H. BLAUBRÜCK

N. Sapošnikov ja N. Valtsev'i

Algebraliste ülesannete kogu

II jao VII, VIII ja X (IX) osade

ülesannete täielikud lahendused

Astmed ja juured. Irratsionaalarvud.
Teise astme võrrandid.

(1020 ülesannet ja 2 joonist.)

Hind 200 marka.

Autori kirjastus / Tartus, 1924.

H. BLAUBRÜCK

N. Šapošnikov ja N. Valtsev'i

Algebraliste ülesannete kogu

II jao VII, VIII ja X (IX) osade

ülesannete täielikud lahendused

Astmed ja juured. Irratsionaalarvud.
Teise astme võrrandid.

(1020 ülesannet)

Autori kirjastus / Tartus, 1924.

J. Raudsepp'a trükk, Tartus 1924.

VII osa.

Astmed ja juured.

§ 1. Üksliikmete astendamine.

1. $(\pm 2)^4 = (\pm 2)(\pm 2)(\pm 2)(\pm 2) = 16$. **2.** $(\pm 5)^3 = (\pm 5)(\pm 5)(\pm 5) = \pm 125$. **3.** $(\pm 10)^3 = (\pm 10)(\pm 10)(\pm 10) = \pm 1000$. **4.** $(\pm 100)^4 = 100000000$. **5.** $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$. **6.** $5^{-1} = \frac{1}{5}$. **7.** $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$. **8.** $(-1)^{-5} = \frac{1}{(-1)^5} = \frac{1}{-1} = -1$. **9.** $(-4)^{-3} = \frac{1}{(-4)^3} = \frac{1}{-64} = -\frac{1}{64}$. **10.** $(-6)^{-1} = \frac{1}{-6} = -\frac{1}{6}$. **11.** $(-1)^{2n} = 1$. **12.** $(-1)^{3n} =$ kui ‚n' paarisarv $+1$, kui ‚n' üksikarv -1. **13.** $(2.3)^3 = 2^3.3^3 = 8.27 = 216$. **14.** $(5.7.3)^2 = 5^2.7^2.3^2 = 25.49.9 = 11025$. **15.** $(ab)^4 = a^4b^4$. **16.** $(-ab)^3 = -a^3b^3$. **17.** $(xyz)^7 = x^7y^7z^7$. **18.** $(abc)^m = a^mb^mc^m$. **19.** $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$. **20.** $\left(\frac{n}{m}\right)^a = \frac{n^a}{m^a}$. **21.** $\left(-\frac{5}{7}\right)^2 = \frac{25}{49}$. **22.** $\left(-1\frac{2}{3}\right)^3 = \left(-\frac{5}{3}\right)^3 = -\frac{125}{27}$. **23.** $(-0,2)^5 = -(0,2)^5 = -0,00032$. **24.** $(-0,01)^4 = 0,00000001$. **25.** $\left(\frac{2}{3}\right)^{-4} = \frac{1}{\left(\frac{2}{3}\right)^4} = \frac{1}{\frac{16}{81}} = \frac{81}{16}$. **26.** $\left(\frac{3}{4}\right)^{-5} = \frac{1}{\left(\frac{3}{4}\right)^5} = \frac{1}{\frac{343}{1024}} = \frac{1024}{343} = 2\frac{338}{343}$. **27.** $(0,3)^{-3} = \frac{1}{(0,3)^3} = \frac{1}{0,027} = 37\frac{1}{27}$. **28.** $(0,02)^{-4} = \frac{1}{(0,02)^4} = \frac{1}{0,00000016} = 6250000$. **29.** $\left(\frac{1}{a}\right)^{-3} = \frac{1}{\left(\frac{1}{a}\right)^3} = \frac{1}{\frac{1}{a^3}} = a^3$. **30.** $\left(\frac{c}{d}\right)^{-6} = \frac{c^{-6}}{d^{-6}} =$

$= \frac{d^6}{c^6}$. **31.** $(a^3)^2 = a^6$. **32.** $(a^5)^4 = a^{20}$. **33.** $(-a^2)^3 = -a^6$. **34.** $(-a^3)^6 = a^{18}$. **35.** $(-a)^{2n} = a^{2n}$. **36.** $(-a^5)^{2n-1} = -a^{10-5}$. **37.** $(-a^2)^{-3} = \frac{1}{(-a^2)^3} = \frac{1}{-a^6} = \frac{1}{a^6}$. **38.** $(-a^7)^{-4} = \frac{1}{(-a^7)^4} = \frac{1}{a^{28}}$. **39.** $(-a^m)^{-6} = \frac{1}{a^{6m}}$. **40.** $(-a^3)^{-2n+1} = \frac{1}{(-a^3)^{2n-1}} = -\frac{1}{a^{6n-3}} = -a^{3-6n}$. **41.** $(a^{-3})^4 = a^{-12} = \frac{1}{a^{12}}$. **42.** $(a^{-5})^{-2} = a^{10}$. **43.** $(a^{-m})^{-n} = a^{mn}$. **44.** $(a^m)^{-n} = a^{-mn}$. **45.** $[(-a)^3]^4 = (-a)^{12} = a^{12}$. **46.** $[(-a)^5]^3 = (-a)^{15} = -a^{15}$. **47.** $[(-b)^5]^m = (-b)^{5m}$ = kui ‚m' paarisarv $+b^{5m}$, kui ‚m' üksikarv $-b^{5m}$. **48.** $[(-b)^5]^{2n} = (-b)^{10n} = b^{10n}$. **49.** $[(-\frac{1}{2})^4]^{-1} = \left[\frac{1}{2^4}\right] = \frac{1}{\frac{1}{2^4}} = 2^4 = 16$. **50.** $\left[\left(-\frac{2}{3}\right)^{-3}\right]^{-2} = \left(-\frac{2}{3}\right)^6 = \frac{2^6}{3^6} = \frac{64}{729}$. **51.** $\left[\left(-\frac{a}{b}\right)^3\right]^{-2} = \left(-\frac{a}{b}\right)^{-6} \frac{1}{\left(-\frac{a}{b}\right)^6} = \frac{1}{\frac{a^6}{b^6}} = \frac{b^6}{a^6}$.

52. $\left[\left(-\frac{b}{a}\right)^5\right]^{-3} = \left(-\frac{b}{a}\right)^{-15} = \frac{1}{\left(-\frac{b}{a}\right)^{15}} = -\frac{a^{15}}{b^{15}}$.

53. $[(-b)^{-3}]^{-2} = (-b)^6 = b^6$. **54.** $\left[\left(-\frac{1}{b}\right)^{-4}\right]^{-5} = \left(-\frac{1}{b}\right)^{20} = \frac{1}{b^{20}}$. **55.** $(2a^3)^4 = 16a^{12}$. **56.** $(5a^2b^3)^3 = 125a^6b^9$. **57.** $(6a^mb^n)^3 = 216a^{3m}b^{3n}$. **58.** $(2a^5b^n)^m = 2^ma^{5m}b^{nm}$. **59.** $\left(\frac{2a}{bc}\right)^4 = \frac{16a^4}{b^4c^4}$.

60. $\left(\frac{4a^2c^5}{5b^3}\right)^3 = \frac{64a^6c^{15}}{125b^9}$. **61.** $(\frac{3}{4}c^7d^2f)^4 = \frac{81}{256}c^{28}d^8f^4$.

62. $(-0{,}2a^pb)^5 = -0{,}00032a^{5p}b^5$. **63.** $(-1\frac{3}{4}a^{2m-1}b) = (-\frac{7}{4}a^{2m-1}b)^3 = -\frac{343}{64}a^{6m-3}b^3$. **64.** $(-0{,}01a^{n-2}b^m)^6 = 0{,}000000000001a^{6n-12}b^{6m}$. **65.** $\left(\frac{2a^7b^8}{c^6d^n}\right)^5 = \frac{32a^{35}b^{40}}{c^{30}d^{5n}}$.

66. $\left(\frac{a^mb^n}{c^{p-1}}\right)^4 = \frac{a^{4m}b^{4n}}{c^{4p-4}}$. **67.** $\left(\frac{a^{2n}b^{n+2}}{c^{mn}}\right)^n = \frac{a^{2n^2}b^{n^2+2n}}{c^{mn^2}}$.

68. $\left(\frac{a^{3m-1}}{b^{3m}}\right)^{3m+1} = \frac{a^{(3m-1)(3m+1)}}{b^{3m(3m+1)}} = \frac{a^{9m^2-1}}{b^{9m^2+3m}}.$

69. $\left(-\frac{a^m b^{n+p}}{c^p}\right)^{2p} = \frac{a^{2mp} b^{2np+2p^2}}{c^{2p^2}}.$ **70.** $\left(-\frac{a^{6n+1}}{b^{2n}c^{n+2}}\right)^{6n-1} =$

$= -\frac{a^{(6n+1)(6n-1)}}{b^{2n(6n-1)}c^{(n+2)(6n-1)}} = -\frac{a^{36n^2-1}}{b^{12n^2-2n}c^{6n^2+11n-2}}.$

71. $(2a^3b^{-2}c^{-1})^2 = 4a^6b^{-4}c^{-2} = \frac{4a^6}{b^4c^2}.$ **72.** $\left(-\frac{2}{3}a^2b^{-1}c^3d^{-2}\right)^{-2} =$

$= \frac{2^{-2}}{3^{-2}}a^{-4}b^2c^{-6}d^4 = \frac{9b^2d^4}{4a^4c^6}.$ **73.** $(-0,5a^{-3}b^{-n}c^{n-1})^{-1} =$

$= -(0,5)^{-1}a^3b^nc^{1-n} = -\frac{a^3b^nc^{1-n}}{0,5} = -2a^3b^nc^{1-n}.$

74. $(-0,04a^{m-1}b^{3-n}c^{-5})^{-2} = (-0,04)^{-2}a^{2-2m}b^{2n-6}c^{10} =$

$= 625a^{2-2m}b^{2n-6}c^{10}.$ **75.** $\left[\left(\frac{a^2b^2}{c^3d^{-2}f}\right)^{-1}\right]^{-m} = \left(\frac{a^2b^2}{c^3d^{-2}f}\right)^m =$

$= \frac{a^{2m}b^{2m}}{c^{3m}d^{-2m}f^m} = \frac{a^{2m}b^{2m}d^{2m}}{c^{3m}f^m}.$ **76.** $\left[\left(\frac{a^{-m}b^n}{c^{m-n}}\right)^{-m}\right]^{-n} =$

$= \left(\frac{a^{-m}b^n}{c^{m-n}}\right)^{mn} = \left(\frac{b^n}{a^mc^{m-n}}\right)^{mn} = \frac{b^{mn^2}}{a^{m^2n}c^{m^2n-mn^2}}.$

77. $\left(\frac{a^3b^{-2}}{3cd^{-3}}\right)^3\cdot\left(\frac{3b^3c^{-2}}{a^5d}\right)^2 = \frac{a^9b^{-6}}{27c^3d^{-9}}\cdot\frac{9b^6c^{-4}}{a^{10}d^2} = \frac{c^{-7}}{3ad^{-7}} = \frac{d^7}{3ac^7}.$

78. $\left(\frac{a^2bd^2}{4c^2f^3}\right)^3 : \left(-\frac{b^3d^3}{2c^3f^2}\right)^3 = \frac{a^6b^3d^6}{64c^9f^6} : -\frac{b^9d^9}{8c^9f^6} = \frac{a^6}{8f^3} : -\frac{b^6d^3}{c^3} =$

$= -\frac{a^6c^3}{8b^6d^3f^3}.$ **79.** $\left(-\frac{a^2bx^2}{y^3}\right)^{2m-1}\cdot\left(-\frac{y^3}{ab^2x^3}\right)^{2m} =$

$= -\frac{a^{4m-2}b^{2m-1}x^{4m-2}}{y^{6m-3}}\cdot\frac{y^{6m}}{a^{2m}b^{4m}x^{6m}} = -\frac{a^{2m-2}b^{-2m-1}x^{-2m-2}}{y^{-3}} =$

$= -\frac{a^{2(m-1)}y^3}{b^{2m+1}x^{2(m+1)}}.$ **80.** $\left(\frac{4a^{n-1}b^3c^{3-x}}{9x^2y^{3n-2}z^6}\right)^2\cdot\left(-\frac{2a^nb^2c^{2-x}}{3xy^{n-1}z^4}\right)^{-3} =$

$= \frac{16a^{2n-2}b^6c^{6-2x}}{81x^4y^{6n-4}z^{12}}\cdot -\frac{27x^3y^{3n-3}z^{12}}{8a^{3n}b^6c^{6-3x}} = -\frac{2c^x}{3xy^{3n-x}a^{n+2}}.$

§ 2. Hulkliikmete astendamine.

81. $(a-b+c)^2=[(a-b)+c]^2=(a-b)^2+2c(a-b)+$ $+c^2=a^2-2ab+b^2+2ac-2bc+c^2=a^2+b^2+c^2-2ab-$ $-2bc+2ac.$

82. $(a^4+a^2-1)^2=[(a^4+a^2)-1]^2=(a^4+a^2)^2-2(a^4+$ $+a^2)+1=a^8+2a^6+a^4-2a^4-2a^2+1=a^8+2a^6-a^4-$ $-2a^2+1.$

83. $(3a^2-2ab-b^2)^2=[(3a^2-2ab)-b^2]^2=(3a^2-$ $-2ab)^2-2b^2(3a^2-2ab)+b^4=9a^4-12a^3b+4a^2b^2-$ $-6a^2b^2+4ab^3+b^4=9a^4-12a^3b-2a^2b^2+4ab^3+b^4.$

84. $(x^4-2ax^3+2a^2x-a^4)^2=[(x^4-2ax^3)+(2a^2x-$ $-a^4)]^2=(x^4-2ax^3)^2+2(x^4-2ax^3)(2a^2x-a^4)+(2a^2x-$ $-a^4)^2=x^8-4ax^7+4a^2x^6+4a^2x^5-2a^4x^4-8a^3x^4+4a^5x^3+$ $+4a^4x^2-4a^6x+a^8=x^8-4ax^7+4a^2x^6+4a^2x^5-$ $-2a^3(a+4)x^4+4a^5x^3+4a^4x^2-4a^6x+a^8.$

85. $(3a^{3x}+2a^{2x}+a^x+1)^2=(3a^{3x})^2+(2a^{2x})^2+$ $(a^x)^2+1+6a^{3x}(2a^{2x}+a^x+1)+4a^{2x}(a^x+1)+2a^x=$ $=9a^{6x}+4a^{4x}+a^{2x}+1+12a^{5x}+6a^{4x}+6a^{3x}+4a^{3x}+$ $+4a^{2x}+2a^x=9a^{6z}+12a^{5x}+10a^{4x}+10a^{3x}+5a^{2x}+$ $+2a^x+1.$

86. $(a^{2n}+a^n-1-a^{-n})^2=a^{4n}+a^{2n}+1+a^{-2n}+$ $+2a^{2n}(a^n-1-a^{-n})+2a^n(-1-a^{-n})-2(-a^{-n})=$ $-a^{4n}+a^{2n}+1+a^{-2n}+2a^{3n}-2a^{2n}-2a^n-2a^n-2+$ $+2a^{-n}=a^{4n}+2a^{3n}-a^{2n}-4a^n-1+2a^{-n}+a^{-2n}.$

87. $(a^3-{}^3/_2a^2b-{}^3/_4ab^2-{}^1/_8b^3)^2=(a^3)^2+(-{}^3/_2a^2b)^2+$ $+(-{}^3/_4ab^2)^2+(-{}^1/_8b^3)^2+2a^3(-{}^3/_2a^2b-{}^3/_4ab^2-{}^1/_8b^3)-$ $-2\,.\,{}^3/_2a^2b(-{}^3/_4ab^2-{}^1/_8b^3)-2\,.\,{}^3/_4ab^2(-{}^1/_8b^3)=a^6+$ $+{}^9/_2a^4b^2+{}^9/_{16}a^2b^4+{}^1/_{64}b^6-3a^5b-{}^3/_2a^4b^2-{}^1/_4a^3b^3+$ $+{}^9/_4a^3b^3+{}^3/_8a^2b^4+{}^3/_{16}ab^5=a^6-3a^5b+{}^3/_4a^4b^2+2a^3b^3+$ $+{}^{15}/_{16}a^2b^4+{}^3/_{16}ab^5+{}^1/_{64}b^6.$

88. $(x^n-{}^1/_2x^3+2{}^1/_2x^{-3}+{}^4/_3x^{-n})^2=(x^n)^2+(-{}^1/_2x^3)^2+$ $+(2{}^1/_2x^{-3})^2+({}^4/_3x^{-n})^2+2x^n(-{}^1/_2x^3+2{}^1/_2x^{-3}+{}^4/_3x^{-n})-$ $-x^3(2{}^1/_2x^{-3}+{}^4/_3x^{-n})+2\,.\,2{}^1/_2\,.\,{}^4/_3x^{-3-n}=x^{2n}+{}^1/_4x^6+$

$+\,{}^{25}/_4x^{-6}+{}^{16}/_9x^{-2n}-x^{n+3}+5x^{n-3}+{}^{8}/_3-{}^{5}/_2-{}^{4}/_3x^{3-n}+$
$+\,{}^{20}/_3x^{-3-n}=x^{2n}+5x^{n-3}+1{}^{7}/_9x^{-2n}-1{}^{1}/_3x^{3-n}-x^{n+3}+$
$+\,6{}^{2}/_3x^{-(n+3)}+{}^{1}/_4x^6+6{}^{1}/_4x^{-6}+{}^{1}/_6.$

89. $(a^4-2a^3+3a^2-2a+1)^2=(a^4)^2+(-2a^3)^2+$
$+(3a^2)^2+(-2a)^2+1+2a^4(-2a^3+3a^2-2a+1)-$
$-4a^3(3a^2-2a+1)+6a^2(-2a+1)-4a=a^8+4a^6+9a^4+$
$+4a^2+1-4a^7+6a^6-4a^5+2a^4-12a^5+8a^4-4a^3-$
$-12a^3+6a^2-4a=a^8-4a^7+10a^6-16a^5+17a^4-16a^3+$
$+10a^2-4a+1.$

90. $(a^x+2a^{x-1}-a^{x-2}-4a^{x-3}-5)^2-(a^x)^2+$
$+(2a^{x-1})^2+(-a^{x-2})^2+(-4a^{x-3})^2+(-5)^2+2a^x(2a^{x-1}-$
$-a^{x-2}-4a^{x-3}-5)+4a^{x-1}(-a^{x-2}-4a^{x-3}-5)-$
$-2a^{x-2}(-4a^{x-3}-5)-8a^{x-3}.-5=a^{2x}+4a^{2x-2}+$
$+a^{2x-4}+16a^{2x-6}+25+4a^{2x-1}-2a^{2x-2}-8a^{2x-3}-$
$10a^x-4a^{2x-3}-16a^{2x-4}-20a^{x-1}+8a^{2x-5}+10a^{x-2}+$
$+40a^{x-3}=a^{2x}+4a^{2x-1}+2a^{2x-2}-12a^{2x-3}-15a^{2x-4}+$
$=8a^{2x-5}+16a^{2x-6}-10a^x-20a^{x-1}+10a^{x-2}+40a^{x-3}.$

91. $(a+b+c)^3=[(a+b)+c]^3=(a+b)^3+3c(a+b)^2+$
$+3c^2(a+b)+c^3=a^3+3a^2b+3ab^2+3+b^3+3a^2c+$
$+6abc+3b^2c+3ac^2+3bc^2+c^3=a^3+b^3+c^3+$
$+3ab(a+b)+3ac(a+c)+3bc(b+c)+6abc.$

92. $(1-x+x^2)^3=1+(-x)^3+(x^2)^3-3x(1-x)+$
$+3x^2(1+x^2)-3x^3(x^2-x)-6x^3=1-x^3+x^6-3x+$
$+3x^2+3x^4-3x^5+3x^4-6x^3=x^6-3x^5+6x^4-7x^3+$
$+6x^2-3x+1.$

93. $(a^2-3a-1)^3=(a^2)^3+(-3a)^3+(-1)^3-9a^3(a^2-$
$-3a)-3a^2(a^2-1)+9a(-3a-1)+18a^3=a^6-27a^3-$
$-1-9a^5+27a^4-3a^4+3a^2-27a^2-9a+18a^3=a^6-$
$-9a^5+24a^4-9a^3-24a^2-9a-1.$

94. $(2a^2+ab-3b^2)^3=(2a^2)^3+(ab)^3+(-3b^2)^3+$
$+6a^3b.(2a^2+ab)-18a^2b^2(2a^2-3b^2)-9ab^3(ab^3-3b^2)-$
$-36a^3b^3=8a^6+a^3b^3-27b^6+12a^5b+6a^4b^2-36a^4b^2+$
$+54a^2b^4-9a^2b^4+27ab^5-36a^3b^3=8a^6+12a^5b-30a^4b^2-$
$35a^3b^3+45a^2b^4+27ab^5-27b^6.$

95. $\left(x^2+2-\frac{3}{x}\right)^3=(x^2)^3+(2)^3+\left(-\frac{3}{x}\right)^3+6x^2(x^2+2)-$
$-9x\left(x^2-\frac{3}{x}\right)-\frac{18}{x}\left(2-\frac{3}{x}\right)-36x=x^6+8-\frac{27}{x^3}+6x^4+$
$+12x^2-9x^3+27-\frac{36}{x}+\frac{54}{x^2}-36x=x^6+6x^4-9x^3+12x^2-$
$-36x+35-\frac{36}{x}+\frac{54}{x^2}-\frac{27}{x^3}.$

96. $\left(a^3b^2-\frac{4a^2}{b}-\frac{b}{2a^2}\right)^3=(a^3b^2)^3+\left(-\frac{4a^2}{b}\right)^3+\left(-\frac{b}{2a^2}\right)^3-$
$-12a^5b\left(a^3b^2-\frac{4a^2}{b}\right)+6\left(-\frac{4a^2}{b}-\frac{b}{2a^2}\right)-\frac{3ab^3}{2}\left(a^3b^2-\frac{b}{2a^2}\right)+$
$+12^3b^2=a^9b^6-\frac{64a^6}{b^3}-\frac{b^3}{8a^6}-12^8b^3+48a^7-\frac{24a^2}{b}-\frac{3b}{a^2}-$
$-\frac{3a^4b^5}{2}+\frac{3b^4}{4a}+12a^3b^2=a^9b^6-12a^8b^3+48a^7-\frac{64a^6}{b^3}-\frac{3a^4b^5}{2}+$
$+12a^3b^2-\frac{24a^2}{b}+\frac{3b^4}{4a}-\frac{3b}{a^2}-\frac{b^3}{8a^6}.$

97. $[(a-1)^2]^2=(a^2-2a+1)^2=a^4+4a^2+1-4a^3+$
$+2a^2-4a=a^4-4a^3+6a^2-4a+1.$

98. $[(2a-1)^3]^2=(8a^3-12a^2+6a-1)^2=(8a^3)^2+$
$+(-12a^2)^2+(6a)^2+(-1)^2+16a^3(-12a^2+6a-1)-$
$-24a^2(6a-1)-12a=64a^6+144a^4+36a^2+1-192a^5+$
$+96a^4-16a^3-144a^3+24a^2-12a=64a^6-192a^5+240a^4-$
$-160a^3+60a^2-12a+1.$

99. $(a+2)^6=[(a+2)^2]^3=(a^2+4a+4)^3=a^6+64a^3+$
$+64+12a^3(a^2+4a)+48a(4a+4)+12a^2(a^2+4)+96a^3=$
$=a^6+64a^3+64+12a^5+48a^4+192a^2+192+12a^4+48a^2+$
$+96a^3=a^6+12a^5+60a^4+160a^3+240a^2+192a+64.$

100. $(2a-3b)^6=[(2-3b)^3]^2=(8a^3-36a^2b+54ab^2-$
$-27a^3)^2=(8a^3)^2+(-36a^2b)^2+(54ab^2)^2+(-27b^3)^2+$
$+16a^3(-36a^2b+54ab^2-27b^3)-72a^2b(54ab^2-27b^3)-$
$-2916ab^5=64a^6+1296a^4b^2+2916a^2b^4+729b^6-576a^5b+$
$+864a^4b^2-432a^3b^3-3888a^3b^3+1944a^2b^4-2916ab^5=$
$=64a^6-576a^5b+2160a^4b^2-4320a^3b^3+4860a^2b^4-$
$-2916ab^5+729b^6.$

101. $(a+b+c+d)^3 = a^3+b^3+c^3+d^3+3a^2(b+$ $+c+d)+3b^2(a+c+d)+3c^2(a+b+d)+3d^2(a+b+c)+$ $+6abc+6abd+6acd+6bcd = a^3+b^3+c^3+d^3+3a^2b+$ $+3a^2c+3a^2d+3ab^2+3cb^2+3db^2+3ac^2+3bc^2+3dc^2+$ $+3ad^2+3bd^2+3cd^2+6abc+6abd+6acd+6bcd.$

102. $(x^3+x^2-x-1)^3 = x^9+x^6-x^3-1+3x^6(x^2-$ $-x-1)+3x^4(x^3-x-1)+3x^2(x^3+x^2-1)+3(x^3+$ $+x^2-x)+6x^5(-x-1)+6x(x^3+x^2) = x^9+x^6-x^3-1+$ $+3x^8-3x^7-3x^6+3x^7-3x^5-3x^4+3x^5+3x^4-3x^2+$ $+3x^3+3x^2-3x-6x^6-6x^5+6x^4+6x^3 = x^9+3x^8-$ $-8x^6-6x^5+6x^4+8x^3-3x-1.$

103. $(x+y+z)^2+(x-y-z)^2+(2z-y)^2 = 2x^2+$ $+3y^2+6z^2;$ $x^2+y^2+z^2+2xy+2xz+2yz+x^2+y^2+$ $+z^2-2xy-2xz-2yz+4z^2+y^2-4zy = 2x^2+3y^2+$ $+6z^2.$

104. $(a+b+c+d)^2+(a-b+c-d)^2+(a-2c)^2+$ $+(2b-d)^2 = 3(a^2+d^2)+6(b^2+c^2)$. Muundame võrrandi pahema poole, teda ruutu võttes ja koondades, saame resultaadis parema poole: $a^2+b^2+c^2+d^2+2ab+2ac+2ad+$ $+2bd+2bc+2cd+a^2+b^2+c^2+d^2-2ab+2ac-2ad+$ $+2bd-2bc-2cd+a^2-4ac+4c^2+4b^2-4bd+d^2 =$ $= 3a^2+6b^2+6c^2+3d^2 = 3(a^2+d^2)+6(b^2+c^2).$

105. $(a^2+b^2+c^2)(m^2+n^2+p^2)-(am+bn+cp)^2 =$ $(an-bm)^2+(ap-cm)^2+bp-cn)^2$. Muundame pahema poole, teda ruutu võttes ja koondades saame resultaadis parema poole: $a^2m^2+a^2n^2+a^2p^2+b^2m^2+b^2n^2+b^2p^2+$ $+c^2m^2+c^2n^2+c^2p^2-a^2m^2-b^2n^2-c^2p^2-2ambn-$ $-2bncp-2amcp = (a^2n^2-2ambn+b^2m^2)+(a^2p^2-$ $-2amcp+c^2m^2)+(b^2p^2-2bncp+c^2n^2) = (an-bm)^2+$ $(ap-cm)^2+(bp-cn)^2.$

106. $(x+y+z)^3-3(x+y+z)(xy+xz+yz)+$ $+3xyz = x^3+y^3+z^3$. Muundame võrrandi pahema poole ja resultaadis saame samasuse: $x^3+y^3+z^3+3x^2y+3xy^2+3x^2z+$

$+ 3xz^2 + 3y^2x + 3yz^2 + 6xyz - 3x^2y - 3x^2z - 3xyz - 3xy^2 - 3xyz - 3y^2z - 3xyz - 3xz^2 - 3yz^2 + 3xyz = x^3 + y^3 + z^3.$

107. $(a + b + c)^2 + (a - b + c)^2 + (a + b - c)^2 + (b + c - a)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc + a^2 + b^2 + c^2 - 2ab + 2ac - 2bc + a^2 + b^2 + c^2 + 2ab - 2ac - 2bc + b^2 + c^2 + a^2 + 2bc - 2ba - 2ac = 4a^2 + 4b^2 + 4c^2 = 4(a^2 + b^2 + c^2).$

108. $(a + b + c)^3 + (b - a - b)^3 + (c - a - b)^3 + (a - b - c)^3 = a^3 + b^3 + c^3 + 3a^2b + 3ab^2 + 3a^2c + 3ac^2 + 3b^2c + 3bc^2 + 6abc + b^3 - a^3 - c^3 - 3b^2a + 3ba^2 - 3b^2c + 3bc^2 - 3a^2c - 3ac^2 + 6abc + c^3 - a^3 - b^3 - 3c^2a + 3ca^2 - 3c^2b + 3cb^2 - 3a^2b - 3ab^2 + 6abc + a^3 - b^3 - c^3 - 3a^2b + 3ab^2 - 3a^2c + 3ac^2 - 3b^2c - 3bc^2 + 6abc = 24abc.$

109. $AB(A^2 + B^2) = (a + b + c + d)(a + b - c - d) . [(a + b + c + d)^2 + (a + b - c - d)^2] = [(a + b) + (c + d)][(a + b) - (c + d)] . \{[(a + b) + (c + d)]^2 + [(a + b) - (c + d)]^2\} = [(a + b)^2 - (c + d)^2] . [(a + b)^2 + 2(a + b) . (c + d) + (c + d)^2 + (a + b)^2 - 2(a + b)(c + d) + (c + d)^2] = 2[(a + b)^2 - (c + d)^2] . [(a + b)^2 + (c + d)^2] = 2[(a + b)^4 - (c + d)^4] = 2[(a^2 + 2ab + b^2)^2 - (c + 2cd + d^2)^2] = 2[a^4 + 4a^2b^2 + b^4 + 4a^3b + 2a^2b^2 + 4ab^3 - c^4 - 4c^2d^2 - d^4 - 4c^3d - 2c^2d^2 - 4cd^3] = 2[a^4 + 6a^2b^2 + b^4 + 4ab(a^2 + b^2) - c^4 + 6c^2d^2 - d^4 - 4cd(c^2 + d^2)]$, kuid tingimise järele, $ab(a^2 + b^2) = cd(c^2 + d^2)$; sellepärast saame: $2[a^4 + 6a^2b^2 + b^4 - c^4 - d^4 - 6c^2d^2] \ldots\ldots (1)$. $CD . (C^2 + D^2) = (a - b + c - d)(a - b - c + d)[(a - b + c - d)^2 + (a - b - c + d)^2] = [(a - b) + (c - d)][(a - b) - (c - d)]\{[(a - b) + (c - d)]^2 + [(a - b) - (c - d)]^2\} = [(a - b)^2 - (c - d)^2][(a - b)^2 + 2(a - b)(c - d) + (c - d)^2 + (a - b)^2 - 2(a - b)(c - d) + (c - d)^2] = 2[(a - b)^2 - (c - d)^2] . [(a - b)^2 + (c - d)^2] = + 2[(a - b)^4 - (c - d)^4] = 2[(a^2 - 2ab + b^2)^2 - (c^2 - 2cd + d^2)^2] = 2[a^4 + 4a^2b^2 + b^4 - 4a^3b - 4ab^3 + 2a^2b^2 - c^4 - 4c^2d^2 - d^4 - 2c^2d^2 + 4c^3d + 4cd^3] = 2[a^4 + 6a^2b^2 + b^4 - 4ab(a^2 + b^2) - c^4 - 6c^2d^2 - d^4 + 4cd(c^2 + d^2)]$, aga

$ab(a^2+b^2)=cd(c^2-d^2)$, sellepärast on meil $2[a^4+6a^2b^2+$ $+b^4-c^4-d^4-6c^2d^2]$..... (2). Võrreldes avaldusi (1) ja (2) näeme, et mõlemad on sarnased.

110. $S_3+p_1S_2 = a^3+b^3+c^3-(a+b+c)(a^2+$ $+b^2+c^2)=a^3+b^3+c^3-a^3-ab^2-ac^2-ba^2-b^3-bc^2-$ $-ca^2-cb^2-c^3=-[ab(a+b)+ac(a+c)+bc(b+c)]$....(1). $p_1p_2-3p_3=-(a+b+c)(ab+ac+bc)+3abc=-a^2b-$ $-a^2c-abc-b^2a-bac-b^2c-cab-ac^2-bc^2+3abc=$ $=-[ab(a+b)+ac(a+c)+bc(b+c)]$..... (2). Avaldus (1) ja (2) on sarnased.

§ 3. Üksliikmete juurimine.

111. $\sqrt{144}=\sqrt{12^2}=\pm 12.$

112. $\sqrt{104.26}=\sqrt{13.8.2.13}=\sqrt{13^2.2^4}=13.2^2=52.$

113. $\sqrt{50.18}=\sqrt{25.4.9}=5.2.3=30.$

114. $\sqrt{180.20}=\sqrt{9.20.20}=\sqrt{3^2.20^{20}}=3.20=60.$

115. $\sqrt{\frac{48.3}{125.5}}=\frac{\sqrt{16.3.3}}{\sqrt{25.5.5}}=\frac{4.3}{5.5}=\frac{12}{25}.$

116. $\sqrt{\frac{847.7}{216.6}}=\frac{\sqrt{121.7.7}}{\sqrt{36.36}}=\frac{11.7}{6.6}=\frac{77}{36}.$

117. $\sqrt{17^2-8^2}=\sqrt{(17+8)(17-8)}=\sqrt{25.9}=5.3=15.$

118. $\sqrt{25^2-7^2}=\sqrt{(25+7)(25-7)}=\sqrt{32.18}=\sqrt{64.9}=$ $=8.3=24.$

119. $\sqrt{\frac{15^2-1}{\sqrt{50^2-48^2}}}=\sqrt{\frac{(15+1)(15-1)}{\sqrt{(50+48)(50-48)}}}=$ $=\sqrt{\frac{16.14}{\sqrt{98.2}}}=\sqrt{\frac{16.14}{7.2}}=\sqrt{16}=\pm 4.$

120. $\sqrt{\frac{\sqrt{113^2-112^2}}{19^2-11^2}}=\sqrt{\frac{\sqrt{(113+112)(113-112)}}{(19+11)(19-11)}}=$ $=\sqrt{\frac{\sqrt{225.1}}{30.8}}=\sqrt{\frac{15}{30.8}}=\sqrt{1/16}=1/4.$

121. $\sqrt[6]{2^{12}} = 2^2 = 4$. **122.** $\sqrt[3]{-a^6} = -a^2$. **123.** $\sqrt[n]{a^{3n}} = a^3$.

124. $\sqrt[n+2]{a^{3n+6}} = a^{\frac{3(n+2)}{n+2}} = a^3$. **125.** $\sqrt[3]{8.3^3} = \sqrt[3]{2^3.3^3} = 2.3 = 6$.

126. $\sqrt[4]{16.81} = \sqrt[4]{2^4.3^4} = 2.3 = 6$. **127.** $\sqrt{\frac{a^4}{9}} = \pm\frac{a^2}{3}$.

128. $\sqrt[5]{-\frac{a^{10}}{b^{15}}} = -\frac{a^2}{b^3}$. **129.** $\sqrt[4]{a^{16}b^8c^4} = a^4b^2c$.

130. $\sqrt[3]{-27a^{-12}b^3} = -3a^4b$. **131.** $\sqrt[-3]{27} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$.

132. $\sqrt[-2]{4/9} = \frac{1}{\sqrt{4/9}} = \frac{1}{2/3} = 3/2$. **133.** $\sqrt[3]{a^{-6}} = a^{-2} = \frac{1}{a^2}$.

134. $\sqrt[5]{-a^{-20}} = -a^{-4} = -\frac{1}{a^4}$. **135.** $\sqrt[-5]{-\frac{1}{32}} = \sqrt[5]{-32} = -2$.

136. $\sqrt[-n]{-\frac{1}{a^{5n}}} = \sqrt[n]{-a^{5n}} = a^5\sqrt[n]{-1}$. **137.** $\sqrt[4]{16a^{-4}b^{12}} =$
$= 2a^{-1}b^3 = \frac{2b^3}{a}$. **138.** $\sqrt[-3]{\frac{8}{125}a^{3n}b^{-6}} = \frac{1}{\sqrt[3]{\frac{8}{125}a^{3n}b^{-6}}} =$
$= \frac{1}{\frac{2}{5}a^n b^{-2}} = \frac{5b^2}{2a^n}$. **139.** $\sqrt{6\frac{1}{4}a^6c^{4m}} = \frac{5}{2}a^3c^{2m}$. **140.** $\sqrt[4]{\frac{16}{81}a^{8n}b^{16}} =$
$= \frac{2}{3}a^{2n}b^4$. **141.** $\sqrt[3]{0,027a^{6n-3}b^{18}c^{-6}} = 0,3a^{2n-1}b^6c^{-2}$.

142. $\sqrt[5]{-10^{10}a^{-20n}b^{5-15m}} = -10^2a^{-4n}b^{1-3m}$.

143. $\sqrt{\frac{4^{-1}a^4b^{-6}}{9^{-1}c^8d^{-2}}} = \sqrt{\frac{9a^4d^2}{4c^8b^6}} = \frac{3a^2d}{2c^4b^3}$.

144. $\sqrt[3]{\frac{343a^{-15}b^{18}}{2^{-6}c^9d^{-3}}} = \sqrt[3]{\frac{343.2^6.b^{18}.d^3}{a^{15}c^9}} = \frac{28b^6d}{a^5c^3}$.

145. $\sqrt[-2]{\frac{a^2b^{2n-6}c^{2m}}{4d^{-6}f^{-4n+2}}} = \frac{a^{-1}b^{3-n}c^m}{2^{-1}d^3f^{2n-1}} = \frac{2b^{3-n}c^m}{ad^3f^{2n-1}}$.

146. $\sqrt[3]{-\frac{1000\,p^{12}q^{-6}r^{3n}}{27\,a^{-3m}\,b^9}} = \frac{10\,p^4\,q^{-2}r^n}{3\,a^{-m}\,b^3} = -\frac{10\,a^m p^4 r^n}{b^3 q^2}$.

147. $\sqrt[9]{2^{36}\,a^{-40}b^7\,\frac{(a+b)^{27}}{a^{-4}b^{-11}}} = \sqrt[9]{2^{36}a^{-36}b^{18}(a+b)^{27}} =$
$= 2^4\,a^{-4}\,b^2\,(a+b)^3 = \frac{16\,b^2\,(a+b)^3}{a^4}$.

148. $2\,ab^2\sqrt{2\,a^3bc^2\sqrt[3]{8\,a^3b^9c^6}} = 2\,a^2b^2c\sqrt{2\,ab\,.\,2\,ab^3c^2} =$
$= 4\,a^3b^4c^2$.

149. $\sqrt[-n]{\frac{(3\,a^3b^{-2})^{2n}\,a^{-(p+n)}\,b^{-(n+np)}\,c^n}{a^{-p}}} =$
$= \sqrt[-n]{(3\,a^3b^{-2})^{2n}a^n\,b^{-n(1+p)}c^n} = (3\,a^3b^{-2})^{-2}a^{-1}\,b^{1+p}c^{-1} = \frac{b^{5+p}}{9\,a^7c}$.

150. $3\,a^{5-n}b^{-4n}\,.\,\sqrt[-3]{\tfrac{27}{64}\,a^{-15}b^{3n}\,c^{6-3n}\,d^9} = 3a^{5-n}\,b^{-4n}\,.$
$.\,\frac{3^{-1}}{4^{-1}}\,a^5\,b^{-n}c^{n-2}\,.\,d^{-3} = \frac{4\,a^{10-4}c^{n-2}}{b^{5n}d^3}$.

§ 4. Hulkliikme ruudu ja kuubi juurimine.

151. $x^2+2ax+b$ on täisruut sel juhtumisel, kui $a=\sqrt{b}$; $x^2+2ax+b = x^2+2\,x\sqrt{b}+b = (x+\sqrt{b})^2$.

152. Kui $p^2 = 2\,aq$, on antud hulkliige täisruut. $a^2x^2 - p^2x + q^2 = a^2x^2 - 2\,aqx + q^2 = (ax \pm q)^2$.

153. Kui $m = 12$ on kolmliige täisruut. $4\,a^2 + mab + 9\,b^2 = 4\,a^2 + 12\,ab + 9\,b^2 = (2\,a + 3\,b)^2$.

154. $x^4 - 4\,x^3 + 10\,x^2 + mx + n = x^4 - 4\,x^3 + 6\,x^2 + 4\,x^2 + mx + n = x^4 - 2\,x^2\,(2\,x - 3) + (4\,x^2 + mx + n)$. Kolmliige on ruut, kui $(4\,x^2 + mx + n)$ on $(2x - 2)$'$\underline{me}$ ruut; see on juhusel, kui $m = -12$ ja $n = 9$. Siis: $x^4 - 2\,x^2\,(2\,x - 3) + (4\,x^2 - 12\,x + 9) = x^4 - 2\,x^2(2\,x - 3) + (2\,x - 3)^2 = [x^2 - (2\,x - 3)]^2 = (x^2 - 2\,x + 3)^2$.

155. Kui $b = a^2 + 2\,c$, saab avaldus „$x^4 + 2\,ax^3 + bx^3 + 2\,acx + c^2$" kuju: $x^4 + 2\,ax^3 + a^2x^2 + 2\,x^2c + 2\,acx + c^2 =$

$= (x^4 + 2ax^3 + a^2x^2) + (2x^2c + 2acx) + c^2 = x^2(x + a)^2 +$
$+ 2xc(x + a) + c^2 = [x(x + a) + c]^2 = (x^2 + ax + c)^2.$

156. Üks järjestikustest arvudest olgu x; teised arvud on siis $(x + 1)$, $(x + 2)$ ja $(x + 3)$. Tõestada, et

$$x(x + 1)(x + 2)(x + 3) + 1 = \text{täisruutuga.}$$

$x(x + 1)(x + 2x)(x + 3) + 1 = (x^2 + x)(x + 2)(x + 3) =$
$= (x^2 + x)(x^2 + 5x + 6) + 1 = x^4 + 6x^3 + 11x^2 + 6x + 1 =$
$+ x^4 + 6x^3 + 9x^2 + 2x^2 + 6x + 1 = (x^2 + 3x)^2 + 2(x^2 + 3x) + 1 =$
$= [(x^2 + 3x) + 1]^2 = (x^2 + 3x + 1)^2.$

157. $\sqrt{4a^4 + 12a^2b + 9b^2} = 2a^2 + 3b$

$$\begin{array}{r|l} & \mp 4a^4 \\ \hline 4a^2 + 3b & 12a^2b + 9b^2 \\ + 3b & \mp 12a^2b \mp 9b^2 \\ \hline & 0 \end{array}$$

158. $\sqrt{\frac{9}{16}a^2b^4 - \frac{3}{5}a^3b^2 + \frac{4}{25}a^4} = \frac{3}{4}ab^2 - \frac{2}{5}a^2.$

$$\begin{array}{r|l} & \mp \frac{9}{16}a^2b^4 \\ \hline \frac{3}{2}ab^2 - \frac{2}{5}a^2 & -\frac{3}{5}a^3b^2 + \frac{4}{25}a^4 \\ -\frac{2}{5}a^2 & \pm\frac{3}{5}a^3b^2 \mp \frac{4}{25}a^4 \\ \hline & 0 \end{array}$$

159. $\sqrt{x^{2n-2}y^2 - 4x^{2n-4}y^3 + 4x^{2n-6}y^4} = x^{n-1}y - 2x^{n-3}y^2.$

$$\begin{array}{r|l} & \mp x^{2n-2}y^2 \\ \hline 2x^{n-1}y - 2x^{n-3}y^2 & -4x^{2n-4}y^3 + 4x^{2n-6}y^4 \\ -2x^{n-3}y^2 & \pm 4x^{2n-4}y^3 \mp 4x^{2n-6}y^4 \\ \hline & 0 \end{array}$$

160. $\sqrt{\frac{1}{4}a^{2m}b^{-6} + 0{,}3a^{m+n} + 0{,}09a^{2n}b^6} = \frac{1}{2}a^mb^{-3} + 0{,}3a^nb^3.$

$$\begin{array}{r|l} & \mp \frac{1}{4}a^{2m}b^{-6} \\ \hline a^mb^{-3} + 0{,}3a^nb^3 & 0{,}3a^{m+n} + 0{,}09a^{2n}b^6 \\ + 0{,}3a^nb^3 & 0{,}3a^{m+n} \mp 0{,}09a^{2n}b^6 \\ \hline & 0 \end{array}$$

161. $\sqrt{4a^4 - 4a^3 + 5a^2 - 2a + 1} = 2a^2 - a + 1.$

$$\pm 4a^4$$

$$\begin{array}{r|l} 4a^2 - a & -4a^3 + 5a^2 \\ -a & \pm 4a^3 \mp a^2 \end{array}$$

$$\begin{array}{r|l} 4a^2 - 2a + 1 & 4a^2 - 2a + 1 \\ -a + 1 & \mp 4a^2 \pm 2a \mp 1 \\ \hline & 0 \end{array}$$

162. $\sqrt{16a^4 - 32a^3 + 24a^2 - 8a + 1} = 4a^2 - 4a + 1.$

$$\pm 16a^4$$

$$\begin{array}{r|l} 8a^2 - 4a & -32a^3 + 24a^2 \\ -4a & \pm 32a^3 \mp 16a^2 \end{array}$$

$$\begin{array}{r|l} 8a^2 - 8a + 1 & 8a^2 - 8a + 1 \\ +1 & \mp 8a^2 \pm 8a \mp 1 \\ \hline & 0 \end{array}$$

163. $\sqrt{9a^4 - 6a^3b + 25a^2b^2 - 8ab^3 + 16b^4} = 3a^2 - ab + 4b^2.$

$$\mp 9a^4$$

$$\begin{array}{r|l} 6a^2 - ab & -6a^3b + 25a^2b^2 \\ -ab & \pm 6a^3b \mp a^2b^2 \end{array}$$

$$\begin{array}{r|l} 6a^2 - 2ab + 4b^2 & 24a^2b^2 - 8ab^3 + 16b^4 \\ +4b^2 & \mp 24a^2b^2 - 8ab^3 \mp 16b^4 \\ \hline & 0 \end{array}$$

164. $\sqrt{\frac{1}{4}a^4 - 2a^3b + \frac{13}{3}a^2b^2 - \frac{4}{3}ab^3 + \frac{1}{9}b^4} = \frac{1}{2}a^2 - 2ab + \frac{1}{3}b.$

$$\mp \frac{1}{4}a^4$$

$$\begin{array}{r|l} a^2 - 2ab & -2a^3b + \frac{13}{3}a^2b^2 \\ -2ab & \pm 2a^3b \mp 4a^2b^2 \end{array}$$

$$\begin{array}{r|l} a^2 - 4ab + \frac{1}{3}b^2 & \frac{1}{3}a^2b^2 - \frac{4}{3}ab^3 + \frac{1}{9}b^4 \\ +\frac{1}{3}b^2 & \mp \frac{1}{3}a^2b^2 \pm \frac{4}{3}ab^3 \mp \frac{1}{9}b^4 \\ \hline & 0 \end{array}$$

165. $\sqrt{a^2+2-2a^{-1}+a^{-2}-2a^{-3}+a^{-4}}=a+a^{-1}-a^{-2}.$

$$
\begin{array}{r|l}
\multicolumn{2}{l}{\mp a^2} \\
\hline
2a+a^{-1} & 2-2a^{-1}+a^{-2} \\
+a^{-1} & \pm 2 \qquad \pm a^{-2} \\
\hline
2a+2a^{-1}-a^{-2} & -2a^{-1} \qquad -2a^{-3}+a^{-4} \\
-a^2 & \pm 2a^{-1} \qquad -2a^{-3}\mp a^{-4} \\
\hline
 & \qquad 0
\end{array}
$$

166. $\sqrt{\frac{16}{9}a^2-\frac{8}{5}-\frac{16}{9a}+\frac{9}{25a^2}+\frac{4}{5a^3}+\frac{4}{9a^4}}=\frac{4}{3}a-\frac{3}{5a}-\frac{2}{3a^2}.$

$$
\begin{array}{r|l}
\multicolumn{2}{l}{\mp\frac{16}{9}a^2} \\
\hline
\frac{8}{3}a-\frac{3}{5a} & -\frac{8}{5}-\frac{16}{9a}+\frac{9}{25a^2} \\
-\frac{3}{5a} & \pm\frac{8}{5} \qquad \mp\frac{9a}{25a^2} \\
\hline
\frac{8}{3}a-\frac{6}{5a}-\frac{2}{3a^2} & -\frac{16}{9a} \qquad +\frac{4}{5a^3}+\frac{4}{9a^4} \\
-\frac{2}{3a^2} & \pm\frac{16}{9a} \qquad \mp\frac{4}{5a^3}\mp\frac{4}{9a^4} \\
\hline
 & \qquad 0
\end{array}
$$

167. $\sqrt{x^6-4x^5-2x^4+22x^3-11x^2-30x+25}=x^3-2x^2-3x+5.$

$$
\begin{array}{r|l}
\multicolumn{2}{l}{\mp x^6} \\
\hline
2x^3-2x^2 & -4x^5-2x^4 \\
-2x^2 & \pm 4x^5\mp 4x^4 \\
\hline
2x^3-4x^2-3x & -6x^4+22x^3+11x^2 \\
-3x & \pm 6x^4\mp 12x^3\mp 19x^2 \\
\hline
2x^3-4x^2-6x+5 & 10x^3-20x^2-30x+25 \\
+5 & \pm 10x^3\pm 20x^2\pm 30x\mp 25 \\
\hline
 & \qquad 0
\end{array}
$$

168. $\sqrt{x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6} =$
$\mp x^6 \qquad = x^3 - 3x^2y + 3xy^2 - y^3 = (x-y)^3.$

$$
\begin{array}{r|l}
2x^3 - 3x^2y & -6x^5y + 15x^4y^2 \\
-3x^2y & -6x^5y + 9x^4y^2 \\
\hline
2x^3 - 6x^2y + 3xy^2 & 6x^4y^2 - 20x^3y^3 + 15x^2y^4 \\
+3xy^2 & \mp 6x^4y^2 \pm 18x^3y^3 \mp 9x^2y^4 \\
\hline
2x^3 - 6x^2y + 3xy^2 - y^3 & -2x^3y^3 + 6x^2y^4 - 6xy^5 + y^6 \\
-y^3 & \pm 2x^3y^3 \mp 6x^2y^4 \pm 6xy^5 \pm y^6 \\
\hline
 & 0
\end{array}
$$

169.

$\sqrt{9a^6 - 12a^5b - 38a^4b^2 + 52a^3b^3 + 33a^2b^4 - 56ab^5 + 16b^6} =$
$\mp 9a^6 \qquad = 3a^3 - 2a^2b - 7ab^2 + 4b^3.$

$$
\begin{array}{r|l}
6a^3 - 2a^2b & -12a^5b - 38a^4b^2 \\
-2a^2b & \pm 12a^5b \mp 4a^4b^2 \\
\hline
6a^3 - 4a^2b - 7ab^2 & -42a^4b^2 + 52a^3b^3 + 33a^2b^4 \\
-7ab^2 & \pm 42a^4b^2 \mp 28a^3b^3 \mp 49a^2b^4 \\
\hline
6a^3 - 4a^2b - 14ab^2 + 4b^3 & 24a^3b^3 - 16a^2b^4 - 56ab^5 + 16b^6 \\
+4b^3 & \mp 24a^3b^3 \pm 16a^2b^4 \pm 56ab^5 \mp 16b^6 \\
\hline
 & 0
\end{array}
$$

170. $\sqrt{x^4 - 4x^2 + 10 - 20x^{-2} + 25x^{-4} - 24x^{-6} + 16x^{-8}} =$
$\pm x^4 \qquad = x^2 - 2 + 3x^{-2} - 4x^{-4}.$

$$
\begin{array}{r|l}
2x^2 - 2 & -4x^2 + 10 \\
-2 & \pm 4x^2 \mp 4 \\
\hline
2x^2 - 4 + 3x^{-2} & +6 - 20x^{-2} + 25x^{-4} \\
+3x^{-2} & \mp 6 \pm 12x^{-2} \mp 9x^{-4} \\
\hline
2x^2 - 4 + 6x^{-2} - 4x^{-4} & -8x^{-2} + 16x^{-4} - 24x^{-6} + 16x^{-8} \\
-4x^{-4} & \pm 8x^{-2} \mp 16x^{-4} \pm 24x^{-6} \mp 16x^{-8} \\
\hline
 & 0
\end{array}
$$

171. $125x^3 - 150x^2 + mx + n = (5x)^3 - 3.(5x)^2.2 + {}$ $+ mx + n$. Et antud hulkliige oleks täiskuup, peab $mx = {}$ $= 3.2^2.(5x)$, s. o. $m = 60$ ja $n = (-2)^3 = -8$. Siis: $(5x)^2 -$ $- 3.(5x)^2.2 + 3.(5x).2^2 - 2^3 = (5x - 2)^3$.

172. $x^3 - 3ax^2 + mx - n = (x)^3 - 3.(x)^2.a + mx - n$; kui $m = 3.a^2$ ja $n = a^3$ saame $(x)^3 - 3.(x)^2.a + 3.(a)^2x -$ $- (a)^3 = (x - a)^3$.

173. Tingimuste määramiseks, mis hulkliikme $x^3 + ax^2 +$ $+ bx + c$ muudaksid täiskuubiks, harutame nii: 1-ne hulkliikme liige on x'i kuup, 2-ne hulkliikme liige peab olema kolmekordne 1-se ruudu (x^2) korrutis teise liikmega, s. o. $ax^2 =$ $= 3.2$-ne liige $. x^2$, kust 2-ne liige $= \frac{a}{3}$. Kolmas hulkliikme liige $bx = 3.x.\left(\frac{a}{3}\right)^2$, kust $b = \frac{a^2}{3}$; neljas liige $c = \left(\frac{a}{3}\right)^2$. Hulkliige saaks järgmise kuju: $x^3 + ax^2 + \frac{a^2x}{3} + \frac{a^3}{27} = \left(x + \frac{a}{3}\right)^3$.
Tingimused on: $b = \frac{a^2}{3}$ ja $c = \left(\frac{a}{3}\right)^2$.

174. Arv, mis tarvis liita kolme naturaalarvu korrutisega $a(a+1)(a+2)$, et saabuks täiskuup, olgu x.

$a(a+1)(a+2) + x = (a^2 + a)(a+2) + x = a^3 + a^2 +$ $+ 2a^2 + 2a + x = a^3 + 3a^2 + 2a + x$. Nagu siit näha, muutub hulkliige täiskuubiks, kui $x = a + 1$. Järjelikult: $a^3 + 3a^2 +$ $+ 2a + (a+1) = a^3 + 3a^2 + 3a + 1 = (a+1)^3$.

175.

$$\sqrt[3]{64x^3 - 144x^2y + 108xy^2 - 27y^3} = 4x - 3x.$$

$$\begin{array}{l|l} & \mp 64x^3 \\ \hline 3.(4x)^2.(-3y) + & -144x^2y + 108xy^2 - 27y^3 \\ +3.(4x)(-3y)^2 + (-3y)^3 & \pm 144x^2y \mp 108xy^2 \pm 27y^3 \\ \hline & 0 \end{array}$$

176.
$$\sqrt[3]{343a^6 - 441a^4b^5 + 189a^2b^{10} - 27b^{15}} =$$
$$\pm 343a^6 \qquad = 7a^2 - 3b^5.$$
$$\begin{array}{l|l} 3.(7a^2)^2.(-3b^5) + & -441a^4b^5 + 189a^2b^{10} - 27b^{15} \\ + 3.7a^2(-3b^5)^2 + (3b^5)^3 & \pm 441a^4b^5 \mp 189a^2b^{10} - 27b^{15} \end{array}$$
$$0$$

177.
$$\sqrt[3]{x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1} =$$
$$\pm x^6 \qquad = x^2 + x + 1.$$
$$\begin{array}{l|l} 3(x^2)^2x + 3(x^2)(x^2) + (x)^3 & 3x^5 + 6x^4 + 7x^3 \\ & \mp 3x^5 \mp 3x^4 \mp x^3 \end{array}$$
$$\begin{array}{l|l} 3(x^2 + x)^2.1 + 3(x^2 + x).1^2 + 1^3 & 3x^4 + 6x^3 + 6x^2 + 3x + 1 \\ & \mp 3x^4 \mp 6x^3 \mp 6x^2 \mp 3x \mp 1 \end{array}$$
$$0$$

178.
$$\sqrt[3]{-8a^6b^6 - 36a^5b^5 - 6a^4b^4 + 117a^3b^3 + 12a^2b^2 - 144ab + 64} =$$
$$\pm 8a^6b^6 \qquad = -2a^2b^2 - 3ab + 4.$$
$$\begin{array}{l|l} 3.(-2a^2b^2)^2.(-3ab) + & -36a^5b^5 - 6a^4b^4 + 117a^3b^3 \\ + 3.(-2a^2b^2)(-3ab)^2 + (-3ab)^3 & \pm 36a^5b^5 \pm 54a^4b^4 \pm 27a^3b^3 \end{array}$$
$$\begin{array}{l|l} 3(-2a^2b^2 - 3ab)^2.4 + & 48a^4b^4 + 144a^3b^3 + 12a^2b^2 - 144ab + 64 \\ + 3(-2a^2b^2 - 3ab).4^2 + 4^3 & \mp 48a^4b^4 \mp 144a^3b^3 \mp 12a^2b^2 \pm 144ab \mp 64 \end{array}$$
$$0$$

179.
$$\sqrt[3]{a^{30} - 9a^{25} + 33a^{20} - 63a^{15} + 66a^{10} - 36a^5 + 8} =$$
$$\mp a^{30} \qquad = a^{10} - 3a^5 + 2.$$
$$\begin{array}{l|l} 3.(a^{10})^2(-3a^5) + & -9a^{25} + 33a^{20} - 63a^{15} \\ + 3.(a^{10})(-3a^5)^2 + (-3^5)^3 & \pm 9a^{25} \mp 27a^{20} \pm 27a^{15} \end{array}$$
$$\begin{array}{l|l} 3.(a^{10} - 3a^5)^2.2 + 3(a^{10} - 3a^5).2^2 + & 6a^{20} - 36a^{15} + 66a^{10} - 36a^5 + 8 \\ + 2^3 & \mp 6a^{20} \pm 36a^{15} \mp 66a^{10} \pm 36a^5 \mp 8 \end{array}$$
$$0$$

180. $$\sqrt[3]{x^9-3x^8+6x^7-10x^6+12x^5-12x^4+10x^3-6x^2+3x-1} = x^3-x^2+x-1.$$

$$\begin{array}{r|l}
 & \mp x^9 \\
\hline
3\,.\,(x^3)^2(-x^2)+ & -3x^8+6x^7-10x^6 \\
+3(x^3)(-x^2)^2+(-x^2)^3 & \pm 3x^8 \mp 3x^7 \pm x^6 \\
\hline
3(x^3-x^2)^2.\,x+3(x^3-x^2)\,x^2+x^3 & 3x^7-9x^6+12x^5-12x^4+10x^3 \\
 & \mp 3x^7 \pm 6x^6 \mp 6x^5 \pm 3x^4 \mp x^3 \\
\hline
3(x^3-x^2+x)^2\,.\,(-1)+ & -3x^6+6x^5-9x^4+9x^3-6x^2+3x-1 \\
+3(x^3-x^2+x)\,.\,(-1)^2+(-1)^3 & \pm 3x^6 \mp 6x^5 \pm 9x^4 \mp 9x^3 \mp 6x^2 \mp 3x \pm 1 \\
\hline
 & 0
\end{array}$$

§ 5. Arvude ruutjuure leidmine.

