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GENERALIZED SYMMETRIES AND HIGH ORDER DEFORMATIONS OF THE DIRECT SUM P $\oplus$ A

V.D.Lyakhovsky

1. Introduction

Difficulties of the group-theoretic approach to the elementary particles theory are often connected with non-relativistic character of usually used internal symmetry groups. This caused numerous attempts of the joint description of the space-time and internal properties. It was supposed that breaking of the internal symmetry is totally described by the inclusion of the internal symmetry algebra into a larger one containing the Poincaré subalgebra. This scheme was intended to describe coupling of the space-time and internal properties of the system of hadrons. But the so called no-go theorems were proved within quite general suppositions /1/,which leaves only one possible scheme of generalized symmetry, i.e. the direct sum of the Poincaré and internal symmetry algebras. The space-time and internal symmetries thus appear to be totally independent and all the particles of the same multiplet have the same mass and spin. On the other hand the mass-splitting as well as connections between the external and internal characteristics of the system are experimentally evident.

Several attempts to revive the old scheme being fruitless the construction of the new model of the symmetry- breaking mechanism appeared to be the most hopefull way /2/.

It seemed very likely that the new description might accumulate the information from both algebraic objects: the precise symmetry algebra U and broken symmetry algebra U_c . Difficulties of the previous scheme constrain one to think the inclusion into a wider symmetry to be not the main reason of breaking of the precise symmetry. It was supposed that the precise internal symmetry algebra A may not be a subalgebra of the generalized algebra U_c . Breaking results not only in connections between the space-time P and internal A algebras but also in transformations of the internal and probably space-time symmetries. When the internal symmetry is precise the system is described by the direct sum of algebras $P \oplus A$ .Taking into account of the nonsymmetric part of the interaction should correspond to the transition from the direct sum $P \oplus A$ to an algebra of a generalised symmetry. So to find the generalized symmetry is to construct the algebra which can be transformed into the direct sum $P \oplus A$ by some limiting procedure. In addition the commutation relations of these two algebras must be "close" in the Segal's terminology /3/ according to the correspondence principle.

The above model was carefully investigated on the basis of the contractions theory /4,5/. However the existence of the generalized symmetry with such properties was not established. It must be taken into account that there is no theory of general contractions. Only some special types of contractions (IW-contractions /6/ Saletan's contractions /7/, p-contractions /8/, singular contractions /9/ and some other) are thoroughly treated. So the results of the investigations /4, 5/ refer to few types of contractions. The method gives no possibility to reconstruct all the Lie algebras which form the neighbourhood of the given algebra.

The theory of deformations of Lie algebras proposed by M.Gerstenhaber /10/ is free from these disadvantages. This theory was first used by M.Levy-Nahas /9/ to find

The deformations of Lie algebras is the theory of inverse contractions and therefore gives the most suitable method to construct the generalized symmetry algebra. Deformations do not depend on the specific form of the limiting transition between two algebras. Over and above the obtained results do not depend upon the algebraic basis chosen. By means of deformations one can obtain all the algebras that are"close" to the given one /11/. The investigation of the generalized symmetry "close" to the direct sum $P \oplus A$ makes it necessary to study the deformations of the Lie algebra $P \oplus A$.

The results of course differ for various algebras A of internal symmetry. First there were considered the semi-simple S and inhomogeneous IS algebras of internal symmetry which were of great physical importance. In the article /12/ it was demonstrated that in these cases all the deformations conserve the structure of the direct sum. Thus in the Lie algebras close to $P \oplus S$ or $P \oplus IS$ no relativization can be achieved. In face of this result the problem is formulated somewhat differently. It should be stated what properties are necessary for the relativization of the finite dimentional Lie algebra. In the previous paper /13/ it was supposed that the internal algebra A has quite general form (finite dimensional Lie algebra with solvable ideal). It was proved that the possible form of the infinitesimal deformations of $P \oplus A$ changing the structure of the direct sum is unique. The restrictions on the possible structure of A are explicitly formulated (they are given in Section 2). But the properties of the generalized symmetry algebra thus obtained cause serious difficulties in its physical applications. It is impossible to solve the problem of mass spectrum by means of the infinitesimal deformation of the direct sum $P \oplus A$, whatever the internal algebra A is. But the possibility remains to construct the generalized algebra using the high order deformations of $P \oplus A$. They are studied in this article.

In Section 2 the main properties of deformations and the theory of uniqueness are briefly discussed. In Section 3 the connections between the high order and repeated deformations are studied. It appears impossible to restrict the investigation to the first order deformations of the structure given in /13/. In Section 4 the theorem of quasistability of this structure is proved for the high order deformations. The second part of the theorem of quasistability (Section 5) refers to the repeated deformations. These theorems state the unique form of possible coupling of the space-time and internal symmetries. Some properties of the generalized symmetry thus obtained are discussed in Section 6. The conclusions are illustrated by simple examples of the first order and high order deformations.

2. General Properties of Deformations and the First Order Deformations of $P \oplus A$.

Let $X = (V, \mu = [\,])$ be the finite dimentional Lie algebra over the field of real numbers R , where V is the algebraic linear vector space and μ is its composition law. The extension of the vector space V by the power series ring of c over R is the linear vector space $V_R = V \underset{R}{\otimes} R((c))$. Lie algebra $X_c = (V_R, f_c)$ is called the deformation of X if

$$f_c(a,b) = [a,b] + c\, f_1(a,b) + c^2 f_2(a,b) + \dots \qquad (1)$$

and $$\sum_{P(a,b,d)} f_c(f_c(a,b),d) \equiv (f_c * f_c)(a,b,d) = 0 \tag{2}$$

where $a,b,d \in V$

$f_i : V \wedge V \longrightarrow V$, and

$P(a,b,d)$ is the cyclic permutation of variables.
It is more comfortable to use a different notation

$$[f,f'] \equiv f * f' + f' * f \tag{3}$$

Then Eq.(2) assumes the form

$$[f_c, f_c] = 0 \tag{4}$$

In the vector space $\mathrm{Hom}_2(E(V), V)$ elements μ satisfying (4) form an algebraic set $\mathcal{M}$.

$$\mathcal{M} = \{ f \in \mathrm{Hom}_2(E(V),V) \mid [f,f] = 0 \} \tag{5}$$

Let L be the neighborhood of $(\cdot)\ \mu \in \mathcal{M}$ on $\mathrm{Hom}_2(E(V), V)$. Then

$$M = \{ \eta \in L \mid [\eta, \eta] = 0 \} \tag{6}$$

is the algebraic set called the set of deformations of the Lie algebra $X = (V, \mu)$.

