DISSERTATIONES MATHEMATICAE UNIVERSITATIS TARTUENSIS

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MATRIX TRANSFORMATIONS OF SUMMABILITY AND ABSOLUTE SUMMABILITY FIELDS OF MATRIX METHODS

by

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ABSTRACT OF THE INVESTIGATIONS PRESENTED TO OBTAIN THE ACADEMIC DEGREE OF A DOCTOR OF MATHEMATICS

by A. Aasma

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* * *

Ants Aasma was born in 1957. Graduated from the Tartu University in 1980, postgraduate student in 1983-1986 in the same university, a teacher in 1987-1992 and since 1992 a lecturer in the Pedagogical University of Tallinn. Author of 8 scientific paper from which 7 about the theory of summability.

INTRODUCTION

The considerable dissertation belongs to the theory of summability of series and sequences . The aim of this work is to study matrix transformations from the summability (or absolute summability) field of a matrix method of summability into the summability (or absolute summability) field of another matrix method of summability. In the case when the transformation matrix has a diagonal form the considerable problem is reduced to the problem of summability (or absolute summability) factors, which have been widely investigated (cf., for example, the S. Baron's monography [36] and the articles [7,8,9,10-14,17-28,34,35,37]).

Up to now for solving the above-mentioned problem both functional analitic methods and the methods which use mainly the results from the classical theory of summability have been considered by different authors. In [15,16] the necessary and sufficient conditions for the matrix that it would transform a sequence space into another sequence space have been obtained. These conditions have been given by the properties of certain kind dual (so called the γ-dual and the second γ-dual) spaces of these sequence spaces. In the case when these sequence spaces are summability fields of summability methods the results of [15,16] are available but to describe the dual space and the second dual space for given summability field of summability method is complicated enough , not to speak about its properties. Therefore, for solving the above-mentioned problem methods which use only the properties of considerable methods of summability and the properties of continuous linear functionals on summability fields of these methods of summability are considered.

The first result in the case of non-diagonal matrix (mainly by the classical methods of theory of summability) obtained L. Alpár in 1978 (cf. [4]). He found the necessary and sufficient conditions for matrix $M = (m_{nk})$ that the transformation

$$y_n = \sum_k m_{nk} x_k \qquad (1)$$

transforms each convergent series $\sum_k x_k$ to C^{α}-summable series $\sum_n y_n$ for $\alpha \geq 0$. After that, in 1980, he found the necessary and sufficient conditions in order that the matrix M transforms each C^{α}-summable series to $C^{\alpha+\beta}$-summable series for $\alpha, \beta \geq 0$ (cf. [5]) and, in 1982, he generalized the above-mentioned result looking now at the method C^{β} instead of the method $C^{\alpha+\beta}$ (cf.[6]). In addition, in 1986, B. Thorpe generalized the result of L. Alpár (also mainly by the classical methods of the theory of summability) considering now instead of the method C^{β} an arbitrary normal method of summability B (cf.[29]). In this paper he found also the necessary and sufficient conditions in order that the matrix M would transform each C^{α}-summable series to B-summable series in the case when $-1 \leq \alpha \leq 0$ and B is a normal method.

In the present dissertation this problem is considered more generally studing matrix transformations from a summability (or an absolute summability) field of a method A into the summability (or an absolute summability) field of another method B in the case when A is a regular perfect or a reversible method and B is an arbitrary triangular or an arbitrary method. The cases when A or A and B both are Cesàro or Riesz methods as applications are considered.

All results of dissertation have been obtained in the period of 1984-1990 and introduced in the seminars of the department of mathematical analysis of Tartu University (in 1984-1986, 1989 and 1991), at the conferences "Problems of pure and applied mathematics" (1985, 1990) and "Methods of algebra and analysis" (1988) and in the seminar of theory of function in Ural State University (1986).

The main results of the present dissertation have been published in [1-3,31-33].

MAIN RESULTS

The present dissertation includes an introduction, two chapters, which both consist of three paragraphs, references and the table of basic symbols. All this material has been presented on 78 pages.

In the introduction a short review of the subject, purposes and the structure of dissertation are given .

In the first chapter basic notations and notions are given which often are used later on. As follows we shall give a short account of it.

