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Some Problems of Electric Spectrometry of Aerosol Particles

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1. Introduction

A principle: If we wish to know numerical values of n independent scalar parameters then we should do at least n scalar measurements.

First conclusion: If our instrument issues n scalar measurements then we cannot find more than n values or scalar parameters of the size distribution.

Second conclusion: Before starting to develop a method of measurement, we should choose a model of size distribution, where the distribution is uniquely determined by a small number n of numerical values.

2. Models of size distribution

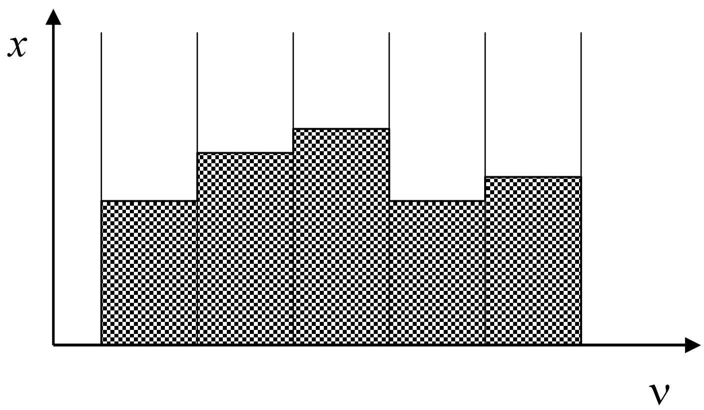
Continuous model: most fundamental in theory, but it has infinite number of degrees of freedom.

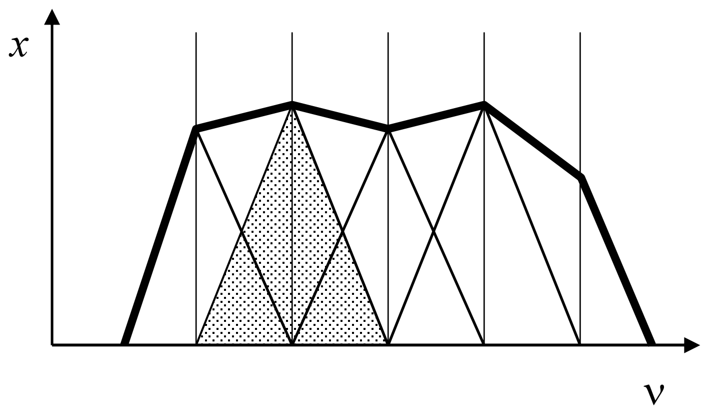
Fraction model: finite number of degrees of freedom, but there is a problem how to associate the set of fraction concentrations with the continuous model.

Finite-dimensional linear model:
Argument (size) ν, density of distribution x. Basis $\{f^i(\nu)\}$, $i = 1..n$.

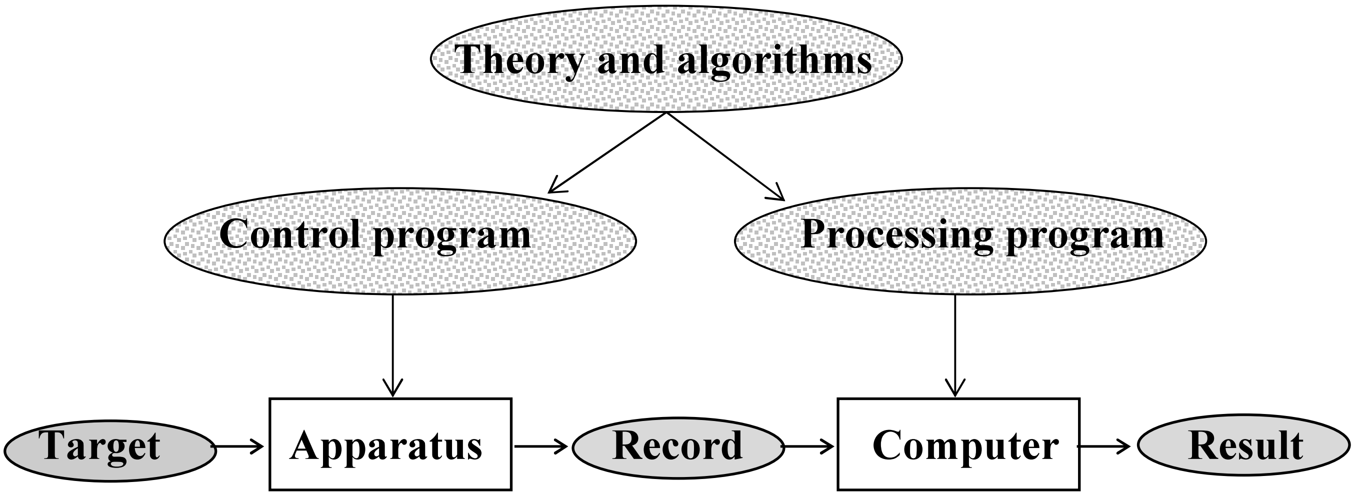
$$x(\nu) = \sum_{i=1}^{n} x_i f^i(\nu)$$