Recalling now the expansion (1), one obtains from Eq.(4)

$$\delta \bar{f}_c + 1/2 [\bar{f}_c, \bar{f}_c] = 0 \tag{7}$$

where $\delta \bar{f}_c \equiv [\bar{f}_c, \mu = [\ ]]$,

$\bar{f}_c \equiv f_c - [\ ]$.

Eq.(7) is called the deformation equation. In particular for f_1 one has relations:

$$\delta f_1 = [f_1, \mu = [\ ]] = 0 \tag{8}$$

$$\tfrac{1}{2}\left[f_1, f_1\right] = -\delta f_2 \qquad (9)$$

Eq.(8) implies that f_1 must be the element of the 2-cocycles group $Z^2(X,X)$. If it is so the left hand side of Eq.(9) is the element of $Z^3(X,X)$ which must belong to $B^3(X,X)$ as indicated by the right hand side of this equation.

The function $\bar{f}_c$ satisfying the deformation equation is called the deforming function. But the deforming function does not necessarily change the initial algebra X , for it may occur that X_c and X are isomorphic. Such deformations are called trivial. The condition

$$\bar{f}_c \in B^2(X,X) \qquad (10)$$

is necessary and sufficient for triviality of the deformation. Two deforming functions $\bar{f}_c$ and $\bar{f}_c'$ referring to the same 2-cohomology class lead to isomorphic Lie algebras X_c and X_c'. Such deformations are called equivalent. It was first stated by Gerstenhaber /10/ that only those members of $Z^2(X,X)$ can give nontrivial deformations which are not cohomologous to zero. Algebras with zero second cohomology group have only trivial deformations and are called rigid. It is often important to know whether any subalgebra of the algebra under consideration is rigid (it is then called stable). The stability may be strong (all the commutators containing elements of this subalgebra are unchanged) or weak (when only commutators in the considered subalgebra are rigid). We shall use only the fact that semi-simple algebras are strongly stable /11/.

Let the parameter c in the deformed composition law (1) be infinitesimally small. Only the relation (8) will be essential in this case. This means that such deformations form the neighbourhood of the initial algebra $X = (V, \mu)$ on the tangent manifold to the set of all Lie composition laws at the point μ. These deformations are called infinitesimal. All nontrivial infinitesimal deformations are thus described by the second cohomology group $H^2(X, X)$. Any investigation of the deformation properties of an algebra impose the detailed study of this group. The calculations are considerably simplified by means of the Serre and Hochschild's formula /14/.

$$H^n(X, M) = \sum_{i+k=n} H^i(X/J, F) \otimes_F H^k(J, M)^X \qquad (11)$$

where M is a free X-modul,
J is an ideal of X-algebra such that X/J is semi-simple,
F is a ring over which X is defined,
$H^k(J, M)^X$ is a group of X-invariant elements in $H^k(J, M)$.

Suppose that all the maps f_i with $i > 1$ are zero in Eq.(1). Then it is necessary to fulfill not only the relation (8) but also the relation (9) in the form

$$[f_1, f_1] = 0 \qquad (12)$$

Such deformations are called the deformations of the first order. If Eq.(11) is not fulfilled Eq.(9) may be still true and some high order deformations may exist with the first nonzero component f_1. But when the composition $[f_1, f_1]$ appears to be the 3-cocycle not cohomologous to zero no deformations other than infinitesimal can exist with such f_1. The deforming component f_1 is then called nonintegrable. The corresponding 3-cocycle is called the obstruction to the deformation with f_1. In this case there may be more than one irreducible component in M (Eq.(6)) and the point μ is not simple on $\mathcal{M}$ /11/.

Our task formulated in Section 1 is to find all the deformations breaking the structure of the direct sum in $P \oplus A$ and to study their properties connected with the mass-spectrum problem.

All the proposed models of the internal symmetry were based on the semi-simple S or the inhomogeneous IS Lie algebras. Let us introduce the following decomposition of the Poincaré algebra $P = T \oplus L$, where L is the Lorentz subalgebra and T - the translational ideal. The algebraic vector space splits accordingly $V_P = V_T \oplus V_L$. In the direct sum $(T \oplus L) \oplus S$ the subalgebras L and S are strongly stable. So the deforming function has V_T as the domain of definition. In /12/ it is shown that its domain of values is V_L. This function describes the autonomous deformation of the Poincaré subalgebra. As a result all the deformations of $P \oplus S$ are of the form $D \oplus S$, where D is one of the De-Sitter algebras.

In case of inhomogeneous algebras there are some specific difficulties. But we arrive at the similar conclusion. It is demonstrated in /12/ that all the deformations of the direct sum $P \oplus IS$ are autonomous deformations of its direct summands.

Nevertheless the possibility of existence of more complicated finite dimentional Lie algebras with proper relativization remained. In /13/ it was required merely that the internal symmetry algebra is a finite dimentional Lie algebra. Its Levy decomposition is the only information used.

$$A = \mathcal{G} \mathbin{\subset\!\!\!\!+} S$$

where $\mathcal{G}$ is the radical and S is semi-simple.

The following theorem proved in /13/ was called the theorem of uniqueness:

"Among the nontrivial infinitesimal deformations of $U = P \oplus A$ there is only one that changes the structure of the direct sum. The corresponding deforming function allows the first order deformation and has the form

$$f_1^h(u_1,u_2) = \begin{cases} h(g)\cdot t & \text{for } u_1 = t,\ u_2 = g \\ -h(g)\cdot t & \text{for } u_2 = t,\ u_1 = g \\ 0 & \text{for all other arguments} \end{cases} \qquad (13)$$

where $u_i \in U$, $t \in T$ - the translational ideal of P, $g \in \mathcal{G}$. $h(g)$ is a constant depending on g, $h(g) = 0$ if $g \in A^1$ - the first commutator subalgebra of A."

Thus an algebra is relativizable if and only if in its radical not all elements belong to A^1. This requirement being fulfilled the structure of the deformed generalized algebra U_c is unique

$$U_c = (L \oplus S) \mathbin{\supset\!\!\!\!+} (\mathcal{G} \mathbin{\supset\!\!\!\!+} T) = P \mathbin{\subset\!\!\!\!+} A,$$

$$[L, \mathcal{G}] = 0;\ [T, S] = 0;\ [T, g^+] = T \qquad (14)$$

where $g^+ \in \dfrac{\mathcal{G}}{\mathcal{G} \cap A^1}$.

