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Modelling Interactions Between Traffic and Crosswalk Agents at Unsignalized Crossings

Master's Thesis (30 ECTS)

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Tartu 2021

Modelling Interactions Between Traffic and Crosswalk Agents at Unsignalized Crossings

Abstract:

Agent behaviour modelling is currently one of the most challenging topics in the autonomous driving industry meaning that more research is needed in order to understand how autonomous cars should be interacting and behaving around pedestrians and cyclists. The research includes modelling vehicle-bicycle interactions based on hierarchical modelling framework [1] generating different features mentioned in literature and some proposed by author, selecting relevant ones using F value and finally creating a simple logistic regression model for estimating the drivers yielding probability. As a result, a model with following features is suggested: bicycle distance from conflict zone when vehicle enters the interaction zone, vehicle mean speed inside interaction zone, minimum speed distance and minimum speed inside interaction zone. The research also indicates that driver yielding probability is reduces with higher vehicle speed and higher bicycle distance. Furthermore, the data indicates that drivers who perceive a conflict but do not yield experience the highest mean speed referring that some vehicles do accelerate in order to avoid giving way. Additionally, the low variation in bicycles' speed indicates cyclists' assertive behaviour and perhaps expectations for the vehicle to yield.

Keywords:

Autonomous driving, vehicle-bicycle interaction, yielding decision

CERCS: P170 Computer science, numerical analysis, systems, control

Mootorsõidukite ja jalgratturite vahelise suhtluse modelleerimine signaliseerimata ülekäigurajal

Lühikokkuvõte:

Liiklusagentide käitumise modelleerimine on autonoomse juhtimise valdkonnas hetkel üks aktuaalsemaid ning väljakutsuvamaid teemasid. Seega, et paremini mõista, kuidas isesõitvad autod jalakäijate ja jalgratturitega suhtlema peaksid on oluline sellele rohkem tähelepanu pöörata. Käesolev uurimus modelleerib esmalt auto ja jalgratturi vahelised interaktsiooni põhinedes hierarhilisele modelleerimis raamistikule [1]. Seejärel genereeritakse erinevad *feature*-id tuginedes enamjaolt kirjandusele ent osaliselt ka autori soovitustele. Enim olulised *feature*-id valitakse kasutades F väärtust. Nende *feature*-te põhjal luuakse *logistic regression* masinõppe mudel hindamaks auto peatumise tõenäosust. Katse tulemusena saavutab mudel järgmiste *feature*-tega parima tulemuse: jalgratta kaugus konflikti tsoonist kui auto on interaktsiooni tsooni sisenemas, auto keskmine kiirus interaktsiooni tsoonis, kaugus konflikti tsoonist, kus saavutati minimaalne kiiruse ning minimaalne kiirus interaktsiooni tsoonis. Lisaks näitab uuring ka seost auto kiiruse ning auto peatumise tõenäosuse vahel. Mida kõrgem auto kiirus on seda madalam on tema peatumise tõenäosus. Lisaks mida kaugemal on jalgrattur seda madalam on auto peatumise tõenäosus. Andmete põhjal võib järeldada, et autojuhid, kes tajuvad konflikti situatsiooni, kuid ei peatu lisavad hoopis kiirust juurde, et hoidud jalgratturile tee andmisest. Lisaks indikeerib jalgratturite ühtlane kiirus olenemata konflikti olukorrast, et jalgratturid, kas eeldavad iseenesest, et neid märgati või püüavad ennast kehtestava käitumisega autojuhti peatuma sundida.

Võtmesõnad:

Isesõitev auto, auto-jalgratta interaktsioon, peatumisotsus

CERCS: P170 Arvutiteadus, arvutusmeetodid, süsteemid, juhtimine (automaatjuhtimisteooria)

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1 Introduction

The autonomous driving field has been growing rapidly in recent years and the potential of it seems greater than ever with new state of the art Advanced Driver Assistance Systems (ADAS) being developed, European Commission funding the field and Waymo being first to commercialise a fully autonomous taxi service in the end of 2018 [1].

1.1 Context

Despite of autonomous vehicles already entering the traffic, it will take years long transition period before adjusting to solely autonomous vehicles. Therefore, the society must adapt to having manually driven, partly automated and fully automated vehicles simultaneously on the roads without jeopardizing the safety of cyclists and pedestrians. [2]

In current traffic system the interactions between road users and crosswalk agents are mainly based on formal rules and regulations. In addition, road users also use mainly non-verbal communication such as nodding or waving in order to express their desires of either following the rules or occasionally suggesting the opposite [2]. The studies show that crosswalk agents following certain rules may depend on multiple factors such as gender, age, group size, traffic characteristics, road conditions etc [2] [3].

However, when interacting with autonomous vehicle these factors might differ. Studies have shown that pedestrians and cyclists tend to be cautious and sceptical of automated vehicles' competence of safely interacting with other road agents. This suggests that the role of the regulations together with communication between pedestrians and automated vehicles will most probably differ from the communication between pedestrians and manually driven vehicles but how exactly is not yet certain. [2]

Pedestrians and cyclists are one of the most vulnerable road agents as they do not have a protective shell surrounding them [2]. In 2018 they made 29% of all recorded road deaths across the European Union out of which 8% were bicycles [4]. Due to a political commitment European Green Deal released in the end of 2019 the usage of bicycles will most probably increase even more in the upcoming years. Since the goal of abolishing net emissions of greenhouse gases by the year 2050 will only be achievable by decarbonizing road transport and therefore strongly supporting and funding cycling as it is the most energy-efficient transport mode currently available. [5]

There are multiple ways for increasing the road safety of cyclists for instance redesigning and adjusting infrastructure and vehicles or managing road user behaviours. To integrate bicyclists into the traffic system it is important that both road and crosswalk agents acted predictably for the other party to be able to react safely. Therefore, in order to better understand both cyclists and drivers, researching their behaviour in different situations is necessary. [6]

1.2 Contribution

The vehicle-bicycle interactions can be divided into three separate categories such as mixed, longitudinal and crossing interactions [7].

Mixed interaction implies vehicles and bicycles interacting within the lanes as there is no barrier separating two travel modes. It could be considered as the most hazardous of all of them. Vehicle and bicycle interacting in side by side lanes is named longitudinal interaction where the main safety concern is vehicles' speed in the traffic flow. A crossing interaction is vehicle and bicycle interaction at the crosswalk and can be divided into signalized and unsignalized crossings. Although priority rule is supposed to regulate the unsignalized crossings there are still serious accidents. [7] For instance, studies have shown that when turning right, drivers tend to focus more on the traffic coming from the left resulting in higher number of accidents happening on vehicle right turn when bicycle crossing from the right. [8]

Therefore, the wider research question tackled in this thesis is how to make autonomous cars act predictably and as human-like as possible on unsignalized crosswalks.

The aim of this research is to study vehicle-bicycle crossing interactions at an unsignalized crosswalk to better understand human drivers' yielding decision and most relevant features that affect it in order to contribute to future treatments, solutions and models for making interaction with autonomous cars as safe and as human-like as possible.

The contribution of this thesis is to determine most important features affecting human driver's yielding decision to bicycle on an unsignalized crosswalk.

Contribution:

- Determining a set of features for estimating drivers yielding decision.
- Using F value for determining relevant features for estimating drivers yielding decision.
- Verifying the relevance of features found from literature.

1.3 Structure

At first the relevant literature review will be provided in the Background section after which the methodology of the research will be elaborated. In the Experiments section the code and course of research will be explained and illustrated followed by the obtained Result. Lastly, the research will be concluded.

2 Terms and Notations

ATD – Arrival Time Difference

CZ – Conflict Zone

IZ – Interaction Zone

Crosswalk agents – pedestrians, bicycles

Yielding decision – driver’s judgement of whether one needs to slow down at the crosswalk and let the bicycle cross or to keep moving and cross the crosswalk before the cyclist.

3 Background

Vehicle-pedestrian interactions on unsignalized crosswalks have been studied widely while the interactions between vehicle and bicycle have not been researched that thoroughly yet.

3.1 Priority Rule

Unsignalized crosswalks are mainly governed by priority rules where driver is expected to slow down if necessary and yield to the pedestrian.

During pedestrian-vehicle interaction the driver pays attention to the crosswalk in order to come to a full stop if necessary, while in case of bicycle-vehicle interaction the driver has to look further to the sidewalk in order to be able to stop [9].

Therefore, as bicycle moves at a greater speed than pedestrian, the approaching vehicle is obliged to observe a larger part of the pedestrian road thus resulting in different interaction dynamics [7].

Räsänen et al. [10] analysed 188 bicycle-car accidents in depth were neither party yielded to the other. The paper concluded that there are two main mechanisms for accidents. Firstly, distraction or allocation of attention resulting in driver failing to detect crosswalk agents. And secondly, having certain expectations towards other drivers which prove to be unjustified. [10]

For instance, T-junctions are a common traffic configuration where drivers often fail to meet crosswalk agents' expectations by not giving way to a pedestrian or cyclist coming from the right while vehicle also turning right [8].

Furthermore, drivers travelling in higher speed have been reported failing to notice the cyclist, therefore not meeting the expectation to yield [11]. In addition, drivers might fail due to lack of attention or misinterpretation. The lack of attention might be caused by limited vision or some distraction such as mobile phone while misinterpretation can be explained by cyclist assuming that he has been noticed while actually driver might not have seen him. Therefore, regardless of the priority rule vehicle might not always yield.

3.2 Vehicle-Bicycle Interactions

In recent years there have been multiple studies looking into cyclist-driver interactions and different aspects that may affect drivers' yielding decision on unsignalized crosswalks [7].

According to a systematic review of the research done in the field of vehicle-pedestrian interactions by Amado et al. [12], the most influential features affecting the pedestrian decision to cross, were the gap size, age of pedestrians and the number of pedestrians waiting.

On the other hand, the most relevant features affecting the driver's yielding decision were **traffic in opposite direction**, **number of pedestrians** waiting and the **vehicle type**. In addition, **the time headway** and **vehicle speed** were also said to have a great impact on the drivers' yielding decision. The review also reveals that drivers tend to yield more to **pedestrians** with **assertive behaviour** such as fast walking pace. [12]

On the contrary, **high vehicle speed**, **deceleration** and **traffic rate** were stated as factors decreasing the probability of vehicle yielding to pedestrian. In addition, **vehicles in the far lane relative to the pedestrian** were said to be less likely to yield. [12]

Moreover, the first study examining the racial bias conducted by Goddard et al. [13] indicates that the drivers' yielding decision is also affected by the **skin colour of the pedestrian** revealing that black participants had to wait 32% longer until driver yielded.

However, while the bicycle-vehicle interactions are somewhat similar to pedestrian-vehicle interactions, as in both cases driver is affected by vehicle speed and relative position of the other agent, there are also some differences.

Similar to pedestrian-vehicle interactions investigated by Amado et al. [12] it is stated that the yielding rate increases as the number of bicycles increases and when yield signs are located before the crossing.

A study conducted by Boda et al. [14] investigating the bicycle and vehicle speed influence on the driver response process indicates that neither car nor bicycle speed or crosswalk configuration directly influences the driver's breaking process. However, it states that the **point in time when bicycle becomes visible** together with the **configuration of the crosswalk** had the largest impact on driver response process. Therefore, as bicycle becoming visible depends on **vehicle speed, bicycle speed** and the **crossing configuration**, these features were stated as relevant only in relation to each other [14].

A study carried out by Bella et al. [15] analysed the effect of counter measures at the crosswalk during the cyclist and vehicle interaction. The study was carried out using a driving simulator in order to reduce risk and have full control over the experiment.

As a baseline condition 3 traffic signs were set 150 meters, 10 meters and at the crosswalk indicating absence of treatment. As a second counter measure in addition to the baseline condition the pavement at the crosswalk was painted red aiming to attract the approaching driver's attention hopefully resulting in speed adjustment and also affecting the yielding decision. Third counter measure was a raised 40 meters long island placed between the vehicle lanes which resulted in narrower lanes and therefore was hoped to influence drivers to adapt the speed accordingly. [15]

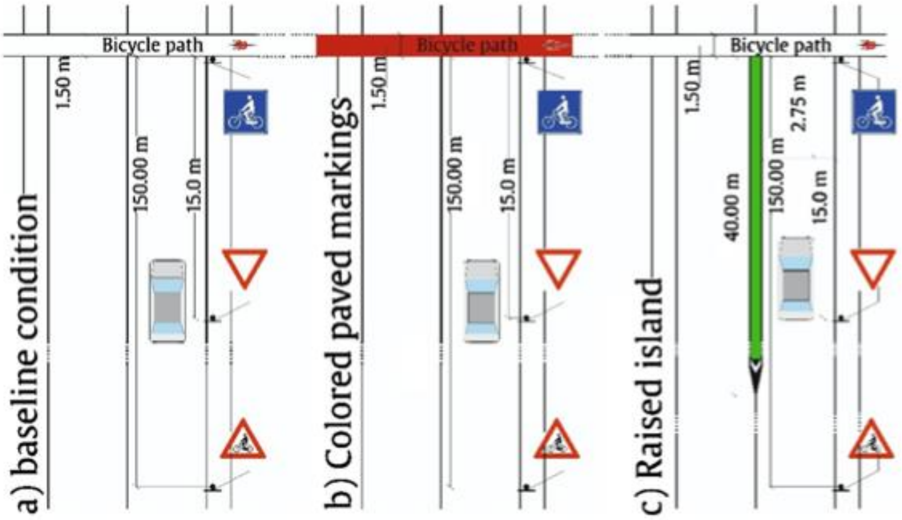

Figure 1. The conditions for experiment – (a) Baseline condition (b) Coloured paved markings (c) Raised island [15].

In total there were 355 observations of cyclist and driver interactions on the crosswalk from which multiple variables were determined. For example, the initial speed (speed at the moment when driver starts to decrease the speed after noticing the crossing bicycle), initial speed distance (the distance from which initial speed is recorder), minimum speed (the minimum speed recorded during driver breaking), minimum speed distance (distance from crossroad at which the minimum speed was recorder) based on which the average deceleration rate was calculated. [15]

After collecting the data statistical analysis were conducted to understand the effect of the counter measures. As a result, the **minimum speed distance** was the main variable affected by the countermeasures especially by the coloured pavement which improved the perception of the crosswalk from larger distance resulting in vehicles stopping further away from the crosswalk therefore making the interaction safer for the cyclists. Furthermore, both **countermeasures** increased the yielding rate of the drivers. [15]

A research conducted by Silvano et al. [7] investigated factors contributing most to the driver’s yielding decision by looking into the bicycle vehicle interactions at an unsignalized crosswalk after exiting a roundabout in Sweden. The data was collected using video cameras and Semi-Automatic Video Analysis program for data extraction.

