the order of those occurring in meteors, the mean penetration into matter of an impinging molecule is from five to eight times greater than at ordinary velocities of gas molecules[1]); thus the air molecule penetrates deep into the solid or liquid surface of the meteor, losing its energy mostly in collisions with the meteor atoms placed at a certain depth below the surface of the nucleus; with decreasing velocity the penetrating power of the air molecule decreases and the molecule (which actually must be dissociated into atoms, as the result of the collision) will be unable to regain the surface of the meteor; or at least, when the air molecules return to the surface, they will possess an average translational energy corresponding to the temperature of the meteor, whereas most of their kinetic energy will be absorbed by the meteor; in such a manner a kind of "energy trap" establishes itself already at velocities of 5 km/sec (for nitrogen), which is much smaller than the lowest possible meteor velocity. On the other hand, sometimes an atom of the meteor material may be "knocked out" when hit by the air molecule at the very surface of the meteor; this atom will carry away some fraction of the kinetic energy of the impinging molecule, which fraction (together with the lattice energy) is no more available for heating the meteor; on the other hand, the energy dissipated beneath the surface will not have much chance to escape. Considering all these circumstances, a rough calculation indicates that the major part of the relative kinetic energy of the impinging molecules is transformed into heat and absorbed by the nucleus. For direct impact into a solid or liquid surface of the meteor nucleus a value of $\varkappa_0 = 0.80$ may be a fair estimate. We may add that we get such a high value of $\varkappa$ even when the individual collisions between the air molecule and the molecules of the meteor substance are perfectly elastic.

Sparrow[4] assumes $\varkappa = 0.37$ to 0.27 (nitrogen), from a consideration of a certain type of inelastic collision between air molecules and meteor atoms [something like our case 2), applied to individual atomic collisions]; with his mechanism of individual collisions, and our "penetration trap", a value close to 1 would have resulted. Direct observations of positive ions support our

1) cf.[1], pp. 16, 23, 29.

conclusions as to the high value of $\varkappa$ at high velocities; thus Voorhis and Compton[6] find (impact into metallic surface) for A^+, $\varkappa = 0.75$ (vel. 10 to 25 km/sec), for Ne^+, $\varkappa = 0.65$ (vel 15 to 40 km/sec), for He^+, $\varkappa = 0.35$ (vel. 30 to 50 km/sec), and for same, $\varkappa = 0.55$ (vel. 80 km/sec).

Table II is the result of approximate calculations of the shielding effect of vapours. The "diffusion" of the kinetic energy by secondary collisions is taken into account.

Table II.

Relative Shielding Effect of a Protecting Layer of Vapours (or air).

Thickness of layer, in units of the half-energy range, d	0	1	2	3	4	$d > 4$
$\varkappa$	(1.00)	(0.77)	0.58	0.43	0.32	$\frac{1.28}{d}$

In this table, the values for $d > 2$ are supposed to represent absolute values of $\varkappa$; for $d = 1$, a value of $\varkappa = \varkappa_0$ instead of 1 must be adopted.

In Section 3. *i* we find that the shielding effect of meteor vapours can never cause $\varkappa$ to fall much below its "unshielded value", $\varkappa_0$, and that $\varkappa = 0.60$ appears to be a fair approximation for all possible cases during the process of vaporization. During the process of heating up to, and including, fusion, $\varkappa = \varkappa_0 = 0.8$ may be a better estimate. Nevertheless, in our first approximation $\bar{\varkappa} = 0.60$ is assumed throughout.

Thus, in the case of naked eye meteors case 3) of aërodynamic resistance appears as the closest to actual circumstances; we assume, for the sake of convention, a certain weighted average between cases 1) and 3):

$$p_s = \varrho w^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4) (4'),$$

and

$$p_n = (2 - \varkappa') \varrho w^2 \cos^2 \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8);$$

Here $\varkappa'$ may be called the "dynamical coefficient of accomodation". The central pressure (for $\alpha = 0$) is

$$p_o = (2 - \varkappa') \varrho w^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

For the sake of convention we take $\varkappa' = 0.60$, $2 - \varkappa' = 1.4$, although $\varkappa'$ may be greater than $\bar{\varkappa}$, the "thermal" coefficient of accomodation.

The work of aërodynamic resistance per unit cross section and second is ϱw^3, and the total heat absorbed by the nucleus per second is (R = radius of meteor)

$$E = \frac{1}{2} \bar{\varkappa} \pi R^2 \varrho w^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10),$$

where $\bar{\varkappa}$ is a value averaged over the cross-section. If dq is an amount of heat absorbed per gram of mass of the nucleus, the deceleration is found from (4) and (10) as

$$dw = -\frac{2\,dq}{\varkappa w} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10^a).$$

e. Stability of liquid non-rotating drops.

The aërodynamic forces are unevenly distributed over the surface of the drop and tend to break it to pieces; surface tension (σ) keeps the drop together. For a given aërodynamic pressure, there may be assigned a certain limiting radius of stability, R_s; a drop smaller than R_s (which denotes the radius of the original undistorted sphere) may keep together, drops larger than R_s certainly break. The exact theory of stability is very complicated; but the transition from stability to instability is more or less abrupt, and R_s may be determined fairly well from an approximate theory. Such a theory [1]) gave:

$$R_s = \frac{4\sigma}{p_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11).$$

As a check of the order of magnitude, at least, we may apply formula (11) to the case of a non-adhering gravitating drop ($g = 981$) supported by a flat surface; we get $R_s = \sqrt{\frac{\sigma}{330\,\delta}}$; this gives for mercury $2R_s = 0.72$ cm, for water $2R_s = 0.94$ cm, or values near the right order of magnitude. We notice that the application of (11) to the case of a drop breaking under its own weight is not quite legitimate because of the different character of the distribution of pressure; also, in the gravitating drop the average pressure decreases with flattening; whereas the drop flattening under aërodynamic pressure will be subject to a more or less constant pressure, and will thus break more easily.

1) Still too complicated to be described here.

f. Shedding off drops.

The conditions under which a partly solid, partly liquid body loses detached drops are still more complicated than the preceding case. To guess the order of magnitude, we worked out approximately the model of a solid circular shield, subject to aërodynamic pressure on the front side, and carrying on the rear side a liquid fraction η of the whole mass (solid + liquid) the effective spherical radius being R (i. e. total mass $= \frac{4}{3}\pi\delta R^3$), assumed equal to the radius of the shield, the condition of stability is found as

$$R \leqslant \frac{\sigma\left(1+\frac{3}{2}\eta\right)}{\eta^2\left(1+\frac{3}{4}\eta\right)p_s} = \frac{R_s\left(1+\frac{3}{2}\eta\right)}{4\eta^2\left(1+\frac{3}{4}\eta\right)} \quad . \quad . \quad . \quad . \quad (12).$$

The formula is a very rough approximation, and must not be used for η near 1, when evidently the shield cannot exist and when formula (11) is to be preferred. In the process of fusion, for a given radius, η cannot exceed the limit set by (12); all the extra liquid is shed off and sprayed by drops which are small as compared with the nucleus.

g. Atmosphere.

The actual dynamical structure of the upper atmosphere, i. e. the form of the density function, is not of fundamental importance for the development of the present theory (cf. Introduction). On the other hand, the composition of the upper atmosphere must be known to some extent; at least, we must know what is the rôle of hydrogen in the upper atmosphere. The hydrogen atom at meteor velocities carries only a small amount of kinetic energy[1]), so that under 20 km/sec no visible impact radiation can be produced; at higher velocities, visible impact radiation can be produced, but with very low efficiency, which means greater masses of meteors of a given brightness[2]), and a greater relative importance of the continuous radiation of the nucleus. Thus the assumed relative amount of hydrogen, as compared with nitrogen and oxygen, may influence our theory to a considerable extent. The relative amount of helium has not so great an influence. On the other hand, the relative

1) cf.[1], p. 13, Table II, also p. 18, form. (12), and pp. 26—32.

2) cf.[1], p. 39.

amount of nitrogen and oxygen, elements of similar atomic weight, or their actual molecular structure (degree of dissociation and ionization), is of no such importance in the development of our theory, although in the applications of the theory such questions may arise.

Not so very long ago the prevailing view on the structure of the upper atmosphere, sponsored by Humphreys[7], Jeans[8], and others, ascribed to hydrogen a predominant rôle at heights over 80 kilometres where meteors appear. Sparrow in his meteor theory[4] accepts this view, but he finds that the amount of hydrogen postulated by Humphreys is too large. On the other hand, Chapman[9] denies any rôle to hydrogen; according to his views, a layer of maximum ozone concentration at 50 km height is followed by layers of increasing content of monatomic oxygen, O and O^+ (produced by the dissociating and ionizing action of the solar short wave radiation); the green (auroral) line of the night sky, λ 5577, is explained as a forbidden transition of monatomic oxygen $({}^1S \rightarrow {}^1D)$[10].

We feel inclined to accept Chapman's view; at least, the amount of hydrogen postulated by Humphreys is contradicted by a number of observational facts; we notice that Humphrey's calculations are based on very uncertain determinations of the hydrogen content near the earth's surface, and on a still less certain assumption of an undisturbed "conductive" equilibrium 1) of the atmosphere from a height of 11 km on. Only in such a form of equilibrium will the lighter constituents such as hydrogen, or helium, show a smaller density gradient, and may outnumber the heavier elements in the upper atmosphere. Let us first consider the probability of such a form of equilibrium.

Ordinarily only two kinds of atmospheric equilibrium (from the standpoint of mixing) are considered: the conductive and the convective equilibrium. The upper atmosphere, at least in its lower strata, is known as more or less isothermal, even with a possible inversion of temperature; it has been justly pointed out by different authors that under such conditions convective currents cannot persist, any rising current being rapidly stopped through adiabatic expansion making the ascending current colder and heavier than the undisturbed strata and vice versa. Thus convective equilibrium must be certainly rejected.

1) Corresponding to pure diffusion.

But there is another form of mixing which may produce almost the same large-scale effects as convection currents: turbulence. Near the earth's surface turbulence is a powerful mixing agent[1]), and it may play an important rôle also in the upper atmosphere; the wave-shaped *cirrus* clouds may be regarded as an indication that turbulence is certainly active at heights up to 10 km.

Turbulence is irregular wave motion. From the standpoint of mixing, only the vertical component of the wave motion is of interest. We may roughly estimate the relative importance of turbulence and diffusion in the following way. Let λ be the mean free path, u' the average vertical component of molecular velocities, ΔH the average amplitude of the turbulent wave, U the mean (absolute) vertical velocity of turbulence. The efficiency, from the standpoint of a transport of mass (or heat) in vertical direction, is proportional to the vertical component of velocity and to the mean depth of direct interchange; thus the relative efficiency of the two processes is

$$\text{turbulence : diffusion} = \frac{U \Delta H}{u' \lambda}.$$

On the other hand, for an isothermal atmosphere the amplitude of turbulence depends upon the maximum vertical velocity $= 2\,U$ (approximately). With $2\,U = 5$ m/sec (which does not seem excessive, taking into account the considerable wind velocities revealed by meteor trains), we find $\frac{1}{2}\,\Delta H = 250$ metres. Further, from our subsequent approximate theory (as well as Sparrow's) it follows that average meteors appear (centre of trajectory) in a stratum of density about $\frac{1}{200\,000}$ to $\frac{1}{2\,000\,000}$ of atmospheric density at sea level, which gives λ from 2 to 20 cm. Taking $u' = 250$ m/sec, the above data give the ratio of efficiency in mixing:

$$\text{turbulence : diffusion} = \text{from } 250 \text{ to } 25.$$

Turbulence at meteor heights may thus be much more efficient than diffusion, in which case the atomic composition of the atmosphere at these heights will be identical with the compos-

1) Its effects may be easily observed on smoke from chimneys; the so-called diffusion of gas clouds in chemical warfare is almost entirely due to turbulence.

ition at sea level (except for water vapour); we should proceed to much greater (130—150 km) heights to find diffusion as the major factor. Only beginning from such great heights a differentiation of the different constituents according to molecular weight may set in; but not before heights of about 200 km are reached can hydrogen (if present at all) become relatively conspicuous. For the meteor theory in such a case the presence of hydrogen may be entirely neglected.

The absence of hydrogen lines in the spectrum of the aurora, and especially of meteors[11], supplies serious evidence against the existence of considerable amounts of hydrogen in the upper atmosphere.

Rough calculations based on the height of appearance of fireballs, such as the well observed Pultusk meteorite (Jan. 30, 1868) which increased by about 6 magnitudes between 260 and 180 km height[1]), give an increment of density of 10:1 for each 33 km decreasing height. Such a figure is incompatible with hydrogen or helium at any reasonable temperature (radiative equilibrium), but very well agrees with monatomic oxygen (or nitrogen).

Even assuming the "conductive" equilibrium to hold at moderate heights, it may be shown that hydrogen in the amounts postulated by Humphreys and Jeans appears extremely improbable. At a certain height hydrogen and oxygen may be found in the right proportion to form an explosive mixture, and if not outnumbered by the inert nitrogen the mixture will explode at the first fireball or meteor penetrating the layer. Assuming that a temperature of 800° C of the mixture is sufficient to support the explosion, we find that an amount of hydrogen not exceeding $^1/_{23}$ fraction of the amount assumed by Humphreys can exist; the explosive layer (partial pressures of $N_2:O_2:H_2 = 17{,}2:1{,}0:2{,}0$) will be found at $H = 82{,}0$ km in such a case. With Humphreys's data, the explosive layer lies between 60 and 70 km height, and the temperature of explosion exceeds 1000° C. We do not believe in the "conductive equilibrium" at these heights; the above figures are quoted merely to show that even in the case of such a sort of equilibrium the possible amount

1) At these heights the radiation of the meteor is probably pure impact radiation at the solid surface before appreciable heating sets in; the brightness in such a case is proportional to the density of the atmosphere. Cf. [1], p. 24.

of hydrogen in the upper atmosphere must be much smaller than that supposed by Humphreys and others.

Thus we seem to be justified in building up our first approximation theory on the assumption of an upper atmosphere composed chiefly of heavy constituents such as nitrogen and oxygen. We note that our former computations referring to meteor radiation[1] were based on the same assumption.

h. Air masses and densities.

The variation of gravity with height and the effect of the curvature of the earth (which sets in only at great angles of incidence) being neglected, the total air mass per unit cross section encountered by the meteor down to a certain level is given by

$$M = \frac{B \sec z_r}{g} \left(\frac{\text{gr}}{\text{cm}^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13);$$

$$\text{also } dB = -g\varrho dH \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14).$$

Here B is the barometric pressure $\left(\frac{\text{dyne}}{\text{cm}^2}\right)$ at that level, z_r the zenith angle of incidence, or the zenith distance of the meteor radiant, g the acceleration of gravity ($\bar{g} = 953$ at 89 km height), H the height.

Formulae (13) and (14) apply to an atmosphere of arbitrary structure. In the particular case of a "uniform" atmosphere, of constant logarithmic decrement of density (or pressure) with height, we have:

$$\varrho = \varrho_0 \, e^{-\frac{H}{H_0}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15),$$

$$B = g\,\varrho_0 H_0 \, e^{-\frac{H}{H_0}} = g H_0 \varrho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16),$$

$$M = \varrho_0 H_0 \sec z_r \, e^{-\frac{H}{H_0}} = \varrho H_0 \sec z_r \quad . \quad . \quad . \quad (17).$$

Here ϱ_0 is the density of atmosphere at $H = 0$; H_0 is the "effective height" of the atmosphere; the density and pressure decrease in the ratio $1 : e$ for a difference of height equal to H_0; if all the mass of the atmosphere above the level H is given a constant density equal to ϱ, the thickness of the layer thus formed is H_0. We have also:

$$H_0 = 8.270 \times 10^7 \frac{T}{\mu g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18),$$

where μ is the average molecular weight. For $g = 981$ and dry air ($\mu = 28.82$) at $T = 273^0$ we find $H_0 = 7.985 \times 10^5$ cm.

The model of a uniform atmosphere, although far from actual conditions when the whole atmosphere is considered, is nevertheless a useful approximation when dealing with a particular layer, more or less limited in height, such as the region of visibility of naked-eye meteors; the model presents the advantage of permitting a very simple analytical treatment; the value of H_0 in such a case is a certain effective, or average quantity, chosen to satisfy equations (17) or (16) over a given limited range in height; H_0 may vary from level to level.

Taking into account that in meteor phenomena the air mass M, traversed by the meteor, is the chief argument (independent variable), a meteor theory built up on the model of a uniform atmosphere is believed to hold also[1]) in the case of a non-uniform atmosphere of the same average parameter H_0, when we assume that all the characteristics of the meteor flight in the non-uniform atmosphere at a given value of M are identical with the characteristics calculated for the same value of M in the uniform atmosphere. Thus, by representing the results of our calculations as a function of M, we make them practically independent of our assumptions regarding the actual structure of the atmosphere.

Section 2.

First Approximation for Slowly Rotating Meteors.

As stated in the Introduction, it appeared useful to work out an approximate theory for the purpose of making estimates of the physical conditions prevailing during the flight of the meteor in our atmosphere; a second approximation is made possible only by the preliminary results of this first approximation. From the standpoint of simplification, our first approximation resembles former theories, notably Sparrow's[4]; the purposes and consequences, however, are different; our first approximation pursues auxiliary purposes and cannot be regarded as a satisfactory theory itself.

The following simplifying assumptions are here made:

1) the temperature is constant throughout all the solid or liquid nucleus; in other words, a sufficiently high conductivity for heat is postulated;

1) Within the limits of uncertainty of any theory whatever.

2) vaporization starts suddenly at a certain temperature above the temperature of fusion; thus the gradual rise of vapour pressure is neglected;
3) after vaporization has started, all the heat absorbed by the nucleus is spent in vaporization; this follows also directly from assumptions 1), 2), and 4);
4) heat losses by radiation are neglected;
5) the time of visibility is assumed equal to the time of vaporization 1);
6) deceleration is neglected [12];
7) the integrity of the liquid drop is assumed.

Condition 2) is artificial, never being fulfilled; its introduction makes the obtained shape of the light curve illusory without however seriously affecting the calculated mean heights and the duration of visibility (or the length of the visible path); on the other hand, assumption 3), although artificial, well represents the actual conditions on the major part of the visible path of an observable slowly rotating meteor (as follows from computations made for the second approximation, which we hope to discuss in one of the next papers of the present series).

Conditions 5) and 6) are practically fulfilled in all cases of observable meteors; 5), because the chief source of visible radiation is the collision of the vapour atoms with the air molecules[1]; 6), because the heat of vaporization is small as compared with the kinetic energy[12].

Condition 4) is a good approximation in most cases, except for faint and slow telescopic meteors, when 6) also fails (because a large fraction of the kinetic energy is spent in radiation).

Condition 1) is well fulfilled in the case of a liquid drop and fails for the solid nucleus, which circumstance, however is of minor importance; for a low viscosity of the liquid, the actual process consists in liquefaction on the front side and the sweeping of the fused material to the rear side; a further rise of temperature does not start before all the nucleus is liquid.

The actual limitations of all these assumptions follow from the approximate theory itself (cf. Section 3).

The total heat intercepted from the beginning by unit cross section of the nucleus when reaching a certain atmospheric

1) cf. [1], pp. 5 and 22—26; also [4] and [12].

level is a fraction $\bar{\varkappa}$ of the relative kinetic energy of the traversed air mass, or

$$F = \frac{1}{2}\bar{\varkappa}Mw^2 \quad \ldots \ldots \ldots \quad (19).$$

The mass of the nucleus per unit of its cross section is

$$\frac{4}{3}\pi\delta R^3 : \pi R^2 = \frac{4}{3}\delta R.$$

Let the total amount of heat required to heat the unit mass of the meteor from a certain initial temperature to vaporization be $q = q_1 + q_2$, where q_1 is the heat up to, and including fusion. Liquefaction is complete when $F = \frac{4}{3}\delta R q_1$, or when the air mass is

$$M_l = \frac{\frac{8}{3}\delta q_1 R_0}{\bar{\varkappa}w^2} \text{ (liquefaction)} \quad \ldots \quad (20);$$

here R_0 is the initial radius. Similarly, vaporization starts when

$$M_a = \frac{\frac{8}{3}\delta q R_0}{\bar{\varkappa}w^2} \text{ (point of appearance)} \quad \ldots \quad (20').$$

After the beginning of vaporization, the amount of heat absorbed by unit cross section of the nucleus when passing an infinitesimal air mass dM is $dF = \frac{1}{2}\bar{\varkappa}w^2\, dM$. This heat is spent in vaporization, i. e. in diminishing the mass per unit cross section, or

$$dF = -h\, d\left(\frac{4}{3}\delta R\right).$$

The following linear equation between radius and air mass results [with the aid of (20)]:

$$M = \frac{\frac{8}{3}\delta R_0}{\bar{\varkappa}w^2}\left[q + h\left(1 - \frac{R}{R_0}\right)\right] \quad \ldots \ldots \quad (21).$$

The equation is valid during vaporization only.

For $R = 0$, or for the point of disappearance we get:

$$M_d = \frac{\frac{8}{3}\delta(q+h)R_0}{\bar{\varkappa}w^2} \quad \ldots \ldots \ldots \quad (22).$$

The ratio of air masses at disappearance and appearance is

$$\frac{M_d}{M_a} = \frac{q+h}{q} \quad \ldots \ldots \ldots \ldots \quad (23).$$

Formula (23) actually determines the effective length of the observable trail.

Assuming a uniform atmosphere, we get the density of the atmosphere at a certain point of the meteor path from

$$\varrho = \frac{M \cos z_r}{H_0} \quad . \quad . \quad (17') \text{ (cf. Section 1.}h.);$$

for a particular point, M may be taken from formulae (20) — (22).

Let us call such points on the trajectories of two meteors for which the ratio $\frac{R}{R_0}$ is the same similar points. From (21'), (17'), and (15) we get the following equations which permit us to draw some general conclusions regarding physical conditions at similar points of the trajectories of different meteors:

$$\varrho = \frac{\frac{8}{3}\,\delta R_0 \cos z_r}{H_0\, \varkappa w^2}\left[q + h\left(1 - \frac{R}{R_0}\right)\right] \quad . \quad . \quad . \quad . \quad (24),$$

$$H = H_{10} \log \left\{ \frac{H_0\, \varkappa w^2 \varrho_0}{\frac{8}{3}\,\delta R_0 \cos z_r \left[q + h\left(1 - \frac{R}{R_0}\right)\right]} \right\} \quad . \quad . \quad (25),$$

where

$$H_{10} = \frac{H_0}{\log e} = 2.303\ H_0 \quad . \quad . \quad . \quad . \quad . \quad (26)$$

is the increment in height for an atmospheric density ratio of 10:1. For meteors of the same composition at similar points of their paths from (24) we infer that the density of atmosphere is proportional 1) to the initial radius, 2) to the cosine of the angle of incidence, and 3) inversely proportional to the square of velocity.

From (25) we find that the difference in height of similar points of the trajectories of two meteors is

$$H_1 - H_2 = H_{10}\left(2 \log \frac{w_1}{w_2} - \log \frac{R_1}{R_2} - \log \frac{\cos z_1}{\cos z_2}\right) \quad . \quad . \quad (27),$$

where w_1, w_2, R_1, R_2 are the initial velocities and radii of the two meteors, z_1, z_2 — the angles of incidence.

For meteors of different composition but of the same radius and velocity ($\varkappa$ and H_0 assumed to be constant) at similar points of their paths we find that the density of the atmosphere is proportional to δq at appearance, and to $\delta\,(q + h)$ at the point of disappearance.

The difference in height is in this case

$$H' - H'' = -H_{10}\left[\log\frac{\delta'}{\delta''} + \log\frac{q'+h'\left(1-\frac{R}{R_0}\right)}{q''+h''\left(1-\frac{R}{R_0}\right)}\right] \quad . \quad . \quad . \quad (28).$$

Formulae (27) and (28) hold not only under the restrictions of the present approximate theory, but partly also in the general case.

From (23), (17), and (15) we get the difference between height of appearance and disappearance:

$$H_a - H_d = H_{10}\log\frac{q+h}{q} \quad . \quad . \quad . \quad . \quad . \quad . \quad (29);$$

thus the range in height depends only upon the heat ratio of the material and upon the vertical density gradient of the atmosphere.

The length of the observable path is evidently

$$L = H_{10}\sec z_r \log\frac{q+h}{q} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30),$$

or it increases with the increasing angle of incidence; this latter statement is confirmed, qualitatively at least, by direct observations[1]). As to the absolute value of L, formula (30) depends greatly upon the validity of our assumption 1); it is not advisable to use the formula for stone meteors.

Meteor observations actually represent a selection by luminosity. It is convenient therefore to replace radius by luminosity in the above equations. Formulae (55) and (54) of Section 3. *h*, together with (30), give with a fair degree of approximation:

$$\operatorname{Log} R_0 = -\frac{2}{15}m_0 - \log w - \frac{1}{3}\log\cos z_r + \text{const.}$$

Substituting this in (27), we get, for two meteors of the same composition:

$$H_1 - H_2 = H_{10}\left[3\log\frac{w_1}{w_2} + \frac{2}{15}(m_1 - m_2) - \frac{2}{3}\log\frac{\cos z_1}{\cos z_2}\right] \quad . \quad . \quad . \quad (27');$$

1) E. g. in the Leonid shower of 1931 as observed in Arizona, when the shower started with long trails (50—80°), the radiant being low; when the radiant was high, the trails became short (5—10°).

here m_1 and m_2 are the absolute magnitudes of the two meteors. Formula (27′) proves to be of considerable value in the analysis of meteor heights. The error of the formula is the smallest for the point of disappearance. As a differential formula it remains more or less valid over a wide range of meteor sizes for which many of our assumptions may fail.

For the heat constants of iron, according to Section 1. *a*, we find (assuming an initial temperature of 280° abs., and an effective temperature of vaporization of 2800°):

$$q_1 = 1.18 \times 10^{10} \text{(erg/gr)};\ q_2 = 7.5 \times 10^9;\ q = 1.93 \times 10^{10};$$
$$h = 6.0 \times 10^{10};\ q + h = 7.93 \times 10^{10};$$
$$\frac{q+h}{q} = 4.11 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iron)}.$$

For stone, the data are less certain; with an initial temperature of 280° and an effective temperature of vaporization of 2300° we find, according to Section 1. *b*:

$$q_1 = 2.01 \times 10^{10};\ q_2 = 6.5 \times 10^9;\ q = 2.66 \times 10^{10};$$
$$h = 5.6 \times 10^{10};\ q + h = 7.86 \times 10^{10};$$
$$\frac{q+h}{q} = 2.95 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(stone)}.$$

The parameter of the atmospheric density gradient, H_{10} (or H_0), can be obtained from observations only; an accurate value can be derived only with the aid of a more refined theory. However, for the purpose of preliminary estimates we must know H_{10}, approximately at least. Approximate values of this parameter may be guessed in different ways.

1) Assuming the upper atmosphere to be in radiative equilibrium with the surface of the earth, $T = \frac{288^0}{\sqrt[4]{2}}$, $g = 953$ cm/sec², from (18) we get, assuming $\mu = 28.82$,

$$H_{10} = 2.303\ H_0 = 15.8 \text{ km}.$$

This may be a minimum, because the average molecular weight must be smaller than assumed (dissociation, ionization) and the temperature may be higher.

2) From the Pultusk meteor, in Section 1. *g* we estimated $H_{10} = 33$ km for the topmost atmosphere (above 180 km).

3) Unpublished results of the Arizona Expedition [1]), referring

1) Based upon the heights of about 3600 meteors.

to the height of disappearance as a function of luminosity and to the mean height as a function of velocity, in connection with formula (27′) gave preliminarily a mean value of

$$H_{10} = 24.6 \pm 1.1 \text{ km}.$$

In the numerical estimates of Section 3., the round value of $H_{10} = 20$ km (or $H_0 = 8.7$ km) is adopted. Taking into account the preliminary character of our estimates and of our first approximation, there can be no serious consequences from a possible error in the adopted value of H_{10}; in any case the error cannot be large.

Section 3.

Estimates of the Physical Conditions, on the Basis of the First Approximation.

We repeat that the first approximation refers to small, slowly rotating meteors of an upper limit of size and rotation to be determined below. Certain results of our estimates, however, are so definite that they may be applied also without the above limitations.

a. Difference in height of stone and iron meteors.

When dealing with differences in the composition of meteors, it appears to be sufficient to consider only two extreme types — the typical iron, and the typical stone. From the observed falls of meteorites we know that stones largely outnumber irons, a circumstance which may be due to better preservation during the flight through the atmosphere of the stone core protected from heat action by the relatively small thermal conductivity; on the other hand, iron meteorites prevail among finds, again an evident result of selection, because irons are more likely to attract attention. Among observed meteor spectra[11] stones and irons seem to appear more or less equal in number, a circumstance which may represent actual conditions. Large meteor showers, such as the Leonids, however, seem to consist almost exclusively of stone[11]. In any case in a meteor theory one should keep in mind both kinds of objects.

For some purposes, it is sufficient to make the detailed computations for iron, with differential estimates of the properties of stone relative to iron.

The ratio in the absolute length of trail, under similar conditions, is given by (30):

$$\frac{L \text{ stone}}{L \text{ iron}} = \frac{\log \dfrac{q'+h'}{q'}}{\log \dfrac{q+h}{q}} = \frac{\log 2.95}{\log 4.11} = 0.765.$$

Small slowly rotating stones are expected to yield shorter visible trajectories than small irons (at the same angle of incidence). According to Section 3. *e* below, this applies to telescopic meteors only. Larger stones must show much longer trails.

From form. (28), with the proper numerical data, we find that stones are expected to appear 0.22 $H_{10} = 4.4$ km higher, and disappear 0.365 $H_{10} = 7.3$ km higher than iron meteors of an equal radius; the middle of the visible path is expected to lie about 5.6 km higher for stones than for iron meteors. If, however, objects of equal mass are considered, the stone must have a radius 1.32 times the radius of the iron, which, according to formula (27), requires by $H_{10} \log 1.32 = 2.4$ km lower height. Finally, for meteors of the same mass, shape, velocity, and angle of incidence, stones are expected to appear on the average by 0.16 $H_{10} = 3.2$ ($\pm$) km higher than iron meteors. We do not yet know the relative luminous efficiency of iron and stone; therefore, when objects of equal luminosity are compared, the difference in height of stone and iron may be considerably changed.

b. Fissure and spraying of the liquid.

For slow rotation, when centrifugal force is insignificant, the condition of fissure of a drop is $R > R_s$, where R_s is given by equation (11). We shall try to investigate the variation of R_s over the meteor path.

From equations (11), (4) (4′), (17′), and (20) we find the condition for the breaking of a drop just at the moment of complete fusion to be independent of velocity and given by

$$R_0 > R_s = \sqrt{\frac{3}{2} \frac{\sigma \bar{z} H_0}{\delta q_1} \sec z_r} = \sqrt{\frac{C}{q_1} \sec z_r} \ldots (31), \text{ where } C = \frac{3}{2} \frac{\sigma \bar{z} H_0}{\delta}.$$

For the point of appearance the same formula holds when q is substituted for q_1.

We find the condition of first rupture during the process of vaporization by setting the variable radius R in (21′) equal to R_s. Let

$$\frac{R}{R_0} = y \quad \ldots \ldots \ldots \quad (32);$$

from (11), (4) (4′), (17′), and (21′) we get for the initial radius of a drop which breaks first when the radius has decreased to $R = y R_0$, the value

$$R_0 = \frac{R_s}{y} \sqrt{\frac{C \sec z_r}{y [q + h (1 - y)]}} \quad \ldots \ldots \quad (33).$$

We notice that y varies only within the restricted limits from 1 to 0. For this interval, the expression (33) reaches a minimum when

$$y = y_0 = \frac{q + h}{2h} \quad \ldots \ldots \ldots \quad (34)$$

if $q < h$, a condition fulfilled for most substances. This minimum value of R_0 is

$$R_0 = \sqrt{\frac{4h}{(q + h)} \frac{C \sec z_r}{(q + h)}} = \sqrt{\frac{2 C \sec z_r}{y_0 (q + h)}} \quad \ldots \quad (35).$$

Meteors of initial radius smaller than (35) never break from aërodynamic pressure. The formula does not apply to the case of detached drops originating from a larger nucleus, or to the fissure products of an originally larger drop.

In deriving (34) and (35) we tacitly assumed σ to be constant; for a surface tension slowly varying with temperature the results hold equally well. Table III contains the limiting radii of fissure at different characteristic points of the trajectory computed from the preceding formulae and the numerical constants adopted above.

Table III.

Limiting Conditions [1]) of Fissure of the Liquid Meteor.

Vertical incidence ($z_r = 0^0$); $\bar{z} = 0.6$; $H = 8.7 \times 10^5$ cm. *a*: first fissure at liquefaction; *b*: first fissure at point of appearance; *c*: first fissure at point of optimum fissure, $y = y_0$, or ultimate limit of fissure

	$\bar{\sigma}$	C	y_0	Limiting initial radius R_0, cm		
				a	b	c
Iron	900	9.02×10^7	0.661	0.087	0.068	0.059
Stone	380	8.75×10^7	0.702	0.066	0.058	0.056

When the radius is smaller than the value given under the heading "*c*" of Table III, the drop is expected to maintain its integrity all over its path. For other angles of incidence, or other values of H_0 the values for R_0 of Table III must be multiplied by $\sqrt{\frac{H_0 \sec z_r}{8{,}7.10^5}}$. The uncertainty in the basic physical data is considerably diminished for R_0 by the square root in formulae (31) and (35). Large drops which burst will show evidently a more violent increase in light (because the surface increases and vaporization becomes more intense), and a shorter path; larger meteors may exhibit a strong and more or less sudden outburst of light due to the spraying of the liquid into millions of drops; bursts and "spindles" observed in bright meteors may be apparently explained in this way.

However, for stone meteors viscosity may prevent fissure (Section 3. *e*); also, it appears that rotational centrifugal force may be a more powerful spraying agent than aërodynamic pressure; in such a case the outbursts of light must be explained in a different way (cf. Sections 3. *d* and 4).

In the case of fissure into smaller drops, each of radius $\sim R_s$, the surface exposed to aërodynamic heating increases in the ratio $\frac{R}{R_s} = \frac{R_0 y}{R_s}$. According to (11), $R_s \sim \frac{1}{p_s}$; according to (24), $p_s \sim \varrho \sim R_0$. Hence $\frac{R}{R_s} \sim R_0^2$; if for case *c*, Table III, $\frac{R}{R_s} = 1$, for iron of $R_0 = 0.236$ cm the surface should increase through fissure

1) Independent of velocity.

$4^2 = 16$ times, whereas the number of fragments is of the order of 4000. For still larger meteors, however, there will be no opportunity for such a sudden outburst; the nucleus will spray the liquid during the whole process of fusion; thus, according to (12), a maximum fraction of liquid mass of the order of $\eta \sim 0.1$ can be retained by the "shield", the rest being sprayed away, when $R = 25\ R_s$, or $R_0 \sim 0.5$ cm; the detached drops are pulverized into pieces of the order of R_s, small as compared with R_0, and therefore practically instantaneously vaporized. Therefore, for such large iron meteors, even when rotation is slow, visibility is synchronous with the fusion of the large nucleus. Evidently the formulae of the first approximation do not apply to the case of such large meteors. Assuming an effective angle of incidence $z_r = 45^0$, the apparent magnitude of meteors at the limit of fissure observed 45^0 from the zenith may be estimated according to Table VIII as follows:

Table IV.

Apparent Magnitude (m) at Limit of Fissure (for slowly rotating meteors).

w, km/sec	16	25	40	60	100
m, Iron	8.3	6.9	5.6	4.4	3.0
m, Stone	9.3	7.9	6.6	5.4	4.0

The first approximation may be assumed to hold, from the standpoint of fissure, still for meteors two magnitudes brighter than those of the table.

The figures of Table IV depend upon certain assumed values of the "heat factor" β, postulated to be the same for iron and stone; as only the order of magnitude of the heat factor can be estimated, our figures in Table IV may be systematically in error; however, these figures are sufficient to show that the effect of fissure and spraying may be important for naked-eye meteors, especially for those of low velocity. Earlier authors did not take into account fissure and spraying in developing theories of meteor phenomena in naked-eye meteors; thus earlier theories are similar to our "first approximation". If the neglect of fissure in Sparrow's theory [4] is not of serious consequences, because from Table IV we conclude that such a theory applies at least to the fainter naked-eye meteors, in Lindemann and

Dobson's theory[13] the neglect leads to a striking contradiction between their postulates and consequences. These authors, by assuming a very small value of $\varkappa$, arrive at very high densities of the atmosphere along an average meteor path; with such densities, violent fissure into smaller drops must occur (even the viscosity of stone cannot prevent fissure in such a case), so that a theory based on the assumption of a meteor unbroken all over its trajectory cannot have much resemblance to what actually would occur. We may add that Lindemann and Dobson's chief postulates, which led to the small value of $\varkappa$ and great atmosphere densities, namely, the assumption of an air cap in front of the meteor, and the assumption of the adiabatic formula for the temperature in such an air cap, do not hold: the first assumption for reasons put forward by Sparrow[4] and Maris[14], and explained in the following section; the second assumption because it violates the fundamental law of the conservation of energy[1]). Hence it follows that Lindemann and Dobson's theory, containing gross errors in the postulates, and leading to consequences contradicting their own postulates, must be rejected.

c. *Formation of an air cap.*

When the velocity of the meteor is large as compared with the velocity of sound in the undisturbed atmosphere, the air mass, or the mass per cm^2, accumulated in front of the meteor is proportional a) to the density of the surrounding atmosphere (independent of velocity, because the velocity of forced escape sidewise from the air cap is proportional to the velocity of the meteor[15,16]), and b) to the radius of the meteor (by reason of geometrical similarity). Thus, the air mass

$$a \sim \varrho R.$$

Neglecting fissure and spraying in the following, we assume evidently maximum values for R and ϱ (because the non-breaking meteor will penetrate into deeper portions of the atmosphere than the actual meteor that breaks); thus we get in such a

1) cf. Sparrow,[4]: "... The use of the adiabatic equation by Lindemann and Dobson is therefore equivalent to the assumption that a elocity of 60 km/sec is small compared to one of 0.5 km/sec". Also [1], p. 5 s.

manner for all possible initial conditions maximum values of the thickness a of the air cap.

A rough way to estimate a is the following. In front of the meteor the compression is so great that in spite of the rise of temperature the linear thickness of the air cap is small as compared with the radius (according to Epstein's data[16] it is of the order of $\frac{R}{4}$ for the "ideal" case and actually smaller on account of cooling through contact with the nucleus and through radiation). We assume simply an average surface density $a\ \frac{\mathrm{gr}}{\mathrm{cm}^2}$, and an effective velocity of sidewise escape kw. The mass of air caught per second by the cross section of the meteor is $\pi R^2 \varrho w$; this must be equal to the mass escaping sidewise, or to $2\pi R a k w$; hence we get

$$a = \frac{1}{2k} R\varrho.$$

In the "ideal" case, k is a large fraction; for 45^0 angle of impact, or for a quadratic profile of the projectile moving along a diagonal, Epstein's data[16] give $a = \frac{3}{2} b\varrho$, where b is one-half of the diagonal; within a close order of magnitude, for a sphere we may assume $b = R$, and

$$a = \frac{3}{2} R\varrho \quad \ldots\ldots\ldots\ldots \quad (36),$$

or $k = \frac{1}{3}$ for the "ideal" case.

With cooling in a ratio $\frac{T}{T_0}$, the velocity of escape sidewise will be smaller in the ratio $\sqrt{\frac{T}{T_0}}$ (neglecting viscosity), or a larger in the ratio $\sqrt{\frac{T_0}{T}}$. On the other hand, the fraction of heat absorbed by the nucleus is evidently close to

$$\bar{\varkappa} = 1 - \frac{T}{T_0},$$

$$\text{or } \frac{T}{T_0} = 1 - \bar{\varkappa} \quad \ldots\ldots\ldots\ldots \quad (37).$$

Hence we get for a cooling air cap

$$a = \frac{3R\varrho}{2\sqrt{1-\bar{\varkappa}}} \quad \ldots\ldots\ldots\ldots \quad (38).$$

According to this formula, a increases with increasing $\bar{\varkappa}$; according to Table II, the inverse relation takes place ($d \sim a$). A certain solution $0 < \bar{\varkappa} < 1$ must satisfy both conditions.

We are interested in the existence of an air cap only so far as it prevents the flow of heat toward the nucleus, i. e. so far as it makes $\bar{\varkappa}$ smaller; on the other hand, from (38) we infer that the largest value of $\bar{\varkappa}$ gives the largest a. Assuming $\bar{\varkappa} = 0.6$ as a maximum value which may be considered as differing sufficiently from 1, we get a maximum estimate of a[1]):

$$a_m = 2\, R\varrho \left(\frac{\text{gr}}{\text{cm}^2}\right) {}^{2)} \quad \ldots \ldots \ldots (38').$$

Substituting $R = R_0\, y$, and ϱ from (17′) and (21′), we get for a_m an expression which contains the product

$$y\,[q + h\,(1 - y)],$$

already investigated in Subsection b. Along the meteor trail it reaches a maximum when $y = y_0$ (cf. form. (34)). At this point of the trajectory the mass per cm^2 of the air cap is the largest for the given meteor; for this maximum maximorum of a we get

$$a_m = C_1\, \frac{R_0^2 \cos z_r}{w^2} \quad \ldots \ldots \ldots (39),$$

$$\text{where } C_1 = \frac{4}{3}\,\frac{\delta\,(q+h)^2}{H_0 \bar{\varkappa} h};$$

with our adopted data $C_1 = 2.09 \times 10^6$ for iron. Our incipient air cap corresponds to the case when a_m is equal to the air mass of the half-energy range, λ_0 ($d = a/\lambda_0 = 1$, cf. above). We must take into account the change of λ_0 with velocity[3]). If λ_0 refers to N. T. P., the transition value of a_m is $1.293\, \lambda_0 \times 10^{-3}$. Substituting this in (39), we calculate R_0 for different velocities and for an average $z_r = 45^0$. The results are[4]) (iron):

1) In other words, we are looking for the condition of an incipient air cap, such that a is of the order of the half-energy range, $d = 1$ in Table II, or $\frac{\varkappa}{\varkappa_0} = 0.77$; with $\varkappa_0 = 0.8$, $\bar{\varkappa} = 0.6$ appears to be close to the maximum possible value in such a case.

2) Round value of the coefficient used.

3) According to [1]; cf. Subsection h.

4) In [1], p. 23, formula (15), a factor of 10^6 is omitted. The calculations were made with the correct formula, but in Table VII, loc. cit., R_0 is given in mm instead of cm and the luminosities are overestimated. Fortunately, even this gross mistake has no effect on the conclusions made there.

w, km/sec	14.8	29.6	59.2	118.4
$10^5 \lambda_0$, cm	8.4	11.7	14.1	21.0
R_0, cm	0.40	0.95	2.14	5.07

Comparing these results with Table III, case a, we see that the air cap starts at values of R_0 much larger than the limit of fissure at fusion. Actually at the moment of complete fusion the original drop is broken into thousands and millions of drops, each of which has too small a radius for the formation of an air cap. Thus the most favourable conditions for the formation of an air cap must be sought during the process of fusion. We still overestimate the air cap if we assume that the meteor remains unbroken up to the moment of complete fusion; the maximum air mass in this case we find from (38′) (with $R = R_0$), (20) and (17′):

$$a_m = C_2 \frac{R_0^2 \cos z_r}{w^2} \quad . \; . \; . \; . \; . \; . \; (40), \text{ where}$$

$$C_2 = \frac{16}{3} \frac{\delta q_1}{H_0 \bar{\varkappa}}.$$

For iron we have

$$C_2 = 9.4 \times 10^5.$$

Table V gives the results of a computation according to (40) for $z_r = 45^0$; although computed for iron, the data may be regarded as representing the conditions for stone equally well, at least with respect to a close order of magnitude. The apparent magnitudes are guessed on the basis of some computations connected with the second approximation.

Table V.

Limiting Radius for the Formation of an Air Cap.

w, km/sec	14.8	29.6	59.2	118.4
Minimum R_0, cm	0.60	1.4	3.2	7.6
Appar. magnitude	0	—5	—11	—16

We notice further that the minimum radii found in this way correspond to an air cap of thickness less than the half-energy range (because shedding off drops and breaking makes the radius, and the air cap smaller); to obtain a noticeable effect in $\bar{\varkappa}$ we should have a layer at least four times thicker than that (cf. Table II) which means the doubling of R_0; the corresponding limiting magnitudes must be set in this case by 2 mag. brighter.

Thus we may repeat our former conclusion that "the formation of air caps is limited to fireballs of quite unusual brightness" [1]). When dealing with the statistical material of visual observations of meteors, we feel safe in neglecting the formation of air caps.

d. Rotation and oscillation of the nucleus.

In the preceding considerations we neglected the dynamical consequences of a possible rotation of the meteor. When the rotation is small, its influence on meteor phenomena is insignificant; rapid rotation, however, causes a spraying of the liquid during the process of fusion into small drops which are vaporized much faster than the integer meteor drop would have been; thus vaporization in this case is practically almost synchronous with fusion; the rotating meteor evaporates at a greater height and in a shorter time than the non-rotating meteor.

In Paper II we tried to evaluate the probable average velocity of rotation of meteors caused by collisions with other meteors; surprisingly high values are obtained; although the estimate of the average velocity of rotation is supposed to give only the order of magnitude, and although not all possible factors are taken into account, it appears highly probable that meteors actually rotate at very high speed, the mean equatorial velocity being of the order of 3000 cm/sec for $R = 0.1$ cm. If this is true, the theory of a slowly rotating meteor applies only to exceptions, whereas the rule is the theory of a meteor in rapid rotation [2]). Let us consider this case more closely.

Centrifugal force compels the fused material to move towards the "equator" of rotation, where it is shed off in drops the size of which depends upon the centrifugal acceleration, which, in notations of *II*, is equal to $\frac{U_e}{R}$. Let the effective

1) Cf. [1], p. 23.

2) For the larger stones the viscosity of the liquid may cause the two cases, of a rotating, and of a non-rotating meteor, to remain similar; cf. Subsections *e* and *f*.

radius of the drop shed off be r; a very close estimate of this quantity may be obtained from the equation

$$\frac{2}{3}\pi r^3 \delta \frac{U_e^2}{R} = 2\pi r \sigma,$$

in known notations. From this equation, for iron, and practically also for stone (with the adopted values of σ at fusion), we get

$$r = \frac{19.1\sqrt{R}}{U_e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41).$$

Substituting for U_e its expression from (9) of *II*, we get

$$r_0 = \frac{1}{60} R_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42).$$

The first drops shed are thus sixty times smaller in diameter, and sixty times sooner vaporized than the integer drop would have been. The proportionality of r and R obtained is curious.

As the process of spraying for a given nucleus goes on, the angular velocity of rotation decreases on account of loss of momentum through the liquid travelling from the "poles" toward the "equator" which it leaves with the highest possible speed. For constant angular velocity $U_e \sim R$; hence (41) yields $r \sim R^{-1/2}$, and $\frac{r}{R} \sim R^{-3/2}$. Because the angular velocity is not constant, but must decrease gradually, the increase of the ratio $\frac{r}{R}$ will proceed faster than estimated above, with a negative power of R larger than $\frac{3}{2}$; we may estimate it roughly at $\frac{r}{R} \sim R^{-5/2}$. Thus, for the validity of (42) as an initial condition, when the shrinking radius of the meteor reaches about one-fifth of its original value, $\frac{r}{R} = 1$, or no spraying or breaking can take place any more. The remnant core of the rotating meteor will move on without further fissure. The maximum diameter of individual drops (not counting fissure from aërodynamic pressure) is equal to this remnant core, or not more than about one-quarter of the initial diameter. It is clear that in the case of rapid rotation, the theory of meteor phenomena is very different from the theory, the basis of which has been exposed in the preceding sections.

The upper limit to what we called "slow rotation", in connection with the first approximation, may be defined now.

An increase, as the result of spraying, of the surface exposed to aërodynamic heating to double the original value may be considered as determining the margin between "slow" and "fast" rotation. This corresponds to $\frac{r}{R} = \frac{1}{2}$; equation (41) yields in this case:

$$U_e' = \frac{38.2}{\sqrt{R}} = 121 \sqrt{\frac{0.1}{R}} \frac{\text{cm}}{\text{sec}} \quad . \quad . \quad . \quad . \quad . \quad (43).$$

The margin equals $\frac{1}{30}^{th}$ of the probable velocity of rotation, estimated in *II*, (9). Thus, for an average naked-eye meteor, an equatorial velocity below 100 cm/sec, or less than about 100 revolutions per second, may be called "slow" from the standpoint of centrifugal force. From the reasoning put forward in *II*, such a low rotational speed appears extremely improbable, unless the space density of hyperbolic meteors is thirty times smaller than that assumed there.

Aërodynamic resistance can hardly stop or change the original rotation of a meteor considerably if it is fast enough. For a symmetrical body (sphere) the effect is practically nil, the relative loss of rotational momentum through friction during the flight in the atmosphere being comparable with the relative loss of linear velocity which is known to be small. For a non-symmetrical body aërodynamic resistance under certain conditions may stop slow rotation. In the interaction of aërodynamic pressure and rotation, two phases may be distinguished: the phase of acceleration (about one-half of the period of rotation), and the phase of retardation. For unchanged external conditions, and when the linear velocity of rotation is small as compared with the velocity of the projectile, the total momentum of acceleration is equal to the total retardation [1]), the sum of the momentum over one revolution being zero. Therefore, in an atmosphere of constant density, an initial rotation may continue indefinitely (at least during all the short time of visibility of the meteor), unless it is stopped during the first negative phase. Actually the meteor moves in an atmosphere of increasing

1) This holds exactly for the particular case of aërodynamic pressure considered here, when hydrodynamic stream lines are not formed, cf. Section 1. *d.* When an air cap is formed, conditions are very different, cf. below.

density, so that the positive and negative momenta also steadily increase. For the estimate of the order of magnitude we assume the period of revolution to be constant, and the duration of each of the two phases equal to one-half of the period; also, we neglect the difference of aërodynamic pressure due to the change of relative velocity during rotation (U_e assumed small as compared with w); actually the relative aërodynamic velocity is greater for the negative phase, and the duration of the negative phase is greater than for the positive phase; thus our assumptions mean underestimating the effect of retardation of rotation for a given density of the atmosphere.

Let the angular velocity of rotation be ω, the period thus $\frac{2\pi}{\omega}$. For a body of a given shape and dimensions the change of angular velocity during one phase is $\Delta\omega = \pm kw^2 \varrho$, the value of the constant k depending upon the shape of the nucleus; according to (15), with $\frac{dH}{dt} = -w \cos z_r$, or $H = -wt \cos z_r + \text{const.}$, we have $\varrho = e^{\frac{wt \cos z_r}{H_0}} \times \text{const.}$ Let us subdivide time into intervals of the half-period of rotation, according to the phases of interaction, and let the ordinal number of a given phase be n; we have

$$t = \frac{\pi n}{\omega}, \text{ and}$$

$$\Delta\omega = \pm kw^2 e^{bn},$$

$$\text{where } b = \frac{\pi w \cos z_r}{\omega H_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44).$$

The sum of all positive impulses up to n is equal to the sum of members of the form (44) for $n = n$; $n-2$; $n-4$; $\dots n = -\infty$, and is equal to $+kw^2 \frac{e^{bn}}{1-e^{2b}}$; the sum of all the negative impulses up to $n+1$ is evidently $-kw^2 \frac{e^{b(n+1)}}{1-e^{-2b}}$; the angular velocity at the end of the $(n+1)^{st}$ impulse (the last one being negative) is thus

$$\omega = \omega_0 - \frac{kw^2 e^{bn} (e^b - 1)}{(1 - e^{-2b})} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45).$$

When (45) yields $\omega < 0$, it means that rotation is stopped and converted into oscillations.

When b is small, which means that the atmospheric density changes little during one half-revolution, we have

$$\omega = \omega_0 - \frac{1}{2}\, kw^2\; e^{bn} \quad . \quad . \quad . \quad . \quad . \quad (45'),\ \text{or}$$

$$\omega = \omega_0 - \frac{1}{2}\, kw^2 \varrho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45''),$$

where the arbitrary constant of the density formula is included in the factor k.

Thus, for given angular and linear velocity, and for a meteor of definite shape and dimensions, there exists a certain value of $\varrho = \varrho_s = \frac{\omega_0}{\frac{1}{2}\, kw^2}$ at which rotation is stopped. The larger ω_0 is, the larger is ϱ_s. Now, a meteor of given size is able to penetrate down only to a certain depth where the atmospheric density attains the largest possible value ϱ_m; when $\varrho_m < \varrho_s$, rotation cannot be stopped. Without giving details, it suffices to mention that a computation was made for a case of "typical asymmetry", — a triangular iron prism of 30^0 angle rotating around its axis and moving at right angles to it. The limiting value of the period of rotation was found to be of the order of $\frac{\sqrt{\sec z_r}}{75}$ seconds (based on the first approximation), independent of velocity and absolute dimensions. Thus, for a speed of rotation of about 100 revolutions per second or more the original rotation will persist all over the visible path of the meteor; curiously enough, this is near the margin of "slow" rotation (43). As in most actual cases we may expect a closer coincidence of the centre of mass with the resultant vector of the aërodynamic force than in the assumed case of the iron prism, even smaller speeds of rotation are likely to persist. But even in the case where rotation is stopped by aërodynamic resistance, it is replaced by oscillations, which maintain a period of the same order of magnitude as the period of the original rotation. From the standpoint of the equalization of the heating effect over the surface of the meteor such oscillations must be almost as efficient as rotation.

We notice that the period of rotation itself, or the mean angular velocity was assumed constant for the whole meteor trail in the preceding formulae; the assumption being admissible with respect to the order of magnitude, it cannot be retained for a more accurate treatment; the period of rotation (when near the critical value), and of the subsequent oscillation of an asymmetrical body, must change steadily along the path of the meteor.

A meteor of asymmetrical shape, if not rotating originally, when moving in a resisting medium will be forced to oscillate by aërodynamic pressure; the period and amplitude of the oscillation decrease with penetration into deeper portions of the atmosphere; the period of forced oscillation, for bodies of not too unusual a shape, may lie between $\frac{1}{10}$ to $\frac{1}{100}$ of a second for the visible portion of the path of the meteor, independent of velocity and size. It is obvious that a completely fused nucleus is not subject to all the above described effects.

All the preceding considerations are valid only in case no trace of an air cap is present, e. g., for meteors of radii considerably smaller than those of Table V; as, according to (40), the air mass of the cap changes as R_0^2, about $\frac{1}{3}$ to $\frac{1}{4}$ of the radii of Table V are sufficiently small to render the aërodynamic effect of the air cap negligible; for average meteor velocities this condition is still satisfied for practically all naked-eye objects. For the brighter meteors, however, the accumulation of air mass in front of the projectile, even when insufficient to protect the nucleus from heating, may cause systematical deviations of the aërodynamic pressure depending upon the curvature of the surface (whereas the pressure produced by independent individual molecular impacts depends only upon the area of the contour projected upon the plane perpendicular to the direction of motion); a concave surface experiences a systematically greater pressure than a convex one. Let us consider first the case of the ideal continuous aërodynamics and the extreme case of the two hemispherical cups of a Robinson anemometer. For such cups, the aërodynamic pressure upon the concave surface is from 2.5 to 2.9 times greater than upon the convex surface; the excess of pressure on the concave surface equals thus about 0.9 of the average pressure. If such a pair of Robinson cups is made to move freely in the air, a rotational impulse about 0.9 of the aërodynamic deceleration of the centre of gravity is acquired. The effective deceleration of a meteor near the end of its path may be estimated from the condition that a fraction $\varkappa$ of the relative kinetic energy lost is spent in vaporizing the meteor; the neglect of the condition of changing mass does not affect the order of magnitude of this effective quantity. Thus, from equation (10^a) we get

$$\Delta w = -\frac{2(q+h)}{\bar{\varkappa} w}.$$

For iron, the following values are obtained ($\bar{\varkappa} = 0.60$):

$$\begin{array}{lll} w, \text{ cm/sec} & 20 \times 10^5 & 80 \times 10^5 \\ -\Delta w, \text{ cm/sec} & 1.32 \times 10^5 & 3.30 \times 10^4. \end{array}$$

The equatorial velocity of rotation of ideal Robinson cups should thus attain about 0.45 Δw, which is a considerable speed.

For meteor projectiles of irregular shape, such a systematical effect must inevitably exist, although in a much smaller degree, perhaps from 1 to 10 per cent of the Robinson cup effect; for the phase of fusion, when a real asymmetry can exist, the impulse Δw must be taken equal to about $\frac{1}{5}$ of the above; hence rotational effects for fireballs result of the order of 200 to 2000 cm/sec ($w = 20.10^5$), or 60 to 600 cm/sec ($w = 80.10^5$).

For ordinary naked-eye meteors the air cap pressure effect is smaller, depending upon the number of elastically reflected atoms hitting a concave surface a second time; the reduction factor is thus smaller than $1 - \varkappa_0 = 0.20$ and may be estimated at $\frac{1}{20}$ th; thus, for such meteors which originally did not rotate, forced velocities of rotation acquired from the "Robinson cup effect" may result, amounting to from 10 to 100 cm/sec ($w = 20.10^5$), or 3 to 30 cm/sec ($w = 80.10^5$), thus "small" as compared with the expected order of magnitude of the initial rotation (3000 cm/sec). These estimates are important so far as they show that it is extremely improbable for a meteor to be devoid of rotation while moving in the terrestrial atmosphere.

e. Heat transfer within a slowly rotating nucleus.

Our theory of the first approximation is considerably simplified by neglecting differences of temperature within different parts of the nucleus. The question arises to what limit such a simplification is permissible.

We notice that from considerations put forward above meteors are supposed to rotate, or to oscillate, with a minimum speed of slow rotation corresponding to about 30 revolutions per second for ordinary naked-eye meteors; thus the period of one revolution is short as compared with the duration of the visibility of the meteor and therefore this slow rotation seems to be quite efficient in producing an equalization of the heating effect of different portions of the surface of the nucleus. If in the following estimates we neglect rotation as an equalizing factor, thus considering a non-rotating meteor, or a meteor moving along its axis of rotation, we evidently exaggerate differences of temperature existing in the nucleus. The least favourable for an equalization of temperature is the case of a solid nucleus where the only means of heat transfer is conductivity.

Let us first consider the phase just before fusion, when the nucleus is solid throughout. With the aid of the formulae of the approximate theory, we find for the difference of temperature between the front and the rear the maximum value

$$\Delta T < \frac{8}{9} \frac{\delta q' R_0^2 w \cos z_r}{k_t H_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (46),$$

where $q' = q_1 - f$ is the heat up to fusion. With the constants adopted as before, for $z_r = 0^0$, $\Delta T < 200^0$, we find, for the moment of starting fusion:

$w =$	20×10^5	80×10^5	
iron, $R_0 <$	0.07	0.035	$(\Delta T = 200^0)$
apparent mag. in $45^0 >$	7	5	
stone, $R_0 <$	0.018	0.009	
appar. mag. $>$	11	9	

The adopted difference of temperature is not small; taking into account that the difference increases as R_0^2, we conclude that in the non-rotating solid nucleus equalization of temperature never takes place in naked-eye meteors. In the case of rotation equalizing the heating over the surface the values of R_0 must be increased four times [(or ΔT is 16 times smaller than in (46)]; thus the margin of equalization at starting fusion is: for iron, $R_0 = 0.28$ to 0.14 cm, for stone, $R_0 = 0.07$ to 0.035 cm, for the same velocities as above. It may be assumed that practical equalization of temperature in the solid nucleus sets in for iron meteors fainter than the 2^{nd}, and for stone meteors fainter than the 6^{th} apparent magnitude. This conclusion, derived for the moment of starting fusion, remains practically valid also for the earlier stages, although the conditions of equalization of temperature are somewhat more favourable then; also, oblique incidence of the meteor trajectory favours equalization ($\Delta T \sim \cos z_r$).

Fusion of the nucleus evidently starts at a spot exposed to the highest aërodynamic pressure, on the front side of the nucleus; the aërodynamic pressure compels the fluid to move and collect on the rear side of the projectile when rotation is slow (the fluid moves to the "equator" in the case of fast rotation); let us consider this effect more closely.

The pressure produced by the mostly inelastic impacts of the air molecules, in the absence of an air cap, is not perpendicular to the surface, but more or less in the direction of motion (cf. Section 1. *d*); therefore a tangential backward directed force acting upon the liquid layer exists, the force resembling more a force of viscosity or friction than true hydrodynamic pressure; such a tangentially dragging force, independent of the thickness of the layer, is much more efficient in propelling a viscous fluid than hydrodynamic normal pressure, the efficiency of which is proportional to the thickness of the liquid layer. To get an estimate of the existing conditions, the following theory may be regarded as sufficient. Instead of using variables in their complicated relationship, we introduce average, or effective quantities. Let ΔR denote the effective thickness of the liquid layer, flowing with a surface velocity u_s from the front hemisphere toward the rear of a nucleus of radius R, and let $\frac{\Delta R}{R}$ be small (only in such a case is the sweeping of the liquid toward the rear efficient, from the standpoint of the meteor theory). The total tangential force of aërodynamic resistance, acting upon the front hemisphere, is equal to $\frac{2}{3}\pi R^2 p_s$; this we set equal to the total viscosity friction in the liquid layer, $2\pi R^2 \nu \frac{u_s}{\Delta R}$; hence we get

$$\frac{1}{3} p_s = \frac{u_s \nu}{\Delta R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a).$$

Further, for pure fusion (without vaporization, or rising temperature), equilibrium conditions require the amount fused per unit of time to be equal to the amount flowing from the front to the rear, or

$$\frac{E}{f} = \frac{u_s \delta}{2} \cdot 2\pi R\, \Delta R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b).$$

Substituting into (b) E from (10), and u_s from (a), we finally get the following expression for the thickness of the layer:

$$\Delta R = \sqrt{\frac{3}{2} \frac{\bar{\varkappa} \nu}{f \delta} RW} \quad . \quad . \quad . \quad . \quad . \quad . \quad (47).$$

This expression does not contain absolute pressure, and, therefore, for a meteor of a given size and velocity, the equilibrium thickness of the liquid layer is a constant all over the portion of its path where fusion takes place (under the restrictions mentioned above).

As, with our assumptions, all the heat is spent in fusion, this heat is transported to the solid surface by conductivity; and whereas the bottom of the liquid is at a constant temperature of fusion (assumed 1800°), the top is hotter by an amount ΔT, so that

$$E = 2\pi R^2 k_t \cdot \frac{\Delta T}{\Delta R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c).$$

With the aid of equations (10), (20), and (17), we get

$$\Delta T = \frac{2}{3} \frac{\delta q_1}{k_t H_0} \cdot R\, \Delta R \,.\, w \cos z_r \quad . \quad . \quad . \quad . \quad . \quad . \quad (48);$$

here ΔR is to be taken from (47).

Equation (48) yields the maximum difference of temperature in the liquid layer, computed conventionally for the moment of complete fusion. We notice a considerable similarity of (48) and (46), as might have been expected. Further, as roughly speaking the luminosity I of a meteor is proportional to $R^3\, W^3$, or $RW \sim I^{1/3}$, we notice that in (48) and (47) the actual argument is luminosity,

$$\Delta R \sim I^{1/6}\, \nu^{1/2}, \text{ and}$$
$$\Delta T \sim I^{1/2}\, \nu^{1/2}.$$

This is not exactly true, because the luminous efficiency changes probably with velocity and mass.

In equations (47) and (48) an important criterion presents itself, allowing us to distinguish between two fundamentally different kinds of meteor phenomena. If ΔR, and ΔT are large, it means that the liquid has not time enough to flow away, and that vaporization at the liquid surface sets in before the nucleus is completely molten. If ΔR and ΔT are small ($\Delta T < 500^0 \pm$ for stone, $< 1000^0$ for iron), the liquid is swept from the front side before it gets heated, and a further rise of temperature or perceptible vaporization does not start before all the nucleus is molten. The decision depends chiefly upon ν, the coefficient of viscosity, and k_t, the thermal conductivity (assumed to be the same for the liquid and the solid).

For stone, at the temperature of fusion, values of ν of about 100 have been observed (cf. Section 1. *b*), and, for the more elevated mean temperature of the liquid layer, a value between the extreme limits $10 > \nu > 0.1$ may be suggested (ibidem). In view of the cold bottom of the liquid layer, the upper limit $\nu = 10$ may be regarded as the more probable. With this value, and with the rest of the constants as assumed before, the following table is computed from (47) and (48):

Table VI.

Characteristics of Fusion, Stone, $\nu = 10$, Slow Rotation.
m = apparent magnitude at $z = 45^0$; $z_r = 0^0$.

	$R = 1$ cm			$R = 0.1$ cm			$R = 0.02$ cm		
	ΔR	ΔT	m	ΔR	ΔT	m	ΔR	ΔT	m
$w = 16$ km/sec	0.035	(24 000^0)	−1	0.011	800^0	7	0.005	70^0	13
$w = 40$ " "	0.054	(92 000)	−5	0.017	(3100)	5	(0.008)	270	10
$w = 90$ " "	0.081	(320 000)	−9	0.026	(10 000)	2	(0.012)	870	7

For $R = 0.02$, ΔR comes out of the order of R itself, and thus has no analytical meaning; in such small stones, the large viscosity prevents any transport of matter; the slower meteors of such a radius ($m = 10$ and 13) are molten through before vaporization starts, whereas for $m = 7$, $\Delta T = 870^0$ suggests boiling (under low pressure) at the surface while the centre is still melting.

For $R = 1$ cm, the values of ΔT are enormous; this means that the layer never reaches the computed thickness ΔR, but starts boiling when its thickness is of the order of $\Delta R \times \frac{1000}{\Delta T}$; such a thin layer sticks to the surface by reason of viscosity. Thus, for $R = 1$ cm, the substance of the nucleus is practically vaporized, so to speak, on the spot, the thickness of the liquid layer being of the order of 0.001 cm only; most of the solid nucleus remains cold inside. This case evidently resembles the phenomena observed in large meteorites reaching the ground. $R = 0.1$ cm represents an intermediate case, although more similar to $R = 1$ cm, than to $R = 0.02$ cm.

The above results may be summarized as follows: if $\nu = 10$ (the most probable effective viscosity for stone), no sensible transport of the liquid under the influence of aërodynamic pressure (rotation assumed to be small) takes place; stone meteors brighter than the 7^{th} apparent magnitude are vaporized from the surface of a thin liquid layer, the nucleus remaining solid; those fainter than the 7^{th} magnitude maintain a more or less uniform temperature throughout, complete fusion preceding the beginning of perceptible vaporization.

If we assume the less probable inferior limit of viscosity $\nu = 0.1$ for stone, the values of both ΔR and ΔT decrease ten times; the characteristics of fusion are then as follows: for stone meteors brighter than the 2^{nd} apparent magnitude the conditions are the same as they were in the case of $\nu = 10$ for $m < 7$; for those fainter than the 4^{th} magnitude, an efficient transport of the liquid under the influence of aërodynamic friction establishes itself, which helps to equalize the temperature of front and rear (or of pole and equator for a rotating projectile); the liquid is collected in the portions less exposed to aërodynamic pressure (rear side, especially rear pole of rotation); all the solid nucleus is fused before a further rise of temperature and sensible vaporization start. For meteors fainter than the 7^{th} magnitude thermal conductivity takes the place of the immediate transport of the liquid.

As to iron, its low viscosity ($\nu = 0.01$) and high thermal conductivity altogether change the picture; the values of ΔR equal $\frac{1}{40}^{th}$, those of ΔT only $\frac{1}{1200}^{th}$ of the corresponding data of Table VI. For all naked-eye and telescopic iron meteors the frictional transport of the liquid is very efficient, ΔT is small during fusion, the nucleus melts completely into a practically isothermal drop before a further rise of temperature and perceptible vaporization start. For the larger projectiles, considerably exceeding the limit of fissure (Table III), a spraying of the liquid (cf. Section 1. *f* and 3. *b*), instead of an accumulation at the rear takes place. Only for iron fireballs brighter than magnitude — 12 the liquid layer cannot attain its equilibrium depth and starts boiling at the surface instead.

As to the liquid drop, convection currents produced by aërodynamic friction start there for the same reason which causes the transport of liquid during the process of fusion; only the transport of matter and heat in the drop is much more efficient than in the former case, because the resistance from viscosity is much smaller in the whole drop than in a thin liquid layer of same radius covering a solid nucleus. It is obvious that in all cases when complete fusion of the nucleus takes place before vaporization, the drop so formed is kept practically isothermal by the frictional convection currents (a backward current around the periphery of the "contour of resistance", balanced by a forward current around the central portion of the contour); in other words, only such liquid isolated drops can be formed which are maintained at a more or less uniform temperature either by mechanical convection, or by thermal conductivity (very small drops). Computations corroborating these conclusions have been made; they are not given here.

f. The process of fusion of a nucleus in fast rotation.

With the average speeds of rotation considered as probable in *II*, and in Section 3. *d*, the liquid formed during fusion is supposed to get immediately sprayed in the form of very small drops; as the centrifugal force in the case considered is much greater than aërodynamic friction, in all cases where the transport of the liquid by aërodynamic friction is found to be efficient the spraying by centrifugal force must be still more efficient; thus, according to the preceding subsection, all fast rotating iron meteors actually spray their substance during a more or less isothermal process of fusion; the vaporization takes place in the minute drops, the products of the spraying. The question arises whether in stone meteors the large viscosity is able to prevent such a spraying.

The liquid is supposed to move, under the action of centrifugal force, toward the equator of rotation, where it separates by drops as considered in Section 3. *d*. By analytical estimates similar to those made above and with the aid of formula (41), assuming that the drop is formed when the liquid layer has a thickness of the order r (same formula), we find the time of separation of the drop approximately equal to

$$\tau = \frac{8\pi r^2 R_0 \nu}{U_e^2 \delta} = \frac{9200\, \nu R_0^2}{U_e^3 \delta} \quad \ldots\ldots \quad (49).$$

$\tau = 0^s.01$ may be regarded as sufficiently short an interval as compared with the life time of the meteor; assuming this as the maximum admissible for efficient spraying, and with $\nu = 10$ we find the condition

$$U_e > 130\ R_0^{2/3}\ \mathrm{cm/sec} \quad \ldots\ldots \quad (50).$$

Equation (9) of *II* gives

$$U_e = 1140\ R_0^{-1/2} \quad \ldots\ldots \quad (51)$$

as a probable mean value. Condition (50) is fulfilled in this case for $R_0 < 6$ cm, thus over more than all the range to be considered.

Considering the rate of supply of the liquid from fusion to be in equilibrium with the amount flowing away toward the equator under the action of the centrifugal force, we find (notations as before) for the effective thickness of the liquid layer

$$\Delta R = \sqrt{\frac{8}{3} \frac{q_1 \nu}{f H_0 \delta} \frac{R^3 w \cos z_r}{U_e^2}} \quad \ldots\ldots \quad (52),$$

and $\frac{\Delta T}{\Delta R}$ the same as in (48). Assuming (51), with $\nu = 10$, for $\Delta T = 500^0$ as an upper admissible limit for fusion without vaporization, we find the following upper limits for the size of the stone:

Table VII.

Size of Fast Rotating Meteors
with $\Delta T = 500^0$ in the liquid layer.
Vertical incidence.

w	16×10^5	40×10^5	90×10^5
R_0	0.43	0.28	0.18
m	2	0	0
ΔR	0.0008	0.0003	0.0001

As in the present case $\Delta T \sim R_0^3\, w^{3/2}\, \nu^{1/2}\, (\cos z_r)^{3/2}$, the limiting radii represent a relatively sharp margin; meteors rotating at the assumed speed and smaller than these limiting radii must spray their substance instantaneously during the process of fusion even if the viscosity is as large as $\nu = 10$; thus there seems to be no doubt that most naked-eye stone meteors except the brightest, if rotating fast, undergo the process of centrifugal spraying. For other values of ν, the limiting radius is $R_0 \sim \nu^{-1/6}$, thus varying slowly. For $\nu = 0.1$, the limiting magnitudes $(I \sim R_0^3 \sim \nu^{-1/2})$ must be set by 2—3 magnitudes brighter than those in Table VII. Stone meteors of greater size than the limiting one, even when rotating fast, are expected to behave like non-rotating meteors (cf. preceding subsection).

g. Radiation from the nucleus and deceleration.

The heat of vaporization being small as compared with the kinetic energy of an average meteor, the meteor is vaporized before it loses much of its initial velocity [cf. formula (10^a), also Section 3. *d*], deceleration thus being negligible for the naked-eye and the brighter telescopic meteors. This would hold also for the smallest objects but for radiation losses. Black body radiation of the nucleus, being comparatively insignificant for most observable objects, grows in relative importance with the decreasing size of the bodies; for very small particles the radiation of the nucleus may absorb all the kinetic energy, and the particle may be decelerated to zero velocity before it reaches a temperature sufficiently high for vaporization.

For a given surface temperature T, the relative importance of vaporization and radiation may be set equal to $\frac{h\xi}{Q}$. For an iron surface, the data of Table I lead to the following values of this ratio:

T	1800°	2000°	2200°	2400°	2600°	2800°	3000°
$\frac{h\xi}{Q}$	0.062	0.31	1.09	3.0	6.9	13.6	24.6

Thus, for temperatures above 2200° vaporization takes the major part of the heat absorbed by an iron surface, whereas below 2200° the chief loss of heat is by radiation. For stone, the corresponding limiting temperature may be set at about 1600°, as a guess.

Let us consider very small meteors for which radiation losses become important; neglecting the heat spent in heating the nucleus, the equilibrium condition between aërodynamic friction and black body radiation, together with the condition of deceleration [cf. (10^a)]

$$4\pi R^2 Q\, dt = \frac{4}{3}\pi\delta R^3 . \frac{1}{2} w\, dw,$$

lead to the following expression (iron):

$$T = 6.86\, w_0^{3/4} \varrho^{1/4} e^{-\frac{H_0 \sec z_r \varrho}{13.9\, R}},$$

w_0 being the initial velocity. This expression yields a maximum

$$T_{max} = 3.18\, w_0^{3/4} \left(\frac{13.9\, R \cos z_r}{H_0}\right)^{1/4} \quad . \quad . \quad . \quad . \quad (53).$$

Setting $T_{max} = 2200^0$, we find the following maximum sizes of iron particles which are stopped in their motion before sensible vaporization sets in:

	$w_0 = 16 \times 10^5$	$w_0 = 64 \times 10^5$
$\sec z_r = 1$	$R = 5 \times 10^{-4}$ cm	$R = 8 \times 10^{-6}$ cm
$\sec z_r = 10$	$R = 5 \times 10^{-3}$ cm	$R = 8 \times 10^{-5}$ cm.

The apparent magnitude of such meteors lies between the 25^{th} and the 40^{th}; thus only such "ultra-telescopic" meteors may reach the ground without losing much of their mass through vaporization.

For stone, and for an upper limit of temperature $T_m = 1600^0$, the limiting radii are found about the same as for iron.

We conclude that for all kinds of observable meteors vaporization exceeds radiation considerably and that the deceleration of the nucleus is comparatively small. On the other hand, fine meteoric dust may be stopped by the atmosphere without getting vaporized; under especially favourable circumstances (low relative velocity and oblique incidence) the dust particles may be as large as 0.01 cm in diameter; for average velocities, particles of the order of 10^{-5} cm will certainly be caught "uninjured".

h. Working tables of absolute magnitudes.

The absolute magnitude of a meteor, m_0, we define as its apparent magnitude seen from a distance of 100 km and free of atmospheric absorption. The zenithal magnitude, m_z (cf. [12]), does not differ much from m_0 for most "ordinary" meteors. With the existing system of stellar magnitudes we may write

$$m_0 = 24.6 - 2.5 \log I \quad \ldots \ldots \quad (54),$$

where I is the "visual" intensity of radiation in ergs per second, "visual" denoting the spectrum interval 4500 to 5700 Å. The average value of I over the whole visible path of a meteor is proportional to the total kinetic energy divided by the duration of visibility; thus we may write

$$I = \frac{2}{3} \beta \delta \frac{\pi R_0^3 w_0^3}{L} \quad \ldots \ldots \quad (55),$$

where L is the absolute length of the visible path, β — the visual efficiency of radiation (cf. A). The chief uncertainty of our estimates lies in the factor β. In A we tried to estimate the order of magnitude of β; the results were there given in Tables XII and XIII; the first table corresponds to the case of a sufficiently massive meteor for which the dissipation of the molecular kinetic energy takes place within the coma, or the cloud of meteor vapours; the second table corresponds to meteors which are too small for the formation of a coma, so that the dissipation of the molecular energy takes place in the free atmosphere. Whereas the absolute values of β in these tables cannot be regarded as established except for the order of magnitude, the difference between these tables is more certain. It is therefore important to know, at what minimum sizes of the meteor nucleus the formation of a coma begins. The treatment can only be rough. In spite of considerable simplifications, the theory is rather complicated and cannot be given here in all its details.

We consider first the case of a non-breaking spherical iron nucleus as in Section 2. At a certain distance behind the nucleus let d_t be the maximum transverse thickness of the coma, measured in units of the half-energy range of an average molecule; the latter, according to Paper A, Table X, changes as $w^{1/2}$, whence $d_t \sim w^{-1/2}$ for a given linear thickness and density of the coma. The fraction s_c of secondary collisions which happen within the coma is estimated as follows:

d_t	0	3	6	9	$\geqslant 12$
s_c	0.00	0.23	0.51	0.72	1.

If β_0 and β_c are the luminous efficiencies corresponding to no coma, and to a compact coma, we may set:

$$\beta = \beta_0 + s_c(\beta_c - \beta_0) \quad \ldots \ldots \ldots \quad (56).$$

Let $2x$ be the diameter of the coma at a distance y behind the nucleus, and γ the mean density over that diameter.

We have

$$d_t = \frac{2x\gamma}{bw^{1/2}} \quad \ldots \ldots \ldots \quad (57),$$

where $bw^{1/2}$ is the half-energy range in $\frac{\mathrm{gr}}{\mathrm{cm}^2}$; $b = 2{,}76 . 10^{-11}$.

The boundary of the coma is formed by the combined effect a) of the expansion, which may be regarded as an adiabatic outflow of gas into empty space (density of coma $>>$ density of atmosphere), with a velocity $\frac{dx}{dt} = \frac{wu^{1/2}}{1+u}$, and b) of the backward motion relative to the nucleus, due to the inertia of the admixed air molecules, $\frac{dy}{dt} = \frac{wu}{1+u}$. Here u denotes the effective proportion of admixed air (cf. A, p. 34);

$$u = \frac{2h}{\varkappa w^2} \frac{x^2}{R^2} \quad \ldots \ldots \ldots \quad (58),$$

in adopted notations.

The paraboloid outline of the effective boundary of the coma results from the above equations as

$$2\sqrt{\frac{\varkappa w^2}{2h}}\frac{y}{R} = \frac{x^2}{R^2} - 1 \quad \ldots \ldots \quad (59).$$

Further we find from the above equations and from the consideration of the total amount of vaporized material flowing backward through the cross-section πx^2 with the velocity $\frac{dy}{dt}$ (except for u very small):

$$\gamma = \varrho\,\frac{(1+u+u^2)}{u^2} \quad \ldots \ldots \ldots \quad (60).$$

From (57), (60), and (58) d_t for given ϱ and u can be determined. An effective value of $u = \bar{u}$ is defined by the condition that one-half of the energy of secondary collisions be liberated for $u \leqslant \bar{u}$. In the notations of Paper A, p. 34, this means

$$\int_0^{\bar{u}} \beta_2 q_2\, d\left(\frac{u}{1+u}\right) = \frac{1}{2}\int_0^{\infty} \beta_2 q_2\, d\left(\frac{u}{1+u}\right).$$

From this, $\bar{u} = 0.20$ is found. Thus, for the effective value of d_t we find generally

$$d_t = \frac{27.6 \sqrt{\frac{\bar{\varkappa}}{2h}}}{b} w^{1/2} \varrho R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61).$$

This formula applies to an arbitrary spherical nucleus of the radius R and velocity w, in an atmosphere of density ϱ; no restrictions are imposed except for the adopted dependence of the mean free path upon velocity.

For a particular meteor d_t varies as ϱR; we have discussed the same product in Section 3. *b*, *c*; along the visible path it reaches a maximum at $\frac{R}{R_0} = \frac{q+h}{2h}$; the mean value of $R\varrho$ may be assumed as equal to its value at $R = R_0$, $\varrho = \varrho_1$ as defined by equation (24); it is even an overestimate when we consider the breaking of the meteor into drops. With the constants assumed for iron we get, for the particular model of a non-breaking nucleus, with $\bar{\varkappa} = 0.6$ and $H_0 = 8{,}68 \cdot 10^5$:

$$d_{t\,eff.} = 1{,}72 \cdot 10^{12} R_0^2 w_0^{-3/2} \cos z_r \quad . \quad . \quad . \quad . \quad (62).$$

If the nucleus breaks into pieces of radius r, d_t must be set smaller in the proportion $\frac{r}{R_0}\frac{\varrho}{\varrho_1}$, where ϱ is the density of the atmosphere at the moment of separation of the fragments. In this way, from $d_{t\,eff.}$ the fraction s_c for formula (56) could be estimated. For an integer iron nucleus we find ($z_r = 45^0$):

	$w_0 = 16.10^5$	$w_0 = 40.10^5$
s_c	R_0 cm	R_0 cm
1.00	$\geqslant 0.12$	$\geqslant 0.24$
0.50	0.085	0.17
0.25	0.06	0.12
0.08	0.033	0.067

Now, from the preceding discussion of the conditions of integrity we find the maximum size of an iron drop $r = 0.059$ cm even without rotation (Section 3. *b*), and ϱ (near fusion) $= 0.6\,\varrho_1$. This makes $s_c = 0$ practically in all cases. $w_0 = 16.10^5$ presents an exception, but in this case (cf. *A*) $\beta_c = \beta_0$ practically, and the actual value of s_c is unimportant. Thus, for iron meteors except very bright fireballs, in any case for $R_0 < 1$ cm, $\beta = \beta_0$: the amount of radiation per atom is the same as for a single atom moving in the atmosphere; for the same reason, "temperature radiation" (*A*, pp. 36—38) must be negligible in iron meteors.

For stone, non-rotating and of high viscosity, $\nu = 10$, the nucleus actually remains integer and the following values may be considered as valid:

	$w_0 = 16.10^5$	40.10^5	100.10^5
s_c	R_0 cm	R_0 cm	R_0 cm
1.0	$\geqslant 0.16$	$\geqslant 0.31$	$\geqslant 0.62$
0.5	0.11	0.22	0.44
0.25	0.08	0.16	0.32
0.08	0.044	0.087	0.17

For fast rotation, the nucleus must actually break into drops, and $s_c = 0$, if the radius is smaller than the limiting value given below:

	$w_0 = 16.10^5$	40.10^5	100.10^5
$\nu = 10$, R cm $\leqslant$	0.43	0.28	0.17
$\nu = 0.1$, R cm $\leqslant$	0.93	0.60	0.37

Having estimated s_c according to these principles, the table of absolute magnitudes given below was calculated, from formulae (54), (55), (56), and Tables XII and XIII of Paper A; these tables were assumed provisionally to apply equally to stone and to iron. An empirical mean value of $L_0 = 18.10^5$ cm was adopted. If L is the actual effective length of the visible path, a correction of $+2.5 \log \frac{L}{L_0}$ must be added to the tabulated values. Larger, but yet unknown systematic corrections, depending upon composition and velocity, must be added to these values of m_0. In the case of an integer nucleus (the larger stone meteors), to m_0 as defined by (54) an additional correction for temperature radiation, in agreement with Paper A, has been added as follows:

R_0, cm	0.25	0.30	0.4	0.5	0.7	1.0
Δm_0	0.0	—0.1	—0.2	—0.3	—0.5	—0.9.

Table VIII.

Provisional Absolute Magnitudes.

a = iron meteors; b_1 = stone, $\nu = 10$, slow rotation; b_2 = stone, $\nu = 10$, fast rotation; b_3 = stone, $\nu = 0.1$, fast rotation.

w_0 km/sec and Case	R_0, cm 1.0	0.7	0.5	0.4	0.3	0.2	0.15	0.10	0.07	0.05	0.03	0.01
	m_0											
16 a	−1.7	−0.5	0.6	1.3	2.2	3.5	4.5	5.8	7.0	8.1	9.7	13.3
„ b_1	−2.0	−0.4	0.9	1.7	2.7	4.1	5.1	6.6	7.8	9.0	10.6	14.2
„ b_2	−2.0	−0.4	0.9*	2.2*	3.1	4.4	5.4	6.7	7.9	„	„	„
„ b_3	−2.0*	+0.4*	1.5	2.2	3.1	4.4	5.4	6.7	„	„	„	„
25 a	−3.0	−1.8	−0.7	0.0	0.9	2.2	3.2	4.5	5.7	6.7	8.4	12.0
„ b_1	−4.1	−2.5	−1.2	−0.4	0.6	2.0*	3.5*	5.1	6.5	7.6	9.3	12.9
„ b_2	−4.1	−2.5	−1.2	−0.4*	1.8*	3.1	4.1	5.4	6.6	„	„	„
„ b_3	−4.1	−2.5*	+0.2*	+0.9	1.8	3.1	„	„	„	„	„	„
40 a	−4.4	−3.2	−2.1	−1.4	−0.5	0.8	1.8	3.1	4.3	5.4	7.0	10.6
„ b_1	−6.3	−4.7	−3.4	−2.6	−1.6*	0.8*	2.3	3.8	5.1	6.3	7.9	11.5
„ b_2	−6.3	−4.7	−3.4	−2.6	−1.6*	1.7*	2.7	4.0	5.2	„	„	„
„ b_3	−6.3	−4.7*	−1.2*	−0.5	+0.4	1.7	„	„	„	„	„	„
60 a	−5.6	−4.4	−3.3	−2.6	−1.7	−0.4	0.6	1.9	3.1	4.2	5.8	9.4
„ b_1	−7.9	−6.3	−5.0	−4.2*	−2.0*	+0.1*	1.3	2.7	3.9	5.1	6.7	10.3
„ b_2	−7.9	−6.3	−5.0	−4.2*	−2.0*	+0.5*	1.5	2.8	4.0	„	„	„
„ b_3	−7.9	−6.3	−5.0*	−1.7*	−0.8	+0.5	„	„	„	„	„	„
100 a	−7.0	−5.8	−4.7	−4.0	−3.1	−1.8	−0.8	0.5	1.7	2.8	4.4	8.0
„ b_1	−9.9	−8.3	−5.9*	−4.4*	−3.0*	−1.2*	−0.1	1.3	2.5	3.7	5.3	8.9
„ b_2	−9.9	−8.3	−5.9*	−4.4*	−3.0*	−1.2*	+0.1	1.4	2.6	„	„	„
„ b_3	−9.9	−8.3	−5.9*	−4.4*	−2.2*	−0.9	+0.1	„	„	„	„	„

In the above table, for stone meteors the transition from β_0 to β_c causes a more or less sudden increase of the luminosity with the radius; the figures which correspond to this sudden change are marked with asterisks. A more or less deep depression in the frequency curve of meteor luminosities may be expected at these transition magnitudes with an excess of frequency at higher luminosities; such an effect may eventually be subjected to an observational test.

i. The shielding effect of meteor vapours.

In Section 1. *d* general mention was made of the shielding effect of the vapours which decrease the flow of heat toward the nucleus; now we can consider this effect in more detail. The shielding effect can be important only when the vapour pressure exceeds the normal component of the aërodynamic pressure, i. e., when

$$P > p_n \quad \ldots\ldots\ldots\ldots \quad (a).$$

It is easy to show that when $P < p_n$ the vapour can only form a layer of a thickness which is small as compared with the half-energy range, of the order of

$$d < \frac{u_0}{w}\,[1],$$

or about $d < 0.01$, which is negligible (cf. Table II).

Let us consider a spherical nucleus for which at a certain angle of incidence, α_0, $P = p_n$. The vapours have free escape only for $\alpha > \alpha_0$, and the total amount vaporized per second is $2\pi(1+\cos\alpha_0)R^2\xi$. With the aid of formulae (1) (with $T = 3000^0$), (8) and (10) we get, independently of the density of the atmosphere and of the radius:

$$\frac{P}{p_n} = \frac{2{,}1.10^4\, w}{(1+\cos\alpha_0)\,h\cos^2\alpha}\cdot\frac{\bar{\varkappa}}{(2-\varkappa')} \quad \ldots\ldots \quad (63);$$

here $\bar{\varkappa}$ is an average value for the whole nucleus, whereas $\varkappa'$ is a constant, independent of the thermal shielding. We assume $2-\varkappa' = 1.4$ (Section 1. *d*).

According to equations (9) and (8), $p_n = p_0 \cos^2\alpha$. This leads to another form for (63):

$$\frac{P}{p_0} = \frac{2{,}1.10^4\, w}{(1+\cos\alpha_0)\,h}\cdot\frac{\bar{\varkappa}}{(2-\varkappa')} \quad \ldots\ldots \quad (63').$$

When $P = p_0$, which means also $\alpha_0 = 0$, the front (maximum) aërodynamic pressure equals the vapour pressure; this is evidently the limiting case for which a complete vapour shell

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1) This follows when we consider that the free length of path of the particle, of the order of $d = 1$, is travelled with a speed $w + u_0$ relative to the atmosphere, whereas the speed of the particle relative to the nucleus is u_0; thus, when losing $\frac{1}{\sqrt{2}}(w+u_0)$, the particle should reach a distance from the nucleus of only $d < \frac{u_0}{w+u_0}$, in units of the half-energy range; actually it has to lose much less, namely $\Delta w = u_0$ before it gets stopped relative to the nucleus.

starts forming around the nucleus. When $P > p_0$, a vapour "microatmosphere" shields the nucleus from the direct impacts of the air molecules. When $P < p_0$, a "bald" cap extending from $\alpha = 0$ to $\alpha = \alpha_0$ is formed subject to direct impacts of the air molecules; α_0 is evidently defined in this case by the equation

$$\cos^2 \alpha_0 = \frac{P}{p_0} \quad \ldots \ldots \ldots \quad (64).$$

Let the "unshielded" value of the accomodation coefficient be $\varkappa_0$. Evidently

$$\bar{\varkappa} < \varkappa_0 \quad \ldots \ldots \ldots \quad (65);$$

$\varkappa_0 = 0.8$ may be estimated.

For the critical case $\frac{P}{p_0} = 1$, $\alpha_0 = 0$, we get from (63′), with $h = 6.10^{10}$:

$$\bar{\varkappa}_c = \frac{8.10^6}{w} \quad \ldots \ldots \ldots \quad (66^a).$$

Of course, we do not yet know under what conditions the critical case sets in; but the above equation, together with (65) furnish us with an inferior limit for the velocity below which a closed vapour shell cannot be formed. For the closed vapour shell we may safely put $\bar{\varkappa}_c < 0.50$ (cf. Table II), whence

$$w > 160 \text{ km/sec.}$$

Most meteor velocities fall without doubt below this limit; thus, already from such general considerations it is obvious that the only case of practical importance is $P < p_0$.

Nevertheless, to have things clarified as much as possible, we start by assuming the opposite case of $P > p_0$, the least favourable for $\bar{\varkappa}$. In formula (63′) $\alpha_0 = 0$ must be set, and we get

$$\bar{\varkappa}_c = \frac{8.10^6}{w} \cdot \frac{P}{p_0} \quad \ldots \ldots \ldots \quad (66).$$

An increasing P, according to this formula, requires an increasing $\bar{\varkappa}$; on the other hand, with P increasing the shielding effect of the vapours must increase, and $\bar{\varkappa}$ must diminish; hence, for a certain value of w, definite solutions for $\bar{\varkappa}_c$ and $\frac{P}{p_0}$ must exist. The maximum shielding effect may be estimated as follows. If D is the density of vapours at the surface of the nucleus, the density at a distance x is less than $\frac{DR^2}{x^2}$ (because velocity of flow and temperature increase with the distance); assuming the latter value means committing an overestimate. Similarly, the pressure may be estimated at $< \frac{PR^2}{x^2}$. On the front side of the meteor, the expansion of the vapours may be considered to proceed until equilibrium with the aërodynamic pressure is reached, or $p_0 = \frac{2\,PR^2}{x^2}$, the factor 2 allowing for partly oblique incidence; hence

$$x_{max} (\widetilde{=}) R \sqrt{\frac{2P}{p_0}}.$$

The effective shielding vapour mass per cm² of cross section may be set equal to

$$a = 2 \cdot \frac{1}{2} \int_R^{x\,max} \frac{DR^2}{x^2}\, dx = \left(1 - \sqrt{\frac{p_0}{2P}}\right) DR \;\left(\frac{\text{gr}}{\text{cm}^2}\right) \quad . \quad . \quad (67).$$

Here, of the two mutually cancelling factors, 2 and $\frac{1}{2}$, the first allows for oblique incidence, the second for the neglect of the acceleration of the expanding and steadily heated meteor vapours.

The "kinetic depth", d, we obtain from (67) dividing it by $2{,}76.10^{-11}\, w^{1/2}$ (cf. preceding subsection). Setting the amount vaporized equal to $\frac{E}{h}$ (cf. form. (10)), and substituting ϱ_1 for the point of appearance (form. (24) with $R = R_0$) as representing average conditions (cf. preceding subsection), we get (with $D = 1{,}92.10^{-5}\xi$, cf. Table I, and different constants as before):

$$d_c = 0.67\, R^2 w^{1/2} \cos\, z_r \left(1 - \sqrt{\frac{p_0}{2P}}\right) \quad . \quad . \quad . \quad . \quad (68),$$

referring to iron.

Further, Table II for $\varkappa < 0.5$, or $d > 2$, is well represented by

$$\varkappa = \frac{1.28}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69),$$

which is actually a formula for the conductivity for heat.

Formulae (66), (68), and (69) lead to the following quadratic equation with respect to $\varkappa_c$ (or to $\frac{1}{\varkappa_c}$):

$$\frac{1.28}{\varkappa_c} = 0.67\, R^2\, w^{1/2} \cos z_r \left(1 - \sqrt{\frac{4.10^6}{w\varkappa_c}}\right).$$

Setting $y = \frac{1}{\varkappa_c}$, $n = \frac{1.92}{R^2\, w^{1/2} \cos z_r}$, $b = \frac{4.10^6}{w}$, we get:

$$y = \frac{(2n+b) \pm \sqrt{b^2+4nb}}{2n^2} \quad . \quad . \quad (70) \quad (y > 2).$$

It is obvious that for a larger R the shielding effect must be greater; thus, to find a minimum value for $\varkappa_c$ we may set $R \sim \infty$, or n small as compared with b; actually, at meteor conditions, $w \sim 4.10^6$, $n = 0.1\; b$ even for R as small as 0.1 cm. In this case (70) yields the following two solutions, valid for practically all meteors with $R > 0.1$ cm:

$$y_1 = \frac{b}{n^2}, \quad \varkappa_1 = \sim \frac{1}{10^6 \cdot R^4};$$

$$\text{and } y_2 = \frac{1}{b}, \bar{\varkappa}_2 = \frac{4.10^6}{w} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71).$$

The first solution has no physical meaning, as it may give for example, for $R \sim 10$ cm, $\varkappa_1 \sim 10^{-10}$, $d \sim 10^{10}$ (equation (69)), or $a \sim 10^{10}.2{,}76.10^{-11}\, w^{1/2} = 500$ gr/cm², which is more than the mass of the nucleus per cm² [1]). The second solution is in dimensional agreement with (66^a) and is the only real one. Thus, with increasing radius, for a completely shielded nucleus the value of $\bar{\varkappa}$ tends to a certain minimum value, inversely proportional to the velocity and depending upon velocity alone, the limit being practically reached already for $R \geqslant 0.1$ cm. From the above (form. 66^a), (71) must be valid only beginning from about $w \geqslant 16.10^6$, when (71) gives $\varkappa_c \leqslant 0.25$, and 66^a gives $\bar{\varkappa}_c \leqslant 0.50$; there is a contradiction, in a ratio 2:1, for the two formulae which agree only for $w = \infty$, $\bar{\varkappa}_c = 0$. In any case, the order of magnitude of $\bar{\varkappa}$ is little affected even at unusually high velocities.

Now we may proceed to the more important case of $P < p_0$. The front side of our spherical nucleus may be divided into

1) In the present subsection a is the mass per cm² formed by the meteor vapours only, without counting the air mass of the air cap.

two zones: the zone of unshielded impact, for $\alpha < \alpha_0$, with a high value of $\varkappa = \varkappa_0$, and with a relative cross section of $\sin^2 \alpha_0$; and the shielded area, with a low value of $\varkappa = \varkappa_1$, with the relative cross-section equal to $\cos^2 \alpha_0$. The shielded area starts with $\frac{P}{p_n} = 1$, and hence $x_{max} = R\sqrt{2}$ (cf. above) may be a fair estimate for this area. By analogy with equation (68) we set as above (the divisor $\cos\ \alpha_0$ allows for oblique incidence):

$$\frac{1{,}28}{\varkappa_1} = \frac{0.67}{\cos \alpha_0} R^2 w^{1/2} \cos z_r \left(1 - \sqrt{\frac{1}{2}}\right) \quad . \quad . \quad . \quad (72),$$

valid for $\varkappa_1 < 0.5$. Further, obviously

$$\bar{\varkappa} = \varkappa_0 \sin^2 \alpha_0 + \varkappa_1 \cos^2 \alpha_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (73).$$

Substituting in this $\bar{\varkappa}$ from (63′), and applying (64), we get with the proper constants

$$\varkappa_1 = \frac{4{,}0.10^6 (1 + \cos \alpha_0)}{w} - \varkappa_0 \operatorname{tg}^2 \alpha_0 \quad . \quad . \quad . \quad (74).$$

On the other hand (71) gives:

$$\varkappa_1 = \frac{6.56 \cos \alpha_0}{R^2 w^{1/2} \cos z_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72').$$

These two equations determine $\varkappa_1$ and α_0. The solution may be simplified. From (72′) it appears that $\varkappa_1$ approaches zero for large R, and is negligible even at as small values as $R = 0.2$ cm. Thus, for large radii we set $\varkappa_1 = 0$. We then get from (74):

$$\sin^2 \frac{\alpha_0}{2} = \frac{b}{2(\varkappa_0 + b)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (75), \text{ or}$$

$$\cos \alpha_0 = \frac{\varkappa_0}{\varkappa_0 + b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75'), \text{ and}$$

$$\bar{\varkappa} = \frac{b(2\varkappa_0 + b)}{(\varkappa_0 + b)^2} \varkappa_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76),$$

$$\text{where } b = \frac{4.10^6}{w} \text{ as before.}$$

On the other hand, for very small radii the shielding effect is nil and we must have $\bar{\varkappa} = \varkappa_1 = \varkappa_0$.

We notice that (75) and (76) are valid for all velocities, and that $\alpha_0 \neq 0$ except for $w = \infty$, in which case (76) becomes identical with (66^a) but for the factor $\varkappa_0$. It appears that the case $P < p_0$ is the only possible real case, whatever the velocity, and that a complete vapour shell can never be formed.

Assuming $\varkappa_0 = 0.80$, from (72′) and (74) the following values of $\bar{\varkappa}$ and α_0 are found for different values of the radius and velocity.

Table IX.

Values of $\bar{\varkappa}$ the Effective Fraction of Heat Absorbed by the Nucleus During Vaporization.
Iron sphere; $z_r = 0^0$; $H_0 = 8{,}68.10^5$ cm.

$w = 16.10^5$			$w = 25.10^5$			$w = 50.10^5$		
R cm	$\bar{\varkappa}$	α_0	R cm	$\bar{\varkappa}$	α_0	R cm	$\bar{\varkappa}$	α_0
(∞)	0.75	76^0	(∞)	0.71	70^0	(∞)	0.60	60^0
0.095	0.76	„	0.10	0.72	„	0.11	0.63	„
0.048	0.78	. . .	0.052	0.76	. . .	0.068	0.68	. . .
$\leqslant$0.037	0.80	. . .	$\leqslant$0.040	0.80	. . .	0.052	0.74	. . .
						$\leqslant$0.043	0.80	. . .

$w = 100.10^5$			$w = 160.10^5$			$w = 250.10^5$		
R cm	$\bar{\varkappa}$	α_0	R cm	$\bar{\varkappa}$	α_0	R cm	$\bar{\varkappa}$	α_0
(∞)	0.44	48^0	(∞)	0.38	40^0	(∞)	0.24	34^0
0.10	0.49	„	0.092	0.45	„	0.083	0.32	„
0.063	0.58	. . .	0.057	0.55	. . .	0.051	0.46	. . .
0.052	0.66	. . .	0.046	0.63	. . .	0.041	0.58	. . .
0.045	0.73	. . .	0.040	0.72	. . .	0.036	0.69	. . .
$\leqslant$0.040	0.80	. . .	$\leqslant$0.036	0.80	. . .	$\leqslant$0.032	0.80	. . .

The same table applies to other values of z_r and H_0 after multiplying R in the table by the factor $\sqrt{\frac{H_0}{8{,}68.10^5 \cos z_r}}$. The upper limit of R for which the table is valid is set by the formation of an air cap (cf. Table V).

For stone, the average molecular weight of the vapours is but slightly smaller than for iron, and so also is the temperature of vaporization (cf. Section 1. *a*, *b*); the ratio $\frac{\xi}{P}$ is therefore practically the same as for iron. More important is the difference in the heat of vaporization; in an integer viscous stone nucleus with its low conductivity for heat, vaporization in the thin surface layer takes place simultaneously with fusion, and the whole heat, $q+h=7{,}7.10^{10}$, for stone must be taken, instead of h alone; for a given velocity and radius the vapour pressure is smaller, and $\bar{\varkappa}$ larger; Table IX may be adapted to this case by increasing the tabular w by 30 per cent, which, however, does not introduce essential changes in $\bar{\varkappa}$. On the other hand, the per-volume heat of vaporization is smaller for stone than for iron, so that, for a given radius, the stone is vaporized at greater heights, with smaller ϱ and D, and with a greater resulting $\bar{\varkappa}$; to account for this, R in Table IX must be increased by 50 per cent.

Table IX refers to the average conditions during the process of the vaporization of a nucleus. During the process of pure fusion, and before it, no perceptible shielding can take place, and $\bar{\varkappa}=\varkappa_0=0.80$ must be assumed for this stage. Due to differences of temperature, the vapour pressure on the front side may be somewhat greater than the pressure on the rear side, which means an increased shielding effect; the extreme case is no vaporization from the rear side; the values of w in Table IX must be halved in this case; however, even a slow rotation will strongly counteract this effect, so that in an average case the effect must be much smaller.

Similarly, the shape of the nucleus must influence to some extent the value of $\bar{\varkappa}$. Deviations from spherical shape mean an increase of the relative surface for a given mass, thus a smaller density of the atmosphere, and of the vapours also; this would require a value of the effective radius in Table IX which exceeds the tabular one by a factor of the order of $\sqrt{\frac{S}{3v}\sqrt[3]{\frac{3v}{4\pi}}}$; here v denotes volume, S — surface.

For a very flat prism with dimensions 1:4:4, the factor is still as small as 1,25. Taking into account that table IX is not very sensitive to changes of radius, or velocity, and that the different effects are partly in opposite directions, we conclude that Table IX apparently represents the variation of $\bar{\varkappa}$ in most imaginable cases well, with an uncertainty that perhaps is less than 10 per cent. The value of $\bar{\varkappa}=0.6$ chosen above for the first approximation seems well to represent average conditions during vaporization. For a constant absolute magnitude (iron, Table VIII) Table IX yields:

$w=$	16.10^5	25.10^5	50.10^5	100.10^5
$m_0=3, \bar{\varkappa}=$	0.75	0.71	0.66	0.70
$m_0=0, \bar{\varkappa}=$	0.75	0.71	0.60	0.47.

Thus, for an average naked-eye meteor ($m_0=3$), the constancy of $\bar{\varkappa}$ for different velocities appears to be a good approximation.

Section 4.

Synopsis.

The estimates of the different physical conditions which are made in this paper permit us to obtain a clearer view of the actual problems set by the meteor theory. A meteor theory must be able to account for the chief observational facts: the display of visible radiation over a certain limited range in height. As for meteors radiation and vaporization are almost inseparable, the major problem consists in describing the rate of the vaporization of the meteor substance along its trajectory in the terrestrial atmosphere. Vaporization may take place in two different ways: 1) directly from the surface of the nucleus; 2) from small drops shed off during the process of fusion. In the second case the drops are almost instantaneously vaporized after their separation from the nucleus and fusion of the nucleus coincides in time with the visible display of the meteor; smaller amounts of heat, thus greater heights of the visible trajectory, correspond to the second case as compared with the first. The two fundamentally different processes of vaporization may have their shades and gradations.

From the variety of cases which may present themselves only a few typical ones seem to be important. These cases are listed below. For the sake of convenience, absolute magnitudes according to Table VIII are partly quoted as defining the limits of the validity of the different cases[1]). The limits can be given only approximately, of course.

1) As noticed before, the marginal cases being actually defined by two variable parameters, R_0 and w_0, are characterized by little variation in $m=f(R_0, w_0)$; the absolute magnitudes quoted here are thus a substitute for

a) Case of an isothermal sphere getting vaporized from the surface. This case is the closest to our first approximation; the difference consists in taking into account the variation of vapour pressure with temperature, the heat losses from black-body radiation, and eventually fissure of the liquid nucleus. Slowly rotating small iron meteors, of $m_0 > 0$, $R_0 < 0.2$ cm, belong to this class; those of $R_0 > 0.06$ cm will burst after liquefaction into smaller more or less equal drops which at $R_0 \sim 0.2$ cm already are small enough to be vaporized 10–20 times faster than the integer nucleus would have been; a real burst at the end of fusion may thus result, preceded, however, by intense spraying of the liquid during fusion; thus, $R_0 = 0.2$ cm represents a transition toward case b). Further, slowly rotating stones ($\nu = 10$), $m_0 > 6$ (fainter than $m_0 = 6$) belong to case a), fissure never occurring for them.

In case a), vaporization requires a supply of heat per gram of the nucleus: q preceding vaporization, h during vaporization.

a′) Case of a non-isothermal sphere with complete fusion preceding vaporization. Only iron meteors of slow rotation $m_0 < 0$, $R_0 < 0.1$ cm may belong to this group (a narrow group with high velocities and intermediate radii, not represented in Table VIII).

The supply of heat required by the nucleus for vaporization is exactly the same as in case a), and from the standpoint of the display of light the case must be closely similar to a).

b) Case of an isothermal sphere with complete spraying during fusion. To this case belong: iron meteors of slow rotation, $m_0 > 0$, $R_0 > 0.2$ cm; iron meteors of fast rotation, $m_0 > 0$, all radii; stone meteors ($\nu = 10$) rotating fast, $m_0 > 6$.

The supply of heat per gram of the nucleus is: $q_1 - f$ preceding spraying, f during spraying. The formulae of the first approximation apply also to this case, with f and $q_1 - f$ substituted for h and q.

c) Case of a non-isothermal sphere with complete spraying during the fusion. To this class belong: iron meteors of slow

the more complex characteristic (R_0, w_0); the values of m_0 need not be, and may not be accurate, because of the lack of knowledge with respect to β; at the same time, these conventional values of m_0 define in a unique manner (R_0, w_0), according to Table VIII. Cf. Section 3. c, f, h.

rotation, $m_0 < 0$, $R_0 > 0.2$ cm; iron meteors of fast rotation, $m_0 < 0$, all radii; stone meteors ($\nu = 10$) rotating fast, $-1 < m_0 < 6$.

Fusion starts at the surface when the inner portions of the nucleus have not yet reached the temperature of fusion. The heat supply per gram of nucleus is: less than $q_1 - f$ preceding spraying; more than f during spraying (q_1 in the limiting case). The formulae of the first approximation cannot be well applied to this case.

d) Case of a non-isothermal sphere with the liquid vaporized "on the spot". Stone meteors ($\nu = 10$) belong to this case, brighter than $m_0 = 6$ for slow rotation, and brighter than $m_0 = -1$ for fast rotation. Also, slowly rotating iron fireballs of $m_0 < -12$, and still larger fast rotating ones may be counted with this case, but for such bright objects complications from an air cap arise, which make them a special class (cf. next).

The display of light in case d) starts early and ends late. The heat supply per gram during vaporization is: more than h, attaining $q + h$ in the limiting case, thus larger than in all the preceding cases. This case permits of simple mathematical treatment (cf.[5]), very different from the first approximation.

e) Same as case d), but with an air cap. Very bright fireballs, and meteorites belong to this class.

In the second approximation, cases a) and a′) may be joined together; the same refers to b) and c). Thus numerical computations are required for three cases, of which d) may be treated analytically, the other two cases by mechanical quadratures.

According to the order of the relative length of the trail (or range in height) the different cases may be classified as follows (order of increasing length): b); c); a); a′); d); e).

The same is the order of the decreasing height of the point of disappearance, for constant radius and velocity. With respect to the length of the trail, however, for large size meteors c) may follow a′).

Bursts may be explained partly by the sudden spraying of the liquid, apt to happen in transition cases d) — c), and a) — b), or a′) — c); minor bursts may be due to the spraying of fusible inclusions (Troïlite); occluded gases will help spraying and favour bursts.

Conclusion.

Our first approximation of the physical theory of meteor phenomena developed above can be applied to the analysis of certain kinds of observational data, such as the average heights of meteors and their relation to velocity, luminosity, and the density gradient of the atmosphere. The first approximation gave us also a means of describing qualitatively and quantitatively the different physical conditions of meteor phenomena; the knowledge of these conditions is required to form a basis for the second approximation, a more detailed theory of meteors, a theory which must explain the variation of luminosity along the trail, as well as a number of other important details without which a complete understanding of meteor phenomena cannot be attained. The second approximation is developed by a purely numerical method, because a satisfactory analytical treatment seems to be impossible, except in a few particular cases, in view of the complexity of the conditions. It may be added that most of the computations required for the second approximation are completed. We hope to give soon the results, together with a comparison of the theory with suitable observational data.

Tartu, February 13, 1937.

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Table of Contents.

KRITIK DER ANSICHTEN VON B. JUNG ÜBER DIE OBERE GRENZDICHTE DER HIMMELSKÖRPER

VON

WILHELM ANDERSON

TARTU 1937

K. Mattiesens Buchdruckerei Ant.-Ges., Tartu 1937.

Unter der Annahme, daß die Eigenenergie einer (homogenen) Kugel nicht negativ sein darf, gelangt A. Haas zu der Formel:

$$Mc^2 - \frac{3}{5}\frac{GM^2}{R} \geqq 0 .$$

Andererseits ist:

$$M = \frac{4}{3}\pi R^3 \varrho .$$

Das Eliminieren von R ergibt:

$$\varrho \leqq \frac{125}{36\,\pi}\,\frac{c^6}{G^3 M^2} . \tag{1}$$

G. I. Pokrowski nimmt an, daß der Grenzwert des Oberflächenpotentials gleich $-c^2$ sei, daß also

$$\frac{GM}{R} \leqq c^2$$

sein müsse. Daraus ergibt sich:

$$\varrho \leqq \frac{3}{4\,\pi}\,\frac{c^6}{G^3 M^2} . \tag{2}$$

Ich verschärfte Pokrowski's Annahme durch die Forderung, daß selbst im Zentrum des Sterns das Gravitationspotential den Grenzwert $-c^2$ nicht überschreiten dürfe. Eine solche Forderung führt zu der Formel:

$$\varrho \leqq \frac{2}{9\,\pi}\,\frac{c^6}{G^3 M^2} . \tag{3}$$

Bedeutend früher (im Jahre 1923) hatte ich die Forderung aufgestellt, daß die Kontraktionsenergie nicht imaginär werden dürfe. Meine damalige Theorie führt zu der Formel:

$$\varrho \leqq \frac{125}{72\,\pi}\,\frac{c^6}{G^3 M^2} . \tag{4}$$

Neuerdings untersuchte N. R. Sen die Beziehung zwischen Masse und Dichte einer homogenen Kugel nach den Methoden der allgemeinen Relativitätstheorie. Für den Grenzfall läßt sich nach Sen's Theorie die Gleichung

$$\varrho = \frac{(0{,}526)^2}{16\,\pi} \, \frac{c^6}{G^3 M^2} \tag{5}$$

ableiten [1]).

Nach Einstein ist

$$\frac{\varkappa \varrho}{2} = \frac{1}{R^2}$$

und

$$M = 4\,\pi^2 \, \frac{R}{\varkappa},$$

wo M die Masse des Universums bedeutet, ϱ seine durchschnittliche Dichte und R den Krümmungsradius des Raumes [2]). Außerdem ist

$$\varkappa = \frac{8\,\pi\,G}{c^2}.$$

Eliminiert man aus diesen drei Gleichungen R und $\varkappa$, so erhält man:

$$\varrho = \frac{\pi}{16} \, \frac{c^6}{G^3 M^2}. \tag{6}$$

Die Formeln (1), (2), (3), (4), (5) und (6) sind bis auf den abweichenden Zahlenfaktor identisch. Sie alle können mit gleichem Rechte sowohl auf das Universum als Ganzes, wie auch auf einzelne Sterne angewandt werden.

Unlängst erhebt B. Jung Einwände gegen die Ableitung der Formeln von Haas und von Pokrowski: „Daß diesen Betrachtungen ein Fehlschluß zugrunde liegt, scheint bisher entgangen zu sein. Das Versehen liegt darin, daß zwar die Eigenenergie, die der Masse M zukommt, richtig (relativistisch) angesetzt wird,

[1]) Vgl. W. Anderson, Publ. de l'Observatoire Astronomique de l'Université de Tartu **29**$_1$ [= Acta et Commentationes Universitatis Tartuensis (Dorpatensis) A **29**$_9$] S. 28 ff., 1936. Dort finden sich auch die entsprechenden Literaturnachweise.

[2]) A. Einstein, Berl. Ber. 1917, S. 152.

nicht aber die Gravitationsenergie"[3]). Jung berechnet vor allem die Schwerebeschleunigung, die eine homogene Kugel von dem Radius R und von der Dichte ϱ auf eine äußere Kugelschale (mit den Radien R und $R+dR$) ausübt. Er weist darauf hin, daß die Gesamtmasse M der Kugel sich zusammensetzt „aus der materiellen Masse M_m und der Masse, die der negativen Gravitationsenergie entspricht". In gleicher Weise muß, nach Jungs Ansicht, auch die Masse dM der Kugelschale berechnet werden, wodurch sich die Differentialgleichung

$$dM = 4\pi\varrho R^2 dR - 4\pi\varrho R^2 \frac{GM}{c^2 R}\, dR \tag{7}$$

ergibt. Die Lösung dieser linearen Differentialgleichung ist:

$$M = 4\pi\varrho \frac{\int R^2 e^{\frac{2\pi\varrho G}{c^2}R^2} dR}{e^{\frac{2\pi\varrho G}{c^2}R^2}}. \tag{8}$$

Jung weist nun darauf hin, daß die Gesamtmasse M (also auch die Gesamtenergie Mc^2) mit wachsendem R dauernd zunehme; von einem Negativwerden des Energieinhaltes sei keine Rede.

Auf Jungs Einwände kann folgendes erwidert werden.

Wenn x die gegenseitige Entfernung zweier Massen M und m bedeutet, so ist die entsprechende Newtonsche Anziehungskraft gleich $\frac{GMm}{x^2}$. Zur Überführung der Masse m aus der Entfernung r bis in eine unendlich große Entfernung muß die Arbeit $\int_r^\infty \frac{GMm}{x^2}\,dx$ geleistet werden. Dann, und nur dann, wenn Mm konstant ist, dürfen wir schreiben:

$$\int_r^\infty \frac{GMm\,dx}{x^2} = GMm \int_r^\infty \frac{dx}{x^2} = \frac{GMm}{r}. \tag{9}$$

Beim freien Fallen der kleinen Masse m auf die große Masse M nimmt die potentielle Gravitationsenergie ab, aber im

[3]) B. Jung, Astron. Nachrichten **261**, 87, 1936.

selben Maße steigt die kinetische Energie an, so daß m (und auch M) unverändert bleibt. Deshalb haben wir auch das Recht, beim freien Fallen die Beziehung (9) anzuwenden. Anders ist es, wenn während des Fallens auch nur ein Teil der entstehenden kinetischen Energie durch Ausstrahlung oder auf irgendeine andere Weise entfernt wird. In einem solchen Falle nimmt die Masse m ständig ab, so daß Mm nicht konstant bleiben kann. Dann darf aber (9) nicht mehr angewandt werden.

Bei Jung fällt die unendlich kleine Masse $m = 4\pi\varrho R^2 dR$ aus der Unendlichkeit bis zur Entfernung R vom Kugelzentrum, wobei eine Umwandlung von potentieller Gravitationsenergie in kinetische stattfindet. Dann, und nur dann, wenn diese kinetische Energie während des Fallens auf keine Weise entfernt wird, bleibt $m = 4\pi\varrho R^2 dR$ konstant, wodurch wir berechtigt werden (9) zu benutzen. Dann, und nur dann, dürfen wir sagen, daß die Gravitationsenergie um $\frac{4\pi\varrho R^2 dR \,.\, GM}{R}$ abgenommen hat, also die kinetische Energie (die anfangs gleich Null gewesen war) um ebensoviel zugenommen. Wenn wir jetzt (d. h. nach Beendigung des Fallens) die gesamte entstandene kinetische Energie (die der Masse $\frac{4\pi\varrho R^2 dR . GM}{Rc^2}$ äquivalent ist) restlos entfernen, so erhalten wir in der Tat Jungs Differentialgleichung (7). Jungs Gleichung setzt also einen solchen Prozeß der Sternbildung voraus, wo jede unendlich dünne Kugelschale durch ein Herabfallen aus der Unendlichkeit der entsprechenden Masse $4\pi\varrho R^2 dR$ gebildet wird, wobei während des Herabfallens kein Energieverlust durch Ausstrahlung (oder auf irgendeine andere Weise) stattfindet. Erst nach Beendigung des Fallens muß die entstandene kinetische Energie $\frac{4\pi\varrho R^2 dR . GM}{R}$ restlos entfernt werden. Erst danach fällt aus der Unendlichkeit die nächste (unendlich kleine) Portion Masse, und so weiter.

Ein solcher Prozeß entspricht aber gar nicht den wirklichen Verhältnissen. Beim wirklichen Kontraktionsprozeß eines Sterns erfolgt ein ununterbrochener Massenverlust durch Ausstrahlung, aber ein Teil der kinetischen Energie verbleibt im Stern. Sogar beim absoluten Nullpunkt der Temperatur verbleibt im Stern kinetische Nullpunktsenergie, die beim Kontraktionsprozeß auf Kosten der potentiellen Gravitationsenergie entstanden ist. Es ist daher klar, daß Jungs Differentialgleichung (7) einem sehr künstlichen und äußerst unnatürlichen Prozeß entspricht.

Will man bei der Kontraktion eines homogenen Sterns den Massenverlust durch Ausstrahlung berücksichtigen, so muß man folgendermaßen verfahren.

Die anfängliche Gesamtmasse M_∞ sei bis zur Unendlichkeit zerstreut. Die endgültige Gesamtmasse M_R bilde eine homogene Kugel vom Radius R und von der Dichte ϱ_R. In einem Zwischenstadium sei die Gesamtmasse M_x und bilde eine homogene Kugel vom Radius x und von der Dicbte ϱ_x. Selbstverständlich ist

$$M_\infty > M_x > M_R\,.$$

Wir fassen ins Auge die Gesamtmasse M_x mit dem Radius x. Eine innere Kugelschale habe die Radien y und $y + dy$. Die Masse dieser Kugelschale ist $4\pi y^2 \varrho_x\,dy$. Die auf diese Masse wirkende Newtonsche Gravitationskraft ist gleich

$$\frac{G \,.\, 4\pi y^2 \varrho_x\,dy \,.\, \frac{4}{3}\pi y^3 \varrho_x}{y^2} = \frac{16\,\pi^2 G \varrho_x^2 y^3 dy}{3}\,.$$

Verkürzt sich der Radius x um dx, so verschiebt sich unsere Kugelschale gegen das Kugelzentrum um $\frac{y\,dx}{x}$, wobei die Newtonsche Gravitationskraft die Arbeit:

$$\frac{16\,\pi^2 G \varrho_x^2 y^3 dy}{3} \,.\, \frac{y\,dx}{x} = \frac{16\,\pi^2 G \varrho_x^2 y^4\,dy\,dx}{3x}$$

leistet. Diese Arbeit verwandelt sich in kinetische Energie. Die gesamte in der Kugel auf diese Weise entstandene kinetische Energie ist gleich

$$\int\limits_{y=0}^{y=x} \frac{16\,\pi^2 G \varrho_x^2 y^4\,dx dy}{3\,x} = \frac{16\,\pi^2 G \varrho_x^2 x^5\,dx}{5\,.\,3\,x} =$$

$$= \frac{3}{5}\left(\frac{4\,\pi x^3 \varrho_x}{3}\right)^2 \frac{G dx}{x^2} = \frac{3}{5}\,\frac{G M_x^2\,dx}{x^2}\,.$$

(Wir könnten diesen Ausdruck auch durch Differenzieren der potentiellen Gravitationsenergie $-\frac{3}{5}\frac{GM_x^2}{x}$ gewinnen.) Nehmen wir an, daß der Bruchteil a dieser kinetischen Energie ausgestrahlt wird (also $1-a$ im Stern verbleibt). Bezeichnen wir den dadurch entstandenen Massenverlust durch $-dM_x$, so können wir schreiben:

$$dM_x = a \cdot \frac{3}{5}\frac{GM_x^2\,dx}{x^2 c^2}.$$

Daraus erhalten wir:

$$\int_{M_R}^{M_\infty} \frac{dM_x}{M_x^2} = \int_R^\infty a \cdot \frac{3}{5}\frac{G dx}{x^2 c^2}.$$

Der Einfachheit halber nehmen wir an, daß a konstant sei. Dann ergibt die Integration:

$$\frac{1}{M_R} - \frac{1}{M_\infty} = \frac{3}{5}\frac{G a}{R c^2},$$

oder:

$$\frac{G M_R}{R} = \frac{5 c^2}{3 a}\left(1 - \frac{M_R}{M_\infty}\right). \tag{10}$$

Das Gravitationspotential im Zentrum einer homogenen Kugel erreicht den kritischen Wert $-c^2$, sobald das Oberflächenpotential den Wert $-\frac{G M_R}{R} = -\frac{2}{3} c^2$ erreicht hat. Eliminiert man $\frac{G M_R}{R}$ aus der letzten Gleichung und aus (10), so erhält man:

$$\frac{2}{3} c^2 = \frac{5 c^2}{3 a}\left(1 - \frac{M_R}{M_\infty}\right),$$

oder:

$$\frac{M_R}{M_\infty} = 1 - \frac{2 a}{5}. \tag{11}$$

Die gesamte während der Kontraktion ausgestrahlte Masse ist also gleich

$$M_\infty - M_R = 0{,}4\, a M_\infty = \frac{0{,}4\, a}{1 - 0{,}4 a} M_R.$$

Erfolgt die Kontraktion ohne Energieverlust durch Ausstrahlung, so ist $\alpha = 0$ zu setzen, und (11) ergibt dann $M_R = M_\infty$, wie dies auch zu erwarten war. Wird hingegen die gesamte bei der Kontraktion entstandene kinetische Energie ausgestrahlt, so ist $\alpha = 1$ zu setzen, und (11) ergibt dann:

$$\frac{M_R}{M_\infty} = 1 - \frac{2}{5} = \frac{3}{5}\,.$$

Es möge z. B. die anfängliche Masse eines (sehr großen) Nebels $M_\infty = 10^{45}\,g$ betragen; dann beträgt $M_R = \frac{3}{5}\,M_\infty = 6\,.\,10^{44}\,g$. Dieser Wert in (3) eingeführt ergibt für den Grenzfall:

$$\varrho = 4{,}83\,.\,10^{-7} g\,.\,cm^{-3}\,.$$

Wenn sich also unser Nebel bis zur Dichte von $4{,}83\,.\,10^{-7} g\,.\,cm^{-3}$ zusammenballt, so erreicht in seinem Zentrum das Gravitationspotential den kritischen Wert $-c^2$.

Wird bei der Kontraktion die Hälfte der entstandenen kinetischen Energie ausgestrahlt, so ist $\alpha = \frac{1}{2}$ zu setzen, und (11) ergibt in diesem Falle:

$$\frac{M_R}{M_\infty} = 1 - \frac{1}{5} = \frac{4}{5}\,.$$

Ist wiederum $M_\infty = 10^{45}\,g$, so wird $M_R = \frac{4}{5}\,M_\infty = 8\,.\,10^{44}\,g$ sein, und dann ergibt (3) für den Grenzfall:

$$\varrho = 2{,}72\,.\,10^{-7}\,g\,.\,cm^{-3}\,.$$

Erfolgt die Kontraktion ganz ohne Ausstrahlung (ist also $\alpha = 0$), so ist $M = M_R = M_\infty = 10^{45}$ in (3) zu setzen, und dies ergibt für den Grenzfall:

$$\varrho = 1{,}74\,.\,10^{-7}\,g\,.\,\mathrm{cm}^{-3}\,.$$

Wir sehen also, daß Massenverlust durch Ausstrahlung die Entstehung des kritischen Gravitationspotentials ($-c^2$) keinesfalls verhindern kann. Ist die Masse des Nebels sehr groß, so entsteht das kritische Gravitationspotential schon bei sehr geringer Dichte.

Bevor wir weitergehen, wollen wir untersuchen, ob (vom Standpunkt unserer elementaren Theorie) sich auch reine strahlende Energie im Gravitationsgleichgewicht befinden könne.

Ein solches Gleichgewicht verlangt, daß in jedem Punkte der Strahlungsdruck gleich dem Gravitationsdruck sei. Der Einfachheit halber nehmen wir statt dessen an, daß im Gleichgewichtsfalle der durchschnittliche Strahlungsdruck gleich dem durchschnittlichen Gravitationsdruck sein muß. Außerdem nehmen wir an, daß die strahlende Energie eine homogene Kugel von der Dichte $\varrho\ g\,.\,\mathrm{cm}^{-3}$ bilde. In einem solchen Falle ist der Strahlungsdruck an allen Punkten ein und derselbe: $p = \frac{1}{3}\,\varrho c^2$; ebenso groß ist natürlich auch der durchschnittliche Strahlungsdruck. Der durchschnittliche Gravitationsdruck einer homogenen Kugel ist bekanntlich gleich $\frac{3}{20\,\pi}\,\frac{GM^2}{R^4}\cdot$ Im Falle des Gleichgewichts muß also

$$\frac{1}{3}\,\varrho c^2 = \frac{3}{20\,\pi}\,\frac{GM^2}{R^4}$$

sein. Andererseits ist

$$\frac{4}{3}\,\pi R^3 \varrho = M\,.$$

Eliminiert man ϱ aus den beiden Gleichungen, so erhält man:

$$R = \frac{3}{5}\,\frac{GM}{c^2}\,. \tag{12}$$

Also existiert eine Gleichgewichtslage tatsächlich, und (12) gibt die Größe des „Gleichgewichtsradius" an. Ist aber dies Gleichgewicht stabil oder labil? — Machen wir den Radius R etwas kürzer als der Gleichgewichtsradius ist, so steigt die Dichte ϱ und also auch der Strahlungsdruck $\frac{\varrho c^2}{3}$ umgekehrt proportional R^3, aber der durchschnittliche Gravitationsdruck $\frac{3}{20\,\pi}\,\frac{GM^2}{R^4}$ umgekehrt proportional R^4, also schneller. Der Gravitationsdruck, der jetzt Übergewicht erhalten hat, wird eine weitere Kontraktion der Kugel verursachen, wodurch das relative Übergewicht des Gravitationsdruckes noch mehr steigen wird, usw. Unsere Kugel muß schließlich in einen Punkt zusammenstürzen.

— Wenn wir den Radius etwas größer machen als der Gravitationsradius ist, so erhält der Strahlungsdruck das Übergewicht; dies bewirkt eine weitere Ausdehnung, wodurch das relative Übergewicht des Strahlungsdruckes ein noch größeres wird, usw.

Wir sehen also, daß das Gravitationsgleichgewicht reiner strahlender Energie ein labiles ist.

Das Volumen einer gegebenen Menge strahlender Energie im freien Raume wird sich mit Lichtgeschwindigkeit ausdehnen, wenn diese Ausdehnung durch keinen Gravitationsdruck verhindert wird. Je mehr sich nun unsere aus strahlender Energie bestehende Kugel ausdehnt (also je mehr sie sich vom Gravitationsgleichgewicht entfernt), desto mehr tritt der Gravitationsdruck in den Hintergund, desto schneller wächst also der Radius der Kugel. Man kann annehmen, daß nach genügend langer Zeit der Radius mit Lichtgeschwindigkeit wächst. Im Gleichgewichtszustande ist nach (12) der Radius gleich $\frac{3\,GM}{5\,c^2}$; t Sekunden nach der Störung des Gleichgewichts sei seine Länge gleich R_t. Es ist klar, daß

$$\frac{3\,GM}{5\,c^2} < R_t < \frac{3\,GM}{5\,c^2} + c\,t \tag{13}$$

sein muß. Wir wollen annehmen, daß unsere expandierende Kugel immer homogen bleibe. Außerdem wollen wir voraussetzen, daß t genügend groß sei. Dann können wir sagen, daß der Radius R_t in jeder Sekunde sich um c vergrößert, daß also jedes cm des Radius um $\frac{c}{R_t}$ cm in der Sekunde zunimmt. Mit anderen Worten: jeder Punkt, der sich in der Entfernung 1 cm vom Zentrum befindet, entfernt sich vom letzteren mit der Geschwindigkeit:

$$a = \frac{c}{R_t}. \tag{14}$$

Beträgt die Entfernung des Punktes vom Zentrum l cm, so ist die zentrifugale Geschwindigkeit gleich:

$$la = \frac{lc}{R_t}.$$

Es ist nicht schwer einzusehen, daß auch wenn der Beobachter sich nicht im Zentrum, sondern in einem anderen (inneren) Punkte der Kugel befindet, er trotzdem genau dieselbe Expansion um sich herum beobachten wird. Wir wollen annehmen, daß der Beobachter sich in irgendeinem inneren Punkte A befindet, dessen Entfernung vom Kugelzentrum C offenbar gleich der Geraden AC ist. Es sei B irgendein anderer innerer Punkt, dessen Entfernung vom Kugelzentrum gleich BC ist. Der Punkt A (mit dem Beobachter) entfernt sich vom Kugelzentrum mit der Geschwindigkeit $AC \,.\, a$, der Punkt B hingegen — mit der Geschwindigkeit $BC \,.\, a$. Dies bedeutet, daß im Verlaufe einer Sekunde sowohl AC als auch BC sich um das $(1 + a)$-fache vergrößern. Mit anderen Worten: AC ändert sich proportional BC. Sowohl A als auch B verschieben sich längs den entsprechenden Radien, weshalb der Winkel $\measuredangle ACB$ konstant bleibt. Unter solchen Bedingungen muß sich aber auch AB proportional AC und BC ändern; also auch AB vergrößert sich im Verlaufe einer Sekunde um das $(1 + a)$-fache. Dem Beobachter in A wird es daher scheinen, daß sich der Punkt B mit der Geschwindigkeit $AB \,.\, a$ von A entferne.

Aus (13) und (14) ergibt sich:

$$\frac{c}{\frac{3}{5}\frac{GM}{c^2}} > a > \frac{c}{\frac{3}{5}\frac{GM}{c^2} + ct},$$

oder:

$$\frac{c^3}{0{,}6\, GM} > a > \frac{c^3}{0{,}6\, GM + c^3 t}. \tag{15}$$

Nehmen wir z. B. an, daß die Masse unserer Kugel gleich der Masse des Universums sei, also gleich $2 \,.\, 10^{55}$ g. Nehmen wir ferner an, daß seit Störung des Gleichgewichts ebensoviel Zeit verflossen ist, als unser Universum existiert (nach der „kurzen" Zeitskala). Auf Grund der Radioaktivität der Meteorite schätzt Öpik das Alter des Universums auf ungefähr 3 Milliarden Jahre[4]), was $9{,}468 \,.\, 10^{16}$ Sekunden ausmacht. Setzt man $M = 2 \,.\, 10^{55}$, $t = 9{,}468 \,.\, 10^{16}$, $c = 3 \,.\, 10^{10}$ und $G = 6{,}66 \,.\, 10^{-8}$ in (15) ein, so erhält man:

$$a < 3{,}378 \,.\, 10^{-17} \text{ cm} \,.\, \text{sec}^{-1} \text{ pro cm},$$
$$a > 8{,}046 \,.\, 10^{-18} \text{ cm} \,.\, \text{sec}^{-1} \text{ pro cm},$$

oder:

$$\left.\begin{aligned} a &< 1041 \text{ km} \,.\, \text{sec}^{-1} \text{ pro megaparsec}, \\ a &> 248 \text{ km} \,.\, \text{sec}^{-1} \text{ pro megaparsec}. \end{aligned}\right\} \tag{16}$$

[4]) E. Öpik, Popular Astronomy **41**, 79, 1933.

Was den Gleichgewichtsradius unserer Kugel anbetrifft, so ist er nach (12) gleich

$$R = \frac{3 \cdot 6{,}66 \cdot 10^{-8} \cdot 2 \cdot 10^{55}}{5 \cdot 9 \cdot 10^{20}} = 8{,}88 \cdot 10^{26} \text{ cm.}$$

Wie wir sehen, ist das Verhalten unserer aus reiner strahlender Energie bestehenden Kugel sehr ähnlich dem Verhalten unseres Universums. So z. B. sei daran erinnert, daß nach den Beobachtungen von Hubble

$$a = 560 \text{ km} \cdot \text{sec}^{-1} \text{ pro megaparsec}$$

beträgt, was sich mit (16) im Einklang befindet.

Trotzdem besteht ein wesentlicher Unterschied zwischen der Expansion unserer Kugel und der Expansion des Universums (wenigstens nach der traditionellen Auffassung). Es wird ja gewöhnlich angenommen, daß der Radius des Universums sich alle 1300 Millionen Jahre verdopple. Der Radius unserer Kugel hingegegen wächst mit t nur einfach linear; deshalb muß sich a (nach unserer Auffassung) im Verlaufe der Zeit mehr und mehr verringern.

Gehen wir jetzt zum allgemeineren Falle eines homogenen Himmelskörpers über. Es sei im letzteren

$$\text{Gasdruck} + \text{Strahlungsdruck} = A\varrho^k,$$

wo A und k konstant sind. Im Gleichgewichtsfalle muß $A\varrho^k$ gleich dem durchschnittlichen Gravitationsdruck sein, es muß also

$$A\varrho^k = \frac{3}{20\,\pi}\,\frac{GM^2}{R^4} = \frac{1}{5}\left(\frac{4\,\pi}{3}\right)^{1/3} GM^{2/3}\varrho^{4/3}$$

sein. Es ist leicht einzusehen, daß bei $k > \frac{4}{3}$ das Gleichgewicht ein stabiles ist, und bei $k < \frac{4}{3}$ ein labiles. Bei $k = \frac{4}{3}$ ist ein Gleichgewicht nur dann möglich, wenn

$$A = \frac{1}{5}\left(\frac{4\,\pi}{3}\right)^{1/3} GM^{2/3}$$

ist. Das entsprechende Gleichgewicht ist dann weder stabil noch labil, sondern indifferent, da es ja von ϱ nicht abhängt.

Nach J. von Neumann ist die ultra-extremste Zustandsgleichung einer jeden Materie $p = \frac{\varrho c^2}{3}$. Eine solche Ansicht ist schon vor Neumann von mir ausgesprochen und begründet worden [5]). Aus dem oben Gesagten folgt, daß das Gravitationsgleichgewicht von Materie im ultra-extremsten Zustande nur ein labiles sein kann.

Nach A. Haas kann die Kontraktion eines (homogenen) Himmelskörpers nur so lange andauern, bis

$$Mc^2 = \frac{3}{5}\frac{GM^2}{R}$$

geworden ist. Die dabei erreichte Dichte entspricht dem Gleichheitszeichen in (1). Wir können diese Gleichung auch folgendermaßen schreiben:

$$\varrho^{1/3} = \frac{5}{3}\left(\frac{3}{4\pi}\right)^{1/3}\frac{c^2}{GM^{2/3}}. \tag{17}$$

Wir fragen nun: wie groß wird in diesem Grenzfalle die Summe des Gasdruckes und des Strahlungsdruckes sein, wenn dabei Gravitationsgleichgewicht besteht? — Die erwähnte Summe muß gleich sein dem durchschnittlichen Gravitationsdruck, es muß also die Gleichung:

$$\text{Gasdruck} + \text{Strahlungsdruck} = \frac{1}{5}\left(\frac{4\pi}{3}\right)^{1/3} GM^{2/3}\varrho^{4/3} =$$

$$= \frac{1}{5}\left(\frac{4\pi}{3}\right)^{1/3} GM^{2/3}\varrho^{1/3}\varrho$$

bestehen. Ersetzt man $\varrho^{1/3}$ durch seinen Wert aus (17), so erhält man:

$$\text{Gasdruck} + \text{Strahlungsdruck} =$$

$$= \frac{1}{5}\left(\frac{4\pi}{3}\right)^{1/3} GM^{2/3}\cdot\frac{5}{3}\left(\frac{3}{4\pi}\right)^{1/3}\frac{c^2}{GM^{2/3}}\cdot\varrho = \frac{\varrho c^2}{3}.$$

Natürlich setzen wir dabei voraus, daß die Kontraktion ohne Übergang von Energie zwischen dem Himmelskörper und der Außenwelt stattfindet.

5) Vgl. W. Anderson, l. c. S. 24 ff.

Ein solches Resultat darf uns nicht verwundern. Wenn die Kontraktion ohne Ausstrahlung erfolgt, so verbleibt die entstandene kinetische Energie im Stern, so daß seine Gesamtmasse

$$\left(M - \frac{3}{5}\frac{GM^2}{Rc^2}\right) + \frac{3}{5}\frac{GM^2}{Rc^2} = M$$

konstant bleibt. Wir können $\frac{3}{5}\frac{GM^2}{Rc^2}$ als kinetische Masse bezeichnen und $\left(M - \frac{3}{5}\frac{GM^2}{Rc^2}\right)$ als Ruhemasse. Je mehr der Stern sich verdichtet, desto größer wird seine kinetische Masse und desto kleiner seine Ruhemasse. Schließlich muß letztere gleich Null werden, so daß der Stern nur aus kinetischer Masse bestehen wird, wie dies bei reiner strahlender Energie der Fall ist. Es ist daher auch nicht verwunderlich, wenn dann der Druck der Sternmaterie mit dem Drucke strahlender Energie identisch wird, nämlich gleich $\frac{\varrho c^2}{3}$. Wir wissen aber schon, daß in einem solchen Falle das Gravitationsgleichgewicht ein labiles sein muß.

Bei den tatsächlich existierenden Himmelskörpern sind die Berechnungen natürlich sehr viel komplizierter. Trotzdem kann nicht daran gezweifelt werden, daß auch bei ihnen eine fortschreitende Verdichtung schließlich zu einem labilen Gravitationsgleichgewichtszustand führen muß. Dieser labile Zustand wird bei desto geringerer Dichte eintreten, je größer die Masse des Sterns ist. Die geringste Störung des erwähnten labilen Gleichgewichts muß entweder zur Expansion oder zur weiteren Verdichtung des Himmelskörpers führen. Im letzteren Falle muß die kinetische Masse weiterwachsen. Damit dabei die Gesamtmasse M unverändert bleibe, sind wir nolens volens gezwungen anzunehmen, daß die Ruhemasse negativ wird. Über die Möglichkeit einer negativen Ruhemasse habe ich mich schon früher geäußert[6]). Schließlich muß der Himmelskörper in einen Punkt zusammenfallen. Was geschieht aber weiter? — Am nächstliegenden wäre wohl anzunehmen, daß im erwähnten Punkte eine Art Reflexion eintrete: die Geschwindigkeiten keh-

[6]) W. Anderson, l. c. S. 59 f.

ren ihre Richtungen um, so daß der Himmelskörper sich wieder ausdehnt. Wir sind aber dabei gezwungen anzunehmen, daß die Dichte im erwähnten Punkte den Wert $\varrho = \infty$ erreicht habe, freilich nur auf unendlich kurze Zeit. Ob ein derartiges „ungewöhnliches" Ereignis auch tatsächlich bei irgendeinem Himmelskörper stattgefunden hat, ist natürlich eine ganz andere Frage. Vielleicht ist seit dem „Anfang der Welt" noch nicht genug Zeit verflossen, damit ein (genügend großer) Himmelskörper sich bis zu dem labilen Gleichgewichtszustande verdichten könnte. Vielleicht aber wird das oben beschriebene Zusammenbrechen der labil gewordenen Himmelskörper durch irgendeinen neuen Faktor verhindert. Man könnte z. B. an eine Art von „kosmischer Repulsion" denken, oder auch an J. Nuut's „kinematische Expansion" des Raumes[7]). Solche Faktoren stellen aber wiederum etwas „Ungewöhnliches" dar[8]).

Früher habe ich meine Ansichten über die Grenzdichte sehr kategorisch formuliert: „In keinem Punkte eines Himmelskörpers kann das Gravitationspotential den kritischen Wert $-c^2$ überschreiten". Jetzt ziehe ich folgende mildere Formulierung vor: „In keinem Punkte eines Himmelskörpers kann das Gravitationspotential den kritischen Wert $-c^2$ überschreiten, ohne daß etwas „Ungewöhnliches" passiert".

Ich möchte noch daran erinnern, daß alles über die Himmelskörper Gesagte auch für das Universum als Ganzes gültig ist. Wir müssen daher annehmen, daß am „Anfang der Welt", wo sich das Universum noch im Gravitationsgleichgewicht befand, die Ruhemasse gleich Null gewesen ist. Dies bedeutet, daß damals alle „materiellen" Teilchen bis zu einem gewissen Grade sich wie Lichtquanten benahmen (die ja auch keine merkliche Ruhemasse, sondern nur kinetische Masse besitzen). In gewissem Sinne haben wir daher das Recht zu behaupten, daß „am Anfang der Welt nur Licht existiert habe".

Wir wissen schon, daß bei einem derartigen „Lichtstadium" der Materie das Gravitationsgleichgewicht labil sein muß: die geringste Störung dieses Gleichgewichts wird zu einer Kontrak-

7) J. Nuut, Acta et Comm. Univ. Tartuensis (Dorpatensis) A **29**$_3$ und **29**$_6$ (1935).

8) Ich möchte noch darauf aufmerksam machen, daß bei allen unseren Berechnungen wir immer nur einen euklidischen Raum vorausgesetzt haben.

tion oder zu einer Expansion führen. Wir wollen letzteres annehmen. Außerdem nehmen wir an, daß zwischen dem expandierenden Universum und dem außerhalb des Universums befindlichen Raume kein Energieübergang stattfindet. Zur Zeit des anfänglichen (labilen) Gleichgewichts war die kinetische Masse des Universums gleich $\frac{3\,GM^2}{5\,Rc^2}$ und die Ruhemasse gleich $M - \frac{3\,GM^2}{5\,Rc^2} = 0$, wo R den Gleichgewichtsradius bedeutet. Bei der Expansion steigt die Ruhemasse auf Kosten der kinetischen Masse. In unendlich ferner Zukunft wird sich unser Universum bis zur Unendlichkeit ausgedehnt haben, wobei sich die kinetische Masse um $\frac{3\,GM^2}{5\,Rc^2}$ verringert haben wird und die Ruhemasse um ebensoviel vergrößert. Dann wird die kinetische Masse gleich

$$\frac{3\,GM^2}{5\,Rc^2} - \frac{3\,GM^2}{5\,Rc^2} = 0$$

sein, und die Ruhemasse gleich

$$\left(M - \frac{3\,GM^2}{5\,Rc^2}\right) + \frac{3\,GM^2}{5\,Rc^2} = M.$$

Während des „Lichtstadiums“ (d. h. während des labilen Gravitationsgleichgewichts) ist die durchschnittliche „Molekulargeschwindigkeit“ der Partikelchen gleich der Lichtgeschwindigkeit. Bei der fortschreitenden Expansion verringert sich diese durchschnittliche Molekulargeschwindigkeit mehr und mehr und wird schließlich bei unendlicher Expansion gleich Null. Wie groß ist aber die augenblickliche durchschnittliche Molekulargeschwindigkeit der Partikelchen des Universums? —

Wir wollen wiederum annehmen, daß seit der Störung des (labilen) Gravitationsgleichgewichts 3 Milliarden Jahre $= 9{,}468 . 10^{16}$ Sekunden verflossen sind. Der Gleichgewichtsradius ist gleich $R = 8{,}88 . 10^{26}$ cm (siehe oben S. 13), daher ist der augenblickliche Radius des Universums

$$R_t < R + ct = 8{,}88 . 10^{26} + 3 . 10^{10} . 9{,}468 . 10^{16} = 3{,}7284 . 10^{26}.$$

Also ist jedenfalls

$$R_t < 4{,}2\, R\,.$$

Die zur Expansion von R bis R_t verbrauchte kinetische Energie ist gleich

$$\frac{3}{5}\frac{GM^2}{R} - \frac{3}{5}\frac{GM^2}{R_t} < \frac{3}{5}\frac{GM^2}{R} - \frac{3}{5\,.\,4{,}2}\frac{GM^2}{R} = \frac{16}{21}\cdot\frac{3}{5}\frac{GM^2}{R} = \frac{16}{21}Mc^2 .$$

Diese verbrauchte kinetische Energie ist natürlich nicht spurlos verschwunden, sondern hat sich in potentielle Gravitationsenergie verwandelt, wodurch die Ruhemasse (die anfangs gleich Null war) sich entsprechend vergrößert hat. Diese Ruhemasse wird jetzt aber immerhin noch kleiner sein als $\frac{16}{21}M$. Die im Universum verbliebene kinetische Energie ist daher größer als

$$Mc^2 - \frac{16}{21}Mc^2 = \frac{5}{21}Mc^2 .$$

Wenn die Masse m eines Partikelchens des Universums im Durchschnitt aus $\frac{16\,m}{21}$ Ruhemasse und $\frac{5\,m}{21}$ kinetischer Masse bestehen würde, so könnte man die entsprechende durchschnittliche Geschwindigkeit v_m aus der Gleichung

$$\frac{5\,mc^2}{21} = \frac{16\,mc^2}{21}\left(\frac{1}{\sqrt{1-\frac{v_m^2}{c^2}}} - 1\right)$$

berechnen. Wir erhalten:

$$\frac{v_m}{c} = 0{,}648 .$$

Doch ist die kinetische Masse nicht gleich, sondern größer als $\frac{5\,m}{21}$; deshalb muß

$$\frac{v_m}{c} > 0{,}648 \qquad (18)$$

sein. Wir sehen also, daß die heutige durchschnittliche Molekulargeschwindigkeit der Partikelchen des Universums zwar nicht mehr (wie „am Anfang der Welt") gleich der Lichtgeschwindigkeit ist, aber immerhin noch recht nahe an sie heranreicht.

Heutzutage dehnt sich unser Universum mit einer Geschwindigkeit aus, die ebenfalls recht nahe an Lichtgeschwindigkeit heranreicht. Doch in ferner Zukunft wird sich die Expansionsgeschwindigkeit (im Gegensatz zur Expansionsgeschwindigkeit einer aus „wahrer" strahlender Energie, d. h. aus Photonen bestehenden Kugel) mehr und mehr verringern. Bei unendlich großer Expansion sinkt die Expansionsgeschwindigkeit (sowie auch die durchschnittliche Molekulargeschwindigkeit der Partikelchen des Universums) bis auf Null [9]).

Natürlich wird es in jedem beliebigen Augenblick Partikelchen geben, deren Geschwindigkeiten größer oder kleiner, sogar sehr viel kleiner als die durchschnittliche Geschwindigkeit sind. Während der Expansion werden die schnelleren Partikelchen größere Wege durchlaufen als die langsameren. Es findet also eine Diffusion der Partikelchen statt, entsprechend ihren Geschwindigkeiten. Dies ist genau dieselbe Erscheinung wie bei dem bekannten „kinematischen" Weltmodell von E. A. Milne.

Bei allen unseren Untersuchungen haben wir der Einfachheit halber immer angenommen, daß die expandierende Kugel homogen bleibt. Dann muß jedes Kubikzentimeter

$$n = \frac{N}{\frac{4}{3}\,\pi R_t^3} = \frac{3\,N}{4\,\pi R_t^3}$$

Partikelchen enthalten, wenn N die Gesamtzahl der im Universum vorhandenen Partikelchen bedeutet. Das Volumen einer

[9]) Es sei hier vor Verwechslung der Expansionsgeschwindigkeit mit der durchschnittlichen Molekulargeschwindigkeit gewarnt. Während des anfänglichen „Lichtstadiums" war die Ruhemasse der Universums gleich Null, und die duchschnittliche Molekulargeschwindigkeit seiner Partikelchen gleich der Lichtgeschwindigkeit. Dagegen war die Expansionsgeschwindigkeit gleich Null, da sich unser Universum damals in einem (freilich labilen) Gravitationsgleichgewicht befand. Nachdem die Expansion begonnen hat, wächst ihre Geschwindigkeit mehr und mehr, während die durchschnittliche Molekulargeschwindigkeit abzunehmen beginnt. Schließlich erreicht die Expansionsgeschwindigkeit ihren Maximalwert, der recht nahe der Lichtgeschwindigkeit ist. Auch die duchschnittliche Molekulargeschwindigkeit wird immer noch recht nahe der Lichtgeschwindigkeit sein. Bei noch weiterer Expansion wird sowohl die Expansionsgeschwindigkeit als auch die durchschnittliche Molekulargeschwindigkeit mehr und mehr sinken, um schließlich bei unendlicher Ausdehnung des Universums gleich Null zu werden.

Wir haben andererseits angenommen, daß die Expansionsgeschwindigkeit einer aus „wahrer" stahlender Energie (d. h. aus Photonen) bestehenden Kugel inneren Kugelschale mit den Radien r und $r+dr$ ist gleich $4\pi r^2 dr$, und dieses Volumen enthält

$$n \, . \, 4\pi r^2 dr = \frac{3N}{4\pi R_t^3} \cdot 4\pi r^2 dr = \frac{3Nr^2 dr}{R_t^3} \tag{19}$$

Partikelchen. Die Expansionsgeschwindigkeiten dieser Partikelchen liegen zwischen $v = ra$ und $v + dv = (r + dr)a = ra + dr \, . \, a$. Aus diesen Gleichungen ergibt sich:

$$r = \frac{v}{a} \quad \text{und} \quad dr = \frac{dv}{a} \cdot$$

Führt man diese Werte in (19) ein, so erhält man:

$$\frac{3Nr^2 dr}{R_t^3} = \frac{3Nv^2 dv}{a^3 R_t^3},$$

oder im Hinblick auf (14):

$$N_v^{v+dv} = \frac{3Nv^2 dv}{a^3 R_t^3} = \frac{3Nv^2 dv}{c^3},$$

wo N_v^{v+dv} die im Universum vorhandene Gesamtzahl jener Partikelchen bedeutet, deren Expansionsgeschwindigkeiten zwischen v und $v+dv$ liegen. Es ist natürlich klar, daß die gewonnene

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selbst nach unendlich großer Ausdehnung gleich der Lichtgeschwindigkeit bleibt. Dies läßt sich erklären, wenn man annimmt, daß im unendlich ausgedehnten Zustande die Ruhemasse der Kugel gleich Null sei und die kinetische Masse gleich M. Zur Zeit des (labilen) Gravitationsgleichgewichts hingegen muß die Ruhemasse negativ sein, nämlich gleich $-\frac{3}{5}\frac{GM^2}{Rc^2}$, und die kinetische Masse gleich $M + \frac{3}{5}\frac{GM^2}{Rc^2}$. Also auch in diesem Falle ist die Gesamtmasse gleich M. Bei einer unendlichen Expansion nimmt die kinetische Masse um $\frac{3}{5}\frac{GM^2}{Rc^2}$ ab und die Ruhemasse um ebensoviel zu. Da auf diese Weise die Ruhemasse der Photonen niemals positiv werden kann, so liegt auch kein Grund vor für das Abrücken ihrer Geschwindigkeiten von der gewöhnlichen Lichtgeschwindigkeit. Ob sich aber eine nur aus Photonen bestehende Kugel auch tatsächlich in alle Ewigkeiten mit Lichtgeschwindigkeit ausdehnen wird, ist natürlich eine andere Frage, auf die wir jedoch nicht näher eingehen wollen.

Man darf auch nicht die Expansionsgeschwindigkeit mit dem Expansionskoeffizienten a verwechseln. Selbst wenn die Expansionsgeschwindigkeit tatsächlich in alle Ewigkeiten konstant und gleich der Lichtgeschwindigkeit bleiben sollte, wird sich a im Verlaufe der Zeit trotzdem mehr und mehr verringern, wie ja dies aus (14) klar hervorgeht.

Gleichung nur so lange gültig bleibt, als das Universum sich annähernd mit Lichtgeschwindigkeit ausdehnt. In genügend ferner Zukunft wird unsere Gleichung schließlich versagen.

Wie verhält sich aber die oben abgeleitete Verteilung der Expansionsgeschwindigkeiten zu der Maxwellschen Verteilung der Molekulargeschwindigkeiten? —

Natürlich kann die gewöhnliche Maxwellsche Formel nicht ohne weiteres angewandt werden bei durchschnittlichen Molekulargeschwindigkeiten, die nahe an Lichtgeschwindigkeit herankommen. Wollen wir aber trotzdem versuchen, die Maxwellsche Formel auf die Partikelchen des Universums anzuwenden, wobei wir ihre durchschnittliche (quadratische) Geschwindigkeit der Einfachheit halber genau gleich der Lichtgeschwindigkeit setzen. Wir erhalten dann:

$$N_v^{v+dv} = \frac{4}{\sqrt{\pi}} \sqrt{\frac{27}{8}} \frac{N v^2 e^{-\frac{3}{2}\frac{v^2}{c^2}} dv}{c^3} .$$

Ist die individuelle Geschwindigkeit v merklich kleiner als die durchschnittliche Geschwindigkeit, so wird $e^{-\frac{3}{2}\frac{v^2}{c^2}}$ nur wenig verschieden von 1 sein. Wir können daher annähernd schreiben:

$$N_v^{v+dv} = \frac{4}{\sqrt{\pi}} \sqrt{\frac{27}{8}} \cdot \frac{N v^2 dv}{c^3} = 1{,}38 \cdot \frac{3\, N v^2 dv}{c^3} .$$

Stellen wir noch einmal die erhaltenen Formeln zusammen:

Verteilungsgesetz der Expansionsgeschwindigkeiten:

$$N_v^{v+dv} = \frac{3\, N v^2 dv}{c^3} .$$

Maxwellsches Verteilungsgesetz der Molekulargeschwindigkeiten:

$$N_v^{v+dv} = 1{,}38 \cdot \frac{3\, N v^2 dv}{c^3} . \qquad (20)$$

Wir sehen, daß die Verteilung der Expansionsgeschwindigkeiten sich von der Maxwellschen Verteilung nur unwesentlich unterscheidet. Der ganze Unterschied besteht nur in einem konstanten Zahlenfaktor.

Gegen Milne's „kinematisches“ Weltmodell macht A. Haas folgenden Einwand: „Die Milnesche Idee begegnet allerdings der Schwierigkeit, daß — unter der Voraussetzung einer gleichmäßigen Verteilung der Nebel in dem teleskopisch erreichbaren Raume — von Anfang an die höheren Geschwindigkeiten viel häufiger als die niedrigeren hätten sein müssen; eine derartige Geschwindigkeitsverteilung wäre völlig von den sonst in der Physik bekannten verschieden“[10]. Vom Standpunkt unserer Theorie jedoch muß dieser Einwand als hinfällig betrachtet werden: wir haben ja soeben gesehen, daß zwischen der Verteilung der Expansionsgeschwindigkeiten und der Maxwellschen Verteilung gar kein besonders großer Antagonismus besteht.

Freilich ist die räumliche Geschwindigkeitsverteilung in beiden Fällen eine völlig verschiedene. Bei der Expansion ordnen sich die Partikelchen so, daß ihre Geschwindigkeiten proportional den Entfernungen vom Zentrum sind, und gerichtet sind diese Geschwindigkeiten vom Zentrum weg. Daraus folgt, daß die Bewegungszustände zweier in der Nachbarschaft befindlicher Partikelchen nur wenig verschieden sind. Mit anderen Worten: die relative Geschwindigkeit eines Partikelchens hinsichtlich der Nachbarpartikelchen ist gering. Keine derartige räumliche Ordnung existiert in einem gewöhnlichen Gase: hier können auch die in der nächsten Nachbarschaft befindlichen Partikelchen die verschiedensten relativen Geschwindigkeiten aufweisen.

Wir haben zwar angenommen, daß bei der Expansion die Homogenität gewahrt bleibt: trotzdem verlangt das Gesetz des Zufalls, daß in jedem Moment bald hier bald dort kleine zufällige Abweichungen von der entsprechenden mittleren Dichte auftreten. An einer Stelle werden die Partikelchen zufällig etwas mehr zusammengedrängt sein, an einer anderen etwas weniger. Wenn nun irgendwo sich zufällig eine beträchtlich stärkere Verdichtung ausbildet, so wird letztere eine merklich verstärkte Newtonsche Anziehungskraft auf die Nachbarpartikelchen ausüben, wobei nicht vergessen werden darf, daß die relativen Geschwindigkeiten dieser Nachbarpartikelchen nur gering sind. Die rein zufällig entstandene Verdichtung kann sich dank den hereinstürzenden Nachbarpartikeln bedeutend vergrößern. Dieser Verdichtungsprozeß hat auf den Bewegungszustand des Schwer-

[10]) A. Haas, Kosmologische Probleme der Physik, Leipzig 1934, S. 58.

punkts der sich verdichtenden Masse natürlich keinen Einfluß. Da wir bei der Expansion annähernde Homogenität voraussetzen, so werden die an verschiedenen Stellen entstandenen Verdichtungen nicht sehr verschiedene Massen besitzen.

Zur Zeit des labilen Gravitationsgleichgewichts (zur Zeit des „Lichtstadiums") des Universums bewegten sich die Partikelchen mit Lichtgeschwindigkeit und völlig ungeordnet hin und her. Eine Entstehung von großen lokalen Verdichtungen war damals undenkbar. Erst während der Expansion bildete sich die bekannte Ordnung in der räumlichen Geschwindigkeitsverteilung aus, wodurch die Entstehung großer lokaler Verdichtungen (Himmelskörper) möglich wurde. Dabei darf man aber natürlich nicht denken, daß die erwähnte Ordnung sich sofort nach Beginn der Expansion in voller Strenge eingestellt habe. In Wirklichkeit wird dieser Prozess allmählich vor sich gegangen sein. Je weiter die Expansion fortschritt, desto größer wurde die Zahl der Partikelchen, die „sich genügend der Ordnung unterworfen hatten", desto geringer wurde die Zahl derer, deren Bewegungen man noch als „ungeordnet" zu betrachten gezwungen war. Auch heutzutage, wo bereits die meisten Partikelchen „sich der Ordnung genügend unterworfen haben" (und sich, als weitere Folge davon, meistens zu Himmelskörpern verdichtet haben), wird es noch eine gewisse Anzahl „ungeordneter" Partikelchen geben, die nach allen Seiten hin und her fliegen[11]. Die durchschnittliche Geschwindigkeit dieser Partikelchen ist nach (18) größer als 0,648 c[12]. Diese Partikelchen sind dem-

[11]) Was die Natur dieser Partikelchen anbetrifft, so wäre am nächstliegenden ein Gemisch von Neutronen, Protonen, Elektronen und Positronen anzunehmen, und vielleicht noch von negativen Protonen (über letztere vgl. Ioan I. Placinteanu, „Sur l'existence du proton à charge négative; constitution du noyau de l'isotope de H_2", Extrait du Bulletin de la Société Roumaine de Physique Vol. 35. Nr. 60. — Août — Octobre 1933).

[12]) Der „geordnete" Zustand unterscheidet sich vom „ungeordneten" nicht durch die Größe der Molekulargeschwindigkeiten, sondern nur durch ihre räumliche Verteilung.

Die kinetische Energie eines mit $0{,}648\ c = 1{,}944 \cdot 10^{10}$ cm $\cdot$ sec^{-1} Geschwindigkeit sich bewegenden Protons (oder Neutrons) ist gleich $4{,}67 \cdot 10^{-4}$ Erg $=$ 294 Millionen Volt-Elektronen. Die kinetische Energie eines mit gleicher Geschwindigkeit sich bewegenden Elektrons (oder Positrons) ist beinahe 2000 mal kleiner.

Es ist jedoch durchaus wahrscheinlich, daß sich unter den „ungeordneten" Partikelchen des Universums bereits eine weitgehende Äquipartition der kinenach als „Überreste des anfänglichen Lichtstadiums des Universums" anzusehen. Es wäre verlockend, diese „ungeordneten" Partikelchen mit den Partikelchen der kosmischen Höhenstrahlung zu identifizieren. Nach P. Kunze kommen kosmische Partikelchen von über einer Milliarde Volt-Elektronen nur selten vor. Das schnellste Partikelchen, welches er noch gut messen konnte, hatte eine Energie von 2,66 Milliarden Volt-Elektronen[13]). Dies sind Zahlen von ähnlicher Größenordnung, wie wir sie für unsere „ungeordneten" Partikelchen berechnet haben (vgl. die Fußnote 12). Dagegen gelangt L. G. H. Huxley zu beträchtlich höheren kinetischen Energien für die kosmische Strahlung[14]). Es darf aber dabei

tischen Energie eingestellt hat. Wir wollen außerdem die vereinfachende Annahme machen, daß im Universum die Zahl der „ungeordneten" leichten Partikelchen (der Elektronen und Positronen) gleich der Zahl der „ungeordneten" schweren (der Protonen und Neutronen) sei. In einem solchen Falle wird die durchschnittliche kinetische Energie einer jeden Art immerhin noch größer als $\frac{294}{2} = 147$ Millionen Volt-Elektronen sein.

Nehmen wir aber an, daß die durchschnittliche kinetische Energie der „ungeordneten" Partikelchen genau 147 Millionen Volt-Elektronen betrage. Setzen wir außerdem die gewöhnliche Maxwellsche Streuung der individuellen kinetischen Energie um ihren Durchschnittswert voraus. Dann werden z. B. unter 10^{12} Partikelchen sich $7{,}4 \cdot 10^9$ solcher finden, deren individuelle kinetische Energien größer sind als 588 Millionen Volt-Elektronen. Bei $5{,}9 \cdot 10^6$ Partikelchen werden diese Energien mehr als 1,32 Milliarden Volt-Elektronen betragen, und bei nur 210 Partikelchen mehr als 2,35 Milliarden. Nehmen wir ferner an, daß auf eine Fläche von 10 cm^2 jede Sekunde 100 „ungeordnete" Partikelchen fallen (was einen Energiestrom von $2{,}3 \,.\, 10^{-3}$ Erg $\cdot$ cm^{-2} $\cdot$ sec^{-1} bedeutet). Außerdem wollen wir noch annehmen, daß die von uns vorausgesetzte normale Maxwellsche Energieverteilung weder durch ein absorbierendes Medium, noch auf irgendeine andere Weise beeinflußt werde. Dann wird durchschnittlich alle 1,4 Sekunden ein Partikelchen von über 588 Millionen Volt-Elektronen auf die erwähnte Fläche fallen. Alle 28 Minuten fällt ein Partikelchen von über 1,32 Milliarden Volt-Elektronen, und alle 6,7 Jahre eins von über 2,35 Milliarden. Man darf jedoch dabei nicht vergessen, daß die tatsächliche durchschnittliche kinetische Energie der Partikelchen des Universums nicht gleich, sondern größer als 147 Millionen Volt-Elektronen ist.

Außer den „geordneten" und den „ungeordneten" wird es im Universum selbstverständlich auch noch solche Partikelchen geben, die man als „halbgeordnet" bezeichnen könnte. Auf die Eigenschaften solcher Partikelchen wollen wir jedoch hier nicht näher eingehen.

[13]) P. Kunze, ZS. f. Phys. **80**, 559, 1933.

[14]) L. G. H. Huxley, Phil. Mag. (7) **18**, 979, 1934.

nicht außer acht gelassen werden, daß Huxley seine hohen Zahlen nicht (wie Kunze) aus direkten Beobachtungen mit der Wilsonkammer erhalten hat, sondern nur auf Grund spekulativer Betrachtungen über die vermutliche Wikung des irdischen Magnetfeldes auf die kosmische Strahlung [15]). Auf eine weitere Durchmusterung der vorhandenen Literatur wollen wir jedoch verzichten.

Wenn „am Anfang der Welt“ das „Lichtstadium“ geherrscht hat, wenn also damals die Ruhemasse gleich Null war, so reicht die damalige kinetische Energie gerade aus, um das Universum vom anfänglichen (labilen) Gleichgewichtszustand bis zur Unendlichkeit auszudehnen. Die selbstverständliche Folge davon ist, daß während der ganzen Expansionsdauer die kinetische Energie eines jeden Partikelchens in jedem Moment genau groß genug ist, um das Partikelchen von seinem augenblicklichen Orte in eine unendliche Entfernung zu tragen. Diese Regel ist selbstverständlich unabhängig von dem Orte und von dem Bewegungszustand des Beobachters. Es möge der Beobachter in irgendeinem inneren Punkte A sich befinden, und das den Beobachter interessierende Partikelchen in einem (ebenfalls inneren) Punkte B. (Zwar können A und B ganz beliebige innere Punkte bedeuten, aber immerhin wollen wir annehmen, daß die Entfernun-

[15]) Es sei hier daran erinnert, was für Überraschungen uns der Sonnenmagnetismus gebracht hat. So z. B. haben wir für den starken Magnetismus der Sonnenflecke immer noch keine absolut befriedigende Erklärung. Außerdem verhält sich das magnetische Feld der Sonne gar nicht so, wie es sich „vernünftigerweise“ verhalten sollte, sondern nimmt mit der Entfernung von der Sonnenoberfläche „viel zu schnell“ ab. Um dies zu erklären, sind ganze Theorien aufgestellt worden (vgl. T. G. Cowling, „On the radial limitation of the Sun's magnetic field“, Monthly Not. Roy. Astron. Soc. **90**, 140, 1929). So suchte man nachzuweisen, daß die stark ionisierte Chromosphäre eine diamagnetische Wirkung ausübe, welche das magnetische Sonnenfeld teilweise neutralisiere. Es sei aber daran erinnert, daß auch unsere Erde von einer stark ionisierten atmosphärischen Oberschicht umgeben ist. Wie es nun mit dem irdischen Magnetfelde außerhalb dieser ionisierten Oberschicht tatsächlich bestellt ist, das kann niemand mit Bestimmtheit wissen. Allen Berechnungen über die Wirkung des irdischen Magnetfeldes auf die kosmische Strahlung sollte daher nicht ohne Mißtrauen begegnet werden. — Das andere Problem: wie groß die minimale kinetische Energie der Elektronen, der Protonen oder der Neutronen sein müsse, damit sie in merklicher Menge die irdische Atmosphäre durchdringen können, scheint bis jetzt ebenfalls noch nicht wirklich befriedigend gelöst worden zu sein.

gen dieser beiden Punkte vom Zentrum C beträchtlich kleiner sind als der augenblickliche Radius des Universums R_t). Wir wissen schon (siehe oben S. 12), daß es dem Beobachter in A scheinen wird, daß sich der Punkt B (mit dem Partikelchen) von A entferne, wobei diese Geschwindigkeit gleich $AB \,.\, a$ sei. Wenn diese Geschwindigkeit beträchtlich kleiner als die Lichtgeschwindigkeit ist, so wird die scheinbare kinetische Energie des Partikelchens gleich $\frac{1}{2}\, m\,(AB \,.\, a)^2$ sein, wo m die Masse des Partikelchens bedeutet. Diese augenblickliche relative kinetische Energie muß gerade ausreichen, um das Partikelchen aus dem augenblicklichen Aufenthaltsort in unendliche Entfernung von dem Beobachter in A tragen zu können[16]). Wir können ohne merklichen Fehler behaupten, daß auf das Partikelchen ausschließlich die Newtonsche Anziehungskraft einer Kugel vom Radius AB und von der Masse $\frac{4}{3}\pi(AB)^3\varrho$ wirke. Wenn sich nun infolge der fortschreitenden Expansion des Universums die Entfernung des Punktes B (mit dem Partikelchen) von A vergrößert, so bleibt trotzdem die auf das Partikelchen wirkende Masse unverändert. Diese Masse wird die ganze Zeit hindurch genau so wirken, als ob sie im Punkte A konzentriert wäre. Unter solchen Umständen ist die zur unendlich weiten Entfernung des Partikelchens notwendige Arbeit gleich

$$\frac{G \cdot \frac{4}{3}\pi\,(AB)^3\varrho \,.\, m}{AB} = \frac{4\,\pi\, G (AB)^2 \varrho m}{3}.$$

[16]) Dies ist freilich nur bei idealer Homogenität des Universums streng richtig. Sollte sich hingegen um den Punkt A eine zufällige Verdichtung ausgebildet haben, so reicht die relative kinetische Energie $\frac{1}{2}\, m\,(AB \,.\, a)^2$ nicht mehr aus, um das Partikelchen B bis zur Unendlichkeit von A zu entfernen. Dehnt sich nun das ganze Universum bis zur Unendlichkeit aus, so werden auch die Entfernungen AC und BC unendlich, nicht aber die Entfernung AB, die in alle Ewigkeiten endlich bleiben muß. Nur wenn auch um einen anderen Punkt D eine Verdichtung entsteht, die auf das Partikelchen B stärker wirkt als die Verdichtung um A, kann die Entfernung AB unendlich werden. Dafür bleibt aber dann BD in alle Ewigkeiten endlich. In einem solchen Falle wird also das Partikelchen B nicht mehr dem System A angehören, sondern dem System D.

Diese Arbeit muß genau gleich der augenblicklichen kinetischen Energie des Partikelchens sein, es muß also die Gleichung

$$\frac{1}{2} m (AB \,.\, a)^2 = \frac{4\pi G (AB)^2 \varrho m}{3}$$

bestehen, oder:

$$a^2 = \frac{8\pi G \varrho}{3} = \frac{8\pi G}{c^2} \,.\, \frac{\varrho c^2}{3},$$

oder:

$$\boxed{a^2 = \frac{1}{3} \varkappa \varrho c^2 \,.} \tag{21}$$

Vor etwa 5 Jahren haben Einstein und de Sitter diese Formel aus den Friedmanschen Gleichungen deduziert[17]). Nun haben wir dieselbe Formel auf ganz elementare Weise aus unserer Expansionstheorie des Universums abgeleitet.

[17]) A. Einstein und W. de Sitter, Proc. Nat. Acad. of. Sciences (Washington) **18**, 213, 1932.

WEITERE BEITRÄGE ZU DER ELEMENTAREN EXPANSIONSTHEORIE DES UNIVERSUMS

VON

WILHELM ANDERSON

TARTU 1937

K. Mattiesens Buchdruckerei Ant.-Ges., Tartu 1937.

In meinem vorigen Aufsatz [1], in dem ich die Einwände von B. Jung zurückwies, habe ich gleichzeitig eine elementare Expansionstheorie des Universums entwickelt. Unter anderem gelang mir eine ganz elementare Ableitung der Einstein-de Sitterschen Gleichung:

$$\alpha^2 = \frac{1}{3}\,\varkappa\varrho c^2 = \frac{8\pi G\varrho}{3}. \tag{1}$$

Inzwischen hat Prof. Milne die Freundlichkeit gehabt, mir einige seiner Schriften zuzusenden, darunter einen mir bis jetzt unbekannt und unzugänglich gewesenen Artikel vom Jahre 1934, wo er die Einstein-de Sittersche Gleichung ebenfalls auf elementare Weise ableitet [2]. Ich bedaure es sehr, diesen Artikel nicht rechtzeitig gelesen zu haben.

Wie ich jetzt sehe, ist Milne's Ableitung identisch mit der unsrigen, bis auf folgenden Unterschied. Milne sagt: „Consider the particular case in which the distant particle has the *parabolic* velocity of escape from the mass contained in the sphere of radius r" [3]. Bei Milne spielt also die parabolische Geschwindigkeit nur die Rolle einer ad hoc gemachten Voraussetzung, in unserer Theorie hingegen erscheint die parabolische Geschwindigkeit als **notwendige Konsequenz** des anfänglichen „Lichtstadiums" des Universums. Wir haben ja angenommen, daß im Anfangsstadium die Ruhemasse des Universums gleich

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[1] W. Anderson, Public. de l'Observ. de l'Univ. de Tartu $\mathbf{29}_6$ = Acta et Comm. Univ. Tartuensis (Dorpatensis) A $\mathbf{33}_2$.

[2] E. A. Milne, The Quarterly Journal of Mathematics (Oxford series) **5**, 70, 1934. — Diese Zeitschrift ist weder in unserer Universitätsbibliothek, noch in irgendeiner anderen hiesigen Institution vorhanden.

[3] Ebenda, S. 67.

Null gewesen ist, so daß seine Gesamtmasse gleich seiner kinetischen Masse war, nach der Gleichung:

$$M = \frac{3\,GM^2}{5\,Rc^2}, \tag{2}$$

wo R den Gleichgewichtsradius bedeutet. Dies Gravitationsgleichgewicht erweist sich aber als ein labiles. Während der Expansion nimmt die kinetische Energie ab, kann dabei jedoch nicht spurlos verschwinden, sondern muß sich in potentielle Energie verwandeln. Diese Vergrößerung der potentiellen Energie muß sich in einer entsprechenden Vergrößerung des Ruhemasse manifestieren. Mit anderen Worten: die Abnahme an kinetischer Masse wird genau kompensiert durch die Zunahme an Ruhemasse, so daß die Gesamtmasse des Universums konstant bleibt. Bei unendlicher Expansion wird das Universum gar keine kinetische, sondern nur Ruhemasse besitzen. Die anfängliche kinetische Energie $\frac{3\,GM^2}{5\,R}$ reicht also g e n a u dazu aus, um das Universum bis zur Unendlichkeit auszudehnen. Die notwendige Konsequenz davon ist, daß die Expansionsgeschwindigkeit eines jeden Partikelchens immer eine parabolische bleiben muß. Nun haben wir im vorigen Aufsatz auf Grund elementarer geometrischer Überlegung gezeigt (S. 12), daß ein jeder Beobachter genau dieselbe Expansion um sich sehen wird. D e s h a l b m u ß f ü r e i n e n j e d e n B e o b a c h t e r d i e E x p a n s i o n s g e s c h w i n d i g k e i t e i n e s j e d e n P u n k t e s i m m e r e i n e p a r a b o l i s c h e b l e i b e n. (Selbstverständlich bezieht sich diese Regel nicht auf die „ungeordneten" Partikelchen.)

Hinsichtlich der „ungeordneten" Partikelchen ist mir im vorigen Aufsatz bei der Berechnung der unteren Energiegrenze eine Ungenauigkeit untergelaufen, die ich gleich berichtigen werde. Ein und derselbe Körper von der Masse m kann dem einen Beobachter als ruhend erscheinen, dem anderen als bewegt. Der erstere wird sagen, daß m nur Ruhemasse sei, der letztere hingegen wird m als Summe von Ruhemasse und von kinetischer Masse ansehen. Sollte sich endlich ein Beobachter mit Lichtgeschwindigkeit hinsichtlich des Körpers bewegen, so wird er m als reine kinetische Masse ansehen[4]). Solange die Ge-

[4]) Somit erscheint einem jeden Beobachter die Gesamtmasse des Körpers immer gleich m, aber die Ruhemasse verschieden. Dieselbe Konstanz der

schwindigkeit eines Protons (oder Neutrons) relativ zu uns gering ist, können wir seine Ruhemasse gleich $1{,}66 . 10^{-24}$ setzen. Dies ist aber bei den schnell bewegten „ungeordneten" Partikelchen nicht mehr zulässig. Das hatte ich nicht in Betracht gezogen, weshalb ich für die untere Energiegrenze einen etwa um $24\,^0/_0$ zu hohen Wert erhalten habe.

Ich will aber jetzt zeigen, daß man nicht nur die untere Grenze, sondern auch den genaueren Wert der durchschnittlichen kinetischen Energie berechnen kann. Der Gleichge-

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Gesamtmasse finden wir auch beim freien Fallen eines Körpers, wo die Zunahme an kinetischer Energie durch die Abnahme an potentieller Energie genau kompensiert wird. Für ein Anwachsen der Gesamtmasse ist hier kein Grund vorhanden, da ja der Körper die entstandene kinetische Energie bereits schon früher in potentieller Form besessen hat. Wenn wir aber denselben Körper etwa mit der Hand in Bewegung setzen (und zwar in einer zum Gravitationsfelde senkrechten Richtung), so führen wir dadurch eine ganz neue Energiemenge dem Körper zu, welche er vorher weder in potentieller, noch in kinetischer Form besessen hatte. Dadurch haben wir die ursprüngliche Gesamtmasse des Körpers vergrößert. Da nun hier die kinetische Masse von außen hinzugekommen und nicht auf Kosten der Ruhemasse entstanden ist, so bleibt letztere unverändert. Wir haben also hier eine Bewegung „mit konstanter Ruhemasse" vor uns, während unser erstes Beispiel eine Bewegung „mit konstanter Gesamtmasse" darstellte. Natürlich sind auch gemischte Fälle möglich.

Ist v die Geschwindigkeit des Partikelchens, m_0 seine Ruhemasse und m seine Gesamtmasse, so müssen die beiden relativistischen Gleichungen:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

und

$$\text{kinetische Energie} = (m - m_0)\, c^2 = m_0\, c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

selbstverständlich immer erfüllt sein, ganz unabhängig davon, ob bei der Bewegung m_0 oder m konstant bleibt.

Ist nun die Bewegung von einer solchen Art, daß die Ruhemasse m_0 konstant bleibt, so muß bei $v = c$ sowohl die kinetische Energie als auch die Gesamtmasse m unendlich werden. Nach der traditionellen Auffassung trifft dies bei einer jeden Art von Bewegung ohne Ausnahme zu, sobald $v = c$ wird. Diese traditionelle Auffassung stellt aber in Wirklichkeit eine unerlaubte Verallgemeinerung eines Spezialfalles dar.

wichtsradius des im „Lichtstadium“ sich befindenden Universums ist nach (12) des vorigen Aufsatzes gleich

$$R = \frac{3\,GM}{5\,c^2} \tag{3}$$

gewesen. Außer diesem anfänglichen Radius R müssen wir noch den augenblicklichen Radius R_t kennen. Wenn a den augenblicklichen Wert des Expansionskoeffizienten bezeichnet, so ist nach (1) die augenblickliche Dichte des Universums gleich $\frac{3\,a^2}{8\pi\,G}$, und daher haben wir:

$$\frac{4}{3}\pi R_t^3 \cdot \frac{3a^2}{8\pi\,G} = M,$$

oder:

$$R_t = \left(\frac{2\,G}{a^2}\right)^{1/3} M^{1/3}. \tag{4}$$

Setzt man $a = 1{,}8.10^{-17}$, $G = 6{,}66.10^{-8}$, $c = 3.10^{10}$, und $M = 2.10^{55}$, so erhält man aus (3) und (4):

$$\left.\begin{aligned} R &= 8{,}88.10^{26}, \\ R_t &= 2{,}018.10^{27}. \end{aligned}\right\} \tag{5}$$

Die vom expandierenden Universum bis jetzt verbrauchte kinetische Masse ist gleich

$$\frac{3\,GM^2}{5\,R\,c^2} - \frac{3\,GM^2}{5\,R_t\,c^2} = \frac{3\,GM^2}{5\,R\,c^2}\left(1 - \frac{R}{R_t}\right) =$$

$$= \frac{3\,GM^2}{5\,R\,c^2}\left(1 - \frac{8{,}88.10^{26}}{2{,}018.10^{27}}\right) = \frac{3\,GM^2}{5\,R\,c^2} \cdot \frac{1130}{2018},$$

oder im Hinblick auf (2) gleich $\frac{1130}{2018} M$. Sie ist natürlich nicht spurlos verbraucht worden, sondern hat sich in Ruhemasse verwandelt. Die im Universum noch verbliebene kinetische Masse ist daher gleich $\frac{888}{2018} M$. Ist nun die Gesamtmasse eines Partikelchens gleich m, so besteht sie durchschnittlich aus $m_0 = \frac{1130}{2018}\, m$

Ruhemasse und $m - m_0 = \frac{888}{2018} m$ kinetischer Masse. Die durchschnittliche Geschwindigkeit v_m kann aus folgender Gleichung gefunden werden:

$$\frac{888}{2018} mc^2 = \frac{1130}{2018} mc^2 \left(\frac{1}{\sqrt{1 - \frac{v_m^2}{c^2}}} - 1 \right).$$

Man erhält:

$$\frac{v_m}{c} = 0{,}828\,. \tag{6}$$

Wenn das Partikelchen ein Proton (oder ein Neutron) mit der Gesamtmasse $1{,}66.10^{-24}$ ist, so beträgt seine kinetische Masse $\frac{888}{2018} \cdot 1{,}66.10^{-24}$ Gramm, was einer kinetischen Energie von $\frac{888}{2018} \cdot 1{,}66.10^{-24}.9.10^{20} = 6{,}57.10^{-4}$ Erg $=$ 413 Millionen e-Volt entspricht. Nimmt man wie früher an, daß die Zahl der „ungeordneten" leichten Partikelchen gleich der Zahl der „ungeordneten" schweren sei, und daß sich eine genügende Äquipartition der Energie eingestellt habe, so beträgt die durchschnittliche kinetische Energie bei beiden Arten der Partikelchen 207 Millionen e-Volt.

Wir wollen wie früher die normale Maxwellsche Streuung der individuellen kinetischen Energien um ihren Durchschnitts wert voraussetzen [5]). Diese „normale" Maxwellsche Energieverteilung soll weder durch ein absorbierendes Medium, noch durch irgendeinen anderen Faktor beeinflußt werden. Wie früher nehmen wir an, daß jede Sekunde 100 Partikelchen auf 10 cm² fallen. Dies bedeutet einen Energiestrom von $3{,}29.10^{-3}$ Erg.

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[5]) Die weiter folgenden Berechnungen sind am einfachsten mit Hilfe einer von K. K. Järvinen zusammengestellten Tabelle durchzuführen (ZS. f. physikalische Chemie **93**, 747 [Tabelle 3], 1919). So z. B. entnehmen wir ihr, daß $5{,}887.10^{-6}$ aller Moleküle individuelle Geschwindigkeiten besitzen, die mehr als um das 3-fache die mittlere (quadratische) Geschwindigkeit übersteigen. Eine 3-fache Geschwindigkeit bedeutet aber eine 9-fache kinetische Energie. Wenn die mittlere Molekulargeschwindigkeit mit Lichtgeschwindigkeit vergleichbar ist, so wird die „normale" Maxwellsche G e s c h w i n d i g k e i t s - verteilung gröblich falsche Resultate ergeben. Die „normale" Maxwellsche E n e r g i e verteilung hingegen wird vielleicht auch in diesen extremen Fällen ihre Gültigkeit beibehalten. Jedenfalls wollen wir eine solche Annahme machen.

$cm^{-2}.sec^{-1}$. Es läßt sich dann berechnen, daß alle 1,4 Sekunden ein Partikelchen von über 828 Millionen e-Volt auf die erwähnte Fläche fällt. Alle 28 Minuten fällt ein Partikelchen von über 1,86 Milliarden e-Volt, und alle 6,7 Jahre eins von über 3,31 Milliarden.

Wollen wir nun aber annehmen, daß die Äquipartition der Energie keine vollständige ist: 10% aller Partikelchen sind solche schwere Partikelchen, die der Äquipartition entgangen sind. Die durchschnittliche kinetische Energie dieser schweren Partikelchen ist also gleich 413 Millionen e-Volt geblieben. Dann fällt (speziell von diesen Partikelchen) alle 14 Sekunden ein Partikelchen von über 1,65 Milliarden e-Volt auf unsere Fläche, alle 280 Minuten eins von über 3,72 Milliarden e-Volt, und alle 67 Jahre eins von über 6,61 Milliarden.

Wir hatten das vorige Mal angenommen, daß seit Beginn der Expansion 3 Milliarden Jahre $= 9{,}468 . 10^{16}$ Sekunden verflossen sind, wobei wir uns auf die Radioaktivität der Meteorite stützten. Diese Zahl muß jedoch vergrößert werden, da nach unserer Theorie Meteorite nicht gleich beim Beginn der Expansion entstanden sein können, sondern erst nachdem die Expansion einige Zeit angedauert hat. Wir wollen daher die Sekundenzahl nach oben abrunden und $t = 10^{17}$ Sekunden annehmen. Der Weltradius hat sich in diesen t Sekunden von R bis R_t vergrößert. Somit ist die durchschnittliche Expansionsgeschwindigkeit gleich

$$u_m = \frac{R_t - R}{t},$$

oder in Hinblick auf (5):

$$u_m = \frac{2{,}018 . 10^{27} - 8{,}88 . 10^{26}}{10^{17}} = 1{,}13 . 10^{10}\,\mathrm{cm.\,sec}^{-1}.$$

Dies bedeutet, daß

$$\frac{u_m}{c} = 0{,}377 \tag{7}$$

ist. Diese Zahl macht keinen besonders unwahrscheinlichen Eindruck. Natürlich darf u_m in (7) nicht mit v_m in (6) verwechselt werden!

Alle diese Zahlen sind unter der Voraussetzung berechnet, daß die Masse des Universums gleich $2 . 10^{55}$ Gramm ist; andere

Voraussetzungen würden zu anderen Zahlen führen. Schreibt man aber der Masse des Universums einen gar zu großen Wert zu, so erhält man ein unwahrscheinlich kleines u_m. Bei einer Masse von etwa $6{,}85 \,.\, 10^{-55}$ Gramm wird $u_m = 0$, was bedeuten würde, daß das Universum überhaupt nicht expandiert. Bei noch größeren Massen erhält man sogar negative Werte für u_m. Ist hingegen die Masse übermäßig klein angesetzt, so erhält man für den augenblicklichen Radius des Universums (R_t) einen unwahrscheinlich kleinen Wert, der in extremen Fällen sogar kleiner ausfallen kann, als die tatsächlich beobachtete Entfernung der weitesten Spiralnebel [6]).

Hier sei noch auf folgende scheinbare Diskrepanz zwischen unserer Theorie und der Beobachtung hingewiesen. Nach den Beobachtungen von Shapley und Miss Ames kann die durchschnittliche Dichte der inneren Metagalaxis gleich 10^{-30} g.cm^{-3} angenommen werden [7]). Wenn wir aber andererseits den beobachteten Expansionskoeffizienten $\alpha = 1{,}8 \,.\, 10^{-17}$ in (1) einführen, so erhalten wir $\varrho = 5{,}8 \,.\, 10^{-28}$ g.cm^{-3}, also einen 580 mal größeren Wert als Shapley und Miss Ames. Wie soll man diese 580-fache Diskrepanz erklären? — Eine Antwort auf diese Frage könnte man vielleicht in der Untersuchung von S. Smith erblicken [8]). Dieser Forscher hat gefunden, daß die Masse des „Virgo cluster" $2 \,.\, 10^{47}$ Gramm oder $10^{14}\, M_\odot$ betrage. Da man im „cluster" 500 Nebel unterscheiden kann, so ist die durchschnittliche Masse eines jeden Nebels $2 \,.\, 10^{11}\, M_\odot$ Dies ist 200 mal mehr als der gewöhnlich angenommene Wert von $10^9\, M_\odot$. Smith hält es für möglich, diese 200-fache Diskrepanz durch Anwesenheit von internebularer Materie zu erklären. Mit anderen Worten: dank dem Vorhandensein von internebularer Materie erweist sich

[6]) „Boötes cluster" und „Ursa Major cluster No. 2" sind etwa 70 megaparsec von uns entfernt. Da ihre Expansionsgeschwindigkeiten etwa 40000 km. sec^{-1} betragen, so ist damit erwiesen, daß selbst bei diesen Entfernungen das Hubble'sche Expansionsgesetz noch seine Geltung beibehält (M. L. Humanson, Astrophys. Journ. **83**, 10, 1936). Man darf jedoch nicht vergessen, daß wir unsere Expansionsformel (1) unter der Voraussetzung abgeleitet haben, daß die Expansionsgeschwindigkeit beträchtlich kleiner als Lichtgeschwindigkeit ist. Daher haben wir nicht das Recht zu erwarten, daß das Hubble'sche Gesetz auch für mit R_t vergleichbare Entfernungen noch gelte.

[7]) Ich zitiere nach Arthur Haas, „Kosmologische Probleme der Physik", Leipzig 1934, S. 10.

[8]) S. Smith, Astrophys. Journ. **83**, 23, 1936.

die tatsächliche Dichte 200 mal größer als die scheinbare. Wenn nun im Falle des „Virgo cluster“ die 200-fache Diskrepanz durch Anwesenheit von internebularer Materie erklärt werden kann, warum könnte nicht auch unsere obenerwähnte 580-fache Diskrepanz auf analoge Weise erklärt werden? — Jedenfalls wäre eine solche Erklärung sehr naheliegend.

Bei unserer Berechnung der kinetischen Energien der „ungeordneten“ Partikelchen sind nur die Gravitationspotentiale des Universums während des Anfangsstadiums und während der Jetztzeit maßgebend. Unsere Theorie der kosmischen Partikelchen könnte man daher als „Gravitationstheorie“ bezeichnen. Es sei hier aber darauf hingewiesen, daß auch E. A. Milne die Energie der kosmischen Partikelchen als Gravitationsenergie auffaßt, wie er dies darlegt in seinem (mir bis jetzt leider noch unzugänglich gebliebenen) Buche: „Relativity, Gravitation and World-Structure“.

Nicht nur Protonen, Neutronen, Elektronen und Positronen unterliegen der Gravitation, sondern auch größere Körper, wie etwa Meteorite. Es taucht nun die scheinbar sehr berechtigte Frage auf: warum auf unsere Erde niemals Meteorite von solchen Geschwindigkeiten fallen, wie sie bei den kosmischen Partikelchen beobachtet werden? — Auf diese Frage gibt unsere Theorie eine sehr einfache Antwort. Das Sichzusammenballen von Partikelchen zu einem größeren Körper ist nur dann denkbar, wenn ihre relativen Geschwindigkeiten gering sind, wie dies bei den „geordneten“ Partikelchen der Fall ist. Wenn sich aber ein größerer Körper aus „geordneten“ Partikelchen gebildet hat, so wird seine relative Geschwindigkeit hinsichtlich eines in gleicher Weise entstandenen Nachbarkörpers gering sein. Die relative Geschwindigkeit eines Meteorits hinsichtlich unserer Erde ist rein „lokaler“ Natur, hervorgerufen durch ungenügende Homogenität in der Verteilung der Materie im Raume. Bei den „ungeordneten“ Partikelchen hingegen ist die Sachlage eine p r i n z i p i e l l verschiedene. Selbst ein in nächster Nähe befindliches „ungeordnetes“ Partikelchen kann hinsichtlich der Erde eine relative Geschwindigkeit aufweisen, die an Lichtgeschwindigkeit grenzt. Die kosmischen Partikelchen sind die bis jetzt „ungeordnet“ gebliebenen Partikelchen jenes anfänglichen „Lichtstadiums“ des Universums, wo eine Bildung komplizierterer Atome noch undenkbar war. Dank der bis jetzt stattgehabten Expansion hat sich die durchschnittliche kinetische Energie der Partikelchen zwar verringert, sie ist aber immer noch sehr viel größer geblieben, als die beim „Einfangen“ eines Neutrons oder Protons durch einen Atomkern frei werdende Energie. Unter solchen Bedingungen ist die Bildung von Elementen undenkbar. (Ganz so, wie in einem gewöhnlichen völlig ionisierten Gase keine Vereinigung der Elektronen mit den positiven Ionen zu bemerken sein wird, wenn die durchschnittliche kinetische Energie das entsprechende Ionisationspotential um ein Vielfaches übertrifft, und wenn dazu noch die Dichte des ionisierten Gases extrem klein ist.) Es ist klar, daß nur Elementarteilchen die Rolle von kosmischen Partikelchen spielen können; kompliziertere Atomkerne hingegen sind ausgeschlossen, und von Meteoriten kann erst recht keine Rede sein.

Die Form der Spiralnebel läßt sich auf Grund des gewöhnlichen Newtonschen Gravitationsgesetzes schwer erklären. Es macht den Endruck, als ob das Gravitationspotential im Spiralnebel zu rasch abnehme, was man als Überlagerung der Gravitationskraft durch eine „innere kosmische Repulsion“ deuten könnte. Es ist der Gedanke aufgetaucht, daß bei den Spiralnebeln ein analoges Expansionsbestreben vorhanden sei wie bei dem Universum als Ganzes[9]). Lambrecht, der diese Frage ebenfalls untersucht, spricht folgende Meinung aus: „Eine neue Theorie von Milne, die Fluchtbewegung mit Hilfe der klassischen Mechanik zu erklären, muß hier unberücksichtigt bleiben, da sie keine Anwendungsmöglichkeit auf das uns in erster Linie interessierende Problem der Entstehung der Spiralarme bietet“ [10]). Lambrecht irrt sich aber: die „elementare“ Expansionstheorie läßt sich ganz ungezwungen auch auf einzelne Spiralnebel anwenden! — Wir haben oben gesehen (S. 4), daß die Expansionsgeschwindigkeit eines jeden Partikelchens für einen jeden Beobachter eine parabolische sein muß. Dies ist aber nur bei absoluter Homogenität streng richtig. Entsteht hingegen eine große lokale Verdichtung (ein Nebel), so tritt eine entsprechende lokale Gravitationssteigerung auf. Die Gravitation dominiert jetzt in kleinen Entfernungen, in größeren hingegen gewinnen die Expansionsgeschwindigkeiten mehr und mehr an Bedeutung, da ja

[9]) Vgl. H. Vogt, Astron. Nachr. **243**, 405, 1931, und andere Stellen.
[10]) H. Lambrecht, Astron. Nachr. **254**, 119, 1935.

eine lokale Gravitationssteigerung nur eine lokale Bedeutung haben kann. Dies muß den Eindruck erwecken, als ob die (lokale) Gravitationskraft von einer mit der Entfernung wachsenden „inneren kosmischen Repulsionskraft“ überlagert würde. Sollten sich im Nebel noch weitere kleine, aber sehr kompakte Verdichtungen (Sterne) ausbilden, so wird dadurch die Sachlage nicht im geringsten geändert: man darf ja nicht vergessen, daß wenn eine gewisse Menge Nebelmaterie sich zu einem Stern verdichtet, dies keinen Einfluß auf den Bewegungszustand des Schwerpunkts dieser Menge haben kann.

Nach unserer „Gravitationstheorie“ beträgt die durchschnittliche kinetische Energie der primären kosmischen Partikelchen 207 Millionen *e*-Volt. Die individuellen kinetischen Energien können dabei natürlich bedeutend größer sein. Aber das Vorhandensein einer merklichen Anzahl solcher Partikelchen, deren individuelle kinetische Energien 10^{10} *e*-Volt beträchtlich übersteigen, läßt sich mit unserer „Gravitationstheorie“ nicht in Einklang bringen. Bei direkter Beobachtung mit der Wilson-Kammer hat man auch keine derartig großen Energien konstatieren können. Auf indirektem Wege hingegen will man das Vorhandensein primärer Partikelchen von 10^{11} *e*-Volt, ja selbst solcher von 10^{12} konstatiert haben. Dies zwingt uns auf die betreffende Frage und auf ihre Literatur näher einzugehen. Außerdem wollen wir noch untersuchen, ob nicht irgendeine andere in der Literatur vorgeschlagene Theorie die beobachteten Tatsachen besser erklären kann als unsere „Gravitationstheorie“. Wir schreiten also zur Übersicht der Literatur, wobei wir aber selbst auf eine entfernte Vollständigkeit unserer Durchmusterung verzichten wollen; außerdem wollen wir nur die neuere Literatur (seit 1929) in Betracht ziehen.

Bothe und Kolhörster fassen kosmische Strahlung als Korpuskularstrahlung auf: „Eine Korpuskularstrahlung könnte ... ihre Energie in sehr schwachen, dafür aber ungeheuer ausgedehnten Kraftfeldern erlangen, rechnet doch beispielsweise die Entfernung der „nichtgalaktischen Nebel“ nach heutigen Vorstellungen nach Millionen Lichtjahren“ [11]).

Carlson und Oppenheimer nehmen (noch vor Entdeckung

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[11]) W. Bothe und W. Kolhörster, ZS. f. Phys. **56**, 777, 1929.

des „Breiteneffekts“) an, daß die kosmischen Partikelchen Neutronen seien [12]). Die gleiche Ansicht äußert auch Swinne [13]).

Lemaître sagt: „If the cosmic rays originated chiefly before the actual expansion of the space, their original energy was even bigger, and it has been reduced by the expansion in the ratio of the change of the radius of the universe . . .“ [14]). Dies steht in vollem Einklange mit unserer „Gravitationstheorie“. Wir können jedoch Lemaître's weitere Gedanken nicht akzeptieren, wenn er sagt: „The only energy we know which is comparable to the energy of the cosmic rays is the matter of the stars. Therefore it seems that the cosmic rays must have originated from the stars“. Da jeder Stern eine Atmosphäre besitzt, die das Entweichen der kosmischen Partikelchen verhindern könnte, so meint Lemaître: „The stars are borne without atmosphere; the atmosphere evolved after the escape of the cosmic rays“. (Dazu sei zu bemerken, daß bei einem Stern „ohne Atmosphäre“ seine äußeren Schichten die Rolle einer Atmosphäre übernehmen würden. Außerdem müßte die Absorption einer aus dem Inneren ausgehenden Strahlung hauptsächlich schon in tieferen Schichten stattfinden, und nicht erst in der Atmosphäre des Sterns.) Die Energie der kosmischen Partikelchen erklärt Lemaître durch super-radioaktive Prozesse. Er hält es sogar für nicht unmöglich, daß das Gewicht eines super-radioaktiven Atoms mit dem Gewichte des ganzen Sterns vergleichbar wäre. Lemaître meint, daß die kosmische Strahlung α-, β- und γ-Strahlen enthalten müsse.

Millikan behauptet, „that there is no atomic transformation whatever that can furnish the necessary energy except an atom-building process“ [15]). Millikan weist außerdem die Unmöglichkeit kosmischer elektrischer Felder mit gewaltiger Potentialdifferenz nach (vgl. oben über Bothe und Kolhörster).

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[12]) J. F. Carlson and J. R. Oppenheimer, Phys. Rev. (2) **38**, 1788, 1931.

[13]) R. Swinne, ZS. f. technische Phys. **13**, 279, 1932.

[14]) G. Lemaître, Nature **128**, 705, 1931. — In einem späteren Artikel (Phys. Rev. (2) **43**, 87, 1933) faßt er die kosmischen Partikelchen als Sprengstücke von der „kosmischen Explosion“ auf, welche die Expansion des Universums verursacht hat.

[15]) R. A. Millikan, Nature **128**, 713, 1931.

A. K. Das[16]) betrachtet die kosmische Strahlung als Temperaturstrahlung von 5.10^{12} Grad. Da eine solche Temperatur nur im Inneren der Sterne existieren kann, nimmt Das an, daß die entsprechenden energiereichen Strahlungsquanten von dort in Form kosmischer Strahlung nach außen gelangen.

Laue[17]) sieht in der Entstehung von Elementen die Ursache der kosmischen Strahlung (vgl. oben Millikans Ansicht).

Wegen des inzwischen entdeckten „Breiteneffekts" nimmt Swann[18]) an, daß die kosmischen Partikelchen Elektronen seien. Swann weist darauf hin, daß die Energie dieser Elektronen mindestens 10^{10} *e*-Volt betragen müsse, damit sie die Erdoberfläche beim magnetischen Äquator erreichen können.

In einem späteren Artikel erklärt Swann die Energie der kosmischen Partikelchen durch Änderung des stellaren lokalen Magnetismus. „It would be difficult to realize energies as 10^{10} volts from the magnetic fields of spots such as occur on the sun. Energies corresponding to 10^9 volts are, however, within the range of possibility, and it is suggested that electrons projected from such spots may play a part in auroral phenomena. For cosmic rays one must, however, probably look to the stars for the necessary conditions" [19]).

Pollard glaubt feststellen zu können, daß 16,7 % der kosmischen Strahlung die Maxwellsche Energieverteilung aufweisen[20]).

Bethe leitet eine Bremsformel für Elektronen relativistischer Geschwindigkeit ab[21]).

C. D. Anderson teilt mit: „Fig. 8 shows an electron of 113 million volts initial energy which loses 27 million volts in passing through 13.4 mm of lead. This corresponds to an energy loss close to 20 million volts per cm which is good accord with the calculated value of 19.4 million volts per cm of lead for electrons of 100 million volts energy given by Bethe. The mean value for energy loss in lead on the basis of our present data is, however, greater than this and is approximately

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16) A. K. Das, Naturwissenschaften **19**, 305, 1931.
17) M. v. Laue, ebenda S. 530.
18) W. F. S. Swann, Phys. Rev. (2) **41**, 540, 1932.
19) W. F. S. Swann, Phys. Rev. (2) **43**, 217, 1933.
20) W. G. Pollard, Phys. Rev. (2) **44**, 703, 1933.
21) H. Bethe, ZS. f. Phys. **76**, 293, 1932.

35 million volts" [22]). Anderson weist noch darauf hin, daß in der kosmischen Strahlung Elektronen und Positronen in gleichen Mengen auftreten. Nach Andersons Meinung wird dadurch die hervorragende Rolle erwiesen, welche die „Paarbildung" bei der Absorption der kosmischen Strahlen spielt.

Heitler [23]) berechnet auf Grund der Diracschen relativistischen Wellengleichung (in erster Annäherung) die Bremsstrahlung eines Partikelchens, dessen Energie groß gegenüber mc^2 ist. Die berechnete Bremsung sei im Einklang mit den Messungen an kosmischer Strahlung. Auch Weizsäcker macht Berechnungen [24]). Sein Resultat sei innerhalb der verwendeten Näherungen mit den früheren Rechnungen identisch. Oppenheimer bestreitet Weizsäckers Ansichten, spricht von „an uncritical application of quantum mechanics to these problems" [25]), und weist darauf hin, daß Weizsäckers Formel mit der Beobachtung nicht im Einklange sei.

Bowen, Millikan und Neher [26]) schätzen die auf die irdische Atmosphäre einfallende kosmische Strahlung auf $3{,}2 \cdot 10^{-3}$ $\mathrm{Erg} \cdot \mathrm{cm}^{-2} \cdot \mathrm{sec}^{-1}$, während sämtliche Sterne der Erde $6{,}91 \cdot 10^{-3}$ $\mathrm{Erg} \cdot \mathrm{cm}^{-2} \cdot \mathrm{sec}^{-1}$ gewöhnlicher strahlender Energie zusenden (also nur etwa doppelt so viel).

Compton und Stephenson [27]) sind zu der Überzeugung gelangt, daß Photonen keine merkliche Rolle in der primären kosmischen Strahlung spielen. Wahrscheinlich bestehe letztere zum größten Teil aus Protonen.

Williams [28]) findet, daß positive und negative Protonen die Rolle der energiereichsten kosmischen Partikelchen spielen.

C. D. Anderson und Neddermeyer [29]) stellen eine große Fluktuation in dem spezifischen Energieverlust der (positiven und negativen) Elektronen fest (bei manchen bis 10^8 e-Volt pro cm). Sie erklären dies durch die Entstehung eines oder mehrerer Photonen.

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[22]) C. D. Anderson, Phys. Rev. (2) **44**, 409, 1933.
[23]) W. Heitler, ZS. f. Phys. **84**, 145, 1933.
[24]) C. F. v. Weizsäcker, ZS. f. Phys. **88**, 612, 1934.
[25]) J. R. Oppenheimer, Phys. Rev. (2) **47**, 44, 1935.
[26]) I. S. Bowen, R. A. Millikan and H. V. Neher, Phys. Rev. (2) **44**, 252, 1933.
[27]) A. H. Compton and R. J. Stephenson, Phys. Rev. (2) **45**, 441, 1934.
[28]) E. J. Williams, Phys. Rev. (2) **45**, 729, 1934.
[29]) C. D. Anderson and S. H. Neddermeyer, Phys. Rev. (2) **46**, 325, 1934.

Steinke veröffentlicht einen zusammenfassenden Bericht[30]. Er weist darauf hin[31], daß eine Wasserschicht von 200 Meter nach Bethe (s. oben) nur von einem solchen Elektron durchdrungen werden könne, dessen Energie wenigstens 10^{11} *e*-Volt beträgt. Es seien aber Partikelchen beobachtet worden, die sogar eine Wasserschicht von 500 Meter durchdrungen haben. „Als abschließendes Ergebnis der bisherigen Betrachtungen können wir feststellen, daß die Ultrastrahlung aus diskreten Komponenten besteht, deren Energien im Falle einer Wellenstrahlung zwischen 10^7 und 10^{10} *e*-Volt, im Falle einer Korpuskularstrahlung zwischen 10^9 und 10^{12} *e*-Volt liegen.“ Weiter sagt er: „Außerdem geht aus dem Vorhandensein von Strahlung in äquatorialen Gegenden hervor, daß diese entweder nicht beeinflußbar ist (Photonen, Neutronen) oder daß sie Energien von mehr als $4 \cdot 10^{10}$ *e*-Volt besitzt[32]. Hinsichtlich der sogen. „Hoffmannschen Stöße“ sagt Steinke: „Die größten bisher gemessenen Stöße (etwa 200 000 000 Ionen) stellen bei Annahme einer Bildungsenergie von 32 *e*-Volt für ein Ionenpaar bereits Energien von $6 \cdot 10^9$ *e*-Volt dar. Hierbei ist in der Kammer nur ein kleiner Teil der Korpuskelreichweite zur Wirksamkeit gekommen. Bei einer Reichweite von 5 cm Pb ($3{,}5 \cdot 10^4$ cm Luft) und einer noch häufig beobachteten mittleren spezifischen Ionisation von 10^5 Ionen/cm Luft ergibt sich bereits eine Energie von $1 \cdot 10^{11}$ *e*-Volt“[33]. Für den gesamten Energiefluß der Ultrastrahlung akzeptiert Steinke die Zahl $3{,}5 \,.\, 10^{-3}$ Erg.cm^{-2}.sec^{-1}[34] (vgl. oben die Schätzung von Bowen, Millikan und Neher).

Auch Geiger hat einen zusammenfassenden Bericht veröffentlicht[35]. Er unterscheidet 5 Arten von Strahlen, wobei er die primären als A-Strahlen bezeichnet. Er hält letztere für geladene Massenteilchen[36]. Er hat auch die mit der Wilson-Kammer beobachtete Energieverteilung der Ultrateilchen in einer

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[30]) E. G. Steinke, „Die kosmische Ultrastrahlung“, Ergebnisse der exakten Naturwissenschaften **13**, 89, 1934.

[31]) Ebenda, S. 106.

[32]) Ebenda, S. 108.

[33]) Ebenda, S. 128 f.

[34]) Ebenda, S. 132.

[35]) H. Geiger, „Die Sekundäreffekte der kosmischen Strahlung“, Ergebn. d. exakten Naturw. **14**, 42, 1935.

[36]) Ebenda, S. 46.

Tabelle zusammengestellt[37]). Da diese Tabelle für uns Interesse hat, reproduzieren wir sie teilweise.

Energien in 10^6 *e*-Volt	P. Kunze		C. D. Anderson	
	Zahl der positiven Partikelchen	Zahl der negativen Partikelchen	Zahl der positiven Partikelchen	Zahl der negativen Partikelchen
unter 500	15	16	14	15
500—1000	10	7	9	11
1000—1500	9	2	5	3
1500—2000	5	1	4	1
2000—2500	1	1	4	3
2500—3000	1	1	3	3
über 3000	—	—	0	1

In dieser Tabelle wird vorausgesetzt, daß die Partikelchen Elektronen seien. Sollten sie aber Protonen sein, so verschiebt sich die Energieskala in folgender Weise[38]):

Elektronen:	500	1000	1500	2000	2500	3000.10^6 *e*-Volt
Protonen:	120	440	840	1270	1740	2210.10^6 *e*-Volt

Bei Kunze sind 6 Strahlen nicht eingetragen, bei denen wegen zu hoher Energie die Krümmung nicht mit Sicherheit meßbar war.

Nach Newman und Walke[39]) lassen sich zwei scharf getrennte Gruppen von kosmischen Partikelchen beobachten: solche, die nur einige Zentimeter Blei durchdringen können, und solche, die viele Meter durchdringen. Da es bis jetzt noch nie gelungen sei eine Entstehung von Partikelchen der „härteren“ Gruppe in der Luft oder in anderer Materie zu beobachten, müßten diese Partikelchen als die primären angesehen werden. Sie seien wahrscheinlich schwere Ionen.

Kolhörster will im Dezember 1934 bei der kosmischen

[37]) Ebenda, S. 72, Tabelle 11.

[38]) Ebenda, S. 73.

[39]) F. H. Newman and H. J. Walke, Phil. Mag. (7) **20**, 263, 1935.

Strahlung „ungewöhnliche Schwankungen" konstatiert haben, und zwar ungefähr gleichzeitig mit dem Aufflammen der Nova Herculis. „Es ist daher nicht ganz unwahrscheinlich, daß die Nova für die beobachtete Zunahme der Höhenstrahlung in Betracht zu ziehen wäre, zumal ja bei diesen Sternen genügend große Energiemengen zur Erzeugung von Höhenstrahlen verfügbar sein sollen. Die Nova Herculis würde also zur Zeit 1 bis 2% der gesamten Höhenstrahlung liefern" [40]). Hess und Steinmaurer finden jedoch, daß „The effect, if it is real, certainly does not exceed 2 per thousand of the total radiation" [41]). Barnóthy und Forró finden, daß der problematische Effekt „does not exceed in any case four times the mean error" [42]).

Schon etwas früher haben Baade und Zwicky [43]) die Frage aufgeworfen, ob nicht in den sogen. „Super-Novae" die Quelle der kosmischen Strahlung zu suchen sei. Durchschnittlich erscheint in einem jeden Nebel alle 1000 Jahre eine Super-Nova. Die sichtbare Strahlung einer Super-Nova übertrifft etwa um das 10^8-fache die Sonnenstrahlung, und die „Lebensdauer" beträgt ungefähr 20 Tage.

McCrea untersucht rein theoretisch Kolhörsters Nova-Hypothese [44]). Er kommt zu dem Schluß, daß eine durchschnittliche Nova, die im Verlaufe ihres „Lebens" (nach Unsöld) 6.10^{44} Erg gewöhnlicher strahlender Energie aussendet, während derselben Zeitspanne etwa 10^{49} Erg an kosmischer Strahlung aussenden muß. In einem weiteren Artikel [45]) sagt McCrea, daß er die Super-Nova-Hypothese von Baade und Zwicky übersehen hatte. Er sagt weiter: „I offered no theory of the origin of cosmic rays... It turned out in point of fact that, on the present knowledge of stellar structure, one cannot definitely exclude the possibility of this source of the radiation, *on energy considerations alone*".

Walke [46]) äußert die Meinung, „that the cosmic ray ions are mainly emitted from the heavier stars".

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[40]) W. Kolhörster, ZS. f. Phys. **93**, 431, 1935.

[41]) F. Hess and R. Steinmaurer, Nature **135**, 618, 1935.

[42]) J. Barnóthy and M. Forró, ebenda.

[43]) W. Baade and F. Zwicky, Phys. Rev. (2) **45**, 138, 1934; **46**, 76, 1934.

[44]) W. H. McCrea, Nature **135**, 371, 1935.

[45]) Ebenda S. 821.

[46]) H. J. Walke, ebenda S. 36.

Milne weist nach, daß die hohe kinetische Energie der kosmischen Partikelchen durch die im Universum wirkenden Gravitationskräfte erklärt werden könne: „The identification accounts for the observed isotropy; and it provides the origin of the high energies, predicting indeed that there is no upper limit to the energy of a single 'ray' ... Lastly, identification removes the old impasse to which other theories of the origin of cosmic radiation have appeared to lead: that if the primary rays were born in the interiors of stars, it is difficult to see how they could ever get out; yet if they were born as a result of multiple collisions in inter-galactic space, it is difficult to see how the inter-galactic density of matter could be high enough" [47]).

H. Nie teilt mit: „Versuche, die ... unter 400 m Gestein angestellt worden sind, zeigen, daß dort keine Stöße mehr auftreten .... Neuere Forschungen lassen auch vermuten, daß die Häufigkeit der Stöße mit zunehmender Höhe über dem Meeresspiegel sehr stark ansteigt" [48]). „Die Neuartigkeit der Erscheinung kommt vor allem in der großen Ionenmenge [nämlich 10^8 bis 10^9] zum Ausdruck ... Selbst im günstigsten Falle aber können Elementarpartikeln auf einer Gasstrecke von 1000 cm LÄ [dies ist die Länge der Ionisationskammer] insgesamt nur rund 10^6 Ionenpaare erzeugen. Aber wenn auch Mitwirkung von schwereren Partikeln (insbesondere bei großen Stößen) nicht ausgeschlossen ist, wird man gezwungen sein, den gleichzeitigen Durchgang von einer großen Partikelzahl (bei großen Stößen bis zu mehreren 1000) anzunehmen. Ob es sich hier um etwas Ähnliches handelt wie bei den später entdeckten Elektronenschauern in Wilson-Kammern, ist eine zunächst noch offene Frage" [49]).

Die beiden Montgomery weisen auf die hohe Energie der erwähnten Schauer hin: „... we see that at least half of the rays of the shower which emerge from the chamber have energies greater than 3×10^8 electron volts, and that the total energy in the showers which we observe must often exceed 3×10^{10} electron volts" [50]).

Rumbaugh und Locher machen Mitteilung über Neutronen

[47]) E. A. Milne, ebenda S. 183.

[48]) H. Nie, ZS. f. Phys. **99**, 454, 1936.

[49]) Ebenda S. 455.

[50]) C. G. and D. D. Montgomery, Phys. Rev. (2) **49**, 711, 1936.

in der Stratosphäre: „If these neutrons are primary constituents of cosmic radiation, as the present evidence indicates, the free neutron must be comparatively stable.... The suggestion that primary cosmic radiation at high altitudes contains a strong component of α-particles whose effects are conspicuous at about 1 meter of water, is inconsistent with our observations... Moreover, the absence of observable primary α-particles cannot be explained by their disappearance through nuclear collisions“ [51]).

Millikan, Neher und Korff sagen: „Airplane measurements on cosmic-ray intensities to altitudes up to more than 26,000 feet have been made both in South America and in Asia, with results which show close agreement on the two sides of the earth, the apparent absorption coefficient in both localities being only slightly different from its value in the temperate latitudes. These results remove one of the chief arguments that has in the past been advanced for the great predominance of the corpuscular component of the incoming cosmic rays“ [52]).

Swann macht auf folgendes aufmerksam: „A charged particle is characterized by the fact that its ionization increases enormously towards the end of its range, so that, in the case of protons and alpha-particles, large and measurable spurts of ionization should be produced in relatively short distances by those rays which are ending their journeys“ [53]).

Compton ist für geladene Partikelchen [54]).

C. D. Anderson und Neddermeyer machen Mitteilung über folgende Beobachtungen: „Evidence is here found for the first time that electrons also can occasionally disintegrate nuclei and eject from them massive particles. In Fig. 10 an electron apparently disintegrates a lead nucleus, ejecting protons from it. Some evidence is found of disintegrations which seem to be produced by neutrons occurring as secondaries in the cosmic rays... Practically all the heavy particles can be interpreted only as secondaries produced within the atmosphere or material surrounding the cloud chamber. Certain types of disintegrations, here-

[51]) L. H. Rumbaugh and G. L. Locher, ebenda S. 855.

[52]) R. A. Millikan, H. V. Neher and S. Korff, ebenda S. 871.

[53]) W. F. G. Swann, ebenda S. 478. Vgl. auch: S. 650; C. G. and D. D. Montgomery, W. E. Ramsey and W. F. G. Swann, ebenda S. 890; W. F. G. Swann, Phys. Rev. (2) **50**, 1103, 1936.

[54]) A. H. Compton, Phys. Rev. (2) **50**, 1119, 1936.

tofore unobserved, in which the summed energies of the ejected particles exceed 1000 MEV show that at these high energies the ejection of several particles is common"[55]).

Sawyer spricht über Absorption der schauererregenden kosmischen Strahlen: „The coefficients of absorption .... are nearly equal to 0.018 cm^{-1} times the density"[56]).

Hsiung bestimmt den Absorptionskoeffizienten mit Hilfe dreier Geiger-Müllerscher Zähler[57]) zu $\mu = 2{,}2 \, . \, 10^{-3} \, cm^{-1}$, während die Ionisationskammer $\mu = 1{,}90 \, . \, 10^{-3} \, cm^{-1}$ ergibt.

Heisenberg sagt: „Die bisherige Quantenelektrodynamik gibt keine Erklärung für die Tatsache, daß sehr energiereiche Teilchen in einem einzigen Akt eine große Anzahl von Sekundärteilchen erzeugen können"[58]). Heisenberg gibt nun eine qualitative Erklärung der Schauerbildung, wobei er sich auf Fermis Theorie des β-Zerfalls stützt.

Aus Anlaß von Ultrastrahlungsmessungen im Bodensee erinnert Weischedel daran, daß schon in 150 m Wassertiefe die Ultrastrahlungsintensität nur noch etwa $1^0/_0$ des Wertes an der Seeoberfläche und nur etwa den 10000-sten Teil des Wertes in großen Höhen beträgt[59]).

Pfotzer berichtet über Messungen bei einem Aufstieg bis 29 km Höhe (10 mm Hg), und sagt unter anderem: „In diesen 115 Teilchen pro 4 min haben wir die Korpuskeln der Ultrastrahlung vor Eintritt in die Atmosphäre vor uns. Es sind dieselben Teilchen, welche außerhalb der Erdatmosphäre der Ablenkung durch das Magnetfeld der Erde unterworfen sind und den Breiteneffekt hervorrufen"[60]).

Messerschmidt sagt: „Bei einer Reichweite der Garben von 5 cm in Blei berechnet sich die Energie der häufigsten Stoßgröße aus Blei zu $5 \cdot 10^9$ e-Volt. Der größte überhaupt beobachtete Stoß hat eine Energie von etwa $5 \cdot 10^{11}$ e-Volt. Da die Energien um zwei bis drei Größenordnungen höher liegen als bei

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55) C. D. Anderson and S. H. Neddermeyer, ebenda S. 271.
56) J. H. Sawyer, ebenda S. 25.
57) D. S. Hsiung, Phys. Rev. (2) **46**, 653, 1934.
58) W. Heisenberg, ZS. f. Phys. **101**, 533, 1936.
59) F. Weischedel, ebenda S. 754.
60) G. Pfotzer, ZS. f. Phys. **102**, 39 f., 1936.

den Schauern, erscheint es nicht mehr gegeben zu sein, die Stöße direkt mit den Schauern in Beziehung zu setzen" [61]).

Regener bringt einen zusammenfassenden Bericht [62]). Er hält es für erwiesen, daß die primären Strahlen aus geladenen Partikelchen bestehen, denn neutrale würden keinen Breiteneffekt hervorrufen. Auch müssen sie aus kosmischen Räumen kommen (und nicht erst in unserer Atmosphäre entstehen), denn auf einer kurzen Strecke könnte sich das irdische Magnetfeld nicht auswirken. Auch müssen sie schon von Anfang an verschiedene kinetische Energien besitzen, denn der Breiteneffekt zeigt ein allmähliches Ansteigen der Ultrastrahlungsintensität vom Äquator bis zu einer Breite von 50°. Die Ultrastrahlung erweist sich als um so durchdringender, je größere Schichtdicken sie passiert hat. „Man findet ... noch Intensitäten in Tiefen, wo man, nach dem Absorptionskoeffizient am Erdboden gerechnet, überhaupt nichts mehr finden sollte. So erhält man in 200 m Wassertiefe noch etwa 4‰ des Wertes an der Meeresoberfläche, und in einer Tiefe gleich 700 m Wasseräquivalent ... der Größenordnung nach etwa 1‰. Rechnet man die formalen Absorptionskoeffizienten aus, so findet man in 700 m Wassertiefe nur den 100. Teil des Absorptionskoeffizienten wie in der Atmosphäre (in mittleren Höhen), d. h. die Strahlung ist 100 mal durchdringender geworden. Oder, besser gesagt, hinter diesen Absorberdicken beobachtet man nur den kleinen Anteil der Strahlung, der ganz durchdringend ist; die weicheren sind schon früher absorbiert worden" [63]). „Das eine kann man jedenfalls von vornherein sagen, daß die härtesten Anteile der Strahlung, soweit sie (wahrscheinlich) aus Elektronen bestehen, außerordentlich nahe an die Lichtgeschwindigkeit heranreichen müssen. Ihre Energie ist mindestens 10^{11} e-Volt" [64]). „Mit ziemlicher Sicherheit kann man ... annehmen, daß die primäre Ultrastrahlung, wenigstens in unseren Breiten, *ohne* Sekundärstrahlung aus dem Weltenraum in die Atmosphäre einfällt. Es ist zwar zu vermuten, daß die Ultrastrahlung entweder an ihrem Entstehungsorte oder aber auf ihrem

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[61]) W. Messerschmidt, ZS. f. Phys. **103**, 55, 1936.

[62]) E. Regener, „Die kosmische Ultrastrahlung", Naturwissenschaften **25**, 1—11, 1937.

[63]) Ebenda S. 2.

[64]) Ebenda S. 3.

Wege durch den Weltenraum bei der Begegnung mit kosmischen Staubmassen Sekundärstrahlen erzeugt hat. Diese Sekundärstrahlen werden aber, da sie wesentlich weicher als die primären Strahlen sind, durch das Magnetfeld der Erde weiter nach den Polen zu abgelenkt, wo in der Tat in größeren Höhen in der Atmosphäre größere Ultrastrahlungsintensitäten gemessen werden als in unseren Breiten"[65]). „Berücksichtigt man, daß bei den Wilson-Aufnahmen, die mit zwar breiten, aber nur ein paar Zentimeter tiefen Kammern gemacht werden, meist nur ein Bruchteil aller Einzelstrahlen eines Schauers erfaßt wird, so kann man schätzen, daß in den strahlenreichsten Garben mehr als 1000 Einzelstrahlen enthalten sind. Rechnet man die mittlere Energie eines Einzelstrahles zu 10^8 e-Volt, so kommt man zu dem Schluß, daß die Energie, die in den strahlenreichsten Garben steckt, bis an 10^{11} e-Volt heranreicht. Auch in der Ionisationskammer machen sich solche großen Schauer als „Hoffmannsche Stöße", als plötzliches Auftreten großer erzeugter Ionenmengen, bemerkbar"[66]). „Wie schon erwähnt, haben Wilson-Kammeraufnahmen in starken Magnetfeldern ergeben, daß ziemlich genau gleich viel positive wie negative Teilchen in der primären Ultrastrahlung enthalten sind.... Ob dieses *Elektronen* oder *Protonen* sind, ist nicht so leicht zu entscheiden.... Trotzdem glaubt man heute, daß die primäre Ultrastrahlung in der Hauptsache aus positiven und negativen Elektronen besteht. Die harte Komponente dagegen, welche bis in tiefe Wasserschichten hinabdringt, wird von vielen Autoren als Protonenstrahlung aufgefaßt. Doch ist das letzte Wort in dieser Angelegenheit noch nicht gesprochen... Die am zahlreichsten auftretenden Teilchen mit geringer Energie, bis etwa $3 \cdot 10^9$ e-Volt, sind dabei noch als sekundäre Teilchen aufzufassen, da Teilchen mit einer kleineren Energie als etwa $3 \cdot 10^9$ e-Volt die Atmosphäre nicht durchdringen können. Im übrigen geht die Verteilung auf die verschiedenen Energiestufen etwa mit der reziproken Energie im Quadrat"[67]). Regener ist der Super-Nova-Theorie der Ultrastrahlung (s. oben über Baade und Zwicky) nicht abgeneigt. „Die Lokalisierung des Entstehungsorte der Ultrastrahlung in die Nova- bzw. Super-Nova-Ausbrüche

[65]) Ebenda S. 4.
[66]) Ebenda S. 6.
[67]) Ebenda S. 6 f.

der Sterne wäre jedenfalls am besten verträglich mit den beobachteten zeitlichen Änderungen der Ultrastrahlungsintensität... Nach freundlicher brieflicher Mitteilung von Prof. Clay gab es sogar am 20. Mai dieses Jahres [Regener meint 1936] in mehreren Apparaten (Ionisationskammern und Zählrohren) gleichzeitig eine ziemlich plötzliche Erhöhung der Ultrastrahlungsintensität von etwa 10%, die im Juni ähnlich abklang wie die Helligkeit einer Nova" [68]).

Die beiden Montgomery sagen in einem neuen Artikel: „The conclusion is reached that the most likely entities are *protons* which lose energy according to the relation

$$-dE/dx = \lambda E + a,$$

where a represents the energy lost per unit path by ionization, and λE the energy which goes into the production of secondaries in the amount that is actually observed in the form of showers" [69]).

Bhabha hält es für erwiesen, daß zu dem Bestande der durchdringlichen primären Strahlen auch negative Partikelchen (Elektronen, oder auch negative Protonen) gehören [70]).

Ganz vor kurzem hat Cosyns die Ionisationsfähigkeit extrem schneller Partikelchen gemessen. Das Resultat ist unerwartet: „Theoretical calculation of the interaction between fast electrons and hydrogen atoms (Bethe, Williams, etc.) predict an increase of primary ionization when the energy of the incident particle exceeds about one million electron-volts ... Contrary to theory, we find that the primary ionization *decreases* for high-energy particles" [71]).

Bevor wir weitergehen, wollen wir einige Bemerkungen zu mehreren in der Literatur hie und da geäußerten Ansichten machen. Regener ist der Ansicht, daß Teilchen mit unter 3.10^9 e-Volt Energie als sekundäre anzusehen seien. Dagegen bemerken wir, daß diese Zahl sich doch offenbar nur auf die ursprüngliche Energie des Partikelchens, bevor noch letzteres die Atmosphäre durchdrungen hat, bezieht. Am Meeres-

[68]) Ebenda S. 10.

[69]) C. G. and D. D. Montgomery, Phys. Rev. (2) **51**, 217, 1937.

[70]) H. J. Bhabha, Nature **139**, 415, 1937.

[71]) M. G. E. Cosyns, Nature **139**, 802, 1937.

niveau hingegen können auch die primären Partikelchen beliebig kleine Geschwindigkeiten aufweisen, bis Null inklusive (letzteres, wenn die ursprüngliche Energie des primären Partikelchens zur Durchdringung der Atmosphäre knapp ausgereicht hat). Wollen wir aber trotzdem für einen Augenblick annehmen, daß alle Partikelchen mit unter 3.10^9 e-Volt Energie als sekundäre zu betrachten seien. Wie kann in einem solchen Falle Regener behaupten, daß die Wilson-Kammer uns Aufklärung über die Zusammensetzung der primären Strahlung gebracht habe? Zuverlässige Messungen kann man mit der Wilson-Kammer nur bei Energien unter 3.10^9 e-Volt ausführen, d. h. (nach Regeners Ansicht) nur bei sekundären Strahlen.

Nach Messerschmidt beträgt die häufigste Energie der Stöße 5.10^9 e-Volt, die maximale beobachtete Energie hingegen 5.10^{11} e-Volt. Andererseits berechnet Regener die Energie eines großen Schauers zu 10^{11} e-Volt. Dies bedeutet, daß die stärksten Stöße und die stärksten Schauer Energien von gleicher Größenordnung aufweisen. Wie ist dies mit Messerschmidts Behauptung in Einklang zu bringen, daß „die Energien [der Stöße] um zwei bis drei Größenordnungen höher liegen als bei den Schauern“?

H. Nie hält es für erwiesen, daß bei den großen Stößen mehrere tausend Partikelchen gleichzeitig in die Ionisationskammer eindringen, denn ein einzelnes Partikelchen wäre zur Erzeugung der beobachteten Ionenzahl völlig ungenügend. Es muß jedoch dazu bemerkt werden, daß jedes ausgelöste Ion vom stoßenden Partikelchen eine gewisse Menge kinetischer Energie übernimmt. Ist letztere groß genug, so kann das ausgelöste Ion von sich aus weitere Ionen auslösen. Auf diese Weise entsteht eine „lawinenförmige“ Ionisation. (Eine soche „Lawine“ stellt der gewöhnliche elektrische Funke dar, mit dem Unterschiede jedoch, daß im letzteren die ausgelösten Ionen ihre kinetischen Energien nicht direkt von den stoßenden Ionen erhalten, sondern vom äußeren elektrischen Felde. Ist die Energie der primären Partikelchen extrem groß, so könnte sogar eine „lawinenförmige“ Zertrümmerung der schwereren Kerne zustande kommen. (Wir wollen jedoch hier auf die Frage nicht eingehen, ob die Schauer mit solchen „Lawinen“ zu identifizieren seien.)

Alle Forscher scheinen darüber einig zu sein, daß die größten Stöße von solchen Partikelchen ausgelöst werden, deren individuelle

Energie 10^{11} e-Volt übersteigt. Ebensogroße Energien werden Partikelchen zugeschrieben, die Wasserschichten von Hunderten von Metern durchdrungen haben (die Intensität der Strahlung nimmt mit der Tiefe ab, aber ihre Härte steigt). Dieser Umstand müßte zur Folge haben, daß das Verhältnis der Stößezahl zu der Strahlungsintensität mit der Tiefe wachse. Wie ist es nun aber zu erklären, daß die Beobachtung das genaue Gegenteil zeigt? —

Nach der beobachteten Energieverteilung soll die Häufigkeit der Partikelchen umgekehrt proportional dem Quadrate der Energie sein. Dies kann aber unmöglich das wahre Energieverteilungsgesetz der primären Partikelchen bedeuten. Wäre letzteres der Fall, so könnten wir

$$dn = \frac{KdE}{E^2}$$

schreiben, wo dn die dem Energieintervall dE entsprechende Korpuskelzahl bedeutet, E die individuelle Energie und K eine Konstante. Ist E_1 eine beliebig gegebene individuelle Energie, und summiert man die Energien aller Partikelchen mit $E \geqq E_1$, so erhält man:

$$\int_{E_1}^{\infty} Edn = \int_{E_1}^{\infty} E \cdot \frac{K\,d\,E}{E^2} =$$

$$= K \int_{E_1}^{\infty} \frac{dE}{E} = K(\log \infty - \log E_1) = \infty,$$

was eine offenbare Unmöglichkeit darstellt. Summiert man hingegen die Energien aller Partikelchen mit $E \leqq E_1$, so erhält man:

$$\int_{0}^{E_1} Edn = K \int_{0}^{E_1} \frac{dE}{E} = K(\log E_1 - \log 0) =$$

$$= K(\log E_1 + \infty) = \infty,$$

also ebenfalls ein unmögliches Resultat.

Energien von etwa 10^{11} e-Volt können mit Hilfe der Wilson-Kammer nicht gemessen werden. Nur auf indirektem Wege will man die Existenz solcher Energien erwiesen haben. So z. B. wird die minimale Energie eines Elektrons berechnet, welches (die Wirkung des irdischen Magnetismus überwindend) am Äquator die Erdoberfläche zu erreichen imstande ist. Die Berechnungen ergeben $4 . 10^{10}$ e-Volt. Dies ist aber nur die minimale Energie; die durchschnittliche Energie der am Äquator einfallenden kosmischen Partikelchen ist natürlich größer, so daß 10^{11} e-Volt sehr plausibel aussieht. Leider wird bei diesen magnetischen Berechnungen ein Faktor nicht in Betracht gezogen, auf den bereits vor etwa 9 Jahren R. Gunn hingewiesen hat: „the layer [nämlich die Kennely-Heaviside-Schicht] is strongly diamagnetic"[72]) Eine diamagnetische Schicht muß aber das irdische Magnetfeld nach außen hin mehr oder weniger abschirmen. In einem späteren Artikel[73]) wendet Gunn diese Idee auf den Sonnenmagnetismus an und sucht dadurch das ungemein schnelle radiale Abfallen des Sonnenmagnetfeldes zu erklären. Dies schnelle radiale Abfallen erklärt Chapman auf eine andere Weise: „It is shown that the combined effect of the gravitational, electrostatic, and magnetic fields existing in the sun's reversing layer will be to produce an eastward 'drift-current' of electrons, and that this current is of the right order of magnitude to explain the radial limitation of the sun's magnetic field"[74]). In Chapmans folgendem Artikel lesen wir: „... it is shown that the strong eastward motion in the chromosphere affords evidence that the general magnetic field of the sun cannot extend appreciably into the chromosphere"[75]). Wenn eine Kombination des elektrostatischen, des magnetischen und des Gravitationsfeldes in der ionisierten Sonnenatmosphäre eine Driftbewegung der Elektronen hervorrufen kann, warum sollte etwas Ähnliches in der ionisierten Heaviside-Schicht unmöglich sein? — Man muß freilich mit der Möglichkeit rechnen, daß die beiden Erklärungen falsch sind, und daß ein noch unbekannter Faktor die radiale Begrenzung des magnetischen Sonnenfeldes

[72]) R. Gunn, Phys. Rev. (2) **32**, 134, 1928.
[73]) Ebenda **33**, 615, 1929.
[74]) S. Chapman, Monthly Not. Roy. Astron. Soc. **89**, 78, 1929.
[75]) Ebenda S. 80.

verursacht. Dies ist nätürlich nicht unmöglich. Woher sollen wir aber unter solchen Umständen die Sicherheit haben, daß dieser unbekannte Faktor nicht auch in der irdischen Atmosphäre seine Wirkung ausübt? — Kiepenheuer sagt: „Die Beobachtung zeigt, daß die magnetische Feldstärke des allgemeinen Magnetfeldes der Sonne mit zunehmender Höhe über der Photosphäre außerordentlich schnell abfällt, und zwar im Verlaufe von einigen hundert Kilometern von etwa 50 auf 10 Ørsted. Kleinere Feldstärken lassen sich mit Sicherheit spektroskopisch nicht nachweisen. Der Druck in diesem Gebiet ist von der Größenordnung 10^{-4} Atm. Das Sonnenfeld scheint also nach außen weitgehend abgeschirmt zu sein. Aus diesem Grunde wird das magnetische Feld der Sonne in der umkehrenden Schicht und in der Chromosphäre im wesentlichen tangentielle Komponenten in Richtung der Meridiane besitzen, da alle nach außen gehenden Feldlinien auf eine dünne, zur Sonne konzentrische Kugelschale zusammengedrängt werden. Auf den Mechanismus der Abschirmung sei hier nicht eingegangen. Eine einwandfreie Deutung steht noch aus“ [76]). An einer weiteren Stelle berechnet Kiepenheuer μ_0, d. h. das Verhältnis der Wirkung des unabgeschirmten Magnetfeldes der Sonne auf eine Protuberanz zu der entsprechenden Wirkung des Gravitationsfeldes, und findet, daß $\mu_0 \sim 10^7$ ist. „D. h. daß die Kraftwirkung des *unabgeschirmten* Magnetfeldes der Sonne auf eine Protuberanz diejenige des Gravitationsfeldes um das etwa 10^7 fache übertreffen würde“ [77]). Es liegt nahe zu erwarten, daß das Magnetfeld der Erde bei weitem nicht so gewaltig abgeschirmt ist wie dasjenige der Sonne, aber selbst eine mäßige Abschirmung müßte sämtliche quantitativen Betrachtungen über die Wirkung des irdischen Magnetfeldes auf die kosmische Strahlung völlig illusorisch machen. Damit wollen wir die Existenz einer solchen Wirkung durchaus nicht bestreiten; was wir beanstanden, ist nur die quantitative Seite der Frage, und keineswegs die qualitative.

Die erwähnte magnetische Wirkung scheint aber Sonderbarkeiten aufzuweisen. So müßten wir doch erwarten, daß der Breiteneffekt nicht nur bis zum 50. Breitengrade reicht, sondern

[76]) K. O. Kiepenheuer, ZS. f. Astrophys. **10**, 265 f., 1935.

[77]) Ebenda S. 273.

bis zu den Polen. Dies widerspricht aber den Beobachtungen. Zuerst glaubte man eine sehr einfache Erklärung gefunden zu haben: die weiter gegen die Pole magnetisch abgelenkten Partikelchen seien so „weich“, daß sie die tieferen Atmosphärenschichten nicht erreichen könnten, und daß sie deshalb nur in den oberen Schichten anzutreffen seien. Die Stratosphärenbeobachtungen haben letzteres jedoch widerlegt. B. Gross sucht dies dadurch zu erklären, daß die primären Strahlen überhaupt keine „weicheren“ Partikelchen enthalten, und daß sich die anfängliche Energieverteilung der primären Partikelchen „überhaupt nur über einen ziemlich schmalen Bereich erstreckt, also fast der einer scharfen Linie entspricht. In diesem Falle müßte man nach dem plötzlich einsetzenden Abfall auch in niedrigen Breiten wieder Konstanz erwarten. In der Tat findet sich auch zwischen 0 und 20^0 nur eine ganz schwache Intensitätsänderung“ [78]). Eine andere Erklärung schlägt Jánossy vor [79]). Er meint, daß die „weichen“ primären Partikelchen durch den Sonnenmagnetismus von der Erde abgelenkt werden. Derselben Ansicht ist auch Vallarta [80]). Wir können jedoch eine solche Erklärung nicht akzeptieren, da das magnetische Sonnenfeld viel zu stark abgeschirmt ist. Auch das Verhalten der Schauer gegenüber dem irdischen Magnetfelde weist unerwartete Sonderbarkeiten auf: „The non-existence of any correlation between shower intensity and the earth's magnetic field can likewise be interpreted in the sense that the primary shower-producing radiation is not composed of electrically charged particles. On the other hand, the experiments indicate a very good agreement between the variation of the shower intensity and that of the temperature of the outer air; both having a maximum in the late afternoon“ [81]).

Als anderer indirekter Beweis für die Existenz von Energien über 10^{11} e-Volt gilt das enorme Durchdringungsvermögen eines (freilich sehr kleinen) Teiles der kosmischen Partikelchen. Dabei wird aber die Möglichkeit eines sogen. „Ramsauer-Effekts“ nicht in Betracht gezogen (wir meinen hier nicht den Raumsauer-Effekt hinsichtlich der Atome als Ganzes, sondern denjenigen

[78]) B. Gross, ZS. f. Phys. **105**, 338, 1937.
[79]) L. Jánossy, ZS. f. Phys. **104**, 335, 1937.
[80]) M. S. Vallarta, Nature **139**, 839, 1937.
[81]) M. Forró, Nature **139**, 633, 1937.

hinsichtlich der Atom k e r n e). Das Vorhandensein eines solchen Effekts müßte für einen gewissen Teil der Partikelchen eine hohe selektive Durchsichtigkeit des Mediums bewirken, wodurch ungeheure Energien der betreffenden Partikelchen vorgetäuscht würden. Da diese Frage für uns sehr wichtig ist, so wollen wir auf die Eigenschaften des Ramsauer-Effekts näher eingehen.

Vor etwa 14 Jahren schrieb Ramsauer: „*Alle* Edelgase ... zeigen gegenüber den sonstigen bisher untersuchten Gasen ... die gemeinsame Eigentümlichkeit, daß der Wirkungsquerschnitt mit abnehmender Elektronengeschwindigkeit ein Maximum erreicht und dann wieder abfällt .... der Wirkungsquerschnitt steigt im Maximum auf das 4 bis 5 fache des gaskinetischen Querschnitts und sinkt bei dem kleinsten untersuchten Geschwindigkeitswert von etwa 0,75 Volt auf unter $^1/_7$ des gaskinetischen Querschnitts herab .... Die Edelgase Ne, A, Kr, X zeigen in bezug auf Lage und Höhe ihrer Maxima einen ausgesprochenen G a n g: mit steigender Ordnungszahl (10; 18; 36; 54) wächst das Maximum absolut genommen stark an .... und rückt außerdem zu immer kleineren Elektronengeschwindigkeiten hin ($Volt_{max}$ = 25; 13,2; 11,3; 6,4)“ [82]). Etwa 4 Jahre später schreibt Brüche: „Das jetzt wesentlich vermehrte Versuchsmaterial zeigt, daß der Ramsauereffekt keine spezielle Eigentümlichkeit der Edelgase ist, sondern bei allen bisher untersuchten Gasen auftritt .... Dagegen zeigt sich, daß bei einer nach äußerer Ähnlichkeit vorgenommenen Anordnung der Querschnittskurven in Gruppen solche Gase zusammenstehen, von denen auch sonst ähnliche Eigenschaften und ähnlicher Bau bekannt sind. Darüber hinausgehend läßt die Ähnlichkeit der schweren Edelgase .... es vermuten, daß für die Gestalt der Querschnittskurven die Anordnung und Zahl der „Außen“-Elektronen des Moleküls eine wesentliche Rolle spielt“ [83]). In späteren Arbeiten untersucht Ramsauer (zusammen mit Kollath) die Wirkungsquerschnittskurve für noch kleinere Elektronengeschwindigkeiten, bis 0,16 Volt. „Die Wirkungsquerschnittskurve des Ne fällt zwischen 1,2 und 0,2 Volt ständig ab. Die Wirkungsquerschnittskurve des X hat bei etwa $0{,}6_5$ Volt ein kräftiges Minimum, wobei der Anstieg nach kleineren Geschwindigkeiten

[82]) C. Ramsauer, Annalen der Phys. (4) **72**, 351, 1923.
[83]) E. Brüche, Annalen der Phys. (4) **83**, 1127, 1927.

steiler ist, als der Anstieg nach größeren Geschwindigkeiten.... Die Wirkungsquerschnittskurve des He verläuft innerhalb des Gebietes von 1,1 — 0,2 Volt im wesentlichen horizontal, wobei das Maximum bei $0{,}7_6$ Volt angedeutet ist“ [84]. „CH_4 schließt sich den schweren Edelgasen auch insofern an, als es unterhalb 1 Volt nach Durchlaufung eines Minimums wieder ansteigt. Die Lage der Minima ist hierbei für Ar, Kr, CH_4 bzw. $0{,}3_9$; $0{,}6_0$; $0{,}4_3$ Volt“ [85]. „Die Wirkungsquerschnittskurven der Edelgase und der Nichtedelgase zeigen bei den kleinsten noch untersuchten Geschwindigkeiten zum Teil einen Anstieg, zum Teil einen Abfall nach 0 Volt hin“ [86]. Nach Ansicht der Verfasser erscheint es aber als möglich, daß bei sämtlichen Elementen der Wirkungsquerschnitt wieder ansteigt bei extrem kleinen Elektronengeschwindigkeiten, die zwischen 0,16 und 0 Volt liegen und experimentell schwer zu erfassen sind. In einem anderen Artikel stellen Ramsauer und Kollath in einer kleinen Tabelle die von verschiedenen Forschern gefundenen Werte des Argonminimums zusammen [87].

Beobachter	Argonminimum bei
Ramsauer-Kollath	0,37 Volt
Normand	0,62 „
Rusch	0,4 „
Townsend-Bailey	0,39 „

Wir reproduzieren teilweise diese Tabelle. Wir sehen, daß Normands Wert aus der Reihe fällt, während die übrigen Werte gut untereinander übereinstimmen.

Im Falle des Ramsauer-Effekts erhält also ein sehr langsames Elektron die scheinbare Fähigkeit, ohne Energieverlust und ohne Ablenkung ein Atom zu durchdringen, als ob das starke elektrische Atomfeld gar keinen Einfluß auf das Elektron ausübte, als ob also das Elektron gegen das Atomfeld „immun“ wäre. F. Hund schlug folgende Erklärung dieser sonderbaren Erscheinung vor. Wenn ein Atom von einem Elektron durchdrungen wird, erhält letzteres seitens des starken elektrischen

[84]) C. Ramsauer und R. Kollath, Annalen der Phys. (5) **3**, 563, 1929.
[85]) C. Ramsauer und R. Kollath, Annalen der Phys. (5) **4**, 101, 1930.
[86]) Ebenda S. 107.
[87]) C. Ramsauer und R. Kollath, Annalen der Phys. (5) **7**, 181, 1930.

Atomfeldes eine Beschleunigung, weshalb es (das Elektron) nach der klassischen Theorie eine der Beschleunigung entsprechende Energiemenge ausstrahlen müßte. „Da bei kleinen Geschwindigkeiten der klassische Energieverlust größer wäre als die kinetische Energie des Elektrons, da es ferner ein Atom ohne Elektronenaffinität wieder verlassen muß, kann das von außen kommende Elektron sich nicht klassisch bewegen, sondern muß das Atom auf einer Bahn mit geringerer Strahlung oder ohne Strahlung [also auch ohne Ablenkung und ohne Energieverlust] durchsetzen"[88]. Hund's Theorie läßt sich jedoch nicht aufrechterhalten, da sie nicht erklärt, warum bei extrem kleinen Elektronengeschwindigkeiten der Wirkungsquerschnitt wieder ansteigt.

Eine ganz andere Erklärung des Ramsauer-Effekts gibt Elsasser. Er weist darauf hin, das die sogen. de Broglie-Wellenlänge gleich

$$\lambda = \frac{h}{mv}$$

ist. Daher müsse man auch erwarten, daß das entsprechende Elektron alle Eigenschaften einer Welle aufweise, deren Länge λ ist. „Versucht man z. B. den merkwürdigen Gang der freien Weglänge von Elektronen, den Ramsauer und nach ihm eine Reihe anderer Autoren gefunden haben, mit der oben besprochenen Hypothese zu deuten, so zeigt sich, daß die Kurven, die diesen Gang wiedergeben, eine ganz auffällige Übereinstimmung aufweisen mit den Kurven, die man in der klassischen Theorie für die Beugung von Licht an kolloidalen Kügelchen erhält. Es sieht danach so aus, als ob die langsamen Elektronen an den Atomen nach Gesetzen gestreut würden, wie sie für Licht der berechneten Wellenlänge [Elsasser meint die de Broglie-Wellenlänge] bei Streuung an Kugeln vom Radius der Atome gelten würden. Naturgemäß ist die Übereinstimmung nur qualitativ"[89]. Faxén und Holtsmark behandeln das Problem des Ramsauer-Effekts wellenmechanisch[90]; desgleichen Mensing[91]. In einem anderen Artikel sagt Holtsmark: „Nach der Wellen-

[88]) F. Hund, ZS. f. Phys. **13**, 249, 1923.
[89]) W. Elsasser, Naturwissenschaften **13**, 711, 1925.
[90]) H. Faxén und J. Holtsmark, ZS. f. Phys. **45**, 307, 1927.
[91]) Lucy Mensing, ebenda S. 603.

mechanik ist die Streuung der Elektronen in einem Gas ein Beugungsphänomen ... Der Wirkungsquerschnitt als Funktion der Geschwindigkeit oder der Wellenlänge des Elektronenstrahls zeigt im allgemeinen Maxima und Minima, die als Beugungsmaxima bzw. -minima erkannt werden sollen“ [92]).

Alle bisherigen Untersuchungen haben sich immer nur auf den Ramsauer-Effekt des ganzen Atoms bezogen. Sollte aber vielleicht auch noch ein Ramsauer-Effekt der Atomkerne existieren? — Diese Frage muß bejaht werden, unter der selbstverständlichen Voraussetzung, daß der Ramsauer-Effekt auch tatsächlich eine Beugungserscheinung darstellt, wie dies jetzt allgemein angenommen wird. In einem solchen Falle muß offenbar die notwendige de Broglie-Wellenlänge ungefähr proportional dem Radius des „Hindernisses“ sein, welches die Elektronenbeugung hervorruft. Das Verhältnis des Atomradius zu dem Kernradius ist von der Größenordnung 10000. Deshalb muß die de-Broglie-Wellenlänge bei dem Kern-Ramsauer-Effekt etwa 10000 mal kleiner sein als bei dem Atom-Ramsauer-Effekt. Nehmen wir beispielsweise an, daß das Minimum des Atomwirkungsquerschnitts bei einer Elektronengeschwindigkeit von 0,5 Volt auftritt, also bei $v_1 = 4{,}22 \,.\, 10^7$ cm . sec^{-1}. Der entsprechende Impuls ist gleich

$$m_0 v_1 = 9 \,.\, 10^{-28} \,.\, 4{,}22 \,.\, 10^7 = 3{,}80 \,.\, 10^{-20}.$$

Die de Broglie-Wellenlänge λ ändert sich umgekehrt proportional dem Impulse. Da nach unserer Schätzung der Kern-Ramsauer-Effekt bei einem 10000 mal kleineren λ auftritt, so muß der entsprechende Elektronenimpuls 10000 mal größer sein, d. h. er muß betragen:

$$mv_2 = 3{,}80 \,.\, 10^{-20} \,.\, 10000 = 3{,}80 \,.\, 10^{-16}.$$

Da nun die lineare Elektronengeschwindigkeit v_2 nur wenig von Lichtgeschwindigkeit verschieden sein wird, kann man ohne großen Fehler $v_2 = c$ setzen, und dann erhalten wir:

$$mc = 3{,}80 \,.\, 10^{-16},$$

und also:

$$m = \frac{3{,}80 \,.\, 10^{-16}}{3 \,.\, 10^{10}} = 1{,}27 \,.\, 10^{-26}.$$

[92]) J. Holtsmark, ZS. f. Phys. **55**, 437, 1929.

Für die kinetische Energie des Elektrons ergibt sich: $(m - m_0) c^2 = (1{,}27 . 10^{-26} - 9 . 10^{-28}) . 9 . 10^{20} = 1{,}06 . 10^{-5}$ Erg $= 6{,}6$ Millionen e-Volt. Diese Zahl hat natürlich nur einen rein orientierenden Charakter. Sie zeigt jedoch, auf welche Größenordnung von Elektronenenergie man bei einem Kern-Ramsauer-Effekt gefaßt sein muß.

Bei noch kleinerer de Broglie-Wellenlänge ist ein Ramsauer-Effekt sogar hinsichtlich eines jeden einzelnen Elementarteilchens zu erwarten. In einem solchen Falle wird das entsprechende Elektron (auch bei einem zentralen Zusammenstoß) scheinbar widerstandslos durch das Proton (oder durch das Neutron) hindurchgehen. Nun besteht die gewöhnliche Materie letzten Endes aus Protonen, Neutronen und Elektronen. Da die „Radien" dieser Elementarteilchen als mehr oder weniger gleich angenommen werden können, so wird für sie alle der Ramsauer-Effekt bei annähernd ein und derselben Elektronenenergie auftreten. Wir müssen daher erwarten, daß die gewöhnliche Materie für Elektronen von bestimmter Energie in hohem Grade durchsichtig ist. Es ist durchaus nicht verwunderlich, wenn man solchen Elektronen phantastische Energien zuschreibt.

Man kann jedoch auch ohne Ramsauer-Effekt sich einen solchen Prozeß vorstellen, wo phantastische Elektronenenergien vorgetäuscht werden. C. D. Anderson hat gezeigt, daß ein genügend energiereiches Elektron auch schwere Teilchen aus dem Kern auslösen kann (s. oben S. 20). Nehmen wir nun an, daß ein genügend energiereiches primäres kosmisches Elektron bereits in der obersten Atmosphärenschicht mit einem Sauerstoff- oder Stickstoffkern zusammenstößt, wobei ein Neutron ausgelöst wird. Das Neutron kann dabei eine beträchtliche kinetische Energie von dem stoßenden Elektron übernehmen. Nun ist die Absorbierbarkeit der Neutronen so gering, daß sie gar keiner phantastischen Energie bedürfen, um gewaltige Schichtdicken zu durchdringen. Nehmen wir an, daß unser obenerwähntes Neutron die ganze Atmosphäre und außerdem noch eine sehr dicke Bleischicht zwar durchdringt, aber schließlich mit einem Kern derart zusammenstößt, daß aus letzterem ein energiereiches Proton ausgelöst wird. Wenn nun jemand zufällig nur das Ende dieser Protonenbahn in der Wilson-Kammer beobachtet, wird er geneigt sein zu glauben, daß das Proton ein primäres sei, welches die ganze Atmosphäre und dazu noch eine sehr dicke Bleischicht durchdrungen habe. Naturgemäß wird der Beobachter diesem „primären“ Proton eine phantastische Energie zuschreiben.

Wenn man also aus der berechneten Wirkung des irdischen Magnetfeldes, oder auch aus dem scheinbaren Absorptionskoeffizienten, Energien von 10^{11} bis 10^{12} e-Volt ableitet, so kann man nicht absolut sicher sein, daß diese Zahlen reell sind. Man kann nicht einmal völlig sicher sein, ob sich nicht die wirklichen Werte von den berechneten um Zehnerpotenzen unterscheiden.

Es gibt noch eine dritte Methode die primäre Energie der kosmischen Partikelchen zu messen. Man schätzt nämlich den Energieinhalt eines Schauers (oder eines Stoßes) mehr oder weniger genau ab. Dieser Energieinhalt muß gleich oder kleiner sein als die primäre Energie des kosmischen Partikelchens, welches den Schauer (oder den Stoß) verursacht hat. Für große Schauer (und große Stöße) erhält man angeblich Energien von über 10^{11} e-Volt, während nach unserer „Gravitationstheorie“ Energien von über 10^{10} e-Volt in der kosmischen Strahlung nicht anzunehmen sind. Jedenfalls könnten Partikelchen mit über 10^{11} e-Volt Energie nach der „Gravitationstheorie“ nur als ein unmessbar kleiner Prozentsatz auftreten. Es besteht also eine mindestens 10-fache Diskrepanz (nicht hinsichtlich der durchschnittlichen, sonder nur hinsichtlich der maximalen Energien der Partikelchen) zwischen unserer „Gravitationstheorie“ und der Beobachtung, vorausgesetzt natürlich, daß man die in den großen Schauern und Stößen auftretenden Energiemengen auch tatsächlich richtig geschätzt hat. Trifft letzteres zu, so sehen wir uns veranlaßt zu untersuchen, ob sich unsere „Gravitationstheorie“ nicht derart modifizieren läßt, daß die obenerwähnte 10-fache Diskrepanz verschwindet.

Unsere Diskrepanz könnte mit Leichtigkeit durch die Annahme beseitigt werden, daß unter den primären Partikelchen auch schwerere Atomkerne sich befinden, und daß dabei die Äquipartition in der Energieverteilung keine vollständige ist. Nach dem früher Gesagten jedoch (vgl. S. 11) müssen wir einen solchen Ausweg ablehnen.

Die Diskrepanz wäre auch sehr leicht behoben, wenn wir uns entschließen könnten, nach traditioneller Weise nicht die Gesamtmasse m, sondern die Ruhemasse m_0 eines fallenden

Körpers als konstant anzunehmen. Dann würde sich aus der relativistischen Formel

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$m \to \infty$ bei $v \to c$ ergeben, und wir hätten die Möglichkeit, das Auftreten beliebig großer Energien ungezwungen zu erklären. Jedoch müssen wir auch diesen Ausweg ablehnen, weil die traditionelle Auffassung mit dem Gesetz der Erhaltung der Energie unverträglich ist. So z. B. müßte man vom Standpunkt der traditionellen Auffassung annehmen, daß die Masse eines sich verdichtenden Sterns größer werde (trotz des Energieverlustes durch Ausstrahlung), da ja bei der Verdichtung Temperatur und durchschnittliche Molekulargeschwindigkeit zunehmen. Entgegen der traditionellen Auffassung nehmen wir daher an, daß bei einem frei fallenden Körper nicht die Ruhemasse m_0, sondern die Gesamtmasse m konstant bleibt. Unsere Auffassung führt zu $m_0 \to 0$ bei $v \to c$. Wir halten es nicht für möglich, unsere Auffassung zugunsten der traditionellen aufzugeben.

Die Diskrepanz könnte auch durch die Annahme beseitigt werden, daß bei extremen Energien die von uns vorausgesetzte normale Maxwellsche Energieverteilung stark von der Wirklichkeit abweiche. Wir können zwar die Unmöglichkeit einer solchen Annahme nicht einwandfrei beweisen, uns scheint aber letztere trotzdem sehr gewagt zu sein.

Wir hatten angenommen, daß die Masse des Universums gleich $2 \cdot 10^{55}$ Gramm sei. Sollten wir uns entschließen, dem Universum eine größere Masse zuzuschreiben (z. B. $3 \cdot 10^{55}$ oder $4 \cdot 10^{55}$ Gramm), so würde dies zu vergrößerten Werten für die Energien der kosmischen Partikelchen führen. Leider erweist sich diese Vergrößerung auch im günstigsten Falle als ungenügend, um unsere 10-fache Diskrepanz zu beseitigen.

Die Ruhemasse eines Protons oder eines Neutrons kann gleich $1{,}66 \cdot 10^{-24}$ angenommen werden. Diese Zahl bezieht sich aber nur auf freie Protonen und Neutronen. Dagegen ist die Masse eines Kernprotons oder eines Kernneutrons (infolge des bekannten Massendefekts von etwa 0,7 bis 0,8 %) kleiner als $1{,}66 \cdot 10^{-24}$, obgleich sich im Kern die Protonen und Neutronen bewegen. Natürlich wird deren Ruhemasse im Kern erst recht kleiner als $1{,}66 \cdot 10^{-24}$ sein. Nehmen wir nun an, daß im Kern sich ein Mikro-Einwohner befindet, der da glaubt, daß die Ruhemasse eines Protons (oder eines Neutrons) auch außerhalb seiner „Welt“ kleiner als $1{,}66 \cdot 10^{-24}$ sei. Von dieser Annahme ausgehend, berechnet der Mikro-Einwohner die Energie, welche bei dem Fallen eines Neutrons aus der Unendlichkeit bis in den Kern frei wird. Wenn nun aber ein Neutron tatsächlich von dem Kern „eingefangen“ wird, so wird die dabei frei werdende Energie den vom Mikro-Einwohner berechneten Wert übersteigen. Der Mikro-Einwohner hatte nämlich nicht in Betracht gezogen, daß innerhalb seiner „Welt“ das Neutron eine kleinere Ruhemasse besitzt als außerhalb. Wir könnten nun in analoger Weise annehmen, daß in der „geordneten“ Welt (d. h. in allen Himmelskörpern) die Neutronen-Ruhemasse kleiner ist als in der „ungeordneten Außenwelt“ der kosmischen Partikelchen. Wenn nun ein „ungeordnetes“ kosmisches Neutron von einem Himmelskörper „eingefangen“ wird, so muß die dabei tatsächlich frei werdende Energie die berechnete übertreffen. Wir halten jedoch eine solche Hypothese zur Beseitigung unserer Diskrepanz für sehr unwahrscheinlich, wenn nicht vom qualitativen, so doch vom quantitativen Standpunkt aus.

Wir sehen also, daß unsere „Gravitationstheorie“ der kosmischen Strahlen auf gewisse Schwierigkeiten stößt (wenn die bei großen Schauern und Stößen auftretenden Energiemengen auch tatsächlich richtig geschätzt worden sind), die sich jedoch nicht auf die durchschnittlichen, sondern nur auf die maximalen Energiewerte der Partikelchen beziehen. Wir können aber nicht mit Bestimmtheit behaupten, daß diese Schwierigkeiten unüberwindlich sind.

Wollen wir nun untersuchen, ob nicht vielleicht irgendeine andere in der Literatur vorgeschlagene Theorie der kosmischen Strahlung geringere Schwierigkeiten biete.

Vor etwa 6 Jahren äußerte Millikan die Meinung, daß „an atom-building process“ die notwendige Energie liefern könne (vgl. S. 13). Millikans Hypothese ist jedoch unhaltbar, da sie selbst dann zu geringe Energien liefert, wenn die gesamte bei der Kernbildung entstandene Energie als ein einziges Strahlungsquant ausgesandt wird. In der hier beigefügten Tabelle, die ich auf Grund von Eastman's [93]) Daten berechnet habe,

[93]) E. D. Eastman, Phys. Rev. (2) **46**, 9, 1934.

sind die Bildungsenergien einiger Kerne angegeben [94]). Vergleicht man diese Tabelle mit derjenigen auf S. 17, so erkennt man sofort die Unmöglichkeit, die größeren in der Wilson-Kammer gemessenen Energien durch Kern-Aufbau-Prozesse zu erklären. Radioaktive Prozesse können erst recht nicht in Frage kommen,

Elemente	Bildungsenergie der Kerne
Ne (Isotop 20)	137,5 Millionen e-Volt
Ne (Isotop 22)	152,0 „ „
Si	204,7 „ „
Ni	443,4 „ „
Sn (Isotop 112)	823,9 „ „
Sn (Isotop 124)	906,8 „ „
Ba	994,1 „ „
Pb (Isotop 206)	1350 „ „
Pb (Isotop 208)	1363 „ „

denn beim radioaktiven Zerfall eines Kerns wird sehr viel weniger Energie frei, als bei dessen Aufbau aus Protonen und Neutronen. Die „Zerstrahlung“ eines Elektrons und Positrons ergibt nur etwa 1 Million e-Volt, kommt also für uns überhaupt nicht in Betracht. Sogar das „Zerstrahlen“ eines Elektrons und Protons wäre ungenügend, da es nur etwa 900 Millionen e-Volt ergeben würde; außerdem glaubt heutzutage kein ernster Forscher an die Möglichkeit eines solchen Prozesses. Es geht auch nicht an, sich damit zu trösten, daß es vielleicht noch unbekannte Elemente von phantastisch hohen Ordnungszahlen gibt, daß vielleicht solche Elemente eine wesentliche Rolle beim Zustandekommen der kosmischen Strahlung spielen. „Schießt man ein positiv geladenes Teilchen gegen einen Atomkern, so muß es zunächst gegen das Coulombfeld anlaufen, das eine um so stärkere bremsende Wirkung auf das Teilchen ausübt, je höher die Ordnungszahl des Kernes ist“ [95]).

[94]) Nebenbei gesagt zeigt diese Tabelle, daß ein Pb-Kern (Isotop 208) durch den Stoß eines Partikelchens in 208 Teilchen (82 Protonen und 126 Neutronen) zersprengt werden kann, wenn die Energie des stoßenden Partikelchens 1363 Millionen e-Volt übersteigt. Bei einem Ne-Kern (Isotop 20) genügt zu dessen Zersprengung (in 10 Protonen und 10 Neutronen) ein stoßendes Partikelchen von über 137,5 Millionen e-Volt.

[95]) S. Flügge und A. Krebs, Phys. ZS. **36**, 471, 1935.

Je höher also die Ordnungszahl eines Elements ist, desto schwerer läßt sie sich (durch Hinzufügung neuer Protonen zum Kern) weiter erhöhen. „Die Packungsanteile (Massendefekte pro Teilchen) der leichteren Kerne (etwa bis Fe) sind nicht streng konstant, sondern nehmen weiter langsam zu ... Die Packungsanteile der schwereren Kerne nehmen nach nahezu konstantem Verlauf wieder langsam ab ... Die [letztere] Tatsache läßt sich, wie schon Gamow bei der Aufstellung des Tröpfchenmodells vermutet hatte, durch die Coulombsche Abstoßung der Protonen erklären“ [96]). Eine Verminderung des Massendefekts pro Teilchen bedeutet aber eine Verminderung der Bindungskraft dieses Teilchens an den übrigen Kernteil. Die Kernstruktur wird also bei wachsender Ordnungszahl immer unstabiler. Will man jedoch dem Kern gar keine Protonen, sondern immer nur neue und neue Neutronen zuführen, so muß auch dies schließlich zu einer Stabilitätsverminderung führen, denn ausschließlich aus Neutronen lassen sich keine Atomkerne aufbauen. Die Existenz eines unbekannten Elements, dessen Atomgewicht *sehr viel größer* als dasjenige des Urans ist, wäre also schon aus rein theoretischen Erwägungen nicht anzunehmen. Auf Grund alles Gesagten glauben wir behaupten zu dürfen, *daß weder bekannte, noch unbekannte subatomare Prozesse bei der Entstehung der primären kosmischen Strahlung eine Rolle spielen können.*

Bothe und Kolhörster erklären die Energien der kosmischen Partikelchen durch ungeheuer ausgedehnte elektrische Felder im Kosmos (vgl. S. 12). Millikan weist die Unmöglichkeit solcher Felder nach (vgl. S. 13). Wir unsererseits finden. daß die Entstehung und die Unterhaltung derartiger Felder jedenfalls kein leichteres Problem darstellen würde, als das Problem der kosmischen Strahlung selbst.

Swann erklärt die Energie der kosmischen Partikelchen durch Änderung des lokalen magnetischen Feldes, zwar nicht in den Sonnenflecken, wohl aber in den „Sternflecken“ (vgl. S. 14). Es darf jedoch nicht vergessen werden, daß wenn die Sterne als Quelle der Ultrastrahlung betrachtet werden, man anzunehmen gezwungen ist, daß sie annähernd ebensoviel Energie an kosmischer Strahlung wie an gewöhnlicher Strahlung emittieren. Un-

[96]) C. F. v. Weizsäcker, ZS. f. Phys. **96**, 431 f., 1935.

ter solchen Bedingungen müßte man einen wesentlich anderen Ionisationszustand der entsprechenden Sternatmosphäre erwarten, als er aus den Beobachtungen zu erschließen ist. Auch müßte bei einem einzelnen Stern während des Maximums der „Sternfleckentätigkeit“ der erwähnte Ionisationszustand völlig verschieden sein von demjenigen während des Minimums. Derartige Schwankungen könnten aber der Beobachtung nicht entgehen. Noch ein anderer Einwand läßt sich gegen Swann's Hypothese erheben. Der lokale Magnetismus der Sonnen- und Sternflecke wird wahrscheinlich durch relativ langsame Driftbewegungen der Elektronen verursacht. Die gesamte magnetische Energie aller gleichzeitig vorhandenen Flecke des Sterns wird sogar während des Maximums wahrscheinlich sehr viel kleiner sein, als die vom Stern im Verlaufe einer Sekunde ausgestrahlte Energiemenge. Die kosmische Strahlung rührt aber nach Swann bloß von der Änderung des lokalen Magnetismus her, besitzt also eine noch viel schwächere Energiequelle. Auf welche Weise sollte unter solchen Bedingungen die vom Stern ausgehende kosmische Strahlungsenergie beinahe gleich sein mit der gleichzeitig ausgehenden gewöhnlichen Strahlungsenergie? — Und dabei wären wir noch gezwungen anzunehmen, daß die uns so nahe Sonne keinen merklichen Beitrag zu der der Erde zugesandten kosmischen Strahlung liefere, denn ein merklicher Beitrag wäre mit der Beobachtung unvereinbar. Man dürfte allerdings annehmen, daß die magnetische Wirksamkeit der Sonne schwächer ist als diejenige vieler Sterne, aber doch nicht in einem derartigen Mißverhältnis! —

A. K. Das betrachtet die kosmische Strahlung als Temperaturstrahlung, die aus dem Inneren der Sterne in den Außenraum gelangt (vgl. S. 14). Nach unseren jetzigen Kenntnissen müßte sich jedoch die erwähnte Temperaturstrahlung auf ihrem langen Wege aus dem Sterninneren zur Oberfläche derart verändern, daß sie als ganz normale Oberflächenstrahlung den Stern verließe.

Baade und Zwicky sehen in den „Super-Novae“ die Quelle der Ultrastrahlung (vgl. S. 18). Auch Regener sympathisiert mit einer solchen Hypothese (vgl. S. 23 f.). Gegen diese Hypothese ist aber Folgendes einzuwenden. Ein durchschnittlicher „Nebel“ besteht aus etwa 10^9 Sternen, die wir, der Einfachheit halber, als unserer Sonne gleich annehmen wollen. Alle tausend

Jahre erscheint im Nebel eine „Super-Nova“ mit einer durchschnittlichen „Lebensdauer“ von etwa 20 Tagen. Dies bedeutet, daß unter $1{,}8.10^{13}$ Sternen immer irgendeine Super-Nova leuchtet. Die von der Super-Nova ausgehende Ultrastrahlungsenergie muß also beinahe $1{,}8.10^{13}$ mal oder rund gerechnet 10^{13} mal größer sein als die von der Sonne ausgehende „gewöhnliche“ strahlende Energie (weil ja die Intensität der kosmischen Strahlung nur etwas kleiner ist als die von allen Sternen unserer Erde zugesandte gewöhnliche Strahlung). Für die von der Super-Nova ausgehende „gewöhnliche“ strahlende Energie hingegen beträgt das erwähnte Verhältnis „bloß“ 10^8. Dies bedeutet, daß eine Super-Nova etwa $\frac{10^{13}}{10^8} = 10^5$ mal mehr Energie an „kosmischer“ als an „gewöhnlicher“ Strahlung emittiert. Die Ausstrahlung unserer Sonne beträgt $3{,}8 \,.\, 10^{33}$ Erg.sec^{-1}; eine durchschnittliche Super-Nova hingegen emittiert $3{,}8 \,.\, 10^{41}$ Erg.sec^{-1} gewöhnlicher Temperaturstrahlung und gleichzeitig $3{,}8 \,.\, 10^{46}$ Erg.sec^{-1} kosmischer Ultrastrahlung. Die Stellarmaterie darf aber höchstens den hunderttausendsten Teil dieser Ultrastrahlungsenergie absorbieren und in Wärmeenergie verwandeln, weil sonst die Temperaturstrahlung der Super-Nova größer als $3{,}8 \,.\, 10^{41}$ Erg.sec^{-1} sein würde. Diese Überlegung zwingt uns, den Erzeugungsort der Ultrastrahlung in die äußerste Oberflächenschicht der Super-Nova zu verlegen. Setzt man den durchschnittlichen Massenabsorptionskoeffizienten der Ultrastrahlung gleich 0,001, so kann die ungeheure Energieerzeugung von $3{,}8 \,.\, 10^{46}$ Erg.sec^{-1} nur in der äußersten Oberflächenschicht stattfinden, deren Dicke kleiner als 0,01 g.cm^{-2} ist. Außerdem müßte die dort erzeugte Ultrastrahlung die wunderliche Eigenschaft besitzen, sich außschließlich in der Richtung nach außen hin zu bewegen. Selbst wenn nur der hunderttausendste Teil der erzeugten Ultrastrahlung nach innen gerichtet (und dort natürlich absorbiert) wäre, müßte die Temperaturstrahlung der Super-Nova größer als $3{,}8 \,.\, 10^{41}$ Erg.sec^{-1} sein. Wie wahrscheinlich dies alles ist, möge der Leser selbst beurteilen. — Regener erwähnt eine briefliche Mitteilung von Clay, wonach am 20. Mai 1936 in mehreren Apparaten eine ziemlich plötzliche Erhöhung der Ultrastrahlungsintensität um etwa 10 % stattgefunden haben soll, die im Juni wie die Helligkeit einer Nova abklang (vgl. S. 24). Nach unserer Meinung müßte aber diese Erscheinung in sämtlichen Apparaten der Welt bemerkbar gewesen sein, und nicht bloß „in mehreren“. Außerdem müßte die Intensität der Ultrastrahlung eine Periode von 24 Stunden aufgewiesen haben (wegen der Erdrotation). Wenn die kosmische Ultrastrahlung durch eine hypothetische Super-Nova um 10 % vergrößert worden wäre, so müßte letztere der Erde etwa $3 \cdot 10^{-4}$ Erg.cm^{-2}.sec^{-1} an Ultrastrahlung zugesandt haben, also gleichzeitig auch $3 \cdot 10^{-4}/10^5 = 3 \cdot 10^{-9}$ Erg.cm^{-2}.sec^{-1} an gewöhnlicher Temperaturstrahlung. Letzteres würde einem Stern ungefähr 9. Größe entsprechen. Ein Stern, der $3{,}8 \cdot 10^{41}$ Erg.sec^{-1} ausstrahlt, wovon auf unsere Erde $3 \cdot 10^{-9}$ Erg.cm^{-2}.sec^{-1} fällt, muß von uns etwa 1 Million parsec entfernt sein. Wir müßten also annehmen, daß am 20. Mai 1936 in einer Entfernung von etwa 1 Million parsec, d. h. in einem der nächsten Spiralnebel, ein neuer Stern von ungefähr 9. Größe aufgeflammt sein. Dies nachzuprüfen dürfte doch nicht übermäßig schwer fallen.

Ganz vor kurzem hat Alfvén eine neue Theorie der Ultrastrahlung entwickelt, wonach ihre Entstehung in die Doppelsterne verlegt wird. Letztere betrachtet Alfvén als magnetische Dipole und zeigt, daß durch ihre Bewegung unter bestimmten Bedingungen Elektronenergien von 10^{11} e-Volt zustande kommen können[97]). Dadurch könne man die Entstehung der primären kosmischen Ultrastrahlen erklären. — Gegen Alfvén's Hypothese läßt sich folgendes einwenden. Die Elektronen erhalten ihre kinetischen Energien letzten Endes auf Kosten der kinetischen Energie und der potentiellen Gravitationsenergie der um den gemeinsamen Schwerpunkt rotierenden Sterne. Der Einfachheit halber nehmen wir an, daß die Masse M des Hauptsterns viel größer als die Masse m des Begleiters sei. Dann können wir ohne großen Fehler sagen, daß m sich um das unbewegliche M mit der linearen Geschwindigkeit v bewege, wobei der Radius der Bahn gleich R sei. Wir haben dann:

$$\frac{GMm}{R^2} = \frac{mv^2}{R},$$

also ist:

$$\frac{GMm}{2R} = \frac{1}{2}mv^2 = \text{kinetische Energie von } m.$$

Macht man die übertrieben günstige Annahme, daß anfänglich m unendlich weit von M entfernt gewesen sei, so ist die gesamte

97) H. Alfvén, ZS. f. Phys. **105**, 319, 1937.

frei gewordene Energie gleich $\frac{GMm}{R}$, wovon die eine Hälfte als kinetische Energie von m auftritt, und also nur die andere Hälfte für die Ultrastrahlung zur Verfügung gestanden hat. Somit kann die Ultrastrahlung auch im besten Falle nur $\frac{GMm}{2R}$ Erg erhalten haben, was $\frac{1}{2}mv^2$ Erg gleich wäre. Es möge der Doppelstern seit t Sekunden „existiert" haben. Wir wollen annehmen, daß zur Bestreitung der gewöhnlichen Temperaturstrahlung sowohl im Hauptstern als auch im Begleiter 2 Erg . g^{-1} . sec^{-1} produziert werden (wie bei unserer Sonne). Da der Doppelstern seit t Sekunden „existiert", so wird er im ganzen $2(M+m)t$ Erg an Temperaturstrahlung emittiert haben. Außerdem wird er ungefähr halb so viel Energie an Ultrastrahlung ausgesandt haben, d. h. $(M+m)t$ Erg. Diese Zahl muß der zur Verfügung gestandenen Gravitationsenergie $\frac{GMm}{2R}=\frac{1}{2}mv^2$ Erg gleich sein. Wir erhalten also:

$$(M+m)t=\frac{1}{2}mv^2,$$

oder:

$$t=\frac{mv^2}{2(M+m)}.$$

Da nun aber $M>m$ ist, so können wir schreiben:

$$t<\frac{mv^2}{2(m+m)},$$

oder:

$$t<\frac{v^2}{4}.$$

Setzt man $v=2 \cdot 10^6$ cm . sec^{-1}, was für einen durchschnittlichen Doppelstern wohl nicht weit von der Wahrheit sein dürfte, so erhält man:

$$t<10^{12} \text{ sec.}$$

Dies bedeutet, daß die Ultrastrahlung höchstens seit ungefähr 30 000 Jahren existiere, was wohl ein völlig unzulässiges Resultat darstellt[98]). Außerdem sei noch erwähnt, daß Alfvén die vermutlich sehr große Abschirmung des magnetischen Feldes der Sterne (vgl. S. 27 f.) bei seinen Berechnungen nicht in Betracht gezogen hat[99]).

Nachträglich wollen wir noch auf folgenden Ausweg zur Beseitigung der oben besprochenen 10-fachen Diskrepanz (vgl. S. 35) hinweisen. Wir haben gesehen, daß genügend energiereiche Ultrateilchen imstande sind die Kerne der Atome völlig zu zertrümmern und in freie Protonen und Neutronen aufzulösen. Diese können von anderen Kernen eingefangen werden, wodurch weiterer Aufbau oder auch Abbau der letzteren zustande kommen kann. Die Kernreaktionen sind teilweise endothermisch, teilweise exothermisch, und es ist schwer a priori zu sagen, welche Reaktionen überwiegen werden. Es ist nicht unmöglich, daß die durch Kernreaktionen in einem Schauer (oder in einem Stoß) frei gewordene Energie (die man manchmal auf 10^{11} bis 12^{12} e-Volt schätzen muß) sehr viel größer ist als die Energie des primären kosmischen Ultrateilchens, der man also bloß eine „auslösende" Wirkung zuschreiben müßte (und die vielleicht viel kleiner als 10^{10} oder sogar kleiner als 10^{9} e-Volt sein mag). Eine derartige Idee ist gar nicht neu: man braucht sich nur daran zu erinnern, daß z. B. Atkinson in den Kernreaktionen sogar die Energiequelle der meisten Sterne sieht[100]).

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98) Wir haben hier vorausgesetzt, daß die von den Doppelsternen als Ultrastrahlung emittierte Energie die Hälfte der Temperaturstrahlung ausmache, weil ungefähr in diesem Verhältnis die beiden Energiestrahlungen auf unsere Erde einfallen, und weil wir außerdem stillschweigend angenommen haben, daß es keine Sterne außer Doppelsternen gebe. In Wirklichkeit aber gibt es außer Doppelsternen noch viele andere Sterne. Zieht man letzteres in Betracht, so erhält man ein noch kleineres t.

99) Vielleicht könnte man aber Alfvén's Gedanken auf ein ganz anderes Gebiet anwenden. Man kann die Sonnenkugel als aus einer unendlichen Zahl paralleler magnetischer Dipole bestehend betrachten, die um die Sonnenachse rotieren (jedoch nicht mit absolut gleichen Winkelgeschwindigkeiten) und ihr annähernd parallel bleiben. Vielleicht könnte dies zur Vergrößerung der Elektronengeschwindigkeiten in Alfvén's Sinne führen. Nach Grotrian's Beobachtungen soll ja die durchschnittliche Elektronengeschwindigkeit in der äußeren Sonnenatmosphäre etwa 4.10^{8} cm. sec^{-1} betragen. Bis jetzt waren wir nicht imstande eine so hohe durchschnittliche Geschwindigkeit zu erklären (vgl. W. Anderson, Publ. de l'Observ. Astron. de l'Univ. de Tartu **29**$_4$, S. 14).

100) R. d'E. Atkinson, Nature **128**, 194, 1931; Astrophys. Journ. **73**, 250 und 308, 1931; **84**, 73, 1936.

Wenn nun (nach Atkinson) Kernreaktionen die Energiequelle der Sterne bilden, warum können diese Reaktionen nicht (wenigstens teilweise) auch die Energiequelle großer Schauer oder Stöße sein? — Wir wollen eine solche Hypothese zwar nicht besonders verteidigen, aber mit ihrer prinzipiellen Möglichkeit müßte man immerhin rechnen.

Hauptergebnis.

Sämtliche zur Erklärung der primären kosmischen Ultrastrahlung bis jetzt vorgeschlagenen Theorien sind völlig indiskutabel, ausgenommen die „Gravitationstheorie". Zwar bietet auch letztere gewisse Schwierigkeiten, doch sind diese „von ganz anderer Größenordnung" als die Schwierigkeiten der übrigen Theorien.

Anhang I.

Der Gleichgewichtsradius des sich im (labilen) „Lichtstadium" befindlichen Universums ist nach (3) gleich

$$R = \frac{3}{5} \frac{GM}{c^2}. \tag{8}$$

Nach (7) haben wir:

$$\frac{u_m}{c} = 0{,}377,$$

wo

$$u_m = \frac{R_t - R}{t}$$

ist, und R_t den augenblicklichen Radius des Universums bedeutet. Wir haben offenbar:

$$R_t = R + u_m t = R + 0{,}377\, ct,$$

und daher ist

$$R_t > 0{,}377\, ct. \tag{9}$$

Wenn ein Beobachter von der Expansion des Universums nichts weiß, so wird er selbstverständlich

$$R = R_t \tag{10}$$

setzen. Wenn wir aber die Übereinstimmung dieser Gleichung mit unseren Formeln (8) und (9) um jeden Preis erzwingen wollen, müssen wir nolens volens schreiben:

$$\frac{3\,GM}{5\,c^2} = R = R_t > 0{,}377\;ct,$$

oder:

$$G > \frac{5}{3} \cdot \frac{0{,}377\;c^3 t}{M},$$

oder:

$$G > 0{,}63\,\frac{c^3 t}{M}. \tag{11}$$

Aus $R_t = R + u_m t$ folgt, daß man bei genügend großem t ohne merklichen Fehler $R_t = u_m t$ schreiben kann. Da nach unserer Theorie die durchschnittliche Expansionsgeschwindigkeit u_m immer kleiner als Lichtgeschwindigkeit bleibt, so muß bei genügend großem t die Ungleichung

$$R_t < ct \tag{12}$$

bestehen. Aus (8), (10) und (12) erhalten wir:

$$\frac{3\,GM}{5c^2} = R = R_t < ct\,,$$

oder:

$$G < \frac{5}{3}\,\frac{c^3 t}{M}. \tag{13}$$

Wir können (11) und (13) folgendermaßen vereinigen:

$$\frac{5}{3}\,\frac{c^3 t}{M} > G > 0{,}63\,\frac{c^3 t}{M}. \tag{14}$$

Milne hat aus seiner Theorie die Gleichung

$$G = \frac{c^3 t}{M} \tag{15}$$

abgeleitet[101]), die unserer Beziehung (14) formell nicht widerspricht, aber von Milne anders interpretiert wird.

Anhang II.

Die volle Schwingungsdauer eines Pendels ist

$$\vartheta = 2\,\pi\sqrt{\frac{l}{g}}\,,$$

101) E. A. Milne, Proc. Roy. Soc. London (A) **154**, 43, 1936, und andere Stellen.

wo die Anziehungsbeschleunigung g selbstverständlich proportional der Gravitationskonstante G ist. Ein Beobachter definiere die Pendellänge l als den N-ten Teil der Entfernung zweier Himmelskörper A und B, wobei diese Entfernung AB sehr groß sein möge. AB ändert sich offenbar proportional mit R_t, wobei letzteres gleich $u_m t$ gesetzt werden kann, wenn t sehr groß ist. Solange dabei u_m annähernd konstant bleibt, kann man ohne großen Fehler R_t und AB als proportional t annehmen. Also muß sich auch die Pendellänge

$$l = \frac{AB}{N}$$

proportional mit t ändern.

Dies bewirkt eine Verlangsamung der Schwingung, weil dann ϑ proportional mit $\sqrt{t}$ wächst. Der Beobachter, der nichts von der Expansion des Universums weiß, und der außerdem glaubt, daß nach (15) die Gravitationskonstante G (und also auch die Anziehungsbeschleunigung g) proportional t sein müsse, würde erwartet haben, daß ϑ nicht nur nicht wachse, sondern proportional mit $\sqrt{t}$ abnehme. Der Beobachter müßte sich also um das $\sqrt{t}\Big/\frac{1}{\sqrt{t}} = t$-fache getäuscht sehen. Hat er die Erzählung von H. G. Wells „The new accelerator" gelesen, so wird er vielleicht auf den Gedanken kommen, daß die „wirkliche" Zeit „immer schneller gehe", und zwar proportional mit t, so daß die „entsprechenden Zeitintervalle" im selben Verhältnis immer kürzer werden. [Wenn sich die gemessene Schwingungsdauer ϑ größer erweist als man erwartet hat, so könnte dies ja in der Tat durch entsprechende Verringerung der Zeiteinheit erklärt werden. Wenn sich aber die Zeiteinheit verringert, so muß sich im selben Verhältnis auch jeder beliebige Teil der Zeiteinheit: $\Delta t = \frac{\text{Zeiteinheit}}{n}$ verringern, also auch $\lim (\Delta t) = dt$.] Wir können daher schreiben:

$$d\tau = K \frac{dt}{t}, \tag{16}$$

wo K den Proportionalitätsfaktor bedeutet, und τ eine neue Variable. Letztere kann man als eine besondere Art von „Zeit" auffassen, deren Bedeutung durch (16) definiert ist. Für die Gegenwart stimmen die Zeiten τ und t überein, so daß man schreiben kann:

$$\left.\begin{aligned} \tau_0 &= t_0 \\ (d\tau)_0 &= (dt)_0. \end{aligned}\right\} \tag{17}$$

In der Zukunft und in der Vergangenheit gehen aber τ und t auseinander. Aus (16) erhalten wir:

$$(d\tau)_0 = K \frac{(dt)_0}{t_0},$$

also nach (17):

$$K = t_0,$$

weshalb wir statt (16) schreiben können:

$$d\tau = t_0 \frac{dt}{t}. \tag{18}$$

Die Intergration von (18) ergibt:

$$\int_{\tau_0}^{\tau} d\tau = \int_{t_0}^{t} t_0 \frac{dt}{t},$$

oder:

$$\tau - \tau_0 = t_0 \log \left(\frac{t}{t_0}\right),$$

oder im Hinblick auf (17):

$$\tau = t_0 \log \left(\frac{t}{t_0}\right) + t_0. \tag{19}$$

Eine solche Gleichung ist auch von Milne abgeleitet, aber anders interpretiert worden [102]).

Zum Schluß mögen drei Druckfehler berichtigt werden. Auf S. 17, Fußnote [39]) muß 264 statt 263 stehen. Auf S. 25, 7. Zeile von unten „. . . . Felde.) Ist . . . .“ statt „. . . Felde. Ist . . .“. Auf S. 29, Fußnote [79]) 430 statt 335.

Herrn Professor E. A. Milne, der mich auf einige von mir übersehene Fakta aufmerksam gemacht hat, sei hier mein aufrichtigster Dank ausgesprochen.

[102]) E. A. Milne, Proc. Roy. Soc. London (A) **158**, 327, 1937, und andere Stellen.

FROM THE DEPARTMENT OF SURGERY (PROFESSOR KARELL) OF TARTU UNIVERSITY, ESTONIA

TUBE FLAP GRAFTING

BY

U. KARELL, M. D.

TARTU 1937

Printed by K. Mattiesen, Ltd., Tartu 1937.

In injuries associated with extensive defects and lesions of the skin, our first task should be to take care of the life of the patient; the reconstruction of tissue defects is of second-rate importance. There is nothing to object to this rule except, perhaps, that more attention should be paid also to the second task if it does not interfere with the first. That is, if it is not dangerous for the life of the patient, we should begin earlier with measures helping the reconstruction of lost tissues. Because, if we examine more closely how the reconstruction of tissue defects is going on, even if we succeed in preventing infection, we must confess that the natural healing process is not at all a perfect one. If we do not hasten the healing of skin defects by grafting, they will fill themselves with granulation tissue and, in the best event, also epithelize during a more or less prolonged time. But such an epithelium covers a hard, stiff, poorly vascularised scar tissue, is thin and immovable on its base, and breaks easily from the least bruise. Besides, owing to the transformation of the granulations into the cicatrix tissue, and the contraction of the same if located on extremities and in the neighbourhood of joints, contractures, and even subluxations, with all their sequelae will follow. Appearances of this kind are not infrequent with patients treated even in well-furnished hospitals, where every care has been taken for their primary lesion but no attempt made to cover the defect with skin grafts.

Large defects of the skin may follow scalds and burns, trauma or infection. In burns the depth of the wound varies. If the epithelium is destroyed in all its thickness there will be, after the sloughing of the coagulum, a granulating surface of the wound. In more superficial burns the epithelium of profound tissues, such as hair follicles, sweat ducts, and sebaceous glands, may be spared to form islets from which the new epithelium will spread and cover the surface of the granulation tissue. In deeper burns the subcutaneous tissue is also destroyed. From these wounds, if no infection follows, the coagulum will separate in the third or fourth week and the wound will be covered with healthy granulations. In case of avulsion of the skin, the deeper layers, such as muscles, fasciae, vessels, nerves and bones, may be uncovered. Such a wound is often primarily infected and a great deal of the tissue so much injured that it will slough. The granulation tissue follows rapidly on muscles, but the bone, cartilage and ligaments usually necrotize if exposed, and if infection is present their healing is slow.

Severe injuries of the extremities are sometimes also followed by large areas of gangrenized skin. The infection of these parts can be prevented by treating them in hot air boxes. In this case the gangrene will dry and resembles the eschar of burns, and will finally separate, although it takes much more time. But much more prolonged is the healing of the sloughing of the skin following infection. In this case, after the separating of the necrotized skin, the aponeurosis, tendons, cartilages and fasciae will be exposed and may slough in their turn, and the healing by granulation, especially in chronic infections, may take years.

Two chief moments may be considered in the process of the healing of an open wound: first from the fibrous cells of the denuded wound-surface the granulations spring and fill the defect and, secondly, from the edges of the wound the epithelium which covers the surface of the granulations develops.

After they have filled the defect of the wound, the granulations will again transform in the connective tissue from which they derived by losing their capillaries and succulent appearance. This process also causes the contraction of the whole area of the wound up to one-third of its size. The larger the wound, the greater will be the contraction of the scar. The amount of the newly-formed scar or connective tissue depends on the amount of the granulations. If, for some reason, the healing is delayed, as in the centre of a large wound, the covering granulations will thicken, causing in their turn the increase of scar tissue. But the formation of granulations is limited. As seen in old ulcerations the granulations lose their former red colour, and become pale, transparent and atrophic. This appearance is caused by the deeper layers becoming contracted and the vascularisation poorer. Such a tissue is also a bad base for healthy epithelization. So we see that the delay in the development of one of the chief moments in the healing of the wound, the retarded formation of granulations, causes the delay of the epithelization, and *vice versa*, if the epithelization is delayed, the contraction of the wound will take the upper hand and avascular granulation tissue will follow, which is a bad base for the good growth of the epithelium. The vicious circle can be broken only by the artificial hastening of the process of epithelization before the contraction of the wound is accomplished and the vascularisation of the wound-surface diminished. In creating, in the early stages of the process of the healing of the wound, an even and tight network of islets of epithelium on its surface, or covering it totally with grafts of the full thickness of the skin, we hasten the process of epithelization and at the same time considerably prevent the unnecessary formation of surplus scar tissue and the annoying contraction of the wound. In other words, by treating the large wounds this way, the contraction of the wound, which is estimated by Carrel & Hartman* as the most important moment in the cicatrization, lessens, and the epithelization, which, in natural healing of the wound, takes place only at the final stage of the contraction, becomes a more active factor. Splinting, traction and other mechanical means to prevent contractures without skin grafting are quite useless, because they do not prevent the formation of the excessive granulation tissue from which the scar develops. The immediate plastic covering of the wound is of course the ideal treatment from every standpoint. In lacerated wounds of the hand Beekman & O'Connel** recommend, as an ideal procedure, the immediate application of a pedicle flap, as the subcutaneous tissue of the flap prevents the scar from adhering to the bones and tendons, and prevents the formation of the scar itself. Unfortunately, this is not always possible, because most wounds which are the result of trauma or burns are infected primarily, and we must wait till the separation of the sloughs or eschars, or the subsidence of the acute infection, before a plastic operation can be made. Only then can we be sure that the grafts will take.

* Carrel, Alexis & Hartman, Alice: Cicatrization of Wounds. The Relation between the Size of the Wound and the Rate of its Cicatrization, Journ. Exper. Med. Vol. XXIV, p. 429, 1916.

** Beekman, F. & O'Connell, R. J.: The Healing of Surface Wounds for the Prevention of Deformities. Ann. Surg. Vol. 98, 1933.

In burns treated by tannic acid the eschar will separate at the end of the fourth week, leaving a healthy, vital, even surface of granulation tissue, an ideal ground for the growth of grafts. In avulsions where the subcutaneous tissue is lost and infection present, the muscles will soon be covered with granulations ready for grafting, but the sloughing of other neighbouring tissues will spoil the taking. Here we must wait several months before all the surface is covered by healthy granulations. For hastening the separation of sloughs as well as the production of good granulation tissue and the control of the infection, Löhr's codliver-oil ointment method proved of great value to us. The grafting must be made immediately after all sloughs have been separated. The grafts will not take if there are still fistulae or deep nests of infection. But the secretion from healthy granulation does not interfere with the taking.

For covering large surfaces it is preferable to use little grafts of full thickness, not larger than 0,5 cm in diameter. Such islets put tightly all over the surface will give a strong epidermis which does not hornify easily, and, as its base is sufficiently vascularized, will form a normally smooth and pliable skin. The grafts will acclimatize before the formation of contracting scar tissue.

For the reconstruction of deeper and smaller defects the use of the tube flap method is indispensable. As so many important inventions have become the common property of everybody, without anybody's remembering the author, it is unknown who used this method first. During the world war the London surgeon Gillies elaborated the tube flap method. By this method it is possible to transplant from a distance about 40 cm away grafts sized 8 × 40 cm and even more to a pedicle. The principle of the method is that a strap of the skin, with the subcutaneous tissue, about 8 cm in width and up to 40 cm in length, is built with both its ends remaining in connection with the body. Both the edges of the strap must be sewed together immediately, forming a tube with the skin surface on the outside. Both the edges of the remaining wound beneath the tube are also undermined and stitched together. Owing to the fact that the tube remains for some time with both its ends in organic connection with the organism, its vascularisation is good and no contraction will develop in its inside. One end of the tube with connection flap can be lifted after some time and used for transplantation purposes. We can develop the vascularisation of the tube voluntarily through one of its bases by clamping the other one temporarily. To be sure of the taking of the flap to be transplanted we must dissect it completely away from the underlying fascia after three weeks aproximately and, after careful hemostasis, replace it and approximate with interrupted dermal sutures. The purpose of this procedure is to permit the formation of new bloodvessels through the other pedicle before the complete severance of the distal end. If after that there occurs a necrosis of the edge of the distal flap, we can be sure that the remaining part will be surely vascularised through the proximal pedicle and will take totally also in its new bed. This next stage can be performed about 2—3 weeks later.

The following case is instructive, not only from the standpoint of the use of this method, but also of the other rules and requirements of plastic surgery.

Patient Karl K., newspaper reporter, 24 years old, 12 years ago had had lupus of the face, which destroyed the top of his nose with *alae nasi* and the *filtrum* of the upper lip. With gold therapy and x-rays the spread of the disease was stopped. An attempt was made in another hospital one year ago to sew the defect, but the refreshed edges of the defect necrotised and, as the patient states when admitted to the I Surgical Clinic, the defect became larger than before the first operation. As seen in Fig. 1 it extended over two-thirds of the lip. The defect reached the gums in the middle of the lip, and the patient could not close the mouth totally and had to wear a bandage. Because the neighbourhood of the defect was cicatrized, no sliding flap came into consideration.

We had the intention to use one and the same graft also for the building of a new top to the nose and nostrils as well as the lip. The patient agreed and the tube flap method was chosen. Accordingly, on the 5. V. 34. an S-shaped strap from the skin of the breast was built with its proximal pedicle above the left, and distal below, the right mammary gland. Stitching the edges of the strap, we got a good mobile tube and succeeded easily in stitching the S-shape defect. The tension of the last stitches was eased by making a lot of 2 mm long punctures with the point of the scalpel on both sides of and parallel to the suture. The last method proved useful to us in several cases of sliding flap applications. The length of the tube bridge was 25 cm, the width or circumference 8 cm. The healing of both sutures was *per primam*, except two little triangles on both ends of the S-shape suture, where no parallel punctures had been made and where the silk

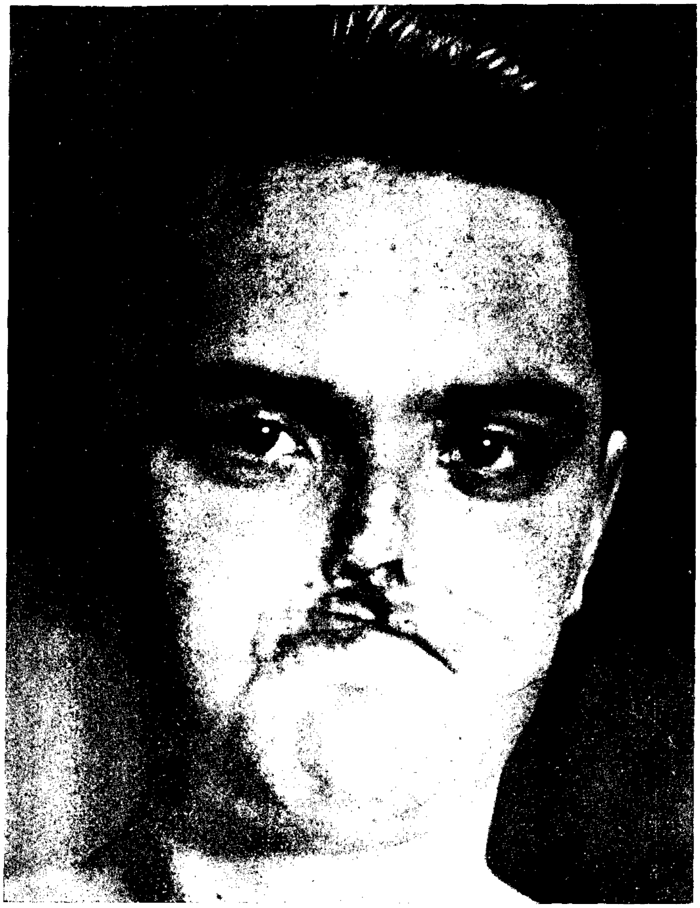

Fig. 1.

stitches had cut through the skin. All the stitches were removed on the 15-th May. The patient temporarily left the clinic and was told to come back when the both granulating triangles had been epithelialized. He was also recommended to pull at the bridge and twist it at home. He came again on the 25. VI. Although the patient had been warned to pull on the tube the same proved shortened and measured now only 20 cm and the circumference scarcely 7 cm. By clamping the distal pedicle the viation proved sufficient through the proximal one. As a doubt arose that the

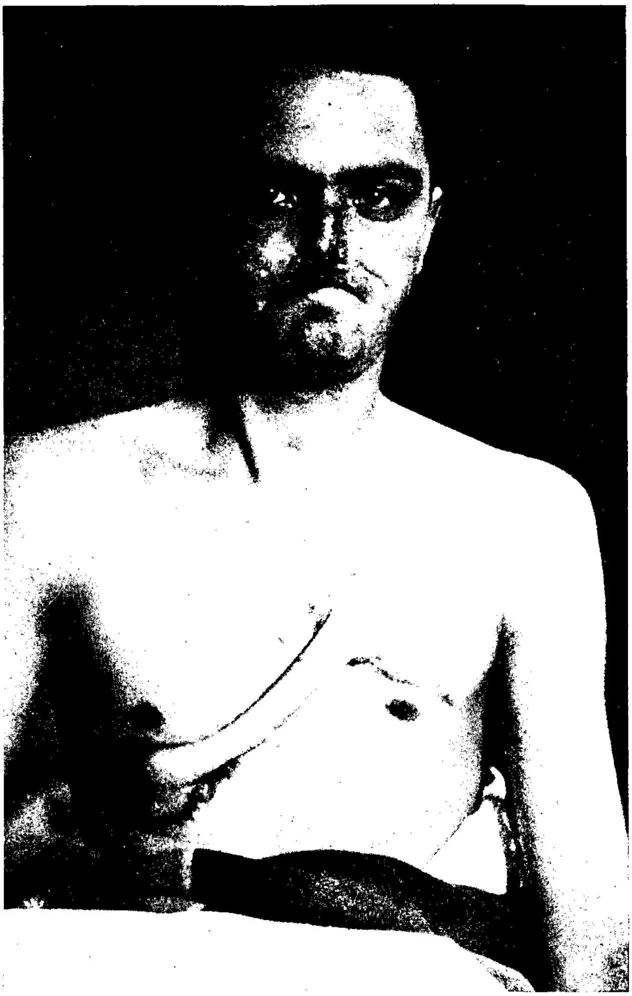

Fig. 2.

shortened tube would reach the nose freely, a round flap about 8 cm in diam. with the distal pedicle was completely dissected away from the underlying fascia, its proximal part turned into a tube — to lengthen the same — and, after careful hemostasis, replaced, and the skin edges approximated with interrupted dermal sutures. In the lateral upper part of the flap little necrosis followed. After removing the stitches on the 8-th day the patient was sent home and told to twist and pull the proximal pedicle.

On 6. X. 34. the defect of the necrotized part of the flap was cured so far that one could now transplant the original raised

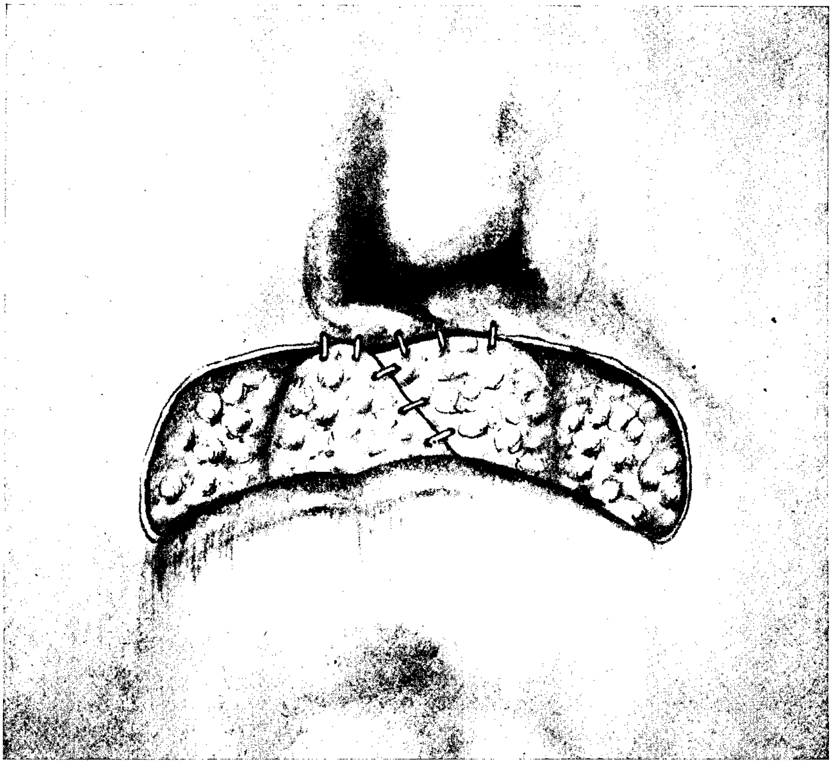

Fig. 3.

flap to its final bed in place of the upper lip (Fig. 2). The patient's back and the back of his head were covered with plaster-of-paris, so as to hold his head firm and keep it bowed down in order to prevent any tension on the tube graft. As the patient had no lip at all in the midline, it was necessary at first to build a base for the transported flap here. For that purpose two wing-shaped flaps about 2 cm in width and 3 cm in length were cut from both sides of the defect with their tops directed to the cheeks. The wings were turned with their raw surface outside and sutured in the midline with interrupted stitches knotted towards the gum. The upper edges were sutured to the refreshed gum just below the septum (Fig. 3). Now the flap of the distal pedicle was punctured with the point of a scalpel and a couple of leeches applied to the bleeding points, the flap ex-

Fig. 4.

cised and transported carefully with the leeches hanging to the newly formed raw bed and sutured here with separate skin sutures (Fig. 4).

No bandage was applied to the transported flap in order to avoid any pressure which might disturb the circulation. The sutures were powdered with calomel. The plaster-of-paris bed proved very inconvenient for the patient in a half-sitting position and it was replaced by pillows. After two weeks the stitches were removed and the wound healed *per primam*. Only in the middle of the upper edge just beyond the *septum nasi* was there a little suppuration, resulting in a fistula (Fig. 5).

On the 27. X. 34. the IV stage could be performed. It was meant only as a temporary lifting of the proximal pedicle of the graft from its place in order to gain material for building the new top of the nose. But the patient now refused categorically owing to economical considerations. So we had to become re-

Fig. 5.

conciled with sadness to the forming of the lip only. The tube was clamped at the left corner of the mouth and, as no change in the colour of the transplant of the lip followed, it was cut through. It needed no anaesthetics, only a faint pain was felt in the tube. The resulting wound was sutured with fine interrupted silk stitches. Although the lip was originally very thick, no attempt was made to make it thinner expecting that the subcutaneous tissue would atrophy in time.

On 3. XI. 34. the edges of the fistula below the septum were refreshed and sutured. The healing followed by granulation. As seen in Fig. 6, the lip is still thick with shining and extended skin.

On the 15. II. 35. the patient presented himself again with complaints that the mouth had become smaller and he could not eat except with a tea-spoon and that the dentist had refused to cure his carious teeth until his mouth had been sufficiently widened. The lip proved thinner and shorter. The external sutures

Fig. 6.

were well cicatrized, the temperature of the skin did not differ from that of the cheeks. Only in the midline the lip was about 1,5 cm thick and a lump of hard tissue protruded into the defect of both incisors. The sensibility to pain in the flap was normal and for the correcture operation local anaesthetics were necessary. An incision was made along the lower border of the lip and the disturbing scar tissue excised. As the inside of the lip was shortened so the opening of the mouth was lengthened towards the left side and a vertical incision was made in the mucosa here and sutured horizontally. A ribbon from the skin of the lip was cut away and the midline between the two frontally dissected parts of the lip burnt with Paquelin in order to get an excavation imitating the filtrum of the lip. The edges of the wound were sutured with interrupted silk stitches. At

Fig. 7.

the same time according to the wish of the patient, the stump of the soft proximal tube pedicle was removed.

In autumn 1936 the patient presented himself at the clinic again. He is now very pleased with the result. The lip is now flat and not at all protruding, sufficiently elastic, and not only allows the opening of the mouth freely but also change of expression. Only under the influence of the summer sun the pig mentation of the lip is more intensive than in other parts of the face (Fig. 7).

This case proves that even in such bad conditions as a base cicatrized by lupus and x-ray treatment good results may be obtained by tube-flap grafting.

Summary.

The natural healing process of great skin defects is not at all a perfect one. Because of the contraction of the deeper parts of the granulations, the epithelization is always delayed and contractions of the scar with bad skin covering result. To avoid this, early covering of the granulations by grafting is emphasized.

Löhr's cod-liver oil procedure hastens the sloughing and makes good healthy granulations for full-thickness skin-grafting. Tube-flap grafting for covering large skin and subcutaneous tissue defects is emphasized. In a case with a large defect of the upper lip after x-ray treatment of lupus, tube-flap-grafting was made.

To ensure the taking of the pedicle the author recommends making with the point of the scalpel a trellis-work of punctures to the depth of ¼ cm along the edges of the flap and the application of leeches to maintain the bleeding.

CHANGE OF CLIMATE IN THE NORTHERN HEMISPHERE

BY

K. KIRDE

TARTU 1938

Printed by C. Mattiesen, Inc., Tartu (Estonia), 1938.

An attempt is made in this paper to analyze the variations of climate in the Northern Hemisphere. For this purpose we have used the temperature observations of meteorological stations whose period of observation extends at least from 45 to 50 years beginning with 1860—1870. In Table 1 are given the names of the observation points with their geographic coordinates and length of their period of observation.

In order to give a better survey the distribution of the meteorological stations is shown in Fig. 1.

The variations of temperature have been deduced from the average monthly temperatures given in the Meteorological Year Books. It must be noted that the average monthly temperatures of the North American Meteorological Stations have been calculated from the daily max. and min. temperatures. The whole period of the observations of each station is divided into two halves and the average temperature for each month has been reckoned for each half-period separately. On the basis of the abovementioned average temperatures for both half-periods we are able to determine the variation of the monthly mean temperature for the whole period with its mean error, which has been reckoned by means of the term

$$\varepsilon = \sqrt{\frac{\sum_{i=1}^{n} e_i^2 + \sum_{j=n+1}^{2n} e_j^2}{n(n-1)}}$$

where e_i denotes the deviations of the monthly mean temperatures from the average for the first half period, e_j — the corresponding deviations from the average temperature of the second half period, and n — the length of the half period in years. Table 2 gives the determined variations of temperature (Δt) with their mean error (ε) for January.

The temperature variations for the North American Stations have been determined separately from the average

Table 1.

Name	Latitude	Longitude	Years	Name	Latitude	Longitude	Years
Tartu	58° 23′ N	26° 43′ E	1866—1930	Bogoslovsk	59° 45′ N	60° 01′ E	1871—1930
Helsingfors	60° 10′ N	24° 57′ E	1881—1934	Barnaoul	53° 20′ N	83° 48′ E	1871—1930
Oulu	65° 01′ N	25° 28′ E	1881—1933	Touroukhansk	65° 55′ N	87° 38′ E	1878—1930
Haparanda	65° 50′ N	24° 09′ E	1873—1932	Verkhoiansk	67° 33′ N	133° 24′ E	1884—1920
Stockholm	59° 21′ N	18° 04′ E	1873—1932	Thorshavn	62° 02′ N	6° 45′ W	1874—1924
Vardö	70° 22′ N	31° 06′ E	1868—1934	Grimsey	66° 33′ N	17° 58′ W	1875—1934
Bergen	60° 24′ N	5° 19′ E	1861—1934	Stykkisholm	65° 05′ N	22° 46′ W	1874—1934
Copenhagen	55° 41′ N	12° 33′ E	1874—1933	Ivigtut	61° 12′ N	48° 10′ W	1875—1932
Königsberg	54° 44′ N	20° 34′ E	1870—1932	Jacobshavn	69° 13′ N	51° 02′ W	1874—1932
Hamburg	53° 38′ N	10° 00′ E	1876—1932	Upernivik	72° 47′ N	56° 07′ W	1874—1932
Munich	48° 09′ N	11° 34′ E	1879—1933	Toronto	43° 40′ N	79° 24′ W	1841—1930
Aberdeen	57° 10′ N	2° 06′ W	1869—1934	Buffalo	42° 53′ N	78° 50′ W	1885—1934
Valentia	51° 56′ N	10° 15′ W	1869—1934	Detroit	42° 21′ N	83° 03′ W	1885—1934
San Fernando	3 ° 28′ N	6° 12′ W	1870—1932	Chicago	41° 47′ N	87° 35′ W	1885—1934
Ponta Delgada . . .	37° 44′ N	25° 40′ W	1867—1934	Savannah	32° 04′ N	81° 08′ W	1885—1934
Vienna	48° 15′ N	16° 22′ E	1864—1929	New Orleans	29° 57′ N	90° 04′ W	1885—1934
Graz	47° 04′ N	15° 28′ E	1886—1929	Bismarck	46° 47′ N	100° 38′ W	1885—1934
Budapest	47° 31′ N	19° 01′ E	1870—1933	Portland	45° 32′ N	122° 41′ W	1885—1934
Milan	45° 27′ N	9° 15′ E	1851—1933	San Francisco . . .	37° 48′ N	122° 26′ W	1885—1934
Prague	50° 05′ N	14° 25′ E	1865—1932	San Diego	32° 43′ N	117° 10′ W	1885—1934
Warsaw	52° 13′ N	21° 02′ E	1870—1931	Bombay	18° 54′ N	72° 49′ E	1847—1933
Moscow	55° 50′ N	37° 33′ E	1879—1930	Batavia	6° 11′ S	106° 50′ E	1866—1930
Sverdlovsk	56° 50′ N	60° 38′ E	1871—1930				

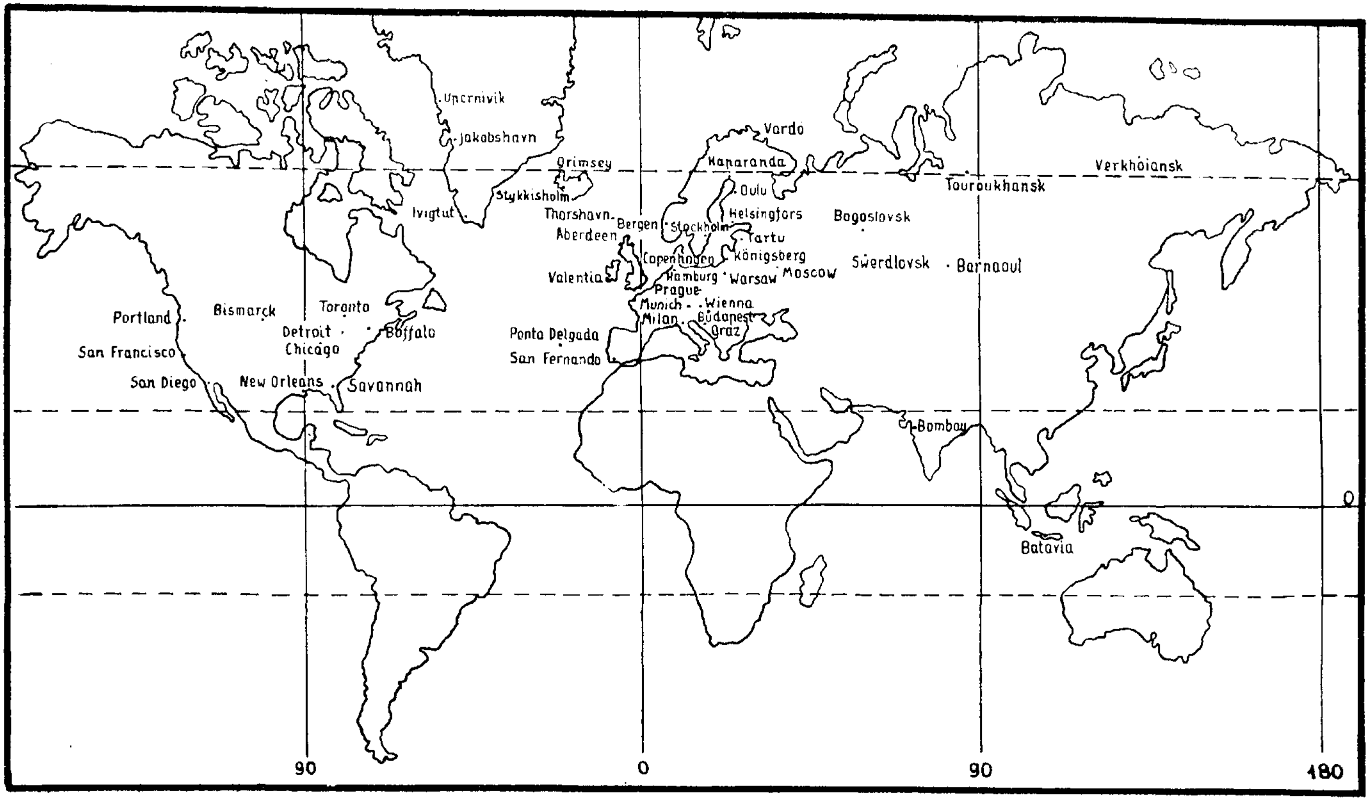

Fig. 1. Distribution of Meteorological Stations.

Table 2.

Name	Δt	ε	Name	Δt	ε
Tartu	0.7	0.5	Bogoslovsk . . .	0.7	0.7
Helsingfors . . .	0.6	0.7	Barnaoul	2.7	0.6
Oulu	0.3		Touroukhansk . .	4.0	0.6
Haparanda . . .	1.6	0.9	Verkhoiansk . .	4.2	1.0
Stockholm	1.2	0.6	Thorshavn . . .	0.1	
Vardö	1.4	0.5	Grimsey	0.9	0.7
Bergen	2.5	0.5	Stykkisholm . . .	1.5	0.6
Copenhagen . . .	1.9	0.5	Ivigtut	1.4	0.7
Königsberg . . .	1.3	0.7	Jacobshavn . . .	1.6	1.3
Hamburg	2.6	0.6	Upernivik	1.7	1.3
Munich	4.4	0.7	Toronto	−0.1	
Aberdeen	1.2	0.3	Buffalo	0.8	0.8
Valentia	0.7	0.3	Detroit	1.1	0.8
San Fernando . .	0.1		Chicago	2.6	1.0
Ponta Delgada . .	0.5	0.2	Savannah	2.3	0.6
Vienna	3.4	0.6	New Orleans . . .	2.0	0.6
Graz	3.1	0.7	Bismarck	3.6	1.4
Budapest	3.5	0.6	Portland	1.2	0.6
Milan	2.6	0.4	San Francisco . .	−0.2	0.4
Prague	2.0	0.6	San Diego	0.2	0.4
Warsaw	2.3	0.7	Bombay	0.4	0.2
Moscow	2.1	1.0	Batavia	0.9	0.1
Sverdlovsk . . .	2.6	0.8			

max. and min. temperatures. In Table 2 as well as in the following tables we find the arithmetical means of the variations determined from the average max. and min. temperatures.

The variations of temperatures for January given in Table 2 are graphically represented in Fig. 2. The regions where the rise in temperature surpasses its double mean error are marked with thin lines, whereas the dots denote the regions where the fall of temperature surpasses its double mean error. The regions where the variation of temperature surpasses its fourfold mean error are marked with thick lines or corresponding thick dots. As seen from the map a considerable rise in temperature is observed in West Europe and North-East Asia. The British Islands, Spain, the Azores, Iceland, Greenland, the

Western Coast of North America as well as the West Indies show but a slight rise in temperature.

Temperature variations can also be determined by means of frequency-curves[1]). The calculation of temperature frequencies being very troublesome the latter have only been reckoned for January for Hamburg (Fig. 3) and Vienna (Fig. 4).

In compiling the distribution of temperature the three daily observations have equally been taken into consideration. One degree C. was taken as a unit for the division of temperatures into groups. In determining the climatic changes the frequencies of air temperature have been composed for both the abovementioned half-periods and expressed in a percentage of the whole number of observations for each period. Both temperature distributions are graphically represented by broken lines. The solid line denotes the frequency for the first half-period, the dotted line — for the second. The mean error of frequency for the first half-period has been calculated by means of the formula

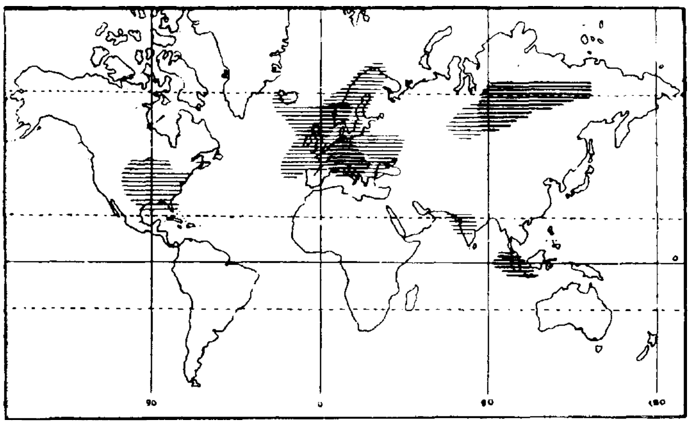

Fig. 2. Variations of Temperatures for January.

$$\sigma = 100 \sqrt{\frac{pq}{S}}$$

where S denotes the whole number of observations, p — the probability that an observation belongs to the i group, q —

1) K. Kirde, Meteorological Elements characterized by Frequency-Curves. Scientific Papers of the Meteorological Observatory of the University of Tartu № 1.

the reverse probability ($q = 1 - p$). In order to obtain a clearer survey of the difference in the distribution of temperatures in both half-periods a four σ broad dotted stripe has been drawn in the figures. We see that the line of frequencies

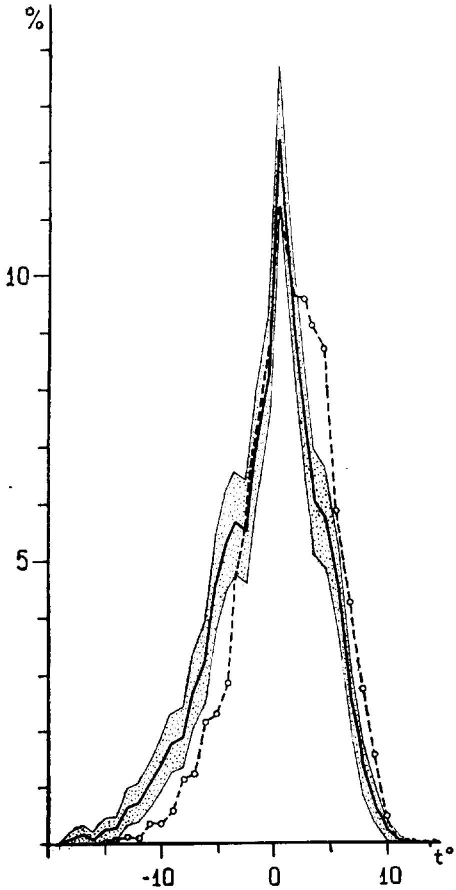

Fig. 3. Frequency of Temperatures for Hamburg in January.

for the second period lies several times beyond the four σ stripe.

These outlying parts are marked by circles in the graphs. We know from the theory of probabilities that 95 % of all the cases must keep within the limits of the four σ and only 5 % of all the cases may lie outside provided that the outward conditions remain unaltered during each trial. As seen from Fig. 3 the distribution of temperature for Hamburg is divided into 33 different groups. The frequencies of the second period can lie outside the four σ but twice. In reality we see

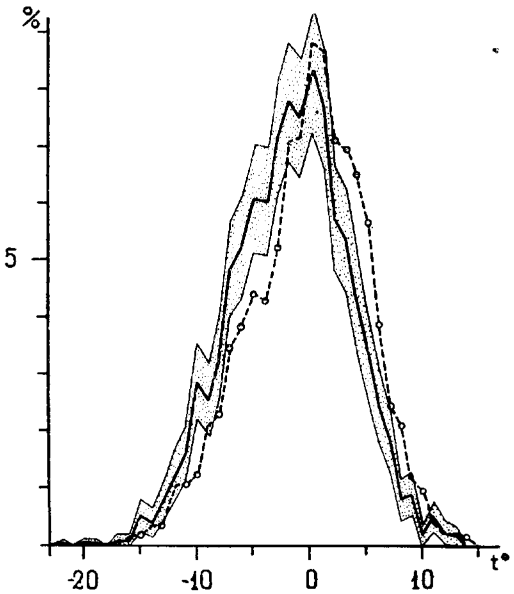

Fig. 4. Frequency of Temperatures for Vienna in January.

that the frequencies of temperature for the second half-period lie 19 times outside the limits of the four σ.

By means of Pearson's term

$$P = \frac{\int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2} \chi^{n-1} \, d\chi}{\int_{0}^{\infty} e^{-\frac{1}{2}\chi^2} \chi^{n-1} \, d\chi}$$

we can easily find the possibility that such a difference in the distribution of temperatures for both half-periods has taken place accidentally without any extraneous influence. In the above term e is the basis of the natural logarithms, n — the number of groups less one,

$$\chi = \sum \frac{d^2}{\mu}$$

where d is the difference between the frequencies for each group and μ the frequency for the corresponding group of the first period. We obtain for the required probability for Hamburg:

$$P = 6.10^{-71}$$

which means that such a difference can take place accidentally only once in a period of 5.10^{71} years. For Vienna the corresponding probability $P = 3.10^{-58}$ and the number of years — 10^{59}.

The variations of temperature for the other months have been calculated in the same way. Table 3 gives the variations of temperature for February.

We see in Fig. 5 a considerable rise of temperature in Iceland, Middle North America, and Java. A fall of temperature is observed in Thorshavn, Ireland, Spain, Eastern

Table 3.

Name	Δt	ε	Name	Δt	ε
Tartu	0.3	0.7	Bogoslovsk . . .	1.1	0.9
Helsingfors . . .	0.6	0.8	Barnaoul	2.4	0.8
Oulu	0.8	1.0	Touroukhansk . .	−1.1	0.7
Haparanda . . .	1.9	1.0	Verkhoiansk . . .	0.0	
Stockholm	2.2	0.7	Thorshavn	−0.6	0.3
Vardö	0.9	0.5	Grimsey	2.8	0.6
Bergen	1.5	0.4	Stykkisholm . . .	2.4	0.6
Copenhagen . . .	1.1	0.5	Ivigtut	0.8	0.9
Königsberg . . .	0.4		Jacobshavn . . .	2.7	1.5
Hamburg	0.4		Upernivik	2.8	1.2
Munich	0.9	0.8	Toronto	−0.6	0.4
Aberdeen	0.7	0.4	Buffalo	0.8	0.8
Valentia	−0.8	0.4	Detroit	2.3	0.8
San Fernando . .	−0.3		Chicago	4.7	0.8
Ponta Delgada . .	0.0		Savannah	2.0	0.6
Vienna	1.2	0.7	New Orleans . .	2.4	0.8
Graz	1.3	0.7	Bismarck	7.2	1.0
Budapest	2.4	0.6	Portland	1.6	0.6
Milan	0.7	0.5	San Francisco . .	1.4	0.4
Prague	0.1		San Diego	0.5	0.4
Warsaw	1.3	0.8	Bombay	−0.6	0.2
Moscow	−2.5	0.9	Batavia	0.7	0.1
Sverdlovsk . . .	−0.2	0.8			

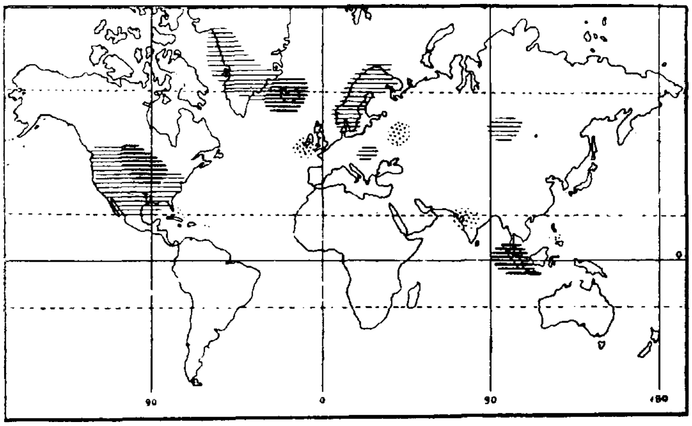

Fig. 5. Variations of Temperatures in February.

Europe, and India. The fall in the average temperature surpasses its fourfold mean error only in Moscow.

The frequency curve of temperature for February has been composed for Stockholm (see Fig. 6).

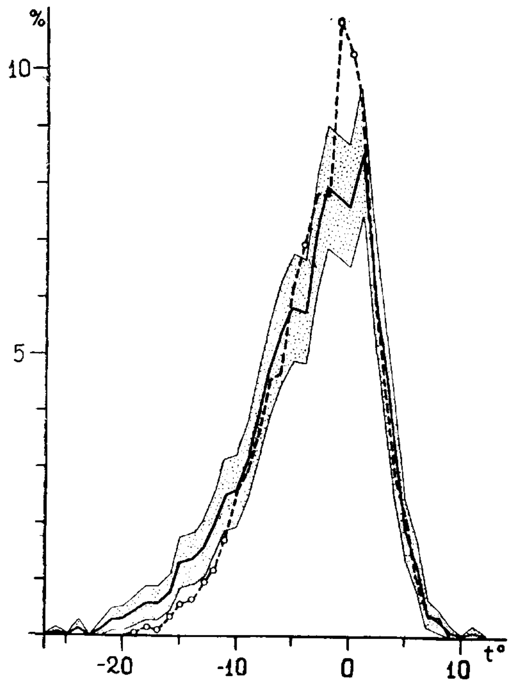

Fig. 6. Frequency of Temperatures for Stockholm in February.

The calculation shows that such a fall in low temperatures can occur accidentally only once in a period of 4.10^{22} years.

As regards the variations of temperature in March, we see from Fig. 7 (Table 4) that a rise of temperature took place in North America (except the Eastern Coast), in Greenland, Iceland, Europe (except the British Islands), and in Java. The greatest rise is observed in Greenland where it surpasses its mean error 6 times. A considerable fall of temperature (-4.7^0) was observed only in Siberia.

Table 4.

Name	Δt	ε	Name	Δt	ε
Tartu	1.2	0.5	Bogoslovsk	0.0	
Helsingfors	1.8	0.7	Barnaoul	0.0	
Oulu	2.0	0.7	Touroukhansk	−4.7	0.6
Haparanda	3.7	0.7	Verkhoiansk	3.7	1.2
Stockholm	2.2	0.6	Thorshavn	0.3	
Vardö	1.2	0.4	Grimsey	4.3	0.7
Bergen	2.1	0.4	Stykkisholm	3.2	0.6
Copenhagen	1.5	0.2	Ivigtut	1.4	0.8
Königsberg	2.0	0.6	Jacobshavn	6.2	1.2
Hamburg	1.6	0.5	Upernivik	5.1	1.1
Munich	1.9	0.6	Toronto	2.0	0.6
Aberdeen	0.6	0.4	Buffalo	0.1	
Valentia	−0.1		Detroit	1.2	0.8
San Fernando	0.0		Chicago	2.1	0.8
Ponta Delgada	−0.4	0.2	Savannah	−0.8	0.6
Vienna	2.2	0.5	New Orleans	−0.8	0.6
Graz	2.1	0.5	Bismarck	5.4	1.2
Budapest	2.8	0.5	Portland	1.4	0.6
Milan	1.6	0.3	San Francisco	2.6	0.4
Prague	2.2	0.5	San Diego	1.6	0.4
Warsaw	2.0	0.6	Bombay	−0.3	0.2
Moscow	1.4	0.7	Batavia	0.6	0.1
Sverdlovsk	0.3	0.7			

The frequency curve (Fig. 8) represents the variations of temperature at Jacobshavn, where a marked warming has been registered.

The reckoning of the corresponding probability ($P = 10^{-54}$) gives for the accidental occurrence of such warming a period of 3.10^{55} years.

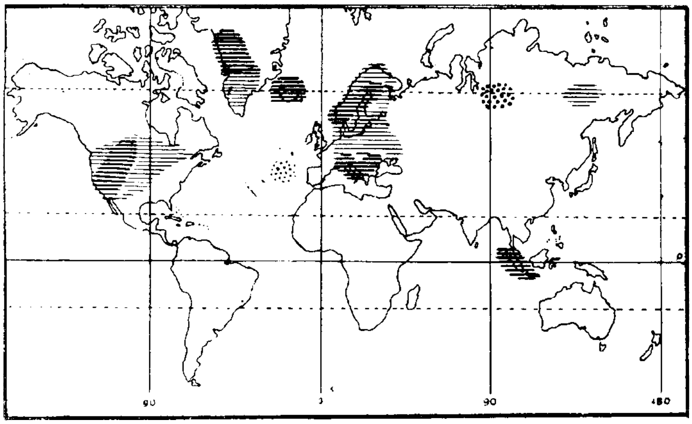

Fig. 7. Variations of Temperatures in March.

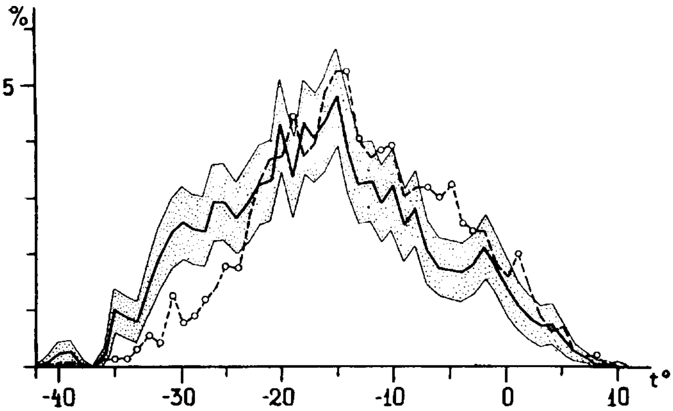

Fig. 8. Frequency of Temperatures for Jacobshavn in March.

The data of the variations of temperature in April differ considerably from those in March (Table 5).

Table 5.

Name	Δt	ε	Name	Δt	ε
Tartu	1.2	0.4	Bogoslovsk . . .	3.7	0.6
Helsingfors . . .	1.2	0.4	Barnaoul	1.4	0.7
Oulu	—0.3		Touroukhansk . .	2.1	0.3
Haparanda . . .	2.0	0.4	Verkhoiansk . . .	2.4	1.1

Name	Δt	ε	Name	Δt	ε
Stockholm	1.2	0.4	Thorshavn	−1.6	0.3
Vardö	0.4	0.4	Grimsey	1.4	0.5
Bergen	0.4		Stykkisholm	0.6	0.5
Copenhagen	1.2	0.3	Ivigtut	0.5	
Königsberg	1.7	0.5	Jacobshavn	3.1	1.0
Hamburg	0.3		Upernivik	2.1	0.8
Munich	0.5	0.4	Toronto	2.4	0.4
Aberdeen	−0.4		Buffalo	−0.6	0.6
Valentia	−1.5	0.2	Detroit	0.4	
San Fernando	0.0		Chicago	2.0	0.6
Ponta Delgada	−1.5	0.2	Savannah	1.3	0.4
Vienna	−0.2		New Orleans	0.8	0.4
Graz	1.0	0.4	Bismarck	0.7	0.8
Budapest	−0.1		Portland	1.2	0.6
Milan	0.2		San Francisco	1.3	0.4
Prague	0.4	0.4	San Diego	1.0	0.4
Warsaw	1.1	0.5	Bombay	0.4	0.2
Moscow	2.5	0.6	Batavia	0.7	0.1
Sverdlovsk	3.7	0.8			

Vast regions of a rise in temperature spread over North-East Europe, North Asia, Java, and Greenland (Fig. 9). The region of decrease takes the form of a stripe stretching from the Azores across the British Islands to the Faroe Isles. In West Europe we observe only a slight warming. The frequency curve of

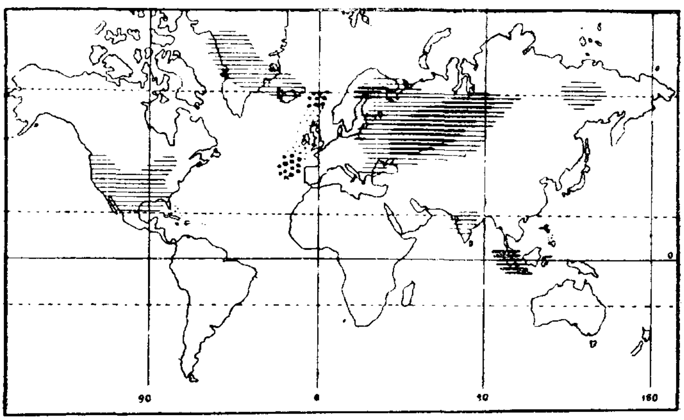

Fig. 9. Variations of Temperatures for April.

Haparanda (Fig. 10) shows a rise of high — and a decrease of low temperatures.

The probability of such a variation is only 2.10^{-24} which shows that the accidental occurrence of such a variation is possible only once in a period of 10^{25} years.

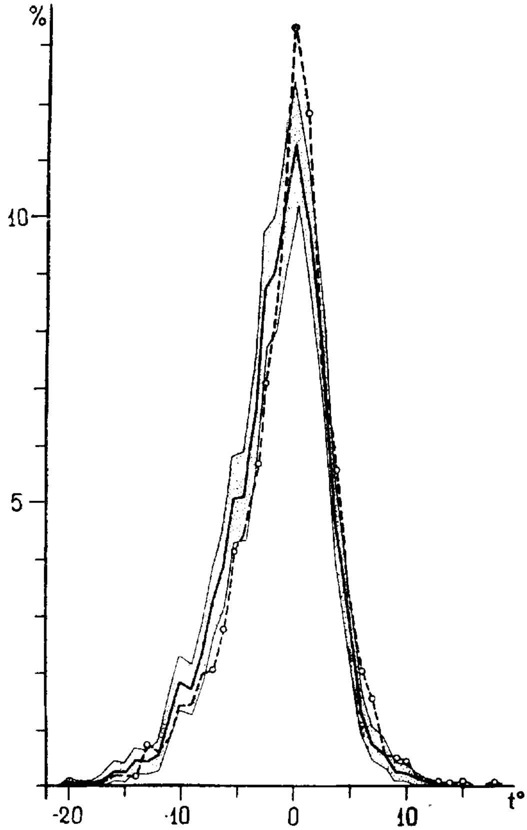

Fig. 10. Frequency of Temperatures for Haparanda in April.

Table 6 and the corresponding Fig. 11 show that the region of the warming in May lies in Middle Europe, where in many places it surpasses its fourfold mean error.

Other regions where the temperature has risen are found in North America, Greenland, and Java. A slight cooling has taken place in the Azores, Faroe Isles, and in East Europe. The variation of temperature on the basis of the frequency curve is given for Hamburg in Fig. 12, which shows a slight diminution of the low — and a rise of the high temperatures.

Table 6.

Name	Δt	ε	Name	Δt	ε
Tartu	1.8	0.4	Bogoslovsk . . .	—0.4	
Helsingfors . . .	0.2		Barnaoul	0.7	0.5
Oulu	—0.6	0.5	Touroukhansk . .	1.2	0.3
Haparanda . . .	1.3	0.4	Verkhoiansk . . .	—0.1	
Stockholm	1.2	0.4	Thorshavn . . .	—1.4	0.3
Vardö	0.6	0.4	Grimsey	1.2	0.4
Bergen	0.4		Stykkisholm . . .	0.6	0.4
Copenhagen . . .	1.7	0.3	Ivigtut	1.4	0.4
Königsberg . . .	3.3	0.5	Jacobshavn . . .	2.5	0.5
Hamburg	2.2	0.4	Upernivik	2.6	0.5
Munich	3.0	0.5	Toronto	2.3	0.4
Aberdeen	0.8	0.2	Buffalo	—0.3	0.4
Valentia	0.0		Detroit	0.4	0.6
San Fernando . .	0.6	0.2	Chicago	2.3	0.6
Ponta Delgada . .	—0.9	0.2	Savannah	—0.4	0.4
Vienna	1.3	0.4	New Orleans . .	0.3	0.4
Graz	1.6	0.5	Bismarck	1.0	0.8
Budapest	2.2	0.4	Portland	1.1	0.6
Milan	1.7	0.3	San Francisco . .	1.3	0.4
Prague	3.0	0.4	San Diego	0.7	0.4
Warsaw	2.5	0.5	Bombay	0.2	
Moscow	—2.0	0.7	Batavia	0.6	0.1
Sverdlovsk . . .	—1.6	0.6			

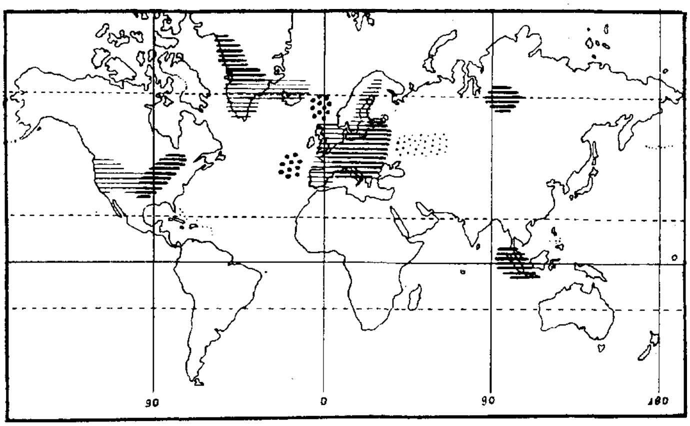

Fig. 11. Variations of Temperatures for May.

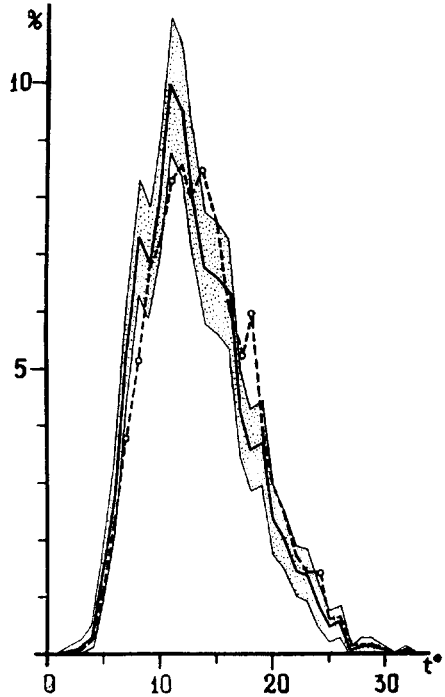

Fig. 12. Frequency of Temperatures for Hamburg in May.

Such a change of temperature can occur without any extraneous influences according to the above-mentioned probability ($P = 3.10^{-16}$) only once in 10^{17} years.

The data of the variations of temperatures in June (see Table 7) differ from those of the preceding month.

Besides the regions where the rise of temperature surpasses its fourfold mean error, there are also some places with an equally intensive diminution of temperature.

Such a cooling is observed in the Azores, Faroe Isles, South Finland, and Estonia. (Fig. 13). The regions with an increase of temperature like that of the preceding month spread over North America, Greenland, Siberia, and Java. Only in Portland, Touroukhansk, and Java (Batavia) does the increase of temperature exceed its fourfold mean error.

The frequency curves for June are given for Tartu (Fig. 14) and Hamburg (Fig. 15).

Table 7.

Name	Δt	ε	Name	Δt	ε
Tartu	—1.7	0.2	Bogoslovsk . . .	1.4	0.4
Helsingfors . . .	—1.8	0.4	Barnaoul	0.1	
Oulu	—1.5	0.5	Touroukhansk . .	2.0	0.4
Haparanda . . .	—0.3	0.4	Verkhoiansk . . .	2.4	0.9
Stockholm	—1.2	0.4	Thorshavn	—1.0	0.2
Vardö	0.4		Grimsey	1.0	0.4
Bergen	—1.2	0.4	Stykkisholm . . .	0.8	0.3
Copenhagen . . .	—0.3	0.3	Ivigtut	0.6	0.4
Königsberg . . .	1.4	0.4	Jacobshavn . . .	0.7	0.4
Hamburg	—0.6	0.4	Upernivik	0.9	0.4
Munich	0.2		Toronto	2.6	0.3
Aberdeen	—0.4	0.3	Buffalo	—1.2	0.4
Valentia	—0.9	0.2	Detroit	0.4	0.6
San Fernando . .	0.0		Chicago	1.4	0.6
Ponta Delgada . .	—1.5	0.2	Savannah	0.1	0.3
Vienna	0.4		New Orleans . . .	0.8	0.2
Graz	0.3		Bismarck	1.7	0.8
Budapest	1.0	0.4	Portland	1.8	0.4
Milan	0.8	0.3	San Francisco . .	1.4	0.4
Prague	0.4	0.4	San Diego	0.6	0.2
Warsaw	—1.5	0.4	Bombay	0.6	0.1
Moscow	—0.8	0.5	Batavia	0.9	0.1
Sverdlovsk . . .	1.4	0.4			

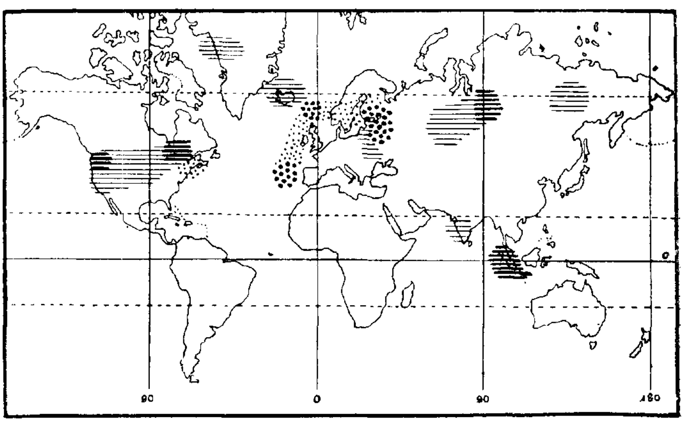

Fig. 13. Variations of Temperatures for June.

Both figures show a considerable increase of the low — and a decrease of the high temperatures.

The lengths of the periods for the accidental occurrence of such changes according to the above mentioned probabilities:

$$\begin{array}{ll} \text{Tartu} & P = 7.10^{-26} \\ \text{Hamburg} & P = 10^{-20} \end{array} \quad \text{and}$$

are 4.10^{26} years for Tartu, and 3.10^{21} years for Hamburg.

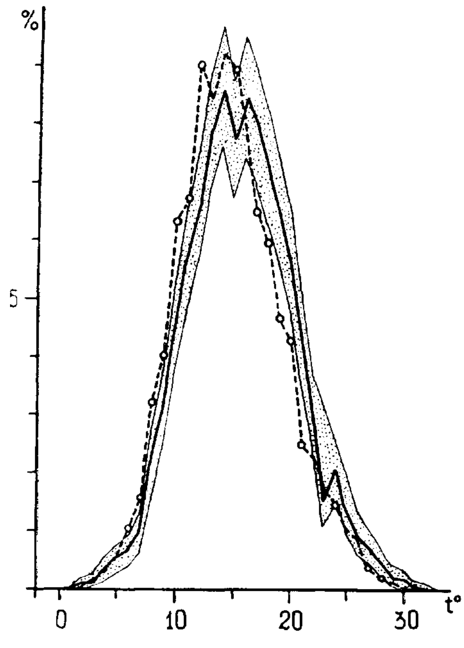

Fig. 14. Frequency of Temperatures for Tartu in June.

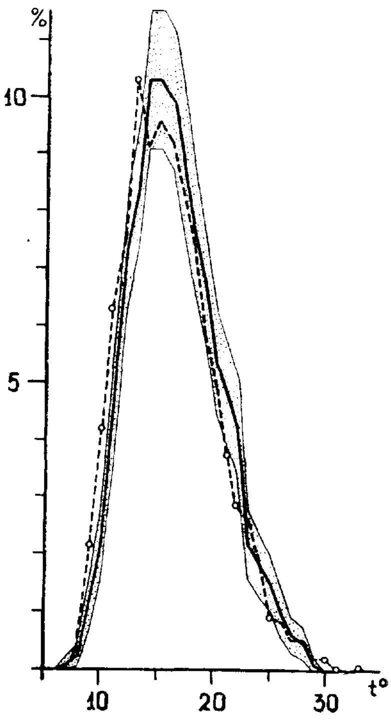

Fig. 15. Frequency of Temperatures for Hamburg in June.

The picture of the variations of temperature for July is similar to that for June (Table 8).

Here also we see some regions with an intensive cooling surpassing its mean error at least four times, but besides the Azores, we must also point out Warsaw and Moscow (Fig. 16). Among the stations with an augmentation of temperature

Table 8.

Name	Δt	ε	Name	Δt	ε
Tartu	—0.1		Bogoslovsk	—0.6	0.5
Helsingfors	1.6	0.5	Barnaoul	—1.0	0.4
Oulu	1.3	0.5	Touroukhansk	0.4	
Haparanda	1.6	0.4	Verkhoiansk	2.2	0.8
Stockholm	—0.4	0,4	Thorshavn	—0.5	0.2
Vardö	1.0	0.3	Grimsey	1.2	0.5
Bergen	—0.3		Stykkisholm	0.6	0.2
Copenhagen	1.0	0.3	Ivigtut	0.2	
Königsberg	2.2	0.4	Jacobshavn	0.1	
Hamburg	1.2	0.3	Upernivik	0.6	0.3
Munich	0.2		Toronto	2.0	0.3
Aberdeen	—0.2		Buffalo	—0.4	0.4
Valentia	—0.1		Detroit	1.0	0.4
San Fernando	0.0		Chicago	1.7	0.4
Ponta Delgada	—1.7	0.2	Savannah	0.2	0.3
Vienna	0.2		New Orleans	1.0	0.2
Graz	0.5	0.3	Bismarck	1.4	0.6
Budapest	0.8	0.3	Portland	0.9	0.4
Milan	0.1		San Francisco	1.6	0.4
Prague	0.7	0.4	San Diego	1.0	0.4
Warsaw	—1.1	0.3	Bombay	0.5	0.1
Moscow	—2.5	0.5	Batavia	1.2	0.1
Sverdlovsk	—1.0	0.5			

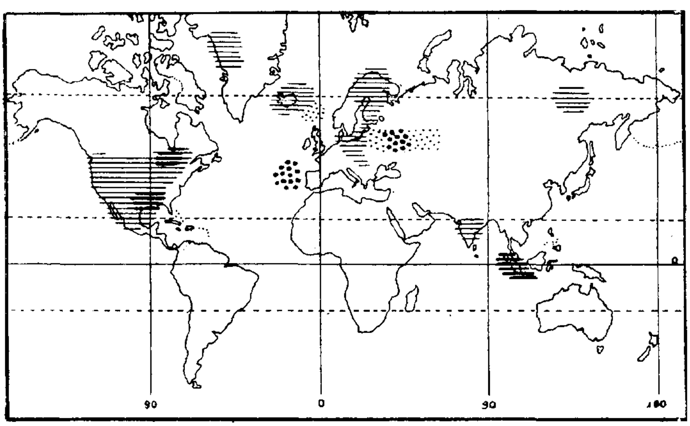

Fig. 16. Variations of Temperatures for July.

Table 9.

Name	Δt	ε	Name	Δt	ε
Tartu	—0.5	0.3	Bogoslovsk . . .	—0.1	
Helsingfors . . .	1.0	0.4	Barnaoul	0.0	
Oulu	0.1		Touroukhansk . .	1.3	0.3
Haparanda . . .	0.6	0.3	Verkhoiansk . . .	3.0	0.6
Stockholm	0.0		Thorshavn . . .	—0.7	0.2
Vardö	0.4		Grimsey	0.5	0.4
Bergen	—0.4		Stykkisholm . . .	0.8	0.3
Copenhagen . . .	0.6	0.3	Ivigtut	1.2	0.2
Königsberg . . .	1.9	0.3	Jacobshavn . . .	1.7	0.3
Hamburg	0.3		Upernivik	2.3	0.4
Munich	0.6	0.3	Toronto	0.9	0.2
Aberdeen	0.2		Buffalo	—0.2	0.6
Valentia	0.0		Detroit	0.8	0.4
San Fernando . .	0.7	0.2	Chicago	0.8	0.4
Ponta Delgada . .	—1.3	0.1	Savannah	0.4	0.4
Vienna	0.7	0.4	New Orleans . .	1.0	0.2
Graz	0.0		Bismarck	0.4	
Budapest	0.9	0.4	Portland	1.6	0.4
Milan	1.0	0.3	San Francisco . .	1.2	0.4
Prague	0.7	0.3	San Diego	—0.2	0.3
Warsaw	—1.2	0.3	Bombay	0.4	0.1
Moscow	—1.0	0.5	Batavia	0.9	0.1
Sverdlovsk . . .	—0.7	0.4			

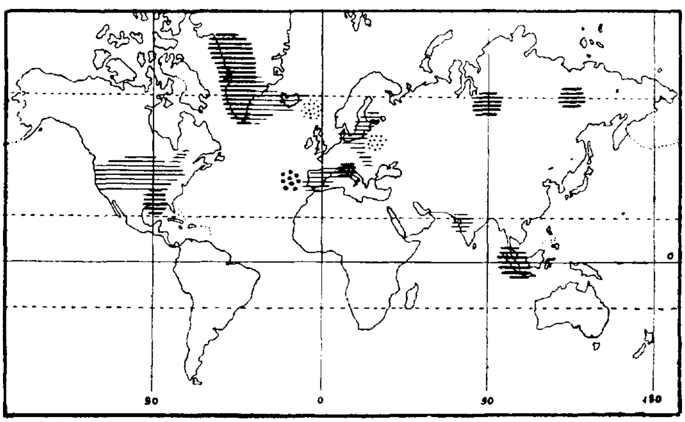

Fig. 17. Variations of Temperatures for August.

only Batavia shows an increase surpassing its fourfold mean error. A slight warming is observed in North America, Iceland, Finland, and Middle Europe.

Table 9 and the corresponding chart for August (Fig. 17) show an intensive warming at a large number of stations, such as Greenland, North America, Siberia, and Java.

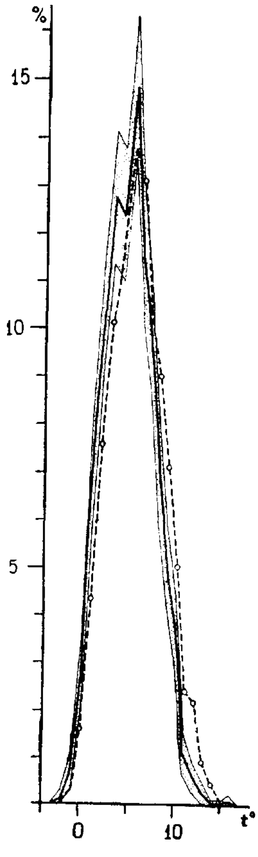

Fig. 18. Frequency of Temperatures for Upernivik in August.

Among the stations which show a fall of temperature, Moscow alone is characterised by a comparatively small mean error. The frequency curve for Upernivik in Greenland (Fig. 18) shows a marked increase of temperature in almost all groups. On the basis of the reckoned probability

$$P = 3.10^{-76}$$

this may happen only once in 10^{77} years.

The data for September are almost identical with those for August (see Table 10).

Table 10.

Name	Δt	ε	Name	Δt	ε
Tartu	—0.2	0.2	Bogoslovsk . . .	0.5	0.5
Helsingfors . . .	0.2		Barnaoul	—0.7	0.3
Oulu	0.1		Touroukhansk . .	2.6	0.3
Haparanda . . .	0.0		Verkhoiansk . . .	—0.2	
Stockholm	—0.8	0.2	Thorshavn	—0.4	0.2
Vardö	0.7	0.3	Grimsey	—0.6	0.4
Bergen	—0.4		Stykkisholm . . .	—0.4	0.3
Copenhagen . . .	0.0		Ivigtut	1.0	0.3
Königsberg . . .	1.3	0.3	Jacobshavn . . .	1.4	0.5
Hamburg	—0.1		Upernivik	1.9	0.3
Munich	0.4		Toronto	2.2	0.4
Aberdeen	—0.2		Buffalo	0.3	0.4
Valentia	—0.1		Detroit	1.0	0.4
San Fernando . .	0.9	0.6	Chicago	1.8	0.4
Porta Delgada . .	—1.0	0.2	Savannah	1.7	0.4
Vienna	—0.5	0.4	New Orleans . . .	1.8	0.3
Graz	—0.5	0.4	Bismarck	—0.7	0.6
Budapest	0.5	0.4	Portland	0.6	0.3
Milan	1.3	0.3	San Francisco . .	1.2	0.4
Prague	—0.2		San Diego	—0.4	0.4
Warsaw	—0.8	0.4	Bombay	0.4	0.1
Moscow	0.6	0.4	Batavia	0.9	0.1
Sverdlovsk . . .	0.3				

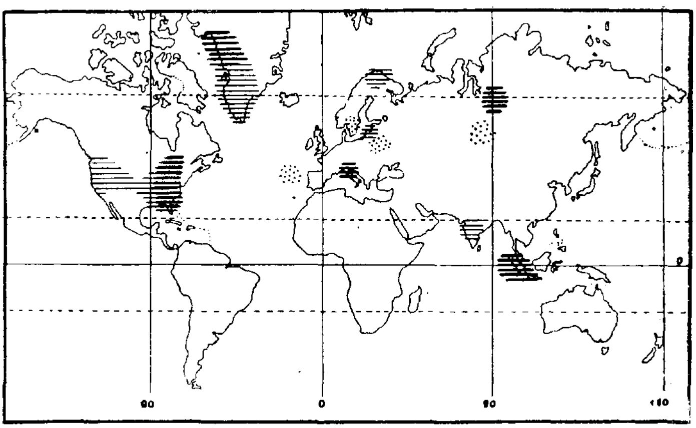

Fig. 19. Variations of Temperatures for September.

In North America, Greenland, Java, and Milan a considerable rise of temperature has taken place, whereas a decrease has been observed only in the Azores and Stockholm (Fig. 19).

A slight cooling is also seen in Iceland, the British Isles, and the Faroe Isles. In Upernivik (Greenland) the augmentation

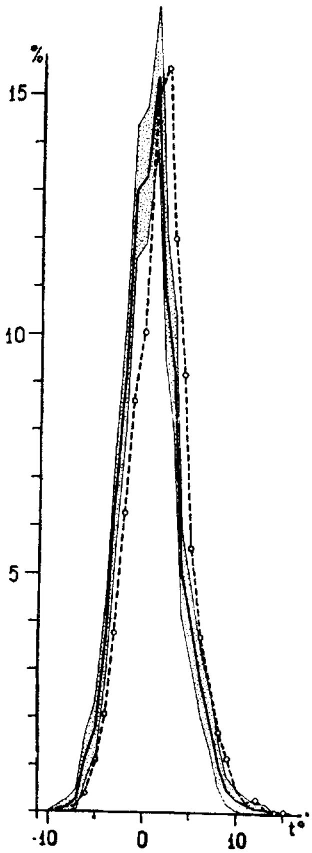

Fig. 20. Frequency of Temperatures for Upernivik in September.

of temperature is as strongly marked in September as it was in August (see Fig. 20).

The reckoning gives for the mentioned probability

$$P = 10^{-68}$$

and for the occurrence of this change, when the outlying conditions remain without alteration, a period of 2.10^{69} years.

Table 11.

Name	Δt	ε	Name	Δt	ε
Tartu	0.2	0.4	Bogoslovsk . . .	—0.5	0.7
Helsingfors . . .	—0.2		Barnaoul	—0.4	0.6
Oulu	—0.1		Touroukhansk . .	2.3	0.4
Haparanda . . .	—0.2		Verkhoiansk . . .	2.3	1.1
Stockholm	0.6	0.4	Thorshavn	0.6	0.4
Vardö	0.0		Grimsey	0.1	
Bergen	0.9	0.3	Stykkisholm . . .	0.4	0.4
Copenhagen . . .	0.9	0.3	Ivigtut	0.5	0.4
Königsberg . . .	1.4	0.4	Jacobshavn . . .	0.1	
Hamburg	0.7	0.4	Upernivik	0.1	
Munich	2.2	0.4	Toronto	2.8	0.4
Aberdeen	1.1	0.3	Buffalo	1.1	0.4
Valentia	—0.9	0.4	Detroit	2.4	0.6
San Fernando . .	1.0	0.3	Chicago	2.8	0.6
Ponta Delgada . .	—0.5	0.2	Savannah . . .	2.2	0.4
Vienna	—0.1		New Orleans . . .	2.7	0.4
Graz	0.9	0.5	Bismarck	0.7	0.8
Budapest	1.2	0.4	Portland	0.7	0.4
Milan	0.5	0.3	San Francisco . .	1.2	0.4
Prague	0.9	0 4	San Diego	0.2	0.3
Warsaw	0.3		Bombay	0.7	0.1
Moscow	—0.5	0.6	Batavia	0.6	0.1
Sverdlovsk . . .	—0.7	0.8			

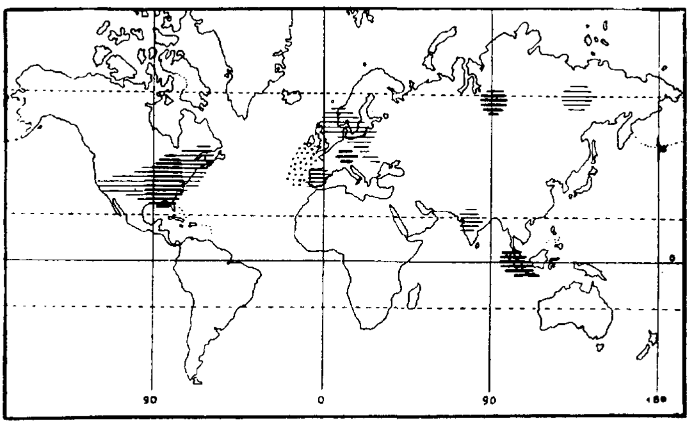

Fig. 21. Variations of Temperatures for October.

The variations of temperature in October differ from those of September and August by the fact that Greenland shows only a slight rise of temperature (Table 11).

Only the stations of the Eastern part of North America, Siberia, and Java are characterised by a rise surpassing its fourfold mean error.

The region of cooling spreads over the Azores and British Isles, but the decrease of temperature surpasses only its double

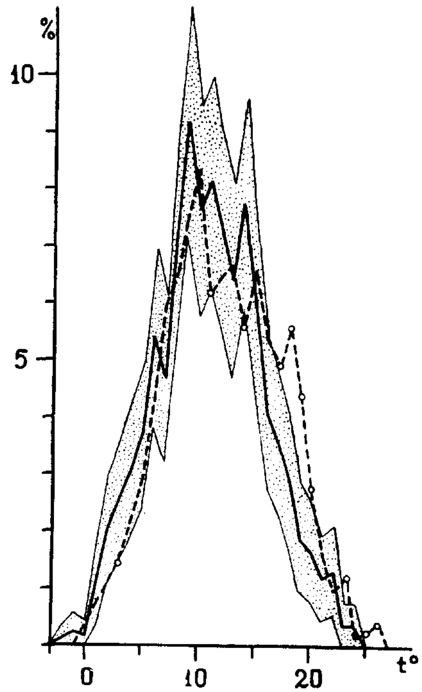

Fig. 22. Frequency of Temperatures for Munich in October.

mean error. The frequency curve of Munich (Fig. 22) shows a slight increase of high temperatures. Such a change can occur according to the reckoned probability

$$P = 2.10^{-14}$$

only once in a period of 10^{15} years.

There is no great difference between the variations of temperature in October and November (Table 12), except for a more marked warming in Greenland.

Table 12.

Name	Δt	ε	Name	Δt	ε
Tartu	1.6	0.4	Bogoslovsk . . .	2.2	0.9
Helsingfors . . .	0.2		Barnaoul	2.6	0.9
Oulu	0.4		Touroukhansk . .	3.0	0.5
Haparanda	1.4	0.8	Verkhoiansk . . .	3.2	1.1
Stockholm	0.4	0.5	Thorshavn	—0.6	0.4
Vardö	1.4	0.4	Grimsey	0.7	0.4
Bergen	1.2	0.4	Stykkisholm . . .	0.6	0.3
Copenhagen . . .	1.0	0.4	Ivigtut	1.1	0.5
Königsberg . . .	0.4		Jacobshavn . . .	1.3	0.7
Hamburg	—0.1		Upernivik . . .	2.3	0.7
Munich	1.1	0.4	Toronto	1.8	0.4
Aberdeen	0.4		Buffalo	0.8	0.4
Valentia	—0.5	0.3	Detroit	0.9	0.6
San Fernando . .	0.2		Chicago	1.9	0.6
Ponta Delgada . .	—0.7	0.2	Savannah	0.1	0.4
Vienna	1.2	0.5	New Orleans . . .	0.6	0.6
Graz	0.3		Bismarck	2.6	1.2
Budapest	1.6	0.5	Portland	0.6	0.4
Milan	1.8	0.3	San Francisco . .	0.9	0.4
Prague	0.0		San Diego	0.8	0.4
Warsaw	0.1		Bombay	0.7	0.2
Moscow	1.0	0.7	Batavia	0.6	0.1
Sverdlovsk . . .	2.2	0.8			

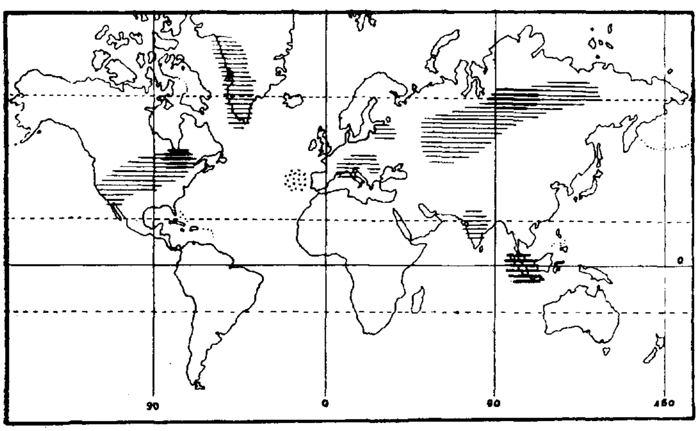

Fig. 23. Variations of Temperatures for November.

The regions showing a considerable rise of temperature lie in North America, Siberia, and Java (see Fig. 23).

The frequency curve of Upernivik (Fig. 24) again presents a strong warming, which on the basis of the corresponding probability

$$P = 8.10^{-59}$$

can accidentally take place only once in 4.10^{59} years.

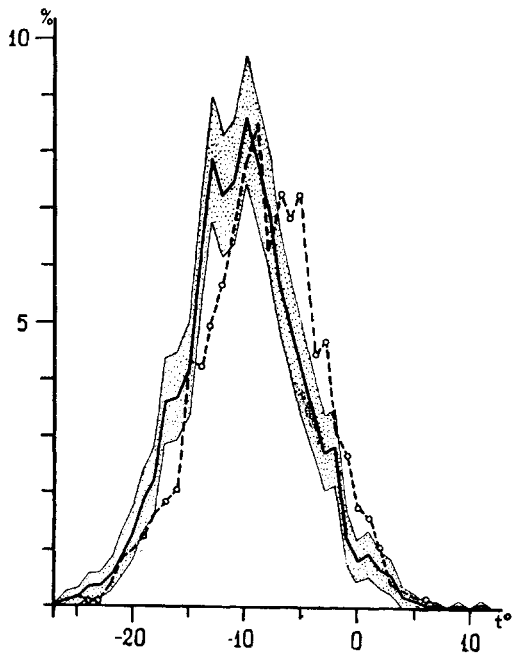

Fig. 24. Frequency of Temperatures for Upernivik in November.

The data for December (Table 13) show a considerable warming in West Europe, North America, Siberia, and Java, where in many stations the rise of temperature surpasses its fourfold mean error.

Table 13.

Name	Δt	ε	Name	Δt	ε
Tartu	1.8	0.4	Bogoslovsk . . .	0.4	
Helsingfors . . .	2.2	0.8	Barnaoul	1.2	1.0
Oulu	1.7	0.9	Touroukhansk . .	−0.4	
Haparanda . . .	3.6	1.0	Verkhoiansk . . .	1.2	1.3

Name	Δt	ε	Name	Δt	ε
Stockholm	2.0	0.6	Thorshavn	0.5	0.4
Vardö	3.2	0.4	Grimsey	0.8	0.5
Bergen	2.4	0.5	Stykkisholm . . .	2.0	0.5
Copenhagen . . .	1.8	0.4	Ivigtut	0.8	0.7
Königsberg . . .	0.8	0.6	Jacobshavn . . .	3.0	0.9
Hamburg	2.5	0.5	Upernivik	2.8	1.0
Munich	3.6	0.7	Toronto	2.6	0.4
Aberdeen	1.5	0.4	Buffalo	0.3	
Valentia	—0.2		Detroit	0.6	0.6
San Fernando . .	1.1	0.3	Chicago	0.8	0.8
Ponta Delgada . .	0.0		Savannah	1.8	0.6
Vienna	2.5	0.6	New Orleans . . .	2.1	0.6
Graz	3.1	0.5	Bismarck	—0.2	
Budapest	4 5	0.7	Portland	—0.4	0.4
Milan	2.7	0.4	San Francisco . .	0.2	0.4
Prague	2.0	0.6	San Diego	—0.7	0.4
Warsaw	2.4	0.6	Bombay	0.2	
Moscow	1.2	0.8	Batavia	0 9	0.1
Sverdlovsk . . .	3.2	1.0			

The regions with a cooling do not contain any stations at which the decrease of temperature exceeds its twofold mean error (Fig. 25).

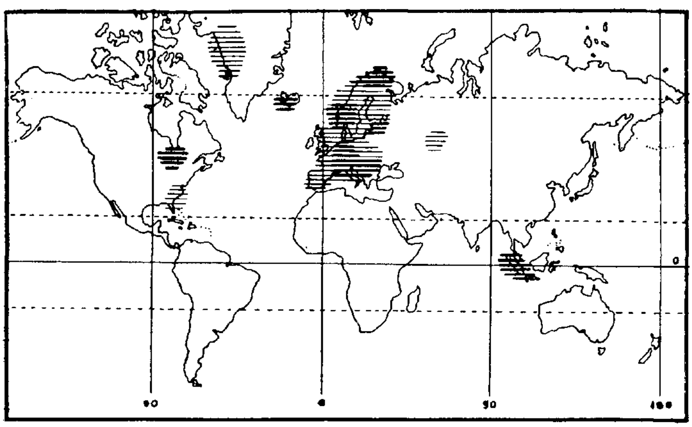

Fig. 25. Variations of Temperatures for December.

The frequency curves are composed for Haparanda and for Jacobshavn (Fig. 26 and Fig. 27).

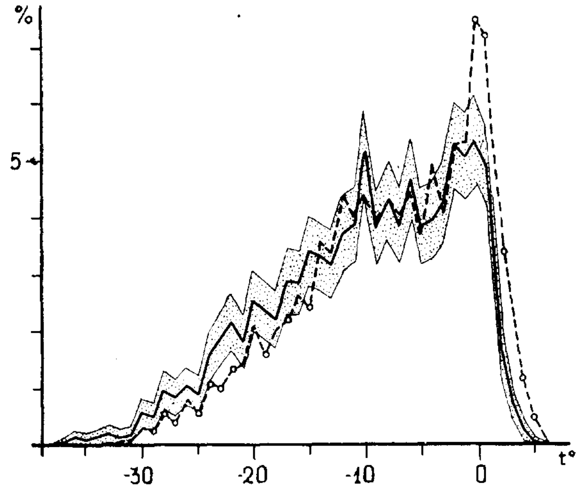

Fig. 26. Frequency of Temperatures for Haparanda in December.

They both show an increase of high temperatures, which gives the corresponding probability for

Haparanda $P = 3.10^{-33}$ and
Jacobshavn $P = 10^{-51}$.

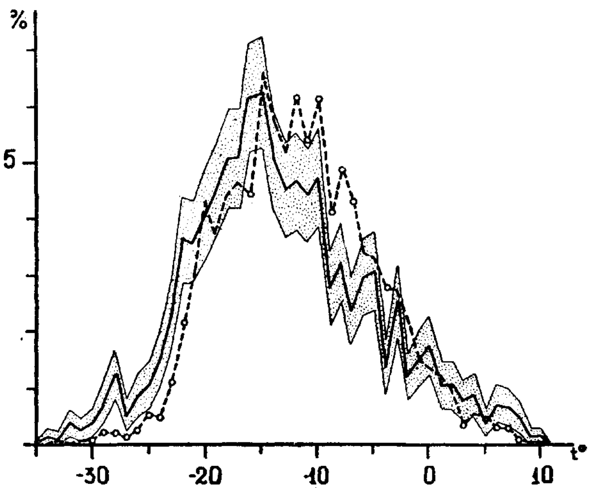

Fig. 27. Frequency of Temperatures for Jacobshavn in December.

The accidental occurrence of these changes requires for Haparanda a period of 10^{34} years, for Jacobshavn — 3.10^{52} years.

In order to determine more exactly the change of temperature for the discussed period, we have reckoned the variations of yearly amplitudes and the shifting of the maximal and minimal points in the yearly range of temperature. For this purpose the whole period of observation was divided into groups of 10 years each. By means of the polynom

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

we have calculated for each ten years' period the moments of the highest and lowest temperature in the yearly range for each station. Harmonic analysis was not applied because it takes all the average monthly temperatures equally into con-

Table 14.

Name	Δd	ε	Name	Δd	ε
Tartu	10.2	1.6	Bogoslovsk . . .	—4.1	2.2
Helsingfors . . .	11.5	1.8	Barnaoul	—0.9	1.8
Oulu	7.3	1.2	Touroukhansk . .	9.6	3.0
Haparanda . . .	3.1	2.0	Verkhoiansk . . .	13.5	2.0
Stockholm	9.6	2.0	Thorshavn	0.0	
Vardö	—9.1	4.2	Grimsey	4.5	2.8
Bergen	15.1	4.0	Stykkisholm . . .	1.3	2.8
Copenhagen . . .	4.1	2.8	Ivigtut	19.6	6.2
Königsberg . . .	11.3	3.0	Jacobshavn . . .	15.7	1.8
Hamburg	15.6	4.2	Upernivik	5.2	1.8
Munich	0.1		Toronto	—4.7	1.4
Aberdeen	7.8	2.0	Buffalo	5.0	1.6
Valentia	—3.0	3.4	Detroit	11.8	1.8
San Fernando . .	7.0	3.0	Chicago	10.0	3.8
Ponta Delgada . .	0.0		Savannah	3.0	1.4
Vienna	3.2	4.2	New Orleans . .	2.0	1.0
Graz	1.1	2.6	Bismarck	4.8	4.0
Budapest	—0.9	1.6	Portland	—0.8	1.6
Milan	9.6	2.0	San Francisco . .	—3.7	4.0
Prague	2.2	2.0	San Diego	—18.2	3.4
Warsaw	16.1	3.0	Bombay	5.0	2.6
Moscow	2.4	3.6	Batavia	0.3	0.7
Sverdlovsk . . .	—9.5	1.6			

Table 15.

Name	Δd	ε	Name	Δd	ε
Tartu	9.6	6.4	Bogoslovsk . . .	9.4	5.2
Helsingfors . . .	—16.8	2.6	Barnaoul	—4.2	2.8
Oulu	3.2	5.8	Touroukhansk . .	—2.2	3.6
Haparanda . . .	—3.8	3.4	Verkhoiansk . . .	4.4	4.8
Stockholm	—11.1	4.4	Thorshavn	—15.0	6.6
Vardö	—2.0	2.2	Grimsey	—43.8	11.2
Bergen	—16.6	6.4	Stykkisholm . . .	—41.1	11.2
Copenhagen . . .	6.3	2.4	Ivigtut	—17.8	4.0
Königsberg . . .	4.7	5.8	Jacobshavn . . .	—17.4	3.0
Hamburg	26.2	4.8	Upernivik	—8.6	6.8
Munich	12.0	4.6	Toronto	—3.5	2.4
Aberdeen	29.6	11.2	Buffalo	—9.6	2.2
Valentia	17.5	15.2	Detroit	—10.5	1.2
San Fernando . .	—4.4	6.0	Chicago	—25.2	4.0
Ponta Delgada . .	0.0		Savannah	0.8	5.0
Vienna	22.9	5.2	New Orleans . . .	0.9	6.4
Graz	7.4	5.2	Bismarck	—31.8	4.2
Budapest . . .	14.4	4.4	Portland	—21.9	2.0
Milan	12.0	3.0	San Francisco . .	—21.0	3.4
Prague	20.4	2.8	San Diego	—0.2	0.8
Warsaw	14.4	5.0	Bombay	7.4	3.2
Moscow	5.7	4.2	Batavia	0.1	0.7
Sverdlovsk . . .	5.2	5.6			

sideration, whereas in our case we are interested only in the maximal and minimal temperatures of the annual range. The coefficients in the abovegiven polynom were determined by means of least squares. We find the moment of the highest temperature from the monthly averages of May, June, July, August, and September; the moment of the lowest temperature is found from the averages of December, January, February, March, and April. On the basis of these moments for each ten years' period the shifting of the highest and lowest points in the yearly range of temperatures has been reckoned for each station by means of least squares. For the stations of America the monthly average max. temperatures have been used for determining the variation of the maximal points and the monthly average minimum temperatures for the variation of the minimal point. These data expressed in days (Δd) are given in tables 14 (Shifting of max. temperature) and 15 (Shifting of min. temperature) with their corresponding mean errors (ε). In both tables a minus sign (—) is put before the number of days showing a premature occurrence of the extreme points of temperature in the yearly range. In order to obtain a better

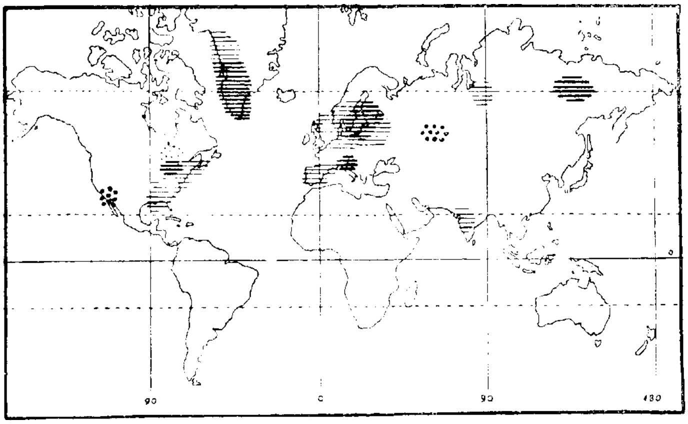

Fig. 28. Shifting of Max. Temperature.

survey the variations of the maximal and minimal points expressed in days are represented graphically. In Fig. 28 the thin lines denote the districts, where the retardation of the max. temperature surpasses its double mean error, and the thick lines — the districts, where it surpasses its fourfold mean

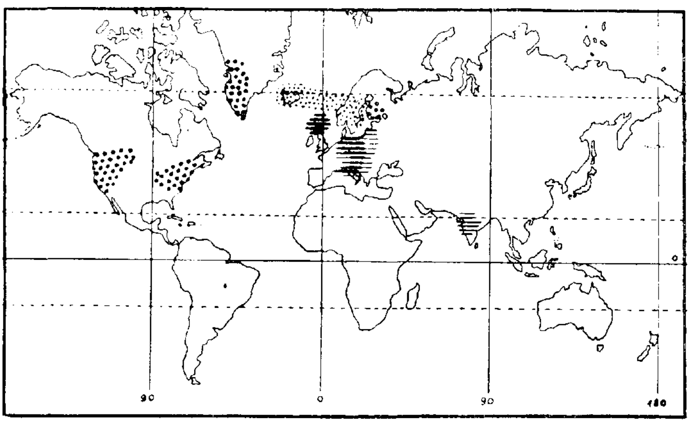

Fig. 29. Shifting of Min. Temperature.

error. The thin dots mark the regions, where the premature occurrence of the max. temperature surpasses its double mean error, the thick dots — the regions, where it surpasses its fourfold mean error. In the same way the shifting of min. temperature is represented in Fig. 29.

As can be seen from the figures the maximal point of the temperature in the yearly range is late in Europe (except Vardö), partly in Siberia, India, Greenland, and on the East Coast of North America. An advance of the maximal point is observed in the Eeast of North America. As regards the shifting of the minimal point, a considerable retardation is seen in Middle Europe and an advance is marked in North America, Greenland, Iceland, and Scandinavia.

The change of the amplitude of the annual range of temperature has been found in the same way by means of

Table 16.

Name	Δt	ε	Name	Δt	ε
Tartu	—2.0	0.7	Bogoslovsk . . .	—1.1	0.8
Helsingfors . . .	—0.2	1.0	Barnaoul	—1.7	1.2
Oulu	—0.2	1.4	Touroukhansk . .	0.2	
Haparanda . . .	1.7	0.8	Verkhoiansk . . .	—1.7	0.4
Stockholm	—0.9	0.6	Thorshavn	—0.4	0.2
Vardö	0.0	0.5	Grimsey	—4.1	0.3
Bergen	—1.6	0.4	Stykkisholm . . .	—1.7	0.2
Copenhagen . . .	—0.7	0.4	Ivigtut	0.6	0.6
Königsberg . . .	1.8	0.7	Jacobshavn . . .	—1.6	1.3
Hamburg	—0.8	0.6	Upernivik	3.4	1.0
Munich	—3.5	0.7	Toronto	2.3	0.4
Aberdeen	—1.7	0.5	Buffalo	—1.8	0.5
Valentia	0.2	0.4	Detroit	—0.6	0.6
San Fernando . .	0.2	0.3	Chicago	—1.9	0.2
Ponta Delgada . .	—1.3	0.2	Savannah	—1.6	0.5
Vienna	—1.0	0.9	New Orleans . . .	—0.8	0.5
Graz	—1.6	0.6	Bismarck	—3.6	0.7
Budapest	—2.8	0.6	Portland	0.4	0.4
Milan	—1.0	0.4	San Francisco . .	1.8	0.2
Prague	—1.4	0.6	San Diego	—0.3	0.4
Warsaw	—2.6	0.6	Bombay	0.2	0.2
Moscow	—1.6	0.9	Batavia	0.0	
Sverdlovsk	—2.0	0.6			

least squares. These data (Δt) with their mean errors ε are given in Table 16.

The corresponding Fig. 30 shows a decrease of the yearly amplitude (marked with dots) in the greater part of North America, Greenland, the Azores, Iceland, Siberia, and East

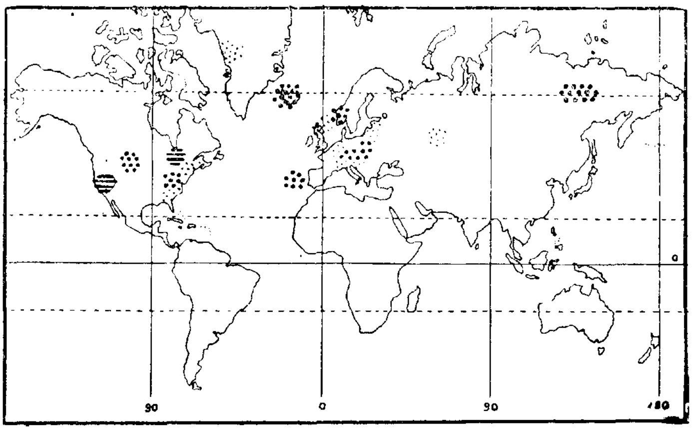

Fig. 30. Change of Yearly amplitude of Temperature.

Europe. An augmentation of amplitude (marked in Fig. 30 with lines) is observed only in North Scandinavia, Königsberg, and the Western Coast of North America.

The aim of this work was only to show the variations of temperatures that have taken place during the last 60—70 years. The causes of the variations are not discussed, because the analysis of that question requires a much thicker net of climatic and hydrologic observations in both hemispheres.

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EESTI TAIMED

K. EICHWALD

III

(101—150)

WITH A SUMMARY:

ESTONIAN PLANTS

TARTU 1938

Printed by K. Mattieseni trükikoda o.-ü., Tartu 1938.

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Liikide kaardistamisel on kasustatud, peale Tartu ülikooli botaanikamuuseumi Eesti Herbaariumi, järgmisi taimekogusid: Tartu ülikooli juures oleva Loodusuurijate Seltsi herbaarium; Eestimaa Kirjanduse Seltsile kuuluv Tallinna provintsiaalmuuseumi herbaarium; K. R. Kupffer'i herbaarium (prof. dr. N. Malta kaudu); R. Lehbert'i herbaarium.

101. Dryopteris filix-mas (L.) Schott. — Maarja-sõnajalg.

[*Polypodium Filix mas* L i n n é; *Nephrodium Filix mas* R i c h a r d; *Polystichum Filix mas* R o t h; *Aspidium Filix mas* S w a r t z.]

Kohati varjukais, niiskeis segametsades ja võsastikes, vahel ka kivistuil.

E s t o n i a s u p e r i o r, V i r u - J a a g u p i khk., Põlula asunduse lähedal kiviaial.

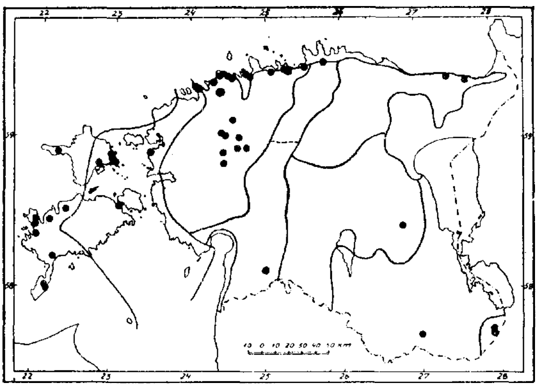
Sporadically in humid, shady mixed woods and shrubberies, sometimes also on stony land.

E s t o n i a s u p e r i o r, parish of V i r u - J a a g u p i, along a stone rampart near the settlement of Põlula.

26. VII. 1933. leg. **G. Mechmershausen.**

102. Dryopteris Robertiana (Hoffm.) C. Christensen. — Pae-sõnajalg.

[*Polypodium Robertianum* H o f f m a n n; *Phegopteris Robertiana* A. B r a u n.]



Lubjataimena paekalda segametsades, paelõhedes ja -paljanditel, *Campanula rotundifolia* — *Cystopteris fragilis*'e uniooni kaaslasena.

E s t o n i a i n f e r i o r, R a p l a khk., Palamulla küla lähedal paekihtide vahelistes lõhedes. Saatjad (Companions): *Asplenium trichomanes* ja *Campanula rotundifolia*.

As a calciphilous species it grows in mixed woods upon the ordovician limestone cliffs, in clefts and denudations, on calcareous rocks as a companion in the *Campanula rotundifolia* — *Cystopteris fragilis* union.

E s t o n i a i n f e r i o r, parish of R a p l a, in clefts of limestone rocks, near Palamulla.

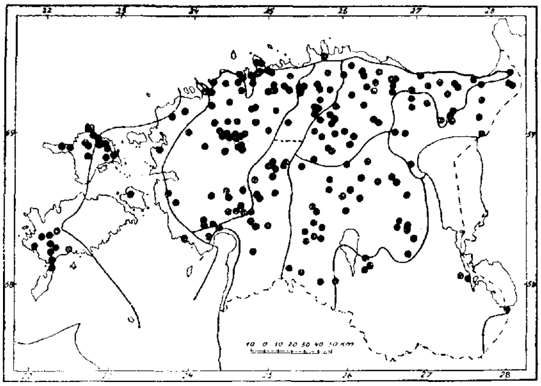
23. VIII. 1934. leg. **H. Aasamaa.**

103. Polygonum viviparum Linné. — Pung-kirburohi.

Arktoalpiinne, Eestis lõunapiiri omav taim, levinud eriti Põhja-Eesti niiskeil, lubjarikkail puisniitudel ja niitudel, kus ta sageli esineb *Scorzonera humilis* — *Melampyrum nemorosum*'i unioonis. Eestis esinevad taimed omavad allküljel enam või vähem karvaseid lehti, kuuludes seega, ka antud juhul, teisendisse: var. *Roessleri* B e c k [R c h b. Ic. XXIV. 84 (1906)], lähenedes lehtedelt, mitte aga õitelt, ka vormile: var. *tatrense* Z a p a ł o -

wicz [Consp. fl. Galic. crit. X, in Rozprawy Akad. Umiejętn. Ser. III Tom VII. B, p. 609 (1907)].

Estonia media, Tartu-Maarja khk., Jänese lähedal Emajõe äärsel niidul.



An arctic-alpine species, reaching in Estonia the southern limit of its northern circumpolar distribution. It is confined mainly to N. Estonia; there it grows in moist meadows and wooded meadows on calcareous soil, where it often occurs in the *Scorzonera humilis — Melampyrum nemorosum* union. The Estonian plants, with leaves more or less pubescent beneath, belong to the var. *Roessleri* Beck [in Rchb. Ic. XXIV. 84 (1906)]; in leaves, not in flowers, it approaches the var. *tatrense* Zapałowicz [Consp. fl. Galic. crit., pars X, in Rozprawy Akad. Umiejętn. Ser. III. Tom VII. B, pag. 609 (1907)].

Estonia media, parish of Tartu-Maarja, in a meadow near Jänese.

25. VI. 1933. leg. **K. Eichwald.**

103-a. Polygonum viviparum Linné. — Pung-kirburohi.

Estonia maritima borealis, Tallinna juures Suhkrumäe puisniidul. Saatjad (Companions): *Melampyrum pratense, Scorzonera humilis, Anthoxanthum odoratum, Orchis maculata.*

Estonia maritima borealis, neighbourhood of Tallinn, in a wooded meadow.

6. VII. 1933. leg. **Juta Rebane.**

104. Cerastium arvense Linné. — Põld-kadakkaer.

Eestis käesoleval ajal aktiivselt leviv antropohoorne taim; esineb põld-umbrohuna, põllu- ja teeservadel, parkides, kraavikallastel jne. Taksonoomiliselt on see peam. madalmaades esinev alaliik, subsp. *commune* Gaudin [= subsp. *arvum* (Schur) Correns].

Estonia media, Tartu, Raadi mõisa pargis.

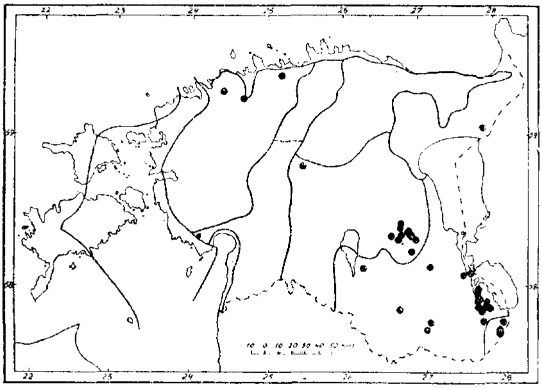
An anthropochore species in Estonia, actively spreading at the present time as a weed of arable land; it grows also along roadsides, ditches and in parks. Taxonomically this is the subsp. *commune* Gaudin [= subsp. *arvum* (Schur) Correns], predominant in lowlands.

Estonia media, Tartu, in the Raadi park.

4. IV. 1932. leg. **E. Lepik.**

105. Gypsophila muralis Linné. — Müür-kipslill.

Paiguti mandril; esineb peamiselt lubjavaesel liivapinnasel — teeservadel ja umbrohuna põldudel.



Estonia orientalis, Rannu khk., Lapetukme küla Lossmanni talu lähedal liivasel teel.

Sporadically on Estonian mainland, especially on sandy soil poor in lime; it occurs along roadsides and as weed on arable land.

Estonia orientalis, parish of Rannu, along a sandy road to the village of Lapetukme.

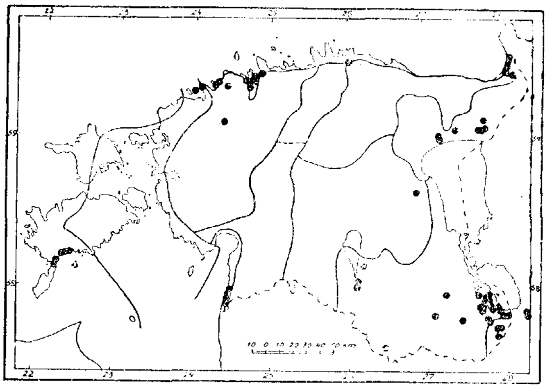
8. VII. 1935. leg. **Ellen Vilbaste.**

106. Dianthus arenarius Linné. — Nõmmnelk.

Eriti mandriosas luidetel, siseluidetel, liivanurmedel ja kuivades liivastes männimetsades, paiguti hulgi.

Estonia maritima borealis, Keila, khk., Nõmme lähedal liivasel lasketiirul avaühinguna, moodustades üksikuid kogumikke; saatja *Festuca polesica* Zapał.

Mainly on Estonian mainland, sporadically abundant on maritime dunes, inland dunes, sandy plains and dry *Pinus silvestris* forests.



Estonia maritima borealis, parish of Keila, in open vegetation of a sandy shooting-place near Nõmme, in isolated colonies, accompanied by *Festuca polesica* Zapał.

5. VIII. 1933. leg. **V. Sirgo.**

106-a. **Dianthus arenarius** Linné. — **Nõmmnelk.**

Estonia maritima occidentalis, Anseküla khk., Järve rannas, luidetel, koos *Koeleria glauca* ja *Festuca polesica*'ga.

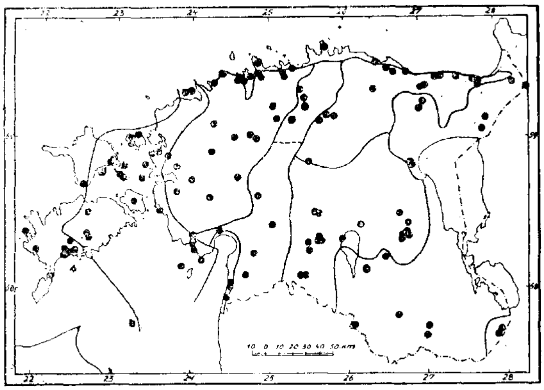
Estonia maritima occidentalis, parish of Anseküla, on the dunes of Järve, associated with *Koeleria glauca* and *Festuca polesica.*

27. VI. 1933. leg. **A. Tamsalu.**

107. **Ranunculus ficaria** Linné. — **Kanakoole.**

[*R. Ficaria* L.; *Ficaria verna* Hudson; *F. ranunculoides* Roth.]

Sageli hulgi jõgede ja ojade äärsetes lehtmetsades ja põõsastikes, puisniitudel, niitudel ja parkides.



Estonia orientalis, Nõo khk., Vapramäe kohal Elva jõe madalal niidulammil.

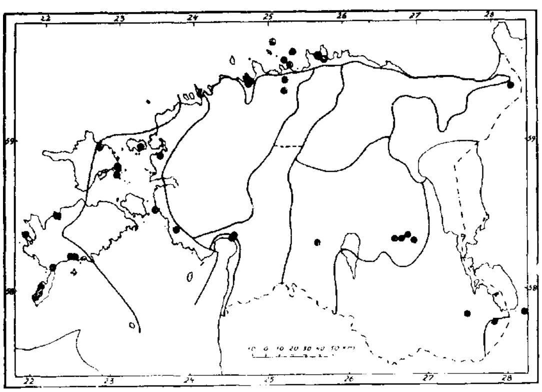
Often abundantly in deciduous woods and bushes along rivers and brooks, in meadows, wooded meadows and parks.

Estonia orientalis, parish of Nõo, in a low riverside meadow at Elva river, against the Vapramäe hill.

17. V. 1931. leg. **V. Sirgo.**

108. **Lepidium ruderale** Linné. — **Haisev kress.**

Antropohoorse liigina prügil, tänavatel, teeservadel, raudteedel, sageli hulgi.



Estonia media, Tartu, Emajõe ääres sadamaraudteel, kohati hulgi.

As an anthropochore species on rubbish, along roads and railroads, often abundantly.

Estonia media, Tartu, here and there abundantly on the railroad.

21. VI. 1933. leg. **K. Eichwald.**

108-a. **Lepidium ruderale** Linné. — **Haisev kress.**

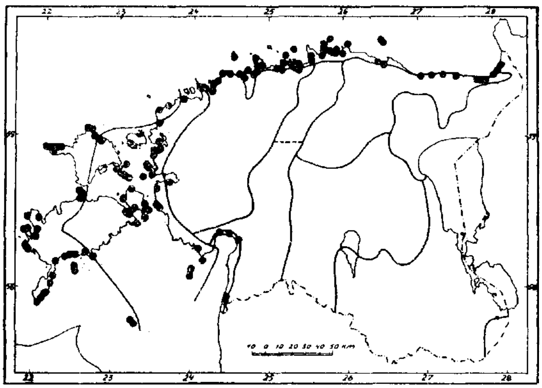
Estonia inferior, Jõelähtme khk., Jägala rahvamaja ümbruses.

Estonia inferior, parish of Jõelähtme, around the People's House at Jägala.

17. V. 1934. leg. **H. Aasamaa.**

109. **Cakile maritima** Scop. — **Merisinep.**

Esineb liivase mereranniku saliinses vöötmes, eriti väljauhetud, kõduneval *Fucus vesiculosus*'el.



Estonia maritima occidentalis, Kihelkonna khk., Harilaiu liivasel lõunakaldal.

In the saline belt of the sandy shores, especially on the drift of mouldy *Fucus vesiculosus*.

Estonia maritima occidentalis, parish of Kihelkonna, on the sandy seashore of the Harilaid peninsula.

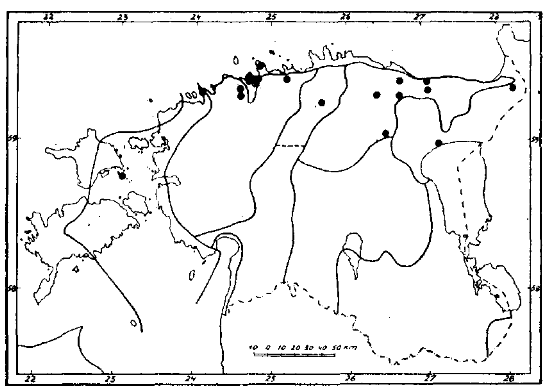
15. VII. 1933. leg. **Elsa Pastak.**

110. **Draba nemorosa** Linné. — **Metskevadik.**

[*D. nemorosa* L. *β. hebecarpa* Lindblom; *D. nemoralis* Ehrhardt.]

Mandri põhjaosas, lubjarikkal siluuri pinnasel, osalt ka saartel; esineb loopealseil, kuivadel liivastel aasadel, teedel ja umbrohuna põldudel.

Estonia maritima borealis, Tallinna juures Lasnamäe loopealsel.



On silurian, calcareous soils of northern Estonia, likewise on islands; it occurs in alvars, dry sandy meadows, along roads and as a weed on arable land.

Estonia maritima borealis, on the alvar soils of Lasnamägi, near Tallinn.

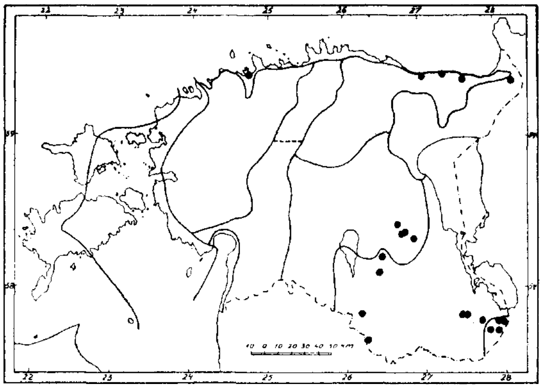
8. VI. 1933. leg. **Juta Rebane.**

111. **Draba nemorosa** L. var. **leiocarpa** Lindblom. — **Metskevadik.**

[*D. nemoralis* E h r h. *β. glabra* F l e i s c h e r et L i n d e m a n n; *D. lutea* G i l i b e r t.]

Esineb peamiselt mandri idaosas loopealseil, teeservadel, kuivadel nõlvadel, vahel ka pioneertaimena mitmesuguseil paljastuil.

E s t o n i a s a r m a t i c a, I r b o s k a varemete all orundi idanõlva loopealsel.



Occurs mainly in the eastern part of the Estonian mainland in alvars, along roadsides, on dry slopes and sometimes as a pioneer plant on various denudations.

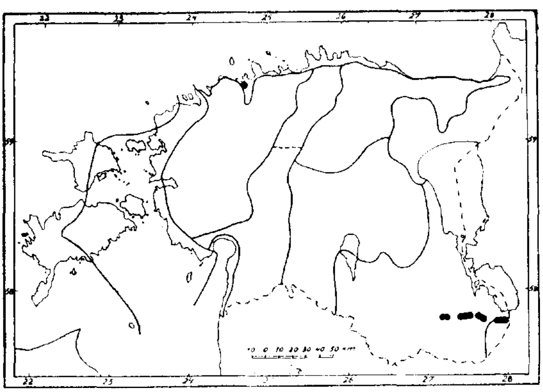
E s t o n i a s a r m a t i c a, underneath the ruins of I r b o s k a, on alvar ground.

29. V. 1933. leg. **Juta Rebane.**

112. **Alyssum desertorum** Stapf. — **Põld-kilbirohi.**

[*A. minimum* W i l l d e n o w.]

Antropohoorse liigina leviv taim, esineb seni harva, peamiselt Kagu-Eestis.



E s t o n i a s a r m a t i c a, Vilo peatuskoha lähedal Irboska-Petseri raudteetammi serval.

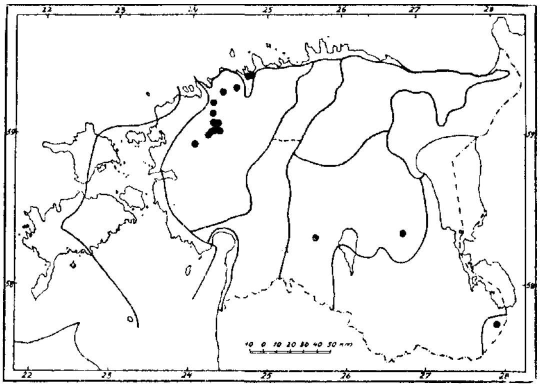
Spreading as a rare anthropochore species, as yet confined mainly to South-Eastern Estonia.

E s t o n i a s a r m a t i c a, on the southern slope of the railroad embankment near Vilo, between the railway stations of Irboska and Petseri.

29. V. 1933 ja 30. V. 1935. leg. **K. Eichwald.**

113. **Reseda lutea** Linné. — **Kollane reseeda.**

Antropohoorne, kohati, peamiselt raudteeliinidel ja raudteejaamades levinud liik, harva ka põldudel ja kuivadel nõlvadel. Eriti sage raudteel Riisipere ja Turba jaama vahel (E i n f), kus esineb kohati hulgi.



E s t o n i a i n f e r i o r, N i s s i khk., raudteetammi nõlval, Riisipere jaamast 2 km Haapsalu suunas, c o p.

As an anthropochore species confined mainly to the railways and railway stations, rarely also in fields and on dry hillsides. It is especially distributed on the railway embankment between the stations of Riisipere and Turba (E i n f).

E s t o n i a i n f e r i o r, parish of N i s s i, on the slope of the railroad embankment 2 km SW. of the Riisipere station.

2. VII. 1933. leg. **L. Sepp.**

113-a. **Reseda lutea** Linné. — **Kollane reseeda.**

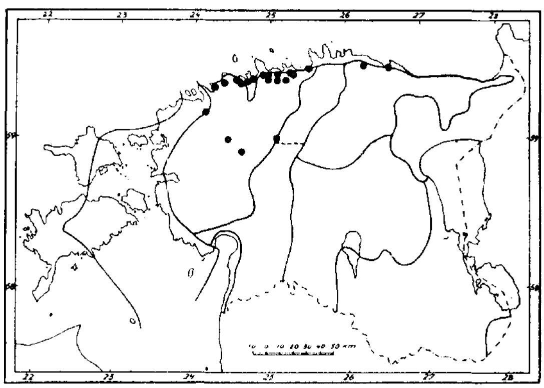
E s t o n i a s a r m a t i c a, Brod'i vesiveskite lähedal, I r b o s k a ürgoru läänenõlval.

E s t o n i a s a r m a t i c a, near the water-mills of Brod, upon the western slope of the I r b o s k a valley.

15. IX. 1935. leg. **V. Sirgo.**

114. **Saxifraga adscendens** Linné. — **Püstkivirik.**

[*S. controversa* S t e r n b e r g; *S. tridactylites* (L.) Engl. et I r m s c h. subsp. *adscendens* (L.) B l y t t.]



Arktoalpiinne, eriti suuruses ja harunemises väga varieeruv liik. Lubjalembene, esineb vaid mandri põhjaosas, peamiselt paekalda loopealseil.

E s t o n i a m a r i t i m a b o r e a l i s, T a l l i n n a lähedal Lasnamäe lool.

An arctic-alpine calciphilous species, variable especially in tall and branching. It is confined to the alvars and limestone cliffs of N. Estonia.

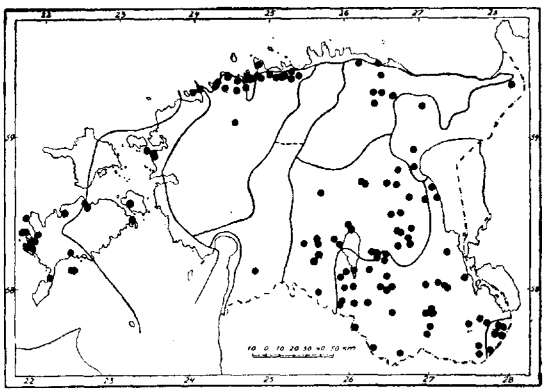
Estonia maritima borealis, in the Lasnamägi alvar E. of Tallinn.

VI. 1933. leg. **Juta Rebane.**

115. Saxifraga granulata Linné. — Harilik kivirik, lambapähkel.

[*S. granulata* L. subsp. *eugranulata* Engler et Irmschler.]

Enam-vähem harilik, kuid katkendilise levikuga ja haruldane või puudub mõnes valdkonnas. Esineb päikesepaistelistel nõlvadel, teeservadel, kuivadel niitudel ja puisniitudel.



Estonia orientalis, Põlva khk., Võru lähedal Võhandu jõe äärsel niidul.

Usually common, nevertheless rare or missing in some parts of Estonian territory. Occurs on dry hillsides, along roads, in dry meadows and wooded meadows.

Estonia orientalis, parish of Põlva, in riverside meadow of the Võhandu river, near Võru.

13. VI. 1927. leg. **W. J. Reinthal.**

115-a. Saxifraga granulata Linné. — Harilik kivirik, lambapähkel.

[*S. granulata* L. subsp. *eugranulata* Engler et Irmschler.]

Estonia orientalis, Rõngu khk., 2 km kirikust idas, kuival niidul.

Estonia orientalis, parish of Rõngu, in a dry meadow, 2 km east of the church.

13. VI. 1930. leg. **K. Eichwald.**

116. Trifolium montanum Linné. — Mägi-ristikhein.

[*T. montanum* L. *α. genuinum* Grenier et Godron.]

Harilik kuival lubja- ja liivapinnasel: kinkudel, liivastel aruniitudel, loopealseil, puisniitudel ja hõredais männimetsades. *Filipendula hexapetala* — *Trifolium montanum*'i uniooni karakterliik.

Litorale heademeesteense, Pärnu khk., Sauga jõe ääres kuival liivasel niidul, pillatult.

Common on dry calcareous and sandy soil: on hills, in sandy meadows, in alvars, in wooded meadows and in thin *Pinus silvestris* forests. A characteristic species of the *Filipendula ulmaria* — *Trifolium montanum* union.

Litorale heademeesteense, parish of Pärnu, in a dry sandy meadow along the Sauga river.

29. VI. 1933. leg. **P. Kaaret.**

117. Lathyrus pratensis Linné. — Aas-seahernes.

Väga harilik ja sageli hulgi aasadel, niiskeil niitudel ja puisniitudel.

Estonia intermedia, Saarde khk., Ristiküla Orumäe talu põldude ääres ja kraavikaldail.

Very common in Estonia; often abundant in meadows and wooded meadows.

Estonia intermedia, parish of Saarde, village of Ristiküla, on borders of fields and along ditches.

20. VII. 1935. leg. **Sylvia Talts.**

118. Lathyrus palustris Linné. — Soo-seahernes.

Esineb pillatult niiskeil niitudel ja puisniitudel, soodes ja lodudes.

Estonia inferior, Jüri khk., Arukülast läänes, Sillaotsa lähedal *Schoenus ferrugineus*'e soos.

Throughout Estonia in moist meadows, wooded meadows, swamps and marshes.

Estonia inferior, parish of Jüri, in a *Schoenus ferrugineus* swamp near Sillaotsa, west of Aruküla.

4. VII. 1933. leg. **A. Üksip.**

119. Geranium pratense Linné. — Aas-kurereha, pistirohi.

Harilik niitudel, aasadel, kaldavõsastikes, põllu- ja teeservadel.

Estonia intermedia, Ambla khk., Jäneda asunduse lähedal teeserval, lubjarikkal pinnasel.

Very common in meadows, dry riverside meadows and riverside shrubberies, along edges of fields and roads.

Estonia intermedia, parish of Ambla, along a roadside on calcareous soil, near Jäneda.

19. VII. 1935. leg. **A. Üksip.**

120. **Helianthemum nummularium** (L.) Miller. — **Kuldkann.**

[*H. Chamaecistus* Miller; *H. vulgare* Gaertner.]

Eestis esinev taim kuulub alaliiki: subsp. *nummularium* (L.) Schinz et Thellung [= subsp. *nummularium* (L.) var. *tomentosum* Grosser]; on väga harilik Põhja-Eesti ja saarte loopealseil, kuivadel lubjarikastel kinkudel ja teeservadel, harvem liivapinnasel. On *Filipendula hexapetala-Trifolium montanum*'i uniooni *Helianthemum nummularium*'i teisendi karakterliik.

Estonia maritima occidentalis, Kihelkonna khk., Kuusnõmme poolsaare loopealsel.

The Estonian plant belongs to the subsp. *nummularium* (L.) Schinz et Thellung [= subsp. *nummularium* (L.) var. *tomentosum* Grosser]; it is widely distributed on the limy alvars of N. Estonia and the islands, as well as on dry hillsides and along roads, rarely on sandy soil. This is a characteristic species of the *Helianthemum nummularium* variant of the *Filipendula ulmaria* — *Trifolium montanum* union.

Estonia maritima occidentalis, parish of Kihelkonna, in an alvar of the Kuusnõmme peninsula.

25. VII. 1933. leg. **A. Vaga.**

121. **Angelica silvestris** Linné. — **Heinputk.**

[*A. silvestris* L. var. *α. vulgaris* Avé-Lallemant.]

Harilik niiskeil niitudel ja puisniitudel, võsastikes, leht-, sega- ja lodumetsades, vahel ka rabastuval pinnal.

Estonia media, Viljandi lähedal, Kleinhofi Mädajärvega piirduval niiskel niidul.

Common in moist meadows and wooded meadows, in shrubberies, marsh woods, deciduous and mixed woods, sometimes even on boggy ground.

Estonia media, in a moist meadow near Viljandi.

29. VII. 1933. leg. **W. J. Reinthal.**

122. **Andromeda polifolia** Linné. — **Küüvits.**

Harilik rabades, turbasoodes ja turbapinnasel kasvavais männimetsades. Rabadel moodustab erilise *Andromeda polifolia* uniooni.

Estonia media, Äksi khk., Pupastvere raba serval. Saatjad (Companions): *Betula nana, Ledum palustre, Vaccinium uliginosum, Chamaedaphne calyculata, Rubus chamaemorus, Carex chordorrhiza, Eriophorum vaginatum.*

Very frequent in bogs throughout Estonia, in peat bogs and in boggy *Pinus silvestris* forests. In bogs it forms a special *Andromeda polifolia* union.

E s t o n i a m e d i a, parish of Ä k s i, on the edge of the Pupastvere bog.

11. VI. 1933. leg. **H. Salasoo.**

123. Calluna vulgaris (L.) Hull. — Kanarbik.

Harilik nooremais liivastes männimetsades ja rabamännikutes. Luidetel, siseluidetel, liivanurmedel ja liivaseil raiesmikkudel moodustab hiljemini männimetsa eest taganeva sekundaarse massivegetatsiooni — *Calluna vulgaris*'e unionni. Enam-vähem primaarse nõmmena vähemal ulatusel Hiiumaa põhjaosa luidetel. — Esitatud materjal on selle liigi teisend, var. *glabra* N e i l r e i c h (= var. *genuina* R e g e l).

E s t o n i a m e d i a, T a r t u lähedal Lohkva küla liivaaugu serval. Saatjad (Companions): *Thymus serpyllum*, *Solidago virga-aurea*, *Peucedanum oreoselinum*, *Jasione montana*, *Hieracium umbellatum*.

Very common in young *Pinus silvestris* forests in Estonia, also in boggy pine forests. On dunes, inland dunes, sandy plains and sandy wood clearings it forms large secondary associations (a *Calluna vulgaris* union), the predecessors of *Pinus silvestris* forests. It forms more or less primary heaths only on the northern dunes of the Estonian island of Hiiumaa. — The material at hand belongs to the var. *glabra* N e i l r e i c h (= var. *genuina* R e g e l).

E s t o n i a m e d i a, vicinity of T a r t u, on the verge of a sand-pit near the village of Lohkva.

30. VII. 1931. leg. **H. Salasoo.**

124. Androsace septentrionalis Linné. — Nõmmekann.

Kasvab liivasel ja rühkmullapinnasel esinevais avaühinguis, loopealseil, ka umbrohuna põldudel. Käesolev materjal on var. *α. typica* K n u t h [F. P a x u. R. K n u t h, Primulaceae in E n g l e r, Pflanzenreich IV. 237 (1905)].

E s t o n i a s a r m a t i c a, Irboska — Petseri raudteel Vilo ja Liivamäe peatuskohtade vahel.

Occurs in open vegetation on dry sandy soils, rihk soils and alvars, also as weed on arable land. The material at hand belongs to the var. *α. typica* K n u t h [F. P a x u. R. K n u t h, Primulaceae in E n g l e r, Pflanzenreich IV. 237 (1905)].

E s t o n i a s a r m a t i c a, on the railroad embankment between the stations of Vilo and Liivamäe.

29. V. 1933. leg. **Juta Rebane.**

124-a. **Androsace septentrionalis** Linné. — **Nõmmekann.**

Estonia intermedia, Ambla khk., Jäneda asunduse kesapõllul umbrohuna.

Estonia intermedia, parish of Ambla, on a fallow field of the settlement of Jäneda.

20. VI. 1933. leg. **A. Miljan.**

125. **Trientalis europaea** Linné. — **Laanelill, metsatäht.**

[*T. europaea* L. var. *eurasiatica* Knuth.]

Harilik okas-, leht- ja segametsades, lodumetsades ja lodudes.

Alutagia, Narva-Jõesuu lähedal samblarohkes liivases männimetsas.

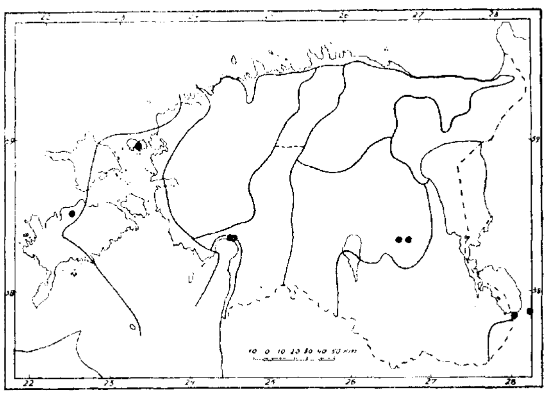
Common in coniferous, deciduous and mixed woods, in marshes and marshy woods.

Alutagia, in a mossy *Pinus silvestris* forest near the town of Narva-Jõesuu.

19. VI. 1935. leg. **V. Viktorov.**

126. **Centunculus minimus** Linné. — **Pisikas.**

Eestis haruldisena niiskeil liivastel põldudel ja jäätmaadel.



Estonia maritima orientalis, Vormsi saar, Hullo küla lähedal niidust küntud jäätmaal, *Juncus bufonius*'e unioonis.

In Estonia as a rare weed in moist sandy fields and waste places.

Estonia maritima orientalis, the island of Vormsi, in a waste place near the village of Hullo, associated with *Juncus bufonius*.

16. VII. 1933. leg. **G. Vilbaste.**

127. **Stachys silvatica** Linné. — **Mets-nõianõges.**

Esineb kogu Eestis niiskeis, varjukais lehtmetsades, kuusesegametsades, võsastikes ja puisniitudel. On võrdlemisi sage *Hepatica triloba* — *Pulmonaria officinalis*'e unioonis.

E s t o n i a s a r m a t i c a, põõsastikus 1 km Irboska alevist lõunas.

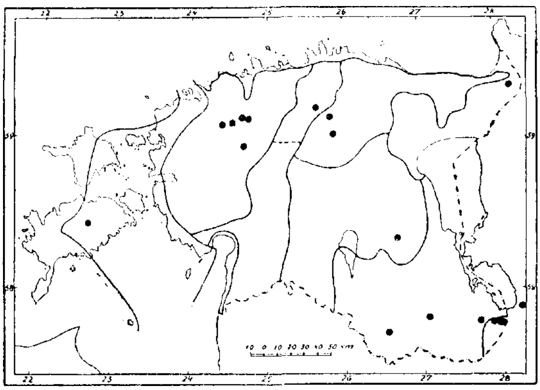
Spread over the whole territory of Estonia in humid, shady deciduous and mixed woods, in thickets and in wooded meadows. It grows relatively often in the *Hepatica triloba* — *Pulmonaria officinalis* union.

E s t o n i a s a r m a t i c a, in bushes 1 km S. of Irboska.

26. VI. 1936. leg. **A. Gahov.**

128. Salvia verticillata Linné. — Männas-salvei.

Eestisse möödunud sajandi lõpul antropohoorina sisse rändama hakanud, kiiresti kohanenud ja käesoleval ajal edasileviv taim. Esineb teeservadel, põllupeenrail, raudteetammi nõlvadel jne.



E s t o n i a i n t e r m e d i a, A m b l a khk., Lehtse-Risti koolimaja lähedal teeserval.

As an anthropochore species introduced at the end of last century, having become quickly naturalized and at present spreading further; it grows along roads, railroads, and also in balks of arable land.

E s t o n i a i n t e r m e d i a, parish of A m b l a, along a roadside near the school of Lehtse-Risti.

