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Master’s thesis

Mechanical Effects of Electoral Systems on Proportionality and Parliament Fragmentation

Kadri Vakmann

Supervisors: Prof. Rein Taagepera
Mihkel Solvak, PhD

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Kadri Vakmann

Contents

Abstract

The following master's thesis studies the mechanical effects of electoral systems on two electoral outcomes – proportionality and parliament fragmentation. The aim of the study is to investigate whether there is a precise universal relationship between proportionality and fragmentation across different electoral systems. The thesis places itself into the general framework of new institutionalism, saying that institutions including electoral systems matter, but their precise effects depend on the context in which they operate. We propose that if context is taken into account and held under control while analysing pure mechanical effects of electoral systems, a clear universal pattern emerges between proportionality and fragmentation. A computational experiment is carried out using constituency level data of 5 countries and 10 elections from the CLEA database. The results show a squared relationship between proportionality and parliament fragmentation, not a linear one that has been a tacit assumption in the debate between the proponents of majoritarian and proportional electoral systems. Testing this squared relationship on broader data proves its validity, especially for PR electoral systems.

Introduction

Electoral systems are crucial institutions for democracy to function. The choice of electoral system determines the main characteristics of the democratic regime as it significantly influences political outputs. In addition to the fact that electoral systems allocate parliamentary seats – who gets in and who is left out – they shape the main features of political system and the nature of party competition more broadly. It is important to analyse electoral system effects in order to better understand the functioning of democratic political systems around the world.

There has been a general consensus in the literature of electoral studies that there is no such thing as the ideal electoral system. It is standard practice to acknowledge the inevitability of trade-offs when discussing the pros and cons of different systems (Carey, Hix 2011: 383). There are two main conceptually conflicting ideals of electoral systems – the ideal of representativeness (i.e. high proportionality) and the ideal of accountability (stable single-party governments, i.e. low fragmentation). The representation ideal is best achieved by proportional representation (PR) electoral systems; accountability ideal by majoritarian electoral systems.

It is widely accepted that these two ideals are contradictory as it is impossible to achieve both highly representative as well as highly accountable parliament. It seems to hold true, because high proportionality allows more actors into the parliament, which by definition means higher parliament fragmentation. One must decide what she cares about most: representation or accountability, because one cannot have both.

We could think that the trade-off between representation and accountability is thus linear. The higher the proportionality, the higher is the fragmentation, and vice versa. Furthermore, as electoral systems are sets of mathematical rules, we would expect to see a clear linear pattern when graphing fragmentation measures against proportionality measures. However, simple descriptive statistics of electoral outcomes show that these expectations do not hold true (see Figures 2.1 and 2.2 in Chapter 2). There is by no means a linear relationship between those two phenomena. Moreover, the picture seems blurry – there is no precise pattern at all between proportionality and fragmentation. So it is important to investigate why there is no clear pattern that would reflect the contradictory nature of the two ideals when comparing proportionality and fragmentation indices.

Building on the new institutionalism framework, we propose that we cannot see a clear pattern in electoral outcomes without taking context into account. According to the new institutionalist logic, institutions (including electoral systems) are important as they affect the political outcomes. But institutions are also endogenous – their functioning depends on the conditions in which they emerge and operate. Thus, when analysing electoral systems' mechanical effects on proportionality and fragmentation, we need to hold context under control in order to see a clear pattern.

The aim of this thesis is to gain better insight into mechanical effects that electoral systems have on proportionality and fragmentation. The first main analytical question is how various electoral systems (i.e. different electoral formulae or district magnitudes) in different contexts affect proportionality and parliament fragmentation and whether there exists a universal relationship between those phenomena. We propose that there is a universal relationship between proportionality and fragmentation, but only when context is held under control by normalising the values of proportionality and fragmentation indices. The second question addressed in this thesis is whether the relationship between proportionality and fragmentation is linear. We expect to see that the trade-off between representation and accountability is non-linear, as Carey and Hix (2011) proposed, and that low-magnitude districts optimise this trade-off.

To answer the proposed questions, a computational experiment is carried out. Using regional level data for 5 countries and 10 elections from the CLEA database, we calculate alternative seat distributions for each election if different electoral rules were used. We vary both the electoral formulae as well as district magnitude in our experiment. This computational experiment gives us at least six different seat distributions for every vote distribution. By holding the input side (vote distributions) constant, the variations in seat distributions can be attributed to electoral systems' mechanical effects only. Normalising the values of proportionality and fragmentation allows us to meaningfully look into the universal relationship between those features. We believe that the initial blurry picture then becomes clearer.

The thesis is divided into four main chapters. The first chapter gives an overview of the importance of electoral systems and their effects. It discusses how in addition to mechanical effects, electoral systems also have psychological effects that cannot be ignored when analysing the former. Psychological effects make it difficult to study "pure" mechanical effects, but there are ways to overcome the challenges. The second chapter theoretically explains the two main concepts of this thesis, proportionality and parliament fragmentation, and discusses the conflicting nature of the two democratic ideals of representation and accountability. The third chapter describes in detail the method used in this thesis and how context is taken into account. Furthermore, operationalisation of the central concepts is provided in this chapter as well as the overview of the cases and data. The final chapter is fully dedicated to the empirical study and its results – the precise relationship between proportionality and fragmentation is analysed and tested.

1. Why and how to study electoral system mechanical effects?

There is a growing body of literature on electoral system effects. The understanding of how different variables such as district magnitude and electoral formula affect electoral outcomes has improved significantly. But there is still much left to discover and explain. Electoral systems are essentially sets of mathematical rules, which means that their effects should be quite accurately predictable. However, many of electoral system effects are not yet so precisely described as they could be due to their mathematical nature. This thesis tries to fill this gap by analysing mechanical effects on proportionality and fragmentation – not an easy task as mechanical effects do not exist separately from the broader context in which they operate. In the following chapter it is explained (1) why it is important to study electoral systems and their mechanical effects, (2) why it is so challenging to ascertain precise electoral system effects and (3) how to possibly overcome these challenges.

1.1. Importance of electoral systems

According to the theory of new institutionalism, institutions are important as they influence norms, actions and beliefs, and thus affect the outcomes. New institutionalism also proposes that institutions are endogenous, which means that their form and functioning depends on the conditions in which they emerge and endure (Przeworski 2004: 527). This work builds on a new institutionalist framework. The notion that institutions significantly affect outcomes, but their functioning and precise effects are affected by the context, in which they operate, is considered as a theoretical basis of this thesis.

So, it is clear that institutions matter. Electoral systems are part of democratic institutions, so they matter too. Of course, like Taagepera (1998) has said, content matters more than containers (i.e. institutions). But the latter is still important. Why? Because institutions affect the outcome and indirectly they may even go as far as to affect the content. According to Taagepera, it is false to ask whether institutions (incl. electoral systems) matter or not, the question should be instead how much they matter and how (Taagepera 1998: 68). The following section tries to briefly open those questions.

There is a connection between institutional inputs and political outputs. One of the important institutional inputs is the electoral system, which influences political outputs, such as the number of parties in the party system. Studying electoral system effects helps to improve our understanding of the connection between these aforementioned inputs and outputs (Taagepera 2007: v). This is the main reason why it is important to discover in quantitative detail how electoral systems work.

First of all, it is necessary to clarify, what an electoral system is. An electoral system is essentially a set of rules. It specifies the ballot structure and the mechanism how votes are translated into seats. Because of the latter, the system must at least specify the electoral districts, the district magnitude and the seat allocation formula. So, when analysing electoral system mechanical effects, all these three components of the system must be taken into account.

Differences in those three seat allocation components, i.e. how votes are translated into seats, can lead to drastically different outcomes (Taagepera 2007: 2). Electoral systems as important institutional inputs can influence the political output in a variety of ways. The most straightforward influence is that electoral systems determine who gets into the representative assembly and into the governing cabinet. This means, electoral systems directly affect the answer to the question "Who governs?" (Taagepera 2007: 1). This can be considered as an electoral system direct influence. As a result of this direct effect, the electoral system can encourage or discourage the rise of new parties, determining the number of parties competing in the elections and thus, in long term, shape the main characteristics of the political system and the nature of party competition.

To go even further, the electoral system has a role to play in shaping the broader political culture in the country, because the nature of party competition and political system characteristics may influence the political culture significantly. As an example, an electoral system that strongly encourages new parties to enter the party competition may lead to excessive parliament fractionalisation. This again may lead to the kind of political system where in every election we see new parties, new winners and a very little continuity between two governments. This instability definitely affects broader attitudes, beliefs and sentiments in the country, which to put it very simply, constitute the political culture. It is important to emphasise that this is a two-way process – the political culture also (maybe even more importantly) influences the political system. This may mean that in our example where the electoral system is extremely favourable to new parties, the political system might still be relatively stable because of a mature political culture where people highly value stability, which reflects in their voting behaviour.

Stable electoral systems, according to Taagepera (1998), consist not only of rules, but also of the way these rules are used in the democratic culture. This democratic culture again includes some concern for stability, but also avoidance of miscalculations that result from limited knowledge about the effects of given electoral rules. This understanding of the effects of electoral rules comes with time (Taagepera 1998: 71). So it is evident that electoral systems in longer term influence significantly voters' behaviour and the political culture of the country.

Electoral systems can be compared and assessed based on many criteria. Most widely used examples of these criteria are electoral system proportionality and two-party or multi-party political system (Gallagher 1992: 469), i.e. the number of political parties in the party system. Proportionality and number of parties are central in this work as well, because they convey in their essence the main importance of the electoral system: electoral system effect on the political system and political competition. Distinction between proportional and majoritarian systems, is considered as one of the main determinants of the nature of political competition. The number of parties in the system, on the other hand, gives us a starting point of what the political system is like.

Electoral systems matter as they influence who governs, determine the nature of political competition, the number of parties in the system and even shape the characteristics of political culture. There are a considerable number of empirical studies that show electoral systems' influences on a wide array of dependent variables. To point out just few, electoral systems have an effect on voter turnout (Singh 2011), voter incentives (Cox 1997), incentives of parties and candidates (Carey, Shugart 1995; Cox

1990), corruption level (Myerson 1993), representation of women and minorities (Rule, Zimmerman 1994), etc.

In the discussion above some major aspects of why electoral systems matter are outlined. This discussion is by no means compete, but should be sufficient to admit that it is well worth to study electoral system effects in detail. By doing that, we can contribute to better understanding of the political system as a whole. As electoral rules are mathematically precise, they should have strongly predictable outcomes. Because of the mathematical nature of electoral rules, as well as the fact that the central data for electoral studies are in quantitative form (numbers of votes and seats), there is a good opportunity to carry out a quantitative analysis of electoral system effects (Taagepera, Shugart 1989: xi). But as outlined before, electoral systems often have indirect effects and the political system is also influenced by many other factors, so it may be tricky to analyse the effects produced by electoral rules only. The rest of this chapter deals with the difficulties of ascertaining pure electoral system mechanical effects and gives some examples of how to overcome these obstacles.

1.2. Psychological effects blur the picture

We are interested in analysing direct electoral system effects on electoral outcomes such as proportionality and parliament fragmentation. But when doing it, it is important to bear in mind that electoral systems do not exist in a vacuum. Electoral outcomes are also significantly influenced by other structures, institutions and broader contextual features (Horowitz 2003). Electoral system direct effects come into play when votes have to be translated into seats. But other factors may influence the composition of votes, i.e. the electoral input, in the first place. Electoral system direct mechanical effects are thus influenced by its indirect effects as well as contextual features. In this section it is briefly explained how these indirect effects together with contextual features blur the pure mechanical effects of electoral systems.

As proposed by Duverger (1951), electoral systems have psychological and mechanical effects. Mechanical effects refer to how votes are translated into seats, psychological effects, on the other hand, shape how parties and voters react to the limits set by electoral rules (Duverger 1959: 224). This means that mechanical effects occur after the vote, but psychological effects before or at the moment of the vote (Blais et al. 2011: 1600).

Taking this Duvergerian distinction between electoral system effects into account, it is evident that the precise mathematical nature of electoral system effects holds true only when speaking of mechanical effects. The definition of electoral systems as sets of rules that determine how votes are translated into seats already refers to this mechanical aspect. Psychological effects of electoral systems, on the other hand, are not mathematically precise and thus also much harder to ascertain.

Logically, it should be relatively easy to ascertain electoral system mechanical effects, because the mathematical rules should produce an accurately predictable outcome. But as the electoral system input, i.e. votes, are influenced by psychological effects, it may blur the picture and overshadow mechanical effects. It definitely is the case when analysing electoral system effects on proportionality, because proportionality is affected both by votes and seats, where former is influenced by psychological effects, but latter is a result of electoral system mechanical effects.

According to Blais and Carty (1991), upon analysing electoral system psychological effects it is useful to distinguish effects on parties and on voters. Electoral system psychological effects on voters mean mainly the influence on voters' strategic considerations. To put it simply – voters make different choices under different electoral rules (Blais et al. 2011: 1600). Yet this claim is only valid when talking about strategic voters, i.e. voters who vote for those parties that they believe have a good chance of getting elected. The other two types of voters (using Marsh and Franklin (1996) typology), sincere and protest voters are not influenced by electoral system psychological effects in the same way. Sincere voters vote for their most favourable party even if it does not have a chance of getting elected – electoral system psychological effects do not come into play. Assessing electoral system psychological effects on protest voters is difficult as their incentives for casting a protest vote may be very different and only one of them is a protest against electoral system.

Electoral system psychological effects also affect parties, i.e. the political supply side. Similarly to (strategic) voters, parties also change their behaviour as a reaction to electoral system mechanical impact (Blais et al. 2011: 1600). While voter behaviour essentially means for which party she casts her vote, parties have more repertoires. Electoral system psychological effects may change party elite incentives to form or dissolve a party, to run or not to run candidates, to stand in some districts but not in others, to form alliances or merge with other parties etc. It is clear that these psychological effects on parties also influence the set of choices available for voters. If there are only two or three parties competing because of the majoritarian electoral system effect, then voters have much less to choose from than in a country where proportional representation system is used and where also many smaller parties decide to run in elections.

To sum up, according to Blais et al. (2011) there are three kinds of electoral system effects. Firstly, mechanical effects, which are further analysed in this thesis, and which determine the distribution of seats. Secondly, there are psychological effects on voters, which influence the distribution of votes. And thirdly, there are psychological effects on parties, which affect the number of competitors. As Blais and others put it, the total effect of electoral system is the following (Blais et al. 2011: 1602):

Total effect = Mechanical effect + Psychological effect (voters)+ Psychological effect (parties)

When psychological effects of electoral systems occur before or at the moment of the vote and mechanical effects after it, it is obvious that for a complete understanding of electoral system effects we must also look "inside" the vote, not only at how votes become into seats. It means we need to study what incentives and calculations are behind voters' choice and what strategies parties use and why. This is in fact an enormous field of study in political science. Furthermore, these incentives and strategies, i.e. psychological effects are constantly changing and somewhat different from one election to another. Thus conducting a comprehensive comparative study of electoral system mechanical effects, with psychological effects accurately included into the analysis, is very ambitious and I would even say impossible.

Resulting from the above discussion, it should be understandable that for analysing electoral system mechanical effects, inevitably some simplifications have to be made. One cannot take into account everything, so concessions are necessary. But these concessions have to be thought-out and they must be kept in mind when interpreting results. In the following section, some ways how scholars have made those simplifications and how they have analysed "pure" mechanical effects are discussed.

1.3. Analysing "pure" mechanical effects

For the previously mentioned reasons, it is not an easy task to ascertain the electoral system mechanical effects. But since we need to do it in some way to analyse the effects on proportionality and fragmentation, we must find the best method. The analysis can be performed by studying individual countries or by comparing electoral outcomes across countries, which is the most common way in the literature. Alternative and much less common methods for this widely used approach are experiments, which can be either natural (e.g. comparing outcomes before and after electoral system change) or lab- or quasi-experiments (e.g. based on questioning voters how they would have voted under different electoral systems) (Blais et al. 2012: 829-830). The following section discusses these options for analysing "pure" mechanical effects, highlights their limitations and proposes an alternative.

Focusing on an individual country and its electoral system is one way to analyse electoral system effects. This essentially means carrying out a comprehensive case study. This way it is possible to concentrate on both the electoral outcomes as well as on the specific context in which the system operates. Furthermore, it is important that someone compiles country's election results and describes electoral system in detail throughout its historical unfolding (Taagepera, Shugart 1989: 61). But the greatest drawback in this type of a study is that it can say hardly anything about how different electoral systems work. Analysing only one country and its system makes it impossible to draw any broader conclusions about electoral system mechanical effects. But it definitely is the first important stage in electoral studies, after which comparative insights can be applied (Ibid.).

Comparing electoral outcomes across countries is the most widely used approach in studying electoral system mechanical effects. It has its merits, but also some significant shortcomings. By comparing electoral outcomes, it is possible to classify the various electoral systems worldwide and make some general conclusions about their mechanical effects. But the most important shortage of this method is that it ignores the second new institutionalist proposition that context is important. By only focusing on electoral outcomes, the conditions in which the electoral system operates are overlooked. Furthermore, as Blais and colleagues have said, the observed differences in outcomes might be due to other characteristics that are correlated with electoral systems (e.g. PR may be more common in more heterogeneous settings) (Blais et al. 2012: 829).

Carrying out a so-called “natural experiment” is a unique but good method to ascertain electoral system mechanical effects. There are two kinds of natural experiments. Firstly, a natural experiment can mean comparing electoral outcomes before and after electoral change. The second option is to analyse electoral outcomes of two simultaneous elections that use different electoral systems. Basically it means that there are some rare instances of electoral change or simultaneous elections and scholars use these golden opportunities for ascertaining electoral system mechanical effects. According to Blais et al. (2012), under a natural experiment of electoral system change one can be confident that the differences in outcomes are not due to other societal factors, because we are dealing with the very same society (Blais et al. 2012: 829). Some problems still remain – one cannot rule out the possibility that there were other factors changing together with electoral system. Furthermore, these kind of electoral changes to electoral system are rare and thus the number of cases is limited (Ibid.).

To overcome these aforementioned shortcomings, there is a possibility not to wait for those “natural experiments” to happen, but to carry out a lab or quasi-experiment. These kinds of experiments are based on questioning voters how they would have voted under different electoral systems (Blais et al. 2012: 829-830). A lab experiment means that people are invited to vote in different elections under different electoral systems and then the outcomes are compared by the researcher. A quasi experiment is similar to the lab experiment, with the difference that people are invited to vote in a number of elections, but in a “real” election campaign context (Ibid.). These methods allow us to hold the context in which electoral systems operate under control, but they still have significant shortcomings. One of the problems is similar to the “natural experiment” and that is data availability – carrying out a considerable amount of lab or quasi experiments among voters (and maybe even parties) takes a lot of effort and money. Furthermore, it is questionable whether people behave in the same way in the real world as under the experiment.

It is evident that electoral system effects (both psychological and mechanical) vary across cases (Bodet et al. 2013: 2). Reasons for those variations are related to contextual features, such as social divisions, the age of democracy, citizen experience with the system, the strength of preferences, the entry of new parties etc.

One of the options to analyse "purer" mechanical and psychological electoral system effects is to hold those aforementioned contextual features constant (Ibid.: 3). It is quite hard to do without carrying out a quasi experiment (as Blais et al. did in 2012), because it is important how voters and parties would behave under different electoral systems in the same context. But another way to hold the context constant it is to carry out a "computational experiment". The logic of this kind of experiment is best summarised by Blais et al. (2011: 1600):

> *"To ascertain the mechanical effect, we determine how different final outcomes (that is, the number of seats won by the various parties) would have been if we let the electoral system vary while keeping constant the vote distribution."*

This method is by no means perfect. However, effectively, it allows us to hold the psychological effects under control. This simplification allows us to presume that context remains constant when the input side (competing parties and the vote distribution between them) is held constant. This approach is described in detail in Chapter 3.

2. Proportionality and fragmentation

Being interested in the relationship between proportionality and fragmentation of electoral outcomes leads us to the very basis of the debate between the proponents of majoritarian and proportional electoral systems. Proportional electoral systems are associated with the ideal of high representativeness and high proportionality, while majoritarian systems lead to the ideal of stable and accountable governments by reducing the number of parties (i.e. fragmentation). At the core of the debate is the trade-off between representation and accountability and the tacit assumption that this trade-off is absolute. This section gives an overview of this trade-off and of the central concepts in this thesis – proportionality and fragmentation.

2.1. Trade-off between representation and accountability

There has been a long history of the debate over evaluating outcomes of electoral systems. The main line in this debate has been related to the differences between plurality and list PR systems and which one of these two is better. The widespread consensus in the literature is that there is no such thing as the ideal electoral system (Carey, Hix 2011: 383). PR and plurality systems have inherently competitive values and ideals, so one must make an inevitable trade-off when choosing an electoral system.

In the plurality-PR debate there are two major arguments. Firstly, plurality rule proponents see as its great advantage that it produces, at least more likely than PR, a firm and accountable government (Lijphart, Grofman 1984: 5). Moreover, with this kind of a firm, one-party government a more broad ideal is pursued – the ideal of stability (Carey, Hix 2011: 383). So, plurality rule leads to a stabile and accountable government by reducing the number of parties that make it into the parliament. The second main argument, proposed by PR proponents, is that a democratic legislature should be a microcosm of the views and interests in the electorate, i.e. representative, and thus proportionality is even considered “virtually synonymous with electoral justice” (Lijphart 1984: 140).

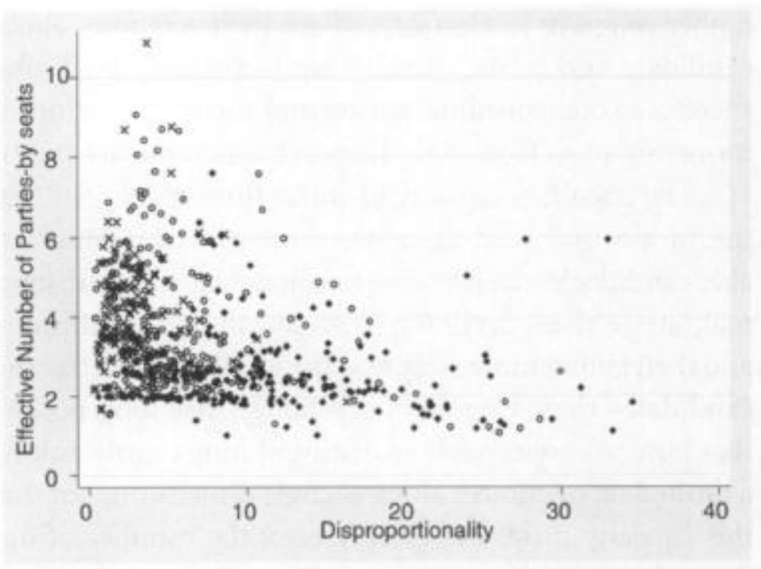
The central trade-off in electoral system design is therefore between the accountability of government and representation of voters' preferences (Carey, Hix 2011; Powell 2000). Every country has to choose which ideal they care about most. If they choose that they want an accountable and stable single-party government, majoritarian system is the best. But if they care most about having a highly representative parliament where the pluralism of opinions in the society is represented, proportional representation electoral system suits best (Carey, Hix 2011: 383).

It seems to be quite straightforward that those two democratic ideals, stabile and accountable government on the one hand and representative legislature on the other hand, cannot coexist. These ideals are indeed inherently competitive. The larger the proportionality, the more fragmented and unstable is the composition of the parliament. This basic logic of trade-off between those ideals is vividly formulated by Carey and Hix (2011: 383): *"You have to choose which you care about most: representation or accountable government. You cannot have both, so the mantra goes."*

There are some doubts in the literature concerning this widely accepted view of clear trade-offs. Not all authors agree with the notion that PR and plurality systems should be regarded as completely different systems. Lijphart and Grofman say that these competing values, such as stability and proportionality, may be differently satisfied by different electoral systems and that the dichotomy between PR and plurality is thus misleading (Lijphart, Grofman 1984: 4). Along the same lines, Carey and Hix doubt that the trade-off between representation and accountability is linear, and they suggest that small multimember districts help to maximise these competing objectives – there exists a so-called electoral "sweet-spot" (Carey, Hix 2011).

Carey and Hix (2011: 386) illustrate graphically their concern that accountability-representation trade-offs are not linear. Their graph (Figure 2.1), where Gallagher's disproportionality index is graphed against the effective number of parties (see Chapters 2.2; 2.3 and 3.2 for details about conceptualisation and operationalisation of representation and accountability), leaves no doubt – the relationship is far from being linear. The data are concentrated along the axes, which mean that there is higher proportionality in elections with lower fragmentation and vice versa. But there are a large number of observations that score relatively low on both variables (Carey, Hix 2011: 387).

Figure 2.1. Trade-off between disproportionality of representation and party system fragmentation



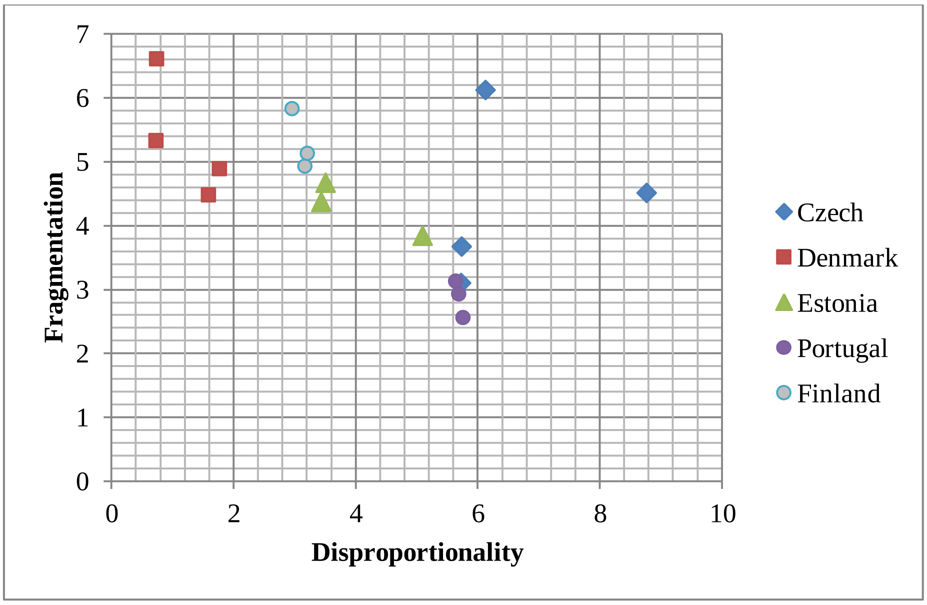
Note: Diamond = system with single-member districts. Circle = system with a median district magnitude between 2 and 10. Cross = system with a median district magnitude greater than 10.

Graph by Carey and Hix (2011: 386)

Below is a similar graph as in the article by Carey and Hix (2011), but it uses the data from the parliamentary elections of 5 countries only (Figure 2.2). These are the same countries that are used in the analysis of this study. Similarly as Carey and Hix concluded, we cannot see in this graph a clear negative linear relationship between disproportionality and parliament fragmentation.

This empirical reality is convincing enough to say that the trade-off between representation and accountability is not as straightforward as it may seem at first glance. It is useful to think of the relationship between accountability and representation "as a convex maximization problem rather than as a straightforward trade-off" (Carey, Hix 2011: 385). So, it is important to study why these intuitively conflicting ideals do not fit into the linear model, what is the precise relationship between those phenomena and why. These questions are considered as the main analytical motivation for carrying out the experiment in this thesis (Chapters 3 and 4).

Figure 2.2. Trade-off between disproportionality of representation and party system fragmentation for 5 sample countries



2.2. Proportionality of electoral systems

Electoral proportionality is one of the most important electoral outputs. As Gallagher has said, when talking about assessment of different electoral systems, the concept of proportionality "always comes to the fore" (Gallagher 1991: 33). As it often happens with widely used concepts, their precise meaning may get fuzzy. For that reason it is important to clarify what exactly we mean by electoral proportionality. Furthermore, this section gives some preliminary insights into the proportionality of various electoral systems. The related topic of how to measure proportionality is discussed in the next chapter (Chapter 3.3).

Proportionality, to put it simply, means a relationship between seat and vote shares of parties (Taagepera 2007: 65). Ideal proportionality would be when all parties competing in the election receive the exact same share of seats in the parliament as they won of the vote - in other words, when seat shares equal vote shares (Ibid: 66). This situation, however, is nearly impossible as it can happen only exceptionally (Gallagher 1991: 33).

Why? The short answer would be that the parliament is always much smaller than the electorate and parties can only have whole number of seats, not fractions. For instance, if we have 1 million votes and 100 seats, we would need votes for every party to be divisible by exactly 10000 for to achieve a perfect proportionality. Otherwise, perfect proportionality becomes impossible. If a party gets 15000 votes, it would mean 1.5 seats in this hypothetical example – but in real life, a party can only have either 1 or 2 seats and this already means that some deviation from perfect proportionality is inevitable.

For the reason that perfect electoral proportionality hardly ever occurs, it is more useful to talk about disproportionality rather than proportionality. Disproportionality means deviation from the perfect proportionality. Even most proportional electoral systems have some deviation from proportionality, which means some degree of disproportionality is inevitable (Gallagher 1991: 33).

Using the concept of disproportionality, different electoral systems can be assessed based on how proportional outcomes they produce on average. It is important to emphasise that the proportionality of the outcome depends on the input (i.e. vote shares), but nevertheless, some generalisations about average proportionalities of various electoral systems can be made. Lijphart 1986 have ranked the most common PR electoral systems based on how proportional outcomes they generally produce. Lijphart's (1986: 178) ranking is the following (systems that are included into analysis of this thesis are highlighted in boldface):

1. **Largest remainders (Hare)**
2. Single-transferable vote
3. **Sainte-Lague**
4. Imperiali largest remainders
5. **D'Hondt**
6. Imperiali highest averages

It is clear that proportionality (or disproportionality) of an electoral system is influenced by the specific seat allocation formula. But it would be misleading to think that all or most of the deviation from proportionality can be attributed to the formula. Other main sources of disproportionality are, according to Gallagher, distribution of votes between the parties, district magnitude, the possibility of malapportionment, and the use of thresholds (Gallagher 1991: 43). Gallagher (1991), Taagepera and Shugart (1989) and many others emphasise most the importance of district magnitude, saying that it is the major determinant of proportionality. District magnitude directly influences the proportionality of the electoral outcome, because the larger the district magnitude, the more closely each party’s seat share tends to correspond to its vote share (Taagepera, Shugart 1989: 19).

Following this logic that the district magnitude is the most important determinant of how proportional outcomes the system produces, there is a clear difference between majoritarian and PR electoral systems. It is widely accepted in the electoral studies literature that PR systems are generally quite successful in achieving reasonably proportional translation of votes into seats, which is essentially their principal goal (Lijphart 1986: 170). This is especially so in comparison with plurality and majority formulae (Ibid.). According to Rae’s proposition, PR formulae tend to allocate seats more proportionally than majority formulae (Rae 1971: 96). So the general conclusion is that PR systems produce lower levels of disproportionality than majoritarian electoral systems. It is also known, both from theoretical literature and empirical analyses, that different PR formulae produce different outcomes of proportionality (Lijphart 1986: 170). This means that not all PR systems are equally proportional.

Deviation from proportionality is an important characteristic of the entire electoral system, and it can be used for comparing different countries and their electoral systems. But the mere deviation from proportionality does not tell us whether the system advantages large or small parties (Taagepera 2007: 70). Even though higher proportionality and whether the system is less favourable towards larger parties usually go hand in hand, it is important to bear in mind that this is not always the case.

Gallagher (1992) has done the analysis comparing 11 different electoral systems and ranked them based on how favourable these systems are to larger parties. From the most favourable to larger parties to the least favourable, the order is the following (systems that are included into analysis of this thesis are highlighted in boldface) (Gallagher 1992: 490):

1. Imperiali highest averages
2. LR-Imperiali
3. **D’Hondt**
4. STV

5. **Largest remainders-Droop**
6. **Largest remainders-Hare/Sainte-Lague**
7. Equal proportions
8. Danish
9. Adams

In this analysis, we might expect (according to Gallagher 1992 and Lijphart 1986 rankings) to see higher proportionality when largest remainders-Hare or Sainte-Lague system is used, compared to d'Hondt or largest remainders-Droop, because the former two are less favourable to winning parties.

2.3. Fragmentation of the parlament

Another crucially important electoral output along with proportionality is the number of parties. This means how many parties there is in a party system. Is there a two-party or a multi-party system in the country? It is a very important question as two-party systems have remarkably different characteristics from systems with many parties.

According to Taagepera (2007), the number of parties is one of the most frequently used numbers in political analysis, and it is central to the study of party systems. It is impossible to describe a party system without giving some idea of how many actors are involved (Taagepera 2007: 47). It is even said that the number of parties is "a most important feature in a county's politics and therefore in comparative studies also" (Taagepera, Shugart 1993: 455). The number of parties is directly related to the concept of fragmentation. The larger the number of parties, the higher the party system fragmentation. This section briefly discusses the importance of the concept of fragmentation as an electoral system output.

From the previous discussion we saw that majority and plurality electoral systems produce firm accountable governments, at least when compared to PR systems. According to Carey and Hix, PR systems can, indeed, produce broad and fractious coalitions (Carey, Hix 2011: 384). Some scholars have said that a large fractionalisation tends to destabilise a political system (Duverger 1954), but some have said it does not (Lijphart 1968). We can see empirical evidence from both sides. So, we cannot judge whether large fragmentation of the party system is generally a good or a bad thing. But according to the proponents of "accountability" ideal, the least fragmented government and parliament is the most desirable electoral outcome.

There are many factors that determine the party system fragmentation. It is influenced by history, institutions, current issues etc. But according to Taagepera and Shugart, the major determinant is the district magnitude, i.e. the number of seats allocated in the single district (Taagepera, Shugart 1993: 455). This argument is coherent with a widely used Duverger rule that one-seat districts tend to lead to a two-party system and multiseat districts tend to go with a multiparty system (Duverger 1954). This Duverger rule is a good starting point, but it is important to bear in mind that multiseat districts can be very different in their size and thus influence the party system fragmentation differently. We might expect that within the multiseat category, a larger magnitude tends to go with a larger number of parties (Taagepera, Shugart 1993: 455).

We can talk about different kinds of political fragmentations: fragmentation of the whole political system, fragmentation of government, fragmentation of parliament. In this thesis, the latter is analysed as it is the most direct electoral system outcome out of those three fragmentations. This is the case because as a result of elections, distribution of parliamentary seats is determined. It is clear that parliament fragmentation is probably correlated to government fragmentation, but there is a mediating mechanism of coalition negotiations that influence the precise composition of the government. For analysing electoral system direct mechanical effect on fractionalisation, the fragmentation of parliament is the most suitable concept.

Parliament fragmentation essentially means how many parties there are in the parliament. The more there are parties in the parliament, the more fragmented it is. But a mere nominal number of parties might not be the best way to measure parliament fragmentation. Why? Because some parties generally have considerably more seats in the parliament than others, which mean they are not equal actors in the parliament. For that reason for measuring the parliament fragmentation, party size should be taken into account. Precise measures how to do it are discussed in Chapter 3.2.

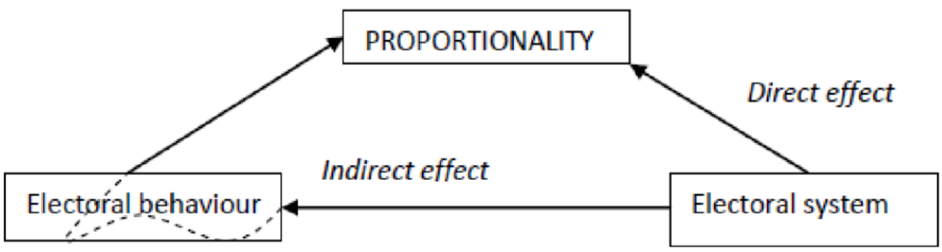
3. Methods, measures and data

From the theoretical analysis it is evident that electoral systems have both mechanical and psychological effects. In other words, electoral systems can affect the outcome both directly (mechanically) and indirectly (psychologically). Both indirect and direct impacts come into play when talking about proportionality and fragmentation. If only interested in direct mechanical effects, one somehow needs to hold the indirect psychological impact under control. This section gives a detailed description of the experimental method how it is done in this thesis. Furthermore, this section introduces the indices for measuring proportionality and fragmentation and the data that is used for carrying out the experiment.

3.1. Ascertaining mechanical effects on proportionality and fragmentation

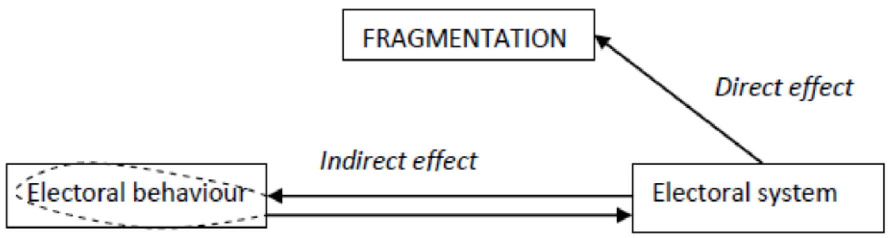
Proportionality is determined by vote shares and seat shares. So, electoral system's direct effect comes into play after votes, i.e. in translation of votes into seats. Electoral system's indirect effect, on the other hand, influences the composition of votes, i.e. what are the vote shares like. This logic is represented in the Figure 3.1. To sum up, electoral systems influence proportionality directly by mechanically making seats out of votes, and indirectly through influencing electoral behaviour which determines the vote shares.

Figure 3.1. Electoral system effects on proportionality



Using the analogy, it seems that parliament fragmentation is only influenced by direct or mechanical effects. As the fragmentation only depends on how seats are distributed among parties and vote shares do not matter, it may lead to the conclusion that only mechanical effects matter. But when starting to think about where those seats come from in the first place, the answer is – from votes. So, vote shares still seem to matter and there is also an indirect psychological effect of electoral systems on parliament fragmentation. It is visualised in Figure 3.2.

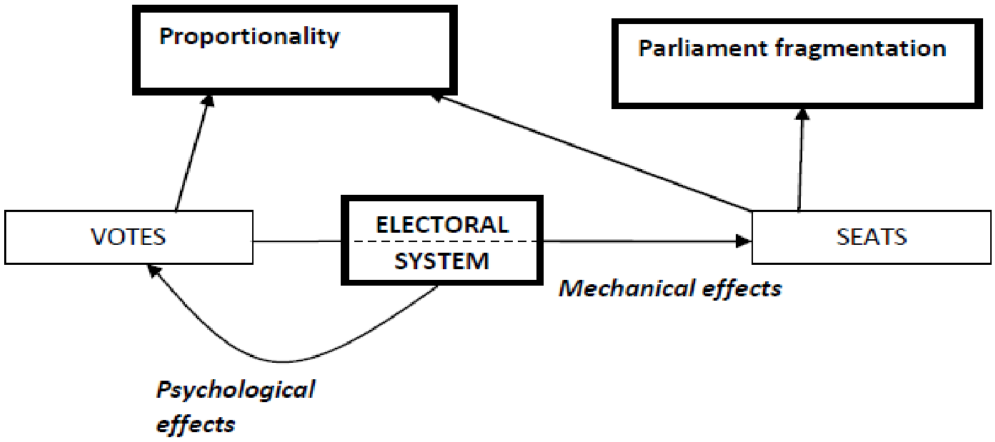
Figure 3.2. Electoral system effects on parliament fragmentation



To make it clearer, it is useful to add to the picture how exactly votes and seats are connected to both variables (fragmentation and proportionality). As we previously saw, votes are problematic – they embody psychological effects that make the study of mechanical effects complicated. But we cannot have seats without votes and thus there would be no mechanical effects without votes. Figure 3.3 explains how votes and seats are exactly related to proportionality and fragmentation. Not surprisingly, proportionality is both influenced by psychological and mechanical effects. But now it is clearly visualised that parliament fragmentation is also influenced by psychological effects, but more indirectly than proportionality and only through the mediation of electoral system.

It should be now visually clear (Figure 3.3) that for analysing "pure" mechanical effects, psychological effects must be held under control. But as psychological effects occur before or at the moment of the vote, they can be held under control when vote shares remain unchanged. So, one way to analyse "pure" mechanical effects is to hold the vote shares constant. By holding vote shares constant and altering the electoral system, we can see, *ceteris paribus*, variations in mechanical effects only.

Figure 3.3. Electoral system effects on proportionality and fragmentation



The method for ascertaining mechanical effects is the computational experiment. The real vote share of an individual election is taken as an input. With this very same input an experiment is carried out – what are the seat shares when different electoral systems are used? As a result of calculations using different electoral rules, we have many different seat distributions for the one vote distribution. And then we repeat the same thing for a number of other individual elections, i.e. vote distributions.

Table 1 contains an example of this computational experiment. The example uses data from Estonian 2007 parliamentary elections. This illustrates vividly how different electoral systems can purely mechanically produce very different outcomes. In this example not a single electoral system out of six produces exactly same outcome with the same input. Some of these systems with this particular input produce more proportional results than others; and some lead to a more fragmented parliament than others.

If we have many different seat distributions for one vote distribution, we can calculate the mechanical effect on proportionality and fragmentation. Precise measures for those indicators are outlined in Chapter 3.2. When input side is held constant then the variations in proportionality and fragmentation indices can be seen as "pure" mechanical effects.

As the psychological side is held under control, it becomes meaningful to graphically analyse the relationship between the indices of proportionality and fragmentation. When the input was not controlled, there seemed to be no precise relationship between proportionality and fragmentation (see again Figures 2.1 and 2.2). But with this computational method, we expect to see a clearer pattern between those variables. In other words, presuming that with this method we can see "pure" mechanical effects, and that those mechanical effects are due to their mathematical nature precisely predictable, we should see a clear relationship between proportionality and fragmentation, as they are conceptually related.

Table 1. Estonian 2007 parliamentary election results and seat distributions under different electoral systems

	Votes (%)	Real seat distribution	D'Hondt	Sainte-Lague	Simple Quota	Droop Quota	M=1
Party 1	28	31	34	27	28	28	68
Party 2	26	29	31	28	27	30	33
Party 3	18	19	19	16	16	20	0
Party 4	11	10	11	12	12	11	0
Party 5	7	6	1	9	9	4	0
Party 6	7	6	5	9	9	8	0
Party 7	2	0	0	0	0	0	0
Party 8	1	0	0	0	0	0	0

3.2. Measuring proportionality and fragmentation

This chapter deals with measuring proportionality and parliament fragmentation. We start with proportionality. As previously discussed, perfectly proportional representation is nearly impossible to achieve. Thus, some deviation from perfect proportionality is inevitable, so it is important to measure the magnitude of this deviation from proportionality. According to Taagepera (2007: 66-67), there are three main indices that measure this deviation from proportionality, i.e. deviation of seat shares from vote shares. But firstly, another index, Rae index, is briefly discussed, as it is the oldest measure of disproportionality.

The index proposed by Rae (1967) is called Rae's deviation (RD). Its formula is the following, where v_i and s_i are the vote share and the seat share of party i:

$$RD = \frac{1}{n}\sum_{i=1}^{n}|v_i - s_i|$$

Rae's index is the oldest measure of deviation from proportionality. The problem with this index is that it is too sensitive for small parties. When there is a presence of small parties, this index underestimates the disproportionality, i.e. produces too low value.

Secondly, there is an index proposed by Loosemore and Hanby (1971) with the following formula:

$$D = \frac{1}{2}\sum_{i=1}^{n}|s_i - v_i|$$

Loosemore-Hanby index takes always higher values than Rae's index. The main problem with Loosemore-Hanby index is its vulnerability to paradoxes. But the Loosemore-Hanby's advantage over Rae's index is that it does not have to disaggregate "other" small parties, i.e. it is not too sensitive for small parties.

Loosemore-Hanby index was a dominant way to measure the deviation from proportionality until Gallagher introduced a new index in 1991. It is often called a "least square" index (LSq):

$$LSq = \sqrt{\frac{1}{2}\sum_{i=1}^{n}(v_i - s_i)^2}$$

In this thesis, Gallagher LSq is used for measuring electoral system disproprotionality. It has been the most widely used measure of proportionality in political science studies after it was introduced in 1991. According to Lijphart (1994), this index is "the most faithful reflection of disproportionality of election results". While some indices produce in some cases too high values (e.g. Loosemore-Hanby) or too low values (e.g. Rae index), this LSq is a "happy medium", as said by Gallagher (1991).

The fourth option for measuring deviation from proportionality is to calculate the largest single difference for any party, let's call it "largest deviation" (LD):

$$LD = \max |s_i - v_i|$$

The beauty of this index is its simplicity. According to Lijphart (1994), who proposed this index, the LD is the simplest way of measuring disproportionality. But for the purposes of this study, this index is not as suitable as Gallagher's, because we need a more comprehensive measurement of disproportionality. Taking into account only the largest single deviation for any party is not enough to make broad conclusions about the electoral system effects.

Another central variable in this analysis is the parliament fragmentation, which means the number of parties in the parliament. How to operationally define the number of parties may sound as an absurd question as the answer seems to be obvious. The number of parties can be simply counted – how many parties are registered in the country, how many parties compete in the elections, how many parties are in the parliament – depending on what we want to know. In this work, we are interested in the latter two. But if there are, for instance, two strong parties and a lot of small marginal parties competing in the elections, it can be misleading to take them all into account as equal. Then it would seem useful to use the number of "relevant" parties instead (Sartori 1976). But how to measure this "relevant" number of parties?

Basically, we want to somehow measure a meaningful number of parties in the parliament, when some parties have a lot of seats and some few. Furthermore, how to measure the number of parties in an election, when some get a considerably larger vote share than others? So, the number of both electoral and parliamentary parties should be measured in a way that takes into account their relative size.

There are some different ways how to take into account parties' relative size when measuring the number of parties. One method is to take into account the relative size of the largest party only. The formula is as follows:

$$N_{s\infty} = \frac{1}{s_1} \quad N_{v\infty} = \frac{1}{v_1}$$

The inverse of the largest party vote share represents the smallest number of "relevant" parties (Taagepera 2007: 48). This is clearly more realistic than a mere nominal number of parties, but it underestimates the number of parties (Ibid.). So, counting the nominal number of parties overestimates the number of "relevant" parties and calculating N based on the relative size of the largest party underestimates this number. The effective number of parties (introduced by Laakso and Taagepera 1979) yields a value of N somewhere in between. Laakso-Taagepera effective number of (1) parliamentary and (2) electoral parties is the following:

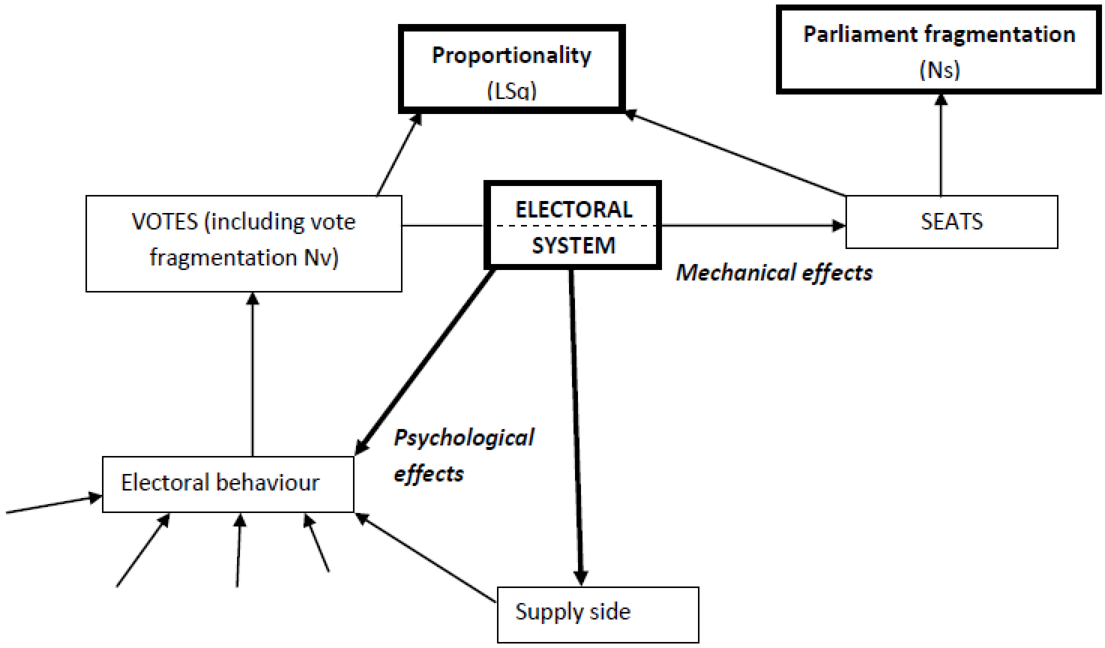
(1) $$N_s = \frac{1}{\sum_{i=1}^{n} s_i^2}$$ (2) $$N_v = \frac{1}{\sum_{i=1}^{n} v_i^2}$$

This measure is most widely used to express the number of parties, as it takes into account the relative size of the components in a meaningful way. Using this measure, one can express the number of components "for any system of qualitatively similar components which differ in size" (Laakso-Taagepera 1979: 23). As it is important to take into account the relative size of parties when analysing parliament fragmentation, this index suits best for operationalising and measuring fragmentation in this thesis.

3.3. Context matters – taking it into account

The concept and measurement of electoral system proportionality are problematic, because they only look at the relationship between vote and seat shares. But this does not entail the whole essence of proportionality. In the literature, when talking about (dis)proportionality, the discussion is mainly focused on the direct effects of electoral systems, i.e. scholars are concerned how proportionally votes are translated into seats (e.g. Gallagher 1991). But I would argue that this kind of (dis)proportionality cannot be presented as a pure "electoral system (dis)proportionality", because it also embodies electoral systems' indirect psychological effects, which are influenced by voters' strategies and preferences and a supply side (see Figure 3.4).

Figure 3.4. Electoral system mechanical and psychological effects on proportionality and fragmentation



There are only some rare instances of scholarly works that do not see (dis)proportionality as a mere vote-seat share relationship. Powell and Vanberg (2000) use a similar logic about the nature of (dis)proportionality as this thesis. They say that the starting point of the analysis of disproportionality should not be votes, but already the voters' preferences (Powell, Vanberg 2000).

This way we can explain why the very same electoral system in the same country can produce a quite different disproportionality index value from one election to another. Not to even mention differences across countries with same electoral system. The explanation lies in the input (vote shares) and in the psychological effects that are behind this particular input. From one election to another and from one country to another, voters' strategic considerations may vary considerably. And these variations lead to differences in electoral outcomes.

Starting the analysis of both proportionality and parliament fragmentation from voters' preferences and strategies takes us too far from our original goal. Moreover, it would certainly be too ambitious for a MA project. For simplification purposes, it seems reasonable to create a model that takes into account the importance of electoral behaviour and contextual features without actually analysing them in detail.

One meaningful way to take context into account is to normalise the values of LSq and Ns. We can do it, because both indicators have their logical minimal and maximal values, or to use Taagepera’s (2008) terminology, anchor points. As maximal values of LSq and Ns depend on the input (vote distribution), then by normalising using these maximal values, the context is taken into account. Logically, it means that for every vote distribution, the maximal values of LSq and Ns are different. Furthermore, by doing normalisation, different electoral inputs and experiments carried out with them become comparable. This means, we can meaningfully put all the cases on the same graph, because their LSq and Ns values are now normalised.

How to normalise LSq and Ns? First, we need to fixate logical anchor points, both maximum and minimum points. With the minimum points, there is no long story. LSq minimal value is 0 when there is a perfect proportionality – seat shares are equal to the vote shares. Ns minimal value is 1, because there must be at least one party in the parliament.

Ascertaining logical maximum values of LSq and Ns is more complex. As previously mentioned, maximum values depend on the context, i.e. vote distribution. So, there are no universal maximum values of Ns and LSq for all cases. Every vote distribution has its own LSq_{max} and Ns_{max}.

Maximal value of disproportionality index (LSq) could be achieved in the situation where a party that received 0% of votes gets 100% of seats and the parties that actually received votes get no seats at all. But this situation is not possible in the case of democratic elections. And as we are interested only in democratic elections and democratic electoral systems, using this maximum would not make sense. To leave this undemocratic extreme out, the highest degree of disporportionality under the democratic rule would be in the situation where the winning party gets all seats and all others are left out. This would be so even if the winning party received only mere 10-20% of the popular vote. In this case, the maximal value of LSq would be very high. But when the winning party receives around 60-70% of the popular vote, the maximal value of LSq will be much lower. So it is clear that the maximal value of LSq depends on the vote distribution, mainly on the winning party's vote share.

Maximal value of the parliament fragmentation (Ns) could be the number of parties that compete in the elections. But as we saw, it is important to take into account the parties' relative strength, not merely nominal number. Competing parties' relative strength can be taken into account when using, again, effective number of parties. Ns is effective number of parties, which reflects how many equal parties there are in the parliament and is thus calculated based on seats. The maximal value of Ns, on the other hand, reflects how many equal parties are competing in the election and is thus calculated based on votes (Nv). To put it differently, if Nv is the maximal value of Ns, then we presume that the output (seats) cannot be more fragmented than the input (votes). Ns measures how fragmented the parliament is and Nv measures how fragmented the input is. We assume that input fragmentation (Nv) sets the upper limit for parliament fragmentation. Ns can in very rare instances be higher than Nv, but usually it does not happen. It can occur only when an electoral system gives disproportionally more seats to the parties with smaller vote shares than for the winning parties. Generally, electoral systems work the opposite way, giving disproportionally more seats to the winners.

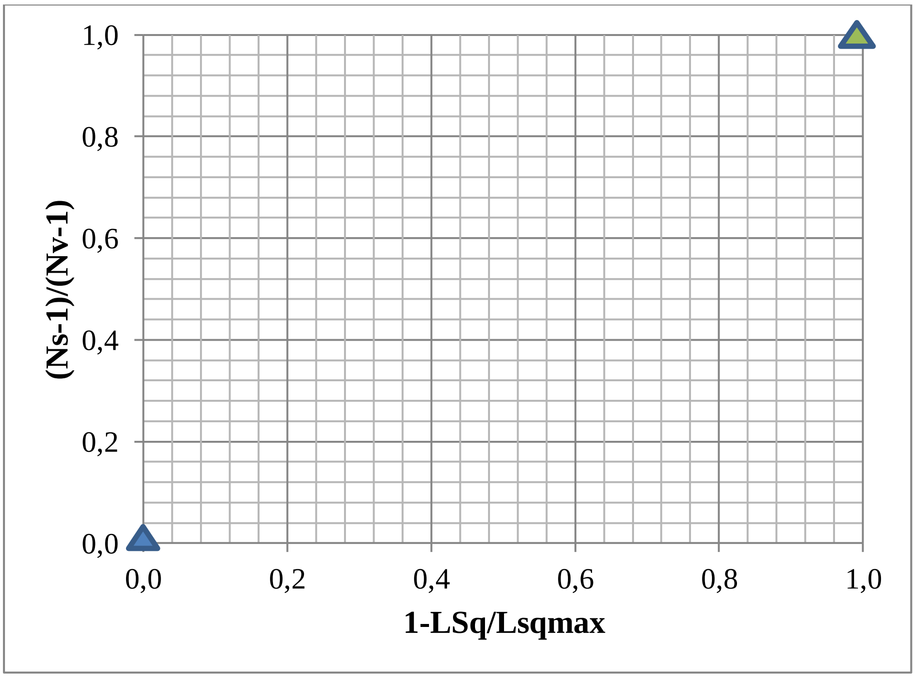
Now we have put in place the anchor points. As a next step we need to create a normalised model using those anchor points. An example of Estonian 2007 parliamentary elections is used to explain this process. Minimum points, as we saw, are universal for all cases, being 0 for LSq and 1 for Ns. Maximum points depend on the input, for Estonian 2007 input, LSq_{max} is 56.7 and Nv (i.e. Ns_{max}) is 5.0. We have two axes, one ranging from 0 to 56.7 and other from 1 to 5.0. But we want to make a normalised model, where both axes range from 0 to 1 and where logical anchor points would be in the points (0; 0) and (1; 1). The normalised LSq (1) and Ns (2) axes are the following:

$$LSq' = 1 - \frac{LSq}{LSq_{max}} \quad (1)$$

$$N_s' = \frac{N_s - 1}{N_v - 1} \quad (2)$$

When LSq equals LSq_{max}, then $1-(LSq/LSq_{max}) = 0$. This means that if we achieve the maximal value of LSq, then we are dealing with the anchor point where the Ns value is minimal. But when LSq equals 0, then $1-(LSq/LSq_{max})$ gets its maximal value of 1. In this case, we are dealing with another anchor point, where also the Ns value is maximal (see Figure 3.5). Upon normalising, we turned LSq around, because we wanted the anchor points to be (0; 0) and (1; 1) instead of (1; 0) and (0; 1) as the former are mathematically easier to work with. Now we have to interpret the LSq' axis in the opposite way – the higher the LSq', the more proportional is the outcome and vice versa.

Figure 3.5. Normalised Ns and LSq axes and anchor points



Now it is possible to calculate for every data point its normalised value. We also know the anchor points, where the graph should start and end. After carrying out the experiment and calculating the normalised values for all data points, we should be able to see the relationship between LSq and Ns more precisely, because the context (i.e. vote shares) are held under control. If we are on a right track with this logic, we should be able to see relatively pure electoral system mechanical effects and thus there should be a clear pattern between LSq and Ns normalised values. The questions when analysing the results of the experiment are the following: Is there a pattern between Ns and LSq normalised values? If yes, what kind of pattern? Why? Those questions are addressed in Chapter 4.

3.4. Cases, data and systems

For carrying out the computational experiment, a real dataset of five counties and 10 elections was used: Czech Republic (2002, 2006), Denmark (2007, 2011), Finland (2003, 2007), Portugal (2009, 2011) and Estonia (2007, 2011). For district level vote shares for all these elections, CLEA (The Constituency-Level Elections Archive) dataset was used. CLEA dataset includes detailed election results at the constituency level for parliamentary elections around the world. For the experiment in this thesis, CLEA provided data about constituency level votes for individual parties, seats won by each party and district magnitudes.

Cases included in the analysis were chosen based on many criteria. The most important criterion was that all countries in the sample should use proportional electoral system. Why? Had we included majoritarian systems, then there might have been some problems carrying out the experiment. For instance, when only 3 parties receive votes, then it might not be meaningful to do calculations with different PR electoral systems, as the results may be too similar.

Secondly, when choosing countries, the main characteristics of their electoral systems and electoral outcomes were taken into account. We mainly looked at the average value of disproportionality index in the last decade and tried to include into the sample countries with low, medium and high disproportionality (see Table 2). Even though all sample countries use PR electoral systems which try to minimise the disproportionality, some systems still achieve more proportional results than others. We can see that average disproportionality index ranges from 1.3 to 7.4, a relatively big variation considering that all those countries use PR electoral systems.

Table 2. Average value of disproportionality index in 1990-2013 in 5 sample countries

Country	Average LSq (1990-2013)
Denmark	1,34
Finland	3,27
Estonia	5,19
Portugal	5,33
Czech Republic	7,37

Source: average LSq is calculated using Gallagher election indices dataset

Finally, but even most importantly, the selection of cases was limited by data availability. For carrying out the computational experiment, it was necessary to get the vote shares for all parties in every district, not just aggregate vote shares. Also, it was necessary to know district magnitudes. The CLEA dataset we used did not allow us to include into the analysis some countries that were initially planned to. That was because of large amount of missing values in the dataset. Collecting these missing values from the websites of national electoral committees would have been too time-consuming.

Also, originally, the plan was to include into the analysis the two latest parliamentary elections from each country, but for many countries there was no latest update in the CLEA database. So instead, last two elections in the database were included. The only exception is Estonian 2011 elections that were not in the database, but because of personal interest, the data were collected from the national election committee website and included into analysis.

Extracting real data from CLEA database for five countries and 10 cases was the starting point for carrying out the experiment. This real input was used to calculate new seat distributions under various electoral rules. For each of the 10 elections, results were calculated under at least six different electoral rules. It means that one vote distribution was used as an input for six different electoral systems. By doing that, we obtained seven seat distributions for one seat distribution (one real and six experimental seat distributions). In many cases all of these seven seat distributions differed from each other, but not always. Still, there were at least 5 different seat distributions for every vote distribution. In total, 79 different seat distributions were obtained from the experiment (including real seat distributions). The seven electoral systems used in the experiment were the following:

1) D'Hondt
2) Sainte-Lague
3) Simple quota and remainders
4) Droop quota and remainders
5) M=1 (single seat districts)
6) M=2 (two-seat districts)[1]

In one case, Czech Republic 2002 election, additionally M=4 and M=7 rules were applied. But since the experiment of manipulating the district magnitude was quite time-consuming, only the more extreme rules of M=1 and M=2 were used for other vote distributions. Nonetheless, M=4 and M=7 in the Czech case fitted the pattern described in the results section (Chapter 4) very well.

The aforementioned list contains two types of rules that were used in the experiment. Firstly, the electoral formula (i.e. the precise method how votes are transformed into seats in the district) and secondly, district magnitude (i.e. how many seats are distributed in one district). The electoral formulae in the list can also be classified – there are divisor methods (d'Hondt and Sainte-Lague) and largest remainders methods (simple and Droop quota).

Variations in both electoral formulae and district magnitudes were intentional. The selection of electoral formulae for the experiment was also influenced by how widely the systems are actually used in the real world. The four formulae included in study are the most widely used PR seat allocation methods. The selection of district magnitudes was guided by the intention to include in the analysis also systems that produce lower proportionality scores. And as we know from the electoral system literature, the lower the district magnitude, the less proportional results the system produces. For that reason, the two lowest possible district magnitudes (1 and 2) were used.

[1] For calculating results under the M=2 rule, all previous electoral formulae were used (d'Hondt, Sainte-Lague, simple and Droop quota), but the results were generally the same in all cases or just one was different from others. So, the M=2 rule sometimes gave us just one seat distribution (same for all formulae), but sometimes two and in one case three seat distributions.

4. Results and analysis

4.1. Non-linear relationship between proportionality and fragmentation

As expected, when context (i.e. vote shares) is held under control, the relationship between proportionality and parliament fragmentation becomes clearer. As we saw in Figures 2.1 and 2.2 in Chapter 2, when the indices of fragmentation and disproportionality for different elections with different vote shares are plotted, there seems to be no clear relationship between proportionality and effective number of parties. But when we hold the vote shares constant and alter only the electoral system, a much cleaner pattern appears.

Figures 4.1 and 4.2 illustrate the results of the experimental calculations with input from Czech 2002 and Estonian 2007 elections. Even though there are few data points, we see that there is a relatively strong negative relationship between disproportionality (LSq) and fragmentation (Ns). The lower the value of LSq, the higher fragmentation (Ns) we get. A similar pattern appears in all other eight cases, too (Appendix 1).

Figure 4.1. Results of the computational experiment for Czech Republic 2002 elections and logical anchor points

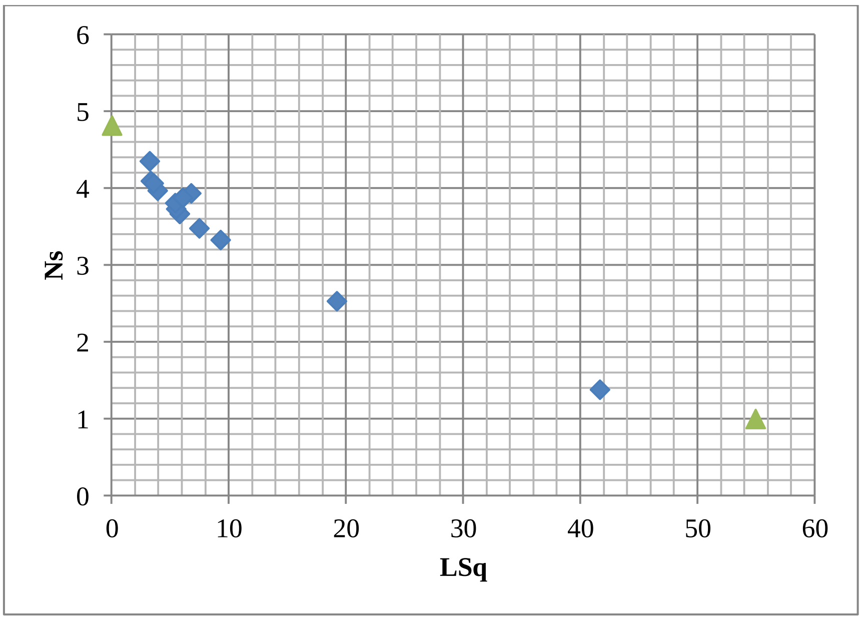
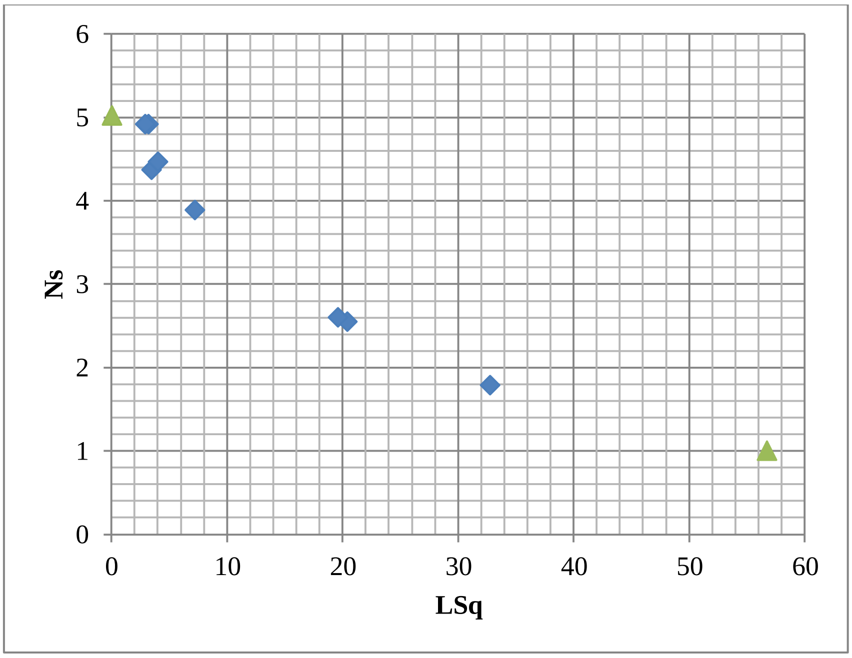


Figure 4.2. Results of the computational experiment for Estonian 2007 elections and logical anchor points

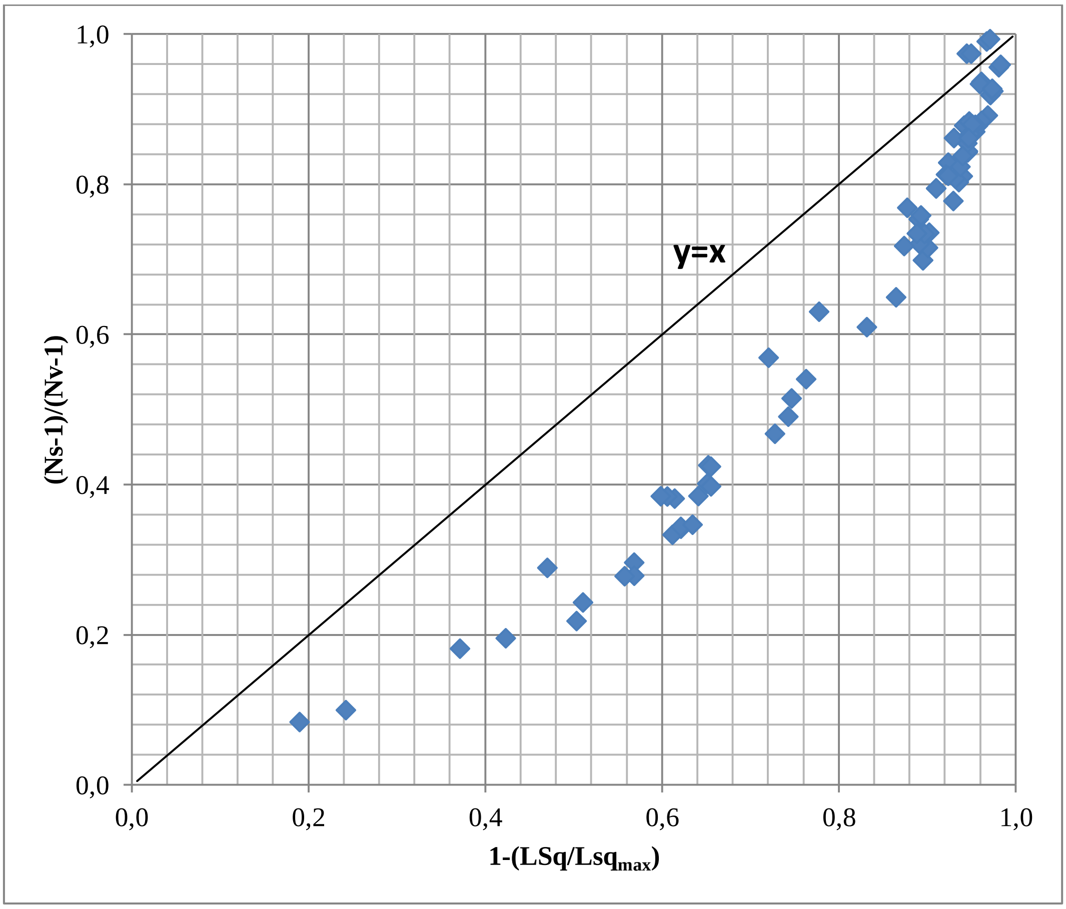


For some of the graphs in the Appendix 1, a linear model seems to fit quite well. But for most of them, a linear fit does not seem appropriate. It is visually clear that data points deviate from the linear model especially when the value of disproportionality is low. In the region of high proportionality and high fragmentation, the graph starts curving up more sharply than the linear fit. Furthermore, after adding logical anchor points and forbidden zones to these graphs, it becomes more likely that data points may fall on a curved model rather than on a straight line.

In the Appendix 1 there are 10 different Ns – LSq graphs. We cannot plot all cases on the same Ns – LSq graph, because then the importance of context would not be taken into account. So, for every vote distribution, there has to be a separate Ns – LSq graph. But if we use Ns and LSq anchor points and calculate normalised values for Ns and LSq, then all the data points can be put on the same graph. It means that normalisation of Ns and LSq takes context into account, making all 10 cases comparable.

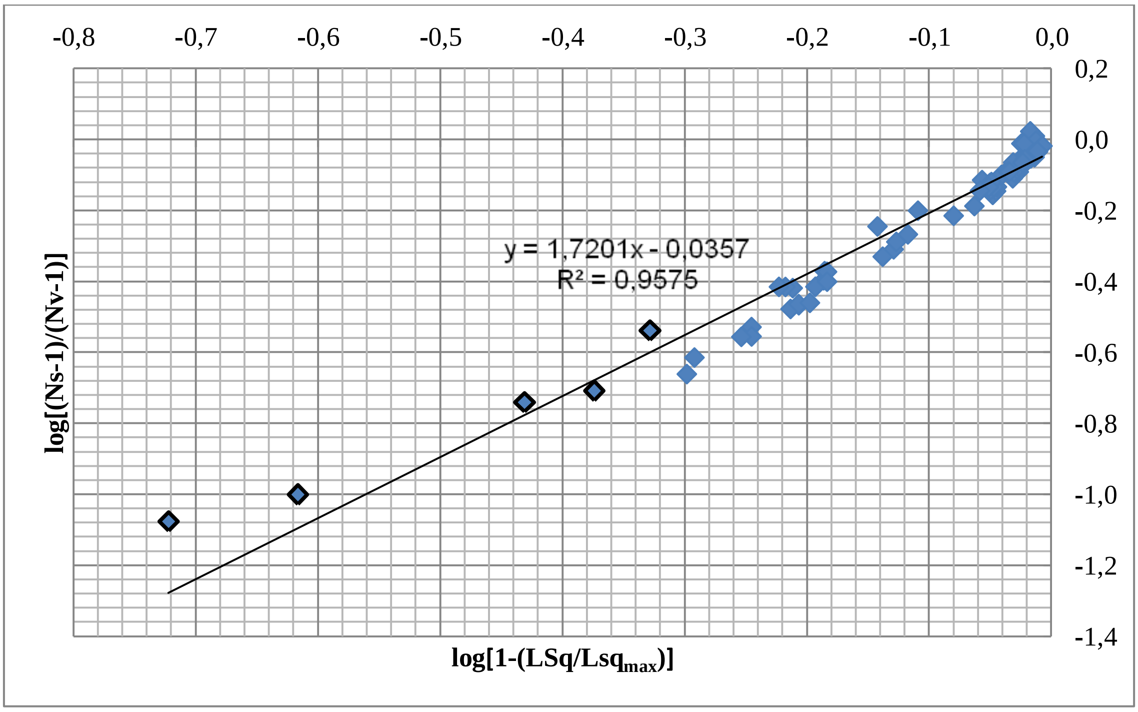
Normalised results of the experiment are shown in Figure 4.3 (for precise results of the experiment see Appendix 2). From that figure, it becomes obvious that the relationship is far from being linear when context is taken into account. In the graph, also a simple linear model (y = x) is shown that connects the logical anchor points (0; 0) and (1; 1). Not a single data point falls on that simple model. Furthermore, most of the data points are below the prediction line – systematically the value of the "y" is smaller than what the linear model would predict. So it is clear that the relationship is not linear, and thus the representation-accountability trade-off is not linear either.

Figure 4.3. Results of the computational experiment, plotted using normalised axes



There are no major outliers in Figure 4.3 and it looks like it would be possible to draw a curve (not a linear line) that goes relatively well through all data points. It is necessary to find the exact equation for the best fitting curve. It seems that the data points might fit on some kind of a $y = x^k$ curve. From the equation $y = x^k$, "k" can be expressed using logarithms: log y = k*log x. So, if we do a log-log plot of normalised Ns and LSq, then the slope of the straight line is the exponent "k" we are looking for. This graph is shown in Figure 4.4.