The interaction area was divided into interaction and conflict zones. The conflict zone was defined by the area where vehicle and bicycle trajectories meet therefore the whole crosswalk was considered as a conflict zone. An interaction zone was defined as starting from the conflict zone and extending 10 meters from it on the road for the vehicle and 30 meters from it on the sidewalk for the bicycle. The bicycle interaction zone was divided into 3 separate zones each being 10 meters long illustrated on the Figure 2. [7]

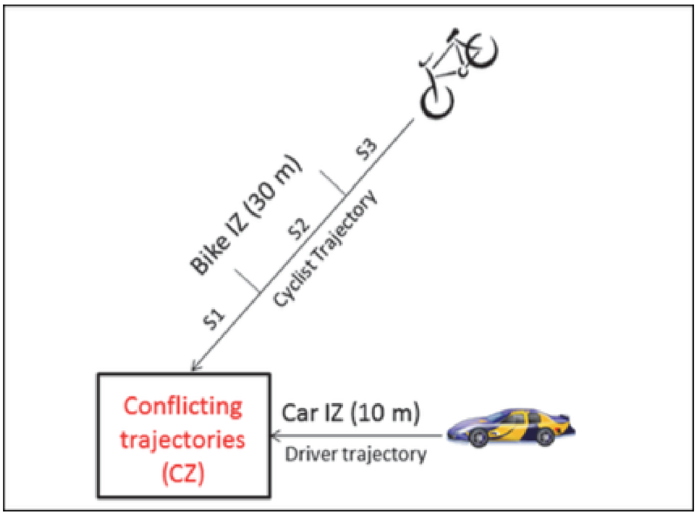

Figure 2. Data collection approach illustrating the definition of conflict and interaction zones [7].

The paper divides interactions into four different events. In case driver does not meet any bicycles and therefore, leaves both interaction and conflict zone without any interaction, is considered a non-conflict event. In case driver encounters cyclist and interacts with one in order to avoid collision, the interaction is considered as a conflict event. Silvano et al. [7] assumes the key variable for detecting the conflict to be the arrival time difference at the boundary of an interaction zone – the smaller the arrival time difference is, the more likely driver is to perceive the situation as a conflict. Therefore, a threshold of 5.5 seconds is identified to distinguish between conflict and non-conflict events.

In case of conflict there are two possible resulting actions – the vehicle either yields and lets the cyclist cross the road or it does not slow down and passes before the bicycle. Therefore, the conflict events were labelled as yield or non-yield events. [7]

The results of Silvano et al. indicate that **vehicle speed** plays an important role as vehicles with **lower speed tend to have a higher yielding rate** while bicycle speed does not really have a significant effect on the vehicle yield decision. Furthermore, the **relative position of the cyclist** is reported to be an important factor as most of the **non-yield events happened when bicycle was in the furthest interaction zone** at least 20 meters from the conflict zone while **vehicle speed being relatively low** at around 18 km/h. In general, the **yielding rate decreased together with higher vehicle speed and cyclist's longer distance from the conflict zone**. However, it was concluded that the importance of **bicycle's relative position** is **rapidly reduced at the distance larger than 20 meters**. [7]

Silvano et al. [7] mentions that the distance from conflict zone should be further investigated as the increasing distance may affect the drivers in an opposite direction, making them add speed instead of yielding.

3.3 Main Findings from Literature

In conclusion, the main factors reported as affecting the vehicles' yielding decision are traffic in opposite direction, number of pedestrians waiting, pedestrian skin colour, the vehicle type, the time headway, vehicle speed, pedestrian's assertive behaviour, vehicle in far lane relative to bicycle, configuration of the crosswalk, minimum speed distance and bicycle distance from the conflict zone.

Based on the literature review the following hypothesis were raised:

- Vehicle yielding rate increases with lower vehicle speed [7].
- Vehicle yielding rate decreases with higher bicycle distance from conflict zone [7].
- Vehicle yielding rate decreases together with higher vehicle speed and cyclist's longer distance from the conflict zone [7].
- The relevance of bicycle relative distance is rapidly reduced from the distance larger than 20 meters [7].

4 Methodology

Unsignalized crosswalks are mainly governed by priority rules or joint obligations. However, there may be some differences between unsignalized pedestrian and bicycle crossings.

According to Swedish regulations there are four different types of unsignalized crossings distinguishable based on the road markings and traffic signs: pedestrian crossing, bicycle passage, bicycle crossing and bicycle path. On pedestrian crossings vehicles are obligated to give way to pedestrians which however is not the case for bicycles. [16]

Bicycle passages are only indicated by discreet road markings and in contrary obligate bicycle to give way to vehicles. On the other hand, bicycle crossings and bicycle paths are indicated by both traffic signs and road markings and obligate the driver to give way respectively to bicycles and pedestrians. [16] Therefore, in case of both agents simultaneously approaching the crosswalk (as illustrated on Figure 3) it is mostly expected that driver yields to the bicycle [9].

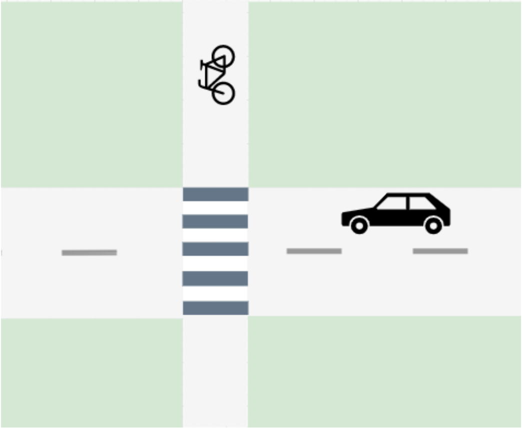

Figure 3. Generalization of vehicle and bicycle approaching the conflict zone.

In addition, according to Swedish Transport Agency [17] the obligations of cyclist and driver are similar on unsupervised bicycle crossings as both are expected to slow down, adjust their speed and communicate in order to keep the crossing as safe as possible.

4.1 Yielding Decision Process

According to Silvano et al. vehicle-bicycle interaction modelling framework [11] an interaction between vehicle and bicycle is provoked by agents perceiving a potential collision. A collision that would occur in case both agents continued their trajectories without adjusting their behaviour such as speed, trajectory etc. Crosswalk is the common area for both vehicle and bicycle and is therefore the region where agents' trajectories intersect resulting in a potential collision. Therefore, the whole crosswalk is defined as a conflict zone. [11] [7]

However, in order to avoid the collision, agents start interacting at some distance from conflict zone. The driver must decide whether to yield or not and therefore, starts negotiating with the cyclist. Depending on various factors such as driver behaviour, agents' speed, attitudes and intersection characteristics the decision point may vary. [11] The point from where the driver starts observing the cyclist can be considered as the beginning of decision process. The area between decision point and conflict zone is defined as interaction zone [7].

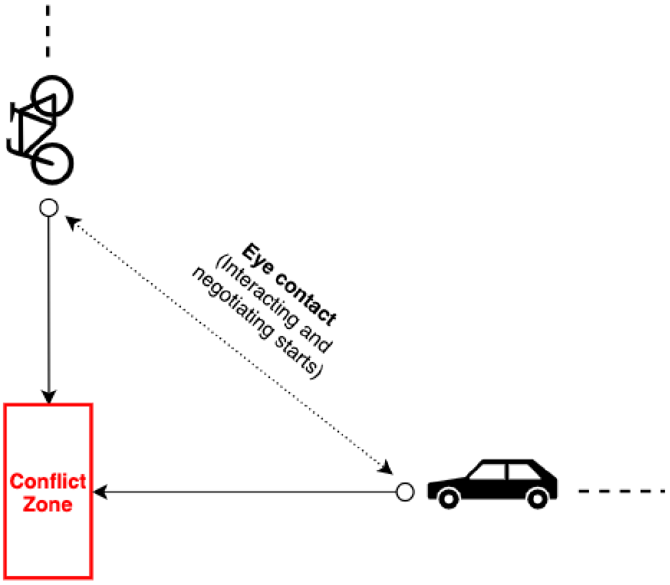

Figure 4. Generalized bicycle vehicle interaction on unsignalized crossing.

Therefore, the yielding decision process during vehicle-bicycle interaction consists of two consecutive events: a) perceiving a conflict b) observing yielding [11].

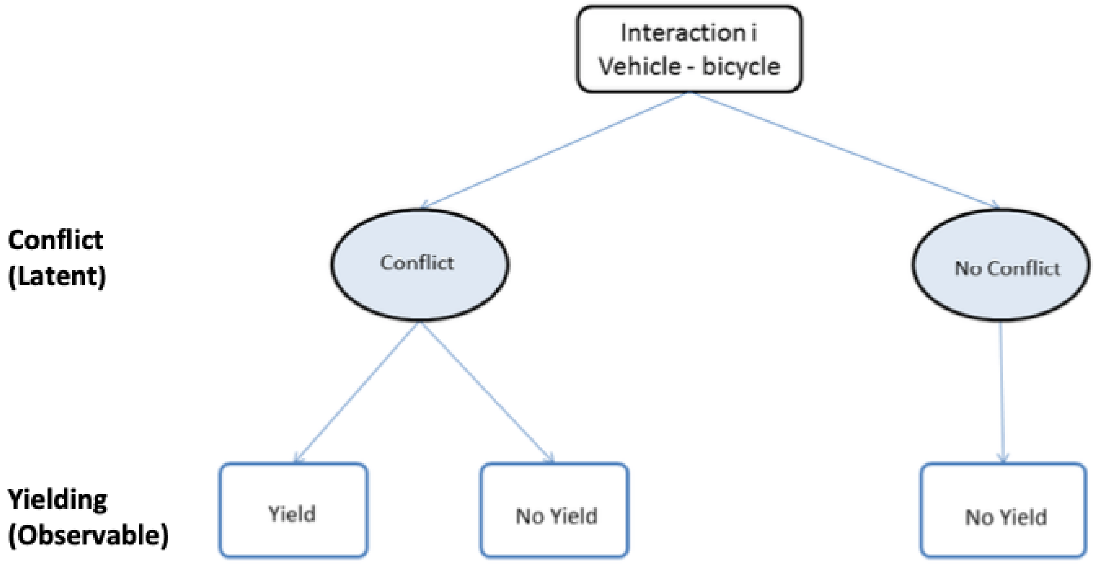

Figure 5. Vehicle yielding decision process described in two consecutive events [11].

Consequently, if driver starts screening a cyclist a conflict situation is perceived. The driver then has to estimate whether it is necessary to yield to the approaching bicycle and whether it is possible for him to come to a stop. Therefore, as visible on Figure 5 the yielding decision process can be described in two levels: latent conflict event and observable yielding event. [11]

Interaction Modelling

In order to model the vehicle-bicycle interaction first the vehicle and bicycle that might have interacted have to be detected. This is done according to the time stamps when agents were monitored. A small threshold of 2 seconds has been chosen to determine whether the bicycle and vehicle might have been on the intersection at a similar time frame and possibly interacted. This threshold allows to also include vehicles and bicycles with a very small chance of interacting, therefore, providing data for understanding the driver behaviour in case it is not affected by the approaching bicycle.

In order to detect driver perceiving a conflict event the observed yielding decision is used. As yielding event is binary (1 = yes, 0 = no), a logit model can be used [11]. A logistic regression model is chosen from the python machine learning library scikit learn[1].

According to Silvano et al. [11] the conflict event can be estimated based on the arrival time difference ($\mathrm{X}^{\mathrm{ATD}}$), which is the time difference between vehicle ($\mathrm{t_v}$) and bicycle ($\mathrm{t_b}$) reaching their respective interaction zone borders.

Furthermore, according to the literature, it can be assumed that the conflict probability is highest in case of both agents entering their respective interaction zones at the same time, thus at arrival time difference 0. Therefore, the arrival time difference is extracted into two separate features:

$$\mathrm{X}_{\mathrm{v}}^{\mathrm{ATD}} = \min(0, \mathrm{t_v} - \mathrm{t_b})$$

$$\mathrm{X}_{\mathrm{b}}^{\mathrm{ATD}} = \max(0, \mathrm{t_v} - \mathrm{t_b})$$

The $\mathrm{X}_{\mathrm{v}}^{\mathrm{ATD}}$ represents the case where vehicle enters the IZ first and $\mathrm{X}_{\mathrm{b}}^{\mathrm{ATD}}$ represents the case where on the contrary bicycle enters the IZ first [11].

By estimating the conflict probability as described above the arrival time difference range used for distinguishing between conflict and non-conflict events is determined.

As discussed in section 4.1, whenever a yielding event happens (yielding is observable) a conflict event must have preceded (conflict is not fully observable). However, in case of no conflict the probability of yielding is zero. Therefore, only conflict events are used for estimating the yielding probability.

The estimation is done based on a subset of features described in section 4.2. Again, a logistic regression model from python machine learning library scikit learn is used [11].

Vehicle-Bicycle Interaction Zones

As mentioned above modelling the two-level vehicle-bicycle interaction on unsignalized crosswalk requires separating the area into two essential zones: interaction and conflict zones [7].

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[1] https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LogisticRegression.html

As crosswalk is the common area for vehicle and bicycle and is therefore the region where vehicle and bicycle trajectories intersect resulting in a potential collision, the whole crosswalk is defined as a conflict zone (illustrated on Figure 6).

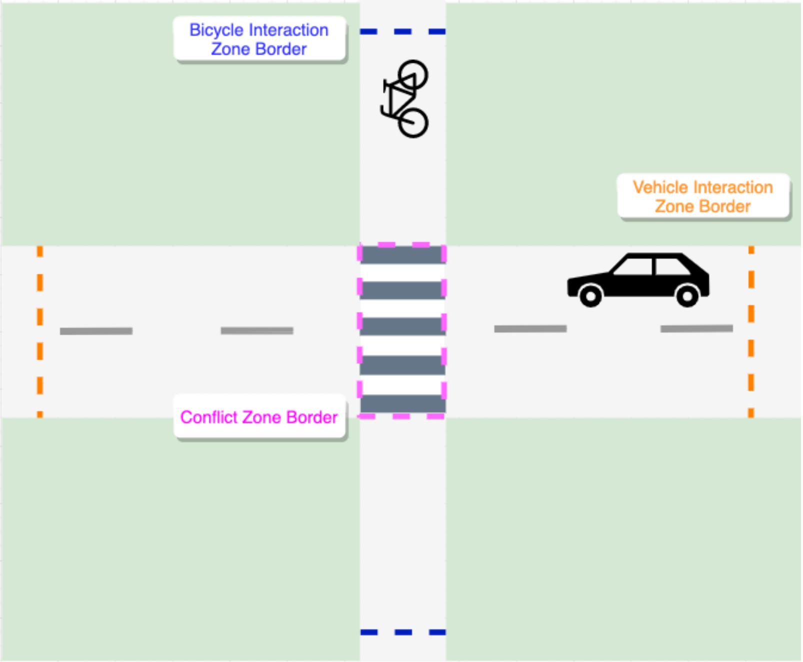

Figure 6. Generalization of conflict and interaction zones (magenta – conflict zone, blue – bicycle interaction zone, orange – vehicle interaction zone).

In order to model the vehicle bicycle interaction, the vehicle decision point must be defined. Vehicle decision point is set at some constant distance from the conflict zone based on vehicle, bicycle characteristics and geometric characteristics of the area, such as visibility. It is the point from where driver starts interacting with the cyclist in order to negotiate and decide whether to yield or not. Therefore, the area between decision point and conflict zone is defined as interaction zone (illustrated on Figure 6). [7] [9]

4.2 Features

As elaborated in literature review scientists have claimed many different features to be relevant for estimating the driver's yielding decision. For instance vehicle speed [7], position of the bicycle [7], arrival time difference [7], crosswalk configuration [15] and distance from which the minimum speed is measured [15] have been stated as significant features.

Most relevant features found from literature, and some proposed by author (denoted with *) have been described in Table 1. The table is followed by the description of different features and the intuition behind their use. All features described in Table 1 will be generated and further investigated based on which the final best performing model is created.

Table 1. Set of investigated features.

Variable	Unit	Description
$\mathrm{X}^{\mathrm{ATD}}$	sec	Time difference between vehicle entering his IZ versus bicycle entering his respective IZ (negative if vehicle enters reaches his IZ border first)
$\mathrm{X}_{\mathrm{v}}^{\mathrm{ATD}}$	sec	Arrival time difference below 0 for cases when vehicle reaches the IZ before bicycle (below or equal to 0)
$\mathrm{X}_{\mathrm{b}}^{\mathrm{ATD}}$	sec	Arrival time difference above 0 for cases when bicycle reaches the IZ before bicycle (below or equal to 0)
V_{v}	km/h	Vehicle mean speed inside IZ
V_{b}	km/h	Bicycle mean speed inside IZ
$\mathrm{V}_{\mathrm{v}}^{\mathrm{IZ}}$	km/h	Vehicle speed at the border of interaction zone
$\mathrm{V}_{\mathrm{b}}^{\mathrm{IZ}}$	km/h	Bicycle speed at the border of interaction zone
$\mathrm{V}_{\mathrm{v}}^{\mathrm{max}*}$	km/h	Maximum vehicle speed measured inside the IZ
$\mathrm{V}_{\mathrm{b}}^{\mathrm{max}*}$	km/h	Maximum bicycle speed measured inside the IZ
$\mathrm{S}_{\mathrm{v}}^{\mathrm{max}}$	meters	Vehicle distance from CZ where the maximum speed inside IZ was measured
$\mathrm{S}_{\mathrm{b}}^{\mathrm{max}}$	meters	Bicycle distance from CZ where the maximum speed inside IZ was measured
$\mathrm{V}_{\mathrm{v}}^{\mathrm{min}}$	km/h	Minimum vehicle speed measured inside the IZ
$\mathrm{V}_{\mathrm{b}}^{\mathrm{min}}$	km/h	Minimum bicycle speed measured inside the IZ
$\mathrm{S}_{\mathrm{v}}^{\mathrm{min}}$	meters	Vehicle distance from CZ where the minimum speed inside IZ was measured
$\mathrm{S}_{\mathrm{b}}^{\mathrm{min}}$	meters	Bicycle distance from CZ where the minimum speed inside IZ was measured
$\mathrm{V}_{\mathrm{b}}^{\mathrm{v10}}$	km/h	Bicycle speed when vehicle is 10 meters from the CZ
X^{b}	number	Number of bicycles simultaneously traversing the CZ
X^{side}	-	1 if both agents entered the IZ from the same side of the road and 0 otherwise
$\mathrm{X}_{\mathrm{v}}^{\mathrm{IZB}}$	-	Vehicle IZ border from where it entered the IZ (IJ/KL/OP)
$\mathrm{X}_{\mathrm{b}}^{\mathrm{IZB}}$	-	Bicycle IZ border from where it entered the IZ (EF/GH/MN)
t_{v}	sec	Vehicle time of entering the IZ

t_b	sec	Bicycle time of entering the IZ
$V_v^{1}*$	km/h	Vehicle speed when agent entering the IZ first entered the IZ (not certain whether the bicycle entered first of the car itself)
$V_b^{1}*$	km/h	Bicycle speed when agent entering the IZ first entered the IZ (not certain whether the bicycle entered first of the car itself)
$S_v^{1}*$	meters	Vehicle distance from CZ when agent who entered the IZ first entered the IZ (not fixed whether the bicycle entered first or the vehicle itself)
$S_b^{1}*$	meters	Bicycle distance from CZ when agent who entered the IZ first entered the IZ (not fixed whether the bicycle entered first or the vehicle itself)
S_v^{CZ}	meters	Vehicle distance from CZ when bicycle enters his IZ
S_b^{CZ}	meters	Bicycle distance from CZ when vehicle enters his IZ
V_v^{bIZ*}	km/h	Vehicle speed when bicycle enters his IZ
V_b^{vIZ*}	km/h	Bicycle speed when vehicle enters his IZ
T_v^{ATexp}	sec	Vehicle expected arrival time at the CZ based on the IZ length and speed when entering IZ
T_b^{ATexp}	sec	Bicycle expected arrival time at the CZ based on the IZ length and speed when entering IZ
T_v^{exp*}	sec	Vehicle expected time needed for traversing the IZ and reaching the CZ
T_b^{exp*}	sec	Bicycle expected time needed for traversing the IZ and reaching the CZ
$T^{ATDexp*}$	sec	Expected arrival time difference between vehicle reaching the CZ versus bicycle reaching the CZ (negative if vehicle reaches the CZ before bicycle)
V^{diff*}	km/h	Speed difference between vehicle speed when vehicle entering his IZ versus bicycle speed when bicycle entering his IZ
V_v^{diff}	km/h	Speed difference between vehicle and bicycle speeds at the moment when vehicle enters IZ
V_b^{diff}	km/h	Speed difference between vehicle and bicycle speeds at the moment when bicycle enters IZ

* indicates features proposed by author

Arrival Time Difference (ATD)

Arrival time difference is the time difference of vehicle and bicycle reaching the interaction zone [7]. Therefore, if time of vehicle entering the interaction zone is denoted by $\mathrm{t_v}$ and time of bicycle entering the interaction zone is denoted by $\mathrm{t_b}$ then their arrival time difference $\mathrm{X^{ATD}}$ is given by:

$$\mathrm{X^{ATD}} = \mathrm{t_v} - \mathrm{t_b}$$

The equation above results in negative arrival time difference in case vehicle enters the interaction zone first and positive in case bicycle enters first.

The calculation of $\mathrm{X_v^{ATD}}$ representing the interactions where vehicle enters the IZ first and $\mathrm{X_b^{ATD}}$ representing the interactions where on the contrary bicycle enters the IZ first are defined above in the section 4.1.

Mean Speed Inside IZ

As mentioned in multiple articles vehicle speed is an important factor affecting the driver's decision whether to yield or not. It is generally proved that higher speed will result in non-yield decision while lower speed will affect driver to slow down even more and let the cyclist cross the road.

Mean speed inside the interaction zone is the average of speeds measured while agent is inside the interaction zone.

Speed at the Border of IZ

Vehicle speed at the border of IZ is one of the features used in the best performing model created by Silvano et al. [7]. Therefore, it will also be investigated. However, the same feature is also generated for bicycle.

Speed at the border of interaction zone is the speed measured when agent enters his respective interaction zone.

Max Speed Inside IZ

Maximum speed inside the IZ is the maximum speed agent reached when being inside the interaction zone.

Max Speed Distance

As minimum speed distance has been mentioned in literature to be a relevant feature the same has been assumed for the maximum speed distance. The maximum speed distance is the distance from conflict zone where the maximum speed of the agent was measured.

Min Speed Inside IZ

Minimum speed inside IZ is the minimum speed agent reached when being inside the interaction zone.

Min Speed Distance

Minimum speed distance is the distance from crosswalk where the minimum speed experienced during the driver breaking manoeuvre, was recorded [15]. The breaking manoeuvre can be defined as speed reduction happening inside the interaction zone as this is the area where driver and cyclist decide whether to yield or not and adjust their behaviour accordingly.

According to Bella et al. [15] noticing the crosswalk from a greater distance resulted in greater minimum speed distance and thus safer crossing for cyclist.

Therefore, it is the distance from conflict zone where the minimum speed was measured.

Bicycle Speed While Vehicle 10 Meters from CZ

This feature was used in one of the models mentioned by Silvano et al. [7] and is similarly generated only for the bicycle. It is the bicycle speed measured while vehicle is 10 meters from the conflict zone.

Number of Bicycles

As stated in chapter 3, the number of pedestrians waiting to cross has reported to be an important aspect affecting drivers yielding decision. Therefore, although pedestrian-vehicle dynamics is slightly different from the bicycle-vehicle dynamics, it might still be relevant for both. Thus, the effect of number of bicycles simultaneously crossing the road on drivers' yielding decision is investigated.

It is the number of bicycles simultaneously inside the interaction zone.

The Entrance IZ

As mentioned above, vehicles in the far lane relative to the pedestrian are said to yield less likely. Although this has been stated about the pedestrians it might also be relevant for vehicle-bicycle interactions. Therefore, looking at the interactions were bicycle and vehicle were approaching from opposite sides of the road will help to prove or disprove the statement.

Therefore, for both vehicle and bicycle the interaction zone border which they crossed first is set as the entrance IZ.

Time of entering IZ

Time of entering the interaction zone is the timestamp in seconds at the moment when agent reaches his respective interaction zone.

Speed When First Agent Entered the IZ

This feature has been proposed by the author and is an experimental one. It is the speed at the moment when the agent closer to the interaction zone reaches the interaction zone. Therefore, it can also be the speed when agent himself reaches the interaction zone.

Distance from CZ When First Agent Entered the IZ

Similarly, to the previous, this feature has been proposed by the author and is an experimental one. It is the distance from conflict zone at the moment when the agent closer to the interaction zone reaches the interaction zone. Therefore, it can also be the distance from conflict zone when agent himself reaches the interaction zone and should in this case be equal to the interaction zone length.

Distance from CZ When Other Agent Entered the IZ

As mentioned in chapter 3, position of the bicycle has been stated as a relevant feature by Silvano et al. [7] defining the position of bicycle as distance from the conflict zone. Furthermore, it has been assumed in the literature, that larger bicycle distance from conflict zone may affect the drivers in an opposite direction making them add speed instead of yielding.

The same feature has also been generated for the vehicle – vehicle distance from conflict zone when bicycle enters his interaction zone.

Speed When Other Agent Entered the IZ

This feature has been proposed by the author and is similar to the distance from conflict zone when other agent entered the interaction zone.

This feature is the instantaneous speed of an agent when other agent enters his interaction zone. Therefore, for instance it is vehicle speed at the moment when bicycle enters his interaction zone.

Expected Time Needed for Traversing IZ

Expected time needed for traversing the interaction zone has been proposed by the author which however is inspired by the expected arrival time difference at the conflict zone.

The expected time needed for traversing the interaction zone is a duration in seconds calculated based on the interaction zone length and the instantaneous speed at the border of interaction zone.

If vehicle interaction zone length is denoted by $\mathrm{l_v}$, bicycle interaction zone length is denoted by $\mathrm{l_b}$ and instantaneous speed at the border of interaction zone ($\mathrm{V_v^{IZ}},\ \mathrm{V_b^{IZ}}$) is converted into meters per second, then expected time needed for traversing interaction zone could be calculated as follows:

$$\mathrm{T_v^{exp}} = \mathrm{l_v}/\mathrm{V_v^{IZ}}$$

$$\mathrm{T_b^{exp}} = \mathrm{l_b}/\mathrm{V_b^{IZ}}$$

Expected Arrival Time at CZ

Expected arrival time at the conflict zone has been very briefly mentioned by Silvano et al. [9] and is the estimated arrival time timestamp in seconds calculated based on the interaction zone length and the instantaneous speed at the border of interaction zone. The feature has been generated for both vehicles and bicycles.

If time of vehicle entering the interaction zone is denoted by $\mathrm{t_v}$ and time of bicycle entering the interaction zone is denoted by $\mathrm{t_b}$ then expected arrival time at CZ could be calculated as follows:

$$\mathrm{T_v^{ATexp}} = \mathrm{t_v} + \mathrm{T_v^{exp}}$$

$$\mathrm{T_b^{ATexp}} = \mathrm{t_b} + \mathrm{T_b^{exp}}$$

Expected ATD at CZ

Expected arrival time difference has been proposed by the author and is inspired from the features arrival time difference and expected arrival time at the conflict zone.

The expected arrival time difference at CZ is the time difference between bicycle's expected arrival time at the conflict zone and vehicle's expected arrival time at the conflict zone.

$$\mathrm{T^{ATDexp}} = \mathrm{T_v^{ATexp}} - \mathrm{T_b^{ATexp}}$$

The expected arrival time difference at the conflict zone is negative in case vehicle is expected to reach the conflict zone first and positive in case bicycle is expected to reach the conflict zone first.

Relative Speed When Entering IZ

Relative speed when entering interaction zone is an experimental feature proposed by the author. It is the speed difference between vehicle instantaneous speed when vehicle enters his interaction zone relative to bicycle instantaneous speed when bicycle enters his interaction zone. It can be calculated as follows:

$$\mathrm{V^{diff}} = \mathrm{V_v^{IZ}} - \mathrm{V_b^{IZ}}$$

Relative Speed When Other Entering IZ

Relative speed when other agent enters the interaction zone is the speed difference between vehicle and bicycle speeds at the moment when one of them enters the interaction zone. The feature can be calculated as follows for: a) relative speed when vehicle enters interaction zone b) relative speed when bicycle enters interaction zone.

$$\mathrm{V_b^{diff}} = \mathrm{V}_{v}^{\mathrm{IZ}} - \mathrm{V_v^{bIZ}}$$

$$\mathrm{V_v^{diff}} = \mathrm{V_v^{bIZ}} - \mathrm{V_b^{IZ}}$$

4.3 Feature Selection

In order to create performant and accurate models only relevant features should be used. Therefore, the relationship between input (feature value) and target variable is statistically estimated.