5. VIII. 1934. leg. **J. Lunts.**

129. Pedicularis palustris Linné. — Soo-kuuskjalg.

Väga harilik soistel niitudel ja puisniitudel, soodes ja rabastuvais soodes. *Carex Goodenowii* — *C. panicea* uniooni karakterliik.

E s t o n i a o r i e n t a l i s, P õ l v a khk., Väimela lähedal soisel niidul.

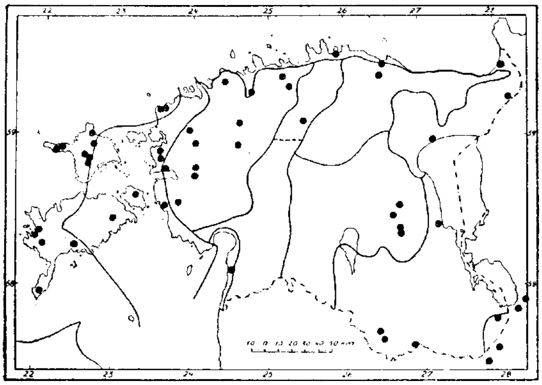
Very common in swampy meadows, wooded meadows, swamps and boggy swamps. A characteristic species of the *Carex Goodenowii* — *C. panicea* union.

E s t o n i a o r i e n t a l i s, parish of P õ l v a, in a swampy meadow of Väimela.

26. VI. 1927. leg. **W. J. Reinthal.**

130. Utricularia minor Linné. — Väike vesihernes.

Seni leitud pillatult ja näib olevat enam-vähem haruldane. Kasvab nii turba kui lubjarohkel mineraalpinnasel esinevais madalaveelistes turbaaukudes, kraavides, lompides ja allikasoodes.



E s t o n i a m a r i t i m a o c c i d e n t a l i s, K i h e l k o n n a khk., Kuusnõmme Bioloogiajaama lähedal lubjarikkas allikasoos, madalas vees *Schoenus ferrugineus*'e unioonis.

Up till now found sporadically and seems to be rare. It occurs on peaty as well as on calcareous mineral soils, and grows in the low water of peat pits, ditches, pools, also on calcareous soils near springs.

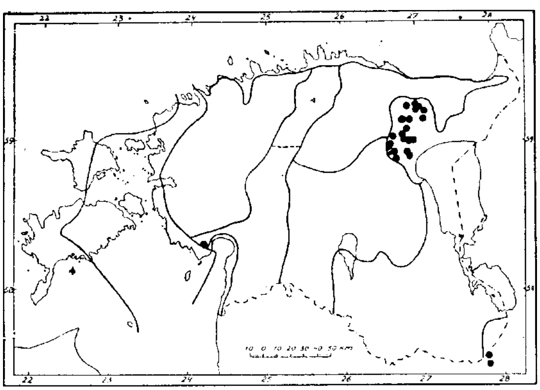
E s t o n i a m a r i t i m a o c c i d e n t a l i s, parish of K i h e l k o n n a, in *Schoenus ferrugineus* union of a calcareous water-logged ground near the Biological research station of Kuusnõmme.

3. VII. 1932. leg. **Elsa Pastak.**

131. Galium triflorum Michaux. — Kolmeõiene madar.

Eestis esmakordselt leitud a. 1931; kasvab lodumetsades, arenedes eriti raiesmikel mõnikord hulgi.

E s t o n i a s u p e r i o r, L ü g a n u s e khk., 2 km Lümatu raudteejaamast idas, kohati massiliselt kuuse-kase lodumetsa raiesmikul. Saatjad (Companions): *Calliergon cordifolium*, *Mnium cuspidatum*, *Rhytidiadelphus triquetrus*, *Dryopteris spinulosa*, *D. Linnaeana*, *Equisetum silvaticum*, *Ranunculus cassubicus*, *Circaea alpina*, *Oxalis acetosella*, *Aegopodium podagraria*, *Crepis paludosa*.



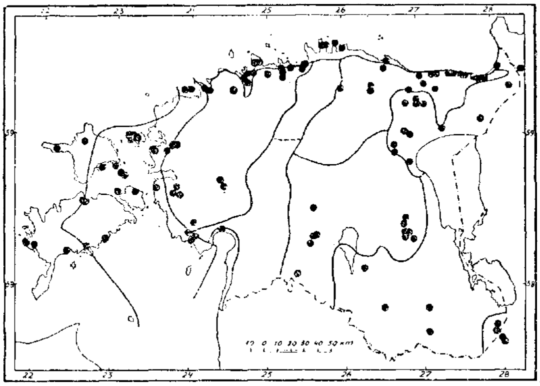
Was for the first time observed in Estonia in 1931; occurs especially on the outskirts of various marshy woods.

Estonia superior, parish of Lüganuse, 2 km E. from the Lümatu railway station, sporadically abundant in the outskirts of *Picea excelsa* — *Betula pubescens* marshy woods.

7. VII. 1935. leg. **A. Rühl.**

132. Adoxa moschatellina Linné. — Muskuslill.

Sageli hulgi varjukais ja veidi niiskeis segametsades, uhtlamm-metsades, kaldavõsastikes ja parkides, vahel ka puisniitudel.



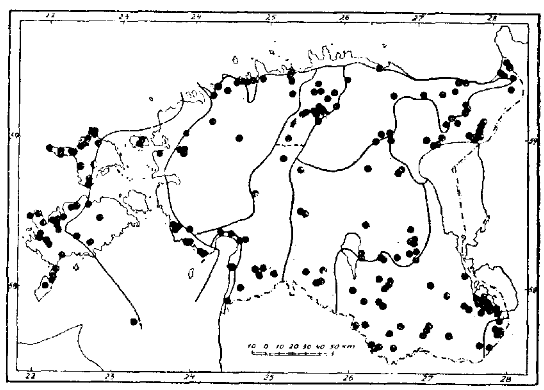
Estonia superior, Rakvere lähedal, Palermo laskeraja niiskel liivakal pinnal, *Hepatica triloba* — *Pulmonaria officinalis*'e unioonis. Saatjad (Companions): *Alnus incana, Betula verrucosa, Anemone ranunculoides, Stellaria holostea, S. media, Lysimachia nummularia, Urtica dioeca.*

In Estonia often in large colonies in shady and slightly moist mixed woods, riverside woods and thickets, in parks and sometimes also in woody meadows.

Estonia superior, near the town of Rakvere, on a moist sandy shooting place, in the *Hepatica triloba* — *Pulmonaria officinalis* union.

2. VI. 1935. ja 20. VI. 1936. leg. **Selma Kaaber.**

133. Jasione montana Linné. — Sininukk.



Käesolev materjal, servast vähe karvaste üldkatelehtedega, läheneb vormile: f. *glabra* Petermann.

Esineb ranniku- ja siseluidetel, liivanurmedel, kuivadel liivakinkudel ja hõredais männimetsades.

Estonia media, Tartu lähedal Lohkva küla Kötsi talu liivaaugu serval. Saatjad (Companions): *Potentilla argentea, Galium mollugo, Sedum acre, S. purpureum, Thymus serpyllum, Trifolium agrarium, T. arvense, Veronica spicata, Peucedanum oreoselinum, Campanula rotundifolia, Artemisia campestris, Hieracium pilosella.*

The plants available, with sparingly pubescent margins of bracts of involucre, approach the f. *glabra* Petermann in form.

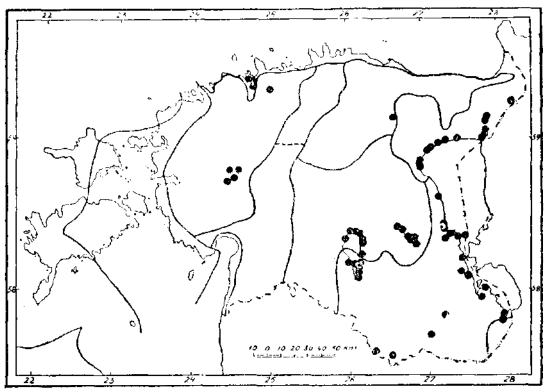
Is present on maritime and inland dunes, dry sandy plains and hills, also in thin *Pinus silvestris* forests.

Estonia media, vicinity of Tartu, on the verge of a sand-pit near the village of Lohkva.

24. VI. 1930. leg. **H. Salasoo.**

134. Inula britannica Linné. — Inglise vaak.

Esineb mandri niiskeil niitudel ja võsastikes, järve- ja jõekallastel.



Estonia orientalis, Võnnu khk., Peipsi rannikul Emajõe suubumisalal. Saatjad (Companions): *Carex elata, C. gracilis, Calamagrostis neglecta, Iris pseudacorus, Ranunculus reptans, Lychnis flos-cuculi, Galium palustre, Mentha arvensis, Sium latifolium.* Mullastiku pH 5,5.

Confined to Estonian mainland, grows in wet meadows and shrubberies, on shores and banks.

Estonia orientalis, parish of Võnnu, shore of Lake Peipsi, in the delta of the river Emajõgi.

30. VII. 1931. leg. **V. Sirgo.**

134-a. Inula britannica Linné. — Inglise vaak.

Estonia inferior, Märjamaa khk., Valgu asunduse lähedal Päärdu jõe kaldal, kogumikuna. Saatjad (Companions): *Deschampsia caespitosa* ja *Odontites serotina.*

Estonia inferior, parish of Märjamaa, in colonies on the banks of the Päärdu river, in the neighbourhood of the settlement of Valgu.

12. VIII. 1934. leg. **H. Aasamaa.**

135. Achillea millefolium Linné. — Raudrohi, verihein.

Laialdaselt levinud ja väga harilik niitudel, puisniitudel, loopealseil, teeservadel ja karjamaadel.

Estonia maritima borealis, Keila khk., Nõmme ümbruses raudtee ääres.

Widely spread and very common in meadows, wooded meadows, alvars, pastures and along roadsides.

Estonia maritima borealis, parish of Keila, along the railway near Nõmme.

1. IX. 1933. leg. **V. Sirgo.**

136. Senecio jacobaea Linné. — Voolme-ristirohi.

Antropohoorne liik, esineb pillatult kogu Eestis teeservadel, põllupeenrail jne., paiguti ka niitudel ja metsades ning nende servadel.

Estonia maritima borealis, Nõmme läheduses Kadaka paemurrus, mullaga kaetud paeprügil.

As an anthropochore species dispersed over the whole territory of Estonia, along roadsides and ridges between fields, in gravel pits etc., sometimes also in forests and meadows and along their margins.

Estonia maritima borealis, in the Kadaka quarry, near Nõmme.

I. IX. 1933. leg. **V. Sirgo.**

137. Scorzonera humilis Linné. — Madal mustjuur.

Eriti sage ja esineb hulgi Põhja-Eesti ja saarte lubjarikkail puisniitudel (*Scorzonera humilis* — *Melampyrum nemorosum*'i unioonis), harvem hõredais liivastes männimetsades, luidetel, siseluidetel ning turbapinnasel.

Estonia inferior, Keila khk., 2 km Keila alevist loodes, „Linnamäe“ nimelises kuivas, päikesepaistelises metsas tammede ja pärnade all.

Very common and abundant on the calcareous soils of N. Estonia and the islands (in the *Scorzonera humilis*—*Melampyrum nemorosum* union); scarce in thin *Pinus silvestris* forests on dry sandy soil, on dunes, inland dunes, also on peaty soil.

Estonia inferior, parish of Keila, in a dry, thin forest of *Quercus robur* and *Tilia cordata*, 2 km NW. of Keila.

19. VI. 1935. leg. **P. Kohava.**

138. Triglochin palustre Linné. — Soo-õisluht.

Väga harilik niiskeil kuni märgadel niitudel ja puisniitudel, allikate ümber, kraavides, oja-, jõe- ja järvekaldail; esineb ka mereranna halofiilseil niitudel.

Estonia inferior, Rapla khk., Lipstu küla Põldmaa talu heinamaa teel, koos *Carex flava* ja *C. contigua*'ga.

Very common on moist as well as on wet meadows and wooded meadows, around springs, in ditches, on banks along brooks, rivers and lakes, also in the saline belt of maritime meadows.

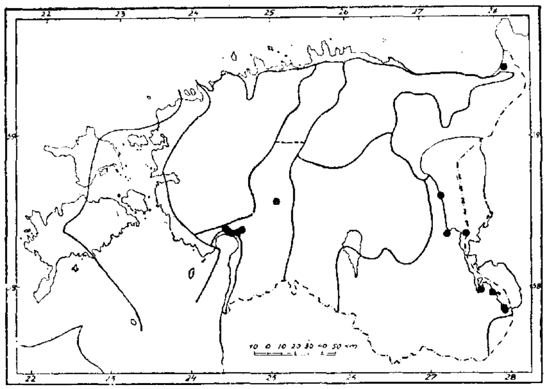
E s t o n i a i n f e r i o r, parish of R a p l a, along a meadow passage to the village of Lipstu, associated with *Carex flava* and *C. contigua*.

13. VIII. 1934. leg. **H. Aasamaa.**

139. Alisma gramineum Gmelin. — Kaarjas konnarohi.

[*A. arcuatum* M i c h a l e t; „*A. graminifolium* E h r h a r t"; *A. Loeselii* G o r s k i.]

Käesolev materjal kuulub vormi: f. *gracilis* K. R. K u p f f e r.



Senini vähe leiukohti Edela- ja Ida-Eestis, tõenäoselt rohkem levinud. Kasvab 0—120 sm sügavuses vees liivasel või mudasel põhjal, omaette või koos liikidega: *Heleocharis acicularis, Limosella aquatica, Elatine hydropiper,* harvemini koos liikidega: *Ranunculus reptans, Subularia aquatica, Isoëtes echinosporum, Callitriche autumnalis.*

L i t o r a l e h e a d e m e e s t e e n s e, P ä r n u jõesuus vasema muuli algusel, 20—50 sm sügavas vees.

The material in question belongs to the f. *gracilis* K. R. K u p f f e r.

Till now a few habitats in SW. and E. Estonia; probably dispersed more widely. It grows on sandy and muddy ground to a depth of 0—120 cm, alone or associated with *Heleocharis acicularis, Limosella aquatica, Elatine hydropiper,* as yet rarely with *Ranunculus reptans, Subularia aquatica, Isoëtes echinosporum* and *Callitriche autumnalis.*

L i t o r a l e h e a d e m e e s t e e n s e, at the mouth of Pärnu river, submerged, to a depth of 20—50 cm at the commencement of the left mole.

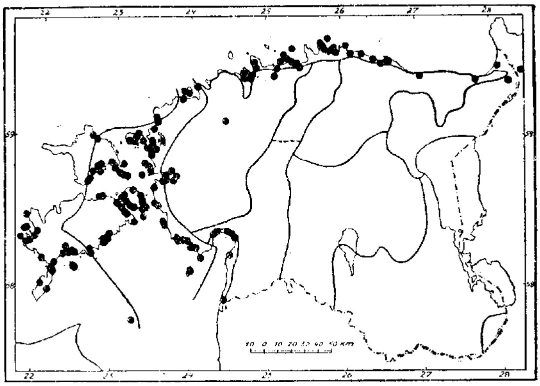
10. VIII. 1936. leg. **Edm. Spohr.**

140. Alopecurus ventricosus Persoon. — Mustjas rebasesaba.

[*A. arundinaceus* P o i r e t; *A. nigricans* H o r n e m a n n; *A. ruthenicus* W e i n m a n n.]

Peamiselt saarte ja mandri läänepoolsemate randniitude saliinses ja suprasaliinses astmes, sageli kogumikkudena.

Estonia maritima orientalis, Suure- ja Väikse-Pakri saare vahel asuva Kapa saarekese halofiilsel randniidul.



Found in Estonia in the saline and supra-saline belt of the maritime meadows, mainly on the islands and the western part of the mainland; it often grows in colonies.

Estonia maritima orientalis, in a salt meadow of the islet of Kapa between Suur- and Väike-Pakri saar.

15. VII. 1931. leg. **Tatjana Amitan-Ruckteschell.**

141. **Poa nemoralis** Linné. — **Salunurmikas.**

Harilik, eriti hõredamais leht- ja segametsades (*Hepatica triloba* — *Pulmonaria officinalis*'e unioonis), puisniitudel ja parkides.

Estonia media, Tartu Raadi pargis, varjus.

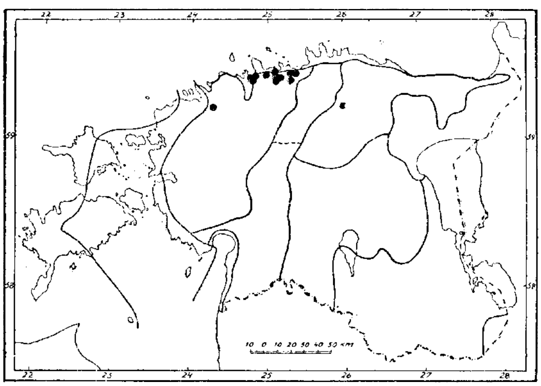
Common especially in thin foliferous and mixed woods (in the *Hepatica triloba* — *Pulmonaria officinalis* union), in wooded meadows and parks.

Estonia media, Tartu, in the shady Raadi park.

18. VII. 1935. leg. **E. Lepik** ja **Salme Käspert.**

142. **Poa alpina** Linné. — **Alpi nurmikas.**

Arkto-alpiinne, Eestis haruldane liik, esineb mandri põhjaosa (Einf, Emb, Esup) kuivadel loopealsetel.



Käesolev materjal on harilik var. *typica* Beck.

Estonia maritima borealis, Tallinna lähedal Maarjamäe kohal loopealsel.

Occurs as a rare arctic-alpine species only in N. Estonia (Einf, Emb, Esup) on dry alvar soils.

The material available belongs to the usual var. *typica* Beck.

Estonia maritima borealis, vicinity of Tallinn, in alvars south of Maarjamägi (Marienberg).

20. VI. 1934. leg. **A. Üksip.**

143. Carex chordorrhiza Ehrhart. — Peenejuurene tarn.

Esineb harilikult hulgi kõrgrabadega piirduvais õõtsrabades, üleminekurabades, rabastuvais lodudes ja lodumetsades

Estonia media, Laiuse khk., Tooma Sookatsejaama lähedal rabas.

In general abundantly in bogs, transition-bogs, boggy marshes and marshy woods.

Estonia media, parish of Laiuse, in a bog near Tooma.

VI. 1931. leg. **P. W. Thomson.**

143-a. Carex chordorrhiza Ehrhart. — Peenejuurene tarn.

Estonia media, Äksi khk., Pupastvere rabas. Saatjad (Companions): *Eriophorum vaginatum, Oxycoccus palustris, Comarum palustre, Menyanthes trifoliata.*

Estonia media, parish of Äksi, in the bog of Pupastvere.

16. VII. 1933. leg. **H. Salasoo.**

144. Carex limosa Linné. — Mudatarn.

Harilik eriti mandril, rabades, soodes ja rabastuvais lodudes.

Estonia media, Laiuse khk., Tooma sookatsejaama ümbruses.

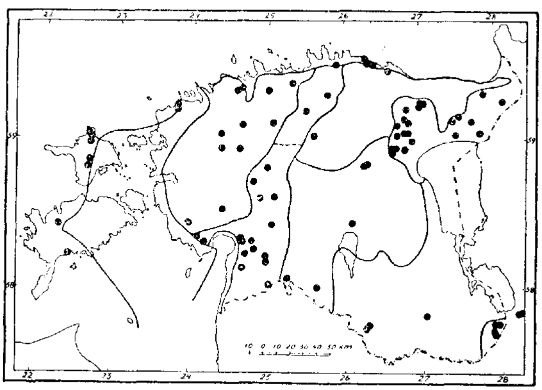
Very common especially on the mainland of Estonia, in bogs, swamps and in boggy marshes.

Estonia media, parish of Laiuse, in the vicinity of Tooma.

VI. 1931. leg. **P. W. Thomson.**

145. Carex magellanica Lamarck. — Sagristarn.

[*C. irrigua* (Wahlenb.) Smith.]



Kohati, peam. mandri rabastuvais lodumetsades ja rabade ääreosades.

Estonia intermedia, Suure-Kõpu khk., Õrdi raba lõunaserval, rabastuvas segametsas.

Confined mainly to the mainland of Estonia, usually in boggy marshy woods and marginal part of peat-bogs.

Estonia intermedia, parish of Suure-Kõpu, on the southern edge of the Ördi peat-bog, in a boggy mixed wood.

4. VI. 1930. leg. **K. Eichwald.**

145-a. **Carex magellanica** Lamarck. — **Sagristarn.**

[*C. irrigua* (Wahlenb.) Smith.]

Estonia maritima occidentalis, Risti khk., Maisoo ja Nõva vahel, rabastuvas lodumetsas.

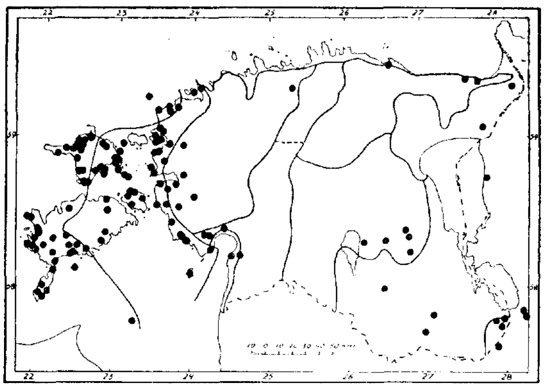
Estonia maritima occidentalis, parish of Risti, in a boggy marshy wood between Maisoo and Nõva.

2. VII. 1933. leg. **K. Eichwald.**

146. **Herminium monorchis** (L.) R. Brown. — **Muguljuur.**

[*Ophrys Monorchis* Linné.]

Lupjaeelistav liik, sage läänepoolse siluuri ala loopealseil, niiskeil niitudel ning randniitude suprasaliinses astmes.



Estonia media, Tartu lähedal Emajõe soisel luhal, 2 km Ropka asundusest idas. Saatjad (Companions): *Betula humilis, Salix rosmarinifolia, Eriophorum latifolium, Polygonum bistorta, Lathyrus palustris* jt.

A calciphilous species, more frequently distributed on western silurian soils, in alvars and humid meadows, also in the suprasaline belt of maritime meadows.

Estonia media, neighbourhood of Tartu, in a marshy riverside meadow of the Emajõgi river, 2 km east of Ropka.

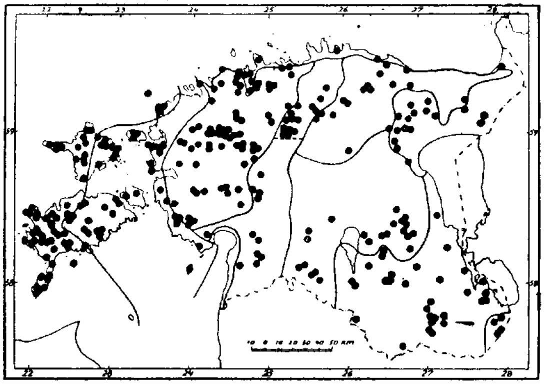
20. VI. 1930. leg. **V. Sirgo.**

147. **Epipactis palustris** (L.) Crantz. — **Soo-neiuvaip.**

[*Helleborine palustris* (Miller) Schrank.]

Lubjarikastes soodes, rabastuvates lodudes ja -segametsades.

Estonia maritima occidentalis, Kihelkonna khk., Sutru metsaga piirduvas Lümanda Suurissoos. Saatjad (Companions): *Sesleria coerulea* * *uliginosa*, *Molinia coerulea*, *Schoenus ferrugineus*, *Carex dioeca*, *C. Davalliana*, *Tofieldia calyculata*, *Orchis Traunsteineri*, *Pinguicula alpina* jt.



Found in swamps on calcareous soil, in boggy marshes and boggy mixed woods.

Estonia maritima occidentalis, parish of Kihelkonna, in the eastern part of Suurissoo swamp, near Lümanda.

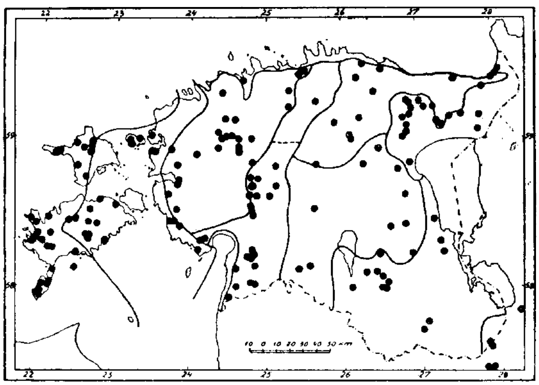
23. VII. 1933. leg. **B. Saarsoo.**

148. **Epipactis latifolia** (L.) Allioni. — **Laialehine neiuvaip.**

[*Helleborine latifolia* (Hudson) Druce.]

Varjukais salumetsades, puisniitudel ja võsades.

Estonia maritima occidentalis, Kihelkonna khk., pillatult kuival puisniidul Himmiste külast läänes. Saatjad (Companions): *Juniperus communis*, *Avena pubescens*, *Sesleria coerulea* **uliginosa*, *Carex panicea*, *C. capillaris*, *Convallaria majalis*, *Epipactis rubiginosa*, *Asperula tinctoria*, *Tetragonolobus siliquosus*, *Laserpitium latifolium*, *Campanula persicifolia*, *Inula salicina*, *Scorzonera humilis* jt.



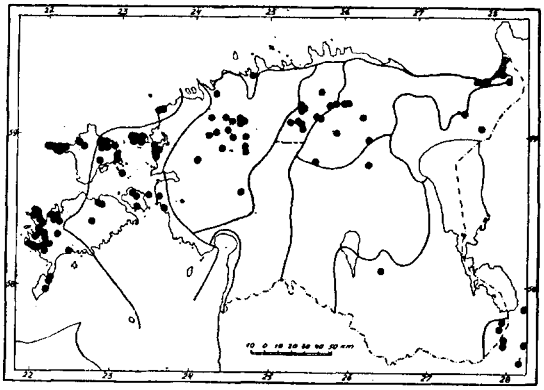
In shady groves, wooded meadows and shrubberies.

Estonia maritima occidentalis, parish of Kihelkonna, dispersed in a wooded meadow, west of the village of Himmiste.

1. VIII. 1933. leg. **B. Saarsoo.**

149. **Epipactis rubiginosa** Crantz. — **Tumepunane neiuvaip.**

[*E. atropurpurea* Rafinesque; *E. atrorubens* Schultes; *Helleborine atropurpurea* (Rafin.) Schinz et Thellung.]



Lupjaeelistava liigina eriti Põhja-Eestis ja saartel, kus kasvab rannikuluidetel ja puisniitudel; sisemaal harva ka hõredais liivastes männimetsades ning siseluidetel.

Estonia maritima occidentalis, Kihelkonna khk., kuival puisniidul Himmiste külast läänes.

As a calciphilous species distributed especially in the West and North of Estonia, — on maritime dunes and in wooded meadows; rarely on inland dunes and in thin inland *Pinus silvestris* forests on sandy soil.

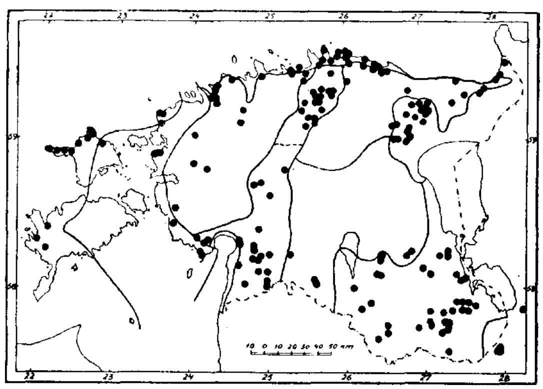
Estonia maritima occidentalis, parish of Kihelkonna, in a wooded meadow west of the village of Himmiste.

19. VII. 1933. leg. **B. Saarsoo.**

150. **Goodyera repens** (L.) R. Brown. — **Öövilge.**

[*Satyrium repens* Linné.]

Esineb Eestis peamiselt mandril, varjurohkeis vanemais kuusemetsades, kuuse-segametsades ning samblarohkeis männimetsades, sageli vähemate kogumikkudena.



Estonia media, Nõo khk., Vapramäe männi-kuuse segametsas. Saatjad (Companions): *Vaccinium myrtillus*, *Pyrola secunda*, *Fragaria vesca*, *Crepis biennis*, *Lactuca muralis*.

Especially on the mainland of Estonia, often in little colonies in shady *Picea excelsa* forests, in mossy *Pinus silvestris* forests and in mixed woods.

Estonia media, parish of Nõo, in the mixed *Pinus silvestris* — *Picea excelsa* forest of Vapramäe.

20. VII. 1930. leg. **H. Salasoo.**

ÜBER DIE ANWENDBARKEIT VON SAHA'S IONISATIONSFORMEL BEI EXTREM HOHEN TEMPERATUREN

VON

WILHELM ANDERSON

TARTU 1937

K. Mattiesens Buchdruckerei Ant.-Ges., Tartu 1937.

Solange ein Gas nicht entartet ist und keine Druckionisation vorliegt, und solange die Geschwindigkeiten der Partikelchen keine relativistischen sind, kann der Ionisationsgrad x des Gases nach Saha's bekannter Formel:

$$\log_{10}\left(\frac{x^2}{1-x^2}P\right) = -\frac{5040\,\psi}{T} + \frac{5}{2}\log_{10} T - 6{,}18 \qquad (1)$$

berechnet werden, wo P den Druck (in Atmosphären) und ψ das Ionisationspotential (in Volt) bedeutet. Diese Formel ist nicht ganz genau; trotzdem wird allgemein angenommen, daß sie annähernd richtige Resultate in allen Fällen ergebe, wo die obenerwähnten Bedingungen erfüllt sind. Ist aber eine solche Ansicht auch richtig, und wird nicht Saha's Formel selbst bei Abwesenheit von Entartung und von relativistischen Geschwindigkeiten in gewissen Fällen zu gröblich falschen Resultaten führen? — Der Untersuchung dieser Frage ist die vorliegende Schrift gewidmet.

Um ein nichtangeregtes Wasserstoffatom zu ionisieren, ist es notwendig, ihm eine Energiemenge von 13,53 e-Volt zuzuführen. Dadurch wird das Elektron von der tiefsten Quantenbahn in unendliche Entfernung übergeführt. Wenn umgekehrt das Elektron aus unendlicher Entfernung auf die tiefste Quantenbahn hinabfällt, werden 13,53 e-Volt ausgestrahlt. Eine solche Energiemenge wird ausgestrahlt, wenn die kinetische Energie des Elektrons in unendlicher Entfernung gleich Null gewesen ist. Sollte aber diese anfängliche kinetische Energie beispielsweise 8 e-Volt betragen haben, so ist die ausgestrahlte Energiemenge gleich $13{,}53 + 8 = 21{,}53$ e-Volt. Denn wäre sie bloß gleich den „normalen" 13,53 e-Volt, so müßte das Elektron auf der tiefsten Quantenbahn eine überschüssige kinetische Energie von 8 e-Volt beibehalten. Diese überschüssige Energie ist jedoch unzureichend, um das Elektron wieder in unendliche Entfernung zu bringen. Diese

8 e-Volt reichen nicht einmal aus, um das Elektron von der Grundbahn auf die nächste Quantenbahn zu heben (denn dazu wären 10,15 e-Volt nötig). Der einzige Ausweg aus der Schwierigkeit besteht darin, daß die überschüssigen 8 e-Volt zusammen mit den „normalen“ 13,53 e-Volt ausgestrahlt werden, so daß die gesamte ausgestrahlte Energie 21,53 e-Volt beträgt

Nehmen wir aber jetzt an, daß die kinetische Anfangsenergie des Elektrons 15 e-Volt betrage (also mehr als 13,53). Auch in einem solchen Falle wäre es denkbar, daß das Elektron unter Ausstrahlung von $13{,}53 + 15 = 28{,}53$ e-Volt auf der tiefsten Quantenbahn eingefangen wird. Es wäre aber auch denkbar, daß das Elektron beim Hinabfallen auf die tiefste Quantenbahn nur die „normalen“ 13,53 e-Volt ausstrahlt, so daß es eine überschüssige Energie von 15 e-Volt beibehält. Von letzterer verbraucht das Elektron 13,53 e-Volt, um sich wieder in unendliche Entfernung zu begeben. Dort wäre die kinetische Energie des Elektrons gleich $15 - 13{,}53 = 1{,}47$ e-Volt. Die anfänglichen 15 e-Volt kinetischer Energie hätten sich auf solche Weise in 13,53 e-Volt strahlender und 1,47 e-Volt kinetischer Energie verwandelt. Welchen von diesen beiden Wegen wird nun das Elektron in Wirklichkeit einschlagen? — Um diese Frage zu beantworten, wollen wir untersuchen, wie sich in einem analogen Falle das klassisch strahlende Elektron verhalten würde.

Wenn ein Elnktron mit sehr großer Anfangsgeschwindigkeit v_a neben einem Atomkern vorbeifliegt, so beschreibt es annähernd eine Hyperbel. Die Geschwindigkeit des Elektrons bleibt nicht konstant, sondern erreicht ihren maximalen Wert v_p im „Perihel“. Doch diese Geschwindigkeitsänderung ist bei großen Anfangsgeschwindigkeiten relativ gering (d. h. $\frac{v_p - v_a}{v_a}$ ist ein kleiner Bruch), so daß man die durchschnittliche Geschwindigkeit v ohne großen Fehler gleich v_p oder auch gleich v_a setzen kann. Die gesamte nach der klassischen Theorie ausgestrahlte Energiemenge ist dann gleich

$$Q = \frac{2\pi Z^4 e^{10}}{c^3 m^4 \sigma^5 v^5}, \tag{2}$$

wo Z die Ordnungszahl des Atomkerns bedeutet, σ das aus dem Brennpunkt auf die Asymptote gefällte Lot, m die Masse des Elektrons, e seine Ladung, und c die Lichtgeschwindigkeit[1]). Je kleiner v ist, desto mehr strahlt das Elektron aus. Bei genügend kleiner Anfangsgeschwindigkeit wird der Energieverlust durch Ausstrahlung so groß sein, daß die kinetische Energie $\frac{1}{2} m v_p^2$ im „Perihel“ nicht mehr ausreicht, um das Elektron wieder in unendliche Entfernung zu bringen. Das Elektron wird daher „eingefangen“. Es muß also (bei gegebenem σ) eine bestimmte „kritische“ Anfangsgeschwindigkeit existieren, unterhalb deren das Elektron eingefangen wird, oberhalb aber nicht. In ganz roher Annäherung könnte man mit Eddington[2]) annehmen, daß die kritische Geschwindigkeit v der Gleichung

$$\frac{1}{2} m v^2 = \frac{2 \pi Z^4 e^{10}}{c^3 m^4 \sigma^5 v^5} \tag{3}$$

entspreche. Dies wäre aber aus verschiedenen Gründen eine äußerst ungenaue Methode, so daß wir auf sie verzichten.

Wenn die (auf eine unendliche Entfernung bezogene) Anfangsgeschwindigkeit des Elektrons unendlich klein ist, und wenn das Elektron nicht die Fähigkeit hätte auszustrahlen, so müßte es um den Kern eine Parabel beschreiben. Die kinetische Energie im „Perihel“ würde genau ausreichen, um das Elektron wieder in unendliche Entfernung zu bringen. Nach der klassischen Theorie jedoch verliert ein beschleunigtes Elektron Energie durch Ausstrahlung, weshalb in unserem Falle die kinetische Energie im „Perihel“ unzureichend wird, um das Elektron wieder in unendliche Entfernung zu bringen. Statt ganz wegzufliegen, beginnt das Elektron um den Kern zu rotieren. Diese Bewegung ist aber nach der klassischen Theorie ebenfalls von Energieverlust durch Ausstrahlung begleitet. Das Elektron wird sich daher mehr und mehr dem Kern nähern, es wird also in Orte mit immer tieferem und tieferem elektrischem Potential gelangen, bis es schließlich auf das denkbar tiefste Potential hinabgesunken ist (das wir durch $-\psi$ Volt bezeichnen wollen) und dort zur Ruhe kommt (denn ein ewiges Rotieren um den Kern läßt die klassische Theorie selbst-

[1]) Vgl. A. S. Eddington, Der innere Aufbau der Sterne, Berlin 1928, S. 279.

[2]) A. S. Eddington, ebenda.

verständlich nicht zu). Die gesamte ausgestrahlte Energie beträgt ψ e-Volt oder $\frac{\psi e}{300}$ Erg.

Ist die Anfangsgeschwindigkeit v_a von Null verschieden, jedoch kleiner als die „kritische“, so wird das Elektron ebenfalls eingefangen, wobei aber der Prozeß des Einfangens länger dauern wird. Die gesamte dabei ausgestrahlte Energiemenge beträgt jetzt $\frac{1}{2}mv_a^2+\frac{\psi e}{300}$, sie ist also größer als früher. (Doch die Ausstrahlung pro Zeiteinheit ist jetzt geringer als früher.) Je größer die Anfangsgeschwindigkeit ist, desto größer ist die gesamte ausgestrahlte Energiemenge. Diese Regel gilt aber nur unterhalb der „kritischen“ Anfangsgeschwindigkeit. Ein Elektron mit „kritischer“ Anfangsgeschwindigkeit wird nicht eingefangen, sondern fliegt weg in unendliche Entfernung, wo es zur Ruhe kommt. Dies bedeutet, daß im Falle der „kritischen“ Anfangsgeschwindigkeit das Elektron dort zur Ruhe kommt, wo das elektrische Potential gleich Null ist; bei nur etwas kleinerer Anfangsgeschwindigkeit aber dort, wo das Potential den denkbar tiefsten Wert $-\psi$ besitzt.

Ist die Anfangsgeschwindigkeit kleiner als die „kritische“, so beginnt der Prozeß mit einer kinetischen Energie gleich $\frac{1}{2}mv_a^2$ und einer potentiellen gleich Null, so daß die anfängliche Gesamtenergie gleich $\frac{1}{2}mv_a^2+0=\frac{1}{2}mv_a^2$ ist. Am Ende des Prozesses ist die kinetische Energie gleich Null und die potentielle gleich $-\frac{\psi e}{300}$, so daß die gesamte Energie gleich $0+\left(-\frac{\psi e}{300}\right)=$ $=-\frac{\psi e}{300}$ ist. Die ausgestrahlte Energiemenge ist gleich der Energiedifferenz am Anfang und am Ende des Prozesses, d. h. gleich $\frac{1}{2}mv_a^2-\left(-\frac{\psi e}{300}\right)=\frac{1}{2}mv_a^2+\frac{\psi e}{300}$.

Wollen wir jetzt aber annehmen, daß die Anfangsgeschwindigkeit v_a gleich der kritischen ist. Der Prozeß beginnt mit einer kinetischen Energie gleich $\frac{1}{2}mv_a^2$ und einer potentiellen gleich Null, also mit einer Gesamtenergie gleich $\frac{1}{2}mv_a^2$. Der

Prozeß endet mit einer kinetischen Energie gleich Null und einer potentiellen gleich Null, also mit einer Gesamtenergie ebenfalls gleich Null. Die ausgestrahlte Energiemenge ist gleich der Energiedifferenz am Anfang und am Ende des Prozesses, d. h. gleich $\frac{1}{2} m v_a^2 - 0 = \frac{1}{2} m v_a^2$. Dicht unterhalb der „kritischen" Anfangsgeschwindigkeit ist aber die ausgestrahlte Energiemenge gleich $\frac{1}{2} m v_a^2 + \frac{\psi e}{300}$. Daraus folgt, daß wenn die Anfangsgeschwindigkeit den „kritischen" Wert überschreitet, die ausgestrahlte Energiemenge sich sprungweise um $\frac{\psi e}{300}$ Erg verringert. Beim weiteren Steigen der Anfangsgeschwindigkeit wird nach (2) die ausgestrahlte Energiemenge weiter abnehmen, jedoch kontinuierlich.

Wir können erwarten, daß die wirklichen Elektronen sich in dieser Hinsicht bis zu einem gewissen Grade ähnlich den „klassischen" verhalten. Danach muß es eine gewisse „kritische" Anfangsgeschwindigkeit (und also auch eine „kritische" Anfangsenergie) geben, bei deren Überschreitung die (wirklichen) Elektronen von den Atomkernen (oder von den Atomionen) nicht mehr eingefangen werden können.

Fällt ein (wirkliches) Elektron mit der anfänglichen kinetischen Energie gleich Null auf die tiefste Quantenbahn so strahlt es $\frac{\psi e}{300}$ Erg aus, wo ψ das Ionisationspotential bedeutet. Ist hingegen die anfängliche kinetische Energie des Elektrons von Null verschieden und genügt sie der Bedingung $\frac{1}{2} m v_a^2 < \frac{\psi e}{300}$, so wird die ausgestrahlte Energiemenge $\frac{1}{2} m v_a^2 + \frac{\psi e}{300}$ betragen. Unser Elektron muß auf solche Weise eingefangen werden, weil nach „normaler" Ausstrahlung von $\frac{\psi e}{300}$ Erg die überschüssige Energie auf der tiefsten Quantenbahn ungenügend ist, um das Elektron wieder in unendliche Entfernung zu bringen. Deshalb muß $\frac{1}{2} m v_a^2$ zusammen mit $\frac{\psi e}{300}$ ausgestrahlt werden.

Ist aber $\frac{1}{2}mv_a^2 = \frac{\psi e}{300}$, so besitzt das Elektron (nach normaler Ausstrahlung von $\frac{\psi e}{300}$ Erg) auf der tiefsten Quantenbahn eine überschüssige Energie von $\frac{\psi e}{300}$ Erg, die genau dazu ausreicht, um das Elektron wieder in unendliche Entfernung zu bringen. Da das Elektron jetzt imstande ist in die Unendlichkeit wegzufliegen, wird es auch tatsächlich wegfliegen (genau so, wie ein klassisch strahlendes Elektron, welches im „Perihel" noch genug kinetische Energie besitzt, um in unendliche Entfernung zu fliegen, auch tatsächlich wegfliegt; keinesfalls wird ein solches Elektron seine überschüssige kinetische Energie dazu benutzen, um sie restlos in strahlende Energie zu verwandeln!). Auf diese Weise wird das Elektron nur die „normalen" $\frac{\psi e}{300}$ Erg ausstrahlen. Dicht unterhalb der „kritischen" Anfangsenergie (die gleich $\frac{1}{2}mv_a^2 = \frac{\psi e}{300}$ zu setzen ist) wird das Elektron eingefangen, wobei $\frac{1}{2}mv_a^2 + \frac{\psi e}{300} = \frac{\psi e}{300} + \frac{\psi e}{300} = \frac{2\,\psi e}{300}$ Erg ausgestrahlt wird. Daraus folgt, daß wenn die Anfangsenergie (und also auch die Anfangsgeschwindigkeit) den „kritischen" Wert überschreitet, die ausgestrahlte Energiemenge sich sprungweise um $\frac{\psi e}{300}$ Erg verringert. Dies ist genau dieselbe Regel wie im „klassischen" Falle (s. oben). Doch im letzteren Falle nimmt die ausgestrahlte Energiemenge bei weiter steigender Anfangsgeschwindigkeit kontinuierlich ab. Dies könnte man quantentheoretisch vielleicht so deuten, daß bei sehr großer Geschwindigkeit des Elektrons die ausgestrahlte Energie nicht Zeit hat sich genügend zu entfernen und deshalb von dem fortfliegenden Elektron teilweise wieder absorbiert wird. Auf diese Weise gelangt ein entsprechend weniger energiereiches Quantum $h\nu$ zur endgültigen Ausstrahlung.

Die durchschnittliche kinetische Energie nichtentarteter freier Elektronen ist gleich $\frac{3}{2}kT$, wenn die Geschwindigkeiten keine relativistischen sind, und gleich $3\,kT$ bei relativistischen Geschwindigkeiten[3]). Soll nun (im nichtrelativistischen Falle) $\frac{3}{2}kT$ gleich der „kritischen" Anfangsenergie sein, so muß die Gleichung

$$\frac{3}{2}kT = \frac{1}{2}mv_a^2 = \frac{\psi e}{300}$$

erfüllt sein. Daraus läßt sich die entsprechende „kritische" Ionisationstemperatur zu

$$T = \frac{\psi e}{450\,k} \tag{4}$$

berechnen. Im relativistischen Falle haben wir:

$$3\,kT = \frac{\psi e}{300}$$

und

$$T = \frac{\psi e}{900\,k}. \tag{5}$$

Setzt man $e = 4{,}77.10^{-10}$ und $k = 1{,}372.10^{-16}$, so erhält man aus (4) und (5):

$$T = 7{,}726.10^3\,\psi \text{ [nichtrelativistisch]}, \tag{6}$$

$$T = 3{,}863.10^3\,\psi \text{ [relativistisch]}. \tag{7}$$

Saha's Formel (1) basiert auf der Annahme eines reversiblen Prozesses:

Neutrales Atom $\rightleftarrows$ Ion + Elektron — Bindungsenergie,

wobei stillschweigend vorausgesetzt wird, daß die Begegnung eines Ions mit einem Elektron unter Ausstrahlung der Bindungsenergie gleichbedeutend sei mit der Entstehung eines neutralen Atoms. In Wirklichkeit aber ist diese stillschweigende Voraussetzung nur unterhalb der „kritischen"

[3]) Vgl. W. Anderson, „Existiert eine obere Grenze für die Dichte der Materie und der Energie?", Acta et Comm. Universitatis Tartuensis (Dorpatensis) A XXIX$_9$, Tartu 1936, S. 90.

Ionisationstemperatur zutreffend[4]), oberhalb hingegen verläuft jede Begegnung eines Ions mit einem Elektron ohne daß letzteres vom ersteren eingefangen werde, selbst wenn bei der Begegnung die Bindungsenergie ausgestrahlt wird. Es ist daher klar, daß Saha's Formel nur unterhalb der „kritischen" Ionisationstemperatur angewendet werden darf.

Die „kritische" Ionisationstemperatur des Wasserstoffs ist nach (6) gleich

$$T = 7{,}726 . 10^3 . 13{,}53 = 105000^0 \text{ (rund gerechnet).}$$

Unterhalb dieser Temperatur kann der Ionisationsgrad x durch genügenden Druck beliebig herabgesetzt werden, entsprechend Saha's Formel. Oberhalb von 105000^0 hingegen bleibt Wasserstoff unter jedem beliebigen Druck völlig ionisiert. Man könnte bildlich sagen, daß bei 105000^0 die Elektronengeschwindigkeiten einen „parabolischen" Charakter haben; bei noch höherer Temperaturen — einen „hyperbolischen". Dieser Charakter hängt ausschließlich von ψ und von der kinetischen Energie, d. h. von der Temperatur ab, und keinesfalls vom Drucke (solange wenigstens keine Druckionisation eingetreten ist). Unterhalb von 105000^0 ist das Einfangen von Elektronen prinzipiell möglich, doch seine Häufigkeit hängt von der Häufigkeit günstiger Begegnungen, d. h. von der Dichte und also auch vom Drucke, ab. Oberhalb von 105000^0 hingegen kann die Häufigkeit der Begegnungen so groß sein, wie sie will: alle müssen „ungünstig" verlaufen wegen des „hyperbolischen" Charakters der Elektronengeschwindigkeiten, wodurch das Einfangen zu einer prinzipiellen Unmöglichkeit wird. Bei der Ableitung von Saha's Formel ist nur der (vom Drucke abhängige) „Häufigkeitsfaktor" der günstigen Begegnungen in Betracht gezogen worden, keinesfalls aber der andere (vom Drucke unabhängige) Faktor, der den Elektronengeschwindigkeiten einen „hyperbolischen" Charakter mitteilt, sobald die Temperatur eine bestimmte Höhe überschreitet. Es ist daher klar, daß Saha's Formel nur unterhalb der „kritischen" Ionisationstemperatur angewendet werden darf. In der beigefügten Tabelle sind einige „kritische" Ionisationstemperaturen zusammengestellt.

[4]) Zusammen mit der Bindungsenergie wird in diesem Falle zwangsweise auch die kinetische Anfangsenergie des Elektrons ausgestrahlt.

	Ionisations-potential	„Kritische" Ionisations-temperatur
$Cs \rightleftarrows Cs_{+}$ + Elektron	3,87 Volt	29900°
$Ca \rightleftarrows Ca_{+}$ + Elektron	6,09 „	47100
$Ca_{+} \rightleftarrows Ca_{++}$ + Elektron	11,82 „	91300
H ⇄ Proton + Elektron	13,53 „	105000
$He \rightleftarrows He_{+}$ + Elektron	24,47 „	189000
$He_{+} \rightleftarrows$ Heliumkern + Elektron	54,14 „	418000
25-faches Eisenion ⇄ Eisenkern + Elektron	9150 „	$7{,}07.10^{7}$
Proton ⇄ Neutron + Positron	$1{,}53.10^{6}$ „	$5{,}9.10^{9}$

Es wird heute ziemlich allgemein angenommen, daß das Proton aus einem Neutron und einem Positron besteht. Es wäre jedoch aus verschiedenen Gründen sehr schwierig, eine Protonenzertrümmerung experimentell nachzuweisen [5]). Den Massendefekt können wir rund gerechnet gleich der 3-fachen Elektronenmasse setzen, was eine Bindungsenergie von $3.9.10^{-28}.9.10^{20} = 2{,}43.10^{-6}$ Erg $= 1{,}53.10^{6}$ e-Volt repräsentiert. Die „kritische" Ionisationstemperatur muß nach (7) berechnet werden, da in unserem Falle die Geschwindigkeiten der Positronen bereits relativistisch sind. Wir erhalten $5{,}9.10^{9}$, oder rund gerechnet 6 Milliarden Grad. [Die Formel (6) würde den doppelten Wert ergeben.] Dies bedeutet, daß bei einer Temperatur von über 6 Milliarden Grad die Neutronen und Positronen sich unter keinem Druck zu Protonen vereinigen können. Ohne Protonen können aber die Atomkerne nicht aufgebaut werden. Daraus folgt, daß bei 6 Milliarden (oder mehr) Grad nur freie Neutronen, Positronen (und Elektronen) existieren können. Bei dieser Temperatur kann die Entstehung von Atomkernen durch keinen Druck erzwungen werden.

Wenn keine „kritische" Ionisationstemperatur existieren würde, müßte Saha's Formel (1) in allen nichtentarteten und

[5]) Vgl. W. Anderson, l. c. S. 66 ff.

nichtrelativistischen Fällen anwendbar sein, also auch bei $\psi = 0$ und $T > 0$. In einem solchen Falle hätten wir:

$$\log_{10}\left(\frac{x^2}{1-x^2}P\right) = \frac{5}{2}\log_{10} T - 6{,}18,$$

also

$$0 < x < 1$$

bei jedem endlichen P. Dies würde bedeuten, daß bei $\psi = 0$ d. h. bei Abwesenheit jeder Bindekraft, ein Teil der Elektronen trotzdem gebunden und die Ionisation unvollständig bliebe. Ein solches Resultat ist offenbar absurd, und es hat seine Ursache darin, daß wir die Existenz der „kritischen" Ionisationstemperatur nicht in Betracht gezogen haben. Nach (6) ist bei $\psi = 0$ die „kritische" Ionisationstemperatur gleich

$$T = 7{,}726\,.\,10^3\,\psi = 0.$$

Nun kann aber Saha's Formel (1) nur unterhalb der „kritischen" Ionisationstemperatur angewendet werden, also in unserem Falle nur unterhalb 0^0 abs. Eine solche Temperatur ist jedoch physisch unmöglich. Daraus folgt, daß bei $\psi = 0$ Saha's Formel überhaupt nicht angewendet werden darf. Wendet man sie aber trotzdem bei genau $T = 0$ an, so erhält man:

$$\log_{10}\left(\frac{x^2}{1-x^2}P\right) = -\frac{5040\,.\,0}{0} + \frac{5}{2}\log_{10} 0 - 6{,}18 = -\infty,$$

also:

$$\frac{x^2}{1-x^2}P = 0.$$

Saha's Formel bezieht sich nur auf ideale Gase, bei denen bei $T = 0$ auch $P = 0$ sein muß. Wir erhalten also:

$$\frac{x^2}{1-x^2} = \frac{0}{P} = \frac{0}{0}.$$

Dies bedeutet, daß x unbestimmt bleibt.

Wir haben die „kritische" Ionisationstemperatur so abgeleitet, als ob bei T die kinetische Energie eines jeden einzelnen Elektrons $\frac{3}{2}kT$ betrüge. In Wirklichkeit stellt $\frac{3}{2}kT$ bloß die durchschnittliche kinetische Energie dar. Deshalb wird die „kritische" Ionisationstemperatur in Wirklichkeit keinen absolut scharfen Temperaturpunkt darstellen, sondern wird mehr oder weniger „verwaschen" erscheinen: ganz so, wie auch die kritische Siedetemperatur einer Flüssigkeit nach genauen Messungen keinen absolut scharfen Temperaturpunkt darstellt, sondern mehr oder weniger „verwaschen" erscheint. „Ohne in die Erscheinungen in der Nähe des kritischen Zustandes tiefer eingedrungen zu sein, sieht man die kritische Isotherme vielfach als die Grenze an, oberhalb welcher der Stoff sich bereits im gasförmigen Zustande befinden soll, obgleich jedes Isothermennetz deutlich erkennen läßt, daß es ein Zwischengebiet gibt, wo die Materie weder gasförmig noch flüssig ist, sondern vielmehr beiden Zuständen zugleich angehört, und zwar noch viele Grade oberhalb der kritischen Temperatur" [6]). „Unzweifelhaft besteht die flüssige Phase fein verteilt noch oberhalb des kritischen Zustandes fort, wie dieses aus der Färbung des Rohrinhalts in der Durchsicht ... hervorgeht" [7]).

Vom Standpunkt der Quantentheorie besteht für jedes Elektron eines ionisierten Gases eine bestimmte Wahrscheinlichkeit, im Verlaufe einer bestimmten Zeit von einem Ion eingefangen zu werden; es sei ω diese Wahrscheinlichkeit. Das Elektron wird nur dann eingefangen, wenn folgende zwei Ereignisse zusammentreffen: 1) das Elektron fällt auf die tiefste Quantenbahn [8]) und 2) es besitzt auf dieser Quantenbahn eine ungenügende überschüssige Energie, um wieder in die Unendlichkeit zu fliegen (die gesamte überschüssige Energie muß daher zusammen mit der Bindungsenergie ausgestrahlt werden). Die Wahrscheinlichkeit des ersteren Ereignisses bezeichnen wir

[6]) J. Wilip, „Experimentelle Studien über die Bestimmung von Isothermen und kritischen Konstanten", Acta et Comm. Universitatis Tartuensis (Dorpatensis) A VI$_2$, Tartu 1924, S. 4.

[7]) Ebenda, S. 73.

[8]) Der Einfachheit halber ignorieren wir die Existenz höherer Quantenbahnen, genau so, wie auch Saha sie ignoriert hat bei der Ableitung seiner Formel.

durch ω_1, diejenige des letzteren durch ω_2. Dann ist die Wahrscheinlichkeit des Einfangens gleich

$$\omega = \omega_1 \omega_2.$$

ω_1 kann sehr verschiedene Werte haben[9]), ω_2 hingegen nur zwei: 1 oder 0. Ist die Anfangsgeschwindigkeit kleiner als die „kritische“, so ist die überschüssige Energie des Elektrons auf der tiefsten Quantenbahn nicht nur wahrscheinlich, sondern sicherlich ungenügend, um das Elektron wieder in unendliche Entfernung zu bringen. Wir haben also in diesem Falle $\omega_2 = 1$ zu setzen. Ist hingegen die Anfangsgeschwindigkeit größer als die „kritische“, so fliegt das Elektron sicherlich weg. In diesem Falle haben wir $\omega_2 = 0$ zu setzen. Unterhalb der „kritischen“ Anfangsgeschwindigkeit ist also die Wahrscheinlichkeit des Einfangens gleich

$$\omega = \omega_1 \omega_2 = \omega_1 . 1 = \omega_1 ;$$

oberhalb der „kritischen“ Anfangsgeschwindigkeit hingegen gleich

$$\omega = \omega_1 \omega_2 = \omega_1 . 0 = 0.$$

Bei der Ableitung von Saha's Formel wird stillschweigend ω mit ω_1 identifiziert, was in Wirklichkeit aber nur unterhalb der „kritischen“ Anfangsgeschwindigkeit der Elektronen zulässig ist.

Wenn die Anfangsgeschwindigkeit des Elektrons[10]) den „kritischen“ Wert überschreitet, so fällt die Wahrscheinlichkeit des Einfangens dieses Elektrons sprungweise auf Null.

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[9]) ω_1 hängt unter anderem von der Dichte des Gases, also auch vom Drucke ab.

[10]) Die (verhältnismäßig kleinen) Geschwindigkeiten der Ionen wollen wir der Einfachheit halber vernachlässigen.

Zusatznote.

Der Verfasser, Dr. phil. nat. Wilhelm Anderson, ist durch eine plötzlich eingetretene schwere Erkrankung zur Zeit verhindert, eine beabsichtigte Ergänzung zu der vorliegenden Arbeit [Acta et Comm. Univ. Tart. (Dorp.) A XXXIII.$_7$ = Publ. de l'Obs. Astr. de l'Univ. de Tartu 30.$_2$] druckfertig zu machen. Es handelt sich dabei hauptsächlich um den auf S. 9 der Arbeit eingeführten und weiterhin angewandten Begriff einer „kritischen" Ionisationstemperatur. Falls nämlich die vom Verfasser abgeleitete kritische oder maximale Geschwindigkeit für die Wiedervereinigung als richtig zu betrachten ist, folgt daraus, daß es eine im strengen Sinne kritische Temperatur nicht geben kann, da infolge der Maxwellschen Geschwindigkeitsverteilung ein gewisser Bruchteil der Geschwindigkeiten unterhalb des kritischen Wertes fallen muß. Die Wiedervereinigung ist bei hohen Temperaturen also wohl behindert, nicht aber ausgeschlossen.

Der Direktor der Sternwarte Tartu.

MISCELLANEOUS ASTROPHYSICAL NOTES

TARTU 1938

K. Mattieseni trükikoda o.-ü., Tartu 1938.

I. On the Empirical Mass-Luminosity Relation[1].

By Jacob Gabovitš.

1. Introduction. Empirical mass-luminosity relations have been established on several occasions; the question of the statistical error in such relations mostly remains obscure. Thus, in a recent paper, Huffer (*Ap. J.* **80,** 269, 1934) published a list of known mass ratios of visual binary stars; from these data and the orbital elements he computes the masses of sixty stars and gives an empirical mass-luminosity relation using the visual absolute magnitudes instead of the bolometric magnitudes. The dispersion of the masses from the mean curve obtained from Huffer's material is considerable, but it is due more probably to observational errors than to a real cosmic spread. The aim of the present note is to confirm this supposition, as well as to establish a reliable empirical mass-bolometric magnitude relation; only the best observed binaries with good trigonometric parallaxes were chosen for this purpose.

2. Selection of Material. The masses of the two components of a visual binary star (μ_1 and μ_2) are given by the formulae:

$$\left.\begin{aligned} \mu_2 &= \frac{k\,a^3}{P^2\,\pi^3} \\ \mu_1 &= \mu_2\left(\frac{1}{k}-1\right) \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1),$$

where a denotes the semi-major axis in seconds of arc, π — the parallax, P — the period in years, and $k = \frac{\mu_2}{\mu_1 + \mu_2}$. As the greatest uncertainty in the determination of mass appears to originate from the error in parallax, we decided to use only stars with a probable error in the parallax not exceeding 8 per cent

[1] Seminar in Astrophysics, 1936/37, conducted by E. Öpik.

of the parallax itself. Further, we rejected stars with unknown spectra, since the bolometric correction in such cases is quite uncertain. In complex systems all visual components which are spectroscopic binaries were also rejected. The following stars were further excluded: 70 Ophiuchi A (invisible companion), 85 Pegasi (unreliable mass ratio), 61 Cygni (uncertain orbital elements), and o^2 Eridani B and Sirius B (white dwarfs). All

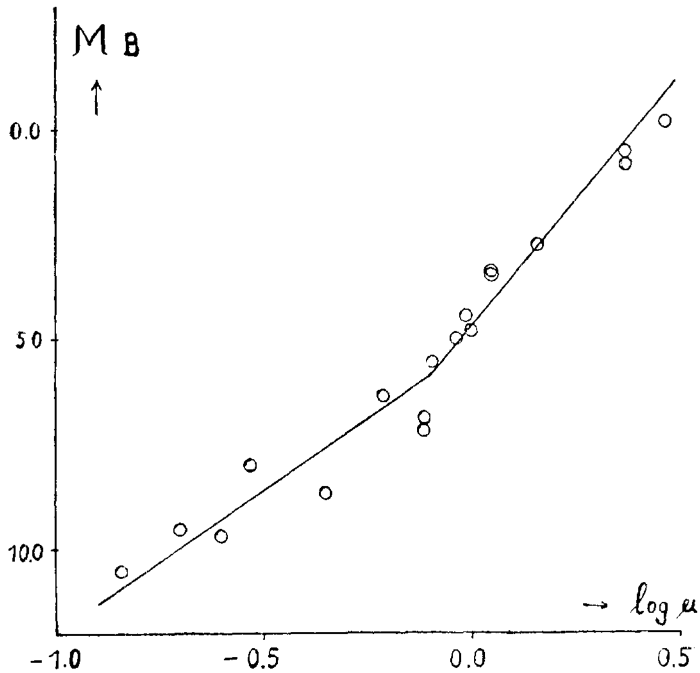

Fig. 1. The mass-luminosity relation. Abscissae — logarithm of mass; ordinates — bolometric absolute magnitude.

unsuitable stars having thus been eliminated, a small but well determined material of masses and bolometric magnitudes is left over.

3. Observational Data. Table I gives the observational data for the stars considered. The consecutive columns give: (1) the name of the star; (2) the spectrum; (3) the apparent visual magnitude (from *T. P.* **25.** 6); (4) the period of revolution in years; (5) the semi-major axis of the orbit in seconds of arc; the orbital elements are taken from Finsen's catalogue,

Table I.

The Masses and Bolometric Absolute Magnitudes of Visual Binary Stars.

1 Star	2 Sp.	3 vis. mag.	4 P	5 a	6 k	7 $\pi \pm \Delta\pi$	8 μ $\odot = 1$	9 M_v	10 C	11 ΔM_B	12 M_B	13 $\log \mu$
η Cas A	F9	3.7	479^y	11″.9	0.24	0″.182 ± 0″.005	0.93	5.0	0.41	0.02	5.0	−0.03
„ B	M0	7.4					0.29	8.6	1.38	0.62	8.0	−0.53
o^2 Eri C	M5	11.0	248	6.89	.31	.202 3	0.20	11.3	2.25	1.75	9.5	−0.70
Capella A	G1	0.74	0.285	0.0536	.44	.071 4	2.97	0.00	0.75	0.13	−0.13	0.47
„ a	F5	1.2					2.33	0.5	0.39	0.02	0.5	0.37
Sirius A	A0	−1.58	49.9	7.62	.32	.373 2	2.33	1.28	(10000°)	0.48	0.80	0.37
Procyon A	F3	0.48	40.2	4.26	.26	.291 4	1.44	2.80	0.28	0.03	2.77	0.16
γ Vir A	F0	3.65	182	3.74	.50	.089 7	1.12	3.40	0.22	0.04	3.36	0.05
„ B	F0	3.68					1.12	3.43	0.22	0.04	3.39	0.05
ξ Boo A	G5	4.80	151	4.87	.49	.147 6	0.81	5.64	0.63	0.08	5.56	−0.09
„ B	K5	6.8					0.78	7.6	1.20	0.46	7.1	−0.11
44 ι Boo A	G1	5.28	205	3.58	.54	.079 5	1.01	4.77	0.50	0.03	4.74	0.00
β 416 A	K5	5.99	42.2	1.83	.43	.147 6	0.61	6.82	1.20	0.46	6.36	−0.21
26 Dra A	G1	5.34	80.6	1.51	.47	.066 5	0.97	4.44	0.47	0.03	4.41	−0.01
μ Her B	M3	10.2	43.0	1.29	.50	.109 6	0.45	9.9	1.85	1.20	8.7	−0.35
70 Oph B	K6	6.0	87.7	4.495	.50	.196 4	0.78	7.5	1.29	0.55	6.9	−0.11
Kr 60 A	M3	9.2	44.5	2.36	0.37	0″.258 ± 0″.004	0.25	10.8	1.85	1.20	9.6	−0.60
„ B	M4	10.7					0.14	12.0	1.99	1.4	10.6	−0.84

Union Observatory Circular No. 91 (except for 26 Draconis; cf. Huffer, *loc. cit.*); (6) the mass ratio from Huffer's list; (7) the trigonometric parallax and its probable error, from Schlesinger's new Parallax Catalogue; (8) the mass, computed from formula (1); (9) the visual absolute magnitude, computed from the data in columns (3) and (7); in the case of the M type stars the TiO correction, as given in *T. P.* **28.**5, Table II, is added; (10) the colour index, taken from *T. P.* **27.**1 (F and G stars), and *T. P.* **28.**5 (K and M stars); as to Sirius A (H. D. spectrum A0), it seems to be advisable in this case to use the ionization temperature instead of the colour temperature; we assume for Sirius A: $T = 10\,000^0$ Abs.; (11) the bolometric correction (for derivation cf. *T. P.* **28.**3, formulae 7 and 9); (12) the bolometric absolute magnitude; (13) the logarithm of mass.

4. The Mass-Luminosity Relation. Fig. 1 shows the correlation between *log*mass and bolometric absolute magnitude, obtained from the data in Table I. Although the material is scanty, the correlation is of a small dispersion and thus well defined. Actually the mass-luminosity law, as given in our graph, may be fairly well represented by two linear relations between $\log \mu$ and M_B. For the A — dK stars we get:

$$\log \mu = 0.40 - 0.085\ M_B \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

and for the M dwarfs:

$$\log \mu = 0.77 - 0.15\ M_B \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Introducing the luminosity L ($\odot = 1$) into the expressions (2) and (3), we obtain:

$$L = 0.90\,\mu^{4.7} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2\text{ a})$$

and

$$L = 0.56\,\mu^{2.7} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3\text{ a}).$$

Thus, the luminosities of the A — K stars vary nearly as the fifth power of the masses, whereas the luminosities of the M-type dwarfs are approximately proportional to the cube of the masses.

The observed dispersion of *log*mass from the mean curve [actually we take the dispersions from the straight lines (2) and (3)] is calculated from $\Delta_0 = \pm \sqrt{\frac{\Sigma \Delta^2}{n-k}}$, where n is the number of stars, and k the effective number of groups, or normal points; we assume $k=3$, because three points determine our pair of intersecting straight lines. We find:

$$\Delta_0 = \pm 0.07.$$

The observational error dispersion (Δ_e) of $\log \mu$ is given by

$$0{,}674\, \Delta_e = \sqrt{\Delta_k^2 + \Delta_a^2 + \left(\Delta_m \cdot \frac{d \log \mu}{d M_B}\right)^2 + (3\, \Delta_\pi)^2},$$

where Δ_k is the p. e. of $\log k$, Δ_a — the p. e. of $\log \frac{a^3}{P^2}$, Δ_m — the photometric error of M_B, and Δ_π — the p. e. of $\log \pi$. As the error of $\log \mu$ is mostly due to the parallax error, we shall consider the "minimum" value of Δ_e, putting $\Delta_k = \Delta_a = \Delta_m = 0$, and taking into account Δ_π only. From column 7 of Table I we get $\Delta_\pi = \pm 0.04$, whence

$$\Delta_e = \pm 0.18.$$

Thus the observed dispersion of $\log \mu$ amounts to only two-fifths of the "minimum" error dispersion, and the true cosmic spread of the masses from the mean curve must be practically zero. We conclude that, as revealed by our selected first-class data, the stars (chiefly of the main sequence) probably follow a strict mass-luminosity law.

II. On the Orientation of the Orbital Planes in Multiple Systems[1].

By Jacob Gabovitš.

Our knowledge of the relative inclinations of the orbits in multiple systems is rather scanty. The question is, whether the plane of revolution of a close pair coincides, at least approximately, with the plane of revolution of the farther component, or whether the two planes are independent of each other. Only in one case do we know with certainty that the two planes of revolution practically coincide, namely in the triple system 44 ι Bootis. The inclination of the visual pair AB is 83° (W. Finsen, *Union Obs. Circ.* No. 91); the fainter component is an eclipsing variable, and thus its inclination must also be near 90°. In order to get more information on the relative inclinations, we propose to treat the problem statistically; for the inclinations we use the catalogue compiled by Finsen (*loc. cit.*).

We consider separately the visual binary stars with two known components only, and the visual systems, where at least one component is a spectroscopic binary. If there exists no correlation between the two planes of revolution in complex systems, the distribution of the visual inclinations must be the same in both classes of systems. If, however, the planes in question coincide, the average visual inclination in complex systems must be greater than the average inclination in simple systems, on account of the well-known fact that, owing to observational selection, the average inclination of spectroscopic binaries is greater than the expected statistical average. The actual averages from our material are as follows:

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[1] Seminar in Astrophysics, 1936/37, conducted by E. Öpik.

Table I.

	$\overline{\sin i}$	Number
Triple (or multiple) systems	0.850 ± 0.022	20
Double systems	0.735 ± 0.020	122
All	0.750 ± 0.017	142

The difference between the two values of $\overline{\sin i}$ seems to be real, which speaks in favour of the coincidence of the orbital planes in multiple systems. The effect is still more pronounced, if we consider the distribution of $\sin i$ in both cases (Table II). Whereas in the simple systems all values of $\sin i$ from 0.0 to 1.0 are present, the values of $\sin i$ in complex systems occur exclusively in the interval from 0.6 to 1.0.

Table II.

Distribution of $\sin i$ in Visual Binaries.

Sin i of the visual binary	.0—.1	.1—.2	.2—.3	.3—.4	.4—.5	.5—.6	.6—.7	.7—.8	.8—.9	.9—1.0
Multiple (spectroscopic) systems (visual binaries with spectroscopic components)	—	—	—	—	—	—	4	3	4	9
Double systems without known sp. components	2	1	2	4	8	9	20	24	20	32

If the law of distribution were the same in both cases, we should expect 5.4 multiple systems in the interval of $\sin i$ from 0.0 to 0.6, whereas the observed number is zero. By Poisson's formula the probability for such a result to be accidental is $e^{-5.4} = 0.005$, or small enough to be considered improbable. On the contrary, the hypothesis of a coincidence of the orbital planes in multiple systems explains our result, since the discovery of spectroscopic binaries is strongly favoured by high inclination.

Returning to Table I, we notice that the average value of $\sin i$ for all computed orbits is slightly smaller than the expected statistical average for a random distribution ($\frac{\pi}{4} = 0.785$). This may be the result of a slight selection in discovery and computation, because, contrary to what happens to spectroscopic binaries, greater inclination of the orbit makes the visual binary a more difficult object.

We might expect à priori some kind of influence of the galactic plane upon the orbital planes. If a galactic orientation exists, in the sense of the galactic plane being a preferential plane for binaries, the average value of $\sin i$ for systems in high galactic latitudes must be smaller, in low galactic latitudes larger than the average. A slight effect in this direction is shown by Table III, although its reality is not certain.

Table III.

Galactic latitude	$\sin i$
0^0 to $\pm 30^0$	0.764 ± 0.023
$\pm 30^0$ to $\pm 90^0$	0.736 ± 0.025

III. On the Mass Ratio of Spectroscopic Binaries with One Spectrum Visible[1].

By Jacob Gabovitš.

The circumstance that the orbital planes in a multiple system are very likely to coincide (at least approximately), as appears probable from the preceding note, provides us with a new method of determining the mass ratios of spectroscopic binaries with one spectrum, if both the spectroscopic and the visual orbits of a complex system are calculated.

The mass function

$$f = \frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

may be written as

$$\frac{(1+k_1)^2}{k_1^3} = \frac{m_1 \sin^3 i}{f} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2),$$

where $k_1 = \frac{m_2}{m_1}$ is the mass ratio to be determined. Assuming for i the inclination of the visual orbit, and determining the mass of the primary (m_1) from the empirical mass-luminosity relation, we get the mass ratio. The results of this computation are given in Table I, where all complex systems with known spectroscopic (all with one spectrum visible) and visual orbits are collected.

The successive columns give: (1) the name of the star; (2) the inclination of the visual orbit (W. Finsen, *Union Obs. Circ.* No. 91); (3) the mass function of the spectroscopic orbit (J. Moore, Fourth Catalogue of Spectroscopic Binaries); (4) the spectrum (J. Moore, *loc. cit.*); (5) the bolometric absolute magni-

[1] Seminar in Astrophysics, 1936/37, conducted by E. Öpik.

tude deduced from the visual abs. magnitude (from Beer's list in *Ver. Berl. Bab.* 5. 6) by the usual bolometric corrections; (6) the logarithm of the mass of the primary obtained from an empirical mass-luminosity relation (cf. the first note in this number); (7) the mass ratio, computed from formula (2).

Table I.

Mass Ratios (k_1) of Spectroscopic Binaries with one Spectrum Visible.

Star	i	f	Sp.	M_B	Log m_1	k_1
13 Cet	52°	0.0108	F7	4.2	0.048	0.33
OΣ 82	136	.0193	F8	3.9	.072	.47
μ Ori	110	.0113	A2	1.7	.258	.23
α^1 Gem	115	.00150	A2	1.0	.317	.11
α^2 Gem	115	.0097	A2	1.9	.240	.23
ξ UMa B	123	.000053	Go	5.2	—.036	.05
ξ UMa A	123	.0214	Go	4.7	.000	.38
$\varkappa$ Peg	102°	0.045	F2	2.6	0.182	0.39

The average mass ratio of the spectroscopic binaries with one spectrum visible, calculated from the data in column 7 of Table I, is 0.27, with an individual dispersion of $\pm$ 0.13. According to the mass-luminosity relation, established in this publication, the corresponding difference in magnitude between the two components is $\Delta m = 4.5$, being thus much larger than suggested by many authors as an average for invisible components of spectroscopic binaries. Thus, we arrive at the important practical conclusion that, in order to obtain the magnitude of the primary, no correction need be added to the combined magnitude of spectroscopic binaries with one spectrum visible; without doubt, the correction hitherto used by many authors, 0.3 mag., must be regarded as too large.

Tartu, May 21, 1937.

IV. Über die Abhängigkeit der interstellaren Absorption von der Wellenlänge[1].

Von G. Kusmin.

Die Erforschung der interstellaren Absorption als Funktion der Wellenlänge bietet ein besonderes Interesse, da es auf diesem Wege möglich erscheint, gewisse Kenntnisse über die Eigenschaften der interstellaren Materie zu erhalten. Es sind gegenwärtig verhältnismässig wenig entsprechende Messungen vorhanden, wobei dieselben, einzeln genommen, auch noch ziemlich ungenau sind (mittlerer Fehler grösser als $0^m.05$). Dennoch steht für die meisten untersuchten Raumrichtungen mehr als eine Messung für jede Wellenlänge zur Verfügung, und sind die Mittelwerte aus diesen Messungen schon viel vertrauenswürdiger. Abb. 1 zeigt zwei Kurven, die solche Mittelwerte darstellen. Die obere Kurve repräsentiert die Abhängigkeit der Absorption (differentielle Absorption) von der Wellenlänge, die auf Grund der Messungen an dem Sternhaufen NGC 6913 erhalten wurde (nach Trümpler[2]). Die Kurve stellt das Resultat einer Vergleichung zwischen drei im obenerwähnten Sternhaufen und zwei viel näher zu uns gelegenen Sternen dar. Die untere Kurve stellt das Mittel aller Messungen der Absorption an 55 Cygni dar (nach 6—4 Messungen von O. Struve, P. Keenan und J. Hynek[3] und nach einer Messung von J. Rudnick[4]). Beide Kurven sind verhältnismässig genau bestimmt, und wir können den mittleren Fehler ihrer Punkte annähernd auf $0^m.02 - 0^m.04$ schätzen.

Wie aus Abb. 1 zu ersehen ist, zeigen die beiden Kurven, die einen ziemlich unglatten, wellenförmigen Verlauf aufweisen,

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[1] Astrophysikalisches Seminar 1936/37, geleitet von E. Öpik.

[2] Publ. A. S. P. **42**, 267 (1930).

[3] Ap. J. **79**, 1 (1934).

[4] Ap. J. **83**, 394 (1936).

eine merkwürdige Ähnlichkeit miteinander. Scheinbar ist der Charakter der Absorption in beiden Raumrichtungen durchschnittlich derselbe. Es ist aber sehr wahrscheinlich, dass die durchschnittliche Absorption auch in der ganzen Milchstrassenebene qualitativ dieselbe ist. Besonders kommt das zutage, wenn wir alle Messungen durch Reduktion zu einer Normalkurve vereinigen. Abb. 2 enthält eine solche Zusammenstellung, wo die Messungen in den einzelnen Richtungen (oder die Mittelwerte der Messungen in verschiedenen Richtungen) durch die Form der Punkte zu unterscheiden sind (s. hierzu die folgende Tabelle); als Normalkurve ist die obere Kurve in Abb. 1 angenommen.

Tabelle zu Abb. 2.

Bezeichnung der Punkte auf Abb. 2	Gemessenes Objekt	Galaktische Koordinaten l	b	Quelle
1	NGC 6910	46°	+1°	R. Trümpler[2]
2	NGC 6913	45	0	„
3	55 Cygni	53	1	1) O. Struve, P. Keenan u. J. Hynek[3]; 2) J. Rudnick[4]
4	13 Cephei	67	1	O. Struve u. a.[3]
5	Verschiedene Sterne, fast über die ganze Milchstrasse verteilt			H. Wilkens[5] (nach den Messungen von S. Thorndike[6])

Wie aus Abb. 2 zu ersehen, ist in der Tat die Übereinstimmung zwischen den einzelnen Punkten eine ziemlich gute. Die Kurve, die durch die Punkte gezogen ist, erscheint als ganz zuverlässig und ihr Verlauf ist den in Abb. 1 dargestellten Kurven ähnlich. Bei kleiner Wellenlänge ($< 400\ \mu\mu$) werden die Ergebnisse wegen verschiedener Wasserstoffabsorption in den Vergleichssternen stark ungenau (Wirkung der Absorptionsumrisse der naheliegenden Einzellinien der Balmerserie). In diesem Bereiche haben wir nur die Messungen von Trümpler berücksichtigt, denn diese Messungen beziehen sich auf B0-Sterne, wo die Wirkung der Wasserstoffabsorption wahrscheinlich eine kleine ist. Bei den anderen Messungen muss aber diese Wirkung beträchtlich sein.

[5] Breslau Mitt. IV, 52 (1937).
[6] Lick Obs. Bull. **17**, 461 (1934).

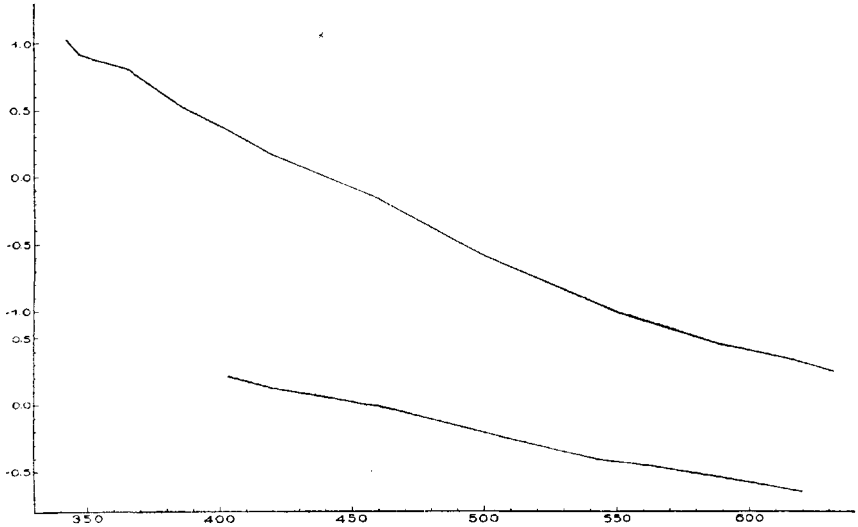

Abb. 1. Abhängigkeit der interstellaren Absorption von der Wellenlänge nach Messungen an NGC 6913 (obere Kurve) und 55 Cygni (untere Kurve). In allen Abbildungen zeigen die Abszissen die Wellenlängen (in $\mu\mu$) und die Ordinaten die Werte der Absorption.

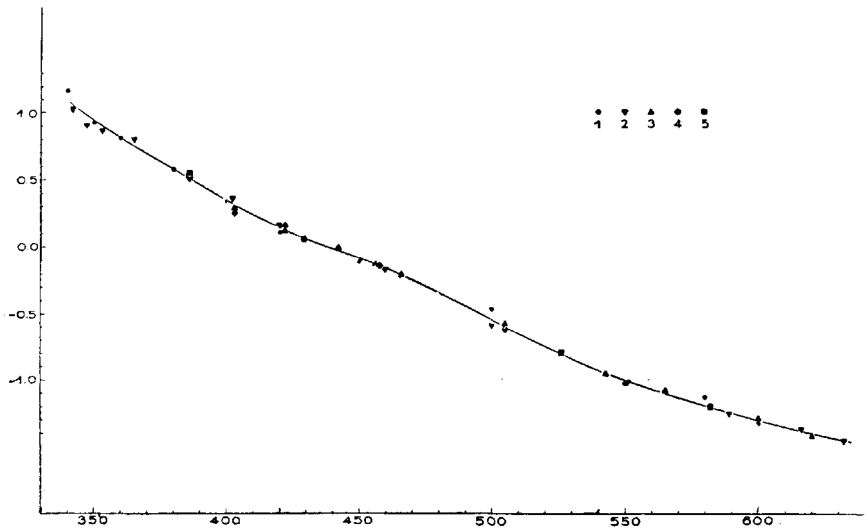

Abb. 2. Abhängigkeit der interstellaren Absorption von der Wellenlänge. Zusammenstellung der Messungen (vgl. Tabelle).

Die dargestellten Kurven zeigen eine verhältnismässig kleine totale Krümmung, welche beinahe dem Gesetze λ^{-1} entspricht (jedenfalls nicht dem Rayleigh'schen Gesetze $\sim \lambda^{-4}$!).

Die Nullinien der Absorptionskurven sind willkürlich gewählt. Die wirkliche Lage der Nullinien ist unbekannt, da alle Messungen nur die differentielle Absorption liefern; den absoluten Wert der Absorption können wir auf Grund der Bestimmungen der Beziehung der allgemeinen Absorption zur selektiven nur annähernd schätzen.

Nun entsteht die Frage: ob wir aus solchen zuverlässigen Kurven, wie sie in den Abbildungen 1 u. 2 dargestellt sind, über die Konstitution des absorbierenden Mediums Schlüsse zu ziehen imstande sind? Diese Frage müssen wir, wie aus dem Folgenden zu ersehen ist, negativ beantworten. Teils wegen der Kleinheit des Intervalls der betrachteten Wellenlänge, teils wegen der Unbestimmtheit der Nullinien kann man den Verlauf der Beobachtungskurven auf ganz verschiedene Weise erklären. Absorbierende Medien von ganz verschiedener Konstitution zeigen bei so engem Intervall von λ, falls die Nullinie und der Parameter des Verteilungsgesetzes der Partikeln geeignet gewählt sind, beinahe denselben relativen Verlauf der Absorption mit der Wellenlänge.

Als Beispiel haben wir einen Vergleich zwischen der Beobachtungskurve und den Kurven, die zwei diametral entgegengesetzten Annahmen über die Verteilung der Partikeldurchmesser des Mediums entsprechen, durchgeführt. Der ersten Annahme nach besteht das Medium aus Partikeln mit konstantem Durchmesser; der zweiten nach aber sind Partikeln mit allen möglichen Durchmessern d kontinuierlich vorhanden, dargestellt durch das einfache Verteilungsgesetz $\sim d^k$ (vgl. d. folg. Aufsatz). Im ersten Falle ist die absorbierende Wirkung der Partikeln für bestimmte Werte des Durchmessers und der optischen Konstanten nach der Theorie von G. Mie[7] berechenbar; solche Berechnungen sind von verschiedenen Autoren durchgeführt worden. Den Absorptionsverlauf bei der Durchmesserverteilung $\sim d^k$, und für von λ und d unabhängige optische Konstanten hat E. Öpik[8] untersucht und

[7] Ann. d. Phys. (4 F.) **25**, 377 (1908).
[8] H. C. **359**, 7 (1931).

dabei folgende Formel abgeleitet:

$$\sigma = \mathrm{const} \times \lambda^p \int_0^\infty y^{p-1}\, \omega(y)\, dy,$$

wo σ den Absorptionsbetrag bei der Wellenlänge λ bedeutet, $y = \frac{d}{\lambda}$, $p = k + 3$, und ω das Verhältnis der effektiven Schattenfläche der Partikel zur geometrischen ist.

Das Integral in dieser Formel ist eine von λ unabhängige Konstante, und die Absorption muss also nach dem Gesetze $\sim \lambda^p$ verlaufen. ω ist nur dann eine reine Funktion von y, wenn die optischen Eigenschaften der Partikeln sich nicht mit der Wellenlänge und dem Durchmesser verändern; sonst gilt das Gesetz $\sim \lambda^p$ nur in gewisser Annäherung.

Im Falle sehr kleiner Werte von y wird ω bei nichtabsorbierenden Partikeln y^4 proportional sein (Rayleigh'sches Ges.), bei absorbierenden aber y; im Falle sehr grosser y-Werte ist dagegen $\omega = 1$. Dementsprechend konvergiert das Integral in der oben gegebenen Formel, wenn $0 > p > -4$ resp. $0 > p > -1$ ist (oder $-3 > k > -7$ resp. $-3 > k > -4$).

Die Abbildungen 3 und 4 enthalten den obenerwähnten Vergleich des beobachteten Verlaufes der Absorption mit dem theoretischen Verlaufe der letzteren. Als Beobachtungskurve ist die Kurve aus Abb. 2 angenommen worden. Die theoretischen Kurven zeigen unseren beiden Annahmen entsprechend den Gang der Absorption bei geeignet gewählten Werten des konstanten Durchmessers, oder bei passendem Werte von k im Verteilungsgesetze $\sim d^k$. Diese Kurven sind mit dünneren Linien dargestellt; die Lage ihrer Nullinien ist durch die angegebenen Werte der Absorption bei $\lambda\lambda$ 440 und 550 $\mu\mu$ bestimmt. Abb. 3 repräsentiert den Fall unveränderlicher optischer Konstanten. Die zwei theoretischen Kurven dieser Abbildung, die der Absorption bei konstantem Durchmesser entsprechen, sind für nichtabsorbierende Partikeln (Absorptionsindex $= 0$) nach Berechnungen von J. Stratton und H. Houghton[9] (obere Kurve) und für absorbierende Partikeln (optische Konstanten des galvanisch zerstäubten Eisens bei λ 550 $\mu\mu$) nach Berechnungen

[9] Phys. Rev. **38**, 159 (1931).

von C. Schalén[10] gegeben (untere Kurve). Die theoretische Kurve für die zweite Annahme der Konstitution des Mediums stellt das Gesetz $\sim\lambda^{-0.7}$ dar, welches der Durchmesserverteilung $\sim d^{-3.7}$ entsprechen muss (das Integral in der Formel von Öpik ist bei $p = -0.7$ jedenfalls konvergent).

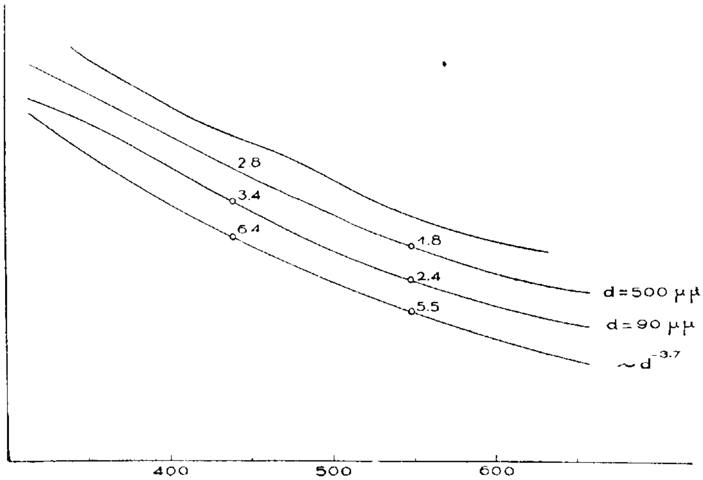

Abb. 3. Vergleich der Beobachtungskurve mit den theoretischen Kurven für den Fall der mit der Wellenlänge nicht veränderlichen optischen Konstanten. Kurven von oben nach unten: 1) Beobachtungskurve; 2) theoretische Kurve für d = const. = 500 $\mu\mu$ und nicht absorbierende Partikeln; 3) theoretische Kurve für d = const. = 90 $\mu\mu$ und absorbierende Partikeln mit unveränderlichen opt. Konstanten; 4) theoretische Kurve für das Verteilungsgesetz der Durchmesser $d^{-3.7}$ und unveränderliche optische Konstanten. Der absolute Wert der Absorption ist in den beigefügten Zahlen angegeben (für 440 u. 550 $\mu\mu$).

Abb. 4 enthält drei Beispiele für den Fall veränderlicher optischer Konstanten. Die zwei oberen theoretischen Kurven entsprechen den optischen Konstanten des galvanisch zerstäubten Eisens, die zwei mittleren denjenigen des elektrolytischen Nickels und die zwei unteren dem massiven Kupfer. Für die Kurven, die den Absorptionsverlauf bei konstantem Durchmesser geben, sind die Berechnungen von C. Schalén[10, 11] (Eisen, Nickel und Kupfer) und von E. Schoenberg und B. Jung[12] (Eisen) benutzt worden. Die Kurven der Verteilung d^k sind auf Grund der Berechnungen von Schalén[10, 11] durch numerische

[10] Upsala Medd. **64**, 1 (1936).

[11] Upsala Medd. **58**, 38 (1934).

[12] Breslau Mitt. IV, 66 (1937).

Integration erhalten. Die Integration wurde für $k = -3^1/_3$, $-3^1/_2$, $-3^2/_3$, $-3^4/_5$ (nur Eisen) und $\lambda\lambda$ 395, 440, 477 (nur Eisen), 550 $\mu\mu$ durchgeführt. Bei Kupfer sind für λ 550 $\mu\mu$ die Schalénschen Berechnungen für die opt. Konstanten 0.84; 2.62 benutzt, die sich nicht viel von den wahren Konstanten bei dieser Wellenlänge (0.89; 2.23) unterscheiden. Für die letzteren ist bei

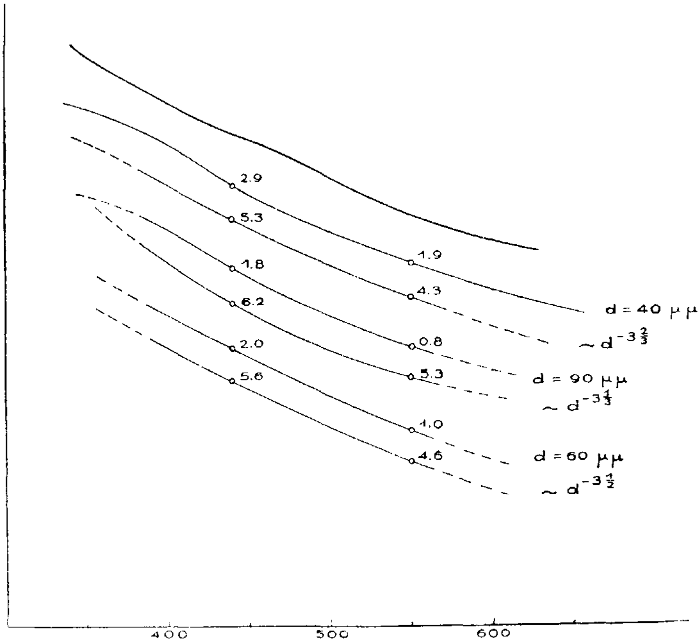

Abb. 4. Vergleich der Beobachtungskurve mit den theoretischen Kurven für den Fall der mit der Wellenlänge veränderlichen optischen Konstanten (Fe, Ni, Cu). Kurven von oben nach unten: 1) Beobachtungskurve; 2) galvanisch zerstäubtes Eisen, $d = 40$ $\mu\mu$ = const.; 3) dasselbe, Verteilung $d^{-3^2/_3}$; 4) elektrolytischer Nickel, $d = 90$ $\mu\mu$ = const.; 5) derselbe, Verteilung $\sim d^{-3^1/_3}$; 6) massives Kupfer, $d = 60$ $\mu\mu$ = const.; 7) dasselbe, Verteilung $\sim d^{-3^1/_2}$.

Schalén die absorbierende Wirkung nicht berechnet. Für grosse Partikeldurchmesser wurde, wo keine direkten Berechnungen vorhanden sind, eine einfache Extrapolationsformel benutzt. Bei (algebraisch) kleinen Werten von k ($k < -3^1/_2$), die für Eisen zutreffend erscheinen, spielt die Wahl dieser Extrapolationsformel keine wesentliche Rolle; bei grösseren Werten dagegen,

z. B. $k = -3^1/_3$, wie er für Nickel gewählt ist[13], kann die ungenaue Extrapolationsformel die Ergebnisse beträchtlich verfälschen, hauptsächlich aber die Lage der Nullinie beeinflussen.

Aus Abb. 3 u. 4 ist nun zu ersehen, dass bei zwei so verschiedenen Annahmen von der Konstitution des absorbierenden Mediums, wie die hier gemachten, eine gleichgute Übereinstimmung mit den Beobachtungsergebnissen erreicht wurde. Es ist aber nicht möglich, bei einem so beschränkten Intervall von λ eine von diesen zwei Annahmen zu bevorzugen. Der Verlauf der theoretischen Kurven für beide Annahmen ist in diesem Intervall fast derselbe; nur in der Nähe der Grenzen des Intervalls (hauptsächlich bei der unteren Grenze) ist meistens ein gewisser Unterschied zu merken.

Bei grösserem Wellenlängenspielraume ist der Verlauf der Absorption für die beiden Annahmen jedoch ein ganz verschiedener. Wenn man die theoretischen Kurven z. B. in der Richtung der kleineren Wellenlängen fortsetzt, so findet man, dass die Absorption im Falle der d^k-Durchmesserverteilung mit der Abnahme der Wellenlänge bis zu sehr kleinen Wellenlängen nach dem Gesetze $\sim \lambda^p$ (wenigstens durchschnittlich) ansteigt, während bei konstantem Durchmesser das Ansteigen der Absorption bald aufhört und eine unregelmässig absteigende Absorption anfängt[14] (s. Abb. 5; diese Abbildung stellt eine Ergänzung der theoretischen Kurven der Abb. 3 dar). Eine solche absteigende Absorption, z. B. bei den theoretischen Kurven in Abb. 3 und 5 (und auch Nickel in Abb. 4), muss schon bei ca λ 250 $\mu\mu$ anfangen.

Hier sei aber bemerkt, dass das Aufhören des Absorptionsanstiegs mit der Abnahme der Wellenlänge im Falle von absorbierenden Partikeln (grosser Absorptionsindex) sich auch nur bei viel kleineren Wellenlängen vorfinden kann. Wegen der etwaigen Veränderung der optischen Konstanten der Partikeln mit der Wellenlänge kann nämlich der Absorptionsverlauf der

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[13] Bei Nickel ist die Übereinstimmung zwischen der d^k und der Beobachtungskurve bei $k = -3^1/_3$ ziemlich ungenügend, bei noch grösseren k-Werten ist eine bessere Übereinstimmung zu erwarten.

[14] Da die Partikeln nicht kleiner als Moleküle sein können, muss das Ansteigen der Absorption immer bei einer gewissen Wellenlänge aufhören, jedoch z. B. im Fall der d^k-Verteilung nur bei einer Wellenlänge von der Grössenordnung der Moleküle.

ganz kleinen absorbierenden Partikeln der Beobachtungskurve entsprechen, ohne dass die Veränderung der opt. Konstanten dabei sehr gross zu sein braucht. Bekanntlich ist bei sehr kleinen absorbierenden Partikeln die Absorption λ^{-1} und einem Faktor, der von den optischen Konstanten abhängt, proportional. Unsere Beobachtungskurven sind ja auch in erster Annäherung an das Gesetz λ^{-1} darstellbar (die Kurve $\lambda^{-0.7} = d^{-3.7}$ auf Abb. 3, die ganz gut mit den Beobachtungskurven übereinstimmt, unter-

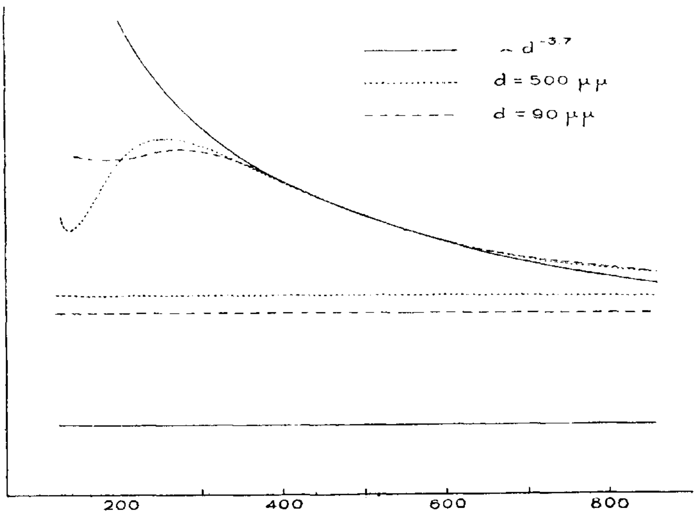

Abb. 5. Die theoretischen Kurven der Abb. 3 auf kleine und grosse Wellenlängen fortgesetzt (die Geraden geben die entsprechenden Nullinien an).

scheidet sich nur wenig von der λ^{-1}-Kurve). Bei sehr kleinen absorbierenden Partikeln ist aber das Aufhören des Absorptionsanstiegs gerade nur bei kleineren Wellenlängen zu erwarten Im Falle des galv. zerst. Eisens z. B., bei dem $d = 40\ \mu\mu$ zutreffend ist, beginnt die abnehmende Absorption vielleicht erst bei etwa λ 150 $\mu\mu$. In solchem Falle zeigen die Kurven, die den beiden Annahmen entsprechen, auch bei ganz kleinen Wellenlängen nur einen kleinen Unterschied.

Bei grossen Wellenlängen ist ein Unterschied zwischen den den beiden Annahmen entsprechenden Kurven auch vorhanden (s. Abb. 5). Bei der d^k-Verteilung muss die Absorption mit der Zunahme der Wellenlänge schneller abnehmen, als bei konstantem Durchmesser; die Absorptionskurven müssen sich asymptotisch ihren Nullinien nähern, wobei die Lage der Nullinien bei den beiden Annahmen, wie man aus den Abbildungen 3, 4 u. 5 ersehen kann, eine ganz verschiedene ist.

Die Fortsetzung der Messungen in der Richtung der kleinen[15] und grossen Wellenlängen bietet also einige Möglichkeiten, um gewisse Schlüsse über die Partikelverteilung ziehen zu können. Doch können die Ergebnisse kaum sehr sicher sein, da es unbekannt ist, wie sich die effektiven optischen Konstanten der Partikeln mit der Wellenlänge und dem Durchmesser verändern; ihre Bestimmung aus den Messungen selbst ist ebenfalls kaum denkbar. Mit entsprechenden Veränderungen der optischen Konstanten ist es aber möglich, bei ganz verschiedenen Durchmesserverteilungen jedes Absorptionsgesetz zu erklären. Man könnte natürlich annehmen, dass die Änderung der effektiven optischen Konstanten eine genügend kleine sei, was aber sehr fraglich ist (unglatter Gang der Absorptionskurven; s. unten). Es ist wichtig, die Messungen in der Richtung der grossen Wellenlängen fortzusetzen, da es ja mit ihrer Hilfe möglich würde, den Minimalwert der allgemeinen Absorption zu bestimmen. Auf Grund der Kurven in Abb. 1 u. 2 kann man schon sagen, dass die allgemeine photographische Absorption wenigstens $^3/_2$ der selektiven Absorption ($\lambda\lambda$ 440, 550 $\mu\mu$) ausmachen muss. Nach den Messungen von J. Rudnick[4] an ζ Persei, die sich bis zu λ 755 $\mu\mu$ erstrecken, ist aber die allgemeine photographische Absorption schon dreimal grösser als die selektive (leider sind diese Messungen ziemlich ungenau; sie sind wegen grosser Dispersion der Messungspunkte in Abb. 2 nicht in Betracht gezogen worden). Auf diesem Wege könnte man natürlich nur eine gewisse untere Grenze der allgemeinen Absorption erreichen; der wahre Wert der allgemeinen Absorption und ihre Beziehung zur selektiven Absorption ist nur auf stellarstatistischem Wege zu bestimmen, wobei das Resultat doch ziemlich unsicher sein dürfte. Die Kenntnis der Werte der allgemeinen Absorption ist aber sehr wichtig, da die Lagen der Nullinien im Falle desselben Verlaufes der differentiellen Absorption bei verschiedenen Annahmen von der Konsti-

[15] In der Richtung der kleinen Wellenlängen kann man die Messungen nur bis auf etwa λ 290 $\mu\mu$ ausdehnen; bei noch kleineren Wellenlängen ist die irdische Atmosphäre infolge der grossen Absorption des Ozons praktisch undurchsichtig.

tution des absorbierenden Mediums ganz verschieden sein können (wie das bei unseren zwei Annahmen der Fall ist; s. die Abbildungen).

Sehr interessant ist eine Einzelheit der Beobachtungskurven — die Welle bei λ 460 $\mu\mu$. Wenn diese Welle wirklich eine Besonderheit der Absorption darstellen sollte, so wäre die Ursache ihrer Erscheinung wahrscheinlich auf die Veränderung der optischen Konstanten der Partikeln des absorbierenden Mediums mit der Wellenlänge zurückzuführen. Diese Ursache könnte ja ganz beliebige Einzelheiten hervorrufen, während andere mögliche Ursachen, nämlich eine besondere Verteilung der Partikeldurchmesser oder eine besondere Veränderung der opt. Konstanten mit dem Durchmesser, nur spezielle Einzelheiten erzeugen könnten, die durch die Form der Kurven der einzelnen Partikeln bedingt wären. Diese letztgenannten Ursachen könnten auch gewisse Wellen in der Kurve hervorbringen, die sich aber wahrscheinlich in einem grösseren Wellenlängenbereiche abspielen müssten als derjenige der in den Kurven der Abbildungen 1 u. 2 sichtbaren Wellen. Leider ist die genaue Form der Absorptionskurven der einzelnen Partikeln mit im Vergleich zur Wellenlänge grossem Durchmesser, besonders bei absorbierenden Partikeln (Metallen), beinahe ganz unbekannt, so dass es nicht möglich ist, ganz sichere Schlüsse über die Entstehung unserer Welle zu ziehen. Jedenfalls dürfte eine Veränderung der optischen Konstanten mit sich verändernder Wellenlänge ihre a priori wahrscheinlichste Ursache bilden.

Die Annahme einer Unveränderlichkeit der optischen Konstanten wäre also sehr riskant. Wohl aber könnte angenommen werden, dass die Veränderung der effektiven optischen Konstanten der Partikeln mit denen eines gewissen wahrscheinlichen Stoffes identisch sei. Es sei jedoch bemerkt, dass die opt. Konstanten, besonders die opt. Konstanten der Metalle, Veränderungen in deren Bestande und in den Bedingungen gegenüber sehr empfindlich sind. Es steht ja sicher, dass im interstellaren Raume kein reines Metall vorhanden sein kann; die gemessenen optischen Konstanten beziehen sich aber gerade auf reine massive, elektrolytische oder galvanisch zerstäubte Metalle. Ausserdem ist die Veränderung der effektiven optischen Konstanten in Fällen, wo im Vergleich zur Wellenlänge die Partikeln gross oder klein sind, fast sicher eine verschiedene da im ersten Falle die absorbierenden (Metalle) und nichtabsorbierenden (Nichtmetalle) Partikeln beinahe gleich wirksam sind, im zweiten Falle aber die absorbierenden Partikeln viel wirksamer sein können.

Zum Schluss sei bemerkt, dass es ziemlich wahrscheinlich ist, dass Partikeln mit einem bestimmten Durchmesser bevorzugt sind, d. h. dass in der Verteilungskurve der Durchmesser ein scharfes Maximum hervortritt. Im letzteren Falle ist die Entstehung der erwähnten Absorptionsbesonderheit (Welle) leichter zu begreifen. Ein bevorzugter Durchmesser entspricht unserer ersten Annahme über die Konstitution des absorbierenden Mediums; solche Wellen in der Absorptionskurve könnten einige Gründe für diese Annahme, die in den Arbeiten von Schalén[10, 11] eine besondere Rolle spielt, liefern. Jedoch ist auch bei der d^k-Verteilung im Falle $k < -4$ (absorbierende Partikeln; bei nichtabsorbierenden $k < -7$) ein genügend bevorzugter Durchmesser — und zwar der kleinste — vorhanden.

V. Über die Partikeldurchmesserverteilung in der interstellaren Materie [1].

Von G. Kusmin.

Wie aus der Statistik der Sternschnuppen folgt, wächst deren Anzahl nach dem d^k-Gesetz an[2] (d — Durchmesser, k — beinahe konstant). Wir können nun (nach Öpik[2]) annehmen, dass solch ein Anwachsen nach dem d^k-Gesetze sich bis zu sehr kleinen, schon absorbierend wirkenden Partikeln der interstellaren Materie erstrecken muss, und dass dieses Gesetz nicht nur in der Sonnenumgebung, sondern auch im ganzen interstellaren Raume mit durchschnittlich derselben Konstante k seine Geltung bewahrt. Die folgenden elementaren Berechnungen zeigen, dass die Beobachtungsergebnisse über die interstellare Absorption dieser Annahme jedenfalls nicht widersprechen.

Wir berechnen auf Grund der Anzahl der sporadischen (interstellaren) Sternschnuppen in cm^3, von dem d^k-Gesetz ausgehend, die allgemeine photographische Absorption. Dazu benutzen wir die in der vorhergehenden Abhandlung vom Verfasser gegebene Formel von Öpik (Seite 17), die gerade für die d^k-Durchmesserverteilung abgeleitet ist. Der Proportionalitätsfaktor in dieser Formel ist der folgende:

$$2.5 \log e \frac{\pi}{4} cH,$$

wo c den Proportionalitätsfaktor des d^k-Gesetzes und H die Länge des Lichtweges in dem absorbierenden Medium bedeutet.

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[1] Astrophysikalisches Seminar 1936/37, geleitet von E. Öpik.

[2] E. Öpik, H. C. **359**, 7 (1931).

Wir nehmen $H = 1$ kpc $= 3.1 \times 10^{21}$ cm an, den Faktor c berechnen wir aus der Anzahl der sporadischen Sternschnuppen:

$$A = c \int_{d_0}^{\infty} d^k \, dd \text{ pro cm}^3.$$

d_0 ist hier die untere Durchmessergrenze der in Betracht genommenen Sternschnuppen. Wenn wir $d_0 = 0.1$ cm wählen, so ist rund $A = 10^{-24}$ pro cm^3, welche Zahl grössenordnungsgemäss als Mittelwert für den interstellaren Raum zu betrachten ist (Schätzung von E. Öpik, dem Verfasser mitgeteilt). Für k nehmen wir verschiedene Werte zwischen — 3.3 (teleskopische Sternschnuppen) und — 4.5 (helle Sternschnuppen) an[2]. Weiter setzen wir $\lambda = 440 \; \mu\mu = 0.44 \times 10^{-4}$ cm an, wonach wir den Wert von λ^p ($p = k + 3$) berechnen. Das Verhältnis des effektiven Querschnittes der Partikeln zum wahren, ω, ist bekanntlich im Falle der kleinen y bei nichtabsorbierenden Partikeln $\sim y^4$ und bei absorbierenden $\sim y$ ($y = \frac{d}{\lambda}$). Als Proportionalitätsfaktoren benutzen wir runde Werte, im ersten Falle 10 und im zweiten 3. Diese Werte erhalten wir aus den bekannten Formeln von Mie[3], wenn wir im ersten Falle den Brechungsexponenten gleich 1.3, und im zweiten gleich $1.5 - 3i$ ansetzen. Bei grösserem y wächst ω mehr oder weniger über 1 und bei weiterem Anwachsen von y rückt ω zu 1 asymptotisch zurück[4]. Dementsprechend setzen wir schematisch $\omega = 10\, y^4$ resp. $3y$, wenn $y < 0.6$ resp. 0.4, und $\omega = 1.5$, wenn $y > 0.6$ resp. 0.4 ist. Tabelle 1 enthält die Ergebnisse der Berechnungen für verschiedene k (I — nichtabsorbierende Partikeln, II — absorbierende Partikeln).

Tabelle 1.

Berechnete Werte der interstellaren Absorption pro kpc (photographisch), von der beobachteten Sternschnuppenhäufigkeit ausgehend.

k	—3.7	—3.8	—3.9	—4.0	—4.1	—4.2	—4.3
I	$0^m.06$	$0^m.12$	$0^m.26$	$0^m.56$	$1^m.3$	$2^m.9$	$6^m.5$
II	$0^m.17$	$0^m.45$	$1^m.3$	$4^m.0$	13^m	43^m	150^m

[3] Vgl. Ann. d. Phys., Fussn. 7 des vorhergehenden Aufsatzes.
[4] Rechenfehler in H. C. **359**: Tab. III, ω mit 70 zu dividieren

Im Falle der absorbierenden Partikeln ist hier als untere Grenze der Integration $y = 10^{-3}$ benutzt (welcher Wert etwa dem Durchmesser der Moleküle entspricht), statt $y = 0$ in der Formel von Öpik. Bei nichtabsorbierenden Partikeln spielt die Wahl der unteren Grenze keine wesentliche Rolle.

Wie aus der Tabelle zu ersehen ist, kann man die auf stellarstatistischem Wege geschätzten Werte der allgemeinen Absorption im interstellaren Raume (von der Grössenordnung 1^m pro kpc) wohl durch die d^k-Verteilungshypothese erklären, und zwar bei nichtabsorbierenden Partikeln durch $k = -4.0$ bis -4.2 und bei absorbierenden Partikeln durch $k = -3.8$ bis -4.0. Die Wirkung der nichtabsorbierenden Partikeln ist, wie diese Tabelle zeigt, beträchtlich kleiner als die Wirkung der absorbierenden Partikeln. Dementsprechend sind in der Tabelle 2 die Werte der Absorption für verschiedene Mischungen der absorbierenden und nichtabsorbierenden Partikeln dargestellt. In dieser Tabelle ist der Anteil der absorbierenden (metallischen) Partikeln in zwei Fällen kleiner als derjenige der nichtabsorbierenden (Mineralstaub) angenommen, was wahrscheinlich der Wirklichkeit entspricht, wie das die Erforschung der Sternschnuppen auch ahnen lässt.

Tabelle 2.
Photographische Absorption pro kpc, für verschiedene Mischungsverhältnisse absorbierender und nichtabsorbierender Partikeln.

Anteil der absorb. Partikeln	k						
	-3.7	-3.8	-3.9	-4.0	-4.1	-4.2	-4.3
1/2	$0^m.11$	$0^m.28$	$0^m.8$	$2^m.3$	7^m	23^m	80^m
1/5	$0^m.08$	$0^m.19$	$0^m.47$	$1^m.2$	$3^m.6$	11^m	35^m
1/10	$0^m.07$	$0^m.15$	$0^m.36$	$0^m.9$	$2^m.5$	7^m	21^m

Das Gesetz und der Betrag der Verfärbung der Absorption entsprechen bei der d^k-Verteilung auch ziemlich genau der Wirklichkeit. Bei nichtabsorbierenden Partikeln muss die Absorption nämlich auf Grund der Formel von Öpik nach dem Gesetze $\sim \lambda^p$ mit $p = k + 3$ vor sich gehen (wenn die optischen Konstanten der Partikeln sich wenig mit dem Durchmesser und der Wellenlänge ändern). Nehmen wir $k = -4$, welcher Wert für verschiedene Mischungen der absorbierenden und nichtabsorbierenden Partikeln die Absorption von $0^m.56$ bis $4^m.0$ pro kpc fordert (Tabellen 1 u. 2), so ist das Verfärbungsgesetz des nichtabsorbierenden Anteils $\sim \lambda^{-1}$. Dieses Gesetz stimmt mit der beobachteten Verfärbung der Absorption mit der Wellenlänge, die in Abb. 1 u. 2 der vorgehenden Abhandlung vom Verfasser gegeben ist, ziemlich gut überein. Die totale Krümmung der Beobachtungskurve unterscheidet sich nur wenig von der theoretischen Krümmung, welche etwas zu gross erscheint (das Gesetz $\sim \lambda^{-0.7}$ stimmt besser mit den Beobachtungen überein, siehe Abb. 3 jener Abhandlung). Die allgemeine Absorption (λ 440 $\mu\mu$) im Vergleich zur selektiven ($\lambda\lambda$ 440, 550 $\mu\mu$) ist für das Gesetz $\sim \lambda^p$ in Tabelle 3 (I) für verschiedene k gegeben. Es variiert bei den betrachteten Werten von k ($= p - 3$), wie die Tabelle zeigt, zwischen 4 und 6.

Bei den absorbierenden Partikeln ist das Gesetz $\sim \lambda^p$ für $-1 < p < -0.7$ wegen des Vorhandenseins einer unteren Grenze der Partikeldurchmesser nicht ganz genau, da im Falle der absorbierenden Partikeln bei solchen Werten von p das Integral in Öpiks Formel sehr langsam konvergiert. Genauer ist das Gesetz:

$$a\,\lambda^p - b\,\lambda^{-1},$$

wo das zweite Glied das Fehlen von sehr kleinen Partikeln (kleiner als Moleküle) bedeutet. Bei $p \leqslant -1$ ($k \leqslant -4$) ist aber im Falle von absorbierenden Partikeln das Integral überhaupt nicht konvergent. In diesem Falle ist die Annahme einer unteren Durchmessergrenze unbedingt nötig. Bei $p = -1$ muss, wie nicht schwer zu ersehen ist, das folgende Gesetz gelten:

$$(b_1 + a_1 \log \lambda)\,\lambda^{-1},$$

bei $p < -1$ aber:

$$b\,\lambda^{-1} - a\,\lambda^p\,.$$

Die Übereinstimmung zwischen den Kurven der oben gegebenen Gesetze für absorbierende Partikeln und der Beobachtungskurve ist noch grösser als bei dem $\sim \lambda^p$-Gesetz mit $p \approx -1$ (nichtabsorbierende Partikeln); sie ist durch eine kleinere Krümmung der Kurven bei Berechnungen nach diesen Gesetzen bedingt, was besser den Beobachtungen entspricht. Jedoch ist auch hier die theoretische Kurve etwas zu sehr gekrümmt. Tabelle 3 (II) bietet für die absorbierenden Partikeln die allgemeine Absorption im Verhältnis zur selektiven und ausserdem Angaben über die

Parameter der Absorptionsgesetze (für λ in μ-Einheiten). Aus der Tabelle ersehen wir, dass das Verhältnis der allgemeinen Absorption zur selektiven bei beiden Arten von Partikeln fast dasselbe ist; bei absorbierenden Partikeln ist die allgemeine Absorption relativ etwas grösser und variiert zwischen 5 und $6^1/_2$.

Tabelle 3.

Verhältnis der photographischen Absorption zur selektiven: I, nichtabsorbierende; II, absorbierende Partikeln.

k	—3.8	—3.9	—4.0	—4.1	—4.2
I	6.1	5.5	5.0	4.6	4.2
II	6.5	6.0	5.7	5.5	5.3
b/a	0.20	0.45	(1.00)	2.2	5.0
b_1/a_1	—	—	3.5 [5]	—	—

Im Falle einer grossen Veränderung der optischen Konstanten der Partikeln kann der Betrag der Verfärbung ganz anders erscheinen und die selektive Absorption mehr oder weniger von den berechneten Beträgen von $^1/_4$ — $^1/_6$ der allgemeinen Absorption sich unterscheiden. Fast alle gemessenen Metalle liefern eine grössere selektive Absorption. Tabelle 4 stellt die allgemeine Absorption im Vergleich zur selektiven für einige Metalle dar. Diese Angaben sind bei $k = -3^2/_3$ auf Grund der in der vorhergehenden Abhandlung erwähnten numerischen Integration, und für $k = -4^1/_2$ auf Grund der Berechnungen nach der Formel für sehr kleine absorbierende Partikeln gegeben (das Absorptionsgesetz bei $k = -4^1/_2$ unterscheidet sich nur wenig von demjenigen bei sehr kleinen Partikeln, denn die relative Wirkung der grösseren Partikeln ist bei diesem k schon verschwindend klein).

Wie aus der Tabelle zu ersehen ist, erweist sich die allgemeine Absorption im Verhältnis zur selektiven wegen der Veränderung der optischen Konstanten von 2 bis 3 mal kleiner als bei unveränderlichen Konstanten. Die Anwesenheit von Partikeln mit veränderlichen optischen Konstanten kann also

[5] Für Log_{10}.

die selektive Absorption verhältnismässig beträchtlich vergrössern. Eine solche grössere Verfärbung kann noch besser den Beobachtungstatsachen entsprechen.

Tabelle 4.

Verhältnis der photographischen Absorption zur selektiven für Metalle.

Metall	k $-3^2/_3$	$-4^1/_2$
Galvanisch zerstäubtes Eisen	5.5	3.0
Elektrolytischer Nickel	4.0	2.1
Massives Kupfer	3.8	1.9
Massiver Stahl	—	2.0
Unveränderliche optische Konstanten	> 7	5.0

Wir sehen also, dass unsere Hypothese über die Verteilung der Partikeldurchmesser in ganz gutem Einklang mit den Beobachtungen der allgemeinen und selektiven Absorption steht. Die etwas kleinere Krümmung der beobachteten Absorptionskurve kann man leicht durch ein kleineres k bei feineren Partikeln oder durch eine Veränderung der opt. Konstanten erklären (von den Ursachen der wellenförmigen Besonderheit in der Kurve ist schon in der ersten Abhandlung die Rede gewesen). Jedenfalls lässt die beobachtete Anzahl der Sternschnuppen sich auf kleine Teilchen durch eine kontinuierliche Verteilung (etwa d^k-Verteilung) extrapolieren, wobei die beobachtete beinahe λ^{-1}-Verfärbung, die das Ähnlichbleiben der Spektralenergiekurve einer Planckschen Kurve hervorruft, eine entsprechende Erklärung findet.

Zu ganz anderen Ergebnissen sind E. Schoenberg und B. Jung[6] gelangt. Sie finden nämlich, dass ein kontinuierlicher Übergang von Sternschnuppen zu feineren

[6] Breslau Mitt. IV, 76 (1937).

Partikeln überhaupt nicht möglich ist. Zu einem solchen Schlusse sind sie durch die Wahl eines konkreten Verteilungsgesetzes gekommen. Von ihren zwei Hypothesen über die Durchmesserverteilung:

$$\sim \frac{1}{d^3} \text{ und } \sim \frac{1}{d^3}\frac{1}{1+h^2 d^2} \quad (h=5 \text{ bei } d \text{ in } \mu)$$

haben sie die letztere gewählt, da diese den Betrag der Verfärbung besser angebe. Auf Grund dieser Hypothese finden sie aber die Gesamtdichte des nichtverfärbenden Anteils der absorbierenden Materie, wenn diese aus Eisen, Nickel und anderen Metallen besteht, als nur 6×10^{-27}, während nach C. Hoffmeister[7] schon die Gesamtdichte nur der teleskopischen Sternschnuppen rund 10^{-24} g/cm^3 beträgt.

Dieser Unterschied zwischen unseren Ergebnissen und denen von Schoenberg und Jung ist erstens natürlich durch die verschiedenen Verteilungshypothesen bedingt. Nach der Hypothese von Schoenberg und Jung entsteht der nichtnichtverfärbende Anteil der Absorption durch die Wirkung durchschnittlich feinerer, weniger massiver Partikeln, als bei der Hypothese der d^k-Verteilung ($k \approx -4$), was durch eine steilere Abnahme der Anzahl der grossen Partikeln (nach dem Gesetze $\sim d^{-5}$) bei der ersten Hypothese bedingt ist. Wie die entsprechenden Berechnungen zeigen, erfordert die erste Hypothese bei gleichem nichtverfärbendem Anteil der Absorption eine 10^3 bis 10^4 mal kleinere Anzahl von massiven Partikeln mit einem Durchmesser > 0.1 cm, als die zweite Hypothese. Auf die Dichte bezüglich kann ein noch etwas grösserer Unterschied entstehen (dieser hängt von der Wahl der oberen Durchmessergrenze ab[8]). Zweitens scheint auch, dass Hoffmeisters Wert der Gesamtdichte der teleskopischen Sternschnuppen stark überschätzt ist (bis 10^2 mal).

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[7] Astr. Abh., Erg.-Hefte z. d. A. N. **4** Nr. 5 (1922).

[8] Z. B. bei der effektiven oberen Grenze 10^5 cm ist der Unterschied schon beträchlich grösser, als dem Verhältnis 10^4 entspricht. Die Gesamtdichte der interstellaren Materie hat bei der d^k-Verteilung (falls $k = -4$, 10^{-7} cm $\leqslant d \leqslant 10$ cm und $A = 10^{-24}$ pro cm^3) den Wert $2{,}9 \times 10^{-26} \times \varrho$ g/cm^3 (ϱ — durchschnittliche Partikeldichte), d. h. eine ca. 10 mal grössere Gesamtdichte als nach Schoenberg und Jung's Hypothese (7.6×10^{-27} g/cm^3).

Welch eine Verteilung der Partikeldurchmesser der interstellaren Materie in der Tat statthat, ist schwer zu entscheiden. Die d^k-Verteilung mit $k \approx -4$ stellt jedenfalls die Durchmesserverteilung der Sternschnuppen gut dar und liefert das richtige Gesetz und einen plausiblen Betrag der Absorption. Diese Verteilung entspricht wenigstens schematisch den realen Verhältnissen, wobei die wirkliche Verteilung einen viel komplizierteren Charakter (mit Extremen) haben kann.

Tartu, Juni 1937.

VI. A Tentative Determination of the Surface Brightness of Dark Nebulae.[1]

By V. Riives.

Absorbing clouds appear on the bright belt of the Milky Way as dark markings. But the dark nebulae doubtlessly possess some surface brightness of their own, because their particles must reflect the starlight. In the following it is attempted to derive a probable value of their surface brightness, and from that to estimate the albedo of interstellar matter.

The total surface brightness of the night sky mainly consists of the following components: the direct starlight, the atmospheric light, the zodiacal light, and the starlight reflected by interstellar matter. Knowing the total amount of the light of an area, and that of the light from other sources, it is possible to determine the amount of the last-mentioned component. On certain assumptions results can be obtained without knowing the amount of terrestrial or zodiacal light. Therefore, it is sufficient to know only the following data: 1. The observed value of the surface brightness of the night sky. 2. For each area the stellar distribution according to apparent magnitude reaching down to the faintest stars recorded.

Photographic photometry is of sufficient exactness for the present purpose. A work belonging here is: Wolf-Pannekoek, "Photographische Photometrie der nördlichen Milchstrasse", Amsterdam Publ. **3**, 1932. The data given there are used in the present investigation. The surface brightness given in their photometric charts includes the light from all stars fainter than about $7^{m}_{.}5$. When calculating the part due to starlight this limit must be taken into account.

[1] Seminar in Astrophysics 1936/37, conducted by E. Öpik.

For the Selected Areas data exist giving the stellar distribution down to sufficiently faint stars. The material used here is taken from Groningen Publ. **43**, tab. 1, which gives the log of star number per square degree down to apparent magnitude $18^{m}_{.}5$. Being confined to the Selected Areas it was impossible to pay attention to other known areas with great absorption.

For each Selected Area the stars were divided into groups, by intervals of one magnitude, and the amount of starlight for each group was calculated. The total light of stars fainter than $18^{m}_{.}5$ was found by extrapolation. The total amount of starlight is set equal to the sum of light from all magnitude groups. The extrapolation may cause a quite considerable error, especially for the areas rich in stars. For this reason it was impossible to find any satisfactory value of the starlight for Selected Areas 64 and 98.

In the Wolf-Pannekoek charts, which are reduced to a common zero point (from plates covering each other), the combined value of atmospheric and zodiacal light appears as a more or less constant correction. This enables us on certain assumptions to determine the quantity of starlight scattered by interstellar matter without knowing the absolute value of the correction.

Subtracting from the mean observed surface brightness, given in the charts, the total light due to the corresponding mean stellar distribution, we obtain a difference which contains the constant correction *plus* a certain mean value of the surface brightness of the absorbing matter for the given area. Table 1 contains mean values of this difference for different galactic latitudes, derived from the entire material of the Wolf-Pannekoek charts.

Table 1.

b	I'	A'	a
0^{0}	136	92	44
5^{0}	132	83	49
10^{0}	112	69	43
15^{0}	97	50	47
		mean	46

b — galactic latitude
I' — mean surface brightness taken from Amst. Publ. **3**
A' — computed mean starlight per square degree
$a = I' - A'$
Unit of surface brightness = the light of a 10^{m} star per square degree

In calculating A', the stellar distribution given in Mount Wilson Contrib. **301** is used, because this and Amst. Publ. **3** refer to the same portion of the sky, namely the northern hemisphere.

Table 2 contains all the material discussed in the present note.

Table 2.

Sel. Area	I	A	H	d	Sel. Area	I	A	H	d
2	107	48	+ 13	+0.5	49	120	75	— 1	+ 1.0
8	160	83	+ 31	+ 1.3	50	130	76	+ 8	+ 1.0
9	111	56	+ 9	+ 2.2	63	120	112	— 38	— 0.1
10	81	36	— 1	+ 1.3	65	140	73	+ 21	+ 0.3
18	118	84	— 12	+ 0.5	73	100	40	+ 14	+ 1.1
19	136	81	+ 9	+ 1.7	74	130	69	+ 15	+ 1.9
22	115	55	+ 14	+ 0.9	75	90	64	— 20	+ 0.9
23	111	55	+ 10	+ 1.1	86	105	70	— 11	— 0.2
24	105	50	+ 9	+ 1.9	87	130	73	+ 11	+ 1.0
25	100	49	+ 5	+ 1.7	88	140	110	— 16	+ 0.1
39	139	127	— 34	— 0.7	97	120	46	+ 28	+ 1.9
40	169	105	+ 18	+ 1.1	109	95	44	+ 5	— 0.1
41	162	119	— 3	+ 0.2	110	95	27	+ 22	+ 3.4
42	115	74	— 5	+ 0.5	111	130	59	+ 25	+ 0.5
48	95	35	+ 14	+ 0.9					

I — total surface brightness taken from Amst. Publ. **3**
A — computed starlight per square degree
$H = I - A - a \ldots.$ — residual surface brightness
$d = 18.^{m}5 - m_N$; m_N — effective limiting magnitude found from starnumber $N_{18.^{m}5}$ of the area, and from the mean stellar distribution in the corresponding galactic latitude as given in Groningen Publ. **43**, tab. 6; thus d is a measure of the excess (—) or defect (+) of starnumber in the given individual area expressed in magnitudes

The relative diminution of light in an absorbing layer of optical depth x equals $1 - e^{-x}$. A part of this quantity is reflected (or diffused) by the absorbing matter. For the purpose of our estimates the amount of reflected light can be assumed to be proportional to the total diminution of light. On this account the surface brightness $K = K_0\ (1 - e^{-x})$. In the simplest case the albedo is proportional to the surface brightness, $a = a_0\ (1 - e^{-x})$; K_0 and a_0 are surface brightness and albedo of a layer with infinite optical depth. Further, denoting $r = \frac{a}{a_0} = 1 - e^{-x}$, we have $K = K_0\ r$.

On the assumption that the interstellar matter is illuminated more or less uniformly by stars, it is possible to use the preceding formulae. Denoting the total absorption in stellar magnitudes by Δm, the following equation is obtained: $2.512^{-\Delta m} = e^{-x}$. Thus the relative albedo $r = 1 - 2.512^{-\Delta m}$ (1).

Supposing that H given in Table 2 includes besides K only a constant term Δ, we have $H = K_0\ r + \Delta$ (2).

Knowing the values of H with the corresponding total absorption Δm [which determines r by (1)], it is possible to derive the correlation (2). For our purposes, it is only important to find K_0 and a_0.

It is impossible to obtain the amount of total absorption of starlight in interstellar space for each area separately. In Table 2 the values of d denote only the excess of starnumber of the area over the mean starnumber in the given galactic latitude. Thus d contains the effect of interstellar absorption together with local irregularities of stellar distribution. Taking the mean value of d from several areas, the effect of local irregularities in starnumber must be partly compensated. However, the accidental error of H seems to be quite considerable. On this account it is necessary to combine the available data into normal groups, and to find the corresponding mean values for each group separately. The whole material we choose to subdivide into four groups according to the value of d.

Further, the mean d represents only a deviation of the local absorption from the mean absorption. Therefore the total absorption is $\Delta m = d + \mu$, μ being a mean value of total absorption for the given galactic latitude. It is possible to obtain minimum values of the surface brightness and albedo of a dark nebula by taking $\mu = 0$ and $\Delta m = d$.

From the distribution of the extragalactic nebulae, Hubble (Mt. W. Contrib. 485) estimates the total absorption in the direction of the galactic pole to be $0^m.25$. In other directions it varies approximately with the cosecant of galactic latitude. On the assumption that this law holds in low galactic latitudes, it is possible to compute the value of μ.

This was found for each group separately by using the mean galactic latitude of each group. In such a manner another set of values of the surface brightness and albedo is obtained.

In Table 3 are given the results for the two cases.

Table 3.

Limits of d	$\bar{H}$	e	$\bar{d}$	n	r_1	$\bar{\lvert b \rvert}$	μ	r_2
≤ 0.5	$-$ 5.0	$\pm$4.1	+ 0.14	11	0.12	$11^{0}.1$	1.30	0.73
0.6 — 1.2	+ 7.6	$\pm$2.6	+ 1.00	9	0.60	$9^{0}.1$	1.51	0.90
1.3 — 2.0	+12.3	$\pm$3.0	+ 1.67	7	0.78	$5^{0}.7$	2.5	0.98
$2.1 \leq$	+ 15.5	$\pm$4.3	+ 2.8	2	0.92	2^{0}	—	1.0

$\bar{H}$ — mean H
e — probable error of $\bar{H}$ derived from the dispersion of individual H
$\bar{d}$ — mean value of d
n — number of Selected Areas in the group
r_1 — relative albedo $\frac{a}{a_0}$ (minimum) computed on the assumption $\mu = 0$
$\bar{\lvert b \rvert}$ — mean absolute galactic latitude
$\mu = 0.25 \operatorname{cosec} \bar{\lvert b \rvert}$ = assumed mean total absorption
r_2 — relative albedo corresponding to the above value of μ

In fig. 1 the values of H and r are represented graphically. According to formula (2), the correlation obtained should be linear; in the figure, I corresponds to the hypothesis $\mu = 0$ (r_1); II — to $\mu = 0.25 \operatorname{cosec} \lvert b \rvert$ (r_2).

As it turns out, in both cases all the four points fall close to a straight line. The agreement is closer than required by the probable errors of the data, thus the good correlation is evidently accidental.

The total amount of starlight according to Seares, Van Rhijn, Joyner, and Richmond, Ap. J. **62**, 373, 1925, equals the light of 577 first magnitude stars or fifty-six 10^m stars per square degree. Assuming the mean surface brightness of the sky due to the starlight in interstellar space to be the same as observed from the earth, it is possible to estimate the albedo.

The correlation I of fig. 1 furnishes a lower limit of surface brightness $K_0 = 25$ and $a_0 = 0.5$, which refers to the albedo of a cloud of infinite optical thickness.

Using Hubble's data for the total absorption of starlight in the Galactic System, i. e., the correlation II of fig. 1, the surface brightness is estimated at $K_0 = 70$ units, and the albedo appears to be unreasonably great, namely, $a_0 > 1$. This second solution is especially uncertain. A deviation from the cosecant law, and a small error in the absorption

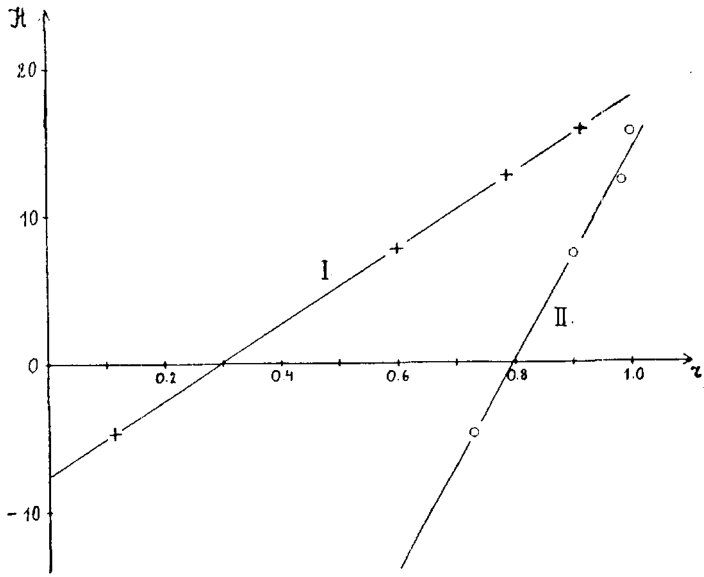

Fig. 1.

coefficient given by Hubble may cause very different results. Taking into account the possible deviation from the cosecant law in low galactic latitudes, the value of K_0 here found may be regarded as a maximum value.

From our results it seems to be certain that the albedo of interstellar matter is comparatively high. It is known that clouds consisting of dielectric particles (water, ice, etc.) have much higher an albedo than mettallic dust. The results obtained seem to indicate that dielectric particles (mineral dust?) are more abundant than absorbing particles (metals) in interstellar space.

Taking into account the provisional nature of the present paper, the results obtained may indicate only the order of magnitude of the effect which we wanted to determine. The uncertainties in the limited observational data and in the hypotheses used are sufficiently great to cover the tiny effect of the surface brightness of dark nebulae.

After this paper had been finished, the author found in Ap. J. **85**, 213, 1937 the same problem treated by C. T. Elvey and F. E. Roach. Remarkably enough, the conclusions reached by these authors coincide entirely with those of the present investigation, though the method and observational data are quite different.

Tartu, June 4, 1937.

VII. The Influence of the Selective Absorption in Space upon a Differential Scale of Stellar Magnitudes.[1]

By V. Riives.

In the catalogue of Tartu Obs. Publ. 28_2 the scale of photographic magnitudes is based upon Harvard visual magnitudes of stars earlier than F8. Assumed mean colour indices are there added to the visual magnitudes, and standard photographic magnitudes are obtained in this manner.

In general the faint stars are farther away than the brighter ones. On account of selective absorption the colour indices of faint stars must be larger than the mean assumed value. This may cause a systematic error in the photometric scale of the catalogue.

In the following an upper limit to the correction for the selective absorption is estimated. The coefficient of the selective absorption is certainly overestimated if we assume $+0.^{m}5$ per 1000 parsec. For each star, used for calibrating the photometric scale, the distance was calculated from the known absolute magnitude, or from a mean value of the absolute magnitude of the given spectral type. The distance being known, it is possible to find the correction for selective absorption.

The stars are divided into groups according to their apparent brightness. For each group the mean value of the correction was computed. The zero point of the colour index has no influence upon the scale correction. Therefore the values of the scale correction are given relative to the mean selective absorption.

The results are presented in the following table and in the accompanying figure.

[1] Seminar in Astrophysics 1936/37, conducted by E. Öpik.

m	A	Δi	n
	m	m	
4.0 — 4.5	0.017	— 0.059	3
4.5 — 5.0	0.04	— 0.036	1
5.0 — 5.5	0.038	— 0.038	4
5.5 — 6.0	0.03	— 0.046	1
6.0 — 6.5	0.062	— 0.014	13
6.5 — 7.0	0.063	— 0.013	14
7.0 — 7.5	0.084	+ 0.008	14
7.5 — 8.0	0.094	+ 0.018	19
8.0 — 8.5	0.087	+ 0.011	14
8.5 — 9.0	0.120	+ 0.044	3
9.0 — 9.5	0.12	+ 0.044	1
Mean	0.076	0.000	....

m — magnitude interval of the group
A — mean computed selective absorption
Δi — relative scale correction
n — number of stars in the group

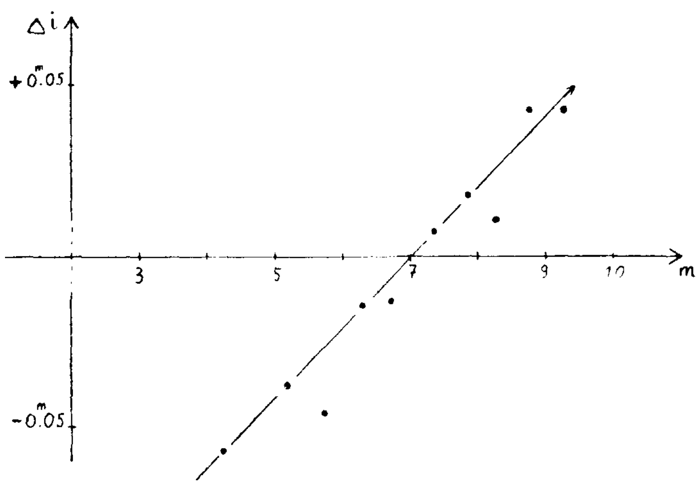

Fig. 2.

Our theoretical scale correction appears to be quite close to a linear function of magnitude. Considering the fact that our assumed value of the coefficient of space absorption amounts to at least the double of the probable value, the actual corrections must be much smaller than those given in the table; they are small enough to be disregarded in the above mentioned catalogue.

Tartu, May 27, 1937.

VIII. On the Upper Limit of Stellar Masses.

By E. Öpik.

A widely adopted opinion, first expressed by Eddington, is that stars of large mass are unlikely to persist for a very long interval of time; because of the large ratio of radiation pressure to gravitation ($1 - \beta$ approaching 1), these stars, under the action of external or internal disturbing factors, may perhaps more easily divide into smaller masses than small stars with a small value of $1 - \beta$.

Alternative speculations, intended to demonstrate the existence of an upper limit to stellar masses, and based upon theoretical considerations of stellar structure, have been put forward by H. Vogt and W. Anderson.

Disregarding the uncertainty of such theoretical speculations, an uncertainty which still appears unavoidable when dealing with stellar interiors, we notice that the theoretical reasons are never physically prohibitive for the existence of large masses; the persistence of such masses, never being regarded as impossible, becomes from the theoretical standpoint a question of mere probability. The considerable attention paid to the question of an upper limit to stellar masses seems to have been stimulated by a general belief, or impression, that observations indicate the existence of such an upper limit. Below we try to demonstrate that the belief is not well founded.

In the first place, the absence of very large stellar masses from our observational records may be a mere statistical phenomenon, referring to the probability of origin, and not to the probability of persistence. A universal law makes large masses less numerous than small masses; the law holds, in broad outline, equally for meteors, planets, and stars; the trivial explanation of the law is that from a limited amount of material there can be made more small bodies than large ones. If V is the total volume (may be proportional to the total number of objects, or to the volume of space) of a sample, $p(m)$ the probability for a mass to exceed m, the expected observable number of masses greater than m is $Vp(m)$. When $p(m)$ is a rapidly decreasing function (such as a Gaussian at large values of the argument), the "observational", or catalogue limit of m is with a mostly low accidental error practically defined by the equation

$$Vp(m) = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1);$$

further, the limit of m defined in such a manner varies but slowly with V, and an apparent "upper limit" of m may result when the different volumes V in different samples remain of the same order of magnitude.

Thus, if our knowledge of the masses of celestial bodies were limited to those directly striking the earth (V = volume swept by the earth in historical time), 100 or 1000 tons might appear to us as an upper limit of mass; with respect to meteoric bodies we know that there is no physical reason for the existence of such an upper limit of mass and that actually no upper limit exists. The apparent upper limit of stellar masses ($\sim 70\odot$ at least) may be of a similar character. To test this, with respect to the order of magnitude we are allowed to assume Kapteyn's Gaussian "Luminosity-Curve" for the frequency of stellar luminosities (which generally represents the high-luminosity branch of the distribution satisfactorily), and to assume luminosity proportional to the cube of the mass,

$$L = m^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

which again is an order-of-magnitude simplification. We get, using a well known approximation for the Gaussian integral at large values of the argument:

$$Vp(m) = \frac{0.045 \quad V e^{-x^2}}{\sqrt{\pi}\, x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where $x = \frac{\log m/m_\odot + 0.36}{0.473}$, in agreement with Kapteyn's figures (Mt. Wilson Contr. **188**); V is the volume of space in cubic parsecs, of constant stardensity equal to the stardensity in the neighbourhood of the sun.

Below are computed upper limits to stellar masses, according to formulae (1) and (3).

Table 1.

Statistical Apparent Upper Limits of Stellar Masses for a Gaussian Distribution

V, cubic parsecs	10^3	10^6	10^9	10^{12}	10^{15}
Total stellar population	45	$4{,}5.10^4$	$4{,}5.10^7$	$4{,}5.10^{10}$	$4{,}5.10^{13}$
$m/m_\odot$ limit	4.7	11.5	32	74	162

The case $V=10^{12}$ corresponds approximately to the stellar population of the whole galaxy; the statistical upper limit, 70—80 solar masses, corresponds approximately to what is believed to have been observed actually[1]. It is interesting to follow the gradual increase of the "limit" of mass as the volume of selection increases; for $V=10^3$ ($\pi > 0''.16$), among the nearest stars a normal giant or an A0 star, of mass 4—5$\odot$ may be found; for $V=10^6$ ($\pi > 0''.016$), B stars, of mass 10—12$\odot$ occur; $V=10^9$ corresponds more or less to the actual selection of our observational data with respect to stars of high luminosity and mass in our galaxy; V Puppis, Y Cygni, with masses ranging up to 20$\odot$, indicate that the tabular limit, 32$\odot$, is not much beside the point. The brightest star in the Large Magellanic Cloud, S Doradus, according to Shapley has a luminosity $\sim 4.10^5\odot$, which with the conventional cubic relation gives a mass of about 70—80$\odot$, which corresponds to $V=10^{12}$ in our table (the true mass of S Doradus is probably greater, as for the Trumpler stars). If there is anything peculiar in the frequency of the more massive stars, it appears that they are observed more readily and in greater numbers than would follow from our assumed statistical law (Trumpler's O stars, loc. cit.); thus, if there is any deviation, it is in the sense that the existence of massive stars may be more secure than the existence of the smaller ones. In any case, the actual data do not imply a physical upper limit to stellar masses; the frequency of large masses seems to be governed practically by the same law as the frequency of small masses (with an excess of large masses); the law is apparently a statistical one connected with

[1] Larger masses as found by Trumpler for O stars, 100—300$\odot$ (P. A. S. P. **47**, 249, 1935), do not interfere yet with our formal way of reasoning, in which luminosity is the cube of mass, and the frequency of luminosities is actually considered. The luminosity of Trumpler's stars is only $\sim 2.10^4\odot$, corresponding to a "luminosity-mass" of $\sim 30\odot$ for our formulae.

the mode of origin of the stars, not with their physical stability. From Table 1, $V = 10^{15}$ corresponding to a population of about 1000 galaxies, we infer that the maximum stellar "cube-law" mass we may some day discover in a spiral nebula may be of the order of 160⊙, corresponding approximately to absolute magnitude — 12. These data, of course, are extrapolations.

From the above it appears that our present state of observational data does not allow us to decide whether there exists an actual upper limit to stellar masses, or whether the observed limit is only the result of a "statistical perspective". But even if such a real limit could ever be observationally established, the interpretation is not necessarily bound to considerations of physical instability. On the contrary, below we try to show that a limit of this order of magnitude, produced by energetic causes, is very likely to exist, even when stellar masses of different size are mechanically equally stable.

According to the modern physical theory of stellar interiors, as first outlined by Eddington, there exists a certain mass-luminosity relation for gaseous stars, such that the luminosity is more or less independent of the degree of compression of the stellar material; for supergiant stars any kind of atomic transmutations (Atkinson) as a source of stellar energy seems to be inadequate even with the short time scale; these sources are soon exhausted and the star (as a whole, or its nucleus only) starts to contract; unless a new source of energy comes into action (annihilation of matter), the star must live at the expense of its gravitational energy — which, after all, represents an ideal mechanism for converting mass into radiant energy (cf. W. Anderson, T. P. **29**$_1$, 1936); a star of sufficiently great mass, $> 1.6\odot$ for matter not containing hydrogen (cf. Chandrasekhar, Zts. f. Astroph. **5**, 321, 1932), cannot become a "degenerate" white dwarf; it is thus compelled to radiate with non-decreasing intensity (with the exhaustion of hydrogen, even an increase of the luminosity may follow). Thus, in one or another way, the supergiant rapidly burns down its mass. With (2) as the conventional law of luminosity ($\odot = 1$), the equation of variable mass (year as unit of time) becomes:

$$\frac{1}{m}\frac{dm}{dt} = -\frac{2}{3} \cdot 10^{-13}\, m^2, \text{ whence}$$

$$\frac{1}{m^2} = \frac{1}{m_0^2} + \frac{4}{3} \cdot 10^{-13}\, t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4);$$

here m_0 is the initial mass, m the mass after the time interval t. Assuming $m_0 = \infty$, (4) yields a maximum value for the masses of stars of an age t. Now, it is probable, although not certain, that most stars are more or less of the same age, $t = 3.10^9$ y (the "short" time scale); substituting this into (4), we get as an upper limit of stellar masses observable at present

$$m = 50\odot.$$

Such is the present mass of a star which started 3.10^9 years ago with an infinite mass.

This, again, is remarkably close to the "observed", as well as to the "statistically predicted" upper limit of the "cube-law" mass. Without doubt, there may exist massive stars of a younger age, and consequently of a greater present mass. For $t = 7{,}5.10^8$ y, $m = 100\odot$ is the maximum. Further, the luminosity for large masses varies less rapidly than with m^3, so that a longer life, or a larger upper limit of the true mass results. On the other hand, $m_0 = \infty$ cannot represent a real case; for a finite initial mass, the final mass is obviously smaller.

It is not advisable to lead our discussion further, because our considerations are based on simplifications and schematizations. We feel contented to point out, however, that by using more detailed laws instead of those expressed by formulae (2) and (3) (cf. Eddington, Internal Constitution of Stars, Cambridge 1926, p. 308 ff.), we do not arrive at essential changes in our conclusions.

Summary. There is no observational evidence for the existence of a definite upper limit to stellar masses; the observed frequency of large masses corresponds more or less to what would occur in a Gaussian distribution of mass logarithms such as suggested by Kapteyn for stellar luminosities; on the other hand, a true upper limit to stellar masses may exist as the result of the radiation of mass during the life time of the stars, if the life time amounts to a large fraction of the supposed age of the universe (3.10^9 y on the short time scale).

Tartu, Sept. 6, 1937.

IX. The Density of the White Dwarf A. C. $+ 70^0$ 8247.

By E. Öpik.

For this star Kuiper finds a density of the order of $> 10^7$ gr/cm^3 (Publ. Astr. Soc. Pacific **47**, 307, 1935); a revision of his data indicates that his estimate cannot be accepted, and that the star must have a density of $\sim 10^5$ gr/cm^3, thus nothing exceptional as compared with the two well determined cases (Sirius B, 3.10^4 gr/cm^3; o^2 Eridani B, 5.10^4 gr/cm^3; cf. Gabovitš and Öpik, T. P. $\mathbf{28}_3$, 1935). The absence of the hydrogen Balmer lines in the spectrum of this star is explained as the result of the exhaustion of hydrogen, the main source of subatomic energy.

Kuiper's estimate is based on a comparison of the colour of A. C. $+ 70^0$ 8247 with 10 Lacertae (H. R. 8622, mag. 4,91; H. D. 214680, sp. Oe 5); he finds the colours of the two stars essentially equal, and concludes that the effective temperature of the white dwarf is equal to $28\,000^0$, the "true" effective temperature of the O star; he thinks that space reddening does not affect the colour of the comparison star, 10 Lacertae. The latter assumption is quite arbitrary. The distance of 10 Lacertae (absol. mag. ~ -5 to -6) exceeds 1000 parsecs and space reddening at such a distance cannot be entirely absent. For the white dwarf, at ~ 15 parsecs distance, the space reddening is negligible. Thus the colour of the white dwarf must be redder than the "true" colour of 10 Lacertae by the whole amount of space reddening over 1000 — 1600 parsecs.

In the general catalogue of stellar colours by the writer (T. P. $\mathbf{27}_1$, 1929), the colour of 10 Lacertae is $C = -0.41 \pm 0.03$; the system of these colours is connected with the effective temperature by the relation

$$\frac{C_2}{T} = 1.47\, C + 1.82 \text{ (cf. Öpik, Ap. J. } \mathbf{81}\text{, 177, 1935),}$$

which gives $T = 11700^0 \pm 300^0$. If the temperature is too low for the O star on account of space reddening, it must be accurate for A. C. + 70° 8247 if the colours are equal. Our empirical formula for the colour temperature cannot be much in error for such a low temperature as 12 000°; Wien's formula which underlies theoretically (not empirically) the linear relation leads to an error in the colour temperature of only — 100°, as compared with Planck's law, and the actual error must be smaller on account of the empirical derivation of the relation. As no spectral lines appear in A. C. + 70° 8247, there can be no question of the spectral energy distribution to be distorted by absorption lines.

On the other hand, there is a direct indication of a not inconsiderable space reddening of 10 Lacertae. In the above mentioned catalogue of stellar colours we find fifteen northern stars which are at least as blue, or bluer than 10 Lacertae:

Harvard Revised Photometry Numbers

Star No.	153	6787	7298	7426	7739	7131	8603
m	3.7	4.3	4.5	4.8	4.8	5.5	5.1
H. D. Sp.	B3	B3	B3	B3	B3	B3	B3 + B5
C	— 0.43	— 0.42	— 0.62	— 0.48	— 0.45	— 0.45	— 0.45
p. e.	± 0.03	± 0.04	± 0.04	± 0.04	± 0.03	± 0.04	± 0.04

Göttingen Aktinometrie Numbers

Star No.	584	720	791	1016	1068	2766	2772	2950
m	7.3	6.4	7.2	7.1	6.8	6.7	7.2	8.0
H. D. Sp.	B3	B3	B8	B2	B5	AO	B3	B5
C	— 0.53	— 0.41	— 0.46	— 0.44	— 0.41	— 0.45	— 0.49	— 0.42
p. e.	± 0.04	± 0.04	± 0.04	± 0.04	± 0.04	± 0.04	± 0.04	± 0.04

The prevalence of spectrum B3 in this list is remarkable. Thus, a number of stars of spectra later than 10 Lacertae show the same colour (and colour temperature) in spite of the well known effect of the wings of hydrogen Balmer lines which tend to make the B and A spectra redder[1]; the ionization temperatures of these stars are about 15 000°; a comparison of the colour of A. C. + 70° 8247 with one of these B3 stars would have led, according to Kuiper's way of reasoning, to a tempe-

[1] Violet — blue colour indices of AO stars are almost equal to those of KO stars, on account of Balmer absorption, cf. T. P. 26_3, 1925.

rature of 15 000° for the white dwarf: but even this must be considered as too high because some space reddening is doubtlessly present already in the B3 stars.

There is no reason to assume that the colour temperature of 11700° for A. C. + 70^0 8247 is systematically in error, as it is from space reddening for the distant B and O stars, and for the B and A stars on account of Balmer absorption. As the estimated density varies with the sixth power of the effective temperature, Kuiper's density must be decreased 190 times; his estimate of mass as a function of radius (Chandrasekhar's relation) must also require a reduction factor of ~ 2.5, and the corrected density of A. C. + 70^0 8247 becomes

$$\frac{3{,}6.10^8}{190.2{,}5} = 70\,000 \text{ gr/cm}^3,$$

or by only 40 per cent greater than for o^2 Eridani B.

The absence of lines (cf. Kuiper, loc. cit.), chiefly of the Balmer lines of hydrogen in the spectrum of A. C. + 70^0 8247 (which should be the most prominent lines at 11 700°, corresponding to a normal B9 — AO star) may be considered as a proof of Atkinson's theory of atomic synthesis; a star cannot become a white dwarf before all its internal store of hydrogen is exhausted. For Sirius B and o^2 Eridani B the mixing is apparently incomplete, so that some hydrogen is preserved at the surface; in A. C. + 70^0 8247 the mixing is probably complete, and hydrogen absent from the atmosphere, as it must be absent from its interior.

Tartu, October 10, 1937.

STELLAR STRUCTURE, SOURCE OF ENERGY, AND EVOLUTION

BY

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TARTU 1938

Printed by K. Mattiesen Ltd., Tartu, 1938.

Abstract.

A critical review of the problem of stellar structure and evolution is given, special attention having been paid to the question of the sources of stellar energy. The most probable picture, obtained from a discussion of alternative hypotheses, is summarized in the present abstract.

A t o m i c s y n t h e s i s at present has become an experimental reality; however, details of the process in stars have not yet been established with certainty. The observed synthesis $C^{12} + H^1$, with the observed amount of carbon in the sun, is able to supply the radiation of the sun for 500 million years, at $T_c \sim 2.10^7$. If the central temperature is allowed to rise above 10^8, various observed nuclear reactions (involving capture of α-particles and the formation of neutrons) suffice to convert all the hydrogen of the sun into heavier elements, providing energy for 10^{10} years. Thus, atomic synthesis at a certain stage of stellar evolution m u s t play, quantitatively, all the role ascribed to it by its advocates. At the most probable central temperature of the sun, $1{,}2.10^7$, corresponding to adiabatic equilibrium and 38 per cent hydrogen, the well-studied nuclear reaction $Li^7 + H^1 \rightarrow 2\,He^4$, with the amount of lithium as in the solar atmosphere, alone provides for one-fifth of the energy of the sun; the small amount of lithium itself, however, cannot present a persistent source of energy, but must be restored by a process of synthesis from hydrogen; thus, the incorporation of one proton in the above mentioned reaction must be preceded by the incorporation of seven more protons required for the synthesis of lithium, and the whole cycle, terminated by the Li + H reaction, must yield eight times more energy, or by 60 per cent exceed the actual needs of the sun. The excess of energy generation thus found lies

within the uncertainty of the estimate; it appears highly probable that a cycle of nuclear reactions, starting from hydrogen, and including the Li + H reaction, represents the main source of energy for the sun, and for the main sequence stars.

The starting reaction of the atomic synthesis is most probably the not yet observed direct synthesis of the deuteron from protons (with the expulsion of a positron). If this is a reaction of protons in their ground state, the observed rate of energy generation in the sun, together with the requirement of equilibrium with the rate of the known Li + H reaction, sets for the probability of proton capture by a proton (after penetration has taken place) as low a limit as $q = 1{,}3.10^{-19}$; in such a case there is no hope of detecting the reaction in the laboratory. The energy generation, ε, should have to vary then with about the sixth power of the temperature, $s = 6$. The correlation of radius and mass for stars of the main sequence leads, however, to $s = 19$ (from 15 to 25). Formal agreement in s can be obtained upon the assumption that one of the reacting protons must be excited to a nuclear level of about 10 000 volts; this yields $q = 10^{-14}$, thus small but more hopeful for laboratory experiments. It might be worth while to try the synthesis of deuteron from protons "activated" by X rays from 10 000 to 50 000 volts, and bombarded by H canal rays of suitable velocity.

A special giant source of energy which works at low temperatures and densities appears to be impossible: whatever assumptions as to the law of energy generation are made, if these assumptions are in harmony with general physical principles, they invariably lead to the conclusion that the central temperatures of the giants must be at least as high, or higher than the central temperatures of the main sequence stars; hence the giants must be differently built (with a greater concentration of mass towards the centre) as compared with the main sequence.

Annihilation of matter as a source of stellar energy probably cannot come into play before the atomic synthesis is exhausted (at least in a central core); in a superdense core the existence of such a process cannot be denied in principle, but its effect is practically the same as for energy generation by gravitational contraction; without losing much of the general character of our discussion, we may altogether disregard annihilation of matter as a possible subatomic process, especially as such a conception does not seem very welcome to the physicist; the gravitational energy of a contracting superdense core renders possible the radiation into space of a large fraction of the stellar mass, and is an adequate substitute for the doubtful process of annihilation of matter.

Stellar structure may exhibit different types, with different degrees of concentration of mass towards the centre: without appealing to unknown physical properties of matter created ad hoc, we have shown that in the course of stellar evolution governed by atomic synthesis and gravitation the stars must arrive at widely different types of structure, characterized by widely different ratios $T_c : \left(\frac{\beta M}{R}\right)$. The difference in the structure of giants and dwarfs is explained in this way, as well as minor differences between the dwarfs themselves.

Convection currents arise at the centre of a star (Kramers opacity) as the result of a law of energy generation $\varepsilon \sim \varrho T^s$, when $s > 6.5 - 3\, n_0$, where n_0 is the adiabatic polytropic index for the temperature-density relation. In the case of atomic synthesis being an important source of energy, convection currents always start at the centre (except when the source of energy exhibits progressive exhaustion towards the centre). Convection in stellar interiors is highly efficient for the transport of heat; therefore, wherever convection starts, adiabatic, instead of radiative, equilibrium sets in: in the sun, the convective transport of heat is provided for by as small relative deviations as 10^{-8} of temperature and pressure from their adiabatic values. Only for a very low density of matter, especially near the boundary of a star, convection becomes inadequate as a means of heat transport. The role of rotational currents is only a subordinate one. According to the extent of the central convectional region, different types of structure of main sequence stars may arise. On account of convection, negative density gradients inward are impossible (except under certain rather peculiar conditions).

The net flux of energy, L_r, cannot be computed directly from the flux of radiation, Q_r, nor can the distribution of the true sources of energy inside a star be derived from Q_r in the presence of convection; in case of a sufficient degree of concentration of the energy sources, their distribution may be quite arbitrary; Q_r in such a case is prescribed by the adiabatic structure, whereas convection undertakes the transport of the excess $L_r - Q_r$; in other words, when adiabatic equilibrium takes place in the central region at least, no functional connection can be established between the type of structure ($n_{eff.}$) and the law of energy generation (s). As, however, heat that has passed outwards of a spherical shell of the radius r cannot be transported back against the temperature gradient, the total luminosity of a star, L, must be closely equal to the maximum flux of radiation, Q_{max}, which corresponds to a certain radius $r = r_0$ depending upon the actual structure. Although L may theoretically exceed Q_{max}, because convection can transport almost an unlimited extra amount of heat, in the normal course of the evolution (contraction) of a star it settles down automatically at an equilibrium radius R, with $L = Q_{max}$; there is no imaginable way of getting the star into the "overcompressed" state required for the generation of the extra amount of energy. Therefore, a normal mass-luminosity relation must hold for adiabatic structures, as it holds in the case of pure radiative equilibrium.

The different probable types of structure of actual stars, in the order of increasing ratio $T_c : \left(\frac{\beta M}{R}\right)$, are listed below.

The complete adiabatic model. This is of homogeneous composition throughout; the "dead zone" of convection at $r = r_0$, separating the central from the marginal convective systems, is overcome by the rotational currents; as mixing is complete, the uniformity of composition is not disturbed by the progress of atomic synthesis. The position of the sun in the radius-mass correlation (Fig. 3), as well as the agreement of the heat output of the sun with the value calculated for this model from the Li + H reaction, render it highly probable that the sun is built according to the present model. The evolutional course of the adiabatic model, with the gradual exhaustion of hydrogen, is characterized by a steady increase in luminosity and a slight secular increase of the radius (if $s > 7.5$), which after reaching a certain maximum ($1.2\ R_\odot$ for $s = 19$ for the sun) begins decreasing again; after the exhaustion of hydrogen, and of all the minor sources of subatomic energy, the collapse begins; smaller masses become white dwarfs, larger masses which cannot become degenerate end their lives as Wolf-Rayet stars of high bolometric, but low visual or photographic luminosity, on account of their high effective temperature. The average temperature of the earth, so far as it depends upon the radiation of the adiabatic sun, should at present increase at the rate of 1^0 C each 150 million years; the increase proceeds at an accelerated rate, rendering life on earth impossible after 4000 million years; the exhaustion of hydrogen in the sun takes place after 10^{10} years, when the surface of the earth is heated to over $+600^0$ C; after that, the collapse of the sun into the white dwarf stage lowers the temperature of the earth to about -150^0 C. As to the past, the geologic history of the earth does not contradict such a conception; disregarding irregular fluctuations such as the ice ages (which are of too short a duration to influence the mean geologic temperature), a gradual increase of the terrestrial temperature at about the theoretical rate seems to be indicated; the greater frequency and extent of the glaciations in the Palaeozoicum and Praecambrium, as compared with the later eras (which contain only one ice age, the recent Diluvial), is one of the most important arguments in favour of the earth getting gradually warmer. From this standpoint, the investigation of glaciation must be considered one of the most important problems of archaean geology.

The composite adiabatic-radiative model of homogeneous composition. This originates when the extent of the central convective region does not reach to r_0; the model is only of genetic interest, because in actual stars the atomic synthesis changes the composition of the core, and transforms the model into the next following.

The composite adiabatic-radiative model of non-homogeneous composition. The exhaustion of hydrogen in the convective core increases the mean molecular weight there, a circumstance which renders impossible any further efficient interchange of matter between core and envelope. The outer layers may be in complete, or in partial radiative equilibrium; in the latter case a zone of radiative equilibrium separates the inner from the marginal convective regions. With progressing exhaustion of hydrogen, the ratio $T_c : \left(\frac{\beta M}{R}\right)$, and the radius increase, producing at an advanced stage of exhaustion of the core a kind of "semi-giant" structure. Procyon is probably in such a state. During the evolution, the range in luminosity is smaller, the time-scale shorter than for the complete adiabatic model. With the complete exhaustion of the subatomic energy sources in the core, the composite model enters the giant stage of evolution, defined by the following model.

The giant model. While the exhausted central core contracts at a rate determined by the supply of gravitational energy, the outer shell containing hydrogen cannot follow indefinitely; the inner portions of the shell settle down automatically at certain moderate effective values of temperature and density, so that the release of subatomic energy does not exceed the amount which the shell is able to transport to the surface; the peculiar distribution of the energy sources creates the typical giant structure, with a contracting superdense core and an extended shell (simple, or composite) of expanding tendency. The ratio $T_c : \left(\frac{\beta M}{R}\right)$ may increase almost indefinitely. The core gradually sucks in the exhausted material of the envelope, and the evolution also ends in a Wolf-Rayet star, rendered almost invisible on account of its high surface temperature.

Condensation of meteoric material at an early (nebular) stage of a star's life may create an initial core of small hydrogen content, favouring evolution towards a composite, eventually a giant model. An initial small exhausted core of such a kind need not disturb the complete adiabatic equilibrium in the outer regions; it is not impossible for the sun to possess such a core.

S t a t i s t i c a l e q u i l i b r i u m o f e v o l u t i o n. The observed relative frequency of main sequence stars and of giants of different luminosities in the average Galactic space (surroundings of the sun, $D < 125$ parsec) and in the Globular Clusters may be accounted for by the following hypotheses: all stars are created as main sequence objects, of which a fraction a remains completely adiabatic, whereas the fraction $1 - a$ is originally composite and develops into giants; the duration of the main sequence stage of a composite model equals 0.3 of the life of a complete adiabatic model of equal mass, and the life of a giant is at least 50 times longer than that; all stars in the Globular Clusters are of the same age of 3.10^9 years, whereas the stars in the average Galactic space have been continually born at a uniform rate during the last 3.10^9 years; the initial hydrogen content equals 40 per cent. The theoretical upper limit of luminosity of a main sequence star 3.10^9 years old, i. e. of an adiabatic model on the edge of collapse, is found equal to $- 2.3$ (bolo. mag.), as compared with the observed limit, $- 2.0$, in Globular Clusters; the corresponding mass is $1.9\odot$. More massive main sequence stars in the Galaxy must be either younger than 3.10^9 years, or contain initially more than 40 per cent hydrogen. The fraction of adiabatic models born is $a = 0.5$ for masses below $4.3\odot$, and 0.94 for larger masses; the latter result is somewhat puzzling. The interpretation of the statistical picture cannot yet be considered as definitive.

The Mount Wilson n and s subdivisions of A and B spectra are tentatively identified as the adiabatic, and the composite (semi-giant) models, respectively.

A p p r e c i a b l e n u c l e a r d i s s o c i a t i o n in a collapsing core does not seem to be possible, the increase of pressure counterbalancing that of temperature; this conclusion seems to be quite definite, as possible departures from the dissociation formula at very high temperatures do not much alter the results of our computations. The formation of a pure neutron core, under conditions of thermodynamic equilibrium, or by diffusion, is impossible.

W h i t e d w a r f s may be explained as collapsed main sequence stars with a small initial hydrogen content (for Sirius B, $\leqq 26$ per cent); a different initial hydrogen content in the components of a binary may be easily explained (cf. above, condensation of meteoric material); if, nevertheless, objections to that effect should be made, we should first want to have a plausible explanation of the origin of binary stars.

The ice ages, due most probably to temporary minima of solar radiation, may be explained by a temporary expansion of the sun; an ice age of one million years' duration can be produced by a total expansion of less than 1 per cent of the solar radius; during the expansion, a fraction of the heat generated is spent on mechanical work, and is withdrawn from the amount radiated into space. The withdrawal is possible only on the expense of the convectional transport of heat in the outer portions of the star, at $r > r_0$.

The spread of luminosities around a mean mass-luminosity relation, due to the hydrogen content changing with age, is not as large as has been anticipated; considering the relative speed of the exhaustion of hydrogen (proportional to the luminosity), and the equalizing effect of electron scattering, the maximum spread $\pm$ 0.46 mag is expected at $M = 2.5\odot$, decreasing for smaller as well as for larger masses.

Introduction.

Below it is attempted to give, on the basis of our present state of knowledge, a picture of stellar structure and evolution. Although guided by a few basic ideas, we have tried, whenever possible, to consider alternative hypotheses, and the conclusions we arrive at were forced upon us by the combined theoretical, experimental, and observational evidence.

We are not going to construct at present a new mathematical theory of stellar interiors, based on fixed premises chosen a priori; the amount of theoretical work already done on this subject is large enough to offer a choice, and to obtain an answer, at least a qualitative one, to the question: what happens to the stars, when the initial physical conditions are given. Stellar structure is a physical, not a mathematical problem. What matters are the premises, not the exact mathematical deductions from given premises; we want to know the actual physical conditions determining stellar structure and evolution; a "correct" mathematical theory may then easily follow*. We believe that a mere qualitative picture, taking into account all the complexity of the conditions in stellar interiors, is still a better approximation to the truth than an exact mathematical theory based on simplifications which do not take into account certain most important factors of stellar structure and evolution.

For our purposes we have to compare the present state of our physical knowledge, experimental as well as theoretical, with individual and statistical observational data referring to the stars. Nuclear physics at present has progressed far enough to lead to certain definite conclusions regarding stellar structure. Thus, it may be considered an experimentally established fact that nuclear transmutations may take

* Cf. Eddington[1], pp. 101—103, where a brilliant discussion of the role of physical premises as compared with mathematical deduction is given.

place at a considerable rate in the interior of stars with a central temperature as low as in Eddington's model, namely, at twenty million degrees, and even less. On the other hand, we cannot affirm that the energy liberated by these atomic transmutations is in all cases the only, or even the chief source, of stellar energy. With respect to the source of stellar energy, as well as in different other respects, we must allow for a vast realm of the unknown; nevertheless, it seems to be possible, by the aid of general physical principles, to set up limitations to the unknown, and to arrive at definite conclusions with respect to stellar structure and evolution.

When dealing with actual stars as they are observed at present, the question of how long they have already existed as individual stars is of primary importance. A maximum age, and in many cases a probable age of the order of 3.10^9 years may be estimated: this is the age of the universe on the short time scale. The arguments in favour of the short time scale are at present so numerous and weighty that the alternative of a much longer time scale need hardly be considered (cf. [2,3]).

Section 1.

Gravitational Energy.

a. Radiation of mass.

Gravitation as a source of stellar energy is doubtlessly inadequate for stars built on something similar to the standard model of Eddington; for white dwarfs, however, gravitation appears to be perfectly sufficient; similarly, gravitation may play an important, perhaps a predominant, role in nuclei of planetary nebulae, and in massive stars with superdense cores. Ultimately, gravitation may turn out to be the most powerful source of energy, a substitute for the annihilation of matter as postulated by Eddington, Jeans and others; in a manner imagined by W. Anderson[4] the star, fed by gravitation only, may radiate away a considerable fraction of its mass (not more than about one-half of it, however, and perhaps much less when the compressibility is limited); the number of individual atoms may remain unchanged in such a case, and the mass of each atom decrease, being converted into radiation which afterwards leaves the star; there is in fact no escape from Anderson's conception, unless the principle of the conservation of energy (mass being a form of energy) is to be abandoned in the case of stars radiating at the expense of their gravitational energy.

b. Limit of degeneracy.

According to a theorem established by Chandrasekhar [5], when the stellar mass exceeds $6.6\,\mu^{-2}\odot$ (μ = molecular weight) "the perfect gas equation of state does not break down, however high the density may become"; there may be limitations to this statement at very high densities; nevertheless, except for such unpredictable limitations, in the absence of other sources of energy, a giant star may live actually on its gravitational energy for intervals of time which are large as compared with the short time scale. The limit of mass for which such an unlimited contraction with practically undimmed luminosity appears to be possible, is for $\mu = 1$ (~ 30 per cent hydrogen), about $6.6\odot$. However, as pointed out below, the contraction and formation of a central core can start only with the exhaustion of hydrogen, thus $\mu \sim 2.1$, and the limiting mass may be estimated at $\sim 1.6\odot$ in agreement with Chandrasekhar [5]. This, however, can be valid only until a disintegration of the atomic nuclei into protons and neutrons starts at very high temperatures, which means a smaller μ again, and a higher limit of mass; however, such a nuclear dissociation cannot come into question for the existing stellar masses (cf. Section 4).

c. Nuclear dissociation and source of energy.

It is interesting to note that such a star, contracting indefinitely, cannot make use of the energy of atomic synthesis in the manner imagined by Sterne [6]; in a state of thermodynamic equilibrium as considered by him (temperatures $> 10^9$), energy is liberated when the material is cooling; the temperature of the contracting gaseous star rises, however, and atomic transmutations must now absorb energy instead; the star must provide energy for radiation *plus* atomic synthesis (which in this case consists in the formation of the lighter elements out of the heavier ones), and gravitational energy must pay for both, unless there is a third source of energy (which is very unlikely). The consideration of variable density, disregarded by Sterne, changes the picture altogether (Section 4); at a certain degree of compression the dissociated material begins to recombine again (Figure 1), and heat is liberated at an increasing temperature; but appreciable dissociation does not happen in such a case, for $M < 50\odot$; thus the mechanism of energy generation imagined by Sterne (and Milne, cf. [23]) does not work.

Section 2.

Annihilation of Matter.

a. The time scale.

The long time scale, which for some while seemed to dominate the minds of astronomers (including that of the writer, cf. [10]), required for the stars a powerful source of energy which, in the terminology of Eddington and Jeans, is called the annihilation of matter. We keep this label to designate processes in which a large fraction of the atomic mass, much larger than the packing fractions of the elements, 100 per cent in the limiting case, is converted into radiation or into kinetic energy, without making use of the potential energy of gravitation. The actual process need not be specified; the original idea of electrons and protons annihilating each other has lost much of its probability after the discovery of the neutron and the positron, but nevertheless it cannot be denied altogether.

With the adoption of the short time scale the hypothesis of the annihilation of matter is forced upon us only when we assume that the more luminous stars are as old, or almost as old, as the universe itself, whereas most stars get along excellently with atomic synthesis; further, if superdense cores are postulated, it becomes, from our present standpoint, practically impossible to discern between annihilation in our restricted sense, and between the gravitational conversion of mass into radiation.

b. Giants without superdense cores.

The necessity of postulating annihilation shows itself only when we consider giant stars to be built more or less according to a "standard" model without a superdense core; we propose to consider the question first from this standpoint. Thus, let us assume giants and dwarfs to be built more or less according to a homologous model (which in the general case need not be a polytropic one), so that the central densities and temperatures follow more or less the order indicated by Eddington's standard model; thus, dwarfs are hotter and denser than giants, and

$$T_c \sim \frac{\beta \mu M}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)^{*}.$$

The process of annihilation may be imagined to belong to one of the following classes: (α) it may be a reaction involving one single corpuscle (an atom with bound electrons, or a nucleus); (β) it may be a process of collision of two (or more) corpuscles, involving a potential barrier and thus stimulated by increasing temperature and density; (γ) it may be a process of collision of two (or more) corpuscles without a potential barrier, possibly with a limitation of the angular momentum; this leads to indifference with respect to temperature, or even to a slight decrease of the rate with increasing temperature, whereas density still favours the process. The origin of cosmic rays may be considered on this occasion, because these have been referred to as possibly connected with annihilation of matter, and as in analogy with hypothetical processes which may occur in stars [7]. It is proposed to consider the different types of processes separately.

Process (α). Radioactive processes belong to this class; the observed processes of radioactivity, however, involve much smaller energies than atomic synthesis, and cannot come into question for our purposes. Spontaneous annihilation of an H atom involves sufficient energy; such a process, however, must occur like radioactivity unaffected by temperature and density, at least when the temperature is below 10^9; it should occur everywhere with an intensity proportional to the concentration of the active material; the secular stability of stars,

* Cf. [1], p. 135.

depending upon the automatic adjustment of the energy generation to the loss prescribed by mass, radius, the law of opacity, and composition (cf. [8, 9, 10]), would be impossible in such a case, unless the stars should obtain the exact amounts of the active material required to maintain their radiation. Also, if cosmic rays could be explained in such a manner, the absence of a process of this sort in the earth's crust would speak against the possibility of it. Thus, if process (α) exists, it cannot play a conspicuous role in the energy generation of stars, and cannot help us out of our difficulties; this process may be left out of consideration in the following discussion. For annihilation of matter as a source of stellar energy, only collision processes need be considered.

Process (β). Collisions of two particles separated by a potential barrier are qualitatively similar to Atkinson's atomic synthesis [11, 12, 13, 6]; the speed of reaction is proportional to density and to an exponential function of the temperature, the latter varying extremely rapidly; although the requirements of secular stability are automatically fulfilled in this case, a single process of annihilation involving particles which are relatively abundant in the universe cannot be accepted, because in such a case dwarfs should produce much more energy than giants. An exhaustion of the energy source in dwarfs through annihilation cannot be postulated, because giants are the ones which must arrive at exhaustion first.

An escape may be found in a hypothesis put forward by Atkinson in connection with his theory of atomic synthesis: let a certain atomic nucleus A be indispensable for the reaction of annihilation, and let A, more or less stable at the low temperature of a giant, disappear by atomic synthesis at the higher temperatures of the dwarfs; obviously A can belong only to the lighter atomic nuclei; if the dwarfs are well mixed by convection currents, A may disappear altogether from the star, and the powerful source of energy will stop working in the dwarfs. If, however, the mixing is incomplete (which probably is often the case), at a certain distance from the centre there will be found a region of intense annihilation; the smaller energy production of the dwarf may in this case be ascribed to the smaller mass involved.

The greatest difficulty is to conceive how a dwarf, with such an enormous store of energy released at low temperatures, can ever get into its present condensed state; and still more so — why diffuse stars of small mass should be entirely absent. A star contracting from infinity must soon reach the state when the central temperature is sufficiently high for annihilation to balance radiation (prescribed by the mass-luminosity law); no further contraction can follow before the exhaustion of the energy source, which for dwarfs cannot come into consideration at all. Thus, on the basis of a condensation theory of the origin of the stars, with our postulated source of energy, the central temperatures of the dwarfs should be lower than those of the giants, which is exactly opposite to the actual state of affairs (on the basis of the standard model, of course).

The hypothesis that A cannot be changed or produced by atomic synthesis in stellar interiors (such elements might be those of high atomic weight), and that A, present in the giants, is absent from dwarfs, must be rejected: it is inconceivable how a certain element, which must be present in the original diffuse matter of which stars are imagined to have been built, should get only into giants (showing an inferior limit of mass), and be entirely absent from stars of small masses.

Cosmic radiation cannot originate in the diffuse interstellar matter from process (β). This process must be considered improbable, and the respective hypothesis useless, as unable to explain the existence of actual stars if their central temperatures are defined by (1).

P r o c e s s (γ). To obtain the greatest contrast with case (β), we assume a constant upper limit of the angular momentum in the nuclear collision, $vr \leq$ const.; thus the target area is $v^{-2} \sim T^{-1}$; further, the probability of the reaction to take place we assume as $\sim v^{-1} \sim T^{-\frac{1}{2}}$ (which, according to Bethe[14], must hold for the capture of neutrons by nuclei); the number of collisions is proportional to the density, ϱ, and to the velocity, $\sim T^{\frac{1}{2}}$; finally we get for the probability of the reaction to happen per unit of time

$$W \sim \varrho\, T^{-1} \quad \dots\dots\dots \quad (2).$$

With (1), setting $\beta = 1$ for not too large masses, we get

$$W \sim \varrho^{\frac{2}{3}} M^{-\frac{2}{3}}.$$

When comparing giants and dwarfs we find that ϱ decreases as M increases; thus smaller values of W result for giants as compared with dwarfs. The difficulty is substantially the same as in process (β), and the situation becomes worse when we consider triple or multiple collisions where the effect of density is enhanced. Our approximation ($\beta = 1$) has practically no influence on the conclusion.

For the explanation of cosmic radiation the situation is slightly more favourable when we forget about the stars: if T is taken small enough, a sufficient probability of reaction may result. The absence of annihilation on the earth may be explained by the shielding effect of bound electrons which make nuclear collisions of small velocity impossible; in interstellar space there is considerable ionization; in fact, ionized hydrogen alone should be regarded as responsible for cosmic radiation from interstellar space in such a case, because complete ionization of other elements appears to be impossible there. For the interstellar space of our Galaxy $T = 3^0$ K may be estimated as a minimum (approximately the temperature of black body equilibrium; gaseous substances should attain a much higher temperature, on account of line absorption), and $\varrho \sim 10^{-24}$. As compared with a supergiant of $T = 3.10^6$ K, $\varrho \sim 10^{-6}$, we find that the star should exhibit a 10^{12} times greater activity than interstellar space with respect to the annihilation of matter. It is known that the total amount of cosmic radiation is about the same order of magnitude as the integrated light from the stars, and that the total mass of diffuse matter is also comparable with the total mass of the stars in the Galaxy. A ratio of activity of 10^{12}, however, means that a process important in stars must have zero intensity in interstellar space, and a process important in interstellar space must be enhanced in a star to approach the intensity of 10 000 Super-Nova explosions. The assumption of a partially high density of interstellar matter, such as exists on the surface of solid bodies (meteors), cannot save the situation, because the number of collisions still would depend upon the density of the surrounding gaseous medium. The conclusion seems to be definite that cosmic radiation cannot be traced to processes of annihilation (or atomic synthesis) happening in the diffuse matter of interstellar space. From the above consideration of the three possible types of the annihilation of matter we conclude that if the stars are built more or less according to Eddington's model, annihilation of matter cannot be an important source of energy. The hypothesis does not serve its purpose: it does not help us to escape from the assumption of central condensations in giant stars. Being also in disharmony with our physical knowledge, this hypothesis may safely be rejected altogether, at least for temperatures below 10^{10} K. The only process of radiation of stellar mass which we need take into account takes place through gravitation as described in Section 1.

Section 3.

Atomic Synthesis.

a. Rate of the reaction.

The theory of atomic synthesis as a source of stellar energy has been put on a sound physical basis by Atkinson[12, 13]; although the actual chain of processes involved in the synthesis of heavier elements out of hydrogen cannot yet be indicated with certainty, some of the reactions such as $Li^7 + H^1 \rightarrow 2\,He^4 + 17$ Mev are well established experimentally. The reaction takes place at a sufficient rate at comparatively low stellar temperatures, in collisions of a high multiple of kT, and thanks to the circumstance that penetration according to wave mechanics (Gamow) is possible when the relative energy of the collision is smaller than the potential barrier. A comprehensive review of the theory is given by B. Strömgren in B. VII (Ergänzungsband) of the Handbuch der Astrophysik. A somewhat more precise theory of transmutation than Atkinson's is given by Wilson[15], but his conclusion that at $T = 4.10^7$ K the rate of reaction is negligible is untenable — the reaction is perceptible even at much lower temperatures; Steensholt[16] made a numerical solution of a stellar model in hydrostatic equilibrium on the basis of Wilson's theory and finds "that it is quite possible to build up stars generating energy by proton capture in the way imagined by Wilson, assuming internal temperatures of the order of 10^7—10^8 K. This is in direct contradiction to the view of Wilson... He failed, however, to inquire closely into what is to be understood by a reasonably large rate of reaction... this fact to some extent strengthens the position of Atkinson's views." For the reaction of two elements of nuclear charges $Z_1 e$ and $Z_2 e$, and of density $\varrho_1 = N_1 m_1$ and $\varrho_2 = N_2 m_2$, respectively, the rate of decay (reciprocal of the life time) of one of the elements is given by

$$\frac{1}{\varrho_1}\,\frac{d\varrho_1}{dt} \sim -\frac{q\,\sigma\,\varrho_2}{m_2\,Z_2}\,F\left(\frac{Z_1^2\,Z_2^2\,m_1}{T}\right) \ldots (3),$$

where F is a certain exponential function (cf.[6], p. 770 f.); the formula is the result of the combination of Gamow's probability of penetration with Maxwell's law of the distribution of molecular velocities; thanks to the extremely strong dependence of the speed of transmutation upon temperature, the important range of temperature for a given reaction is rather limited; for this limited range $F = T^s$ may be assumed with sufficient approximation, thus

$$\frac{1}{\varrho_1}\,\frac{d\varrho_1}{dt} \sim -q\,T^s \ \ldots (3'),$$

with s of the order of 10 to 20. $\sigma = 10^{-25}$ cm^2 is the assumed (constant) cross-section of the target[17]; q = const. $\sim$ 0.01 to 0.1, for intense reactions observed in the laboratory, is the probability of capture when penetration has taken place; for uncommon reactions q may be much smaller. Atkinson apparently disregarded the importance of the factor q, assuming it to be always of the same order of magnitude which is not the case.

b. Overstability.

The only serious objection to atomic synthesis as an energy source has been the danger of pulsational instability, or "overstability", which seemed to follow when $s > 3$ (cf.[1], p. 201 f.); as the stars do not usually pulsate, Eddington and Atkinson tried to escape from this danger by supposing that the energy is produced in two steps, so that only the first step depends upon the temperature, whereas the second step during which the major part of the energy is released is independent of temperature and covers a time interval which is large as compared with the period of the pulsation of the star. Now, however, it appears that the danger of overstability has been exaggerated, on account of imperfect analysis; Cowling[18] has shown that if there is no convection, the star is vibrationally stable save when γ, the ratio of specific heats, is nearly equal to $^4/_3$; if there is convection, the lower limit of γ for which stability begins increases from the minimum value $^4/_3$ with the increase of the exponent s in (3′); for example, for $s = 20$, $\gamma \geqslant 1.44$ is the condition for stability.

The effective value of γ for the sun may be estimated according to Eddington[1], p. 191, in the following way. The heat content, inasmuch as it depends upon the temperature, consists of the following components: the kinetic energy of the particles which are all monatomic, with $\gamma = {}^5/_3$; the imprisoned radiation, with $\gamma = {}^4/_3$; the energy of ionization and excitation, with γ near 1. Let the effective ratio of specific heat for the ionized material alone, without radiation, be Γ. Eddington gives formulae for γ as a function of Γ and β (cf.[1], p. 191, Table 28). The average value of Γ for the whole interval of temperature from 0 to T may be computed by assigning to the separate values (kinetic and ionization) of γ weights proportional to the corresponding heat contents.

The kinetic energy may be taken according to[1], p. 289; the ionization depends upon composition. A considerable hydrogen content of the stars seems to be at present highly probable[19, 20], as this removes the discrepancy between the astronomical and the physical values of the opacity. We assume, therefore, as a combination of the data of B. Strömgren[21] and Russell[22], the following schematical mean composition for the sun:

Table 1.

Mean composition, and Energy of Ionization for the Sun.

Element	H	He	O	Fe	Other metals	All
Proportion by weight	0.37	0.05	0.29	0.06	0.23	1.00
Energy of ionization, volts	13.5	78	2020	17000 *	8000 *	—
„ „ „ , 10^{-12} erg : gr	13.1	18.9	121	293	241	114

* Without the two inner K-electrons.

For the components of the heat content and the mean value of γ (not a very defiutite conception) we have the following data:

Kind of energy	Kinetic	Radiant	Ionization	Γ	$\bar{\gamma}$
γ	1,667	1,333	($\geqslant$)1,000	—	—
Amount, erg: gr. a) $(1-\beta)=0.003$	$14{,}1.10^{14}$	$0{,}09.10^{14}$	$1{,}14.10^{14}$	1,615	1,613
„ „ b) $(1-\beta)=0.05$	$13{,}5.10^{14}$	$1{,}5.10^{14}$	$6{,}0.10^{14}$	1,462	1,448

Case a) corresponds to the composition according to Table 1, with $T_c = 1{,}91.10^7$ K, $\bar{\mu} = 0.98$ [21]; $\bar{\gamma} = 1{,}613$ indicates, according to Cowling, vibrational stability even with $s \geqslant 20$. Case b) is computed for Eddington's "standard" data, $\bar{\mu} = 2.11$, $T_c = 3{,}95.10^7$ K, on the assumption of 100 per cent of completely ionized iron, including the two K electrons (35 000 volts altogether); $\bar{\gamma} = 1.448$ is already near Cowling's limit of instability for $s = 20$. When we allow a star of given composition to contract, the content of kinetic energy changes as R^{-1}, the radiant energy per unit mass $\left(= \frac{R^{-4}}{\varrho}\right)$ changes in the same proportion (thus $1-\beta = \text{const.}$), whereas ionization remains practically constant; as a consequence $\bar{\gamma}$ increases (approaching the limiting value for $\Gamma = {}^5/_3$), and the star gets farther away from overstability, especially because the exponent s decreases rapidly with increasing temperature[6]. Hence we conclude that the stars can very well be vibrationally stable with Atkinson's mechanism of energy generation — as stable as they are pictured by observation. Pulsations maintained by energy generation may be expected only for special values of the central temperature and density, especially when the ionization is in a stage of transition so as to be oversensitive to moderate changes of temperature; Eddington ([1], p. 203 f.) has shown that a peculiar behaviour of the coefficent of opacity at such transition phases of ionization may be itself a cause of vibrational instability; both causes of pulsation may perhaps be expected to cooperate in the Cepheids.

c. Giant and dwarf energy generation.

To account for the puzzle of giants which produce more energy at a low temperature than dwarfs at a high one Atkinson[12] assumes two main processes of energy generation by transmutations; one working intensely at low temperatures (hypothetically identified as He → Li → Be^8 → He) and stopped at high temperatures (rapid transmutation of Be^8 into heavier nuclei), when a second, or several, new sources come automatically into action.

The situation is exactly the same as discussed in Section 2.*b*, "process (β)", and the conclusion is identical*:

from the genetic standpoint it is inconceivable how the dwarfs (all dwarfs!), in the process of contraction, could ever reach the second, hot-temperature stage; the contraction must stop as soon as the first, "giant" source of energy comes into action, a source which, on the short time scale, should be inexhaustible for dwarfs; for giants and dwarfs the same group of processes of atomic synthesis must exist, which come into action step by step as the temperature increases; the start is made at approximately the same T_c (in giants higher than in dwarfs), and in giants exhaustion begins earlier and the temperature has to rise in order to open up the next source of energy. Giants cannot get enough energy at all unless their central temperatures are higher than those of the dwarfs, this can never be attained for a homologous structure of giants and dwarfs. Thus, giants must possess superdense cores, probably formed by the collapse of the central portions after the exhaustion of the original source of energy (exhaustion of hydrogen in the central region, cf. below). We thus arrive at the conception of Milne's superdense cores, only in giants of course, but on a different basis of reasoning than Milne's[23], who thought that subatomic energy can be efficiently released only at temperatures exceeding 10^{10} degrees; we know that this is not correct. As to Russell's "giant stuff", it may be most probably identified with gravitation.

* At stellar temperatures Be^8 is apparently never formed directly from Li, and Be^8 seems to be stable (cf. below); thus regeneration of He^4 follows without delay, and Atkinson's mechanism, by which dwarfs had to escape the regeneration of He, does not work.

d. *The lithium-hydrogen reaction.*

The possible chain of processes of atomic synthesis leading to the liberation of subatomic energy has been discussed by Atkinson[12, 13]; however, there is little certainty in the details, although the general picture is more or less clear; experimental data could only settle the question.

A transmutation of considerable energy generation, well investigated in the laboratory, is the hydrogen-lithium reaction. We may inquire into the probable importance of this reaction in the energy budget of the sun. In the reaction $Li^7 + H^1 = 2\,He^4$, the rate of energy generation is proportional to the amount of hydrogen present multiplied by its rate of decay; the rate of decay is given by equation (3) when ϱ_2 is the density of lithium. According to Russell[22], the abundance of lithium amounts to $3{,}8.10^{-8}$ of mass in the solar atmosphere, which is 1 000 times less than in the earth's crust. If the scarcity of lithium in the sun is caused by atomic transmutations, the relative amount of this metal in the solar atmosphere cannot be less than in the interior of the sun; assuming Russell's value, we possibly overestimate the rate of the generation of energy. On the other hand, by assuming a minimum value $q = 0.01$ for the probability of nuclear capture[24], we underestimate the rate of the reaction. Further, although we consider the capture of only one proton in our reaction, it is clear that the supply of lithium must be continually replenished, unless lithium were allowed to disappear within a rather short interval of time ($\sim 10^5$ years, for the actual sun); whatever the chain of the synthesis of lithium, if the synthesis starts from hydrogen it means seven protons more captured; the total energy released for each capture $Li^7 + H^1 = 2\,He^4$ is thus equivalent to the mass defect $8\,H^1 - 2\,He^4$, which corresponds to $6{,}4 \,.\, 10^{18}$ erg per gram of hydrogen. Similarly, for the rate of decay of hydrogen we have to take eight times the value which directly follows from (3); for the latter we used Sterne's table (cf.[6], p. 774) of the values

$$\varrho_2 \left[\frac{1}{\varrho_1}\,\frac{d\varrho_1}{dt}\right]^{-1}$$

for the hydrogen-lithium reaction.

According to B. Strömgren[21], we assume 37 per cent of hydrogen and $\bar{\mu} = 0.98$ (corresponding to $a = 2.5$ for the mass-luminosity relation). The temperature and density, for the first approximation, we take according to the polytrope $n = 3$ (Emden's tables; also cf.[1], pp. 83, 85 and 136). The computations are as follows:

Fraction of internal mass $\frac{Mr}{M}$	T 10^6 deg.	δ density gr/cm^3	$\log\left(\frac{1}{\varrho_1}\frac{d\varrho_1}{dt}\right)$ sec^{-1}	Life of hydrogen, years	Energy, erg per gr of solar mass and second
0.0000	19.1	76	−13.38	8.10^5	250
0.0025	19.0	74	−13.42	8.10^5	1300
0.0192	18.3	67	−13.62	$1{,}3.10^6$	1600
0.0595	17.4	58	−13.97	3.10^6	1000
0.125	16.3	47	−14.37	8.10^6	520
0.212	15.0	37	−14.88	$2{,}4.10^7$	170
0.312	13.7	28	−15.44	9.10^7	42
0.418	12.4	21	−16.10	4.10^8	6
0.498	11.1	15	−16.83	2.10^9	1
1.000	0	0	. . .	. . . .	
Sum . . .	. . .	. . .	. . . .	. . . .	4889

For $\frac{1}{\varrho_1}\frac{d\varrho_1}{dt}$ is assumed the value $8\,q\,\varrho \cdot \frac{1}{P}$, where P is the life time as tabulated by Sterne (graphical interpolation used); here $8\,q\,\varrho = 8.0{,}01.3{,}8.10^{-8}\,\delta = 3.10^{-9}\,\delta$ is taken (the factor eight allows for the eight protons finally bound after the Li + H reaction is accomplished). The contribution to the energy per gram of solar mass is $0{,}37.6{,}4.10^{18}\,.\,\Delta\left(\frac{M_r}{M}\right)\cdot\left[\frac{1}{\varrho_1}\frac{d\varrho_1}{dt}\right]$, for a given shell containing the fraction $\Delta\left(\frac{M_r}{M}\right)$ of the total mass.

The computations give a total of 4 900 erg per gram of solar mass and second as the energy developed by the hydrogen-helium synthesis with the H + Li reaction as the final phase for a polytropic model $n = 3$. This is 2500 times larger than the actual radiation of the sun; a reduction of the internal abundance of lithium in such a ratio would lead to agreement, but it appears from the following that so large a reduction is not necessary. From the table we infer that 50 per cent of the energy is developed by the central fraction 0.03 of the whole mass, at an effective temperature of energy generation $T_\varepsilon = 17{,}4.10^6$ K $= 0{,}94\ T_c$. Such a concentration of the energy sources corresponds closely to the mathematical point-source case; practically this leads to convective adiabatic equilibrium, with $n = \frac{1}{\gamma - 1}$ (cf. below), for which T_ε is lower than for the "standard" model $n = 3$. Assuming $\gamma = 1.615$, $n = 1.63$ (cf. Subsection *b*), the temperature (according to Table 2 below) amounts to 0,651 of the above. A second correction in the same direction is required to account for the different energy output (at $n = 1.63$, $a = 2.2$, cf. Table 2, interpolation of $\frac{1}{a}$); as compared with the case $a = 2.5$, the polytrope $n = 1.63$ requires a slightly larger hydrogen content, corresponding to a further reduction of T_c by 2.0 per cent. Thus, for T_ε, the effective "working" temperature of the sun, we get at $n = 1.63$ only 0.638 of the first adopted value, or $T_\varepsilon = 17{,}9 \,.\, 10^6 \,.\, 0{,}638 = 11{,}4.10^6$ ($T_c = 12{,}1.10^6$; 38,5 per cent hydrogen instead of 37), which, according to Sterne's table, leads to a decrease in the rate of the reaction in the ratio 200 : 1 (the exponent s in (3′) becomes 13,3). Further, the central density for $n = 1.63$ is 7.6 times less than for $n = 3$. On the other hand, a 4 per cent increase follows from the increased hydrogen content. The corrected rate of energy generation in the sun from the Li + H reaction becomes now $\frac{4889.1{,}04}{200.7{,}6} = 3{,}2$ erg/gr. sec., which is very close to the actual rate (1,9 erg/gr. sec.).

Thus, the helium synthesis through lithium is able to account for the energy generation of the sun; the difference between the computed and observed figures is smaller than the uncertainty involved in the computation. This does not preclude the possibility of further synthesis; e. g., if oxygen is the last step, sixteen protons instead of eight are incorporated, and the Li synthesis step will share only about one-half of the total energy output; in such a case, however, the central temperature must be higher (central condensation), and the internal abundance of lithium much smaller than assumed (cf. Section 7).

The $Li + H \rightarrow 2\,He$ reaction was considered by Atkinson[12] as a low temperature reaction, characteristic of the "giant" source of energy: he, of course, postulated Be^8 as the first product, but Be^8 seems to be produced as a resonance effect only at high relative energies of the collision, of $4{,}5.10^5$ and 9.10^5 volts, energies which cannot come into question at $T \sim 2.10^7$ K. Atkinson's "giant stuff" actually works in dwarfs; as we have seen above from general considerations, the existence of a special "giant stuff" is improbable.

e. Probability of the direct deuteron synthesis.

In the chain leading to the synthesis of Li, some of the reactions must be extremely rare; such should be the reaction $H^1 + H^1 = H^2 + \beta_+$, which Atkinson[13] considered as the most probable starting reaction of the synthesis; to keep pace with the formation and decay of lithium, this reaction should occur at an equal absolute rate with the $Li^7 + H^1$ reaction; considering the comparative easiness of the reaction ($Z_1 = Z_2 = 1$, cf. (3)), the speed of which is comparable to the lithium reaction at a temperature nine times higher, a value of $q < 1{,}3.10^{-19}$ must be postulated: such a small upper limit to the probability of the capture of a proton by another is indicated by the fact that the sun actually does not explode from this reaction. In such a case, there is no hope of discovering the reaction in the laboratory: for canal rays the fraction of energy dissipated in central collisions being of the order of 10^{-4} of all collisions, 1 in 10^{23} collisions of protons with hydrogen atoms is expected to yield a deuteron nucleus; a whole gram of hydrogen canal rays yields less than six atoms of H^2.

f. Equilibrium of abundance for intermediate steps.

As to the intermediate members of the chain of atomic synthesis, no such limitations of the value of q can be derived for them from solar observations; the equilibrium abundances of the different intermediate elements will automatically settle themselves in ratios inversely proportional to the speeds of the corresponding reactions (Atkinson's theory of abundance); and there is apparently no certain way of testing the abundances even when the speeds of reaction are known from experiment, because the observed abundances refer to the solar atmosphere, not to the interior; although, as follows from the case of lithium, perhaps in the sun (if not in all stars) mixing may be complete, and the observed composition is indicative of the conditions in the interior. Only the first element in the chain (H^1), and the last effective one (He^4, or O^{16}, perhaps both), will not possess equilibrium abundance: the first will gradually disappear, the last will steadily accumulate. Some of the light nuclei which are known to react easily in the laboratory (such are H^2, Li^6, Li^7, also the neutron), must be extremely rare in the interior of the sun. The neutron must be practically absent (cf. below).

g. The starting reaction.

The starting point of the synthesis is the most important one; the approximate constancy of the central temperatures of the main sequence stars must be the direct consequence of the laws governing this first reaction, without regard to the following steps (the rate of the subsequent reactions is completely determined by the rate of the first reaction, cf. above, equilibrium of abundance). The dependence of the average energy generation upon temperature is also entirely determined by the first reaction. However, for short period fluctuations of the temperature (pulsations), during which the absolute abundance of the elements has not time enough to change considerably, all the links of the chain of synthesis contribute to the dependence of energy generation upon temperature. Unfortunately, we do not know anything definite with respect to this first step. Atkinson[13, 12] has made certain hypotheses which may remain unproved for a long while. There are practically three possibilities to be considered.

(α) The reaction ${}_1H^1 + {}_1H^1 = {}_1H^2 + \beta_+ + (0.41 \pm 0.05)$ Mev, already mentioned above. The probability of capture must be as small as $q \leqslant 1{,}3.10^{-19}$, to account for the rate of the generation of solar energy, and there is no hope of ever detecting the reaction in the laboratory; we are here confronted with a dilemma: if the reaction were detectable experimentally, the sun would blow up from the immense energy generation; or, rather, the sun could exist only as a diffuse star of spectrum

M of about nine times its present radius (Atkinson [13] finds that the probability of the reaction "should be much too high at main sequence temperatures"); if, however, the reaction takes place in the sun, there would be an upper limit to the probability of the reaction, which makes it undetectable experimentally. The well-known mass defects of H^1 and H^2 cannot allow of much change in the energy developed by the reaction; and even if this were zero (which is impossible), the liberated positron would soon combine with an electron, thus releasing about one million volts of energy; thus the energy of the reaction alone would be sufficient to make the sun explode if q is not as small as assumed above, not counting the much larger energy released in the subsequent steps of the synthesis after H^2 is formed.

Another possibility is the absence of hydrogen in the interior of the sun; with q of the order of from 0.01 to 10^{-6} for our reaction, hydrogen must be absent from 95 to 90 per cent of the internal mass; in this case, of course, not much energy from atomic synthesis could be obtained; however, such a possibility must be ruled out by the hydrogen content known to be large, from 0.30 to 0.37 of the whole mass [19, 20, 21].

(β) The reaction ${}_1H^1 + \beta_- = {}_0n^1 - E$, or the formation of neutrons in the collision of a proton with an electron at the expense of a yet not very accurately known energy E. This is an endothermic reaction and, as such, it can occur only when the relative energy of the collision exceeds E; on the other hand, apparently there exists no potential barrier, and the rate of the reaction is proportional to the Maxwell frequency of kinetic energies exceeding E, multiplied by an unknown probability q. Negative results of some experiments (cf. [13], p. 79 f.) are not conclusive, at most they show that q is small. The value of E, which depends upon the mass excess of the neutron, is not well determined. From recent data we may try to estimate it anew. Some of the atomic weights of the lightest atoms determined by different methods are as follows*:

* The weights refer to the neutral atoms; to get weights of the nuclei, such as they occur in atomic transmutations, 0.00054 must be subtracted for each bound electron; failure to observe this may lead to misunderstandings (such as in [4], p. 65).

	H^1	H^2	H^3	He^3
Oliphant, Kempton and Rutherford [25]	1.00807 ± .00007	2.0142 ± .0002	3.0161 ± .0003	3.0172 ± .0003
Bethe [26]	1.0081 ± .0001	2.0142 ± .0002	3.0161 ± .0003	3.0170 ± .0005
Bainbridge and Jordan * [27] . .	1.00815 ± .00002	2.01478 ± .00003	. . . . . .	. . . . . .
Weighted mean adopted	1.00814 ± .00002	2.01476 ± .00003	3.0161 ± .0002	3.0171 ± .0003

	He^4	Li^6	Li^7
Oliphant, Kempton and Rutherford [25]	4.0034 ± .0004	6.0163 ± .0006	7.0170 ± .0007
Bethe [26]	4.0034 ± .0002	6.0161 ± .0005	7.0169 ± .0005
Bainbridge and Jordan * [27] . .	4.00395 ± .00007	. . . . . .	7.01822 ± .00014
Weighted mean adopted	4.00390 ± .00007	6.0162 ± .0004	7.01811 ± .00014

Similarly, for the other nuclei:

Nucleus	Be^9	B^{10}	B^{11}	C^{12}
Atomic weight . .	9.0150 ± .0002	10.0161 ± .0001	11.0127 ± .0001	12.0040 ± .0001

Nucleus	C^{13}	N^{14}	N^{15}	O^{16}
Atomic weight . .	13.0078 ± .0002	14.0076 ± .0002	15.0050 ± .0003	16.0000

Nucleus	O^{17}	F^{19}	Ne^{20}	Ne^{21}	Ne^{22}
Atomic weight . .	17.0040 ± .0010	19.0000 ± .001	19.9992 ± .0002	21.0001 ± .0003	21.9987 ± .0004

The data of Bainbridge and Jordan are of much greater weight than the rest, as indicated by their probable errors. The weighted mean values may be used with more confidence in the computations of mass defects. The mass of the proton is thus 1.00760 ± 0.00002, and of the deuteron $_1H^2$ (nucleus of H^2) 2.01422 ± 0.00003. The mass of the neutron cannot be determined with present methods directly; it has been computed several times from energy considerations. A recent determination by Livingston and Hoffmann [28], partly based on their own experimental data ($_3Li^6 + _0n^1 \rightarrow _2He^4 + _1H^3 +$ 4.67 ± 0.05 Mev), assigns to the neutron 1.00884, with $H^1 = 1.00798$. In such calculations, however, it is advisable to use directly determined atomic weights for the charged particles, in order to reduce the error of the indirect method as much as possible.

* From measures of doublets.

We take the well studied reaction, which also served as a basis for Livingston and Hoffman,

$$_{1}H^{1} + {}_{0}n^{1} \rightarrow {}_{1}H^{2} + (2.22 \pm 0.06) \text{ Mev.}$$

With the mean atomic weights cited above this gives for the mass of the neutron $1.00899 \pm .00007$, and for E in the hypothetical reaction

$$E = (0.79 \pm 0.07) \text{ Mev.}$$

This is only slightly less than the value considered by Atkinson[13], and his conclusion that the neutron formation cannot proceed at 2.10^{7} K remains valid; our computations point to a temperature about $1{,}2.10^{8}$ K, with $s \sim 60$—70 in formula (3′). Thus, neutrons may be generated in such a way in overdense cores only, but not in the interior of stars if these are built more or less according to a polytrope $n \leqslant 3$.

(γ) Reactions starting from a heavier nucleus which must be assumed to have existed in sufficient amount "from the beginning", or to be continually regenerated by disintegration of nuclei of higher order (cf. Atkinson[12]). Considering the fact that lithium has not completely disappeared from the sun, and that from calculations made above it is very likely that lithium reactions form part of the main process of energy generation in the sun (if built more or less polytropically), the nucleus starting the chain of reactions must be lighter than Li^{7}. The only possible one is evidently He^{4} (pointed out by Atkinson already), as may be inferred from its abundance, and its tendency towards regeneration revealed in many nuclear reactions. $Li^{7} + H^{1} \rightarrow 2\, He^{4}$ would in this case represent the regenerative process for helium. However, for He^{4} to be a starting point at $T \sim 10^{7}$ without the interference of lighter nuclei except protons, it should be able to form sufficiently stable atoms by proton capture; now, He^{5} is unstable, having a life of $\sim 6.10^{-20}$ sec. (cf. [29]), too short to form a step in the continued synthesis; also, the energy (2,93 Mev, computed from $_{2}He^{5} \rightarrow {}_{2}He^{4} + {}_{0}n^{1} + 0.93$ Mev, cf. [29]) * is far too high for He^{5} to be formed at stellar temperatures (10^{7}). Li^{5} does not seem more promising[13] (there may still be a loophole on the

* The mass of He^{5} is thus 5.01389, according to our standard masses of $_{2}He^{4}$ and $_{0}n^{1}$.

assumption of the existence of the unknown Li^5). In such a case, the synthesis cannot start from He^4 without a steady supply of deuterons [13], and we have to go back to cases (α) or (β) (neutrons, of course, easily lead to the formation of deuterons). Another possibility is $He^4 + He^4 \rightarrow Be^8$ as the starting reaction, which however requires $T \sim 3{,}5.10^7$ (cf. Section 7), thus no longer a polytropic ($n < 3$) structure for main sequence stars.

From the preceding discussion we conclude that, if at least the main sequence stars are built more or less according to Eddington's model, i. e. without superdense cores, the only starting reaction for atomic synthesis can be the formation of a deuteron from two protons, with the expulsion of a positron. The law of energy generation (3′) in dwarf stars must in this case be represented by $s = 6.4$ (at $T_\varepsilon = 1{,}12.10^7$), thus by a much slower dependence upon temperature than hitherto supposed.

If, however, the main sequence stars possess central condensations and higher central temperatures, the process of neutron formation may come into question; in this case the direct deuteron synthesis from protons would be prohibited.

Atkinson supposed that the deuteron synthesis might be characteristic of giants only, being for some reason prohibited at the higher temperatures of the main sequence (cf. [13], p. 81); such prohibition may happen only in a process of thermodynamic equilibrium, when the reaction becomes reversible; now, for the reaction ${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + \beta_+ + E$, the atomic weights assumed above give for the energy of reaction

$$E = 0.41 \pm 0.047 \text{ Mev.}$$

Reversibility of the reaction at $T \sim 2.10^7$ may be expected only when E is small, i. e., about 0.02 — 0.03 Mev. The smallness of the probable error above practically excludes such a possibility. Besides, the reverse reaction would require a supply of free positrons, which is rather unlikely to exist at $T \sim 2.10^7$, because the positrons would be absorbed by free electrons and converted into radiation. In the present case, too, the "giant stuff" and "dwarf stuff" hypothesis is not supported by our physical knowledge.

Section 4.

Abundance of Elements and Mixing.

a. Equilibrium of atomic synthesis.

There have been a few attempts to explain the relative abundance of the elements theoretically. Atkinson[12] considers the abundance to be the result of the equilibrium of atomic synthesis reactions and, without doubt, for the intermediate members of the chain of reactions such an equilibrium must happen (cf. above), except for nuclei of a large initial abundance (oxygen, carbon?) for which the time scale may be too short for equilibrium to be established. For the starting (hydrogen?), and the final (helium, carbon, or oxygen?) members the age also comes into play, of course. At $T \sim 2.10^7$, however, only the relative abundance of the lighter nuclei can be explained in such a manner. It is true that neutrons, which are probably transiently formed in the process of the synthesis, may react with heavy nuclei; but the increase of the atomic weight by neutron capture alone is rather limited, and there cannot be seen any possibility for the formation of heavy nuclei from the lighter ones without the capture of positively charged particles; such a capture, however, is practically impossible at values of $Z > 8$, when $T \sim 2.10^7$.

b. Dissociative equilibrium.

Sterne[6] considers the abundance of elements as the result of thermodynamic (dissociative) equilibrium at high temperatures, of the order of 3.10^9 K. The relative abundance is determined by temperature, density, and by the energies of formation of different nuclei (corresponding to the packing fractions, or mass defects). Increasing temperature favours the abundance of nuclei with smaller mass defects, thus of the lighter nuclei, and vice versa. At $T = 2.10^9$, or lower, the equilibrium condition is practically all iron (or similar nuclei); at $T \geqslant 4.10^9$, it is all hydrogen (according to our present views, all neutron); oxygen must remain scarce ($\sim 10^{-16}$) under any circumstances. It is clear that matter in stellar atmospheres is not in thermodynamic equilibrium, because here we observe an abundance of hydrogen and oxygen, instead of their complete absence, and instead of 100 per cent of Fe and related elements. The explanation is that the rate of approach towards thermodynamic equilibrium is too slow at low temperatures (the rate being exactly the speed of Atkinson's transmutations). If we assume high temperatures inside the stars, and dissociative equilibrium there, we must have at $T_c \sim 2.10^9$ an iron core extending outwards until $T_c \sim 4.10^8$ is reached, when the rate of reaction becomes too slow; the formation of such an iron core, from material which originally contained plenty of hydrogen (as stellar atmospheres and nebulae do), could have been attained only after the lapse of a sufficiently long interval of time on an atomic synthesis basis, to allow all the mass excess of hydrogen to be radiated into space; a further rise of the central temperature ($T_c > 4.10^9$), which can be attained only through continued contraction, inverts the process: an amount of energy equal to the amount formerly spent on radiation would have to be regenerated by gravitation (during an actual collapse) and absorbed in the process of the dissociation of iron back again into hydrogen; there would be now a hydrogen (actually neutron) core, surrounded by a pure iron shell where the temperature would be $\sim 2.10^9$ with intermediate composition in between. It is inconceivable how a single lighter nucleus from the central region could ever reach the boundary of a star without being incorporated in the heavier elements of the intermediate shell, as the rate of reaction for Li + H is defined by a life of the order of 10^{-6} sec, and even for Fe + H it is about 1 hour, at $T \sim 2.10^9$ (cf.[6], p. 774, and our formula (3)), and for $\varrho \sim 10$ (Sterne's assumption; the density in the collapsed core must be much higher and thus the reaction much faster). Thus Sterne's dissociative equilibrium would lead to pure iron (or something similar) at the surface, as in the case of $T_c \sim 2.10^9$, contrarily to what is observed. *

Sterne's considerations need to be corrected so far as the neutron instead of hydrogen, being the particle of highest

* There is no danger of an atomic explosion of the collapsing nucleus, if the history of the stars is as described here: the gravitational "pit" into which the star has contracted and radiated itself, is deeper than the subatomic energy of the dissociated mixture, by an amount equal to the total energy radiated into space; thus the star cannot "jump out of the pit" (no matter whether it contracted as a whole, or only in its central core).

internal energy, is the dominating nucleus at very high temperatures. The energy of binding, $E = 0.79$ Mev (cf. above), being relatively low, protons with electrons are rapidly transformed into neutrons in dissociative equilibrium at temperatures of the order of 7.10^8 K; at such temperatures, however, protons must have been already completely absorbed by atomic synthesis (reactions observed in the laboratory suffice to accomplish this, cf. Section 7), thus there is no chance for such an "early" formation of free neutrons to happen.

There is another interesting point to be considered in connection with the dissociative equilibrium of the elements. During the transition phase, when the state of dissociative equilibrium changes rapidly with the temperature, the ratio of specific heats, $\bar{\gamma}$, is chiefly determined by the enormous amount of energy involved in the atomic reactions (at 4.10^9 K, the translatory energy of the particles is $\sim 5.10^5$ volts, whereas the subatomic energy is $\sim 8.10^6$ volts), for which $\gamma \sim 1.0$. Therefore $\bar{\gamma} < \frac{4}{3}$, and an unstable state is reached: the contraction is rapidly converted into a real collapse (cf.[1], p. 142), which is stopped only after dissociation has been completed; the value of $\gamma > \frac{4}{3}$ is expected to remain after that still close to $\frac{4}{3}$ on account of the closeness to dissociative equilibrium; in that case the star must be vibrationally unstable[18], and we may safely suppose that non-pulsating stars cannot have collapsed cores of the dissociative type ($T > 2.10^9$). Curiously enough, the argument of vibrational instability, which Sterne[6] thought to speak against Atkinson's mechanism of energy generation, turns out to be harmless to the transmutation theory of stellar energy, presenting instead an argument against Sterne's dissociative energy generation (disregarding other difficulties, cf. Section 1), as well as against the existence of collapsed cores in stars of the main sequence which are known to be, as a rule, vibrationally stable. In any case, the dissociative equilibrium hardly determines the internal composition of these stars — simply because the time scale is too short for dwarfs to have radiated away all their supply of subatomic energy stored in free hydrogen, a radiation which must have been accomplished before the first "iron" stage ($T \leqslant 2.10^9$) is reached.

So far as to Sterne's qualitative picture, where the effect of pressure (or density) upon dissociation is disregarded, as a first approximation. Surprising conclusions, however, are reached when the influence of density is considered.* We limit ourselves to the two-phase dissociation Fe → He; from Sterne's calculations for a more complex case it appears that the most important dissociation He → H follows soon the dissociation Fe → He, so that the start of Fe → He is actually the start of rapid complete dissociation and collapse if such can happen. From Sterne's figures we estimate that a degree of dissociation $x = 0.01$ is reached at $T = 2{,}2.10^9$, $\varrho = 10$ gr/cm³; we use the Saha formula for order-of-magnitude extrapolation starting from this point (the approximation is much better than might appear at first glance; the dissociation does not happen suddenly according to ${}_{26}Fe^{56} \rightarrow 14_2\,He^4 + 2\beta_-$, but gradually, as ${}_{26}Fe^{56} \rightarrow {}_2He^4 + {}_{24}Cr^{52}$, with the absorption of about 3.5 Mev; thus it is not only a two-phase reaction, but the number of reacting particles is the same as in the Saha case of ionization); the effective "ionization potential" results as $I = 3.0$ Mev (in good agreement with the packing fraction of one incorporated α-particle, cf. above), and the dissociation formula for $x = 0.01 =$ const. becomes:

$$\log \varrho/_{10} = -1{,}5.10^{10}\left(\frac{1}{T} - \frac{1}{2{,}2.10^9}\right) + \frac{3}{2}\log\frac{T}{2{,}2.10^9} \quad \ldots \text{(a)}.$$

Now let us consider a contracting superdense nucleus of mass M, central density ϱ_c, molecular weight $\bar{\mu} = 2.11$ (exhausted, no free hydrogen, all ∼ iron), and $\beta =$ ratio of gas pressure to total pressure; the source of energy is gravitation, and the nucleus is so dense in comparison with the rest of the star that it behaves like an independent polytropic model $n = 3$ (cf. below). The central temperature (cf.[1]) is then given by

$$T_c = 4{,}2.10^7\,\beta\left(\frac{M}{M_\odot}\right)^{\frac{2}{3}}\left(\frac{\varrho_c}{76}\right)^{\frac{1}{3}} \quad \ldots\ldots \text{(b)},$$

where β is determined from Eddington's quartic equation (cf.[1], p. 137, Table 14).

* At $T = 2{,}2.10^9$, the black-body radiation has a material density of ∼ 200 gr/cm³; the energy stored in radiation equals the subatomic energy of transmutations at a density of matter ∼ 3.10⁴ gr/cm³. Sterne's calculations of the equilibrium of transmutations, for a total density of only 10 gr/cm³ (!) neglecting radiation, are an example of mathematical abstraction which disregards physical realities (cf.[6], p. 715).

By solving the pair of equations (a)+(b), with $T=T_c$, $\varrho=\varrho_c$ in (a), the unknowns, T_c and ϱ_c, corresponding to the degree of dissociation $x=0.01$ are found. Now, equation (b) is equivalent to the linear equation

$$\log \varrho_c = 3 \log T_c + \text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (b'),$$

the constant depending solely upon the mass, whereas (a) approaches asymptotically the straight line

$$\log \varrho = \tfrac{3}{2} \log T + \text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (a').$$

This pair of equations, (a) and (b), as shown by the accompanying figure, yields either two solutions (b_3 on the figure),

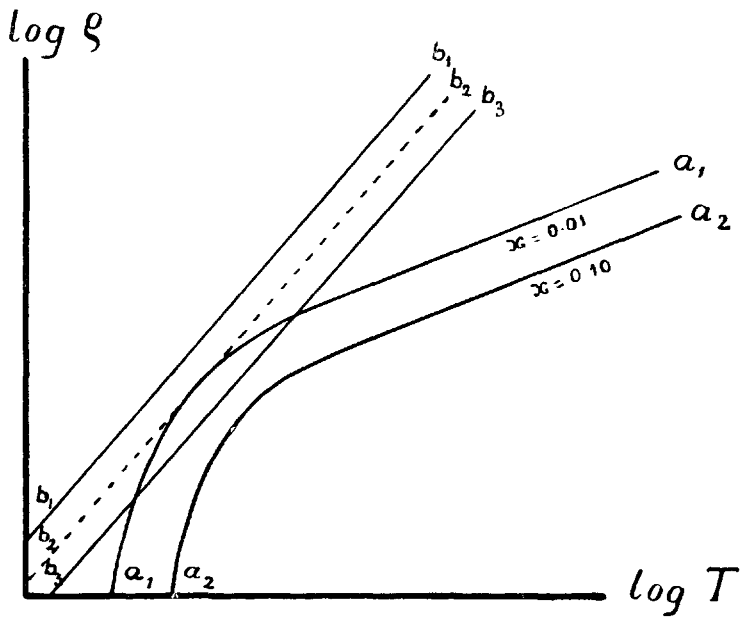

Fig. 1. Conditions of nuclear dissociation. b_1, b_2, b_3 = equation of state for different stars ($\log \varrho_c = 3 \log T_c + \text{const.}$); a_1, a_2 = equation of state for x = const.

or none (b_1). The case is similar to the conditions of degeneracy investigated by Chandrasekhar[5]. In our case, the condition of maximum dissociation attaining $x=0.01$ (b_2 on the figure) is given by

$$4{,}2.10^{7} \left(\frac{M}{M_\odot}\right)^{\frac{2}{3}} \beta = 1{,}25.10^{8}.$$

This gives $M = 19_{\odot}$; for collapse, $x \geqslant 0.01$ is required; thus only masses much larger than $19_{\odot}$ arrive, in the process of contraction, at a sufficient degree of dissociation to cause a collapse. For other masses, the maximum attainable degree of dissociation ($x_{max.}$) is*:

Mass $M/M_{\odot}$ . . .	50	19	10	8	5	3	2	1.5
x_{max}	0.014	0.01	0.008	0.007	0.005	0.004	0.003	0.002
Energy of dissoci. erg: gr	8.10^{15}	6.10^{15}	5.10^{15}	4.10^{15}	3.10^{15}	$2{,}4.10^{15}$	$1{,}8.10^{15}$	$1{,}2.10^{15}$
ϱ_c, gr/cm³	3.10^{8}	6.10^{8}	9.10^{8}	$1{,}2.10^{9}$	2.10^{9}	4.10^{9}	6.10^{9}	$1{,}2.10^{10}$
T_c	$2{,}4.10^{10}$	$2{,}4.10^{10}$	$2{,}4.10^{10}$	$2{,}5.10^{10}$	$2{,}5.10^{10}$	$2{,}6.10^{10}$	$2{,}6.10^{10}$	$2{,}7.10^{10}$

We see that the maximum energy absorbed by the dissociation is only a small fraction of the energy lost during the preceding contraction $\left[= 1{,}2.10^{15}\left(\frac{M}{M_{\odot}}\right)^{\frac{2}{3}}\left(\frac{\varrho_c}{76}\right)^{\frac{1}{3}} \text{erg/gr}\right]$ and that the "collapse" thus amounts to only a negligible decrease in the radius: no real collapse thus takes place from nuclear dissociation for existing stellar masses, and no appreciable amount of dissociated material, to feed further radiation, can be formed. Possible degeneration (for $M < 1.6$) is an additional factor to prevent dissociation.

In the process of dissociation described, the small, but perceptible equilibrium content of the neutron must collect by diffusion at the centre; unfortunately, no pure neutron core can be formed in such a manner as imagined by Anderson[4], because the same law of thermodynamic equilibrium, which led to the formation of the small percentage of neutron, will provide for the constancy of the percentage and convert the excess of neutron collected at the centre into the heavier nuclei. There is no escape from the conclusion that nuclear dissociation as well as pure neutron cores cannot play an appreciable role in the energy balance, stability, and structure of actual stars. (Dissociation by pressure, of course, is pos-

* All our conclusions, derived from Saha's formula $\log \frac{x^2}{1-x}\varrho = -\frac{A}{T} + p \log T + C$ with $p = \frac{3}{2}$, remain essentially correct also for the case of possible deviations from the formula at very high temperatures, if these deviations are such that $\bar{p} \leqslant 3$.

sible, as it does not involve absorption of energy; the formation of electron-positron pairs from radiation is more likely to occur in the collapsing core; involving an energy of only 10^6 volts, and depending upon the density of radiation, instead of the material density, thus solely upon the temperature, it may be an important process in the core, especially as it increases the opacity, and lowers the molecular weight, thus reducing the luminosity).

c. *Initial distribution of abundance.*

It is the question whether we have any right at all to derive the observed abundance of elements from conditions prevailing at present in stellar interiors. A more or less similar distribution of the elements is revealed by the earth, the meteorites, and the stellar atmospheres (cf. [30], and [22]), with exceptions which are easily explained by the history of the celestial bodies (e. g., escape of hydrogen from small bodies), without recourse to transmutations becoming necessary. Even if the earth was formed from ejected portions of the primitive sun (Chamberlain and Moulton), the relative abundance of the elements in it cannot correspond to equilibrium conditions inside the present sun. There seems to be no escape from the conclusion that the meteorites, the earth, and to all appearance the stellar atmospheres reflect the composition of primordial matter which must have been well mixed; present processes in stellar interiors may influence the abundance of the less abundant lighter elements (e. g., lithium, beryllium, boron in the sun, cf. [22], and Section 3. *d*, *f*). In some stars (if not in the sun), on account of imperfect mixing of the stellar material, the internal changes in the composition may but slowly (perhaps not at all) become reflected in their atmospheres (giant stars; "composite" adiabatic-radiative model, cf. below). The observed abundance of the more abundant lighter elements, such as oxygen, and (especially) the abundance of the heavy ones, must have its origin in conditions which prevailed in the universe before the present stars were formed (neutrons if formed at all can influence the heavy elements only to a limited extent).

d. White dwarfs.

White dwarfs, such as Sirius B, o^2 Eridani, represent cases where we are forced to conclude that the mixing of their material is rather inefficient. Possessing internal temperatures doubtlessly higher than those of the main sequence stars, and densities that are much higher, these stars should develop much more energy than they actually do, if there is a trace of hydrogen in the interior; hydrogen must be completely absent from the interior of the white dwarfs (where the temperature exceeds $\sim 7.10^6$ K); on the other hand, spectroscopic evidence points to a not inconsiderable abundance of hydrogen at the surface of these stars. We are forced to conclude that there is practically no mixing in these white dwarfs. If red giants exist for 3.10^9 years, the hydrogen in their central portions must have become exhausted (cf. below), whereas their atmospheres seem to show a rather abnormal abundance of hydrogen; thus mixing must be incomplete in the giants, too. The white dwarf A. C. + 70° 8247 according to Kuiper[65] is devoid of spectral lines; the writer has shown[66] that its colour as estimated by Kuiper implies an effective temperature of 12 700°; the absence of Balmer lines in such a case can be explained only by the absence of hydrogen; AC + 70° 8247 is thus an instance where the mixing has been complete (as in the sun) and where the internal exhaustion is reflected in the atmosphere.

e. Calcium.

The remarkable constancy of the relative abundance of calcium in stellar atmospheres[31] seems to speak in favour of the stellar atmospheres reflecting the composition of a primordial uniform mixture, although the equality of calcium content for stars of similar spectrum and absolute magnitude may be the result of similar conditions and history (the absolute amount of calcium cannot change much from transmutations, but its relative amount may change when the amount of hydrogen changes). The comparison of the relative abundance in giants and dwarfs made by the writer[31] generally leads to an ambiguous interpretation: the colour-absolute magnitude effect, partly or entirely due to a pressure broadening of Ca 4227, may partly, to an unknown extent, be due to a real variation of mean composition with absolute magnitude. However, there is one case when the ambiguity disappears: for a weak spectrum line, on the margin of appearance, the pressure effect must be absent. For Ca 4227, this corresponds to spectrum F0; the absence of the colour-absolute magnitude effect at this spectrum indicates that at least the atmospheres of F0 giants and dwarfs possess an almost equal relative calcium content (ratio of Ca to hydrogen *plus* all the other elements), within ± 5 per cent; such a coincidence of the expected and the observed disappearance of the colour effect is not very likely to be accidental; thus it appears highly probable that there is little systematic difference in the composition of the atmospheres of giants and dwarfs, although the internal composition is very likely to be different: Strömgren[21] finds for the interior of giants a smaller average hydrogen content than for the main sequence stars, whereas the data of the writer[31] (if the pressure effect is disregarded) would require an increased hydrogen content in the atmospheres of the giants (smaller relative abundance of calcium), except F0. The same was found by Russell[22] for red giants. Thus, whatever the interpretation of the observed colour-absolute magnitude effect, the conclusion is the same, namely, that the composition of stellar atmospheres is not always determined by the composition of the interior. The mixing of the stellar material may in some cases be rather inefficient.

f. Rotational currents and convection.

This conclusion appears to be at the first sight in conflict with certain theories requiring vertical circulation. Thus, in rotating stars convection currents inevitably arise, as shown by von Zeipel[32]. Biermann, Rosseland, Steensholt have shown that in stars generating energy by atomic synthesis, or generally by a process of a speed rapidly increasing with the temperature, the mathematical theory of the model leads to negative density gradients inwards (similar to the point-source model), which of course cannot persist as such, and give rise to convection currents instead[33,16]. Thus there cannot be the least doubt as to the existence of convection currents in stars. And, nevertheless, the mixing of the material may be incomplete. Following a suggestion made by Bjerknes, Eddington admits "that a circulation of this kind tends to become stratified, so that instead of one circulation between the centre and the outside we may have two or three layers of circulation. Each layer will then be thoroughly mixed, but there will be little interchange between consecutive layers" (cf.[1], p. 286). Rosseland[33] remarks, with regard to the central convective zone: "If . . . the stars are convectively unstable even without rotation, the role of the rotation is less that of instigating convection than that of determining the type of the ensuing currents". In fact, the vertical currents in a rotating star are deflected horizontally by the difference of the linear velocity of rotation in different zones, exactly as happens to the wind on the earth; the deflection increases with decreasing friction (which is relatively much smaller in stars than on the earth), and is already considerable when the difference of rotational velocity (between the given and the "starting" point of the current) is comparable to the velocity of the current. For von Zeipel's effect Eddington[47] estimates a velocity of vertical convection less than 2.10^{-4} cm/sec for the sun, which would require about ten million years to travel from the centre to the surface if undeflected; appreciable deflection starts, however, already after a path of 100 cm only.

For the central unstable region the calculated negative density gradient is, of course, only a mathematical fiction; it means the breakdown of the assumption of radiative equilibrium there; adiabatic equilibrium and a heat transfer by convection supplementing radiation take place (radiative equilibrium may persist in the outer portions of some stars, giants for example; probably not in the sun). The region of the convectional transfer of heat may extend beyond the computed region of a negative density gradient, in some cases over the whole star (cf. Section 5). The equations of adiabatic equilibrium must be almost strictly fulfilled in the convective region because, as shown in Section 5, a sufficient transfer of heat takes place at such low velocities of the current that the ensuing deviations of pressure and density from their static equilibrium values are negligible. Indeed, we obtain a maximum stream velocity of convection by assuming that the transfer of heat is accounted for by the mere kinetic energy of the current, disregarding the heat transfer by an excess of temperature (only in the absence of viscosity or turbulence, and for adiabatic changes of state, our assumption would cease to be an overestimate); for the central region of the sun, the maximum velocity we find $< 10^4$ cm/sec (Section 5. *a*). An ascending current of $v = 10^4$ cm/sec will experience sensible deflection only after travelling the considerable fraction ~ 0.05 of the sun's radius; it is therefore probable that the entire inner core of convectional instability forms one system of circulation, with complete and rapid mixing. On the other hand, in the outer shell of radiative equilibrium (probably absent from the sun) only the weak currents due to rotation can exist; these must form a large number of superposed shells (thickness of the order of 10^3 cm) with a more or less complete circulation in each; the mixing in such a case is practically nil, if the interchange between surface and interior is considered. However, peculiarities of ionization $\left(\text{reducing the value of } \gamma = \frac{c_p}{c_v}\right)$ may produce local instabilities and convection zones (cf.[34]), such as we probably observe in sun-spots; probably all the mixing in the outer layers of the sun is due to such causes. Thus, rotation favours mixing only to a negligible extent, and prevents it efficiently by deflecting the more powerful central and ionization currents; however, in completely adiabatic stars (cf. Section 5) the role of rotation, by overcoming a certain "dead zone", may be important, leading to complete mixing.

For dwarfs with their slow rate of energy generation a considerable change in composition with age may be expected only among the brighter ones, of classes A and B; "composite" models (Sirius A, Procyon?, cf. Section 7) should show an unchanged original composition of their atmospheres, whereas complete adiabatic models should exhibit a considerably deviating distribution of the lighter nuclei (up to oxygen) (such as exemplified by the low abundance of lithium and beryllium in the sun). Giant stars with radiative shells conceal their inner composition. Perhaps giants of exceptional composition (such as R — N stars) are those where the mixing is (or has been more or less recently) complete, so that their atmospheres reflect the internal changes. In this respect we may suppose that the relative excess of carbon, as compared with oxygen, postulated by Curtiss and investigated by Russell in the N stars ("carbon" stars), is the result of atomic synthesis in the stellar interiors. The equlibrium of abundance, for intermediate members of the atomic synthesis, can be reached only when the life-time of the atom is short as compared with the life-time of both the star, and the hydrogen in it. The last nucleus which has a life comparable to, but greater than, the life of the hydrogen in the star, will accumulate from the more rapid transmutations of the lighter nuclei; the abundance of the next heavier nucleus, however, will not appreciably increase before the life of the star has approached the life of the nucleus. From Sterne's tables we derive the following figures, for $\varrho = 10$ gr/cm^{3} *, $q = 0.01$ (except hydrogen, for which $q = 1{,}3.10^{-19}$ is assumed, cf. above):

Life time for proton capture

	Hydrogen	Boron	Carbon	Nitrogen	Oxygen
$T_\varepsilon = 13{,}8.10^6$ K	10^{10} years	2.10^9 years	3.10^{11} years	10^{14} years	10^{16} years
$T_\varepsilon = 18{,}6.10^6$ K	10^9 „	2.10^7 „	1.10^9 „	2.10^{11} „	3.10^{13} „

Both temperatures are higher than the probable adiabatic temperature of the sun (cf. above); $T_\varepsilon = 18{,}6.10^6$ gives on the Li+H synthesis several hundred times more energy per gram than is produced by the sun, thus this case may correspond to a supergiant in its early, non-collapsed (main sequence) stage (cf. below); $T_\varepsilon = 13{,}8.10^6$ gives 35 times more energy per gram than the sun and may correspond to the case of a normal giant.

We see here that the lives of carbon and hydrogen are of the same order of magnitude; also, that for the normal giant (first case) boron falls below the limit of 3.10^9 years, whereas carbon exceeds considerably the limit and may be considered the last element accumulating in the chain of the synthesis, whereas nitrogen, and still more so oxygen are "inert". Nitrogen must accumulate in the supergiant. Thus, it is very likely that in the interior of giant stars carbon has

* For another value of ϱ, the figures little change if T_ε is chosen so as to keep ε, the rate of energy generation per unit mass constant.

increased in amount, whereas oxygen has remained unchanged; if the original mixture contained more oxygen, the final composition may show an excess of carbon; in stars where the mixing is efficient* the excess of carbon may extend to the atmospheres.

g. Neutron.

There have been attempts to attribute to the neutron an important role in the structure of all stars (W. Anderson, cf.[4]). In superdense cores the neutron as shown above cannot play a conspicuous role. Outside the superdense cores the neutron must possess a rather short life (cf. Atkinson[13]) and can never have a chance to accumulate; the penetrability of matter by neutrons has been largely overestimated by Flügge[36] (observed target $\sim 10^{-24}$ cm^2, instead of his 10^{-27} cm^2 for elastic collisions of fast neutrons; slow neutrons have a 100 times larger target), and his conclusions as to the possibility of a diffusional separation and concentration of the neutrons at "ordinary" stellar centres are untenable. Thus, the only role of the neutron in stellar interiors is that of a short-lived, highly active link in atomic synthesis; the amount of free neutron must be vanishingly small, just on account of its high activity.

Section 5.

The Composite Adiabatic-Radiative, and the Complete Adiabatic Stellar Models; Giant and Dwarf Structure.

a. Transfer of heat by convection.

It is known that the point-source model leads to a decreasing density towards the centre (cf.[1], p. 126); physically this means that convection currents arise, and that the calculated point-source state of equilibrium is replaced by another kind of equilibrium. Rosseland, Biermann, and Steensholt have

* But still incomplete; with complete mixing, according to our views exposed below, planetary nebulae and Wolf-Rayet stars are likely to be formed, instead of giant N stars. The mixing may perhaps be considered the result of some catastrophic event (as perhaps the "swallowing" of a companion by the expanding giant, cf. Section 7. *i*).

shown that, for the law of energy generation $\varepsilon \sim \varrho^k T^s$ (ϱ = density, T = temperature), there will be formed a convective core for as low a value of s as 3 (with $k = 0$)[33]. As Cowling[18] truly remarks, the transfer of energy in the interior of the star is in this case by convection, not by radiation. Actually convection takes place wherever the temperature gradient tends to exceed the adiabatic value, ξ_a (absolute values); in such a case adiabatic equilibrium sets in, with a slight excess of the gradient $\Delta \xi = \xi - \xi_a$, sufficient to keep convection going. The extent of the convective core is thus larger than would appear from the extent of the calculated negative density gradient.

The convective transfer of heat (per unit of time and cross section) between two surfaces may be set equal to

$$Q_c \sim v \varrho c_p \Delta T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4),$$

where v is the velocity, ϱ the density, c_p the specific heat, ΔT the excess temperature of the current. The transfer by radiation is

$$Q_r \sim \frac{(T_1^4 - T_2^4)}{k \varrho x},$$

where T_1 and T_2 are the temperatures, k the coefficient of absorption, ϱ the density (supposed to be constant), x the depth. For surfaces separated by a large $k\varrho x$ the advantage of convection, as compared with radiation, is obvious; if the depth of the convection current is of the order of the radius of the star, convection is much more efficient than radiation. Take for the sun, at one-sixth of the radius from the centre, a net transfer of 5.10^{12} erg/sec per cm^2 of the convection current [this is supposed to give one-half of the energy output of the sun, provided by a central fraction of 0.03 of its mass, corresponding to a highly concentrated source of energy about $\varepsilon \sim \varrho T^{12}$ (cf. Section 3. *d*), if the rising current covers one-quarter of $4\pi r^2$]; further, take $\varrho = 10$ gr/cm^3, $c_p = 3.10^8 \dfrac{\text{erg}}{\text{gr. deg}}$. Formula (4) then gives: $v \Delta T = 1700$. *

* For negligible friction (viscosity and turbulence), such as must obtain in the case of a large-scale current in a star, v^2 is of the order of $2 c_p \Delta T$; this gives $v^3 \sim \dfrac{Q_c}{\varrho}$, or, with our adopted data, $v \sim 8000$ cm/sec, and $\Delta T \sim 0^0.2$: the deviations from adiabatic hydrostatic equilibrium are of the order of 10^{-8} for T, and about the same for the pressure.

For $\Delta T = 100^0$ (or $\sim 10^{-5}\, T_1$), $v = 17$ cm/sec. It is obvious that convectional transfer is very efficient, and that it requires such small deviations of temperature and pressure from the static values that it is legitimate to assume for them adiabatic equilibrium values. Thus, the convective region may be assumed to be built according to the polytrope $n = \frac{1}{\gamma - 1}$, where γ is the ratio of specific heats, for almost any concentration of the energy sources. An exception is presented by the true point-source, for which convection becomes inadequate at a small distance from the centre, at about 10^{-6} of the radius (for the sun). Also, in the outer layers of a star (the sun), when $\varrho < 2.10^{-5}$ gr/cm^3, convection may be incapable of transporting the net flux of heat.

b. The net flux of radiation in a polytrope.

Heat that has escaped from a shell of radius r inside a star, containing a fixed mass M_r, cannot get back (because free convection cannot transport heat in the direction of the gravitational force; convection forced by rotation is too slow, and too weak, to work against the excess of the adiabatic temperature gradient required for a reversal of the transport of heat). Also, there are no subatomic processes able to absorb energy at temperatures below 10^9 K. Therefore, all the net flux of heat (radiation + convection) which has once passed outside of r, must make its way through to the surface (with the exception of a mostly small or zero fraction spent upon the heating of an expanding star). On the other hand, the temperature gradient cannot perceptibly exceed its adiabatic value ξ_a; if radiation at the maximum possible value $\xi = \xi_a$ is incapable of transporting all the heat, convection comes into play to supply the difference.

The net flux of radiation passing outwards through a shell of radius r is

$$Q_r = \frac{4\pi ac\, r^2}{3k\varrho}\left(-\frac{dT^4}{dr}\right) \quad . \quad . \quad . \quad . \quad . \quad (5)\ (\text{cf.}^1,\ \text{p. } 101);$$

here $\frac{1}{4}\, ac$ = Stefan's constant of radiation.

In the case of pure radiative equilibrium, this is also equal to the net flux of the energy, L_r; in the presence of convection, however,

$$Q_r < L_r.$$

For Kramers' law of opacity $k = k_0 \varrho T^{-\frac{7}{2}}$ . . . (6) (k_0 = intrinsic opacity depending upon composition, primarily upon the hydrogen content), and for a polytropic model

$$\varrho = \varrho_c u^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7),$$

where $u = \frac{T}{T_c}$ (ϱ_c and T_c = central density and temperature). Equation (5) becomes, after appropriate substitution:

$$Q_r = A\, u^{\frac{13}{2} - 2n} \left(- z^2 \frac{du}{dz} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8),$$

where

$$z = \frac{R'}{R} r, \quad - z^2 \frac{du}{dz} = M' \frac{Mr}{M}, \text{ and}$$

$$A = \frac{16 \pi ac R T_c^{\frac{15}{2}}}{3 k_0 R' (\varrho_c/\varrho_m)^2 \varrho_m^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

R is the radius, ϱ_m the mean density, M the mass of the star; R' and M' are constants of Emden's tables (final values of z and $-z^2\frac{du}{dz}$), depending as well as $\frac{\varrho_c}{\varrho_m}$ upon the polytropic index, and

$$T_c = \frac{R'}{(n+1) M'} \frac{G}{\Re} \frac{\bar{\beta} \mu M}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

(cf. Eddington [1], pp. 79—85); μ is the molecular weight, G the constant of gravitation, $\Re$ the gas constant, β is the effective ratio of gas pressure to total pressure which for a given n differs slightly from the value given by Eddington's quartic equation (derived for a particular model, $n = 3$).*

* If β_3 is the value for $n = 3$, the average value for (10) is given by the following table:

n	3	2.5	2.33	2.0	1.5
$\frac{1-\beta}{1-\beta_3}$	1.00	0.82	0.82	0.83	0.90

c. Condition for convection to start at the centre.

For one and the same star A is a constant; equation (8) determines the distribution of energy sources in the star only in the absence of convection, in which case $\eta \sim \int \varepsilon\, dM \sim u^{\frac{13}{2}-2n}$. In the case of convection, however, the distribution of energy sources is independent of Q_r, and is determined by L_r; for convection to start it is evidently necessary that

$$Q_r < L_r \quad \ldots\ldots\ldots \quad (11),$$

which means that radiation alone is unable to transport all the energy liberated inside a given shell. The inequality (11) determines the minimum degree of concentration of the true energy sources for producing convection*. Setting

$$L_r = \bar{\varepsilon} M_r, \quad Q_r \sim T^{\frac{13}{2}-2n} M_r,$$

$$\text{and } \bar{\varepsilon} \sim \varrho T^s \sim T^{s+n} \text{**}, \quad \text{for } n < 3.25,$$

(11) leads to the "minimum law of energy generation" for convection (for this purpose, ε must increase inwards faster than Q_r/M_r):

$$s > \frac{13}{2} - 3n \quad \ldots\ldots\ldots \quad (12).$$

(The use of a mean value, $\bar{\varepsilon}$, instead of ε, introduces little difference.)

For $n = 1.5$ (minimum value), $s > 2$ appears to be sufficient. Actually, for the mixture of ionized gas + radiation, $n = 1.63$ seems to be a fair estimate for the sun (with $T_c \sim 1{,}9.10^7$, $\mu = 0.98$, cf. Section 3. *b*); $s > 1.6$ is only required in this case. There cannot be any doubt that any kind of subatomic processes will satisfy this condition (Atkinson estimates $s \sim 20$), and that convection is inevitable in such a case.

* When $Q_r > L_r$ (subatomic), the balance is made up by gravitational contraction; the contraction soon stops, when the rapidly increasing L_r exceeds Q_r (which varies little).

** We notice that on account of convection the structure is almost exactly polytropic (cf. above), and an objection of J. Tuominen, Zeitschr. f. Astrophysik **9**, 260, 1936, against using $\varrho \sim T^n$ in the formula for ε is thus invalidated.

For gravitational contraction, $\varepsilon \sim T$ (cf.[1], p. 123); convection would start when $1 > \frac{13}{2} - 2\,n$, or

$$\frac{1}{\gamma - 1} = n > 2.75, \text{ or } \gamma < 1.364.$$

This can happen, except under peculiar conditions of ionization, only when $1 - \beta > 0.81$, thus in stars of exceptionally large mass; the gravitational source of energy for "normal" stars probably never leads to convection.

d. The luminosity of a polytrope.

Let us consider a complete polytropic model of constant n. For all values of $n < 3.25$, Q_r as given by (8) increases from the centre outwards to a certain maximum value, Q_{max}, at $r = r_0$, and then drops down to zero at the surface $(r = R)$. By reason of our postulate (Subsection *b*, non-reversibility of the flux of heat), the luminosity of the star, L_0, must satisfy the inequality

$$L_0 \geqslant Q_{max} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13).$$

The minimum luminosity is $L_{min} = Q_{max}$. On account of the probable absence (weakness) of energy sources outside r_0, the true luminosity must be very near its minimum value. The mass-luminosity function of the complete polytrope is then determined by

$$L = Q_{max} = \frac{Q_{max}}{A} \cdot A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14),$$

where $\frac{Q_{max}}{A}$ is computed from (8) by the aid of Emden's tables (Gaskugeln, Leipzig u. Berlin 1907; cf.[1], loc. cit.); for a constant polytropic index, Eddington's mass-luminosity function results, with a certain divisor α of the luminosity (cf.[1], p. 124) depending upon the polytropic index. The following table contains the data for different polytropic indices:

Table 2.

Divisor of luminosity, a $[L \sim \frac{1}{a} f(M, R)]$.

$n =$	3.25	3.0	2.5	2.0	1.5	0.0
$r_0/R =$	1.00	0.457	0.378	0.375	0.383	0.433
$M_{r_0}/M =$	1.00	0.85	0.54	0.37	0.25	0.24
$Q\,max/A =$	1.96	0.996	0.480	0.302	0.211	0.309
$a =$	1.63	2.64	3.67	3.31	2.02	0.057
$T_c\,(M, R, \mu = \text{const.}) =$	1.12	1.000	0.822	0.705	0.632	0.584 *

For constant mass, radius, intrinsic opacity, and molecular weight, the luminosity is inversely proportional to a. The standard value of $a = 2.5$ chosen by Eddington (loc. cit.) fits well for $1.5 < n < 2.0$.

e. The adiabatic model.

Now it is proposed to prove in a rather simple way that the complete adiabatic polytrope ($n < 3.25$) may actually be a persistent form of stellar structure. For $r < r_0$, $Q < Q_{max}$, conditions (11) and (12) are fulfilled, convection takes place, and the state is one of adiabatic equilibrium. For $r > r_0$, $Q_r < Q_{max} < L_{min}$, the same holds as far as formula (8) is valid, or as far as a polytropic distribution may be used for a satisfactory (not necessarily exact) representation of the state of stellar matter. The extra heat, $Q_{max} - Q_r$, is transported by convection, and convection maintains the adiabatic structure almost rigorously. Thus, a star can be built completely on the adiabatic model with $n = \frac{1}{\gamma - 1}$ (n may, of course, be slightly variable), but for a thin outer layer which more or less "shines through to space", and where the density becomes too small for an efficient transport of heat by convection. At $r = r_0$, the radiative, and the adiabatic states of equilibrium coincide, and the transport of heat is by radiation only. Nevertheless, the mixing of the stellar material which is complete inside and outside of r_0, may be efficient also in the narrow "dead" zone at $r = r_0$, on ac-

* The central temperature has a minimum:

$n =$	1.5	1.0	0.5	0.0
$T_c =$	0.632	0.584	0.567	0.584

count of von Zeipel's rotational currents which, although weak for the star as a whole, may be sufficient to fill out the narrow link required for complete mixing of the whole stellar material. There may be cases, however, where the interchange of matter through the shell r_0 is weak, or absent; with progressing atomic synthesis in the interior the molecular weight becomes larger inside r_0 than it is outside, a circumstance of great importance in stellar evolution (cf. below).

f. The composite model.

Another possible form of the equilibrium of a star with a concentrated source of energy is an adiabatic-convectional core inside, surrounded by an envelope (of considerable mass and extent) in radiative equilibrium. Which form of equilibrium a model actually assumes can be settled only by computations of stellar models by the method of trial and error; some general principles and qualitative criteria are formulated below.

The condition of the stability of radiative equilibrium is

$$n > \frac{1}{\gamma - 1} = n_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15),$$

where n is the "local" polytropic index, n_0 the adiabatic polytropic index of the material. We choose to define n by

$$n + 1 = \frac{T}{P}\frac{dP}{dT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

(which follows from $P \sim T^{n+1}$), where P is the total pressure. If computations on the basis of the formulae of radiative equilibrium lead to

$$n < n_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17),$$

radiative equilibrium is unstable, and convectional-adiabatic "equilibrium" governs instead *.

The possibility of fitting a shell in radiative equilibrium on the top of a convective shell of radius r depends upon $\frac{dn}{dr}$ calculated from the formulae of radiative equilibrium: when

* In actual computations, the index $1 + \frac{1}{n} = \frac{\varrho}{P}\frac{dP}{d\varrho}$ . . . (16′) may be preferred as a criterion of convectional stability. On account of the presence of radiation, the two indices (16) and (16′) are not identical.

the derivative is positive at $n = n_0$, (15) holds, and radiative equilibrium is possible; when the derivative is negative, (17) holds just outside, and adiabatic equilibrium continues; a transition to radiative equilibrium at the given value of r for which (17) holds is impossible.

From (8), together with (16) and the well known equations of radiative equilibrium,

$$\frac{dP}{dr} = -\frac{G\,M_r\,\varrho}{r^2},\; P = \frac{\Re\,\varrho\,T}{\beta\,\mu},\; \frac{dM_r}{dr} = 4\,\pi\,\varrho\,r^2,$$

and $P \sim T^{n+1}$ (because the structure is polytropic with $n = n_0$ at the value of r chosen), we find

$$n + 1 = \frac{M_r}{\beta}\,T^{\frac{13}{2} - 2n_0} \times \text{ const., and}$$

$$\frac{1}{(n_0 + 1)}\,\frac{dn}{dr} = \frac{(\frac{13}{2} - 2\,n_0)}{T}\,\frac{dT}{dr} + \frac{1}{M_r}\,\frac{dM_r}{dr} + \frac{\beta\,d\left(\frac{1}{\beta}\right)}{dr} \quad . \quad . \quad (18).$$

For $r = r_0$, $\dfrac{dn}{dr} = 0$, because there radiative and adiabatic equilibrium merge one into the other (cf. above). For $r < r_0$, calculations give $\dfrac{dn}{dr} > 0$, whereas for $r > r_0$, $\dfrac{dn}{dr} < 0\;(n_0 < 3.25)$.

Thus, for a star of uniform composition, a radiative equilibrium envelope can be fitted to an adiabatic core only when the radius of the core is smaller than the radius r_0 of the maximum radiation, Q_{max}. In such a case, for $r < r_0$, as convection is absent from the envelope, $L = Q_r = L_r < Q_{max}$; the luminosity of the whole star must be less than the luminosity of the complete adiabatic polytrope (of which the core represents the central portion).

The condition $r < r_0$ is necessary, but not sufficient for the fitting of a radiative equilibrium shell to an adiabatic core; this question will be discussed in more detail in another paper.

g. Regulation of luminosity for the adiabatic model.

For the complete adiabatic model the minimum luminosity is Q_{max}; on the other hand, a larger luminosity is also possible, because convection currents can transport an almost unlimited amount of excess energy to the surface. However, it is extremely improbable for a star to get into a state of being actually able to radiate into space more than Q_{max}. In the process of original gravitational contraction the star settles down at a certain equilibrium radius, when the subatomic energy sources exactly balance the amount directly required, Q_{max} (the total heat is balanced; the impossibility of detailed balancing of radiation and subatomic sources leads to convection). To obtain more, the star must be compressed further, so that T_c, ϱ_c and ε (subatomic) may increase, and the star must be kept in the "overcompressed" state until the convection currents transport the extra liberated heat to the surface, and until the extra potential energy of contraction $\left(\sim \frac{3\beta\, G\, M^2\, \Delta R}{2\,(5-n)\, R^2}\right)$ is radiated into space: if the latter does not happen, the star rebounds to its original equilibrium state $L = Q_{max}$. Now, the time during which a star can be kept without interruption in an "overcompressed" state is of the order of one-half of the period of its free pulsation. The extra amount which may be radiated into space during so short an interval (a few hours, or days) is extremely small, corresponding for δ Cephei to a relative decrease in the radius, or an increase of T_c of about 10^{-7}, which is too small to be of a perceptible influence upon the energy generation. There cannot be a cumulative effect for a whole pulsation period: the star sets its average luminosity in balance with the average energy generation during the whole pulsation. An equally important hindrance for a pulsating star to settle into an overcompressed state is the delay in the transport of heat to the surface: it is obvious from elementary mechanical considerations (even disregarding the numerical estimates of the velocity of the current as made above) that the period of the circulation of a convection current must be large as compared with the period of the pulsation of a star; therefore the extra heat of compression has no chance to get to the surface of the star. To obtain a permanent overcompression of 0.1 of the radius (which would increase the rate of subatomic energy generation $\varrho\, T^s$ by $1.0 - 2.5$ mag, for $s = 6 - 20$), δ Cephei should be kept in the overcompressed state for 7 000 years, with free radiation into space allowed. Evidently, overcompressed stars are practically impossible; a monotone mass-luminosity relation ($L = Q_{max}$) for adiabatic stars of the same composition must hold, as it holds for the purely radiative model.

h. Model of non-homogeneous composition.

So far stars of uniform composition throughout have been considered. It is, however, not likely that all stars are of a homogeneous composition (we do not consider here, as of minor importance, the change of mean molecular weight due to the variable degree of ionization considered by Eddington). In the composite, but originally homogeneous model exhaustion of hydrogen leads to an increase of molecular weight in the core; for the purely adiabatic model (massive stars), the link between the central, and the marginal convective systems may in some cases be inefficient, leading to the same result. A core containing less hydrogen (condensation of meteoric material, cf. later on) than the envelope may appear in some stars from the very "beginning"; perhaps a very small core of such an origin is present in all stars. It is easy to show that quite a small differentiation of composition would prevent any further convectional interchange of matter between the core and the envelope, leading thus to further differentiation, the extreme case of which is a complete absence of hydrogen from the core. For the surface of demarkation between core and envelope (we assume schematically a sudden change of molecular weight), the conditions of mechanical stability and of the finiteness of the flux of energy require equality of temperature and total pressure (thus also of radiation pressure and β) on both sides of the surface; there is a discontinuity in density,

$$\frac{\varrho_i}{\varrho_e} = \frac{\mu_i}{\mu_e} \quad \ldots\ldots\ldots\ldots \quad (19),$$

and in the temperature gradient. Here the index i refers to the inner, e to the outer side of the surface of demarkation. Assuming first that there is no convection, and that the opacity is given by (6) (k_0 is the intrinsic opacity, depending upon composition), the ratio of the temperature gradients becomes

$$\frac{\left(\frac{dT}{dr}\right)_i}{\left(\frac{dT}{dr}\right)_e} = \left(\frac{\varrho_i}{\varrho_e}\right)^2 \frac{k_i}{k_e} \quad (20). \text{ Further,}$$

$$\frac{\left(\frac{dP}{dr}\right)_i}{\left(\frac{dP}{dr}\right)_e} = \frac{\varrho_i}{\varrho_e} \quad \ldots \ldots \ldots \ldots \quad (21).$$

Hence, from (16), the ratio of the polytropic indices is

$$\frac{n_i + 1}{n_e + 1} = \frac{\varrho_e k_e}{\varrho_i k_i} \quad \ldots \ldots \ldots \ldots \quad (22).$$

The intrinsic opacity, defined differently from the usual definition, for variable hydrogen content varies as follows*:

Hydrogen content, X	0	0.25	0.333	0.50	0.75	0.90	0.99
μ	2.24	1.21	1.063	0.836	0.631	0.546	0.500
$10^{-23}\ k_0$	288	146	116	71.3	26.3	9.27	0.853

The actual opacity is then given by

$$k = k_0\, \varrho\, T^{-\frac{7}{2}} + 0.2\,(1 + X) \quad \ldots \ldots \ldots \quad (23),$$

including the correction for electron scattering.

For 25 per cent hydrogen in the envelope and none in the core, (21), (22), and the above table give

$$(n_e + 1) = 3.6\,(n_i + 1).$$

In other words, $n_e \gg n_i$; as the latter cannot fall below the adiabatic value, the adjacent envelope is always in stable radiative equilibrium.

If the margin of the core has definitely settled to radiative equilibrium ($n_i \sim 3$), $n_e \sim 13$: the envelope starts almost isothermally.

If the core is adiabatic up to the surface of demarkation, $n_i > 1.5$, $n_e > 8$, which makes little difference as compared with the preceding case. In addition to the radiation emerging from the core, the adjacent envelope containing

* Computed from Eddington's data in [19],

$$k_0 = \frac{k_c}{g} \cdot \frac{T_c^{\frac{7}{2}}}{\varrho_c}.$$

25 per cent hydrogen is able to take up and transport by radiation a much greater amount of heat supplied by convection from the core, without starting convection itself *. Assuming $\gamma_i = \gamma_e \sim 1.5$, $n_i = n_e = 2$, the maximum amount which the envelope is able to transport by radiation alone equals 3.6 times the radiation from the core [cf. (16), $n_e + 1 \sim \left(\frac{dT}{dr}\right)_e^{-1}$, as $\left(\frac{dP}{dr}\right)_e$ = const.]; thus, such an envelope may be fitted to an adiabatic core with $r > r_0$. Eventual convection currents reaching the boundary of the core cannot rise by inertia any farther into the envelope, because of the difference of density in the core and the envelope **. Our conclusions remain unchanged in principle also in the case of the existence of a gradual transition (stratification) from core to envelope, instead of that of a sharp boundary.

i. Collapse of the exhausted core of a composite model and giant structure.

A core devoid of hydrogen, thus presumably devoid of subatomic sources of energy, is doomed to collapse on a "Kelvin" time scale, i. e., with gravitation as the source of energy; high densities can be attained, and a super-dense core *** may be formed. The hydrogen-containing envelope cannot be sucked into the core as long as traces of hydrogen are present, because the corresponding immense increase of temperature and density would lead to an instantaneous release of the whole store of subatomic energy, sufficient to disperse all the envelope into space. Actually no such

* Actually such an extra supply of heat is necessary; as shown in another paper, the necessary condition (assuming Kramers' opacity) for a finite solution is $n_e < 3.25$.

** For a velocity $< 10^4$ cm/sec (cf. above), and $\frac{\varrho_i}{\varrho_e} = 2$, the height to which the current mouth intrudes into the envelope is of the order of $< 10-20$ meters, for the interior of the sun.

*** We avoid the term "centrally condensed", as it has been attached by Milne [23] to a certain mathematical model which is a mathematical, and probably also a physical impossibility, leading to the central singularity which can be removed only by an appeal to unknown physical properties created ad hoc, cf. [5].

catastrophe happens *, the contraction of the core being a gradual one; instead of blowing up, the envelope gradually expands and adjusts itself to such low values of the effective density and temperature that the release of subatomic energy remains more or less normal (it may be even less than the "normal", as the gravitational energy of the core supplies now a large fraction of the star's needs). In spite of the high gravitational force exerted by the core, the transition from the superdense core to the envelope of "normal" density and temperature is made possible by the peculiar distribution of the energy sources, and the smaller molecular weight of the envelope **; the presence of subatomic energy sources suddenly beginning to work outside the core creates radiation pressure that "blows away" the matter of the shell, leaving a small density of matter just sufficient for the subatomic sources to work. The conditions are similar to those in the mathematical point-source model (cf. [1], p. 126), except that here the subatomic source of energy is not concentrated in exactly one point, and that an additional point-source of energy and a considerable point-mass complicate the problem.

At a certain distance from the core adiabatic equilibrium may set in; it may be shown that for the distance R_ε, which halves the subatomic energy sources, convection is always effective, and adiabatic equilibrium takes place. We omit the proof, as this result obviously follows from the fact that R_ε and ϱ_ε are of the order of $R_\odot$, and $\varrho_\odot$, for which case the numerical estimates made at the beginning of this section apply. As to R_ε, the effective radius of energy generation, and ϱ_ε, the effective density, they may be estimated in the following way. Let L_1 be the energy output of the core, L_2 the additional energy from the subatomic energy sources outside the core; M_c and $M - M_c$, the mass in the core, and outside the core respectively; T_ε the temperature at R_ε. For constant β, the temperature represents the potential; for a small distance from the centre, and a considerable mass in

* Except for the "dissociative collapse" discussed in the preceding section, where this was shown to be insignificant for all actual stars.

** Without admitting such peculiar conditions, the central density and temperature of a star of fixed outer dimensions cannot exceed certain "moderate" limits, cf. Eddington [48].

the core, the potential is close to $\frac{M_c}{R_\varepsilon}$, thus $T_\varepsilon \sim \frac{M_c}{R_\varepsilon}$. The law of energy generation we assume to be $\varrho\, T^4$ (cf. Section 3.*g*, where $s \sim 6.5$ is estimated for the deuteron synthesis at $T_\varepsilon \sim 1{,}1.10^7$; $s = 4$ seems to be better a value, allowing for higher temperatures $\sim 5.10^7$ and more in our present case, even when the deuteron synthesis is replaced by a reaction of higher order). Approximately, for a constant fraction of active mass, we have

$$L_2 \sim (M - M_c)\, \varrho_\varepsilon\, T_\varepsilon^4 \sim (M - M_c)\, \varrho_\varepsilon\, M_c^4\, R_\varepsilon^{-4}\,.$$

In Section 3.*d* we estimated for the active mass an effective density of $\varrho_\varepsilon = 10$ gr/cm³ ,and $R_\varepsilon = 0.16\, R_\odot$ in the sun. If mass and radius are measured in units of the sun, for a constant ratio of active mass to total mass we have

$$\varrho_\varepsilon = \frac{10\,(M - M_c)}{\left(\frac{R_\varepsilon}{0.16}\right)^3} = \frac{M - M_c}{25\, R_\varepsilon^3}\,.$$

This gives for L_2, in units of the sun's luminosity,

$$L_2 = \frac{(M - M_c)^2\, M_c^4}{25\, R_\varepsilon^7}\ \frac{\varrho_\varepsilon}{10}; \text{ hence we find}$$

$$R_\varepsilon = 0.46\,(M - M_c)^{\frac{2}{7}}\, M_c^{\frac{4}{7}}\, L_2^{-\frac{1}{7}} \quad . \quad . \quad . \quad . \quad (24),$$

all in units of the sun,

$$\text{and}\ \ \varrho_\varepsilon = 0.4\,(M - M_c)^{\frac{1}{7}}\, M_c^{-\frac{12}{7}}\, L_2^{\frac{3}{7}} \quad . \quad . \quad . \quad . \quad (25),$$

the density in gr/cm³.

For a typical case of $M_c = 0.25\, M$ (when the non-collapsed no-hydrogen core has approximately a luminosity equal to the luminosity of a hydrogen-containing star of mass M), and for extreme limits of L_2

$$0.01\, M^3 < L_2 < M^3$$ (the subatomic energy sources cannot give much more than the "normal" luminosity of a star which is $\sim M^3$ empirically); we have

$$0.25\, M^{\frac{3}{7}} > R_\varepsilon > 0.125\, M^{\frac{3}{7}}\,,$$

and

$$2\, M^{-\frac{2}{7}} < \varrho_\varepsilon < 16\, M^{-\frac{2}{7}}\,,$$

thus rather narrow limits for such a wide range (100:1) in the subatomic energy output, L_2. For $M = 1 - 10 \odot$ we have $R_\varepsilon \sim 0.2 - 0.6\ R_\odot$, thus large and almost invariable, as compared with the small and widely variable radius of a superdense core ($10^{-2} - 10^{-4}\ R_\odot$); $\varrho_\varepsilon = 2 - 5$ gr/cm^3. As shown in the following section, the outer radius of the star in such a case may be subject to considerable variation (for small variations in the luminosity), and may be large in some cases (when L_t is large) as compared with R_ε. A typical giant structure results, consisting of a vast extended envelope of low density in radiative or adiabatic equilibrium, an intermediate zone in adiabatic (convectional) equilibrium, of a density about the central density of main sequence stars, containing active sources of subatomic energy, and a contracting superdense core of zero hydrogen content and no subatomic energy. The intermediate zone, with active atomic synthesis, is supposed to contain a decreased amount of hydrogen and to get in this way definitely separated from the outer envelope (cf. above); if not, the whole outer mass except the core may be stirred by convection currents (as in the purely adiabatic model), and the outer radius becomes little sensitive to eventual changes in the luminosity (corresponding to the changing mass of the core which must increase with the progress of time from the exhausted material of the shell, and decrease as the result of energy losses)*, in which case an apparently "main sequence" star with a superdense core may result.

Section 6.

The Course of Stellar Evolution.

a. Presumptions.

Let us consider the course of stellar evolution determined by the most probable conditions which follow from our preceding discussion: atomic synthesis and gravitation as the only sources of stellar energy; absence of complete mixing in some

* Some kind of equilibrium for the mass of the core may result: increasing mass leads to rapidly increasing energy output and radiation pressure at the boundary of the core, which resists the flow of exhausted material inwards.

stars; complete mixing for all stars (without superdense cores) in a central portion of considerable extent, without necessarily an efficient interchange of material with the outer shell; origin from condensation of diffuse matter (nebula), which also determines the original composition.

b. Condensation from a diffuse state.

The first stage of a star's life consists of a comparatively short interval of contraction from a diffuse state; the structure of the star approaches closely Eddington's radiative model (polytrope $n = 3$; $\varepsilon \sim T$), the rate of generation of gravitational energy is automatically equal to the "prescribed" loss by radiation; convection currents are practically absent (the rotational currents are too much stratified); the central temperature increases during contraction inversely to the radius, and $\varrho_c = 54\,\bar{\varrho}$ (with slight uncertainty as to the definition of the boundary of a star); the first stage may last $\sim 10^7$ years for $M = \odot$, $\sim 10^5$ years for $M \sim 10\odot$.

c. Stage of atomic synthesis.

As the central temperature rises, processes of atomic transmutation come gradually into play; a second stage of the star's life starts when an outwardly steady state is reached, subatomic energy balancing the "prescribed" losses by radiation; contraction becomes extremely slow (just enough to balance exhaustion of hydrogen by an increase of the central temperature); for the sun, this stage may last for 10^{10} years, for $M \sim 10\odot$ perhaps 10^8 years.

d. Evolution of the adiabatic model.

If the star is of the completely adiabatic structure, with complete mixing (sufficient rotation to overcome the dead zone at $r = r_0$, cf. preceding section), it remains a "main sequence" star of more or less "normal" density; with the gradual exhaustion of hydrogen its luminosity increases (cf.[19,20] and Section 7). At the same time slow changes in the radius occur which may be estimated in the following way.

For the energy generated by atomic synthesis we assume further the expression

$$L \sim X^2 M \varrho T_c^s \text{ (cf. Section 3 and 5).}$$

With the aid of (10) and $\varrho \sim M R^{-3}$ this becomes

$$L \sim \frac{X^2 (\beta \mu)^s M^{s+2}}{R^{s+3}} \quad \ldots \ldots \ldots \quad (26);$$

here X is the relative proportion of hydrogen (for a composite model the formula holds when X refers to the core); the second power of X follows for the reaction $H^1 + H^1 \rightarrow H^2$; for the neutron synthesis the first power of X should be used.

On the other hand, with Kramers' law of opacity (disregarding electron scattering) we have

$$L \sim k_0^{-1} R^{-\frac{1}{2}} M^{5.5} (\beta \mu)^{\frac{15}{2}} \quad \ldots \ldots \quad (27)\,^*.$$

(26) and (27) lead to

$$R \sim k_0^{\frac{1}{s+2.5}} X^{\frac{2}{s+2.5}} (\beta \mu)^{\frac{s-\frac{15}{2}}{s+2.5}} M^{\frac{s-3.5}{s+2.5}} \quad \ldots \quad (28).$$

With Eddington's quartic equation this becomes

$$R \sim k_0^{\frac{1}{s+2.5}} X^{\frac{2}{s+2.5}} (1-\beta)^{\frac{2s-15}{8s+20}} M^{\frac{2s+1}{4s+10}} \quad \ldots \quad (28').$$

The deviation from the prescribed radius may be considered a measure of deviations from uniform homologous structure.

For the interval $0 < X < 0.50$, the intrinsic opacity as tabulated in Section 5. *h* is satisfactorily represented by $k_0 \sim (1-X)^2$.

Thus, for constant mass and changing hydrogen content the radius changes according to

$$R \sim [(1-X)X]^{\frac{2}{s+2.5}} (1-\beta)^{\frac{2s-15}{8s+20}}.$$

In Section 3.*g* we estimated $s = 6.5$ for the most probable process of atomic synthesis. With that, $1-\beta$ influences the radius but slightly $[\sim (1-\beta)^{-0.03}]$, and in the same direction as X. With sufficient approximation the change of radius for

* This equation is equivalent to Eddington's mass-luminosity relation; it is independent of the polytropic index, when homologous structure is presumed.

an adiabatic star is then $R \sim [(1-X)X]^{0.22}$. The change is rather slow. For $X \sim 1$ per cent, the radius is about one-half of its original value at $X = 33\frac{1}{3}$ per cent. Thus for $s = 6.5$ a slow contraction proceeds during the atomic synthesis; after its exhaustion, the star starts rapid contraction, relying upon gravitational energy alone. A superdense O-type or Wolf-Rayet star results.

For $s = 19$ (cf. Section 7), $R \sim [(1-X)X]^{0.10}\ (1-\beta)^{0.14}$; with increasing molecular weight $(1-\beta)^{0.14}$ increases faster than $X^{0.10}$ decreases, and the radius starts very slowly expanding; after reaching a maximum (for the sun, at $X = 0.069$, $R = 1.2\ R_\odot$) just before exhaustion, the radius begins to decrease and ends in a collapse as described before.

e. Evolution of the composite model.

If the star possessed originally a core of smaller hydrogen content, or should acquire such a core as the result of incomplete circulation and atomic synthesis, or if it originally settled into a compound radiative-adiabatic state, it will, during the second stage of its life, maintain the typical compound structure; in this stage the star is supposed to consist of a more or less extended convective core, built adiabatically according to a polytrope of $n = \frac{1}{\gamma - 1}$ (cf. Section 5), above which an outer shell in complete or partial radiative equilibrium is placed; at first no energy is produced in the outer shell. For such stars there is little, or perhaps no interchange of matter between the inner core and the surface. With the short time scale, composite main sequence stars of solar mass and less may at present still be in this stage of evolution; if larger masses also should be found in this stage (Procyon, cf. Section 7), this could be explained by their age being less than 3.10^9 years.

With the exhaustion of hydrogen in the convective core the third stage of evolution for the compound model starts: the contraction of the core which gradually is transformed into a superdense nucleus. An inner core devoid of subatomic sources of energy assumes a structure very similar to an incomplete polytrope $n = 3$ (built up from the centre), generating gravitational energy according to $\varepsilon \sim T$; the persistent contraction of such a nucleus is unavoidable (the only non-collapsing form would be an isothermal structure, where the loss of energy is zero; this, however, could not maintain itself: with the first increase ΔP of the external pressure over its original equilibrium value the configuration departs from isothermity, and the net flux of energy which arises then stimulates progressive contraction and progressive departure from isothermity, until the polytrope $n = 3$ is approximately reached). The change of the radius r of the nucleus with time may be represented by

$$\frac{1}{r} = \frac{1}{r_0} + ct \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29).$$

Outside the nucleus the material is not exhausted; with the progress of the central condensation the temperature of the shell adjacent to the nucleus rises, and subatomic energy is released in an intermediate shell; the rapid increase of energy generation with increasing temperature and density in the intermediate shell prevents it and the rest of the star from being drawn into the overdense nucleus; on the contrary, if the outmost shell is in radiative equilibrium, by a process described below, it is forced to expand, and a giant star is formed. (For adiabatic equilibrium in the outer shell, a giant structure is also possible.) Let Fig. 2 show the scheme of a giant star; C is the exhausted nucleus, of radius r, mass M_c, and a net output of gravitational energy L_1 transported by radiation; A is the region of release of the subatomic energy and of convective circulation (at least in its outer portion), of radius R_1, mass $M_1 - M_c$, and a net output of energy L_2, transported to the top of the shell partly by convection; B is the region of undisturbed radiative equilibrium *, with a temperature T_1 at the bottom, extending to the surface of the star of radius R and mass M; no energy is generated in B (gravitational energy at eventual changes of radius being there negligible). The condition for secular stability is

$$L_1 + L_2 = L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30),$$

* The alternative of adiabatic equilibrium we do not consider here now.

where L is the luminosity. The violation of this condition, leading to an increase or decrease of the energy content, affects primarily the outer shell, B. The nucleus liberates automatically the practically fixed amount which it spends, and it is doomed to gradual collapse: there is no secular stability for the nucleus; but, if r/R is small, changes in r do not much reflect directly upon R, and the star may keep

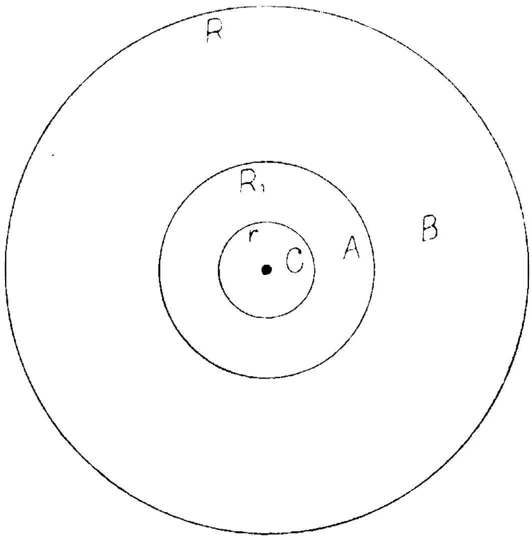

Fig. 2. Scheme of giant structure.

the outer appearance of being unchanged, which we describe as secular stability. The intermediate shell A, with convectional transport of heat, transports, under all circumstances, all the heat $L_1 + L_2$ to its top (cf. Section 5. a, b), and no accumulation is possible there. The shell B in a given fixed state, however, with its radiative transfer of energy, is able to transport, and actually does transport, a fixed amount, equal to L, and unless L is apt to vary with the radius, secular stability cannot take place. The expansion of B, of course, causes mechanically the expansion of A, and in a minor degree of C*. For homologous changes of structure, L can indeed change but slowly with the radius, the change being of a

* For C, only a slowing down of the contraction is actually imaginable.

non-stabilizing character ($\sim R^{\frac{1}{2}}$, thus opposite to the required direction, cf. [1]), so that secular stability can be attained only by an automatic adaptation of the energy sources. For our model, L_1 is not much apt to vary; as to L_2, although an increase of it is able to prevent collapse in the case $L_1 + L_2 < L$, it is unable to prevent expansion if alone $L_1 > L$. In spite of that, as shown below, secular stability may be attained, because the changes in our complex model are not homologous, and L may vary so as to fit almost an arbitrary amount of energy generated in the interior.

To derive the luminosity we apply a simple method sufficient for our qualitative purposes. The flux of radiation between two spherical surfaces R_1, M_1, T_1 and R_2, M_2, T_2, may be represented with sufficient approximation by

$$L \sim \frac{(T_1^4 - T_2^4)\ R_1\ R_2}{k\varrho\,(R_2 - R_1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31).$$

This follows from the equation of radiative transfer with $k\varrho = \text{const.}$, when sources of energy inside the shell $R_2 - R_1$ are absent; when such sources are present, the proportionality remains valid, with a variable proportionality factor depending upon $\frac{M_1}{M}$ and $\frac{M_2}{M}$ (relative internal masses), and upon the relative amount of energy developed inside the shell. For k, the coefficient of absorption, and ϱ, the density, certain mean, or effective values for the given shell must be assumed.

For stars of a homologous series, when $\frac{M_1}{M}$ and $\frac{M_2}{M}$ are kept constant, we have: $T_1 \sim T_2 \sim \frac{\beta\mu M}{R}$ (cf. [1]); $R_1 \sim R_2 \sim R$; $k \sim \varrho T^{-\frac{7}{2}}\mu^{-1}$ (Kramers); $\varrho \sim R^{-3}$; $1 - \beta \sim M^2\mu^4\beta^4$ (Eddington's quartic equation for the radiation pressure *); $L \sim R^2 T_e^4$; with these proportionalities, (31) is easily transformed into

$$L \sim M^{\frac{7}{5}}(1 - \beta)^{\frac{3}{2}}\, T_e^{\frac{4}{5}}\, \mu^{\frac{4}{5}},$$

which is exactly Eddington's mass-luminosity relation [37],

* Valid for homologous stars, with the coefficient of proportionality depending upon structure.

derived of course on the assumption $1 - \beta = \text{const.}$ throughout the star. This may be considered as a check of reliability of formula (31).

With the more general law of absorption

$$k \sim \varrho\, T^{-3+\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (32),$$

the mass-luminosity relation for stars of a homologous series (not necessarily polytropic) becomes (the influence of the molecular weight upon k and luminosity is not considered now, as it is a more complicated function of the composition, cf. [19, 20] and Section 5; it cannot be represented by simple proportionality):

$$L \sim \beta^{7-\alpha} M^{5.5} R^{\alpha},$$

where β is an effective value for the whole star, depending upon its mass. For the Kramers formula $\alpha = -\frac{1}{2}$, for relativistic non-degenerate matter $\alpha = 0$. To degenerate matter the formula does not apply.

The "giant" model of Fig. 2 cannot undergo homologous changes; the nucleus, an incomplete polytrope ($n \sim 3$), when having reached a sufficient degree of compression, is practically independent of changes occurring in the outer shells. For an example take a nucleus of about the mass of the sun, $\mu = 2.1$ (no hydrogen), $T_c = 1,6.10^9$, $\beta \sim 1$; the radius of the complete polytrope is $0.025\ R_\odot$. The boundary of the incomplete polytrope coincides with the effective "centre" of the outer configuration*, for which a temperature of $1,6.10^7 < T' < 7.10^7$ may be estimated; for these "boundary" temperatures from Emden's tables ($n = 3$) we find: the radius of the incomplete polytrope equal to from 0.94 to 0.87 of the "complete" radius; the mass equal to 1.000 of the "complete" mass. If $T' \sim \frac{1}{R}$, our estimated figures correspond more or less to the following relation

* Which means that the central temperature and density of a polytropic configuration, devoid of a superdense nucleus, may be assumed to be of the same order of magnitude as the temperature and density are at the boundary of the superdense nucleus, when the mass and external radius in both cases are equal.

between the radius of the core, and the radius of the star:

$$r \sim R^{0.05}.$$

The exponent here decreases with increasing density of the core. Thus, from the standpoint of external changes, the nucleus is almost incompressible. On the other hand, for regions near the outer boundary of the star the changes of radius in a shell must follow closely the changes in R. Generally, for an intermediate shell like A, we may write the intermediate formula

$$R_1 \sim R^{1-p} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (33),$$

with $(0.05) < 1 - p < 1$. Substituting (33) and (32) into (31), with $R_2 = R$, $T_2 = 0$, $\varrho \sim (R^3 - R_1^3)^{-1}$, $T_1 \sim R_1^{-1}$ (the approximation is sufficient, as for inner points the major part of the potential is due to the central condensation, if it is considerable, thus $T_1 \sim \frac{Mc}{R_1}$), and assuming with sufficient precision $R^3 - R_1^3 \sim R^3$, $R - R_1 \sim R$ (R_1/R and p being small, justify this simplification), we get for the dependence of luminosity upon the radius:

$$L \sim R^{6.5\,p + \alpha} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (34).$$

A comparatively small deviation p of the exponent in (33) from unity suffices to make our model secularly stabilized by changes in the radius; (34) must for this purpose give increasing luminosity with increasing radius, thus the stabilizing condition is

$$6.5\,p > -\alpha.$$

For $\alpha = -\frac{1}{2}$ (Kramers), $p > 0.077$ is required (for the non-degenerate relativistic coefficient of absorption $\alpha = 0, p > 0$ *). For an increase of the luminosity in the ratio 2 : 1, different changes in the radius are met by different values of p as follows:

Ratio of R	2	10	50	∞
Ratio of density $\bar{\varrho}$	0.125	0.001	8.10^{-6}	0
p_1 ($\alpha = -\frac{1}{2}$)	0.23	0.123	0.104	0.077
p_2 ($\alpha = 0$; or $k =$ const.)	0.15	0.046	0.027	0.000
Ratio of R_1	1.70	7.5	33	∞

* The same condition $p > 0$ is valid for $k =$ const., which corresponds to a predominant role of electron scattering in the opacity (massive stars).

Thus, for a small value of p, or for a slight deviation from proportionality of R_1 and R, the outer radius may be extremely sensitive to changes in the energy generation; the doubling of the internal source of energy may produce a typical giant, of any degree of diffusion, from an originally dense star. Now, a progressive increase of the internal generation of heat, above its original normal (original regulated) value may be actually expected, when the exhausted central core starts its collapse. This core, practically a complete polytrope as far as mass and rate of heat generation are concerned, being devoid of hydrogen, radiates more energy than if hydrogen in the normal proportion were present (cf. [19, 20]) (up to 100 times more for a solar mass; the difference, however, is greatly reduced for large masses and high central temperatures, on account of electron scattering).

If L_0 is the "prescribed" luminosity of the original "main sequence" star, no great expansion of the radius can start before L_1 is a considerable fraction of L_0 (cf. formula (30)), because a moderate expansion reduces the subatomic energy L_2 and makes a balance; but when the energy output of the nucleus L_1 approaches, or even exceeds, L_0, the expansion of R must be large, and the star enters the giant stage (especially because L_2 can never drop to zero, or even to a very low value; at the boundary of the nucleus a rather peculiar zone exists where the exhausted material of the adjacent outer shell continually is driven into the nucleus, adding the released gravitational energy to L_2; the nucleus thus increases steadily, probably until a certain equilibrium size is reached, when the outwards directed resultant of the radiation pressure, being large on account of the sudden increase of the energy sources outwards, produces at the boundary of the central core a sufficiently small material density, so that the amount of inflowing material becomes equal to the radiation losses of the nuclear mass). For the exhausted core, containing no hydrogen, the amount L_1 may be read off from Eddington's table in[37] (or [1], p. 137), with the mass M_c of the nucleus as argument, and by making the bolometric magnitude 4.6 magnitudes brighter than tabulated (except when the mass is large; cf. [19, 20]). For a core of high density, and thus of large theoretical effective temperature T_e, the correction $2 \log T_e$ is unnecessary on account of electron scattering. Take $M = 5.0\odot$ (Eddington's case of the point-source, cf.[1], p. 126), for which the central region of the convectional transfer of heat ($Q < Q_{max}$, cf. Section 5) during the original "main-sequence" stage may be estimated to reach to $\sim 0.3\,R$, including 0.25 of the total mass; taking this as the mass of the future central core, $M_c = 1.25\odot$, we have: "normal" absolute magnitude of the star, $m_0 = -0.8$; absolute magnitude of the core, $m' = 3.5 - 4.6 = -1.1$.

Thus, the original core alone, without further accretion of mass, may produce more energy per unit of time than the original non-exhausted star (the collapsed core may be even more efficient); in our case, $L_1 \sim 1.3\,L_0$, so that $L_1 + L_2 \sim 2\,L_0$, or more or less as in the above given table.

Our discussion is but qualitative: a more definite picture can be obtained only with the aid of laborious calculations. Nevertheless, it appears highly probable that the structure of the giants, and the riddle of their energy generation, can be explained as above, where only such physical laws as are known at least in principle have been applied. The picture does not change essentially if at high temperatures and pressures something like annihilation of matter occurs; this would put a limit to the collapse of the central core, whereas the external phenomena would remain the same. In the process of the collapse, there may indeed be opened up an auxiliary source of energy, by the transmutation of helium and other light nuclei into heavier ones (chiefly into the group of iron), which may yield perhaps 10 per cent of the energy of the original synthesis of hydrogen. After that, if not stopped by the interference of some unknown source of energy, the core enters the stage of perpetual and gradual contraction, without actual collapse: as shown above (Section 4. *b*), nuclear dissociation can never start appreciably; the contraction ends when the upper limit of density of matter and radiation is reached, if such a limit exists ($\varrho \sim 10^{15}$ gr/cm^3 may be such a limit, corresponding to a close packing of the atomic nuclei). The exhausted material of the envelope is gradually absorbed by the core, and there will be reached a stage when no envelope is left: Wolf-Rayet stars, or planetary nebulae with their overdense central stars perhaps represent the final stage of the giant evolution, too.

f. The imaginary catastrophic collapse.

Although we have shown that nuclear dissociation cannot be of importance in actual stars ($M < 200 \odot$), we might for a moment forget this conclusion, and try to imagine the consequences if dissociation existed. The stage of nuclear dissociation, as already mentioned above (cf. Section 4. *b*), must assume the form of an actual cataclysm; the subatomic energy radiated previously must now be paid back at the expense of the gravitational energy (all the preceding radiation into space being thus a "credit account" to be cleared at the collapse); the energy of contraction is absorbed by the nuclear dissociation, and the collapse stops only when all the matter is transformed into neutrons; the duration of the collapse must be very short, of the order of one second. As a consequence the non-exhausted outer shell A (Fig. 2) follows, the temperature and density suddenly increase there, so that an actual atomic explosion (sudden transformation of hydrogen into heavier elements) follows, releasing (in a small fraction of the mass, of course) almost instantaneously an amount of energy which normally might have lasted for millions of years; a sudden expansion of the outer shell starts, and a part of the shell is dispersed into space: thus, the phenomenon of a Nova may be well explained as a secondary effect, following the collapse of the inner nucleus. We notice that this picture of a Nova phenomenon bears some outer resemblance to Milne's theory [23], but actually the two conceptions have little in common. Unfortunately, by reasons explained in Section 4. *b*, this picture of a Nova must also be abandoned. Although we probably have to do here with a subatomic explosion, we do not find any better explanation than an old hypothesis of ours (cf.[10], p. 35 f.), according to which the "explosive" (hydrogen) mixture is driven into the interior by some external force, or by the upset of radiative equilibrium.

g. Effect of degeneracy.

All the above said refers to massive stars, for which the core is supposed to remain non-degenerate; for smaller masses, the core may become partly, or completely, degenerate, which in the first place means a decreased energy output of the core. The value of L_t approaches zero, and thus the cause which led to the formation of the diffuse envelope in massive stars is greatly reduced, or even absent, for the smaller ones. The chief reason for the absence of diffuse stars among small masses seems to be, however, a question of the speed of evolution. During 3.10^9 years, the hydrogen in dwarfs did not get exhausted, whereas in giants it did.

h. Semi-giant stage of the composite model.

After having sketched the supposed course of the evolution of a giant in broad outline, a few details may be considered. During the "main sequence" stage of a composite model, the gradual exhaustion of hydrogen in the central convective (adiabatic) region increases the mean molecular weight there, as the result of which the amount radiated outwards increases considerably (up to a maximum of about 100 times the initial value for a solar mass of the core; actually much less, because the imaginary mass of the "complete" polytrope ot the core decreases *); as shown in Section 5. *h*, the increased internal radiation never disturbs the radiative equilibrium of the outer shell; the excess of heat accumulating in this shell at first produces expansion and re-adjustment to the new conditions of radiative equilibrium. Formula (28) above was derived for a complete polytrope, but the formula practically represents the tendency of the change in radius for an incomplete polytrope, as well; thus, for $s \sim 6$ a very slow contraction with increasing molecular weight (exhaustion) follows; for $s = \frac{15}{2}$ the radius remains constant. We assume for a moment the constancy of the radius of the core, and imagine its molecular weight to have been increased suddenly in a ratio $\frac{\mu_2}{\mu_1}$; to support the external pressure the temperature in the entire core, thus also at the surface of demarkation, should increase in the ratio $\frac{\beta_2 \mu_2}{\beta_1 \mu_1}$. Thus, the

* A problem to be considered in another paper.

temperature at the bottom of the envelope (with unchanged $\mu = \mu_1$) increases, too, and the temperature gradient decreases [cf. Section 5. h, formula (22) ff.] by the condition of radiative equilibrium; to meet the change of temperature, the condition of mechanical equilibrium requires the density at the bottom of the envelope to decrease in the ratio $\frac{\mu_1}{\mu_2}$ (because the internal density of the core has remained the same), which by itself means an expansion of the envelope; but the expansion must proceed further, as the result of the decreased weight of the envelope (after the first imaginary expansion), which no longer balances the pressure at the surface of demarkation. As a result, the surface of demarkation must also expand somewhat, and a steady state will be reached when the temperature at the surface of demarkation exceeds by less than $\frac{\beta_2 \mu_2}{\beta_1 \mu_1}$ times its original value (which it had before the change of μ). Thus, with progressing exhaustion, the outer layers of the star show a definite expanding tendency, whereas the core cannot adequately follow (form. 28), because its subatomic energy sources work only at a certain degree of compression; the picture is in principle the same as that described for the giant model above. With a maximum estimated increase of $\sim 20:1$ in the heat output of the convective core due to exhaustion, and a corresponding expansion of the envelope and decrease in the mean density, "semi-giants" may be produced; the maximum luminosity*, for a given mass, of such stars may be higher by about 3 magnitudes than the "normal", or the apparent (computed) hydrogen content may be smaller (RT Lacertae, cf.[21]; Procyon, β Aurigae, cf. Section 7); of course, according to our conceptions, typical giants with a collapsing core may also show a similar excess of luminosity, though smaller (because of electron scattering) and depending upon the fraction of mass in the core.

With progressing exhaustion of the core the luminosity increases; therefore, exhaustion proceeds more and more rapidly

* The luminosity here is chiefly prescribed by the structure of the inner core without much influence of the surrounding envelope; a more limited case of the influence of an atmosphere has been considered by Eddington[49], with similar conclusions.

and the last transition phase towards a real giant, when the luminosity excess is conspicuous, cannot last for long, and the representatives of the "semi-giants" cannot be very numerous *. When the giant stage finally is reached, the luminosity may decrease again (although the extent of the envelope, with decreasing r and p, may increase considerably), because the core, now an almost complete polytrope, has a smaller net output of radiation than the former, very incomplete, adiabatic polytrope, which radiates almost at the rate of a much larger mass, of which it is an imaginary portion.

i. White dwarfs.

To explain the low rate of energy generation in white dwarfs, we are forced to conclude (as Atkinson does, cf.[12]) that their interior is devoid of hydrogen (and of neutron, too, of course); the hydrogen observed in their atmospheres must be a superficial feature, and cannot reach into regions where the temperature exceeds 10^7 K (cf. Section 4. *d*). On the short time scale it would be hard to understand how a star of less than the solar mass could ever get exhausted, unless it contained from the beginning a relatively smaller proportion of hydrogen (about 26 per cent of original hydrogen content for Sirius B, according to Table 4 below, which would have made the star originally by 1.0 mag. more luminous than the sun, thus speeding exhaustion; o^2 Eridani B would require an initial amount of 16 per cent of hydrogen). In double stars the components might have got such widely different proportions of hydrogen (Sirius A > 0.40; Sirius B $= 0.26$, etc.) perhaps as the result of the unequal differentiation of meteoric material at an early stage of the primordial nebula (cf. Section 7. *l*).

Another suggestion is that the white dwarfs are perhaps remnant cores of composite models after Nova explosions, where the greater portion of the original mass (the hydrogen containing shell) had been thrown away.

* The same must hold for the complete adiabatic model of an advanced degree of exhaustion, before its final collapse.

Section 7.

Theory and Observation.

a. Hydrogen content and mass-luminosity relation.

A mass-luminosity relation based on Kramers' general opacity corrected for electron scattering according to (23), may be written as follows:

$$L = \frac{C\left(\frac{M}{M_\odot}\right)^{\frac{16}{3}}\beta^{\frac{15}{2}}}{\left(\frac{\varrho}{\varrho_\odot}\right)^{-\frac{1}{6}} + F\beta^{\frac{7}{2}}\left(\frac{M}{M_\odot}\right)^{\frac{7}{3}}} \cdot \frac{2{,}5}{\alpha} \quad . \quad . \quad . \quad . \quad (35),$$

where C and F depend upon composition and structure. For $F = 0$, (35) corresponds exactly to Eddington's mass-luminosity relation; the formula differs from Eddington's by the factor $\dfrac{1}{1 + \dfrac{0.2\,(1+X)}{\bar{k}}}$, which is mostly close to 1 except for massive stars. For the mean opacity of a star, the following expressions may be assumed: $\dfrac{\bar{k}}{k_0} = 1.31\ \varrho_c T_c^{-\frac{7}{2}}$ for $n = 3$, and $2.42\ \varrho_c T_c^{-\frac{7}{2}}$ for $n = 2$. When the sun is taken as the unit of luminosity ($m_{bol.} = 4.65$), the constants are (in agreement with [19]): for $X = 0$ (no hydrogen), $n = 3$, $\alpha = 2.64$ (gravitational collapse), $C = 107$, $F = 0.033$; for $X = 33\frac{1}{3}$ per cent hydrogen, $n = 2$, $\alpha = 3.31$ (adiabatic model), $C = 1.02$, $F = 0.0054$. It is understood that the formula refers to homologous models of homogeneous composition, more or less of a polytropic structure; it may be applied to centrally situated incomplete polytropes, in which a correction factor $\dfrac{Q}{Q_{max}}$ (cf. Section 5. *b*) is required, when $r < r_0$ and none, when $r \geqslant r_0$. Thus, for the collapsing core formula (35) applies (until the relativistic change of the absorption coefficient comes into play) with $X = 0$; in this case $\left(\dfrac{\varrho}{\varrho_\odot}\right)^{-\frac{1}{6}}$ may be neglected, and the limiting formula for the luminosity of the collapsing core at high density becomes (in solar units of luminosity):

$$L_1 = 2{,}63 . 10^3 \left(\frac{M}{M_{\odot}}\right)^3 \beta^4 \quad . \quad . \quad . \quad . \quad . \quad (36),$$

where β may be taken from Eddington's table in[37]; the relativistic absorption coefficient at $T \sim 2.10^{10}$ leads to practically the same result. The luminosity may be reduced by the increased opacity due to "Paarbildung" and increased electron scattering, which, however, may have only a small influence, because the density of radiation remains small as compared with the density of matter (verified by computations of nuclear dissociation, cf. Section 4.*b*). Thus it appears that the collapsing core exhibits an enormous luminous efficiency, and that only a small core can persist without making the outer shell expand to infinity (cf. Section 6.*e*; as explained already, the size of the core is probably subject to automatic regulation). To get a total luminosity of the order of the empirical, or more or less like Eddington's mass-luminosity curve, the size of the core (M_c) according to (36) must be as follows:

M	1	2	4	9	20 $\odot$
$L_1 \sim$	1	13	100	600	2500 $\odot$
M_c	0.08	0.19	0.33	0.61	0.99 $\odot$
$M_c : M$	0.08	0.095	0.08	0.07	0.05

Thus, the core, if it exists and if it is not degenerate, can amount to only a few per cent of the stellar mass. It must be emphasized that Chandrasekhar's criterion of degeneracy[5], for our complete polytropic core which behaves almost like an independent star, applies to the mass of the core, not to that of the whole star. These cores are so small that they can be degenerate, even when increasing in mass ($M_1 < 1.6$); L_1 must be in this case much smaller, formula (36) applying only to a transition phase (contraction before degeneracy is reached). We notice that Biermann[50] has considered somewhat similar stellar models.

Below are given some examples (all computed with the same $\alpha = 2.5$ for the sake of simplicity) of the application of formula (35). For ζ Aurigae br. (not in Strömgren's list), an eclipsing binary consisting of a K1 supergiant and a B companion, $T_e = 3360^0$ according to the colour-index (cf.[55], Table I, *H. R.* 1612, $C = 1.66$ = corrected colour of the K1 star), and $\varrho/\varrho_{\odot} = 2{,}1.10^{-6}$ have been adopted.

Star	$\odot$	Sirius A	Algol	U Oph. br.	U Her. br.	Y Cygni br.	ζ Aur. br.
$m_{bol.}$ observed . .	4.6	0.8	−1.1	−2.0	−3.6	−5.3	−4.5
Mass	1.0	2.45	4.7	5.4	10.0	17.3	15.5
$m_{bol.}$, $X = 0\%$ . .	0.0	−3.8	−5.7	−6.2	−7.7	−8.8	−7.2
$m_{bol.}$, $X = 33^1/_3\%$.	4.6	−0.2	−3.3	−4.0	−6.4	−8.0	−6.1
Difference	4.6	3.6	2.4	2.2	1.3	0.8	1.1
$X\%$, Strömgren[21]	37	40	53	49	(70 ±)	(80 ±)	. . . .
Spectrum	G0	A0	B8	B5	B3	O9.5	K1

Star	*H. D.* 1337 br.	Trumpler's typical O star*
$m_{bol.}$ observed	−8.5	−6.0
Mass	36	91
$m_{bol.}$, $X = 0\%$	−9.2	−11.6
$m_{bol.}$, $X = 33^1/_3\%$ . .	−8.6	−12.9
Difference	0.6	−1.3
$X\%$, Strömgren . . .	(30 ±)	. . .
Spectrum	O8.5	O9

For massive stars, the influence of hydrogen content upon calculated luminosity is small (cf. also Strömgren[21]), and even may change the sign (Trumpler's star): the hydrogen content for massive stars cannot therefore be determined with confidence; mostly the observed luminosity is found to be rather low for the mass, which requires a high hydrogen content (Y Cygni, sp. O9; V Puppis, sp. B1, cf. Strömgren[21]; the K1 supergiant ζ Aurigae behaves in the same way, opposite to what Strömgren finds for the fainter giants in his list). Certainly no hydrogen content can satisfy Trumpler's O stars; there must be some reason for the strongly reduced luminosity of very massive stars (at least of those of early spectra), perhaps an increase of opacity from an unforeseen

* Cf.[51]. The typical star is just an estimate, on the basis of the seven O stars of large mass. Trumpler's average mass is greater than our adopted ypical.

source ("Paarbildung" in a collapsing core?), of which Trumpler's stars present an extreme case; the apparently high hydrogen content found by Strömgren[21] for a number of massive eclipsing binaries (*H. D.* 1337 br. for which data are given above is an accidental exception to the general rule, within the observational uncertainty) may therefore be illusory, the depressed luminosity of these stars presenting perhaps the start (at $M = 5 \odot$ already, cf. Algol in the preceding table) of a phenomenon which for Trumpler's extreme masses already amounts to 6—7 magnitudes. Nothing forces us to accept the suggested high hydrogen values of other massive stars, and the most simple hypothesis seems to be at present the following: the stars are originally built of a material containing ~ 40 per cent hydrogen; the amount may decrease with time, but it can never exceed the original value (except by stratification in the original nebula, cf. below condensation of meteoric material).

For the few giants (semigiants) occurring in Strömgren's list[21], the hydrogen content falls below the normal value:

Star	Capella A	Capella B	U Sge ft	Z Vul ft	RS Vul ft	TV Cas ft	Z Her ft	RT Lac ft	RT Lac ft
Sp. . . .	G 0	F 5	G 2	(F 2)	(F 4)	(F 5)	(G 5)	(G 9)	(K 0)
$m_{bol.}$ . .	−0.4	+0.2	+1.5	+0.2	+0.3	+1.9	+2.8	+2.7	+2.9
$R : R_{\odot}$.	12.6	6.6	5.4	5.6	6.0	2.9	3.3	4.9	4.9
Mass . .	4.2	3.3	2.0	3.0	1.7	1.2	1.3	1.0	1.9
X% . .	30	31	19	27	3	7	15	2	28

[However, as already mentioned, ζ Aur. br., mass $15.5\odot$, requires a high hydrogen content ($\sim$ 70—80 per cent).]

In other words, the luminosities of most of these giants are too high as compared with the usual one for their masses; this exactly may be expected from our conception of the giant structure, where the outer shell is forced to expand by excessive luminosity from an "independent" core; in which case, however, the mass-luminosity formula does not hold any more: these giants, therefore, may — or may not — contain a more normal amount of hydrogen; apparently we find again that there is no reliable method of determining the hydrogen content in giants. Only "normal" main sequence stars, built according to a more or less polytropic model, permit of the determination of the hydrogen content from the observed mass-luminosity relation. How many such stars exist? In Strömgren's list[21] we find the following figures for main sequence stars fainter than absolute magnitude zero:

$m_{bol.}$ . .	0.8	4.6	5.3	4.8	2.9	1.1	1.4	0.5	0.9	1.3	2.2	2.6
X% . .	40	37	28	37	22	32	28	24	24	35	34	32

The spread is comparatively small, which explains the fact that normal dwarfs fit excellently into a single-valued mass-luminosity relation; in these stars, with their moderate luminosity, the exhaustion of hydrogen cannot as yet have proceeded very far, whence their relative uniformity of apparent composition. Thus, for dwarfs the small spread around the mass-luminosity relation is explained by the shortness of the time scale. If the more massive stars "do not play havoc" with the mass-luminosity relation, it is because the influence of the hydrogen content upon luminosity decreases with increasing mass (cf. above); if all stars are of the same age (3.10^9 y.), an equal degree of exhaustion of hydrogen may be expected for equal masses, which would result in an almost exact mass-luminosity relation for the massive stars as well. This apparently is not the case; the apparent hydrogen content is variable (cf. Strömgren's data[21], also above; unfortunately, for $M > 10\odot$, the variability cannot be well detected), thus these stars show a sensible spread around the mean mass-luminosity relation. Either the stars are not all of the same age; or the deviations are due to difference of internal structure.

The effect of variable hydrogen content upon the dispersion of the mass-luminosity relation is not so great as it seems at the first glance. If the change in hydrogen content is the result of evolution, in a steady statistical state the number of the representatives of a given mass must be inversely proportional to the luminosity; therefore, large deviations of the luminosity, corresponding to a small hydrogen content, are rare. For a solar mass starting with 36 per cent hydrogen we have:

X%	36 — 30	30 — 24	24 — 18	18 — 12	12 — 6	6 — 0
$m_{bol.}$	4.6	4.2	3.4	2.5	1.6	0.5
relative frequency .	100	63	33	14	6	2

The arithmetical mean luminosity is $m_b = 4.0$, and the individual mean deviation from this luminosity is only ± 0.59 mag.; actually a solar mass cannot have passed through such a complete evolution during the short time scale, but larger masses can. For these the mean deviations from an average empirical mass-luminosity relation, due to evolution on a hydrogen synthesis basis, are:

$M/M_{\odot}$	2.5	5	10	20
deviation . . .	± 0.46	± 0.28	± 0.17	± 0.09

The deviations are so small that Eddington's fear that a widely variable hydrogen content might "play havoc" with the mass-luminosity relation is not justified. It is a trick well known to observers: the probable error of the observations seems to be surprisingly small as compared with the extreme range of the measures.

b. Atomic synthesis and stellar structure.

M. Schwarzschild[52] recently made a purely formal attempt to explain the stellar energy generation by a single process, in correlation with the polytropic ($n = 3$) central temperature, density, and the apparent hydrogen content. He sets $\varepsilon \sim M^p X^q \varrho^m T^n$ (X = hydrogen content) and derives empirical correlations from forty stars (Strömgren's data[21]):

$$p = 2.29 + 1.34\, m - 0.05\, n;$$
$$q = -0.77 - 1.64\, m - 0.32\, n.$$

If we abandon the formal procedure, and consider the problem from the physical standpoint, it is almost beyond doubt that $p = 0$ and $m = 1$. Hence $n = -73$, and $q = -25$ (!). Now, q also should be equal either to 2,1, or to 0. The result for q is absurd. Further, from these values of the exponents a ratio of luminosity of Capella : sun $= 10^{-24}$ (!) results, which needs no further comment. If we set $q = 1$ as a more reasonable value, we get two values for n: $n = +73$, and $n = -11$; thus enormously contradictory results for the temperature dependence of ε. In view of such catastrophic discrepancies* we do

* Which cannot be removed by other, even unreasonable combinations of the exponents: the observational data are intrinsically contradictory.

not think that even pure mathematical reasons can justify the absence of the most primitive physical insight from the above mentioned paper. There is, nevertheless, one useful conclusion (which the author failed to draw): for a single process of energy generation the stars cannot be homologous polytropic structures; the calculated polytropic central temperatures may differ considerably from the true central temperatures.

We may invert the problem; the laws of atomic synthesis are perhaps better known than the internal structure of the stars; having adopted a law of energy generation we may pick out stars of similar structure, and mark those of a different one.

For stars of the main sequence, especially for the less massive ones, we may expect a priori a more or less uniform structure resembling a polytropic one. The radius in this case is given by formula (28), wich holds for homologous stars when the law of energy generation $\varrho\, T^s$, and Kramers' opacity are valid. In (28) the main change in radius is due to the mass, whereas for any probable value of s the influence of X is very small, and that of $\beta\mu$ is also small. Further, for non-massive main sequence stars ($M < 5$), β is so near to 1 ($\beta > 0.98$), and the hydrogen content and μ change so little (cf. Strömgren's data [21], and above), that the mass alone determines the radius. We may thus write

$$R \sim M^{\frac{s-3.5}{s+2.5}} \quad \ldots \ldots \ldots \quad (37),$$

for the "main sequence".

Densities of visual binaries furnish the most extensive statistical material for the test of relation (37). A list of such densities based upon our actual knowledge of the colour temperatures has been published by Gabovitš and the writer [58]. As the density of a visual binary is very sensitive to the adopted colour, deviations from a normal spectral energy distribution due to line or band absorption may introduce systematic errors into the calculated densities. For the spectral interval F0-M0, no serious influence of line absorption upon colour index (λ 440—550 μ) exists, and for this interval the directly determined highly accurate colour temperatures are certainly to be preferred to average estimated values (such as those given by Russell-Dugan-Stewart, partly based on the ionization temperatures which contain the hypothetical element of pressure; besides, the effective temperatures of Russell-Dugan-Stewart are somewhat too high for giant stars, which has led to not inconsiderable systematic errors in computations based upon them) (cf. [31, 53]). For spectra later than M0, the effect of TiO absorption has been taken into account in [53] (cf. also Gabovitš, [54]); for spectra earlier than F0 the effect of the crowding of the hydrogen Balmer lines (wings of the lines) towards the violet must produce a depression of the colour index, which, as well as the space reddening of the distant B stars, is not taken into account in [53] (the effect of the Balmer lines is practically the only one to be considered in the case of the brighter A stars). Systematic corrections are estimated from the following data:

H. D. Sp.	F0	A5	A3	A2	A0	B9	B8	B5
Observed colour, C_0	0.19	0.04	0.03	−0.07	−0.14	−0.18	−0.24	−0.25
Assumed T_e	7300	8700	8900	10000	11000	12000	13000	15000
True colour, C_1 . .	0.09	−0.12	−0.15	−0.27	−0.36	−0.43	−0.49	−0.59
ΔC	−0.10	−0.16	−0.18	−0.20	−0.22	−0.25	−0.25	−0.34
$\Delta \log \varrho$	+0.17	+0.27	+0.31	+0.34	+0.38	+0.43	+0.43	+0.58

Here the second line gives the mean observed colour index (in a special system) for naked-eye stars (mostly brighter than 5.0 mag, cf. [55], p. 50); the third line — the adopted "true" radiation temperature, assumed to be 15 000° at B5, and made to approach gradually the colour temperature towards F0; the fourth line gives the "true" colour index, $C_t = \frac{9730}{T} - 1.24$ (cf. [31, a], p. 180); the fifth, $\Delta C = C_t - C_0$; the last line gives the resulting correction of the *log* density of a binary computed with the apparent colour:

$$\Delta \log \varrho = -1.71\, \Delta C \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)^*.$$

After applying these corrections, the mean density logarithms and other data for main sequence visual binaries, according to [53], Table IV, are as follows:

* Cf. [53], p. 4, form. (8).

Table 3.

Mean Densities and Radii of Main Sequence Visual Binaries.

Mean sp.	M 2	K 4	G 9	G 4	F 9	F 5	F 1	A 5	A 1	B 3
n	6	13	9	13	31	28	24	10	21	2
$\log \varrho/\varrho_\odot$	0.22	0.09	0.27	0.25	−0.21	−0.24	−0.55	−0.19	−0.55	(−0.67)
$\log M/M_\odot$	−0.40	−0.22	−0.12	−0.05	0.04	0.15	0.20	0.28	0.36	(0.74)
$\log R/R_\odot$	−0.21	−0.10	−0.13	−0.10	0.08	0.13	0.25	0.16	0.30	(0.47)
p. e.	±0.04	±0.03	±0.03	±0.03	±0.02	±0.02	±0.02	±0.03	±0.02	±0.07

The last line gives the observational probable error in the mean log R. The data of the table are well represented by the linear correlation

$$\log R = (0.72 \pm 0.06) \log M + 0.03 \quad . \quad . \quad . \quad (39)^{*}.$$

The individual spread around this correlation is considerably greater than expected from the observational probable error, indicating real causes of the deviation ("inflation" of the atmospheres at F1, "deflation" at G 9, G 4, and A 5). Comparing (37) and (39), we find $s = 19$, within the probable limits from 15 to 25, thus an exponent for the temperature variation of subatomic energy close to the value suggested by Atkinson, but in conflict with the hypothesis of $H^1 + H^1 \rightarrow H^2$ as the basic process, which requires $s = 6.5$.

With this value of the exponent ($s = 19$), formula (28) is transformed into

$$R \sim [(1-X)X]^{0.10} (\beta\mu)^{0.54} M^{0.72} \quad . \quad . \quad . \quad . \quad . \quad . \quad (40).$$

Also, as

$$(\beta\mu)^4 \sim (1-\beta) M^{-2}$$

(Eddington's quartic equation), we have

$$R \sim [(1-X)X]^{0.10} (1-\beta)^{0.14} M^{0.45} \quad . \quad . \quad . \quad . \quad . \quad (40').$$

Fig. 3 represents the correlation; in addition to the mean data of Table 3, individual values for five nearby binaries and the sun are given; for Sirius A the correction of log ϱ for Balmer wings is applied. The radii for Sirius A, Procyon, and α Centauri A are based on directly observed colour indices, and

* We take the opportunity to point out that the correlation of mass and radius has been already successfully studied by K. Lundmark, cf.[67].

for α Centauri B on a mean adopted colour index. Further, in Fig. 3, all "main sequence" individual components of eclipsing binaries are plotted which occur in Strömgren's list [21] (for equal components, one point representing the average is plotted): for these the masses and radii are directly observed quantities, independent of the adopted temperature and the data are especially valuable for the correlation *.

As shown by Fig. 3, the individual data including even the massive stars V Puppis and Y Cygni, agree excellently with the linear correlation derived from the visual binaries. Some stars, such as Procyon (F 5), Sirius A (A 0), β Aurigae (A 0), TV Cas br (B 9), u Her ft (B 8.5) show definitely inflated radii (from 25 to 60 per cent, and 2.5 times for u Her ft), which perhaps represent a transition toward giant structure. A slight depression at large masses (B stars) is indicated, which however is well accounted for (with Strömgren's data) by the factor $(\beta\mu)^{0.54}$ which becomes important at these masses; however, we did not make a correction for this factor, because we do not consider the hydrogen content, nor μ and β, as being well established for the massive stars (cf. above). Trumpler's typical O star falls decidedly below the line of correlation, which, however, must be explained by the failure of our luminosity formula (with Kramers' + electron scattering opacity), in which case formulae (28) or (40) are no longer valid.

Thus, practically all main-sequence stars from $M = 0.2$ to $M = 20\odot$ appear to form one continuous sequence, corresponding to a homologous structure and to a law of energy generation with $s = 19$. In such a case the hypothesis of $H^1 + H^1 \rightarrow H^2 + \beta_+$ being the starting reaction of the atomic

* The hydrogen content, X, computed by Strömgren [21] is based on luminosities derived from mean adopted temperatures, and, therefore, X has lost somewhat of its individual value. If we keep T_e constant for a given spectrum, a larger radius (for eclipsing binaries) leads to a greater luminosity and thus to an apparently smaller hydrogen content; such a correlation is prominent in Strömgren's data, and part of it may be spurious: for constant mass, a larger radius means smaller pressure, and lower T_e for a given spectrum; thus, the luminosity is overestimated, X underestimated in such a case.

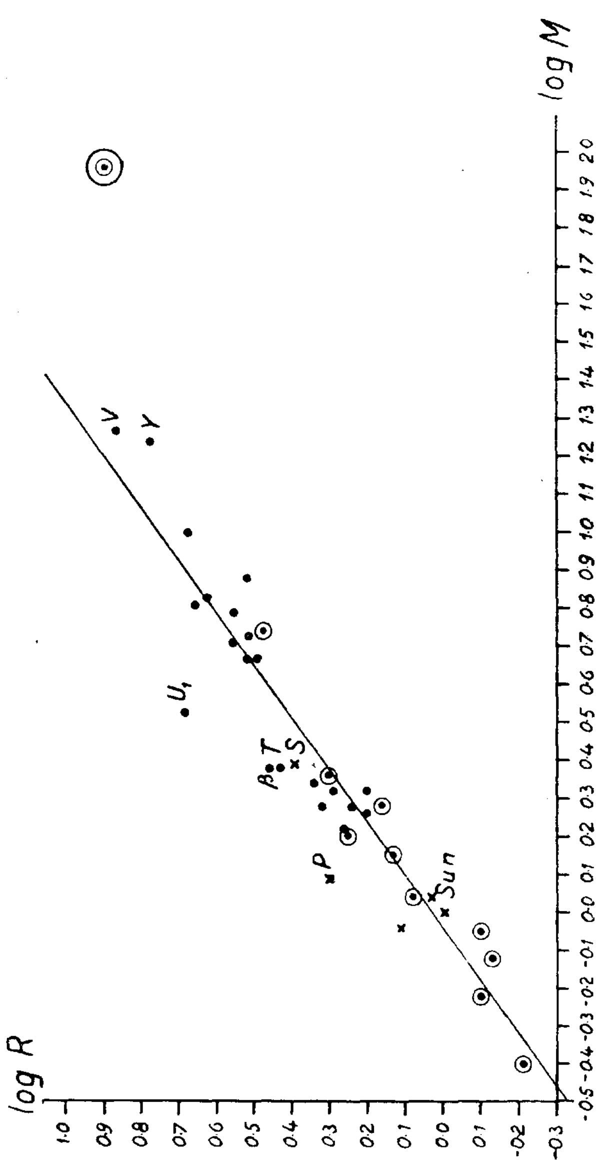

Fig. 3. Correlation of *log* radius and *log* mass for main sequence stars. The straight line is $\log R = 0.72 \log M + 0.03$; encircled dots = group mean values for visual binaries; crosses = individual stars of large parallax (Sun, Procyon (P), Sirius A (S), α Centauri A and B); dots = eclipsing binaries ($\beta = \beta$ Aurigae; T = TV Cas br; U_1 = u Her ft; V = V Puppis; Y = Y Cygni); double circle with dot = typical O star.

synthesis must be abandoned; for the main sequence, this reaction requires $s \sim 6.5$; assuming this, we should have, instead of (40), the correlation:

$$R \sim [1 - X) X]^{0.22} (\beta \mu)^{-0.11} M^{0.33} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(a)},$$

which cannot be made to satisfy the correlation of Fig. 3. An escape may be found in the assumption that the correlation of R and M in Fig. 3 is the combined effect of a law of energy generation with $s \sim 6.5$ and a progressive inflation of the radius with increasing mass; the cause of the inflation may be the formation of an exhausted core causing an expansion of the envelope, as considered in Sections 5 and 6. The sudden increase of log R at about the sun's mass (cf. Table 3 and Fig. 3) may perhaps be regarded as an indication of the starting of such a core, which would seem to be in agreement with estimates of the time ($\sim 3.10^9$) required for the exhaustion of a limited central region (cf. below). If a superdense core, it need not be large, a few per cent of the total mass, and probably cannot be very large (cf. the table in Section 7. *a*). Such a small core need not be in contradiction to the small effective degree of concentration of mass $\left(\frac{\varrho_c}{\varrho_m} \sim 16\right)$ derived by Russell[46] for Y Cygni, or by Walter[56] and Kopal[58] for eclipsing binaries from their ellipticity, and by Walter[57] from librations in β Lyrae and W Ursae Majoris; if most of the mass is little concentrated, the influence of the small core is imperceptible. The observed small apparent concentration of mass in the B type eclipsing binaries indicates for them either a complete mixing with complete adiabatic structure throughout, or practically the same with a small superdense nucleus at the centre. An effective polytropic index $n \sim 2$ would result in a good agreement with Russell's data for Y Cygni[46].

The interpretation of Fig. 3 as partly the result of "inflation" is not very attractive; it has been proposed to save the hypothesis of hydrogen — deuteron synthesis by setting $s = 6.5$, but the postulated superdense core becoming an independent source of energy makes this escape illusory: formula (28), as based on a purely subatomic source of energy, loses its meaning in such a case, and the regular correlation shown by Fig. 3 cannot bear the theoretical interpretation which we have given; in this case the value of s would probably have little influence upon the correlation of R and M, which should be chiefly determined by the properties of the core.

A few words with regard to Kopal's paper[58] as containing the most numerous data referring to the ellipticities of eclipsing binaries. It has been pointed out rightly[59] that Kopal's material is not homogeneous. Indeed, little definite meaning can apparently be attributed to the conception of the mean ellipticity of a binary with largely differing radii, masses, and luminosities. Nevertheless, when retaining only the homogeneous data in Kopal's list, the change of the calculated concentration of mass with spectral class remains the same as found by him from the entire material: the concentration increases for the later spectra. As the later eclipsing variables are chiefly giants, this fact seems to be in agreement with our conceptions of giant structure (formation of a superdense core and inflated envelope). Kopal's absolute values of the effective polytropic index seem to be systematically in error, as for the early type stars he finds persistently $n = 0$ to 0.5, which is less than the minimum adiabatic value ($n = 1.5$), and would lead to catastrophic convection; we have seen that convection in stellar interiors (except quite near a superdense core) is of such high efficiency in transporting heat that no perceptible deviation from adiabatic equilibrium can occur. Therefore Kopal's figures can be regarded only as of a more or less qualitative character. To get reliable absolute data for the ellipticities, the elements of the eclipsing variables should be rediscussed with such a special purpose in view; the limb darkening should be taken as variable according to effective temperature, as follows from Schwarzschild's theory of the radiative equilibrium of the outer layers of a star.

Our above result with respect to the probable value of the exponent $s = 19$ (15 to 25) forces us to attempt some revision of our former a priori views concerning the basic process of subatomic energy generation. The hydrogen — deuteron synthesis requires $s \sim 6.5$ only; further, it must have as small a probability per collision as $q \leqslant 1{,}3.10^{-19}$, to work reluctantly enough at the actual adiabatic (minimum!) temperatures of the main sequence stars, and it can never be detected in the laboratory; if the probability of the reaction is even smaller than that — the reaction loses its importance in the stellar energy budget (the reaction is theoretically possible, but the probability may be too small to exert a practical influence). From considerations put forward in Section 3.g, the $He^4 \rightarrow Li^6 \rightarrow Li^7 + H^1 \rightarrow He^4$ regenerative process cannot very well be considered as the starting point of the atomic synthesis (although it may occur as a lateral reaction), because it requires deuterons or neutrons which must be supplied by a reaction of higher order which is split necessarily into different branches and yields, therefore, too small a number of deuterons for $He^4 + H^2 \rightarrow Li^6$. $s = 15 - 25$ corresponds, according to the theory of atomic synthesis, to proton capture of a probability $q \sim 0.1$ to 0.01 (as observed in the laboratory) by a nucleus of charge $z \sim 4 - 8$, thus from beryllium to oxygen. The process must be regenerative in Atkinson's sense, if the original abundance of the starting phase is not very great. Existing physical experimental data (we cite from a compilation made by Fleischmann and Bothe[60]) seem favourable to this suggestion — indeed, there are a number of observed reactions which must occur with great intensity; stars in any case cannot settle down to higher central temperatures before the possibility of these reactions is exhausted.

Observed proton captures with a release of energy:

Reaction	$Li^6 + H^1 \rightarrow He^4 + He^3$ $Li^7 + H^1 \rightarrow 2\,He^4$	$Be^9 + H^1 \rightarrow B^{10}$ $Be^9 + H^1 \rightarrow Li^6 + He^4$
s for $T_\varepsilon = 1{,}4.10^7$ . . .	12.5	15.0
T_ε required for the sun .	$1{,}1.10^7$	$1{,}8.10^7$

Reaction	$B^{11} + H^1 \rightarrow 3\,He^4$ $B^{11} + H^1 \rightarrow He^4 + Be^8$	$C^{12} + H^1 \rightarrow N^{13}$ $N^{13} \rightarrow C^{13} + \beta +$	N^{14}, O^{16} . . .
s for $T_\varepsilon = 1{,}4.10^7$ . . .	17.2	19.6	. . .
T_ε required for the sun .	$> 2{,}4.10^7$	$2{,}1.10^7$	. . .

In the last line, the temperature $T_\varepsilon = 0.94\ T_c$ is given, for which the reaction alone is able to cover the radiation of the sun, on the assumption of a relative abundance as found by Russell[22] for the solar atmosphere. Of course, the abundance may be quite different in the interior; if carbon is 100 times more abundant in the interior, T_ε becomes $1{,}9.10^7$ for the 4th reaction, instead of $2{,}1.10^7$. The abundances having been assumed as for the above table, only the carbon reaction represents an important store of energy, lasting for about 5.10^8 years in the case of the sun; if this is insufficient even for the short time scale, it is important to realize that the $C^{12} + H^1$ reaction makes the atomic synthesis an observed reality, and not a mere speculation based on probabilities. The stars must have gone through this stage of atomic synthesis, at least. The central temperature of the sun for 37 per cent hydrogen content and adiabatic structure with $n = 1.63$ is $1{,}2.10^7$ (cf. Section 3.*d*), thus too low for the carbon, boron, and beryllium reactions; if these are to happen in the main sequence stars, the temperatures of the latter must be higher, which can be accounted for only by attributing to them a non-homogeneous structure (some degree of condensation towards the centre). It is disappointing to realize that at the temperatures of these reactions no exothermic processes of generation of neutrons or deuterons can occur, except the $H^1 + H^1 \rightarrow H^2$ reaction, and that without the help of deuterons and neutrons no experimental continuous chain of atomic synthesis can be traced; the above reactions stand thus at present isolated; neither are regenerative processes not leading to helium indicated.

With helium, and without the intervention of neutrons or deuterons, we must admit still higher temperatures; a small central core, due to slight exhaustion of hydrogen and complete exhaustion of the $C + H$ reaction (if no regenerative process exists) may be formed, with temperatures of $\sim 3{,}5.10^7$ which may render the following reaction important (cf. Atkinson):

$$_2He^4 + {}_2He^4 \rightarrow {}_4Be^8 + 0.36 \text{ Mev.}$$

It has been the belief that this reaction is endothermic, and that Be^8 is therefore unstable (cf. Kronig [61]); however, with the more accurate atomic weights (cf. Section 3.*g* and [27]) the result is different. The observed reaction $_4Be^9 + 1{,}3.10^6$ volts (γ radi.) $\rightarrow {}_4Be^8 + {}_0n^1$ gives $_4Be^8 = 8.0074$, against $2_2\,He^4 = 8.0078$: thus it appears that the nucleus of Be^8 is stable, after all, with a small binding energy of $360\,000 \pm$ volt. At $T \sim 3{,}5.10^7$, Be^8 leads to a rapid cycle of reactions (those not definitely observed are in brackets, and not observed nuclei in parentheses) (all exothermic):

$$[{}_2He^4 + {}_2He^4 \rightarrow {}_4Be^8];\ [{}_4Be^8 + {}_1H^1 \rightarrow ({}_5B^9);\ ({}_5B^9) \rightarrow {}_4Be^9 + \beta_+];$$

hence the principal branch follows:

$$_4Be^9 + {}_1H^1 \rightarrow {}_5B^{10};\ [{}_5B^{10} + {}_1H^1 \rightarrow {}_6C^{11};\ {}_6C^{11} \rightarrow {}_5B^{11} + \beta_+];$$

$${}_5B^{11} + {}_1H^1 \rightarrow 3_2\,He^4;\ {}_5B^{11} + {}_1H^1 \rightarrow {}_4Be^8 + {}_2He^4;$$

$$[{}_5B^{11} + {}_1H^1 \rightarrow {}_6C^{12}];\ {}_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13} \rightarrow {}_6C^{13} + \beta_+,\ \text{etc.};$$

latera branch:

$${}_4Be^9 + {}_1H^1 \rightarrow {}_3Li^6 + {}_2He^4;\ {}_3Li^6 + {}_1H^1 \rightarrow {}_2He^3 + {}_2He^4.$$

Thus, the cycle involves an efficient regeneration of helium (cf. Atkinson), and heavier elements are also formed through C^{12}.

If this mechanism does not work, at still higher temperatures observed reactions involving α-particles and resulting partly in neutron synthesis come into play, such as

$${}_4Be^9 + {}_2He^4 \rightarrow {}_6C^{12} + {}_0n^1 + 5.5 \text{ Mev (at } T > 10^8);$$

direc neutron synthesis, however, should also begin at this stage, and with neutrons the atomic synthesis is well provided for.

From all the preceding it is clear that the synthesis of heavier elements out of hydrogen must inevitably occur at a certain stage of the evolution of the star; observed reactions are able to absorb all the hydrogen content of a star when the central temperature is allowed to rise over 10^8. However, the apparent homogeneity of structure of the main sequence (correlation and Fig. 3) does not seem to favour such high temperatures and condensations; it is more likely that the important cycle of processes takes place at $T_c \sim 2.10^7$; on account of lack of experimental data we cannot decide yet upon this question. After all, it is not impossible that $H^1 + H^1 \rightarrow H^2$ is still the basic reaction, and that our correlation of R and M reflects chiefly the effect of a progressive inflation of the stellar envelope with increasing mass, so that $s \sim 6.5$ may still be valid.

Another escape able to remove some contradictions may be proposed. The direct reaction $H^1 + H^1$ may be prohibited for the protons in their ground states, but may become possible when one of the protons is excited to a certain level of H volts. The fraction of excited nuclei is $f = 10^{-\frac{5040\,H}{T}}$ (thermodynamic equilibrium); therefore, the rate of the reaction is equal to the rate computed from the absolute abundance of hydrogen multiplied by f. The apparent probability of capture, $q = 1{,}3.10^{-19}$, which we estimated in Section 3.g, is in this case equal to the product qf.

Thus, $q.10^{-\frac{5040H}{T}} = 1{,}3.10^{-19}$. With $q = 0.01$, $-\frac{5040\,H}{T} =$ $= -16.9$. This holds upon the assumption that the energy generated in the sun at $T_\varepsilon = 1{,}1.10^7$ equals its observed heat output. Thus, with $T = 1{,}1.10^7$, $H = \frac{16{,}9.1{,}1.10^7}{5040} = 36000$ volts. Such should be the hypothetical nuclear excitation potential of hydrogen to account for the energy generation of the sun (adiabatic model) and of the main sequence stars, and which is in harmony with the observed absolute abundance of lithium in the sun: the hypothetical rate of the deuteron synthesis in the sun is then in equilibrium with the rate of the Li + H reaction, calculated from experimental data.

The rate of the reaction as given by formula (3) must be multiplied by f in this case. For $T = 1{,}1.10^7$, the effective value of the exponent in the temperature dependence of ε is then $s = 44$, which is now too high for the trend of Fig. 3. With a smaller value of q, s may be reduced; thus, for $q = 10^{-8}$, $H = 24\,000$ v., $s = 32$; for $q = 10^{-16}$, $H = 6300$ v., $s = 13$. An agreement with the "observed" value ($s = 19$) may be obtained by a suitable choice of the constant q. Although the procedure is somewhat artificial, and the question quite problematic, the importance of the above speculations consists in demonstrating that an apparent disagreement between the "observed" and the theoretical values of s cannot be a reason for denying the possibility of the direct deuteron synthesis in stellar interiors. It might be worth while to attempt the synthesis in the laboratory, in a hydrogen medium "activated" by radiation of 10 000—50 000 volts (for the lower limit, the yield of the reaction may be too small to be detected in the laboratory if our calculations as to q are correct).

c. *Stellar statistics and stellar evolution.*

Under the above heading fifteen years ago the writer published an attempt to explain the observed frequency function of stellar luminosities on the basis of a recurrent cycle of stellar evolution, by assuming statistical equilibrium between the "rate of cooling" and the frequency of Nova catastrophes, which were supposed to bring the star back to the start of a new evolutional course with high initial luminosity and gradual "cooling". At present it stands without doubt that such a conception does not represent the state of our present stellar universe: the stars do not change much in luminosity (as once considered by Eddington, on the basis of a "Van der Waals degeneracy" of dwarf stars), except the white dwarfs; for an evolution of mass, the time scale is too short (studies of double stars by the writer[62], cf. also[2]). The Russell-Hertzsprung diagram is not a diagram of evolution, but a diagram of stellar structure.

Now, from the fact that the hydrogen content is smaller in giants than in main sequence stars, Strömgren[21] concludes that the course of evolution with practically constant mass is at right angles to the main sequence line of the diagram: main sequence stars change into giants. Our preceding theoretical discussion points to the same possibility. Below, a comparison with statistical data seems to impose certain restrictions upon this type of evolution also: some main sequence stars become giants; many, however, cannot.

d. Evolution of the sun and geologic temperatures.

Evolution with decreasing hydrogen content requires increasing luminosity, and in the case of the sun some vague information regarding such an evolution can be obtained from the geological history of the earth. The mean temperature of the earth must be chiefly determined by the intensity of solar radiation; unfortunately, local conditions during past ages obscure the general climatic picture too much; but certain conclusions can nevertheless be drawn. There seems to be no doubt that, during the Cambrian and Ordovician, the mean temperature was about the same as it is now; it was considerably warmer since the Silurian, up to the late Tertiary. The last Diluvial relapse of temperature, the ice age, which lasted with interruptions for about half a million years, is so short as compared even with the Tertiary (60 million years) that its influence upon the mean geological temperature is negligible; we may consider $+20^{\circ}$ C at present as the normal mean temperature of the earth, corresponding more or less to the conditions prevailing at the end of the Tertiary; the present actual temperature ($+15^{\circ}$) is still below the normal, not having as yet recovered from the relapse of the ice age. Allowing for the cooling effect of an ice + snow-covered area, an ice age with glaciation reaching to about 45° latitude (zero annual isotherm with sufficient snow in winter) should correspond to as high a mean temperature of the whole earth as + 9° C. There can be hardly any doubt that the last ice age, which was bipolar, was caused by a decrease in solar radiation (speculations upon the eccentricity of the earth's orbit cannot produce a bipolar effect, and the unipolar effect must be rather small even for the affected hemisphere), and probably most preceding ice ages were, too [there were mighty glaciations* in the Algonkian (South Australia; synchronous in South Africa and Canada), in the lower Cambrian (bipolar, Australia and Greenland), some glaciation in the Middle Ordovician, in the lower Devonian, and an enormous glaciation in the Permian (perhaps the late Carbon already), apparently restricted to the southern hemisphere, with traces left in Australia, Africa, East India, Brazil, and the Falkland islands; absence of Permian glaciation from the northern hemisphere may perhaps be explained by special circumstances of the distribution of continents and mountains]. Between the Permian and the Diluvian there are no ice ages known. Although Wegener's theory of continental drift, postulating a corresponding displacement in latitude, cannot explain the last ice age, and the amount of the drift required for such a purpose is not verified by astronomical observations, there can hardly be any doubt about the reality of large-scale horizontal displacements (Alpine foldings) in the earth's crust; these displacements cannot reach the scale of Wegener's theory, but, nevertheless, they summon to caution with respect to a generalization of local peculiarities of a geological climate: the latitude where at present a fossil is found may considerably differ from the latitude of its origin. With due allowance for all such circumstances, the general trend of temperature seems to be an increase, highly irregular, interrupted by sudden minima of short duration, the ice ages; the apparent absence of these from the Mezozoicum and Tertiary is perhaps due to the increase of the "normal" temperature of the earth (or of

* Professor A. Öpik, geologist, has checked upon these geological data.

the mean solar radiation), so that only rare exceptionally deep minima, such as the Diluvial, lead now to glaciation; whereas in the Palaeozoicum and Praecambrium, with their lower normal temperature, moderate and therefore more numerous minima of the solar radiation already produced glaciation; thus, the relatively greater frequency and extent of glaciation in these early periods seems to be in agreement with the gradual warming up of the earth following the increase of solar radiation with the gradual exhaustion of hydrogen. Judging from the oldest Archaic rocks, of an age $\sim 2.10^9$ years, in spite of changes from metamorphism, it seems to be certain that a permanent ice age could not have taken place even at this early age: the traces of ice in the Archaicum are scarce, probably mostly destroyed, but still suggestive of intermittent ice ages, as observed in later ages *. A minimum estimate for the mean "normal" temperature of the earth.

Table 4.

Luminosity of the Sun (Adiabatic Model, Complete Mixing), and Terrestrial Temperatures.

Hydrogen content, $X\%$	40	39	38	37,5	. . .	. . .	. . .	. . .
Age, 10^8 years .	−31.5	−20.2	−9.7	−4.8 (Ordovician)	−3.2 (Silur.)	−1.2 (Jurassic)	−0.6 (Palaeocene)	−0.2 (Miocene)
$m_{bol.}\odot$, computed	4.84	4.75	4.67	4.62	. . .	. . .	. . .	. . .
t^0 C Earth, computed	$+3^0$	$+9^0$	$+14^0$	$+17^0$	$+18^0$	$+19^0$	$+20^0$	$+20^0$
t^0 C Earth, geological estimate	. . .	$\geqslant +9^0$	$\geqslant +9^0$	$+15^0$:	$+22^0$	$+17^0$	$+25^0$	$+20^0$

Hydrogen content, $X\%$	. . .	37,0	37	36	30	24	18	12	6	0
Age, 10^8 years .	−0.01 (Diluv.)	0.00	0	+9.0	+49.0	+74.0	+87.2	+92.8	+95.2	+96.0
$m_{bol.}\odot$, computed	. . .	4.58	4.58	4.49	3.93	3.28	2.53	1.58	0.58	−0.47
t^0 C Earth, computed	$+20^0$	$+20^0$	$+20^0$	$+26^0$	$+62^0$	$+117^0$	$+191^0$	$+305^0$	$+458^0$	$+655^0$
t^0 C Earth, geological estimate	$+8^0$	$+15^0$	—	—	—	—	—	—	—	—

* Here a most important problem for Archaean geology presents itself.

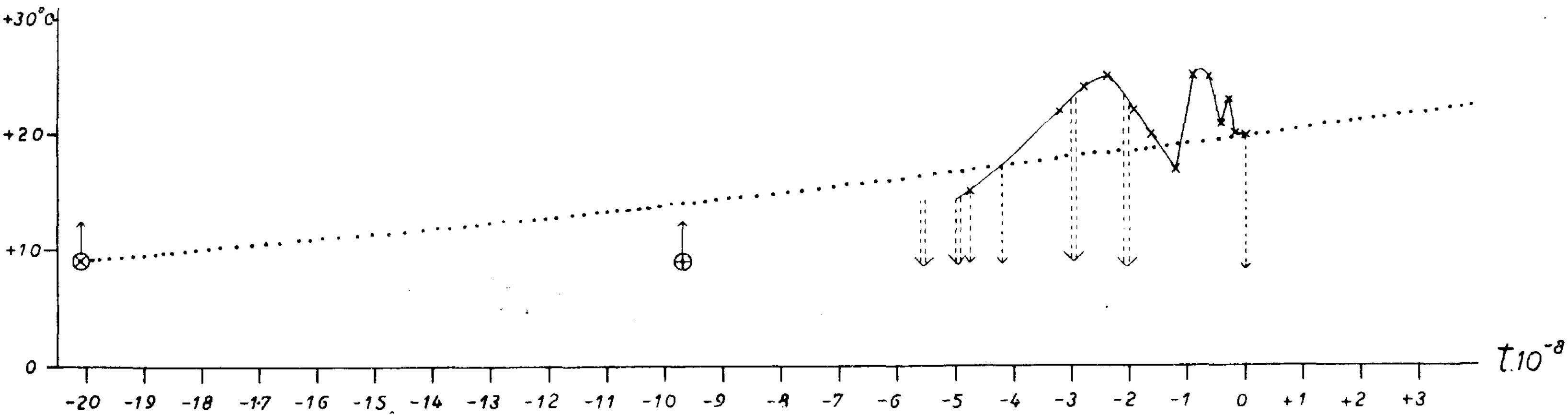

Fig. 4. Mean temperature of the earth (ordinates), and geologic time (abscissae, unit = 100 million years). Crosses and full curve = estimated mean temperature in the past; broken arrows directed vertically downwards = ice ages; crossed circles with arrows directed upwards = estimated minimum mean temperature during the Archaean. Dotted line = theoretical "normal" temperature of the earth, for complete mixing and atomic synthesis in the sun.

2.10^9 years ago may thus be $\geqslant + 9^0$ C. Above, the table gives the theoretical change of the luminosity of the sun on a hydrogen synthesis basis with complete mixing, with the computed and "observed" mean terrestrial temperatures. The present hydrogen content is assumed to be 37 per cent, conventionally, and the present "normal" absolute magnitude (4.58) by 0.07 brighter than the present observed one (4.65). The data for the past and the "near" future are represented in Fig. 4; the theory is not contradicted by the observational evidence; the latter hardly admits of a more rapid increase in luminosity than the computed one, but the actual increase cannot be determined with certainty. Thus, on the basis of the complete adiabatic model the sun must have started $\sim 3.10^9$ years ago with a hydrogen content $X = 40$ per cent, and has used up to present 3 per cent of it; the accelerated rate of evolution on the atomic synthesis basis will come to an end after $9{,}6.10^9$ years, but life on earth will be destroyed by heat long before that. The total life of the sun on the hydrogen basis is $12{,}6.10^9$ years, after which a collapse follows towards the white dwarf stage (when the temperature of the earth may fall to $- 180^0$ C).

From the table we also infer that a solar mass in complete adiabatic equilibrium containing originally less than 26 per cent hydrogen must already have become, within 3.10^9 years, a white dwarf (Sirius B? cf. Section 6. *i*).

The recurrence of the ice ages — i. e., of short minima of solar radiation — seems to be rather difficult to explain on the completely adiabatic model of the sun, with its complete mixing. A composite adiabatic-radiative model, with its changing extent of the adiabatic region, progressive stratification at the boundary of the convective core as the central exhaustion increases, and the formation of a small collapsing kernel, is more likely to lead to fluctuations of luminosity. The answer can probably be obtained from numerical computations. Qualitatively, however, it seems likely that the collapsing kernel ($\sim$ 7 per cent of the mass, cf. Subsection *a*, thus small enough to become exhausted during $\sim 10^9$ years), passing through different stages of degeneracy, and gradually acquiring mass from exhausted material of the outer shell, may produce an uneven variation in the luminosity. For example, let us suppose that the adaptation to the changing condition of equilibrium requires an intermittent expansion of the outer shell; during the expansion, a fraction of the heat output is stored in the form of potential energy of gravitation, and the radiation into space is temporarily reduced by a corresponding amount. To explain in this way an ice age of one million years' duration with a depression of the terrestrial temperature by 11^0 C, a total expansion (of the shell containing $\sim$ 90 per cent of the solar mass) of less than one per cent of the radius is required for an unchanged internal heat output. The plausibility of such an explanation adds some probability in favour of a complex model of the sun.

In the case of a superdense core, however, the luminosity cannot be calculated in such a simple manner as was done above; the degenerate core will not add much to the heat output, thus the atomic synthesis will be still the main source of energy; the luminosity, however, will not rise much with exhaustion, because the exhausted material joins the degenerate core. A longer age on the atomic synthesis basis must result.

e. *Duration of evolution for adiabatic models.*

Here we cannot as yet decide the question of the probable structure of the sun and the main sequence stars. As a standard of comparison for probable ages we below consider the complete adiabatic model for main sequence stars, with an initial hydrogen content of 40 per cent as found for the sun (cf. Table 4, $X = 40\,^0/_0$ at $t = -3{,}15.10^9$ years).

Allowing for the unknown cause of a depressed luminosity in massive stars (which Strömgren [21] ascribed to a large hydrogen content, and which we as yet hesitate to accept, for reasons given above), we take an empirical mass-luminosity relation, or assume luminosities of actual typical stars as corresponding conventionally to $X = 33\frac{1}{3}$ per cent, and use the hydrogen content only for a differential correction of the luminosity in Table 5. We need not bother about the rigour of these assumptions, as they are needed only for our order-of-magnitude comparison. Our table in any case corresponds

Table 5.

Duration of Atomic Synthesis Stage, Complete Mixing.

$M/M_{\odot}$		1.0	1.5	2.0	2.5	3.0	4.0
$m_{bol.}$	begin., m_0	4.8	3.5	2.2	1.2	0.4	— 0.6
	mean, $\overline{m}$	4.0	2.7	1.4	0.4	— 0.4	— 1.4
	end, m_1	— 0.5	— 1.5	— 2.4	— 2.9	— 3.3	— 3.8
Relative duration		1.000	0.479	0.208	0.115	0.0730	0.0437
Duration, years		$1,3.10^{10}$	$6,2.10^{9}$	$2,7.10^{9}$	$1,5.10^{9}$	$9,5.10^{8}$	$5,7.10^{8}$
$M/M_{\odot}$		5.0	7.0	10 *	17 **	36 ***	90 ****
$m_{bol.}$	begin., m_0	— 1.4	— 2.3	— 3.3	— 5.1	— 8.4	— 6.0
	mean, $\overline{m}$	— 2.1	— 2.9	— 3.8	— 5.5	— 8.7	— 6.0
	end, m_1	— 4.1	— 4.4	— 4.9	— 6.1	— 9.1	— 6.0
Relative duration		0.0301	0.0219	0.0148	0.0060	0.0007	(0.021 :)
Duration, years		$3,9.10^{8}$	$2,8.10^{8}$	$1,9.10^{8}$	$7,8.10^{7}$	$9,1.10^{6}$	$2,7.10^{8}$

* u Her. br.; ** Y Cygni; *** *H. D.* 1337 br.; **** Trumpler's typical O type star; no effect of hydrogen content assumed.

to the actual luminosities of the stars. The relative duration is computed with sufficient precision from the formula

$$t_{rel.} = \frac{5.34}{(m_0 - m_1)} \frac{M}{M_{\odot}} \cdot 2.512^{-(4.8 - m_0)} \left[1 - 2.512^{(m_0 - m_1)} \right],$$

and the mean bolometric magnitude as in Subsection *a* from

$$\overline{m} = m_0 + \frac{1}{t} \int (m - m_0)\, dt = m_0 - 1.085 + \frac{m_0 - m_1}{2.512^{m_0 - m_1} - 1} + \varDelta,$$

where $\varDelta$ is a small correction $= 0.2 - 0.0$ mag, and where m_0 and m_1 are the initial and the final bolometric magnitudes; the first formula implies a linear change of the magnitude with X (hydrogen content), which is not quite correct, but sufficient for the relative duration; the absolute duration for $M = 1$, however, is taken from Table 4 where it was computed step by step, without simplifying assumptions. The most interesting feature of the table is the still considerable relative duration for massive stars: the smaller increase of luminosity with exhaustion partly balances here the greater initial speed of evolution. Nevertheless, for masses exceeding 2.0, there seems to be a shortage of the subatomic energy source even with the short time scale. There exists, of course, a possibility of increasing the figures: allowing for uncertainties in the internal structure, the initial hydrogen content may be increased to about 50 per cent, which lengthens the duration of the "dwarf" stage by 25 per cent in proportion to the store of hydrogen (the theoretically fainter luminosity for the increased hydrogen content does not lengthen our time scale, because we started from the observed luminosity of the sun, and computed the rest in a differential way). On the other hand, for incomplete mixing (composite model), only a fraction of the mass is involved in atomic synthesis, so that the duration of the dwarf stage may be shorter. The composite model may be supposed to enter the giant stage after the exhaustion of perhaps 0.25 of the internal mass, thus after one-quarter of the time interval of Table 5. With 3.10^9 years as the maximum age, the minimum mass of a giant equals in this case $1.2\odot$, which more or less corresponds to the observed limit (RT Lacertae, 1.0 and $1.9\odot$; ZHer ft, $1.3\odot$; etc.).

f. Semi-giants.

It is tempting to identify the Mt. Wilson *s* (sharp) and *n* (nebulous) classification of B and A type stars [58] as corresponding to our two subdivisions of stars without collapsed cores: *s*, about 0.8 mag brighter than *n* (for the same spectrum), with the "sharp" lines corresponding to a smaller surface gravity, may be identified with the "semi-giant" stage, and *n* with the dwarf stage. From certain complicated considerations here omitted we estimate the difference of luminosity for equal mass, at $0.8 \times \frac{3}{2} = 1.2$ mag. From Table 5 it appears that the duration of the non-collapsed stage at $M \sim 2.5\odot$ (A stars) is certainly shorter than 3.10^9 years for the composite model ($\sim$ 0.25 of the tabulated durations); as non-collapsed stars are still observed, we must suppose that these stars are continually born in the place of those which have become giants; in such a case, the relative number of the *n* and *s* stars must be proportional to their relative lives, or inversely proportional to their luminosities, or 3 : 1 for $\Delta m = 1.2$, as found above. For northern stars brighter than the fifth magnitude (complete selection) we find indeed 69 stars B8*n* — A3*n*, of a mean absolute magnitude 1.0, against 24 stars B8*s* —

A3s, $\overline{m}_{abs.} = 0.5$ *; for equal volumes of space, the number of s stars must be counted to $m = 4.5$ and is 17; this requires a ratio of the relative lives about $n:s = 4:1$, or slightly greater than the theoretical ratio (3:1). The agreement in relative number and luminosity is hopeful indeed. If A and B stars can be identified as non-collapsed, i. e., as dwarfs or semi-giants, their presence in the Galaxy, when confronted with the figures of Table 5, and allowance made for the most generous increase of these figures, would require a steady supply of such stars, to replace those entering the giant stage. The production of new stars (not Novae) cannot, of course, be limited only to these classes, but must be in this case a more general phenomenon.

The Mount Wilson s and n subdivisions may be looked upon also as different stages of exhaustion of the adiabatic model; from formula (40′), for $M = \text{const.}$, $R \sim [(1 - X)X]^{0.10}(1 - \beta)^{0.14}$, we find for $M = 2.5$, and an interval of luminosity $\Delta m = 1.5$ mag, $X_1 = 0.40$, $X_2 = 0.21$, $\mu_1 = 0.92$, $1 - \beta_1 = 0.0145$, $\mu_2 = 1.26$, $1 - \beta_2 = 0.042$, and $\log \frac{R_1}{R_2} = -0.049$. This gives an effective relation $L \sim R^{12}$, thus a slow increase of radius with luminosity (for X near 0 the radius begins to decrease again). In such a case, the difference of luminosity between $n - s$ is 0.8, practically the same for equal mass, as it is for equal spectrum, and the relative number of $n:s$ to be expected is 2:1, thus a greater deviation from the observed ratio (4:1); the difference, however, is not serious. More serious is the question of surface gravity which for the adiabatic model varies as $g \sim L^{-0.16}$, and requires a ratio $g_n:g_s = 1.12$ only, too small to influence sensibly the appearance of spectral lines; whereas for the "semi-giant" theory of the As stars the ratio of surface gravity we estimate at $g_n:g_s = 2.3$, which may be considered as sufficient **. It is, therefore, possible that the Mount Wilson As stars are "semi-giants", on the point of becoming giants. The An stars may thus be supposed to

* The difference $s - n = 0.5$ mag is smaller than the difference for a given spectrum (0.8 mag), because of a preponderance of early n subclasses and late s subclasses in our sample.

** Computations referring to the semi-giant model will be given in another paper.

contain: all complete adiabatic models, and the younger composite models; whereas As contain only the advanced composite models; in such a case the relative number $n:s$ should be greater than the expected, 3:1, as actually is the case (4:1).

g. Statistical equilibrium of evolution.

Let us consider now the possibility of an evolution from the main sequence towards giants with a continuous supply of main sequence stars. Taking 3.10^9 as the age of our universe, and assuming a uniform rate of the generation of new stars (not Novae) — the order of magnitude may be right although more stars may be supposed to be generated at the beginning — further assuming that the dwarfs after the lapse of a time interval t_d become giants, — the relative number of giants and dwarfs (including semi-giants) of a given mass will be proportional to

$$\frac{n_g}{n_d}=\frac{3.10^9-t_d}{t_d} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (41),$$

where t_d is the total duration of the non-collapsed stage; the formula holds for $t_d < 3.10^9$ only, and for the case of an infinite (sufficiently long) life of the giants; for $t_d > 3.10^9$, $n_g = 0$.

If, however, giants are also doomed to a short life, t_g, (41) holds for $t_g > 3.10^9 - t_d$, whereas for $t_g < 3.10^9 - t_d$ we have

$$\frac{n_g}{n_d}=\frac{t_g}{t_d} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (42).$$

The hypothetical "disappearance" of a giant, at the end of its life, may consist in it becoming a Wolf-Rayet star (similar to a nucleus of a planetary nebula) of high effective temperature and density; the difference of visual *minus* bolometric magnitude for high temperatures runs as follows:

Table 6.

Visual *minus* Bolometric Magnitude.

T_e	3.10^4	6.10^4	10^5	2.10^5	3.10^5	5.10^5	10^6
$m_v - m$	+2.3	4.3	5 6	7.9	9.3	10.9	13.2

Thus, if the surface temperature of the former giant attains 10^5 (such a temperature has been actually observed for nuclei of planetary nebulae, cf. [39]), and if its bolometric magnitude remains unaltered, it will appear fainter by about five magnitudes, so that less than one per cent of the former giants can be observed. Still higher effective temperatures may occur; for $T_e > 10^6$ this would mean complete disappearance of all former giants from our catalogues (such high temperatures have not yet been observed, but perhaps for the same reason — they cannot be observed; the high temperature stars are practically invisible).

Table 7 contains data which may be used as a crucial test for the question of the persistence of the giants. The first half of the table refers to the distribution in our Galaxy, based on the data derived by the writer and his collaborators [40] from proper motions of the Boss catalogue; the original figures, referring to a fixed limiting apparent magnitude, are reduced to equal volumes of space with the aid of Kapteyn's mean density function. The second half of the table represents the mean distribution for three globular clusters, derived from Shapley's data [41]. The "main sequence" is supposed to include the dwarfs and the semi-giants. Actually, the giants are distinguished by their mean density, and there is a possibility for the main sequence to contain collapsed stars with a giant structure, but of high mean density. The line of demarkation is not very definite, but this introduces little uncertainty into the data for the Galaxy, because stars with transition spectra are few in number there. For the globular cluster, the stars near $m_{bol.} = 0$ are most numerous for transition spectra, and the relative numbers depend more upon the choice of the line of demarkation between giants and main sequence stars.

On the assumption that all main sequence stars are transformed into giants within time intervals comparable to about one-quarter of those of Table 5, the distribution "around the sun" in Table 7 can be satisfied only by assuming comparatively short lives for the giants; a trial-and-error solution gave a rough representation, according to formulae (41) and (42), of the observed frequency of giants, with the following assumptions: $t_d = t_g = 0.3$ times the total ages of Table 5; $\bar{m}_d = \bar{m}_g$ *.

* The approximate equality of the average luminosity of a giant and a

Table 7.

$m_{\text{bolom.}}$	Distribution of Bolometric Absolute Magnitudes							
	Around the sun, $D < 125$ parsec				Globular cluster (Shapley)			
	Main sequence		Giants		Main sequence		Giants	
	Spectrum	Number	Spectrum	Number	Colour class	Number	Colour class	Number
−8	O	0.018	B0 — M	0.005	. . . .	0	. . . .	0
−7	O	0.045	B0 — M	0.047	. . . .	0	. . . .	0
−6	O	0.10	B0 — M	0.11	. . . .	0	. . . .	0
−5	O — B2	2.6	B3 — M	0.75	. . . .	0	b5 — m	0.7
−4	O — B5	2.5	B8 — M	2.5	. . . .	0	„	12.7
−3	O — B5	18	B8 — M	23	. . . .	0	„	18.0
−2	B0 — A0	105	A2 — M	51	$\leqq$ a0	0.3	a0 — m	46
−1	B0 — A5	122	F0 — M	420	$\leqq$ f0	8.1	f0 — m	109
0	B3 — A5	460	F0 — M	910	$\leqq$ f0	143	„	174
+1	A0 — F2	1380	F5 — M	890	$\leqq$ f0	60	„	155
+2	A0 — G0	4400	G5 — K5	2400	. . . .	. . .	. . . .	. . .
+3	A3 — G5	34000	K0 — K5	4300	. . . .	. . .	. . . .	. . .
+4	F5 — G5	8000	. . . .	0	. . . .	. . .	. . . .	. . .

The same assumptions, however, do not in the least fit the distribution in the globular clusters; unless all these are less than 6.10^7 years old (thus less than the time of one revolution or oscillation in the Galactic system), the existence there of red giants, unmatched by main sequence stars, can be explained only upon the assumption of a great persistence of the giant stars, $t_g > 50\, t_d$. The discrepancy can be removed only by rejecting our first assumption: we conclude, that not all main sequence stars can be transformed into giants; this conclusion agrees with our theoretical considerations (Sections 5 and 6: complete adiabatic structure).

The absence of bright main sequence stars in globular clusters may be explained by assuming that no "new" stars are born in the cluster, all its stars being of equal age; in such a case the more massive stars have either become giants, or collapsed into superdense Wolf-Rayet stars which have become

main sequence star of the same mass is an observed fact, cf. ζ Aurigae and Y Cygni above; the smaller hydrogen content of the giants is partly a computational result counterbalancing Eddington's correction $-2 \log T_e$.

invisible on account of their high effective temperature. For an age of 3.10^9 years, Table 5 sets the limit of collapse for the adiabatic model at $M/M_\odot = 1.9$ (thus above Chandrasekhar's limit of degeneracy for zero hydrogen content), initial magnitude $m_0 = 2.3$; the bolometric magnitude at the end of the main sequence stage, at complete exhaustion, then is: $m_1 = -2.3$ (cf. Table 5). Shapley's data according to Table 7 indicate a limit at -2.0; a better agreement one could hardly expect. Assuming a monotonous distribution of the initial magnitudes, the theoretical distribution of the final magnitudes of main sequence stars in a globular cluster is found to be as follows (cf. Tables 5 and 4; the magnitude excess $m - m_0$ as a function of relative age is computed with the aid of the latter table, with a reduction of the difference in a proportion of $\frac{m_1 - m_0}{5.3}$; only the adiabatic model, i. e., the durations of Table 5 are assumed, as only this model determines the "top" of the residual frequency of luminosities in the clusters, the composite model main sequence stars having changed into giants much earlier):

Frequency of Main Sequence Magnitudes (bol.) in Globular Clusters, and Final Magnitudes.

Initial mag., m_0 . .	3.8	2.8	2.7	2.6	2.5	2.4	2.35	2.325	2.31	2.30
Assumed frequency		. . .	51	46	42	38	19	10	6.4	3.6
Duration of main sequ. stage, $t_0 =$	$7,1.10^9$	$3,8.10^9$	...	...	...	...	...	...	...	$3,0.10^9$
Relative age, $(3.10^9)/t_0 =$ . .	0.42	0.79	0.84	0.88	0.92	0.96	0.98	0.990	0.996	1.000
Final (present) hydrogen content, X %	34	25	23	21	19	14	10	7	4	0
Final (present) mag., $m =$ . .	3.3	1.5	1.2	0.9	0.5	−0.2	−0.75	−1.28	−1.69	−2.3

The resulting distribution, arranged according to the magnitude limits of Table 7, is:

Bolom. mag. m, limits . . .	1.5 ... 0.5	0.5 ... −0.5	−0.5 ... −1.5	−1.5 ... −2.5	< -2.5
Number, computed	139	48	22	6.6	0
Number, observed . . .	60	143	8	0.3	0

The disagreement between the computed and observed distributions lies chiefly in the observed excess about zero magnitude which is near the median magnitude of the cluster type variables (cf. Shapley[41]); if this maximum is smoothed out, the agreement between observed and computed distributions may become complete; the maximum, and perhaps the cluster type variability, may be intimately connected with some peculiarity in stellar structure where the exhaustion of hydrogen has attained a certain degree (about 15 per cent, as follows from the table).

Thus, the presence of massive and luminous main sequence stars in the Galactic System in general we choose to ascribe to stars being continually formed in the place of those which become giants, or which collapse. The distribution of the ages of meteorites (probably mostly interstellar) as determined by Paneth[64] (cf. also[2]) from the helium-radium ratio is suggestive of a continuous condensation of diffuse matter at a uniform rate (no matter whether the meteorites are products of direct condensation, or fragments of larger bodies):

Age, millions of years	0—500	500—1000	1000—1500	1500—2000	2000—2500
Number of meteorites	4	4	6	3	4

Age, millions of years	2500—3000	> 3000	—	—
Number of meteorites	3	0	—	—

Thus, the assumption that stars in the Galaxy are also born at a uniform rate does not seem quite arbitrary; in any case, there seems to be enough diffuse matter still left for such a purpose. Our other assumptions, based upon all the preceding theoretical and observational evidence, are: that all stars start as main sequence objects with a conventionally constant hydrogen content of 40 per cent; that a fraction a of them are adiabatic models, to which the figures of Table 5 apply, and which disappear observationally after the exhaustion of hydrogen and the following collapse (Wolf-Rayet stars for $M > 1.6 \odot$, cf. Table 6; white dwarfs for $M < 1.6 \odot$); that a fraction $1 - a$ of them are compound adiabatic-radiative models, which after an intermediate semi-giant stage of shorter duration (cf. above) become giants after the lapse of 0.3 the durations of Table 5, with a mean luminosity equal to $\bar{m}$ of that table (this is an empirical fact, although the decimal of the mean magnitude is not warranted); that the giant stage for any giant lasts for more than 3.10^9 years (cf. globular clusters).

Upon these assumptions, and with $3{,}0.10^9$ years as the maximum age, the frequency table for the Galactic System (Table 7, 1-st half) is analysed below. Unlike the globular clusters, where all stars are of the same age, all ages are represented in the Galactic System, and it is therefore permissible to use the average magnitudes ($\overline{m}$ of table 5) as representative of the average masses. The statistical equilibrium number n_1 of compound main sequence stars is computed from the observed number of giants by formula (41) ($n_d = n_1$, $t_d = 0.3\ t$); the number n_2 of existing adiabatic models is $n_2 = n_{dw} - n_1$, where n_{dw} is the total number of observed main sequence stars; the number of collapsed (invisible, in any case not counted in Table 5) adiabatic models n_c is given by the same formula (41), with $t_d = t$ (duration from Table 5), and with n_c for n_g, n_2 for n_d. The total number of adiabatic models which have come into existence since the creation of the Galactic System is $N_a = n_2 + n_c$; the corresponding number of composite models is $N_r = n_1 + n_g$. The relative frequency of composite models born is

$$1 - a = \frac{N_r}{N_a + N_r},$$ whereas the relative frequency of composite models among observed main sequence stars is $q = \frac{n_1}{n_{dw}}$. On these lines, Table 8 has been computed.

The most remarkable feature of this table is, for $\overline{m} \geqslant -1.0$, the steady behaviour of $1 - a$ (last line) fluctuating around 0.5, and the sudden drop for greater luminosities. If our interpretation is correct, this means that masses below $4.3\odot$ have an equal chance to become adiabatic, or composite; whereas for larger masses the probability to become composite is small, about 0.06; the considerable frequency of luminous giants is, from this standpoint, explained by their longer life; all, of course, depends upon our assumptions.

As to the residual small fraction $(1 - a)$ of composite models for $M > 4.3\,M\odot$, it may be due to an original difference in composition between the central region and the rest of the star (cf. Section 5. *h*, and below).

Table 8.

Analysis of the Frequency of Main Sequence and Giant Stars in the Neighbourhood of the Sun.

$D < 125$ parsec. $1-a$ = fraction of compound radiative-adiabatic models born (a = fraction of adiabatic models).

m_{bol} ($= m_{observed}$)	4.0	3.0	2.0	1.0	0.0	—1.0
$M/M_{\odot}$	1.0	1.4	1.8	2.2	2.8	3.6
t, years	$1{,}3.10^{10}$	$7{,}1.10^{9}$	$3{,}8.10^{9}$	$2{,}2.10^{9}$	$1{,}2.10^{9}$	$7{,}4.10^{8}$
n_g	0	4300	2400	890	910	420
n_1	(< 8000)	12600	1400	250	120	33
n_{dw}	8000	34000	4400	1380	460	122
$q = n_1/n_{dw}$	. . .	0.37	0.32	0.18	0.26	0.27
n_2	< 8000	21400	3000	1130	340	89
n_c	0	0	0	410	510	270
N_a	< 8000	21400	3000	1540	850	359
N_r	< 8000	16900	3500	1140	1030	453
$1-a$	. . .	0.44	0.54	0.42	0.55	0.56

$m_{bol.}$ ($= m_{observed}$)	—2.0	—3.0	—4.0	—5.0	$\leqq -6.0$
$M/M_{\odot}$	4.9	7.3	10.8	14.9	25:
t, years	$4{,}3.10^{8}$	$2{,}8.10^{8}$	$1{,}7.10^{8}$	$9{,}7.10^{7}$	$5{,}6.10^{7}$
n_g	51	23	2.5	0.75	0.16
n_1	2.3	0.66	0.04	0.008	0.001
n_{dw}	105	18	2.5	2.6	0.16
$q = n_1/n_{dw}$	0.022	0.037	0.016	0.003	0.006
n_2	103	17	2.5	2.6	0.16
n_c	650	160	40	75	8
N_a	753	177	42	78	8.2
N_r	53	24	2.5	0.8	0.16
$1-a$	0.066	0.12	0.056	0.010	0.020

With respect to the Mount Wilson n and s subdivisions of A — B stars, Table 8 opens up a different prospect; for the typical A star, the table suggests a ratio of $n_2 : n_1 \sim 3:1$, or approximately equal to the observed ratio of $n:s$ (cf. above); in such a case it may seem that the s subdivision comprises the entire composite model class, whereas n corresponds to the adiabatic model. It is, however, not advisable to go more deeply into these details.

Our general results seem to be in favour of our original assumptions made in connection with Table 8; it seems that these assumptions may be used as a working hypothesis in future research. As to the correlation of R and M found in Subsection b, in view of the apparently homogeneous material for the most important range of the correlation (for $M < 4.3 \odot$, constant small fraction $\bar{q} = 0.28$ of composite models in Table 8), corresponding chiefly to the typical adiabatic model, the conclusions drawn with respect to the law of energy generation ($\bar{s} = \sim 19$) may be considered corroborated: for the adiabatic model a considerable progressive inflation of the radius with mass does not seem possible. The chances for the reaction $H^1 + H^1 \rightarrow H^2$, with the protons in their ground states governing the atomic synthesis are thus rather low.

h. Probable structure of the sun and the main sequence stars.

With respect to the structure of the sun and the main sequence stars somewhat more definite statements can now be made. From Table 8 it appears probable that of the main sequence stars born about 50 per cent remain adiabatic models, the other half being transformed into composite models. Now, a composite model for a given mass and radius must possess a higher central temperature than the adiabatic model (cf. Table 2: the complex model must have a central temperature between the temperatures corresponding to $n = \frac{1}{\gamma - 1} \sim 1.7 - 2$, and $n = 3$); on the other hand, for a given luminosity (constant, or approximately constant) the subatomic energy sources require an almost constant central temperature; the composite model adjusts itself, therefore, from the beginning, to a radius by 30—40 per cent larger than the radius of an adiabatic model of the same mass, and progressive inflation with the gradual exhaustion of the core increases the difference (semi-giant stage). In fig. 3 the radius of the sun is 7 per cent less than the average, thus the sun is apparently an adiabatic model, and Table 4 probably refers to the actual sun; the ice ages may perhaps be explained by assuming a perturbing effect of a very small central core which is too small to disturb the general adiabatic equilibrium. A number of individuals in Fig. 3 may be classified as composite models:

Star	Procyon	Sirius A	β Aurigae av.	TV Cas. br.
Mass	1.2	2.4	2.4	2.4
Excess of radius, per cent	66	23	44	38

α Centauri B seems to belong to the same class, but this result depends upon estimated colour temperature and is therefore uncertain. The radius of Procyon agrees more or less with Strömgren's figure, whereas Sirius A differs on account of the discrepancy between the measured and Strömgren's mean adopted temperatures.

i. Close binaries and expanding envelopes.

If a main sequence component of a binary is going to become a giant, its expanding envelope may touch and enclose the companion; the event must lead to catastrophic consequences (even when the period of rotation has time to settle itself equal to the period of revolution), perhaps to some kind of a Nova phenomenon. The frequency cannot be great: about one-tenth of the total number of giants in the Galaxy per 3.10^9 years, or about one in 100 years. Distant companions of large eccentricity may be also "swallowed" by the expanding giant, in which case the effect may be especially violent (revolution and rotation cannot be equalized). Perhaps Supernovae may be explained in such a manner (atomic synthesis explosion stimulated by the collision).

k. Coexistent faint giants.

Thus, main sequence stars may become giants, but need not. The considerable number of eclipsing binaries with an early primary, and a considerably fainter typical giant secondary, in spite of the strongly favoured selection of such eclipsing systems, is still an argument against the supposed evolution of the more massive main sequence stars either into giants or into collapsed nuclei: during the time when the fainter component has become a giant, the brighter must have long ago finished its course of evolution. The argument is, however, a weak one: from among visual binaries, from which selection of this kind is absent, the main sequence — giant pairs are also practically absent (the rule being: primary = giant; secondary = main sequence); the eclipsing binaries may represent exceptional cases. Perhaps for these the giant structure of the companion has developed at an early date of the star's life, the giant structure being due to the formation of a core with little hydrogen content at the very beginning, as considered below.

l. Central core of meteoric material.

A giant structure, i. e., an extended outer shell, and with an excessive concentration of mass towards the centre, can be produced by an increase of the mean molecular weight towards the centre (cf.[1], p. 130), and by a peculiar distribution of the energy sources, produced by a central exhausted core (Sections 5 and 6); both conditions are attained early by making hydrogen originally less abundant near the centre than in the outer regions of a star. Let us inquire into factors other than exhaustion which might have led to a non-uniform distribution of hydrogen inside the star.

The diffusion of electrons, with the electrostatic field forcing protons to follow (cf. Rosseland,[43]), can hardly be made responsible for the differentiation, simply because of rotational mixing alone; at an early stage of the contracting nebulosity, when the rotation must have been slow (conservation of angular momentum), the differentiation might have taken place, but the time intervals in the most favourable case of feebly ionized matter (largest $\bar{\mu}$ and temperature, large free path of the electrons undeflected by ionic fields) are still too large; we find the time of relaxation (cf.[1], p. 277 f.; $\bar{\mu}=20$): $t \sim 3.10^{11}\sqrt{\frac{M}{R}}$, thus decreasing with increasing radius; for $M = 10^{34}$ gr ($5\odot$), $R = 10^{18}$ cm (one light year, $T_c = 100^0$ only), $t \sim 3.10^{19}$ sec $= 10^{12}$ years: during the short early history of the contracting star no electrostatic differentiation could have taken place.

But, at the low temperature of the primordial nebula, meteor particles may start condensing and increasing in size (cf. Lindblad,[44]); as shown by Jung[45], electrostatic forces due to ionization in interstellar space would actually resist such an accretion of meteoric mass except when the gas density ϱ exceeds $\sim 10^{-22}$ g/cm^3. In our example, $\varrho \sim 10^{-21}$, and increases with contraction, thus the process is quite possible. The meteoric dust particles, instead of continuing their rotation with the rest of the gas, are forced by gravitation to fall towards the centre (because they are no more supported by gas pressure, and because the velocity of rotation of a nebula which is able to contract at all must be negligible as compared with the circular orbital velocity); resistance of the medium (proportional to the mass acquired by the meteor) decreases the major axis and the ellipticity of the originally very elongated ellipse; as a result, the condensed meteoric dust (it may also have been partly captured from interstellar space) collects near the centre of the future star (our present solar meteors are the last remnants of the nebula; they no longer suffer much from the resistance of the medium, that is why most of them have still almost parabolic orbits). Now, as taught by the observed composition of meteors and by chemical considerations, in the process of condensation hydrogen (free or water vapour), helium, and nitrogen are not included, whereas oxygen is partly chemically bound (stone meteors) and collects with iron, nickel, and other metals near the centre. Thus, in the process of condensation, a strong differentiation in the required direction must take place.

It is probable, therefore, that a nucleus poor in hydrogen and soon exhausted is present in each star from the very beginning; but the nucleus is mostly so small that the radius, luminosity, and evolutional trend of the star remain practically the same as they would be without the nucleus; in rare cases ($\leqslant 6$ per cent of all, judging from $1-\alpha$ of Table 8 for $M > 4.3\odot$), the original nucleus is large enough to force upon the star an evolution towards the giant model, which in some cases may start from the beginning. Our conclusion is that the stars from the very beginning may possess somewhat distended atmospheres and concentrated central masses, due to the deficiency of hydrogen at the centre; rotational mixing, which sets in powerfully enough at an early stage of contraction (when the central convectional currents have not yet started), is apparently unable to upset the differentiation (cf. Section 4. *f*, and 5. *h*), especially as the larger molecular weight at the centre increases the convectional stability of the distribution. Further, as the result of concentration and greater molecular weight, the central temperature must be higher than for uniform composition. We further call the model a differentiated one. In the differentiated model it is convenient to distinguish schematically a core of greater molecular weight (~ 2), surrounded by an outer shell of $\mu \sim 1$. Let us inquire whether giant structure can be produced without the intervention of the subatomic energy. If subatomic sources of energy are absent, and rotational convection is slow, the whole differentiated model is in radiative equilibrium (cf. Section 5. *h*); the energy production is chiefly determined by the central core which is an incomplete polytrope. The outer shell, fitted to it, takes little part in the energy production. If a large proportion of the mass is in the core, the luminosities of the stars must be much greater than observed, unless we assume the hydrogen content in the core as high as now assumed for the envelope ($\sim$ 40 per cent); a small core does not imply such a difficulty (cf. Subsection *a*). The energy is furnished by gravitational contraction; subatomic sources being absent in- and outside, there is no such obstacle to the outer shell following the collapsing inner core as considered in Sections 5. *f* and 6. *e*. Therefore the contracting model may reveal only moderate inflation (semi-giant structure), and the origin of diffuse giants remains a mystery as before.

Thus, we are forced to accept the efficiency of the subatomic sources of energy even in the case of the originally differentiated model; exhaustion of the central source, and intense subatomic energy starting only at a certain distance from the centre, give rise to distending forces through which an actual giant comes into being*.

Tartu, October 19, 1937.

* These qualitative considerations will be discussed mathematically in a paper to follow, where examples of the corresponding stellar models are given.

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Table of Contents.

Eelmiste köidete sisu. — Contenu des volumes précédents.

A I (1921). **1.** A. Paldrock. Ein Beitrag zur Statistik der Geschlechtskrankheiten in Dorpat während der Jahre 1909—1918. — **2.** K. Väisälä. Verallgemeinerung des Begriffes der Dirichletschen Reihen. — **3.** C. Schlossmann. Hapete mõju kolloiidide peale ja selle tähtsus patoloogias. (L'action des acides sur les colloïdes et son rôle dans la pathologie.) — **4.** K. Regel. Statistische und physiognomische Studien an Wiesen. Ein Beitrag zur Methodik der Wiesenuntersuchung. — **5.** H. Reichenbach. Notes sur les microorganismes trouvés dans les pêches planctoniques des environs de Covda (gouv. d'Archangel) en été 1917. — **Misc.** F. Bucholtz. Der gegenwärtige Zustand des Botanischen Gartens zu Dorpat und Richtlinien für die Zukunft.

A II (1921). **1.** H. Bekker. The Kuckers stage of the ordovician rocks of NE Estonia. — **2.** C. Schlossmann. Über die Darmspirochäten beim Menschen. — **3.** J. Letzmann. Die Höhe der Schneedecke im Ostbaltischen Gebiet. — **4.** H. Kaho. Neutraalsoolade mõjust ultramaksimum-temperatuuri peale *Tradescantia zebrina* juures. (Über den Einfluss der Neutralsalze auf die Temperatur des Ultramaximums bei *Tradescantia zebrina*.)

A III (1922). **1.** J. Narbutt. Von den Kurven für die freie und die innere Energie bei Schmelz- und Umwandlungsvorgängen. — **2.** A. Томсонъ (A. Thomson). Значеніе аммонійныхъ солей для питанія высшихъ культурныхъ растеній. (Der Wert der Ammonsalze für die Ernährung der höheren Kulturpflanzen.) — **3.** E. Blessig. Ophthalmologische Bibliographie Russlands 1870—1920. I. Hälfte (S. I—VII und 1—96). — **4.** A. Lüüs. Ein Beitrag zum Studium der Wirkung künstlicher Wildunger Helenenquellensalze auf die Diurese nierenkranker Kinder. — **5.** E. Öpik. A statistical method of counting shooting stars and its application to the Perseid shower of 1920. — **6.** P. N. Kogerman. The chemical composition of the Esthonian M.-Ordovician oilbearing mineral „Kukersite". — **7.** M. Wittlich und S. Weshnjakow. Beitrag zur Kenntnis des estländischen Ölschiefers, genannt Kukkersit. — **Misc.** J. Letzmann. Die Trombe von Odenpäh am 10. Mai 1920.

A IV (1922). **1.** E. Blessig. Ophthalmologische Bibliographie Russlands 1870—1920. II. Hälfte (S. 97—188). — **2.** A. Valdes. Glükogeeni hulka vähendavate tegurite mõju üle südame spetsiifilise lihassüsteemi glükogeeni peale. (Über den Einfluss der die Glykogenmenge vermindernden Faktoren auf das Glykogen des spezifischen Muskelsystems des Herzens.) — **3.** E. Öpik. Notes on stellar statistics and stellar evolution. — **4.** H. Kaho. Raskemetallsoolade kihvtisusest taimeplasma kohta. (Über die Schwermetallgiftwirkung in bezug auf das Pflanzenplasma.) — **5.** J. Piiper und M. Härms. Der Kiefernkreuzschnabel der Insel Ösel *Loxia pityopsittacus estiae* subsp. nov. — **6.** L. Poska-Teiss. Zur Frage über die vielkernigen Zellen des einschichtigen Plattenepithels.

A V (1924). **1.** E. Öpik. Photographic observations of the brightness of Neptune. Method and preliminary results. — **2.** A. Lüüs. Ergebnisse der Krüppelkinder-Statistik in Eesti. — **3.** C. Schlossmann. Culture in vitro des protozoaires de l'intestin humain. — **4.** H. Kaho. Über die physiologische Wirkung der Neutralsalze auf das Pflanzenplasma. — **5.** Y. Kauko. Beiträge zur Kenntnis der Torfzersetzung und Vertorfung. — **6.** A. Tammekann. Eesti diktüoneema-kihi uurimine tema tekkimise, vanaduse ja levimise kohta. (Untersuchung des Dictyonema-Schiefers in Estland nach Entstehung, Alter und Verbreitung.) — **7.** Y. Kauko. Zur Bestimmung des Vertorfungsgrades. — **8.** N. Weiderpass. Eesti piparmündi-õli (*Oleum menthe esthicum*). (Das estnische Pfefferminzöl.)

A VI (1924). **1.** H. Bekker. Mõned uued andmed Kukruse lademe stratigraafiast ja faunast. (Stratigraphical and paleontological supplements on the Kukruse stage of the ordovician rocks of Eesti (Estonia).) — **2.** J. Wilip. Experimentelle Studien über die Bestimmung von Isothermen und kritischen Konstanten. — **3.** J. Letzmann. Das Bewegungsfeld im Fuss einer fortschreitenden Wind- oder Wasserhose. — **4.** H. Scupin. Die Grundlagen paläogeographischer Karten. — **5.** E. Öpik. Photometric measures on the moon and the earth-shine. — **6.** Y. Kauko. Über die Vertorfungswärme. — **7.** Y. Kauko. Eigentümlichkeiten der H_2O- und CO_2-Gehalte bei der unvollständigen Verbrennung. — **8.** M. Tilzen und Y. Kauko. Die wirtschaftlichen Möglichkeiten der Anwendung von Spiritus als Brennstoff. — **9.** M. Wittlich. Beitrag zur Untersuchung des Öles aus estländischem Ölschiefer. — **10.** J. Wilip. Emergenzwinkel, Unstetigkeitsflächen, Laufzeit. — **11.** H. Scupin. Zur Petroleumfrage in den baltischen Ländern. — **12.** H. Richter. Zwei Grundgesetze (Funktion- und Strukturprinzip) der lebendigen Masse.

A VII (1925). **1.** J. Vilms. Kõhreglükogeeni püsivusest mõnesuguste glükogeeni vähendavate tegurite puhul. (Über die Stabilität des Knorpelglykogens unter verschiedenen das Glykogen zum Verschwinden bringenden Umständen.) — **2.** E. Blessig. Ophthalmologische Bibliographie Russlands 1870—1920. Nachtrag. — **3.** O. Kuriks. Trachoma Eestis (eriti Tartus) möödunud ajal ja praegu. (Das Trachom in Estland (insbesondere in Dorpat) einst und jetzt.) — **4.** A. Brandt. Sexualität. Eine biologische Studie. — **5.** M. Haltenberger. Gehört das Baltikum zu Ost-, Nord- oder zu Mitteleuropa? — **6.** M. Haltenberger. Recent geographical work in Estonia.

A VIII (1925). **1.** H. Jaakson. Sur certains types de systèmes d'équations linéaires à une infinité d'inconnues. Sur l'interpolation. — **2.** K. Frisch. Die Temperaturabweichungen in Tartu (Dorpat) und ihre Bedeutung für die Witterungsprognose. — **3.** O. Kuriks. Muutused leeprahaigete silmas Eesti leprosooriumide haigete läbivaatamise põhjal. (Die Lepra des Auges.) — **4.** A. Paldrock. Die Senkungsreaktion und ihr praktischer Wert. — **5.** A. Öpik. Beiträge zur Kenntnis der Kukruse-(C_2-)Stufe in Eesti. I. — **6.** M. Wittlich. Einiges über den Schwefel im estländischen Ölschiefer (Kukersit) und dessen Verschwelungsprodukten. — **7.** H. Kaho. Orientierende Versuche über die stimulierende Wirkung einiger Salze auf das Wachstum der Getreidepflanzen. I.

A IX (1926). **1.** E. Krahn. Über Minimaleigenschaften der Kugel in drei und mehr Dimensionen. — **2.** A. Mieler. Ein Beitrag zur Frage des Vorrückens des Peipus an der Embachmündung und auf der Peipusinsel Pirisaar in dem Zeitraum von 1682 bis 1900. — **3.** M. Haltenberger. Der wirtschaftsgeographische Charakter der Städte der Republik Eesti. — **4.** J. Rumma. Die Heimatforschung in Eesti. — **5.** M. Haltenberger. Der Stand des Aufnahme- und Kartenwesens in Eesti. — **6.** M. Haltenberger. Landeskunde von Eesti. I. — **7.** A. Tammekann. Die Oberflächengestaltung des nordostestländischen Küstentafellandes. — **8.** K. Frisch. Ein Versuch das Embachhochwasser im Frühling für Tartu (Dorpat) vorherzubestimmen.

A X (1926). **1.** M. Haltenberger. Landeskunde von Eesti. II—III. — **2.** H. Scupin. Alter und Herkunft der ostbaltischen Solquellen und ihre Bedeutung für die Frage nach dem Vorkommen von Steinsalz im baltischen Obersilur. — **3.** Th. Lippmaa. Floristische Notizen aus dem Nord-Altai nebst Beschreibung einer neuen *Cardamine*-Art aus der Sektion *Dentaria*. — **4.** Th. Lippmaa. Pigmenttypen bei Pteridophyta und Anthophyta. I. Allgemeiner Teil. — **5.** E. Pipenberg. Eine städtemorphographische Skizze der estländischen Hafenstadt Pärnu (Pernau). — **6.** E. Spohr. Über das Vorkommen von *Sium erectum* Huds. und *Lemna gibba* L. in Estland und über deren nordöstliche Verbreitungsgrenzen in Europa. — **7.** J. Wilip. On new precision-seismographs.

A XI (1927). **1.** Th. Lippmaa. Pigmenttypen bei Pteridophyta und Anthophyta. II. Spezieller Teil. — **2.** M. Haltenberger. Landeskunde von Eesti. IV—V. — **3.** H. Scupin. Epirogenese und Orogenese im Ostbaltikum. — **4.** K. Schlossmann. Mikroorganismide kui bioloogiliste reaktiivide tähtsusest keemias. (Le rôle des ferments microbiens dans la chimie.) — **5.** J. Sarv. Ahmese geomeetrilised joonised. (Die geometrischen Figuren des Ahmes.) — **6.** K. Jaansoon-Orviku. Beiträge zur Kenntnis der Aseri- und der Tallinna-Stufe in Eesti. I.

A XII (1927). **1.** E. Reinwaldt. Beiträge zur Muriden-Fauna Estlands mit Berücksichtigung der Nachbargebiete. — **2.** A. Öpik. Die Inseln Odensholm und Rogö. Ein Beitrag zur Geologie von NW-Estland. — **3.** A. Öpik. Beiträge zur Kenntnis der Kukruse-(C_2-)Stufe in Eesti. II. — **4.** Th. Lippmaa. Beobachtungen über durch Pilzinfektion verursachte Anthocyaninbildung. — **5.** A. Laur. Die Titration des Ammoniumhydrosulfides mit Ferricyankalium. — **6.** N. King. Über die rhythmischen Niederschläge von PbJ_2, Ag_2CrO_4 und AgCl im kapillaren Raume. — **7.** P. N. Kogerman and J. Kranig. Physical constants of some alkyl carbonates. — **8.** E. Spohr. Über brunsterzeugende Stoffe im Pflanzenreich. Vorläufige Mitteilung.

A XIII (1928). **1.** J. Sarw. Zum Beweis des Vierfarbensatzes. — **2.** H. Scupin. Die stratigraphische Stellung der Devonschichten im Südosten Estlands. — **3.** H. Perlitz. On the parallelism between the rate of change in electric resistance at fusion and the degree of closeness of packing of mealltic atoms in crystals. — **4.** K. Frisch. Zur Frage der Luftdruckperioden. — **5.** J. Port. Untersuchungen über die Plasmakoagulation von *Paramaecium caudatum.* — **6.** J. Sarw. Direkte Herleitung der Lichtgeschwindigkeitsformeln. — **7.** K. Frisch. Zur Frage des Temperaturansteigens im Winter. — **8.** E. Spohr. Über die Verbreitung einiger bemerkenswerter und schutzbedürftiger Pflanzen im Ostbaltischen Gebiet. — **9.** N. Rägo. Beiträge zur Kenntnis des estländischen Dictyonemaschiefers. — **10.** C. Schlossmann. Études sur le rôle de la barrière hémato-encéphalique dans la genèse et le traitement des maladies infectienses. — **11.** A. Öpik. Beiträge zur Kenntnis der Kukruse-(C_2-C_3-)Stufe in Eesti. III.

A XIV (1929). **1.** J. Rives. Über die histopathologischen Veränderungen im Zentralnervensystem bei experimenteller Nebenniereninsuffizienz. — **2.** W. Wadi. Kopsutuberkuloosi areng ja kliinilised vormid. (Der Entwicklungsgang und die klinischen Formen der Lungentuberkulose.) — **3.** E. Markus. Die Grenzverschiebung des Waldes und des Moores in Alatskivi. — **4.** K. Frisch. Zur Frage über die Beziehung zwischen der Getreideernte und einigen meteorologischen Faktoren in Eesti.

A XV (1929). **1.** A. Nõmmik. The influence of ground limestone on acid soils and on the availability of nitrogen from several mineral nitrogenous fertilizers. — **2.** A. Öpik. Studien über das estnische Unterkambrium (Estonium). I—IV. — **3.** J. Nuut. Über die Anzahl der Lösungen der Vierfarbenaufgabe. — **4.** J. Nuut. Über die Vierfarbenformel. — **5.** J. Nuut. Topologische Grundlagen des Zahlbegriffs. — **6.** Th. Lippmaa. Pflanzenökologische Untersuchungen aus Norwegisch- und Finnisch-Lappland unter besonderer Berücksichtigung der Lichtfrage.

A XVI (1930). **1.** A. Paris. Über die Hydratation der Terpene des Terpentinöls zu Terpinhydrat durch Einwirkung von Mineralsäuren. — **2.** A. Laur. Die Anwendung der Umschlagselektroden bei der potentiometrischen Massanalyse. Die potentiometrische Bestimmung des Kaliums. — **3.** A. Paris. Zur Theorie der Strömungsdoppelbrechung. — **4.** O. Kuriks. Pisarate toimest silma mikrofloorasse. (Über die Wirkung der Tränen auf die Mikroflora des Auges.) — **5.** K. Orviku. Keskdevoni põhikihid Eestis. (Die untersten Schichten des Mitteldevons in Eesti.) — **6.** J. Kopwillem. Über die thermale Zersetzung von estländischem Ölschiefer Kukersit.

A XVII (1930). **1.** A. Öpik. Brachiopoda Protremata der estländischen ordovizischen Kukruse-Stufe. — **2.** P. W. Thomson. Die regionale Entwickelungsgeschichte der Wälder Estlands.

A XVIII (1930). **1.** G. Vilberg. Erneuerung der Loodvegetation durch Keimlinge in Ost-Harrien (Estland). — **2.** A. Parts. Über die Neutralsalzwirkung auf die Geschwindigkeit der Ionenreaktionen. — **3.** Ch. R. Schlossmann. On two strains of yeast-like organisms cultured from diseased human throats. — **4.** H. Richter. Die Relation zwischen Form und Funktion und das teleologische Prinzip in den Naturphänomenen. — **5.** H. Arro. Die Metalloxyde als photochemische Sensibilatoren beim Bleichen von Methylenblaulösung. — **6.** A. Luha. Über Ergebnisse stratigraphischer Untersuchungen im Gebiete der Saaremaa-(Ösel-)Schichten in Eesti (Unterösel und Eurypterusschichten). — **7.** K. Frisch. Zur Frage der Zyklonenvertiefung. — **8.** E. Markus. Naturkomplexe von Alatskivi.

A XIX (1931). **1.** J. Uudelt. Über das Blutbild Trachomkranker. — **2.** A. Öpik. Beiträge zur Kenntnis der Kukruse-(C_2-C_3-)Stufe in Eesti. IV. — **3.** H. Liedemann. Über die Sonnenscheindauer und Bewölkung in Eesti. — **4.** J. Sarw. Geomeetria alused. (Die Grundlagen der Geometrie.)

A XX (1931). **1.** J. Kuusk. Glühaufschliessung der Phosphorite mit Kieselsäure zwecks Gewinnung eines citratlöslichen Düngmittels. — **2.** U. Karell. Zur Behandlung und Prognose der Luxationsbrüche des Hüftgelenks. — **3.** A. Laur. Beiträge zur Kenntnis der Reaktion des Zinks mit Kaliumferrocyanid. I. — **4.** J. Kuusk. Beitrag zur Kalisalzgewinnung beim Zementbrennen mit besonderer Berücksichtigung der estländischen K-Mineralien. — **5.** L. Rinne. Über die Tiefe der Eisbildung und das Auftauen des Eises im Niederungsmoor. — **6.** J. Wilip. A galvanometrically registering vertical seismograph with temperature compensation. — **7.** J. Nuut. Eine arithmetische Analyse des Vierfarbenproblems. — **8.** G. Barkan. Dorpats Bedeutung für die Pharmakologie. — **9.** K. Schlossmann. Vanaduse ja surma mõistetest ajakohaste bioloogiliste andmete alusel. (Über die Begriffe Alter und Tod auf Grund der modernen biologischen Forschung.)

A XXI (1931). **1.** N. Kwaschnin-Ssamarin. Studien über die Herkunft des osteuropäischen Pferdes. — **2.** U. Karell. Beitrag zur Ätiologie der arteriellen Thrombosen. — **3.** E. Krahn. Über Eigenschwingungszahlen freier Platten. — **4.** A. Öpik. Über einige Karbonatgesteine im Glazialgeschiebe NW-Estlands. — **5.** A. Thomson. Wasserkulturversuche mit organischen Stickstoffverbindungen, angestellt zur Ermittelung der Assimilation ihres Stickstoffs von seiten der höheren grünen Pflanze.

A XXII (1932). **1.** U. Karell. An observation on a peculiarity of the cardiac opening reflex in operated cases of cardiospasmus. — **2.** E. Krahn. Die Wahrscheinlichkeit der Richtigkeit des Vierfarbensatzes. — **3.** A. Audova. Der wirkliche Kampf ums Dasein. — **4.** H. Perlitz. Abstandsänderungen nächster Nachbaratome in einigen Elementen und Legierungen bei Umordnung aus der kubischen flächenzentrierten Anordnung in die kubische raumzentrierte oder die hexagonale dichteste Anordnung.

A XXIII (1932). **1.** J. Port. Untersuchungen über die Wirkung der Neutralsalze auf das Keimlingswachstum bezüglich der Abhängigkeit von ihrer Konzentration. — **2.** E. Markus. Chorogenese und Grenzverschiebung. — **3.** A. Öpik. Über die Plectellinen. — **4.** J. Nuut. Einige Bemerkungen über Vierpunktaxiome. — **5.** K. Frisch. Die Veränderungen der klimatischen Elemente nach den meteorologischen Beobachtungen von Tartu 1866—1930.

A XXIV (1933). **1.** M. Gross. In der Butter vorkommende Sprosspilze und deren Einwirkung auf die Butter. — **2.** H. Perlitz. Bemerkungen zu den Regeln über Valenzelektronenkonzentrationen in binären intermetallischen Legierungen. — **3.** A. Öpik. Über *Scolithus* aus Estland. — **4.** T. Lippmaa. Aperçu général sur la végétation autochtone du Lautaret (Hautes-Alpes). — **5.** E. Markus. Die südöstliche Moorbucht von Lauge. — **6.** A. Sprantsman. Über Herstellung makroskopischer Thalliumkristalle durch Elektrolyse. — **7.** A. Öpik. Über Plectamboniten.

A XXV (1933). **1.** A. Öpik. Über einige Dalmanellacea aus Estland. — **2.** H. Richter. Ergänzungen zu: „Die Relation zwischen Form und Funktion und das teleologische Prinzip in den Naturphänomenen". Die Rolle, welche „Spirale" und „Wirbel" in den biologischen Phänomenen spielt, besonders auch in bezug auf die feinere Struktur des lebendigen Protoplasmas. — **3.** T. Lippmaa ja K. Eichwald. Eesti taimed. (Estonian plants.) I (1—50). — **4.** E. Piipenberg. Die Stadt Petseri in Estland. — **5.** A. Miljan. Vegetationsuntersuchungen an Naturwiesen und Seen im Otepääschen Moränengebiete Estlands. I. — **6.** R. Livländer. On the colour of Mars. — **7.** A. Tudeberg. Über die Theorie und die Anwendungsmethoden der Quadraturreihen.

A XXVI (1934). **1.** E. Blessig. Index ophthalmologiae Balticus. — **2.** E. Öpik. Atomic collisions and radiation of meteors. — **3.** J. Tehver und A. Kriisa. Zur Histologie des Harnleiters der Haussäugetiere. — **4.** H. Kaho. Leelissoolade toimest taimeraku deplasmolüüsile. (Über den Einfluss von Alkalisalzen auf die Deplasmolyse der Pflanzenzellen.) — **5.** A. Öpik. Über Klitamboniten. — **6.** A. Tudeberg. Über die Beweisbarkeit einiger Anordnungsaussagen in geometrischen Axiomensystemen.

A XXVII (1934). **1.** K. Lellep. Simulation von Geisteskrankheiten und deren Grenzzuständen. — **2.** M. Tiitso. Hingamise ergulisest regulatsioonist. I teadaanne: Stenoosi toime inimese hingamisele. (Über die nervöse Atemregulation. I. Mitteilung: Der Einfluss der Stenose auf die menschliche Atmung.) — **3.** M. Tiitso. Hingamise ergulisest regulatsioonist. II teadaanne: Inimese hingamisfrekvents kopsude erineva täitumise korral. (Über die nervöse Atemregulation. II. Mitteilung: Die Atemfrequenz des Menschen bei abnormen Lungenfüllungen.) — **4.** M. Tiitso. Hingamise ergulisest regulatsioonist. III teadaanne: Propriotseptiivsete aferentside toimest hingamisele. (Über die nervöse Atemregulation. III. Mitteilung: Über die Auswirkung der propriozeptiven Afferenzen auf die Atmung.) — **5.** J. Tehver and M. Keerd. The number of ribs in the ox and pig. — **6.** A. Kärsna. Über das Problem der Vorhersage des nächtlichen Temperaturminimums. — **7.** K. Schlossmann. A study of bacterial carbohydrates with special reference to the tubercle bacillus. — **8.** A. Öpik. *Ristnacrinus*, a new ordovician crinoid from Estonia. — **9.** A. Kipper. Variation of surface gravity upon two Cepheids — δ Cephei and η Aquilae. — **10.** E. Lepik. Fungi Estonici exsiccati. Uredinaceae. [I.] — **11.** H. Perlitz. The structure of the intermetallic compound Au_2Pb.

A XXVIII (1935). **1.** T. Lippmaa. Une analyse des forêts de l'île estonienne d'Abruka (Abro) sur la base des associations unistrates.

— **2.** J. Sarv. Foundations of arithmetic. — **3.** A. Tudeberg. Orthogonalsysteme von Polynomen und Extremumprobleme der Interpolationsrechnung. — **4.** T. Lippmaa. Eesti geobotaanika põhijooni. (Aperçu géobotanique de l'Estonie.)

A XXIX (1936). **1.** A. Öpik. *Hoplocrinus* — eine stiellose Seelilie aus dem Ordovizium Estlands. — **2.** A. Kärsna. Vereinfachte Methoden zur Berechnung des Korrelationskoeffizienten bei normaler Korrelation. — **3.** J. Nuut. Eine nichteuklidische Deutung der relativistischen Welt. — **4.** H. Kaho. Das Verhalten der Eiweissstoffe gesunder und abbaukranker Kartoffelknollen gegen Salze. — **5.** T. Lippmaa ja K. Eichwald. Eesti taimed. (Estonian plants.) II (51—100). — **6.** J. Nuut. Ansätze zu einer expansionistischen Kinematik. — **7.** A. Lüüs. Données anthropologiques sur les nouveaux-nés estoniens. — **8.** A. Tudeberg. Energieverluste im Eisenblech bei niederfrequenter Ummagnetisierung. — **9.** Wilh. Anderson. Existiert eine obere Grenze für die Dichte der Materie und der Energie?

A XXX (1936). **1.** E. Öpik. Researches on the physical theory of meteor phenomena. I. II. — **2.** J. Gabovitš. The *TiO* colour effect, and the densities of *M* stars. — **3.** J. Wilip. Über Lichtstrahlung während der Sonnenfinsternis am 21. August 1914 in Üxküll. — **4.** E. Lepik. Fungi Estonici exsiccati. Uredinaceae. II. — **5.** E. Markus. Geographische Kausalität. — **6.** K. Schlossmann. Einige Gedanken über die Ausbildung des praktischen Arztes. — **7.** U. Karell. Aneurism of the internal carotid and the ligation of the carotids. — **8.** K. Kirde. Meteorological elements characterized by frequency-curves.

A XXXI (1937). **1.** V. Ridala. Inquiries into the pathogenic effects produced by *Brucella Abortus* in the udder and certain other organs of the cow. — **2.** Wilh. Anderson. Zu H. Vogts Ansichten über die obere Grenze der Sternmassen. — **3.** J. Gabovitš. The pulsation theory of Mira Ceti. — **4.** T. Lippmaa. E. V. Tartu Ülikooli Botaanikaaia süstemaatilised ja taimegeograafilised kogud. (Les collections systématiques et phytogéographiques de l'Université estonienne à Tartu.) I (p. 1—192).

A XXXII (1937). **1.** Wilh. Anderson. Kritische Bemerkungen zu S. Rosselands und W. Grotrians Ansichten über die Sonnenkorona. — **2.** T. Lippmaa. E. V. Tartu Ülikooli Botaanikaaia süstemaatilised ja taimegeograafilised kogud. (Les collections systématiques et phytogéographiques de l'Université estonienne à Tartu.) II (p. 193—375). — **3.** A. Öpik. Trilobiten aus Estland.

A XXXIII (1939). **1.** E. Öpik. Researches on the physical theory of meteor phenomena. III. — **2.** Wilh. Anderson. Kritik der Ansichten von B. Jung über die obere Grenzdichte der Himmelskörper. — **3.** Wilh. Anderson. Weitere Beiträge zu der elementaren Expansionstheorie des Universums. — **4.** U. Karell. Tube flap grafting. — **5.** K. Kirde. Change of climate in the northern hemisphere. — **6.** K. Eichwald. Eesti taimed. (Estonian plants.) III

(101—150). — **7.** Wilh. Anderson. Über die Anwendbarkeit von Saha's Ionisationsformel bei extrem hohen Temperaturen. — **8.** Miscellaneous astrophysical notes. (I. J. Gabovitš. On the empirical mass-luminosity relation. — II. J. Gabovitš. On the orientation of the orbital planes in multiple systems. — III. J. Gabovitš. On the mass ratio of spectroscopic binaries with one spectrum visible. — IV. G. Kusmin. Über die Abhängigkeit der interstellaren Absorption von der Wellenlänge. — V. G. Kusmin. Über die Partikeldurchmesserverteilung in der interstellaren Materie. — VI. V. Riives. A tentative determination of the surface brightness of dark nebulae. — VII. V. Riives. The influence of selective absorption in space upon a differential scale of stellar magnitudes. — VIII. E. Öpik. On the upper limit of stellar masses. — IX. E. Öpik. The density of the white dwarf A. C. + 70° 8247. — **9.** E. Öpik. Stellar structure, source of energy, and evolution.

B I (1921). **1.** M. Vasmer. Studien zur albanesischen Wortforschung. I. — **2.** A. v. Bulmerincq. Einleitung in das Buch des Propheten Maleachi. 1. — **3.** M. Vasmer. Osteuropäische Ortsnamen. — **4.** W. Anderson. Der Schwank von Kaiser und Abt bei den Minsker Juden. — **5.** J. Bergman. Quaestiunculae Horatianae.

B II (1922). **1.** J. Bergman. Aurelius Prudentius Clemens, der grösste christliche Dichter des Altertums. I. — **2.** L. Kettunen. Lõunavepsa häälik-ajalugu. I. Konsonandid. (Südwepsische Lautgeschichte. I. Konsonantismus.) — **3.** W. Wiget. Altgermanische Lautuntersuchungen.

B III (1922). **1.** A. v. Bulmerincq. Einleitung in das Buch des Propheten Maleachi. 2. — **2.** M. A. Курчинскій (M. A. Kurtschinsky). Соціальный законъ, случай и свобода. (Das soziale Gesetz, Zufall und Freiheit.) — **3.** A. R. Cederberg. Die Erstlinge der estländischen Zeitungsliteratur. — **4.** L. Kettunen. Lõunavepsa häälik-ajalugu. II. Vokaalid. (Südwepsische Lautgeschichte. II. Vokalismus.) — **5.** E. Kieckers. Sprachwissenschaftliche Miscellen. [I.] — **6.** A. M. Tallgren. Zur Archäologie Eestis. I.

B IV (1923). **1.** E. Kieckers. Sprachwissenschaftliche Miscellen. II. — **2.** A. v. Bulmerincq. Einleitung in das Buch des Propheten Maleachi. 3. — **3.** W. Anderson. Nordasiatische Flutsagen. — **4.** A. M. Tallgren. L'ethnographie préhistorique de la Russie du nord et des États Baltiques du nord. — **5.** R. Gutmann. Eine unklare Stelle in der Oxforder Handschrift des Rolandsliedes.

B V (1924). **1.** H. Mutschmann. Milton's eyesight and the chronology of his works. — **2.** A. Pridik. Mut-em-wija, die Mutter Amenhotep's (Amenophis') III. — **3.** A. Pridik. Der Mitregent des Königs Ptolemaios II Philadelphos. — **4.** G. Suess. De Graecorum fabulis satyricis. — **5.** A. Berendts und K. Grass. Flavius Josephus: Vom jüdischen Kriege, Buch I—IV, nach der slavischen Übersetzung deutsch herausgegeben und mit dem griechischen Text verglichen. I. Lief.

(S. 1—160). — **6.** H. Mutschmann. Studies concerning the origin of "Paradise Lost".

B VI (1925). **1.** A. Saareste. Leksikaalseist vahekordadest eesti murretes. I. Analüüs. (Du sectionnement lexicologique dans les patois estoniens. I. Analyse.) — **2.** A. Bjerre. Zur Psychologie des Mordes.

B VII (1926). **1.** A. v. Bulmerincq. Einleitung in das Buch des Propheten Maleachi. 4. — **2.** W. Anderson. Der Chalifenmünzfund von Kochtel. (Mit Beiträgen von R. Vasmer.) — **3.** J. Mägiste. Rosona (Eesti Ingeri) murde pääjooned. (Die Hauptzüge der Mundart von Rosona). — **4.** М. А. Курчинскій (M. A. Kurtschinsky). Европейскій хаосъ. Экономическія послѣдствія великой войны. (Das europäische Chaos.)

B VIII (1926). **1.** A. M. Tallgren. Zur Archäologie Eestis. II. — **2.** H. Mutschmann. The secret of John Milton. — **3.** L. Kettunen. Untersuchung über die livische Sprache. I. Phonetische Einführung. Sprachproben.

B IX (1926). **1.** N. Maim. Parlamentarismist Prantsuse restauratsiooniajal (1814—1830). (Du parlementarisme en France pendant la Restauration.) — **2.** S. v. Csekey. Die Quellen des estnischen Verwaltungsrechts. I. Teil (S. 1—102). — **3.** A. Berendts und K. Grass. Flavius Josephus: Vom jüdischen Kriege, Buch I—IV, nach der slavischen Übersetzung deutsch herausgegeben und mit dem griechischen Text verglichen. II. Lief. (S. 161—288). — **4.** G. Suess. De eo quem dicunt inesse Trimalchionis cenae sermone vulgari. — **5.** E. Kieckers. Sprachwissenschaftliche Miscellen. III. — **6.** C. Vilhelmson. De ostraco quod Revaliae in museo provinciali servatur.

B X (1927). **1.** H. B. Rahamägi. Eesti Evangeeliumi Luteri usu vaba rahvakirik vabas Eestis. (Die evangelisch-lutherische freie Volkskirche im freien Eesti. Anhang: Das Gesetz betreffend die religiösen Gemeinschaften und ihre Verbände.) — **2.** E. Kieckers. Sprachwissenschaftliche Miscellen. IV. — **3.** A. Berendts und K. Grass. Flavius Josephus: Vom jüdischen Kriege, Buch I—IV, nach der slavischen Übersetzung deutsch herausgegeben und mit dem griechischen Text verglichen. III. Lief. (S. 289—416). — **4.** W. Schmied-Kowarzik. Die Objektivation des Geistigen. (Der objektive Geist und seine Formen.) — **5.** W. Anderson. Novelline popolari sammarinesi. I.

B XI (1927). **1.** O. Loorits. Liivi rahva usund. (Der Volksglaube der Liven.) I. — **2.** A. Berendts und K. Grass. Flavius Josephus: Vom jüdischen Kriege, Buch I—IV, nach der slavischen Übersetzung deutsch herausgegeben und mit dem griechischen Text verglichen. IV. Lief. (S. 417—512). — **3.** E. Kieckers. Sprachwissenschaftliche Miscellen. V.

B XII (1928). **1.** O. Loorits. Liivi rahva usund. (Der Volksglaube der Liven.) II. — **2.** J. Mägiste. *oi̯-, ei̯*-deminutiivid läänemeresoome keelis. (Die *oi̯-, ei̯*-Deminutiva der ostseefinnischen Sprachen.)

B XIII (1928). **1.** G. Suess. Petronii imitatio sermonis plebe qua necessitate coniungatur cum grammatica illius aetatis doctrina. —

2. С. Штейн (S. v. Stein). Пушкин и Гофман. (Puschkin und E. T. A. Hoffmann.) — 3. A. V. Kõrv. Värsimõõt Veske „Eesti rahvalauludes“. (Le mètre des „Chansons populaires estoniennes“ de Veske.)

B XIV (1929). **1.** Н. Майм (N. Maim). Парламентаризм и суверенное государство. (Der Parlamentarismus und der souveräne Staat.) — **2.** S. v. Csekey. Die Quellen des estnischen Verwaltungsrechts. II. Teil (S. 103—134). — **3.** E. Virányi. Thalès Bernard, littérateur français, et ses relations avec la poésie populaire estonienne et finnoise.

B XV (1929). **1.** A. v. Bulmerincq. Kommentar zum Buche des Propheten Maleachi. 1 (1, 2—11). — **2.** W. E. Peters. Benito Mussolini und Leo Tolstoi. Eine Studie über europäische Menschheitstypen. — **3.** W. E. Peters. Die stimmanalytische Methode. — **4.** W. Freymann. Platons Suchen nach einer Grundlegung aller Philosophie.

B XVI (1929). **1.** O. Loorits. Liivi rahva usund. (Der Volksglaube der Liven.) III. — **2.** W. Süss. Karl Morgenstern (1770—1852). I. Teil (S. 1—160).

B XVII (1930). **1.** A. R. Cederberg. Heinrich Fick. Ein Beitrag zur russischen Geschichte des XVIII. Jahrhunderts. — **2.** E. Kieckers. Sprachwissenschaftliche Miscellen. VI. — **3.** W. E. Peters. Wilson, Roosevelt, Taft und Harding. Eine Studie über nordamerikanisch-englische Menschheitstypen nach stimmanalytischer Methode. — **4.** N. Maim. Parlamentarism ja fašism. (Parliamentarism and fascism.)

B XVIII (1930). **1.** J. Vasar. Taani püüded Eestimaa taasvallutamiseks 1411—1422. (Dänemarks Bemühungen Estland zurückzugewinnen 1411—1422.) — **2.** L. Leesment. Über die livländischen Gerichtssachen im Reichskammergericht und im Reichshofrat. — **3.** А. И. Стендер-Петерсен (Ad. Stender-Petersen). О пережиточных следах аориста в славянских языках, преимущественно в русском. (Über rudimentäre Reste des Aorists in den slavischen Sprachen, vorzüglich im Russischen.) — **4.** М. Курчинский (M. Kourtchinsky). Соединенные Штаты Европы. (Les États-Unis de l'Europe.) — **5.** K. Wilhelmson. Zum römischen Fiskalkauf in Ägypten.

B XIX (1930). **1.** A. v. Bulmerincq. Kommentar zum Buche des Propheten Maleachi. 2 (1, 11—2, 9). — **2.** W. Süss. Karl Morgenstern (1770—1852). II. Teil (S. 161—330). — **3.** W. Anderson. Novelline popolari sammarinesi. II.

B XX (1930). **1.** A. Oras. Milton's editors and commentators from Patrick Hume to Henry John Todd (1695—1801). I. — **2.** J. Vasar. Die grosse livländische Güterreduktion. Die Entstehung des Konflikts zwischen Karl XI. und der livländischen Ritter- und Landschaft 1678—1684. Teil I (S. 1—176). — **3.** S. v. Csekey. Die Quellen des estnischen Verwaltungsrechts. III. Teil (S. 135—150).

B XXI (1931). **1.** W. Anderson. Der Schwank vom alten Hildebrand. Teil I (S. 1—176). — **2.** A. Oras. Milton's editors and commentators from Patrick Hume to Henry John Todd (1695—1801). II. — **3.** W. Anderson. Über P. Jensens Methode der vergleichenden Sagenforschung.

B XXII (1931). **1.** E. Tennmann. G. Teichmüllers Philosophie des Christentums. — **2.** J. Vasar. Die grosse livländische Güterreduktion. Die Entstehung des Konflikts zwischen Karl XI. und der livländischen Ritter- und Landschaft 1678—1684. Teil II (S. I—XXVII. 177—400).

B XXIII (1931). **1.** W. Anderson. Der Schwank vom alten Hildebrand. Teil II (S. I—XIV. 177—329). — **2.** A. v. Bulmerincq. Kommentar zum Buche des Propheten Maleachi. 3 (2, 10 — 3, 3). — **3.** P. Arumaa. Litauische mundartliche Texte aus der Wilnaer Gegend. — **4.** H. Mutschmann. A glossary of americanisms.

B XXIV (1931). **1.** L. Leesment. Die Verbrechen des Diebstahls und des Raubes nach den Rechten Livlands im Mittelalter. — **2.** N. Maim. Völkerbund und Staat. Teil I (S. 1—176).

B XXV (1931). **1.** Ad. Stender-Petersen. Tragoediae Sacrae. Materialien und Beiträge zur Geschichte der polnisch-lateinischen Jesuitendramatik der Frühzeit. — **2.** W. Anderson. Beiträge zur Topographie der „Promessi Sposi". — **3.** E. Kieckers. Sprachwissenschaftliche Miscellen. VII.

B XXVI (1932). **1.** A. v. Bulmerincq. Kommentar zum Buche des Propheten Maleachi. 4 (3, 3—12). — **2.** A. Pridik. Wer war Mutemwija? — **3.** N. Maim. Völkerbund und Staat. Teil II (S. I—III. 177—356).

B XXVII (1932). **1.** K. Schreinert. Johann Bernhard Hermann. Briefe an Albrecht Otto und Jean Paul (aus Jean Pauls Nachlass). I. Teil (S. 1—128). — **2.** A. v. Bulmerincq. Kommentar zum Buche des Propheten Maleachi. 5 (3, 12—24). — **3.** M. J. Eisen. Kevadised pühad. (Frühlingsfeste.) — **4.** E. Kieckers. Sprachwissenschaftliche Miscellen. VIII.

B XXVIII (1932). **1.** P. Põld. Üldine kasvatusõpetus. (Allgemeine Erziehungslehre.) Redigeerinud (redigiert von) J. Tork. — **2.** W. Wiget. Eine unbekannte Fassung von Klingers Zwillingen. — **3.** A. Oras. The critical ideas of T. S. Eliot.

B XXIX (1933). **1.** L. Leesment. Saaremaa halduskonna finantsid 1618/19. aastal. (Die Finanzen der Provinz Ösel im Jahre 1618/19.) — **2.** L. Rudrauf. Un tableau disparu de Charles Le Brun. — **3.** P. Ariste. Eesti-rootsi laensõnad eesti keeles. (Die estlandschwedischen Lehnwörter in der estnischen Sprache.) — **4.** W. Süss. Studien zur lateinischen Bibel. I. Augustins Locutiones und das Problem der lateinischen Bibelsprache. — **5.** M. Kurtschinsky. Zur Frage des Kapitalprofits.

B XXX (1933). **1.** A. Pridik. König Ptolemaios I und die Philosophen. — **2.** K. Schreinert. Johann Bernhard Hermann. Briefe an Albrecht Otto und Jean Paul (aus Jean Pauls Nachlass). II. Teil S. I—XLII + 129—221). — **3.** D. Grimm. Zur Frage über den Begriff der Societas im klassischen römischen Rechte. — **4.** E. Kieckers. Sprachwissenschaftliche Miscellen. IX.

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B XXXII (1934). **1.** A. Anni. F. R. Kreutzwaldi „Kalevipoeg". I osa: Kalevipoeg eesti rahvaluules. (F. R. Kreutzwalds „Kalevipoeg". I. Teil: Kalevipoeg in den estnischen Volksüberlieferungen.) — **2.** P. Arumaa. Untersuchungen zur Geschichte der litauischen Personalpronomina. — **3.** E. Kieckers. Sprachwissenschaftliche Miscellen. XI. — **4.** L. Gulkowitsch. Die Entwicklung des Begriffes Ḥāsīd im Alten Testament. — **5.** H. Laakmann und W. Anderson. Ein neues Dokument über den estnischen Metsik-Kultus aus dem Jahre 1680.

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B XXXIV (1935). **1.** W. Anderson. Studien zur Wortsilbenstatistik der älteren estnischen Volkslieder. — **2.** P. Ariste. Huulte võnkehäälik eesti keeles. (The labial vibrant in Estonian.) — **3.** P. Wieselgren. Quellenstudien zur Vǫlsungasaga. I (S. 1—154).

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B XXXVI (1935). **1.** A. Pridik. Berenike, die Schwester des Königs Ptolemaios III Euergetes. II. Hälfte (S. I—VIII. 177—305). — **2.** J. Taul. Kristluse jumalariigi õpetus. (Die Reich-Gottes-Lehre des Christentums.) II pool (lk. 161—304).

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B XXXVIII (1936). **1.** J. Mägiste. Einiges zum problem der *oi-*, *ei*-deminutiva und zu den prinzipien der wissenschaftlichen kritik. — **2.** P. Wieselgren. Quellenstudien zur Volsungasaga. III (S. 239—430). — **3.** W. Anderson. Zu Albert Wesselski's Angriffen auf die finnische folkloristische Forschungsmethode. — **4.** A. Koort. Beiträge zur Logik des Typusbegriffs. Teil I (S. 1—138). — **5.** E. Kieckers. Sprachwissenschaftliche Miscellen. XIII.

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C IV—VI (1929). **IV 1.** Ettelug. kava 1924. a. I poolaastal. — **IV 2.** Ettelug. kava 1924. a. II poolaastal. — **V 1.** Ettelug. kava 1925. a. I poolaastal. — **V 2.** Ettelug. kava 1925. a. II poolaastal. — **VI 1.** Ettelug. kava 1926. a. I poolaastal. — **VI 2.** Ettelug. kava 1926. a. II poolaastal.

C VII—IX (1929). **VII 1.** Ettelug. kava 1927. a. I poolaastal. — **VII 2.** Ettelug. kava 1927. a. II poolaastal. — **VIII 1.** Loengute ja praktiliste tööde kava 1928. a. I poolaastal. — **VIII 2.** Loeng. ja prakt. tööde kava 1928. a. II poolaastal. — **IX 1.** Loeng. ja prakt. tööde kava 1929. a. I poolaastal. — **IX 2.** Loeng. ja prakt. tööde kava 1929. a. II poolaastal. — **IX 3.** Eesti Vabariigi Tartu Ülikooli isiklik koosseis 1. detsembril 1929.

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C XI—XIII (1934). **XI 1.** Loeng. ja prakt. tööde kava 1930. a. I poolaastal. — **XI 2.** Loeng. ja prakt. tööde kava 1930. a. II poolaastal. — **XI 3.** E. V. T. Ü. isiklik koosseis 1. dets. 1930. — **XII 1.** Loeng. ja prakt. tööde kava 1931. a. I poolaastal. — **XII 2.** Loeng. ja prakt. tööde kava 1931. a. II poolaastal. — **XII 3.** E. V. T. Ü. isiklik koosseis 1. dets. 1931. — **XIII 1.** Loeng. ja prakt. tööde kava 1932. a. I poolaastal. — **XIII 2.** Loeng. ja prakt. tööde kava 1932. a. II poolaastal. — **XIII 3.** E. V. T. Ü. isiklik koosseis 1. dets. 1932. — **XIII 4.** K. Schreinert. Goethes letzte Wandlung. Festrede. — **XIII 5.** R. Mark. Dotsent Theodor Korssakov †. Nekroloog.

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C XV (1932). L. Villecourt. L'Université de Tartu 1919—1932.

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C XIX—XXI (1939). **XIX 1.** Loeng. ja prakt. tööde kava 1936. a. I poolaastal. — **XIX 2.** Loeng. ja prakt. tööde kava 1936. a. II poolaastal. — **XIX 3.** E. V. T. Ü. isiklik koosseis 1. dets. 1936. — **XIX 4.** V. Paavel. Inseneri tegevus, selle eesmärk, iseärasused, alused ja tulevikusihid. — **XX 1.** Loeng. ja prakt. tööde kava 1937. a. I poolaastal. — **XX 2.** Loeng. ja prakt. tööde kava 1937. a. II poolaastal. — **XX 3.** E. V. T. Ü. isiklik koosseis 1. dets. 1937. — **XXI 1.** Loeng. ja prakt. tööde kava 1938. a. I poolaastal. — **XXI 2.** Loeng. ja prakt. tööde kava 1938. a. II poolaastal. — **XXI 3.** E. V. T. Ü. isiklik koosseis 1. dets. 1938. — **XXI 4.** Vakantsele Tartu Ülikooli kirurgia-õppetoolile kandideerijate teaduslikkude tööde arvustused. — **XXI 5.** Vak. T. Ü. farmakoloogia-õppetoolile kandideerijate tead. tööde arvustused. — **XXI 6.** Vak. T. Ü. õpetatud sepa kohale kandideerija tead. tööde arvustused. — **XXI 7.** Vak. T. Ü. Eesti ja naabermaade muinasteaduse õppetoolile kandideerija tead. tööde hinnang. — **XXI 8.** T. Ü. vak. günekoloogia ja sünnitusabi professuurile kandideerija tead. tööde arvustused. — **XXI 9.** T. Ü. vak. eugeenika professuurile kandideerija tead. tööde arvustused. — **XXI 10.** T. Ü. vak. eripatoloogia, diagnostika ja teraapia (polikliiniku) professuurile kandideerijate tead. tööde arvustused. — **XXI 11.** T. Ü. vak. füsioloogia ja füsioloogilise keemia professuurile kandideerija tead. tööde arvustused. — **XXI 12.** Arvustajate hinnangud ja arvamused E. V. T. Ü. majandusteaduskonna vak. panganduse ja kindlustusasjanduse õppetoolile kandideerija tead. tööde ja sobivuse kohta. — **XXI 13.** T. Ü. vak. loomaarstiteaduskonna anatoomia prosektuurile kandideerija tead. tööde arvustused.

CXXII (1937). Teise Balti riikide vaimse koostöö kongressi toimetis **29.** ja **30.** nov. 1936 Tartus. (Actes du Deuxième Congrès Interbaltique de Coopération Intellectuelle tenu à Tartu les 29 et 30 novembre 1936.)

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