Examples: A simple fraction model and a piecewise linear model





2. Mathematical model of a linear spectrometer



Target: Aerosol
Record: The set of channel signals $\boldsymbol{y} = \{y_1, y_1, ..., y_m\}$, m is number of channels.
Result: The asked distribution presented by n numbers $\boldsymbol{x} = \{x_1, x_1, ..., x_n\}$

Equation of the linear spectrometer:

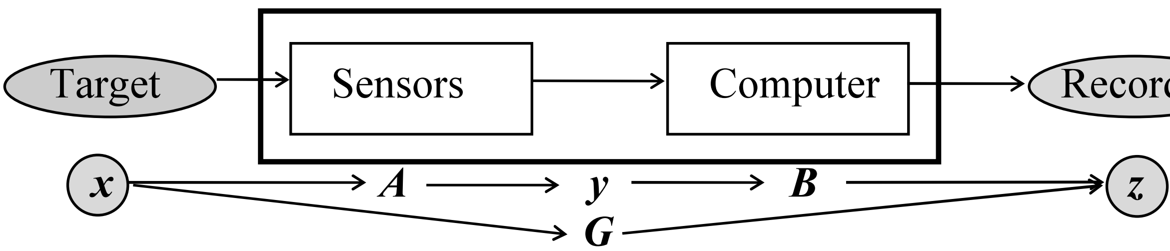
$$y_i = \sum_{j=1}^{n} g_{ij} x_j + \xi_i , \qquad i = 1...m, \qquad \xi \text{ is measuring noise}$$

$$\boldsymbol{y} = \{y_i\}, \ \boldsymbol{G} = \{g_{ij}\}, \ \boldsymbol{x} = \{x_j\}, \xi = \{\xi_j\},$$

$$\boxed{\boldsymbol{y} = \boldsymbol{Gx} + \xi}$$

Applications:

Apparatus record → mobility distribution
Mobility distribution → size distribution
Apparatus record → size distribution



$$\boldsymbol{y} = \boldsymbol{Ax} \qquad \boldsymbol{z} = \boldsymbol{By} \qquad \boldsymbol{z} = \boldsymbol{B}(\boldsymbol{Ax}) = (\boldsymbol{BA})\boldsymbol{x}$$

Conclusion: $\boldsymbol{z} = \boldsymbol{Gx}$, where $\boldsymbol{G} = \boldsymbol{BA}$