Since all features are numeric, a correlation statistic f-regression[2] together with the select-k-best[3] function from the python machine learning library scikit learn is used.

The null hypothesis of F value is that all of the regression coefficients are equal to zero. The F value test compares whether adding the coefficient to an intercept only model (model that assumes that means for all samples are the same) improves the model or not. If the results obtained are significant then it can be concluded that added coefficient improve the model's fit. [18]

Therefore, only features whose input has a strongest relation with target variable are selected for further research.

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[2] https://scikit-learn.org/stable/modules/generated/sklearn.feature_selection.f_regression.html

[3] https://scikit-learn.org/stable/modules/generated/sklearn.feature_selection.SelectKBest.html

5 Experiments

In order to understand what affects human drivers' yielding to bicycle on unsignalized crosswalk multiple such cases were investigated. In order to find the most relevant aspects affecting the yielding decision multiple features elaborated in chapter 4 were generated and their influence on driver's behaviour was then estimated.

All code created and used for the experiments can be found from the public GitHub repository[4].

5.1 Data

In order to investigate the driver yielding decision to bicycle both agents' trajectories, speeds and yielding decision are necessary. Therefore, a dataset meeting those requirements was chosen.

Dataset

The data used in the research was collected from a crosswalk on a two-lane road approaching a roundabout in a small city named Linköping in Sweden visible on the Figure 7. The data was collected in September 2017 during the whole day.

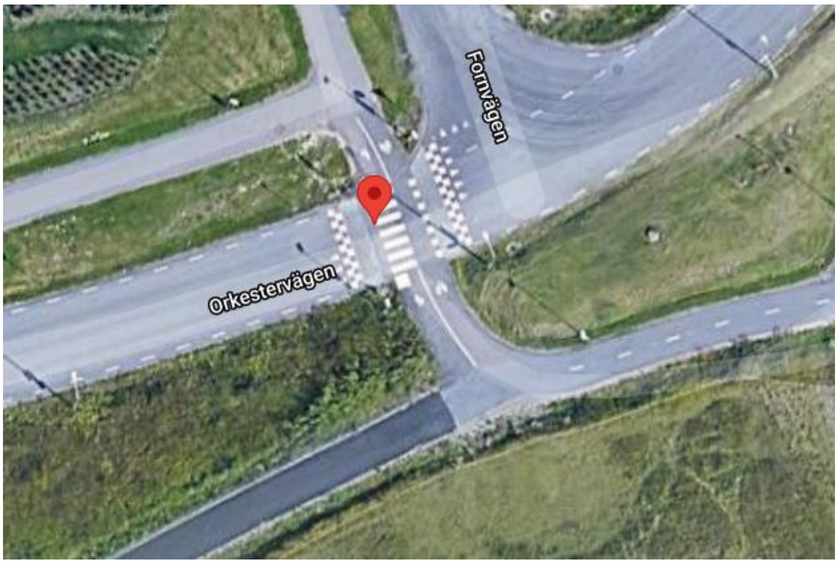

Figure 7. Real life map of the location from where the data was collected (coordinates: 58°22'00.8"N 15°41'24.3"E).

The data consists of 6640 vehicle and bicycle trajectories out of which 5116 are cars and 1524 bicycles. The data includes agent id, agent type (vehicle/bicycle), arrays of x-coordinates, y-coordinates, speed and timestamps of the exact moment when previously mentioned array datapoints were collected.

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[4] https://github.com/lauraliism/traffic-and-crosswalk-agents-interaction

Therefore, agents' location and speed at every timestamp is known based on which their movement can be reconstructed. Vehicle and bicycle trajectories plotted from data are visible on Figure 8.

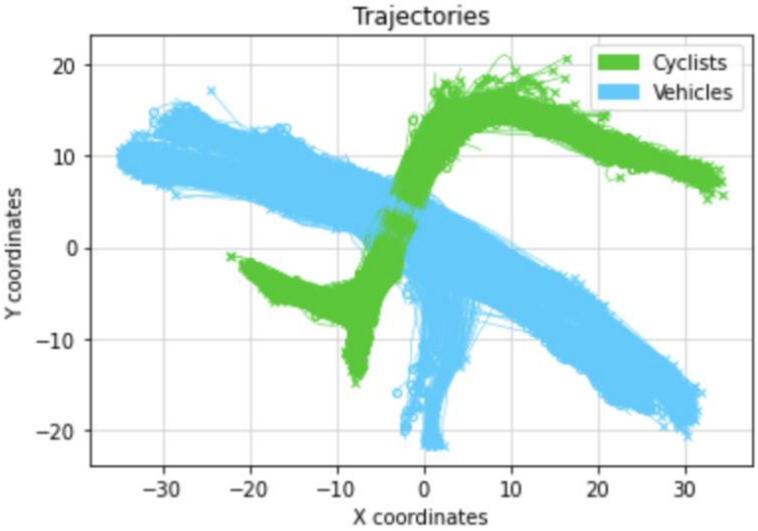


Figure 8. Vehicle and bicycle trajectories of all 6640 instances (units in meters).

In order to understand the quality of data the instances were briefly investigated. Naturally, the more frequent timestamps trajectory has the smoother and more accurate it is. However, the number of timestamps and also the time between timestamps varies a lot between instances. For example, all instances have at least 7 timestamps and 4382 instances out of 6640 have more than 20 timestamps. For 3990 instances the time differences between measurements are less than 1 second and for 3028 instances out of them the time differences are even less than 0.5 seconds.

Similarly, the distances between measured datapoints vary significantly and seem to include some exceptions. For 2468 instances the distances between each datapoint is less or equal to 1 meter, while for 637 instances the distance difference between at least two datapoints is more than 15 meters resulting in an intermittent trajectory.

Grouping Instances

In order to investigate vehicle bicycle interaction dynamics, it was firstly necessary to combine the vehicles with bicycles who might have interacted when approaching the crosswalk. Therefore, as mentioned in section 4.1 vehicle and all bicycles which were observed during the same time ±2 seconds were stated as possible interaction. The small added time of 2 seconds was chosen in order to generate interactions with more variety by possibly also including interactions where agents might not have influenced each other.

Out of 6640 instances 862 possible interactions were defined each of them consisting of one vehicle and one to many bicycles. As grouping was done based on vehicles it is possible that same bicycles are included in multiple possible interactions.

All trajectories included in possible interactions are visible on Figure 9.

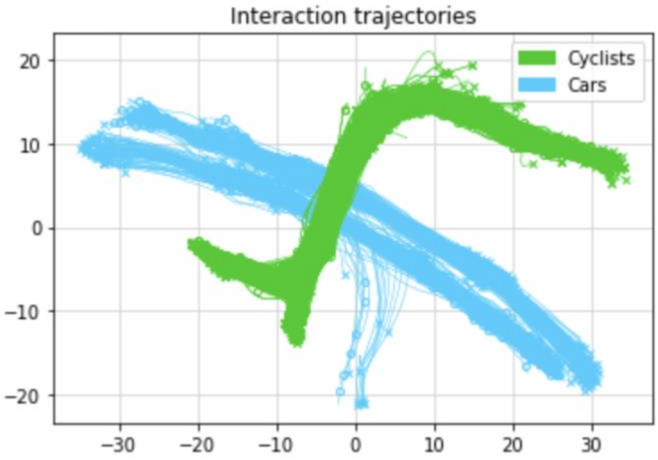


Figure 9. Vehicle and bicycle trajectories from all 862 possible interactions (units in meters).

Furthermore, trajectories of all possible interactions were animated and saved as videos based on which the data was later labelled as yield or non-yield (1 = yield, 0 = non-yield).

In order to be able to use the data for generating features, some possible interactions including too short trajectories had to be discarded.

As a result, 250 interactions out of 862 were left.

5.2 Interactions Modelling

Conflict Zone

As elaborated in chapter 4, crosswalk is where vehicle and bicycle trajectories intersect resulting in a potential collision. Thus, the whole crosswalk can be defined as a conflict zone. Therefore, the vehicle and bicycle trajectories intersection area visible on Figure 10 (which according to the Figure 7 is the crosswalk), was defined as a conflict zone.

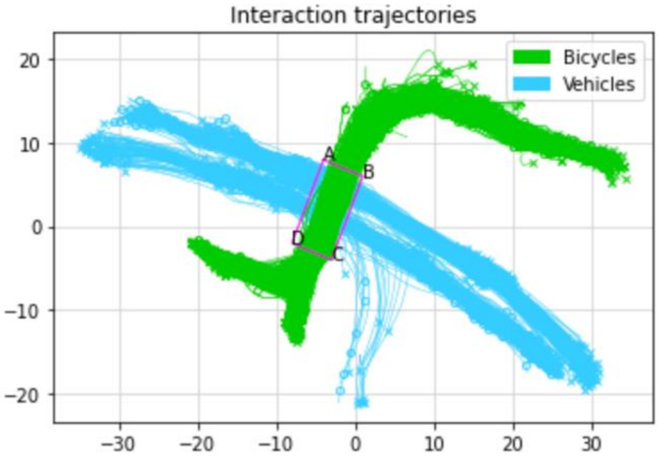


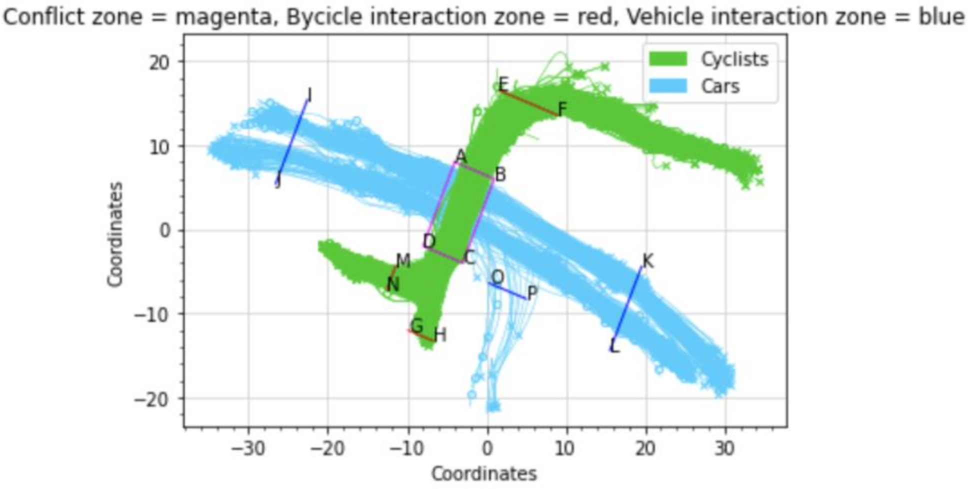
Figure 10. Conflict zone defined according to interaction trajectories (units in meters).

The conflict zone was defined manually according to the vehicle and bicycle trajectories, and it is 10,8 meters long and 5,4 meters wide and has been denoted with letters A, B, C, D.

Interaction Zone

As described in chapter 4 interaction zone is where agents negotiate on who will yield in order to avoid the potential collision. The interaction zone is defined based on vehicle, bicycle characteristics and geometric characteristics of the area, such as visibility.

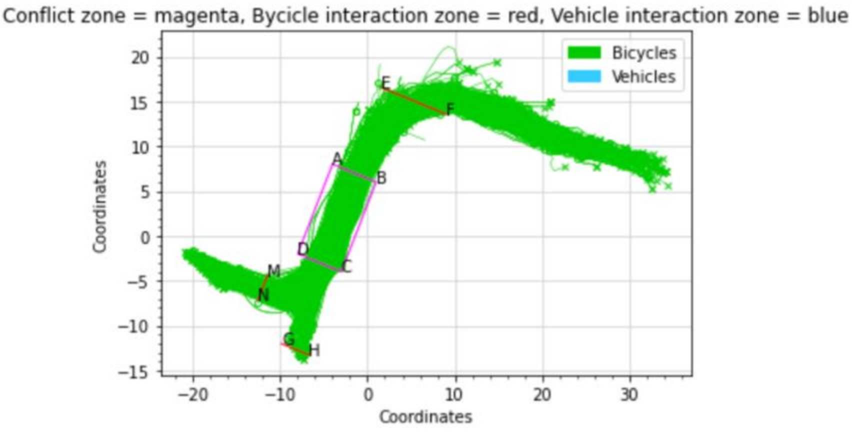
As it is not directly measured where did the driver’s decision process start, the interaction zones have been defined based on observations done when looking at the generated videos of interactions and based on the geometric attributes of the location visible from Google Maps (conflict zone and both vehicle and bicycle interaction zones are visible on Figure 11).



Conflict zone = magenta, Bycicle interaction zone = red, Vehicle interaction zone = blue

Figure 11. Vehicle and bicycle interaction zones together with conflict zone (units in meters).

As visible from the Figure 12 the bicycle interaction zone is marked with 3 separate lines EF, GM and MN based on the characteristics of bicycle trajectories.



Conflict zone = magenta, Bycicle interaction zone = red, Vehicle interaction zone = blue

Figure 12. Bicycle interaction zone (units in meters).

All bicycke interaction zone borders are located 10 meters from the conflict zone marking the location from where vehicle and bicycle assumably start interacting. The 10 meters distance has been defined as this is where the road turns and cyclist has a straight way to the conflict zone making it possible to notice vehicles approaching from the same direction and also enabling eye contact with the driver.

As visible from Figure 13 the vehicle interaction zone is marked with 3 separate borders denoted with IJ, KL and OP based on the characteristics of vehicle trajectories.

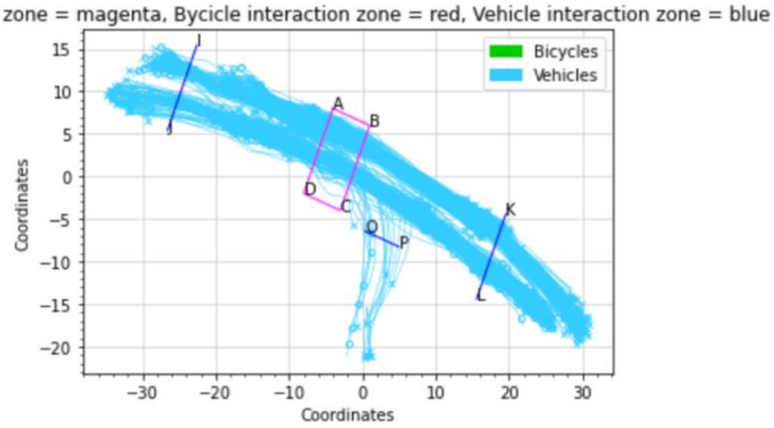


Figure 13. Vehicle interaction zone (units in meters).

The interaction zone borders IJ and KL have been defined at 20 meters from the conflict zone. This is because the road is straight, enabling good visibility to detect the bicycle from a long distance and act accordingly. This is also noticeable from the generated videos.

However, due to the turn the visibility might not be as good for the vehicles coming from the direction of interaction zone OP and might not be able to notice bicycles approaching from behind them (from interaction zone GH and MN). Therefore, the third vehicle interaction zone border OP has been defined at 10 meters from the conflict zone.

Conflict event

As elaborated in section 4.1 the arrival time at the boundary of interaction zone is the key factor affecting the probability of conflict. It is assumed that the smaller the arrival time difference is the more probable that driver perceives a conflict.

Therefore, in order to estimate the threshold time difference based on which the conflict and non-conflict events can be distinguished the yielding event was estimated with a simple logistic regression model using the arrival time difference X_v^{ATD} and X_b^{ATD} described in Table 2.

Table 2. Description of variables used in the conflict probability estimation model.

Features	Unit	Description
X_v^{ATD}	sec	Arrival time difference at IZ border which is above 0 if vehicle enters the IZ first
X_b^{ATD}	sec	Arrival time difference at IZ border which is above 0 if bicycle enters the IZ first

As visible from the Figure 14 the conflict is most probable in case both vehicle and bicycle reach the IZ border at the same time (arrival time difference 0) which was assumed based on the literature.

In addition, the model results indicate that the conflict probability is above 50% in case vehicle reaches the IZ about 2.5 seconds before or up to 5 seconds after bicycle reaches his respective IZ.

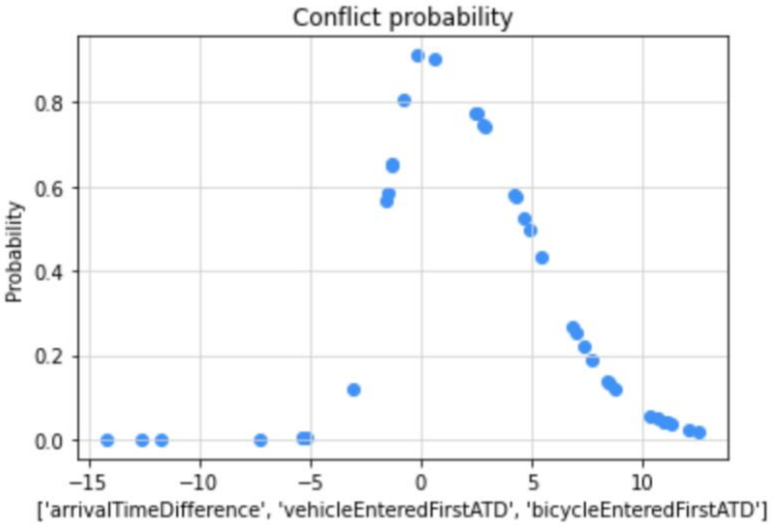


Figure 14. Conflict probability based on arrival time difference (X_v^{ATD}, X_b^{ATD}) (negative ATD indicates that vehicle entered the IZ first, and negative indicates the opposite) (units in seconds).

Therefore, based on the conflict probability estimation using a simple logistic regression model it was determined that a threshold of -2.5 and +5 seconds is sufficient for distinguishing conflict and non-conflict events. Meaning that all interactions where vehicle reached his IZ border 2.5 seconds before or up to 5 seconds after the bicycle reached his IZ border are considered as interactions where driver perceived a conflict.

Table 3. Conflict and non-conflict events out of interactions.

	Event samples
Interactions	150
Non-Conflict	98
Conflict	52

As visible from Table 3 out of 150 interactions, 52 were determined as conflict events based on the -2.5 and +5 second arrival time difference between agents.

Yielding event

As yielding event is observable it is possible to detect it by looking at the video of interaction. Therefore, videos of all conflict events were generated based on which it was possible to label each interaction as yield or non-yield (1 = yield, 0 = non-yield). All interactions where vehicle slowed down and gave way to the bicycle were labelled as yield and all interactions were vehicle passed through the interaction and conflict zones without adjusting its behaviour because of the bicycle were labelled as non-yield.

Table 4. Yield and non-yield events out of conflict events.

	Event samples
Conflict	52
Yield	38
Non-Yield	14

As visible from Table 4 for 38 conflict events out of 52 the vehicle yielded to the bicycle while in 14 interactions vehicle passed conflict zone without giving way.

5.3 Features

In order to investigate how different factors, affect the drivers yielding decision all features elaborated in section 4.2 were implemented. Implementation details are elaborated below. As stated in detailed feature descriptions most of the features were generated for both vehicle and bicycls.

As already mentioned in section 5.1, the initial data included agent type, id and multiple datapoints per each instance. Per each datapoint the x-coordinate, y-coordinate, timestamp and speed is known.

The Entrance IZ

Since for both vehicle and bicycle there are only 2 CZ borders (for vehicle: AD, BC and for bicycle: AB, CD) compared to 3 possible IZ borders, the CZ border from where the agent enters the CZ was identified first. According to the datapoints the movement of agent was reconstructed step by step based on which it was possible to determine which CZ border did the agent intersect first. Based on the CZ border it was possible to eliminate the IZ on the opposite side of the road. Therefore, again increasing the trajectory step by step the intersection with IZ border was found. The corresponding IZ border was set as the initial IZ.

Time of Entering IZ

Index of Entering and Exiting IZ

In order to avoid any confusion, it should be stated that entering interaction zone is defined by crossing the interaction zone border, while exiting interaction zone is marked by exiting the conflict zone.

First the IZ from where the agent entered was determined. After determining the correct IZ border the agent's distance from it at every timestep was calculated. The index at which agent was closest to the IZ border was set to be the index of entering the IZ.

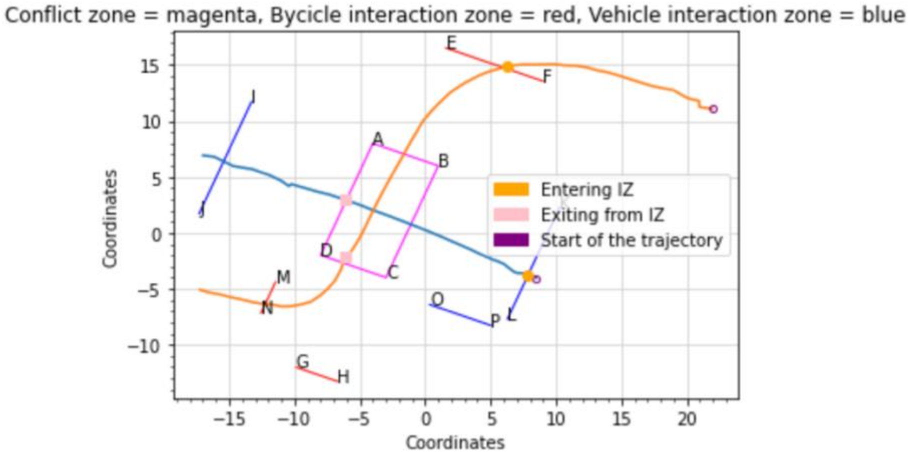


Figure 15. An interaction with orange symbol indicating the timestamp considered as entering the interaction zone and pink symbol indicating the exiting (units in meters).

In order to find the index of exiting conflict zone the entrance IZ border was taken into account. Based on the entrance border it was already clear from which direction will the agent exit. However, in order to determine the index similarly to entering, the distance from exiting CZ border was calculated per each timestep and again the index of the smallest distance was set as the index of exiting CZ.

Figure 15 indicates the timestamps considered as entering and exiting the interaction zone based on which the exact timestamp was found.

Time of Entering IZ

Time of entering interaction zone was determined based on the index of entering, described above. As data includes timestamps per each trajectory datapoint the time on the index of entering IZ was set as the time of entering IZ. Time of entering IZ was generated for both vehicles and bicycles.

Arrival Time Difference

After determining the timestamp of both vehicle and bicycle entering the interaction zone the time difference between vehicle and the bicycle entering the interaction zone was calculated according to the equation mentioned in section 4.2. Thus, resulting in negative arrival time difference when vehicle reached the IZ first and positive in case bicycle reached the IZ first.

Mean Speed Inside IZ

Mean speed inside the interaction zone was generated based on the datapoints between entering and exiting the IZ. The mean was calculated based on the speed measured in each timestamp inside the IZ for both vehicles and bicycles.

Speed at the Border of IZ

The instantaneous speed at the border of IZ was generated based on the index of entering IZ. As speed in every datapoint is known, it could easily be extracted based on the index.

Max Speed Inside IZ

Maximum speed inside the interaction zone was determined from the datapoints between entering and exiting the interaction zone. The datapoint with highest speed was set as maximum speed inside IZ.

Max Speed Distance

Maximum speed distance is the distance from conflict zone from which the maximum speed inside interaction zone was measured. Thus, firstly the datapoint index of maximum speed was determined. As a result, the distance from conflict zone at that index was set as maximum speed distance.

Min Speed Inside IZ

Minimum speed inside the interaction zone was determined from the datapoints between entering and exiting the interaction zone. The datapoint with smallest speed was set as minimum speed inside IZ.

Minimum Speed Distance

Minimum speed distance is the distance from conflict zone from which the minimum speed inside interaction zone was measured. First the datapoint index of minimum speed was determined. As a result, the distance from conflict zone at that index was set as minimum speed distance.

Bicycle Speed While Vehicle 10 Meters from CZ

Distance from CZ

Distances from conflict zone was calculated per each trajectory datapoint. As a first step relative distances between each datapoint were calculated based on the x and y-coordinates. Secondly, the distance from conflict zone was set to be 0 for all datapoints of agent being inside the conflict zone. In order to obtain the distance from CZ for the next datapoints the distances between datapoints (found as a first step) were sequentially added up increasing as moving further from the CZ.

Bicycle Speed While Vehicle 10 Meters from CZ

After calculating the distance from CZ per each datapoint the index of distance closest to 10 meters was determined based on which it was possible to determine the timestamp of that datapoint.

The bicycle datapoint with timestamp closest to the one found previously (when vehicle is approximately 10 meters away) was identified. Based on its index the bicycle speed at that moment was identified.

Number of Bicycles

The number of bicycles simultaneously crossing was determined manually by looking at the generated videos of interactions. Only bicycles who were simultaneously in the CZ were counted, however they might have come from different interaction zones and from different sides of the road.

Speed When First Agent Entered the IZ

As arrival time difference is negative in case vehicle enters the IZ first, it was used for determining the agent who entered IZ first. Secondly, as the time of agent entering the IZ had already been found previously the other agents' datapoint with timestamp closest to it was determined. Based on the found datapoint's index the speed at the same index was determined and set as the speed when first agent enters the IZ.

Distance from CZ When First Agent Entered the IZ

The distance from CZ was found similarly to the speed when first agent enters the IZ described above. The only difference is that instead of speed the distance from CZ on found index was determined.

Distance from CZ When Other Agent Entered the IZ

Distance from CZ when other agent enters the IZ was found similarly to the distance from CZ when first agent entered the IZ, however the major difference was that the order of entering the IZ was not important. For bicycles it is always their distance from CZ when vehicle enters the IZ and for vehicles it is always their distance when bicycle enters the IZ.

Speed When Other Agent Entered the IZ

Speed when other agent enters the IZ was found similarly to the speed when first agent entered the IZ, however the major difference was that the order of entering the IZ was not important. For bicycles it was always their speed when vehicle enters the IZ and for vehicles it was always their speed when bicycle enters the IZ.

Expected Time Needed for Traversing IZ

Expected arrival time at CZ was calculated based on the IZ length (for vehicles 20 meters, for bicycles 10 meters) and agent's speed at the border of IZ (previously already determined) according to the exact formula mentioned in section 4.2.

Expected Arrival Time at CZ

The expected arrival time at CZ was calculated based on previously determined time of entering IZ and expected time needed for traversing IZ according to the exact formula mentioned in section 4.2.

Expected ATD at CZ

The expected arrival time difference at CZ was calculated based on the previously found expected arrival time of both agents according to the exact formula mentioned in section 4.2.

Vehicle expected to reach the interaction zone first will result in negative ATD while bicycle expected to reach the interaction zone first will result in positive ATD.

Relative Speed When Entering IZ

The relative speed when entering IZ was calculate based on the vehicle instantaneous speed when vehicle enters his interaction zone and bicycle instantaneous speed when bicycle enters his interaction zone according to the exact formula mentioned in section 4.2.

Relative Speed When Other Entering IZ

For calculating relative speed when one agent is entering IZ the previously found instantaneous speed when agent enters the interaction zone and speed when other agent enters the IZ were used.

Both relative speed when vehicle enters the IZ and relative speed when bicycle enters the IZ were calculated according to the exact formula mentioned in section 4.2.

Feature Selection

As described in section 4.3 most relevant features are selected using a feature selection algorithm f regression determining the most relevant features improving the model fit.

Only features with the highest scores were selected to be used in the yielding probability models.

Based on the importance scores visible on Figure 16 the vehicle mean speed ($\mathrm{V_v}$), minimum speed distance ($\mathrm{S_v^{min}}$), minimum speed inside interaction zone ($\mathrm{V_v^{min}}$) and bicycle distance from CZ when vehicle enters the IZ ($\mathrm{S_b^{CZ}}$) were chosen for estimating driver’s yielding decision.