Let $M = (m_{nk})$ be a matrix over $\mathbb{C}$. We shall often write (1) in the form $y = Mx$ or $y = (M_n x)$ where $M_n x = y_n$ as usual. Moreover, let ω be the set of all number sequences, in which the algebraic operations have been defined coordinate-wise, $\varkappa$ and μ be the subsets of ω and $A = (\alpha_{nk})$ be a matrix over $\mathbb{C}$. We use the following notations:

$c = \{ x = (x_n) |\ x \in \omega$ and there exists finite limit $\lim_n x_n \}$,

$c^0 = \{ x = (x_n) |\ x \in c$ and $\lim_n x_n = 0 \}$,

$cs = \{ x = (x_n) |\ x \in \omega$ and the series $\sum_n x_n$ is convergent$\}$,

$bv = \{ x = (x_n) |\ x \in \omega,\ \sum_n |x_n - x_{n-1}| < \infty$ and $x_{-1} = 0\}$,

$bv^0 = \{ x = (x_n) |\ x \in bv$ and $\lim_n x_n = 0 \}$,

$\varkappa_A = \{ x = (x_n) |\ x \in \omega$ and $(A_n x) \in \varkappa \}$,

$c^0_A = \{ x = (x_n) |\ (A_n x) \in \omega$ and $\lim_n A_n x = 0\}$,

$b_M = \{ x = (x_k) |\ (M_n x) \in \omega$ and $\sum_{k=1}^{\infty} m_{nk} x_k = O(1) \}$.

and

$(\varkappa, \mu) = \{ M = (m_{nk}) |\ m_{nk} \in \mathbb{C}$ and $Mx \in \mu$ for each $x \in \varkappa \}$.

The spaces c_A and bv_A are usually called a summability field and an absolute summability field of method A respectively.

Definition. *Let $M = (m_{nk})$ be a matrix. We say that two methods $A = (\alpha_{nk})$ and $B = (\beta_{nk})$ are M-consistent if*

$$\lim_n A_n x = \lim_n \sum_k \beta_{nk} M_k x$$

for each $x \in c_A$.

It is easy to see what M-consistency of methods of summability coincides with the ordinary consistency of them if M is an identity matrix.

The main results of the dissertation are proved in §2 and §3 of Chapter I . The necessary and sufficient conditions for the matrix transformations from c_A into c_B are obtained by the method of Peyerimhoff (cf., for example, [24]), but for the matrix transformations from c_A into c_B ,from c_A into bv_B ,from bv_A into c_B and from bv_A into c_B - by the inverse transformation method (cf., for example, [34]).

Now we shall give the results of Chapter I in greater detail. For that let $e = (1,1,\dots)$ and $e^k = (0,\dots,0,1,0,\dots)$ where 1 is in k-th position. In §2 it is assumed that $A = (\alpha_{nk})$ is a series-to-sequence and $\mathfrak{A} = (a_{nk})$ is a sequence-to-sequence transformation. If $\Delta = \{e^0, e^1, \dots\}$ and $\Delta \cup \{e\}$ are fundamental sets for c_A and $c_{\mathfrak{A}}$ respectively then the methods A and $\mathfrak{A}$ are called perfect. Here it is assumed that $c^0_{\mathfrak{A}}$ (or equivalently with it $c_{\mathfrak{A}}$) and c_A are BK-spaces (i.e. the Banach spaces where coordinate-wise convergence holds). The topologies in c_A and in $c^0_{\mathfrak{A}}$ are defined respectively by the norm $\|x\|_{c_A} = \sup_n |A_n x|$ (for each $x \in c_A$) and by the norm $\|x\|_{c^0_{\mathfrak{A}}} = \sup_n |\mathfrak{A}_n x|$ (for each $x \in c^0_{\mathfrak{A}}$). It is assumed that $B = (\beta_{nk})$ is a triangular matrix over $\mathbb{C}$, $M = (m_{nk})$ is an arbitrary matrix over $\mathbb{C}$ and $G = (g_{nk})$ is the product of above-mentioned matrices, i.e.

$$g_{nk} = \sum_{s=0}^{n} \beta_{ns} m_{sk} .$$

Using the method of Peyerimhoff (which is based on the properties of continuous linear functionals on $c^0_{\mathfrak{A}}$ and c_A) the matrix transformations from $c_{\mathfrak{A}}$ into c_B and from c_A into c_B are studied in §2 in the case when $\mathfrak{A}$ and A are the regular perfect methods.

Theorem 1. *Let $\mathfrak{A} = (a_{nk})$ be such a regular perfect method that $c^0_{\mathfrak{A}}$ is a BK-space, $B = (\beta_{nk})$ be a triangular method and $M = (m_{nk})$ be an arbitrary matrix. Then $M \in (c_{\mathfrak{A}}, c_B)$ if and only if $(m_{sk}) \in cs$ for each $s \in \mathbb{N}$,*

1) *there exist finite limits $\lim_n g_{nk} = g_k$,*

2) *there exists finite limit* $\lim_n \sum_k g_{nk} = g$ *and there exist such functionals* $f_{sl} \in (c^o_{\mathfrak{A}})'$ *that*

$$f_{sl}(e^k) = \begin{cases} m_{sk}, & \text{if } k \le l, \\ 0, & \text{if } k > l, \end{cases}$$

$$\|f_{sl}\|_{(c^o_{\mathfrak{A}})'} = O_s(1)$$

and

$$\|F_n\|_{(c^o_{\mathfrak{A}})'} = O(1)$$

where the functionals F_n *have been defined on* $c^o_{\mathfrak{A}}$ *by*

$$F_n(x^o) = \sum_{s=0}^{n} \beta_{ns} f_s(x^o)$$

and

$$f_s(x^o) = \lim_l f_{sl}(x^o) .$$

Moreover, if in addition $g_k \equiv 0$ *and* $g = 1$, *then the methods* $\mathfrak{A}$ *and B are M-consistent.*

An analog of Theorem 1 for a regular perfect method *A* too is presented in §2 .

Let now $\mathfrak{A}$ be such a regular method for which $c^o_{\mathfrak{A}}$ is a BK-AK-space. It means that $c^o_{\mathfrak{A}}$ is simultaneously a *BK*-space and an AK-space (i.e. $\Delta \subset c^o_{\mathfrak{A}}$ and $\lim_n \|x^{[n]} - x\| = 0$ for each $x = (x_k) \in$ $\in c^o_{\mathfrak{A}}$ where $x^{[n]} = (x_o,\dots,x_n,0,\dots)$ or (cf. [30], p., 176) in $c^o_{\mathfrak{A}}$ the weak convergence by the sections is valid , i.e. $\lim_n |f(x^{[n]}) - f(x)| = 0$ for each $x = (x_k) \in c^o_{\mathfrak{A}}$ and $f \in (c^o_{\mathfrak{A}})'$ where $(c^o_{\mathfrak{A}})'$ is a topological dual space of $c^o_{\mathfrak{A}}$). It is easy to see that a regular method $\mathfrak{A}$ is perfect when $c^o_{\mathfrak{A}}$ is a *BK-AK*-space, but for each regular perfect method $\mathfrak{A}$ the space $c^o_{\mathfrak{A}}$ is not necessarily an *AK*-space (cf., for example, [30], p. 214 - 215).

Theorem 2. *Let* $\mathfrak{A} = (a_{nk})$ *be such a regular method that* $c^o_{\mathfrak{A}}$ *is a BK-AK-space,* $B = (\beta_{nk})$ *be a normal method and* $M = (m_{nk})$ *be an arbitrary matrix. Then* $M \in (c_{\mathfrak{A}}, c_B)$ *if and only if conditions* 1) *and* 2) *of Theorem* 1 *hold and there exist functionals* $F_n \in (c^o_{\mathfrak{A}})'$ *such that*

$$g_{nk} = F_n(e^k)$$

and

$$\|F_n\|_{(c^o_{\mathfrak{A}})'} = O(1).$$

Moreover, if, in addition, $\delta_k \equiv 0$ *and* $\delta = 1$*, then the methods* $\mathfrak{A}$ *and* B *are* M*-consistent.*

Using the general form of continuous linear functional on $c^o_{\mathfrak{A}}$ and on c_A for reversible methods $\mathfrak{A}$ and A (i.e. for such methods $\mathfrak{A}$ and A for which the systems of equations $z = \mathfrak{A}x$ and $z = Ax$ have a unique solution for each $z \in c$) it is easy to find conditions that $M \in (c_{\mathfrak{A}}, c_B)$ and $M \in (c_A, c_B)$ in the case when $\mathfrak{A}$ and A are regular perfect reversible methods.

Corollary 1. *Let* $\mathfrak{A} = (a_{nk})$ *be such a regular reversible method that* $c^o_{\mathfrak{A}}$ *is an AK-space,* $B = (\beta_{nk})$ *be a normal method and* $M = (m_{nk})$ *be an arbitrary matrix. Then* $M \in (c_{\mathfrak{A}}, c_B)$ *if and only if the conditions* 1) *and* 2) *of Theorem* 1 *have been fulfilled and there exist series* $\sum_r \tau_{nr}$ *with the property* $\sum_r |\tau_{nr}| = O(1)$ *such that*

$$\delta_{nk} = \sum_{r=0}^{\infty} \tau_{nr} a_{rk} .$$

Corollary 2. *Let* $\mathfrak{A} = (a_{nk})$ *be a regular reversible perfect method,* $B = (\beta_{nk})$ *be a triangular method and* $M = (m_{nk})$ *be an arbitrary matrix. Then* $M \in (c_{\mathfrak{A}}, c_B)$ *if and only if conditions* 1) *and* 2) *of Theorem* 1 *and conditions*

3) *there exist series* $\sum_r \tau^s_{lr}$ *with the property* $\sum_r |\tau^s_{lr}| = O_s(1)$ *such that*

$$\sum_r \tau^s_{lr} a_{rk} = \begin{cases} m_{sk}, & \text{if } k \le l , \\ 0 , & \text{if } k > l \end{cases}$$

and

4) $\sum_r |D_{nr}| = O(1)$ *where numbers* D_{nr} *have been defined by*

$$D_{nr} = \sum_{s=0}^{n} \beta_{ns} \tau^s_r$$

and

$$m_{sk} = \sum_r \tau^s_r a_{rk}$$

(here the existence and absolute convergence of series $\sum_r \tau^s_r$ *have been quaranteed by condition* 3))

have been fulfilled.

The matrix transformations from c_A into c_B, from c_A into bv_B, from bv_A into c_B and from bv_A into bv_B are studied by inverse transformation method in §3 of chapter I in the case when A is a reversible method and B is a triangular method or an arbitrary method. To describe these matrix transformations for the case of triangular method B the necessary and sufficient (but for the case of an arbitrary method B only the sufficient) conditions are found. It is well-known that c_A is a BK-space if A is a reversible method. Therefore in this case the members x_k of each sequence $x = (x_k) \in c_A$ are continuous linear functionals on c_A. Thus the members x_k of each sequence $x = (x_k) \in c_A$ may be represented in the form

$$x_k = \eta_k \mu + \sum_l \eta_{kl}(z_l - \mu) \tag{2}$$

where $z_l = A_l x$, $\mu = \lim z_l$ and the sequences (η_{kn}) (for fixed n) and (η_k) are the solutions of the system $z = Ax$ for $z_l = \delta_{ln}$ and $z_l = \delta_{ll}$ respectively (here $\delta_{ln} = 1$ for $l = n$ and $\delta_{ln} = 0$ for $l \neq n$).

Let

$$\gamma^n_{pk} = \sum_{l=0}^{p} g_{nl}\eta_{lk} .$$

We shall consider the case when B is a triangular method. Then

$$B_n y = G_n x \tag{3}$$

for each $x \in c_A$ where $y = (y_k) = (M_k x)$. By (2), (3) and some well-known results from the theory of summability (for example, the theorems of Koijima-Schur, Hahn and Knopp-Lorentz) the necessary and sufficient conditions for the above-mentioned four types of matrix transformations are proved. Here we present some of them.

Theorem 3. *Let $A = (\alpha_{nk})$ be a reversible method, $B = (\beta_{nk})$ be a triangular method and $M = (m_{nk})$ be an arbitrary matrix. Then $M \in (c_A, c_B)$ if and only if*

1) *there exist finite limits* $\lim_p M^n_{pk} = M_{nk}$,

2) *series* $\sum_l m_{nl}\eta_l$ *are convergent,*

3) $\sum_k |M^n_{pk}| = O_n(1)$,

4) *there exists finite limit* $\lim_n \sum_k g_{nk}\eta_k = g$.