As it was stressed in /13/ the main property of U_c is that P and A subalgebras survive the deformation. They remain not only subalgebras of the generalized algebra but there conserve also internal composition laws of P and A. So the no-go

theorems are still valid for such constructions.

It remains to be solved whether there is any nontrivial deformation of higher order which can change the structure of P and A subalgebras and thus give physically interesting results.

3. High Order Deformations.

There are standard mathematical methods of constructing first order deformations $X_c^I = (V, \mu + c f_1)$ of an arbitrary Lie algebra $X = (V, \mu)$. The direct computation of high order deformations in general case is much more difficult. On the other hand one can always consider X_c^I to be the initial algebra and obtain its first order deformations if there are any. The question is in what way the result of repeated deformations is connected with high order deformations of the initial algebra X .(The most interesting is the case when all infinitesimal deformation functions $f_1 \in Z^2(X, X)$ allow the first order deformation.)

Let us consider the n-th order deforming function $\overline{f}_c^n$ where the first component f_1 allow the first order deformation. Then for f_2 we have following equations

$$\begin{aligned} [\mu, f_2] &= \delta_\mu f_2 = 0 \\ [f_1, f_2] &= -\delta_\mu f_3 \end{aligned} \qquad (15)$$

Suppose f_c' to be the deforming function of the algebra $X_c^I = (V, \mu + c f_1)$. For the first component f_1' of the corresponding relations differ from that of Eq.(15).

$$\begin{aligned} [\mu + c f_1, f_1'] &= \delta_{\mu + c f_1} f_1' = 0 \\ [f_1', f_1'] &= -\delta_{\mu + c f_1} f_2 \end{aligned} \qquad (16)$$

The class of solutions $\{ f_1' \}$ of the Eq.(16) may be wider than the corresponding class $\{ f_2 \}$ in case of Eq.(15). So repeated deformations in general can give algebras unobtainable by high order deformations.

The inverse situation is also possible. Suppose for example that $f_1' \in Z^2(X,X)$. Then it follows from Eq.(16) that $[f_1, f_1'] = 0$. Comparing this restriction with Eq.(15) one can see that the class of "integrable " deforming functions $\{ f_2 \}$ becomes wider than that of $\{ f_1' \}$. Hence there may also exist high order deformations unequivalent to repeated first order transitions.

The above arguments touch upon second order deformations only. The full investigation of an arbitrary order equivalence between the "pure" and repeated deformations is too complicated. But in some cases the problem can be easily solved. These situations are described in the following Lemma I.

Lemma I

Let $X = (V, \mu)$ be the Lie algebra with the first order deformation X_c' (performed by the deforming function $f_1 \in Z^2(X,X)$ such that $[f_1, f_1] = 0$).

1) If there is a 2-cocycle f_2 such that

$$B^3(X,X) \not\ni [f_1, f_2] \neq 0$$

then X has no deformations of order n>2 with first two components f_1 and f_2. The point μ is singular on the set of all Lie composition laws $\mathcal{M}$.

2) If there is a two-cocycle f_2 such that

$$B^3(X,X) \ni [f_1, f_2] \neq 0$$

there exist the high order deformations of X unequivalent to the repeated first order deformations (whatever is the character of the point μ).

3) If there is a subset $\{ f \} \subset Z^2(X,X)$ such that

$$[f_1, f] = 0$$

if all the components f_i of the deforming function $\bar{f}_c$ are the elements of $\{f\}$ and the following relations are true

$$[f_{2n+1}, f_{2n+1}] = 0, \ [f_{2n+1}, f_{2m}] = 0$$

then all such deformations of X can be considered as deformations of X_c' (whatever is the character of μ).

4) If there is a subgroup $\{f\} \in Z^2(X,X)$ such that

$$[f_j, f_j] = 0$$

for every $f_j \in \{f\}$ and all the components f_i of the deforming function $\bar{f}_c$ belong to this subgroup then all such deformations can be considered as repeated first order deformations of X (whatever is the character of μ).

5) If all the elements $f_j \in Z^2(X,X)$ meet the requirement $[f_j, f_j] = 0$, then all deformations of X of the higher order are equivalent to repeated first order deformations of X and the point μ is simple.

Proof.

1) It follows from Eq.(7) that

$$[f_1, f_2] = \delta f_3 \qquad (17)$$

The conditions of the first part of the Lemma are incompatible with this requirement. The Jacoby identity will be fulfilled only if one neglects the powers of the deformation parameter n≥3. The corresponding finite deformation do not exist.

If f_1 and $f_2 \in Z^2(X,X)$, the composition $[f_1, f_2]$ is an element of 3-cocycles group. Being not a coboundary it must be a nonzero member of the 3-cohomology group. So dim $H^3(X, X) \neq 0$ and μ is a singular point on $\mathcal{M}$.

2) Due to the conditions of this part it is always possible to find the third component f_3 of the deforming function for which Eq.(17) is true. This gives the third order deformation (though not necessarily finite) of X. But with respect to X_c the function f_2 cannot be considered as f_1' - the first component of the deforming function $\bar{f}_c'$. In fact it is not the element of $Z^2(X_c^I, X_c^I)$ for

$$[f_2, c f_1 + \mu] \neq 0$$

3) Let f_n be the n-th component of the deforming function $\bar{f}_c$, then it follows immediately from the deformation equation (7) that

$$[f_n, \mu] = -[f_1, f_{n-1}] - [f_2, f_{n-2}] - \cdots$$

$$\cdots - \begin{cases} \tfrac{1}{2}[f_{n/2}, f_{n/2}] & (\text{ for even } n \text{ }) \\ [f_{\frac{n-1}{2}}, f_{\frac{n+1}{2}}] & (\text{ for odd } n \text{ }) \end{cases} \qquad (18)$$

In face of conditions of the third part of Lemma one obtains for an arbitrary n the relation

$$[f_n, \mu] = [f_n, c f_1 + \mu]$$

and in addition for every odd n the following one

$$[f_n, f_n] = 0$$

Each odd component of $\bar{f}_c$ performs the first order deformation of X_c^I (as well as X). Let us change the indices n of the even components of $\bar{f}_c$ by $m = n/2$. The corresponding function will then obey the usual deformation equation (7), where the initial composition

law is substituted by $\mu + c f_1$. As a result every high order deformation of X can be considered as a deformation of the algebra $X'_c = (V, \mu + c f_1)$.
4) From the symmetry properties of the condition (3) it follows that in a group $\{f\}$ of selfcommuting 2-cocycles all the elements commute with each other

$$[f_i, f_j] = 0 ; \qquad f_i, f_j \in \{f\}$$

This provides the decomposition of the deformation equation (7) into a system of separate conditions equal for all the components of the deforming function. Each component describes the first order deformation and each f_{i+1} is the element of $Z^2(X^i_c, X^i_c)$, where $X^i_c = (V, \mu + c f_1 + \ldots + c^i f_i)$. So in this case the deformation of any order splits into a sequence of first order deformations.
5) Remember the relation (18) for the n-th component f_n of an arbitrary deforming function. If all elements of $Z^2(X, X)$ are selfcommuting then it follows immediately from (18) that every deforming function $\bar{f}_c$ may have only those components which belong to $Z^2(X,X)$. Consequently this case is reduced to the previous one.

All the 2-cocycles are selfcommuting here and thus all the infinitesimal deformations are integrable. So the simplicity of the point μ is guaranteed.

This concludes the proof of the Lemma.

As we shall see later (Section 4) the direct sum $P \oplus A$ has physically interesting high order deformations unequivalent to any of repeated deformations of U_c . But the new Lie algebras thus obtained will not differ considerably from the U_c .

4. Theorem of Quasistability I

If certain above mentioned properties of components of the high order deforming function are known one can use the results of the previous Lemma I. So every possibility to narrow the domain of candidates for the components of $\bar{f}_c$ is valuable.

Lemma II

Let f_c be the deforming function of the Lie algebra

$$f_c = \mu + c^{n_1} f_{n_1} + c^{n_2} f_{n_2} + \dots + c^{k} f_{k} + \cdots \qquad (19)$$

Then f_c is equivalent to the deforming function f_c' where no component f_{n_i} or any of its part can be identified with the 2-coboundary $b \in B^2(X,X)$.

Proof

The first nonzero term of $\bar{f}_c$ can be considered as an element of $Z^2(X,X)$ not cohomologous to zero /10/. Moreover it cannot be presented as a linear combination of cocycles and coboundaries. Let f_m be the first component in Eq.(19) having the form

$$f_m = \tilde{f}_m + \delta \varphi_m \qquad (20)$$

Suppose $\Phi_c = 1 - c^m \varphi_m$ to be the element of $GL(V)$ group. Algebras X and $(\Phi_c \cdot X)$ are equivalent. Let us find what changes do occur in the deformation function f_c under such transformation.

$$\Phi_c^{-1} f_c(\Phi_c u_1, \Phi_c u_2) = f_c(u_1,u_2) - c^m f_c(\varphi_m(u_1),u_2) -$$

$$- c^m f_c(u_1,\varphi_m(u_2)) + c^m \varphi_m(f_c(u_1,u_2)) + \dots =$$

$$= \mu(u_1,u_2) + c^{n_1} f_{n_1}(u_1,u_2) + \dots + c^m f_m(u_1,u_2) - \qquad (21)$$

$$- c^m (\delta \varphi_m)(u_1,u_2) + \cdots$$

Thus in every component f_m of the type presented in Eq.(20) the $\delta\varphi_m$ term can be considered as an additional isomorphic transformation of the initial algebra X. Such functions have no reference to the nontrivial deformations of X and can be compensated by the appropriate isomorphic transition from X to $\widetilde{X}$. The regular operator Φ_c transform those components of f_c that come after the f_m. But whatever these changes are one can always step by step eliminate 2-coboundaries from all the elements of f_c.

The above Lemma will help us in constructing high order deformations.

Theorem I (of quasistability)

The only difference between the nontrivial first order coupling f_1^h (Eq.(13)) of the space-time $P = T \oplus L$ and internal $A = G \oplus S$ symmetry algebras and the corresponding coupling obtained in high order deformations is that in the latter case all the elements of the radical G can commute with $t \in T$ nontrivially.

Proof

Let us first consider the problem of correspondence between the high order and repeated deformations of $P \oplus A$.

Proposition 1. There are physically important high order deformations of $P \oplus A$ unequivalent to any of repeated deformations.

Suppose $f_2^A : G \wedge G \to A$ is the element of $Z^2(U,U)$ with the following restrictions

$$f_2^A(g_1, g_2) = \begin{cases} s \in S \text{ for commuting } g_1 \text{ and } g_2 \\ g' + g^{\dagger} \text{ for noncommuting } g_1 \text{ and } g_2 \end{cases} \qquad (22)$$

where $g' \in G \cap A'$ and $f_1^h(t, g^{\dagger}) \neq 0$ (see Eq.(13)). The function f_2^A thus defined do not commute with f_1^h and what is more one can find such a function $f_3^{\tilde{h}}$ that

$$[f_1^h, f_2^A] = -\delta f_3^{\tilde{h}} \qquad (23)$$

Using the definitions (2) and (3) it is possible to reformulate Eq.(23).

$$f_1^h(t, f_2^A(g_1, g_2)) = -f_3^{\tilde{h}}(t, \mu(g_1, g_2)) \quad (24)$$

Consequently

$$f_3^{\tilde{h}}(t, g') = \tilde{h}(g' = \mu(g_1, g_2))\, t =$$

$$= h(f_2^A(g_1, g_2) = g' + g^{+}) \cdot t \quad (25)$$

where $\tilde{h}$ is a constant depending on g'.

Thus the functions $f_2^A \in Z^2(U,U)$ and $f_3^{\tilde{h}}$ can be constructed so that Eq.(23) with f_1^h - the first order deforming component is valid. This immediately reduces the whole problem (of high order deformations of $P \oplus A$) to the second part of Lemma I. So it is necessary to study the high order and repeated deformations of U separately.

Two facts in the proposition 1 are worth noticing. The first is connected with the physical meaning of this or that deforming component. Eq.(23) is important not only due to the existence of nonzero functions f_2^A and $f_3^{\tilde{h}}$ but because $f_3^{\tilde{h}}$ couples P and A and differs from the known function f_1^h.

The second significant fact is that one can add to $f_3^{\tilde{h}}$ determined by (25) any element of $Z^2(U,U)$ (excluding the coboundaries, see Lemma II) not violating Eq.(23). So $f_3^{\tilde{h}}$ is determined with an accuracy of a 2-cocycle not cohomologous to zero.

Proposition 2. Only three types of functions f_n^A, f_n^h and $f_n^{\tilde{h}}$ can exist in any physically nontrivial high order deforming function $\bar{f}_c$ of $P \oplus A$.

Suppose f_c presented in Eq.(19) is a deforming function of $P \oplus A$ with only those nonzero components which are essential for the P-A-coupling. The first nonzero function f_{n_1} may be of the type $f_{n_1}^P : P \wedge P \to P$, $f_{n_1}^A : A \wedge A \to A$, or $f_{n_1}^h$ (Eq.(13)). According to

Lemma II all coboundaries are excluded. The function of the first type $f_{n_1}^P$ will lead to the semi-simple De-Sitter algebras /9/ which are strongly stable. This puts the end to any attempt of violating the direct sum structure. Such functions as f^P are incompatible with any P-A-coupling on any step of high order deformation. So the nontrivial deformations of the Poincaré subalgebra are excluded from the very beginning. The reason will be carefully investigated in Section 5.

Let us put $f_{n_1} = f_{n_1}^A$ and rewrite the relation (9) in the detailed form

$$\sum_{P(1,2,3)} f_{n_1}^A(u_1, f_{n_1}^A(u_2,u_3)) =$$

$$= -\sum_{P(1,2,3)} \left[\mu(u_1, f_{2n_1}(u_2,u_3)) + f_{2n_1}(u_1, \mu(u_2,u_3))\right] \qquad (26)$$

where $u_i \in U$. It is easy now to check that only those f_{2n_1} meet the requirement (26) nontrivially (with nonzero right hand side of Eq.(26)) which are the internal maps $A \wedge A \rightarrow A$. And on the left hand side of Eq.(26) one can put a composition $[f^A, f'^A]$ of any two different $A \wedge A \longrightarrow A$ homomorphisms. The result will be just the same. It was mentioned above (proposition 1) that any 2-cocycle may be included in f_{2n_1}. Thus the following decomposition is true.

$$f_{2n_1} = f_{2n_1}^A + f_{2n_1}^h \qquad (27)$$

where the possible $f^A \in Z^2(A,A)$ is joint with the $f_{2n_1}^A$ determined by (26). If f_{2n_1} in (19) is zero the first order deformation of A must exist and the next nonzero component f_{n_2} is again an element of $Z^2(U,U)$. If the left hand side of (26) equals zero f_{2n_1} is the 2-cocycle. In both cases the corresponding function has the form analogous to (27) with the cocyclic first term.

In all cases the next deforming component to be investigated f_{3n_1} or $f_{n_1+n_2}$ is determined by compositions of the two following types

$$[f^A, f'^A], \quad [f^A, f^h] \qquad (28)$$

The first commutator, if not zero, may be equal to δf^A. As it was shown in the proposition 1 the second composition may give rise to the function $f^{\tilde h}$ defined by (25). Again the 2-cocycles f^h and f^A must be taken into consideration and as the result we have

$$\left.\begin{matrix} f_{3n_1} \\ \\ f_{n_1+n_2} \end{matrix}\right\} = f^A + f^h + f^{\tilde h} \qquad (29)$$

On the next step besides the compositions (28) one must consider also the following

$$[f^h, f^{\tilde h}], \quad [f^{\tilde h}, f^{\tilde h}], \quad [f^A, f^{\tilde h}] \qquad (30)$$

Using the explicit form of compositions (30) and definitions (13) and (25) it is easy to check that first two types of compositions equal zero. One can prove that the last one is a 3-coboundary.

$$[f^A, f^{\tilde h}](t, g_1, g_2) \equiv f^{\tilde h}(t, f^A(g_1, g_2)) =$$

$$= \delta f'^{\tilde h}(t, g_1, g_2) = f'^{\tilde h}(t, \mu(g_1, g_2)) \qquad (31)$$

The composition is nontrivial only if the function f^A of noncommuting arguments from $\mathcal{G}$ gives an element $g' \in A'$. In this case the corresponding coboundary exists with the function $f'^{\tilde h}$ which is of the same type as $f^{\tilde h}$

$$\tilde h'(\mu(g_1, g_2)) = \tilde h(f^A(g_1, g_2) = g' + g^t) \qquad (32)$$

Thus the deforming components on this level have the same decomposition as defined for the previous level in Eq.(29). No new types of functions can appear in any subsequent component of $\bar{f}_c$.

If one suppose $f_{n_1} = f^h_{n_1}$ the result will be just the same: first functions (27) will appear and then that of the decomposition (29). One can obtain the full number of different types of functions on the second stage supposing the first nonzero component f_{n_1} to be linear combination of f^h- and f^A-types.

The proposition 2 proved above has the following important consequence. In any nontrivial high order deformation of $P \oplus A$ there may be only two types of coupling between P and A subalgebras.

$$f^h(t,g) = \begin{cases} 0 & \text{for } g = g' \in A' \\ h(g)\cdot t & \text{for } g = g^\dagger \notin A' \end{cases} \qquad (33)$$

$$f^{\tilde{h}}(t,g) = \begin{cases} \tilde{h}(g)\cdot t & \text{for } g = g' \in A' \\ 0 & \text{for } g = g^\dagger \notin A' \end{cases}$$

Functions f^h and $f^{\tilde{h}}$ together can make all the elements of the radical $\mathcal{G}$ noncommutative with the translations $t \in T$. The only difference between f^h and $f^{\tilde{h}}$ is that they couple $t \in T$ with the diverse subspaces of $V_{\mathcal{G}}$. The numerical coefficients are intimately connected through the relation (25). But f^h plays in our problem more significant role because it is necessarily the first function to break the direct sum structure of U. $f^{\tilde{h}}$ may appear only in the presence of f^h. Thus it is essential that the initial internal symmetry algebra A has the elements of radical which do not belong to A'. The nonzero component of the f^A-type is also necessary for $f^{\tilde{h}}$-coupling. The f^A functions are connected with both f^h and $f^{\tilde{h}}$ by relations (32) and (25)

$$\begin{array}{ccc} h\,(f^A(g_1,g_2) = g' + g^\dagger) & = & \tilde{h}(f^A(g_1,g_2) = g' + g^\dagger) \\ \| & & \| \\ h(g^\dagger) & = & \tilde{h}(g') \end{array} \qquad (34)$$

The function f^A of commuting arguments cannot be equal to other than $s \in S$ elements. On noncommuting g_1 and g_2 this function can have arbitrary values. But there are no components of $f^{\tilde{h}}$-type if $f^A(g_1, g_2)|_{\mu(g_1,g_2)\neq 0} \notin G \cap A/A'$ and $f^{\tilde{h}}$ functions will appear only once in $\bar{f}_c$ if $f^A(g_1, g_2)|_{\mu(g_1,g_2)\neq 0} \notin G \cap A'$

Thus we have determined general form of the high order physically nontrivial deformations of $U = P \oplus A$.

$$f_c = \mu + \sum_{i=1} c^i \left(f_i^h + f_i^A \right) + \sum_{\ell=2} c^\ell f_\ell^{\tilde{h}} \qquad (35)$$

This function gives all the characteristics of high order deformations of $P \oplus A$ which are important for the construction of the generalized symmetry.

The theorem I is thus proved.

5. Theorem of Quasistability II

It was mentioned in Section 3 that the high order deformations do not necessarily cover all the algebras "close" to the given one. In the neighbourhood of the initial algebra X there may be some algebras that can be presented only as repeated deformations of X. The existence of such deformations depends mainly upon the properties of the 2-cocycles group $Z^2(X_c, X_c)$ where X_c is the first order deformation of X. In this section we shall investigate the subgroup of $Z^2(U_c, U_c)$ whose elements can couple P and A in U_c nontrivially. Some relations obtained in the article /13/ will be used.

Theorem II (of quasistability)

If the independent deformations of the internal symmetry algebra A are not taken into account all the repeated deformations of U_c (14) are trivial.

Proof

The 2-cohomology group $H^2(U_c, U_c)$ describing the nontrivial infinitesimal deformations of U_c can be simplified due to Serre and Hochschild's formula (11).

$$H^2(U_c,U_c) = H^2(T \oplus G, U_c)^{U_c} = H^2(\mathcal{J}_c, U_c)^{U_c} \qquad (36)$$

where $H^2(\mathcal{J}_c,U_c)^{U_c}$ is the group of U_c-invariant elements in $H^2(\mathcal{J}_c,U_c)$.

As usual we reformulate the U_c-invariance for the elements of 2-cocycles group $Z^2(\mathcal{J}_c,U_c)$

$$\begin{aligned}(u_c \cdot f)(j_c,j_c') \equiv [u_c, f(j_c,j_c')] - f([u_c,j_c],j_c') - \\ - f(j_c,[u_c,j_c']) = [j_c, \rho(j_c')] - \\ - [j_c', \rho(j_c)] - \rho([j_c,j_c'])\end{aligned} \qquad (37)$$

where $j_c, j_c' \in \mathcal{J}_c$, $u_c \in U_c$ and ρ denotes the map $\mathcal{J}_c \longrightarrow U_c$.

<u>Proposition 1</u>. The condition of U_c-invariance for the elements of $Z^2(\mathcal{J}_c,U_c)$ can be presented in the following form

$$(u_c \cdot f)(t_1,t_2) = [t_1, \rho(t_2)] - [t_2, \rho(t_1)] \qquad (38)$$

$$\begin{aligned}(u_c \cdot f)(g_1,g_2) = [g_1, \rho(g_2)] - [g_2, \rho(g_1)] \\ - \rho([g_1,g_2])\end{aligned} \qquad (39)$$

$$\begin{aligned}(u_c \cdot f)(t,g) = [t, \rho(g)] - [g, \rho(t)] - \\ - \rho([t,g])\end{aligned} \qquad (40)$$

These relations as well as the corresponding equations in /13/ (/13/-5,6,7) are the direct consequence of the condition (37) and the linearity of all the operators used.

We shall use the following decomposition of the deforming function $f = f_L + f_S + f_T + f_G$.

<u>Proposition 2</u>. Functions $f_L(t_1,t_2)$, $f_S(t_1,t_2)$ and $f_G(t_1,t_2)$ are noninvariant. The function $f_T(t_1,t_2)$ is U_c-invariant and as a consequence of G-invariance it is a

2-coboundary.

Let us put $u_c = g \in G$ in Eq.(38) then using the explicit form of the commutator

$$[t_i, g]\Big|_{g \notin G \cap A'} = h(g) \cdot t_i \cdot c \qquad (41)$$

we have

$$2h(g) \cdot c \cdot f_L(t_1, t_2) = 0 \qquad (42)$$

As distinct from the algebra $U = P \oplus A$ it is impossible to deform the subalgebra P of U_c nontrivially.

Let $u_c = s \in S$ in Eq.(38), then it is easy to see that $f_S(t_1, t_2)$ is not S-invariant. The corresponding relation coincides with those obtained for the $P \oplus A$ infinitesimal deformations (Eq./13/-8).

When $u_c = \ell \in L$ we have for the function $f_G(t_1, t_2)$ the following relation

$$f_G([\ell, t], t_2) + f_G(t_1, [\ell, t_2]) = 0 \qquad (43)$$

In /13/ it was demonstrated that such equations can be solved only for zero homomorphisms f_G. They are thus not L-invariant.

Eq.(38) gives for $u_c = g \in G$ the coboundary condition for $f_T(t_1, t_2)$

$$h(g) \cdot c \cdot f_T(t_1, t_2) = [t_1, \rho(t_2)] - [t_2, \rho(t_1)] \qquad (44)$$

where ρ is a homomorphism $J_c \to U_c$. In contrast with $f_L(t_1, t_2)$ the deformed structure of U_c has not changed the properties of $f_T(t_1, t_2)$.

<u>Proposition 3</u>. Functions $f_L(g_1, g_2)$ and $f_T(g_1, g_2)$ are coboundaries.

If $u_c = \ell \in L$ in Eq.(39) one obtains for the homomorphisms $f_L(g_1, g_2)$ and $f_T(g_1, g_2)$ the following equation

$$[\ell, f_{L,T}(g_1, g_2)] = -\rho_U([g_1, g_2]) \qquad (45)$$

As it was proved in /13/ for U this relation could be fulfilled only if

$$f_{L,T}(g_1,g_2) = -\tilde{\rho}_{L,T}([g_1,g_2]) \qquad (46)$$

Eq.(46) is the coboundary condition for the left hand side. The same arguments are true also for U_c.

<u>Proposition 4</u>. Functions $f_L(t,g)$, $f_S(t,g)$ and $f_{\mathcal{G}}(t,g)$ are coboundaries.

Let us put in Eq.(40) $u = \ell \in L$, then for the homomorphisms $f_L(t,g)$ we come to the following restriction

$$[\ell, f_L(t,g)] - f_L([\ell,t],g) = -\rho_L([t,g]) \qquad (47)$$

For $g = g' \in A'$ the left hand side of Eq.(47) equals zero. Such relation was investigated in /13/ and it was demonstrated using the internal structure of P subalgebra that all the L-invariant functions are zero. The Poincaré subalgebra remains unchanged in U_c. So the same is true in our case, when the function $f_L(t,g')$ is studied.

Consider now Eq.(40) for $u_c = g_2 \in \mathcal{G}$ and let $g_1 = g^\dagger \in \mathcal{G} \cap A/A'$ then for $f_L(t,g^\dagger)$ we have

$$-f_L([g_2,t],g^\dagger) - f_L(t,[g_2,g^\dagger]) = -\rho_L([t,g^\dagger]) \qquad (48)$$

It was demonstrated above that the second term in Eq.(48) must be zero. Using the explicit form of the commutator in the first term we obtain the coboundary condition for $f_L(t,g^\dagger)$

$$h(g_2)\cdot c\cdot f_L(t,g^\dagger) = -\rho_L([t,g^\dagger]) \qquad (49)$$

Let again $u_c = \ell \in L$ in Eq.(40) and consider maps $f_S(t,g)$ and $f_{\mathcal{G}}(t,g)$. One comes to the following relation

$$f_S([\ell,t],g') = 0 \qquad (50)$$

The properties of the Poincaré subalgebra allow only zero maps f_S to fulfill Eq.(50). For $f_S(t,g^\dagger)$ and $f_{\mathcal{G}}(t,g)$ with an arbitrary g the restrictions obtained from Eq.(40) are similar to those investigated in /13/ for $f_G(t,g)$. It is possible to repeat all the arguments used in /13/ to find that $f_S(t,g^\dagger)$ and $f_G(t,g)$ are coboundaries.

<u>Proposition 5</u>. The L-invariant maps $f_T(t,g)$ can be devided into two parts. The diagonal part $f_T^d(t,g)$ for $g=g^\dagger$ and the nondiagonal part $f_T^{nd}(t,g)$ for arbitrary arguments are coboundaries.

The L-invariance condition for $f_T(t,g)$ has the following explicit form

$$[l, f_T(t,g)] - f_T([l,t],g) = [t, \rho(g)] - \\ -[g, \rho_T(t)] - \rho_T([t,g]) \qquad (51)$$

Using the commutation relations (41) one can easily verify that the last two terms in (51) mutually compensate. Here the Poincaré algebra conventional commutation relations are necessary

$$\begin{aligned} &[l_{\mu\nu}, t_\gamma] = -\bar{g}_{\mu\gamma} t_\nu + \bar{g}_{\nu\gamma} t_\mu \\ &[l_{\mu\nu}, l_{\gamma\sigma}] = -\bar{g}_{\mu\gamma} l_{\nu\sigma} - \bar{g}_{\nu\sigma} l_{\mu\gamma} + \bar{g}_{\nu\gamma} l_{\mu\sigma} + \bar{g}_{\mu\sigma} l_{\nu\gamma} \qquad (52) \\ &[t_\rho, t_\sigma] = 0 \end{aligned}$$

In this basis the following matrix forms of the functions f_T and ρ will be used

$$f_T(t_\gamma, g) = d_\gamma^\sigma(g) \cdot t_\sigma \qquad (53)$$

$$\rho(g; l) = \rho^{\alpha\beta}(g; l)\, l_{\alpha\beta} \;; \quad \alpha < \beta \qquad (54)$$

As a consequence of Eq.(51) for matrix elements (53) and (54) one has conditions

$$-d_\gamma^\sigma(g) \cdot \bar{g}_{\alpha\sigma} \cdot \delta_{\beta\mu} + d_\gamma^\sigma(g) \cdot \bar{g}_{\beta\sigma} \cdot \delta_{\alpha\mu} + d_\beta^\mu(g) \cdot \bar{g}_{\alpha\gamma} - \\ -d_\alpha^\mu(g) \cdot \bar{g}_{\beta\gamma} = \rho^{\nu\mu}(g; l_{\alpha\beta}) \bar{g}_{\nu\gamma} - \rho^{\mu\eta}(g; l_{\alpha\beta}) \bar{g}_{\eta\gamma} + \\ + \rho(g_i; l_{\alpha\beta}) \cdot h(g_i) \cdot c\, \delta_{\gamma\mu} \qquad (55)$$

where g_i - the basic elements of G and summation is performed over all the repeated indices including i .

The essential property of Eq.(55) is that for the diagonal matrix elements d^μ_μ the last term of this equation is zero. So the restrictions obtained in /13/ for d^μ_μ are also true here

$$d^\mu_\mu(g) = d^\nu_\nu(g) = h_2(g); \quad \mu,\nu = 0,1,2,3 \qquad (56)$$

If only the nondiagonal part of $f_T(t,g)$ is considered the coefficients $\rho(g_i;\ell_{\alpha\beta})$ in Eq.(55) vanish. Thus the L-invariance condition for $f^{nd}_T(t,g)$ coincides with that defined in /13/

$$\bar{g}_{\alpha\alpha}/\bar{g}_{\beta\beta} \cdot d^\alpha_\beta(g) = -d^\beta_\alpha(g) \qquad (57)$$

This condition was used in /13/ to prove that $f^{nd}_T(t,g)$ is a coboundary. Here the situation is just the same.

The diagonal function $f^d_T(t,g^\dagger)$ for $g^\dagger \in G \cap A/A'$ defined by condition (56) coinsides with the deforming function of the first order deformation of U (Eq. (13)). The deformation of the algebra U_c with $f^d_T(t,g^\dagger)$ may change only the constants $h(g^\dagger)$ in the commutation relations (41) of U_c with respect to newly defined $h_2(g)$ in Eq.(56). This procedure is equivalent to the redefinition of the basis of U_c . For example it is easy to find the map ρ necessary for the coboundary condition

$$f^d_T(t,g^\dagger) = [t, \rho(g^\dagger)]$$

$$\rho(g^\dagger) = \frac{h_2(g^\dagger)}{c \cdot h(g^\dagger)} \cdot g^\dagger$$

The proposition 5 is thus proved.

The following reasoning concludes the proof of the Theorem.

The conditions of G- and S-invariance do not provide for $f^d_T(t,g')$ (where $g' \in A'$) any simple restrictions. But the explicit form of the cocyclic condition (8) makes it possible to verify the following

$$f_T^d(t, g' \in G') \notin Z^2(J_c, U_c)$$

$$f_T^d(t, g' \in [S, G]) \notin Z^2(U_c, U_c) \qquad (58)$$

So the functions under consideration can not be deforming.