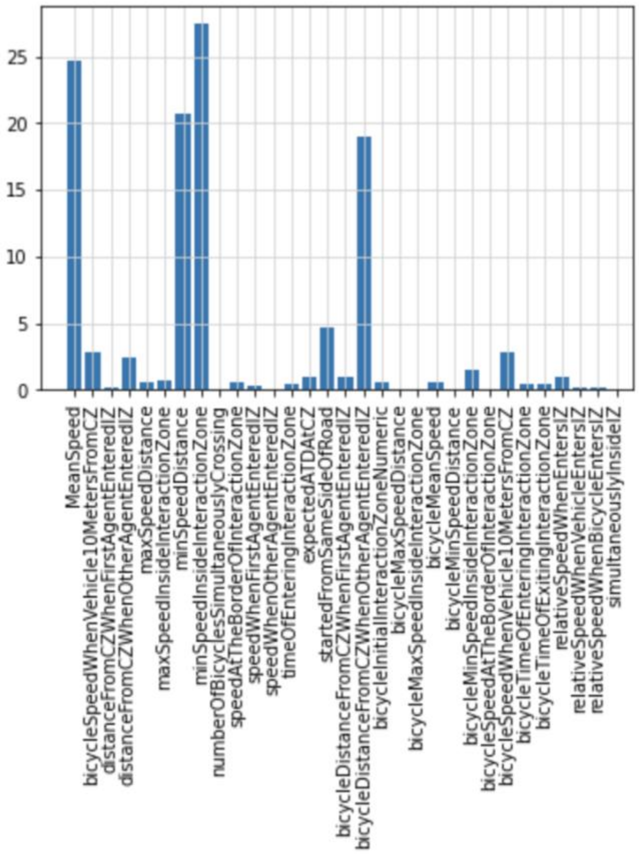


Figure 16. Relevant features based on the correlation scores.

After choosing the relevant features a p-value (visible in Table 5) was also calculated to understand their significance and relation to target variable.

Table 5. Features with highest correlation scores.

Variable	Correlations Score	p-value
Vehicle mean speed ($\mathrm{V_v}$)	24.749242	**0.0298**
Minimum speed distance ($\mathrm{S_v^{min}}$)	20.732487	**0.0032**
Minimum speed inside interaction zone (V_v^{min})	27.454329	0.1946
Bicycle distance from CZ when vehicle enters the IZ ($\mathrm{S_b^{CZ}}$)	18.988224	0.2607

P-values of both, vehicle mean speed ($\mathrm{V_v}$) and minimum speed distance ($\mathrm{S_v^{min}}$) indicate that the null-hypothesis of features having the same means does not hold (with 5% level of significance). Therefore, these should be the most significant feature from the selected ones.

6 Result

Based on the experiments described above the following results were obtained.

Firstly, the analysis of separate event groups (conflict, non-conflict, yield, non-yield) is conducted after which the experimental results will be presented. Lastly a discussion and improvement points will be elaborated.

6.1 Data Analysis

Looking at the data based on most relevant features gives insight about the drivers and cyclists behaviour.

For instance, as visible from Table 6 the vehicle mean speed inside the IZ (V_v) for all yield events was measured to be 14.8 km/h (with a standard deviation of 5.8), being the lowest mean speed of all the vehicle event groups. For non-yield events the vehicle mean speed inside IZ was the highest being 27.8 km/h. Therefore, the difference between yielding vehicles and non-yielding vehicles mean speed was 13 km/h.

Additionally, 80% of all yielding vehicles had a mean speed less than 20 km/h while 80% of all non-yielding vehicles had a mean speed less than 30 km/h.

On the other hand, the bicycles speed (V_b) difference for yield and non-yield groups was only 0.3 km/h. In addition, the mean speed varied much less between all the bicycle event groups (less than 1 km/h) than it did for all the vehicle event groups.

Table 6. Vehicle and bicycle mean speed inside IZ per every event group.

	Vehicle Speed Inside IZ (km/h)		**Bicycle Speed Inside IZ (km/h)**		
Event Group	**Mean**	**Standard deviation**	**Mean**	**Standard deviation**	**Event sample**
All data	21.6	8	12.5	3.8	150
Non-Conflict	23.3	7.1	12.2	3.6	98
Conflict	18.3	8.8	13.0	4.1	52
Yield	14.8	5.8	13.1	3.9	38
Non-Yield	27.8	8.7	12.8	4.8	14

Moreover, when investigating the yield and non-yield events relative to bicycle and vehicle mean speed visible on Figure 17 it appears that most of the non-yield events occur with vehicle speed above 20 km/h while bicycle speed varies between 5 to 25 km/h for both yield and non-yield events.

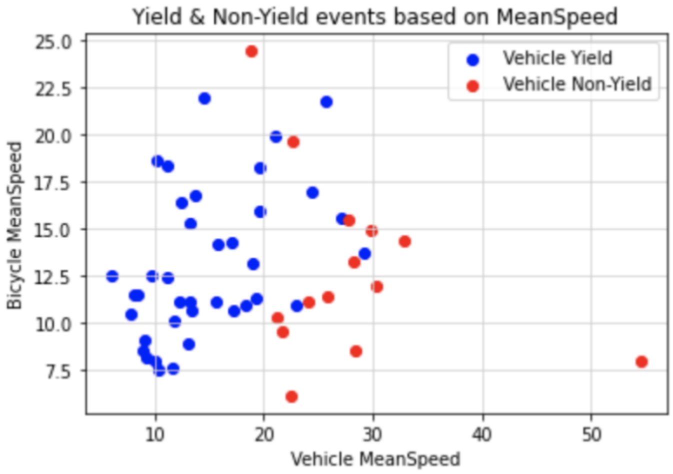


Figure 17. Vehicle yield and non-yield events based on vehicle (V_v) and bicycle (V_b) mean speed inside IZ (units in km/h).

Additionally, when looking at the maximum (S_b^{max}) and minimum speed distance (S_b^{min}) it is evident that most of the bicycles reached their maximum speed inside the conflict zone independent of whether the vehicle yielded or not.

However, when looking at the yield and non-yield events (Figure 18) based on the bicycle and vehicle minimum speed distance, it appears that yielding vehicle reaches his minimum speed further away from the crosswalk at around 5 or more meters away.

On the contrary, vehicles which do not give way tend to reach the minimum speed while inside the conflict zone or very close to it.

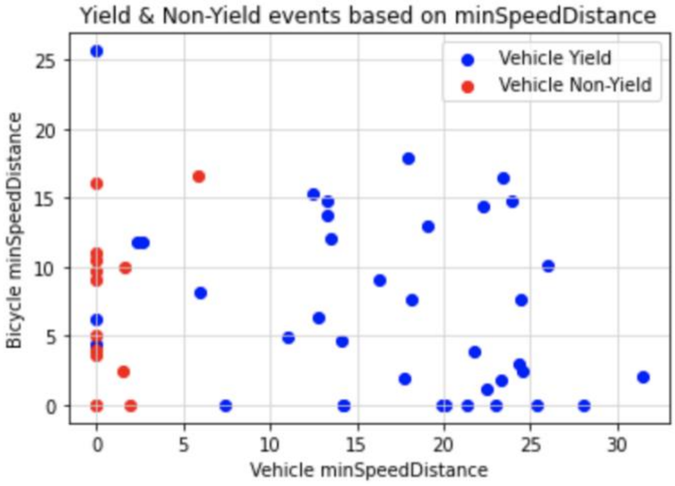


Figure 18. Vehicle yield and non-yield events based on vehicle and bicycle minimum speed distance (units in meters).

Moreover, the minimum speed inside interaction zone (V_v^{min}) is on average 7.5 km/h for vehicles who yielded and 20.2 km/h for vehicles who did not yield. Therefore, the minimum speed inside IZ is 12.7 km/h lower for vehicles that yield to the bicycle. As visible from Figure 19 the yielding events take place independent of the bicycle minimum speed inside the IZ.

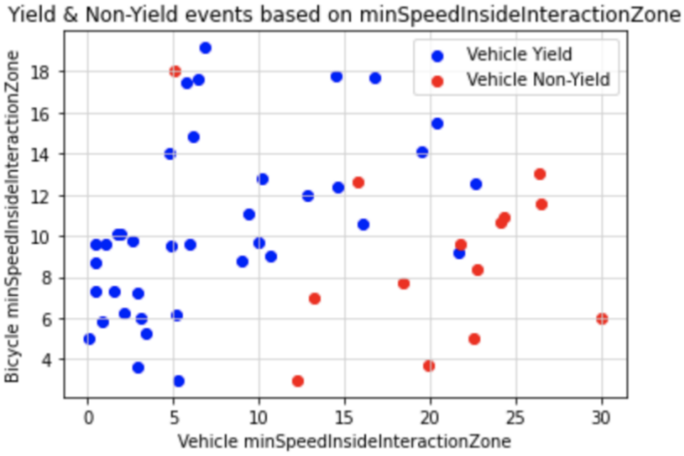


Figure 19. Vehicle yield and non-yield events based on vehicle and bicycle minimum speed inside IZ (units in meters).

The mean bicycle distance from CZ when vehicle enters the IZ (20 meters from CZ) (S_b^{CZ}) is 13.9 meters (standard deviation 5.6) for interactions where vehicle yielded to the bicycle and 14.4 meters (standard deviation 7.6) for interactions where driver did not yield. This results in only 0.5 meters distance difference between yield and non-yield events.

Additionally, when investigating the vehicle mean speed (V_v) together with the bicycle distance from CZ when vehicle enters the IZ (S_b^{CZ}) it appears that both, yield and non-yield events happen independent of the bicycle distance from the CZ.

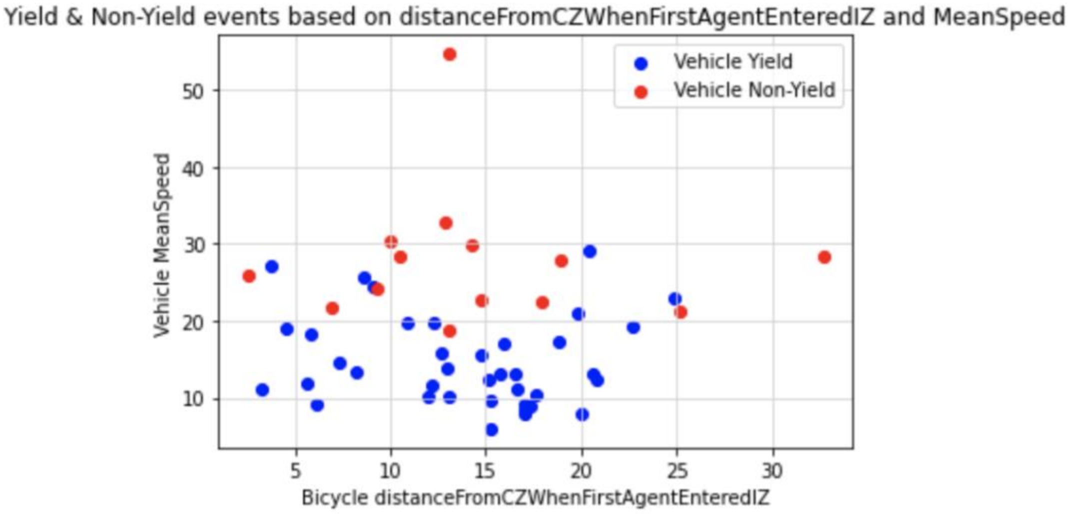


Figure 20. Vehicle yield and non-yield events based on vehicle mean speed inside IZ (V_v) (units inkm/h) and bicycle distance from CZ when vehicle enters the IZ (S_b^{CZ}) (units in meters).

The same is evident when looking at the vehicle distance from conflict zone when bicycle entered the IZ (S_v^{CZ}) and vehicle mean speed (V_v). Similarly, it appears that vehicle speed affects the yielding decision more as the vehicle distance from CZ varies from 4 to 40 meters for all yield events.

As visible from Figure 21 most of the yielding events happen if vehicle speed is below 20 km/h independent of the vehicle distance from CZ. The results indicate that even if bicycle is 10 meters from the CZ and vehicle is 5 meters from the CZ and has a relatively low speed (less than 20 km/h) the vehicle yields.

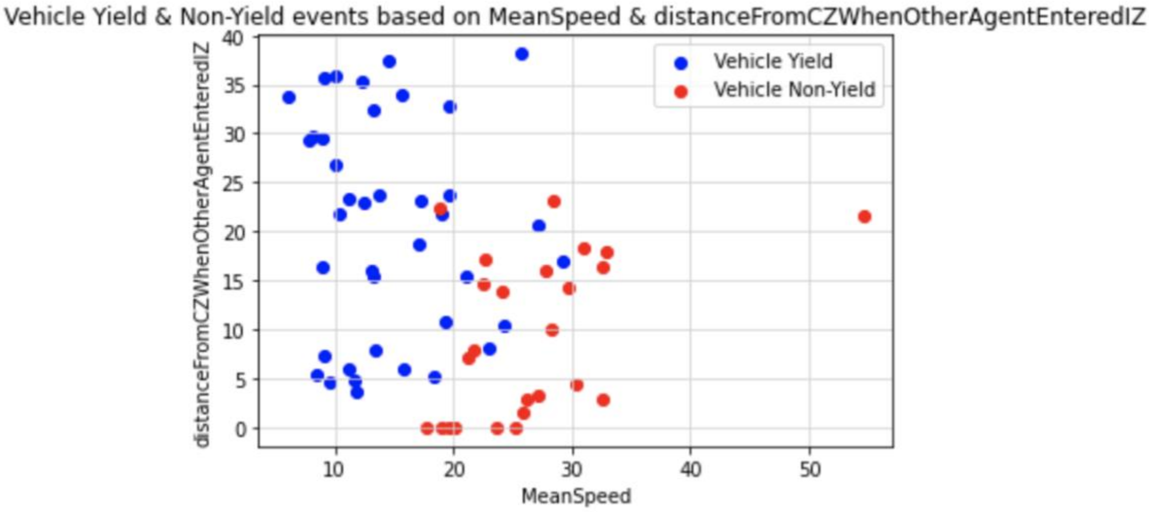


Figure 21. Vehicle yield and non-yield events based on vehicle mean speed (V_v) (units in km/h) and vehicle distance from CZ when bicycle enters IZ (S_v^{CZ}) (units in meters).

6.2 Experimental results

In order to estimate the yielding probability multiple different combinations of features were tested on a simple logistic regression model. Therefore, to determine the most important features for predicting driver's yielding decision, 7 simple logistic regression models were created and compared.

The best model described by Silvano et al. [7] uses instantaneous vehicle speed at the border of IZ and because of data limitations dummy variables for understanding whether the bicycle is 0 to 10, 11 to 20 or 21 to 30 meters from the CZ when vehicle reaches the IZ.

In order to take Silvano's best model as a baseline a model using similar features was recreated (Model I in Table 7). However, as the data used for the research is continuous, instead of dummy variables a specific bicycle distance from CZ at the moment vehicle enters the IZ (S_b^{CZ}) was used.

The models described in Table 7 have been created based on the features selected in the section 5.3 and features determined to be the most relevant one by Silvano et al. [7].

Table 7. Models created based on the selected features and most significant features used by Silvano et al [7].