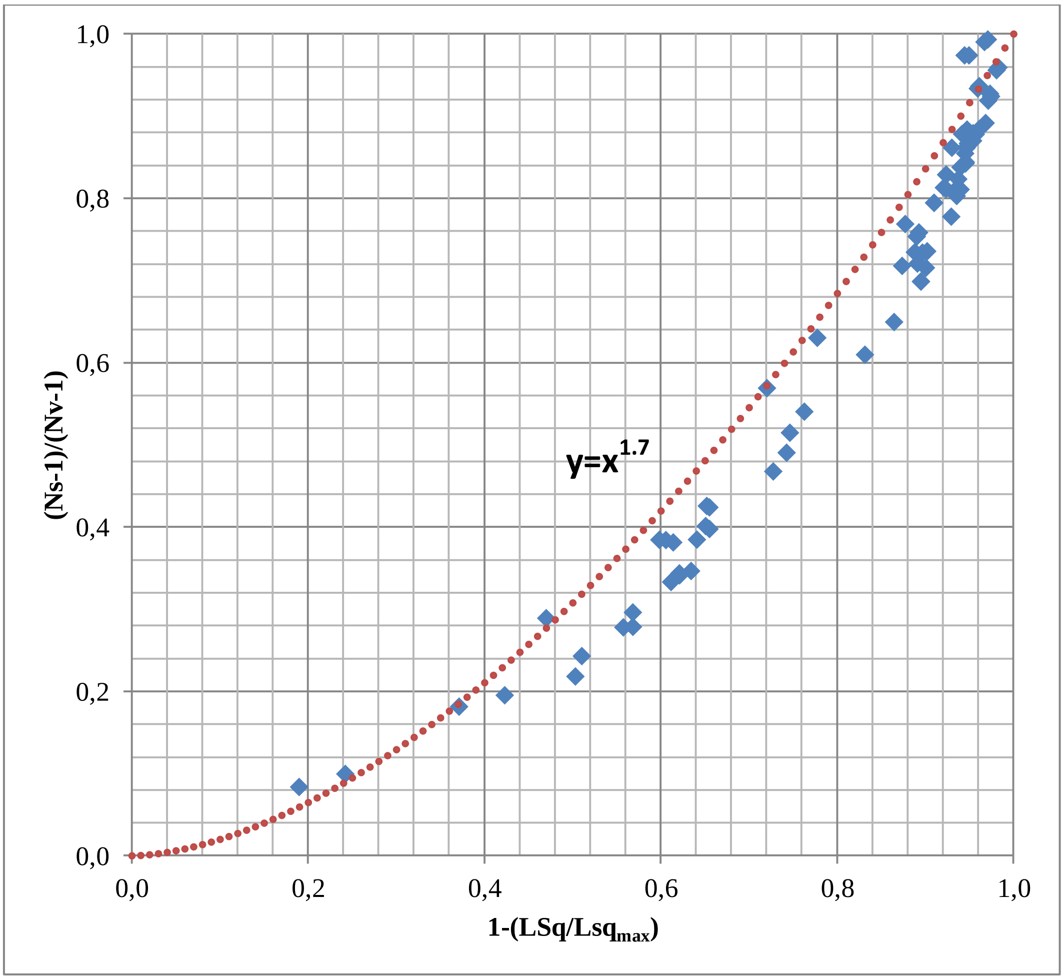
Figure 4.4. Log-log graph of normalised LSq and Ns



As we see, the log-log linear fit goes almost through point (0; 0), which is a necessary condition for saying that the initial curve was in the form of $y = x^k$. Furthermore, R^2 is very high, 0.96, which is another necessary condition for the validity of $y = x^k$ form. The slope of the straight line is 1.7, which means that the relationship between LSq and Ns normalised values is $y = x^{1.7}$.

In the Figure 4.5, it can be seen that the $x^{1.7}$ graph fits well for the data points with lowest or highest values on both axes, but not so well for the ones that lie somewhere in between. If we look at the log-log graph more closely, then we see that five data points flatten the fitted line significantly (these points are highlighted in the Figure 4.4). If these data points were excluded from the graph, the linear fit would be steeper, i.e. the slope would be higher. But we cannot simply ignore some data points, just for the purpose of getting a better fit. So, it would be wise to look up what these five data points possibly have in common.

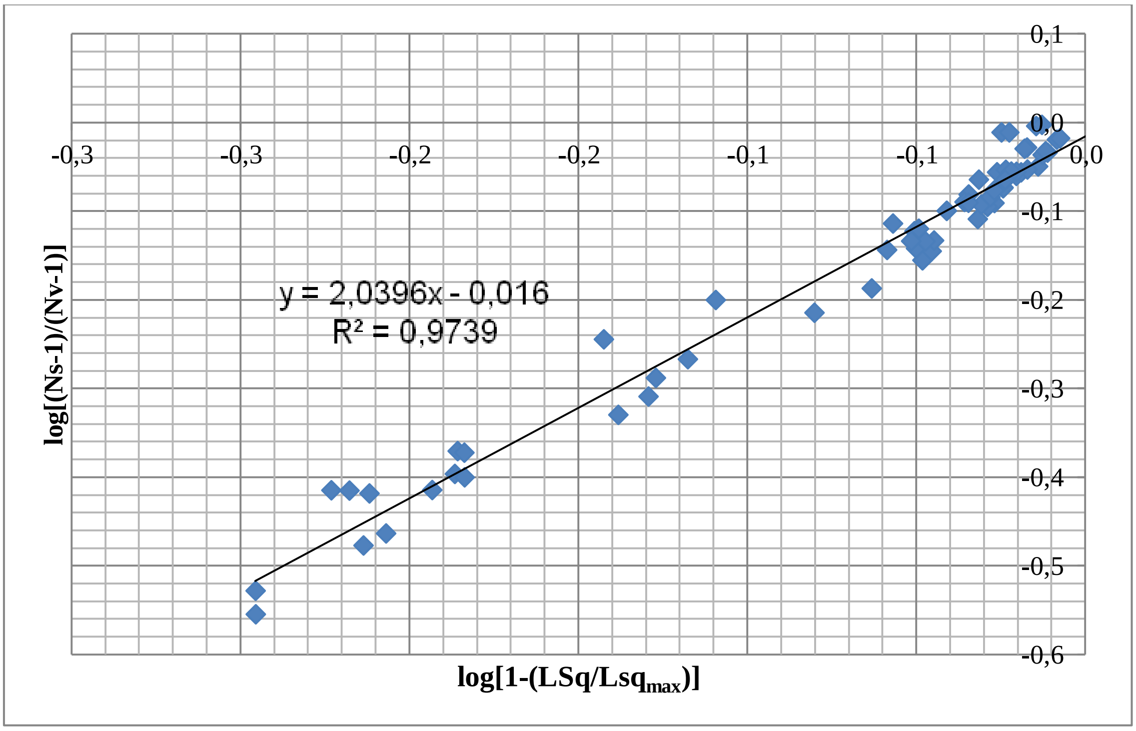
Figure 4.5. Mathematically best-fitting curve for normalised LSq and Ns values



It turns out that all these five data points that are highlighted in Figure 4.4 are the results of the M=1 experiment. These points are obtained by calculating the alternative seat shares for countries that in real life use a PR electoral system if they had used single-member districts. So it seems that single-member districts produce slightly different results in terms of the proportionality-fragmentation relationship. It would then be reasonable to check what the log-log graph would look like if all M=1 data points (altogether 10 data points, including the five previously highlighted points) were excluded (see Figure 4.6).

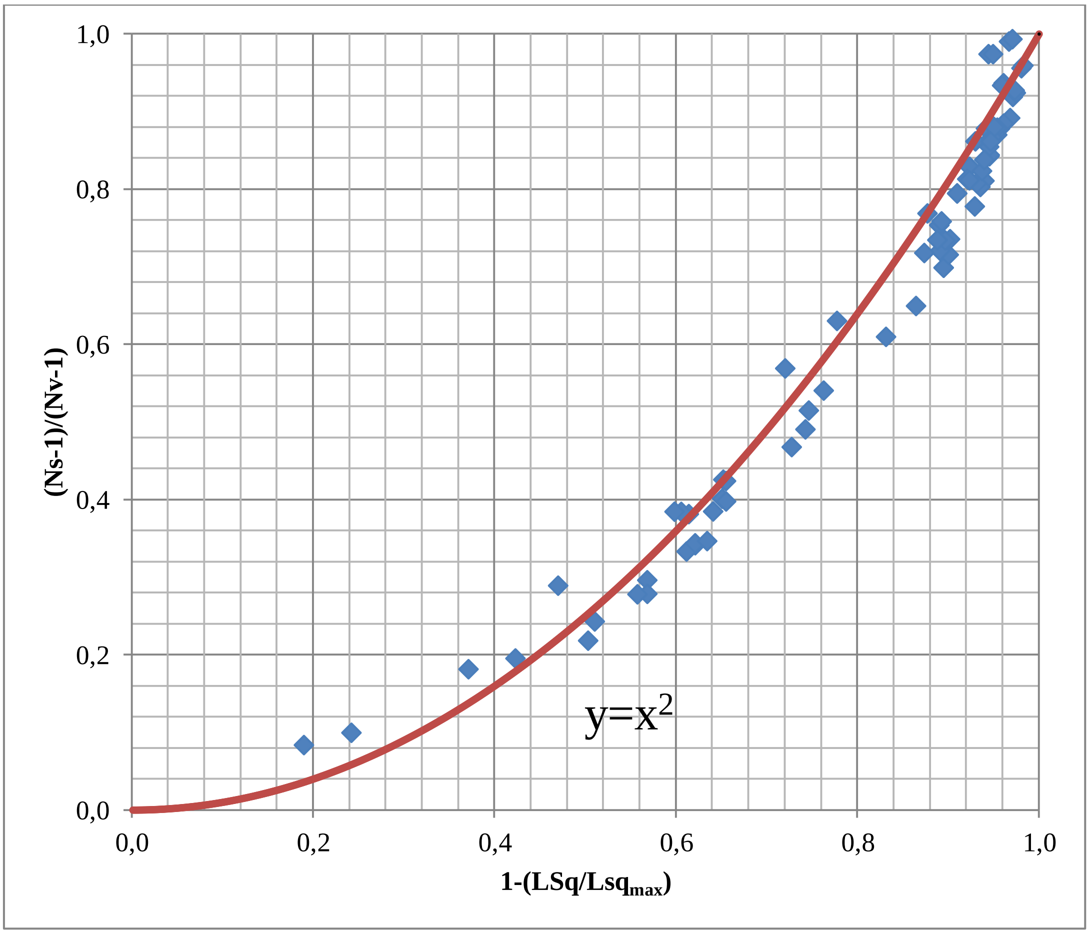
The exclusion of M=1 data points for determining the value of exponent “k” is justified. Using the input (vote shares) from cases that actually use proportional representation (PR) electoral system for M=1 calculations is not very realistic – the input would certainly not be so fragmented if single-member districts were used. In real life, if district magnitude was 1, then vote shares would be much less fragmented, Nv would be smaller and as a result, parliament would be less fragmented. It means that if we used "realistic" M=1 input, data points should have lower values on the Ns axis than in our experiment in Figure 4.5. So in this thesis, the $y = x^2$ graph is used as a best fit, because it is the best fit for PR systems. Even though exponent 2 is not the best fit for M=1 data points, these points still do not deviate from the $y = x^2$ graph significantly, as can be seen in Figure 4.7.

Figure 4.6. Log-log graph for normalised LSq and Ns, M=1 excluded



When the exponent 2 is used instead of 1.7, the fitted curve $y = x^2$ seems to work better (see Figure 4.7). The curve is now a better fit for the data points that are in the middle or at the higher ends of the axes. The fit is a little less suitable for M=1 data points, as expected. But this deviation is not significant. All in all, taking the whole dataset into account, $y = x^2$ is the best model.

Figure 4.7. Best-fitting model for normalised LSq and Ns



For simplicity, we say that the relationship is y = x^2. But of course, y actually refers to normalised Ns and x to normalised LSq:

$$y = \frac{N_s - 1}{N_v - 1} \qquad x = 1 - \frac{LSq}{LSq_{max}}$$

To be precise, the relationship is therefore the following:

$$\frac{N_s - 1}{N_v - 1} = \left(1 - \frac{LSq}{LSq_{max}}\right)^2$$

From this relationship, we can express the formula for both (1) fragmentation Ns and (2) disproportionality LSq:

$$N_s = 1 + (N_v - 1)\left(1 - \frac{LSq}{LSq_{max}}\right)^2$$

(1)

$$LSq = LSq_{max}\left(1 - \sqrt{\frac{N_s - 1}{N_v - 1}}\right)$$

(2)

There are four variables in these formulae: Ns, Nv, LSq and $LSq_{max.}$ We are interested in the relationship between Ns and LSq, but it is impossible to explain this relationship without knowing the context – LSq_{max} and Nv, which depend on the input. So, the exact relationship between LSq and Ns depends on the context. Without knowing the maximum values of Ns and LSq, which are different for every vote distribution, the LSq – Ns relationship cannot be precisely determined. If the Ns and LSq values are normalised (we call their normalised values Ns' and LSq'), then there is a square relationship: Ns' = LSq'^2

Ns, Nv and LSq are all widely used measures in describing electoral outcomes. LSq_{max}, on the other hand, is not a common measure, it has to be calculated from vote shares if one wants to use or test the proposed formula. But it turns out that LSq_{max} can be expressed in terms of Nv and v_1 (winning party's vote share) only. v_1 is a much better measure than LSq_{max}, because one can simply look up the winning party's vote share and does not have to calculate the value of LSq_{max} from scratch, which would take significantly more time. LSq_{max} can be expressed in terms of Nv and v_1 in the following way:

$$\begin{aligned} LSq_{max} &= \sqrt{\frac{1}{2}(v_1 - 1)^2 + \frac{1}{2}\sum_{i=2}^{n}(v_i - 0)^2} \\ &= \sqrt{\frac{1}{2}(v_1^2 - 2v_1 + 1) + \frac{1}{2}\sum_{i=2}^{n} v_i^2} \\ &= \sqrt{\frac{1}{2}(1 - 2v_1) + \frac{1}{2}\sum_{i=1}^{n} v_i^2} \end{aligned}$$

$$
\begin{aligned}
&= \sqrt{\frac{1}{2}(1-2v_1)+\frac{1}{2}\cdot\frac{1}{N_v}} \\
&= \sqrt{\frac{N_v(1-2v_1)+1}{2N_v}}
\end{aligned}
$$

In the initial formula, it is useful to express the value of LSq_{max} in terms of Nv and v_1, because Nv and v_1 are much more widely used and easier to find than LSq_{max}.

To sum up, the relationship between parliament fragmentation and proportionality is $Ns'=LSq'^2$. And since we know that between non-normalised Ns and LSq there is no precise relationship, this squared relationship between Ns' and LSq' proves that the context matters. The fact that this $Ns'=LSq'^2$ model holds true convinces us that the relationship between proportionality and fragmentation is not linear – instead there appears to be a square relationship. Furthermore, this formula improves our knowledge about how exactly input influences the Ns – LSq relationship.

But the usefulness of the formula would increase significantly if we could explain why the best fit is achieved with the exponent 2, not 2.5, 1.5 or even 1 (i.e. why the relationship is not linear). LSq, LSq_{max}, Ns and Nv can all be expressed in terms of votes and seats – this would suggest that the exponent 2 and the LSq-Ns relationship more broadly can be somehow explained through vote-seat relationship. We have not been able to find such an explanation yet. Because of the summations in the LSq; LSq_{max} and Ns; Nv formulae, this approach is mathematically challenging. But it is something that certainly needs to be done in future.

4.2. Testing the relationship on broader data

Even though we cannot yet explain why the relationship is what it is ($Ns' = LSq'^2$), we can still examine its validity. We arrived at this formula using the data from only five countries and 10 elections. But does this formula still work when tested on broader data?

Carrying out a computational experiment with 10 election inputs took quite some time and effort. So, it would be best to test this formula without having to go through all that trouble again. M. Gallagher has created an election indices dataset where three main indices (the values of Ns, Nv and LSq) have been computed for elections all over the world. This is a comprehensive dataset and its last updated version (March 4, 2014) is used to test the validity of the Ns'= LSq'^{2} formula.

There are four variables in the formula that we are testing. From Gallagher's dataset we get three. But for testing the validity of the formula we need all four variables. LSq_{max} is the only one that we do not know. As discussed in the previous section, it can be expressed in terms of v_1 and Nv. The winning party's vote share (v_1) can be looked up relatively easily, but if we have a large number of elections, it still takes quite some time. In Gallagher's dataset there are more than 130 countries and indices for more than 1000 elections. Searching for the winning party's vote share for a thousand elections would be really time-consuming.

But if we look at the formula (Ns - 1)/(Nv - 1) = (1 - LSq/Lsq_{max})^2, it seems that a simplification can be made regarding the value of LSq_{max}. LSq/LSq_{max} ranges from 0 to 1 and it takes the maximal value when the electoral system produces maximal disproportionality. Conceptually, when PR electoral systems are considered, their goal is to minimise disproportionality (LSq), i.e. make LSq value much smaller than LSq_{max}. In our 10 cases, all of which used PR electoral systems, real LSq values ranged from 3 to 5%, but LSq_{max} values varied only from 48 to 59%. Thus, the value of LSq/LSq_{max} would have been between 3/59 = 0.051 and 5/48 = 0.104, which are both at the lower end of the continuum 0 – 1. It means that the LSq/LSq_{max} is much closer to 0 for PR systems and the precise value of LSq_{max} does not influence the value of LSq/LSq_{max} significantly. We can even say that mechanical effects of PR electoral systems are more determined by the value of Nv rather than LSq_{max} as the value of LSq/LSq_{max} ratio is always low and the value of (1 - LSq/LSq_{max})^2 is always close to 1.

Because of this characteristic of PR systems, we can take the average LSq_{max} of our 10 cases and use it for making calculations for all PR systems in Gallagher's dataset. The average LSq_{max} for our elections turned out to be roughly 50. The formula for PR systems is thus the following:

$$\frac{N_s - 1}{N_v - 1} = \left(1 - \frac{LSq}{50}\right)^2$$

To test its validity, we express Ns from it:

$$N_s = 1 + (N_v - 1)\left(1 - \frac{LSq}{50}\right)^2$$

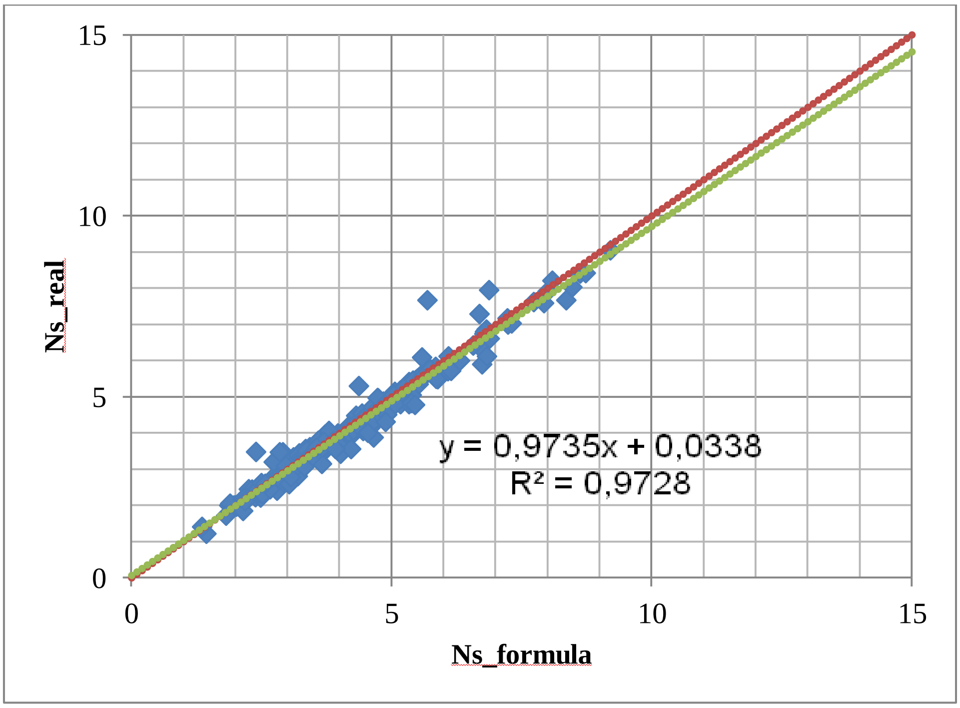
Using Gallagher's Nv and LSq, we can now calculate our formula's prediction for Ns for every PR election in the dataset. Then we can compare our computational Ns to the real Ns in the dataset. We extracted from the dataset most of the European countries that use a PR electoral system: a total of 28 countries and 340 elections. For all 340 elections, we calculated the predicted value of Ns. The comparison of real and computational Ns is visualised in Figure 4.8.

Figure 4.8 shows that our formula works very well for PR systems – even the simplified version of it, where LSq_{max} value was simply taken to be the average of our 10 cases. If our formula had worked perfectly, then the relationship between the real and computational Ns would have been Ns_real = Ns_formula (red line in the Figure 4.8 symbolises this desired relation). The green line refers to the actual relationship between two Ns's. As we see, the red and green lines almost coincide: Ns_formula = 0.97 * Ns_real + 0.03. As 0.97 ≈ 1 and 0.03 ≈ 0, the relationship is roughly Ns_formula = Ns_real. It proves that our formula makes almost perfectly correct predictions for PR systems. Furthermore, green line's R^2 is very high – 0.97, which means that the data points are all very close to this line.

Upon testing the validity of the formula for plurality electoral systems, the value LSq_{max} needs to be known. For these systems LSq/LSq_{max} is much higher than for PR systems and thus it influences the overall Ns value more significantly. So it is important to use the real LSq_{max} value, not just average value. For that, we use the LSq_{max} formula:

$$LSq_{max} = \sqrt{\frac{N_v(1 - 2v_1) + 1}{2N_v}}$$

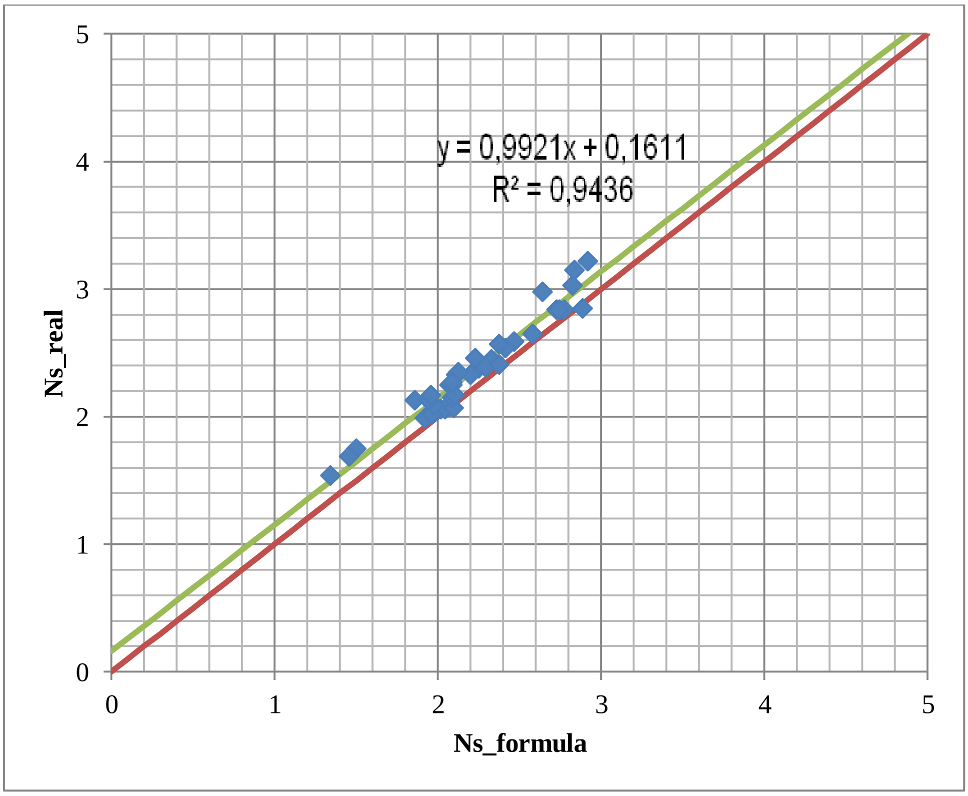
Figure 4.8. Comparison of real and computational Ns for PR systems



To calculate the value of LSq_{max}, we need to know the values of v_1, which are not included into Gallagher's dataset. As looking up v_1 for every election takes time, we use only two countries that employ a plurality electoral system for testing purposes. These countries are Canada and the United Kingdom. In Gallagher's dataset the election indices for these countries have been calculated for elections since 1945 to present. It means that for two countries there are 40 elections, a decent amount for testing the formula. After looking up the values of v_1, we can calculate the LSq_{max} for each of the 40 elections. And then it is possible to calculate the Ns value – our model's prediction what the Ns would be.

Comparing the results of computational Ns and real Ns shows that our model $Ns' = LSq'^2$ works really well for plurality systems, too (Figure 4.9). Again, the red line symbolises the perfect situation of Ns_formula = Ns_real and the green line shows the best linear fit for our data points. We see that those two lines are very close to one another and the R^2 is very high. But we also see that our formula tends to predict slightly lower Ns values than they are in real life. This is coherent with our initial results – we saw that the model $Ns' = LSq'^2$ is a best fit for PR systems and that M=1 experiments deviated slightly from this curve.

Figure 4.9. Comparison of real and computational Ns for plurality systems

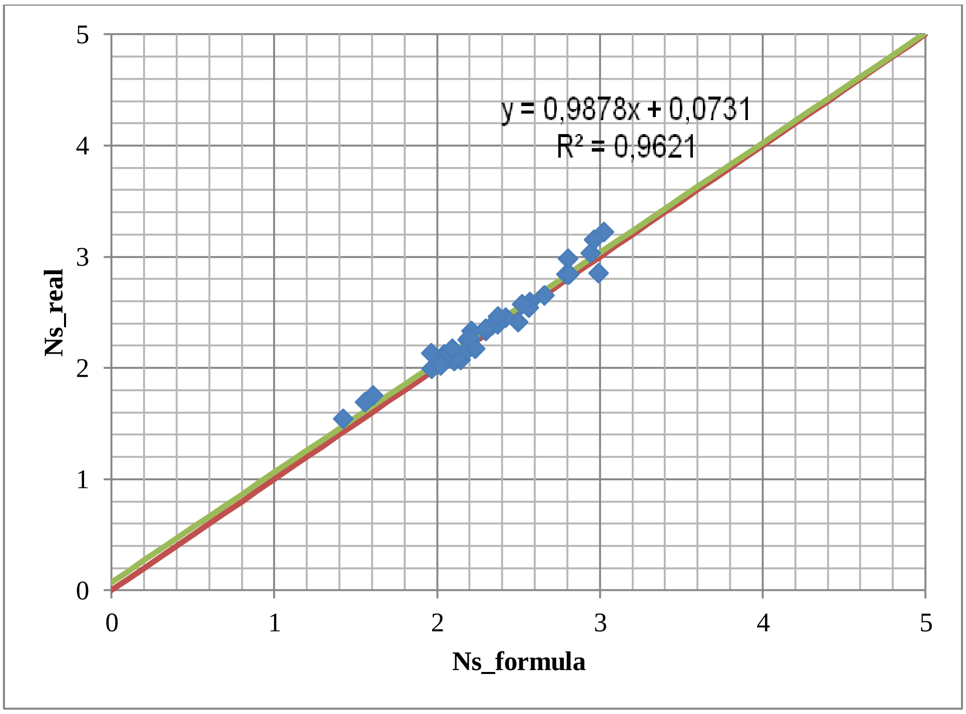


According to our results in Chapter 4.1., the model $Ns' = LSq'^{1.7}$ was the best fit for M=1 experiments. Because of that we calculate the Ns value using the exponent 1.7 instead of 2 and compare the results again with the real Ns value (see Figure 4.10). In the graph, red and green lines coincide with each other almost perfectly. Therefore exponent 1.7 really works better than 2 for plurality systems. This claim should certainly be further looked into, as currently the testing was done using data from only two countries and 40 elections. However, if it really is so and the exponent 2 holds true for PR and the exponent 1.7 for plurality systems, it should be investigated why.

So far, testing the validity of $Ns' = LSq'^2$ relationship has suggested that this formula predicts the real Ns value quite accurately both for PR and plurality electoral systems. However, the formula $Ns' = LSq'^2$ should be regarded as a first approximation rather than a universal rule. Due to slight differences in the effects of the formula for PR and plurality systems, we think that between Ns' and LSq' there is actually not a simple squared relationship. In the region of low fragmentation and disproportionality, the exponent in the formula $Ns' = LSq'^k$ appears to be lower (around 1.7) and in the region of high fragmentation and disproportionality, it appears to be higher (around 2). As the difference between those exponents is not overly significant, the simplified approximation Ns'= LSq'^{2} can be used to good precision.

An advanced model that takes into account deviations from Ns' = LSq'^{2} approximation for plurality systems should be developed in future. But our first approximate model is still probably better for making generalisations about the overall relationship between proportionality and fragmentation.

Figure 4.10. Comparison of real and computational Ns for plurality systems using the exponent 1.7



4.3. Is there an electoral sweet-spot?

In section 2.1 we discussed how representation and accountability ideals are inherently competitive. It means that high proportionality (i.e. high representativeness) goes hand in hand with high parliament fragmentation (i.e. low accountability) and vice versa. This conflicting nature is also clear from the results of this thesis – there are no data points in the region of low Ns' and high LSq'. In sections 4.1 and 4.2 we saw that the trade-off between those competitive ideals is not linear – there is a squared relationship instead. Carey and Hix (2011) also took the position that this trade-off is not linear. They believed that it is possible to achieve a representative but still highly accountable parliament and called it an "electoral sweet-spot".

Conceptually, an "electoral sweet-spot" does not mean high proportionality and low fragmentation, which is impossible, but rather the situation where these ideals are optimised together. If the trade-off between proportionality and fragmentation was linear, it would be impossible to talk about a "sweet-spot" as Ns' and LSq' would always increase at the same rate. But since we are dealing with a non-linear squared relationship, an "electoral sweet-spot" can be found.

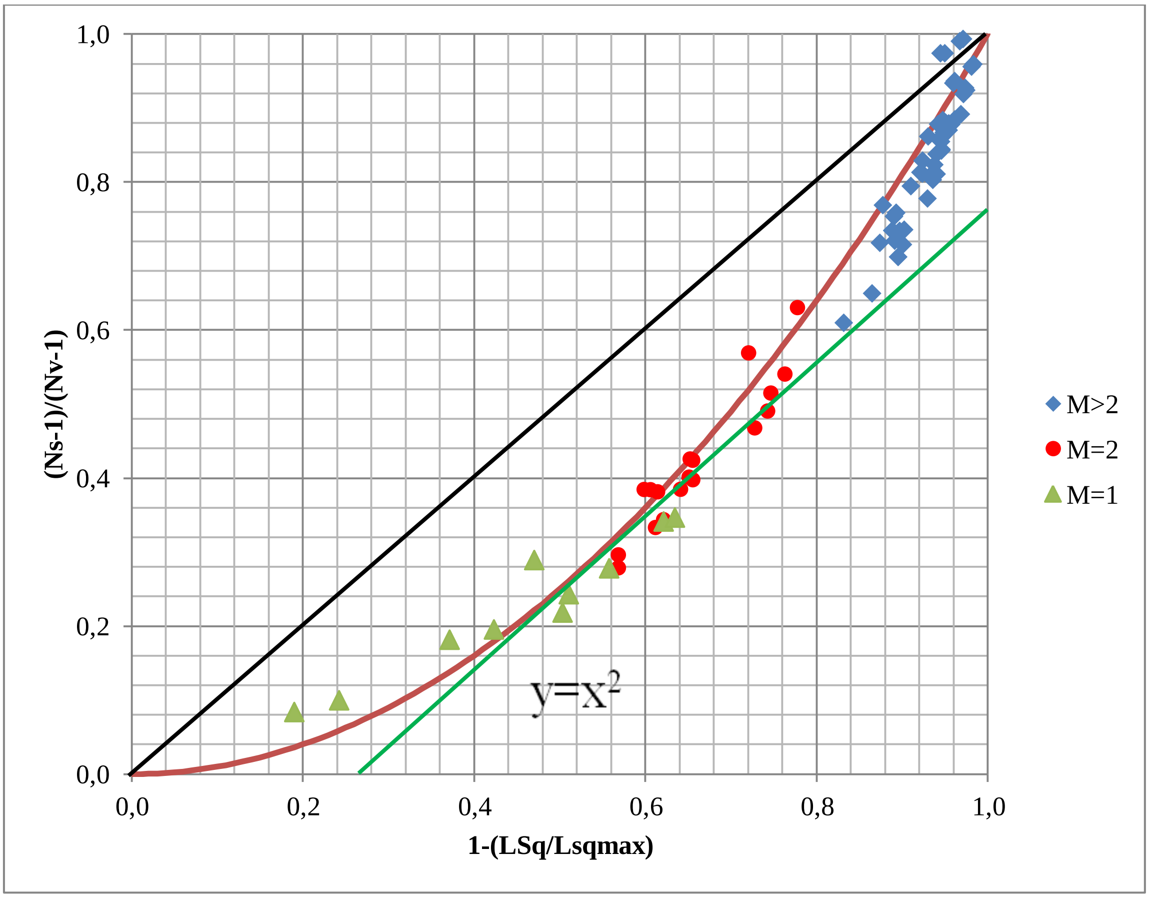
This "sweet-spot" can be best described graphically. It is situated where the $y=x^2$ curve is farthest from the y=x line (see Figure 4.11). In other words, electoral "sweet-spot" is where the line that is parallel to y=x line touches the $y=x^2$ curve (green line in Figure 4.11). The same graphical logic is used by Carey and Hix (2011: 386).

Our ideals are optimised when LSq' is the largest and Ns' is the lowest. Starting from point (0; 0) and following the x^2 curve, the x value increases more than y (desired in terms of optimising the ideals) until the point where green line touches the x^2 line. After that point, x starts increasing more slowly than y (not desired). So the optimal balance is reached in the "sweet-spot" – i.e. where green line touches x^2 curve.

In the Figure 4.11 we see that the "sweet-spot" is not really a "spot" but rather an "optimal region". Yes, mathematically there is a precise "spot" where the green line touches the $y=x^2$ curve. But practically speaking, in the region where the $y=x^2$ curve is farthest from y=x line, the two curves are nearly parallel for a while, so it would be meaningful to talk about an optimal region instead. Let us call it an electoral "sweet-region". The LSq' and Ns' are optimised in the region where LSq' is around 0.3 to 0.8. Most of the M=1 and all of the M=2 cases fall into this range.

Carey and Hix have suggested that the electoral "sweet-spot" would be achieved by the use of low-magnitude multimember districts. According to their results the optimal district magnitude is in the range of three to eight (Carey, Hix 2011: 384). In addition to Carey and Hix, there are other scholars as well who point to the advantages low-magnitude districts. Taagepera and Shugart also recommend using low-magnitude multi-seat districts – 3 or 5, but not 4 (Taagepera, Shugart 1989: 236). Samuels (1999) say that voters under low-magnitude open-list systems are more able to hold their representatives accountable than when other systems are used.

Figure 4.11. Determining electoral "sweet-spot"



Our results do not agree with what Carey and Hix (2011) expect – the sweetest region seems to be achieved when district magnitude is one or two, not in the range of 3-8. However, our results may be influenced by how we combine Ns and LSq and are possibly slightly different if we normalise Ns and LSq in a different way (e.g. (Nv-Ns)/Nv and LSq/LSq_{max}). Nevertheless, it is clear that the "sweet-spot" cannot be achieved using magnitudes as high as 6-8 (as Carey and Hix suggested), but rather in the range of 1-2 (according to our results) or 3-5, excluding 4 (according to Taagepera and Shugart 1989). Carey and Hix were looking for a combination of low disproportionality and moderate number of parties (3 to 8), so the emphasis was more on the proportionality ideal – keeping the number of parties not minimal, but moderate. But when trying to maximise both ideals equally, the “sweet-spot” seems to be achieved using much lower district magnitudes.

Conclusion

This thesis focused on analysing the mechanical effects of electoral systems on two important electoral outcomes: proportionality and parliament fragmentation. Proportionality and parliament fragmentation refer to the ideals of representation and accountability, respectively. These two democratic ideals are at the core of the debate between proponents of PR and majoritarian electoral systems. Conceptually, these ideals are indeed conflicting: the better the representation (more proportionality), the less accountable is the government (more fragmented) and vice versa. Our aim was to ascertain the precise relationship between proportionality and fragmentation, as it has not been done before.

The presumption of new institutionalism that institutions matter but their precise effect depends on the conditions in which they operate helped us to arrive at a universal approximate model for the proportionality-fragmentation relationship. We carried out a computational experiment on data from 5 countries and 10 elections with the intention to hold context under control. Using the same vote distribution (input) and altering electoral rules allowed us to see variations in mechanical effects only. As we had 10 different inputs in our experiment, the variables were normalised to find a universal relationship. By using the values of logical anchor points of each vote distribution (0 and LSq_{max}; 1 and Nv) for normalising, we arrived at the following universal relationship:

$$\frac{N_s - 1}{N_v - 1} = \left(1 - \frac{LSq}{LSq_{max}}\right)^2$$

In other words, we found that there is a squared relationship between the normalised values of disproportionality (LSq') and parliament fragmentation (Ns'): $Ns' = LSq'^2$. This is an important finding as the trade-off between high proportionality and low fragmentation is often tacitly assumed to be linear. The results of this thesis show clearly that the relationship is by no means linear; instead the best first approximation is a squared model.

It is important to emphasise that the precise relationship between proportionality and fragmentation depends on the context, i.e. on the vote distribution. LSq_{max} and Nv, which were used for normalising, are different for every vote distribution. One needs to know the vote distribution to make accurate predictions about electoral systems' mechanical effects on proportionality and parliament fragmentation. Without taking context into account, there is no clear pattern or universal relationship between LSq and Ns and no precise predictions can be made.

Testing the relationship on broader data confirmed the validity of our approximation $Ns' = LSq'^2$. With the help of the simplification that LSq_{max} for PR electoral systems is roughly 50%, we were able to test the validity our formula for 340 PR elections. We expressed the Ns from our formula and compared the results to real Ns values. For PR systems, our formula predicts the value of real Ns very accurately, almost perfectly. For majoritarian systems, the simplification regarding LSq_{max} is not justified: the precise value of LSq_{max} is small enough to influence formula significantly, which was not the case for PR systems. To make progress, we expressed LSq_{max} in terms of Nv and v_1 (winning party's vote share):

$$LSq_{max} = \sqrt{\frac{N_v(1-2v_1)+1}{2N_v}}$$

The real Ns was again compared to the computational one, this time using the results of 40 plurality elections. Once again, our formula predicted the value of Ns very accurately. However, for plurality systems, it tends to give slightly lower Ns values than they are in real life. This suggests that the formula $Ns' = LSq'^2$ holds true for PR electoral systems, but for majoritarian systems the exponent in the formula is slightly lower (around 1.7).

These results point to the fact that the formula $Ns' = LSq'^2$ should be regarded as a first approximation rather than a universal rule. Due to slight differences in the effects of the formula for PR and plurality systems, there is not always a simple squared relationship between Ns' and LSq'. A more advanced model that takes into account these minor deviations should be developed in future. Nevertheless, for making generalisations about the relationship between proportionality and fragmentation, this first approximation is good enough and perhaps even better than a more complex model would be.

As the relationship between Ns' and LSq' is nonlinear, the trade-off between LSq and Ns is not absolute and thus a “sweet-spot” can be found. An electoral sweet-spot means optimising both ideals – having high proportionality and low fragmentation. We found that the relationship is optimised in the region where district magnitude is either 1 or 2. However, this result needs further investigation and interpretation as this may be influenced by the particular normalisation of LSq and Ns. Nevertheless, our results suggest that the two democratic ideals are optimised at lower district magnitudes than expected by other scholars.

This thesis leaves an important question open. We found that the approximation $Ns' = LSq'^2$ holds true relatively well for both PR and majoritarian systems. But why? All the variables in the formula can be expressed using vote and seat shares – we suggest that there exists some kind of an underlying vote-seat relationship that explains the validity of our squared model. It should be definitely further investigated as it may also reveal why our electoral “sweet-spot” turned out to be in a region of surprisingly low district magnitude. Furthermore, if a more comprehensive model of Ns' and LSq' is developed in future, it may also shed some light to this question.

In conclusion, this thesis revealed a squared relationship between normalised proportionality and fragmentation. We also showed that this relationship holds when tested on broader data, especially for PR electoral systems. This work is a first step towards finding and explaining the precise relationship between LSq and Ns, which is something that has remained elusive in previous studies. Given the many new questions that naturally arise from this work, the approximate model $Ns' = LSq'^2$ forms a good starting point for further investigations. Establishing this squared relationship is a strong contribution towards better understanding of mechanical effects of electoral systems on political outcomes.

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Datasets:

- CLEA database:

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- Gallagher’s database:

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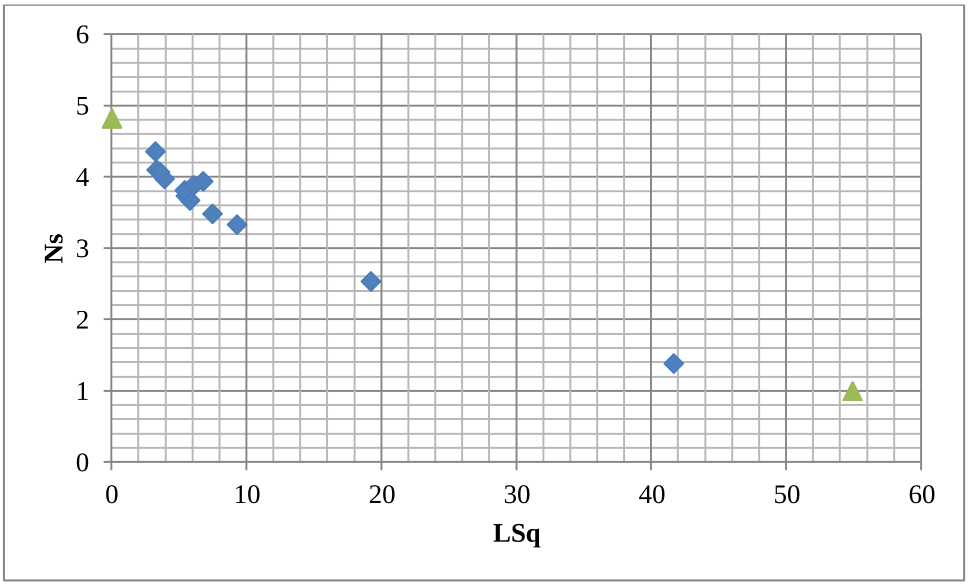
http://www.tcd.ie/Political_Science/staff/michael_gallagher/ElSystems/index.php, accessed March 14, 2014.

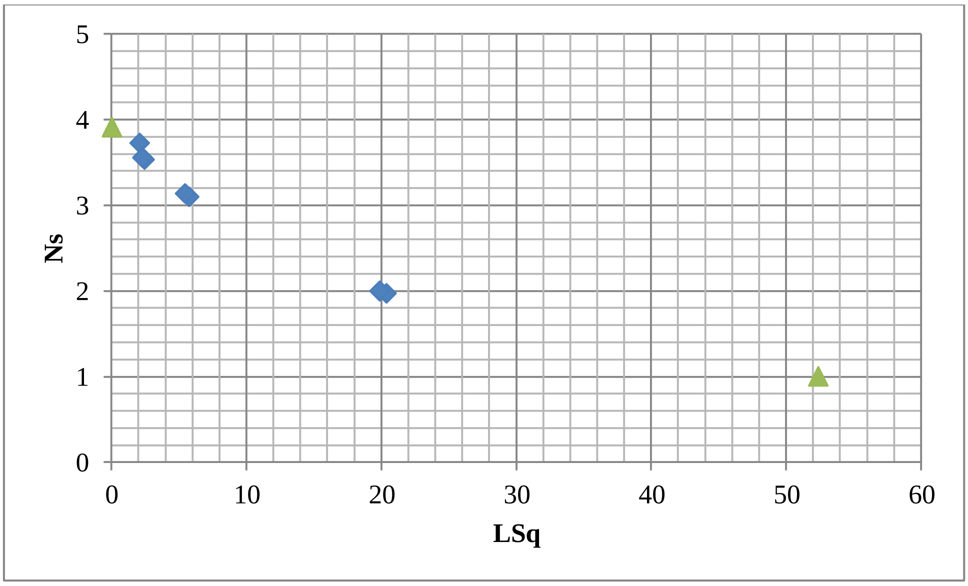
Appendixes

Appendix 1. Results of the computational experiments and logical anchor points for each sample country and its elections

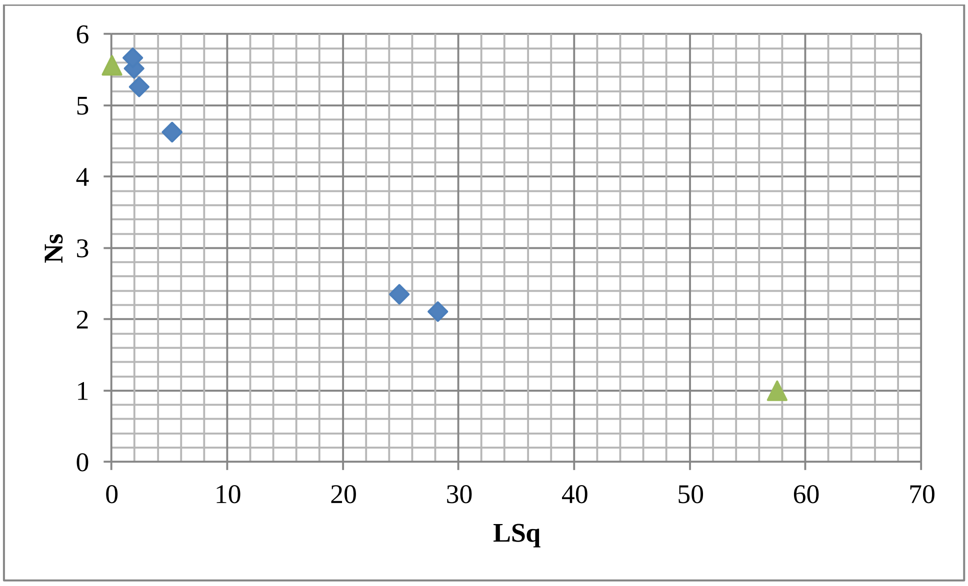
Czech Republic 2002



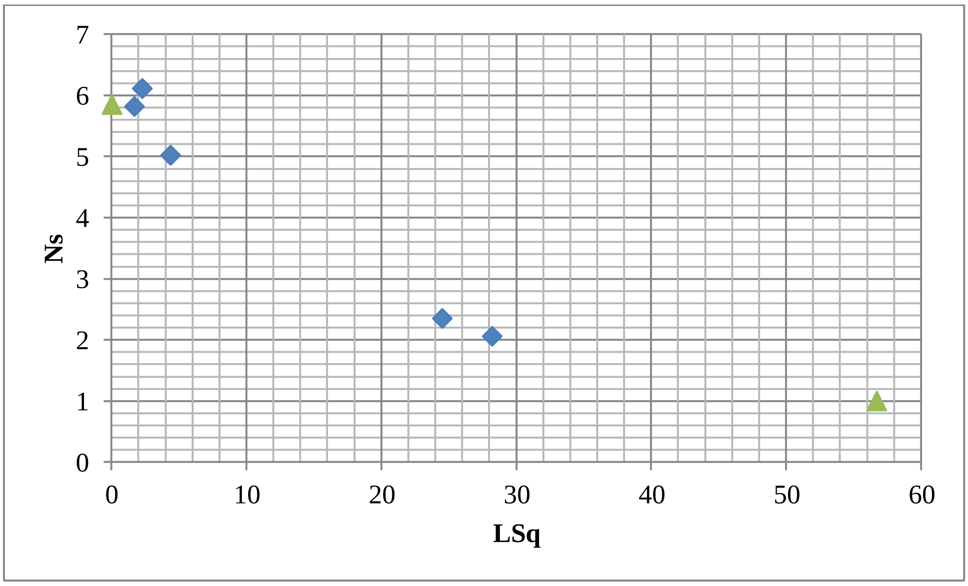
Czech Republic 2006



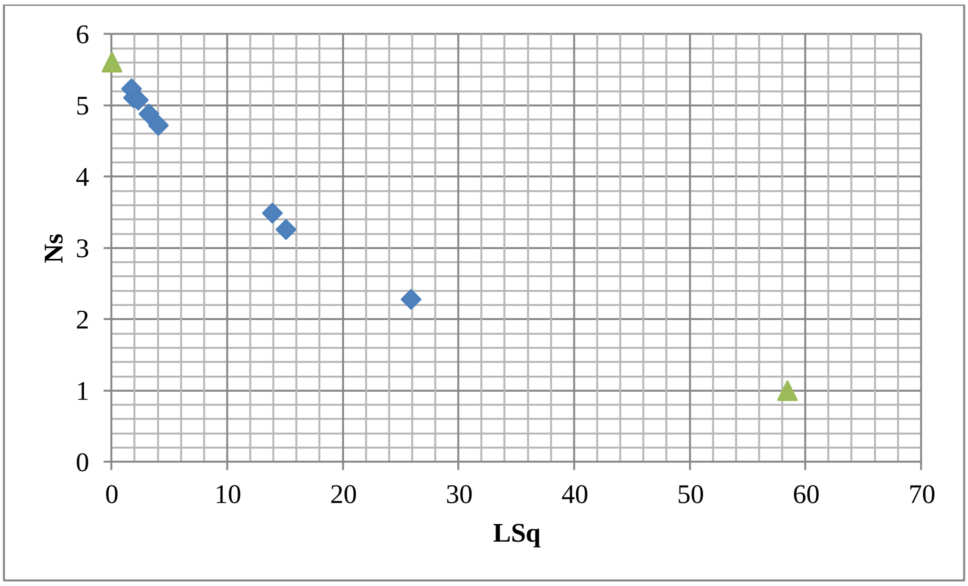
Denmark 2007



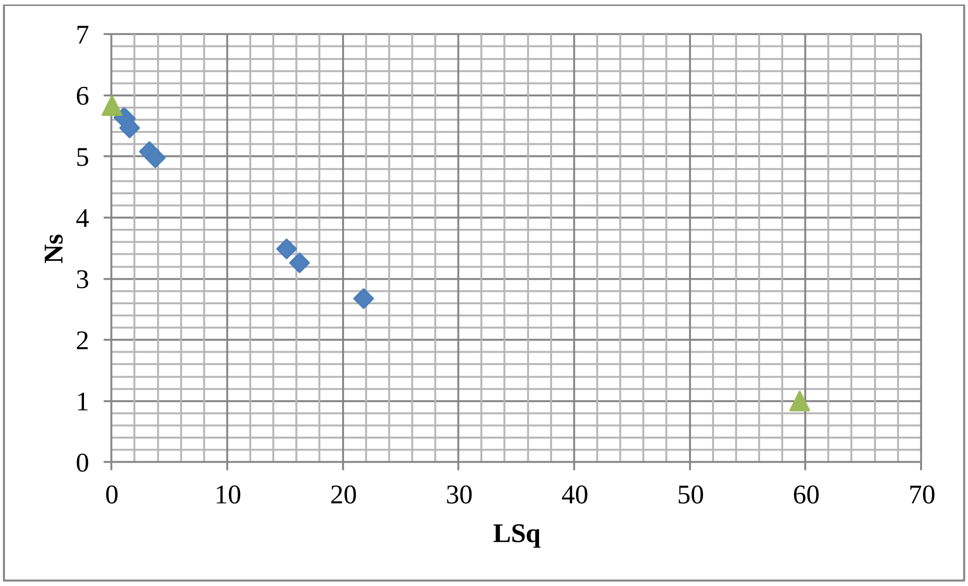
Denmark 2011



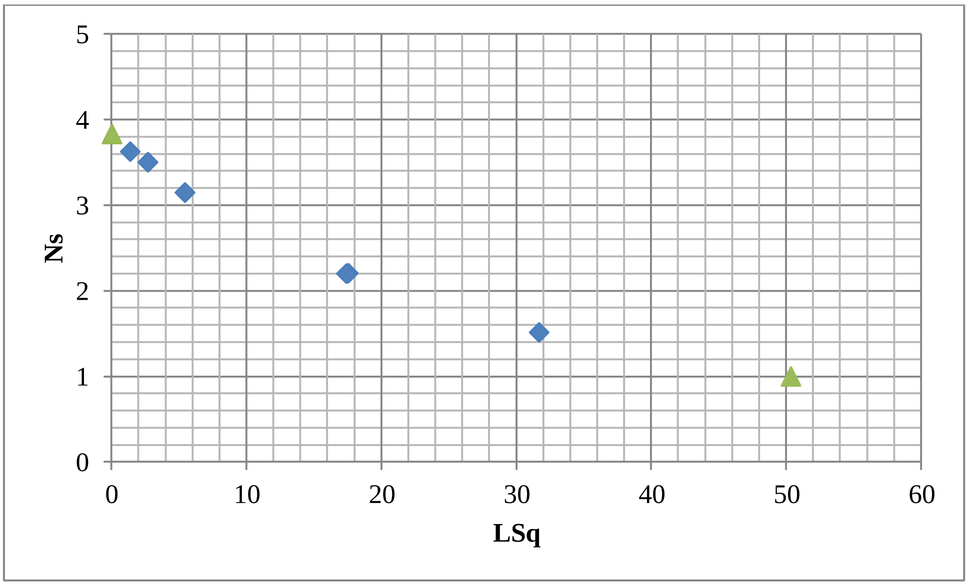
Finland 2003



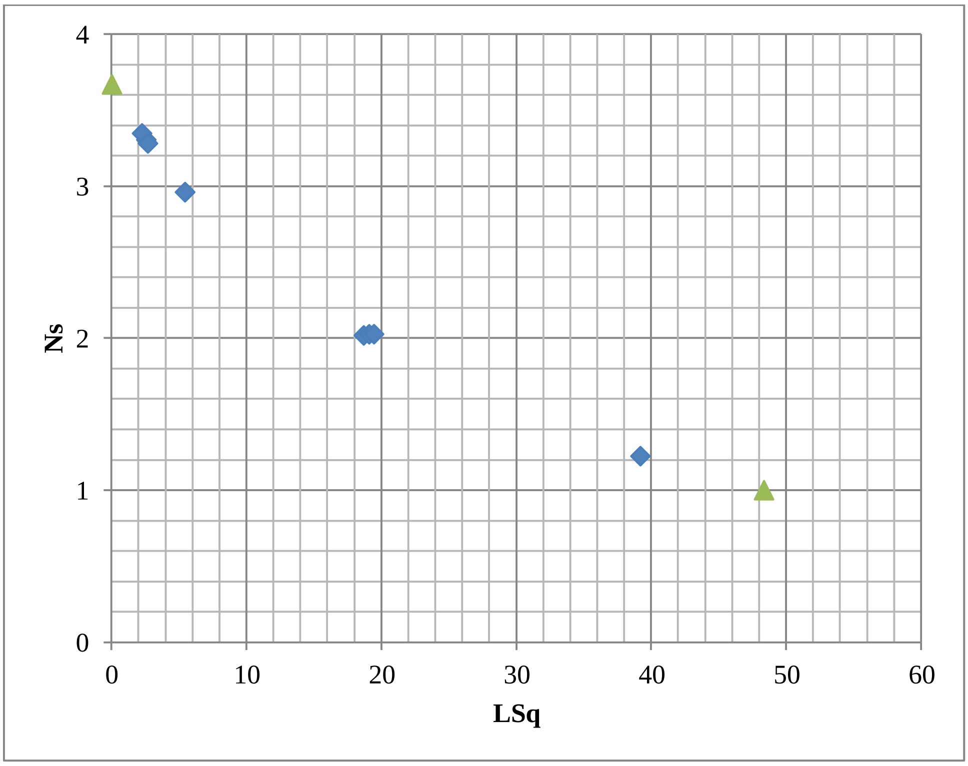
Finland 2007



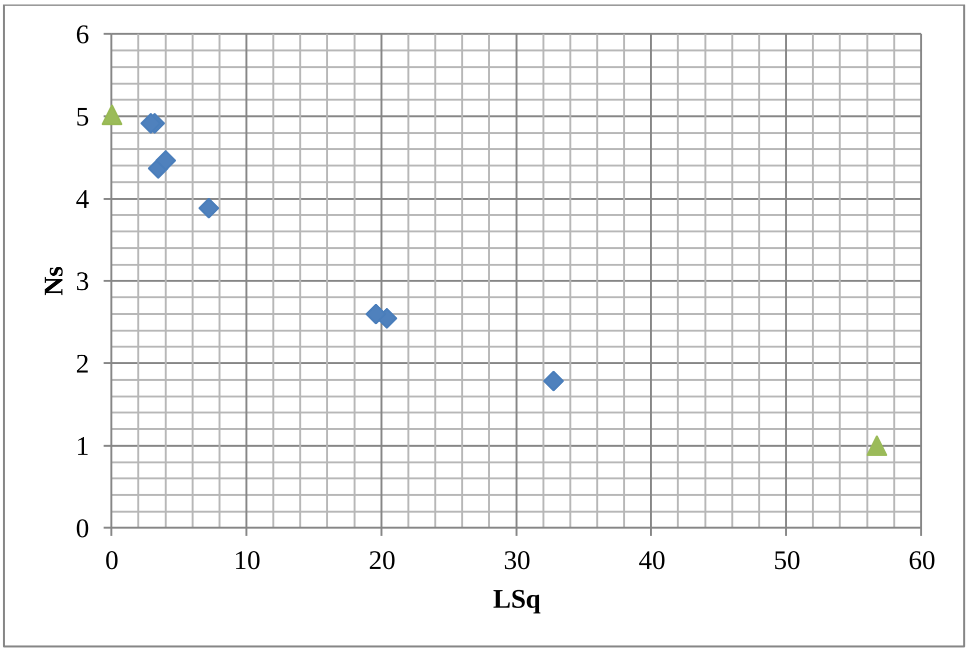
Portugal 2009



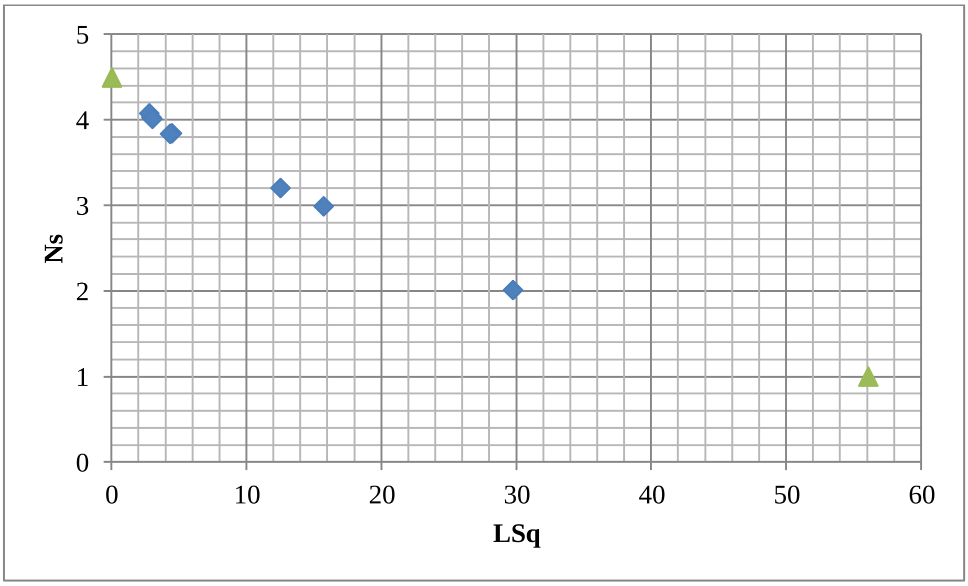
Portugal 2011



Estonia 2007



Estonia 2011



Appendix 2. Results of the computational experiment by electoral system

			Ns	LSq	Nv	LSq max	(N-1)/(Nmax-1)	1-(LSq/Lsqmax)	log((N-1)/(Nmax-1))	log(1-(LSq/Lsqmax))
CZECH REPUBLIC	**2002**	Real	3,668	5,782	4,817	54,930	0,699	0,895	-0,156	-0,048
		d'hondt	3,668	5,782			0,699	0,895	-0,156	-0,048
		saint-lague	3,969	3,895			0,778	0,929	-0,109	-0,032
		simple quota	4,353	3,221			0,878	0,941	-0,056	-0,026
		droop quota	4,095	3,311			0,811	0,940	-0,091	-0,027
	M=7	d'hondt	3,479	7,452			0,650	0,864	-0,187	-0,063
		saint-lague	3,731	5,473			0,716	0,900	-0,145	-0,046
		simple quota	4,065	3,555			0,803	0,935	-0,095	-0,029
		droop quota	3,808	5,393			0,736	0,902	-0,133	-0,045
	M=4	d'hondt	3,328	9,269			0,610	0,831	-0,215	-0,080
		saint-lague	3,877	6,047			0,754	0,890	-0,123	-0,051
		simple quota	3,935	6,760			0,769	0,877	-0,114	-0,057
		droop quota	3,877	6,047			0,754	0,890	-0,123	-0,051
	M=2	all 4	2,532	19,191			0,401	0,651	-0,397	-0,187
	M=1		1,380	41,642			0,100	0,242	-1,002	-0,616
	2006	Real	3,096	5,719	3,909	52,356	0,721	0,891	-0,142	-0,050
		d'hondt	3,135	5,418			0,734	0,897	-0,134	-0,047
		saint-lague	3,531	2,417			0,870	0,954	-0,060	-0,021
		simple quota	3,724	2,052			0,937	0,961	-0,028	-0,017
		droop quota	3,554	2,255			0,878	0,957	-0,056	-0,019
	M=1		1,993	19,848			0,341	0,621	-0,467	-0,207
	M=2	d'hondt	1,969	20,348			0,333	0,611	-0,477	-0,214
		other 3	2,000	19,852			0,344	0,621	-0,464	-0,207

DENMARK	**2007**	Real	4,626	5,198	5,563	57,522	0,795	0,910	-0,100	-0,041
		d'hondt	4,626	5,198			0,795	0,910	-0,100	-0,041
		saint-lague	5,519	1,911			0,990	0,967	-0,004	-0,015
		simple quota	5,668	1,799			1,023	0,969	0,010	-0,014
		droop quota	5,260	2,347			0,934	0,959	-0,030	-0,018
	M=1		2,109	28,178			0,243	0,510	-0,614	-0,292
	M=2	all 4	2,351	24,849			0,296	0,568	-0,528	-0,246
	2011	Real	5,020	4,345	5,850	56,690	0,829	0,923	-0,082	-0,035
		d'hondt	5,020	4,345			0,829	0,923	-0,082	-0,035
		saint-lague	6,112	2,247			1,054	0,960	0,023	-0,018
		simple quota	6,112	2,247			1,054	0,960	0,023	-0,018
		droop quota	5,818	1,666			0,993	0,971	-0,003	-0,013
	M=1		2,059	28,183			0,218	0,503	-0,661	-0,299
	M=2	all 4	2,351	24,485			0,279	0,568	-0,555	-0,246
FINLAND	**2003**	Real	4,881	3,196	5,607	58,413	0,843	0,945	-0,074	-0,024
		d'hondt	4,721	4,012			0,808	0,931	-0,093	-0,031
		saint-lague	5,108	1,864			0,892	0,968	-0,050	-0,014
		simple quota	5,233	1,691			0,919	0,971	-0,037	-0,013
		droop quota	5,075	2,275			0,885	0,961	-0,053	-0,017
	M=1		2,281	25,864			0,278	0,557	-0,556	-0,254
	M=2	d'hondt and sainte-lague	3,260	15,049			0,491	0,742	-0,309	-0,129
		simple and droop	3,490	13,871			0,541	0,763	-0,267	-0,118
	2007	Real	5,080	3,238	5,834	59,470	0,844	0,946	-0,074	-0,024
		d'hondt	4,981	3,743			0,823	0,937	-0,084	-0,028
		saint-lague	5,621	1,160			0,956	0,981	-0,020	-0,009
		simple quota	5,637	1,031			0,959	0,983	-0,018	-0,008

		droop quota	5,467	1,528			0,924	0,974	-0,034	-0,011
	M=1		2,676	21,762			0,347	0,634	-0,460	-0,198
	M=2	d'hondt	3,261	16,220			0,468	0,727	-0,330	-0,138
		other 3	3,489	15,099			0,515	0,746	-0,288	-0,127
PORTUGAL	**2009**	Real	3,149	5,411	3,833	50,331	0,758	0,892	-0,120	-0,049
		d'hondt	3,149	5,411			0,758	0,892	-0,120	-0,049
		saint-lague	3,503	2,666			0,884	0,947	-0,054	-0,024
		simple quota	3,627	1,357			0,927	0,973	-0,033	-0,012
		droop quota	3,503	2,666			0,884	0,947	-0,054	-0,024
	M=1		1,514	31,662			0,182	0,371	-0,741	-0,431
	M=2	d'hondt	2,206	17,523			0,426	0,652	-0,371	-0,186
		other 3	2,201	17,372			0,424	0,655	-0,373	-0,184
	2011	Real	2,9609	5,4183	3,670	48,332	0,735	0,888	-0,134	-0,052
		d'hondt	2,9609	5,4183			0,735	0,888	-0,134	-0,052
		saint-lague	3,3059	2,5423			0,864	0,947	-0,064	-0,023
		simple quota	3,3475	2,2285			0,879	0,954	-0,056	-0,021
		droop quota	3,2817	2,663			0,855	0,945	-0,068	-0,025
	M=1		1,2237	39,171			0,084	0,190	-1,077	-0,722
	M=2	d'hondt	2,0182	18,664			0,381	0,614	-0,419	-0,212
		saint-l. ja droop	2,0258	19,066			0,384	0,606	-0,415	-0,218
		simple quota	2,0265	19,424			0,385	0,598	-0,415	-0,223
ESTONIA	**2007**	Real	4,369	3,426	5,020	56,696	0,838	0,940	-0,077	-0,027
		d'hondt	3,886	7,176			0,718	0,873	-0,144	-0,059
		saint-lague	4,916	3,161			0,974	0,944	-0,011	-0,025
		simple quota	4,916	2,878			0,974	0,949	-0,011	-0,023
		droop quota	4,464	3,980			0,862	0,930	-0,065	-0,032

	M=1		1,786	32,729			0,195	0,423	-0,709	-0,374
	M=2	simple quota	2,547	20,373			0,385	0,641	-0,415	-0,193
		other 3	2,599	19,563			0,398	0,655	-0,400	-0,184
	2011	Real	3,842	4,444			0,813	0,921	-0,090	-0,036
		d'hondt	3,836	4,305			0,811	0,923	-0,091	-0,035
		saint-lague	4,030	2,917			0,867	0,948	-0,062	-0,023
		simple quota	4,076	2,771	4,495	56,062	0,880	0,951	-0,056	-0,022
		droop quota	4,011	3,000			0,862	0,946	-0,065	-0,024
	M=1		2,011	29,725			0,289	0,470	-0,539	-0,328
	M=2	simple quota	2,989	15,690			0,569	0,720	-0,245	-0,143
		other 3	3,203	12,490			0,630	0,777	-0,200	-0,109

Kokkuvõte: "Valimissüsteemide mehaanilised efektid proportisionaalsusele ja parlamendi killustatusele"

Valimissüsteemide näol on tegemist ülioluliste institutsioonidega demokraatia toimimise aspektist, sest nad mõjutavad suurel määral poliitilisi väljundeid. Lisaks sellele, et valimissüsteemid muundavad valijate hääled parlamendikohtadeks, mõjutavad nad ka laiemalt poliitilise süsteemi põhilisi tunnusjooni ning poliitilist kultuuri. Valimissüsteemide efektide uurimine on oluline mõistmaks paremini demokraatlike poliitiliste süsteemide toimimist üle maailma.

Kirjanduses valitseb üldine konsensus, et ideaalset valimissüsteemi ei eksisteeri. Tunnistatakse, et igal süsteemil on omad eelised ja puudused ning et valimissüsteemi valikul on teatavate järeleandmiste tegemine möödapääsmatu. Valimissüsteemi valikul saab rääkida kahest põhilisest vastandlikust ideaalist – esindatuse ideaal (ehk kõrge proportsionaalsus) ning vastutuse/aruandekohustuse ideaal (stabiilsed ühepartei valitsused ehk vähene parlamendi killustatus). Esindatuse ideaal käib käsikäes proportsionaalse esindatuse (PR) valimissüsteemidega ning vastutuse ideaal majoritaarsete süsteemidega. Need kaks ideaali on omavahel vastandlikud, sest mida proportsionaalsem on esindatus, seda rohkem tegutsejaid on parlamendis, mis aga omakorda tähendab suurt parlamendi fragmenteeritust. Seega ei ole võimalik korraga püüelda nii esindatuse kui vastutuse ideaali poole, vaid tuleb valida neist üks.

Võiks arvata, et proportsionaalsuse ja fragmenteerituse vahel on lineaarne seos – mida suurem proportsionaalsus, seda suurem on killustatus ja vastupidi. Empiirilised andmed aga seda oletust ei toeta, proportsionaalsuse ja fragmenteerituse vaheline seos ei ole kindlasti mitte lineaarne (vt. jooniseid 2.1 ja 2.2 peatükis 2). Veelgi enam, pilt tundub kirju – proportsionaalsuse ja fragmenteerituse vahel justkui ei olekski selget seost. Seega on oluline välja selgitada, miks ei ole näha selget seost muutujate vahel, mis peaksid olemuslikult olema omavahel vastandlikud.

Käesoleva magistritöö eesmärgiks oli uurida lähemalt valimissüsteemide mehaanilisi efekte proportsionaalsusele ja killustatusele. Tuginedes uue institutsionalismi teooriale, võeti uurimuse aluseks tõdemus, et valimissüsteemide väljundites selgete seoste nägemine ei ole võimalik ilma konteksti arvesse võtmata. Kui aga hoida kontekst uurimuses kontrolli all, on võimalik näha selget mustrit proportsionaalsuse ja killustatuse vahel.

Leidmaks võimalikku universaalset seost ebaproportsionaalsuse (LSq) ja parlamendi killustatuse (Ns) vahel, viidi läbi arvutuslik eksperiment. Eksperimendis kasutati ringkonnatasandi valimistulemuste andmeid 5 riigi ja 10 valimise jaoks, mis pärinesid CLEA andmebaasist. Kasutades sama häälte sisendit, arvutati iga valimise jaoks välja mitmeid alternatiivseid kohtade jaotusi erinevate valimisreeglite korral. See tähendab, eksperimendis muudeti vaid valimissüsteemi, sisendi poolt hoiti konstantsena. Nii tekkis võimalus näha variatsioone vaid valimissüsteemide mehaanilistes efektides. Kuna eksperimendis oli aga 10 erinevat sisendit, uuritavad muutujad normaliseeriti universaalse seose leidmise eesmärgil. Kasutades normaliseerimiseks iga häältejaotuse loogiliste ankrupunktide väärtusi (0 ja LSqmax; 1 ja Nv), jõuti järgneva universaalse seoseni:

$$\frac{N_s - 1}{N_v - 1} = \left(1 - \frac{LSq}{LSq_{max}}\right)^2$$

Teisiti öeldes leiti käesolevas töös, et normaliseeritud ebaproportsionaalsuse (LSq') ja parlamendi fragmenteerituse (Ns') vahel on ruutseos: $Ns' = LSq'^2$. See on oluline avastus, sest üldjuhul on vaikimisi eeldatud, et proportsionaalsuse ja fragmenteerituse vaheline suhe on absoluutne, st lineaarne. Antud analüüsi tulemused aga näitavad, et see nii ei ole ning parimaks sobituseks osutus hoopiski ruutmudel.

On oluline rõhutada, et täpne seos proportsionaalsuse ja fragmenteerituse vahel sõltub kontekstist (häälte jaotusest). LSq_{max} ja Nv, mida kasutati normaliseerimiseks, on erinevad iga häälte jaotuse jaoks. Tegemaks täpseid ennustusi valimissüsteemide mehaaniliste efektide kohta proportsionaalsusele ja killustatusele, on vajalik teada häälte jaotust. Ilma konteksti arvesse võtmata, ei ole võimalik selget seost LSq ja Ns vahel välja tuua.

Ruutseose $Ns' = LSq'^2$ testimine laiemal andmestikul tõestas seose paikapidavust. PR valimissüsteemide jaoks oli testimise eesmärgil võimalik teha valemiline lihtsustus – Lsq_{max} on PR süsteemide jaoks ligikaudu 50%. Lihtsustuse abil sai valemi paikapidavust testida ühtekokku 340 PR valimise puhul, avaldades algsest valemist Ns-i ning võrreldes seda reaalse Ns-ga. Tulemused näitasid, et PR süsteemide puhul ennustab valem $Ns' = LSq'^2$ Ns-i väärtust äärmiselt täpselt. Testimaks seose paikapidavust majoritaarsete süsteemide puhul, sarnast lihtsustust ei olnud võimalik teha, sest LSq_{max}-i täpne väärtus mõjutab nendes süsteemides Ns-i väärtust olulisel määral. Selle asemel avaldati LSqmax-i väärtus Nv ja v1 (võitva partei häälte saak) kaudu:

$$LSq_{max} = \sqrt{\frac{N_v(1-2v_1)+1}{2N_v}}$$

Taaskord võrreldi tegelikku Ns-i arvutusliku Ns-ga, kasutades tulemusi 40 majoritaarse süsteemiga valimistelt. Ruutvalem ennustas jällegi Ns-i tegelikku väärtust väga täpselt. Selgus aga, et majoritaarsete süsteemide puhul ennustas valem süsteemselt pisut madalamaid Ns-i väärtusi võrreldes tegelikuga. See tulemus näitab meile, et valem $Ns' = LSq'^2$ peab paika PR süsteemide puhul, kuid majoritaarsete süsteemide peaks valemis kaaluma veidi väiksema eksponendi väärtuse kasutamist (ligikaudu 1,7).

Testimise tulemused näitavad, et valemi $Ns' = LSq'^2$ näol on tegemist pigem esimese lähenduse, mitte aga universaalse reegliga. Selgus, et valem omab pisut erinevaid efekte PR ja majoritaarsete süsteemide jaoks, mistõttu tegelikkuses ei eksisteeri Ns' ja LSq' vahel mitte lihtne ruutseos, vaid pisut keerulisem seos. Tulevikus tuleks kindlasti see täpsem mudel ka välja töötada. Esimene lähendus $Ns' = LSq'^2$ töötab aga piisavalt hästi ning on sobivaim tegemaks üldistusi LSq ja Ns vahelise suhte kohta.

Magistritöö tulemused tõestasid, et suhe Ns' ja LSq' vahel ei ole lineaarne, mistõttu kahe vastandliku ideaali, kõrge proportsionaalsuse ja vähese killustatuse, optimeerimine on võimalik. Töös leiti, et suhe Ns' ja LSq' vahel on kõige optimaalsem, kui ringkonnamagnituud on 1 või 2. See tulemus vajab aga kindlasti täiendavat uurimist ja tõlgendamist, sest see võib olla teataval määral mõjutatud konkreetsest viisist, kuidas

Ns ja LSq normaliseeriti. Sellegipoolest viitavad töö tulemused sellele, et kahe demokraatliku ideaali optimum on saavutatav tunduvalt väiksemate ringkonnamagnituudide korral kui varasemalt arvatud.

Käesolev töö jätab avatuks väga olulise küsimuse. Leiti, et seos $Ns' = LSq'^2$ peab võrdlemisi hästi paika nii PR kui ka majoritaarsete süsteemide puhul. Küsimus on: miks? Miks on LSq' ja Ns' vahel just ruutseos? Kõik muutujad valemis on avaldatavad häälte ja kohtade jaotuste kaudu – võib eeldada, et just häälte-kohtade suhe peaks aitama selgitada eelpool väljatoodud ruutseose kehtivust. Seda tuleks aga tulevikus kindlasti täiendavalt uurida.

Lõpetuseks, magistritöö selgitas välja ruutseose normaliseeritud proportsionaalsuse ja parlamendi fragmenteerituse vahel. Näidati, et see seos peab paika, kui testida seda laiemal andmestikul. Käesolev töö on esimene samm LSq ja Ns vahelise täpse seose leidmise ja selgitamise suunas, mida varasemalt tehtud ei ole. Lihtsustatud mudel $Ns' = LSq'^2$ on heaks stardipunktiks edasistele uuringutele, mida võiks ja tuleks tulevikus läbi viia töös kerkinud küsimuste tõttu. Ruutseose avastamine Ns' ja LSq' vahel on aga oluliseks panuseks mõistmaks paremini valimissüsteemide mehaanilisi efekte poliitilistele väljunditele.