5) *there exist finite limits* $\lim_n \gamma_{nk} = \gamma_k$

and

6) $\sum_k |\gamma_{nk}| = O(1)$

where

$$M^{\eta}_{pk} = \sum_{l=0}^{p} m_{nl}\eta_{lk} .$$

Moreover, if $\gamma_k \equiv 0$ *and* $g = 1$ *then the methods* A *and* B *are* M*-consistent.*

Theorem 4. *Let* $A = (\alpha_{nk})$ *be a reversible method* , $B = (\beta_{nk})$ *be a triangular method and* $M = (m_{nk})$ *be an arbitrary matrix. Then* $M \in (bv_A, bv_B)$ *if and only if*

1) *there exist finite limits* $\lim_p M^{\eta}_{pk} = M_{nk}$,

2) series $\sum_l m_{nl}\eta_l$ *are convergent*,

3) $\sum_{k=0}^{l} M^{\eta}_{pk} = O_n(1)$,

4) $(\eta_k) \in bv_G$

and

5) $\sum_{n=1}^{\infty}\left|\sum_{l=0}^{k}(\gamma_{nl} - \gamma_{n-1,l})\right| + \left|\sum_{l=0}^{k}\gamma_{ol}\right| = O(1)$.

Some analogs of Theorems 3 and 4 for transformations from c_A into bv_B and from bv_A into c_B are also proved.

At the end of this chapter we consider the case when the method B is arbitrary. Let $G = (g_{nk})$ where

$$g_{nk} = \sum_s \beta_{ns} m_{sk} .$$

Then (3) is not necessarily valid for each $x \in c_A$ or $x \in bv_A$ where $y = (M_k x)$. In the present dissertation the necessary and sufficient conditions for it are found. Using these conditions we have

Theorem 5. *Let* $A = (\alpha_{nk})$ *be a reversible method* , $B = (\beta_{nk})$ *be an arbitrary method and* $M = (m_{nk})$ *be an arbitrary matrix. Moreover, let* $\sum_k |\beta_{nk}| < \infty$, $m_{nk} = O_k(1)$, *exist finite limits* $\lim_p M^{\eta}_{pk} = M_{nk}$,

$$\sum_{k=0}^{p} m_{nk}\eta_k = O(1)$$

and one of the conditions

$$\sum_k |H^n_{pk}| = O(1)$$

or

$$\sum_{k=0}^{l} H^n_{pk} = O(1)$$

holds. Then there exist finite limits $\lim_m \gamma^n_{mk} = \gamma_{nk}$. *Here* $H \in (c_A, c_B)$ *if conditions 4) – 6) of Theorem 3 and* $H \in (bv_A, bv_B)$ *if conditions 4) and 5) of Theorem 4 have been fulfilled.*

The same kind of analogs of Theorem 5 hold for the classes (c_A, bv_B) and (bv_A, bv_B) too.

APPLICATIONS

Now we shall consider the cases when A or A and B both are Cesàro (§1 of Chapter II) or Riesz (§2 and §3 of Chapter II) methods. Let $A^\alpha_n = \binom{n+\alpha}{n}$ for each $\alpha \in \mathbb{C}$ and $n \in \mathbb{N}$. We keep in mind that the series-to-sequence method of Cesàro of order α ($\alpha \in \mathbb{C}\setminus\{-1,-2,\dots\}$), shortly C^α method, is defined by the matrix (α_{nk}) where

$$\alpha_{nk} = \begin{cases} A^\alpha_{n-k} / A^\alpha_n & \text{if } k \le n, \\ 0 & \text{if } k > n, \end{cases}$$

and the sequence-to-sequence method of Cesàro of order α, shortly $\mathbb{C}^\alpha$ method, is defined by the matrix (a_{nk}) where

$$a_{nk} = \begin{cases} A^{\alpha-1}_{n-k} / A^\alpha_n & \text{if } k \le n, \\ 0 & \text{if } k > n. \end{cases}$$

For the description of the main results of §1 of Chapter II we put

$$\Delta^{\alpha+1}\varepsilon_k = \sum_l A^{-\alpha-2}_l \varepsilon_{k+l}$$

for each bounded number sequence (ε_k) and for each $\alpha \in \mathbb{C}$. If $\mathrm{Re}\,\alpha > -1$ or $\alpha = -1$ then $\sum_l |A^{-\alpha-2}_l \varepsilon_{k+l}| < \infty$. By Corollary 2 we have

Theorem 6. *Let* $B = (\beta_{nk})$ *be a triangular method* , $M = (m_{nk})$ *be an arbitrary matrix and* α *be such a complex number for which* $Re\alpha > 0$ *or* $\alpha = 0$. *Then* $M \in (c_{\mathbb{C}^{\alpha}}, c_B)$ *if and only if*

$$m_{nk} = O_n(k^{-Re\alpha}),$$

$$\sum_k (k+1)^{Re\alpha} \left|\Delta_k^{\alpha} m_{nk}\right| < \infty, \tag{4}$$

$$\sum_k (k+1)^{Re\alpha} \left|\Delta_k^{\alpha} g_{nk}\right| = O(1)$$

and conditions 1) *and* 3) *of Theorem* 1 *have been fulfilled* .

In addition, by Theorem 3 we have

Theorem 7. *Let* $B = (\beta_{nk})$ *be a triangular method,* $M = (m_{nk})$ *be an arbitrary matrix and* α *be such a complex number for which* $Re\alpha > 0$ *or* $\alpha = 0$. *Then* $M \in (c_{C^{\alpha}}, c_B)$ *if and only if*

$$m_{nk} = O_n(k^{-Re\alpha}),$$

$$\sum_k (k+1)^{Re\alpha} \left|\Delta_k^{\alpha+1} m_{nk}\right| < \infty, \tag{5}$$

$$\sum_k (k+1)^{Re\alpha} \left|B_k^{\alpha+1} g_{nk}\right| = O(1)$$

and condition 1) *of Theorem* 1 *has been fulfilled.*

Moreover , *if* $g_k \equiv 1$ *then the methods* C^{α} *and* B *are* M*-consistent.*

For a normal method B the condition (4) in Theorem 6 and the condition (5) in Theorem 7 are redundant. Some generalizations of the results of [4-6,29] follow from Theorem 7 in particular .

Furthermore , let (p_n) be a sequence of non-zero complex numbers, $P_n = p_0 + \ldots + p_n \neq 0$ for each $n \in \mathbb{N}$, $P_{-1} = 0$, $(R, p_n) = (\alpha_{nk})$ and $(\mathfrak{R}, p_n) = (a_{nk})$ be respectively the series-to-sequence and sequence-to-sequence Riesz methods generated by (p_n), i.e.

$$\alpha_{nk} = \begin{cases} 1 - P_{k-1}/P_n & \text{if } k \le n , \\ 0 & \text{if } k > n \end{cases}$$

and

$$\alpha_{nk} = \begin{cases} \rho_k / P_n & \text{if } k \le n, \\ 0 & \text{if } k > n. \end{cases}$$

Moreover, let

$$\Delta\alpha_{nk} = \alpha_{nk} - \alpha_{n,k+1}$$

and $B = (\beta_{nk})$ be an arbitrary triangular method. By Corollary 1 and Theorems 3 and 4 hold

Theorem 8. *Let* $(\mathfrak{R}, \rho_n)$ *be a regular method*, $B = (\beta_{nk})$ *be a normal method and* $M = (m_{nk})$ *be an arbitrary matrix. Then* $M \in (c_{(\mathfrak{R},\rho_n)}, c_B)$ *if and only if*

$$\mathfrak{G}_{nk} = \sigma_n(\rho_k),$$

$$\sum_k \left| P_k \Delta \frac{\mathfrak{G}_{nk}}{\rho_k} \right| = O(1)$$

and conditions 1) and 2) of Theorem 1 have been fulfilled.

Theorem 9. *Let* (R, ρ_n) *be a conservative method*, $B = (\beta_{nk})$ *be a triangular method and* $M = (m_{nk})$ *be an arbitrary matrix. Then* $M \in (c_{(R,\rho_n)}, c_B)$ *if and only if*

$$\sum_k \left| P_k \Delta \frac{\Delta m_{nk}}{\rho_k} \right| < \infty, \tag{6}$$

$$P_k m_{nk} = O_n(\rho_k),$$

$$\sum_k \left| P_k \Delta \frac{\Delta \mathfrak{G}_{nk}}{\rho_k} \right| = O(1)$$

and condition 1) of Theorem 1 has been fulfilled.

Moreover, if $\mathfrak{G}_k = 1$ *then the methods* (R, ρ_n) *and* B *are* M*-consistent.*

Theorem 10. *Let* (R, ρ_n) *be an absolutely conservative method*, $B = (\beta_{nk})$ *be a triangular method and* $M = (m_{nk})$ *be an arbitrary matrix. Then* $M \in (bv_{(R,\rho_n)}, bv_{\bar{B}})$ *if and only if*

$$P_k m_{nk} = O_n(\rho_k),$$

$$P_k \Delta m_{nk} = O_n(\rho_k), \tag{7}$$

$$\sum_n |\beta_{nk} - \beta_{n-1,k}| = O(1)$$

and

$$P_k \sum_n |\Delta(\beta_{nk} - \beta_{n-1,k})| = O(p_k)$$

where $\beta_{-1,k} \equiv 0$.