181.

$$\sqrt{5'76} = 26.$$

$$\begin{array}{r|r}
 & 4 \\
\hline
46 & 17'6 \\
6 & 176 \\
\hline
 & 0
\end{array}$$

182.

$$\sqrt{3'61} = 19.$$

$$\begin{array}{r|r}
 & 1 \\
\hline
29 & 26'1 \\
9 & 261 \\
\hline
 & 0
\end{array}$$

183.

$$\sqrt{18'49} = 43.$$

$$\begin{array}{r|r}
 & 16 \\
\hline
83 & 24'9 \\
3 & 249 \\
\hline
 & 0
\end{array}$$

184.

$$\sqrt{60'84'00} = 780.$$

$$\begin{array}{r|r}
 & 49 \\
\hline
148 & 118'4 \\
8 & 1184 \\
\hline
1560 & 00 \\
0 & 00 \\
\hline
 & 0
\end{array}$$

185.

$$\sqrt{13'69} = 37.$$

$$\begin{array}{r|r}
 & 9 \\
\hline
67 & 46'9 \\
7 & 469 \\
\hline
 & 0
\end{array}$$

186.

$$\sqrt{28'09'00'00} = 5300.$$

$$\begin{array}{r|r}
 & 25 \\
\hline
103 & 30'9 \\
3 & 309 \\
\hline
 & 0
\end{array}$$

187. $\sqrt{46'24} = 68.$

	36
128	102'4
8	1024
	0

188. $\sqrt{94'09'00'00'00} = 97000.$

	81
187	1309
7	1309
	0

189. $\sqrt{65'61 \cdot 10^4} = 81 \cdot 10^2 = 8100.$

	64
161	16'1
1	161
	0

190. $\sqrt{96'04 \cdot 10^6} = 98 \cdot 10^3 = 98000.$

	81
188	150'4
8	1504
	0

191. $\sqrt{5'47'56} = 234.$

	4
43	14'7
3	129
464	185'6
4	1856
	0

192. $\sqrt{5'61'69} = 237$

	4
43	16'1
3	129
467	326'9
7	3269
	0

193. $\sqrt{83'17'44} = 912.$

	81
181	21'7
1	181
1822	3644
2	3644
	0

194. $\sqrt{25'90'81} = 509.$

	25
1009	908'1
9	9081
	0

195. $\sqrt{7'6'7\,3'7\,6} = 876.$

```
  64
 167 | 127'3
   7 | 1169
1736 | 1047'6
   6 | 1047'6
          0
```

196. $\sqrt{4\,6'3\,7'6\,1} = 681.$

```
  36
 128 | 103'7
   8 | 1024
1361 | 136'1
   1 | 1361
         0
```

197. $\sqrt{1'8\,2'2\,5} = 135.$

```
  1
 23 | 8'2
  3 | 69
265 | 132'5
  5 | 1325
       0
```

198. $\sqrt{7\,2'5\,9'0\,4} = 852.$

```
  64
 165 | 85'9
   5 | 825
1702 | 340'4
   2 | 3404
        0
```

199.

$\sqrt{2\,2'5\,6'2\,5'0\,0} = 4750.$

```
 16
 87 | 656
  7 | 609
945 | 472'5
  5 | 4725
       0
```

200.

$\sqrt{9'4\,2'4\,9'0\,0'0\,0} = 30700.$

```
  9
607 | 424'9
  7 | 4249
       0
```

201.

$\sqrt{4'5\,6'2\,4'9\,6} = 2136.$

```
   4
  41 | 5'6
   1 | 41
 423 | 152'4
   3 | 1269
4266 | 2559'6
   6 | 25596
         0
```

202.

$\sqrt{9'9\,6'0\,3'3\,6} = 3156.$

```
   9
  61 | 9'6
   1 | 61
 625 | 350'3
   5 | 3125
6306 | 3783'6
   6 | 37836
         0
```

203.

$\sqrt{1'01'40'49} = 1007.$

	1
2007	1404'9
7	14049
	0

204.

$\sqrt{4'04'81'44} = 2012.$

	4
401	48'1
1	401
4022	804'4
2	8044
	0

205.

$\sqrt{49'12'60'81} = 7009.$

	49
14009	12608'1
9	126081
	0

206.

$\sqrt{56'32'50'25} = 7505.$

	49
145	73'2
5	725
15005	7502'5
5	75025
	0

207.

$\sqrt{72'69'26'76} = 8526.$

	64
165	86'9
5	825
1702	442'6
2	3404
17046	10227'6
6	102276
	0

208.

$\sqrt{89'90'83'24} = 9482.$

	81
184	89'0
4	736
1888	1548'3
8	15104
18962	3792'4
2	37924
	0

209.

$\sqrt{19'74'91'36} = 4444.$

16

84 | 37'4
4 | 336

884 | 389'1
4 | 3536

8884 | 3553'6
4 | 35536

0

210.

$\sqrt{37'31'98'81} = 6109.$

36

121 | 13'1
1 | 121

12209 | 10988'1
9 | 109881

0

211.

$\sqrt{12'26'96'07'84} = 35028.$

9

65 | 32'6
5 | 325

7002 | 1960'7
2 | 14004

70048 | 56038'4
8 | 560384

0

212.

$\sqrt{28'31'72'97'96} = 53214.$

25

103 | 331
3 | 309

1062 | 227'2
2 | 2124

10641 | 1489'7
1 | 10641

106424 | 42569'6
4 | 425696

0

213.

$\sqrt{49'19'71'77'96'49} = 701407.$

49

1401 | 197'1
1 | 1401

14024 | 5707'7
4 | 56096

1402807 | 981964'9
7 | 9819649

0

214.

$$\sqrt{1'0\,2'4\,2'1\,2'8\,1'7\,1'5\,6} = 1012034.$$

$$\begin{array}{r|l}
\multicolumn{2}{l}{1} \\
\hline
201 & 24'2 \\
1 & 201 \\
\hline
2022 & 411'2 \\
2 & 4044 \\
\hline
202403 & 68817'1 \\
3 & 607209 \\
\hline
2024064 & 809625'6 \\
4 & 8096256 \\
\hline
\multicolumn{2}{r}{0}
\end{array}$$

215. $\sqrt{\frac{49}{81}} = \frac{7}{9}.$ **216.** $\sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3} = 1\frac{2}{3}.$

217. $\sqrt{\frac{256}{2809}} = \frac{\sqrt{256}}{\sqrt{2809}} = \frac{16}{53}.$ **218.** $\sqrt{\frac{441}{17424}} = \frac{21}{132} = \frac{7}{44}.$

219. $\sqrt{552\frac{1}{4}} = \sqrt{\frac{2209}{4}} = \frac{47}{2} = 23\frac{1}{2}.$

220. $\sqrt{10955\frac{1}{9}} = \sqrt{\frac{98596}{9}} = \frac{314}{3} = 104\frac{2}{3}.$

221. $\frac{343}{700} = \frac{49}{100};$ $\sqrt{\frac{49}{100}} = \frac{7}{10}.$

222. $\frac{867}{14283} = \frac{289}{4761};$ $\sqrt{\frac{289}{4761}} = \frac{17}{69}.$

223.

$$\sqrt{0,3\,3'6\,4} = 0,5\,8.$$

$$\begin{array}{r|l}
\multicolumn{2}{l}{25} \\
\hline
108 & 86'4 \\
8 & 866 \\
\hline
\multicolumn{2}{r}{0}
\end{array}$$

224.

$$\sqrt{0,0\,0'3\,9'6\,9} = 0,063.$$

$$\begin{array}{r|l}
\multicolumn{2}{l}{36} \\
\hline
123 & 36'9 \\
3 & 369 \\
\hline
\multicolumn{2}{r}{0}
\end{array}$$

225.

$\sqrt{0{,}26'41'96} = 0{,}514$

	25
101	14'1
1	101
1024	409'6
4	4096
	0

226.

$\sqrt{0{,}'00'00'86'49} = 0{,}0093.$

	81
183	54'9
3	549
	0

227.

$\sqrt{2{,}37'16} = 1{,}54.$

	1
25	13'7
5	125
304	121'6
4	1216
	0

228.

$\sqrt{15{,}05'44} = 3{,}88.$

	9
68	60'5
8	544
768	614'4
8	6144
	0

229.

$\sqrt{0{,}00{,}00'25'80'64} = 0{,}00508.$

	25
1008	806'4
8	8064

230.

$\sqrt{40{,}99'84'09} = 6{,}403.$

	36
124	49'9
4	496
12803	3840'9
3	38409
	0

§ 6. Ruutjuure ligikaudsed tähendused.

231.

$\sqrt{9'69} = 31.$

	9
61	69
1	61
	8

232.

$\sqrt{72'69} = 85.$

	64
165	86'9
5	825
	44

233.

$\sqrt{5'37'80} = 231.$

	4
43	13'7
3	129
461	880
1	461
	419

234.

$\sqrt{81'30'00'00} = 9016.$

	81
1801	300'0
1	1801
18026	11990'0
6	108156
	11744

235. $\sqrt{7}\ (\text{kunni}\ \frac{1}{5}) = \frac{\sqrt{7 \cdot 5^2}}{5} = \frac{\sqrt{175}}{5} = \frac{13}{5} = 2\frac{3}{5};\ \sqrt{1,75} = 13.$

	1
23	75
3	69
	6

236. $\sqrt{46}\ (\text{kunni}\ \frac{1}{4}) = \frac{\sqrt{46 \cdot 4^2}}{4} = \frac{\sqrt{736}}{4} = \frac{27}{4} = 6\frac{3}{4}.$

237. $\sqrt{568}\ (\text{kunni}\ \frac{1}{20}) = \frac{\sqrt{568 \cdot 20^2}}{20} = \frac{\sqrt{227200}}{20} = \frac{476}{20} = 23\frac{4}{5}.$

238. $\sqrt{213}\ (\text{kunni}\ \frac{1}{15}) = \frac{\sqrt{213 \cdot 15^2}}{15} = \frac{\sqrt{47925}}{15} = \frac{218}{15} = 14\frac{8}{15}.$

239. $\sqrt{5}\ (\text{kunni}\ \frac{1}{200}) = \frac{\sqrt{5 \cdot 200^2}}{200} = \frac{\sqrt{200000}}{200} = \frac{447}{200} = 2\frac{47}{200}.$

240. $\sqrt{19}\ (\text{kunni}\ \frac{1}{300}) = \frac{\sqrt{19 \cdot 300^2}}{300} = \frac{\sqrt{1710000}}{300} = \frac{1307}{300} = 4\frac{107}{300}.$

241. $\sqrt{3} = 1'73.$

	1
27	20'0
7	189
343	110'0
3	1029
	71

Täpsus = 0,01.

Leiame juure täpsusega kunni 1, saame 1. Ülejäägile kirjutame juurde kaks nulli ja leiame juure kümnendikud (7). Teisele ülejäägile kirjutame jälle kaks nulli juurde, ning leiame juure sajandikud jne.

242. $\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$; $\sqrt{5} = 2{,}23$ (kunni $\frac{1}{100}$); $\frac{\sqrt{5}}{3} = \frac{2{,}23}{3} =$

$$\begin{array}{r|l} & 4 \\ 42 & 10'0 \\ 2 & 84 \\ 443 & 160'0 \\ 3 & 1329 \\ & \;\;271 \end{array}$$

$= 0{,}74$ $\left(\text{kunni } \frac{1}{100 \, . \, 3}\right)$.

243. $\sqrt{\frac{5}{8}} = \sqrt{\frac{10}{16}} = \frac{\sqrt{10}}{4}$; $\sqrt{10} = 3{,}16$ (kunni $\frac{1}{100}$);

$$\begin{array}{r|l} & 9 \\ 61 & 10'0 \\ 1 & 61 \\ 626 & 390'0 \\ 6 & 3756 \\ & \;\;144 \end{array}$$

$\frac{\sqrt{10}}{4} = \frac{3{,}16}{4} =$
$= 0{,}79$
$\left(\text{kunni } \frac{1}{100 \, . \, 4}\right)$.

244. $\sqrt{\frac{7}{24}} = \sqrt{\frac{42}{144}} = \frac{\sqrt{42}}{12}$; $\sqrt{42} = 6{,}48$ (kunni $\frac{1}{100}$);

$$\begin{array}{r|l} & 36 \\ 124 & 60'0 \\ 4 & 496 \\ 1288 & 1040'0 \\ 8 & 10304 \\ & \;\;\;\;96 \end{array}$$

$\frac{\sqrt{42}}{12} = \frac{6{,}48}{12} = 0{,}54$ $\left(\text{kunni } \frac{1}{100 \, . \, 12}\right)$.

245. $\sqrt{3\frac{1}{5}} = \sqrt{3{,}20} = 1{,}78.$

$$\begin{array}{r|l} & 1 \\ 27 & 22'0 \\ 7 & 189 \\ 348 & 310'0 \\ 8 & 2784 \\ & \;\;316 \end{array}$$

246. $\sqrt{11\frac{4}{7}}=\sqrt{\frac{81}{7}}=\frac{\sqrt{567}}{7}$; $\sqrt{5'67}=23{,}8$ (kunni $\frac{1}{10}$);

$$\begin{array}{r|l}
 & 4 \\
\hline
43 & 16'7 \\
3 & 129 \\
\hline
468 & 380'0 \\
8 & 3744 \\
\hline
 & 56
\end{array}$$

$\frac{\sqrt{567}}{7}=\frac{23{,}8}{7}=$
$=3{,}4$
$\left(\text{kunni } \frac{1}{10 \cdot 7}\right)$.

247. $\sqrt{7\frac{1}{12}}=\sqrt{\frac{85}{12}}=\frac{\sqrt{255}}{6}$; $\sqrt{2'55}=15{,}9$ (kunni $\frac{1}{10}$);

$$\begin{array}{r|l}
 & 1 \\
\hline
25 & 15'5 \\
 & 125 \\
\hline
309 & 300'0 \\
9 & 2781 \\
\hline
 & 219
\end{array}$$

$\frac{\sqrt{255}}{6}=\frac{15{,}9}{6}=$
$=2{,}6$
$\left(\text{kunni } \frac{1}{10 \cdot 6}\right)$.

248. $\sqrt{11\frac{5}{49}}=\sqrt{\frac{544}{49}}=\frac{\sqrt{544}}{7}$; $\sqrt{5'44}=23{,}3$ (kunni $\frac{1}{10}$);

$$\begin{array}{r|l}
 & 5 \\
\hline
43 & 14'4 \\
3 & 129 \\
\hline
463 & 150'0 \\
3 & 1389 \\
\hline
 & 111
\end{array}$$

$\frac{\sqrt{544}}{7}=\frac{23{,}3}{7}$
$=3{,}3$
$\left(\text{kunni } \frac{1}{10 \cdot 7}\right)$.

249. $\sqrt{74{,}'12}=8{,}609$ (kunni 0,001).

$$\begin{array}{r|l}
 & 64 \\
\hline
166 & 10\,1'2 \\
16 & 9\,96 \\
\hline
17209 & 16000'0 \\
9 & 154881 \\
\hline
 & 5119
\end{array}$$

250. $\sqrt{9{,}26'47} = 3{,}043$ (kunni 0,001).

```
       9
 604 | 264'7
   4 | 2416
 6083 | 2310'0
    3 | 18249
          4851
```

251.

$\sqrt{0{,}40} = 0{,}632$ (kunni 0,001).

```
    36
 123 | 40'0
   3 | 369
 1262 | 310'0
    2 | 2524
         576
```

252.

$\sqrt{6{,}'72} = 2{,}592$ (kunni 0,001).

```
     4
 45 | 2 7'2
  5 | 2 25
 509 | 47'00
   9 | 4581
 5182 | 1190'0
    2 | 10364
         1536
```

253.

$\sqrt{43{,}35'60} = 6{,}58$ (kunni 0,01).

```
    36
 125 | 735
   5 | 625
 1308 | 1106'0
    8 | 10464
          996
```

254.

$\sqrt{0{,}00'80} = 0{,}089$ (kunni 0,001).

```
     64
 169 | 160'0
   9 | 1521
         79
```

255.

$\sqrt{2,'05'34'70} = 1,433$ (kunni 0,001).

	1
24	1 0'5
4	9,6
283	934
3	849
2863	857'0
3	8589
	—19

256.

$\sqrt{12,50} = 3,53$ (kunni 0,01).

	9
65	35'0
5	325
703	250'0
3	2108
	391

257.

$\sqrt{64,25} = 8,015$ (kunni 0,001).

	64
1601	250'0
1	1601
16025	8990'0
5	80125
	9775

258.

$\sqrt{0,62'50} = 0,79$ (kunni 0,01).

	49
149	135'0
9	1341
	9

259.

$\sqrt{0,23'56'78'97} = 0,4854$ (kunni 0,0001).

	16
88	75'6
8	704
965	5278
5	4825
9704	4539'7
4	38816
	6581

260.

$\sqrt{6,'00'05'78'10} = 2,4495$ (kunni 0,0001).

	4
44	2 0'0
4	1 76
484	240'5
5	1936
4889	4697'8
9	44001
49986	29771'0
6	293916
	3794

§ 7. Kuupjuurte leidmine.

261.

$$\sqrt[3]{4'913}=17$$

$$\begin{array}{r|l} & 1 \\ \hline 3\cdot1^2=3 & 39'13 \\ \hline 3\cdot1^2\cdot7= & 21 \\ 3\cdot1\cdot7^2= & 147 \\ 7^3= & \widehat{1}348 \\ \hline & 3913 \\ \hline & 0 \end{array}$$

262.

$$\sqrt[3]{32'768}=32$$

$$\begin{array}{r|l} & 27 \\ \hline 3\cdot3^2=27 & 57'68 \\ \hline 3\cdot3^2\cdot2= & 54 \\ 3\cdot3\cdot2^2= & \;\;36 \\ 2^3= & \;\;\;\;\;8 \\ \hline & 5768 \\ \hline & 0 \end{array}$$

263.

$$\sqrt[3]{21'952}=28.$$

$$\begin{array}{r|l} & 8 \\ \hline 3\cdot2^2=12 & 139'52 \\ \hline 3\cdot2^2\cdot8= & 96 \\ 3\cdot2\cdot8^2= & 384 \\ 8^3= & \widehat{1}512 \\ \hline & 13952 \\ \hline & 0 \end{array}$$

264.

$$\sqrt[3]{74'088}=42.$$

$$\begin{array}{r|l} & 64 \\ \hline 3\cdot4^2=48 & 100'88 \\ \hline 3\cdot4^2\cdot2= & 96 \\ 3\cdot4\cdot2^2= & \;\;48 \\ 2^3= & \widehat{1}\;\;\;\;8 \\ \hline & 10088 \\ \hline & 0 \end{array}$$

265.

$$\sqrt{132'651}=51$$

$$\begin{array}{r|l} & 125 \\ \hline 3\cdot5^2=75 & 7651 \\ \hline 8\cdot5^2\cdot1= & 75 \\ 3\cdot5\cdot1^2= & \;\;15 \\ 1^3= & \;\;\;\;\;1 \\ \hline & 7651 \\ \hline & 0 \end{array}$$

266.

$$\sqrt[3]{551'368}=82.$$

$$\begin{array}{r|l} & 512 \\ \hline 3\cdot8^2=192 & 39'368 \\ \hline 3\cdot8^2\cdot2= & 384 \\ 3\cdot8\cdot2^2= & \;\;96 \\ 2^3= & \widehat{1}\;\;\;\;8 \\ \hline & 39368 \\ \hline & 0 \end{array}$$

267.

$$\sqrt[3]{753'571}=91.$$

$$\begin{array}{r|r}
 & 729 \\
\hline
3.9^2=243 & 245'71 \\
3.9^2.1= & 243 \\
3.9.1^2= & 27 \\
1^3= & 1 \\
\hline
 & 24571 \\
\hline
 & 0
\end{array}$$

268.

$$\sqrt[3]{884'736'000}=960.$$

$$\begin{array}{r|r}
 & 729 \\
\hline
3.9^2=243 & 1557'36 \\
3.9^2.6= & 1458 \\
3.9.6^2= & 972 \\
6^3= & 216 \\
\hline
 & 155736 \\
\hline
 & 0
\end{array}$$

269.

$$\sqrt[3]{157'464}=54.$$

$$\begin{array}{r|r}
 & 125 \\
\hline
3.5^2=75 & 324'64 \\
3.5^2.4= & 300 \\
3.5.4^2= & 240 \\
4^3= & 64 \\
\hline
 & 32464 \\
\hline
 & 0
\end{array}$$

270.

$$\sqrt[3]{85'184'000}=440.$$

$$\begin{array}{r|r}
 & 64 \\
\hline
3.4^2=48 & 211'84 \\
3.4^2.4= & 192 \\
3.4.4^2= & 192 \\
4^3= & 64 \\
\hline
 & 21184 \\
\hline
 & 0
\end{array}$$

271.

$$\sqrt[3]{3'652'264}=154.$$

$$\begin{array}{r|r}
 & 1 \\
\hline
3.1^2=3 & 26'52 \\
3.1^2.5= & 15 \\
3.1.5^2= & 75 \\
5^3= & 125 \\
\hline
 & 2375 \\
\hline
3.15^2=675 & 2772'64 \\
3.15^2.4= & 2700 \\
3.15.4^2= & 720 \\
4^3= & 64 \\
\hline
 & 277264 \\
\hline
 & 0
\end{array}$$

272.

$$\sqrt[3]{30'959'144}=314.$$

$$\begin{array}{r|r}
 & 27 \\
\hline
3.3^2=27 & 39'59 \\
3.3^2.1= & 27 \\
3.3.1^2= & 9 \\
1^3= & 1 \\
\hline
 & 2791 \\
\hline
3.31^2=2883 & 11681'44 \\
3.31^2.4= & 11532 \\
3.31.4^2= & 1488 \\
4^3= & 64 \\
\hline
 & 1168144 \\
\hline
 & 0
\end{array}$$

273.

$$\sqrt[3]{8'741'816}=206.$$

$$\begin{array}{r|l}
 & 8 \\
\hline
3.20^2=1200 & 741'816 \\
\hline
3.20^2.6= & 7200 \\
3.20.6^2= & \;\;2160 \\
6^3= & \;\;\;\;216 \\
\hline
 & 741816 \\
\hline
 & 0
\end{array}$$

274.

$$\sqrt[3]{137'388'096}=516.$$

$$\begin{array}{r|l}
 & 125 \\
\hline
3.5^2=75 & 12'388 \\
\hline
3.5^2.1= & 75 \\
3.5.1^2= & \;\;15 \\
1^3= & \;\;\;\;1 \\
\hline
 & 7651 \\
\hline
3.51^2=7803 & 47370'96 \\
\hline
3.51^2.6= & 46818 \\
3.51.6^2= & \;\;5508 \\
6^3= & \;\;\;\;216 \\
\hline
 & 4737096 \\
\hline
 & 0
\end{array}$$

275.

$$\sqrt[3]{539'353'144}=814.$$

$$\begin{array}{r|l}
 & 512 \\
\hline
3.8^2=192 & 27'353 \\
\hline
3.8^2.1= & 192 \\
3.8.1^2= & \;\;24 \\
1^3= & \;\;\;\;1 \\
\hline
 & 19441 \\
\hline
3.81^2=19683 & 79121'44 \\
\hline
3.81^2.4= & 78732 \\
3.81.4^2= & \;\;3888 \\
4^3= & \;\;\;\;64 \\
\hline
 & 7912144 \\
\hline
 & 0
\end{array}$$

276.

$$\sqrt[3]{139'798'359} = 519.$$

	125
$3.5^2 = 75$	147'98
$3.5^2.1 =$	75
$3.5.1^2 =$	15
$1^3 =$	1
	7651
$3.51^2 = 7803$	71473'59
$3.51^2.9 =$	70227
$3.51.9^2 =$	12393
$9^3 =$	${}_2729$
	7147359
	0

277.

$$\sqrt[3]{622'835'864} = 854.$$

	512
$3.8^2 = 192$	1108'35
$3.8^2.5 =$	960
$3.8.5^2 =$	600
$5^3 =$	125
	102125
$3.85^2 = 21675$	87108'64
$3.85^2.4 =$	86700
$3.85.4^2 =$	4080
$4^3 =$	64
	8710864
	0

278. $\sqrt[3]{849'278'123} = 947.$

	729
$3.9^2 = 243$	120278
$3.9^2.4 =$	972
$3.9.4^2 =$	432
$4^3 =$	64
	101584
$3.94^2 = 26508$	18694123
$3.94^2.7 =$	185556
$3.94.7^2 =$	13818
$7^3 =$	343
	18694123
	0

279. $\sqrt[3]{134'453'795'867} = 5123.$

	125
$3.5^2 = 75$	94'53
$3.5^2.1 =$	75
$3.5.1^2 =$	15
$1^3 =$	1
	7651
$3.51^2 = 7803$	18027'95
$3.51^2.2 =$	15606
$3.51.2^2 =$	612
$2^3 =$	8
	1566728
$3.512^2 = 786432$	236067867
$3.552^2.3 =$	2359296
$3.512.3^2 =$	13824
$3^3 =$	27
	236067867
	0

280.

$$\sqrt[3]{15{,}888{,}972{,}744} = 2514.$$

	8
$3 \cdot 2^2 = 12$	78′88
$3 \cdot 2^2 \cdot 5 =$	60
$3 \cdot 2 \cdot 5^2 =$	150
$5^3 =$	125
	7625
$3 \cdot 25^2 = 1875$	2639,72
$3 \cdot 25^2 \cdot 1 =$	1875
$3 \cdot 25 \cdot 1^2 =$	75
$1^3 =$	1
	188251
$3 \cdot 251^2 = 189003$	757217,44
$3 \cdot 251^2 \cdot 4 =$	756012
$3 \cdot 251 \cdot 4^2 =$	12048
$4^3 =$	64
	75721744
	0

281. $\sqrt[3]{\frac{27}{125}} = \frac{3}{5}.$ **282.** $\sqrt[3]{\frac{343}{729}} = \frac{7}{9}.$

283. $\sqrt[3]{15\frac{5}{8}} = \sqrt[3]{\frac{125}{8}} = \frac{5}{2} = 2\frac{1}{2}.$

284. $\sqrt[3]{\frac{729}{1000000}} = \frac{9}{100}.$

285. $\sqrt[3]{1\frac{1178}{2197}} = \sqrt[3]{\frac{3375}{2197}} = \frac{15}{13} = 1\frac{2}{13}.$

286. $\sqrt[3]{72\frac{73}{216}} = \sqrt[3]{\frac{15625}{216}} = \frac{25}{6} = 4\frac{1}{6}.$

287.

$$\sqrt[3]{0,'004,096} = 0,16.$$

	1
$3 . 1^2 = 3$	30,96
$3 . 1^2 . 6 =$	18
$3 . 1 . 6^2 =$	108
$6^3 =$	216
	3096
	0

288.

$$\sqrt[3]{68,921} = 4,1.$$

	64
$3 . 4^2 . = 48$	49,21
$3 . 4^2 . 1 =$	48
$3 . 4 . 1^2 =$	12
$1^3 =$	1
	4921
	0

289.

$$\sqrt[3]{0,000,005,832} = 0,018.$$

	1
$3 . 1^2 = 3$	48'32
$3 . 1^2 . 8 =$	24
$3 . 1 . 8^2 =$	192
$8^3 =$	512
	4832
	0

290.

$$\sqrt[3]{0,000,030,664,297} = 0,0313.$$

	27
$3 . 3^2 = 27$	36,64
$3 . 3^2 . 1 =$	27
$3 . 3 . 1^2 =$	9
$1^3 =$	1
	2791
$3 . 31^2 = 2883$	8732,97
$3 . 31^2 . 3 =$	8649
$3 . 31 . 3^2 =$	837
$3^3 =$	27
	873297
	0

§ 8. Ligikaudne kantjuurte leidmine.

291. $\sqrt[3]{4}$ (kunni $\frac{1}{5}$) $= \frac{\sqrt[3]{4 \cdot 5^3}}{5} = \frac{\sqrt[3]{500}}{5} = \frac{7}{5} = 1\frac{2}{5}$.

292. $\sqrt[3]{21}$ (kunni $\frac{1}{6}$) $= \frac{\sqrt[3]{21 \cdot 6^3}}{6} = \frac{\sqrt[3]{4536}}{6} = \frac{16}{6} = 2\frac{2}{3}$.

293. $\sqrt[3]{2}$ (kunni $\frac{1}{100}$) $= \frac{\sqrt[3]{2 \cdot 100^3}}{100} = \frac{\sqrt[3]{2000000}}{100} = \frac{125}{100} = 1{,}25$.

294. $\sqrt[3]{40}$ (kunni $\frac{1}{25}$) $= \frac{\sqrt[3]{40 \cdot 25^3}}{25} = \frac{\sqrt[3]{625000}}{25} = \frac{85}{25} = 3\frac{2}{5}$.

295. $\sqrt[3]{2\frac{1}{4}}$ (kunni $\frac{1}{10}$) $= \frac{\sqrt[3]{\frac{9 \cdot 10^3}{4}}}{10} = \frac{\sqrt[3]{\frac{18}{8} \cdot 10^3}}{10} =$

$= \frac{\frac{1}{2} \cdot \sqrt[3]{18000}}{10} = \frac{1}{20}\sqrt[3]{18000} = \frac{26}{20} = 1{,}3.$

296. $\sqrt[3]{\frac{25}{9}}$ (kunni $\frac{1}{100}$) $= \frac{1}{100}\sqrt[3]{\frac{75}{27} \cdot 100^3} =$

$= \frac{1}{300}\sqrt[3]{75000000} = \frac{421}{300} = 1\frac{121}{300}$.

297.

$$\sqrt[3]{0{,}215} = 0{,}59.$$

$$\begin{array}{r|l}
 & 125 \\ \hline
3 \cdot 5^2 = 75 & 900'00 \\
3 \cdot 5^2 \cdot 9 = & 675 \\
3 \cdot 5 \cdot 9^2 = & 1215 \\
9^3 = & 729 \\ \hline
 & 80379 \\ \hline
 & 9621
\end{array}$$

298

$$\sqrt[3]{0{,}360} = 0{,}71.$$

$$\begin{array}{r|l}
 & 343 \\ \hline
3 \cdot 7^2 = 147 & 170'00 \\
3 \cdot 7^2 \cdot 1 = & 147 \\
3 \cdot 7 \cdot 1^2 = & 21 \\
1^3 = & 1 \\ \hline
 & 14911 \\ \hline
 & 2089
\end{array}$$

299.

$$\sqrt[3]{,0,51,364} = 0,37.$$

	27
$3.3^2 = 27$	243.64
$3.3^2.7 =$	189
$3.3.7^2 =$	241
$7^3 =$	11 343
	21658
	2711

300.

$$\sqrt[3]{0{,}009,560} = 0,212.$$

	8
$3.2^2 = 12$	15,60
$3.2^2.1 =$	12
$3.2.1^2 =$	6
$1^3 =$	1
	1261
$3.21^2. = 1323$	2990,00
$3.21^2.2 =$	2646
$3.21.2^2 =$	252
$2^3 =$	1 8
	267128
	31872

VIII osa.

Irrationaalsed avaldused.

§ 1. Radikaali alt välja toomine ja radikaali alla viimine.

1. $\sqrt{8}=\sqrt{2.4}=2\sqrt{2}.$ **2.** $\sqrt{75}=\sqrt{3.25}=5\sqrt{3}.$

3. $\sqrt[3]{81}=\sqrt[3]{3.27}=3\sqrt[3]{3}.$ **4.** $\sqrt[3]{-108}=\sqrt[3]{-27.4}=-3\sqrt[3]{4}.$

5. $\sqrt[4]{48}=\sqrt[4]{3.16}=2\sqrt[4]{3}.$ **6.** $\sqrt[4]{1250}=\sqrt[4]{625.2}=5\sqrt[4]{2}.$

7. $\sqrt[5]{486}=\sqrt[5]{2.243}=3\sqrt[5]{2}.$ **8.** $\sqrt[5]{-224}=\sqrt[5]{-32.7}=-2\sqrt[5]{7}.$

9. $2\sqrt{405}=2\sqrt{81.5}=18\sqrt{5}.$ **10.** $\frac{2}{3}\sqrt[4]{243}=\frac{2}{3}.3\sqrt[4]{3}=2\sqrt[4]{3}.$

11. $\sqrt[4]{a^8c^3}=a^2\sqrt[4]{c^3}.$ **12.** $\sqrt[5]{a^{15}b^6}=a^3b\sqrt[5]{b}.$ **13.** $\sqrt[3]{x^4y^5}=xy\sqrt[3]{xy^2}.$

14. $\sqrt[4]{a^5b^6}=ab\sqrt[4]{ab^2}.$ **15.** $\sqrt{4a^4b}=2a^2\sqrt{b}.$ **16.** $\sqrt[3]{64x^6y^4}=$
$=4x^2y\sqrt[3]{y}.$ **17.** $3\sqrt{80c^4d^2}=3\sqrt{16.5c^4d^2}=12c^2d\sqrt{5}.$

18. $2\sqrt{\frac{a^5}{4}}=2.\frac{a^2}{2}\sqrt{a}=a^2\sqrt{a}.$ **19.** $\sqrt[3]{\frac{a^8}{b^9}}=\frac{a^2\sqrt[3]{a^2}}{b^3}.$

20. $\sqrt[6]{\frac{a^5}{b^{18}}}=\frac{\sqrt[3]{a^5}}{b^3}.$ **21.** $a\sqrt{\frac{0,54z}{a^2x^2}}=a\sqrt{\frac{0,09.6.z}{a^2x^2}}=$
$=\frac{0,3}{x^2}\sqrt{6z}.$ **22** $\sqrt[3]{\frac{-0,729m}{a^6}}=\frac{-0,9}{a^2}\sqrt[3]{m}.$ **23.** $\sqrt{\frac{(a^2-2ab+b^2)y}{25}}=$

$= \sqrt{\frac{(a-b)^2 y}{25}} = \frac{a-b}{5}\sqrt{y}.$ **24.** $\sqrt{\frac{a}{b^2} - \frac{1}{b}} = \sqrt{\frac{a-b}{b^2}} = \frac{1}{b}\sqrt{a-b}.$

25. $\sqrt[3]{\frac{(y^2-x^2)^4}{8(x+y)}} = \sqrt[3]{\frac{(y^2-x^2)^3(y+x)(y-x)}{8(x+y)}} = \frac{y^2-x^2}{2}\sqrt[3]{y-x}.$

26. $a\sqrt[3]{\frac{b^3}{a^4} - \frac{b^5}{a^6}} = a\sqrt[3]{\frac{b^3a^2 - b^5}{a^6}} = a\sqrt[3]{\frac{b^3(a^2-b^2)}{a^6}} = \frac{b}{a}\sqrt[3]{a^2-b^2}.$

27. $\sqrt[m]{2^{m+1}a^{5m}b^{m+n}c^{mp+1}} = \sqrt[m]{2^m . 2 . a^{5m}b^m . b^n c^{mp} c} =$
$= 2a^5bc^p\sqrt[m]{2b^n c}.$

28. $x^2y\sqrt[2r+1]{-x^{2r+2}y^{6r+5}z^2} = x^2y . \sqrt[2r+1]{-x^{2r+1} . x . y^{6r+3}y^2z^2} =$
$= -x^3y^4\sqrt[2r+1]{xy^2z^2}.$ **29.** $\frac{ac}{b}\sqrt[n]{3^{n+2}a^{n+5}b^{2n-1}c^{1-3n}} =$

$= \frac{ac}{b}\sqrt[n]{3^n . 3^2 . a^n . a^5b^{2n}b^{-1}c . c^{-3n}} =$

$= \frac{ac}{b} . 3 . a b^2c^{-3}\sqrt[n]{3^2 . a^5 b^{-1}c} = \frac{3a^2b}{c^2}\sqrt[n]{\frac{9a^5c}{b}}.$

30. $5a^{-3}c^2x^3\sqrt[3]{108a^5b^{7n}c^{-4}x^{-8}d^6} =$

$= 5a^{-3}c^2x^3 . 3ab^{2n}c^{-1}x^{-2}d^2 . \sqrt[3]{4a^2b^n c^{-1}x^{-2}} =$

$= \frac{15cxb^{2n}d^2}{a^2}\sqrt[3]{\frac{4a^2b^n}{cx^2}}.$

31. $2\sqrt{3} = \sqrt{4 . 3} = \sqrt{12}.$ **32.** $6\sqrt{5} = \sqrt{36 . 5} = \sqrt{180}.$

33. $3\sqrt[3]{2} = \sqrt[3]{27 . 2} = \sqrt[3]{54}.$ **34.** $5\sqrt[3]{3} = \sqrt[3]{125 . 3} = \sqrt[3]{375}.$

35. $2\sqrt[5]{5} = \sqrt[5]{32 . 5} = \sqrt[5]{160}.$ **36.** $a\sqrt{5} = \sqrt{5a^2}.$

37. $x\sqrt[4]{2} = \sqrt[4]{2x^4}.$ **38.** $5\sqrt[4]{a} = \sqrt[4]{625a}.$

39. $-m\sqrt[3]{n} = \sqrt[3]{-m^3n}.$ **40.** $-n^2\sqrt{a} = -\sqrt{an^4}.$

41. $3a\sqrt{ax}=\sqrt{9a^3x}$. **42.** $m^2\sqrt[3]{mn}=\sqrt[3]{m^7n}$.

43. $\frac{1}{2}\sqrt{a}=\sqrt{\frac{a}{4}}$. **44.** $\frac{x}{y}\sqrt[3]{y^2}=\sqrt[3]{\frac{x^3}{y^3}\cdot y^2}=\sqrt[3]{\frac{x^3}{y}}$.

45. $-\frac{a}{b}\sqrt[3]{-\frac{b^4}{a^5}}=\sqrt[3]{-\frac{a^3}{b^3}\cdot-\frac{b^4}{a^5}}=\sqrt[3]{\frac{b}{a^2}}$.

46. $m\sqrt[5]{1-\frac{1}{m^5}}=\sqrt[5]{m^5\left(1-\frac{1}{m^5}\right)}=\sqrt[5]{m^5-1}$.

47. $(m+n)\sqrt{\frac{1}{m^2-n^2}}=\sqrt{\frac{(m+n)^2}{m^2-n^2}}=\sqrt{\frac{(m+n)(m+n)}{(m+n)(m-n)}}=$
$=\sqrt{\frac{m+n}{m-n}}$.

48. $2ac^3\sqrt[3]{3abc^2}=\sqrt[3]{8a^3c^9\cdot 3abc^2}=\sqrt[3]{24a^4bc^{11}}$.

49. $3a^nb\sqrt[m]{3a^2b}=\sqrt[m]{3^ma^{nm}b^m\cdot 3a^2b}=\sqrt[m]{3^{m+1}a^{nm+2}b^{m+1}}$.

50. $3a^2c^3\sqrt[4]{2a^nb^{-3}}=\sqrt[4]{81a^8c^{12}\cdot 2a^nb^{-3}}=\sqrt[4]{\frac{162a^{n+8}c^{12}}{b^3}}$.

§ 2. Juurenäitaja ja juuritava arvu astmenäitaja koondamine ja samanimelised juured.

51. $\sqrt[9]{a^6}=\sqrt[3]{a^2}$. **52.** $\sqrt[8]{a^{10}b^{12}}=\sqrt[4]{a^5b^{3n}}$ **53.** $\sqrt[3n]{a^{2n}b^{3n}}=$
$=\sqrt[3]{a^2b^3}$. **54.** $\sqrt[mn]{a^mb^{2m}}=\sqrt[n]{ab^2}$. **55.** $\sqrt[6]{9a^4b^6}=\sqrt[6]{3^2a^4b^6}=$
$=\sqrt[3]{3a^2b^3}$. **56.** $\sqrt[9]{27a^{3m}b^6}=\sqrt[9]{3^3a^{3m}b^6}=\sqrt[3]{3a^mb^2}$.

57. $\sqrt[12]{64a^4b^{2n}}=\sqrt[12]{8^2a^4b^{2n}}=\sqrt[6]{8a^2b^n}$.

58. $\sqrt[6n]{\frac{16a^{10}b^{-6}}{9c^{18}}}=\sqrt[6n]{\frac{4^2a^{10}}{3^2b^6c^{18}}}=\sqrt[3n]{\frac{4a^5}{3b^3c^9}}$.

59. $\sqrt[12]{\frac{1000a^{-6}}{729b^9c^{-3}}}=\sqrt[12]{\frac{10^3a^{-6}}{9^3b^9c^{-3}}}=\sqrt[4]{\frac{10a^{-2}}{9b^3c^{-1}}}=\sqrt[4]{\frac{10c}{9a^2b^3}}$.

60. $\sqrt[-4]{a^{-8} b^{10} c^{-2}} = \sqrt[4]{a^8 b^{-10} c^2} = \sqrt{a^4 b^{-5} c}$.

61. $\sqrt[6]{a^5} = \sqrt[12]{a^{10}}$ ja $\sqrt[4]{a^3} = \sqrt[12]{a^9}$. **62.** $\sqrt[3]{2a^2} = \sqrt[6]{4a^4}$ ja $\sqrt{ab^5}$. **63.** $\sqrt[3]{2a^2b} = \sqrt[12]{16a^8b^4}$ ja $\sqrt[4]{3a^3b} = \sqrt[12]{27a^9b^3}$.

64. $\sqrt{\frac{3a^5}{b^3}} = \sqrt[18]{\frac{3^9a^{45}}{b^{27}}}$ ja $\sqrt[9]{\frac{10b^2}{a}} = \sqrt[18]{\frac{100b^4}{a^2}}$

65. $\sqrt[m^2]{\frac{3a^2}{bc^3}} = \sqrt[m^2n]{\frac{3^na^{2n}}{b^nc^{3n}}}$ ja $\sqrt[mn]{\frac{2ab^2}{c^3}} = \sqrt[m^2n]{\frac{2^ma^mb^{2m}}{c^{3m}}}$.

66. $\sqrt[12]{a^2b^3} = \sqrt[24]{a^4b^6}$, $\sqrt[4]{a} = \sqrt[24]{a^6}$ ja $\sqrt[8]{a^3} = \sqrt[24]{a^9}$.

67. $\sqrt[6]{a^2b} = \sqrt[150]{a^{50}b^{25}}$, $\sqrt[15]{a^3b^4} = \sqrt[150]{a^{30}b^{40}}$ ja $\sqrt[50]{a^{10}b^{20}} = \sqrt[150]{a^{30}b^{60}}$.

68. $\sqrt{\frac{x}{y}} = \sqrt[30]{\frac{x^{15}}{y^{15}}}$, $\sqrt[5]{\frac{y^3}{z^2}} = \sqrt[30]{\frac{y^{18}}{z^{12}}}$ ja $\sqrt[3]{\frac{a^2}{b}} = \sqrt[30]{\frac{a^{20}}{b^{10}}}$.

69. $\sqrt[4]{\frac{x-1}{x+1}}$, $\sqrt[2n]{\frac{x+1}{x-1}}$ ja $\sqrt[n]{\frac{x}{y}} = \sqrt[4n]{\left(\frac{x-1}{x+1}\right)^n}$, $\sqrt[4n]{\left(\frac{x+1}{x-1}\right)^2}$ ja $\sqrt[4n]{\frac{x^4}{y^4}}$.

70. $\sqrt[n]{(a+b)^m}$, $\sqrt[n^2]{a^m}$ ja $\sqrt[nm]{\frac{a-b}{(a+b)^2}} = \sqrt[n^2m]{(a+b)^{nm^2}}$, $\sqrt[n^2m]{a^{m^2}}$ ja $\sqrt[n^2m]{\frac{(a-b)^n}{(a+b)^{2n}}}$.

§ 3. Juurte normaalkuju.

71. $\frac{3xy^2}{2}\sqrt[3]{\frac{8}{xy}} = \frac{3 \cdot 2 \cdot xy^2}{2}\sqrt[3]{\frac{1}{xy}} = 3xy^2\sqrt[3]{\frac{x^2y^2}{x^3y^3}} = \frac{3xy^2}{xy}\sqrt[3]{x^2y^2} = 3y\sqrt[3]{x^2y^2}$.

72. $a^2\sqrt{\frac{2ab^3}{3c^2d}} = a^2\sqrt{\frac{6ab^3d}{9c^2d^2}} = \frac{a^2b}{3cd}\sqrt{6abd}$.

73. $\frac{1}{a}\sqrt[3]{a^8 - a^6b^2} = \frac{1}{a}\sqrt[3]{a^6(a^2 - b^2)} = a\sqrt[3]{a^2 - b^2}.$

74. $a^2\sqrt[4]{\frac{1}{a^3} - \frac{b}{a^4}} = a^2\sqrt[4]{\frac{a-b}{a^4}} = \frac{a^2}{a}\sqrt[4]{a-b} = a\sqrt[4]{a-b}.$

75. $5\,n^x\sqrt[3]{\frac{ab^5}{25\,n^{3x+1}}} = 5\,n^x\sqrt[3]{\frac{5\,ab^5 . n^2}{125\,n^{3x+3}}} =$

$= \frac{5\,n^x . b}{5\,n^{x+1}}\sqrt[3]{5\,ab^2n^2} = \frac{b}{n}\sqrt[3]{5\,ab^2n^2}.$

76. $\sqrt{\frac{18}{25\,a} - \frac{9\,b^2}{25\,a^3}} = \sqrt{\frac{18\,a^2 - 9\,b^2}{25\,a^3}} = \sqrt{\frac{9\,(2\,a^2 - b^2)\,a}{25\,a^4}} =$

$= \frac{3}{5\,a^2}\sqrt{a\,(2\,a^2 - b^2)}.$

77. $\frac{c^{n-m}}{a^m}\sqrt[m+n]{\frac{a^{m^2-n^2}\,b^{3m+6n}}{c^{m+2n}}} =$

$= \frac{c^{n-m}}{a^m}\sqrt[m+n]{\frac{a^{(m+n)\,(m-n)}\,b^{3(m+n)}\,b^{3n}\,c^m}{c^{2(m+n)}}} = \frac{c^{n-m} . a^{m-n} . b^3}{c^2a^m}\sqrt[m+n]{b^{3n}c^m} =$

$= c^{n-m-2}\,a^{-n}\,b^3\sqrt[m+n]{b^{3n}c^m}.$

78. $\frac{a+b}{a}\sqrt[3]{\frac{a^{13} - a^{12}b}{(a-b)^2}} = \frac{a+b}{a}\sqrt[3]{\frac{a^{12}\,(a-b)\,(a-b)}{(a-b)^3}} = \frac{a+b}{a}\,.$

$\cdot\,\frac{a^4}{a-b}\sqrt[3]{(a-b)^2} = \frac{a^3\,(a+b)}{a-b}\sqrt[3]{(a-b)^2}.$

79. $\frac{a}{c}\sqrt{\frac{a^3b - 4\,a^2b^2 + 4\,ab^3}{c^2}} = \frac{a}{c}\sqrt{\frac{ab\,(a^2 - 4\,ab + 4\,b^2)}{c^2}} =$

$= \frac{a}{c^2}\sqrt{ab\,(a - 2\,b)^2} = \frac{a\,(a - 2b)}{c^2}\sqrt{ab}.$

80. $\frac{a}{2}\sqrt[4]{(a+1)\,(a^2 - 1)\,(1 + 2\,a + a^2)} =$

$= \frac{a}{2}\sqrt[4]{(a+1)\,(a-1)\,(a+1)\,(a+1)^2} = \frac{a\,(a+1)}{2}\sqrt[4]{a-1}.$

§ 4. Sarnased juured.

81. $\sqrt{3}$ ja $\sqrt{12} = 2\sqrt{3}$ on sarnased.

82. $\sqrt{63} = 3\sqrt{7}$ ja $\sqrt{28} = 2\sqrt{7}$ on sarnased.

83. $\sqrt[3]{54} = 3\sqrt[3]{2}$ ja $2\sqrt[3]{2}$.

84. $\sqrt[4]{80} = 2\sqrt[4]{5}$ ja $\sqrt[4]{405} = 3\sqrt[4]{5}$.

85. $\sqrt{18} = 3\sqrt{2}$, $\sqrt{128} = 8\sqrt{2}$ ja $\sqrt{32} = 4\sqrt{2}$.

86. $\sqrt[3]{54} = 3\sqrt[3]{2}$, $\sqrt[3]{16} = 2\sqrt[3]{2}$ ja $\sqrt[3]{432} = 6\sqrt[3]{2}$.

87. $\sqrt{\frac{4}{3}} = \frac{1}{3}\sqrt{12}$ ja $\sqrt{12}$.

88. $\sqrt{\frac{2}{5}} = \frac{1}{5}\sqrt{10}$ ja $\sqrt{\frac{2}{45}} = \frac{1}{15}\sqrt{10}$.

89. $\frac{1}{4}\sqrt{0{,}2} = \frac{1}{4}\sqrt{\frac{1}{5}} = \frac{1}{20}\sqrt{5}$ ja $\frac{1}{5}\sqrt{5}$.

90 $\sqrt[3]{\frac{8}{3}} = 2\sqrt[3]{\frac{1}{3}} = 2\sqrt[3]{\frac{9}{27}} = \frac{2}{3}\sqrt[3]{9}$ ja $\sqrt[3]{\frac{9}{8}} = \frac{1}{2}\sqrt[3]{9}$.

91. $\sqrt{\frac{1}{2}+\frac{3}{4}} = \sqrt{\frac{5}{4}} = \frac{1}{2}\sqrt{5}$ ja $\sqrt{\frac{8}{9}-\frac{1}{3}} = \sqrt{\frac{5}{9}} = \frac{1}{3}\sqrt{5}$.

92. $\sqrt[3]{\frac{6}{25}-\frac{1}{4}} = \sqrt[3]{-\frac{1}{100}} = -\frac{1}{10}\sqrt[3]{10}$ ja $\sqrt[3]{\frac{1}{27}-\frac{1}{32}} =$
$= \sqrt[3]{\frac{5}{864}} = \frac{1}{12}\sqrt[3]{10}$.

93. $\sqrt[6]{a^7b} = a\sqrt[6]{ab}$ ja $\sqrt[6]{a^{13}b^7} = a^2b\sqrt[6]{ab}$.

94. $\sqrt[3]{0{,}027\,xy^2} = 0{,}3\sqrt[3]{xy^2}$ ja $\sqrt[3]{0{,}064\,\frac{x}{y}} = \frac{0{,}4}{y}\sqrt[3]{xy^2}$.

95. $\sqrt{a - \frac{1}{a^2}} = \sqrt{\frac{a^3-1}{a^2}} = \frac{1}{a}\sqrt{a^3-1}$ ja $\sqrt{\frac{a^3-1}{a^4}} =$
$= \frac{1}{a^2}\sqrt{a^3-1}$.

96. $\sqrt{\frac{1}{b} - a} = \sqrt{\frac{(1-ab)\,b}{b^2}} = \frac{1}{b}\sqrt{b\,(1-ab)}$ ja
$\sqrt{\frac{bd^2 - ab^2d^2}{c^2}} = \sqrt{\frac{bd^2(1-ab)}{c^2}} = \frac{d}{c}\sqrt{b\,(1-ab)}$.

97. $\sqrt{\left(\frac{a^2-b^2}{a+b}\right)^3} = \sqrt{\left(\frac{(a+b)\,(a-b)}{a+b}\right)^3} = (a-b)\sqrt{a-b}$;

$$\sqrt{\frac{(a^2+b^2)^2}{a-b}}=(a^2+b^2)\sqrt{\frac{1}{a-b}}=\frac{a^2+b^2}{a-b}\sqrt{a-b};\ \sqrt{a^3-a^2b}=$$
$$=\sqrt{a^2(a-b)}=a\sqrt{a-b}.$$

98. $\frac{x}{y}\sqrt{x^2y\left(\frac{x}{y}-1\right)}=\frac{x^2}{y}\sqrt{y\left(\frac{x-y}{y}\right)}=\frac{x^2}{y}\sqrt{x-y};$

$$x\sqrt{\frac{z}{xz-yz}}=x\sqrt{\frac{z}{z(x-y)}}=x\sqrt{\frac{1}{x-y}}=\frac{x}{x-y}\sqrt{x-y};$$
$$\sqrt{\frac{4x}{y^2}-\frac{4}{y}}=\sqrt{\frac{4x-4y}{y^2}}=\frac{2}{y}\sqrt{x-y}.$$

99. $\sqrt[3]{8a^5-16a^3b^2}=\sqrt[3]{8a^3(a^2-2b^2)}=2a\sqrt[3]{a^2-2b^2};$

$$ab\sqrt[3]{\frac{1}{a}-\frac{2b^2}{a^3}}=ab\sqrt[3]{\frac{a^2-2b^2}{a^3}}=b\sqrt[3]{a^2-2b^2};\ \sqrt[3]{\frac{2}{a^3b}-\frac{1}{ab^3}}=$$
$$=\sqrt[3]{\frac{2b^2-a^2}{a^3b^3}}=\sqrt[3]{-\frac{a^2-2b^2}{a^3b^3}}=-\frac{1}{ab}\sqrt[3]{a^2-2b^2}.$$

100. $\frac{x^2}{y}\sqrt[n]{x^{-3(n-1)}y^{2n+1}}=\frac{x^2}{y}\sqrt[n]{x^{-3n}.x^3.y^{2n}.y}=$

$$=\frac{x^2.x^{-3}.y^2}{y}\sqrt[n]{x^3y}=\frac{y}{x}\sqrt[n]{x^3y};\ \frac{1}{xy}\sqrt[n]{x^{n+3}y^{n+1}}=\frac{1}{xy}\sqrt[n]{x^n.x^3.y^n.y}=$$
$$=\frac{x.y}{xy}\sqrt[n]{x^3y}=\sqrt[n]{x^3y};\qquad (2x-y)\sqrt[n]{x^{3-n}y}=(2x-y).$$
$$.x^{-1}\sqrt[n]{x^3y}=\frac{2x-y}{x}\sqrt[n]{x^3y}.$$

§ 5. Juurte liitmine ja lahutamine.

101. $(5\sqrt{2}-4\sqrt[3]{3})+(3\sqrt{2}+6\sqrt[3]{3})=5\sqrt{2}-4\sqrt[3]{3}+$
$+3\sqrt{2}+6\sqrt[3]{3}=8\sqrt{2}+2\sqrt[3]{3}.$

102. $(10\sqrt[4]{7}+\sqrt[5]{3})-(5\sqrt[5]{3}+2\sqrt[4]{7})=10\sqrt[4]{7}+\sqrt[5]{3}-$
$-5\sqrt[5]{3}-2\sqrt[4]{7}=8\sqrt[4]{7}-4\sqrt[5]{3}.$

103. $(a\sqrt{b}-b\sqrt{c})-(3a\sqrt{b}-5b\sqrt{c})=a\sqrt{b}-b\sqrt{c}-$
$-3a\sqrt{b}+5b\sqrt{c}=4b\sqrt{c}-2a\sqrt{b}.$

104 $(a\sqrt[5]{b^4} - 2c\sqrt[4]{d}) - (-5c\sqrt[4]{d} + 3a\sqrt[5]{b^4}) = a\sqrt[5]{b^4} -$
$- 2c\sqrt[4]{d} + 5c\sqrt[4]{d} - 3a\sqrt[5]{b^4} = 3c\sqrt[4]{d} - 2a\sqrt[5]{b^4}$.

105. $\sqrt{2} + 3\sqrt{32} + \frac{1}{2}\sqrt{128} - 6\sqrt{18} = \sqrt{2} + 12\sqrt{2} +$
$+ 4\sqrt{2} - 18\sqrt{2} = -\sqrt{2}$.

106. $20\sqrt{245} - \sqrt{5} + \sqrt{125} - 2\frac{1}{2}\sqrt{180} = 140\sqrt{5} -$
$- \sqrt{5} + 5\sqrt{5} - 15\sqrt{5} = 129\sqrt{5}$.

107. $\frac{1}{2}\sqrt[3]{5} - 2\frac{1}{4}\sqrt[3]{40} + 10\sqrt[3]{135} - \sqrt[3]{320} = \frac{1}{2}\sqrt[3]{5} -$
$- \frac{9}{2}\sqrt[3]{5} + 30\sqrt[3]{5} - 4\sqrt[3]{5} = 22\sqrt[3]{5}$.

108. $\sqrt{\frac{45}{4}} - \sqrt{20} - 5\sqrt{\frac{1}{18}} - \frac{1}{6}\sqrt{245} - \sqrt{\frac{49}{2}} = \frac{3}{2}\sqrt{5} -$
$- 2\sqrt{5} - \frac{5}{6}\sqrt{2} - \frac{7}{6}\sqrt{5} - \frac{7}{2}\sqrt{2} = -1\frac{2}{3}\sqrt{5} - 4\frac{1}{3}\sqrt{2}$.

109. $3\frac{1}{2}\sqrt{24} - \frac{\sqrt[3]{54}}{4} + 2\frac{\sqrt{99}}{3} - 1\frac{1}{2}\sqrt{44} + 3\sqrt[3]{2} = 7\sqrt{6} -$
$- \frac{3}{4}\sqrt[3]{2} + 2\sqrt{11} - 3\sqrt{11} + 3\sqrt[3]{2} = 7\sqrt{6} - \sqrt{11} + 2\frac{1}{4}\sqrt[3]{2}$.

110. $5\sqrt{8} - 8\sqrt{\frac{1}{3}} + \sqrt{4\frac{1}{2}} + 6\sqrt{\frac{5}{3} - \frac{13}{9}} + \frac{4\sqrt{3}}{3} = 10\sqrt{2} -$
$- \frac{8}{3}\sqrt{3} - \frac{3}{2}\sqrt{2} + 2\sqrt{2} + \frac{4}{3}\sqrt{3} = 10\frac{1}{2}\sqrt{2} - 1\frac{1}{3}\sqrt{3}$.

111. $\sqrt{a^3} + b\sqrt{a} - \sqrt{9a} = a\sqrt{a} + b\sqrt{a} - 3\sqrt{a} =$
$= (a + b - 3)\sqrt{a}$.

112. $\sqrt[3]{27a^4} - 3\sqrt[3]{8a} + \sqrt{125a^7} = 3a\sqrt[3]{a} - 6\sqrt[3]{a} +$
$+ 5a^2\sqrt[3]{a} = (3a - 6 + 5a^2)\sqrt[3]{a}$.

113. $3\sqrt{125a^3b^2} + b\sqrt{20a^3} - \sqrt{500a^3b^2} = 15ab\sqrt{5a} +$
$+ 2ab\sqrt{5a} - 10ab\sqrt{5a} = 7ab\sqrt{5a}$.

114. $\frac{1}{a^2c}\sqrt{3a^8c^4d} - \frac{2}{ac^2}\sqrt{12a^6c^6d} - a^4c^2\sqrt{\frac{3d}{a^4c^2}} =$
$= a^2c\sqrt{3d} - 4a^2c\sqrt{3d} - a^2c\sqrt{3d} = -4a^2c\sqrt{3d}$.