Thus we have proved that the structure of the U_c algebra is stable with an accuracy of independent deformations of A .

The important consequence of Theorem II is the stability of the Poincaré subalgebra. As subalgebra of $U = P \oplus A$ it is definitely deformable. The corresponding deforming function has the following form /9/

$$f_L(t_i, t_k) = \delta_{in}\,\delta_{km}\,l^{nm} \qquad (59)$$

This function allows the first order deformation and transforms P into the semi-simple De-Sitter algebras $\mathfrak{D}$. Its strong stability makes any further nontrivial coupling between $\mathfrak{D}$ and A impossible. But in U_c the Poincaré subalgebra is strongly stable. This may occur only if there is an obstruction to such deformations. The composition $[f_L, f_L]$ cannot lead to any obstruction. So it is the $[f_L, f_T^d]$ composition which must be equal to the element of $Z^3(U,U)$ not cohomologous to zero.

$$\tfrac{1}{2}[f_L, f_T^d](t_\alpha + g_\alpha, t_\beta + g_\beta, t_\mu + g_\mu) = h(g_\alpha)\,\bar{g}_{\mu\mu}\,\bar{g}_{\beta\beta}\,l^{\mu\beta} - h(g_\beta)\,\bar{g}_{\mu\mu}\,\bar{g}_{\alpha\alpha}\,l^{\alpha\mu} - h(g_\mu)\,\bar{g}_{\alpha\alpha}\,\bar{g}_{\beta\beta}\,l^{\beta\alpha} \qquad (60)$$

It is impossible to construct the whole group $H^3(U,U)$ because the properties of the direct summand A are not fixed. None the less in /13/ the following 3-cocycle was determined

$$f_L(t_\alpha, t_\beta, g_\mu) = k(g_\mu)\,\bar{g}_{\alpha\alpha}\,\bar{g}_{\beta\beta}\,l^{\beta\alpha} \qquad (61)$$

The right hand side of Eq.(60) is a linear combination of such functions and therefore is the element of $H^3(U,U)$. So in the deforming function the components f_L and f_T^d cannot appear together. Over and above Eq.(60) indicates that the infinitesimal deforming function $f_1 = f_L + f_T^d$ is not integrable. The point μ corresponding to the algebra U on $\mathcal{M}$ is singular (Section 2) and one can formulate the conclusion: "If in the direct sum $P \oplus A$ the radical G of A is solvable and includes elements not belonging to A' then the corresponding point of the set of all Lie compositions is not simple and the set of all nontrivial nonequivalent deformations splits into at least two irreducible components: the deformation of P and that of the direct sum structure in U ."

6. Discussion and Examples.

The high order deformations investigated in Section 4 and the repeated deformations studied in Section 5 show that functions (33) totally define the admissable form of coupling between space-time and internal algebraic symmetries. The repeated deformations of U_c can provide only the additional transformation of the internal symmetry. The high order deformation can change the subspace of internal symmetry generators directly coupled with translations.

As it was indicated in /13/ the first order deformations of $P \oplus A$ give no possibility to construct the mass-spectrum. The principal objection is the stability of both P and A subalgebras. Or, more generally , the fact that both P and A remain subalgebras of U_c . The investigation carried out in Sections 4 and 5 shows that this main difficulty still remains. If any P-A-coupling appears during the deformation of arbitrary order or type the Poincaré algebra as well as A or any of its deformations A_c remain subalgebras of U_c .

But it is possible to consider only A to be the precise internal symmetry algebra. Then the no-go theorems cannot be used for the deformed internal symmetries A_c . If one consider the redefinition of invariant operators /15/ physically admissible the mass-splitting appears to be possible.

To illustrate the situation we present a very simple example. Let A be the algebra of 2-dimensional pseudoeuclidian transformations with 1-dimensional centre, $A = G$.

$$[g_1, g_3] = g_2; \qquad [g_1, g_2] = 0;$$
$$[g_2, g_3] = g_1; \qquad [g_i, g_4] = 0; \quad i = 1,2,3$$

This algebra has the second order Cazimir operator $(g_1^2 - g_2^2)$. It is possible to deform $P \oplus G$ with the function $\bar{f}_c = c\left(f_1^h + f_1^A\right)$

where $f_1^h(t_\alpha, g_4) = c \cdot h(g_4) \cdot t_\alpha$

$$f_1^A(g_i, g_j) = -c \cdot h(g_4) \sum_{k=1,2} g_k \left(\delta_{ik}\delta_{j4} - \delta_{i4}\delta_{jk}\right)$$

We shall write down only those commutators that change

$$\underset{k=1,2}{[g_k, g_4]} = -h(g_4) \cdot c \cdot g_k; \quad [t_\alpha, g_4] = h(g_4) \cdot c \cdot t_\alpha$$

Consider now the operator $M^2(g_1^2 - g_2^2)$. It is the Cazimir operator of the U_c algebra. In the limit $c \to 0$ the operators M^2 and $(g_1^2 - g_2^2)$ are both invariant in U . If one considers the system that might be characterized by the invariant $(g_1^2 - g_2^2)$ the corresponding mass-splitting will appear.

Besides the problem of mass-splitting the obtained information about the relativizability of various internal symmetries and the unique character of possible coupling betweem space-time and internal symmetries is of considerable physical importance. The investigation shows that our knowledge about the principles of symmetry breaking is not yet adequate. The discontinuity of the process of switching off the uninvariant parts of interactions seems to be the most probable feature which must be taken into consideration in the detailed model of symmetry breaking. The author is grateful to Dr H. Ŏiglane of the Institute of Physics and Astronomy for indicating this possibility.

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Резюме

В работе рассматриваются деформации высших порядков прямой суммы алгебр Пуанкаре и внутренней симметрии. Установлено, что существуют деформации высших порядков, неэквивалентные последовательности деформаций первого порядка (повторным деформациям). Теоремы квазистабильности нетривиальной связи пространственно-временных и внутренних свойств системы (установленные для бесконечно малых деформаций) доказаны как для повторных деформаций, так и для деформаций высших порядков. Применение полученных обобщенных алгебр симметрии в проблеме спектра масс встречает серьезные трудности.

Contents

В. ЛЯХО́ВСКИЙ

ОБОБЩЕННЫЕ СИММЕТРИИ И ДЕФОРМАЦИИ ВЫСШИХ
ПОРЯДКОВ ПРЯМОЙ СУММЫ $P \oplus A$

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