It is shown that for a normal method B the condition (6) in Theorem 9 and the condition (7) in Theorem 10 are redundant. Moreover, for the case of the triangular method B the necessary and sufficient conditions for transformations from $c_{(R,p_n)}$ into bv_B and from $bv_{(\bar{R},p_n)}$ into c_B are found too .

By Theorem 5 we have

Theorem 11. *Let* (R,p_n) *be a conservative method* , $B = (\beta_{nk})$ *be a method which satisfies the condition* $\sum_k |\beta_{nk}| < \infty$, *and* $M = (m_{nk})$ *be an arbitrary matrix. If*

$$P_k m_{nk} = O(p_k),$$

$$\sum_k \left| P_k \Delta \frac{m_{nk}}{p_k} \right| = O(1),$$

$$\sum_k \left| P_k \Delta \frac{\beta_{nk}}{p_k} \right| = O(1)$$

and condition 1) *of Theorem* 1 *has been fulfilled then* $M \in (c_{(R,p_n)}, c_B)$.

Theorem 12. *Let* (R,p_n) *be an absolutely conservative method,* $B = (\beta_{nk})$ *be a method which satisfies the condition* $\sum_k |\beta_{nk}| < \infty$ *and* $M = (m_{nk})$ *be an arbitrary matrix. If*

$$P_k m_{nk} = O(p_k),$$

$$P_k \Delta m_{nk} = O(p_k),$$

$$\sum_n |\beta_{nk} - \beta_{n-1,k}| = O(1)$$

and

$$P_k \sum_n |\Delta(\beta_{nk} - \beta_{n-1,k})| = O(p_k)$$

where $\beta_{-1,k} \equiv 0$ *then* $M \in (bv_{(R,p_n)}, bv_B)$.

The sufficient conditions for the transformations from $c_{(R,p_n)}$ into bv_B and from $bv_{(R,p_n)}$ into c_B are also given . The case when B is a Riesz method is considered separately.

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MAATRIKSMENETLUSTE SUMMEERUVUSVALJADE JA ABSOLUUTSE SUMMEERUVUSE VÄLJADE MAATRIKSTEISENDUSED

Ants Aasma

RESOMEE

Antud väitekirjas vaadeldav probleem kuulub summeeruvusteooria valdkonda. Olgu $A = (\alpha_{nk})$ ja $B = (\beta_{nk})$ maatriksmenetlused üle $\mathbb{C}$ ning $M = (m_{nk})$ maatriks üle $\mathbb{C}$. Peale selle, olgu c_A ja bv_A vastavalt menetluse A summeeruvusväli ning absoluutse summeeruvuse väli. Lisaks eeldatakse, et c_A on BK-ruum. Väitekirjas uuritakse maatriksteisendusi ruumidest c_A või bv_A ruumidesse c_B või bv_B. Peyerimhoffi meetodiga leitakse tarvilikud ja piisavad tingimused selleks, et maatriks M teisendaks ruumi c_A ruumi c_B juhul, kui A on regulaarne perfektne menetlus ja B on kolmnurkne menetlus. Seejuures jada-jada teisendusega antud menetluse A jaoks vaadeldakse eraldi juhtu, kus c_A^0 (menetlusega A nulliks summeeruvate jadade ruum) on AK-ruum ja B on normaalne menetlus.

Pöördteisenduse meetodiga leitakse aga tarvilikud ja piisavad tingimused selleks, et maatriks M teisendaks ruumid c_A või bv_A ruumidesse c_B või bv_B juhul, kui A on reversiivne menetlus ja B on kolmnurkne menetlus. Suvalise menetluse B korral leitakse nimetatud nelja tüüpi teisenduste jaoks ainult piisavad tingimused.

Rakendustena vaadeldakse juhtumeid, kus menetlus A või menetlused A ja B mõlemad on kas Rieszi kaalutud keskmiste menetlused või Cesàro menetlused.

TÜ 93.52.120.0,89.1,0.