Variable	**Model I** (Based on literature [7])	**Model II**	**Model III**	**Model IV**	**Model V**	**Model VI**	**Model VII**
Vehicle speed at the border of IZ ($\mathbf{V_v^{IZ}}$)	-0.0721	0.5357	-	-	-	-	-
Bicycle distance from CZ when vehicle enters the IZ ($\mathbf{S_b^{CZ}}$)	-0.0853	-0.1888	-0.2332	-	-0.1974	-0.1912	-0.1266
Vehicle mean speed inside IZ ($\mathbf{V_v}$)	-	-0.9724	-0.4568	-0.0397	-	-	-
Min speed distance ($\mathbf{S_v^{min}}$)	-	0.4993	0.5747	0.2736	0.7252	0.8210	-
Min speed inside IZ ($\mathbf{V_v^{min}}$)	-	0.0686	0.1950	-0.0911	-0.0938	-	-0.2487
-2LL (log likelihood)	21.3756	5.9191	**4.3503**	7.8949	5.4811	6.0484	9.6969
$\mathbf{R^2}$ (Efron's)	0.1180	0.7731	**0.8259**	0.7092	0.7673	0.7494	0.6135

Based on the comparison of the models, the Model I, using similar features as reported by Silvano et al. [7] seems to result in worst performance compared to other models listed in the table. The highest -2LL score and lowest R^2 statistic indicate a poor fit of the data.

On the contrary, Model III indicates a good fit and correlation between predicted and true values suggesting that Model III is the preferred model to use for estimating the vehicle yielding decision.

Model III uses features such as bicycle distance from CZ when vehicle enters the IZ ($\mathrm{S}_{\mathrm{b}}^{\mathrm{CZ}}$), minimum speed distance ($S_{\mathrm{v}}^{\mathrm{min}}$), vehicle mean speed inside IZ (V_{v}) and vehicle minimum speed inside IZ ($\mathrm{V}_{\mathrm{v}}^{\mathrm{min}}$). Model III results are visible on Figure 22.

Model III results indicate that the vehicle yielding probability is generally highest for vehicles with higher minimum speed distance, lower bicycle distance from CZ, somewhat lower mean speed and also lower minimum speed inside the IZ (the size of instances on Figure 22).

Additionally, the figure indicates that minimum speed distance above 5 meters significantly improves the yielding probability as all instances with minimum speed higher than 5 meters from the CZ show a yielding probability of about 90% or more.

Furthermore, as visible from Figure 22 for vehicles with minimum speed distance 0 (indicating that they obtained their lowest speed when already on the crosswalk) it appears that the closer the bicycle is to the CZ the higher the chance of vehicle yielding. However, the yielding probability seems to drop below 50% if bicycle is more than 10 meters away from the CZ with a vehicle mean speed higher than 20 km/h.

Moreover, the mean speed appears to be an important factor in case vehicle is on the border of IZ and bicycle is at about more than 20 meters away from CZ. This is evident from the fact that one group of drivers obtains minimum speed at more than 20 meters from the CZ. Furthermore, their minum speed inside IZ is about 2 km/h. While the other group meets the bicycle at about the same distance but does not yield and the minimum speed is measured when vehicle is already inside the CZ being 24 km/h. However, the mean speed remains below 10 km/h for the vehicles who yielded while it is above 27 km/h for the vehicles who did not yield.

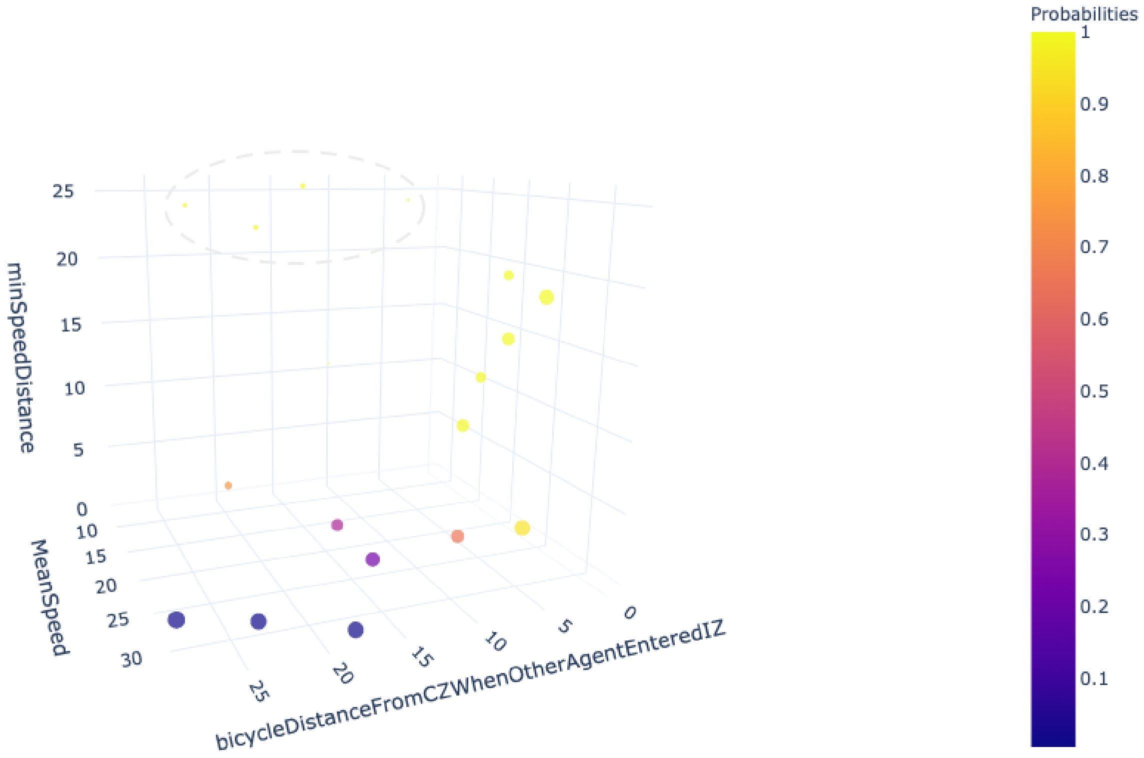


Figure 22. Model III results. Yielding probability relative to bicycle distance from CZ when vehicle enters the IZ (S_b^{CZ}) (units in meters), vehicle mean speed inside IZ (V_v) (units in km/h), minimum speed distance (S_v^{min}) (units in meters) and minimum speed inside IZ (V_v^{min}) (denoted with instance size) (units in km/h,).
The dashed line has been added to draw attention to datapoints with very small minimum speed distance and high yielding probability.

In order to get more insights to driver yielding decision relative to features used in the Model III, all relevant features were also investigated separately.

Vehicle Speed

When looking at the vehicle mean speed (V_v) relative to vehicle yielding decision (Figure 23) it appears that the vehicle yielding probability stays above 50% while vehicle mean speed inside the IZ is less than 25 km/h. However, the yielding probability drops with vehicle mean speed being more than 20 km/h.

The yielding event is very unlikely for vehicles travelling at the speed of 30 km/h or more.

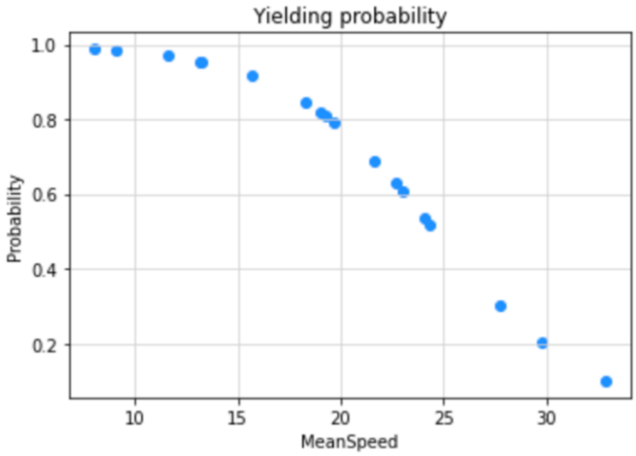


Figure 23. Yielding probability relative to mean speed inside the interaction zone (V_v) (units in km/h).

In addition, the vehicle minimum speed inside interaction zone (V_v^{min}) was explored. The results visible on Figure 24 indicate that the lower the minimum speed is the higher the chance of vehicle yielding.

According to the logit model predictions the vehicle minimum speed of around 17 km/h will result in 50% of yielding chance.

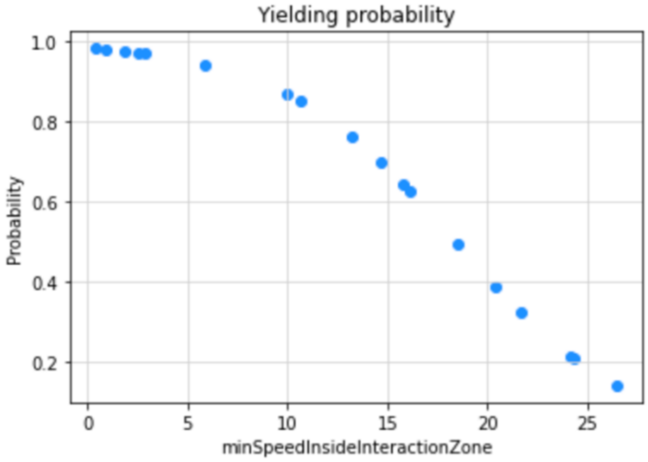


Figure 24. Yielding probability relative to vehicle minimum speed inside the interaction zone (V_v^{min}) (units in km/h).

Moreover, when looking at the distance from CZ where the vehicle minimum speed was measured (S_v^{min}) it appears that the larger the distance from conflict zone where vehicle minimum speed was measured the higher the probability of vehicle yielding to bicycle (Figure 25).

The yielding probability appears to rapidly increase while minimum speed distance stays below 10 meters. In case the vehicle minimum speed is measured more than 10 meters from the conflict zone, the probability of vehicle yielding is more than 90%.

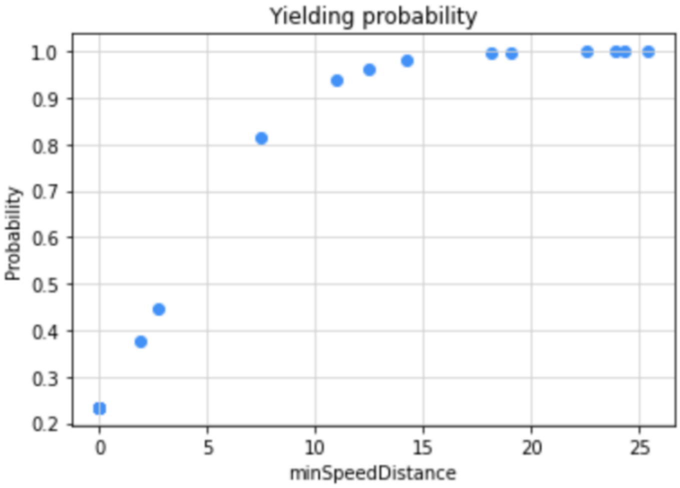


Figure 25. Yielding probability relative to minimum speed distance (S_v^{min}) (units in meters).

In addition, the vehicle speed at the border of interaction zone (V_v^{IZ}) was examined showing an almost linear relation between the speed and yielding probability. The results visible on Figure 26 indicate that higher speed will decrease the probability of vehicle yielding to bicycle which was also assumed based on the literature review. Thus, the results indicate that the lower the vehicle speed when entering IZ the higher the chance of vehicle yielding.

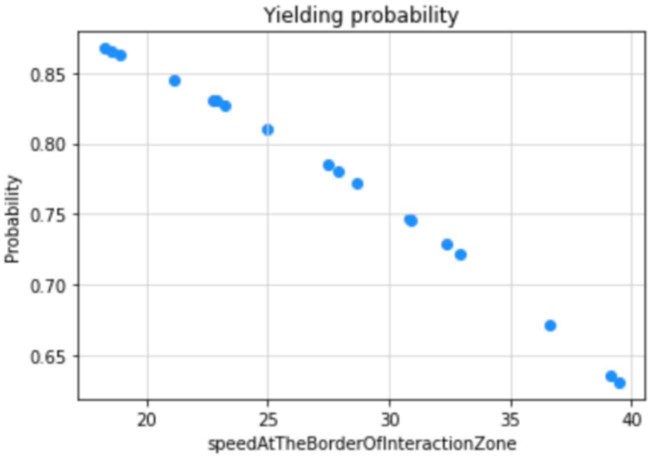


Figure 26. Yielding probability relative to vehicle speed ta the border of interaction zone (V_v^{IZ}) (units in km/h).

Therefore, investigating the vehicle speed relative to yielding probability indicates that the **initially set hypothesis of vehicle yielding rate increasing with lower vehicle speed has proven to be accurate**.

Bicycle Distance from Conflict Zone

When looking solely at bicycle distance from conflict zone at the moment of vehicle entering the interaction zone (S_b^{CZ}) it appears that the vehicle yielding probability reduces with the increasing bicycle distance from conflict zone which was not evident from the data analysis. As visible on Figure 27 in case of bicycle being 27 meters away from the CZ, there is 50% chance of vehicle yielding.

A vehicle would give way with approximately 82% probability if bicycle was on the interaction zone border at the same time as vehicle (bicycle IZ is 10 meters from crosswalk).

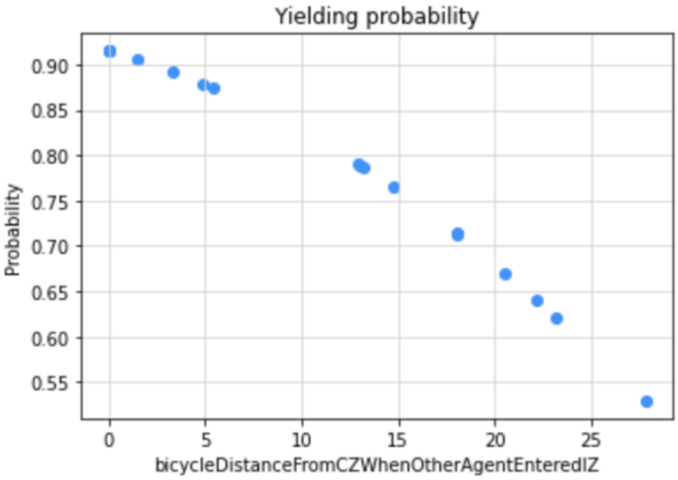


Figure 27. Yielding probability relative to bicycle distance from conflict zone at the moment of vehicle entering the interaction zone (S_b^{CZ}) (units in meters).

The same is evident from looking at the vehicle distance from conflict zone while bicycle is on the border of interaction zone (S_b^{CZ}) (Figure 27). The vehicle yielding probability increases whit vehicle being further from IZ border while bicycle is on the IZ border.