115. $5\sqrt[3]{x^2y^5} + 4y^2\sqrt[3]{\frac{x^2}{y}} + \frac{4y}{x^2}\sqrt[3]{-x^8y^2} - 6xy\sqrt[3]{\frac{y^2}{x}} -$

$$-\frac{3}{2}xy^2\sqrt[3]{-\frac{8}{xy}} = 5y\sqrt[3]{x^2y^2} + 4y\sqrt[3]{x^2y^2} - 4y\sqrt[3]{x^2y^2} -$$
$$-6y\sqrt[3]{x^2y^2} + 3y\sqrt[3]{x^2y^2} = 2y\sqrt[3]{x^2y^2}.$$

116. $\sqrt{m^3 - m^2n} - \sqrt{(m+n)(m^2-n^2)} - \sqrt{mn^2 - n^3} =$
$$= \sqrt{m^2(m-n)} - \sqrt{(m+n)^2(m-n)} - \sqrt{n^2(m-n)} =$$
$$= m\sqrt{m-n} - (m+n)\sqrt{m-n} - n\sqrt{m-n} =$$
$$= (m - m - n - n)\sqrt{m-n} = -2n\sqrt{m-n}.$$

117. $\sqrt{1-\frac{x}{2}} - 3\sqrt{4-2x} - \sqrt{16-8x} + 8\sqrt{\frac{1}{4}-\frac{x}{8}} =$
$$= \tfrac{1}{2}\sqrt{4-2x} - 3\sqrt{4-2x} - 2\sqrt{4-2x} + 2\sqrt{4-2x} =$$
$$= -2\tfrac{1}{2}\sqrt{4-2x}.$$

118. $(a^4 - 2b^4)\sqrt{\frac{a+b}{a-b}} - (a^2+b^2)\sqrt{(a+b)^3(a-b)} +$
$$+\frac{b^2}{a-b}\sqrt{a^2b^4-b^6} = \frac{a^4-2b^4}{a-b}\sqrt{a^2-b^2} - (a^2+b^2)(a+b)\sqrt{a^2-b^2} +$$
$$+\frac{b^4}{a-b}\sqrt{a^2-b^2} = \left[\frac{a^4-2b^4}{a-b} - (a^2+b^2)(a+b) + \frac{b^4}{a-b}\right]\sqrt{a^2-b^2} =$$
$$= \left[\frac{a^4-2b^4-(a^2+b^2)(a^2-b^2)+b^4}{a-b}\right]\cdot\sqrt{a^2-b^2} =$$
$$= (a^4 - 2b^4 - a^4 + b^4 + b^4)\sqrt{a^2-b^2} = 0\sqrt{a^2-b^2} = 0.$$

119. $\frac{x}{2}\sqrt[4]{(1+2x+x^2)(x+1)(x^2-1)} - \sqrt[4]{x^5(1-x^{-1})} +$
$$+\tfrac{1}{2}x^3\sqrt[4]{x^{-3}-x^{-4}} = \frac{x}{2}\sqrt[4]{(1+x)^2(x+1)(x^2-1)} -$$
$$-\sqrt[4]{x^5\frac{(x-1)}{x}} + \tfrac{1}{2}x^3\sqrt[4]{\frac{x-1}{x^4}} = \frac{x}{2}\sqrt[4]{(x+1)^4(x-1)} -$$
$$-\sqrt[4]{x^4\cdot x\cdot\frac{x-1}{x}} + \frac{x^3}{2}\sqrt[4]{\frac{x-1}{x^4}} = \frac{x(x+1)}{2}\sqrt[4]{x-1} - x\sqrt[4]{x-1} +$$
$$+\frac{x^2}{2}\sqrt[4]{x-1} = \left[\frac{x(x+1)}{2} - x + \frac{x^2}{2}\right]\sqrt[4]{x-1} = \frac{x}{2}(2x-1)\sqrt[4]{x-1}.$$

120. $\sqrt[3]{8x^9 - 8x^6y^3} + x\sqrt[3]{x^3y^3 - x^6} + \sqrt[3]{1 - x^3y^{-3}} +$
$$+\frac{x^2}{y^2}\sqrt[3]{x^{-3}y^3 - x^{-6}y^6} = \sqrt[3]{8x^6(x^3-y^3)} + x\sqrt[3]{x^3(y^3-x^3} +$$

$$+\sqrt[3]{\frac{y^3-x^3}{y^3}}+\frac{x^2}{y^2}\sqrt[3]{\frac{y^3}{x^6}(x^3-y^3)}=2x^2\sqrt[3]{x^3-y^3}-x^2\sqrt[3]{x^3-y^3}-$$
$$-\frac{1}{y}\sqrt[3]{x^3-y^3}+\frac{1}{y}\sqrt[3]{x^3-y^3}=\left(2x^2-x^2-\frac{1}{y}+\frac{1}{y}\right)\sqrt[3]{x^3-y^3}=$$
$$=x^2\sqrt[3]{x^3-y^3}.$$

§ 6. Juurte korrutamine ja jagamine.

121. $\sqrt{3}.\sqrt{27}=\sqrt{81}=9.$ **122.** $\sqrt[3]{2}.\sqrt[3]{16}=\sqrt[3]{32}=$ $=2\sqrt[4]{4}.$ **123.** $3\sqrt[3]{18}.\frac{5}{6}\sqrt[3]{-6}=-\frac{5}{2}\sqrt[3]{108}=-\frac{15}{2}\sqrt[3]{4}.$

124. $\frac{1}{3}\sqrt[4]{27}.\frac{1}{9}\sqrt[4]{243}=\frac{1}{27}\sqrt[4]{3^3.3^5}=\frac{1}{3}.$ **125.** $\sqrt[3]{-108}.$ $.\sqrt[3]{50}.\sqrt[3]{40}=-\sqrt[3]{108.50.40}=-\sqrt[3]{27.8.1000}=$ $=-\sqrt[3]{3^3.2^3.10^3}=-60.$ **126.** $2\sqrt[4]{32}.\sqrt[4]{216}.3\sqrt[4]{60}=$ $=6\sqrt[4]{32.216.60}=6\sqrt[4]{2^5.6^3.6.2.5}=6\sqrt[4]{2^4.6^4.20}=$ $=72\sqrt[4]{20}.$ **127.** $(4\sqrt{8}+\frac{1}{12}\sqrt{12}-\frac{1}{4}\sqrt{32}).8\sqrt{32}=$ $32\sqrt{32.8}+\frac{2}{3}\sqrt{32.12}-2\sqrt{32.32}=512+\frac{16}{3}\sqrt{6}-64=$ $=448+5\frac{1}{3}\sqrt{6}.$

128. $(\sqrt[3]{9}-7\sqrt[3]{72}+6\sqrt[3]{1125}).4\sqrt[3]{\frac{1}{9}}=4\sqrt[3]{\frac{9}{9}}-28\sqrt[3]{\frac{72}{9}}+$ $+24\sqrt[3]{\frac{1125}{9}}=4-56+120=68.$

129. $(3\sqrt{\frac{5}{6}}-5\sqrt{30}-2\sqrt{\frac{15}{2}}).2\sqrt{\frac{3}{2}}=6\sqrt{\frac{5}{6}.\frac{3}{2}}-$ $-10\sqrt{30.\frac{3}{2}}-4\sqrt{\frac{15}{2}.\frac{3}{2}}=6\sqrt{\frac{5}{4}}-10\sqrt{45}-4\sqrt{\frac{45}{4}}=3\sqrt{5}-$ $-30\sqrt{5}-6\sqrt{5}=-33\sqrt{5}.$

130. $(2\sqrt{6}-3\sqrt{5}).(\sqrt{3}+2\sqrt{2})=2\sqrt{6}.\sqrt{3}+2\sqrt{6}.$ $.2\sqrt{2}-3\sqrt{5}.\sqrt{3}-3\sqrt{5}.2\sqrt{2}=2\sqrt{18}+4\sqrt{12}-$ $-3\sqrt{15}-6\sqrt{10}=6\sqrt{2}+8\sqrt{3}-3\sqrt{15}-6\sqrt{10}.$

131. $(\sqrt[3]{16}-2\sqrt[3]{2}+4\sqrt[3]{54}).(5.\sqrt[3]{4}-3\sqrt[3]{\frac{1}{2}})=$ $=(2\sqrt[3]{2}-2\sqrt[3]{2}+12\sqrt[3]{12}).(5\sqrt[3]{4}-\frac{3}{2}\sqrt[3]{4})=12\sqrt[3]{2}.$ $.\frac{7}{2}\sqrt[3]{4}=42\sqrt[3]{8}=84.$

132. $(3\sqrt{\frac{2}{3}} - \sqrt{12} - \sqrt{6}) \cdot (2\sqrt{\frac{2}{3}} - 8\sqrt{\frac{3}{8}} + 3\sqrt{\frac{3}{2}}) =$
$= (\sqrt{6} - \sqrt{12} - \sqrt{6})\ (\frac{2}{3}\sqrt{6} - 2\sqrt{6} + \frac{3}{2}\sqrt{6}) = -\sqrt{12}\ \cdot$
$\cdot\ \frac{1}{6}\sqrt{6} = -\frac{1}{6}\sqrt{72} = -\sqrt{2}.$

133. $\sqrt{a^3b} \cdot \sqrt{a^5b^2} = \sqrt{a^8b^3} = a^4b\sqrt{b}.$

134. $a^2\sqrt[3]{2x} \cdot \frac{1}{a}\sqrt[3]{4x} = a\sqrt[3]{8x^2} = a\sqrt[3]{8x^2} = 2a\sqrt[3]{x^2}.$

135. $2\sqrt[3]{25a^5} \,.\, 3\sqrt[3]{15a^4} = 6\sqrt[3]{25\,.\,15a^9} = 30a^3\sqrt[3]{3}.$

136. $3\sqrt{\frac{5a}{b^2}} \cdot 2\sqrt{\frac{4b^4}{5a^3}} = 6\sqrt{\frac{5a\ \ 4b^4}{5a^3b^2}} = 6\sqrt{\frac{4b^2}{a^2}} = \frac{12b}{a}.$

137. $\frac{x}{a}\sqrt[3]{\frac{a^2}{x}} \cdot \frac{1}{4}a\sqrt[3]{\frac{8a}{x^4}} = \frac{x}{4}\sqrt[3]{\frac{8a\,.\,a^2}{x^4\,.\,x}} = \frac{x}{4} \cdot \frac{2a}{x^2}\sqrt[3]{x} = \frac{a}{2x}\sqrt[3]{x}.$

138. $\frac{12a^3}{5x^2}\sqrt[4]{\frac{a^7x}{32}} \cdot \frac{10x^3}{3a^2}\sqrt[4]{\frac{4}{a^3x}} = 8ax\sqrt[4]{\frac{4a^7x}{32a^3x}} = 8ax\sqrt[4]{\frac{2a^4}{16}} =$
$= 4a^2x\sqrt[4]{2}.$

139. $a^{-3}b\sqrt[4]{a^3b^2} \,.\, 2a^2\sqrt[4]{a^{-5}b^3} \,.\, \frac{1}{2}ab^{-2}\sqrt[4]{a^{10}b^7} =$
$= \frac{a^{-3}b\,.\,2a^2\,.\,ab^{-2}}{2} \,.\, \sqrt[4]{a^3b^2\,.\,a^{-5}b^3\,.\,a^{10}b^7} = \frac{1}{b}\sqrt[4]{a^8b^{12}} = a^2b^2.$

140. $\sqrt[3]{\frac{3a^{-2}b^5}{5a^4b^{-5}}} \cdot \sqrt[3]{\left(\frac{6a^{-2}}{5b^3}\right)^{-2}} \cdot \sqrt[3]{-60a^5b^2} =$
$= \sqrt[3]{\frac{3a^{-2}b^5\,.\,6^{-2}a^4\,.\,-60a^5b^2}{5a^4b^{-2}\,.\,5^{-2}\,.\,b^{-6}}} = \sqrt[3]{-\frac{3\,.\,60\,.\,25a^7 \cdot b^5\,.\,b^8\,.\,b^2}{5\,.\,36\,.\,a^4}} =$
$= -ab^5\sqrt[3]{25}.$

141. $\left(\sqrt{a} + \sqrt{ab} - \sqrt{\frac{a}{b}}\right) \cdot \sqrt{\frac{a}{b}} = \sqrt{a\,.\,\frac{a}{b}} +$
$+ \sqrt{ab\,.\,\frac{a}{b}} - \sqrt{\frac{a}{b} \cdot \frac{a}{b}} = \frac{a}{b}\sqrt{b} + a - \frac{a}{b} = a\left(\frac{\sqrt{b}}{b} - \frac{1}{b} + 1\right).$

142. $\left(a\sqrt[6]{\frac{a^4}{x^5}} + x\sqrt[6]{a^5x} - \sqrt[6]{\frac{x^4}{a^3}}\right) \cdot \sqrt[6]{\frac{x^3}{a^2}} = a\sqrt[6]{\frac{a^4}{x^5} \cdot \frac{x^3}{a^2}} +$
$+ x\sqrt[6]{\frac{a^5x\,.\,x^3}{a^2}} - \sqrt[6]{\frac{x^4}{a^3} \cdot \frac{x^2}{a^2}} = \frac{a}{x}\sqrt[6]{a^2x^4} + x\sqrt[6]{a^3x^4} - \frac{x}{a}\sqrt[6]{ax}.$

143. $\left(\sqrt{a} + \sqrt{\frac{b}{a}}\right)\left(\sqrt{ab} - \sqrt{\frac{a}{b}}\right) = \sqrt{a^2b} - \sqrt{\frac{a^2}{b}} +$
$+ \sqrt{\frac{ab^2}{a}} - \sqrt{\frac{b}{a} \cdot \frac{a}{b}} = a\sqrt{b} - \frac{a}{b}\sqrt{b} + b - 1.$

144. $(\sqrt[3]{a^2b}+\sqrt[3]{ab^2})(\sqrt[3]{a}-\sqrt[3]{b})=\sqrt[3]{a^3b}+\sqrt[3]{a^2b^2}-$
$-\sqrt[3]{a^2b^2}-\sqrt[3]{ab^3}=a\sqrt[3]{b}-b\sqrt[3]{a}.$

145. $\sqrt{3}\cdot\sqrt[3]{2}-\sqrt[6]{27}\cdot\sqrt[6]{4}=\sqrt[6]{108}.$

146. $\sqrt[5]{\frac{3}{8}}\cdot\sqrt[3]{\frac{2}{3}}=\sqrt[15]{\frac{3^3}{2^9}}\cdot\sqrt[15]{\frac{2^5}{3^5}}=\sqrt[15]{\frac{3^3\cdot 2^5}{2^9\cdot 3^5}}=\sqrt[15]{\frac{1}{2^4\cdot 3^2}}=$
$=\sqrt[15]{\frac{1}{144}}.$

147. $\sqrt[6]{54}\cdot\sqrt{6}\cdot\sqrt[3]{2}=\sqrt[6]{54}\cdot\sqrt[6]{6^3}\cdot\sqrt[6]{2^2}=\sqrt[6]{54\cdot 6^3\cdot 2^2}=$
$=\sqrt[6]{3^3\cdot 6^3\cdot 2^3}=\sqrt[6]{6^6}=6.$

148. $\sqrt[9]{\frac{9}{4}}\cdot\sqrt[4]{\frac{2}{3}}\cdot\sqrt[2]{2}\cdot\sqrt[12]{3}=\sqrt[36]{\frac{3^8}{2^8}}\cdot\sqrt[36]{\frac{2^9}{3^9}}\cdot\sqrt[36]{2^6}\cdot$
$\cdot\sqrt[36]{3^3}=\sqrt[36]{\frac{3^8\cdot 2^9\cdot 2^6\cdot 3^3}{2^8\cdot 3^9}}=\sqrt[36]{3^2\cdot 2^7}=\sqrt[36]{1152}.$

149. $(3\sqrt{10}-2\sqrt[3]{4}+6\sqrt[6]{25})\cdot\sqrt[4]{2}=3\sqrt{10}\cdot\sqrt[4]{2}-$
$-2\sqrt[3]{4}\cdot\sqrt[4]{2}+6\sqrt[6]{25}\cdot\sqrt[4]{2}=3\sqrt[4]{100}\cdot\sqrt[4]{2}-2\sqrt[12]{4^4}\cdot\sqrt[12]{2^3}+$
$+6\sqrt[12]{25^2}\cdot\sqrt[12]{2^3}=3\sqrt[4]{200}-2\sqrt[12]{256\cdot 8}+6\sqrt[12]{625\cdot 8}=$
$=3\sqrt[4]{200}-2\sqrt[12]{2048}+6\sqrt[12]{5000}.$

150. $(2\sqrt[7]{10}+3\sqrt{2}-4\sqrt[3]{5})\cdot\sqrt[4]{10}=2\sqrt[7]{10}\cdot\sqrt[4]{10}+$
$+3\sqrt{2}\cdot\sqrt[4]{10}-4\sqrt[3]{5}\cdot\sqrt[4]{10}=2\sqrt[28]{10^4\cdot 10^7}+3\sqrt[4]{2^2\cdot 10}-$
$-4\sqrt[12]{5^4\cdot 10^3}=2\sqrt[28]{100000000000}+3\sqrt[4]{40}-4\sqrt[12]{625000}.$

151. $(3\sqrt{2}+4\sqrt[3]{3})\cdot(\sqrt{2}-2\sqrt[3]{3})=3\cdot\sqrt{2}\cdot\sqrt{2}-$
$-6\sqrt{2}\cdot\sqrt[3]{3}+4\sqrt[3]{3}\cdot\sqrt[2]{2}-8\sqrt[3]{3}\cdot\sqrt[3]{3}=6-6\sqrt[6]{2^3 3^2}+$
$+4\sqrt[6]{2^3 3^2}-8\sqrt[3]{3^2}=6-2\sqrt[6]{72}-8\sqrt[3]{9}.$

152. $(6\sqrt[3]{2}-\sqrt[6]{32})(\tfrac{3}{2}\sqrt[3]{2}-2\sqrt[6]{\tfrac{1}{2}})=9\sqrt[3]{2}\cdot\sqrt[3]{2}-$
$-12\sqrt[3]{2}\cdot\sqrt[6]{\tfrac{1}{2}}-\tfrac{3}{2}\sqrt[6]{32}\cdot\sqrt[3]{2}+2\sqrt[6]{32}\cdot\sqrt[6]{\tfrac{1}{2}}=9\sqrt[3]{4}-$
$-12\sqrt[6]{2}-\tfrac{3}{2}\sqrt[6]{128}+2\sqrt[6]{16}=9\sqrt[6]{16}-12\sqrt[6]{2}-3\sqrt[6]{2}+$
$+2\sqrt[6]{16}=11\sqrt[6]{16}-15\sqrt[6]{2}=11\sqrt[3]{4}-15\sqrt[6]{2}.$

153. $\sqrt[4]{a^3b} \cdot \sqrt[6]{ab^4} = \sqrt[12]{a^9b^3} \cdot \sqrt[12]{a^2b^8} = \sqrt[12]{a^{11}b^{11}}$.

154. $3a^2b\sqrt{3bc} \cdot 5ab\sqrt[3]{2a^2c} = 15a^3b^2\sqrt[6]{3^3b^3c^3} \cdot$
$\cdot \sqrt[6]{2^2a^4c^2} = 15a^3b^2\sqrt[6]{108a^4b^3c^5}$.

155. $a^2\sqrt[4]{a^5b^2} \cdot b\sqrt[3]{\frac{a^5}{b}} \cdot \sqrt[4]{a^6b^7} \cdot ab\sqrt[3]{a^4b^7} = a^3b^2\sqrt[12]{a^{15}b^6} \cdot$
$\cdot \sqrt[12]{\frac{a^{20}}{b^4}} \cdot \sqrt[12]{a^{18}b^{21}} \cdot \sqrt[12]{a^{16}b^{28}} = a^3b^2 \cdot \sqrt[12]{a^{15}b^6 \cdot \frac{a^{20}}{b^4} \cdot a^{18}b^{21} \cdot a^{16}b^{28}} =$
$= a^3b^2\sqrt[12]{a^{69}b^{51}} = a^8b^6\sqrt[12]{a^9b^3} = a^8b^6\sqrt[4]{a^3b}$.

156. $2a\sqrt[4]{a^5b^2} \cdot \sqrt[6]{\frac{a^5}{b^3}} \cdot 3b\sqrt[3]{\frac{b^4}{a^2}} \cdot \sqrt{\frac{1}{ab}} = 6ab\sqrt[12]{a^{15}b^6} \cdot$
$\cdot \sqrt[12]{\frac{a^{10}}{b^6}} \cdot \sqrt[12]{\frac{b^{16}}{a^8}} \cdot \sqrt[12]{\frac{1}{a^6b^6}} = 6ab\sqrt[12]{a^{15}b^6 \cdot \frac{a^{10}}{b^6} \cdot \frac{b^{16}}{a^8} \cdot \frac{1}{a^6b^6}} =$
$= 6ab\sqrt[12]{a^{11}b^{10}}$.

157. $(\sqrt[3]{a^2} - 2\sqrt[4]{b^2} - a\sqrt[6]{b^5}) \cdot a^2\sqrt{ab} = \sqrt[3]{a^2} \cdot a^2\sqrt{ab} -$
$- 2\sqrt[4]{b^2} \cdot a^2\sqrt{ab} - a\sqrt[6]{b^5} \cdot a^2\sqrt{ab} = a^2\sqrt[6]{a^4} \cdot \sqrt[6]{a^3b^3} -$
$- 2a^2\sqrt[4]{b^2} \cdot \sqrt[4]{a^2b^2} - a^3\sqrt[6]{b^5} \cdot \sqrt[6]{a^3b^3} = a^2\sqrt[6]{a^7b^3} - 2a^2\sqrt[4]{a^2b^4} -$
$- a^3\sqrt[6]{a^3b^8} = a^3\sqrt[6]{ab^3} - 2a^2b\sqrt[4]{a^2} - a^3b\sqrt[6]{a^3b^2} = a^3\sqrt[6]{ab^3} -$
$- 2a^2b\sqrt{a} - a^3b\sqrt[6]{a^3b^2}$.

158. $(\sqrt[5]{a^2} - \sqrt[3]{a^4} + a\sqrt{a^3}) \cdot -2a\sqrt[3]{a^2} = -2a\sqrt[5]{a^2} \cdot$
$\cdot \sqrt[3]{a^2} + 2a\sqrt[3]{a^4} \cdot \sqrt[3]{a^2} - 2a^2\sqrt{a^3} \cdot \sqrt[3]{a^2} = -2a\sqrt[15]{a^6} \cdot \sqrt[15]{a^{10}} +$
$+ 2a\sqrt[3]{a^6} - 2a^2\sqrt[6]{a^9} \cdot \sqrt[6]{a^4} = -2a\sqrt[15]{a^{16}} + 2a^3 - 2a^2\sqrt[6]{a^{13}} =$
$= -2a^2\sqrt[15]{a} + 2a^3 - 2a^4\sqrt[6]{a}$.

159. $\left(a\sqrt[3]{b} - 2b\sqrt[6]{\frac{1}{b}}\right) \cdot (a\sqrt[6]{b} + \sqrt[3]{b^2}) = a\sqrt[3]{b} \cdot$
$\cdot a\sqrt[6]{b} + a\sqrt[3]{b} \cdot \sqrt[3]{b^2} - 2b\sqrt[6]{\frac{1}{b}} \cdot a\sqrt[6]{b} - 2b\sqrt[6]{\frac{1}{b}} \cdot \sqrt[3]{b^2} =$
$= a^2\sqrt[6]{b^3} + a\sqrt[3]{b^3} - 2ab\sqrt[6]{\frac{1}{b} \cdot b} - 2b\sqrt[6]{\frac{1}{b} \cdot b^4} = a^2\sqrt{b} +$
$+ ab - 2ab - 2b\sqrt{b} = (a^2 - 2b)\sqrt{b} - ab$.

160. $(\sqrt{a} + \sqrt[3]{a^2} + \sqrt[4]{a^3}) \cdot (\sqrt{a} - \sqrt[12]{a^5}) = \sqrt{a} \cdot \sqrt{a} +$
$+ \sqrt[3]{a^2} \cdot \sqrt{a} + \sqrt[4]{a^3} \cdot \sqrt{a} - \sqrt{a} \cdot \sqrt[12]{a^5} - \sqrt[3]{a^2} \cdot \sqrt[12]{a^5} - \sqrt[4]{a^3} \cdot$

$\cdot \sqrt[12]{a^5} = a + \sqrt[6]{a^4} \cdot \sqrt[6]{a^3} + \sqrt[4]{a^3} \cdot \sqrt[4]{a^2} - \sqrt[12]{a^6} \cdot \sqrt[12]{a^5} - \sqrt[12]{a^8} \cdot$
$\cdot \sqrt[12]{a^5} - \sqrt[12]{a^9} \cdot \sqrt[12]{a^5} = a + \sqrt[6]{a^7} + \sqrt[4]{5} - \sqrt[12]{a^{11}} - \sqrt[12]{a^{13}} -$
$- \sqrt[12]{a^{14}} = a + a\sqrt[6]{a} + a\sqrt[4]{a} - \sqrt[12]{a^{11}} - a\sqrt[12]{a} - a\sqrt[6]{a} =$
$= a + a\sqrt[4]{a} - \sqrt[12]{a^{11}} - a\sqrt[12]{a}.$

161. $\sqrt{28} : \sqrt{7} = \sqrt{28 : 7} = \sqrt{4} = 2.$

162. $\dfrac{\sqrt[3]{81}}{\sqrt[3]{3}} = \sqrt[3]{\dfrac{81}{3}} = \sqrt[3]{27} = 3.$

163. $\sqrt{\dfrac{12}{35}} : \sqrt{\dfrac{7}{5}} = \sqrt{\dfrac{12}{35} : \dfrac{7}{5}} = \sqrt{\dfrac{12}{49}} = \frac{2}{7}\sqrt{3}.$

164. $\frac{3}{2}\sqrt[3]{96} : 3\sqrt[3]{\frac{3}{4}} = \frac{1}{2}\sqrt[3]{96 : \frac{3}{4}} = \frac{1}{2}\sqrt[3]{128} = \frac{1}{2}\sqrt[3]{2^7} =$
$= 2\sqrt[3]{2}.$

165. $(5\sqrt[3]{5} - 6\sqrt[3]{10} + 15\sqrt[3]{16}) : 3\sqrt[3]{\frac{1}{2}} = \frac{5}{3}\sqrt[3]{4 : \frac{1}{2}} -$
$- 2\sqrt[3]{10 : \frac{1}{2}} + 5\sqrt[3]{16 : \frac{1}{2}} = \frac{5}{2}\sqrt[3]{8} - 2\sqrt[3]{20} + 5\sqrt[3]{32} =$
$= \frac{10}{3} - 2\sqrt[3]{20} + 10\sqrt[3]{4}.$

166. $(\frac{2}{3}\sqrt[3]{90} + 3\sqrt[3]{10} - \sqrt[3]{\frac{5}{6}}) : -2\sqrt[3]{\frac{5}{3}} = -\frac{1}{3}\sqrt[3]{90 : \frac{5}{3}} -$
$- \frac{3}{2}\sqrt[3]{10 : \frac{5}{3}} + \frac{1}{2}\sqrt[3]{\frac{5}{6} : \frac{5}{3}} = -\frac{1}{3}\sqrt[3]{54} - 1\frac{1}{2}\sqrt[3]{6} + \frac{1}{2}\sqrt[3]{\frac{1}{2}} =$
$= -\sqrt[3]{2} - 1\frac{1}{2}\sqrt[3]{6} + \frac{1}{4}\sqrt[3]{4}.$

167. $\sqrt{5a} : \sqrt{a} = \sqrt{\dfrac{5a}{a}} = \sqrt{5}.$

168. $\sqrt[3]{4a^8} : \sqrt[3]{2a^2} = \sqrt[3]{\dfrac{4a^8}{2a^2}} = \sqrt[3]{2a^6} = a^2\sqrt[3]{2}.$

169. $\sqrt[3]{27a^3} : \sqrt[4]{\dfrac{a^2}{3}} = \sqrt[4]{27a^3 : \dfrac{a^2}{3}} = \sqrt[4]{81a} = 3\sqrt[4]{a}.$

170. $\sqrt[4]{\dfrac{8a^5}{3b}} : \sqrt[4]{\dfrac{6a}{b^3}} = \sqrt[4]{\dfrac{8a^5}{3b} : \dfrac{6a}{b^3}} = \sqrt[4]{\dfrac{4a^4b^2}{9}} =$
$= \frac{a}{3}\sqrt[4]{36b^2} = \frac{a}{3}\sqrt{6b}.$

171. $(ab^2\sqrt{x} - x\sqrt{a}) : \sqrt{bx} = ab^2\sqrt{x : bx} - x\sqrt{b : bx} =$
$= ab^2\sqrt{\dfrac{1}{b}} - x\sqrt{\dfrac{1}{x}} = ab^2\sqrt{\dfrac{b}{b^2}} - x\sqrt{\dfrac{x}{x^2}} = ab\sqrt{b} - \sqrt{x}.$

172. $(\sqrt[4]{a^3x^3} - x\sqrt[4]{a^3} - 4a\sqrt[4]{ax^2}) : \sqrt[4]{ax^3} = \sqrt[4]{\frac{a^3x^3}{ax^3}} - x\sqrt[4]{\frac{a^3}{ax^3}} - 4a\sqrt{\frac{a}{ax^3}} = \sqrt[4]{a^2} - x\sqrt[4]{\frac{a^2}{x^3}} - 4a\sqrt[4]{\frac{1}{x}} = \sqrt{a} - \sqrt[4]{a^2x} - \frac{4a}{x}\sqrt[4]{x^3}.$

173. $\left(2\sqrt[4]{x^3y} - 3\sqrt[4]{\frac{xy^3}{2}} + \sqrt[4]{\frac{1}{x}}\right) : \frac{1}{xy}\sqrt[4]{x^3y^2} = 2xy\sqrt[4]{\frac{x^3y}{x^3y^2}} - 3xy\sqrt[4]{\frac{xy^3}{2x^3y^2}} + xy\sqrt[4]{\frac{1}{x^4y^2}} = 2xy\sqrt[4]{\frac{1}{y}} - 3xy\sqrt[4]{\frac{y}{2x^2}} + y\sqrt[4]{\frac{1}{y^2}} = 2x\sqrt[3]{y^3} - \frac{3y}{2}\sqrt[4]{8x^2y} + \sqrt[4]{y^2} = 2x\sqrt[4]{y^3} - 1\tfrac{1}{2}y\sqrt[4]{8x^2} + \sqrt{y}.$

174. $\left(\frac{4x}{25}\sqrt[5]{\frac{x^2}{y}} + \frac{3x}{50y}\sqrt[5]{\frac{x^3}{y^4}} - \frac{x}{y}\sqrt[5]{x^4}\right) : \frac{4x}{5y}\sqrt[5]{\frac{x^6}{y}} = \frac{y}{5}\sqrt[5]{\frac{x^2}{y}:\frac{x^6}{y}} + \frac{3}{40}\sqrt[5]{\frac{x^3}{y^4}:\frac{x^6}{y}} - \frac{5}{4}\sqrt[5]{x^4:\frac{x^6}{y}} = \frac{y}{5}\sqrt[5]{\frac{1}{x^4}} + \frac{3}{40}\sqrt[5]{\frac{1}{x^3y^3}} - \frac{5}{4}\sqrt[5]{\frac{y}{x^2}} = \frac{5x}{y}\sqrt[5]{x} + \frac{3}{40xy}\sqrt[3]{x^2y^2} - \frac{5}{4x}\sqrt[5]{x^3y}.$

175. $\frac{(\sqrt[3]{a^2} - \sqrt[3]{b^2})}{\sqrt[3]{a} + \sqrt[3]{b}} = \frac{(\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{a} - \sqrt[3]{b})}{\sqrt[3]{a} + \sqrt[3]{b}} = \sqrt[3]{a} - \sqrt[3]{b}.$

Ülesannet võib ka nii lahendada:

$$\begin{array}{l|l}
\sqrt[3]{a^2} \qquad\qquad - \sqrt[3]{b^2} & \sqrt[3]{a} + \sqrt[3]{b} \\ \hline
\underline{\mp \sqrt[3]{a^2} \mp \sqrt[3]{ab}} & \sqrt[3]{a} + \sqrt[3]{b} \\
\qquad - \sqrt[3]{ab} - \sqrt[3]{b^2} & \\
\qquad \underline{\pm \sqrt[3]{ab} \pm \sqrt[3]{b^2}} & \\
\qquad\qquad 0 &
\end{array}$$

176.
$$\begin{array}{l|l}
\sqrt[3]{a^2b} - 2\sqrt[3]{2ab^2} + b\sqrt[3]{4} & \sqrt[3]{a} - \sqrt[3]{2b} \\ \hline
\underline{\mp \sqrt[3]{a^2b} \pm \sqrt[3]{2ab^2}} & \sqrt[3]{ab} - \sqrt[3]{2b^2} \\
\qquad - \sqrt[3]{2ab^2} + b\sqrt[3]{4} & \\
\qquad \underline{\pm \sqrt[3]{2ab^2} \pm b\sqrt[3]{4}} & \\
\qquad\qquad 0 &
\end{array}$$

177.

$$\sqrt[4]{8a^3} - b\sqrt[4]{27b^2} \;\Big|\; \sqrt[4]{2a} - \sqrt[4]{3b^2}$$

$$\sqrt{2a} + \sqrt[4]{6ab^2} + b\sqrt{3}$$

$$\begin{aligned}
&\mp\sqrt[4]{8a^3} \pm \sqrt[4]{12a^2b^2}\\
&\sqrt[4]{12a^2b^2}\\
&\mp\sqrt[4]{12a^2b^2} \pm b\sqrt[4]{18a}\\
&b\sqrt[4]{18a} - b\sqrt[4]{27b^2}\\
&\mp b\sqrt[4]{18a} \pm b\sqrt[4]{27b^2}\\
&0
\end{aligned}$$

178.

$$a^2\sqrt[4]{a} + b^2\sqrt[4]{8b} \;\Big|\; \sqrt[4]{a^3} + \sqrt[4]{2b^3} =$$

$$= \sqrt[4]{a^9} + \sqrt[4]{8b^9} \;\Big|\; \sqrt[4]{a^3} + \sqrt[4]{2b^3}$$

$$\sqrt[4]{a^6} - \sqrt[4]{2a^3b^3} + \sqrt[4]{4b^6} =$$

$$= a\sqrt{a} - \sqrt[4]{2a^3b^3} + b\sqrt{2b}.$$

$$\begin{aligned}
&\mp\sqrt[4]{a^9} \mp \sqrt[4]{2a^6b^3}\\
&-\sqrt[4]{2a^6b^3} +\\
&\mp\sqrt[4]{2a^6b^3} \pm \sqrt[4]{4a^3b^6}\\
&\sqrt[4]{4a^3b^6} + \sqrt[4]{8b^9}\\
&\mp\sqrt[4]{4a^3b^6} \mp \sqrt[4]{8b^9}\\
&0
\end{aligned}$$

179.

$$x^2\sqrt[3]{x^2} + xy\sqrt[3]{xy} + y^2\sqrt[3]{y^2} \;\Big|\; x\sqrt[3]{x} + \sqrt[3]{x^2y^2} + y\sqrt[3]{y}$$

$$x\sqrt[3]{x} - \sqrt[3]{x^2y^2} + y\sqrt[3]{y}$$

$$\begin{aligned}
&\pm x^2\sqrt[3]{x^2} \mp x^2\sqrt[3]{y^2} \mp xy\sqrt[3]{xy}\\
&-x^2\sqrt[3]{y^2} + y^2\sqrt[3]{y^2}\\
&\pm x^2\sqrt[3]{y^2} \pm xy\sqrt[3]{xy} \pm y^2\sqrt[3]{x^2}\\
&xy\sqrt[3]{xy} + y^2\sqrt[3]{x^2} + y^2\sqrt[3]{y^2}\\
&\pm xy\sqrt[3]{xy} \mp y^2\sqrt[3]{x^2} \mp y^2\sqrt[3]{y^2}\\
&0
\end{aligned}$$

180. $\frac{1}{y}\sqrt[5]{\frac{x^2}{y^3}} - xy\sqrt[5]{4xy} + 2y\sqrt[5]{\tfrac{1}{2}x^2y^2} - \tfrac{1}{2}\sqrt[5]{\frac{2y^8}{x^2}} \;\Big|\; \sqrt[5]{\frac{x}{y^4}} + \sqrt[5]{2x^3y^3} - \sqrt[5]{\frac{y^4}{4a}} =$

$$= \sqrt[5]{\frac{x^2}{y^8}} - \sqrt[5]{4x^6y^6} + 2\sqrt[5]{\tfrac{1}{2}x^2y^7} - \tfrac{1}{2}\sqrt[5]{\frac{2y^8}{x^2}} \;\Big|\; \sqrt[5]{\frac{x}{y^4}} + \sqrt[5]{2x^3y^3} - \sqrt[5]{\frac{y^4}{4x}}$$

$$\mp\sqrt[5]{\frac{x^2}{y^8}} \mp \sqrt[5]{\frac{2x^4}{y}} \pm \sqrt[5]{\frac{1}{4}} \qquad\qquad \sqrt[5]{\frac{x}{y^4}} - \sqrt[5]{2x^3y^3} + \sqrt[5]{\frac{y^4}{4x}}$$

$$-\sqrt[5]{\frac{2x^4}{y}} + \sqrt[5]{\frac{1}{4}} - \sqrt[5]{4x^6y^6} + 2\sqrt[5]{\tfrac{1}{2}x^2y^7} - \tfrac{1}{2}\sqrt[5]{\frac{2y^8}{x^2}}$$

$$\pm\sqrt[5]{\frac{2x^4}{y}} \pm \sqrt[5]{4x^6y^6} \mp \sqrt[5]{\tfrac{1}{2}x^2y^7}$$

$$\sqrt[5]{\frac{1}{4}} + \sqrt[5]{\tfrac{1}{2}x^2y^7} - \tfrac{1}{2}\sqrt[5]{\frac{2y^8}{x^2}}$$

$$\mp\sqrt[5]{\frac{1}{4}} \mp \sqrt[5]{\tfrac{1}{2}x^2y^7} - \tfrac{1}{2}\sqrt[5]{\frac{2y^8}{x^2}}$$

$$0$$

Jagu $\sqrt[5]{\frac{x}{y^4}} - \sqrt[5]{2x^3y^3} + \sqrt[5]{\frac{y^4}{4x}} = \frac{1}{y}\sqrt[5]{xy} - \sqrt[5]{2x^3y^3} + \frac{1}{2x}\sqrt[5]{8x^4y^4}.$

181. $\sqrt[3]{9} : \sqrt{3} = \sqrt[6]{81} : \sqrt[6]{27} = \sqrt[6]{\frac{81}{27}} = \sqrt[6]{3}.$

182. $\sqrt[5]{\frac{4}{5}} : 2\sqrt[5]{\frac{1}{400}} = \sqrt[5]{\frac{4}{5}} : \frac{2}{20} = \sqrt[5]{\frac{4}{5}} : \frac{1}{10} = 10\sqrt[5]{\frac{4}{5}} = 2\sqrt[5]{4 . 5^4} = 2\sqrt[5]{2500}.$

183. $(\sqrt[4]{6} - 2\sqrt{3} + \sqrt[3]{6}) : \frac{1}{2}\sqrt{6} = (\sqrt[4]{6} : \frac{1}{2}\sqrt{6}) - (2\sqrt{3} : \frac{1}{2}\sqrt{6}) + (\sqrt[3]{6} : \frac{1}{2}\sqrt{6}) = 2\sqrt[4]{6 : 6^2} - 4\sqrt{\frac{3}{6}} + 2\sqrt[6]{6^2 . 6^3} = 2\sqrt[4]{\frac{1}{6}} - 4\sqrt{\frac{1}{2}} + 2\sqrt[6]{\frac{1}{6}} = \frac{1}{3}\sqrt[4]{216} - 2\sqrt{2} + \frac{1}{3}\sqrt[6]{7776}.$

184. $(\sqrt{3} - 3\sqrt[3]{6} - \frac{1}{2}\sqrt[4]{12}) : \frac{3}{8}\sqrt[4]{3} = (\sqrt[12]{3^6} - 3\sqrt[12]{6^4} - \frac{1}{2}\sqrt[12]{12^3}) : \frac{3}{8}\sqrt[12]{3^3} = \frac{8}{3}\sqrt[12]{3^3} - 8\sqrt[12]{2^4} . 3 - \frac{4}{3}\sqrt[12]{4^3} = \frac{8}{3}\sqrt[4]{3} - 8\sqrt[12]{48} - \frac{4}{3}\sqrt{2}.$

185. $\sqrt{a} : \sqrt[3]{a^2} = \sqrt[6]{a^3} : \sqrt[6]{a^4} = \sqrt[6]{\frac{1}{a}} = \frac{1}{a}\sqrt[6]{a^5}.$

186. $\sqrt[3]{4a^2} : \sqrt[6]{2a^3} = \sqrt[6]{16a^4} : \sqrt[6]{2a^3} = \sqrt[6]{8a}.$

187. $\sqrt{6a^5} : \sqrt[6]{27a^{-9}} = \sqrt{6a^5} : \sqrt{3a^{-3}} = \sqrt{2a^8} = a^4\sqrt{2}.$

188. $10a\sqrt{a} : \sqrt[3]{a^2} = 10a\sqrt[6]{a^3} : \sqrt[6]{a^4} = 10a\sqrt[6]{\frac{1}{a}} = 10\sqrt[6]{a^5}.$

189. $6a^2\sqrt{3a^{-1}b} : 2a^3\sqrt[3]{2ab^{-1}} = 6a^2\sqrt[6]{27a^{-3}b^3} : 2a^3\sqrt[6]{4a^2b^{-2}} = \frac{3}{a}\sqrt[6]{\frac{27}{4}a^{-5}b^5} = \frac{3}{a}\sqrt[6]{\frac{27b^5}{4a^5}} = \frac{3}{2a^2}\sqrt[6]{432ab^5}.$

190. $5x^2y : \sqrt{25xy^4} = \sqrt[3]{125x^6y^3} : \sqrt[3]{25xy^4} = \sqrt[3]{5x^5y^{-1}} = x\sqrt[3]{\frac{5x^2}{y}} = \frac{x}{y}\sqrt[3]{5x^2y^2}.$

191. $\frac{24a^5b^2}{d^2}\sqrt[5]{\frac{a^2b^7}{c^2}} : \frac{4a^2}{b}\sqrt[3]{\frac{a^4b^7}{cd^5}} = \frac{24a^5b^2}{d^2}\sqrt[15]{\frac{a^6b^{21}}{c^6}} : \frac{4a^2}{b}\sqrt[15]{\frac{a^{20}b^{35}}{c^5d^{25}}} = \frac{6a^3b^3}{d^2}\sqrt[15]{\frac{d^{25}}{a^{14}b^{14}c}} = \frac{6a^2b^2}{d}\sqrt[15]{abc^{14}d^{10}}.$

192. $(a^2b + ax^2)\sqrt[3n]{\frac{x}{a^{n-1}c^3}} : ax\sqrt[2n]{\frac{x^4}{a^n c^2}} =$

$= a(ab + x)\sqrt[6n]{\frac{x^2}{a^{2n-2}c^6}} : ax\sqrt[6n]{\frac{x^{12}}{a^{3n}c^6}} = \frac{ab + x^2}{x}\sqrt[6n]{\frac{x^2}{a^{2n-2}c^6} : \frac{x^{12}}{a^{3n}c^6}} =$

$= \frac{ab + x^2}{x}\sqrt[6n]{\frac{a^{n+2}}{x^{10}}} = \frac{ab + x^2}{x}\sqrt[6n]{\frac{a^{n+2}x^{6n-10}}{x^{6n}}} = \frac{ab + x^2}{x^2}\sqrt[6n]{a^{n+2}x^{6n-10}}.$

193. $(x + y) : \frac{1}{3}\sqrt{x^2 - y^2} = \sqrt{(x + y)^2} : \frac{1}{3}\sqrt{x^2 - y^2} =$

$= 3\sqrt{\frac{(x + y)(x + y)}{(x + y)(x - y)}} = 3\sqrt{\frac{x + y}{x - y}} = \frac{3}{x - y}\sqrt{x^2 - y^2}.$

194. $(x^2 - y^2) : \frac{a}{x}\sqrt[3]{\frac{2a}{(x + y)^2}} = \sqrt[3]{(x + y)^3(x - y)^3} :$

$: \frac{a}{x}\sqrt[3]{\frac{2a}{(x + y)^2}} = \frac{x}{a}\sqrt[3]{\frac{(x + y)^5(x - y)^3}{2a}} =$

$= \frac{x(x + y)(x - y)}{2a^2}\sqrt[3]{4(x + y)^2a^2} = \frac{x(x^2 - y^2)}{2a^2}\sqrt[3]{4a^2(x + y)^2}.$

195. $\left(\sqrt[4]{8a^6b^9} - ab\sqrt[6]{8a^4b^5} + ab^2\sqrt[4]{2a^4b}\right) : \sqrt[4]{2b} =$

$= \left(\sqrt[12]{2^9a^{18}b^{27}} - ab\sqrt[12]{2^6a^8b^{10}} + ab^2\sqrt[12]{2^3a^{12}b^3}\right) : \sqrt[12]{2^3b^3} =$

$= \sqrt[12]{2^6a^{18}b^{24}} - ab\sqrt[12]{2^3a^8b^7} + ab^2\sqrt[12]{a^{12}} = ab^2\sqrt{2a} -$

$- ab\sqrt[12]{8a^8b^7} + a^2b^2.$

196. $\left(\sqrt[9]{a^5b^4} - 4a^3b\sqrt[4]{a^2b^2} + \frac{a^3}{b^4}\sqrt[12]{ab}\right) : \frac{a}{b^2}\sqrt[12]{ab^2} =$

$= \left(\sqrt[36]{a^{20}b^{16}} - 4a^3b\sqrt[36]{a^{27}b^{18}} + \frac{a^3}{b^4}\sqrt[36]{a^{18}b^{18}}\right) : \frac{a}{b^2}\sqrt[36]{a^3b^6} =$

$= \frac{b^2}{a}\sqrt[36]{a^{17}b^{16}} - 4a^2b^3\sqrt[36]{a^{24}b^{12}} + \frac{a^2}{b^2}\sqrt[36]{a^{15}b^{12}} = \frac{b^2}{a}\sqrt[36]{a^{17}b^{10}} -$

$- 4a^2b^3\sqrt[3]{a^2b} + \frac{a^2}{b^2}\sqrt[12]{a^5b^4}.$

197.

$$\begin{array}{l|l}
\sqrt[5]{8x^3} \qquad\qquad\qquad -3\sqrt{3} & \sqrt[5]{2x} - \sqrt{3} \\
\hline
\mp\sqrt[5]{8x^3} \pm \sqrt[5]{4x^2}\sqrt{3} & \sqrt[5]{4x^2} + \sqrt[5]{2x}\sqrt{3} + 3 \\
\hline
\qquad\sqrt[5]{4x^2}\sqrt{3} & \\
\qquad\mp\sqrt[5]{4x^2}\sqrt{3} \pm 3\sqrt[5]{2x} & \\
\hline
\qquad\qquad 3\sqrt[5]{2x} - 3\sqrt{3} & \\
\qquad\qquad \mp 3\sqrt[5]{2x} \pm 3\sqrt{3} & \\
\hline
\qquad\qquad\qquad 0 &
\end{array}$$

198. $2a\sqrt[3]{ax^2} - a\sqrt[6]{ax^5} - ax \;\Big|\; \sqrt[3]{a^2x} - \sqrt{ax} =$

$= 2\sqrt[6]{a^8x^4} - \sqrt[6]{a^7x^5} - \sqrt[6]{a^6x^6} \;\Big|\; \sqrt[6]{a^4x^2} - \sqrt[6]{a^3x^3}$

$\mp 2\sqrt[6]{a^8x^4} \pm 2\sqrt[6]{a^7x^5}$ $\qquad 2\sqrt[6]{a^4x^2} + \sqrt[6]{a^3x^3} =$

$\sqrt[6]{a^7x^5} - \sqrt[6]{a^6x^6}$ $\qquad = 2\sqrt[3]{a^2x} + \sqrt{ax}.$

$\mp\sqrt[6]{a^7x^5} \pm \sqrt[6]{a^6x^6}$

0

199. $x^2\sqrt[4]{27xy^3} + 2xy\sqrt{2xy} \;\Big|\; \sqrt[4]{3x^3y} + \sqrt{2xy} =$

$= \sqrt[4]{27x^9y^3} + 2\sqrt[4]{2^2x^6y^6} \;\Big|\; \sqrt[4]{3x^3y} + \sqrt[4]{2^2x^2y^2}$

$\mp\sqrt[4]{27x^9y^3} \mp \sqrt[4]{9.2^2x^8y^4}$ $\qquad \sqrt[4]{9x^6y^2} - \sqrt[4]{3\;2^2x^5y^3} + 2\sqrt[4]{x^4y^4} =$

$-\sqrt[4]{9.2^2x^8y^4} + 2\sqrt[4]{2^2x^6y^6}$ $\qquad = x\sqrt{3xy} - x\sqrt[4]{12xy^3} + 2xy.$

$\pm\sqrt[4]{9.2^2x^8y^4} \pm 2\sqrt[4]{3x^7y^5}$

$2\sqrt[4]{3x^7y^5} + 2\sqrt[4]{2^2x^6y^6}$

$\mp 2\sqrt[4]{3x^7y^5} \mp 2\sqrt[4]{2^2x^6y^6}$

0

200.

$x^3y^{-3} - x^3 - y^3 + 2xy\sqrt{xy} \;\Big|\; \sqrt{xy^{-1}}\sqrt{xy^{-1}} + x\sqrt{x} - y\sqrt{y} =$

$= \sqrt{x^6y^{-6}} - \sqrt{x^6} + 2\sqrt{x^3y^3} - \sqrt{y^6} \;\Big|\; \sqrt{x^3y^{-3}} + \sqrt{x^3} - \sqrt{y^3}$

$\mp\sqrt{x^6y^{-6}} \mp \sqrt{x^6y^{-3}} \pm \sqrt{x^3}$ $\qquad \sqrt{x^3y^{-3}} - \sqrt{x^3} + \sqrt{y^3}$ ehk

$-\sqrt{x^6y^{-3}} - \sqrt{x^6} + 2\sqrt{x^3y^3} + \sqrt{x^3} - \sqrt{y^6}$ $\qquad \sqrt{\frac{x^3}{y^3}} - x\sqrt{x} + y\sqrt{y}$ ehk

$\pm\sqrt{x^6y^{-3}} \pm \sqrt{x^6} \mp \sqrt{x^3y^3}$

raamatu vastus

$\sqrt{x^3} + \sqrt{x^3y^3} - \sqrt{y^6}$ $\qquad \frac{x}{y^3}\sqrt{xy} - x\sqrt{x} + y\sqrt{y}.$

$\mp\sqrt{x^3} \mp \sqrt{x^3y^3} \pm \sqrt{y^3}$

0

§ 7. Juurte astendamine ja juurimine.

201. $(\sqrt[4]{a^3})^4 = \sqrt[4]{a^{3\cdot 4}} = a^3.$

202. $(\sqrt[3]{a^2})^2 = \sqrt{a^4} = a\sqrt[3]{a}.$

203. $(\sqrt[4]{2x^3})^5 = \sqrt[4]{2^5x^{15}} = 2x^3\sqrt[4]{2x^3}.$

204. $(-a\sqrt[8]{a^2b^3})^7 = -a^7\sqrt[8]{a^{14}b^{21}} = -a^8b^2\sqrt[8]{a^6b^5}.$

205. $(a^2x\sqrt[3]{3a^2x})^4 = a^8x^4\sqrt[3]{3^4a^8x^4} = 3a^{10}x^5\sqrt[3]{3a^2x}.$

206. $\left(-2a\sqrt[6]{\frac{3}{a^4}}\right) = 16a^4\sqrt[6]{\frac{81}{a^{16}}} = 16a\sqrt[6]{81a^2} = 16a\sqrt[3]{9a}.$

207. $\left(\sqrt[5]{(x-y)^2}\right)^4 = \sqrt[5]{(x-y)^8} = (x-y)\sqrt[5]{(x-y)^3}.$

208. $\left(\frac{\sqrt[4]{a^{-3}b^2}}{a^{-2}c^3}\right)^{-3} = \frac{\sqrt[4]{a^9b^{-6}}}{a^6b^{-9}} = \frac{a^2b^{-1}\sqrt[4]{ab^{-2}}}{a^6b^{-9}} = \frac{b^8}{a^4}\sqrt[4]{\frac{a}{b^2}} =$
$= \frac{b^7}{a^4}\sqrt[4]{ab^2}.$

209. $\left(a^{-1}b^2\sqrt[3]{4a^nb^{-2}}\right)^{-2} = a^2b^{-4}\sqrt[3]{4^{-2}a^{-2n}b^4} =$
$= a^2b^{-3}\sqrt[3]{\frac{b}{16a^{2n}}} = \frac{a^2}{b^3}\sqrt[3]{\frac{b}{16a^{2n}}\cdot\frac{4a^n}{4a^n}} = \frac{a^2}{b^3}\sqrt[3]{\frac{4a^nb}{64a^{3n}}} =$
$= \frac{a^2}{4a^nb^3}\sqrt[3]{4a^nb} = \frac{a^{2-n}}{4b^3}\sqrt[3]{4a^nb}.$

210. $\left(\sqrt[n]{(x^2+y^2)^m}\right)^{np} = \sqrt[n]{(x^2+y^2)^{mnp}} = (x^2+y^2)^{mp}.$

211. $(\sqrt{3}-\sqrt{2})^2 = 3-2\sqrt{6}+2 = 5-2\sqrt{6}.$

212. $(\frac{1}{2}+2\sqrt{2})^2 = \frac{1}{4}+2\cdot\frac{1}{2}\cdot2\sqrt{2}+(2\sqrt{2})^2 =$
$= \frac{1}{4}+2\sqrt{2}+8 = 8\frac{1}{4}+2\sqrt{2}.$

213. $(\sqrt[3]{4}+\sqrt{2})^2 = (\sqrt[3]{4})^2+2\sqrt[3]{4}\cdot\sqrt{2}+(\sqrt{2})^2 =$
$= \sqrt[3]{16}+2\sqrt[6]{16\cdot8}+2 = 2\sqrt[3]{2}+4\sqrt[6]{2}+2.$

214. $(\sqrt{3}-2\sqrt[3]{2})^3 = (\sqrt{3})^3-3(\sqrt{3})^2\cdot2\sqrt[3]{2}+3(\sqrt{3})\cdot$
$\cdot(2\sqrt[3]{2})^2-(2\sqrt[3]{2})^3 = 3\sqrt{3}-18\sqrt[3]{2}+12\sqrt[6]{432}-16.$

215. $(\sqrt{2}-\sqrt{3}+\sqrt{6})^2 = (\sqrt{2})^2+(-\sqrt{3})^2+(\sqrt{6})^2+$
$+2\sqrt{2}(\sqrt{6}-\sqrt{3})-2\sqrt{3}\cdot\sqrt{6} = 2+3+6+4\sqrt{3}-2\sqrt{6}-$
$-6\sqrt{2} = 11+4\sqrt{3}-2\sqrt{6}-6\sqrt{2}.$

216. $(3\sqrt{2}-2\sqrt{5}-\sqrt{10})^2=(3\sqrt{2})^2+(-2\sqrt{5})^2+$
$+(-\sqrt{10})^2-2.3\sqrt{2}.2\sqrt{5}-2.3\sqrt{2}.\sqrt{10}+2.2\sqrt{5}.\sqrt{10}=$
$=18+20+10-12\sqrt{10}-6\sqrt{20}+4\sqrt{50}=48-12\sqrt{10}-$
$-12\sqrt{5}+20\sqrt{2}.$

217. $\left(\sqrt{3+\sqrt{5}}+\sqrt{3-\sqrt{5}}\right)^2=\sqrt{3+\sqrt{5}}\,)^2+$
$+2\sqrt{3+\sqrt{5}}.\sqrt{3-\sqrt{5}}+\left(\sqrt{3-\sqrt{5}}\right)^2=3+\sqrt{5}+$
$+2\sqrt{3^2-(\sqrt{5})^2}+3-\sqrt{5}=3+\sqrt{5}+2\sqrt{9-5}+3-\sqrt{5}=$
$=6+2\sqrt{4}=6+4=10.$

218. $\left(\sqrt{11+6\sqrt{2}}-\sqrt{11-6\sqrt{2}}\right)^2=(11+6\sqrt{2})-$
$-2\sqrt{(11+6\sqrt{2})(11-\sqrt{2})}+(11-6\sqrt{2})=11+6\sqrt{2}-$
$-2\sqrt{121-72}+11-6\sqrt{2}=22-2\sqrt{49}=22-14=8.$

219. $\left(\frac{b}{a}\sqrt{ab}-\frac{a}{\sqrt{a}}\right)^2=\left(\frac{b}{4}\sqrt{ab}\right)^2-2.\left(\frac{b}{4}\sqrt{ab}\right)\left(\frac{2}{\sqrt{a}}\right)+$
$+\left(\frac{2}{\sqrt{a}}\right)^2=\frac{ab^3}{16}-b\sqrt{b}+\frac{4}{a}.$

220. $(a\sqrt{a}+a\sqrt{2a})^3=(a\sqrt{a})^3+3(a\sqrt{a})^2(a\sqrt{2a})+$
$+3(a\sqrt{a})(a\sqrt{2a})^2+(a\sqrt{2a})^3=a^4\sqrt{a}+3a^4\sqrt{2a}+6a^4\sqrt{a}+$
$+2a^4\sqrt{2a}=7a^4\sqrt{a}+5a^4\sqrt{2a}=a^4\sqrt{a}(7+5\sqrt{2}).$

221. $\sqrt{\sqrt[3]{a^2}}=\sqrt[6]{a^2}=\sqrt[3]{a}.$

222. $\sqrt[3]{\sqrt[5]{a^4}}=\sqrt[15]{a^4}.$

223. $\sqrt[3]{\sqrt{125}}=\sqrt[6]{125}=\sqrt{5}.$

224. $\sqrt[4]{\sqrt{256a^{10}}}=\sqrt[4]{16a^5}=2a\sqrt[4]{a}.$

225. $\sqrt{a\sqrt[4]{a^3}}=\sqrt{\sqrt[4]{a^7}}=\sqrt[8]{a^7}.$

226. $\sqrt[4]{a^2\sqrt[3]{a^4}}=\sqrt[4]{\sqrt[3]{a^{10}}}=\sqrt[12]{a^{10}}=\sqrt[6]{a^5}.$

227. $\sqrt{\sqrt[4]{a^{10}b^2c^8}} = \sqrt[8]{a^{10}b^2c^8} = ac\sqrt[8]{a^2b^2} = ac\sqrt[4]{ab}.$

228. $\sqrt{\sqrt[3]{a^2\sqrt{b}}} = \sqrt{\sqrt[6]{a^4}\sqrt[6]{b^3}} = \sqrt{\sqrt[6]{a^4b^3}} = \sqrt[12]{a^4b^3}.$

229. $\sqrt{x^3\sqrt[3]{x\sqrt[4]{x}}} = \sqrt{\sqrt[3]{x^{10}\sqrt[4]{x}}} = \sqrt{\sqrt[3]{\sqrt[4]{x^{41}}}} =$

$= \sqrt[24]{x^{41}} = x\sqrt[24]{x^{17}}.$

230. $\sqrt{x\sqrt[3]{\frac{x^2}{y}\sqrt{\frac{y}{x}}}} = \sqrt{\sqrt[3]{\frac{x^5}{y}\sqrt{\frac{y}{x}}}} =$

$= \sqrt{\sqrt[3]{\sqrt{\frac{x^{10}y}{y^2x}}}} = \sqrt[12]{\frac{x^{10}y}{y^2x}} = \sqrt[12]{\frac{x^9}{y}} = \frac{1}{y}\sqrt[12]{x^9y^{11}}.$

231. $\sqrt[4]{2x\sqrt[3]{2x^2y.3y\sqrt{3xy^3}}} = \sqrt[4]{\sqrt[3]{8x^3.2x^2y.3y\sqrt{3xy^3}}} =$

$= \sqrt[4]{\sqrt[3]{\sqrt{64\,x^6\,.\,4\,x^4y^2\,.\,9\,y^2\,.\,3\,xy^3}}} = \sqrt[4]{\sqrt[3]{\sqrt{2^8 3^3 x^{11}y^7}}} =$

$= \sqrt[24]{2^8 3^3\,.\,x^{11}y^7}.$

232. $\frac{3}{2}\sqrt{\frac{1}{2}x^4y^2\sqrt{\frac{x}{2}\sqrt{\frac{1}{4x}}}} =$

$= \frac{3}{2}\sqrt{\sqrt{\frac{1}{4}x^8y^3\cdot\frac{x}{2}\sqrt{\frac{1}{4x}}}} = \frac{3}{2}\sqrt{\sqrt{\sqrt{\frac{1}{16}x^{16}y^8\cdot\frac{x^2}{4}\cdot\frac{1}{4x}}}} =$

$= \frac{3}{2}\sqrt{\frac{1}{2^8}\cdot x^{17}y^8} = \frac{3}{4}x^2y\sqrt[8]{x}.$

233. $\sqrt[4]{20736} = \sqrt{\sqrt{20736}} = \sqrt{144} = 12.$

$\sqrt{2,07,36} = 144$

```
        1
   ─────────
  24 │ 10́7
   4 │  96
  ──────────
 284 │ 113́6
   4 │ 1136
  ──────────
         0
```

234. $\sqrt[10]{59049}=\sqrt[5]{\sqrt{59049}}=\sqrt[5]{243}=\sqrt[5]{3^5}=3.$

$$\sqrt{5'90'49}=243$$

$$\begin{array}{r|l} & 4 \\ \hline 44 & 190 \\ 4 & 176 \\ \hline 483 & 144'9 \\ 3 & 1449 \\ \hline & 0 \end{array}$$

235. $\sqrt[12]{4096}=\sqrt[6]{\sqrt{4096}}=\sqrt[6]{64}=\sqrt[6]{2^6}=2.$

$$\sqrt{4096}=64$$

$$\begin{array}{r|l} & 36 \\ \hline 124 & 496 \\ 4 & 496 \\ \hline & 0 \end{array}$$

236. $\sqrt[9]{262144}=\sqrt[3]{\sqrt[3]{262144}}=\sqrt[3]{64}=4.$

$$\sqrt[3]{262144}=64$$

$$\begin{array}{r|r} & 216 \\ \hline 3.6^2=108 & 461'44 \\ 3.6^2.4= & 432 \\ 3.6.4^2= & 288 \\ 4^3= & 64 \\ \hline & 46144 \\ \hline & 0 \end{array}$$

237. $\sqrt[4]{a^4+4a^3+6a^2+4a+1} = \sqrt{\sqrt{a^4+4a^3+6a^2+4a+1}} =$
$= \sqrt{a^2+2a+1} = \sqrt{(a+1)^2} = a+1.$

$$\sqrt{a^4+4a^3+6a^2+4a+1} = a^2+2a+1.$$

	$-a^4$
$2a^2+2a$	$4a^3+6a^2$
$+2a$	$\mp 4a^3 \mp 4a^2$
$2a^2+4a+1$	$2a^2+4a+1$
$+1$	$\mp 2a^2 \mp 4a \mp 1$
	0

238. $\sqrt[4]{16a^4-48a^3b+54a^2b^2-27ab^3+\frac{81}{16}b^4} =$
$= \sqrt{\sqrt{16a^4-48a^3b+54a^2b^2-27ab^3+\frac{81}{16}b^4}} = \sqrt{4a^2-6ab+\frac{9b^2}{4}} =$
$= \sqrt{\frac{16a^2-24ab+9b^2}{4}} = \frac{1}{2}\sqrt{(4a-3b)^2} = \frac{4a-3b}{2} = 2a - \frac{3b}{2}.$

	$16a^4$
$8a^2-6ab$	$-48a^3b+54a^2b^2$
$-6ab$	$\pm 48a^3b \pm 36a^2b^2$
$8a^2-12ab+\frac{9b^2}{4}$	$18a^2b^2-27ab^3+\frac{81}{16}b^4$
$+\frac{9b^2}{4}$	$\pm 18a^2b^2-27ab^3+\frac{81}{16}b^4$
	0

239. $\sqrt[3]{\sqrt{x^6+6x^5y+15x^4y^2+20x^3y^3+15x^2y^4+6xy^5+y^6}} = \sqrt[3]{x^3+3x^2y+3xy^2+y^3}.$

$$-x^6$$

$$2x^3+3x^2y \quad \Big| \quad +6x^5y+15x^4y^2$$
$$3x^2y \quad \Big| \quad -6x^5y\mp 9x^4y^2$$

$$2x^3+6x^2y+3xy^2 \quad \Big| \quad 6x^4y^2+20x^3y^3+15x^2y^4$$
$$3xy^2 \quad \Big| \quad -6x^4y^2\mp 18x^3y^3\mp 9x^2y^4$$

$$2x^3+6x^2y+6xy^2+y^3 \quad \Big| \quad 2x^3y^3+6x^2y^4+6xy^5+y^6$$
$$y^3 \quad \Big| \quad -2x^3y^3\mp 6x^2y\mp 6xy^5\mp y^6$$

$$0$$

240. $\sqrt[3]{\sqrt{64x^{12}-96x^{10}+60x^8-20x^6+\frac{15}{4}x^4-\frac{3}{8}x^2+\frac{1}{64}}} = \sqrt[3]{8x^6-6x^4+\frac{3}{2}x^2-\frac{1}{8}}.$

$$-64x^{12}$$

$$16x^6-6x^4 \quad \Big| \quad -96x^{10}+60x^8$$
$$-6x^4 \quad \Big| \quad \mp 96x^{10}\mp 36x^8$$

$$16x^6-12x^4+\tfrac{3}{2}x^2 \quad \Big| \quad 24x^8-20x^6+\tfrac{15}{4}x^4$$
$$+\tfrac{3}{2}x^2 \quad \Big| \quad -24x^8\pm 18x^6+\tfrac{9}{4}x^4$$

$$16x^6-12x^4+3x^2-\tfrac{1}{8} \quad \Big| \quad -2x^6+\tfrac{3}{2}x^4-\tfrac{3}{8}x^2+\tfrac{1}{64}$$
$$-\tfrac{1}{8} \quad \Big| \quad \mp 2x^6\mp\tfrac{3}{2}x^4\pm\tfrac{3}{8}x^2\mp\tfrac{1}{64}$$

$$0$$

§ 8. Irratsionaalsuse kaotamine murru nimetajast.

241. $\frac{a}{\sqrt{a}} = \frac{a\sqrt{a}}{\sqrt{a}\cdot\sqrt{a}} = \frac{a\sqrt{a}}{a} = \sqrt{a}.$

242. $\frac{m}{\sqrt{m^3}} = \frac{m}{m\sqrt{m}} = \frac{1}{\sqrt{m}} = \frac{\sqrt{m}}{m}.$

243. $\frac{a}{\sqrt[3]{a^2}} = \frac{a\sqrt[3]{a}}{\sqrt[3]{a^2}\sqrt[3]{a}} = \frac{a\sqrt[3]{a}}{a} = \sqrt[3]{a}.$

244. $\frac{m+n}{\sqrt{m-n}} = \frac{(m+n)\sqrt{m-n}}{\sqrt{m-n}\cdot\sqrt{m-n}} = \frac{m+n}{m-n}\sqrt{m-n}.$

245. $\frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}.$

246. $\frac{3}{\sqrt[3]{6}}=\frac{3\sqrt[3]{36}}{6}=\frac{1}{2}\sqrt[3]{36}.$

247. $\frac{6}{\sqrt[4]{8}}=\frac{6\sqrt[4]{2}}{\sqrt[4]{16}}=\frac{6\sqrt[4]{2}}{2}=3\sqrt[4]{2}.$

248. $\frac{\sqrt[6]{49}}{\sqrt[3]{21}}=\frac{\sqrt[6]{7^2}}{\sqrt[3]{7.3}}=\frac{\sqrt[3]{7}}{\sqrt[3]{7}.\sqrt[3]{3}}=\frac{1}{\sqrt[3]{3}}=\frac{\sqrt[3]{9}}{\sqrt[3]{27}}=\frac{1}{3}\sqrt[3]{9}.$

249. $\frac{a^2-b^2}{\sqrt[3]{a-b}}=\frac{(a^2-b^2)\sqrt[3]{(a-b)^2}}{\sqrt[3]{(a-b)^3}}=\frac{(a+b)(a-b)\sqrt[3]{(a-b)^2}}{a-b}=$

$=(a+b)\sqrt[3]{(a-b)^2}.$

250. $\frac{a-b}{\sqrt[3]{a^2-b^2}}=\frac{(a-b)\sqrt[3]{(a^2-b^2)^2}}{\sqrt[3]{(a^2-b^2)^3}}=\frac{a-b}{a^2-b^2}\sqrt[3]{(a^2-b^2)^2}=$

$=\frac{1}{a+b}\sqrt[3]{(a^2-b^2)^2}.$

251. $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\frac{(\sqrt{a}-\sqrt{b})(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})}=\frac{(\sqrt{a}-\sqrt{b})^2}{(\sqrt{a})^2-(\sqrt{b})^2}=$

$=\frac{a-2\sqrt{ab}+b}{a-b}.$

252. $\frac{a}{1-\sqrt{a}}=\frac{a(1+\sqrt{a})}{(1-\sqrt{a})(1+\sqrt{a})}=\frac{a+a\sqrt{a}}{1-a}.$

253. $\frac{\sqrt{3}}{2+\sqrt{3}}=\frac{\sqrt{3}(2-\sqrt{3})}{(2+\sqrt{3})(2-3\sqrt{3})}=\frac{2\sqrt{3}-3}{4-3}=2\sqrt{3}-3.$

254. $\frac{2\sqrt{2}}{3\sqrt{2}-2\sqrt{3}}=\frac{2\sqrt{2}(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})}=$

$=\frac{12+4\sqrt{6}}{(3\sqrt{2})^2-(2\sqrt{3})^2}=\frac{12+4\sqrt{6}}{18-12}=\frac{4(3+\sqrt{6})}{6}=\frac{2}{3}(3+\sqrt{6}).$

255. $\frac{1-a}{\sqrt{1-\sqrt{a}}}=\frac{(1-a)\sqrt{1-\sqrt{a}}}{\sqrt{(1-\sqrt{a})^2}}=\frac{(1-a)\sqrt{1-\sqrt{a}}}{1-\sqrt{a}}=$

$=\frac{(1-a)(1+\sqrt{a})\sqrt{1-\sqrt{a}}}{(1-\sqrt{a})(1+\sqrt{a})}=\frac{(1-a)(1+\sqrt{a})\sqrt{1-\sqrt{a}}}{1-a}=$

$$= (1+\sqrt{a})\sqrt{1-\sqrt{a}} = \sqrt{(1+\sqrt{a})^2}\cdot\sqrt{1-\sqrt{a}} =$$
$$= \sqrt{(1+\sqrt{a})(1+\sqrt{a})(1-\sqrt{a})} = \sqrt{(1-a)(1+\sqrt{a})}.$$

256. $$\frac{n}{\sqrt[3]{a}-\sqrt[3]{b}} = \frac{n\left(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}\right)}{\left(\sqrt[3]{a}-\sqrt[3]{b}\right)\left(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}\right)} =$$
$$= \frac{n\left(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}\right)}{a-b}.$$

257. $$\frac{12}{3+\sqrt{2}-\sqrt{3}} = \frac{12\,[(3+\sqrt{2})+\sqrt{3}]}{[(3+\sqrt{2})-\sqrt{3}]\,[3+\sqrt{2})+\sqrt{3}]} =$$
$$= \frac{12\,(3+\sqrt{2}+\sqrt{3})}{(3+\sqrt{2})^2-(\sqrt{3})^2} = \frac{12\,(3+\sqrt{2}+\sqrt{3})}{9+6\sqrt{2}+2-3} = \frac{12\,(3+\sqrt{2}+\sqrt{3})}{8+6\sqrt{2}} =$$
$$= \frac{6\,(12-9\sqrt{2}+4\sqrt{2}-6+4\sqrt{3}-3\sqrt{6})}{16-18} = -3\,(6-5\sqrt{2}+$$
$$+4\sqrt{3}-3\sqrt{6}) = -18+15\sqrt{2}-12\sqrt{3}+9\sqrt{6}.$$

258. $$\frac{2+\sqrt{30}}{\sqrt{5}+\sqrt{6}-\sqrt{7}} = \frac{(2+\sqrt{30})(\sqrt{5}+\sqrt{6}+\sqrt{7})}{(\sqrt{5}+\sqrt{6}-\sqrt{7})(\sqrt{5}+\sqrt{6}+\sqrt{7})} =$$
$$= \frac{(2+\sqrt{30})\,(\sqrt{5}+\sqrt{6}+\sqrt{7})}{(\sqrt{5}+\sqrt{6})^2-(\sqrt{7})^2} = \frac{(2+\sqrt{30})\,(\sqrt{5}+\sqrt{6}+\sqrt{7})}{5+2\sqrt{30}+6-7} =$$
$$= \frac{(2+\sqrt{30})\,(\sqrt{5}+\sqrt{6}+\sqrt{7})}{2\,(2+\sqrt{30})} = \tfrac{1}{2}\,(\sqrt{5}+\sqrt{6}+\sqrt{7}).$$

259. $$\frac{1+3\sqrt{2}-2\sqrt{3}}{\sqrt{2}+\sqrt{3}+\sqrt{6}} = \frac{(1+3\sqrt{2}-2\sqrt{3})[(\sqrt{2}+\sqrt{3})-\sqrt{6}]}{[(\sqrt{2}+\sqrt{3})+\sqrt{6}][(\sqrt{2}+\sqrt{3})-\sqrt{6}]} =$$
$$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{6}+6+3\sqrt{6}-3\sqrt{12}-2\sqrt{6}-6+2\sqrt{18}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{6})^2} =$$
$$= \frac{\sqrt{2}+\sqrt{3}-3\sqrt{12}+2\sqrt{18}}{2+3+2\sqrt{6}-6} = \frac{\sqrt{2}+\sqrt{3}-6\sqrt{3}+6\sqrt{2}}{2\sqrt{6}-1} = \frac{7\sqrt{2}-5\sqrt{3}}{2\sqrt{6}-1} =$$
$$= \frac{(7\sqrt{2}-5\sqrt{3})(2\sqrt{6}+1)}{(2\sqrt{6}-1)\,(2\sqrt{6}+1)} = \frac{14\sqrt{12}+7\sqrt{2}-10\sqrt{18}-5\sqrt{3}}{(2\sqrt{6})^2-1} =$$
$$= \frac{28\sqrt{3}+7\sqrt{2}-30\sqrt{2}-5\sqrt{3}}{24-1} = \frac{23\sqrt{3}-23\sqrt{2}}{23} = \sqrt{3}-\sqrt{2}.$$

260. $$\frac{n}{\sqrt{\sqrt{2}+\sqrt[3]{3}}}=\frac{n\sqrt{\sqrt{2}+\sqrt[3]{3}}}{\sqrt{(\sqrt{2}+\sqrt[3]{3})^2}}=\frac{n\sqrt{\sqrt{2}+\sqrt[3]{3}}}{\sqrt{2}+\sqrt[3]{3}}=$$

$$=\frac{n\sqrt{\sqrt{2}+\sqrt[3]{3}}\,(\sqrt{2^2}-\sqrt{2}\,.\,\sqrt[3]{3}+\sqrt[3]{3^2})}{(\sqrt{2}+\sqrt[3]{3})(\sqrt{2^2}-\sqrt{2}\,.\,\sqrt[3]{3}+\sqrt[3]{3^2})}=$$

$$=\frac{n\sqrt{\sqrt{2}+\sqrt[3]{3}}+(2-\sqrt[9]{72}+\sqrt[3]{9})}{(\sqrt{2})^3+(\sqrt[3]{3})^3}=$$

$$=\frac{n\sqrt{\sqrt{2}+\sqrt[3]{3}}(2-\sqrt[6]{72}+\sqrt[3]{9})}{2\sqrt{2}+3}=$$

$$=\frac{n\sqrt{\sqrt{2}+\sqrt[3]{3}}(2-\sqrt[6]{72}+\sqrt[3]{9})\,(2\sqrt{2}-3)}{(2\sqrt{2}+3)(2\sqrt{2}-3)}=$$

$$=\frac{n\sqrt{\sqrt{2}+\sqrt[3]{3}}(2-\sqrt[6]{72}+\sqrt[3]{9})\,(2\sqrt{2}-3)}{8-9}=$$

$$=-n\sqrt{\sqrt{2}+\sqrt[3]{3}}(2-\sqrt[6]{72}+\sqrt[3]{9})\,.\,(2\sqrt{2}-3)=$$

$$=n\sqrt{\sqrt{2}+\sqrt[3]{3}}(2-\sqrt[6]{72}+\sqrt[3]{9})\,(3-2\sqrt{2}).$$

§ 9. Irratsionaalsete kaks- ja hulkliigete juurimine.

261. $$\sqrt{2+\sqrt{3}}=\sqrt{\frac{2+\sqrt{2^2-3}}{2}}+\sqrt{\frac{2-\sqrt{2^2-3}}{2}}=$$

$$=\sqrt{\frac{2+1}{2}}+\sqrt{\frac{2-1}{2}}=\frac{\sqrt{3}+1}{\sqrt{2}}=\frac{\sqrt{2}}{2}(\sqrt{3}+1).$$

262. $$\sqrt{6+4\sqrt{2}}=\sqrt{6+\sqrt{32}}=\sqrt{\frac{6+\sqrt{36-32}}{2}}+$$

$$+\sqrt{\frac{6-\sqrt{36-32}}{2}}=\sqrt{\frac{6+2}{2}}+\sqrt{\frac{6-2}{2}}=\sqrt{4}+\sqrt{2}=$$

$$=2+\sqrt{2}.$$

263. $\sqrt{5-\sqrt{21}}=\sqrt{\frac{5+\sqrt{25-21}}{2}}-\sqrt{\frac{5-\sqrt{25-21}}{2}}=$
$=\sqrt{\frac{5+2}{2}}-\sqrt{\frac{5-2}{2}}=\frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{2}(\sqrt{7}-\sqrt{3})}{2}=$
$=\frac{1}{2}(\sqrt{14}-\sqrt{6})$.

264. $\sqrt{7+4\sqrt{3}}=\sqrt{7+\sqrt{48}}=\sqrt{\frac{7+\sqrt{49-48}}{2}}+$
$+\sqrt{\frac{7-\sqrt{49-48}}{2}}=\sqrt{\frac{7+1}{2}}+\sqrt{\frac{7-1}{2}}=2+\sqrt{3}$.

265. $\sqrt{4\sqrt{2}+2\sqrt{6}}=\sqrt{2\sqrt{2}(2+\sqrt{3})}=\sqrt{2\sqrt{2}}\,\cdot$
$\cdot\sqrt{2+\sqrt{3}}=\sqrt[4]{8}\cdot\left(\sqrt{\frac{2+\sqrt{4-3}}{2}}+\sqrt{\frac{2-\sqrt{4-3}}{2}}\right)=$
$=\sqrt[4]{8}\left(\sqrt{\frac{3}{2}}+\sqrt{\frac{1}{2}}\right)=\frac{\sqrt[4]{8}\,(\sqrt{3}+1)}{\sqrt{2}}=\frac{\sqrt[4]{8}\,(\sqrt{3}+1)}{\sqrt[4]{4}}=$
$=\sqrt[4]{2}(\sqrt{3}+1)$.

266. $\sqrt{4\sqrt{5}-2\sqrt{15}}=\sqrt{2\sqrt{5}(2-\sqrt{3})}=\sqrt{2\sqrt{5}}\,-$
$-\sqrt{2-\sqrt{3}}=\sqrt[4]{20}\left(\sqrt{\frac{2+\sqrt{4-3}}{2}}-\sqrt{\frac{2-\sqrt{4-3}}{2}}\right)=$
$=\sqrt[4]{20}\left(\sqrt{\frac{3}{2}}-\sqrt{\frac{1}{2}}\right)=\frac{\sqrt[4]{20}(\sqrt{3}-1)}{\sqrt{2}}=\frac{\sqrt[4]{20}\,(\sqrt{3}-1)}{\sqrt[4]{4}}=$
$=\sqrt[4]{5}(\sqrt{3}-1)=\sqrt[4]{45}-\sqrt[4]{5}$.

267. $\sqrt{\sqrt{14+6\sqrt{5}}}=\sqrt{\sqrt{14+\sqrt{180}}}=$
$=\sqrt{\sqrt{\frac{14+\sqrt{196-180}}{2}}+\sqrt{\frac{14-\sqrt{196-180}}{2}}}=$
$=\sqrt{\sqrt{\frac{14+4}{2}}+\sqrt{\frac{14-4}{2}}}=\sqrt{3+\sqrt{5}}=$

$$=\sqrt{\frac{3+\sqrt{9-5}}{2}}+\sqrt{\frac{3-\sqrt{9-5}}{2}}=\sqrt{\frac{3+2}{2}}+$$
$$+\sqrt{\frac{3-2}{2}}=\frac{\sqrt{5}+1}{2}=\frac{\sqrt{2}(\sqrt{5}+1)}{2}=\tfrac{1}{2}(\sqrt{10}+\sqrt{2}).$$

268. $\sqrt{17+6\sqrt{4-\sqrt{9+4\sqrt{2}}}}=$

$$=\sqrt{17+6\sqrt{4-\left(\sqrt{\frac{9+\sqrt{81-32}}{2}}+\sqrt{\frac{9-\sqrt{81-32}}{2}}\right)}}=$$
$$=\sqrt{17+6\sqrt{4-(\sqrt{8}+1)}}=\sqrt{17+6\sqrt{3-\sqrt{8}}}=$$
$$=\sqrt{17+6\left(\sqrt{\frac{3+\sqrt{9-8}}{2}}-\sqrt{\frac{3-\sqrt{9-8}}{2}}\right)}=$$
$$=\sqrt{17+6(\sqrt{2}-1)}=\sqrt{17+6\sqrt{2}-6}=\sqrt{11+6\sqrt{2}}=$$
$$=\sqrt{11+\sqrt{72}}=\sqrt{\frac{11+\sqrt{121-72}}{2}}+\sqrt{\frac{11-\sqrt{121-72}}{2}}=$$
$$=\sqrt{\frac{11+7}{2}}+\sqrt{\frac{11-7}{2}}=3+\sqrt{2}.$$

269. $\sqrt{a+b-2\sqrt{ab}}=\sqrt{(\sqrt{a})^2+(\sqrt{b})^2-2\sqrt{ab}}=$

$$=\sqrt{(\sqrt{a}-\sqrt{b})^2}=\sqrt{a}-\sqrt{b}.$$

270. $\sqrt{2a^2+2\sqrt{a^4-b^2}}=\sqrt{2a^2+\sqrt{4a^4-4b^2}}=$

$$=\sqrt{\frac{2a^2+\sqrt{4a^4-(4a^4-4b^2)}}{2}}+\sqrt{\frac{2a^2-\sqrt{4a^4-(4a^4-4b^2)}}{2}}=$$
$$=\sqrt{\frac{2a^2+2b}{2}}+\sqrt{\frac{2a^2-2b}{2}}=\sqrt{a^2+b}+\sqrt{a^2-b}.$$

271. $\sqrt{4a+5b-4\sqrt{5ab}}=$

$$=\sqrt{4(\sqrt{a})^2+(\sqrt{5b})^2-4\sqrt{5ab}}=\sqrt{(2\sqrt{a}-\sqrt{5b})^2}=$$
$$=2\sqrt{a}-\sqrt{5b}.$$

272. $\sqrt{a^3 + 2ab\sqrt{ab} + b^3} =$
$= \sqrt{a^2(\sqrt{a})^2 + 2ab\cdot\sqrt{a}\cdot\sqrt{b} + b^2(\sqrt{b})} = \sqrt{(a\sqrt{a}+b\sqrt{b})^2} =$
$= a\sqrt{a} + b\sqrt{b}.$

273. $\sqrt{25 - 10\sqrt[4]{3} + \sqrt{3}} = \sqrt{25 - 10\sqrt[4]{3} + (\sqrt[4]{3})^2} =$
$= \sqrt{(5 - \sqrt[4]{3})^2} = 5 - \sqrt[4]{3}.$

274. $\sqrt{4\sqrt[3]{9} + 2\sqrt{3} + \frac{1}{4}\sqrt[3]{3}} = \sqrt{4\sqrt[6]{3^4} + 2\sqrt[6]{3^3} + \frac{1}{4}\sqrt[6]{3^2}} =$
$= \sqrt{(2\sqrt[6]{3^2} + \frac{1}{2}\sqrt[6]{3})^2} = 2\sqrt[6]{3^2} + \frac{1}{2}\sqrt[6]{3} = 2\sqrt[3]{3} + \frac{1}{2}\sqrt[6]{3}.$

275. $\sqrt{a^2 + a\sqrt{a} - \frac{13}{12}a - \frac{2}{3}\sqrt{a} + \frac{4}{9}} = a + \frac{\sqrt{a}}{2} - \frac{2}{3}.$

$$\begin{array}{r|l}
 & a^2 \\ \hline
2a + \frac{\sqrt{a}}{2} & a\sqrt{a} - \frac{13}{12}a \\
+\frac{\sqrt{a}}{2} & \mp a\sqrt{a} \mp \frac{1}{4}a \\ \hline
2a + \sqrt{a} - \frac{2}{3} & -\frac{16}{12}a - \frac{2}{3}\sqrt{a} + \frac{4}{9} \\
-\frac{2}{3} & \pm\frac{4}{3}a \pm \frac{2}{3}\sqrt{a} \mp \frac{4}{9} \\ \hline
 & 0
\end{array}$$

276. $\sqrt{x^2\sqrt[3]{y^2} - 4x\sqrt[3]{x^2y} + 4x\sqrt[3]{x} + 2xy^2\sqrt[3]{y} - 4y^2\sqrt[3]{x^2} + y^4} =$
$= x\sqrt[3]{y} - 2\sqrt[3]{x^2} + y^2.$

$$\begin{array}{r|l}
 & x^2\sqrt[3]{y^2} \\ \hline
2x\sqrt[3]{y} - 2\sqrt[3]{x^2} & -4x\sqrt[3]{x^2y} + 4x\sqrt[3]{x} \\
-2\sqrt[3]{x^2} & \pm 4x\sqrt[3]{x^2y} \mp 4x\sqrt[3]{x} \\ \hline
2x\sqrt[3]{y} - 4\sqrt[3]{x^2} + y^2 & 2xy^2\sqrt[3]{y} - 4y^2\sqrt[3]{x^2} + y^4 \\
+y^2 & \mp 2xy^2\sqrt[3]{y} \pm 4y^2\sqrt[3]{x^2} \mp y^4 \\ \hline
 & 0
\end{array}$$

277. $\sqrt[3]{a+3\sqrt[3]{a^2b}+3\sqrt[3]{ab^2}+b}=$

$=\sqrt[3]{(\sqrt[3]{a})^3+3(\sqrt[3]{a})^2(\sqrt[3]{b})+3(\sqrt[3]{a})(\sqrt[3]{b})^2+(\sqrt[3]{b})^3}=$

$=\sqrt[3]{(\sqrt[3]{a}+\sqrt[3]{b})^3}=\sqrt[3]{a}+\sqrt[3]{b}.$

278. $\sqrt[3]{2x\sqrt{2x}-6x\sqrt[3]{y^2}+3y\sqrt[6]{8x^3y^2}-y^2}=$

$=\sqrt[3]{(\sqrt{2x})^3-3(\sqrt{2x})^2\cdot\sqrt[3]{y^2}+3\sqrt{2x}(\sqrt[3]{y^2})^2-(\sqrt[3]{y^2})^3}=$

$=\sqrt[3]{(\sqrt{2x}-\sqrt[3]{y^2})^3}=\sqrt{2x}-\sqrt[3]{y^2}.$

279. $\sqrt[3]{ab^2\sqrt{a}+6ab^2\sqrt[12]{a^3b^4}+12ab^2\sqrt[3]{b^2}+8b^3\sqrt[4]{a^3}}=$

$=\sqrt[3]{\sqrt[12]{a^{18}b^{24}}+6\sqrt[12]{a^{15}b^{28}}+12\sqrt[12]{a^{12}b^{32}}+8\sqrt[12]{a^9b^{36}}}=\sqrt[12]{a^6b^8}+2\sqrt[12]{a^3b^{12}}=$

$=\sqrt[6]{a^3b^4}+2b\sqrt[4]{a}.$

$$\begin{array}{l|l}
 & \sqrt[12]{a^{18}b^{24}} \\ \hline
3\left(\sqrt[12]{a^6b^8}\right)^2\left(2\sqrt[12]{a^3b^{12}}\right)+ & 6\sqrt[12]{a^{15}b^{28}}+12\sqrt[12]{a^{12}b^{32}}+8\sqrt[12]{a^9b^{36}} \\
+3\left(\sqrt[12]{a^6b^8}\right)\left(2\sqrt[12]{a^3b^{12}}\right)^2+ & 6\sqrt[12]{a^{15}b^{28}}+12\sqrt[12]{a^{12}b^{32}}+8\sqrt[12]{a^9b^{36}} \\ \cline{2-2}
+\left(2\sqrt[12]{a^3b^{12}}\right)^3 & 0
\end{array}$$

280.

$$\sqrt[3]{\frac{x}{y^2}\sqrt{x}-\frac{2}{y}+\frac{4}{3x\sqrt{x}}-\frac{8y}{27x^3}}=\frac{\sqrt{x}}{\sqrt[3]{y^2}}-\frac{2\sqrt[3]{y}}{3x}.$$

$$\frac{x}{y^2}\sqrt{x}$$

$$-3\left(\frac{\sqrt{x}}{\sqrt[3]{y^2}}\right)^2\left(\frac{2\sqrt[3]{y}}{3x}\right)+3\left(\frac{\sqrt{x}}{\sqrt[3]{y^2}}\right)\cdot\left(\frac{2\sqrt[3]{y}}{3x}\right)^2-\left(\frac{2\sqrt[3]{y}}{3x}\right)^3 \quad \Big| \quad -\frac{2}{y}+\frac{4}{3x\sqrt{x}}-\frac{8y}{27x^3}$$

$$\pm\frac{2}{y}\mp\frac{4}{3x\sqrt{x}}\pm\frac{8y}{27x^3}$$

$$0$$

§ 10. Kordamisnäitused.

281. $\sqrt{ab}+\sqrt{a}=\sqrt{a}\,(\sqrt{b}+1).$

282. $\sqrt[3]{a^2}-\sqrt[3]{ab}=\sqrt[3]{a}\,(\sqrt[3]{a}-\sqrt[3]{b}).$

283. $\sqrt{a+b}-\sqrt{a^2-b^2}=\sqrt{a+b}-\sqrt{(a+b)(a-b)}=$
$=\sqrt{a+b}\,(1-\sqrt{a-b}).$

284. $\sqrt{a^2-b^2}+a-b=\sqrt{(a+b)(a-b)}+\sqrt{(a-b)^2}=$
$=\sqrt{a-b}\,(\sqrt{a+b}+\sqrt{a-b}).$

285. $a^2-\sqrt[3]{b^2}=a^2-(\sqrt[3]{b})^2=(a+\sqrt[3]{b})(a-\sqrt[3]{b}).$

286. $\sqrt[3]{a^2}-\sqrt[5]{4}=(\sqrt[3]{a})^2-(\sqrt[5]{2})^2=$
$=(\sqrt[3]{a}+\sqrt[5]{2})(\sqrt[3]{a}-\sqrt[5]{2}).$

287. $\sqrt[6]{a^5}+\sqrt[4]{a^3}=\sqrt[12]{a^{10}}+\sqrt[12]{a^9}=\sqrt[12]{a^9}(\sqrt[12]{a}+1)=$
$=\sqrt[4]{a^3}(\sqrt[12]{a}+1).$

288. $a^2+\sqrt{a}-\sqrt[4]{a^3}=\sqrt[4]{a^8}+\sqrt[4]{a^2}-\sqrt[4]{a^3}=$
$=\sqrt[4]{a^2}\,(\sqrt[4]{a^6}+1-\sqrt[4]{a})=\sqrt{a}\,(a\sqrt{a}+1-\sqrt[4]{a}).$

289. $a+b+2\sqrt{ab}=(\sqrt{a})^2+(\sqrt{b})^2+2\sqrt{ab}=$
$=(\sqrt{a}+\sqrt{b})^2.$

290. $\sqrt[3]{a^2}-2\sqrt[3]{ab^2}+b\sqrt[3]{b}=(\sqrt[3]{a})^2-2\sqrt[3]{a}\,.\,\sqrt[3]{b^2}+$
$+(\sqrt[3]{b^2})^2=(\sqrt[3]{a}-\sqrt[3]{b^2})^2.$

291. $a^2 - \sqrt[5]{b^4} = a^2 - (\sqrt[5]{b^2})^2 = (a+\sqrt[5]{b^2})(a-\sqrt[5]{b^2}) =$
$= (a+\sqrt[5]{b^2})(\sqrt{a}+\sqrt[5]{b})(\sqrt{a}-\sqrt[5]{b})$

292. $\sqrt[3]{a^2} - \sqrt[3]{b} = (\sqrt[3]{a})^2 - (\sqrt[6]{b})^2 =$
$= (\sqrt[3]{a}+\sqrt[6]{b})(\sqrt[3]{a}-\sqrt[6]{b}).$

293. $a^3 - \sqrt[5]{b^3} = a^3 - (\sqrt[5]{b})^3 =$
$= (a-\sqrt[5]{b})(a^2 + a\sqrt[5]{b}+\sqrt[5]{b^2}).$

294. $a\sqrt{a}+b\sqrt{b} = \sqrt{a^3}+\sqrt{b^3} = (\sqrt{a})^3+(\sqrt{b})^3 =$
$= (\sqrt{a}+\sqrt{b})(a-\sqrt{ab}+b).$

295. $a-b = (\sqrt{a})^2-(\sqrt{b})^2 = (\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}).$

296. $a^2+b = (\sqrt[3]{a^2})^3+(\sqrt[3]{b})^3 =$
$= (\sqrt[3]{a^2}+\sqrt[3]{b})(\sqrt[3]{a^4}-\sqrt[3]{a^2b}+\sqrt[3]{b^2}) =$
$= (\sqrt[3]{a^2}+\sqrt[3]{b})(a\sqrt[3]{a}-\sqrt[3]{a^2b}+\sqrt[3]{b^2}).$

297. $a-\sqrt[3]{ab^2}+\sqrt[3]{a^2b}-b = \sqrt[3]{a^3}-\sqrt[3]{ab^2}+\sqrt[3]{a^2b}-$
$-\sqrt[3]{b^3} = \sqrt[3]{a}(\sqrt[3]{a^2}-\sqrt[3]{b^2})+\sqrt[3]{b}(\sqrt[3]{a^2}-\sqrt[3]{b^2}) =$
$= (\sqrt[3]{a^2}-\sqrt[3]{b^2})(\sqrt[3]{a}+\sqrt[3]{b}) = (\sqrt[3]{a}+\sqrt[3]{b})(\sqrt[3]{a}+\sqrt[3]{b})\cdot$
$\cdot(\sqrt[3]{a}-\sqrt[3]{b}).$

298. $ab - a\sqrt{a} - \sqrt{ab} + b\sqrt{b} = (ab - a\sqrt{a})+(b\sqrt{b} -$
$- \sqrt{ab}) = a(b-\sqrt{a})+\sqrt{b}(b-\sqrt{a}) = (b-\sqrt{a})(a+\sqrt{b}).$

299. $\sqrt[3]{a^4}+\sqrt[3]{a^2b^2}-2a\sqrt[3]{b} = (\sqrt[3]{a^2})^2+(\sqrt[3]{ab})^2 -$
$-2\sqrt[3]{a^3b} = (\sqrt[3]{a^2})^2+(\sqrt[3]{ab})^2-2\sqrt[3]{a^2}\cdot\sqrt[3]{ab} = (\sqrt[3]{a^2}-\sqrt[3]{ab})^2 =$
$= (\sqrt[3]{a^2}(\sqrt[3]{a}-\sqrt[3]{b})^2.$

300. $a\sqrt{ab}+2a\sqrt[4]{b^3}+b\sqrt{a} = \sqrt{a^3b}+2\sqrt[4]{a^4b^3}+$
$+\sqrt{ab^2} = \sqrt[4]{a^6b^2}+2\sqrt[4]{a^4b^3}+\sqrt[4]{a^2b^4} = (\sqrt[4]{a^3b})^2+2\sqrt[4]{a^3b}\cdot$
$\cdot\sqrt[4]{ab^2}+(\sqrt[4]{ab^2})^2 = (\sqrt[4]{a^3b}+\sqrt[4]{ab^2})^2 = (\sqrt[4]{ab})^2(\sqrt[4]{a^2}+\sqrt[4]{b})^2 =$
$= \sqrt{ab}(\sqrt{a}+\sqrt[4]{b})^2.$

301. $\frac{3}{5-\sqrt{5}}-\frac{1}{3+\sqrt{5}}=\frac{3(5+\sqrt{5})}{(5-\sqrt{5})(5+\sqrt{5})}-$

$-\frac{(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}=\frac{15+3\sqrt{5}}{25-5}-\frac{3-\sqrt{5}}{9-5}=\frac{15+3\sqrt{5}}{20}-$

$-\frac{3-\sqrt{5}}{4}=\frac{15+3\sqrt{5}-15+5\sqrt{5}}{20}=\frac{8\sqrt{5}}{20}=\frac{2\sqrt{5}}{5}.$

302. $\frac{5}{4-\sqrt{11}}-\frac{4}{\sqrt{11}-\sqrt{7}}-\frac{2}{3+\sqrt{7}}=\frac{5(4+\sqrt{11})}{(4-\sqrt{11})(4+\sqrt{11})}-$

$-\frac{4(\sqrt{11}+\sqrt{7})}{(\sqrt{11}-\sqrt{7})(\sqrt{11}+\sqrt{7})}-\frac{2(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})}=\frac{20+5\sqrt{11}}{16-11}-$

$-\frac{4\sqrt{11}+4\sqrt{7}}{11-7}-\frac{6-2\sqrt{7}}{9-7}=\frac{20+5\sqrt{11}}{5}-\frac{4\sqrt{11}+4\sqrt{7}}{4}-$

$-\frac{6-2\sqrt{7}}{2}=4\sqrt{11}-\sqrt{11}-\sqrt{7}-3+\sqrt{7}=1.$

303. $a\sqrt{\frac{a+b}{a-b}}-b\sqrt{\frac{a-b}{a+b}}-\frac{2b^2}{\sqrt{a^2-b^2}}=\frac{a\sqrt{a+b}}{\sqrt{a-b}}-$

$-\frac{b\sqrt{a-b}}{\sqrt{a+b}}-\frac{2b^2}{\sqrt{(a+b)(a-b)}}=\frac{a\sqrt{(a+b)^2}-b\sqrt{(a-b)^2}-2b^2}{\sqrt{(a+b)(a-b)}}=$

$=\frac{a(a+b)-b(a-b)-2b^2}{\sqrt{a^2-b^2}}=\frac{a^2+ab-ab+b^2-2b^2}{\sqrt{a^2-b^2}}=\frac{a^2-b^2}{\sqrt{a^2-b^2}}=$

$=\frac{\sqrt{(a^2-b^2)^2}}{\sqrt{a^2-b^2}}=\sqrt{a^2-b^2}.$

304. $\left(\frac{1}{\sqrt{1+x}}+\sqrt{1-x}\right):\left(\frac{1}{\sqrt{1-x^2}}+1\right)=$

$=\frac{(1+\sqrt{1-x^2})}{\sqrt{1+x}}:\frac{(1+\sqrt{1-x^2})}{\sqrt{1-x^2}}=\frac{\sqrt{1-x^2}}{\sqrt{1+x}}=\frac{\sqrt{(1+x)(1-x)}}{\sqrt{1+x}}=$

$=\sqrt{1-x}.$

305. $\frac{a(x+a+\sqrt{x^2-a^2})}{(x+a)-\sqrt{x^2-a^2}}=\frac{a(x+a+\sqrt{x^2-a^2})(x+a+\sqrt{x^2-a^2})}{[(x+a)-\sqrt{x^2-a^2}][(x+a)+\sqrt{x^2-a^2}]}=$

$=\frac{a(x+a+\sqrt{x^2-a^2})^2}{(x+a)^2-(x^2-a^2)}=\frac{a[\sqrt{(x+a)^2}+\sqrt{(x+a)(x-a)}]^2}{x^2+2ax+a^2-x^2+a^2}=$

$$=\frac{a\sqrt{(x+a)^2}[\sqrt{x+a}+\sqrt{x-a}]^2}{2a(a+x)}=\frac{a(x+a)[x+a+2\sqrt{x^2-a^2}+x-a]}{2a(a+x)}=$$

$$=\frac{2x+2\sqrt{x^2-a^2}}{2}=x+\sqrt{x^2-a^2}.$$

306. $$\frac{a+\sqrt{a^2-x^2}}{a-\sqrt{a^2-x^2}}-\frac{a-\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}=$$

$$=\frac{(a+\sqrt{a^2-x^2})^2-(a-\sqrt{a^2-x^2})^2}{(a-\sqrt{a^2-x^2})(a+\sqrt{a^2-x^2})}=$$

$$=\frac{a^2+2a\sqrt{a^2-x^2}+(a^2-x^2)-a^2+2a\sqrt{a^2-x^2}-(a^2-x^2)}{a^2-(a^2-x^2)}=$$

$$=\frac{4a\sqrt{a^2-x^2}}{x^2}.$$

307. $$\sqrt{\frac{a+x^2}{2x}-\sqrt{a}}+\sqrt{\frac{a+x^2}{2x}+\sqrt{a}}=\sqrt{\frac{a+x^2-2x\sqrt{a}}{2x}}+$$

$$+\sqrt{\frac{a+x^2+2x\sqrt{a}}{2x}}=\frac{\sqrt{(\sqrt{a}-x)^2}+\sqrt{(\sqrt{a}+x)^2}}{\sqrt{2x}}=$$

$$=\frac{\sqrt{a}-x+\sqrt{a}+x}{\sqrt{2x}}=\frac{2\sqrt{a}}{\sqrt{2x}}=\frac{2\sqrt{2ax}}{2x}=\frac{\sqrt{2ax}}{x}=\frac{1}{x}\sqrt{2ax}.$$

308. $$\sqrt{\frac{3x+a^3}{2a}+\sqrt{3ax}}-\sqrt{\frac{3x+a^3}{2a}-\sqrt{3ax}}=$$

$$=\sqrt{\frac{3x+a^3+2a\sqrt{3ax}}{2a}}-\sqrt{\frac{3x+a^3-2a\sqrt{3ax}}{2a}}=$$

$$=\sqrt{\frac{(\sqrt{3x})^2+(a\sqrt{a})^2+2a\sqrt{3ax}}{2a}}-$$

$$-\sqrt{\frac{(\sqrt{3x})^2+(a\sqrt{a})^2-2a\sqrt{3ax}}{2a}}=$$

$$=\frac{\sqrt{(\sqrt{3x}+a\sqrt{a})^2}-\sqrt{(\sqrt{3x}-a\sqrt{a})^2}}{\sqrt{2a}}=$$

$$=\frac{\sqrt{3x}+a\sqrt{a}-\sqrt{3x}+a\sqrt{a}}{\sqrt{2a}}=\frac{2a\sqrt{a}}{\sqrt{2a}}=\frac{2a\sqrt{2a^2}}{2a}=$$

$$=\sqrt{2a^2}=a\sqrt{2}.$$

309. $\left(\sqrt[3]{3-\sqrt[4]{5}}-\sqrt[3]{\sqrt[4]{5}-3}\right)\cdot\sqrt[3]{9-\sqrt{5}}=$

$=\left(\sqrt[3]{3-\sqrt[4]{5}}+\sqrt[3]{3-\sqrt[4]{5}}\right)\cdot\sqrt[3]{9-\sqrt{5}}=$

$=2\sqrt[3]{3-\sqrt[4]{5}}\cdot\sqrt[3]{9-\sqrt{5}}=2\sqrt[3]{(3-\sqrt[4]{5})(9-\sqrt{5})}=$

$=2\sqrt[3]{(3-\sqrt[4]{5})(3+\sqrt[4]{5})(3-\sqrt[4]{5})}=$

$=2\sqrt[3]{(3-\sqrt[4]{5})^2(3+\sqrt[4]{5})}.$

310. $\left(\sqrt[6]{9-4\sqrt{2}}+\sqrt[3]{\frac{1}{8}-\frac{\sqrt{2}}{4}}\right)\cdot\sqrt[3]{1+2\sqrt{2}}=$

$=\left(\sqrt[6]{9-4\sqrt{2}}+\frac{1}{2}\sqrt[3]{1-2\sqrt{2}}\right)\cdot\sqrt[3]{1+2\sqrt{2}}=$

$=\sqrt[6]{9-4\sqrt{2}}\cdot\sqrt[3]{1+2\sqrt{2}}+\frac{1}{2}\sqrt[3]{1-2\sqrt{2}}\cdot$

$\cdot\sqrt[3]{1+2\sqrt{2}}=\sqrt[6]{9-4\sqrt{2}}\cdot\sqrt[6]{(1+2\sqrt{2})^2}+$

$+\frac{1}{2}\sqrt[3]{(1-2\sqrt{2})(1+2\sqrt{2})}=$

$=\sqrt[6]{(9-4\sqrt{2})(1+4\sqrt{2}+8)}+\frac{1}{2}\sqrt{(1-8)}=$

$=\sqrt[6]{(9-4\sqrt{2})(9+4\sqrt{2})}+\frac{1}{2}\sqrt[3]{-7}=\sqrt[6]{81-32}-$

$-\frac{1}{2}\sqrt[3]{7}=\sqrt[6]{49}-\frac{1}{2}\sqrt[3]{7}=\sqrt[3]{7}-\frac{1}{2}\sqrt[3]{7}=\frac{1}{2}\sqrt[3]{7}.$

311. $5a\sqrt{a\sqrt{a\sqrt{a}}}-2\sqrt{a^3\sqrt[4]{a^3}}+3\sqrt[-2]{a^{-5}\sqrt[4]{a^5}}-$

$-4a^2\sqrt[-4]{a\sqrt{\frac{1}{a}}}=5a\sqrt{\sqrt{a^3\sqrt{a}}}-2\sqrt{\sqrt[4]{a^{15}}}+3\sqrt[-2]{\sqrt[4]{a^{-15}}}-$

$-4a^2\sqrt[-4]{\sqrt{a}}=5a\sqrt{\sqrt{\sqrt{a^7}}}-2\sqrt[8]{a^{15}}+3\sqrt[-8]{a^{-15}}-$

$-4a^2\sqrt[-8]{a}=5a\sqrt[8]{a^7}-2a\sqrt[8]{a^7}+3\sqrt[8]{a^{15}}-4a\sqrt[-8]{a^{-7}}=$

$=5a\sqrt[8]{a^7}-2a\sqrt[8]{a^7}+3a\sqrt[8]{a^7}-4a\sqrt[8]{a^7}=2a\sqrt[8]{a^7}.$

312. $$\left(-4a\sqrt[3]{a^{-2}\sqrt{ax}}\right)^3+\left(-10a\sqrt{x}\cdot\sqrt[4]{\frac{1}{ax}}\right)^2-$$
$$-\left[5\left(\sqrt[3]{a\sqrt[-4]{\frac{a}{x}}}\right)^3\right]^2=-64a^3\left(\sqrt[3]{a^{-2}\sqrt{ax}}\right)^3+$$
$$+100a^2x\left(\sqrt[4]{\frac{1}{ax}}\right)^2-25\left(\sqrt[3]{a\sqrt[-4]{\frac{a}{x}}}\right)^6=-64a^3\cdot a^{-2}\sqrt{ax}+$$
$$+100a^2x\sqrt{\frac{1}{ax}}-25\left(a\sqrt[-4]{\frac{a}{x}}\right)^2=-64a\sqrt{ax}+100a\sqrt{ax}-$$
$$-25\left(a\sqrt[4]{\frac{x}{a}}\right)^2=36a\sqrt{ax}-25a^2\sqrt{\frac{x}{a}}=36a\sqrt{ax}-$$
$$-25a\sqrt{ax}=11a\sqrt{ax}.$$

313. $$\left\{\sqrt[12]{\left[\left(-\frac{a}{b}\right)^3\right]^{-4}}\cdot\sqrt[5]{\frac{a^2}{b^3}}:\sqrt[3]{\frac{a^3}{b^2}}\right\}^6=\sqrt{\left[\left(-\frac{a}{b}\right)^3\right]^{-4}}\cdot$$
$$\cdot\sqrt[5]{\frac{a^{12}}{b^{18}}}:\sqrt[3]{\frac{a^{18}}{b^{12}}}=\left[\left(-\frac{a}{b}\right)^3\right]^{-2}\cdot\frac{a^2}{b^3}\sqrt[5]{\frac{a^2}{b^3}}:\frac{a^6}{b^4}=\left(-\frac{a}{b}\right)^{-6}\cdot$$
$$\cdot\frac{a^2}{b^4}\sqrt[5]{a^2b^2}:\frac{a^6}{b^4}=\left(-\frac{b}{a}\right)^6\cdot\frac{a^2}{b^4}\sqrt[5]{a^2b^2}:\frac{a^6}{b^4}=\frac{b^6}{a^6}\cdot\frac{a^2}{b^4}\sqrt[5]{a^2b^2}:\frac{a^6}{b^4}=$$
$$=\frac{b^2}{a^4}\sqrt[5]{a^2b^2}:\frac{a^6}{b^4}=\frac{b^6\sqrt[5]{a^2b^2}}{a^{10}}.$$

314. $$\left[\left(x\sqrt{\frac{a}{b^2a}}-\frac{x}{\sqrt{bx}}\right):\frac{\sqrt{x}}{b}-\sqrt{a}\right]:\sqrt[n]{\frac{1}{b^{-m}}}=$$
$$=\left[\left(\frac{1}{b}\sqrt{ax}-\frac{1}{b}\sqrt{bx}\right):\frac{\sqrt{x}}{b}-\sqrt{a}\right]:\sqrt[n]{b^m}=$$
$$=[(\sqrt{ax}-\sqrt{bx}):\sqrt{x}-\sqrt{a}]:\sqrt[n]{b^m}=[(\sqrt{a}-\sqrt{b})-\sqrt{a}]:$$
$$:\sqrt[n]{b^m}=-\sqrt{b}:\sqrt[n]{b^m}=-\frac{\sqrt{b}}{\sqrt[n]{b^m}}=-\frac{\sqrt[2n]{b^n}}{\sqrt[2n]{b^{2m}}}=-\sqrt[2n]{\frac{b^n}{b^{2m}}}=$$
$$=-\sqrt[2n]{b^{n-2m}}.$$

315. $$\sqrt[-2]{\frac{1}{4}a^2\sqrt{\frac{a}{x}}}\cdot\left[\frac{a}{2\sqrt{2}}\cdot\frac{a}{2\sqrt{x}}:\left(\sqrt[3]{\frac{a^2}{x}}\cdot a^{-1}\sqrt{x}\right)^6\right]=$$
$$=\sqrt[-2]{\sqrt{\frac{a^5}{16x}}}\cdot\left[\frac{a^2}{4\sqrt{ax}}:\left(\frac{a^4}{x^2}\cdot a^{-6}\cdot x^3\right)\right]=\sqrt[-4]{\frac{a^5}{16x}}\cdot\frac{a^2}{4\sqrt{ax}}:\frac{x}{a^2}=$$

$$= \sqrt[4]{\frac{16x}{a^5}} \cdot \frac{a^4}{4x\sqrt{ax}} = \frac{2}{a^2}\sqrt[5]{xa^3} \cdot \frac{a^4}{4x\sqrt{ax}} = \frac{a^2\sqrt[4]{a^3x}}{2x\sqrt{ax}} =$$

$$= \frac{a^2\sqrt[4]{a^3x}\cdot\sqrt{ax}}{2ax^2} = \frac{a^2\sqrt[4]{a^5x^3}}{2ax^2} = \frac{a^3\sqrt[4]{ax^3}}{2ax^2} = \frac{a^2}{2x^2}\sqrt[4]{ax^3}.$$

316. $$\left[\sqrt[6]{\frac{(1-a)^3\sqrt{1+a}}{a^3}}\cdot\sqrt[3]{\frac{3a^2}{4-8a+4a^2}}\right]^{-1}:$$

$$:\sqrt[-3]{\frac{3a\sqrt{a}}{2\sqrt{1-a^2}}} = \left[\sqrt[6]{\frac{(1-a)^3(1+a)}{a^3}}\cdot\sqrt[6]{\frac{9a^4}{4^2(1-a)^4}}\right]^{-1}:$$

$$:\sqrt[3]{\frac{2\sqrt{1-a^2}}{3a\sqrt{a}}} = \left[\sqrt[6]{\frac{(1-a)^3(1+a).9a^4}{16a^3(1-a)^4}}\right]^{-1}:\sqrt[3]{\frac{2\sqrt{1-a^2}}{3a\sqrt{a}}} =$$

$$= \left[\sqrt[6]{\frac{9a(1+a)}{16(1-a)}}\right]^{-1}:\sqrt[6]{\frac{4(1-a^2)}{9a^3}} = \sqrt[6]{\frac{16(1-a)}{9a(1+a)}}:$$

$$= \sqrt[6]{\frac{9a^3.16(1-a)}{9a.4(1+a)^2(1-a)}} = \sqrt[6]{\frac{4a^2}{(1+a)^2}} = \sqrt[3]{\frac{2a}{1+a}} =$$

$$= \frac{1}{1-a}\sqrt[3]{2a(1+a)^2}.$$

317. $$\sqrt{3+\sqrt{5+\sqrt{3}-\sqrt{27+8\sqrt{4-2\sqrt{3}}}}} =$$

$$= \sqrt{3+\sqrt{5+\sqrt{3}-\sqrt{27+8\left(\sqrt{\frac{4+\sqrt{16-12}}{2}}-\sqrt{\frac{4-\sqrt{16-12}}{2}}\right)}}} =$$

$$= \sqrt{3+\sqrt{5+\sqrt{3}-\sqrt{27+8(\sqrt{3}-1)}}} =$$

$$= \sqrt{3+\sqrt{5+\sqrt{3}-\sqrt{19+8\sqrt{3}}}} =$$

$$=\sqrt{3+\sqrt{5+\sqrt{3}-\left(\sqrt{\frac{19+\sqrt{361-192}}{2}}+\sqrt{\frac{19-\sqrt{361-192}}{2}}\right)}}=$$

$$=\sqrt{3+\sqrt{5+\sqrt{3}-4-\sqrt{3}}}=\sqrt{3+\sqrt{1}}=\sqrt{3+1}=2.$$

318.

$$\sqrt{6+2\sqrt{2}.\sqrt{3-\sqrt{\sqrt{2}+\sqrt{12}+\sqrt{18-\sqrt{128}}}}}=$$

$$=\sqrt{6+2\sqrt{2}.\sqrt{3-\sqrt{\sqrt{2}+\sqrt{12}+\left(\sqrt{\frac{18+\sqrt{324-128}}{2}}-\sqrt{\frac{18-\sqrt{324-128}}{2}}\right)}}}=$$

$$=\sqrt{6+2\sqrt{2}\sqrt{3-\sqrt{\sqrt{2}+\sqrt{12}+4-\sqrt{2}}}}=$$

$$=\sqrt{6+2\sqrt{2}\sqrt{3-\sqrt{4+\sqrt{12}}}}=$$

$$=\sqrt{6+2\sqrt{2}\sqrt{3-\left(\sqrt{\frac{4+\sqrt{16-12}}{2}}+\sqrt{\frac{4-\sqrt{16-12}}{2}}\right)}}=$$

$$=\sqrt{6+2\sqrt{2}\sqrt{3-\sqrt{3}-1}}=\sqrt{6+2\sqrt{2}\sqrt{2-\sqrt{3}}}=$$

$$=\sqrt{6+2\sqrt{2}\left(\sqrt{\frac{2+\sqrt{4-3}}{2}}-\sqrt{\frac{2-\sqrt{4-3}}{2}}\right)}=$$

$$=\sqrt{6+2\sqrt{2}\,.\,\frac{\sqrt{3}-1}{\sqrt{2}}}=\sqrt{6+2\sqrt{3}-2}=\sqrt{4+2\sqrt{3}}=$$

$$=\sqrt{\frac{4+\sqrt{16-12}}{2}}+\sqrt{\frac{4-\sqrt{16-12}}{2}}=\sqrt{3}+1.$$

319. $$\frac{1+x}{1+\sqrt{1+x}}+\frac{1-x}{1-\sqrt{1-x}}=\frac{(1+x)(1-\sqrt{1+x})}{(1+\sqrt{1+x})(1-\sqrt{1+x})}+$$

$$+\frac{(1-x)(1+\sqrt{1-x})}{(1-\sqrt{1-x})(1+\sqrt{1-x})}=\frac{(1+x)(1-\sqrt{1+x})}{1-(1+x)}+\frac{(1-x)(1+\sqrt{1-x})}{1-(1-x)}=$$

$$=\frac{1-\sqrt{1+x}+x-x\sqrt{1+x}}{-x}+\frac{1+\sqrt{1-x}-x-x\sqrt{1-x}}{x}=$$

$$=\frac{-1+\sqrt{1+x}-x+x\sqrt{1+x}+1+\sqrt{1-x}-x-x\sqrt{1-x}}{x}=$$

$$=\frac{-2x+(1+x)\sqrt{1+x}+(1-x)\sqrt{1-x}}{x}=\frac{-2x+\sqrt{(1+x)^3}+\sqrt{(1-x)^3}}{x}=$$

$$=\frac{-\sqrt{3}+\sqrt{\left(1+\frac{\sqrt{3}}{2}\right)^3}+\sqrt{\left(1-\frac{\sqrt{3}}{2}\right)^3}}{\frac{\sqrt{3}}{2}}=$$

$$=\frac{-2\sqrt{3}+2\sqrt{\frac{(2+\sqrt{3})^3}{8}}+2\sqrt{\frac{(2-\sqrt{3})^3}{8}}}{\sqrt{3}}=$$

$$=\frac{-2\sqrt{3}+\sqrt{\frac{(2+\sqrt{3})^3}{2}}+\sqrt{\frac{(2-\sqrt{3})^3}{2}}}{\sqrt{3}}=$$

$$=\frac{-2\sqrt{6}+\sqrt{(2+\sqrt{3})^3}+\sqrt{(2-\sqrt{3})^3}}{\sqrt{6}}=$$

$$=\frac{-2\sqrt{6}+\sqrt{26+15\sqrt{3}}+\sqrt{26-15\sqrt{3}}}{\sqrt{6}}=$$

$$=\frac{-2\sqrt{6}+\sqrt{\frac{26+\sqrt{676-675}}{2}}+\sqrt{\frac{26-\sqrt{676-675}}{2}}+\sqrt{\frac{26+\sqrt{676-675}}{2}}-\sqrt{\frac{26-\sqrt{676-675}}{2}}}{\sqrt{6}}=$$

$$= \frac{-2\sqrt{6} + \sqrt{\tfrac{27}{2}} + \sqrt{\tfrac{25}{2}} + \sqrt{\tfrac{27}{2}} - \sqrt{\tfrac{25}{2}}}{\sqrt{6}} = \frac{-2\sqrt{6} + 2\sqrt{\tfrac{27}{2}}}{\sqrt{6}} =$$

$$= \frac{-2\sqrt{6} + \sqrt{54}}{\sqrt{6}} = \frac{-2\sqrt{6} + 3\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{\sqrt{6}} = 1.$$

320. $$\frac{2a\sqrt{1+x^2}}{x+\sqrt{1+x^2}} = \frac{2a\sqrt{1+x^2}(x-\sqrt{1+x^2})}{(x+\sqrt{1+x^2})(x-\sqrt{1+x^2})} =$$

$$= \frac{2ax\sqrt{1+x^2} - 2a\sqrt{(1+x^2)^2}}{x^2-(1+x^2)} = \frac{2ax\sqrt{1+x^2} - 2a(1+x^2)}{x^2-1-x^2} =$$

$= 2a(1+x^2) - 2ax\sqrt{1+x^2}$. x'i asemele $\frac{1}{2}\left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right)$ ehk $\frac{1}{2}\left(\frac{a-b}{\sqrt{ab}}\right)$ pannes saame: $2a(1+x^2) - 2ax\sqrt{1+x^2} =$

$$= 2a\left[1 + x^2 - x\sqrt{1+x^2}\right] =$$

$$= 2a\left[1 + \tfrac{1}{4}\cdot\frac{(a-b)^2}{ab} - \tfrac{1}{2}\cdot\frac{a-b}{\sqrt{ab}}\sqrt{1+\frac{(a-b)^2}{4ab}}\right] =$$

$$= 2a\left[1 + \frac{(a-b)^2}{4ab} - \frac{a-b}{2\sqrt{ab}}\cdot\sqrt{\frac{4ab+a^2-2ab+b^2}{4ab}}\right] =$$

$$= 2a\left[\frac{4ab+a^2+b^2-2ab}{4a} - \frac{a-b}{2\sqrt{ab}}\sqrt{\frac{(a+b)^2}{4ab}}\right] =$$

$$= 2a\left[\frac{(a+b)^2}{4ab} - \frac{(a-b)(a+b)}{2\sqrt{ab}\cdot 2\sqrt{ab}}\right] =$$

$$= 2a\left[\frac{(a+b)^2}{4ab} - \frac{(a-b)(a+b)}{4ab}\right] = 2a\,\frac{(a+b)(a+b-a+b)}{4ab} =$$

$$= \frac{4ab(a+b)}{4ab} = a+b.$$

§ 11. Murruliste näitajatega astmed ja juured.

321. $\sqrt[3]{a^2} = a^{\frac{2}{3}}$. **322.** $\sqrt[4]{a^{-3}} = a^{-\frac{3}{4}}$.

323. $\sqrt[5]{a^{-3}b^4} = a^{-\frac{3}{5}}b^{\frac{4}{5}}$. **324.** $\sqrt[-2]{a^{-3}} = a^{\frac{-3}{-2}} = a^{\frac{3}{2}}$.

325. $\sqrt{a^2+b^2} = (a^2+b^2)^{\frac{1}{2}}$.

326. $\sqrt[-3]{\frac{a^3-b^3}{a^{-1}b^2}}=\frac{(a^3-b^3)^{-\frac{1}{3}}}{a^{\frac{1}{3}}b^{-\frac{2}{3}}}=\frac{b^{\frac{2}{3}}}{a^{\frac{1}{3}}(a^3-b^3)^{\frac{1}{3}}}.$

327. $a^{\frac{5}{6}}=\sqrt[6]{a^5}.$ **328.** $a^{-\frac{3}{4}}=\sqrt[4]{a^{-3}}.$

329. $(a+b)^{\frac{2}{3}}=\sqrt[3]{(a+b)^2}.$

330. $3a^{-\frac{1}{2}}(a-b)^{\frac{3}{8}}=3\sqrt{a^{-1}}\sqrt[8]{(a-b)^3}=3\sqrt{\frac{1}{a}}\cdot$
$\cdot\sqrt[8]{(a-b)^3}=\frac{3\sqrt{a}}{a}\sqrt[8]{(a-b)^3}.$

331. $4^{\frac{1}{2}}=\sqrt{4}=\pm 2.$

332. $81^{\frac{3}{4}}=\sqrt[4]{81^3}=\sqrt[4]{3^{12}}=3^3=27.$

333. $16^{-\frac{5}{4}}=\sqrt[4]{16^{-5}}=\sqrt[4]{\frac{1}{16^5}}=\frac{1}{16}\sqrt[4]{\frac{1}{16}}=\frac{1}{32}.$

334. $(-8)^{\frac{2}{3}}=\sqrt[3]{(-8)^2}=\sqrt[3]{64}=4.$

335. $\left(\frac{25}{36}\right)^{-\frac{1}{2}}=\sqrt{\left(\frac{25}{36}\right)^{-1}}=\sqrt{\frac{36}{25}}=\frac{6}{5}=1\frac{1}{5}.$

336. $\left(-3\frac{3}{8}\right)^{-\frac{2}{3}}=\left(-\frac{27}{8}\right)^{-\frac{2}{3}}=\sqrt[3]{\left(-\frac{27}{8}\right)^{-2}}=$
$=\sqrt[3]{\left(-\frac{8}{27}\right)^2}=\sqrt[3]{\left(\frac{8}{27}\right)^2}=\sqrt[3]{\frac{2^6}{3^6}}=\frac{2^2}{3^2}=\frac{4}{9}.$

337. $(0,64)^{0,5}=(0,64)^{\frac{1}{2}}=\sqrt{0,64}=0,8.$

338. $81^{-0,75}=81^{-\frac{3}{4}}=\sqrt[4]{81^{-3}}=\sqrt[4]{\frac{1}{81^3}}=\sqrt[4]{\frac{1}{3^{12}}}=\frac{1}{3^3}=\frac{1}{27}.$

339. $8^{\frac{2}{3}}-16^{\frac{1}{4}}+9^{\frac{1}{2}}=\sqrt[3]{8^2}-\sqrt[4]{16}+\sqrt{9}=4-2+3=5.$

340. $16^{0,5}+\left(\frac{1}{16}\right)^{-0,75}-\left(\frac{1}{2}\right)^{-6}=16^{\frac{1}{2}}+\left(\frac{1}{16}\right)^{-\frac{3}{4}}-2^6=$
$=\sqrt{16}=16^{\frac{3}{4}}-2^6=4+\sqrt[4]{16^3}-2^6=4+8-64=-52.$

341. $a^{\frac{2}{3}}b^{\frac{3}{5}}\cdot a^{\frac{3}{4}}b^{\frac{2}{3}}=a^{\frac{2}{3}+\frac{3}{4}}\cdot b^{\frac{3}{5}+\frac{2}{3}}=a^{1\frac{5}{12}}b^{1\frac{4}{15}}=\sqrt[12]{a^{17}}\cdot$
$\cdot\sqrt[15]{b^{19}}=ab\sqrt[12]{a^5}\cdot\sqrt[15]{b^4}=ab\sqrt[60]{a^{25}b^{16}}.$

342. $a^{\frac{7}{12}}b^{\frac{5}{6}} : a^{\frac{2}{3}}b^{\frac{3}{4}} = a^{\frac{7}{12}-\frac{2}{3}} . b^{\frac{5}{6}-\frac{3}{4}} = a^{-\frac{1}{12}}b^{\frac{1}{12}} = \sqrt[12]{a^{-1}b} = \sqrt[12]{\frac{b}{a}}$.

343. $\left(a^{\frac{3}{2}} + b^{\frac{3}{2}}\right)\left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right) = a^{\frac{3}{2}+\frac{1}{2}} - a^{\frac{3}{2}}b^{\frac{1}{2}} + b^{\frac{3}{2}}a^{\frac{1}{2}} - b^{\frac{3}{2}+\frac{1}{2}} = a^2 - \sqrt{a^3b} + \sqrt{ab^3} - b^2 = a^2 - a\sqrt{ab} + b\sqrt{ab} - b^2 = a^2 - b^2 (a - b)\sqrt{ab} = (a + b)(a - b) - (a - b)\sqrt{ab} = (a - b)(a + b - \sqrt{ab})$.

344. $\left(a^{\frac{1}{2}} + a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}} - a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}}\right) = \left[\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right) + a^{\frac{1}{4}}b^{\frac{1}{4}}\right] . \left[\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right) - a^{\frac{1}{4}}b^{\frac{1}{4}}\right] = \left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right)^2 - \left(a^{\frac{1}{4}}b^{\frac{1}{4}}\right)^2 = a + 2a^{\frac{1}{2}}b^{\frac{1}{2}} + b - a^{\frac{1}{2}}b^{\frac{1}{2}} = a + a^{\frac{1}{2}}b^{\frac{1}{2}} + b = a + b + \sqrt{ab}$.

345.

$$
\begin{array}{l|l}
a^{\frac{2}{3}} - b^{-\frac{5}{4}} & a^{\frac{2}{9}} - b^{-\frac{5}{12}} \\
\mp a^{\frac{2}{3}} \pm a^{\frac{4}{9}}b^{-\frac{5}{12}} & a^{\frac{4}{9}} + a^{\frac{2}{9}}b^{-\frac{5}{12}} + b^{-\frac{5}{6}} = \sqrt[9]{a^4} + \\
a^{\frac{4}{9}}b^{-\frac{5}{12}} - b^{-\frac{5}{4}} & + \sqrt[9]{a^2} . \sqrt[12]{\frac{1}{b^5}} + \sqrt[6]{\frac{1}{b^5}} = \\
\mp a^{\frac{4}{9}}b^{-\frac{5}{12}} \pm a^{\frac{2}{9}}b^{-\frac{5}{6}} & = \sqrt[9]{a^4} . \frac{\sqrt[9]{a^2}}{\sqrt[12]{b^5}} + \frac{1}{\sqrt[6]{b^5}}. \\
a^{\frac{2}{9}}b^{-\frac{5}{6}} - b^{-\frac{5}{4}} & \\
\mp a^{\frac{2}{9}}b^{-\frac{5}{6}} \pm b^{-\frac{5}{4}} & \\
0 &
\end{array}
$$

346. $\left(a^{\frac{3n}{2}} + b^{-\frac{3n}{2}}\right) : \left(a^{\frac{n}{2}} + b^{-\frac{n}{2}}\right) = \left[\left(a^{\frac{n}{2}}\right)^3 + \left(b^{-\frac{n}{2}}\right)^3\right] : \left(a^{\frac{n}{2}} + b^{-\frac{n}{2}}\right) = a^n - a^{\frac{n}{2}}b^{-\frac{n}{2}} = a^n - \sqrt{\frac{a^n}{b^n}} + \frac{1}{b^n}$.

347. $(a^{\frac{4}{3}} + 4a^{\frac{2}{3}}b^{\frac{2}{3}} + 16b^{\frac{4}{3}}) : (a^{\frac{2}{3}} + 2a^{\frac{1}{3}}b^{\frac{1}{3}} + 4b^{\frac{2}{3}}) =$

$$
\begin{array}{l|l}
= a^{\frac{4}{3}} + 4a^{\frac{2}{3}}b^{\frac{2}{3}} + 16b^{\frac{4}{3}} & a^{\frac{2}{3}} + 2a^{\frac{1}{3}}b^{\frac{1}{3}} + 4b^{\frac{2}{3}} \\
\mp a^{\frac{4}{3}} \mp 2ab^{\frac{1}{3}} \mp 4a^{\frac{2}{3}}b^{\frac{2}{3}} & a^{\frac{2}{3}} - 2a^{\frac{1}{3}}b^{\frac{1}{3}} + 4b^{\frac{2}{3}} = \\
-2ab^{\frac{1}{3}} & = \sqrt[3]{a^2} - 2\sqrt[3]{ab} + 4\sqrt[3]{b^2}. \\
\pm 2ab^{\frac{1}{3}} \pm 4a^{\frac{2}{3}}b^{\frac{2}{3}} \pm 8a^{\frac{1}{3}}b & \\
4a^{\frac{2}{3}}b^{\frac{2}{3}} + 8a^{\frac{1}{3}}b + 16b^{\frac{4}{3}} & \\
\mp 4a^{\frac{2}{3}}b^{\frac{2}{3}} \mp 8a^{\frac{1}{3}}b \mp 16b^{\frac{4}{3}} & \\
0 &
\end{array}
$$

348. Märkus. Avaldusi $a^{\frac{1}{2}}$ ja $b^{\frac{1}{2}}$ võib kirjutada ka nii $\left(a^{\frac{1}{4}}\right)^2$, ning $\left(b^{\frac{1}{4}}\right)^2$, sest $\left(a^{\frac{1}{4}}\right)^2 = a^{\frac{1}{4}\cdot 2} = a^{\frac{1}{2}}$;

$$\left(a^{\frac{1}{2}} - b^{\frac{1}{2}} - c^{\frac{1}{2}} + 2b^{\frac{1}{4}}c^{\frac{1}{4}}\right) : \left(a^{\frac{1}{4}} + b^{\frac{1}{4}} - c^{\frac{1}{4}}\right) =$$

$$= \left[a^{\frac{1}{2}} - \left(b^{\frac{1}{2}} + c^{\frac{1}{2}} 2b^{\frac{1}{4}}c^{\frac{1}{4}}\right)\right] : \left(a^{\frac{1}{4}} + b^{\frac{1}{4}} - c^{\frac{1}{4}}\right) =$$

$$= \left[\left(a^{\frac{1}{4}}\right)^2 - \left(b^{\frac{1}{2}} + c^{\frac{1}{2}} - 2b^{\frac{1}{4}}c^{\frac{1}{4}}\right)\right] : \left(a^{\frac{1}{4}} + b^{\frac{1}{4}} - c^{\frac{1}{4}}\right) =$$

$$= \left[\left(a^{\frac{1}{4}}\right)^2 - \left(b^{\frac{1}{4}} - c^{\frac{1}{4}}\right)^2\right] : \left(a^{\frac{1}{4}} + b^{\frac{1}{4}} - c^{\frac{1}{4}}\right) =$$

$$= \left(a^{\frac{1}{4}} + b^{\frac{1}{4}} - c^{\frac{1}{4}}\right)\left(a^{\frac{1}{4}} - b^{\frac{1}{4}} + c^{\frac{1}{4}}\right) : \left(a^{\frac{1}{4}} + b^{\frac{1}{4}} - c^{\frac{1}{4}}\right) =$$

$$= a^{\frac{1}{4}} - b^{\frac{1}{4}} + c^{\frac{1}{4}} = \sqrt[4]{a} - \sqrt[4]{b} + \sqrt[4]{c}.$$

349. $\left(a^{\frac{3}{2}} - a^{\frac{1}{6}}b^{\frac{1}{2}} + b^{\frac{3}{2}}\right)^2 = \left(a^{\frac{3}{2}}\right)^2 + \left(-a^{\frac{1}{6}}b^{\frac{1}{2}}\right)^2 + \left(b^{\frac{3}{2}}\right)^2 - 2 \,.\, a^{\frac{3}{2}} \,.\, a^{\frac{1}{6}}b^{\frac{1}{2}} + 2a^{\frac{3}{2}}b^{\frac{3}{2}} - 2a^{\frac{1}{6}} \,.\, b^{\frac{1}{2}} \,.\, b^{\frac{3}{2}} = a^3 + a^{\frac{1}{3}}b + b^3 - 2a^{\frac{5}{3}}b^{\frac{1}{2}} + 2a^{\frac{3}{2}}b^{\frac{3}{2}} - 2a^{\frac{1}{6}}b^2 = a^3 + b\sqrt[3]{a} + b^3 - 2a\sqrt[6]{a^4b^3} + 2ab\sqrt{ab} - 2b^2\sqrt[6]{a}.$

350. $\left(a^{\frac{1}{2}}b^{\frac{2}{3}} - 2a^{\frac{2}{3}}b^{\frac{1}{4}}\right)^3 = \left(a^{\frac{1}{2}}b^{\frac{2}{3}}\right)^3 - 3 \,.\, \left(a^{\frac{1}{2}}b^{\frac{2}{3}}\right)^2 \,.\, \left(2a^{\frac{2}{3}}b^{\frac{1}{4}}\right) + 3\left(a^{\frac{1}{2}}b^{\frac{2}{3}}\right) \,.\, \left(2a^{\frac{2}{3}}b^{\frac{1}{4}}\right)^2 - \left(2a^{\frac{2}{3}}b^{\frac{1}{4}}\right)^3 = a^{\frac{3}{2}}b^2 - 6a^{\frac{5}{3}}b^{\frac{19}{12}} + 12a^{\frac{11}{6}}b^{\frac{7}{6}} - 8a^2b^{\frac{3}{4}} = ab^2\sqrt{a} - 6ab\sqrt[12]{a^8b^7} + 12ab\sqrt[6]{a^5b} - 8a^2\sqrt[4]{b^3}.$

351. $\left[\left(a^{-\frac{3}{2}}b\right) . \left(ab^{-2}\right)^{-\frac{1}{2}} . \left(a^{-1}\right)^{-\frac{2}{3}}\right]^3 =$

$$= \left[a^{-\frac{3}{2}}b \,.\, a^{-\frac{1}{2}} \,.\, b \,.\, a^{\frac{2}{3}}\right]^3 = \left[a^{-\frac{3}{2}-\frac{1}{2}+\frac{2}{3}}b^2\right]^3 = \left(a^{-\frac{4}{3}}b^2\right)^3 =$$

$$= a^{-4}b^6 = \frac{b^6}{a^4}.$$

352. $\sqrt{\frac{3a^{-\frac{7}{2}}b^8}{a^{\frac{2}{3}}b^{-\frac{1}{2}}}\sqrt{4a^{-10}b^6\cdot\frac{1}{\left(a^{-\frac{1}{2}}b\right)^3}}}=$

$$=\sqrt{3a^{-\frac{7}{2}-\frac{2}{3}}b^{8\frac{1}{2}}\sqrt[4]{4a^{-10}b^6\cdot\left(a^{-\frac{1}{2}}\right)^{-3}}}=$$

$$=\sqrt{3a^{-\frac{25}{6}}a^{\frac{17}{2}}\sqrt[4]{4a^{-10}b^6\cdot a^{\frac{3}{2}}b^{-3}}}=$$

$$=\sqrt{3a^{-\frac{25}{6}}b^{\frac{17}{2}}\cdot\sqrt[4]{4a^{-\frac{17}{2}}b^3}}=$$

$$=\sqrt{3a^{-\frac{25}{6}}b^{\frac{17}{2}}\sqrt[4]{4}\cdot a^{-\frac{17}{8}}b^{\frac{3}{4}}}=\sqrt{3\sqrt{2}\cdot a^{-\frac{151}{24}}b^{\frac{37}{4}}}=$$

$$=\sqrt{3\sqrt{2}\cdot\sqrt[24]{a^{-151}}\sqrt[4]{b^{37}}}=\sqrt{3\sqrt{2}\cdot\frac{1}{\sqrt[24]{a^{151}}}\cdot b^9\sqrt[4]{b}}=$$

$$=\sqrt{\frac{3\sqrt{2}\cdot b^9\sqrt[4]{b}}{a^6\sqrt[24]{a^7}}}=\frac{b^4}{b^3}\sqrt{\frac{3\sqrt{2}\,b\sqrt[4]{b}}{\sqrt[24]{a^7}}}.$$

353. $\frac{a-b}{a^{\frac{1}{2}}-b^{\frac{1}{2}}}-\frac{a^{\frac{3}{2}}-b^{\frac{3}{2}}}{a-b}=\frac{a-b}{a^{\frac{1}{2}}b^{\frac{1}{2}}}-$

$$-\frac{a^{\frac{3}{2}}-b^{\frac{3}{2}}}{\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)}=\frac{(a-b)\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)-a^{\frac{3}{2}}+b^{\frac{3}{2}}}{\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)}=$$

$$=\frac{a^{\frac{3}{2}}-ba^{\frac{1}{2}}+ab^{\frac{1}{2}}-b^{\frac{3}{2}}-a^{\frac{3}{2}}+b^{\frac{3}{2}}}{\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)}=\frac{ab^{\frac{1}{2}}-ba^{\frac{1}{2}}}{\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)}=$$

$$=\frac{a^{\frac{1}{2}}b^{\frac{1}{2}}\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)}{\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)}=\frac{a^{\frac{1}{2}}b^{\frac{1}{2}}}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}=\frac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}}.$$

354. $\sqrt{a^{\frac32}b^{-2} - 6a^{\frac34}b^{-\frac13} + 9b^{\frac43}} =$
$= \sqrt{\left(a^{\frac34}b^{-1}\right)^2 - 2\,.\,3b^{\frac23}\,.\,a^{\frac34}b^{-1} + \left(3b^{\frac23}\right)^2} =$
$= \sqrt{\left(a^{\frac34}b^{-1} - 3b^{\frac23}\right)^2} = a^{\frac34}b^{-1} - 3b^{\frac23} = \frac{\sqrt[4]{a^3}}{b} -$
$- 3\sqrt[3]{b^2}.$

355. $\sqrt[-\frac23]{a\sqrt{b^3}\,.\,b^{-2}\sqrt[\frac12]{a^{\frac13}b}} = \sqrt[-\frac23]{ab^{\frac32}\,.\,b^{-2}a^{\frac23}b^2} =$
$= \sqrt[-\frac23]{a^{\frac53}b^{\frac32}} = \sqrt[-2]{a^5b^{\frac94}} = \frac{1}{\sqrt{a^5b^{\frac94}}} = \frac{1}{a^2b^2\sqrt{a}\sqrt[4]{b}}.$

356. $\sqrt[0,4]{\frac{a^{-2}b^3\sqrt{2a^6b^{-3}}}{\left(\sqrt{a^{-5}b^3}\right)^{\frac4{15}}}} = \sqrt[\frac25]{\frac{a^{-2}b^3\,2^{\frac12}a^3b^{-\frac32}}{\left(a^{-\frac52}b^{\frac32}\right)^{\frac45}}} =$
$= \sqrt[\frac25]{\frac{2^{\frac12}ab^{\frac32}}{\left(a^{-\frac52}b^{\frac32}\right)^{\frac4{15}}}} = \frac{2^{\frac54}a^{\frac52}b^{\frac{15}4}}{\left(a^{\frac52}b^{\frac32}\right)^{\frac23}} = \frac{2^{\frac54}a^{\frac52}b^{\frac{15}4}}{a^{-\frac53}b} = 2^{\frac54}a^{\frac{25}6}b^{\frac{11}4} =$
$= \sqrt[4]{2^5}\,.\,\sqrt[6]{a^{25}}\,.\,\sqrt[4]{b^{11}} = 2\sqrt[4]{2}\,.\,a^4\sqrt[6]{a}\,.\,b^2\sqrt[4]{b^3} =$
$= 2a^4b^2\sqrt[4]{2b^3}\,.\,\sqrt[6]{a}.$

357. $\left(\sqrt[\frac32]{\frac{\sqrt{a}}{b^2}} + \sqrt[\frac23]{b\sqrt[3]{a}}\right)^2 = \left(\sqrt[\frac32]{\frac{\sqrt{a}}{b^2}}\right)^2 +$
$+ \sqrt[\frac32]{\frac{\sqrt{a}}{b^2}}\,.\,\sqrt[\frac23]{b\sqrt[3]{a}} + \left(\sqrt[\frac23]{b\sqrt[3]{a}}\right)^2 = \sqrt[\frac32]{\frac{a}{b^4}} +$
$+ 2\sqrt[\frac32\cdot\frac23]{\left(\frac{\sqrt{a}}{b^2}\right)^{\frac23}}\,.\,\sqrt[\frac23\cdot\frac32]{\left(b\sqrt[3]{a}\right)^{\frac32}} + \sqrt[\frac23]{b^2\sqrt[3]{a^2}} = \frac{a^{\frac23}}{b^{\frac83}} +$

$$+2\left(\frac{\sqrt{a}}{b^2}\right)^{\frac{2}{3}} \cdot \left(b\sqrt[3]{a}\right)^{\frac{3}{2}} + b^3a = \sqrt[3]{\frac{a^2}{b^8}} + \frac{2a^{\frac{1}{3}}b^{\frac{3}{2}}a^{\frac{1}{2}}}{b^{\frac{4}{3}}} + b^3a =$$

$$= \frac{\sqrt[3]{a^2b}}{b^3} + 2a^{\frac{5}{6}}b^{\frac{1}{6}} + ab^3 = \frac{\sqrt[3]{a^2b}}{b^3} + 2\sqrt[6]{a^5b} + ab^3.$$

358. $\sqrt[\frac{2}{3}]{a^{\frac{4}{3}} + a - 2a^{\frac{7}{6}}} = \left(a^{\frac{4}{3}} + a - 2a^{\frac{7}{6}}\right)^{\frac{3}{2}} =$

$$= \sqrt{\left(a^{\frac{4}{3}} + a - 2a^{\frac{7}{6}}\right)^3} = \sqrt{\left[a\left(a^{\frac{1}{3}} + 1 - 2a^{\frac{1}{6}}\right)\right]^3} =$$

$$= a\left(a^{\frac{1}{3}} + 1 - 2a^{\frac{1}{6}}\right)\sqrt{a\left(a^{\frac{1}{3}} + 1 - 2a^{\frac{1}{6}}\right)} = a\left(a^{\frac{1}{6}} - 1\right)^2 \cdot$$

$$\cdot \sqrt{a\left(a^{\frac{1}{6}} - 1\right)^2} = a\left(a^{\frac{1}{6}} - 1\right)^3\sqrt{a} = a\sqrt{a}\left(a^{\frac{1}{2}} - 3a^{\frac{1}{3}} +\right.$$

$$\left.+ 3a^{\frac{1}{6}} - 1\right) = a\sqrt{a}\left(\sqrt{a} - 3\sqrt[3]{a} + 3\sqrt[3]{a} - 1\right) = a^2 -$$

$$- 3a\sqrt[3]{a} . \sqrt{a} + 3a\sqrt[6]{a} . \sqrt{a} - a\sqrt{a} = a^2 - 3a\sqrt[3]{a} \, .$$

$$. \sqrt{a} + 3a^2\sqrt[3]{a^2} - a\sqrt{a}.$$

359. $\left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right) : \left(\sqrt[\frac{2}{3}]{\frac{a\sqrt[3]{b}}{b\sqrt{a^3}}} : \sqrt[\frac{1}{2}]{\frac{\sqrt{a}}{a\sqrt[8]{b^3}}}\right) =$

$$= \left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right) : \left(\sqrt[\frac{2}{3}]{a^{1-\frac{3}{2}}b^{\frac{1}{3}-1}} + \sqrt[\frac{1}{2}]{a^{\frac{1}{2}-1}b^{-\frac{3}{8}}}\right) =$$

$$= \left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right) : \left(\sqrt[\frac{2}{3}]{a^{-\frac{1}{2}}b^{-\frac{2}{3}}} + \sqrt[\frac{1}{2}]{a^{-\frac{1}{2}}b^{-\frac{3}{8}}}\right) =$$

$$= \left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right) : \left(a^{-\frac{3}{4}}b^{-1} + a^{-1}b^{-\frac{3}{4}}\right) =$$

$$= \left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right) : a^{-1}b^{-1}\left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right) = \frac{1}{a^{-1}b^{-1}} = ab.$$

360. $\sqrt[\frac{3}{2}]{a^2b\sqrt{b} - 6a^{\frac{5}{3}}b^{\frac{5}{4}} + 12ab\sqrt[3]{a} - 8ab^{\frac{3}{4}}} =$

$$= \sqrt[\frac{3}{2}]{a^2b^{\frac{3}{2}} - 6a^{\frac{5}{3}}b^{\frac{5}{4}} + 12a^{\frac{4}{3}}b - 8ab^{\frac{3}{4}}} =$$

$$= \sqrt[\frac{3}{2}]{\left(a^{\frac{2}{3}}b^{\frac{1}{2}}\right)^3 - 3\left(a^{\frac{2}{3}}b^{\frac{1}{2}}\right)^2\left(2a^{\frac{1}{3}}b^{\frac{1}{4}}\right) + 3\,.\left(a^{\frac{2}{3}}b^{\frac{1}{2}}\right)\left(2a^{\frac{1}{3}}b^{\frac{1}{4}}\right)^2 - \left(2a^{\frac{1}{3}}b^{\frac{1}{4}}\right)^3} =$$

$$= \sqrt[\frac{3}{2}]{\left(a^{\frac{2}{3}}b^{\frac{1}{2}} - 2a^{\frac{1}{3}}b^{\frac{1}{4}}\right)^2} = \left(a^{\frac{2}{3}}b^{\frac{1}{2}} - 2a^{\frac{1}{3}}b^{\frac{1}{4}}\right)^2 =$$

$$= a^{\frac{4}{3}}b - 4ab^{\frac{3}{4}} + 4a^{\frac{2}{3}}b^{\frac{1}{2}} = ab\sqrt[3]{a} - 4a\sqrt[4]{b^3} + 4\sqrt[3]{a^2}\,.\sqrt{b}.$$

§ 12. Imaginaarsuurused.

361. $(\sqrt{-1})^6 = i^6 = i^2 = -1.$

362. $(\sqrt{-1})^{21} = i^{21} = i = \sqrt{-1}.$

363. $(\sqrt{-1})^7 = i^7 = i = -i = -\sqrt{-1}.$

364. $(\sqrt{-1})^{56} = i^{56} = i^4 = 1.$ **365.** $i^{40} = i^4 = 1.$

366. $i^{37} = i = \sqrt{-1}.$ **367.** $i^{68} = i^2 = -1.$

368. $i^{4n+2} = i^2 = -1.$

369. $i^{4n-1} = \frac{i^{4n}}{i} = \frac{1}{\sqrt{-1}} = \frac{\sqrt{-1}}{-1} = -\sqrt{-1}$

370. $i^{8n+5} = i^{8n+4}\,.\,i = i = \sqrt{-1}.$

371. $\sqrt{-4} = \sqrt{4}\,.\,i = 2i.$

372. $\sqrt{-81} = \sqrt{-1\,.\,81} = \sqrt{81}\,.\,i = 9i.$

373. $\sqrt{-a^2} = \sqrt{a^2}\,.\,i = ai.$

374. $\sqrt{-b^6} = \sqrt{b^6}\,.\,i = b^3 i.$

375. $\sqrt{-\frac{9}{4}} = \sqrt{\frac{9}{4}}\,.\,i = \frac{3}{2}i.$

376. $\sqrt{-\frac{a^4}{b^8}} = \sqrt{\frac{a^4}{b^8}}\,.\,i = \frac{a^2}{b^4}i.$

377. $\sqrt{-a} = \sqrt{a}\,.\,i.$ **378.** $\sqrt{-9x} = \sqrt{9x}\,.\,i = 3\sqrt{x}\,.\,i.$

379. $\sqrt{-a^2 - b^2} = \sqrt{-(a^2 + b^2)} = \sqrt{a^2 + b^2}\,.\,i.$

380. $\sqrt{-x^2 - y^2 + 2xy} = \sqrt{-(x^2 + y^2 - 2xy)} =$
$= \sqrt{-(x-y)^2} = \sqrt{(x-y)^2}\,.\,i = (x-y)\,i.$

381. $\sqrt{-25} + \sqrt{-49} - \sqrt{-64} + \sqrt{-1} =$
$= \sqrt{25}\,.\,i + \sqrt{49}\,.\,i - \sqrt{64}\,.\,i + i = 5i + 7i - 8i + i = 5i.$

382. $3\sqrt{-4} + 5\sqrt{-27} - 3\sqrt{-16} - 5\sqrt{-3} =$
$= 3.2.i + 5.3\sqrt{3}.i - 3.4.i - 5\sqrt{3}.i = 6i + 15\sqrt{3}.i -$
$- 12i - 5\sqrt{3}.i = 10\sqrt{3}.i - 6i = (10\sqrt{3} - 6)i.$

383. $3 + 2i + (4 - 3i) - [(8 - 5i) - (5 + 13i)] =$
$= 3 + 2i + 4 - 3i - 8 + 5i + 5 + 13i = 4 + 17i$

384. $a + bi - (2a - 3bi) + [(a - 4bi) + (5a - 2bi)] =$
$= a + bi - 2a + 3bi + a - 4bi + 5a - 2bi = 5a - 2bi.$

385. $\sqrt{-16}.\sqrt{-9} = 4i.3i = 12i^2 = -12.$

386. $\sqrt{-a}.\sqrt{-b} = \sqrt{a}.i.\sqrt{b}.i = \sqrt{ab}.i^2 = -\sqrt{ab}.$

387. $i\sqrt{-x^2} = i\sqrt{x^2}.i = i.x.i = xi^2 = -x.$

388. $\sqrt{a-b}.\sqrt{b-a} = \sqrt{a-b}.\sqrt{a-b}.i = (a-b)i.$

389. $(2 - 5i)(8 - 3i) = 16 - 6i - 40i + 15i^2 = 16 -$
$- 46i + 15.(-1) = 16 - 46i - 15 = 1 - 46i.$

390. $(5 + 2\sqrt{-7})(5 - 5\sqrt{-7}) =$
$= (5 + 2\sqrt{7}.i)(6 - 5\sqrt{7}.i) =$
$= 30 + 12\sqrt{7}.i - 25\sqrt{7}.i - 70i^2 = 30 - 13\sqrt{7}.i + 70 =$
$= 100 - 13\sqrt{7}.i.$

391. $(\sqrt{a} - \sqrt{-b})(\sqrt{a} + 3\sqrt{-b}) =$
$= (\sqrt{a} - \sqrt{b}.i)(\sqrt{a} + 3\sqrt{b}.i) =$
$= a + 3\sqrt{ab}.i - \sqrt{ab}.i - 3bi^2 = a + 2\sqrt{ab}.i + 3b =$
$= (a + 3b) + 2\sqrt{ab}.i.$

392. $(3\sqrt{-5} - 2\sqrt{-7}).(2\sqrt{-7} + 3\sqrt{-5}) =$
$= (3\sqrt{5}.i - 2\sqrt{7}.i).(3\sqrt{5}.i + 2\sqrt{7}.i) = (3\sqrt{5}.i)^2 -$
$- (2\sqrt{7}.i)^2 = 45i^2 - 28i^2 = 17i^2 = -17.$

393. $a : \sqrt{-a} = a : \sqrt{a}.i = \frac{\sqrt{a}.\sqrt{a}}{\sqrt{a}.i} = \frac{\sqrt{a}}{i} = \frac{\sqrt{a}}{\sqrt{-1}} =$

$= \frac{\sqrt{a}.\sqrt{-1}}{-1} = -\sqrt{a}.i.$

394. $\sqrt{-ax} : \sqrt{-x} = \frac{\sqrt{ax} . i}{\sqrt{x} . i} = \frac{\sqrt{ax}}{\sqrt{x}} = \sqrt{a}.$

395. $\frac{a^2 + b^2}{a - bi} = \frac{(a^2 + b^2)(a + bi)}{(a - bi)(a + bi)} = \frac{(a^2 + b^2)(a + bi)}{a^2 - b^2 i^2} =$

$= \frac{(a^2 + b^2)(a + bi)}{a^2 + b^2} = a + bi.$

396. $\frac{x - y}{x + yi} = \frac{(x - y)(x - yi)}{(x - yi)(x - yi)} = \frac{(x - y)(x - yi)}{x^2 - y^2 i^2} =$

$= \frac{(x - y)(x - yi)}{x^2 + y^2} = \frac{x^2 - xy + y^2 i - xyi}{x^2 + y^2} = \frac{(x^2 - xy) + y(y - x)i}{x^2 + y^2}.$

397. $\frac{4}{1 + \sqrt{-3}} = \frac{4}{1 + \sqrt{3} . i} = \frac{4(1 - \sqrt{3} . i)}{(1 + \sqrt{3} . i)(1 - \sqrt{3} . i)} =$

$= \frac{4(1 - \sqrt{3} . i)}{1 - (\sqrt{3} . i)^2} = \frac{4(1 - \sqrt{3} i)}{1 - 3 i^2} = \frac{4(1 - \sqrt{3} . i)}{4} = \sqrt{3} . i.$

398. $\frac{3 - 5i\sqrt{8}}{3 + 5i\sqrt{8}} = \frac{(3 - 5i\sqrt{8})(3 - 5i\sqrt{8})}{(3 + 5i\sqrt{8})(3 - 5i\sqrt{8})} = \frac{(3 - 5i\sqrt{8})^2}{9 - 200 i^2} =$

$= \frac{9 - 30i\sqrt{8} + 200 i^2}{9 + 200} = \frac{9 - 30i\sqrt{8} - 200}{209} = \frac{191 + 60i\sqrt{2}}{209}.$

399. $\frac{36 - \sqrt{-2}}{2 + 3i\sqrt{2}} = \frac{(36 - i\sqrt{2})(2 - 3i\sqrt{2})}{(2 + 3i\sqrt{2})(2 - 3i\sqrt{2})} =$

$= \frac{72 - 2\sqrt{2} . i - 108\sqrt{2} . i + 6 i^2}{22} = \frac{72 - 110\sqrt{2} . i - 6}{22} =$

$= \frac{66 - 110\sqrt{2} . i}{22} = 3 - 5\sqrt{2} . i.$

400. $\frac{2 - \sqrt{-7}}{3 + \sqrt{-21}} = \frac{2 - i\sqrt{7}}{3 + i\sqrt{21}} = \frac{(2 - i\sqrt{7})(3 - i\sqrt{21})}{(3 + i\sqrt{21})(3 - i\sqrt{21})} =$

$= \frac{6 - 2i\sqrt{21} - 3i\sqrt{7} + 7\sqrt{3} . i^2}{9 - 21 i^2} = \frac{6 - 2i\sqrt{21} - 3i\sqrt{7} - 7\sqrt{3}}{9 + 21} =$

$= \frac{(6 - 7\sqrt{3}) - \sqrt{7}(2\sqrt{3} + 3) i}{30}.$

401. $(a + bi)^2 = a^2 + 2abi + b^2 i^2 = a^2 - b^2 + 2abi.$

402. $(3-\sqrt{-2})^2=(3-i\sqrt{2})^2=9-6i\sqrt{2}+2i^2=$
$=9-6i\sqrt{2}-2=7-6i\sqrt{2}.$

403. $\left(\frac{1+\sqrt{-3}}{2}\right)^2=\left(\frac{1+i\sqrt{3}}{2}\right)^2=\frac{1+2i\sqrt{3}+3i^2}{4}=$
$=\frac{1+2i\sqrt{3}-3}{4}=\frac{2i\sqrt{3}-2}{4}=\frac{i\sqrt{3}-1}{2}=-\frac{1}{2}+\frac{\sqrt{3}}{2}.i.$

404. $(3\sqrt{-5}+2\sqrt{-1})^2=(3i\sqrt{5}+2i)^2=45i^2+$
$+12\sqrt{5}.i^2+4i^2=(49+12\sqrt{5})i^2=-(49+12\sqrt{5}).$

405. $(2-3\sqrt{-2})^2=(2-3\sqrt{2}.i)^2=4-12\sqrt{2}.i+$
$+18i^2=4-12\sqrt{2}.i-18=-14-12\sqrt{2}.i.$

406. $\left(\frac{-1+2\sqrt{-2}}{2}\right)^2=\left(\frac{-1+2\sqrt{2}.i}{2}\right)^2=\frac{(2\sqrt{2}.i-1)^2}{4}=$
$=\frac{8i^2-4\sqrt{2}.i+1}{4}=\frac{-8-4\sqrt{2}.+1}{4}=\frac{-7-4\sqrt{2}.i}{4}=-\frac{7}{4}-$
$-\sqrt{2}.i.$

407. $(a-bi)^3=a^3-3a^2bi+3ab^2i^2-b^3i^3=a^3-$
$-3a^2bi-3ab^2+b^3i=(a^3-3ab^2)-(3a^2b-b^3)i.$

408. $(3+\sqrt{-2})^3=(3+\sqrt{2}.i)^3=27+27\sqrt{2}.i+$
$+18i^2+2\sqrt{2}.i^3=27+27\sqrt{2}.i-18-2\sqrt{2}.i=$
$=9+25\sqrt{2}.i.$

409. $(\sqrt{-3}-2\sqrt{-1})^3=(i\sqrt{3}-2i)=3\sqrt{3}.i^3-$
$-18i^3+12\sqrt{3}.i^3-8i^3=(15\sqrt{3}-26)i^3=-(15\sqrt{3}-26)i=$
$=(26-15\sqrt{3})i.$

410. $\left(\frac{-1+\sqrt{-3}}{2}\right)^3=-\left(\frac{1-i\sqrt{3}}{2}\right)^3=$
$=-\frac{1-3i\sqrt{3}+9i^2-3\sqrt{3}.i^3}{8}=-\frac{1-3i\sqrt{3}-9+3i\sqrt{3}}{8}=$
$=-\frac{-8}{8}=1.$

411. $\sqrt{3+4\sqrt{-1}}=\left[\sqrt{\frac{3+\sqrt{3^2+4^2}}{2}}+\right.$
$\left.+\sqrt{\frac{-3+\sqrt{3^2+4^2}}{2}}\sqrt{-1}\right]=\left[\sqrt{\frac{3+5}{2}}+\sqrt{\frac{-3+5}{2}}\sqrt{-1}\right]=$
$=(2+\sqrt{-1})=2+i.$

412. $\sqrt{-3-4i}=i\sqrt{3+4i}=\left[\sqrt{\frac{3+\sqrt{3^2+4^2}}{2}}+\right.$
$\left.+\sqrt{\frac{-3+\sqrt{3^2+4^2}}{2}}\,i\right]i=\left[\sqrt{\frac{3+5}{2}}+\sqrt{\frac{-3+5}{2}}\,i\right]i=$
$=(2+i)i=2i=i^2=-1+2i.$

413. $\sqrt{1+4\sqrt{-3}}=\sqrt{1+4\sqrt{3}\,.\,i}=$
$=\left[\sqrt{\frac{1+\sqrt{1^2+(4\sqrt{3})^2}}{2}}+\sqrt{\frac{-1+\sqrt{1^2+(4\sqrt{3})^2}}{2}}\,i\right]=$
$=\left[\sqrt{\frac{1+\sqrt{1+48}}{2}}+\sqrt{\frac{-1+\sqrt{1+48}}{2}}\,i\right]=$
$=\sqrt{\frac{1+7}{2}}+\sqrt{\frac{-1+7}{2}}\,i=2+\sqrt{3}\,i.$

414. $\sqrt{2-3\sqrt{-5}}=\sqrt{2-3\sqrt{5}\,i}=$
$=\left[\sqrt{\frac{2+\sqrt{4+(3\sqrt{5})^2}}{2}}-\sqrt{\frac{-2+\sqrt{4+(3\sqrt{5})^2}}{2}}\,i\right]=$
$=\left[\sqrt{\frac{2+\sqrt{4+45}}{2}}-\sqrt{\frac{-2+\sqrt{4+45}}{2}}\,i\right]=$
$\left[\sqrt{\frac{2+7}{2}}-\sqrt{\frac{-2+7}{2}}\,i\right]=\frac{3\sqrt{2}}{2}-\frac{\sqrt{10}}{2}\,i.$

415. $\sqrt{20-4\sqrt{-11}}=\sqrt{20-4\sqrt{11}\,.\,1}=$
$=\left[\sqrt{\frac{20+\sqrt{20^2+(4\sqrt{11})^2}}{2}}-\sqrt{\frac{-20+\sqrt{20^2+(4\sqrt{11})^2}}{2}}\,i\right]=$

$$=\left[\sqrt{\frac{20+\sqrt{20^2+176}}{2}}-\sqrt{\frac{-20+\sqrt{20^2+176}}{2}}\,i\right]=$$

$$=\sqrt{\frac{20+21}{2}}-\sqrt{\frac{-20+24}{2}}\,i=\sqrt{22}-\sqrt{2}\,i.$$

416. $\sqrt{6+\sqrt{-13}}=\sqrt{6+\sqrt{13}\,i}=$

$$=\left[\sqrt{\frac{6+\sqrt{36+13}}{2}}+\sqrt{\frac{-6+\sqrt{36+13}}{2}}\,i\right]=\sqrt{\frac{6+7}{2}}+$$

$$+\sqrt{\frac{-6+7}{2}}\,i=\frac{\sqrt{26}}{2}+\frac{\sqrt{2}}{2}\,i=\frac{1}{2}(\sqrt{26}+\sqrt{2}\,i).$$

417. $\sqrt{\sqrt{-1}}=\sqrt{i}=\sqrt{0+1\,.\,i}=\sqrt{\frac{0+\sqrt{0+1^2}}{2}}+$

$$+\sqrt{\frac{-0+\sqrt{0+1^2}}{2}}\,i=\sqrt{\frac{1}{2}}+\sqrt{\frac{1}{2}}\,i=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\,i=$$

$$=\frac{1+i}{2}.$$

418. $\sqrt[8]{-1}=\sqrt{\sqrt{\sqrt{-1}}}=\sqrt{\frac{1+i}{\sqrt{2}}}=$

$$=\sqrt{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\,i}=\sqrt{\frac{\frac{1}{\sqrt{2}}+\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}}{2}}=$$

$$=\sqrt{\frac{-\frac{1}{\sqrt{2}}+\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}}{2}}\,.\,i=$$

$$=\sqrt{\frac{\frac{1}{\sqrt{2}}+\sqrt{\frac{1}{2}+\frac{1}{2}}}{2}}+\sqrt{\frac{-\frac{1}{\sqrt{2}}+\sqrt{\frac{1}{2}+\frac{1}{2}}}{2}}\,i=$$

$$=\sqrt{\frac{\frac{1}{\sqrt{2}}+1}{2}}+\sqrt{\frac{-\frac{1}{\sqrt{2}}+1}{2}}\,.\,i=\sqrt{\frac{1+\sqrt{2}}{2+\sqrt{2}}}+$$

$$+\sqrt{\frac{-1+\sqrt{2}}{2\sqrt{2}}}\,i=\frac{\sqrt{1+\sqrt{2}}+\sqrt{-1+\sqrt{2}}\,i}{\sqrt{2\sqrt{2}}}=$$

$$=\frac{\sqrt{\sqrt{2}+1}+\sqrt{\sqrt{2}-1}\,.\,i}{\sqrt[4]{8}}=\frac{\sqrt[4]{2}}{2}\left(\sqrt{\sqrt{2}+1}+\sqrt{\sqrt{2}-1}\,.\,i\right).$$

419. Võttes $n=3x$, kus x on täisarv, võime kirjutada:

$$\left(\frac{-1+\sqrt{-3}}{2}\right)^n+\left(\frac{-1-\sqrt{-3}}{2}\right)^n=\left(\frac{-1+\sqrt{-3}}{2}\right)^{3x}+$$

$$+\left(\frac{-1-\sqrt{-3}}{2}\right)^{3x}=\left[\left(\frac{-1+\sqrt{-3}}{2}\right)^3\right]^x+\left[\left(\frac{-1-\sqrt{-3}}{2}\right)^3\right]^x=$$

$$=\left[\left(\frac{-1+\sqrt{3}\,.\,i}{2}\right)^3\right]^x+\left[\left(\frac{-1-\sqrt{3}\,.\,i}{2}\right)^3\right]^x=$$

$$=\left[\frac{-1+3\sqrt{3}\,.\,i-9i^2+3\sqrt{3}\,i^3}{8}\right]^x+\left[\frac{-1-3\sqrt{3}\,.\,i-9i^2-3\sqrt{3}\,.\,i^3}{8}\right]^x=$$

$$=\left[\frac{-1+3\sqrt{3}\,i+9-3\sqrt{3}\,i}{8}\right]+\left[\frac{-1-3\sqrt{3}\,i+9+3\sqrt{3}\,i}{8}\right]^x=$$

$$=\left[\frac{8}{8}\right]^x+\left[\frac{8}{8}\right]^x=1^x+1^x=1+1=2.$$

420. Võttes $n=2x$, võime kirjutada: $\left(\frac{1+i}{\sqrt{2}}\right)^{2x}+$

$$+\left(\frac{1-i}{\sqrt{2}}\right)^{2x}=\left[\left(\frac{1+i}{\sqrt{2}}\right)^2\right]^x+\left[\left(\frac{1-i}{\sqrt{2}}\right)^2\right]^x=\left[\frac{1+2i+i^2}{2}\right]^x+$$

$$+\left[\frac{1-2i+i^2}{2}\right]^x=\left[\frac{1+2i-1}{2}\right]^x+\left[\frac{1-2i-1}{2}\right]^x=i^x+(-i)^x.$$

Kui $x=1$, on avaldus $1^x+(-i)^x$ võrdne 0'iga; kui $x=2$ on nimetatud avaldus $=1^2+i^2=2i^2=-2$; kui $x=3$, on avaldus $-i+i=0$; võttes x'i tähenduseks 4, saame $1+1=2$. Võttes x'i mistahes astmel saame avalduse $1^x+(-i)^x$ jaoks tähendused ± 2 ja 0.

X (IX) osa.

Teise astme võrrandid.

§ 1. Arvuliste teise astme ehk ruutvõrrandite lahendus.

1. $x^2 - 7x = 0$; $x(x - 7) = 0$. Selleks, et kahe teguri korrutis võrduks nulliga, peab üks kahest tegurist võrduma nulliga. Järjelikult, kas $x = 0$ ehk $x - 7 = 0$; ekvatsiooni juured on siis: $x_1 = 0$; $x_2 = 7$.

2. $4x^2 = -9x$; $4x^2 + 9x = 0$; $x(4x + 9) = 0$; $x_1 = 0$ ja $4x + 9 = 0$ kust $x_2 = -\frac{9}{4}$.

3. $7x^2 - 8x = 5x^2 - 13x$; $2x^2 + 5x = 0$; $x(2x + 5) = 0$; $x_1 = 0$ ja $2x + 5 = 0$, kust $x_2 = -\frac{5}{2}$.

4. $5x^2 + 4x = 11x^2 - 8x$; $6x^2 - 12x = 0$; $x(6x - 12) = 0$; $x_1 = 0$; $6x - 12 = 0$, kust $x_2 = 2$.

5. $(2x + 5)^2 - (x - 3)^2 = 16$; $4x^2 + 20x + 25 - x^2 + 6x - 9 = 16$; $3x^2 + 26x = 0$; $x(3x + 26) = 0$; $x_1 = 0$ ja $3x + 26 = 0$ kust $x_2 = -\frac{26}{3} = -8\frac{2}{3}$.

6. $(2x + 7)(7 - 2x) - x(x + 2) = 49$; $49 - 4x^2 - x^2 - 2x = 49$; $5x^2 + 2x = 0$; $x(5x + 2) = 0$; $x_1 = 0$; $5x + 2 = 0$ kust $x_2 = -\frac{2}{5}$.

7. $\frac{x + 5}{2x + 1} = \frac{x + 15}{3 - x}$ nimetajaid ära kaotades saame: $(x + 5)(3 - x) = (2x + 1)(x + 15)$; $3x - x^2 + 15 - 5x =$

$2x^2 + 30x + x + 15$; $3x^2 + 33x = 0$; $x(3x + 33) = 0$; $x_1 = 0$; $3x + 33 = 0$; $x_2 = -11$.

8. $\frac{x+3}{x+2} + \frac{x-3}{x-2} = \frac{2x-3}{x-1}$; nimetajaid ära kaotades saame: $(x + 3)(x - 2)(x - 1) + (x - 3)(x + 2)(x - 1) = (2x - 3)(x^2 - 4)$; $(x^2 + 3x - 2x - 6)(x - 1) + (x^2 - 3x + + 2x - 6)(x - 1) = 2x^3 - 3x^2 - 8x + 12$; $(x^2 + x - 6)(x - 1) + + (x^2 - x - 6)(x - 1) = 2x^3 - 3x^2 - 8x + 12$; $x^3 + x^2 - 6x - - x^2 - x + 6 + x^3 - x^2 - 6x - x^2 + x + 6 = 2x^3 - 3x^2 - 8x + + 12$; $x^2 - 4x = 0$; $x(x - 4) = 0$; $x_1 = 0$ ja $x - 4 = 0$, kust $x_2 = 4$.

9. $\frac{x\sqrt{3}}{x - 2\sqrt{3}} = \frac{2x}{x\sqrt{3} - 5}$; $x\sqrt{3}(x\sqrt{3} - 5) = 2x(x - 2\sqrt{3})$; $3x^2 - 5x\sqrt{3} = 2x^2 - 4x\sqrt{3}$; $x^2 - x\sqrt{3} = 0$; $x(x - \sqrt{3}) = 0$; $x_1 = 0$; $x - \sqrt{3} = 0$; $x_2 = \sqrt{3}$.

10. $\sqrt[4]{2} \,.\, x + 2 = \frac{3\sqrt[4]{2} \,.\, x - \sqrt{5} \,.\, x - 2}{\sqrt[4]{2} \,.\, x + 1}$;

$(\sqrt[4]{2} \,.\, x + 2)(\sqrt[4]{2} \,.\, x + 1) = 3\sqrt[4]{2} \,.\, x - \sqrt{5} \,.\, x - 2$;

$\sqrt{2} \,.\, x^2 + \sqrt[4]{2} \,.\, x + 2\sqrt[4]{2} \,.\, x + 2 = 3\sqrt[4]{2} \,.\, x - \sqrt{5} \,.\, x - 2$;

$\sqrt{2} \,.\, x^2 + 3\sqrt[4]{2} \,.\, x + 2 = 3\sqrt[4]{2} \,.\, x - \sqrt{5} \,.\, x - 2$;

$\sqrt{2} \,.\, x^2 + \sqrt{5} \,.\, x = 0$; $x(\sqrt{2} \,.\, x - \sqrt{5}) = 0$; $x_1 = 0$;

$\sqrt{2} \,.\, x + \sqrt{5} = 0$; $x_2 = -\frac{\sqrt{5}}{\sqrt{2}} = -\frac{\sqrt{10}}{2}$.

11. $x^2 - 25 = 0$; $(x + 5)(x - 5) =$; $x + 5 = 0$, kust $x_1 = -5$ ja $x - 5 = 0$, kust $x_2 = 5$.

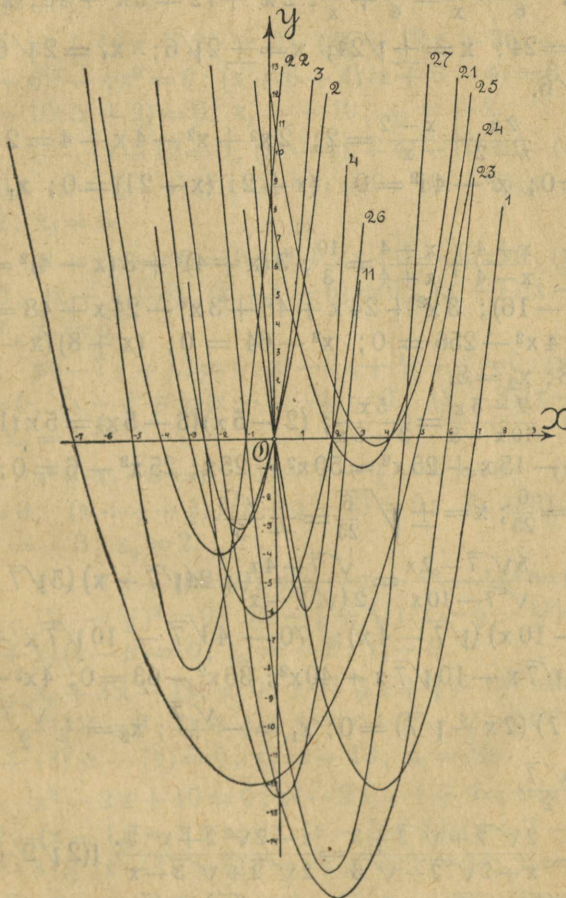
12. $9x^2 = 16$; $9x^2 - 16 = 0$; $(3x + 4)(3x - 4) = 0$; $3x + 4 = 0$; $x_1 = -\frac{4}{3}$; $3x - 4 = 0$; $x_2 = \frac{4}{3}$.

13. $\frac{5x^2}{6} = \frac{6}{125}$; $x^2 = \frac{36}{625}$; $x^2 - \frac{36}{625} = 0$;

$\left(x + \frac{6}{25}\right)\left(x - \frac{6}{25}\right) = 0$; $x + \frac{6}{25} = 0$; $x_1 = -\frac{6}{25}$;

$x - \frac{6}{25} = 0$; $x_2 = \frac{6}{25}$.

Ruutvõrrandite graafiline lahendamine.



Märkus: Ülesandel nr. 24 on y teljemõõt $^1/_3$ x telje mõõdust. Ülesannetel nr. 11 ja 22 on mõõt poole lühem kui teistel.

Seletus. Ruutvõrrandite juurte leidmiseks tuleb joonisel ära lugeda kõverate lõikepunktid x teljega koordinaatide alguspunktist arvates.

14. $x^2+13=4$; $x^2+9=0$; $x^2-9i^2=0$; $(x+3i)(x-3i)=0$; $x_1=-3i$; $x_2=3i$.

15. $\frac{x}{6}+\frac{6}{x}=\frac{x}{4}+\frac{4}{x}$; $2x^2+72=3x^2+48$; $x^2-24=$ $=0$; $x^2=24$; $x=\pm\sqrt{24}$; $x=\pm 2\sqrt{6}$; $x_1=2\sqrt{6}$; $x_2=$ $=-2\sqrt{6}$.

16. $\frac{2x}{x-2}+\frac{x-2}{x}=2$; $2x^2+x^2-4x+4=2x^2-4x$; $x^2+4=0$; $x^2-4i^2=0$; $(x+2i)(x-2i)=0$; $x_1=-2i$; $x_2=2i$.

17. $\frac{x+4}{x-4}+\frac{x-4}{x+4}=\frac{10}{3}$; $3(x+4)^2+3(x-4)^2=$ $=10(x^2-16)$; $3x^2+24x+48+3x^2-24x+48=10x^2-$ -160; $4x^2-256=0$; $x^2-64=0$; $(x+8)(x-8)=0$; $x_1=-8$; $x_2=8$.

18. $\frac{2-5x}{10x-5}=\frac{5x}{3-5x}$; $(2-5x)(3-5x)=5x(10x-5)$; $6-10x-15x+25x^2=50x^2-25x$; $25x^2-6=0$; $25x^2=$ $=6$; $x^2=\frac{6}{25}$; $x=\pm\sqrt{\frac{6}{25}}=\pm\frac{\sqrt{6}}{5}$.

19. $\frac{5\sqrt{7}-2x}{\sqrt{7}-10x}=\frac{\sqrt{7}-4x}{2(\sqrt{7}-x)}$; $2(\sqrt{7}-x)(5\sqrt{7}-2x)=$ $=(\sqrt{7}-10x)(\sqrt{7}-4x)$; $70-4\sqrt{7}-10\sqrt{7}x+4x^2=$ $=7-4\sqrt{7}x-10\sqrt{7}x+40x^2$; $36x^2-63=0$; $4x^2-7=0$; $(2x+\sqrt{7})(2x-\sqrt{7})=0$; $x_1=-\frac{\sqrt{7}}{2}$; $x_2=+\frac{\sqrt{7}}{2}$;

$x=\pm\frac{\sqrt{7}}{2}$.

20. $\frac{2\sqrt{2}+\sqrt{3}+3}{x+2\sqrt{2}-\sqrt{3}}=\frac{x-2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}-x}$; $[(2\sqrt{2}+\sqrt{3})+$ $+x][(2\sqrt{2}+\sqrt{3})-x]=[x+(2\sqrt{2}-\sqrt{3}][x-(2\sqrt{2}-\sqrt{3})]$; $(2\sqrt{2}+\sqrt{3})^2-x^2=x^2-(2\sqrt{3}-\sqrt{3})^2$; $8+4\sqrt{6}+3-x^2=$ $=x^2-8+4\sqrt{6}-3$; $2x^2-22=0$; $x^2-11=0$; $(x+\sqrt{11})(x-\sqrt{11})=0$; $x=\pm\sqrt{11}$.

21. $x^2 - 6x + 8 = 0$; $(x^2 - 6x + 9) - 1 = 0$; $(x - 3)^2 - 1 = 0$; $(x - 3 + 1)(x - 3 - 1) = 0$; $(x - 2)(x - 4) = 0$; $x_1 = 2$; $x^2 = 4$.

22. $x^2 + 12x + 20 = 0$; $(x^2 + 12x + 36) - 16 = 0$; $(x + 6)^2 - 4x^2 = 0$; $(x + 6 + 4)(x + 6 - 4) = 0$; $(x + 10)(x + 2) = 0$; $x_1 = -10$; $x_2 = -2$.

23. $x^2 - 4x - 12 = 0$; $(x^2 - 4x + 4) - 16 = 0$; $(x - 2)^2 - 4^2 = 0$; $(x - 2 + 4)(x - 2 - 4) = 0$; $(x + 2)(x - 6) = 0$; $x_1 = -2$; $x_2 = 6$.

24. $x^2 + 2x - 35 = 0$; $x^2 + 2x + 1 - 36 = 0$; $(x + 1)^2 - 6^2 = 0$; $(x + 1 + 6) \cdot (x + 1 - 6) = 0$; $(x + 7)(x - 5) = 0$; $x_1 = -7$; $x_2 = 5$.

25. $x^2 - 7x + 12 = 0$; $x^2 - 7x + \frac{49}{4} - \frac{1}{4} = 0$; $(x - \frac{7}{2})^2 - (\frac{1}{2})^2 = 0$; $(x - \frac{7}{2} + \frac{1}{2})(x - \frac{7}{2} - \frac{1}{2}) = 0$; $(x - 3)(x - 4) = 0$; $x_1 = 3$; $x_2 = 4$.

26. $x^2 + x - 6 = 0$; $x^2 + x + \frac{1}{4} - \frac{25}{4} = 0$; $(x + \frac{1}{2})^2 - (\frac{5}{2})^2 = 0$; $(x + \frac{1}{2} + \frac{5}{2}) \cdot (x + \frac{1}{2} - \frac{5}{2}) = 0$; $(x + 3)(x - 2) = 0$; $x_1 = -3$; $x_2 = 2$.

27. $x^2 - 7x - 18 = 0$; $x^2 - 7x + \frac{49}{4} - \frac{121}{4} = 0$; $(x - \frac{7}{2})^2 - (\frac{11}{2})^2 = 0$. $(x - \frac{7}{2} + \frac{11}{2})(x - \frac{7}{2} - \frac{11}{2}) = 0$; $(x + 2)(x - 9) = 0$; $x_1 = -2$; $x_2 - 9$.

28. $x_2 + 3x - 130 = 0$; $x^2 + 3x + \frac{9}{4} - \frac{529}{4} = 0$; $(x + \frac{3}{2})^2 - (\frac{23}{2})^2 = 0$; $(x + \frac{3}{2} + \frac{23}{2})(x + \frac{3}{2} - \frac{23}{2}) = 0$; $(x + 13)(x - 10) = 0$; $x_1 = -13$; $x_2 = 10$.

29. $x^2 - 2x + 10 = 0$; $x^2 - 2x + 1 + 9 = 0$; $(x - 1)^2 + 9 = 0$; $(x - 1)^2 - 9i^2 = 0$; $(x - 1 + 3i)(x - 1 - 3i) = 0$; $[x - (1 - 3i)] \cdot [x - (1 + 3i)] = 0$; $x_1 = 1 - 3i$; $x_2 = 1 + 3i$.

30. $x^2 - 6x + 34 = 0$; $x^2 - 6x + 9 + 25 = 0$; $(x - 3)^2 + 25 = 0$; $(x - 3)^2 - 25i^2 = 0$; $(x - 3 + 5i)(x - 3 - 5i) = 0$; $[x - (3 - 5i)]\,[x - (3 + 5i)] = 0$; $x_1 = 3 - 5i$; $x_2 = 3 + 5i$.

31. $(x - 1)(x - 2) = 6$; $x^2 - x - 2x + 2 = 6$; $x^2 - 3x - 4 = 0$; $x^2 - 3x + \frac{9}{4} - \frac{25}{4} = 0$; $(x - \frac{3}{2})^2 - (\frac{5}{2})^2 = 0$;

$(x-\frac{3}{2}+\frac{5}{2})(x-\frac{3}{2}-\frac{5}{2})=0$; $(x+1)(x-4)=0$; $x_1=-1$; $x_2=4$.

32. $(x-2)^2=2(3x-10)$; $x^2-4x+4=6x-20$; $x^2-10x+24=0$; $x^2-10x+25-1=0$; $(x-5)^2-1=0$; $(x-5+1)(x-5-1)=0$; $(x-4)(x-6)=0$; $x_1=4$; $x_2=6$.

33. $4x^2-4x=3$; $4x^2-4x-3=0$; $x^2-x-\frac{3}{4}=0$; $x^2-x+\frac{1}{4}-1=0$; $(x-\frac{1}{2})^2-1=0$; $(x-\frac{1}{2}+1)(x-\frac{1}{2}-1)=$ $=0$; $(x+\frac{1}{2}).(x-\frac{3}{2})=0$; $x_1=-\frac{1}{2}$; $x_2=\frac{3}{2}$.

34. $9x^2-5=12x$; $9x^2-12x-5=0$; $9x^2-12x+$ $+4-9=0$; $(3x-2)^2-3^2=0$; $(3x-2+3)(3x-2-3)=$ $=0$; $(3x+1)(3x-5)=0$; $3x+1=0$; $x_1=-\frac{1}{3}$; $3x-5=$ $=0$; $x_2=\frac{5}{3}=1\frac{2}{3}$.

35. $2x^2-7x+3=0$; $x_2-\frac{7}{2}x+\frac{3}{2}=0$; $x^2-\frac{7}{2}x+$ $+\frac{49}{16}-\frac{25}{16}=0$; $(x-\frac{7}{4})^2-(\frac{5}{4})^2=0$; $(x-\frac{7}{4}+\frac{5}{4})(x-\frac{7}{4}-\frac{5}{4})=$ $=0$; $(x-\frac{1}{2})(x-3)=0$; $x_1=\frac{1}{2}$; $x_2=3$.

36. $4x^2+x-3=0$. $x=\frac{-1\pm\sqrt{1+48}}{8}=\frac{-1\pm7}{8}$; $x_1=\frac{-1+7}{8}=\frac{6}{8}=\frac{3}{4}$; $x_2=\frac{-1-7}{8}=-1$.

37. $(2x-3)^2=8x$; $4x^2-12x+9-8x=0$; $4x^2-20x+9=0$. $x=\frac{20\pm\sqrt{20^2-144}}{8}=\frac{20\pm\sqrt{400-144}}{8}=$ $=\frac{20\pm16}{8}$; $x_1=\frac{20+16}{8}=\frac{36}{8}4{,}5$; $x_2=\frac{20-16}{8}=0{,}5$.

38. $(3x+2)^2=3(x+2)$; $9x^2+12x+4=3x+6$; $9x^2+9x-2=0$. $x=\frac{-9\pm\sqrt{81+72}}{18}=\frac{-9\pm\sqrt{153}}{18}=$ $=\frac{-9\pm\sqrt{9.17}}{10}=\frac{-9\pm3\sqrt{17}}{18}=\frac{3(-9\pm\sqrt{17})}{18}=\frac{-9\pm\sqrt{17}}{6}$; $x_1=\frac{-9+\sqrt{17}}{6}$; $x_2=\frac{-9-\sqrt{17}}{6}$.

39. $x^2-x+1=0$; $x=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm\sqrt{-3}}{2}=$ $=\frac{1\pm\sqrt{3}i}{2}$; $x_1=\frac{1+\sqrt{3}.i}{2}$; $x_2=\frac{1-\sqrt{3}.i}{2}$.

40. $x^2 + 3x + 9 = ;\quad x = \frac{-1 \pm \sqrt{9-36}}{2} = \frac{-3 \pm \sqrt{-27}}{2} =$

$= \frac{-3 \pm 3\sqrt{3}.i}{2};\quad x_1 = \frac{-3+3\sqrt{3}.i}{2};\quad x_2 = \frac{-3-3\sqrt{3}.i}{2}.$

41. $x^2 - 22x + 25 = 2x^2 - 20x + 1;$

$x^2 - 2x^2 - 22x + 20x + 25 - 1 = 0.\qquad -x^2 - 2x + 24 = 0;$

$x^2 + 2x - 24 = 0;\quad x = \frac{-2 \pm \sqrt{4+96}}{2} = \frac{-2 \pm 10}{2};$

$x_1 = \frac{-2+10}{2} = 4;\quad x_2 = \frac{-2-10}{2} = -6.$

42. $2 - 8x + 3x^2 = -4 + 2x^2 - 3x;\ x^2 - 5x + 6 = 0.$

$x = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5 \pm 1}{2};\ x_1 = \frac{5 \pm 1}{2} = 3;\ x_2 = \frac{5-1}{2} = 2.$

43. $(3x-2)^2 = 8(x+1)^2 - 100;\ 9x^2 - 12x + 4 =$

$= 8x^2 + 16x + 8 - 100;\ x^2 - 28x + 96 = 0;$

$x = \frac{28 \pm \sqrt{784-384}}{2} = \frac{28 \pm 20}{2};\quad x_1 = \frac{28+20}{2} = 24;\quad x_2 =$

$= \frac{28-20}{2} = 4.$

44. $(3-x)(4-x) = 2x^2 - 20x + 48;\ 12 - 3x - 4x +$

$+ x^2 = 2x^2 - 20 + 48;\ x^2 - 13x + 36 = 0.$

$x = \frac{13 \pm \sqrt{160-144}}{2} = \frac{13 \pm 25}{2};\ x_1 = 19;\ x_2 = -6.$

45. $\frac{x^2}{2} - \frac{x}{3} + 7\frac{3}{8} = 8;\ 12x^2 - 8x + 177 = 192.$

$12x^2 - 8x - 15 = 0;\quad x = \frac{8 \pm \sqrt{64+720}}{24} = \frac{8 \pm 28}{24};\ x_1 =$

$= \frac{8+28}{24} = 1\frac{1}{2};\ x^2 = \frac{8-28}{24} = -\frac{5}{6}.$

46. $\frac{x+1}{x-2} = \frac{3x-7}{x-1};\ (x+1)(x-1) = (3x-7)(x-2);$

$x^2 - 1 = 3x^2 - 6x - 7x + 14;\quad 2x^2 - 13x + 15 = 0;$

$x = \frac{13 \pm \sqrt{169-120}}{4} = \frac{13 \pm 7}{4};\ x_1 = 5;\ x_2 = 1\frac{1}{2}.$

47. $\frac{x-7}{2(x+3)}=\frac{x-6}{x+24}$; $(x-7)(x+24)=(x-6)2(x+3)$;

$x^2+24x-7x-168=2x^2+6x-12x-36.$

$x^2+17x-168=2x^2-6x-36.$ $\quad x=\frac{23\pm\sqrt{529-529}}{2}=$

$-x^2+23x-132=0$; $x^2-23x+132=0$; $\quad =\frac{23\pm 1}{2}$;

$x_1=12$; $x_2=11.$

48. $\frac{x}{4}+\frac{2}{x}+\frac{(x+1)^2}{x}=\frac{(x+2)(x+1)}{x}$;

$x^2+8+4(x+1)^2=4(x+2)(x+1)$

$x^2-4x+4=0.$ $\quad x=\frac{4\pm\sqrt{16-16}}{2}=\frac{4}{2}=2.$

49. $\frac{x+1}{3}+\frac{3(x-1)}{4}=(x-3)^2+1.$

$4x+4+9x-9=12x^2-72x+108+12.$

$12x^2-85x+125=0.$

$x=\frac{85\pm\sqrt{7225-6000}}{24}=\frac{85\mp 35}{24}$; $\quad x_1=\frac{85+35}{24}=\frac{120}{24}=5$;

$x_2=\frac{85-35}{24}=\frac{50}{24}=2\frac{1}{12}.$

50. $\frac{3(3x-1)}{12x+1}=\frac{2(3x+1)}{15x+8}$; $\quad \frac{9x-3}{12x+1}=\frac{6x+2}{15x+8}.$

$(9x-3)(15x+8)=(6x+2)(12x+1)$;

$135x^2+72x-45x-24=72x^2+6x+24x+2.$

$63x^2-3x-26=0.$

$x=\frac{3\pm\sqrt{9+6552}}{126}=\frac{3\pm 81}{126}$; $\quad x_1=\frac{2}{3}$; $\quad x_2=-\frac{13}{21}.$

51. $\frac{(x-12)^2}{6}-\frac{x}{9}+\frac{x(x-9)}{18}=\frac{(x-14)^2}{2}+5.$

$3(x-12)^2-2x+x(x-9)=9(x-14)^2+90.$

$3x^2-72x+432-2x+x^2-9x=9x^2-252x+1764+90.$

$4x^2-83x+432=9x^2-252x+1854.$

$\pm 5x^2\mp 169x\pm 1422=0$; $\quad x=\frac{169\pm\sqrt{169^2-4.5.1422}}{10}=$

$$=\frac{169\pm\sqrt{28561-28440}}{10}=\frac{169\pm11}{10};\quad x_1=\frac{169+11}{10}=18.$$

$$x_2=\frac{169-11}{10}=\frac{158}{10}=15{,}8.$$

52. $\frac{(x-20)(x-10)}{10}-\frac{(34-x)(40-x)}{2}+\frac{(30-x)(5-x)}{3}=0.$

$3(x-20)(x-10)-15(34-x)(40-x)+10(30-x)(5-x)=0.$

$3x^2-30x-60x+600-20400+510x+600x-15x^2+$

$+1500-300x-50x+10x^2=0.$

$\pm 2x^2\mp 670x\pm 18300=0.$

$$x=\frac{335\pm\sqrt{112225-36600}}{2}=\frac{335\pm\sqrt{75625}}{2}=\frac{335\pm275}{2};$$

$x_1=305;\quad x_2=30.$

53. $\frac{6}{x^2-1}-\frac{2}{x-1}=2-\frac{x+4}{x+1}.$

Ühine nimetaja: x^2-1.

$6-2(x+1)=2(x^2-1)-(x+4)(x-1).$

$6-2x-2=2x^2-2-x^2+x-4x+4.$

$\pm x^2\mp x\mp 2=0;\quad x=\frac{1\pm\sqrt{1+8}}{2}=\frac{1\pm3}{2};\quad x_1=\frac{1+3}{2}=2;$

$x_2=\frac{1-3}{2}=-1.$

54. $\frac{2x+1}{x+3}-\frac{x-1}{x^2-9}=\frac{x+3}{3-x}-\frac{4+x}{3+x};$

$(2x+1)(x-3)-x+1=-(x+3)^2-(4+x)(x-3);$

$2x^2-6x+x-3-x+1=-x^2-6x-9-4x+12-x^2+3x;$

$2x^2-6x-2=-2x^2-7x+3;\quad 4x^2+x-5=0.$

$$x=\frac{-1\pm\sqrt{1+80}}{8}=\frac{-1\pm9}{8}.$$

$x_1=\frac{-1+9}{8}=1;\quad x_2=\frac{-1-9}{8}=-1\frac{1}{4}.$

55. $\frac{x}{2x-1}+\frac{25}{4x^2-1}=\frac{1}{27}-\frac{13}{1-2x}.$

Ühine nimetaja: $27(4x^2-1)$.

$27x(2x+1)+25.27=4x^2-1+13(2x+1)27.$

$54x^2 + 27x + 675 = 4x^2 - 1 + 702x + 351.$

$50x^2 - 675x + 325 = 0.$

$2z^2 - 27x + 13 = 0.$

$$x = \frac{27 \pm \sqrt{729 - 104}}{4} = \frac{27 \pm 25}{4};$$

$$x_1 = \frac{27 + 25}{4} = 13. \quad x_2 = \frac{27 - 25}{4} = \frac{1}{2}.$$

56. $\frac{x+1}{x-1} + \frac{x+2}{x-1} - \frac{2x+13}{x+1} = 0.$

$(x + 1)(x - 2)(x + 1) + (x + 2)(x - 1) \cdot (x + 1) -$
$- (2x + 13)(x - 1)(x - 2) = 0.$

$x^3 - 3x - 2 + x^3 + 2x^2 - x - 2 - 2x^3 - 7x^2 + 35x - 26 = 0.$

$$\pm 5x^2 \mp 31x \pm 30; \quad x = \frac{31 \pm \sqrt{961 - 600}}{10} = \frac{31 \pm 19}{10};$$

$$x_1 = \frac{31 + 19}{10} = 5; \quad x_2 = \frac{31 - 19}{10} = 1\tfrac{1}{5}$$

57. $\frac{1}{x^2 - x - 6} + \frac{2}{x^2 + x - 6} = \frac{4}{x^2 - 9} - \frac{2}{x^2 - 4};$

$x^2 - x - 6 = (x + 2)(x - 3);$ Ühine nimetaja:

$x^2 + x - 6 = (x - 2)(x + 3);$ $(x + 2)(x - 2)(x + 3)(x - 3).$

$x^2 - 9 = (x + 3)(x - 3);$ $(x-2)(x+3) + 2(x+2)(x-3) =$

$x^2 - 4 = (x + 2)(x - 2).$ $= 4(x^2 - 4) - 2(x^2 - 9).$

$x^2 + 3x - 2x - 6 + 2x^2 - 6x + 4x - 12 = 4x^2 - 16 - 2x^2 + 18.$

$$x^2 - x - 20 = 0. \quad x = \frac{1 \pm \sqrt{1 + 80}}{2} = \frac{1 \pm 9}{2}; \; x_1 = 5; \; x_2 = -4.$$

58. $\frac{x^2 + 10x}{x^4 - 1} + \frac{4}{x + 1} = \frac{4x^2 + 21}{x^3 + x^2 + x + 1} + \frac{1}{x^3 - x^2 + x - 1}.$

$x^4 - 1 = (x^2 + 1)(x + 1)(x - 1).$ Ühine nimetaja:

$x^3 + x^2 + x + 1 = (x + 1)(x^2 + 1).$ $(x^2 + 1)(x + 1)(x - 1).$

$x^3 - x^2 + x - 1 = (x - 1)(x^2 + 1).$ $x^2 + 10x + 4(x^2 + 1)(x - 1) =$
$= (4x^2 + 21)(x - 1) + 1(x + 1).$

$x^2 + 10x + 4x^3 - 4x^2 + 4x - 4 = 4x^3 - 4x^2 + 21x - 21 +$
$+ x + 1.$ $x^2 + 14x - 4 = 22x - 20.$ $x^2 - 8x + 16 = 0.$

$$x = \frac{8 \pm \sqrt{64 - 64}}{2} = \frac{8}{2} = 4.$$

59. $\frac{x+6}{x-1}-\frac{x^2+17}{x^2+x+1}=\frac{x+36}{x^3-1}-\frac{x+1}{x^2+x+1}$. Ühine nimetaja: $(x-1).(x^2+x+1)$; $(x+6)(x^2+x+1)-$
$-(x^2+17)(x-1)=x+36-(x+1)(x-1)$.
$x^3+x^2+x+6x^2+6x+6-x^3+x^2-17x+17=x+36-$
$-x^2+x-x+1$.
$9x^2-11x-14=0$.
$x=\frac{11\pm\sqrt{121+504}}{18}=\frac{11\pm25}{18}$; $x_1=\frac{11+25}{18}=2$. $x_2=\frac{11-25}{18}=$
$=-\frac{14}{18}=-\frac{7}{9}$.

60. $\frac{12}{x^2+11x+30}-\frac{1}{x^2-11x+30}=\frac{20}{x^2+x-30}-$
$-\frac{15}{x^2-x-30}$;
$x^2+11x+30=(x+5)(x+6)$; Ühine nimetaja:
$x^2-11x+30=(x-5)(x-6)$; $(x+5)(x-5)(x+6)(x-6)$.
$x^2+x-30=(x-5)(x+6)$;
$x^2-x-30=(x+5)(x-6)$;
$12(x-5)(x-6)-1(x+5)(x+6)=20(x+5)(x-6)-$
$-15(x-5)(x+6)$.
$12x^2-72x-60x+360-x^2-6x-5x-30=20x^2-$
$-120x+100x-600-15x^2-90x+75x+450$;
$11x^2-143x+330=5x^2-35x-150$.
$6x^2-108x+480=0$; $x=\frac{108\pm\sqrt{108^2-4.6.480}}{12}=$
$=\frac{108\pm\sqrt{11664-11520}}{12}=\frac{108\pm\sqrt{144}}{12}=\frac{108\pm12}{12}$; $x_1=\frac{108+12}{12}=$
$=10$; $x_2=\frac{108-12}{12}=\frac{96}{12}=8$.

§ 2. Täheliste ruutvõrrandite lahendamine.

61. $\frac{x-a}{a}=\frac{a}{x-a}$; $(x-a)^2=a^2$; $x^2-2ax+a^2=a^2$;
$x^2-2ax=0$; $x(x-2a)=0$; $x_1=0$; $x-2a=0$; $x_2=2a$

62. $\frac{x+a}{x+b}=\frac{a-x}{x-b}$; $(x+a)(x-b)=(a-x)(x+b)$; $x^2+ax-bx-ab=ax-x^2+ab-bx$; $2x^2-2ab=0$; $2x^2=2ab$; $x^2=ab$; $x=\pm\sqrt{ab}$.

63. $\frac{a-x}{x}-\frac{x}{a+x}=\frac{a}{x}$; $(a-x)(a+x)-x^2=a(a+x)$; $a^2-x^2-x^2=a^2+ax$; $2x^2+ax=0$; $x(2x+a)=0$; $x_1=0$; $2x+a=0$; $x_2=\frac{a}{2}$.

64. $\frac{x+a}{x-a}+\frac{x-a}{x+a}=\frac{x(3x+2a)}{x^2-a^2}$; $(x+a)^2+(x-a)^2=$ $=a(3x+2a)$; $x^2+2ax+a^2+x^2-2ax+a^2=3ax+2a^n$; $2x^2-3ax=0$. $x(2x-3a)=0$; $x_1=0$; $2x-3a=0$; $x_2=\frac{3a}{2}=1\frac{1}{2}$.

65. $ax^2-b^3=a^3-bx^2$; $ax^2+bx^2=a^3+b^3$; $x^2(a+b)=(a+b).(a^2-ab+b^2)$; $x^2=a^2-ab+b^2$; $x=\pm\sqrt{a^2-ab+b^2}$.

66. $\frac{ax}{a+1}=\frac{a+1}{ac}$; $a^2x^2+(a+1)^2$; $x^2=\frac{(a+1)^2}{a^2}$; $x=\pm\frac{a+1}{a}$.

67. $\frac{c}{ab}-2x^2=\frac{a}{b}x^2+\frac{b}{a}x^2$; $\frac{a}{b}x^2+\frac{b}{a}x^2+2x^2=\frac{c}{ab}$; $x^2\left(\frac{a}{b}+\frac{b}{a}+2\right)=\frac{c}{ab}$; $x^2(a^2+b^2+2ab)=c$; $x^2(a+b)^2=c$; $x^2=\frac{c}{(a+b)^2}$; $x=\pm\frac{\sqrt{c}}{a+b}$.

68. $(x+13a)^2+9(x+3a)^2=4(x+10a)^2$; $x^2+26ax+$ $+169a^2+9x^2+54ax+81a^2=4x^2+8ax+400a^2$; $6x^2-$ $-150a^2=0$; $x^2-25a^2=0$; $x^2=25a^2$; $x=\pm 5a$.

69. $\frac{2a+b+x}{x+2a-b}=\frac{x-2a+b}{2a+b-x}$; $[(2a+b)+x][(2a+b)-$ $-x]=[x+(2a-b)].[x-(2a-b)]$; $(2a+b)^2-x^2=$ $=x^2-(2a-b)^2$; $2x^2=(2a+b)^2+(2a-b)^2$; $2x^2=4a^2+$ $+4ab+b^2+4a^2-4ab+b^2$; $2x^2=8a^2=2b^2$; $x^2=4a^2+$ $+b^2$; $x=\pm\sqrt{4a^2+b^2}$.

70. $\frac{x^2+2ax}{x^3-a^3}+\frac{x}{(x+a)^2-ax}=\frac{1}{x-a}$; $\frac{x^2+2ax}{(x-a)(x^2+ax+a^2)}+$ $+\frac{x}{x^2+ax+a^2}=\frac{1}{x-2}$; $x^2+2ax+x(x-a)=x^2+ax+a^2$; $x^2+2ax+x^2-ax=x^2+ax+a^2$; $x^2=a^2$; $x=\pm a$.

71. $x^2-4ax+3a^2=0$; $x=2a\pm\sqrt{4a^2-3a^2}=$ $=2a\pm a$; $x_1=3a$; $x_2=a$.

72. $x^2+2a^3x-35a^6=0$; $x=-a^3\pm\sqrt{a^6+35a^6}=$ $=-a^3\pm\sqrt{36a^6}=-a^3\pm 6a^3$; $x_1=-a^3+6a^3=5a^3$; $x^2=-a^3-6a^3=-7a^3$.

73. $x^2-2ax+a^2-b^2=0$; $x=a\pm\sqrt{a^2-(a^2-b^2)}=$ $=a\pm\sqrt{b^2}=a\pm b$: $x_1=a+b$; $x^2=a-b$.

74. $x^2+2bx-a^2+8ab-15b^2=0$; x^2+2bx- $-(a^2-8ab+15b^2)=0$; $x=-b\pm\sqrt{b^2+a^2-8ab+15b^2}=$ $=-b\pm\sqrt{a^2-8ab+16b^2}=-b\pm\sqrt{(a-4b)^2}=$ $-b\pm(a-4b)$; $x_1=-b+a-4b=a-5b$; $x_2=-b-$ $-a+4b=36-a$.

75. $2x^2-3ax-2a^2=0$; $x=\frac{3a\pm\sqrt{9a^2+16a^2}}{4}=$ $=\frac{3a\pm 5a}{4}$; $x_1=\frac{3a+5a}{4}=2a$; $x_2=\frac{3a-5a}{4}=-\frac{a}{2}$.

76. $6x^2+5ax+a^2=0$; $x=\frac{-5a\pm\sqrt{25a^2-24a^2}}{12}=$ $=\frac{-5a\pm a}{12}$; $x_1=\frac{-5a+a}{12}=-\frac{a}{3}$; $x_2=\frac{-5a-a}{12}=-\frac{a}{2}$.

77. $3b^2x^2+10abx+3a^2=0$; $x=$ $=\frac{-10ab\pm\sqrt{100a^2b^2-36a^2b^2}}{6b^2}=\frac{-10ab\pm 8ab}{6b^2}$; $x_1=\frac{-10ab+8ab}{6b^2}=$ $=-\frac{a}{3b}$; $x_2=\frac{-10ab-8ab}{6b^2}=-\frac{3a}{b}$.

78. $20b^2x^2-9abx-20a^2=0$;
$x=\frac{9ab\pm\sqrt{81a^2b^2+1600a^2b^2}}{40b^2}=\frac{9ab\pm\sqrt{1681a^2b^2}}{40b^2}=\frac{9ab\pm 41ab}{40b^2}$;
$x_1=\frac{5a}{4b}$; $x_2=\frac{4a}{5b}$.

79. $(mx + n)(nx - m) = 0$; $mnx^2 - m^2x + n^2x +$ $+ n^2x - mn = 0$; $mnx^2 - (m^2 - n^2)x - mn = 0$;

$$x = \frac{(m^2 - n^2) \pm \sqrt{(m^2 - n^2) + 4m^2n^2}}{2mn} =$$

$$= \frac{(m^2 - n^2 \pm \sqrt{m^4 + n^4 - 2m^2n^2 + 4m^2n^2}}{2mn} = \frac{(m^2 - n^2) \pm \sqrt{(m^2 + n^2)}}{2mn} =$$

$$= \frac{(m^2 - n^2) \pm (m^2 + n^2)}{2mn}; \quad x_1 = \frac{m^2 - n^2 + m^2 + n^2}{2mn} = \frac{2m^2}{2mn} = \frac{m}{n};$$

$$x_2 = \frac{m^2 - n^2 - m^2 - n^2}{2mn} = -\frac{n}{m};$$ ehk, ekvatsiooni $(mx + n)(nx - m) = 0$ võib ka lahendada, vaadeldes tema pahemat poolt kui kahe teguri kasvatist, milles $mx + n = 0$, kust $x = -\frac{m}{m}$ ehk $nx - m = 0$, kust $x = \frac{m}{n}$.

80. $ab(x^2 + 1) - (a^2 + b^2)x = 0$;
$abx^2 - (a^2 + b^2)x + ab = 0$;

$$x = \frac{(a^2 + b^2) \pm \sqrt{(a^2 + b^2)^2 - 4a^2b^2}}{2ab} = \frac{(a^2 + b^2) \pm \sqrt{(a^2 - b^2)^2}}{2ab} =$$

$$= \frac{(a^2 + b^2) \pm (a^2 - b^2)}{2ab}; \quad x_1 = \frac{a^2 + b^2 + a^2 - ^2}{2ab} = \frac{a}{b};$$

$$x_2 = \frac{a^2 + b^2 - a^2 + b^2}{2ab} = \frac{b}{a}.$$

81. $bx^2 - a = (a - b)x$; $bx^2 - (a - b)x - a = 0$;

$$x = \frac{(a - b) \pm \sqrt{(a - b)^2 + 4ab}}{26} = \frac{(a - b) \pm \sqrt{(a + b)^2}}{26} =$$

$$= \frac{(a - b) \pm (a + b)}{26}; \quad x_1 = \frac{a}{b}; \quad x_2 = -1.$$

82. $(a^2 - b^2)x^2 + ab = (a^2 + b^2)x$; $(a^2 - b^2)x^2 -$ $- (a^2 + b^2)x + ab = 0$; $x = \frac{(a^2 + b^2) \pm \sqrt{(a^2 + b^2)^2 - 4ab(a^2 - b^2)}}{2(a^2 - b^2)} =$

$$= \frac{(a^2 + b^2) \pm \sqrt{(a^2 - 2ab - b^2)^2}}{2(a^2 - b^2)} = \frac{(a^2 + b^2) \pm (a^2 - 2ab - b^2)}{2(a^2 - b^2)}; \quad x_1 =$$

$$= \frac{a^2 + b^2 + a^2 - 2ab - b^2}{2(a^2 - b^2)} = \frac{2a(a - b)}{2(a + b)(a - b)} = \frac{a}{a + b}; \quad x_2 =$$

$$= \frac{a^2 + b^2 - a^2 + 2ab + b^2}{2(a + b)(a - b)} = \frac{b}{a - b}.$$

83. $x - \frac{1}{x} = \frac{a}{b} - \frac{b}{a}$; $abx^2 - ab = a^2x - b^2x$; $abx^2 -$
$- (a^2 - b^2)x - ab = 0$; $x = \frac{(a^2 - b^2) \pm \sqrt{(a^2 - b^2) + 4a^2 + b^2}}{2ab} =$
$= \frac{(a^2 - b^2) \pm \sqrt{(a^2 + b^2)^2}}{2ab} = \frac{(a^2 - b^2) \pm (a^2 + b^2)}{2ab}$; $x_1 = \frac{a^2 + b^2 + a^2 + b^2}{2ab} =$
$= \frac{a}{b}$; $x_2 = \frac{a^2 - b^2 - a^2 - b^2}{2ab} = -\frac{b}{a}$.

84. $\frac{a}{a+x} + \frac{a-x}{x} = \frac{10}{11}$; $10ax + 10(a^2 - x^2) = 11x(a + x)$;
$10ax + 10a^2 - 10x^2 = 11ax + 11x^2$; $21x^2 + ax - 10a^2 = 0$,
kust $x = \frac{-a \pm \sqrt{a^2 + 840a^2}}{42} = \frac{-a \pm \sqrt{841a^2}}{42} = \frac{-a \pm 29a}{42}$; $x_1 =$
$= \frac{2a}{3}$; $x_2 = -\frac{5a}{7}$.

85. $\frac{x+a}{x-a} - \frac{x+b}{x-b}$; $(x + a)(x - b) - (x + b)(x - a) =$
$= (x - a)(x - b)$; $x_2 + ax - bx - ab - x^2 + ax - bx + ab =$
$= x^2 - bx - ax + ab$; $x^2 - 3ax + bx + ab = 0$;
$x^2 - (3a - b)x + ab = 0$. $x = \frac{(3a - b) \pm \sqrt{9a^2 - 6ab + b^2 - 4ab}}{2} =$
$= \frac{3a - b \pm \sqrt{9a^2 + b^2 - 10ab}}{2}$; $x_1 = \frac{3a - b + \sqrt{9a^2 + b^2 - 10ab}}{2}$;
$x_2 = \frac{3a - b - \sqrt{9a^2 + b^2 - 10ab}}{2}$.

86. $\frac{a + 4b}{x + 2b} - \frac{a - 4b}{x - 2b} = \frac{4b}{a}$; $a(a + 4b)(x - 2b) -$
$- a(a - 4b)(x + 2b) = 4b(x^2 - 4b)^2$; $a^2x + 4abx - 2a^2b -$
$- 8ab^2 - a^2x + 4abx - 2a^2b + 8ab^2 = 4bx^2 - 16b^3$;
$4bx^2 - 8abx + 4a^2b - 16b^3 = 0$; $4bx^2 - 8abx + 4a^2b -$
$- 166^3 = 0$; $x^2 - 2ax + a^2 - 4b^2 = 0$; $x = a \pm \sqrt{a^2 - a^2 + 4b^2} =$
$= a \pm 2b$; $x_1 = a + 2b$; $x_2 = a - 2b$.

87. $\frac{1}{a} + \frac{1}{a+x} + \frac{1}{a+2x} = 0$; $(a + x)(a + 2x) +$
$+ a(a + 2x) + a(a + x) = 0$; $a^2 + 2ax + ax + 2x^2 + a^2 +$
$+ 2ax + a^2 + ax = 0$; $2x^2 + 6ax + 3a^2 = 0$;

$$x = \frac{-6a \pm \sqrt{36a^2 - 24a^2}}{4} = \frac{-3a \pm \sqrt{9a^2 - 6a^2}}{2} = \frac{-3a \pm a\sqrt{3}}{2};$$

$$x_1 = \frac{-a(3 - 3\sqrt{3})}{2}; \quad x_2 = \frac{-a(3 + \sqrt{3})}{2}.$$

88. $\frac{x}{x+a} + \frac{2x}{x-a} = \frac{5a^2}{4(x^2 - a^2)}$; $4x(x-a) + 8x(c+a) = 5a^2$; $4x^2 - 4ax + 8x^2 + 8ax = 5a^2$; $12x^2 + 4ax - 5a^2 = 0$; kust

$$x = \frac{-2a \pm \sqrt{4a^2 + 60a^2}}{12} = \frac{-2a \pm 8a}{12}; \quad x_1 = \frac{a}{2}; \quad x_2 = -\frac{5a}{6}.$$

89. $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$; $\frac{1}{a+b+x} = \frac{bx + ax + ab}{abx}$;

$abx = (a + b + x)(bx + ax + ab)$; $abx = abx + a^2x + a^2b + b^2x + abx + ab^2 + bx^2 + ax^2 + abx$; $(a + b)x^2 + a^2x + b^2x + 2abx + a^2b + ab^2 = 0$;

$(a + b)x^2 + x(a + b)^2 + ab(a + b) = 0$;

$(a + b)$'ga koondades saame $x^2 + (a + b)x + ab = 0$;

$$x = \frac{-(a + b) \pm \sqrt{(a + b)^2 - 4ab}}{2} = \frac{-(a + b) \pm \sqrt{a^2 + b^2 - 2ab}}{2} =$$

$$= \frac{-(a + b) \pm \sqrt{(a - b)^2}}{2} = \frac{-(a + b) \pm (a - b)}{2}; \quad x_1 = -a; \quad x_2 = -b.$$

90. $\frac{a(x-2)}{b} + \frac{a}{bx}\left(1 - \frac{b^2}{a^2}\right) = \frac{b(x-2)}{a}$; $\frac{a(x-2)}{b} + \frac{a^2 - b^2}{abx} = \frac{b(x-2)}{a}$;

$a^2x(x - 2) + a^2 - b^2 = b^2x(x - 2)$; $a^2x - 2a^2x + a^2 - b^2 = b^2x^2 - 2b^2x$; $a^2x^2 - b^2x^2 - 2a^2x + 2b^2x + a^2 - b^2 = 0$;

$(a^2 - b^2)x^2 - 2(a^2 - b^2)x + (a^2 - b^2) = 0$; ehk $x^2 - 2x = 1 = 0$;

$x = 1 \pm \sqrt{1 - 1} = 1$; $x_1 = x_2 = 1$.

91. $(a + b)(a - b)x^2 = ab(2ax - ab)$;

$(a + b)(a - b)x^2 = 2a^2bx - a^2b^2$; $(a^2 - b^2)x^2 - 2a^2bx + a^2b^2 = 0$;

kust $x = \frac{a^2b \pm \sqrt{a^4b^2 - a^2b^2(a^2 - b^2)}}{a^2 - b^2} = \frac{a^2b \pm \sqrt{a^4b^2 - a^4b^2 + a^2b^4}}{a^2 - b^2} =$

$$= \frac{a^2b \pm \sqrt{a^2b^4}}{a^2 - b^2} = \frac{a^2b \pm ab^2}{a^2 - b^2}; \quad x_1 = \frac{a^2b + ab^2}{a^2 - b^2} = \frac{ab(a + b)}{(a + b)(a - b)} = \frac{ab}{a - b};$$

$$x_2 = \frac{a^2b - ab^2}{a^2 - b^2} = \frac{ab(a - b)}{(a + b)(a - b)} = \frac{ab}{a + b}.$$

92. $x^2 - \frac{cx}{a+b} - \frac{2c^2}{(a+b)^2} = 0$; $(a+b)^2x^2 - c(a+b)x - 2c^2 = 0$, kust $x = \frac{c(a+b) \pm \sqrt{(a+b)^2c^2 + 8c^2(a+b)^2}}{2(a+b)^2} = \frac{c(a+b) \pm \sqrt{(a+b)^2 \cdot 9c^2}}{2(a+b)^2} = \frac{c(a+b) \pm 3c(a+b)}{2(a+b)^2}$; $x = \frac{c \pm 3c}{2(a+b)}$; $x_1 = \frac{2c}{a+b}$; $x_2 = -\frac{c}{a+b}$.

93. $\frac{2a+b-x}{2b+a-x} = \frac{a}{b} \cdot \frac{x+b}{x+a}$; $b(x+a)(2a+b-x) = a(x+b)(2b+a-x)$; $2abx + 2a^2b + b^2x + ab^2 - bx^2 - abx = 2abx + 2ab^2 + a^2x + a^2b - ax^2 - abx$; $(a-b)x^2 - (a^2-b^2)x + ab(a-b) = 0$; ekvatsiooni $(a-b)$'ga koondades saame $x^2 - (a+b)x + ab = 0$, kust $x = \frac{(a+b) \pm \sqrt{(a+b)^2 - 4ab}}{2} = \frac{(a+b) \pm \sqrt{(a-b)^2}}{2} = \frac{(a+b) \pm (a-b)}{2}$, $x_1 = a$; $x_2 = b$.

94. $\frac{4a+3b-x}{4b+3a-x} = \frac{2a+b}{2b+a} \cdot \frac{2a+3b+x}{2b+3a+x}$;

$(2b+a)(2b+3a+x)(4a+3b-x) = $
$= (2a+b)(2a+3b+x)(4b+3a-x)$;

$(2b+a)(8ab + 6b^2 - 2bx + 12a^2 + 9ab - 3ax + 4ax + 3bx - x^2) = (2a+b)(8ab + 6a^2 - 2ax + 12b^2 + 9ab - 3bx + 4bx + 3ax - x^2)$; $(2b+a)(12a^2 + 6b^2 + 17ab - x^2 + ax + bx) = (2a+b)(6a^2 + 12b^2 + 17ab - x^2 + ax + bx)$; $24a^2b + 12b^3 + 34ab^2 - 2bx^2 + 2abx + 2b^2x + 12a^3 + 6ab^2 + 17a^2b - ax^2 + a^2x + abx = 12a^3 + 24ab^2 + 34a^2b - 2ax^2 + 2abx + 6a^2b + 12b^3 + 17ab^2 - bx^2 + abx + b^2x$; $ax^2 - bx^2 - a^2x + b^2x + a^2b - ab^2 = 0$ ehk $(a-b)x^2 - (a^2-b^2)x + ab(a-b) = 0$, $(a-b)$'ga koondades saame $x^2 - (a+b)x + ab = 0$, kust $x = \frac{(a+b) \pm \sqrt{(a+b)^2 - 4ab}}{2} = \frac{(a+b) \pm \sqrt{(a-b)^2}}{2} = \frac{(a+b) \pm (a-b)}{2}$; $x_1 = a$; $x_2 + b$.

95. $\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{a}{b}+\frac{b}{a}$; $\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{a^2+b^2}{ab}$;

$ab(x+a)(x-b)+ab(x+b)(x-a)=(a^2+b^2)(x-a)(x-b)$;

$ab(x^2+ax-bx-ab)+ab(x^2+bx-ax-ab)=$

$=(a^2+b^2)(x^2-ax-bx+ab)$; $ab(x^2+ax-bx-ab+x^2+$

$+bx-ax-ab)=(a^2+b^2)[x^2-(a+b)x+ab]$;

$2ab(x^2-ab)=(a^2+b^2)x^2-(a^2+b^2)(a+b)x+ab(a^2+b^2)$;

$2abx^2-2a^2b^2-(a^2+b^2)x^2+(a^2+b^2)(a+b)x-ab(a^2+b^2)=0$;

$-x^2(-2ab+a^2+b^2)+(a^2+b^2)(a+b)x-[2a^2b^2+ab(a^2+b^2)]=0$;

$(a-b)^2x^2-(a^2+b^2)(a+b)x+ab(2ab+a^2+b^2)=0$;

$(a-b)^2x^2-(a^2+b^2)(a+b)x+ab(a+b)^2=0$ kust $x=$

$$=\frac{(a^2+b^2)(a+b)\pm\sqrt{(a^2+b^2)^2(a+b)^2-4ab(a+b)^2(a-b)^2}}{2(a-b)^2}=$$

$$=\frac{(a^2+b^2)(a+b)\pm\sqrt{(a+b)^2[a^4+b^4+2a^2b^2-4a^3b+8a^2b^2-4ab^3]}}{2(a-b)^2}=$$

$$=\frac{(a^2+b^2)(a+b)\pm(a+b\sqrt{a^4-4a^3b+10a^2b^2-4ab^3+b^4}}{2(a-b)^2}=$$

$$=\frac{a+b}{2(a-b)^2}(a^2+b^2)\pm\sqrt{a^4-4a^3b+10a^2b^2-4ab^3+b^4}.$$

96. $\frac{1}{x-a}+\frac{1}{x-b}+\frac{1}{x-c}=0$; $(x-b)(x-c)+$

$+(x-a)(x-c)+(x-a)(x-b)=0$; $x^2-(b+c)x+bc+$

$+x^2-(a+c)x+ac+x^2-(a+b)x+ab=0$; $3x^2-$

$-(b+c+a+c+a+b)x+bc+ac+ab=0$; $3x^2-$

$-2(a+b+c)x+bc+ac+ab=0$, kust $x=$

$$=\frac{(a+b+c)\pm\sqrt{(a+b+c)^2-3(bc+ac+ab)}}{3}=$$

$$=\frac{(a+b+c)\pm\sqrt{a^2+b^2+c^2+2ab+2ac+2bc-3bc-3ac-3ab}}{3}=$$

$$=\tfrac{1}{3}(a+b+c\pm\sqrt{a^2+b^2+c^2-bc-ac-ab}).$$

97. $\frac{a+b-x}{a-b-x}=\frac{a-c+x}{a-c-x}$; esimese suhte liikmete summa suhtub liikmete vahele nii, kui teise suhte liikmete summa suhtub nende vahele, sellepärast: $\frac{a+b-x+a-b-x}{a+b-x-a+b+x}=$

$= \frac{a - c + x + a - c - x}{a - c + x - a + c + x}$ ehk $\frac{2a - 2x}{2b} = \frac{2a - 2c}{2x}$ ehk $\frac{a - x}{b} = \frac{a - c}{x}$, kust $ax - x^2 = ab - bc$; $x^2 - ax - b(c - a) = 0$; $x = \frac{a \pm \sqrt{a^2 + 4b(c - a)}}{2}$.

98. $19(a - x)(a - b) + 19(x - b)^2 = 49(a - x)^2 + 49(2x - a - b)(x - b)$; sulgusid avades saame $19a^2 - 19ax - 19ab + 19bx + 19x^2 - 38bx + 19b^2 = 49a^2 - 98ax + 49x^2 + 98x^2 - 49ax - 49bx - 98bx + 49ab + 49b^2$; $128x^2 - 128ax - 128bx + 30a^2 + 68ab + 30b^2 = 0$; $64x^2 - 64(a + b)x + 15a^2 + 15b^2 + 34ab = 0$, kust $x =$

$$= \frac{32(a + b) \pm \sqrt{1024(a + b)^2 - 64(15a^2 + 15b^2 + 34ab)}}{64} =$$

$$= \frac{32(a + b) \pm \sqrt{1024a^2 + 2048ab + 1024b^2 - 960a^2 - 960b^2 - 2176ab}}{64} =$$

$$= \frac{32(a + b) \pm \sqrt{64a^2 - 128ab + 64b^2}}{64} = \frac{32(a + b) \pm \sqrt{64(a - b)^2}}{64} =$$

$$= \frac{32(a + b) \pm 8(a - b)}{64} = \frac{4(a + b) \pm (a - b)}{8}; \quad x_1 = \frac{5a + 3b}{8}; \quad x_2 = \frac{3a + 5b}{8}.$$

99. $x(a - 2cx)[a + c(a + x)] + (a + x)(a - 2cx)[a + c(a - x)] = ax[a + c(a - x)]$; $x(a - 2cx)[a + c(a + x)] + (a - 2cx)[a(a + x) + c(a^2 - x^2)] = ax[a + c(a - x)]$;

$(a - 2cx)(ax + acx + x^2c + a^2 + ax + a^2c - x^2c) = a^2x + a^2cx - x^2ac$; $(a - 2cx)(2ax + acx + a^2 + a^2c) = a^2x + a^2cx - x^2ac$; $2a^2x + a^2cx + a^3 + a^3c - 4x^2ac - 2x^2c^2a - 2a^2cx - 2a^2c^2x - a^2x - a^2x - a^2cx + x^2ac = 0$; $-3x^2ac - 2x^2c^2a + a^2x - 2a^2cx - 2a^2c^2x + a^3 + a^3c = 0$; $ac(2c + 3)x^2 - a^2(1 - 2c - 2c^2)x - a^3(c + 1) = 0$;

$c(2c + 3)x^2 - a(1 - 2c - 2c^2)x - a^2(c + 1) = 0$ kust

$$x = \frac{a(1 - 2c - 2c^2) \pm \sqrt{a^2(1 - 2c^2)^2 + 4a^2c(c + 1)(2c + 3)}}{2c(2c + 3)} =$$

$$= \frac{a(1 - 2c - 2c^2) \pm a\sqrt{1 + 4c^2 + 4c^4 - 4c - 4c^2 + 8c^3 + 8c^3 + 12c^2 + 12c + 8c^2}}{2c(2c + 3)} =$$

$$= \frac{a(1 - 2c - 2^2) \pm a\sqrt{1 + 4c^4 + 16c^2 + 4c^2 + 8c + 16c^3}}{2c(2c+3)} =$$

$$= \frac{a(1 - 2c - 2c^2) \pm a\sqrt{(1 + 2c^2 + 4c)^2}}{2c(2c+3)} =$$

$$= \frac{a(1 - 2c - 2c^2) \pm a(1 + 4c + 2^2)}{2c(2c+3)}; \quad x_1 = -a; \quad x_2 = \frac{a(c+1)}{c(2c+3)}.$$

100. $(x + a)(x - b)(x - c) + (x + b)(x - a)(x - c) + (x + c)(x - a)(x - b) = 3(x - a)(x - b) \,.\, (x - c);$

$(x + a)[x^2 - (b + c)x + bc] + (x + b)[x^2 - (a + c)x + ac] + (x + c)[x^2 - (a + b)x + ab] = 3(x - a)[x^2 - (b + c)x + bc];$
$x^2 - (b + c)x^2 + bcx + ax^2 - a(b + c)x + abc + x^3 - (a + c)x^2 + acx + bx^2 - b(a + c)x + abc + x^3 - (a + b)x^2 + abx + cx^2 - c(a + b)x + abc - 3x^3 + 3(b + c)x^2 - 3bcx + 3ax^2 - 3a(b + c)x + 3abc = 0;$

$(a + b + c)x^2 - 2(ab + bc + ac)x + 3abc = 0$, kust

$$x = \frac{(ab + bc + ac) \pm \sqrt{(ab + bc + ac)^2 - 3abc(a + b + c)}}{(a + b + c)} =$$

$$= \frac{(ab+bc+ac) \pm \sqrt{a^2b^2+b^2c^2+a^2c^2+2ab^2c+2a^2bc+2abc^2-3a^2bc-3ab^2c-3abc^2}}{a + b + c} =$$

$$= \frac{ab + bc + ac \pm \sqrt{a^2b^2 + b^2c^2 + a^2c^2 - a^2b^2 - ab^2c - abc^2}}{a + b + c}.$$

§ 3. Ruutvõrrandite teooria tarvitamine lihtsamail juhuseil.

101. Et $6^2 > 4 \,.\, 5$ s. o. $36 > 20$, siis on ekvatsiooni juured reaalsed ja võrratud.

102. Et $10^2 = 4 \,.\, 25$; $100 = 100$, siis on ekv. juured võrdsed.

103. Et $4^2 < 4 \,.\, 5$; $16 < 20$, siis on ekv. juured imaginaarsed (mõeldavad).

104. Et $8^2 < 4 \,.\, 25$; $64 < 100$, siis on ekv. juured imaginaarsed.

105. Et $2^2 > -4 \,.\, 120$; $4 > -420$; siis on ekv. juured reaalsed ja võrratud.

106. Et $24^2 = 4 . 144$; $576 = 576$, siis on ekv. juured võrdsed.

107. Et $7^2 > -4 . 12 . 12$; $49 > -576$ siis on ekv. juured reaalsed ja võrratud.

108. Et $4^2 > 4 . 4 . 13$; $16 < 208$, siis on ekv. juured imaginaarsed.

109. Et $30^2 = 4 . 25 . 9$; $900 = 900$, siis on ekv. juured võrdsed.

110. Et $18^2 < 4 . 2 . 65$; $324 < 520$, siis on ekv. juured imaginaarsed.

111. Juured on reaalsed ja positiivsed, sest $8^2 < 4 . 15$ ja kordaja x'i 1-se astme ees, mis kujutab juurte summat vastupidise märgiga, on negatiivne, ja tuntud liige on positiivne.

112. Juured on reaalsed; märgi poolest vastupidised, sest tuntud liige on negatiivne.

113. Juured on reaalsed; märgi poolest vastupidised, sest tuntud liige on negatiivne.

114. Juured on imaginaarsed, sest $5^2 < 4 . 130$.

115. Juured on võrdsed, sest $26^2 = 4 . 169$; positiivsed, sest x'i ees olev kordaja on negatiivne.

116. Juured on reaalsed; märgi poolest vastupidised, sest tuntud liige on negatiivne.

117. Juured on reaalsed; negatiivsed, sest x'i ees on kordaja positiivne, ja tuntud liige ka positiivne.

118. Juured on imaginaarsed, sest $5^2 < 4 . 6 . 6$.

119. Juured on imaginaarsed, sest $2^2 < 4 . 4 . 1$.

120. Juured on reaalsed; vastupidiste märkidega, sest tuntud liige on negatiivne.

121. Juurte summa $2 + 3 = 5$, nende korrutis $2 . 3 = 6$; otsitav ekvatsioon $x^2 - 5x + 6 = 0$.

122. Summa $-4 + 6 = 2$; korrutis $-4 . 6 = -24$; $x^2 - 2x - 24 = 0$.

123. Juurte summa $-5+0=5$; korrutis $-5.0=0$; ekvatsioon; $x^2+5x=0$.

124. Juurte summa $3+(-3)=0$; korrutis $3.-3=9$; $x^2-9=0$.

125. Juurte summa $\frac{1}{2}+(-\frac{1}{4})=\frac{1}{4}$; korrutis $\frac{1}{2}\cdot-\frac{1}{4}=-\frac{1}{8}$; $x^2-\frac{1}{4}x-\frac{1}{8}=0$; ehk $8x^2-2x-1=0$.

126. Juurte summa $-\frac{2}{3}+(-\frac{3}{2})=-\frac{13}{6}$; korrutis $-\frac{3}{2}\cdot-\frac{2}{3}=1$; 0. ekv. $x^2+\frac{13}{6}x+1=0$; ehk $6x^2+13x+6=0$.

127. Juurte summa $\sqrt{6}=(-\sqrt{3})=\sqrt{6}-\sqrt{4}$; korrutis $\sqrt{6}.-\sqrt{3}=-3\sqrt{2}$; otsitav ekvatsioon. $x-(\sqrt{6}-\sqrt{3})x-$ $-3\sqrt{2}=0$ ehk $x^2+(\sqrt{3}-\sqrt{6})x-3\sqrt{2}=0$.

128. Juurte summa $(4+\sqrt{3})+(4-\sqrt{3})=8$; korrutis $4^2-(\sqrt{3})^2=13$; otsitav ekvatsioon. $x^2-8x+13=0$.

129. Juurte summa $(-3+\sqrt{-15})+$ $+(-3-\sqrt{-15})=-6$; korrutis $(-3)^2-(\sqrt{-15})^2=$ $=9-(-15)=24$; $x^2+6x+24=0$.

130. Juurte summa $(1+\sqrt{-10})+(1-\sqrt{-10})=2$; korrutis $1^2-(\sqrt{-10})^2=1-(-10)=11$; ekvatsioon. x^2- $-2x+11=0$;

131. Juurte summa $3a+(-2b)=3a-2b$; korrutis $3a.-2b=-6ab$; $x^2-(3a-2b)x-6ab=0$; x^2+ $+(2b-3a)x-6ab=0$.

132. Juurte summa $(2a-b)+(a-2b)=3a-3b$; korrutis $(2a-b)(a-2b)=2a^2-4ab-ab+2b^2=2a^2-$ $-5ab+2b^2$; otsitav ekvatsioon $x^2-3(a-b)x+2a^2-$ $-5ab+2b^2=0$.

133. Juurte summa $-\frac{a}{3}+\frac{a}{2}=\frac{a}{6}$; korrutis $-\frac{a}{3}\cdot\frac{a}{2}=$ $=-\frac{a^2}{6}$; ekvatsioon $x^2-\frac{a}{b}x-\frac{a^2}{6}$; $6x-ax-a^2=0$.

134. Juurte summa $(a+b)+(a-b)=2a$; korrutis $(a+b)(a-b)=a^2-b^2$; ekvatsioon ise $x^2-2ax+a^2-b^2=0$.

135. Juurte summa $\frac{a}{b}+\frac{b}{a}=\frac{a^2+b^2}{ab}$; korrutis $\frac{a}{b}\cdot\frac{b}{a}=$ $=1$; ekvatsioon: $x^2-\frac{a^2+b^2}{ab}x+1=0$; $abx^2-(a^2+b^2)x+$ $+ab=0$.

136. Juurte summa $\frac{a-b}{a+b}+1=\frac{a-b+a+b}{a+b}=\frac{2a}{a+b}$; kasv. $\frac{a-b}{a+a}\cdot 1=\frac{a-b}{a+b}$; ekvatsioon $x^2-\left(\frac{2a}{a+b}\right)x+\frac{a-b}{a+b}=0$; $(a+b)x^2-2ax+a-b=0$.

137. Juurte summa $\frac{ab}{a+b}+\frac{ab}{a-b}=\frac{ab(a-b)+ab(a+b)}{a^2-b^2}=$ $=\frac{ab(a-b+a+b)}{a^2-b^2}=\frac{2a^2b}{a^2-b^2}$; korrutis $\frac{ab}{a+b}\cdot\frac{ab}{a-b}=\frac{a^2b^2}{a^2-b^2}$; otsitav ekvatsioon $x^2-\frac{2a^2b}{a^2-b^2}x+\frac{a^2b^2}{a^2-b^2}=0$; ehk $(a^2-b^2)x-$ $-2a^2bx+a^2b^2=0$.

138. Juurte summa $\frac{b}{1-a}+\frac{a}{1-b}=\frac{b(1-b)+a(1-a)}{(1-a)(1-b)}=$ $=\frac{b-b^2+a-a^2}{(1-a)(1-b)}=\frac{(a+b)-(a^2+b^2)}{(1-a)(1-b)}$; korrutis $\frac{b}{1-a}\cdot\frac{a}{1-b}=$ $=\frac{ab}{(1-a)(1-b)}$; otsitav ekvatsioon $x^2+\frac{(a^2+b^2)-(a+b)}{(1-a)(1-b)}x+$ $+\frac{ab}{(1-a)(1-b)}=0$; ehk $(1-a)(1-b)x^2+[(a^2+b^2)-$ $-(a+b)]x+ab=0$.

139. Juurte summa $(a+\sqrt{b})+(a-\sqrt{b})=2a$; korrutis a^2-b. Ekvatsioon: $x^2-2ax+a^2-b=0$.

140. Juurte summa $(\sqrt{a}+\sqrt{-b})+(\sqrt{a}-\sqrt{-b})=$ $=2\sqrt{a}$. korrutis $(\sqrt{a}+\sqrt{-b})(\sqrt{a}-\sqrt{-b})=(\sqrt{a})^2-$ $-(\sqrt{-b})^2=a-(-b)=a+b$. Ekvatsioon: $x^2-2\sqrt{a}\,.$ $.\,x+(a+b)=0$.

141. Lahendame antud ekvatsiooni $x-7x+12=0$; $x=\frac{7\pm\sqrt{49-48}}{2}=\frac{7\pm1}{2}$; $x_1=4$; $x_2=3$. Teguriteks lahutatuld on ekv. $(x-4)(x-3)$.

142. $x^2 + 3x - 108 = 0$; $x = \frac{-3 \pm \sqrt{9 + 432}}{2} = \frac{-3 \pm 21}{2}$; $x_1 = -12$; $x_2 = 9$. Tegurid: $(x + 12)(x - 9)$.

143. $6x^2 + 5x - 6 = 0$; $x = \frac{-5 \pm \sqrt{25 + 144}}{1 . 2} = \frac{-5 \pm 13}{12}$; $x_1 = -\frac{3}{2}$; $x_2 = \frac{2}{3}$. Tegurid: $6(x + \frac{3}{2})(x - \frac{2}{3})$ ehk $\frac{6(2x + 3)(3x - 2)}{6}$ ehk $(2x + 3)(3x - 2)$.

144. $30x^2 + 37x + 10 = 0$; $x = \frac{-37 \pm \sqrt{1369 - 1200}}{60} = \frac{-37 \pm 13}{60}$; $x_1 = -\frac{5}{6}$; $x_2 = -\frac{2}{5}$. Tegurid: $30(x + \frac{5}{6})(x + \frac{2}{5})$; $(6x + 5)(5x + 2)$.

145. $x^2 - 6x + 11 = 0$; $x = 3 \pm \sqrt{9 - 11} = 3 \pm \sqrt{-2} = 3 \pm \sqrt{2} . i$; ekv. juured on: $x_1 = 3 + \sqrt{2} . i$; $x_2 = 3 - \sqrt{2} . i$. Tegurid $(x - 3 - \sqrt{2} . i)(x - 3 + \sqrt{2} . i)$.

146. Lahendame ekvatsiooni $x^2 + 15x + 44 = 0$; $x = \frac{-15 \pm \sqrt{225 - 176}}{2} = \frac{-15 \pm 7}{2}$; $x_1 = -4$; $x_2 = -11$; tegurid: $(x + 4)(x + 11)$.

147. Lahendame ekvatsiooni $x^2 - ax - 6a^2 = 0$; $x = \frac{a \pm \sqrt{a^2 + 24a^2}}{2} = \frac{a \pm 5a}{2}$; ekv. juured: $x_1 = \frac{a + 5a}{2} = 3a$; $x_2 = \frac{a - 5a}{2} = -2a$; $(x - 3a)(x + 2a)$.

148. Lahendame ekvatsiooni: $abx^2 - 2ax + a^2 - b^2 = 0$; $x = \frac{a \mp \sqrt{a^2 - ab(a^2 - b^2)}}{ab}$; $x_1 = \frac{a + \sqrt{a^2 - ab(a^2 - b^2)}}{ab}$;

$$x_2 = \frac{a - \sqrt{a^2 - ab(a^2 - b^2)}}{ab}.$$ Tegurid:

$$ab\left(x + \frac{a + \sqrt{a^2 - ab(a^2 - b^2)}}{ab}\right)\left(x - \frac{a - \sqrt{a^2 - ab(a^2 - b^2)}}{ab}\right)$$ ehk

$$\frac{1}{ab}[abx - a - \sqrt{a^2 - ab(a^2 - b^2)}]\,[abx - a + \sqrt{a^2 - ab(a^2 - b^2)}].$$

149. Lahendame ekvatsiooni: $x^2 - ax - a\sqrt{b} - b = 0$;

$$x = \frac{a \pm \sqrt{a^2 + 4a\sqrt{b} + 4b}}{2} = \frac{a \pm \sqrt{(a + 2\sqrt{2})^2}}{2} = \frac{a \pm (a + 2\sqrt{2})}{2},$$

$x_1 = (a + \sqrt{b})$; $x_2 = -\sqrt{b}$. Tegurid: $(x - a - \sqrt{b})(x + \sqrt{b})$.

150. Lahendame võrrandi: $abx^2 - 2a\sqrt{ab}.x + a^2 - b^2 = 0$;

$$x = \frac{a\sqrt{ab} \mp \sqrt{a^3b - ab(a^2 - b^2)}}{ab} = \frac{a\sqrt{ab} \pm b\sqrt{ab}}{ab};\ x_1 = \frac{(a+b)\sqrt{ab}}{ab};$$

$x_2 = \frac{(a-b)\sqrt{ab}}{ab}$. Tegurid: $ab\left(x - \frac{(a+b)\sqrt{ab}}{ab}\right)\left(x - \frac{(a-b)\sqrt{ab}}{ab}\right)$ ehk $ab\left(x - \frac{a+b}{\sqrt{ab}}\right)\left(x - \frac{a-b}{\sqrt{ab}}\right)$ ehk $(\sqrt{ab}.x - a - b)(\sqrt{ab}.$ $.x - a + b)$.

151. Et ruutvõrrandi juurte summa võrdub x'i esimese astme vastasmärgilise kordajaga ja nende (juurte) korrutis on võrdne vabaliikmega, siis võib antud juurtest $\frac{1}{x_1}$ ja $\frac{1}{x_2}$ järgmine võrrand kokku seada: $x^2 - \left(\frac{1}{x_1} + \frac{1}{x_2}\right)x + \frac{1}{x_1x_2} = 0$; ehk $x^2 - \frac{x_1 + x_2}{x_1x_2}.x + \frac{1}{x_1x_2} = 0$. Tingimise järele on x_1 ja x_2 võrrandi „$x^2 + px + q = 0$" juured, sellepärast $x_1 + x_2 = -p$; ja $x_1x_2 = q$. Neid tähendusi kokkuseatud võrrandisse asemele pannes saame: $x^2 - \frac{-p}{q}x + \frac{1}{q} = 0$; $qx^2 + px + 1 = 0$.

152. Olgu antud võrrandi „$x^2 + px + q = 0$" jnured α ja β; otsitava võrrandi juured on siis $m\alpha$ ja $m\beta$; ruutvõrrandi juurte omaduste põhjal on otsitav võrrand: $x^2. - (m\alpha + m\beta)x + m^2\alpha\beta = 0$ ehk $x^2 - m(\alpha + \beta)x + m^2\alpha\beta = 0$. Et $\alpha + \beta = -p$ ja $\alpha\beta = q$ siis $x^2 - m.(-p)x + m^2q = 0$ ehk $x^2 + mpx + m^2q = 0$.

153. Kui antud võrrandi „$x^2 + px + q = 0$" juured on α ja β, siis otsitava võrrandi juured on $\left(\alpha + \frac{p}{2}\right)$ ja $\left(\beta + \frac{p}{2}\right)$; otsitav võrrand ise $x^2 - \left(\alpha + \frac{p}{2} + \beta + \frac{p}{2}\right)x + \left(\alpha + \frac{p}{2}\right)\left(\beta + \frac{p}{2}\right) = 0$

ehk $x^2 - (\alpha + \beta + p)x + \left[\alpha\beta + \frac{p}{2}(\alpha + \beta) + \frac{p^2}{4}\right] = 0$; antud võrrandist leiame, et $x + \beta = -p$, $\alpha\beta = q$, sellepärast $x^2 - (-p + p)x + \left[q + \frac{p}{2} \cdot (-p) + \frac{p^2}{4}\right] = 0$; ehk $x^2 + q - \frac{p^2}{2} + \frac{p^2}{2} = 0$ ehk $4x^2 + 4q - p^2 = 0$.

154. Antud võrr. juurte summa on $\alpha + \beta = -p$, korrutis $\alpha\beta = q$. Tingimise järele on $-p$ ja q otsitava võrrandi juured; nende summa „$(-p + q)$" ja korrutise „$-pq$" järele saame võrrandi $x^2 - (-p + q)x - pq = 0$ ehk $x^2 + (p - q)x - pq = 0$.

155. Olgu antud võrr. juured α ja β. Nende summa $\alpha + \beta = -p$, korrutis $\alpha\beta = q$; järjelikult $(\alpha + \beta)^2 = p^2$ ehk $\alpha^2 + \beta^2 + 2\alpha\beta = p^2$, kust $\alpha^2 + \beta^2 = p^2 - 2\alpha\beta = p^2 - 2q$.

156. Antud võrrandi juuri α ja β'-ga ära tähendades, saame $\alpha + \beta = -p$ ja $\alpha\beta = q$; siit $(\alpha + \beta)^3 = -p^3$ ehk $\alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta) = -p^3$ ehk $\alpha^3 + \beta^3 + 3q(-p) = -p^3$; järjelikult $\alpha^3 + \beta^3 = 3pq - p^3 = p(q - 3p^2)$.

157. $x^2 + px + q = 0$ laadi võrrandi juurte ruutude summa (v. ü. nr. 155) $= p^2 - 2q$; võrrandis $x^2 - 2x - 15 = 0$ on meil $p = -2$, $q = -15$; sellepärast $\alpha^2 + \beta^2 = p^2 - 2q = 4 + 30 = 34$; kolmandate astmete summa (v. ü. 156) $\alpha^3 + \beta^3 = p(3q - p^2) = -2(-45 - 4) = 98$.

158. Summa $\alpha^3 + \beta^3 = p^2 - 2q$ (v. ü. 155) ja $\alpha^3 + \beta^3 = p(3q - p^2)$ (ü. nr. 155); v-ist saame $p = \frac{7}{3}$; $q = \frac{2}{3}$; sellepärast $\alpha^2 + \beta^2 = \frac{49}{9} - \frac{4}{3} = \frac{37}{9} = 4\frac{1}{9}$ ja $\alpha^3 + \beta^3 = \frac{7}{3}(2 - \frac{49}{9}) = -\frac{217}{27} = -8\frac{1}{27}$.

159. Juurte ruutude summa ü. 155 järele $= p^2 - 2q$; tiugimise järele $p^2 - 2q = 34$; antud võrrandis $p = -8$, sellepärast $64 - 2q = 34$, kust $-2q = -30$; $q = 15$; lahendame antud võrrandi $x^2 - 8x + 15 = 0$; $x = 4 \pm \sqrt{16 - 15} = 4 \pm 1$; $x_1 = 5$; $x_2 = 3$.

160. Tingimise järele $(\alpha-\beta)^2=144$ ehk $\alpha^2+\beta^2-2\alpha\beta=$ $=144$; $\alpha^2+\beta^2+2\alpha\beta-4\alpha\beta=144$ ehk $(\alpha+\beta)^2-4\alpha\beta=144$; antud võrraudis $\alpha\beta=45$, sellepärast $(\alpha+\beta)^2-180=144$; $(\alpha+\beta)^2=324$; $\alpha+\beta=\pm 18$ s. o. $p=\pm 18$. Meil on kaks võrrandit: I-ne: $x^2+18x+45=0$; $x=-9\pm\sqrt{81-45}=$ $=-9\pm 6$; $x_1=-3$; $x_2=-15$; II-ne: $x_2-18x+45=$ $=0$; $x=9\pm\sqrt{81-45}=9\pm 6$; $x_1=15$; $x=3$.

161. Üldise võrrandi $ax^2+bx+c=0$ juured on võrdsed kui $b^2=4ac$; käesoleval juhtumisel $a=4$, $c=64$ ja sellepärast $b^2=4.4.64$; $b=2.2.8=32$.

162. Kui $b^2=4ac$, siis $b=2\sqrt{ac}$. Antud kolmliikmesse b asemele tema tähendust pannes saame $ax^2+2\sqrt{ac}$. $.x+c$ ehk $(x\sqrt{a})^2+2x\sqrt{a}.\sqrt{c}+(\sqrt{c})^2=(x\sqrt{a}+\sqrt{c})^2$; siit on selge, et kolmliige ax^2+bx+c muutub täieliseks ruuduks $(x\sqrt{a}+\sqrt{c})^2$ kui $b^2=4ac$.

163. Üldise võrrandi $ax^2+bx+c=0$ juured on reaalsed, kui $b^2\geqslant 4ac$ ja immaginaarsed, kui $b^2<4ac$; antud võrrandis „$3x^2-18x+c=0$" on juured reaalsed, kui $18^2\geqslant 4.3.c$, kust $c\leqslant 27$; juured on imaginaarsed, kui $18^2<4.3.c$ s. o. $c>27$.

164. $x=\frac{-b\pm\sqrt{b^2-4a.0}}{2a}=\frac{-b\pm\sqrt{b^2}}{2a}=\frac{-b\pm b}{2a}$;

$x_1=0$; $x_2=-\frac{b}{a}$.

165. Võrrandis $x^2-6x+q=0$ juurte summa $x_1+x_2=6$; tingimise järele $3x_1+2x_2=20$. I-st võrrandit 3'ga korrutades ja temast II-st ära võttes saame $x_2=-2$; tähendab $x_1=8$; sellepärast $q=x_1x_2=-2.8=-16$.

166. ax^2-bx+c laadi kolmliige on täielik ruut kui $b^2=4ac$ (v. ü. № 162); analoogiliselt on kolmliige $(a+b)x^2-$ $-(a+b)x+(a-b)$ täielik ruut, kui $(a+b)^2=4(a-b)^2$; $a+b=\pm 2.(a-b)$; $a+b=2a-2b$; kust $a=3b$; ehk $a+b=-2a+2b$, kust $b=3a$.

167. Selleks, et võrrandi mõlemad juured oleks positiivsed, peab I-se astme tundmatu märk olema miinus (—), b peab olema < 0, ja vabaliikme c märk peab olema $+$, c peab olema > 0; võrrand peab olema $ax^2 - bx + c = 0$.

168. $p = k + \frac{q}{k}$ ehk $\frac{k^2+q}{k}$ antud võrrandisse p asemele pannes, saame $x^2 + \frac{k^2+q}{k} x + q = 0$ ehk $kx^2 + + (k^2+q) = 0$. Siit $x = \frac{-(k^2+q) \pm \sqrt{(k^2+q)^2 - 4k^2q}}{2k} = \frac{-(k^2+q) \pm \sqrt{k^4+q^2+2k^2q-4k^2q}}{2k} = \frac{-(k^2+q) \pm \sqrt{(k^2-q)^2}}{2k} = \frac{-(k^2+q) \pm (k^2-q)}{2k}$; $x_1 = -\frac{q}{k}$; $x_2 = -k$; juured x_1 ja x_2 on alati ühismõõtsed kui q ja k ühismõõtsed on.

169. Antud võrrandi juurte üldine avaldus on järgmine: $x = \frac{-p \pm \sqrt{p^2-4q}}{2}$; $x_1 = \frac{-p + \sqrt{p^2-4q}}{2}$; $x_2 = = \frac{-p - \sqrt{p^2-4q}}{2}$. Et nende lugejaid ratsionaalseteks teha tuleb lugejat ja nimetajat x_1 avalduses $(p + \sqrt{p^2 - 4q})$ ja x_2 avalduses $(p - \sqrt{p^2 - 4q})$'ga korrutada.

$$x_1 = \frac{(-p + \sqrt{p^2-4q})(p + \sqrt{p^2-4q})}{2(p + \sqrt{p^2-4q})} = \frac{(\sqrt{p^2-4q})^2 - p^2}{2(p+\sqrt{p^2-49})} =$$

$$= \frac{p^2 - 4q - p^2}{2(p + \sqrt{p^2-4q})} = \frac{-2q}{p + \sqrt{p^2-4q}};$$

$$x_2 = \frac{-(p + \sqrt{p^2+4a})(p - \sqrt{p^2-4a})}{2(p - \sqrt{p^2-4q})} = \frac{p^2 - (p^2-4q)}{2(p - \sqrt{p^2-4q})} =$$

$$= -\frac{2q}{p - \sqrt{p^2-4q}}.$$

170. Võrrandist $ax^2 - bx + c = 0$, $x_1 = \frac{2c}{b + \sqrt{b^2-4ac}}$ ja $x_2 = \frac{2c}{b - \sqrt{b^2-4c}}$ (v. ü. 169). Kui a lõpmatult väheneb, läheb juur $\sqrt{b^2 - 4ac}$ vähe b'st lahku, ja sellepärast on nimetaja $b + \sqrt{b^2 - 4ac}$ peaaegu $= 2b$; järjelikult läheb x_1 vähe lahku $\frac{2c}{2b}$ ehk $\frac{c}{b}$'st; nimetaja $b - \sqrt{b^2 - 4ac}$ lõpmata väikse a juures võrdub ligikaudu 0'ga ja sellepärast läheb x_2 vähe lahku $\frac{2c}{2} = \infty$.

§ 4. Ruutvõrrandite kokkuseadmine.

171. Tähendame ühe kaateti x'iga ära; siis on teine kaatet $(17 - x)$. Pitagorase teoreem annab meile võrrandi: $x^2 + (17 - x)^2 = 13^2$ ehk $x^2 + 289 - 34x + x^2 = 169$; $x^2 - 17x + 60 = 0$. Võrrandit lahendades saame,

$$x = \frac{17 \pm \sqrt{289 - 240}}{2} = \frac{17 \pm 7}{2};\ x_1 = 12;\ x_2 = 5.$$

171. Kui täisnelinurga üks külg on x, siis on tema teine külg $(21 - x)$, sest pool perimeetrit on 21. Täisnurksest kolmnurgast, mis on moodustatud diagoonali ja 2-he küljega, saame: $x^2 + (21 - x)^2 = 15^2$; $x^2 - 21x + 108 = 0$;

$$x = \frac{21 \pm \sqrt{441 - 432}}{2} = \frac{21 \pm 3}{2};\ x_1 = 12;\ x_2 = 9.$$

172. Tähendame ära ühe, üksteisele järgnevatest arvudest x'iga, siis on teised $(x + 1)$ ja $(x + 2)$. Ülesandest saame võrrandi: $x^2 + (x + 1)^2 + (x + 2)^2 = 365$; $x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 365$; $3x^2 + 6x - 360 = 0$; $x^2 + 2x - 120 = 0$; $x = \frac{-2 \pm \sqrt{4 + 480}}{2} = \frac{-2 \pm 22}{2}$; $x_1 = 10$; $x_2 = -12$. Kui üks arv on 10, siis on teised 11 ja 12.

172. Kolm üksteisele järgnevat paaris arvu on $2x$, $(2x+2)$ ja $(2x+4)$. Saame võrrandi: $(2x)^2 + (2x + 2)^2 + (2x + 4)^2 = 116$; $12x^2 + 24x - 96 = 0$; $x^2 + 2x - 8 = 0$; $x = -1 \pm \sqrt{1 + 8} = -1 \pm 3$; $x_1 = 2$; $x_2 = -4$. Otsitavad arvud oleks 4; 6; 8 sest esimese otsitava tähendasime 2x'iga ära.

173. Olgu üks ruudu külg x, teine $(x+10)$. Ülesandest leiame, et $\frac{(x+10)^2}{x^2}=\frac{25}{9}$ ehk $9(x^2+20x+100)=25x^2$; $16x^2-180x-900=0$; $4x^2-45x-225=0$;

$x=\frac{45\pm\sqrt{2025+3600}}{8}=\frac{45\pm75}{8}$; $x_1=15$; $x_2=-3\frac{3}{4}$. Üks ruudu külg on 15, teine $15+10=25$ jalga.

173. I-se kolmnurga külg olgu x, teise oma $(x+14)$. Geomeetriast teame, et „sarnaste kolmnurkade pinnad suhtuvad nii nagu vastavate külgede ruutud". Selle teoreemi põhjal võime kirjutada: $\frac{x^2}{(x+14)^2}=\frac{25}{49}$; $49x^2=25(x+14)^2$;

$6x^2-175x-1225=0$. $x=\frac{175\pm\sqrt{30625+29400}}{12}=\frac{175\pm245}{12}$; $x_1=35$; $x_2=-5\frac{5}{6}$.

174. Kui puudade arv on x, siis on puuda hind $(x-2)$. Võrrand: $x(x-2)=120$; $x^2-2x-120=0$; $x=1\pm\sqrt{1+120}=1\pm11$; $x_1=12$; $x_2=-10$. Negatiivne vastus (-10) ei vasta ülesande küsimusele.

174. Puudade arv x; puuda hind $(x+3)$. Võrrand: $x(x+30)=270$; $x^2+3x-270=0$;

$x=-\frac{3}{2}\pm\sqrt{\frac{9+1080}{4}}=\frac{-3\pm33}{2}$; $x_1=\frac{-3+33}{8}=15$; $x_2=-18$.

175. Tähendame ära üheliste arvu x'iga, kümneliste arv on siis $(x-2)$; otsitav arv on $[10(x-2)+x]$; arvu ristsumma on $(x-2)+x$ ehk $2(x-1)$; ülesande tingimise järele $2[10(x-2)+x](x-1)=144$. $(22x-40)(x-1)=144$; ehk $22x^2-22x-40x+40-144=0$; $22x^2-62x-104=0$; $11x^2-31x-52=0$; $x=\frac{31\pm\sqrt{961+2288}}{22}=\frac{31\pm57}{22}$; $x_1=4$; $x_2=-\frac{13}{11}$. Ülesande küsimisele vastab ainult positiivne täisarv 4; otsitav arv $=10(4-2)+4=24$.

175. Kümneliste arv x; üheliste oma $(x-2)$. Eelmise ülesande eeskujul saame $[10x+(x-2)][x+(x-2)]=640$;

$(11x - 2)(2x - 2) = 640$; $22x^2 - 22x - 4x + 4 = 640$;
$22x^2 - 26x - 636 = 0$; $11x^2 - 13x - 318 = 0$.

$$x = \frac{13 \pm \sqrt{169 + 13992}}{22} = \frac{13 \pm 119}{22};\quad x_1 = 6;\quad x_2 = -\tfrac{53}{11};$$

arv $= 10 . 6 = (6 - 2) = 64$.

176. Oletame, et esimest sorti kaupa osteti x naela; teist sorti osteti siis $(x + 2)$ naela. I-se sordi naela hind on x mk. II-se sordi naela hind $(x + 2)$ mk. Kõik I-ne sort maksab x^2 mk.; II-ne sort $(x + 2)^2$ mk. Võrrand: $x^2 + (x + 2)^2 = 130$; $x^2 + x^2 + 4x + 4 = 130$; $x^2 + 2x - 63 = 0$; $x = -1 \pm \sqrt{1 + 63} = -1 \pm 8$; $x_1 = 7$; $x_2 = -9$. Ülesande mõttele vastab ainult x'i positiivne tähendus. Teist sorti osteti $7 + 2 = 9$ naela.

176. I sorti osteti x naela hinnaga á x mk.; II-st sorti $(x - 3)$ naela hinnaga á $(x - 3)$ mk. Võrrand:

$x^2 + (x - 3)^2 = 117$; $x^2 + x^2 - 6x + 9 = 117$;
$2x^2 - 6x - 108 = 0$; $x^2 - 3x - 54 = 0$.

$$x = \tfrac{3}{2} \pm \sqrt{\frac{9 + 216}{4}} = \frac{3 + 15}{2};\quad x_1 = 9;\quad x_2 = -6.$$

177. Tähendame ära ühe kaateti x'iga; teine kaatet on siis $(x + 1)$ ja hüpotenuus $(x + 2)$. Pitagorase teoreemi järele saame võrrandi $(x^2 + (x + 1)^2 = (x + 2)^2$; $x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$; $x^2 - 2x - 3 = 0$; $x = 1 \pm \sqrt{1 + 3} = 1 \pm 2$; $x_1 = 3$; $x_2 = -1$. Ülesande mõttele vastavalt peame võtma, et x'iga tähendatud kaatet on 3; teine kaatet $3 + 1 = 4$ ja hüpotenuus $3 + 2 = 5$. Niisugune $\triangle$ on võimalik.

177. Üks kaatet on 2x, teine $(2x + 2)$ ja hüpotenuus $(2x + 4)$; võrrand: $(2x + 4)^2 = 4x^2 + (2x + 2)^2$; $4x^2 + 16x + 16 = 4x^2 + 4x^2 + 8x + 4$; $x^2 - 2x - 3 = 0$;

$x = 1 \pm \sqrt{1 + 3} = 1 \pm 2$; $x_1 = 3$; $x_2 = -1$. Otsitavas $\triangle$'as on üks kaatet $2 . 3 = 6$, teine kateet $2 . 3 + 2 = 8$ ja hüpotenuus $2 . 3 + 4 = 10$.

178. Inimesi oli x, ja igaüks neist pidi maksma $\frac{72}{x}$ marka. Kui inimesi oleks olnud $(x - 3)$, siis oleks tulnud igal ühel maksta $\frac{72}{(x-3)}$. Tingimise järele võime kirjutada: $\frac{72}{x-3} - \frac{72}{x} = 4$, ehk $72x - 72x + 216 = 4x^2 - 12x$; $4x^2 - 12x - 216 = 0$; $x^2 - 3x - 54 = 0$; $x = \frac{3 \pm \sqrt{9+216}}{2} = \frac{3 \pm 15}{2}$; $x_1 = 9$; $x_2 = -6$. Inimesi oli 9.

178. Inimeste arv on x, igaüks pidi maksma $\frac{60}{x}$ mk.; kui neid oleks olnud $(x+3)$, siis oleks igaüks $\frac{60}{x-3}$ maksnud. $\frac{60}{x} - \frac{60}{x+3} = 1$; $60x + 180 - 60x = x^2 + 3x$; $x^2 + 3x - 180 = 0$; $x = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + 180} = -\frac{3}{2} \pm \sqrt{\frac{729}{4}} = \frac{-3 \pm 27}{2}$; $x_1 = 12$; $x_2 = -15$.

179. Ütleme, et tasapinnal on x punkti. Nemad määravad ära x'nurga. Igast x'nurga tipust võib tõmmata $(x-3)$ diagonaali. Kõiki diagonaale peaks olema siis x korda $(x-3)$, kuid siin on iga kahe tipu vahel olev diagonaal kaks korda arvatud (näit. A'st — B'sse ja teinekord B'st — A'sse). Tõelik diagonaalide arv on $\frac{x(x-3)}{2}$; hulknurga külgi on x; kõiki sirgjooni on $x + \frac{x(x-3)}{2}$ ehk tingimise järele 10. $x + \frac{x(x-3)}{2} = 10$; $x^2 - x - 20 = 0$; $x = \frac{1 \pm \sqrt{1+80}}{2} = \frac{1 \pm 9}{2}$; $x_1 = 5$; $x_2 = -4$. Punkte on 5.

179. Niisama kui eelmises ülesandes, $x + \frac{x(x-3)}{2} = 15$; $x^2 - x - 30 = 0$; $x = \frac{1 \pm \sqrt{1+120}}{2} = \frac{1 \pm 11}{2}$; $x_1 = 6$; $x_2 = -5$.

180. Ütleme, et I-ne toru üksinda täidab vesistu x'tunniga; II-ne $(x+5)$ tunniga. I-ne toru täidab iga tunniga $\frac{1}{x}$ osa vesistust ja 6 tunniga $\frac{6}{x}$ vesistust.

II-ne toru täidab igas tunnis $\frac{1}{x+5}$ osa vesistust ja 6-e tunniga $\frac{6}{x+5}$ osa. Ülesande tingimise järele saab vesistu 6 tunniga täis. Sellepärast $\frac{6}{x}+\frac{6}{x+5}=1$; $6x+30+6x=x^2+$ $+5x$; $x^2-7x-30=0$; $x=\frac{7\pm\sqrt{49+120}}{2}=\frac{7\pm 13}{2}$; $x_1=$ $=10$; $x_2=-3$. x'i negatiivne tähendus ei vasta ülesande küsimusele, siis jääb, et I-ne toru täidab täis vesistu 10 tunniga ja II-ne 15 tunniga.

180. I-ne toru täidab vesistu x tunniga; II-ne toru $(x+3)$ tunniga; nagu eelmiseski ülesandes: $\frac{3,6}{x}+\frac{3,6}{x+3}=1$;

$5x^2-21x-54=0$; $x=\frac{21\pm\sqrt{21^2+4.5.54}}{5.2}=\frac{21\pm\sqrt{1521}}{10}=$ $=\frac{21\pm 39}{10}$; $x_1=\frac{21+39}{10}=6$; $x_2=\frac{21-39}{10}=-1,8$; I-ne toru täidab vesistu täis 6 tunniga, II-ne toru $6+3=9$ tunniga.

181. Ütleme, et kell maksis x marka; müümise juures saadi x % kasu, tähendab iga marga pealt sai keegi $\frac{x}{100}$ mk.; x marga pealt sai ta $x.\frac{x}{100}$ marka kasu. Tingimise järele:

$x+\frac{x^2}{100}=39$; $100x+x^2=3900$; $x^2+100x-3900=0$; $x=-50\pm\sqrt{2500+3900}=-50\pm\sqrt{6400}=-50\pm 80$; $x_1=30$; $x_2=-130$. Kell maksis 30 marka.

181. Kell maksab x marka; kahju saadi x %. Harutades niisama, kui eelmises ülesandes saame: $x-\frac{x^2}{100}=24$;

$100x-x^2=2400$; $x^2-100x+2400=0$;

$x=50\pm\sqrt{2500-2400}=50\pm 10$; $x_1=60$; $x_2=40$.

182. Kaupmees sai päranduseks x sada marka ehk 100 x marka; iga aasta x % ära kulutades, s. o. iga saja marga pealt x marka ära kulutades, kulutas ta aastas x . x ehk x^2 mk.;

nelja aastaga kulutas ta $4x^2$ marka. Järele jäi $(100x - 4x^2)$, mis ülesande järele 400 marka peab olema. $100x - 4x^2 = 400$; $x^2 - 25x + 100 = 0$; $x = \frac{25 \pm \sqrt{625 - 400}}{2} = \frac{25 \pm 15}{2}$; $x_1 = 20$; $x_2 = 5$. Tema kapitaal võis olla, kas $20 \cdot 100 = 2000$ mk. ehk $5 \cdot 100 = 500$ mk.

182. Kapitaal on $100x$ marka; iga aasta sai kaupmees $x\%$ kasu; ühe aasta kasu on $x \cdot x = x^2$ marka; 10 aastaga $10x^2$. Kümne aasta pärast oli tema kapitaal $(100x + 10x^2)$ mk. ehk 2640 marka; $100x + 10x^2 = 2640$; $x^2 + 10x - 264 = 0$; $x = -5 \pm \sqrt{25 + 264} = 5 \pm 17$; $x_1 = 12$; $x^2 = -22$. Kapitaal on $12 \cdot 100 = 1200$ marka.

183. Hulknurgal on x külge. Ühest tipust on võimalik $(x - 3)$ diagonaali tõmmata. Kõiki diagonaale on võimalik tõmmata $\frac{x(x-3)}{2}$ (vaata ülesanne 179). Tingimise järele: $\frac{x(x-3)}{2} = 10$; $x^2 - 3x - 20 = 0$; $x = \frac{3 \pm \sqrt{9 + 80}}{2} = \frac{3 \pm \sqrt{89}}{2}$; x_1 ja x_2 tähendused on irratsionaalsed ja ei vasta ülesande küsimusele. Tähendab, niisugune hulknurk pole võimalik.

183. Külgede arv $= x$; diagonaale võib tõmmata $\frac{x(x-3)}{2}$ (v. ü. 183 I). Tingimise järele: $\frac{x(x-3)}{2} = 5$; $x^2 - 3x - 10 = 0$; $x = \frac{3}{2} \pm \sqrt{\frac{9}{4} + 10} = \frac{3}{2} \pm \frac{7}{2}$; $x_1 = 5$; $x_2 = -2$. Hulknurgal on 5 külge.

184. Ütleme, et osteti x puuda I-st sorti ja $(x + 3)$ puuda II-st sorti. Iga puud I-st sorti maksab $\frac{156}{x}$ mk., II-st sorti $\frac{210}{x+3}$ marka. Tingimise järele: $\frac{210}{x+3} - \frac{156}{x} = 1$; $210x - 156x - 468 = x^2 + 3x$; $x^2 - 51x + 468 = 0$;

$$x = \frac{51 \pm \sqrt{2601 - 1872}}{2} = \frac{51 \pm 27}{2};\ x_1 = 39;\ x_2 = 12.$$

Tähendab, I-st sorti osteti 39 ehk 12 puuda, II-st sorti 42 ehk 15 puuda.

184. Ostetud x puuda I-st sorti hinnaga $\frac{240}{x}$ mk. puud ja $(x-4)$ puuda II-st sorti hinnaga $\frac{320}{x-4}$ mk. puud. Ülesande järele $\frac{320}{x-4}-8=\frac{240}{x}$; $320x-8x^2+32x=240x-$ -960; $8x^2-112x-960=0$; $x^2-14x-120=0$; $x=$ $=7\pm\sqrt{49+120}=7\pm13$; $x_1=20$; $x_2=-6$. I-st sorti osteti 20 puuda ja teist 16 puuda.

185. Kui I-se tunnikiirus on x, siis on II-se tunnikiirus $(x-1)$. I-ne sõitis kõik maa ära $\frac{56}{x}$ tunniga, II-ne $\frac{56}{x-1}$ tunniga; et tingimise järele I-ne tund aega varem kohale jõuab kui II-ne, siis $\frac{56}{x-1}-\frac{56}{x}=1$; $56x-56x+56=x^2-x$; x^2- $-x-56=0$; $x=\frac{1\pm\sqrt{1+224}}{2}=\frac{1\pm15}{2}$; $x_1=8$; $x_2=-7$. Kõlbab ainult x'i positiivne tähendus $x=8$. I-se kiirus on 8 klm tunnis, II-se kiirus on $8-1=7$ klm tunnis.

185. I-se kiirus on x, II-se kiirus on $(x-2)$. $\frac{24}{x-2}-$ $-\frac{24}{x}=1$; $x^2-2x-48=0$; $x=1\pm\sqrt{1+48}=1\pm7$; $x_1=$ $=8$; $x_2=-6$. I-se kiirus on 8 klm tunnis, II-se kiirus on $8-2=6$ klm tunnis.

186. Ütleme, et laen tehti x %-ga; tähendab 1-he marga pealt saadakse aastas $\frac{x}{100}$ marka. Iga mark muutub aasta pärast $\left(1+\frac{x}{100}\right)$ margaks ja kõik võlg 820 marka muutub aasta pärast $820\left(1+\frac{x}{100}\right)$ margaks. Kui 441 mk. ära maksetakse jääb veel maksta $\left[820\left(1+\frac{x}{100}\right)-441\right]$ mk. Ülejäänud võla iga mark muutub II-se aasta jooksul jälle $\left(1+\frac{x}{100}\right)$ mk. ja kogu maksta jäänud võla osa II-se aasta lõpuks on: $\left[820\left(1+\frac{x}{100}\right)-441\right]\left(1+\frac{x}{100}\right)$. Kui teise aasta lõpus vee

441 mk. ära maksetakse, siis on võlg tasutud; sellepärast:
$\left[820\left(1+\frac{x}{100}\right)-441\right].\left(1+\frac{x}{100}\right)-441=0$;
$\left(379+\frac{820x}{100}\right)\left(1+\frac{x}{100}\right)-441=0$; $379+\frac{820x^2}{10000}+\frac{820x}{100}+\frac{379x}{100}-$
$-441=0$; $\frac{820x^2}{10000}+\frac{1199x}{100}-62=0$; $820x^2+119900x-$
$-620000=0$; $41x^2+5995x-31000=0$;

$$x=\frac{-5995\pm\sqrt{35940025+5084000}}{82}=\frac{-5995\pm\sqrt{41024025}}{82}=$$
$=\frac{-5995\pm6405}{82}$; $x_1=5$; $x_2=-\frac{6200}{41}$. Laen on tehtud 5%-ga.

186. Laen on tehtud x%-ga. I-se aasta lõpuks on võlg $2100\left(1+\frac{x}{100}\right)$; kui 1210 marka ära maksetud, on võlga veel $\left[2100\left(1+\frac{x}{100}\right)-1210\right]$ marka. II-se aasta lõpuks on see

$$\left[2100\left(1+\frac{x}{100}\right)-1210\right]\left(1+\frac{x}{100}\right);$$

$\left[2100\left(1+\frac{x}{100}\right)-1210\right]\left(1+\frac{x}{100}\right)=1210$. Peale lihtsustamist saame: $21x^2+2990x-32000=0$; $x=\frac{-2990\pm\sqrt{11628100}}{42}=$
$=\frac{-2990\pm3410}{42}$; $x_1=10$; $x_2=-\frac{6400}{42}$. Laen on tehtud 10%-ga.

187. Kui ütleme, et I töötas x päeva, siis II töötas (x—6) p. I sai päevas $\frac{48}{x}$ marka; II $\frac{x-6}{27}$ mk. Kui I oleks töötanud (x—6) päeva ja II x päeva, siis oleks nad ühepalju saanud, sellepärast $\frac{48}{x}(x-6)=\frac{x-6}{27}.x$ ehk $48(x-6)=27x^2$.

$$48x^2-576x+1728=27x^2;\ 21x^2-576x+1728=0;$$

$7x^2-192x+576=0$; $x=\frac{192\pm\sqrt{36864-16128}}{14}=\frac{192\pm144}{14}$;
$x_1=24$; $x_2=3\frac{3}{7}$. Murdarvuline x'i tähendus ei rahulda ülesannet sest, kui I töötas $3\frac{3}{7}$ päeva, siis II pidi töötama $3\frac{3}{7}-6=-2\frac{4}{7}$ päeva, s. o. negatiivse arvu päevi. Kõlbab

ainult positiivne tähendus $x = 24$. Kui I töötas 24 päeva, siis II töötas $24 - 6 = 18$ päeva.

187. I töötas x päeva; II $(x + 6)$ päeva; I sai $\frac{45}{x}$ mk.; II sai $\frac{80}{x+6}$ mk. Võrrand: $\frac{45}{x}(x + 6) = \frac{80}{x+6} \cdot x$, ehk pääle lihtsustamist $7x^2 - 108x - 324 = 0$; $x = \frac{108 \pm \sqrt{11664 + 9072}}{14}$; $x_1 = 18$; $x_2 = -36$. I töötas 18 päeva; II 24 päeva.

188. Ütleme, et I kaupleja müüs x õuna; II $(100 - x)$ õuna. Kui I oleks müünud oma hinnaga $(100 - x)$ õuna, siis oleks ta saanud iga õuna eest $\frac{180}{100-x}$ mk., ja kui II oleks müünud oma hinna eest x õuna, oleks ta iga õuna eest saanud $\frac{80}{x}$ mk. Tähendab I kaupleja müüs oma x õuna $\frac{180x}{100-x}$ marga eest; II oma $(100 - x)$ õuna $\frac{80(100-x}{x}$ marga eest. Ülesande tingimise järele said mõlemad müümisest ühe palju raha; sellepärast $\frac{180x}{100-x} = \frac{80(100-x)}{x}$ ehk $180x^2 = 80(100 - x)$; $9x^2 = 4(100 - x)^2$; $9x^2 = 40000 - 800x + 4x^2$; $x^2 + 160x - 8000 = 0$; $x = -80 \pm \sqrt{6400 + 8000} = -80 \pm 120$; $x_1 = 40$; $x_2 = -200$. Õunte arvu, mis neil oli tuleb muidugi positiivses mõttes võtta, sellepärast kõlbab ainult $x = 40$. I müüs 40 õuna, II $100 - 40 = 60$ õuna.

188. I müüs x õuna, II $(110 - x)$ õuna. Nagu eelmises ülesandes $\frac{75x}{110-x} = \frac{108(110-x)}{x}$; $75x^2 = 108(110 - x)^2$; sulgusid avades ja koondades saame $x^2 - 720x + 39600 = 0$; $x_1 = 60$; $x_2 = 660$. x'i teine tähendus 660 ei kõlba, sest õunu oli kõigest 110. I müüs 60 õuna, II $110 - 50 = 50$ õuna.

189. Ütleme, et töölised on palgatud x päeva peale. I töötas ainult $(x - 1)$ päeva ja sai 18 marka; tähendab iga päeva eest sai tema $\frac{18}{x-1}$ marka; II töötas ainult $(x - 3)$

päeva ja sai 21 marka, seega päevas $\frac{21}{x-3}$ marka. Kui I oleks töötanud (x — 3) päeva, siis ta oleks saanud $\frac{18(x-3)}{x-1}$ marka; kui II oleks töötanud (x — 1) päeva, oleks ta saanud $\frac{21(x-1)}{x-3}$ marka. Tingimise järele võime kirjutada: $\frac{21(x-1)}{x-3} - \frac{18(x-3)}{x-1} = 13$ ehk $21(x-1)^2 - 18(x-3)^2 = 13(x-1)(x-3)$. Sulgusid avades ja koondades saame $5x^2 - 59x + 90 = 0$; $x = \frac{59 \pm \sqrt{3481-1800}}{10} = \frac{59 \pm 41}{10}$; $x_1 = 10$; $x_2 = 1{,}8$. x'i murruline tähendus ei kõlba, sest siis oleks II töötanud $1{,}8 - 3 = -1{,}2$ päeva, mis võimata. Töölised on palgatud 10 päeva peale.

189. Töölised on palgatud x päeva peale. I töötas ainult (x — 2) päeva ja sai päevas $\frac{27}{x-2}$ mk., II töötas (x — 3) päeva ja sai päevas $\frac{30}{x-3}$. Võrrand: $\frac{27(x-3)}{x-2} - \frac{30(x-2)}{x-3} = 3$. Lihtsustades saame $2x^2 + 9x - 35 = 0$;
$x = \frac{-9 \pm \sqrt{81+280}}{4} = \frac{-9 \pm 19}{4}$; $x_1 = 2\frac{1}{2}$; $x_2 = -7$.

Mõlemad vastused ei kõlba; negatiivne üldse ei vasta ülesande mõttele, kuna murruline sellepärast ei kõlba, et siis II tööline $2\frac{1}{2} - 3 = -\frac{1}{2}$ päeva oleks töötanud. Ülesanne on võimatu.

190. Ütleme, et I-ne kapitaali osa kandis x %; II-ne osa (x + 1) %. Kui kapitaali esimese osa pealt x mk. saadakse 100 marga pealt, siis 1 mark saadakse $\frac{100}{x}$ mk. pealt ja 90 mk. saadakse $\frac{100}{x} \cdot 90$ mk. pealt. Tähendab I-ne osa kapitaalist $= \frac{9000}{x}$ mk. Kui kapitaali II-se osa pealt (x + 1) mk. saadakse 100 mk. pealt, siis 1 mk. saadakse $\frac{100}{x+1}$ mk. pealt ja 200 mk. saadakse $\frac{100}{x+1}.200$ pealt. Tähendab II-ne kapitaali osa $=\frac{20000}{x+1}$ mk. Võrrand: $\frac{9000}{x}+\frac{20000}{x+1}=8000$; $\frac{9}{x}+\frac{20}{x+1}=8$; $9x+9+20x=8x^2+8x$; $8x^2-21x-9=0$; $x=\frac{21\pm\sqrt{441+288}}{16}=\frac{21\pm27}{16}$; $x_1=3$; $x_2=-\frac{3}{8}$. Ülesandele vastab ainult positiivne tähendus $x=3\,\%$. II-ne osa annab 4%.

190. Ütleme, et I-ne osa annab $(x+1)\%$; II-ne osa $x\,\%$. $\frac{24000}{x+1}+\frac{10000}{x}=6000$; ehk $\frac{12}{x+1}+\frac{5}{x}=3$; $3x^2-14x-5=0$; $x=\frac{14\pm\sqrt{256}}{6}=\frac{14\pm16}{6}$; $x_1=5$; $x_2=\frac{1}{3}$. I-ne osa on antud kasu kandma $5\,\%$-ga, II-ne $6\,\%$-ga.

191. Olgu tagumise ratta ringjoon x jalga, siis on esimese ratta ringjoon 4 x jalga. Kui esimese ratta ringjoon oleks olnud $(4x-2)$ jalga ja tagumise ringjoon $(x+1)$ jalga, siis oleks esimene ratas 120 jala maa peal $\frac{120}{4x-2}$ ehk $\frac{60}{2x-1}$ korda ümber pööranud, tagumine $\frac{120}{x+1}$ korda. Tingimise järele $\frac{120}{x+1}-\frac{60}{2x-1}=18$; $\frac{20}{x+1}-\frac{10}{2x-1}=3$; $20(2x-1)-10(x+1)=3(x+1)(2x-1)$; $40x-20-10x-10=6x^2-3x+6x-3$; $6x^2-27x+27=0$ ehk $2x^2-9x+9=0$; $x=\frac{9\pm\sqrt{81-72}}{4}=\frac{9\pm3}{4}$; $x_1=3$; $x_2=1\frac{1}{2}$. Tagumise ratta ringjoon $=3$ ehk $1\frac{1}{2}$ jalga, esimese ringjoon 12 ehk 6 jalga.

191. Tagumise ratta ringjoon $=x$ jalga, esimese ringjoon $=3x$ jalga. Muundatult: esimese ringjoon $(3x+3)$ jalga, tagumise $(x+2)$ jalga. Ülesande tingimise järele: $\frac{108}{x+2}-\frac{108}{3x+3}=15$; $5x^2-9x-2=0$; $x=\frac{9\pm\sqrt{81+4.5.2}}{10}=$

$=\frac{9\pm 11}{10}$; $x_1=2$; $x_2=-\frac{1}{5}$. Tagumise ratta ringjoon on 2 jalga, esimese ringjoon 6 jalga.

192. Olgu kilomeetrite arv M ja N linna vahel x, siis on B ühe päeva kiirus $\frac{x}{30}$ klm; aeg mis B teel viibis kunni A'ga kohtamiseni on $\frac{x}{30}$ päeva. Tähendab, kunni A'ga kohtamiseni käis B ära $\frac{x}{30}\cdot\frac{x}{30}$ ehk $\frac{x^2}{900}$ klm. A käis $\frac{x}{30}$ päevaga $\frac{12x}{30}$ klm, peale selle käis ta veel enne 65 klm. M'st kunni kohtamise kohani käis A $\left(65+\frac{12x}{30}\right)$ klm. Kõik tee on $\frac{x^2}{900}+$ $+\left(65+\frac{12x}{30}\right)$ ehk x klm. $\frac{x^2}{900}+\left(65+\frac{12x}{30}\right)=x$; $x^2-540x+$ $+585000=0$; $x=270\pm\sqrt{72900-58500}=270\pm 120$; $x_1=$ $=390$; $x_2=150$. M ja N'ni vahe on, kas 390 ehk 150 klm.

192. Kilomeetrite arv M ja N vahel on x. B päeva kiirus $=\frac{x}{20}$; aeg, mis ta teel oli kunni A'ga kohtamiseni $=$ $=\frac{x}{20}$ päeva. Nagu eelmises ülesandes: $\frac{x^2}{400}+\left(27+\frac{8x}{20}\right)=x$; $x^2-240x+10800=0$; $x=120\pm\sqrt{14400-10800}=120\pm$ ± 60; $x_1=180$; $x_2=60$.

193. Olgu kilomeetrite arv A ja B vahel x, siis kaugus C'st B'ni on (x + 20) klm. Kuller A'st, sõidab x kilomeetrit 5 tunniga ehk 300 minutiga ära. 1 klm sõidab ta $\frac{300}{x}$ minutis; maa (x + 20) klm sõidab kuller C-st 300 minutiga ära; 1 klm sõidab ta $\frac{300}{x+20}$ minutiga. Tingimise järele sõidab C'st sõitev kuller iga kilomeetri $1\frac{1}{4}$ minutit rutem, kui esimene kuller; sellepärast võime kirjutada $\frac{300}{x}-\frac{300}{x+20}=\frac{5}{4}$ ehk $\frac{60}{x}-\frac{60}{x+20}=\frac{1}{4}$; $240(x+20)-240x=x^2+20x$; $x^2+20x-4800=0$; kust $x=-10\pm\sqrt{100+4800}=-10\pm 70$; $x_1=60$; $x_2=-80$. Küsimisele vastab x positiivne tähendus x = 60 klm.

193. A ja B vaheline kaugus on x kilomeetrit; C ja B vahe $(x-12)$ klm; $\frac{360}{x-12}-\frac{360}{x}=1$; $360x-360x+4320=$ $=x^2-12x$; $x^2-12x-4320=0$; $x=6\pm\sqrt{36+4320}=$ 6 ± 66; $x_1=72$; $x_2=-60$.

194. Ütleme et A'st sõitev rong tarvitab A ja B vahelise maa maha sõiduks x tundi. 1 tunnis sõidab tema siis $\frac{600}{x}$ klm ja poole teed käib tema $\frac{x}{2}$ tunniga ära; B'st sõitev tarvitab poole tee sõiduks $\left(\frac{x}{2}+1\frac{1}{2}\right)$ ehk $\frac{x+3}{2}$ tundi; kõik tee käib tema ära $2.\frac{x+3}{2}$ ehk $x+2$ tunniga, igas tunnis $\frac{600}{x+3}$ kilomeetrit sõites. Kui mõlemad rongid oleks ühel ajal välja sõitnud, siis A rong käib kuue tunniga $\frac{6.600}{x}$ ehk $\frac{3600}{x}$ klm, B rong $\frac{6.600}{x+3}$ ehk $\frac{3600}{x+3}$ klm; ühtekokku käivad nad $\left(\frac{3600}{x}+\frac{3600}{x+3}\right)$ klm ja nende vahe on $\frac{1}{10}.600=60$ klm.

$$600-\left(\frac{3600}{x}+\frac{3600}{x+3}\right)=60;\ 540-\frac{3600}{x}-\frac{3600}{x+3}\Big)=0;$$

$540x(x+3)-3600(x+3)-3600x=0$; $3x(x+3)-$ $-20x=0$; $3x^2+9x-20x-60-20x=0$; $3x^2-31x-60=0$; $x=\frac{31\pm\sqrt{961+720}}{6}=\frac{31\pm41}{6}$; $x_1=12$; $x_2=-1\frac{2}{3}$. Negatiivne tundmatu tähendus ei vasta ülesandele, sellepärasi tarvitab A'st sõitev rong A ja B vahelise maa ära käimiseks 12 tundi. B rong sellesama maa käimiseks $(12+3)=15$ tundi.

194. Ütleme, et A rong tarvitab x tundi; pool teed käib $\frac{x}{2}$ tunniga; tunnis käib ta $\frac{480}{x}$ klm; B rong käis pool teed $\left(\frac{x}{2}-2\frac{1}{2}\right)$ ehk $\frac{x-5}{2}$ tunniga ära. Kõik tee käib tema $2.\frac{x-5}{5}=x-5$ tunniga, käies tunnis $\frac{480}{x-5}$ klm. 5-e tunniga käivad mõlemad rongid ühtekokku $\left(\frac{5.480}{x-5}+\frac{5.480}{x}\right)$ klm; nende vaheline kaugus on $\frac{1}{6}.480 = 80$ klm. Võrrand: $480 - \left(\frac{5.480}{x-5} + \frac{5.480}{x}\right) = 80$; $x^2 - 17x + 30 = 0$; $x = \frac{17}{2} \pm \sqrt{\frac{289 - 120}{4}} = \frac{17 \pm 13}{2}$; $x_1 = 15$; $x_2 = 2$. Kui esimene rong linnade vahemaa 15 tunniga ära käib, siis käib teine rong ta $15 - 5 = 10$ tunniga.

195. Kaugus A'st kunui kohtamise kohani olgu x, siis, tingimise järele, kaugus B'st kunni kohtamise kohani on $(x - 6)$; kogu kaugus A'st — B'ni on $(x + x - 6)$ ehk $(2x - 6)$. Peale kohtamise käib B'st tulnud isik x klm, mis A'st tulnud isik kunni kohtamiseni käis, ja tarvitab selleks 9 tundi; järjelikult käib tema ühes tunnis $\frac{x}{9}$ klm. Peale kohtamise käib A'st tulnud isik $(x - 6)$ klm, mis B'st tulnud isik kunni kohtamiseni käis ja tarvitab selleks 4 tundi. Tunnis käib tema $\frac{x-6}{4}$ klm. Kui A'st tulnud isik tunnis käib $\frac{x-6}{4}$ klm, siis käis tema x klm kunni kohtamise kohani $\left[x : \frac{x-6}{4}\right]$ ehk $\frac{4x}{x-6}$ tunniga; kui B'st tulnud isik käib tunnis $\frac{x}{9}$ klm, siis $(x - 6)$ klm kunni kohtamiseni käis tema $\left[(x-6) : \frac{x}{9}\right]$ ehk $\frac{9(x-6)}{x}$ tunniga. Kunni kohtamiseni olid mõlemad mehed ühepalju aega teel, sellepärast $\frac{4x}{x-6} = \frac{9(x-6)}{x}$ ehk $4x^2 = 9(x-6)^2$; $4x^2 = 9x^2 - 108x + 324$; $5x^2 - 108x + 324 = 0$; $x = 54 \pm \sqrt{2916 - 1620} = 54 \pm 36$; $x_1 = 18$; $x_2 = 90$. Kui $x = 18$, siis on kogu maa $2.18 - 6 = 30$ klm.

195. Tähendame x'iga kilomeetrite arvu A'st kunni kohtamise kohani; kilomeetrite arv B'st kunni kohtamise kohani on siis $(x + 4)$. Peale kohtamise käib A'st tulnud isik $(x + 4)$ klm $4\frac{4}{5}$ tunniga ja ühes tunnis $\frac{x+4}{4\frac{4}{5}}$ ehk $\frac{5(x+4)}{24}$ klm; B'st tulnud isik käib x klm, mis A kunni kohtamise kohani käis,

$3\frac{1}{3}$ tunniga; tunnis käib tema $\frac{x}{3\frac{1}{3}}$ ehk $\frac{3x}{10}$ klm. Kunni kohtamiseni käis A'st tulev isik A'st kunni kohtamise kohani x klm $\left[x:\frac{5(x+4)}{24}\right]$ ehk $\frac{24x}{5(x+4)}$ tunniga; B'st tulev isik käis $x+4$ klm $\left[(x+4):\frac{3x}{10}\right]$ ehk $\frac{10(x+4)}{3x}$ tunniga $\frac{24x}{5(x+4)}=\frac{10(x+4)}{3x}$; $72x^2=50(x+4)^2$ ehk $36x^2=25(x+4)^2$; $36x^2=25x^2+$ $+200x+400$; $11x^2-200x-400=0$;

$$x=\frac{200\pm\sqrt{40000+17600}}{22}=\frac{200\pm\sqrt{57600}}{22}=\frac{200\pm240}{22};$$

$x_1=20$; $x_2=-\frac{40}{22}$. Kui $x=20$, s. o. maa A'st kunni kohtamise kohani $=20$, siis B'st kunni kohtamise kohani on 24 klm ja $\overline{AB}=44$ klm.

196. Ütleme, et I kord valati nõu seest x pange piiritust välja, siis jääb nõusse alles $(64-x)$ pange puhast piiritust. Nõu valati veega uuesti täis ja saadud segus 64 pange segu peale tuleb $(64-x)$ pange piiritust; 1 pange segu peale tuleb $\frac{64-x}{64}$ pange piiritust. Valades uuesti x pange segu nõust välja valati ühes sellega II kord $\frac{64-x}{64}\,.\,x$ pange piiritust. Peale I-st valamist jäi puhast piiritust $(64-x)$ pange, peale II-st valamist jääb puhast piiritust $(64-x)-\frac{(64-x)x}{64}$ ehk $\frac{64(64-x)-(64-x)x}{64}$ ehk $\frac{(64-x)^2}{64}$ pange. Tingimise järele jäi nõusse 49 pange piiritust; sellepärast $\frac{(64-x)^2}{64}=49$; $(64-x)^2=49\,.\,64$; $x^2-128x+960=0$;

$$x=64\pm\sqrt{4096-960}=64\pm56;\quad x_1=8;\quad x_2=120;$$

Vastus $x=120$ ei kõlba, sest anum mahutas ainult 64 pange ja selge on, et võimata on temast I kord 120 pange välja valada. I kord valati 8 pange välja; II kord valati $\frac{(64-x)x}{64}$ ehk $\frac{56\,.\,8}{64}=7$ pange.

196. Anumast valati I kord x pange piiritust välja ja täideti anum veega; anumas on nüüd x pange vett ja (40—x) pange piiritust; 1 segu pange peale tuleb $\frac{40-x}{40}$ pange puhast piiritust. II kord valatakse x pange segu välja; x panges segus on $\frac{(40-x)x}{40}$ pange puhast piiritust ja $\left[x-\frac{(40-x)x}{40}\right]$ pange vett; järjelikult jääb anumasse $(40-x)-\frac{(40-x)x}{40}$ ehk $\frac{(40-x)^2}{40}$ pange puhast piiritust; vett jääb anumasse $x-\left[x-\frac{(40-x)x}{40}\right]$ pange ja x pange valati veel juurde sellepärast on vett temas üleüldse $2x-\left[x-\frac{(40-x)x}{40}\right]$. Tingimise järele jääb piiritust 3 korda vähem järele, kui vett ja sellepärast $\frac{3(40-x)^2}{40}=2x-\left[x-\frac{(40-x)x}{40}\right]$ ehk $\frac{3(40-x)^2}{40}=$ $=x+\frac{(40-x)x}{40}$; $3(40-x)^2=40x+40x-x^2$; $4800-240x+$ $+3x^2=40x+40x-x^2$; $4x^2-320x+4800=0$; x^2-80x+ $+1200=0$; $x=40\pm\sqrt{1600-1200}=40\pm20$; $x_1=60$; $x_2=20$. I kord valati välja 20 pange, II kord $\frac{(40-20)20}{10}=$ $=10$ pange.

197. Tähendame panka pandud kapitaali x'iga ära. x marga pealt saadi aastas 120 marka kasu, 1 marga pealt saadi siis $\frac{120}{x}$ mk. Kapitaal ühes protsentidega (x + 120) mk. jäeti panka. Iga kapitaali mark muutus aasta lõpuks $\left(1+\frac{120}{x}\right)$ mk. ja (x + 120) mk. muutusid $\left(x+120\right)\left(1+\frac{120}{x}\right)$ margaks, mis tingimise järele = 2646 mk. $\left(x+120\right)\left(1+\frac{120}{x}\right)=2646$ ehk $(x+120)^2=2646x$; $x^2+240x+14400=2646x$; x^2- $-2406x+14400=0$, kust $x=1203\pm\sqrt{1447209-14400}=$ $=1203\pm\sqrt{1432809}=1203\pm1197$; $x_1=2400$; $x_2=6$.

197. Esimeseks ettevõtteks tarvitas kaupmees x marka, mis temale 240 mk. kasu andis; tähendab see ettevõte andis temale $\frac{24000}{x}$ %. Teine ettevõte andis temale $\left(\frac{24000}{x}+20\right)$%. Selle %-ga mahutas ta ärisse (x + 240) mk. Iga mark andis temale $\frac{\left(\frac{24000}{x}\right)+20}{100}$ mk. ehk $\frac{1200+x}{5x}$ mk., tähendab iga mark muutus aasta lõpuks $\left(1+\frac{1200+x}{5x}\right)$ ehk $\left(\frac{6x+1200}{5x}\right)$ mk. Kõik kapitaal (x + 240) mk. muutus $(x+240)\left(\frac{6x+1200}{5x}\right)$ mk., mis tingimise järele = 3432 mk. $(x+240)\left(\frac{6x+1200}{5x}\right)=3432$ ehk $(x+240)(6x+1200)=17160x$; $6x^2+1200x+1440x+ +288000=17160x$; $6x^2-14520x+288000=0$; $x^2- -2420x+48000=0$; $x=1210\pm\sqrt{1464100-48000}= =1210\pm1190$; $x_1=1210+1190=$ **2400**; $x_2=20$.

