Limits of Particle Mobility Resolution in Electric Aerosol Particle Separators

HANNES TAMMET

Department of Environmental Physics

Tartu University

Estonia

Tartu University

(University of Tartu)

Grounded 1632 as the second (after Uppsala) university in BaltoScandia.

From the old history:

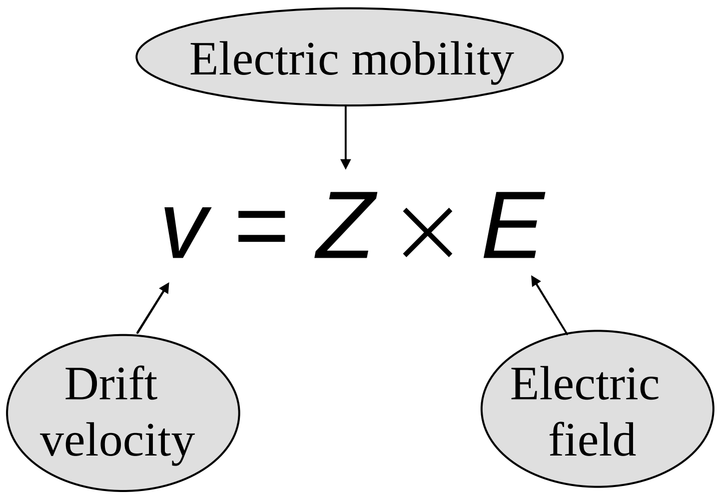
Physicists: Lenz, Jacobi
Environmentalist: Baer

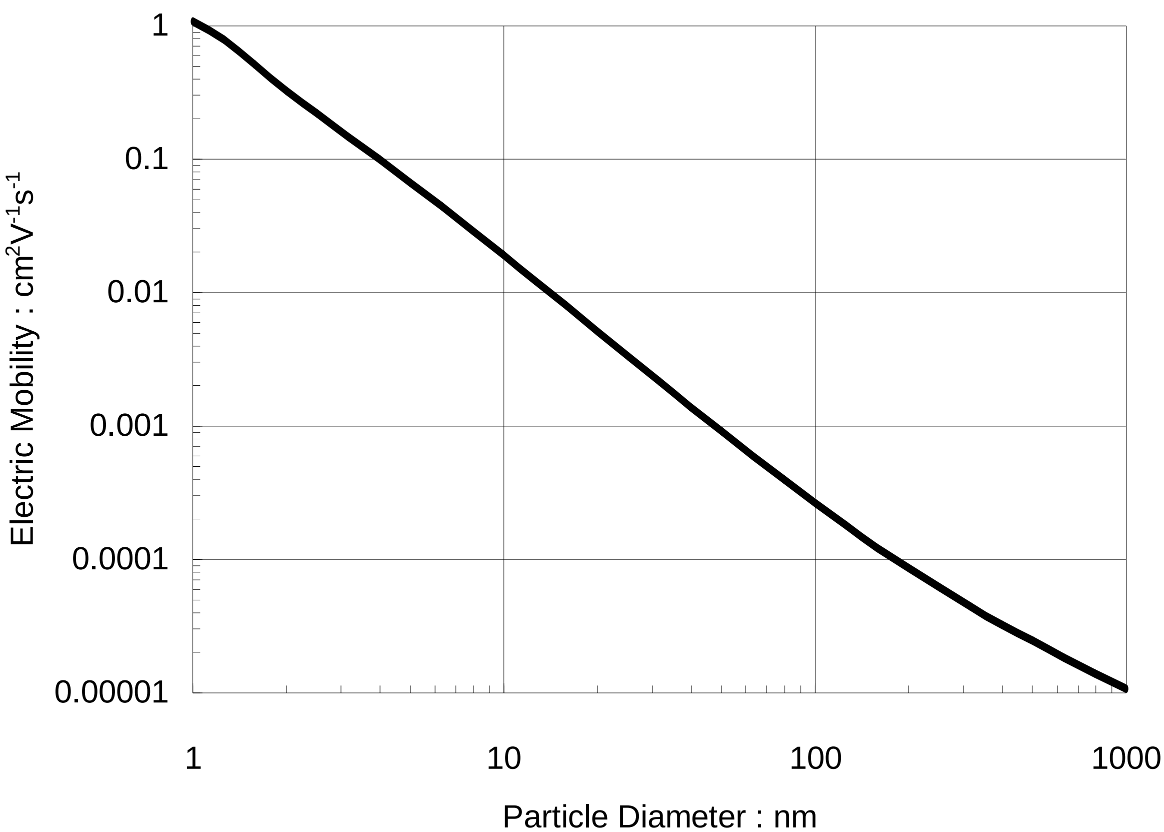
Today:

7560 students
10 faculties

- Faculty of Physics and Chemistry
  - Division of Physics
    - Department of Environmental Physics
      - Air Electricity Laboratory

ELECTRIC MOBILITY





Diameter-mobility relation for single charged particles
(Air, standard pressure, 20ºC)

LAGRANGIAN METHOD

Eulerian method:

reference system tied to the instrument

(a riverbank frame).

Lagrangian method in *hydrodynamics*:

reference system drifting downstream

(a drifting raft frame).

Lagrangian method in *DMA theory*:

reference system drifting with particles

as influenced simultaneously by

air flow and electric field

(a powered boat frame).

A THEOREM ABOUT MONOMOBILE AIR ION CONCENTRATION

1. From Eulerian position:

charge conservation & convection current

$$\frac{\partial\rho}{\partial t} = -\mathrm{div}\,\mathbf{j} \quad \& \quad \mathbf{j} = \mathbf{v}_{ion}\rho \quad \Rightarrow \quad \frac{\partial\rho}{\partial t} = -\rho\,\mathrm{div}\,\mathbf{j} - \mathbf{v}_{ion}\,\mathrm{grad}\,\rho$$

2. From Lagrangian position:

$d\rho$ – complete differential in a point drifting along with air ions.

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \mathbf{v}_{ion}\,\mathrm{grad}\,\rho = -\rho\,\mathrm{div}\,\mathbf{v}_{ion}$$

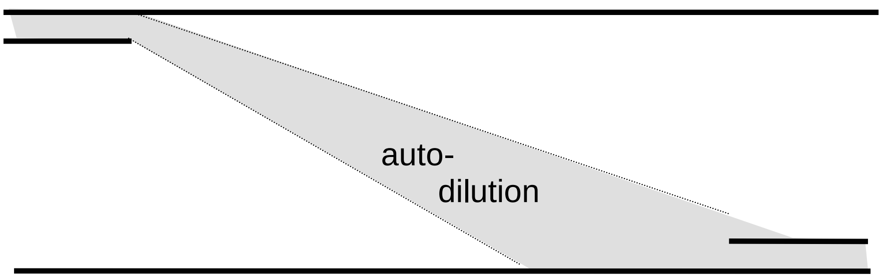
$$\mathbf{v}_{ion} = \mathbf{v}_{air} + Z\mathbf{E} \quad \Rightarrow \quad \mathrm{div}\,\mathbf{v}_{ion} = \mathrm{div}\,\mathbf{v}_{air} + Z\,\mathrm{div}\,\mathbf{E} = Z\frac{\rho}{\varepsilon_\circ}$$

Equation: $\dfrac{d\rho}{dt} = -\dfrac{Z}{\varepsilon_\circ}\rho^2$ Solution: $\rho = \dfrac{\rho_\circ}{1 + \dfrac{Z\rho_\circ}{\varepsilon_\circ}t}$

Autodilution factor: $1 + \dfrac{Z\rho_\circ}{\varepsilon_\circ}t$ Critical time: $t_\circ = \dfrac{\varepsilon_\circ}{Z\rho_\circ}$

Example: 10^5 single-charged particles per ccm
$Z = 1\ \mathrm{cm^2V^{-1}s^{-1}}$
$t_o = 5.5$ s

EFFECT OF SPACE CHARGE



Apparent Width of Inlet Split =
= Dilution Factor × Physical Width

Double-Broadening Limit: $\rho_\circ Zt = \varepsilon_\circ$

In a plain DMA $Zt = \dfrac{d}{E} = \dfrac{d^2}{V}$ and
double-broadening maximum:

$$\rho_{\max 2} = \frac{\varepsilon_\circ V}{d^2} \qquad n_{\max 2} = \frac{\varepsilon_\circ V}{q d^2}$$

Example: $d = 1$ cm & $V = 1$ V $\Rightarrow$ $n_{\max 2} = 550\,000\ \text{cm}^{-3}$

Total yield of particles in a cylindrical DMA $\sim n\, d^2 \sim V$.

EFFECT OF MOLECULAR DIFFUSION

Alternative theoretical models:

1) **Eulerian** – Fick equation
2) **Lagrangian** – Brownian motion

1. TOF (Time Of Flight) spectrometer. Ions are drifting in uniform electric field in calm air. Distance is given and time is measured. Metrological quasiequivalent: time is given and distance is measured.

Distance $d = ZEt$ relative error $E_{ZD} = E_d$.

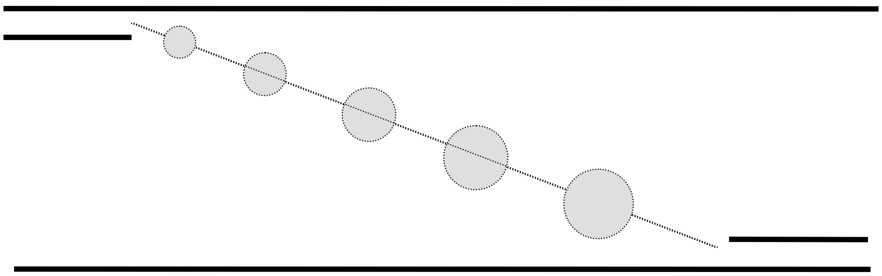
As shown by Einstein, Brownian deflection of the distance is distibuted according to Gauss law with a standard deviation of

$$\sigma_d = \sqrt{2\frac{kTZ}{q}t} \ .$$

Distance can be expressed as $d = \sqrt{ZVt}$. It follows

$$E_{ZD} = E_d = \frac{\sigma_d}{d} = \sqrt{\frac{2kT}{qV}}$$

2. Plain DMA.



The effect of Brownian motion along the electric field is the same as in TOF. It is combined with the effect of Brownian motion along the air flow

$$E_{ZD} = \sqrt{E_d^2 + E_l^2}$$

$$\sigma_l = \sigma_d \quad \Rightarrow \quad E_l = \frac{\sigma_d}{l} = \frac{d}{l} E_d$$

$$E_{ZD} = \sqrt{\left(1 + \frac{d^2}{l^2}\right) \frac{2kT}{qV}}$$

or

$$\mu_D = 1 + \frac{d^2}{l^2} \qquad \& \qquad E_{ZD} = \sqrt{\mu_D \frac{2kT}{qV}}$$

3. Cylindrical DMA.

d – distance between electrodes $d = r_2 - r_1$
L – relative length $L = l / d$
R – flatness $R = r_1 / d$
V – voltage
Re – Reynolds number $\mathrm{Re} = \frac{\overline{v_{air}} d}{\nu} = \frac{\Phi}{\nu\pi(r_1 + r_2)}$

Critical mobility (independent of air flow profile):

$$Z = \frac{\nu\,\mathrm{Re}}{VL} \mathrm{f}_Z(R) \qquad \mathrm{f}_Z(R) = \left(\frac{1}{2} + R\right) \ln\left(1 + \frac{1}{R}\right)$$

Brownian relative error (flat air flow pofile):

$$E_{ZD} = \sqrt{\mu_D \frac{2kT}{qV}} \qquad \mu_D = \left(1 + \frac{1}{(1 + 2R)^2} + \frac{1}{L^2}\right) \mathrm{f}_Z(R)$$

MINIMIZATION OF THE EFFECT OF MOLECULAR DIFFUSION

$$E_{ZD} = \sqrt{\mu_D \frac{2kT}{qV}} \qquad \mu_D = \left(1 + \frac{1}{(1+2R)^2} + \frac{1}{L^2}\right) f_Z(R)$$

$$Z = \frac{\nu \, \mathrm{Re}}{VL} f_Z(R)$$

NB – Z is fixed.

– Increase in voltage: should simultaneously increase Re/L,
– Decrease in μ_D: limited resources, when increasing L, should simultaneosly increase Re/V.

Fixed R & Re problem: $V \sim L \Rightarrow E_{ZD}^2 \sim \left(L + \frac{L}{(1+2R)^2} + \frac{1}{L}\right)$

$$L_{opt} = \frac{1+2R}{\sqrt{1+(1+2R)^2}}$$

R	L_{opt}
∞	1.00
1	0.95
0.5	0.89

Effect of Turbulent Diffusion

$$E_{ZT} = \mu_T \xi$$

ξ – a specific measure of turbulence intensity

Rough empirical model: $\xi \approx c_T \sqrt[4]{\mathrm{Re} - 1200}$

$c_T \approx 0.0025$???

$$\mu_T = \frac{1 + 2R}{6L^2} \left\{ \left[1 + \frac{4L^2 (1+R)^2}{(1+2R)^2} \right]^{3/2} - \left[1 + \frac{4L^2 R^2}{(1+2R)^2} \right]^{3/2} \right\}$$

If R$\to\infty$ then $\mu_T \to \sqrt{1 + L^2}$.

DEPENDENCE OF DIFFUSIVE BROADENING ON CONSTRUCTIONAL PARAMETERS OF DMA

Function – a measure of mobility resolution:

$$\mathrm{Res} = \frac{Z}{\Delta Z} = \frac{1}{2E_Z} = \frac{1}{2\sqrt{E_{ZD}^2 + E_{ZT}^2}}$$

Arguments – constructional parameters of DMA:

- d – distance between electrodes $d = r_2 - r_1$
- L – relative length $L = l / d$
- R – flatness $R = r_1 / d$
- V – voltage
- Re – Reynolds number $\mathrm{Re} = \frac{\overline{v_{air}}d}{\nu} = \frac{\Phi}{\nu\pi(r_1 + r_2)}$

d has no effect on the critical mobility and the resolution.
Effect of R on mobility and resolution is weak.
Three essential arguments L, V, and Re are related by one constraint:

$$Z = \frac{\nu\,\mathrm{Re}}{VL}\,\mathrm{f}_Z(R).$$

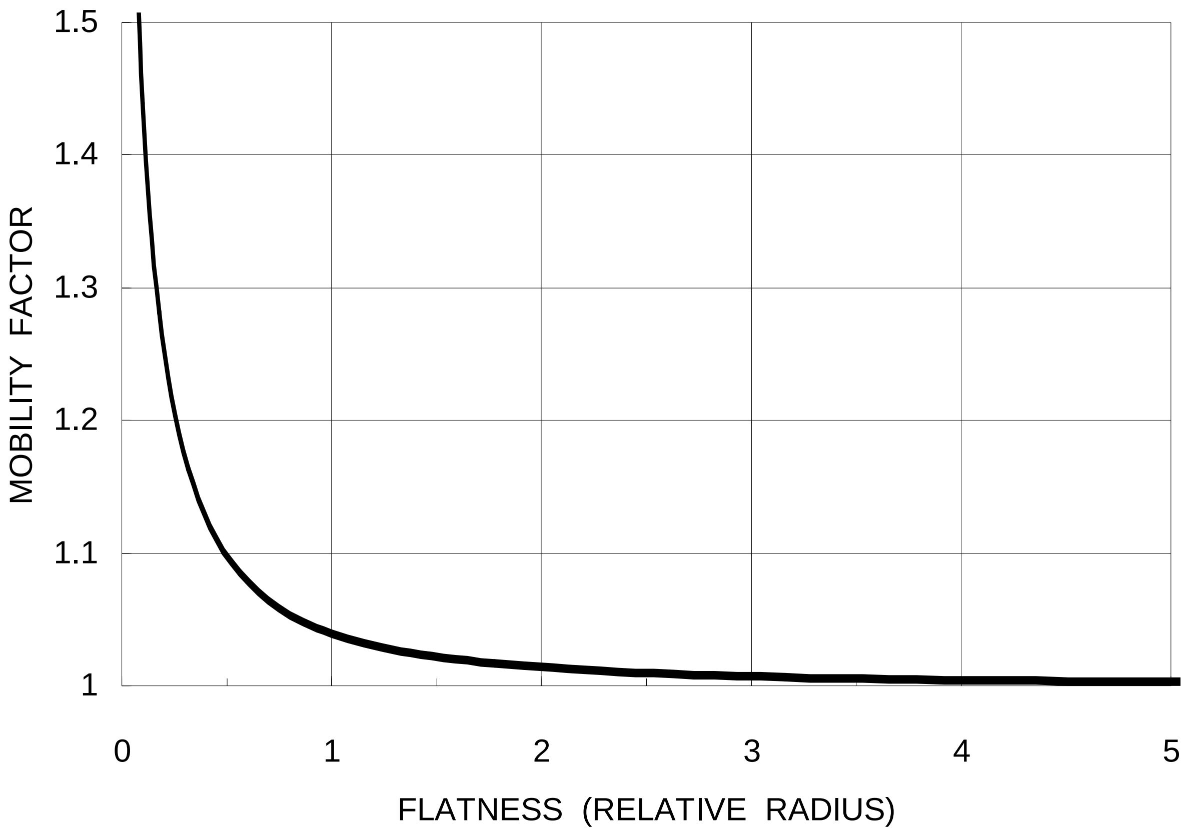
Two free arguments could be:

L & V,
L & Re,
V & Re.

```
Program DifResLR; {H. Tammet, Dec 1995}

CONST
  k_div_q = 8.617E-5; {Boltzmann div elementary charg : V/K}
     ln10 = 2.302585;

     LI : array [1..5] of real = (0.5,  1   , 2,    4,     8);
     VI : array [1..5] of real = (1,   10,  100, 1000, 10000);
    ReI : array [1..5] of real = (75, 300, 1200, 2400,  9600);

{GLOBAL} VAR

      T, {temperature : K}
   visc, {air kinematic viscosity : cm2/s}
     ct, {dimensionless coefficient of the turbulence model}

      R, {dimensionless flatness r1 / (r2 - r1)}
      L, {dimensionless length    l / (r2 - r1)}
     Re, {Reynolds number}
      V, {voltage : V}
      Z, {mobility : cm2/Vs}

     mD, {dimensionless coefficient for molecular diffusion}
     mT, {dimensionless coefficient for turbulence}
     fZ: {curvature factor in the expression of mobility}

          real;

   i, j : integer;

      f : text;   {table of resolving power}
   name : string; {name of the file for table}
```

```
Function Res : real;
   Var
     a, b, p, q, sd, st : real;

   Begin
      p := 1 + R; q := p + R;
      mD := (1 + 1 / sqr (q) + 1 / sqr (L)) * fZ;
      sD := sqrt (mD * 2 * k_div_q * T / V);
      a := 1 + sqr (2 * L * p / q);
      b := 1 + sqr (2 * L * R / q);
      mT := q * (a * sqrt (a) - b * sqrt (b)) / (6 * sqr (L));
      if   Re <= 1200
      then sT := 0
      else sT := mT * ct * sqrt (sqrt (Re - 1200));
      Res := 0.5 / sqrt (sqr (sD) + sqr (sT));
   End;
```

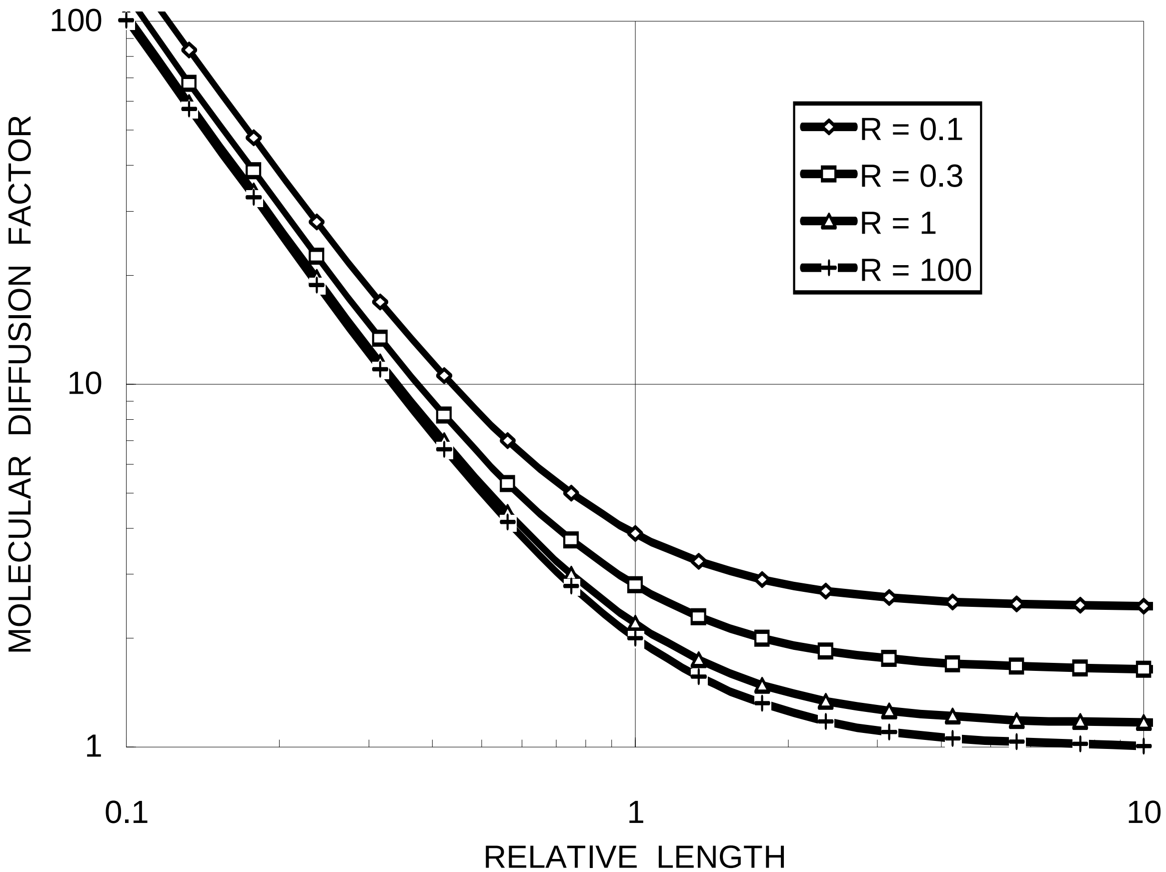
```
BEGIN                             {Z * V * L = visc * Re * fZ}
  name := '\a\lr0';

     T := 293;
  visc := 5.5E-6 * exp (1.8 * ln (T));

    ct := 0.0025;
     R := 1;
     Z := 0.5;

    fZ := (0.5 + R) * ln (1 + 1 / R);

   assign (f, name + '.tab'); rewrite (f);

   for i := 0 to 40 do begin
      L := exp (ln10 * (i - 20) / 20);
      write (f, L:9:3);

      for j := 1 to 5 do begin
         Re := ReI [j];
          V := visc * Re * fZ / (Z * L);
         write (f, Res:9:3);
      end;
      writeln (f, '         ');
   end;

   V := 1000;
   for j := 1 to 5 do begin
      Re := ReI [j];
       L := visc * Re * fZ / (Z * V);
      writeln (f, L:9:3, Res:54:3);
   end;

   close (f);
   writeln (name, ' OK'); readln;
END.
```



$$\text{Factor} \quad f_Z(R) = \left(\frac{1}{2} + R\right) \ln\left(1 + \frac{1}{R}\right)$$

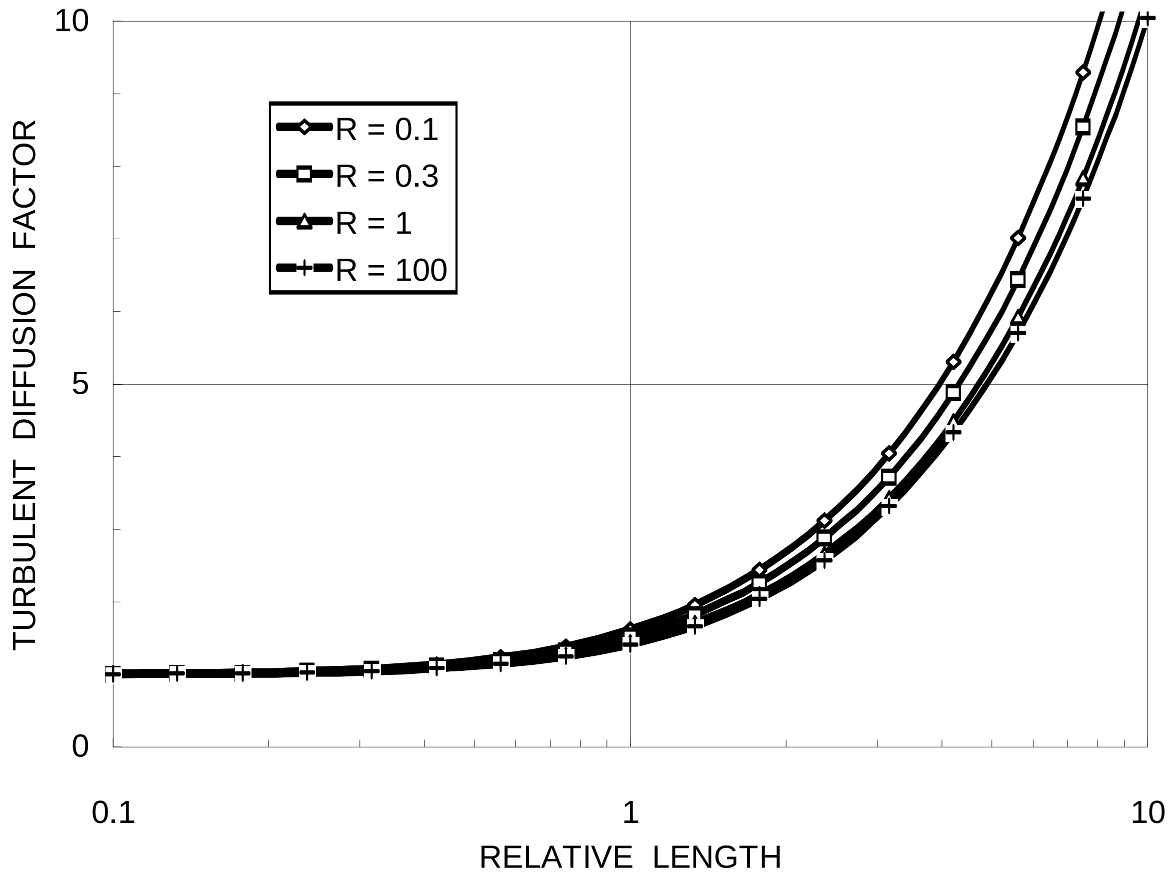
in the equation of cylindrical DMA critical mobility

$$Z = \frac{\nu \, \mathrm{Re}}{VL} f_Z(R)$$



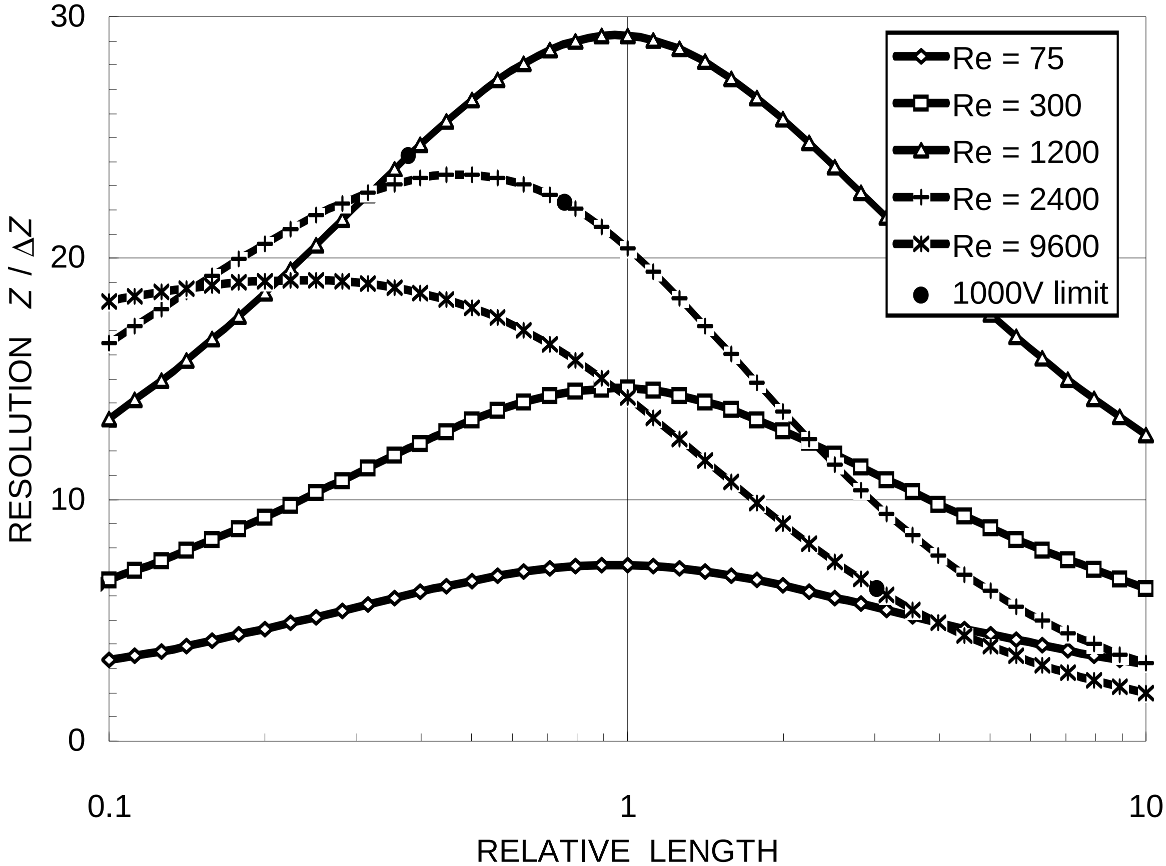
Factor of molecular diffusion

$$\mu_D = \left(1 + \frac{1}{(1+2R)^2} + \frac{1}{L^2}\right) f_Z(R)$$



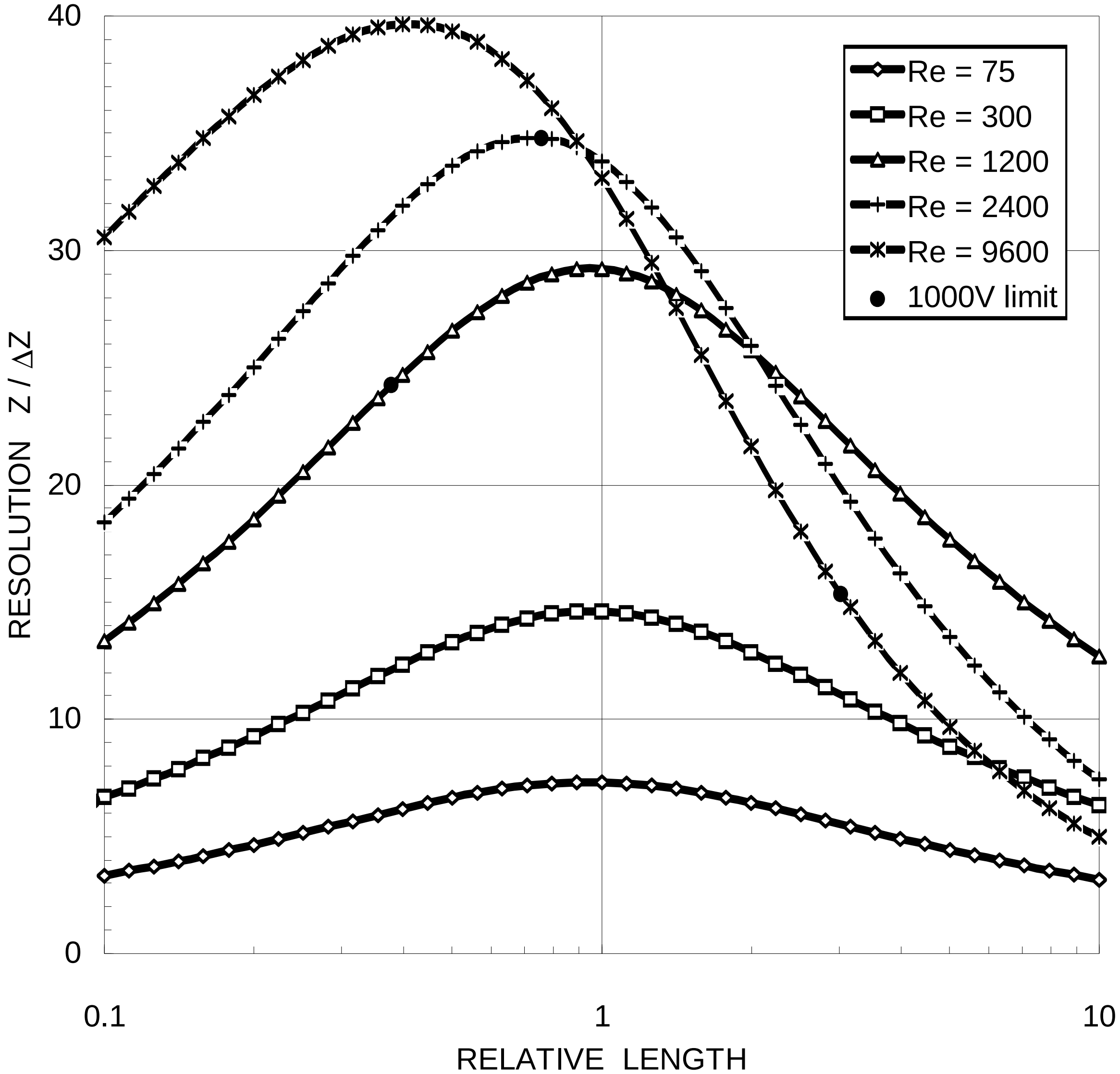
Factor of turbulent diffusion

$$\mu_T = \frac{1+2R}{6L^2}\left\{\left[1+\frac{4L^2(1+R)^2}{(1+2R)^2}\right]^{3/2} - \left[1+\frac{4L^2R^2}{(1+2R)^2}\right]^{3/2}\right\}$$



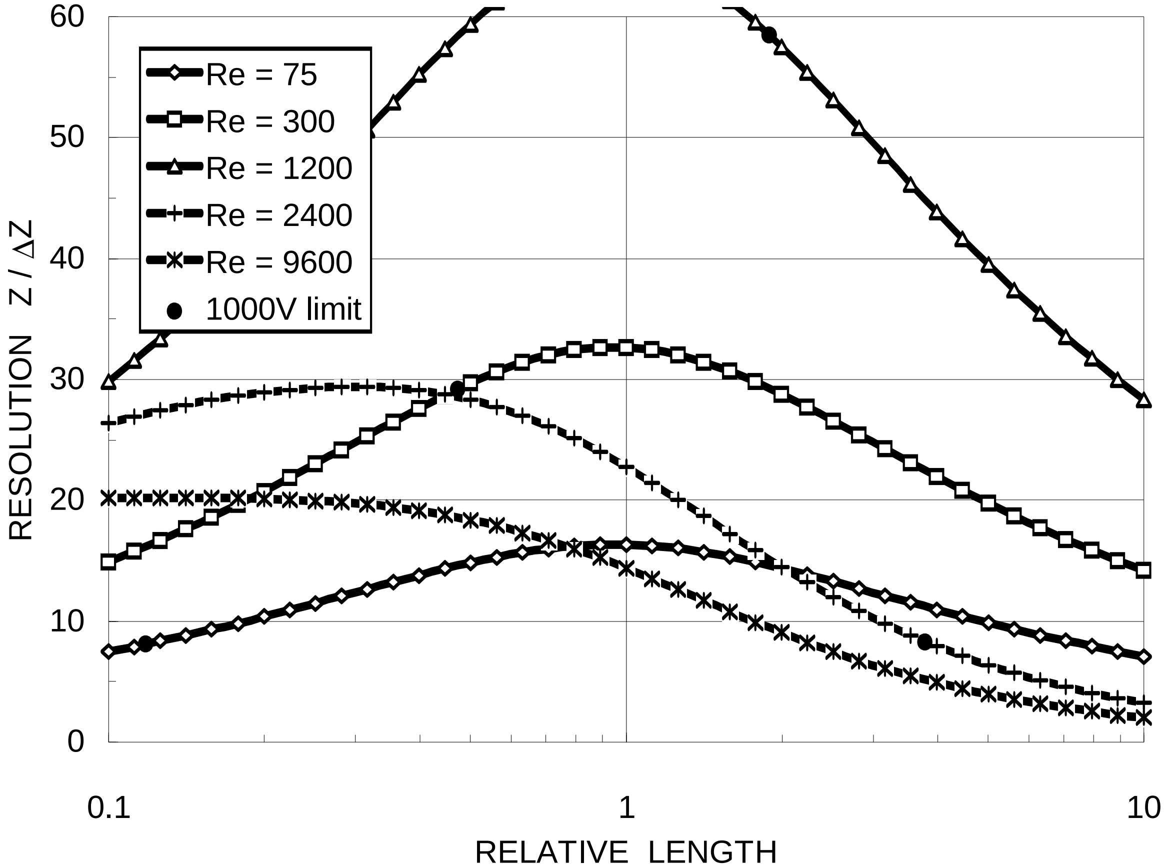
$$\text{Res} = \text{f}\,(L, \text{Re})$$

Parameter	Value	Comment
c_T	0.0025	standard value
R	1	standard value
Z	0.5 $cm^2V^{-1}s^{-1}$	standard value



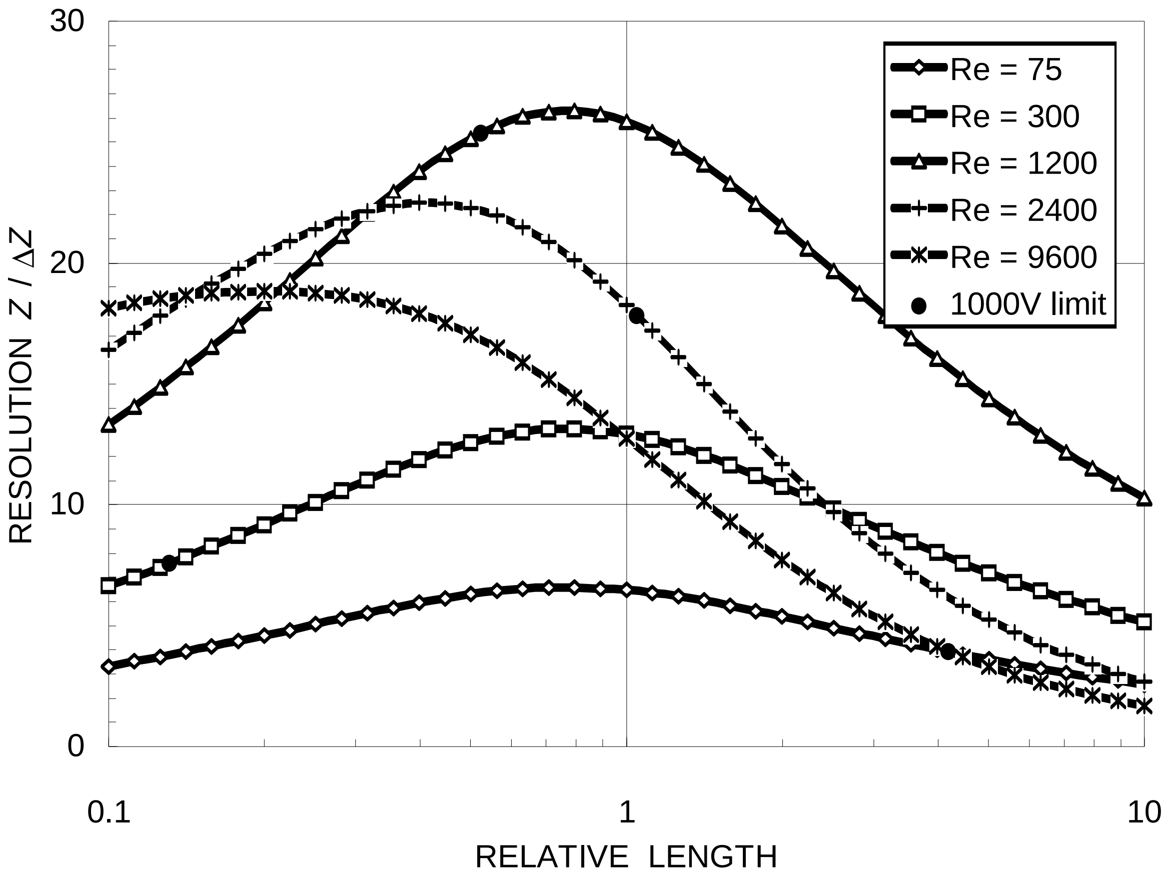
Res = f (L, Re)

Parameter	Value	Comment
c_T	0.001	40% of standard
R	1	standard value
Z	0.5 $cm^2V^{-1}s^{-1}$	standard value



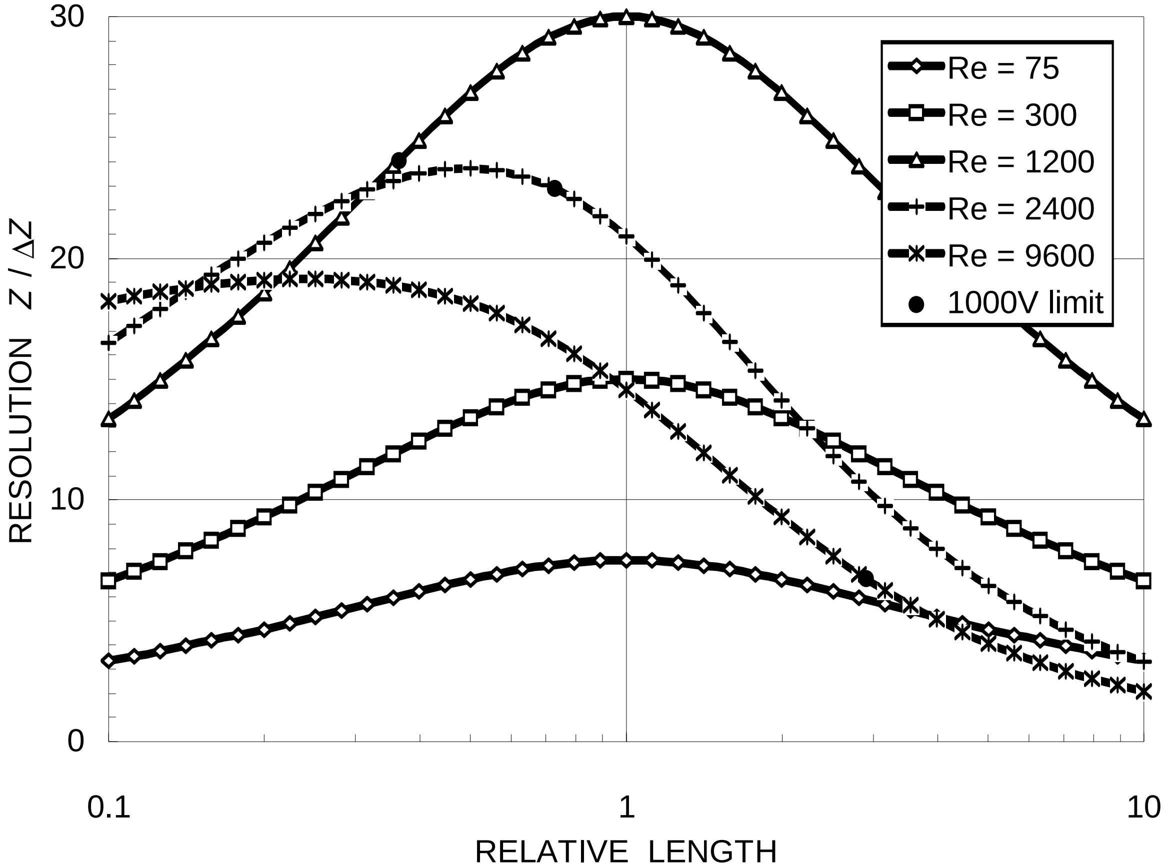
$$\text{Res} = \text{f}\ (L, \text{Re})$$

Parameter	Value	Comment
c_T	0.0025	standard value
R	1	standard value
Z	0.1 $cm^2V^{-1}s^{-1}$	20% of standard



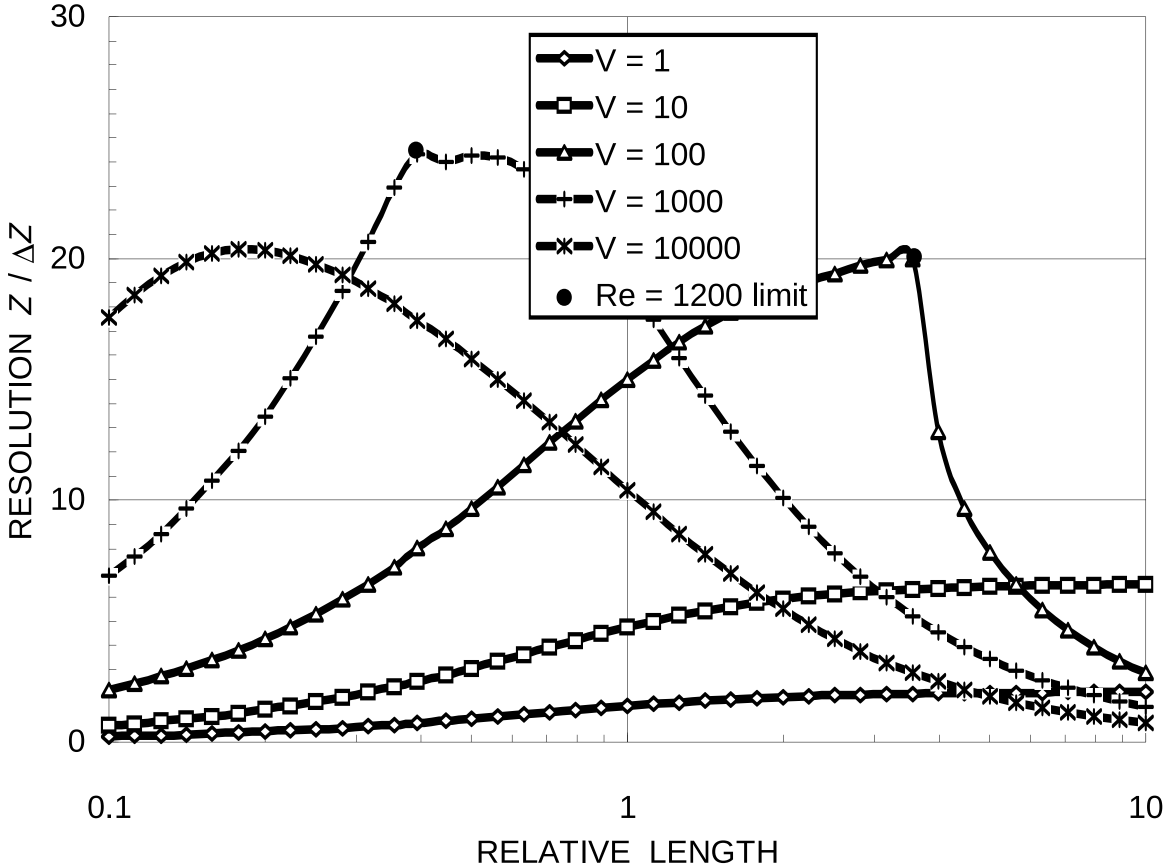
Res = f (L, Re)

Parameter	Value	Comment
c_T	0.0025	standard value
R	0.1	10% of standard
Z	0.5 $cm^2V^{-1}s^{-1}$	standard value



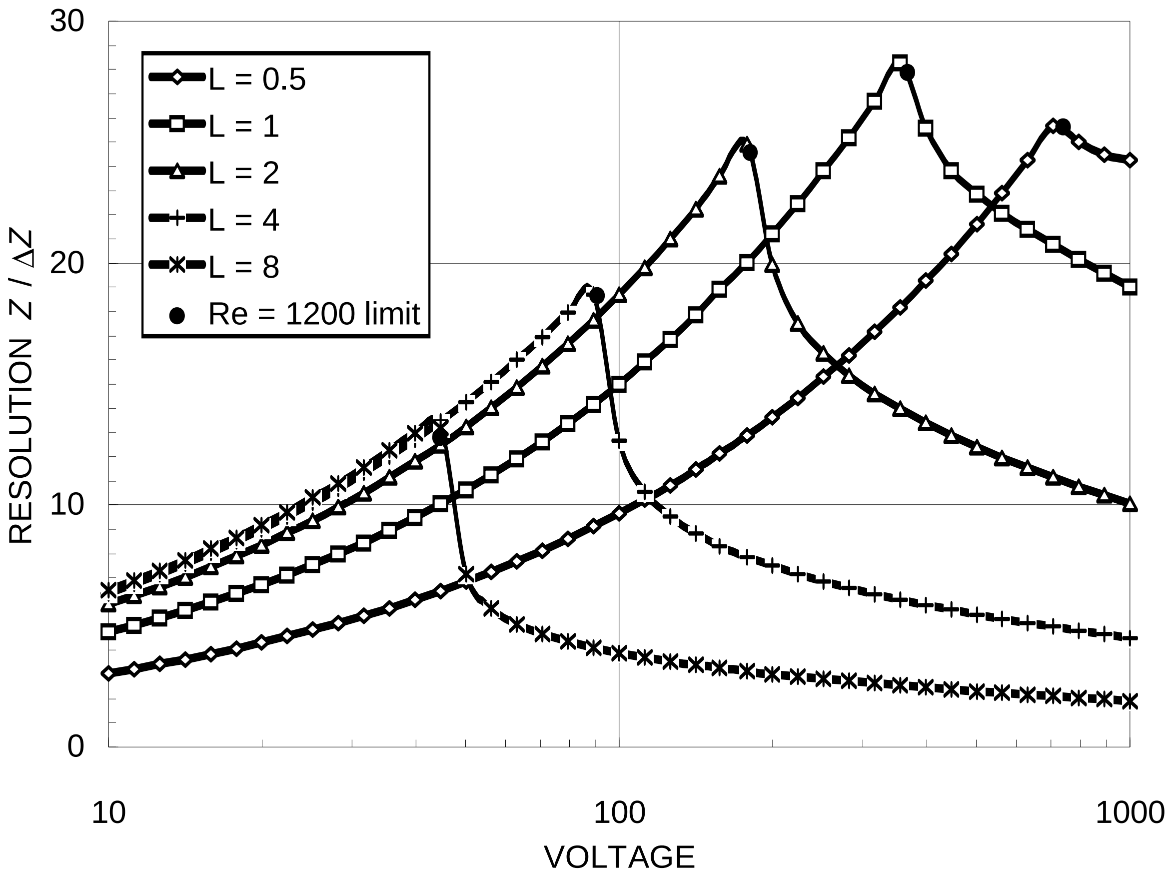
Res = f (L, Re)

Parameter	Value	Comment
c_T	0.0025	standard value
R	100	flat DMA
Z	0.5 $cm^2V^{-1}s^{-1}$	standard value



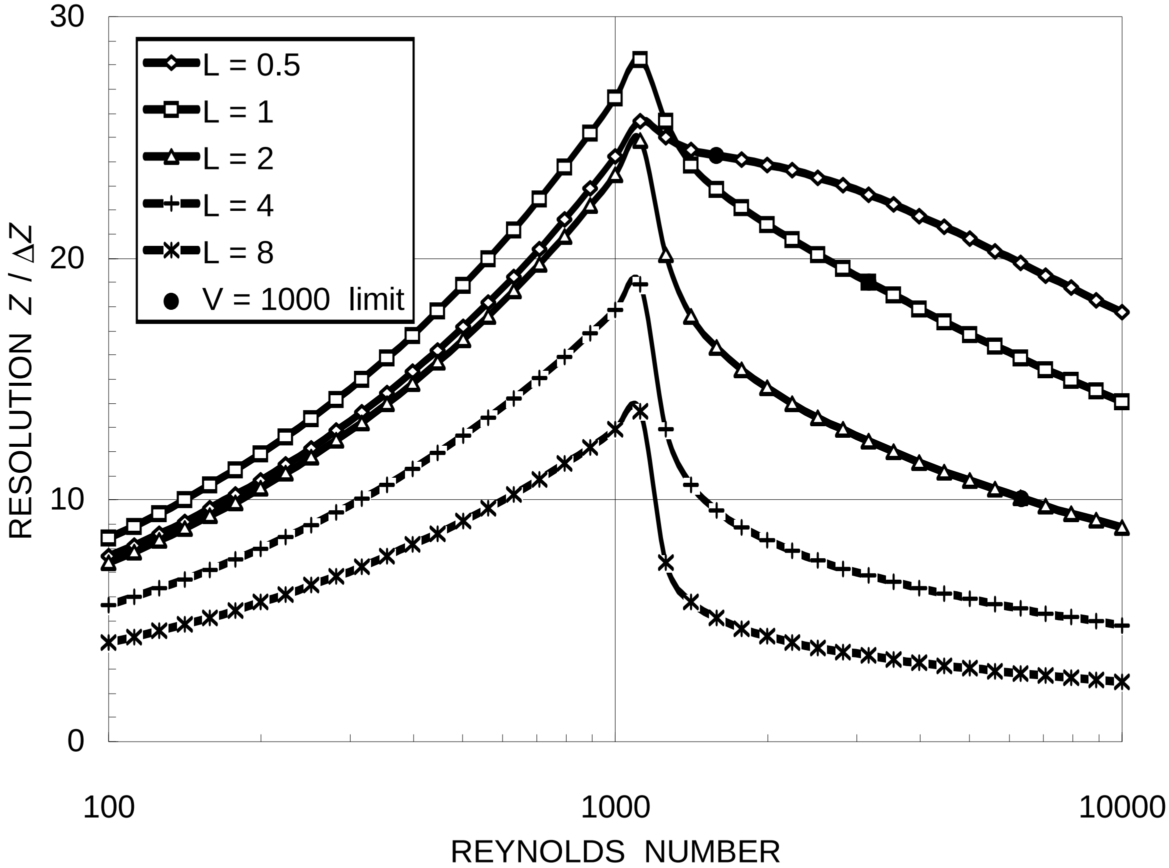
Res = f (L, V)

Parameter	Value	Comment
c_T	0.0025	standard value
R	1	standard value
Z	0.5 $cm^2V^{-1}s^{-1}$	standard value



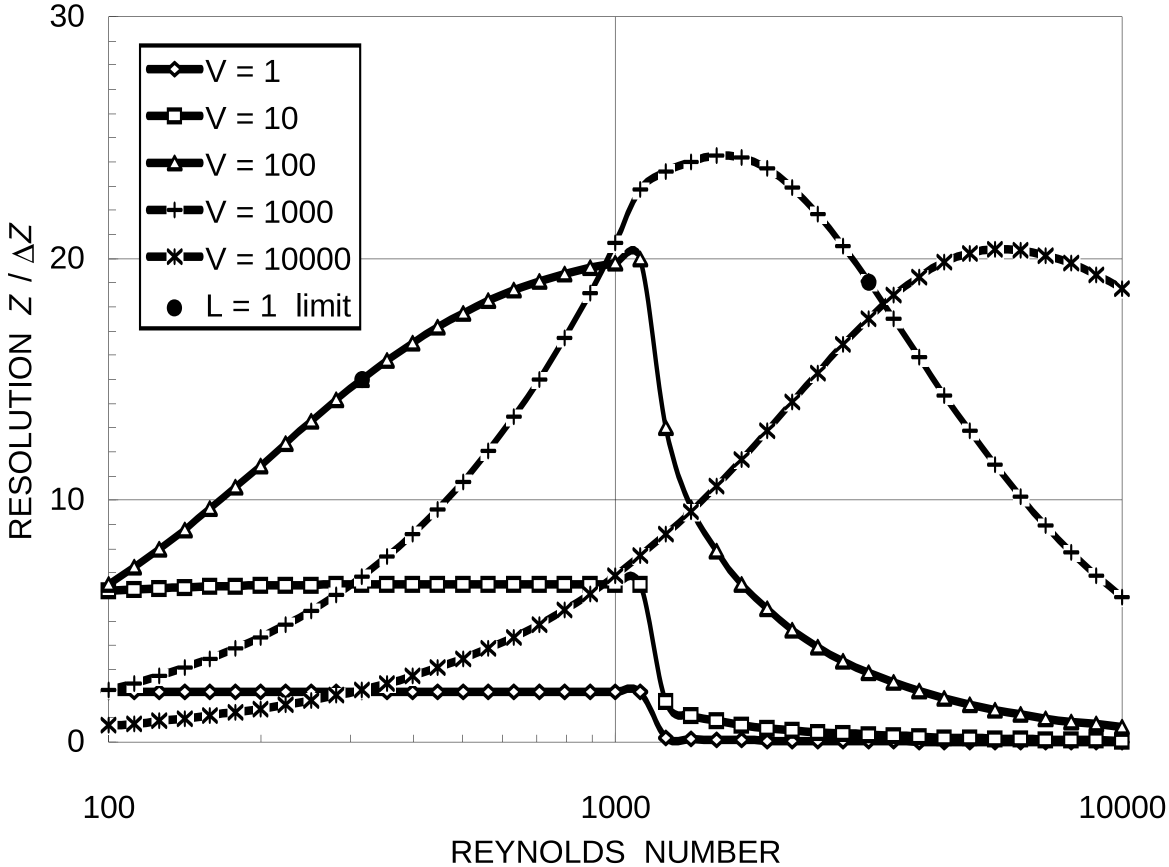
Res = f (V, L)

Parameter	Value	Comment
c_T	0.0025	standard value
R	1	standard value
Z	0.5 $cm^2V^{-1}s^{-1}$	standard value



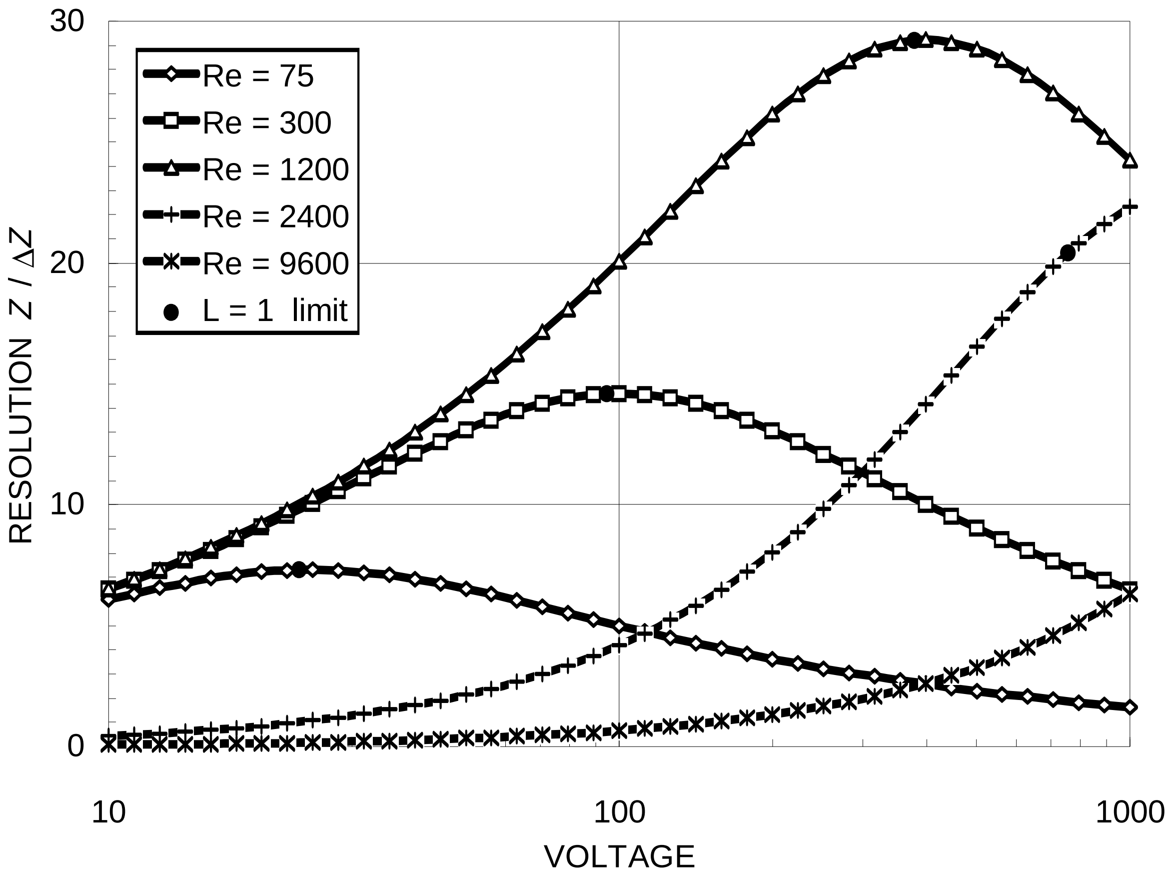
Res = f (Re, L)

Parameter	Value	Comment
c_T	0.0025	standard value
R	1	standard value
Z	0.5 $cm^2V^{-1}s^{-1}$	standard value



Res = f (Re, V)

Parameter	Value	Comment
c_T	0.0025	standard value
R	1	standard value
Z	0.5 $cm^2V^{-1}s^{-1}$	standard value



Res = f (V, Re)

Parameter	Value	Comment
c_T	0.0025	standard value
R	1	standard value
Z	0.5 $cm^2V^{-1}s^{-1}$	standard value