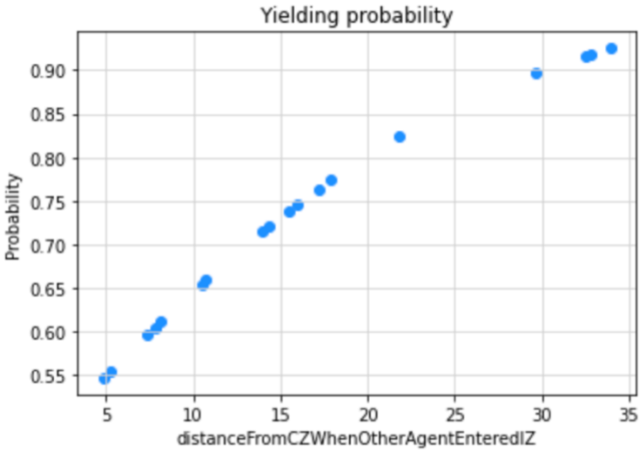


Figure 28. Yielding probability relative to vehicle distance from conflict zone at the moment of bicycle entering the interaction zone (S_v^{CZ}) (units in meters).

Therefore, the further from CZ bicycle is and the closer the vehicle is the lower the chance of vehicle yielding.

Thus, the **hypothesis of yielding rate decreasing with higher bicycle distance from conflict zone proves to hold**.

When investigating the vehicle speed at the border of interaction zone (V_v^{IZ}) together with the bicycle distance from CZ (while vehicle enters the IZ) (S_b^{CZ}) it appears that the yielding probability decreases while vehicle speed at the IZ border and bicycle distance from CZ increase (Figure 29).

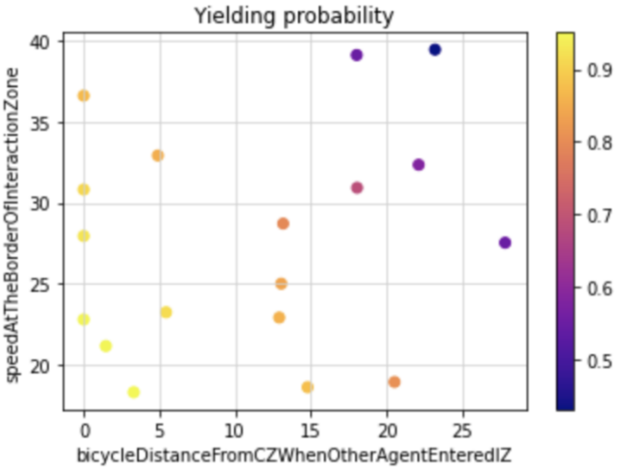


Figure 29. Yielding probability relative to vehicle speed at the border of IZ (V_v^{IZ}) (units in km/h) and bicycle distance from the CZ when bicycle enters the IZ (S_b^{CZ}) (units in m).

Similar findings are evident when estimating the yielding probability based on vehicle mean speed inside IZ (V_v) and bicycle distance from CZ when vehicle enters the IZ S_b^{CZ}).

As visible from Figure 30 the yielding probability seems to decrease with higher vehicle speed and higher bicycle distance from the CZ.

According to the model estimation it appears that there is about 50% chance of vehicle yielding in case vehicle speed is 22 km/h or higher and bicycle is more than 15 meters from the CZ while vehicle has just entered the IZ.

However, the bicycle distance seems to have a smaller effect on the yielding decision than the speed. Similar, to Model III results it appears that the bicycle distance becomes an important factor if vehicle mean speed is more than 20 km/h.

This is visible on Figure 30 where yielding probability stays above 80% while vehicle mean speed is below 20 km/h independent of the bicycle distance from CZ. However, for vehicles with speed about 22 km/h the yielding probability seems to drop to about 50% while bicycle is more than 15 meters from the CZ.

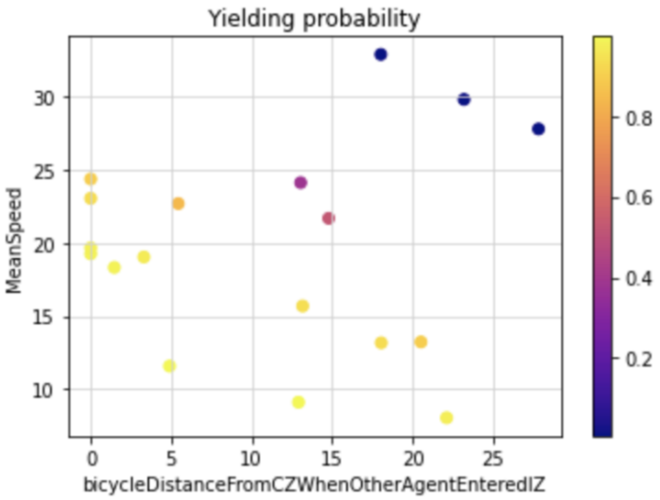


Figure 30. Yielding probability relative to vehicle speed (V_v) (units in km/h) and bicycle distance from the CZ when bicycle enters the IZ (S_b^{CZ}) (units in m).

A similar change in yielding decision probability is also visible when investigating the vehicle minimum speed inside IZ and bicycle distance from CZ visible on Figure 31. It is more probable that the vehicle will yield in case vehicle has shown a lower speed inside IZ and bicycle is further away from the CZ.

However, as stated above the bicycle distance appears to start affecting the decision from a certain vehicle minimum speed as the yielding probability stays around 90% for all vehicles who obtained minimum speed of less than 5 km/h inside IZ independent of the bicycle distance from CZ.

The yielding probability stays above 80% while vehicle minimum speed measured inside IZ is lower than 13 km/h and bicycle is less than 25 meters from the CZ.

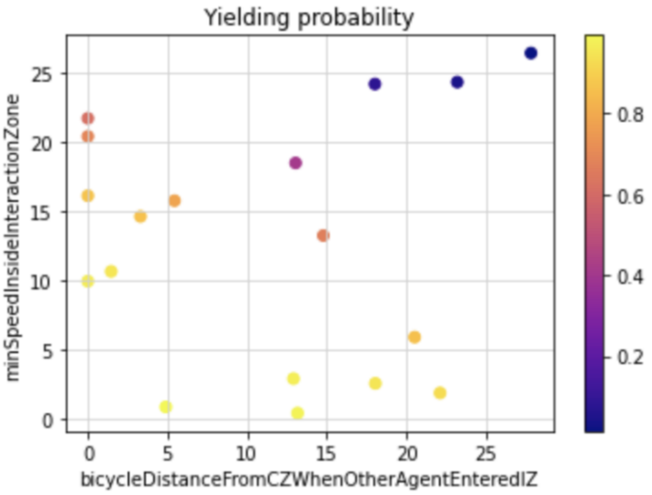


Figure 31. Yielding probability relative to vehicle minimum speed inside IZ (V_v^{min}) (units in km/h) and bicycle distance from the CZ when bicycle enters the IZ (S_b^{CZ}) (units in m).

Therefore, the **hypothesis of yielding rate decreasing with higher vehicle speed and cyclist's longer distance from conflict zone prove to hold**, however the importance of bicycle distance from CZ appears to become more relevant from certain vehicle speed.

Furthermore, it is not evident from the result that **the relevance of bicycle relative distance is rapidly reduced from the distance larger than 20 meters.**

6.3 Discussion

As mentioned in section 6.1 the vehicle mean speed difference between yield and non-yield event groups was the highest being 13 km/h. The speed difference between these groups is expected as yielding vehicles have to slow down in order to give way to the cyclist, however, the size of the gap is rather surprising. It was expected that vehicles who do not perceive a conflict would have the highest mean speed as they don't have to adjust their speed because of approaching bicycles. However, the highest mean speed being measured for the vehicles that did not yield, might indicate that those drivers added speed in order to pass the CZ before bicycle entering it.

In addition, as proved with the experiments, the yielding rate decreases with higher vehicle speed which as mentioned in literature [11] might indicate that vehicles travelling at a lower speed have more time to look around, and therefore notice the approaching cyclist.

Furthermore, as evident from the Model III results, there appear to be two types of driver reactions when bicycle is about 20 meters from the CZ while vehicle enters the IZ. It might be that in such situation drivers travelling at a higher speed tend to add even more speed in order to pass the CZ before the bicycle while drivers already travelling at a lower speed tend to slow down and yield. This however is not fully confirmed by the results as the lower mean speed does not fully prove that the yielding vehicle had the lower speed when already entering the IZ and should therefore be investigated further.

In addition, it appears that bicycle distance from CZ affects vehicles travelling at higher speed more. The reason why vehicles travelling less than 20 km/h on average appear to make the yielding decision independent of the bicycle distance from CZ might be that they are already prone to yield because of the lower speed.

On the contrary, the bicycles speed variation is almost negligible as it stays between 12.2 and 13.1 km/h independent of the event group. This indicates that cyclists expect the vehicle to notice them and therefore give way. Furthermore, cyclist might deliberately demonstrate assertive behaviour to make vehicles yield to them.

As mentioned in the literature [7] and concluded from the research, the bicycle mean speed does not seem to have a significant effect on the drivers yielding decision. This might be due to the fact that bicycle speed is around 12 km/h, and the small fluctuations are not perceived by the driver.

6.4 Possible Improvements

In order to gain more accurate results a bigger dataset should be used for experiments. Additionally, a larger dataset could reduce the risk of model overfitting. Furthermore, using a smaller set of features or some regularisation such as L1 (Lasso) or L2 (Ridge) might also reduce the risk of overfitting.

A dataset with more continuous data would improve the quality and accuracy of the results. As mentioned in section 5.1 the dataset used for the researched included multiple instances with too sparce locations resulting in imprecise time and location marked as entering/exiting the interaction zone visible on Figure 32.

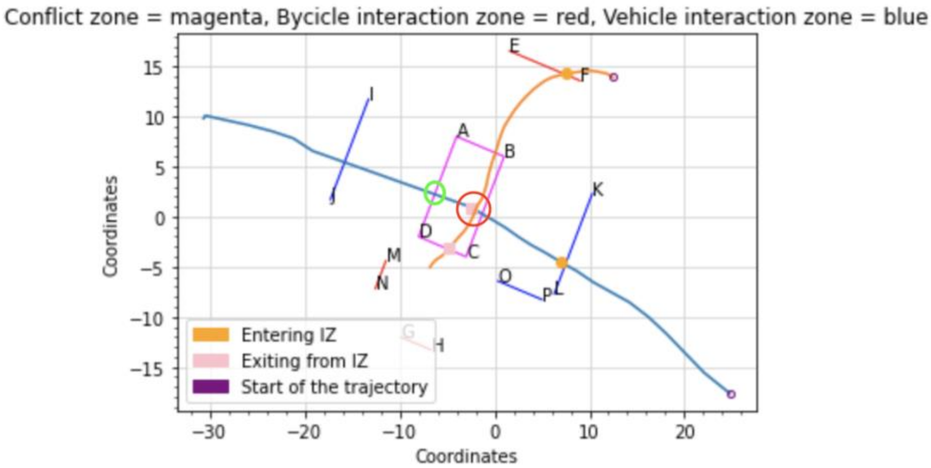


Figure 32. An interaction with vehicle trajectory having imprecise exiting timestamp indicated with the red circle (green circle marking the correct exit point) (units in meters).

Furthermore, all vehicles and bicycles simultaneously present near the crossing should be investigated as one interaction as they might influence each other. As all possible interactions in this research were generated based on vehicles, some interactions might have included exact same bicycles which might have had an influence on the results.

Additionally, although this research used interactions with agents coming from opposites sides of the road, it should be investigated more thoroughly as this topic was not specifically tackled in this thesis. The yielding probability relative to the direction bicycle is coming from versus the side of the road vehicle is travelling should be further investigated.

Moreover, features including driver, cyclist characteristics and also traffic configuration should also be investigated. As this kind of data was not accessible for this research it was not investigated.

In addition, the feature selection could be improved and possibly done based on some other algorithm such as recursive feature elimination. Also, the significance of each feature in a model could be estimated to gain an even better insight of the model.

7 Conclusion

As autonomous driving is rapidly advancing, and automated vehicles are already on the roads the interaction between such vehicles and other crosswalk agents should be as safe and smooth as possible. In order for such interactions to be safe it is necessary to understand both agents' behaviour to be able to predict it.

This thesis investigated vehicle-bicycle interaction at an unsignalized crossing. The purpose of the research was to determine the most relevant features in order to create a model for estimating drivers yielding decision. The wider purpose was to contribute to future treatments and solutions for making interaction with autonomous cars as safe and as human-like as possible.

Vehicle-bicycle interactions were modelled using hierarchical modelling framework for vehicle-bicycle interactions [11]. Multiple features were implemented and investigated. A set of features was chosen based on feature selection algorithm to investigate further. Based on different combinations of these features multiple models were created and compared out of which the most performant was chosen. Additionally, a model was created based on the best performing model found from literature [7] in order to use as a base line.

As a result the research concluded that best features to be used for estimating the vehicle yielding decision to bicycle on an unsignalized crossing are bicycle distance from CZ when vehicle enters the interaction zone ($\mathrm{S_b^{CZ}}$), vehicle mean speed inside interaction zone ($\mathrm{V_v}$), minimum speed distance ($\mathrm{S_v^{min}}$) and minimum speed inside interaction zone ($\mathrm{V_v^{min}}$).

Furthermore, the research determined that the vehicle yielding rate does increase with lower vehicle speed and with higher bicycle distance from conflict zone as assumed based on the literature review. In addition, the results indicated that vehicles travelling at less than 20 km/h on average tend to be less affected by the bicycle distance from conflict zone as they tend to yield independent of it. A more detailed overview can be found from chapter 6.

The results obtained in this research could be applied in autonomous driving field. For instance, the model could be used as a starting point for creating a model to be used by autonomous vehicle for estimating whether to yield to the bicycle on an unsignalized crosswalk or not. Furthermore, the data analysis and analysis of features affecting the driver yielding decision could be used for urban planning in order to reduce risk of bicycle-vehicle conflict.

Avenues for future improvements have been discussed in section 6.4. In summary, the research could be continued by using a larger dataset for validating the models' performance. Additionally, some new features could be generated based on driver, bicycle characteristics or traffic configuration and used for possibly improving and adjusting the model. Furthermore, some specific aspect of yielding decision could be tackled, for instance, the effect of multiple bicycles simultaneously approaching the crosswalk on vehicle yielding decision which was not precisely tackled in this thesis due to lack of data.

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