198. Tähendame I-se kapitaali x'iga; II-se kapitaal on siis (200 — x). Kapitaal x marka oli äris 10 kuud ja tõi (130 — x) marka kasu. 10 kuuga saadi kasu (130 — x) mk., 12 kuuga ehk 1 aastaga on kasu $\frac{(130-x).12}{10}$ marka, et 1 % x margalise kapitaali pealt on $\frac{x}{100}$ mk., siis $\frac{(130-x).12}{10}$ marka moodustavad $\left[\frac{(130-x).12}{10}:\frac{x}{100}\right]$ ehk $\frac{120(130-x).12}{x}$ %. Kapitaal (200 — x) mk. oli kasu kandmas 15 kuud ja tõi [90 — — (200 — x] ehk (x — 110) mk. kasu. 15 kuuga saadi (x — 110) mk. kasu; 12 kuu ehk 1 aasta eest on kasu $\frac{(x-110)}{15}.12$ mk.; et 1 % kapitaalist (200 — x) on $\frac{200-x}{100}$ mk., siis $\frac{(x-110).12}{15}$ mk. moodustavad $\left[\frac{(x-110)12}{15}:\frac{200-x}{100}\right]$ ehk $\frac{1200(x-110)}{15(200-x)}$ %. Et ettevõte ühine oli, siis on ka % üks ja seesama; sellepärast $\frac{120(130-x)}{x}=\frac{1200(x-110)}{15(200-x)}$ ehk $3(130-x)(200-x)=$

$= 2x(x-110)$; $78000 - 390x - 600x + 3x^2 = 2x^2 - 220x$; $x^2 - 770x + 78000 = 0$, kust $x = 385 \pm \sqrt{148225 - 78000} = 385 \pm 265$; $x_1 = 650$; $x_2 = 120$. Kõlbab ainult vastus $x = 120$, sest ei võinud ometi üks 650 mk. ärisse mahutada, kui mõlemate kapitaal kokku oli 200 mk. I-se kapitaal on 120 mk., II-se $200 - 120 = 80$ mk.

198. Ütleme, I-ne pani x marka; II-ne $(500 - x)$ mk. I-se kasu on $(450 - x)$; II-se kasu on $[450 - (500 - x)]$ ehk $(x - 50)$ mk. Ärutades nagu eelmises ülesandes leiame, et $\frac{1200(450-x)}{15x} = \frac{1200(x-50)}{6(500-x)}$; $2(450 - x)(500 - x) = 5x(x - 50$; $x^2 + 550x - 150000 = 0$; $x_1 = 200$; $x_2 = -750$. I-se kapital on 200 marka, II-se oma 300 marka.

199. Ütleme, et I-ne kord valati esimesest anumast teise x pange piiritust; siis jääb I anumasse $(20 - x)$ p. piiritust. II-se anumasse sai peale vee juure valamist segu, milles $(20 - x)$ pange vee peale tuleb x pange piiritust, tähendab 1 segu pange peale tuleb $\frac{x}{20}$ pange piiritust. Et I-st anumat täita, võetakse x pange seda segu, milles on $x \cdot \frac{x}{20}$ ehk $\frac{x^2}{20}$ p. puhast piiritust; II-se anumasse jääb $\left(x - \frac{x^2}{20}\right)$ p. p. piiritust; I-sse anumasse saab nüüd $\left[(20x -) + \frac{x^2}{20}\right]$ p. p. piiritusu 20 pange segu peale; 1 pange segu peale tuleb siis $\frac{\left[(20-x) + \frac{x^2}{20}\right]}{20}$ pange piirust. I-sest anumast valatakse välja $6\frac{2}{3}$ ehk $\frac{20}{3}$ pange segu, mis sisaldavad $\frac{\left[(20-x) + \frac{x^2}{20}\right]}{20} \cdot \frac{20}{3}$ ehk $\frac{\left[(20-x) + \frac{x^2}{20}\right]}{3}$ pange puhast piiritust; I-sse anumasse jääb $\left\{\left](20-x) + \frac{x^2}{20}\right] - \frac{\left[(20-x) + \frac{x^2}{20}\right]}{3}\right\}$ ehk $\frac{2\left[(20-x) + \frac{x^2}{20}\right]}{3}$

pange piiritust. II-s anumas on endist $\left(x-\frac{x^2}{20}\right)$ pange piiritust ja veel nüüd juurde valatud $\frac{\left[(20-x)+\frac{x^2}{20}\right]}{3}$ pange,

kokku $\left\{\left(x-\frac{x^2}{20}\right)+\frac{\left[(20-x)+\frac{x^2}{20}\right]}{3}\right\}$ pange piiritust. Tingimise järele on peale teist valamist mõlemis anumais ühepalju piiritust, sellepärast

$$\frac{2\left[(20-x)+\frac{x^2}{20}\right]}{3}=\left(x-\frac{x^2}{20}\right)+\frac{\left[(20-x)+\frac{x^2}{20}\right]}{3} \text{ ehk}$$

$$\frac{\left[(20-x)+\frac{x^2}{20}\right]}{3}=\left(x-\frac{x^2}{20}\right) \text{ ehk } 20-x+\frac{x^2}{20}=3x-\frac{3x^2}{20};$$

$$\frac{x^2}{5}-4x+20=0;\ x^2-20x+100=0;\ x=10\pm\sqrt{100-100}=10.$$

199. Esialgu valati x pange piiritust ära. Arutades niisama kui eelmises ülesandes leiame, et peale viimast valamist jääb I-sse anumasse $\left[30-x+\frac{x^2}{30}-\frac{\left(30-x+\frac{x^2}{30}\right)12}{3}\right]$ pange. II-se anumasse saab $\left[x-\frac{x^2}{30}+\frac{\left(30-x+\frac{x^2}{30}\right)12}{30}\right]$ p. piiritust. Tingimise järele $\left[30-x+\frac{x^2}{30}-\frac{\left(30-x+\frac{x^2}{30}\right)12}{30}\right]-$

$$-\left[x-\frac{x^2}{30}+\frac{\left(30-x+\frac{x^2}{30}\right)12}{30}\right]=2.\ x^2-30x+200=0;$$

$$x=15\pm\sqrt{225-200}=15\pm5;\ x_1=20;\ x=10.$$

200. Ütleme, et esimese ratta ringjoon on x arssinat; 36 sülla maa peal teeb ratas $\frac{36}{x}$ tiiru. Selle sama maa peal tagumine ratas $\left(\frac{36}{x}-6\right)$ ehk $\frac{6(6-x)}{x}$ tiiru, tähendab tagumise ratta ringjoon on $\left[36 : \frac{6(6-x)}{x}\right]$ ehk $\frac{6x}{6-x}$ arss. Kui esimese ratta ringjoon oleks $(x+1)$ arssinat ja tagumise ratta ringjoon $\left(\frac{6x}{6-x}+1\right)$ ehk $\left(\frac{5x+6}{6-x}\right)$ arssinat, siis teeks 36 arssina maa peal esimene ratas $\frac{36}{x+1}$ ja tagumine ratas $\left[36 : \frac{5x+6}{6-x}\right]$ ehk $\frac{36(6-x)}{5x+6}$ tiiru. Tingimise järele: $\frac{36}{x+1} - \frac{36(6-x)}{5x+6} = 3$ ehk $\frac{12}{x+1} - \frac{12(6-x)}{5x+6} = 1$; $60x + 72 - 12(6-x)(x+1) = (x+1)(5x+6)$; $60x + 72 - 72x - 72 + 12x^2 + 12x = 5x^2 + 6x + 5x + 6$; $7x^2 - 11x - 6 = 0$; $x = \frac{11 \pm \sqrt{121+168}}{14} = \frac{11 \pm 17}{14}$; $x_1 = 2$; $x_2 = -\frac{3}{7}$. Ringjoont võib tähendada ainult positiivse arvuga, sellepärast on esimese ratta ringjoon 2 arssinat, tagumisel $\frac{6 \cdot 2}{6-2} = 3$ arss.

200. Esimese ratta ringjoon $= x$ jalga; 120 jala maa peal teeb tema $\frac{120}{x}$ tiiru; tagumine ratas teeb $\left(\frac{120}{x} - 2\right)$ ehk $\frac{2(60-x)}{x}$ tiiru, järjelikult on tema ringjoon $\left[120 : \frac{2(60-x)}{x}\right]$ ehk $\frac{60x}{60-x}$ jalga. Muudetud kujul on esimese ringjoon $= (x-4)$ jalga, tagumisel $\left(\frac{60x}{60-x} + 5\right)$ ehk $\frac{55x+60}{60-x}$ jalga; esimene ratas teeb $\frac{120}{x-4}$ tiiru; tagumine $\left(120 : \frac{55x+300}{60-x}\right)$ ehk $\frac{4(60-x)}{11x+60}$ tiiru. Tingimise järele: $\frac{120}{x-4} - \frac{24(60-x)}{11x+60} = 9$; $5x^2 + 24x - 1008 = 0$; $x = \frac{-24 \pm \sqrt{576+20160}}{10} = \frac{-24 \pm 144}{10}$; $x_1 = 12$; $x_2 = -16{,}8$.

§ 5. Võrrandite astendamine ja juurimine.

201. $x^2 = 4$; $x^2 - 4 = 0$; $(x+2)(x-2) = 0$; $x_1 = -2$; $x_2 = 2$; lisajuur $x_1 = -2$.

202. $4x^2 = 9$; $4x^2 - 9 = 0$; $(2x + 3)(2x - 3) = 0$; $x_1 = -\frac{3}{2}$; $x_2 = \frac{3}{2}$ (lisajuur).

203. $x = 5$; $x^2 = 25$; $x^2 - 25 = 0$; $(x + 5)(x - 5) = 0$; $x_1 = -5$; $x_2 = 5$. Lisajuur $x_1 = -5$.

204. $x = -3$; $x^2 = 9$; $x = \pm\sqrt{9} = \pm 3$; $x_1 = -3$; $x_2 = 3$ (lisajuur).

205. $5x = 7$; $25x^2 = 49$; $25x^2 - 49 = 0$; $(5x + 7)(5x - 7) = 0$; $x_1 = -\frac{7}{5}$; $x_2 = \frac{7}{5}$. Lisajuur $x_1 = -\frac{7}{5}$.

206. $x = -2{,}7$; $x^2 = 7{,}29$; $x^2 - 7{,}29 = 0$; $(x + 2{,}7)(x - 2{,}7) = 0$; $x_1 = -27$; $x_2 = 2{,}7$. Lisajuur $x_2 = 2{,}7$.

207. $4x = -5$; $16x^2 = 25$; $16x^2 - 25 = 0$; $(4x + 5)(4x - 5) = 0$; $x_1 = -\frac{5}{4}$; $x_2 = \frac{5}{4}$. Lisajuur $x_2 = \frac{5}{4}$.

208. $3x = -\frac{13}{4}$; $x = -\frac{13}{12}$; $x^2 = \frac{169}{144}$; $x^2 - \frac{169}{144} = 0$; $(x + \frac{13}{12})(x - \frac{13}{12}$; $x_2 = \frac{13}{12}$. Lisajuur $x_2 = \frac{13}{12}$.

209. $ax - bx = -c$: $x(a - b) = -c$; $x = -\frac{c}{a-b}$; $x^2 = \frac{c^2}{(a-b)^2}$; $x^2 - \frac{c^2}{(a-b)^2} = 0$; $\left(x + \frac{c}{a-b}\right)\left(x - \frac{c}{a-b}\right) = 0$; $x_1 = -\frac{c}{a-b}$; $x_2 = \frac{c}{a-b}$. Lisajuur $x_2 = \frac{c}{a-b}$.

210. $ax - cx = -b - d$; $x(a - c) = -(b + d)$; $x = -\frac{b+d}{a-c}$; $x^2 = \frac{(b+d)^2}{(a-c)^2}$; $x^2 - \frac{(b+d)^2}{(a-c)^2} = 0$; $\left(x + \frac{b+d}{a-c}\right)\left(x - \frac{b+d}{a-c}\right) = 0$; $x_1 = -\frac{b+d}{a-c}$; $x_2 = \frac{b+d}{a-c}$.

Lisajuur $x_2 = \frac{b+d}{a-c}$.

211. $x = 1$; $x^3 = 1$; $x^3 - 1 = 0$; $(x - 1)(x^2 + x + 1) = 0$; $x_1 = 1$; $x^2 + x + 1 = 0$; kust $x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}\,.\,i}{2}$; $x_2 = \frac{-1 + \sqrt{3}\,.\,i}{2}$; $x_3 = \frac{-1 - \sqrt{3}\,.\,i}{2}$. Lisajuured x_2 ja x_3 paneme uude võrrandisse $x^3 = 1$, x'i asemele, saame samasugused:

1) $\left(\frac{-1 + \sqrt{3}\,.\,i}{2}\right)^3 = 1$ ehk $-\frac{1}{8}(1 - 3\sqrt{3}\,.\,i + 9i^2 - 3\sqrt{3}\,.\,i^3) = 1$ ehk $-\frac{1}{8}(1 - 3\sqrt{3}\,.\,i - 9 + 3\sqrt{3}\,.\,i) = 1$ ehk $-\frac{1}{8}\,.\,-8$ ehk $1 = 1$.

2) $\left[\frac{-1+\sqrt{3}.i}{2}\right]^3=1$ ehk $-\frac{1}{8}(1+3\sqrt{3}.i+9i^2+3\sqrt{3}.i^3)=1$; $-\frac{1}{8}(1+3\sqrt{3}.i-9-3\sqrt{3}.i)=1$ ehk $-\frac{1}{8}.-8=1$; $1=1$.

212. $x=-2$; $x^3=-8$; $x^3+8=0$; $(x+2)(x^2-2x+4)=0$; $x_1=-2$; $x_2=1+\sqrt{1-4}=1+\sqrt{3}.i$; $x_3=1-\sqrt{3}.i$; asemele pannes saame $(1\pm\sqrt{3}.i)^3=1\pm3\sqrt{3}.i+9i^2\pm3\sqrt{3}.i^3=1\pm3\sqrt{3}.i-9\mp3\sqrt{3}.i=1-9=-8$. Tähendab $-8=-8$.

213. $2x=3$; $8x^3=27$; $8x^3-27=0$; $(2x-3)(4x^2+6x+9)=0$; $x_1=\frac{3}{2}$; (antud juur); $4x^2+6x+9=0$; $x_{2,3}=\frac{-3\pm\sqrt{9-36}}{4}=\frac{-3\pm3\sqrt{3}.i}{4}=-\frac{3}{4}(1\mp\sqrt{3}.i)$. Lisajuured $x_{2,3}$ paneme võrrandisse $8x^3=27$ x'i asemele, saame $x^3=[-\frac{3}{4}(1\mp\sqrt{3}.i)]^3=-\frac{27}{64}.(1\mp3\sqrt{3}.i+9i^2\pm3\sqrt{3}.i^3)=-\frac{27}{64}(1\mp3\sqrt{3}.i-9\pm3\sqrt{3}.i)=-\frac{27}{64}.-8=\frac{27}{8}$; $8.\frac{27}{8}=27$.

214. $3x=-4$; $27x^3=-64$; $27x^3+64=0$; $(3x+4)(9x^2-12x+16)=0$; $3x+4=0$; $x_1=-\frac{4}{3}$; (see on antud juur). Võrrandis $9x^2-12x+16=0$ saame $x_{2,3}=\frac{6\pm\sqrt{36-144}}{9}=\frac{6\pm\sqrt{-108}}{9}=\frac{6\pm\sqrt{3}.i}{9}=\frac{2}{3}(1\pm\sqrt{3}.i)$.
Lisajuuri x'i asemele võrrandisse $27x^3=-64$ pannes saame: $x^3=[\frac{2}{3}(1\pm\sqrt{3}.i)]^3-\frac{8}{27}(1\pm3\sqrt{3}.i+9i^2\pm3\sqrt{3}.i^3)=\frac{8}{27}(1\pm3\sqrt{3}.i-9\mp3\sqrt{3}.i)=\frac{8}{27}.-8=-\frac{64}{27}$; $27.x^3=27.-\frac{64}{27}$; s. o. $-64=-64$.

215. $x+2=1$; $x=-1$; $x^3+1=0$; $(x+1)(x^2-x+1)=0$; $x_1=-1$, see on antud võrrandi juur; $x^2-x+1=0$, millest $x_{2,3}=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm\sqrt{3}.i}{2}=-\frac{1}{2}(1\pm\sqrt{3}.i)$. Paneme lisajuured $x_{2,3}$ võrrandisse $x^3=-1$ x-i asemele, saame $x^3=[\frac{1}{2}(1\pm\sqrt{3}.i)]^3=\frac{1}{8}(1\pm\sqrt{3}.i-9\mp3\sqrt{3}.i)=\frac{1}{8}.-8=-1$; $-1=-1$.

216. $2x-3=x$; $x=3$; $x^3=27$; $x^3-27=0$; $(x-3)(x^2+3x+9)=0$; $x_1=3$ on antud juur; x^2+3x+

$+9=0$; kust $x_{2,3}=\frac{-3\pm\sqrt{9-36}}{2}=\frac{-3\pm3\sqrt{3}.i}{2}=$ $=-\frac{3}{2}(1\pm\sqrt{3}.i)$. Paneme lisajuured võrrandisse $x^3=27$ x-i asemele, saame $x^3=[-\frac{3}{2}(1\mp\sqrt{3}.i)]^3=-\frac{27}{8}(1\pm3\sqrt{3}.i-$ $-9\mp3\sqrt{3}.i)=-\frac{27}{8}.-8=27$; s. o. $27=27$.

217. $x=a$; $x^3=a^3$; $x^3-a^3=0$; $(x-a)(x^2+ax+a^2)=$ $=0$; $x_1=a$, antud juur; $x^2+ax+a^2=0$;

$$x_{2,3}=\frac{-a\pm\sqrt{a^2-4a^2}}{2}=\frac{-a\pm a\sqrt{3}.i}{2}=\frac{a}{2}(1\mp\sqrt{3}.i).$$

Lisajuuri $x_{2,3}$ võrrandisse $x^3=a^3$ asemele pannes saame $x^3=$ $=[-\frac{a}{2}(1\mp\sqrt{3}.i)]^3=-\frac{a^3}{8}(1\pm3\sqrt{3}.i-9\pm3\sqrt{3}.i=$ $-\frac{a^3}{8}.-8=a^3$; $a^3=a^3$.

218. $x-b=a$; $x=a+b$; $x^3=(a+b)^3$; $x^3-(a+b)^3=$ $=0$; $[x-(a+b)][x^2+(a+b)x+(a+b)^2]=0$; $x-(a+b)=$ $=0$; kust $x_1=a+b$, antud juur; $x^2+(a+b)x+(a+b)^2=0$;

millest $x_{2,3}=\frac{-(a+b)\pm\sqrt{(a+b)^2-4(a+b)^2}}{2}=$

$=-\frac{a+b}{2}(1\mp\sqrt{3}.i)$. Paneme lisajuured x_2 ja x_3 võrrandisse $x^3=(a+b)^3$ x-i asemele, saame: $x^3=\left[-\frac{a+b}{2}(1\mp\right.$ $\left.\mp\sqrt{3}.i)\right]^3=-\frac{(a+b)^3}{8}(1\pm3\sqrt{3}.i-9\mp3\sqrt{3}.i)=-\frac{(a+b)^3}{8}.$ $.-8=(a+b)^3$; s. o. $(a+b)^3=(a+b)^3$.

219. $ax=-b$; $a^3x^3=-b^3$; $a^3x^3+b^3=0$; $(ax+b)(a^2x^2-abx+b^2)=0$; $ax+b=0$; $x_1=-\frac{b}{a}$ (see on antud juur); $a^2x^2-abx+b^2=0$; $x_{2,3}=\frac{ab\pm\sqrt{a^2b^2-4a^2b^2}}{2a^2}=$ $=\frac{ab\pm ab\sqrt{3}.i}{2a^2}=\frac{b}{2a}(1\pm\sqrt{3}.i)$. Lisajuuri x_2 ja x_3 võrrandisse $a^3x^3=-b^3$ asemele pannes saame $x^3=\left[\frac{b}{2a}(1\pm\sqrt{3}.i)\right]^3=$ $=\frac{b^3}{8a^3}(1\pm3\sqrt{3}.i-9\mp3\sqrt{3}.i)=\frac{b^3}{8a^3}.-8=-\frac{b^3}{a^3}$; $a^3.-$ $-\frac{b^3}{a^3}=b^3$; $-b^3=-b^3$.

220. $ax - b = cx$; $ax - cx = b$; $(a - c)x = b$; $(a - c)^3 x^3 = b^3$; $[(a-c)x - b][(a-b)^2 x^2 + b(a - c)x + b^2] = 0$; $(a - c)x - b = 0$; $x_1 = \frac{b}{a - c}$; see on antud võrrandi juur. $(a - c)^2 x^2 + b(a - c)x + b^2 = 0$; kust x_2 ja $x_3 = \frac{-b(a - c) \pm \sqrt{b^2(a - c)^2 - 4b^2(a - c)^2}}{2(a - c)^2} = \frac{-b(a-c) \pm \sqrt{-3b^2(a-c)^2}}{2(a - c)^2} = -\frac{b}{2(a - c)} \cdot (1 \mp \sqrt{3} \cdot i)$. Lisajuuri x_2 ja x_3 võrrandisse $(a - c)^3 x^3 = b^3$ asemele pannes, saame $x^3 = \left[-\frac{b}{2(a - c)}(1 \mp \sqrt{3} \cdot i)\right]^3 = -\frac{b^3}{8(a - c)^3} \cdot (1 \pm \sqrt{3} \cdot i - 9 \pm \mp \sqrt{3} \cdot i) = -\frac{b^3}{8(a - c)^3} \cdot -8 = \frac{b^3}{(a - c)^3}$; tähendab $(a - c)^3 \cdot \frac{b^3}{(a - c)^3} = b^3$; $b^3 = b^3$.

221. $x = \pm 3$. **222.** $x = \pm\sqrt{-4} = \pm 2i$.

223. $x^2 = -a^2$; $x = \pm ai$.

224. $x^2 = a^2 + b^2$; $x = \pm\sqrt{a^2 + b^2}$.

225. (Seda ülesannet, kui ka kõiki järgnevaid ülesandeid võib ruutvõrrandi vormelite abil lahendada. Ülesannete kogu nõuab, et neid peab juure leidmise abil lahendama). $x^2 - 14x + 49 = 16$; $(x - 7)^2 = 16$; $x - 7 = \pm 4$; $x = 7 \pm 4$; $x_1 = 11$; $x_2 = 3$.

226. $x^2 - x - 2x + 2 = 6$; $x^2 - 3x - 4 = 0$; $x - 3x + \frac{9}{4} - \frac{25}{4} = 0$; $(x - \frac{3}{2})^2 = \frac{25}{4}$; $x - \frac{3}{2} = \pm\frac{5}{2}$; $x = \frac{3}{2} \pm \frac{5}{2}$; $x_1 = 4$; $x_2 = -1$.

227. $(x - a)^2 = b^2$; $x - a = \pm b$; $x = a \pm b$; $x_1 = a + b$; $x_2 = a - b$.

228. $2x^2 - 2x + \frac{1}{2} = 2$; $2(x^2 - x + \frac{1}{4}) = 2$; $x^2 - x + \frac{1}{4} = 1$; $(x - \frac{1}{2})^2 = 1$; $x - \frac{1}{2} = \pm 1$; $x = \frac{1}{2} \pm 1$; $x_1 = \frac{3}{2}$; $x_2 = -\frac{1}{2}$.

229. $x^2 - \frac{a - b}{b}x = \frac{a}{b}$; $x^2 - \frac{a - b}{b}x + \frac{(a - b)^2}{4b^2} = \frac{a}{b} + \frac{(a-b)^2}{4b^2}$ (võrrand selleläbi ei muutu, kui mõlemile poolele ühe ja sama avalduse $\frac{(a-b)^2}{4b^2}$ juure paneme. Selle läbi saame pahemale poole kahe arvu vahe teise astme vormeli). $x^2 = \frac{a-b}{b}x + \frac{(a-b)^2}{4b^2} = \frac{4ab+(a-b)^2}{4b^2}$; $\left(x - \frac{a-b}{2b}\right)^2 = \frac{(a+b)^2}{4b^2}$; $x - \frac{a-b}{2b} = \pm\frac{a+b}{2b}$; $x = \frac{a-b}{2b} \pm \frac{a+b}{2b}$; $x_1 = \frac{a}{b}$; $x_2 = = 1$.

230. $16x^2 - 24x + 9 = 8x$; $16x^2 - 32x + 9 = 0$; $16x^2 - 32x + 16 - 7 = 0$; (viimasesse võrrandisse kirjutasime vaba liikme $(+9)$ asemele temale võrdse vahe $(16-7)$; $16(x^2 - 2x + 1) = 7$; $(x-1)^2 = \frac{7}{16}$; $x - 1 = \pm\frac{\sqrt{7}}{4}$; $x = 1 \pm \frac{\sqrt{7}}{4}$; $x_1 = 1 \pm \frac{\sqrt{7}}{4}$; $x^2 = 1 - \frac{\sqrt{7}}{4}$.

231. $x = \sqrt[3]{-1} = -1$ ehk $x^3 + 1 = 0$; $(x+1)(x^2 - x + 1) = 0$; $x_1 = -1$; $x^2 - x + 1 = 0$; $x_{2,3} = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{3}.i}{2} = \frac{1}{2}(1 \pm \sqrt{3}.i)$.

232. $x = \sqrt[3]{8} = 2$ ehk $x^3 - 8 = 0$; $(x-2)(x^2 + 2x + 4) = 0$; $x_1 = 2$; $x^2 + 2x + 4 = 0$; $x_{2,3} = -1 \pm \sqrt{3}.i$.

233. $x^3 = -27$; $x = \sqrt[3]{-27} = -3$ ehk $(x+3)(x^2 - 3x + 9) = 0$; $x_1 = -3$; $x^2 - 3x + 9 = 0$, kust $x_{2,3} = \frac{3 \pm \sqrt{9-36}}{2} = \frac{3 \pm 3\sqrt{3}.i}{2} = \frac{3}{2}(1 \pm \sqrt{3}.i)$

234. $x^3 = a^3$; $x = \sqrt[3]{a^3} = a$ ehk $(x-a)(x^2 + ax + a^2) = 0$; $x_1 = a$; $x^2 + ax + a^2 = 0$; $x_{2,3} = \frac{-a \pm \sqrt{a^2 - 4a^2}}{2} = \frac{a}{2}(-1 \pm \sqrt{3}.i)$.

235. $x^4 - 16 = 0$; $(x^2+4)(x^2-4) = 0$; ehk $(x^2 - 4i^2)(x^2 - 4) = 0$; ehk $(x+2i).(x-2i)(x+2)(x-2) = 0$; kust $x_1 = -2i$; $x_2 = -2$; $x_3 = -2$; $x_4 = 2i$. (Neljandama astme juurel mingist positiivsest arvust on neli tähendust: 1) aritmeetiline juur; 2) aritmeetilise juure korrutis -1'ga; 3) arit. juur $\times i$; 4) aritm. juur $\times -i$).

236. $x^4+81=0$; $x^4+18x^2+81-18x^2=0$; $(x^2+9)^2-18x^2=0$ ehk $(x^2+9+3\sqrt{2}.x)(x^2+9-$ $-3\sqrt{2}.x)=0$. Edasi kas $x^2+9+3\sqrt{2}.x=0$ ehk $x^2+9-3\sqrt{2}.x=0$. Lahendades saame: 1) $x^2+3\sqrt{2}.x+9=0$; $x_{1,2}=\frac{-3\sqrt{2}\pm\sqrt{18-36}}{2}=\frac{-3\sqrt{2}\pm\sqrt{2}.i}{2}=\frac{3\sqrt{2}}{2}(-1\pm i)$.
2) $x^2-3\sqrt{2}.x+9=0$; $x_{3,4}=\frac{3\sqrt{2}\pm\sqrt{18-36}}{2}=$ $=\frac{3\sqrt{2}\pm3\sqrt{2}.i}{3}=\frac{3\sqrt{2}}{2}(1\pm i)$.

Märkus: $\sqrt[4]{-81}$'l on neli imaginaarset tähendust.

237. $x^6-2^6=0$; $(x^3+2^3)(x^3-2^3)=0$; $(x+2)(x-2)(x^2-2x+4)(x^2+2x+4)=0$; $x_1=2$; $x_2=-2$; $x^2-2x+4=0$ kust $x_{3,4}=1\pm\sqrt{1-4}=1\pm\sqrt{3}.i$ a $x^2+2x+4=0$ kust $x_{5,6}=-1\pm\sqrt{1-4}=-1\pm\sqrt{3}.i$.

238. $x^6+729=0$; $x^6+3^6=0$; ütleme et $x=z\sqrt{-1}$ ja et $(\sqrt{-1})^6=-1$, leiame $x^6+3^6=(z\sqrt{-1})^6+3^6=$ $=-z^6+3^6$; $z^6-3^6=0$; $(z^3+3^3)(z^3-3^3)=0$ ehk $(z+3)(z-3)(z^2-3z+9)(z^2+3z+9)=0$; $z_1=3$; $z_2=-3$; pääle selle $z^2-3z+9=0$ kust $z_{3,4}=\frac{3}{2}(1\pm\sqrt{3}.i)$ ja $z^2+3z+9=0$ kust $z_{5,6}=\frac{3}{2}(-1\pm\sqrt{3}.i)$; et $x=z\sqrt{-1}$, siis on temal kuus tähendust; $x_1=3\sqrt{-1}$, $x_2=-3\sqrt{-1}$; $x_{3,4}=\frac{3}{2}(1\pm\sqrt{3}.i).\sqrt{-1}$ ja $x_{5,6}=\frac{3}{2}(-1\pm\sqrt{3}.i).\sqrt{-1}$; ehk $x_1=3i$; $x_2=-3i$; $x_{3,4}=\frac{3}{2}(i\pm\sqrt{3})$; $x_{5,6}=\frac{3}{2}(-i\pm\sqrt{3}$; ehk $x_{12}=\pm3i$; $x_{3,4}=\frac{3(\sqrt{3}\pm i)}{2}$; $x_{5,6}=\frac{3(-\sqrt{3}\pm i)}{2}$.

239. $b^8x^8-a^8=0$; $(b^4x^4+a^4)(b^4x^4-a^4)=0$; $(b^4x^4=a^4)(b^2x^2+a^2)(b^2x^2-a^2)=0$; $(b^4x^4+a^4)(b^2x^2-$ $-a^2i^2)(b^2x^2-a^2)=0$ ehk $(b^4x^4+a^4)(bx+ai)bx-ai)$.

$.(bx + a)(bx - a) = 0$, siit $x_1 = -\frac{a}{b}.i$; $x_2 = \frac{a}{b}.i$; $x_3 = -\frac{a}{b}$; $x_4 = \frac{a}{b}$. Peale selle $b^4x^4 + a^4 = 0$; $b^4x^4 + 2a^2b^2x^2 + a^4 - 2a^2b^2x^2 = 0$. $(b^2x^2 + a^2)^2 - (abx\sqrt{2})^2 = (b^2x^2 - ab\sqrt{2}.x + a^2).(b^2x^2 + ab\sqrt{2}.x + a^2) = 0$; $b^2x - ab\sqrt{2}.x + a^2 = 0$, kust $x_{5,6} = \frac{ab\sqrt{2} \pm \sqrt{2a^2b^2 - 4a^2b^2}}{2b^2} = \frac{ab\sqrt{2} \pm ab\sqrt{2}.i}{2b^2} = \frac{a\sqrt{2}}{2b}(1 \pm i)$, ja $b^2x^2 + ab\sqrt{2}.x + a^2 = 0$; $x_{7,8} = \frac{-ab\sqrt{2} \pm \sqrt{2a^2b^2 - 4a^2b^2}}{2b^2} = \frac{-ab\sqrt{2} \pm ab\sqrt{2}.i}{2b^2} = \frac{a\sqrt{2}}{2b}(-1 \pm i)$.

240. $a^8x^8 + 2a^4b^4x^4 + b^8 - 2a^4b^4x^4 = 0$; ehk $(a^4x^4 + b^4)^2 - (a^2b^2x^2\sqrt{2})^2 = 0$, kust $(a^4x^4 - a^2b^2x^2\sqrt{2} + b^4)(a^4x^4 + a^2b^2x^2\sqrt{2} + b^4) = 0$, siit: 1) $a^4x^4 - a^2b^2\sqrt{2}.x^2 + b^4 = 0$; $x_1^2 = \frac{a^2b^2\sqrt{2} \pm \sqrt{2a^4b^4 - 4a^4b^4}}{2a^4} = \frac{a^2b^2\sqrt{2} \pm a^2b^2\sqrt{2}.i}{2a^4}$; $x_1^2 = \frac{b^2\sqrt{2}}{2a^2}(1 \pm i)$; $x_{1,2,3,4} = \pm\frac{b}{a}\sqrt{\frac{1 \pm i}{\sqrt{2}}}$; 2) $a^4x^4 + a^2b^2\sqrt{2}.x^2 + b^4 = 0$; $x_2^2 = \frac{-a^2b^2\sqrt{2} \pm \sqrt{2a^4b^4 - 4a^4b^4}}{2a^4} = \frac{-a^2b^2\sqrt{2} \pm a^2b^2\sqrt{2}.i}{2a^4}$; $x_2^2 = \frac{b^2\sqrt{2}}{2a^3}(-1 \pm i)$; $x_{5,6,7,8} = \pm\frac{b}{a}\sqrt{\frac{-1 \pm i}{\sqrt{2}}}$.

§ 6. Irratsionaalsete võrrandite lahendus.

241. 5't teisele poole võrduse märki viies saame $\sqrt{6 - x} = 2$; saadud võrrandit ruutu võttes, leiame et $6 - x = 4$; $x = 2$. Saadud juurt antud võrrandisse x'i asemele pannes leiame, et ta võrrandit rahuldab.

242. $\sqrt{5 + \sqrt{x - 4}} = 3$; $\left(\sqrt{5 + \sqrt{x - 4}}\right)^2 = 3^2$; $5 + \sqrt{x - 4} = 9$; $\sqrt{x - 4} = 9 - 5 = 4$. Uuesti ruutu võttes saame $x - 4 = 16$; $x = 20$. Asemele pannes leiame, et $x = 20$ rahuldab antud võrrandit.

243. Viime radikaali $\sqrt{2x+3}$ teisele poole võrduse märki, saame $\sqrt{x+1} = 1 - \sqrt{2x+3}$. Mõlemaid pooli ruutu võttes saame: $x + 1 = 1 - 2\sqrt{2x+3} + 2x + 3$ ehk $2\sqrt{2x+3} = x + 3$. Uuesti ruutu võttes saame: $4(2x+3) = x^2 + 6x + 9$ ehk $8x + 12 = x^2 + 6x + 9$ ehk $x^2 - 2x - 3 = 0$; $x = 1 \pm \sqrt{1+3} = 1 \pm 2$; $x_1 = 3$; $x_2 = -1$. Antud võrrandit rahuldab ainult juur $x = -1$, kuna teine juur $x = 3$ kõrvale tuleb heita.

244. Radikaali $\sqrt{x+2}$ teisele poole viies saame $\sqrt{3x+4} = 8 - \sqrt{x+2}$; ruutu võttes saame $3x + 4 = 64 - 16\sqrt{x+2} + x + 2$; $16\sqrt{x+2} = 62 - 2x$; $8\sqrt{x+2} = 31 - x$. Teistkorda ruutu võttes saame $64(x+2) = 961 - 62x + x^2$; $64x + 128 = 961 - 62x + x^2$; $x^2 - 126x + 833 = 0$. $x = 63 \pm \sqrt{3969 - 833} = 63 \pm \sqrt{3126} = 63 \pm 56$; $x_1 = 119$; $x_2 = 7$. Antud võrrandti rahuldab ainult juur $x = 7$.

245. Radikaali $\sqrt{10 - x}$ teisele poole viies saame $\sqrt{22 - x} = 2 + \sqrt{10 - x}$; ruutu võttes saame $22 - x = 4 + 4\sqrt{10 - x} + 10 - x$ ehk $4\sqrt{10 - x} = 8$; $\sqrt{10 - x} = 2$; uuesti ruutu võttes saame $10 - x = 4$; $x = 6$.

246. Radikaali $\sqrt{4x - 3}$ teisele poole viies saame $2\sqrt{x + 18} = 15 - \sqrt{4x - 3}$, ruutu võttes saame $4(x + 18) = 223 - 30\sqrt{4x - 3} + 4x - 3$; $30\sqrt{4x - 3} = 150$; $\sqrt{4x - 3} = 5$; teist kord ruutu võttes leiame $4x - 3 = 25$; $4x = 28$; $x = 7$.

247. Anname antud võrrandile järgmise kuju $\sqrt{2x+1} = 2\sqrt{x} - \sqrt{x - 3}$ ja võtame mõlemad võrrandi pooled ruutu saame $2x + 1 = 4x - 4\sqrt{x(x-3)} + x - 3$ ehk $4\sqrt{x(x-3)} = 3x - 4$; uuesti ruutu võttes saame $16x(x-3) = 9x^2 - 24x + 16$; ehk $16x^2 - 48x = 9x^2 - 24x + 16$; $7x^2 -$

$-24x-16=0$; $x=\frac{12\pm\sqrt{144+112}}{7}=\frac{12\pm16}{7}$; $x_1=4$; $x_2=-\frac{4}{7}$. Antud võrrandit rahuldab ainult juur $x=4$.

248. Ruutu võttes saame $3x-3+2\sqrt{(3x-3)(5x-19)}+$ $+5x-19=3x+4$ ehk $2\sqrt{(3x-3)(5x-19)}=26-5x$; võtame uuesti ruutu, saame $4(3x-3)(5x-19)=676-$ $-260x+25x^2$; $60x^2-288x+228=676-260x+25x^2$; $35x^2-28x-448=0$; $5x^2-4x-64=0$; $x=\frac{2\pm\sqrt{4+320}}{5}=$ $=\frac{2\pm18}{5}$; $x_1=4$; $x_2=-\frac{16}{5}$. Antud võrrandit rahuldab ainult juur $x=4$.

249. Ruutu võttes saame $1+x\sqrt{x^2+12}=1+2x+x^2$ ehk $x\sqrt{x^2+12}=x(2+x)$ ehk x-iga koondates, $x_1=0$ saame $\sqrt{x^2+12}=2+x$. Uuesti ruutu võttes saame $x^2+12=4+$ $+4x+x^2$; $x_2=2$. Antud võrrandit rahuldab $x=2$, kuna $x=0$ kõrvale tuleb heita.

250. -2-hte pahemale poole viies, saame $x+2=$ $\sqrt{4+x\sqrt{36+x^2}}$. Ruutu võttes saame $x^2+4x+4=4+$ $+x\sqrt{36+x^2}$ ehk $x\sqrt{36+x^2}=x(x-4)$; koondame x-ga; $x_1=0$; $\sqrt{36+x^2}=x+4$; uuesti ruutu võttes saame $36+$ $+x^2=x^2+8x+16$; $8x=20$; $x_2=\frac{5}{2}=2\frac{1}{2}$.

251. Mõlemaid võrrandi pooli ruutu võttes saame: $\frac{4}{x^2}+\frac{8}{x}+4=4+\frac{1}{x}\sqrt{64+\frac{144}{x^2}}$ ehk $\frac{4}{x^2}+\frac{8}{x}=\frac{1}{x}\sqrt{64+\frac{144}{x^2}}$; koondame $\frac{1}{x}$-iga; $\frac{1}{x}=0$; kui $\frac{1}{x}=0$, siis $x_1=\infty$; $\frac{4}{x}+8=$ $=\sqrt{64+\frac{144}{x^2}}$; uuesti ruutu võttes saame: $\frac{16}{x^2}+\frac{64}{x}+64=64+$ $+\frac{144}{x^2}$; $\frac{128}{x^2}=\frac{64}{x}$; $64x=128$; $x_2=2$. Mõlemad juured rahuldavad võrrandit.

252. Ruutu võttes saame: $1 - \frac{2}{3} + \frac{1}{x^2} = 1 - \frac{1}{x}\sqrt{4 - \frac{7}{x_2}}$ ehk $\frac{1}{x}\sqrt{4 - \frac{7}{x^2}} = \frac{2}{x} - \frac{1}{x^2}$; koondades $\frac{1}{x}$-iga, kusjuures $\frac{1}{x} = 0$, saame $x_1 = \infty$ ja $\sqrt{4 - \frac{7}{x^2}} = 2 - \frac{1}{x}$; uuesti ruutu võttes saame $4 - \frac{7}{x^2} = 4 - \frac{4}{x} + \frac{1}{x^2}$; $\frac{7}{x^2} = \frac{4}{x} - \frac{1}{x^2}$, $\frac{8}{x^2} - \frac{4}{x} = 0$; $\frac{4}{x}\left(\frac{2}{x} - 1\right) = 0$; $\frac{4}{x} = 0$; $x^2 = \infty$ ja $\frac{2}{x} - 1 = 0$; $x^3 = 2$. Antud võrrandit rahuldavad juured ∞ ja 2.

253. Kaotame võrrandil nimetajad ära: $5(x - \sqrt{5 + x^2}) - 5(x + \sqrt{5 + x^2}) = 6(x + \sqrt{5 + x^2})(x - \sqrt{5 + x^2})$; $5x - 5\sqrt{5 + x^2} - 5x - 5\sqrt{5 + x^2} = 6(x^2 - 5 - x^2)$; $-10\sqrt{5 + x^2} = -30$; $\sqrt{5 + x^2} = 3$; ruutu võttes saame $5 + x^2 = 9$; $x^2 = 4$; $x = \pm 2$.

254. Kaotame võrrandis nimetajad ära: $28x - 28\sqrt{4 - x^2} + 28x + 28\sqrt{4 - x^2} = 12(x + \sqrt{4 - x^2})(x - \sqrt{4 - x^2})$; $14x = 3(x^2 - 4 + x^2)$; $14x = 6x^2 - 12$; $3x^2 - 7x - 6 = 0$, kust $x = \frac{7 \pm \sqrt{47 + 72}}{6} = \frac{7 + 11}{6}$; $x_1 = 3$; $x_2 = -\frac{2}{3}$.

255. Märgates, et $\frac{x - 1}{1 + \sqrt{x}} = \frac{(x - 1)(\sqrt{x} - 1)}{x - 1} = \sqrt{x} - 1$; anname võrrandile järgmise kuju $\sqrt{x} - 1 = 4 - \frac{1 - \sqrt{x}}{2}$; $2\sqrt{x} - 2 = 8 - 1 + \sqrt{x}$; $\sqrt{x} = 9$; $x = 81$.

256. Vabastades võrrandi nimetajast saame $\sqrt{5x(3x + 1)} - 4 = 3x + 1$ ehk $\sqrt{5x(3x + 1)} = 3x + 5$; teisel astmel võttes leiame $5x(3x + 1) = 9x^2 + 30x + 25$; $15x^2 + 5x = 9x^2 + 30x + 25$; $6x^2 - 25x - 25 = 0$; $x = \frac{25 \pm \sqrt{625 + 600}}{12} =$

$=\frac{25 \pm 35}{12}$; $x_1 = 5$; $x_2 = -\frac{5}{6}$. Antud võrrandit rahuldab juur $x = 5$, kuna juur $x = -\frac{5}{6}$ kõrvale tuleb heita.

257. Proportsioonist $\frac{a}{b} = \frac{p}{q}$ võime teise proportsiooni saada: $\frac{a+b}{a-b} = \frac{p+q}{p-q}$. Seda proportsiooni omadust ära kasutades saame: $\frac{\sqrt{2x^2+1} + \sqrt{x-1} + \sqrt{2x^2+1} - \sqrt{x-1}}{\sqrt{2x^2+1} + \sqrt{x-1} - \sqrt{2x^2+1} + \sqrt{x-1}} = \frac{2+1}{2-1}$;

ehk $\frac{2\sqrt{2x^2+1}}{2\sqrt{x-1}} = 3$; $\frac{\sqrt{2x^2+1}}{\sqrt{x-1}} = 3$; ruutu võttes saame: $\frac{2x^2+1}{x-1} = 9$ ehk $2x^2 + 1 = 9x - 9$; $2x^2 - 9x + 10 = 0$, kust $x = \frac{9 \pm \sqrt{81-80}}{4} = \frac{9 \pm 1}{4}$; $x_1 = \frac{5}{2} = 2{,}5$; $x_2 = 2$.

258. Muudame ülesannet nii nagu seda tegime ülesande nr. 257-ga. Saame: $\frac{\sqrt{3x^2+1} - \sqrt{2x+1} + \sqrt{3x^2+1} + \sqrt{2x+1}}{\sqrt{3x^2+1} - \sqrt{2x+1} - \sqrt{3x^2+1} - \sqrt{2x+1}} =$ $= \frac{2+5}{2-5}$ ehk $\frac{2\sqrt{3x^2+1}}{-2\sqrt{2x+1}} = \frac{7}{-3}$ ehk $\frac{\sqrt{3x^2+1}}{\sqrt{2x+1}} = \frac{7}{3}$; ruutu võttes saame $\frac{3x^2+1}{2x+1} = \frac{49}{9}$; $27x^2 + 9 = 98x + 49$; $27x^2 - 98x -$ $- 40 = 0$, kust $x = \frac{49 \pm \sqrt{2401 + 1080}}{27} = \frac{49 \pm 59}{27}$; $x_1 = 4$; $x_2 = -\frac{10}{27}$.

259. Võrrandit ühise nimetaja juure tuues ja seda ära jättes saame: $2\left(\sqrt{x + \sqrt{x}}\right)^2 - 2\left(\sqrt{x + \sqrt{x}}\right)\left(\sqrt{x - \sqrt{x}}\right) =$ $= 3\sqrt{x}$ ehk $2(x + \sqrt{x}) - 2\sqrt{x^2 - x} = 3\sqrt{x}$ ehk $2x + 2\sqrt{x} -$ $- 2\sqrt{x^2 - x} = 3\sqrt{x}$; $2\sqrt{x^2 - x} = 2x - \sqrt{x}$; peale ruutu võtmist saame $4(x^2 - x) = 4x^2 - 4x\sqrt{x} + x$; $4x - 4x =$ $= 4x^2 - 4x\sqrt{x} + x$; $5x - 4x\sqrt{x} = 0$; $x(5 - 4\sqrt{x}) = 0$; $x_1 = 0$ ja $5 - 4\sqrt{x} = 0$; $4\sqrt{x} = 5$; $16x = 25$; $x_2 = \frac{25}{16}$.

260. Muudame ülesannet samuti kui nr. 257.

$$\frac{x + 1 - \sqrt{2x+1} + x + 1 + \sqrt{2x+1}}{x + 1 - \sqrt{2x+1} - x - 1 - \sqrt{2x+1}} = \frac{\sqrt{2x+1} + 1 + \sqrt{2x+1} - 1}{\sqrt{2x+1} + 1 - \sqrt{2x+1} - 1}$$

ehk $\frac{2(x+1)}{-2\sqrt{2x+1}} = \frac{2\sqrt{2x+1}}{2}$; $-\frac{x+1}{\sqrt{2x+1}} = \sqrt{2x+1}$; nimetajat ära kaotades saame $-(x+1) = 2x+1$; $3x = -2$; $x = -\frac{2}{3}$.

261. $\sqrt{2ax+x^2} = a-x$. Ruutu võttes saame $2ax + x^2 = a^2 - 2ax + x^2$; $4ax = a^2$; $x = \frac{a^2}{4a} = \frac{a}{4}$.

262. $\sqrt{a-x} = \sqrt{a} - \sqrt{x}$. Ruutu võttes, saame $a - x = a - 2\sqrt{ax} + x$; $2\sqrt{ax} = 2x$; $\sqrt{ax} = x$; uus ruutu võtmine annab $ax = x^2$; $x^2 - ax = 0$; $x(x-a) = 0$; $x_1 = 0$; $x_2 = a$.

263. Ruutu võttes saame $3x + a + 2b - 2\sqrt{(3x+a+2b)(3x+a-2b)} + 3x + a - 2b = 4(x-a)$ ehk $2\sqrt{(3x+a)^2 - 4b^2} = 2x + 6a$; $\sqrt{(3x+a)^2 - 4b^2} = x + 3a$; uuesti ruutu võttes saame $(3x+a)^2 - 4b^2 = x^2 + 6ax + 9a$; $9x^2 + 6ax + a^2 - 4b^2 = x^2 + 6ax + 9a^2$; $8x^2 = 8a^2 + 4b^2$; $2x^2 = 2a^2 + b^2$; $x^2 = \frac{2a^2 + b^2}{2}$; $x = \pm \frac{1}{2}\sqrt{4a^2 + 2b^2}$.

264. Võrrandi mõlemaid pooli ruutu võttes saame: $a - bx + 2\sqrt{(a-bx)(c-dx)} + c - dx = a + c - (b+d)x$ ehk $2\sqrt{(a-bx)(c-dx)} = 0$; $(a-bx)(c-dx) = 0$, kust $a - bx = 0$; $x_1 = \frac{a}{b}$; $c - dx = 0$; $x_2 = \frac{c}{b}$.

265. Nimetajaid võrrandis ära kaotades saame $a + x + \sqrt{(2a+x)(a+x)} = a$; $\sqrt{(2a+x)(a+x)} = -x$; ruutu võttes saame $(2a+x)(a+x) = x^2$ ehk $2a^2 + 3ax + x^2 = x^2$ ehk $3ax = -2a^2$; $x = -\frac{2a}{3}$.

266. Võrrandi mõlemaid pooli ruutu võttes saame: $a + \sqrt{x} - 2\sqrt{(a+\sqrt{x})(a-\sqrt{x})} + a - \sqrt{x} = a$ ehk $2\sqrt{a^2 - x} = a$; uuesti ruutu võttes saame $4a^2 - 4x = a^2$, kust $4x = 3a^2$; $x = \frac{3a^2}{4}$.

267. Võtame ruudus $\frac{1}{a^2}-\frac{2}{ax}+\frac{1}{x^2}=\frac{1}{a^2}-\sqrt{\frac{4}{a^2x^2}-\frac{7}{x^4}}$ ehk $\frac{1}{x^2}-\frac{2}{ax}+\frac{1}{x}\sqrt{\frac{4}{a^2}-\frac{7}{x^2}}=0$ ehk $\frac{1}{x}\left(\frac{1}{x}-\frac{2}{a}+\sqrt{\frac{4}{a^2}-\frac{7}{x^2}}\right)=0$; siit $\frac{1}{x}=0$, $x_1=\infty$ Siis $\frac{1}{x}-\frac{2}{a}+\sqrt{\frac{4}{a^2}-\frac{7}{x^2}}=0$ ehk $\sqrt{\frac{4}{a^2}-\frac{7}{x^2}}=\frac{2}{a}-\frac{1}{x}$; ruutu võttes, leiame $\frac{4}{a^2}-\frac{7}{x^2}=\frac{4}{a^2}-\frac{4}{ax}+$ $+\frac{1}{x}$; ehk $\frac{8}{x^2}-\frac{4}{ax}=0$; ehk $\frac{2}{x^2}-\frac{1}{ax}=0$; $\frac{1}{x}\left(\frac{2}{x}-\frac{1}{a}\right)=0$; kust $\frac{1}{x}=0$; $x_2=\infty$ ja $\frac{2}{x}-\frac{1}{a}=0$; $x^3=2a$.

268. Muundame selle võrrandi niisamuti kui seda tegime nr. 257-ga. Saame: $\frac{\sqrt{a+x}+\sqrt{a-x}+\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}-\sqrt{a+x}+\sqrt{a-x}}=\frac{a+x}{a-x}$ ehk $\frac{2\sqrt{a+x}}{2\sqrt{a-x}}=\frac{a+x}{a-x}$ ehk $\frac{\sqrt{a+x}}{\sqrt{a-x}}=\frac{a+x}{a-x}$ ehk $(a-x)\sqrt{a+x}=$ $=(a+x)\sqrt{a-x}$ ehk $\sqrt{(a-x)^2}\cdot\sqrt{a+x}-\sqrt{(a+x)^2}\cdot\sqrt{a-x}=$ $=0$; ehk $\sqrt{a+x}\cdot\sqrt{a-x}\,(\sqrt{a-x}-\sqrt{a+x})=0$, kust
1) $\sqrt{a+x}\cdot\sqrt{a-x}=0$; ehk $\sqrt{a^2-x^2}=0$; $a^2=x^2$; $x=\pm a$;
2) $\sqrt{a-x}=\sqrt{a+x}$ ehk $a-x=a+x$; $x=0$.

269. Võrrandis nimetajaid ära kaotades, saame $(\sqrt{ax+b}+$ $+\sqrt{ax})(1-\sqrt{ax-b})=(\sqrt{ax+b}-\sqrt{ax})(1+\sqrt{ax-b})$ ehk $\sqrt{ax+b}-\sqrt{a^2x^2-b^2}+\sqrt{ax}-\sqrt{ax(ax-b)}=\sqrt{ax+b}+$ $+\sqrt{a^2x^2-b^2}-\sqrt{ax}-\sqrt{ax(ax-b)}$; koondades saame $2\sqrt{a^2x^2-b^2}=2\sqrt{ax}$ ehk $\sqrt{a^2x^2-b^2}=\sqrt{ax}$; ruutu võttes leiame $a^2x^2-b^2=ax$; ehk $a^2x^2-ax-b^2=0$, kust
$x=\frac{a\pm\sqrt{a^2+4a^2b^2}}{2a^2}=\frac{a\pm a\sqrt{1+4b^2}}{2a^2}=\frac{1\pm\sqrt{1+4b^2}}{2a}$.

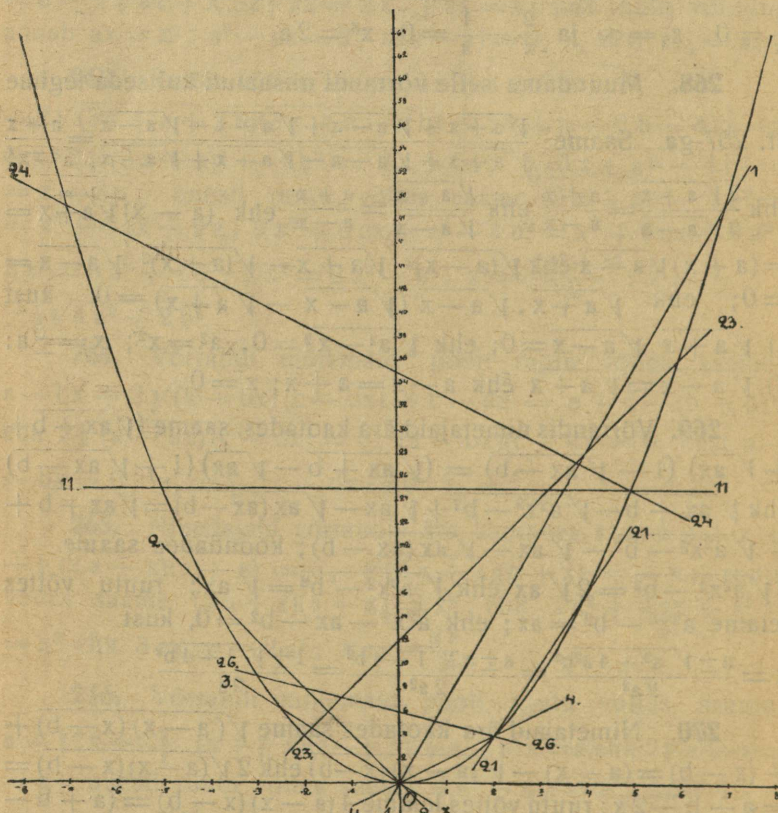
270. Nimetajaid ära kaotades saame $\sqrt{(a-x)(x-b)}+$ $+(x-b)=(a-x)-\sqrt{(a-x)(x-b)}$ ehk $2\sqrt{(a-x)(x-b)}=$ $=a+b-2x$; ruutu võttes leiame $4(a-x)(x-b)=(a+b-$ $-2x)^2$ ehk $4ax-ab-4x^2+4bx=a^2+b^2+4x^2+2ab-$

$-4ax - 4bx$; $8x^2 - 8ax - 8bx + 6ab + a^2 + b^2 = 0$; ehk
$8x^2 - (8a + 8b)x + (a^2 + 6ab + b^2) = 0$;

kust $x = \frac{8a + 8b \pm \sqrt{(8a + 8b)^2 - 32(a^2 + 6ab + b^2)}}{16} =$

$= \frac{8a + 8b \pm \sqrt{32a^2 - 64ab + 32b^2}}{16} = \frac{(8a + 8b) \pm 4(a - b)\sqrt{2}}{16} =$

$= \frac{2a + 2b \pm (a - b)\sqrt{2}}{4} = \frac{a + b}{2} \pm \frac{(a - b)\sqrt{2}}{4}$.



Lisa.

Ruutvõrrandite graafiline lahendamine teistviisi.

K. Veski ja J. Grünthali väljaantud Šapošnikov ja Valtsev'i „Algebraliste ülesannete kogu“ II jaos, lhk. 50, 51, 52 a 54 näidatud ruutvõrrandite graafilise lahendamise viisil, nagu käesolevast raamatust lhk. 99 näha, on see paha omadus, et iga võrrandi jaoks eri kõvera (parabooli) saame, mida võrdlemisi raske joonistada. Sellepärast olgu siin teine viis antud, mis niisamuti, kuid üheainsa parabooliga sihile viib. Mõni näitus. Ruutvõrrand nr. 26: $x^2 + x - 6 = 0$. Kirjutame tema nii $x^2 = - x + 6$ ja võrrutame pahemat ning pahemat poolt y-ga, saame: 1) $y = x^2$ ning 2) $y = - x + 6$. Esimese võrrandi geomeetriliseks kujuks on parabool, mis koordinaatide alguspunktist läbi läheb, kuna teise võrrandi kujuks sirge on. Et joonis liiga pikaks ei veniks on selle parabooli joonestamise juures soovitav x teljel pikkuse üksus 4—5 korda pikem võtta, kui y teljel. x-i ning y-i vastavuste tabel kõvera $y = x^2$ jaoks:

x	−8	$-7\frac{1}{2}$	−7	$-6\frac{1}{2}$	−6	$-5\frac{1}{2}$	−5	$-4\frac{1}{2}$	−4	$-3\frac{1}{2}$	−3	$-2\frac{1}{2}$	−2	$-1\frac{1}{2}$	−1	0	0	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	−4	jne.
y	64	$56\frac{1}{4}$	49	$42\frac{1}{4}$	36	$30\frac{1}{4}$	25	$20\frac{1}{4}$	16	$12\frac{1}{4}$	9	$6\frac{1}{4}$	4	$2\frac{1}{4}$	1	0	0	1	$2\frac{1}{4}$	4	$6\frac{1}{4}$	9	$12\frac{1}{4}$	16	

sirge $y = - x + 6$ jaoks

x	−3	0	2
y	9	4	4

Mõlemaid joonistades näeme, et nad teineteist lõikavad. Võrrandi juurte leidmiseks on tarvis parabooli ja lõikepunktide abstsissid ära lugeda. Käesoleva võrrandi jaoks on nad -3 ja $+2$. Et igale täielikule ruutvõrrandile $x^2 + px + q = 0$ kuju võib anda, siis on selge, et parabool ikka ühesuguseks jääb, kuna ainult sirge oma asukohta teljestikus muudab. Asjaolu ei muutu ka ebatäielikkude ruutvõrrandite juures. Näitus I. Võrrand nr. 11: $x^2 - 25 = 0$; $x^2 = 25$; $y = x^2$; $y = 25$. Parabool on jälle endine, kuna $y = 25$ sirget kujutab, mis x teljele paralleelne ja mille ordinaat 25. Tema ja parabooli lõikepunktide abstsissid $= +5$ ja -5. Näitus II. Võrrand nr. 1: $x^2 - 7x = 0$; $x^2 = 7x$; $y = x^2$; $y = 7c$. Kõver on jälle endine.

Õiendus.

Lhk. 97. Ülesanne nr. 2 peab olema $x^2 + 4x = 0$; $x(x + 4) = 0$; $x_1 = 0$; $x_2 = 4$.