Example: Apparatus matrix of EAS according to Aadu Mirme

9803	**242**	**20**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
24067	**2703**	**107**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
8000	**9032**	**296**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
8339	**4996**	**2080**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
202	**1506**	**6836**	**103**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
0	**34**	**2117**	**1695**	**6.9**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
0	**260**	**737**	**3272**	**507**	**20**	**0**	**0**	**0**	**0**	**0**	**0**	**6.9**
0	**60**	**278**	**1151**	**2062**	**273**	**20**	**0**	**0**	**0**	**0**	**0**	**23.7**
0	**0**	**40**	**741**	**1713**	**1315**	**93**	**0**	**0**	**0**	**0**	**4.9**	**67.3**
0	**0**	**15**	**402**	**537**	**1859**	**588**	**56.4**	**0**	**0**	**8.0**	**90.1**	**61.3**
0	**0**	**0**	**158**	**250**	**625**	**1497**	**247**	**136**	**169**	**174**	**138**	**8.0**
0	**0**	**0**	**78.3**	**120**	**200**	**1622**	**747**	**475**	**482**	**358**	**70.6**	**0**
0	**0**	**0**	**0**	**60**	**105**	**403**	**1262**	**655**	**634**	**320**	**11.7**	**0**
0	**0**	**0**	**0**	**0**	**20.0**	**60.0**	**1171**	**691**	**504**	**154**	**0**	**0**
0	**0**	**0**	**116.5**	**195**	**673**	**622**	**692**	**0**	**0**	**0**	**0**	**0**
0	**0**	**0**	**147.4**	**323**	**651**	**337**	**884**	**450**	**0**	**0**	**0**	**0**
0	**0**	**0**	**123.7**	**464**	**305**	**626**	**548**	**1039**	**0**	**0**	**0**	**0**
0	**0**	**269**	**28.0**	**704**	**200**	**416**	**350**	**1318**	**95**	**0**	**0**	**0**
0	**0**	**584**	**4.4**	**360**	**78.6**	**147**	**131**	**472**	**589**	**0**	**0**	**0**
0	**0**	**70**	**394**	**130**	**40.2**	**0**	**0**	**107**	**733**	**62**	**0**	**0**
0	**0**	**167**	**14**	**46.8**	**0**	**0**	**0**	**0**	**251**	**494**	**0**	**0**
0	**74**	**275**	**68**	**13.8**	**0**	**0**	**0**	**0**	**20**	**711**	**0**	**0**
0	**0**	**329**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**321**	**94.9**	**0**
0	**0**	**162**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**659**	**17.8**
916	**0**	**364**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**228**	**472**
0	**1998**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**232**

3. Data processing

Traditionally, the infinite dimensional model is used in the theory. The infinite dimensional apparatus equation

$$y(w) = \int_{v_{\min}}^{v_{\max}} g(w,v)x(v)\,\mathrm{d}v$$

is a Fredholm equation of the first kind. It is incorrectly set up and cannot solved without additional information.

Our equation is an algebraic linear equation

$$\boldsymbol{y} = \boldsymbol{Gx} + \xi \quad \text{or} \quad y_j = \sum_{i=1}^{n} G_{ji}x_i + \xi_j$$

It can be solved using Gauss-Markoff least squares algorithm:

$$\hat{\boldsymbol{x}} = \boldsymbol{A}^{-1}\boldsymbol{b} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{D}^{-1}\boldsymbol{G})^{-1}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{D}^{-1}\boldsymbol{y}$$

The algorithm:

$$\begin{aligned} \boldsymbol{H} &:= \boldsymbol{G}^{\mathrm{T}}\boldsymbol{D}^{-1} \\ \boldsymbol{C} &:= (\boldsymbol{HG})^{-1} \\ \boldsymbol{x} &:= \boldsymbol{CHy} \end{aligned}$$

D is the covariance (or dispersion) matrix of record noise. Its diagonal elements are the variances σ^2 and non-diagonal elements are covariances $\rho_{i\varphi}\sigma_\iota\sigma_j$. ***C*** is the covariance matrix of the spectrum and the estimates of the measuring errors can be found as its diagonal elements.

Example: Let $G_{ji} = c\exp\left(-c^2 (j-i)^2\right)$

Then less the coefficient *c* then better is the resolution. What happens if we will reduce the value of c? Let us calculate the coefficient of noise amplification as the square root of the ratio of measurement errors in the result and in the record: