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The Analysis of the Redundancy of the Dutch Book Argument:  
Separability of Degrees of Belief and Preferences

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## INTRODUCTION\*

The most famous justification for the Bayesian thesis that degrees of belief should obey the probability calculus is provided by the Dutch Book Argument (DBA). This argument assumes that one's degrees of belief match one's certain betting prices and then makes use of the following mathematical result: if a set of betting prices violate the probability calculus, then there is a set of bets bought or sold at these prices that guarantees a net loss. This is used to establish a connection between rationality and the probability axioms and thereby support the plausibility of the formal probabilistic constraint on degrees of belief i.e. subjective interpretation of probability.

Lately, however, DBA has fallen into disfavor and there seems to exist a virtual consensus that this approach has been rendered redundant by the relative success of the utility theory. The crucial factor is the question of value-additivity: it has been highlighted that DBA assumes that the values attributed to bets are additive although this is far from obvious—someone might for example easily value separate bets more highly and give a lower value to those bets taken together. In comparison, the utility theory is thought to give us both value additivity and probabilism. Thus it is natural to conclude that DBA is invalid as it stands and other justifications for probabilism should be preferred.

Indeed, according to a popular view, DBA-s are only “useful illustrations or dramatizations of deeper truths about rational preference, truths stated more precisely by the representation theorems of axiomatic expected utility theory,

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upon which the case for probabilism is supposed to be properly grounded” (Zynda 2000: 46). So there is a general agreement that the utility theory has rendered DBA redundant. In this dissertation I will defy this wide-spread conclusion and in order to do so it is needed to compare the contributions of both arguments for probabilism.

Whether the utility theory renders DBA unnecessary depends on its own success to show what DBA purports to show and do it in a better and more convincing way. The utility theory is a general decision theory that proceeds from one's preferences between options and results in a numeric account of one's values and degrees of belief (a more elaborate description is given right in the next chapter). It involves the expected utility principle and value-additivity and also establishes the probability axioms as rationality constraints on degrees of belief. Thus it clearly seems to achieve and exceed the goal of DBA.

The comparison of DBA and the utility theory will be accompanying us all through this dissertation with relevant insights and analyses offered in due course. It must be stressed though that my main interest lies in DBA-s and this dissertation is not meant to offer a thorough overview—let alone analysis—of all the aspects of the utility theory. The results of the utility theory are very impressing and I am far from denying it. I also acknowledge that value-additivity constitutes a serious problem for DBA as usually understood. But I nevertheless claim that this does not amount to enough reason to prefer the utility theory to DBA or conclude that it renders DBA redundant.

My alternative understanding of the redundancy matter stems from recognizing and appreciating the differences of several versions of DBA. Although all DBA-s make use of the same mathematical theorem, their interpretations of it are notably different and consequently DBA comes in many different forms. Roughly said,

the theorem establishes that if degrees of belief do not satisfy the probability axioms, then something undesirable happens.

According to the nature of this undesirable thing it is meaningful to distinguish three main forms of the DBA: firstly, the standard kind of arguments which focus on the possibility of losing money; secondly, the preference-based approaches which concentrate on the defect of preferences; and thirdly, de pragmatized DBA-s that identify an inconsistency of degrees of belief. Although all those types have a common name, their contexts range from plainly practical to highly abstract and they are in fact demonstrating a different irrationality. So despite their apparent similarity, they actually have a different conclusion and therefore also contribution.

Since recognizing the disparity of different DBA-s holds the key to my standpoint, I will bestow a lot of consideration upon canvassing the versions of DBA in offering and distinguishing them from one another. As a result, this forms a relatively large part of my dissertation. After shortly introducing the classical versions of both rival views—DBA and the utility theory—in the first part of the dissertation, I will turn to a more thorough canvassing of the modifications of classical DBA: preference-based DBA (Chapter 2), and de pragmatized DBA (Chapter 3). These prefatory chapters serve as a base for the more important analysis in the remaining two chapters, which both rely heavily on the distinctions drawn beforehand.

Chapter 4 focuses on the general objections to the model that is being used by DBA. While assessing the contribution of DBA-s one cannot bypass the concerns raised about using the betting scenario and it is brought out that preference-based DBA and de pragmatized DBA actually specify a different betting model. This chapter also raises the other problematic question if the arguments for probabilism succeed to establish a link to rationality. Although no concept of

rationality is defended, it is exposed that many different interpretations of the word are being used in this context: different versions of DBA refer to a different meaning of the term and the utility theory in turn reveals a very specific type of irrationality. Although all above-mentioned arguments purport to establish a connection between the formal probabilistic constraint and rationality, they in fact strive at different goals.

Finally, in Chapter 5 I address the value-additivity problem and show that its destructiveness depends largely on the underlying metaphysical convictions about separability of degrees of belief and preferences. The issue of separability is present all through the dissertation and in the end it also leads to an answer to the redundancy question. The result turns out to be significantly different for preference-based DBA and de pragmatized DBA. Although my main thesis is about redundancy, this dissertation is not aimed at solving the strife between the utility theory and DBA. Rather than making a case for any argument or any version, I want to argue that one commits to substantial metaphysical assumptions while doing so. Preferring either of the rivals brings along a significant presumption about the nature of degrees of belief, a presumption that cannot be conclusively argued for.

In short, this dissertation offers an overview of three different types of DBA, analyses their separate and common problems and assesses their contribution to the endeavor of justifying why rational degrees of belief should be constrained by the probability axioms. It explicates the important role of certain underlying metaphysical presumptions and argues that contrary to a wide-spread conclusion the DBA is not made redundant by the utility theory.

## 1 OUTLINES OF THE ALTERNATIVES

The thesis of this dissertation is about the redundancy of DBA, but it concerns both the utility theory and DBA. The general utility theory is an extensive decision-theory and this dissertation can not be the place to give a satisfactory overview of it. Discussing all the benefits and problems of this theory is also not the goal since my position on the redundancy matter is based on differentiating between different versions of DBA. It is the distinction and analysis of the different versions of DBA that clarifies the matter substantially and enables me to reach the overall conclusion. But before turning to this important canvassing, I will shortly outline both of the alternatives.

Firstly, the Representation Theorem Argument (RTA) must be introduced, so that different versions of DBA could later be compared with it. RTA forms the core of the utility theory and is exactly the part that competes with DBA since it connects both value additivity and the axioms of probability to rational degrees of belief.

Secondly, the classical version of DBA should be presented, so that it would be made clear what are its disadvantages and why was DBA dismissed by many grand thinkers already a long time ago. Outlining the classical DBA also gives the chance to introduce the logic and framework of this argument and this forms the base of canvassing and distinguishing different modifications in Chapters 2 and 3.

Before offering the outlines of RTA and classical DBA, I will present the probability axioms that both arguments claim to be a rationality constraint on degrees of belief.

1. *Non-negativity*:  $P(X) \geq 0$  for all  $X$  in a set of propositions  $S$ .
2. *Normalization*:  $P(T) = 1$  for any tautology  $T$  in  $S$ .



3. *Finite additivity*:  $P(X \vee Y) = P(X) + P(Y)$  for all  $X, Y$  in  $S$  such that  $X$  is incompatible with  $Y$ .<sup>1</sup>

## 1.1 The Utility Theory

### 1.1.1 Classical Approach

Although the thesis about redundancy concerns both the utility theory and DBA, it should be clear from the start that the focus of this dissertation will clearly be on the latter. It is the distinction and analysis of DBA-s that leads to the conclusion that the supremacy of the utility theory is not straightforward. Although several flaws of the utility theory are also connected to this conclusion, a complete analysis of its strengths and weaknesses would be a topic too large to comprise in this dissertation. Thus a selection must be made and only the most relevant questions are brought forward. While these discussions will come up in due course, here is the place to present the result which forms the core of the utility theory.

The foundations of the general utility theory were laid down by Ramsey (1926) who is in fact also the author of DBA. In his account, utilities (desirabilities) of outcomes (worlds), their probabilities, and rational preferences are all intimately linked. More precisely, he was the first to show how the agent's probabilities and utilities can both be derived from one's rational preferences alone.

Ramsey's argument starts out with the preference-relation: it makes sense to compare and order the outcomes according to their preferability to the agent.<sup>2</sup> It is

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1 Although Kolmogorov extends his theory to infinite sets and accordingly posits countable additivity, this is a matter of controversy in the context of probabilism—most probabilists contend themselves with finite additivity although countable additivity clearly permits significant technological convenience. However, this issue is quite extraneous to my more specific context.

2 'The agent' is used to refer to the person under consideration.

assumed that the agent prefers outcome  $E$  to  $F$  and then defined that a proposition  $X$  is ethically neutral for her just as she is indifferent between options “ $E$  if  $X$  is true,  $F$  if not” and “ $F$  if  $X$  is true,  $E$  if not”. An ethically neutral proposition is Ramsey's basic tool and essentially it reflects the situation where the agent is indifferent between the proposition being true or false.

The next step is to assign  $E$  and  $F$  any two real numbers  $u(E)$  and  $u(F)$  so that  $u(E) > u(F)$ , thought of as the desirabilities of  $E$  and  $F$  respectively. If we take  $Y$  to be ethically neutral then we can assign  $u(G) = (u(E) + u(F))/2$  to an outcome  $G$  that is as desirable to the agent as “ $E$  if  $Y$  is true,  $F$  if not”. Now we can add further utility points midway between  $u(E)$  and  $u(G)$  and  $u(G)$  and  $u(F)$  and do so indefinitely.

Ramsey goes on to prove an important representation theorem. He shows that there are utility functions that map one's preferences between outcomes into the real numbers in an order-preserving way. These functions give us the measurements of value, attaching a number to each outcome in the domain. Since each utility function is a positive linear transformation of the other (i.e. of the form  $u_1 = au_2 + b$ , where  $a > 0$ ), the ratios of utility differences do not depend on which representative utility function is chosen. Degree of belief  $p(X)$  in  $X$  is defined as  $(u(E) - u(G))/(u(F) - u(G))$  given that the agent is indifferent between  $E$  and the option “ $F$  if  $X$ ,  $G$  if not”. Ramsey proves that the utility of the option “ $F$  if  $X$ ,  $G$  if not” is equal to its expected utility  $p(X)u(F) + (1 - p(X))u(G)$  and if degrees of belief are so defined then they meet the requirements given by probability axioms.

Obviously, Ramsey does not prove these important results out of thin air but needs substantial presumptions. For example, above we simply assumed that there is an ethically neutral proposition  $X$  believed to degree  $\frac{1}{2}$  and that such a  $G$  that is as desirable to the agent as “ $E$  if  $X$  is true,  $F$  if not” exists. In addition to

several assumptions about the richness of the preference space (outcomes), he also introduces certain consistency assumptions, i.e. rules that a rational person is supposed to satisfy when making a decision. A suitable example of such rule is Ramsey's axiom that the subject's value differences are transitive (if the difference in value between  $E$  and  $F$  is equal to the difference between  $G$  and  $H$ , and the difference between  $G$  and  $H$  is equal to that between  $I$  and  $J$  then the difference between  $E$  and  $F$  is equal to that between  $I$  and  $J$ ).

These consistency assumptions are of crucial importance since they make it possible to link the theorem to rationality and thereby make the mathematical result relevant for philosophy. Only if the assumptions can be plausibly defended as principles of rational preference, is it admissible to continue with the philosophically significant RTA: if agent's preferences are rational then her degrees of belief have to obey the probability calculus. Thus RTA is an interpretation of the Representation theorem, an interpretation that suits for the purpose of justifying probabilism. The general utility theory clearly operates in a broader domain than the specific RTA, but they are quite equal for our context here so no great meaning should be attached to the usage of one term instead of the other.

Of course there are many axiomatic developments of utility on the offering. It seems that most decision theorists have tended to prefer Savage's (1954) more sophisticated axiomatization to the relatively straight-forward account that Ramsey gives. In Savage's system the ordering of events is likewise determined by preferences between options like the one's highlighted by Ramsey. The preferences are similarly constrained by certain rules of consistency that partly match the corresponding axioms of Ramsey. But while Ramsey virtually postulates that preferences have a structure isomorphic to reals, Savage's axioms appeal more explicitly to rationality considerations.

But analogically with Ramsey's account, Savage's axioms also generate a class of utility functions determined up to a positive linear transformation, and a unique probability function. And again, the expected utility, a certain probability-weighted average of utilities, is said to represent the agent's rational preferences. Jeffrey (1965) refines the method still further and others have also worked on the presentation, but these slight differences are not important in the context of this dissertation. In our context there are far more relevant questions that should be dealt with—most importantly, the question if there are reasons for a probabilist to renounce these powerful results and lean on DBA instead.

### 1.1.2 Problems

There are many questions one could raise about the general utility theory and quite surely there are also many answers to these questions. This section will draw attention to one of these issues that might raise some doubt about the opinion that RTA offers a rigorous proof of everything that a probabilist might need.

In the previous section we saw that the outline of RTA consists of three steps:

- Principles of rational preference: constraints that posit certain formal properties of the preference ordering.
- Representation theorem: any such suitably constrained preference ordering can be represented by two functions  $B$  and  $U$ , where  $B$  satisfies the probability axioms,  $U$  preserves the preference ordering, and both  $B$  and  $U$  conform to an expected utility principle.
- Philosophically significant conclusion: if agent's preferences are rational then her degrees of belief  $B$  have to obey the probability calculus.

On a careful reading of this presentation one can see that rational preferences are only shown to be *representable* in a favorable way, but as Hájek (2008) puts it

—“merely being representable some way or other is cheap”. Christensen (2001) formulates the missing link in the above argument:

- Representation accuracy: If agent's preferences can be so represented as the Representation theorem states then the agent's actual utilities are  $U$  and actual degrees of belief are  $B$ .

Representation accuracy is needed to establish a tight enough connection between rational preferences and incoherent degrees of belief, without it the argument fails.

If an argument draws on one favorable representation, then the question of other representations becomes important. The answer to this question is not reassuring as Lyle Zynda (2000) has demonstrated. Namely, if we have a person, whose preferences can be represented as utilities  $U$  and such partial beliefs  $B$  that satisfy the probability axioms, then there will be another belief function  $B'$  that violates the axioms, but can be combined with  $U$  to yield a valuation function fitting the person's preference ordering equally well.<sup>3</sup> Thus the agent can have incoherent degrees of belief without violating the principles of rational preferences.

We have seen that our agent can be interpreted in a way desirable for a probabilist but she could also be interpreted in a substantially different way. A way to answer such criticism would be to find reasons to single out the probabilistic representation and privilege it to others. One could for example try to demonstrate that the probabilistic interpretation makes better sense of the person's preferences than any competing interpretation does (Maher 1993). Or one can try to justify the choice of probabilistic representation by referring to considerations of simplicity, elegance, fruitfulness, consilience or some other such theoretical virtue (Zynda 2000). But neither of these approaches considers *all* of the other possible alternatives and thus the matter remains vague.

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<sup>3</sup> The violation is compensated with some nonstandard rule for combining one's credences with one's utilities.

As to the vagueness, it deserves to be pointed out that RTA might also need an additional argument that the measures falling out of the principles of rational preference should be interpreted as degrees of belief in the first place. As we saw above, agent's preferences can be represented both in terms of quantities that obey probability theory (function  $B$ ) and in terms of quantities that do not (function  $B'$ ). Now, one might as well doubt if either of these representations—probabilistic and non-probabilistic alike—should be identified as partial belief. Indeed, it is not self-evident for all that degrees of belief have such a strong connection to preferences (see Section 2.2).

The above concerns were not meant to impugn the utility theory: it is clearly not in the scope of this dissertation to give a thorough canvassing of all objections to this large theory and in addition consider all relevant answers to criticism. However, I wanted to bring attention to the fact that RTA has not rigorously demonstrated everything that a probabilist might wish for and thus other paths to justifying probabilism are also still worthy of exploring. So let us now turn to see how far the path of DBA-s can take us.

## **1.2 Classical DBA**

### **1.2.1 Believing and Betting**

The goal of all DBA-s is to prove that the mathematical axioms of the probability theory form a suitable rationality constraint on degrees of belief. Before we can start to delve deeper into different forms of DBA, it is necessary to take a short look at the overall framework that we use for theorizing about degrees of belief. The widespread solution relies heavily on the observation that the extent of believing some proposition is reflected by the betting odds taken to be fair for a bet on this proposition. For a quick example, if a certain agent Maria believes the proposition “Martin will bring home flowers today.” up to degree 0.4, then a bet

on this proposition would seem fair to her if the odds are 0.4:0.6, that is, she considers these odds to be equalizing the prospects of both sides of such bet.

Drawing on such betting attitudes is not an easily admissible solution and using the betting scenario has received heavy criticism. This issue is taken up in Section 4.1 where it is argued that the betting scenario should be taken as a model that enables us to theorize about otherwise quite inaccessible degrees of belief. In the current introductory chapter I will slide over this issue and first give an outline of the argument.

Presentation of the argument demands a more precise construction of the betting model, but it must be noted that its construction varies slightly from one author to another and from one kind of DBA to another. In this section I present the classical version of the argument and significant differences from this version will be referred to in due course. The classical DBA aims at proving practical economical irrationality: if one violates the probability axioms then one can be made to lose money. The betting scenario that is used is accordingly construed in a behavioristic way: agent's degree of belief is the betting quotient which she uses in a specified betting situation. It is time to present the argument more precisely and clarify the necessary betting terminology,

Agent's degree of belief in proposition  $X$  is  $p$  iff she is prepared to buy or to sell a bet that pays  $S$  dollars (the 'stake') for  $pS$  (agent's fair price). In other words, the agent is prepared to bet about proposition  $X$  at her fair odds  $p:(1-p)$ , at any stake and on either side. Let us take our former example, where a certain agent called Maria decides that a bet on "Martin will bring home flowers today." seems fair to her if the odds are set at 0.4:0.6. Such decision is highly subjective of course and depends on all the background information that Maria has about Martin and the situation. When she has considered all the information available to her and posted her fair odds, then it means that she is willing to actually bet at those odds in the

somewhat contrived betting setting where the opponent chooses the stake and decides if Maria has to bet on or against this proposition. Such setting is not meant to reflect the usual betting context but is instead constructed with the aim of eliciting the odds deemed fair by the agent. If Maria decides that with odds 0.4:0.6 she sees no advantage on either side of the bet and is willing to bet without knowing if she bets on or against “Martin will bring home flowers today.”, then it is concluded that she believes this proposition with degree 0.4.

Now that the concept of a fair bet is explicated, we can move on to betting constructions concerning those fair bets. More precisely, we need to clarify the concept of a *Dutch Book*, since it is central for the mathematical theorem used by DBA. Shortly put, a *Dutch Book* is such a set of bets, that each one individually is considered fair by the agent, but all of them collectively guarantee her loss. That is, there is a betting strategy using those fair betting quotients that gives the other side sure net gain no matter what the outcomes of events in question.

In our example, if Maria would be vulnerable to a Dutch Book, then she could be made to lose money no matter if Martin brings home flowers today or not. For instance, this could happen if Maria decides that her fair odds for a bet on the proposition “Martin will bring home flowers today or Martin will not bring home flowers today.” are 0.5:0.5. With this decision Maria is willing to place money on a bet that cannot win, since this proposition is true no matter what Martin will do. Similarly, if Maria would post 0.2:0.8 for “Today Martin will bring home flowers or chocolate.” and at the same time 0.4:0.6 for “Today Martin will bring home flowers.”, then she would be vulnerable to the following Dutch Book:

1. Maria would be willing to pay 8 euros for the chance of gaining 10 euros if “Today Martin will bring home flowers or chocolate.” is false.
2. Maria would be willing to pay 4 euros for the chance of gaining 10 euros if “Today Martin will bring home flowers.” is true.



Now, if it happens Martin brings home neither flowers nor chocolate, then Maria wins 2 euros from the first bet (pays 8, gains 10). But at the same time she necessarily loses the second bet and has to pay 4 euros while gaining nothing. Thus in this case she would lose 2 euros altogether. If on the other hand “Today Martin will bring home flowers or chocolate.” is true then Maria loses 8 euros with the first bet and this is already more than she could win with the second bet.

Let us now move on to the question of how could such situation be avoided. Here the *Dutch Book theorem* becomes relevant, since it shows which conditions bring along the existence of a Dutch Book. Namely, this mathematical result proves that if agent's degrees of belief violate the probability axioms, then there exists a Dutch Book against her. The converse theorem also establishes that if agent's degrees of belief do not violate the probability axioms, then there does not exist a Dutch Book against her. Thus non-negativity, normalization and additivity of degrees of belief are necessary and sufficient to exclude the possibility of Dutch Book. Another way to say that is that the probability axioms are necessary and sufficient conditions of *coherence*.

It must be noted that the mathematical theorem only concerns the existence of abstract bets with certain properties, and in itself claims or proves nothing about degrees of belief or rationality. It takes a philosophical argument DBA that interprets these mathematical results to offer a philosophical thesis of probabilism. More precisely, DBA makes a normative claim about degrees of belief: degrees of belief which violate the probability calculus are *irrational*. Thus DBA basically relies on two connections: firstly the connection between degrees of belief and fair betting odds and secondly the connection between irrationality and the existence of a Dutch Book. The next section will focus on the corresponding problems with the classical DBA, but both of these connections receive a more thorough explication in Chapter 4.

### 1.2.2 Problems

The classical form of the argument presumes an extremely tight connection between degrees of belief and betting behavior. It assumes that degrees of belief entail a willingness to make actual bets according to one's degrees of belief; that the opponent is allowed to choose the side of the bet after the fair betting quotients have been posted; that the theoretical possibility of Dutch Book will be carried out in reality; that irrelevant factors (like one's attitude towards gambling) do not affect agent's choice etc. The complicated nature of real betting behavior is in alarming contrast with assumptions like this and the critics of DBA have used this efficiently. One possible answer is to stipulate the most necessary conditions (for example oblige the agent to accept the bets), but then it becomes highly questionable if the betting quotients that are posted under such conditions, can still be identified with degrees of belief.

This clearly renders the necessary connection between Dutch Book and irrationality questionable. But even if it would be passable to take obedience to the probability calculus as a principle of economic rationality, it is still questionable if this approach would succeed to give a satisfying account of the rationality of degrees of belief. Many authors hold that degrees of belief are epistemic entities and it should be possible to discuss their rationality or irrationality independently from the action they may or may not lead to. According to this view, the irrationality of losing money or some other practical liability, even if it could be passably defended, is just not enough to explain what is irrational about these degrees of belief themselves. As put by Christensen (1991), if the Bayesian Thought Police would torture those who violated probability theory, that might motivate us to try to avoid it, but this motivation is not the one that should matter to the probabilist. Joyce (1998) is one of the authors who has argued forcibly against the prudential nature of DBA-s and

claimed that the pragmatical context they appeal to makes them irrelevant to probabilism construed as a thesis in epistemology.

An attempt to answer this challenge is the view that the classical DBA is essentially just a vivid pragmatic illustration of a deeper underlying flaw. According to this approach, practical troubles that constitute a problem for the standard argument are not troublesome, since the possibility of a Dutch Book is only an indicator of a basal inconsistency. This move is certainly a step forward since inconsistency is more tightly connected with the notion of irrationality than monetary loss. Moreover, inconsistency is no longer a question of mere pragmatic irrationality but can reasonably be argued to be part of epistemic rationality. Therefore it seems to be a promising solution or rather, these seem to be promising solutions—not surprisingly there are many ways to explicate how vulnerability to Dutch Books involves inconsistency. This dissertation offers an overview of these different ways, analyses their separate and common problems and assesses their contribution to the endeavor of justifying why rational degrees of belief should be constrained by the probability axioms.

## 2 INCONSISTENCY OF PREFERENCES

The idea, that incoherence of partial beliefs involves some decision-theoretic kind of inconsistency, was introduced by Ramsey and the following passage from him is extensively quoted in this context:

“If anyone’s mental condition violated these laws [of probability], his choice would depend on the precise form in which the options were offered him, which would be absurd. He could then have book made against him by a cunning bettor and would stand to lose in any event.” (Ramsey 1926: 182)

Ramsey also refers to the cunning bettor as a dramatic device and the possibility of a Dutch Book as a striking symptom of a deeper inconsistency. Thus violating the probability axioms gives rise to inconsistent preferences, which amounts to irrationality.

It is stated by Skyrms (1984), for example, that this direction pointed out by Ramsey is clearly the one that merits investigation: the Dutch Book is merely an illustration but the defect lies underneath. The enterprise of indicating an underlying defect is a different inquiry than the plainly pragmatic one of the standard argument. Skyrms emphasizes that these two enterprises should not be confused as is quite usual in criticism. Thus locating the problem deeper provides a basis for renouncing the objections that operate on the surface of the dramatic device.

It is true that opponents of the DBA do not always acknowledge the difference and continue to operate on the pragmatic level, emphasize the behavioristic problems and the complexity of pragmatic rationality. But it is also true that the alleged underlying inconsistency is often left unexplicated although it is supposed to be the core of the argument. It remains insufficient to present the standard argument, call it a dramatic device and only hint at the deeper meaning of DBA.

For example, Skyrms (1984) refers to Ramsey's quote above and indicates the inconsistency as *evaluating* the same betting arrangement differently under different descriptions. As the problem seems to be different evaluations to the same option, the defect is inconsistent valuing of bets. But surely such inconsistency-in-valuing deserves to be further expanded upon, for example, how exactly such inconsistency is tied to degrees of belief that violate the probability axioms. In the words of Kaplan (1996: 160): “the consistency condition that is basic to the argument is neither articulated in [Skyrms'] argument itself nor obviously at work in all its three proofs”.

Thus the proponents of this version of DBA need to give a further explication of the alleged inconsistency-in-valuing of bets and its role in the argument. They cannot continue to rely on Ramsey on this point since he relates to a very specific interpretation of inconsistency as we saw in Section 1.1. To remind:

“...any definite set of degrees of belief which broke [the probability axioms] would be inconsistent in the sense that it violated the laws of preference between the options, such as that preferability is a transitive asymmetrical relation, and that if  $\alpha$  is preferable to  $\beta$ ,  $\beta$  for certain cannot be preferable to  $\alpha$  if  $p$ ,  $\beta$  if not- $p$ .” (Ramsey 1926: 182)

As noted by Hájek (2009), these arguments have different layouts: while inconsistency-in-valuing DBA only tries to establish the probability axioms, Ramsey is making a more controversial point that laws of preference are also to be taken as rationality constraints. If DBA would succeed to explicate the inconsistency of evaluating the same betting arrangement differently under different descriptions in a satisfying way then it would have the advantage of not needing the axioms of preference, which are not all so plausible. For the argument to succeed, this explication of inconsistency should connect it to irrationality and better yet show its relation to the traditional concept of inconsistency.

## 2.1 Divided-Mind Inconsistency

Brad Armendt (1993) purports to explicate the role and nature of the inconsistency-in-valuing of bets further while being inspired by the ideas of Ramsey and relying heavily on Skyrms' work. The result is the notion of divided-mind inconsistency—giving two different choice-guiding evaluations to the same thing at the same time. The normative claim that divided-mind inconsistency is to be avoided is not a pragmatic rule but “is instead a norm that regulates how we should conceive of, or specify, our interests (*i.e.* consistently)” (Armendt, 1993: 5).

Armendt's (1993) explication of DBA starts with degrees of belief that violate the probability axioms and draws on their action-guiding character. It does so by supposing an ideal scenario in which those guides are operative, a scenario where the agent bets at her fair prizes. It then uses the Dutch Book theorem to demonstrate that exchanges constructed only by reference to those action-guides yield a pragmatically defective outcome (*i.e.* sure loss). The agent is susceptible to such exploitation because she displays pragmatic divided-mind inconsistency—gives conflicting evaluations to the same options.

We see that Armendt's argument is based on the connection between divided-mind inconsistency and violation of the probability axioms. While trying to establish a tight connection, Armendt (1993) focuses on the proof of the additivity axiom  $p(X_1 \vee X_2) = p(X_1) + p(X_2)$  where propositions  $X_1$  and  $X_2$  are mutually exclusive. Appeal to giving two different choice-guiding evaluations to the same thing at the same time is offhand quite plausible for this case. For when this axiom is violated and  $p(X_1 \vee X_2) < p(X_1) + p(X_2)$  then a Dutch Book can be made by buying a bet on the disjunction and selling bets on  $X_1$  and  $X_2$  (if it is violated in the other direction, then the directions of betting deals should be reversed that is buying replaced by selling and vice versa). Since a bet on

mutually exclusive propositions  $X_1$  and  $X_2$  separately is equivalent to a bet on the disjunction  $X_1$  or  $X_2$  but the agent is evaluating them differently, the bookie can assure herself a profit no matter how the bets turn out.

But the prima facie look turns out to be too cursory, since in addition to the equivalence of bets one also needs to assume additivity of values. Namely, to reach the desired result of proving the additivity axiom it is not enough that  $BX_1 + BX_2 = B(X_1 \vee X_2)$  but it is also needed that  $V(BX_1) + V(BX_2) = V(BX_1 + BX_2)$ , where  $V$  is the agent's fair price (see Section 5.1 for a more thorough presentation). This is the famous value-additivity principle, which is unproblematic if we measure value in only money but far from self-evident in a more realistic situation where other values interfere. For a quick example, it might be stressed that bets are usually placed sequentially and one is clearly permitted to revise one's betting prices when the world has changed since the prices were initially posted.<sup>4</sup>

While a more thorough presentation of the value-additivity problem is given in Chapter 5, it deserves to be stressed here that the defect of giving two different choice-guiding evaluations to the same thing cannot be demonstrated without assuming this principle. Thus the value-additivity issue is a serious problem for inconsistency-in-valuing DBA—without this principle it cannot be shown that the *same* betting arrangement is evaluated in different ways, but there might be nothing wrong in evaluating different arrangements in different ways. For this reason Vineberg (2001) finds the value-additivity problem to be a crucial shortcoming of Armendt's argument.

Armendt is obviously aware of the importance of the additivity problem and discusses it at length, his main point being that some simplifying presuppositions should be granted to the illustrating model of inconsistency-in-valuing DBA. One

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4 This objection has been made by many, Hájek (2009) has made the point recently.

relevant presumption is that the bets in question should not be considered sequential; we do not regard beliefs as states incapable of enduring over time and thus we can presume sufficient stability in order to develop a model, that should be applied to simultaneous bets. His further presumptions also exclude the cases that bring along failures of value-additivity.

Since other matters must be clarified before the value-additivity issue can be addressed, an analysis of Armendt's (1993) solution is put over to Chapter 5 (Section 5.3.1). For now, it can be said that value-additivity constitutes a significant obstacle for Armendt's DBA and we can continue with pointing out that it is not the only obstacle.

In addition to having trouble with additivity, Armendt (1993) also does not establish the existence of the divided-mind defect for the other two probability axioms. In fact, he does not pay much attention to the axioms of normativity and non-negativity; his brief footnote-suggestions are far from offering an explicit account of how the violation of these axioms amounts to giving different values to the same thing. Thus we must conclude that Armendt does not succeed in establishing that violating the probability axioms necessarily brings along divided-mind inconsistency.

In addition, it is questionable if this result would be enough for the goal of DBA. Namely, Hájek (2008) brings forth that Armendt (1993) concentrates on only one side of the matter: he tries to show that violating the probability axioms is sufficient for the occurrence of divided-mind inconsistency. But the opposite direction is just as important for the conclusion of DBA: is violation of the axioms necessary for conflicting evaluations to occur? If not, then the axioms do not guarantee consistency and the conclusion of the argument loses much of its significance.



We have seen that Armendt's account has serious problems with establishing a firm enough link between the axioms and consistency. While this might seem as a reason to dismiss DBA and turn to RTA, let us first look at one more objection that will play a big role in the comparison of different arguments.

## 2.2 Degrees of Belief and Preferences

This section will call attention to one problematic issue that in the end turns out to be very significant for the thesis—the question of the connection between degrees of belief and preferences. Namely, Armendt (1993) tries to highlight an inconsistency of *evaluations* that stems from the agent's flawed *preference-system*, but the conclusion of his DBA is about incoherent *degrees of belief*. Although it is evident that degrees of belief affect our preferences and values, it is nevertheless open to discussion if the connection is tight enough for such argument.

The main advocate of this line of criticism is Christensen:

“How plausible is it, after all, that the intellectual defect exemplified by an agent's being more confident in  $P$  than in  $(P \vee Q)$  is, at bottom, a defect in that agent's *preferences*? It is only plausible to the extent that we take seriously and literally the proposal that particular degrees of belief are defined by certain preferences, or, perhaps more precisely, that degrees of belief reduce to (or necessarily include) certain preferences.” (Christensen 1996: 453)

Seeing degrees of belief and preferences as deeply connected is intrinsic to both Armendt's and Skyrms's DBA which take the defect of preferences to constitute the defect of degrees of belief. Thus in the following this approach is referred to as preference-based DBA.

Since preference-based DBA is built upon Ramsey's work, it is not surprising that such underlying view is also intrinsic to RTA where preference are taken to be the primary notion and probabilities (degrees of belief) are used as a device for

interpreting preferences. Thus both approaches presume the existence of a strong constitutive connection between degrees of belief and preferences and the subsequent criticism pertains to both of these preference-based arguments.

Many things can be said about the dubiousness of such metaphysical view. In general, this understanding of degrees of belief does not fit well with our intuitive understanding of degrees of belief. For example, it leaves out important parts of our pretheoretic notion: degrees of belief are also intimately connected with all sorts of other aspects of psychology and it is highly suspicious to settle on one of these connections as definitional (Christensen 2001). To be sure, degrees of belief are often connected to preferences and they can certainly help to explain preference-based behavior, but this does not justify reduction since the connection is much more complex. Degrees of belief interact with many other psychological states and often the connection with preferences is not important at all. For example, it is general knowledge that having low degree of belief in one's success makes one less successful but the role of preferences does not seem to be decisive here.

The prospects for reduction of belief—or degrees of it—to preference are also threatened by their different directions of fit, as noted by Eriksson and Hájek:

“The goal of a credence is to conform to the way the world is; in the case of a mismatch, a rational agent will typically strive to make appropriate changes in her credence. The goal of a preference is that the world should conform to *it*; in the case of a mismatch, a rational agent will typically strive to make appropriate changes in the world.” (Eriksson and Hájek 2007: 14, italics in original)

Eriksson and Hájek conclude that although DBA and RTA depend on reduction, they fail to establish a tight enough connection between the two, let alone a necessary connection.

It is important to notice that the view under question can be much more complex and credible than old-fashioned operationalism. Christensen (2004) identifies the more serious candidate as a holistic scientific definition, which takes degrees of belief to be something like functional properties of people, defined by their causal connections to the agent's utilities, other beliefs, and preferences.<sup>5</sup> Thus the theory may focus on revealing the interconnections which does not involve a straightforward reduction of two of them to one. Nevertheless, Christensen finds this kind of definition unsuitable for the purpose of DBA or RTA.

The problem is that such complex causal interconnections do not simply require that a certain belief state necessarily gives rise to certain preferences. Beliefs are individuated not only by their connections to particular betting preferences, but also by their connections to other psychological states including other beliefs. But if we grant that one's strong belief in  $P$  is also partially constituted by its connections to one's strong belief that  $P \vee Q$  then we have a problem:

„The entire interest of taking the probability calculus as a normative constraint on belief depends on countenancing the real possibility that the second sort of connection might fail to measure up to probabilistic correctness: I might strongly believe  $P$  but not have a sufficiently strong belief in  $(P \vee Q)$ . But once we countenance this possibility, do we have any justification for refusing to countenance the following possibility: that I strongly believe  $P$  but do not have a sufficiently strong preference for receiving a prize conditional on  $P$ 's truth? It seems to me that we do not. We have been given no reason to think that having certain appropriate betting preferences is somehow more essential to having a given belief than having appropriate other beliefs is.“ (Christensen 2004: 113)

Thus Christensen holds that a defect in the agent's preference-system is not enough to establish a defect of degrees of belief and that this remains so even if a functionalistic definition of degrees of belief is presumed.

An adequate account of degrees of belief must recognize the possibility that the ideal connection with preferences breaks down in certain circumstances. But

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<sup>5</sup> See Maher (1993) for a presentation of such a holistic system.

according to the preference-based view there seems to be nothing wrong with incoherent degrees of belief if they fail to give rise to inconsistent preferences. This is certainly unintuitive, the defect of taking  $P$  to be more likely than  $P \vee Q$  seems obvious even if the agent has no related preferences. We can even go to the limit with ignoring preferences and imagine someone who has partial beliefs but no preferences at all, a Zen Buddhist monk perhaps. Eriksson and Howson (2007) claim that if such a monk is conceptually possible then any account that conceptually ties credences to preferences is refuted.

Consequently, more epistemologically-minded philosophers tend to conclude that preference-based justifications of probability do not surmount the problem that was already introduced in Section 1.2.2: the prudential context of DBA makes it unclear how is this argument relevant to probabilism construed as a thesis in epistemology. I gladly join the ranks of Joyce, Howson and Christensen, who stress that both RTA and preference-based DBA assume a dubious metaphysical view and fail to identify an epistemic defect. Moreover, they do not even strive at identifying an epistemic defect since they operate in the prudential domain, where preferences are of great importance. But a defect of preferences can at best be considered a flaw that *indicates* the deeper defect of degrees of belief but it does not *constitute* one itself and is therefore not directly relevant to epistemology.

It is not so surprising that many traditional epistemologists have ignored these preference-based arguments and with it probabilism, since it seems to be founded on such arguments. Indeed, a look at the most popular arguments for probabilism might suggest that it involves an unacceptable account of graded belief and hence the result does not relate to epistemological issues. In contrast, to answer the epistemological challenge, it would be more appropriate to demonstrate that violating the probability axioms brings along an inconsistency of beliefs understood as purely epistemic entities. The next version of DBA that we will

turn to takes this step deeper and purports to show that the underlying logic of the DBA involves an epistemic defect analogous to the inconsistency of full beliefs.

### 3 INCONSISTENCY OF DEGREES OF BELIEF

DBA-s that purport to identify an epistemic defect of incoherent degrees of belief are commonly labeled as 'depragmatized' since they strive to divorce the epistemic issue from practical concerns. Although this is a significant difference, not much attention has been paid to them.<sup>6</sup> This is rather surprising since their different goal and setup is likely to bring along new solutions to old problems and new problems that are not usual in the context of DBA-s.

Previous two chapters served precisely the purpose of describing the usual context of DBA-s and this was needed to enable the comparison with depragmatized DBA-s. For only recognizing the conceptual disparity enables one to see the relative merits and specific problems of this approach. In the course of this dissertation I will argue that this approach has the potential of overcoming the problem of value additivity, but its main impediment is the strongly pragmatic connotation of the betting terminology. But first let us see, how exactly do these two different arguments propose to depragmatize DBA.

#### 3.1 Howson and Urbach's Argument

The first depragmatized DBA that we will look at is Howson and Urbach's (1989, 1993) version of the DBA. In the last chapter we saw that the divided-mind DBA gives the pragmatic betting setting the role of a dramatic device, but now we will observe how Howson and Urbach (1989) explicitly regenerate the setting itself. Their theory of betting odds does not lean on any real betting situation to elicit the agent's fair degrees of belief. Hence they do not need any such specifications

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<sup>6</sup> Even the authors of overview-articles on DBA-s either leave depragmatized DBA-s unmentioned or confine themselves to some references. See for example Hájek (2009: 22-23) that devotes much more space on depragmatized arguments than usual, but the account is still very superficial and no conclusions are drawn or appraisals made.

as the agent having to choose the quotient while being unaware of the side she is on. Instead the agent participates in a thought experiment where she has to decide upon a quotient that—relative to her information-base—seems to equalize the prospects of both sides of the bet. That is, agent's subjectively fair odds are those odds on a hypotheses  $X$  which the agent *believes* to confer no advantage or disadvantage to either side of the bet on  $X$  at those odds.

Howson and Urbach (1989) emphasize that in contrast with the standard or preference-based argument this definition does not need to posit anything about the nature of the link between belief and action. In other words, it is not assumed that one's intellectual judgment of fairness commits one to any behavioral display whatsoever. Also it does not need to assume that any odds are fair *in fact*, except for the extreme cases. What it does assume, is that people rightly or wrongly *think* that some odds are fair and have a notion of advantageous and disadvantageous odds.

The next step of the argument is to look at the consequences what *would* happen if anyone *were* to bet according to the betting odds judged fair. The Dutch Book theorem proves that if the betting odds do not satisfy the probability axioms then there is a betting strategy based on those odds that ensures a net loss. But the well-known mathematical result receives a different interpretation here. Howson and Urbach (1989) state as its corollary that betting quotients which do not satisfy the probability axioms cannot consistently be regarded as fair. The logic of this argument is more precisely described with three steps:

1. Fair odds offer zero advantage to either side of the bet.
2. The sum of zeros is a zero; hence the net advantage of a set of bets at fair odds is zero.
3. If there exists a betting strategy, that assures positive net gain or loss then the net advantage in betting at those odds cannot be zero.

Howson and Urbach conclude that if one's degrees of belief are measured by the betting quotients one thinks fair, then consistency demands that they satisfy the probability axioms.

It is clear that Howson and Urbach's argument also rests on a kind of value-additivity assumption. Namely they assume that if the advantage of bets on  $X_1$  and  $X_2$  separately are both assessed to be zero, then the advantage of the compound bet on  $X_1$  and  $X_2$  is the sum of those zeros and hence also zero (3<sup>rd</sup> step above). This move is again the main target of objections and unfortunately Howson and Urbach (1989, 1993) do not discuss the matter or give any reasons why we should assume it to hold. The later works of Howson (1997b, 1997c, 2001, 2003) refer to this assumption as intuitively plausible and reasonable. As the value-additivity question is addressed in Chapter 5, let us now turn to see how Christensen proposes to deprecaturize DBA.

### 3.2 Christensen's Argument

We saw in Section 2.2 that Christensen has made a strong case against reducing degrees of belief to preferences. As he is also a probabilist, he is thus motivated to formulate an argument that would not need such implausible metaphysical assumption. Instead of seeing degrees of belief to be defined via preferences, he proposes to see the connection as normative.

According to Christensen it is initially plausible that “a degree of belief of, for example,  $2/3$  that of certainty *sanctions as fair*—in one relatively pretheoretic, intuitive sense—a bet at 2:1 odds” (Christensen 1996: 456). Or put another way, degrees of belief provide corresponding betting odds with *ceteris paribus* justification and thus the agent evaluates these odds as fair. Christensen also finds it plausible that if there is a defect in these fair odds—as the Dutch Book theorem demonstrates—then there is something amiss with those degrees of belief that sanctioned the bets fair.



The above formulation depends on the *ceteris paribus* condition which opens the way for different interpretations.<sup>7</sup> For that reason Christensen (2001) reformulated his argument as applying directly to the restricted case of a simple agent, an agent who values money positively, in a linear way, and does not value anything else. Thus a simple agent is defined so as to capture the meaning of the *ceteris paribus* clause that Christensen had in mind in his first presentation of the argument. Now the argument is presentable in the following steps:

1. If agent's degrees of belief are probabilistically incoherent then there exists a Dutch Book against him (the Dutch Book theorem).
2. A simple agent's degrees of belief sanction as fair monetary bets at odds matching her degrees of belief.
3. If the simple agent's set of degrees of belief is Dutch Bookable then it is pragmatically defective.
4. If a simple agent's beliefs sanction as fair each of a set of bets and that set is defective, then this agent's beliefs are rationally defective.

Thus we have the result that incoherent degrees of belief are rationally defective. That is, they are so for a simple agent—the above steps do not hold in general, but only for the simple agents.

So now we have to ask if Christensen's argument concerning the simple agents can be relevant to us, much more complex agents. Christensen (2001) refers to the simple agent case as particularly revealing circumstances that allow us to determine the existence of an epistemic defect and illustrate it.

“The power of the thought experiment depends on its being plausible that the epistemic defect we see so clearly when incoherent beliefs are placed in the value-context of the simple agent is also present in agents whose values are more complex. To me, this is quite plausible. There is no reason to think

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<sup>7</sup> For example, Maher (1997) argues that in case of defective odds there does not need to be a defect in beliefs since the defectiveness can be due to the failure of the *ceteris paribus* conditions.

that the defect is somehow an artefact of the imagined agent's unusually simple value structure" (Christensen 2001: 374)

The question of relevance is again taken up in Section 5.3.1 in the context of value-additivity, but now let us turn to a comparison of the two de pragmatized arguments that were outlined.

### **3.3 Comparison**

Both of these two versions of DBA try to indicate an epistemic defect in incoherent degrees of belief. In the literature they are both accordingly called de pragmatized DBA-s and mostly deemed to be similar in trying to escape the practical context of preference-based DBA-s. However, the outlines offered above are not straightforwardly similar and it is unclear how do these arguments relate to each other. As de pragmatized DBA-s are of crucial importance to the thesis, let us delve deeper into the matter in order to be able to judge the so-called de pragmatizedness of both arguments. Since both authors claim to indicate an epistemic defect in incoherent degrees of belief, let us start with looking closer at this defect.

#### **3.3.1 The Epistemic Defect**

Howson and Urbach's argument explicitly identifies the defect as an inconsistency: the bets that are taken to be fair by the agent are demonstrably not fair. Thus there is a close analogy between this specific inconsistency and traditional logical inconsistency: both have the common feature that the property ascribed to a set of beliefs separately cannot possibly be true for all the members of the set taken together.

At first sight it seems that Christensen refers to the same thing:

"Dutch Book vulnerability is philosophically significant because it reveals a certain inconsistency in some system of beliefs, an inconsistency which itself constitutes an epistemic defect." (Christensen 1991: 239)

But further investigation discloses that Christensen does not specify the nature of the epistemic defect at all and 'inconsistent' is used in a very loose sense meaning something like 'bad' or 'deficient':

“We need not reduce or assimilate consistency of graded beliefs to some previously understood kind of consistency (such as consistency of all-or-nothing beliefs or of preferences). We are seeking intuitive support for taking a certain set of principles as the best candidate for a formal constraint which plays a role similar to deductive consistency, but what applies to graded beliefs.” (Christensen 1996: 457)

Thus the inconsistency indicated by Christensen's argument is not in any clear sense analogical to the inconsistency of formal logic. The justification of using the term might be in what follows:

“Now one would not expect a consistent set of beliefs to sanction a set of bets that would lose no matter what the world turned out to be. Yet the Dutch Book arguments show that if a set of beliefs violates the axioms of the probability calculus, then it does sanction such a set of bets. Thus the Dutch Book arguments support our taking the probability axioms as criteria of consistency.” (Christensen 1991: 457-458)

Hence it seems that the epistemic defects of those two de pragmatized DBA's are to be understood as different. All the more so, because Christensen was well aware of Howson and Urbach's argument when he formulated his own i.e. he knew of the possibility to reduce consistency of degrees of belief to consistency of all-or-nothing beliefs when he declared that this need not be done.<sup>8</sup>

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8 The only place where Christensen expresses some opinion about Howson and Urbach's argument is in a footnote in Christensen (1996). There he focuses on a sentence from Howson and Franklin (1994) that goes “The latter [the axioms of probability] are demonstrably consistency constraints: infringe them and you are implicitly making deductively inconsistent claims about the fair odds on some hypothesis.” and refers that a related account is given in Howson and Urbach (1989). Christensen seems to think that Howson and Franklin (and Howson and Urbach) hold that the real problem with incoherent degrees of belief lies in the claims about bets with which they are ideally correlated and argues that such view is faulty because the connection between degrees of belief and correlated claims is not unbreakable. I,

It must be noted that contending with the *existence* of an epistemic defect and leaving its nature unexplicated, brings along some questions about the connection between the defect and epistemic irrationality. Namely, Maher (2006) raises the question of the nature of the irrationality in Christensen's argument: the argument relies on a set of bets being rationally defective but bets cannot be defective in epistemic rationality. Thus the transition from the irrationality of bets to the epistemic irrationality of degrees of belief should be more explicitly argued for. This reference to the practical nature of Christensen's argument brings us right to the next topic.

### 3.3.2 The Diagnostic Device

Let us now pay attention to the fact that Christensen continues to use the practical betting scenario as a diagnostic device. This approach does not relate to Howson and Urbach's DBA, but is instead reminiscent of Armendt's divided-mind argument.

Of course Christensen stresses that the connection between degrees of belief and preferences is normative and not metaphysical, but the structure of the argument still resembles the structure of Armendt's DBA. This is so especially because Armendt fails to identify the divided-mind inconsistency and consequently his argument boils down to the following:

1. An agent's degrees of belief give rise to evaluations that involve dispositions to choose and act.
2. If agent's dispositions to choose and act (inherent in her evaluations) are Dutch Bookable then they are pragmatically defective.

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however, hold that claims and claiming is not such an essential part of Howson and Urbach's (1989, 1993) treatise of degrees of belief and neither is it formulated so by themselves.

3. If agent's degrees of belief give rise to pragmatically defective dispositions to choose and act then she exhibits an imperfection of practical rationality.

Although Christensen (2001) explicitly rejects Armendt's DBA as too pragmatic, his own argument also uses the pragmatic betting setting as a diagnostic device and thus his argument strongly depends on the pragmatically defective outcome of the Dutch Book. Thus it seems that both Armendt and Christensen rely on the dramatic device of Dutch Bookability while Howson and Urbach's argument refrains from it.

For clarifying the matter, let us look at a device that helps to distinguish between arguments that depend on the sure loss of money and the more abstract kind of arguments. Namely, Hájek (2005) presents a parody of DBA, which claims to show that rational agents must violate the probability axioms. The Czech Book Argument (as labeled later by Hájek 2008) concentrates on the sure gain and shows that adherence to the probability calculus is practically undesirable because it shields one from a corresponding desirable betting arrangement.<sup>9</sup> This parodical argument is constructed to illustrate a difference between these arguments where the dramatic device does real work and versions where it does not play an important role.

Now, from the point of view of Howson and Urbach's version of DBA, Dutch Book and Czech Book are on a par: whether the agent would surely lose money or surely win money if she would bet on her fair betting quotients is not important, both possibilities show equally well that the net advantage of her degrees of belief cannot be zero for those bets. The traditional behavioral DBA is

<sup>9</sup> Such a symmetric construction is possible if DBA focuses on bets sold or bought exactly at the agent's fair prices. Hájek (2005, 2008) is well aware that the Czech Book does not go through if all fair-or-favourable bets are considered. He also stresses that the replacement of fair prices with fair-or-favourable prices is not needed if the dramatic effect of losing money does no real work.

clearly on the other side of the line. However, Armendt's and Christensen's versions of DBA resist such straightforward answer and demand some contemplation.

Armendt's DBA is explicitly analyzed by Hájek (2008) and he concludes that divided-mind inconsistency does not depend on the dramatic device of losing money. Such conclusion makes sense—giving two conflicting choice-guiding evaluations to the same proposition does not seem to be essentially about monetary loss. But recall that we found in Section 2.1 that the divided-mind inconsistency is not established for the violations of any of the probability axioms. This cannot be ignored and the fact that the only detected defect is the pragmatically defective outcome must be taken into account.

Thus Hájek's conclusion reflects only one side of the matter, which is in fact twofold. On the one side the actual argument offered by Armendt (1993) is highly dependent on the dramatic device of losing money. On the other side the specific kind of inconsistency that Armendt tries to highlight is not essentially connected to monetary loss. Let us now see how this conclusion differs from what we can say about Christensen's specific kind of inconsistency.

Well, the obvious problem is that Christensen (1996, 2001, 2004) does not explicate the inconsistency but only refers that it must exist. Hence part of the conclusion is the same as for Armendt's account—the only defect that is in fact detected is the pragmatically defective outcome. As for the second part, the situation is different—Christensen does not present any specific kind of inconsistency that would be independent of the dramatic device of losing money.

This result is rather surprising since Christensen is generally thought to be in the same boat with Howson and Urbach and differ substantially from pragmatic preference-based arguments. Yet the analysis shows that the Czech Book Argument draws a different distinction line, which separates Christensen's DBA

clearly from Howson and Urbach's DBA. Furthermore, we have come to realize that Christensen's argument is somewhat similar to divided-mind DBA and in fact relies even more heavily on the pragmatically defective outcome of being Dutch Bookable.

This raises the question of the role of preferences and values in Christensen's argument. Although he strives to distance his argument from preference-related considerations, the notion of 'sanctioning as fair' might nevertheless entail them. In fact, similar doubts have also been raised about Howson and Urbach's 'taking as fair'. Let us thus bring those concepts into focus and try to figure out their meaning and relation to each other.

### 3.3.3 The Role of Preferences and Values: Basic Notions

One possibility is that both 'sanctioning as fair' and 'taking as fair' simply mean full belief at bottom. Although Howson and Urbach (1989, 1993) do not stress the role of full belief, Vineberg (2001) concludes that their argument is based on this notion and its validity depends on its explanation. However, it can be argued that although a more thorough analysis of acceptance might be needed overall<sup>10</sup>, the context of this particular argument is simple enough to take full belief or acceptance to be primitive concepts.<sup>11</sup> And if the concept 'believe to be fair' still creates too much confusion, we can instead focus on the corresponding judgments and hold that “whether these judgments are actually believed is beside the point in judging whether they are consistent” (Howson 2007: 8).

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10 See Maher (1993), who offers arguments supporting such view and proposes a specific account of acceptance.

11 This line of thought is advocated by Howson (1997a) while replying to the criticism of Chihara (1994). This criticism is targeted at Howson and Urbach's (1989) account of conditionalization and the fact that they do not offer an explication of acceptance. Howson (1997a) replies that acceptance can be taken as a primitive notion.

The notion 'sanctioning as fair' also creates some confusion: Maher (1997) finds it to be somewhat vague and Schütte (2007) judges that this notion is at bottom an appeal to full beliefs. The explanation that an agent's degree of belief sanctions possible bets as fair iff it provides justification for evaluating those bets as fair, might indeed arouse temptation to interpret 'sanctioning as fair' just as we did for Howson and Urbach's account—as a judgment of fairness. However, Christensen (1996, 2001, 2004) also stresses that sanctioning as fair is an informal, intuitive normative connection between an agent's beliefs and her preferences and this explanation seems to point at another direction.

It is important to notice that Christensen sees his sanctioning as fair to be connected to values, while Howson and Urbach (1989, 1993) do not seem to have this opinion about their taking to be fair. In Christensen's system evaluating bets as fair is equal to being indifferent between bets and thus “if an agent values roast ducks more than boiled turnips, her belief that a coin is unbiased will not sanction as fair a bet in which she risks a roast duck for a chance of gaining a boiled turnip on the next coin flip” (Christensen 2001: 370). This example shows explicitly how preferences and values are inseparable from Christensen's argument.

In fact, the concept of sanctioning as fair seems to be the reason why Christensen's argument is so connected to values that he has to restrict its scope to agents with simple preference structures. The need for this presumption reflects the fact that values and preferences play an important role in his argument. A set of bets is pragmatically defective, if it is guaranteed to leave the agent worse off according to the agent's own *values*. If agent's have more complex *values* than simple agent's then degrees of belief must not sanction as fair monetary bets at odds matching these degrees of belief. And a defect of the bets is not generally enough to establish a defect of degrees of belief that sanctioned those bets as fair. That is so since sanctioning as fair is a connection between degrees of belief and *preferences*, but one bet can alter the agent's *preferences* concerning the other bet.



Thus we see that Christensen's interpretation of 'evaluating as fair' leaves the argument open to concerns related to values and preferences.

While Christensen finds his case on a normative connection between degrees of belief and preferences, Howson and Urbach (1989, 1993) try to abstract away from any preference-related concerns. Although 'taking as fair' could also be defined as 'evaluating as fair', this concept must then be interpreted in a value-independent way. In Howson and Urbach's thought experiment values are irrelevant and there is no reason why the agent who wants to elicit her degree of belief, should imagine a practical and complex-valued betting situation with a roast duck and boiled turnip. This would not serve the purpose of the thought experiment but would only complicate things without any benefit. Instead, it would be more appropriate to imagine a substance that is not at all valuable for the agent and perhaps also not preference-connected and use this substance to weigh and balance the advantages.<sup>12</sup>

It seems that although 'taking as fair' and 'sanctioning as fair' can both be argued to mean 'evaluating as fair', the word 'evaluate' is used with different connotations. In Christensen's case it should be interpreted as placing a value on, judging the worth of something. Thus in case of fair bets the agent *values* different possibilities equally. In Howson and Urbach's approach 'evaluating' takes the meaning of assessing, forming an estimate. Thus in case of fair bets the agent forms an intellectual decision about the fairness of the bet. And making this decision about fairness does not inherently mean judging the worth of the stakes but rather finding the point that balances the advantages. A connected issue is the meaning of 'advantage' in such treatise, but I think enough has been said here for the purpose of comparison.<sup>13</sup>

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<sup>12</sup> Helmann's (1997) proposal of ideal fluid is presented in Section 5.3.2.

<sup>13</sup> An analysis of advantage is offered in Section 5.3.2 in the value-additivity context.

I conclude that Christensen's version of DBA is not as de pragmatized as Howson and Urbach's version. While it is de pragmatized in the sense that it aims at indicating an epistemic inconsistency of degrees of belief, it is not de pragmatized in the sense of value-independence, because it continues to depend on preferences and the pragmatic context. This is the most significant difference from Howson and Urbach's fully de pragmatized DBA which purports to abstract away from these connections.

From this point of view, Christensen's DBA is more similar to the preference-based DBA differing only in making the metaphysical or definitional connection into a normative one. With this modification, Christensen surmounts the problem of reduction, but at the same time he fails to identify the epistemic defect his argument is designed to indicate. He rejects both Armendt's and Howson and Urbach's explications of inconsistency, but he does not identify—let alone explicate—the nature of the epistemic defect that he favors. Consequently Christensen's argument relies heavily on the pragmatically defective outcome of the dramatic device, something that plays no role in Howson and Urbach's argument. Thus the arguments that are mostly deemed similar, turned out to have significant differences that we can take into account in the following analysis.

## 4 PRESUPPOSITIONS OF THE MODEL

Since now I have introduced, compared and analyzed different arguments: RTA and several versions of DBA (from Armendt, Christensen and Howson and Urbach). Based on the distinctions made above we can now move on and start to look for answers to some problematic questions, that might lead one to reject DBA.

On the whole, criticism of DBA can be divided into two categories. Since every argument needs a general framework as a starting point, one type of criticism can attack the presuppositions that the argument uses. A different sort of criticism agrees with the presuppositions but indicates a flaw in the reasoning of the argument. In accordance with this distinction, the criticism of DBA-s will be considered in two parts: in this chapter I analyze the objections that attack the overall setup and premises of the argument; and in the next chapter I will finally take a look at the strongest objection against the cogency of the argument itself, the value-additivity problem.

Admittedly, the line between those two types of objections is not clear: what is taken as part of the argument by some authors may be declared as a prerequisite by others. Indeed, value additivity can also be separated from the argument and viewed as an independent principle. Paradoxically I even see such separation of the value-additivity issue as illuminating and helpful. Nevertheless, there are many discussions that object to the result of DBA without even directly discussing the argument itself and discussions of value-additivity are usually not such. So let us follow this somewhat arbitrary distinction and postpone the issue of avoiding the value-additivity problem to the last chapter.

## **4.1 General Framework: Degrees of Belief and Fair Betting Quotients**

All DBA-s are based on the presupposition that partial beliefs can be represented by fair betting quotients, which are ratios of payoffs. Many critics of DBA do not want to grant this and argue that degrees of belief need not be equal to fair betting quotients. DBA completely loses its point if it is denied its general framework, hence the discussion of this criticism is necessary.

To start with, it has to be granted that we have degrees of belief at all, more precisely, that we can use the concept of degrees of belief as an explication of our pretheoretic understanding of partial belief. This does not seem problematic since comparing the strength of different beliefs is part of our everyday life. On reflection, operating with partial beliefs is at least as fundamental to human mind as operating with categorical beliefs. Although measuring this partial belief in degrees and assigning numbers to our vague notion raises many issues, the overall idea is intuitively plausible enough.

More importantly, there comes the question of the connection between degrees of belief and fair betting quotients. Are degrees of belief really at bottom fair betting quotients? It is not difficult to imagine situations, where those two differ, in fact, it is even easy to imagine someone who has degrees of belief but knows absolutely nothing of betting. It seems we have to admit that degrees of belief are not equivalent to fair betting quotients. This state of things is not very surprising, since the relationship between credences and corresponding betting prices is surely more complex than indistinguishableness. The question is: what can we say about their connection and how can we use this knowledge?

It might be said that all attempts to answer this question are doomed since we do not even have a clear definition of degrees of belief. This issue is taken up by Eriksson and Hájek (2007) who first paint a gloomy picture of the situation but

then draw a bright conclusion: the concept of partial belief can be taken as a primitive notion that—similarly to the notion of full belief—is not in need of reduction. While this might be seen as a shortcoming by some, Eriksson and Hájek (2007) emphasize that it does not preclude us from saying many informative things about credences. The important role of partial beliefs is in need of explication and one thing we do know about these entities is that they are connected to one's assessment of fair betting quotients. Although this connection is not definitional, it seems to be tight-enough to use it as a basis of theorizing.

If we want to use this insight, we should not concentrate on attacking the naive definitional view, but try a more complicated question: can degrees of belief be *represented* by fair betting quotients? If the answer is negative, then we can continue with asking, if there are any better proposals to theorize about degrees of belief. For clearly we *can* represent credences with betting quotients, the point is rather if it deserves to be done. So it becomes highly relevant to ask: what other device could we use to represent our degrees of belief more befittingly? But if the answer is positive then we can move on to the question what is the nature of such representation and what are the conditions of success.

#### 4.1.1 Measuring and Modeling

Credences do not usually come in precise numbers and the information we get of them through introspection is rather vague. However, it clearly makes sense to compare the strength of different beliefs and this indicates that we do operate with vague degrees of belief. For advancement of a corresponding theory, it is natural to assign precise numbers to partial belief and this is where the concept of fair betting quotients may be of use to us.

Precise degrees of belief are clearly not available through introspection and we need a method for determining numerical values to our vague credences. The betting scenario is constructed so, as to capture the virtues that a betting situation

can offer to gain better access to one's own beliefs, that is, to help the agent pinpoint her degree of belief. Hence we can think of eliciting our fair betting quotients as a way of measuring our degrees of belief and this is clearly far from claiming their identity.

Admittedly, our actual degrees of belief are vague and are not easy to extract. True enough, that eliciting fair betting quotients is also not foolproof and so our measurement is not absolutely precise either. But isn't all that a common practice in measuring and modeling? We take our best measurements and construct a numerically precise model of real vague entities in order to be able to fruitfully theorize about them. This approach is advocated by Armendt (1993, 2007) who stresses that we do not defend our model as an accurate description of matters of fact, but claim instead, that the model is a valuable illustration and, more precisely, that its inaccuracy is mainly a question of generality and precision and not a complete lack of similarity between our model and the reality.

“Any account of what beliefs are and what they do (or what we do with them) that we can actually construct and present will be an incomplete one; it will be a model, more or less successful, of some feature(s) of beliefs.”  
(Armendt 2007: 3)

It seems that the model-view is not widely accepted, since the gist of opponents' criticism is quite often the demonstration, that degrees of belief can be different from fair betting quotients. But this seems rather trivial after accepting the model-view—we already know that the model is incomplete, but this fact is not significant by itself. While it might be stressed that the disparity between the object and its representative should not be too large, this demands an explication of what counts as 'too large' and why. As argued by Armendt (1993), we do not even have to claim that our model is the best one and no other treatment could be as good. We have only followed one promising trail and succeeded to build a fruitful model. Other fruitful models of credences are welcome!

As can be seen, it is useful to differentiate between two questions. Some critics of DBA tend to focus on the question how exact is the betting model. But it is another question if the model is satisfactory i.e. if it serves our purpose relatively well. The answer is partly dependent on the purpose of our model and there are indeed two different agendas that deserve to be pointed at (see Section 4.2); but here the focus is on the other factor—the fact that the value of the betting model is relative to the goodness of other available models. Interpreting the betting scenario as an illustrative model does by no means render it immune to criticism. But admissible criticism should not focus on constructing examples where the model differs from reality, but rather assess the fruitfulness of the theorizing, that is, significant criticism should compare model's exactness, congruity and usefulness with other available models. The exactness-criticism is much more substantial if it also has a constructive side to it and the best way to attain that is to offer a more exact and fruitful model to deal with degrees of belief.

#### 4.1.2 Constructive Criticism

It was stressed above, that pointing out the imperfections of the betting model is not constructive if some other more fruitful model is not offered. There is a simple reason why this was stressed—fertility is the trump card that probabilists can always play out. In the words of Eriksson and Hájek (2007: 35) “Probabilism codifies the laws of (normative) epistemology: remarkably simple theory achieves tremendous strength in unifying our epistemological intuitions.” In fact, there is a strong temptation to call it the best systematization of epistemology, since there are no alternatives with a comparable level of fruitfulness. But although the success of probabilism justifies dismissing the models that do not enable it, it does not yet settle the issue of the betting model unequivocally. The general betting framework has many modifications that are all leading to probabilism and this opens the way for comparison and constructive criticism.

For starters we must clarify the fact that RTA—an argument that is claimed to offer the most reliable path to probabilism—uses a somewhat different model of degrees of belief than DBA. At first sight, it might even seem that this argument, which defines credences as ratios of utility differences, has nothing to do with betting and is thus an alternative model that is possibly even more fruitful. However, deriving probabilities and utilities from preferences among options is equivalent to defining them in terms of utility-based betting odds.<sup>14</sup> So we see that RTA is based on the same cornerstone as DBA—the relationship between degrees of belief and assessments of fair betting odds.

As for the comparison of these two approaches, this matter is more complex. The overall goal of this dissertation is to explicate that there is another cornerstone, a metaphysical presumption that degrees of belief reduce to preferences. I argue that this presumption is shared by the model of RTA and some particular models of DBA, but that there are versions of DBA that are incompatible with such metaphysical foundation. According to my analysis this other cornerstone should be the main basis of deciding over the vantage of RTA over DBA or the other way around. But as this is the subject of the whole dissertation, I cannot here give an exhaustive answer to the question of choosing between the more precise models of RTA and DBA. Instead, let us turn back to the question of constructive criticism with an eye on other alternatives.

As already said, the best way to offer constructive criticism of some betting model is to provide a better alternative. While the alternative of a particular

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<sup>14</sup> Connection with betting is readily apparent in Ramsey's (1926) theory which starts with presuming the existence of an ethically neutral proposition and defining it as follows. A proposition  $A$  is believed with degree 0.5 if the agent is indifferent between options “ $X$  if  $A$  is true,  $Y$  if not” and “ $Y$  if  $A$  is true,  $X$  if not” while she prefers outcome  $X$  to  $Y$ . These options have the form of gambles and it was also observed by Ramsey that his approach is equivalent to defining degrees of belief via generalized betting odds. This also holds good for Savage's (1954) and other axiomatizations based on preferences between options of the above form.



model of DBA might be RTA, it might also be a betting model that is used for some other version of DBA. In this dissertation several different betting settings have been examined, starting from behavioristic scenarios to ideal abstract thought-experiments. The object that is being modeled is the same (degrees of belief) and its representative is called by the same name (fair betting quotients), but the meaning behind the name is different due to a different betting scenario. Correspondingly, the nature of the betting model varies from one type to another from modeling the choice-guiding character of degrees of belief to concentrating on the purely epistemological side. Naturally, the illustrative qualities of these models also differ and therefore it is reasonable to consider which one should be preferred and why.

There are also alternative betting-based models that strive at a more realistic representation of the actual state of things. For example, there are models where imprecise probability assignments are allowed.<sup>15</sup> Nevertheless, the vast majority prefers to make the simplification in order to have a clear and fruitful model. So the exactness should not be overvalued: while constructing a model, several ideals are taken into consideration and deliberate simplifications and idealizations are made in order to get a fruitful model.

Assessing the appropriateness of models is partly based on their fertility and DBA can itself be part of the fruitful theorizing that can boost betting scenario's value as a model. Thus even the opponents of DBA should first grant the betting scenario if only for the sake of the argument—the admissibility of a betting scenario should be granted to DBA so that the discussion of the argument itself could begin. We should first see where using the model leads us and only then consider disapproving it.

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<sup>15</sup> For a model with sets of precise probability functions see Levi (1974) or Jeffrey (1983), for one with lower and upper probability functions see Walley (1991).

## 4.2 Enabling Probabilism

We saw that the betting scenario can be used to elicit degrees of belief and thus it enables the development of an idealized model of our vague partial beliefs, a model, which uses fair betting quotients as representatives of degrees of belief. These fair betting quotients are radically subjective and the general betting framework that we have been considering does not yet specify if some sets of degrees of belief are more recommendable than others. Thus, if we want to get to the fruitful results of probabilism, then the connection to probability axioms must be established in addition. A move, that is common to RTA and DBA is to argue for probability axioms as rationality constraints.

Offering a model of rational degrees of belief is the prevailing purpose seen for the betting models. It is a substantial addition to the general betting framework and thus one can deny this step while granting the possibility of representing credences via fair betting quotients. The critics do not even have to engage in the discussion of the argument itself, since they can simply argue that while DBA might establish some recommendations of how to assign degrees of belief so that certain undesirable betting situations (Dutch Books) could be avoided, this shows nothing about rational degrees of belief. The protest underneath such claim is that neither Dutch Books nor probability axioms fit together with the concept of rationality as it is usually understood. Available counterexamples give strong intuitive support to such protest since they make it easy to demonstrate that one can violate the probability axioms but nevertheless be clearly rational.

Here the focus of the critics (e.g. Foley 1992, Vineberg 2001) is mostly on our limited logical abilities: there are situations when we know that a proposition is either necessarily true or necessarily false, but it is nevertheless not rational to assign an extreme degree of belief. For example, although I am aware of the overwhelming numerical evidence that speaks in favor of the truth of Goldbach's

conjecture, the mathematical proof is not available and thus it does not seem justified to ascribe an extreme degree of belief to this conjecture. I know, that the axiom of normativity demands it, but it does not seem like a rational thing to do. Thus the critics argue that it is not realistic to demand logical omniscience and thus all violations of the probability axioms are not failures of rationality in the sense of someone committing an error of reasoning.<sup>16</sup>

Naturally, the rationality-dispute between the proponents and the critics of probabilism is considerably complicated and confused by the blurred notion of rationality. In the subsequent discussion of rationality I would like to differentiate between two issues. Firstly, the question, how can the high standard set by the probability axioms relate to real agent's at all. And secondly, the question of explicating the precise meaning of irrationality via some defect that accompanies the violation of probability axioms. While the second question is more important for answering the redundancy question, a quick view on the first issue is also needed.

#### 4.2.1 Real and Ideal Rationality

The probability axioms can be seen as an elaboration of a certain familiar standard of rationality, namely, the standard of coherence. It is generally accepted that beliefs should ideally form a coherent system i.e. they should fit together. When we speak of categoric beliefs then we have the rules of deductive logic that function as a guarantee of fitting together. Now, probabilists argue that if we

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<sup>16</sup> When it comes to unrealistic assumptions, normalization is not the only problematic axiom. Chihara (1994) points out that in the proof of additivity axiom there is no justification for excluding the possibility of both propositions being true. The axiom concerns exclusive propositions, but the exclusiveness may be unknown to the agent and everyone else. Thus from the point of view of all human beings there may be no error of reasoning in violating the additivity axiom.

acknowledge the existence and role of partial beliefs then it is natural to turn to probability axioms to have some guarantee that credences also fit together.

Indeed, it can even be argued that we should skip all the messy talk of rationality and focus on this analogy since the proper purpose for the general betting scenario is the construction of inductive logic.<sup>17</sup> This is a different enterprise that enables us to brush the rationality-issue aside and is thus left untouched by the logical omniscience problem. However, taking this viewpoint means denouncing our epistemological ambitions of modeling rational degrees of belief. Presuming that we do not want to do that, let us see how we can use the analogy with logic for our purposes.

Perhaps we can shed light on the matter by differentiating between real rationality and ideal rationality: no one doubts that an *ideally rational agent* should always conform to the laws of logic but it is clear that no *real person* can ever live up to such ideal. The situation is the same for probabilism: the set of degrees of belief can be defective relative to ideal rationality, while the judgments of the individual, who has this set, must not necessarily be defective relative to real

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<sup>17</sup> This idea goes back to Ramsey (1926), but is recently powerfully endorsed by Howson (1997b, 1997c, 2001, 2007, 2008), who argues that both classical logic and probability theory provide the conditions regulating consistent assignments of values, truth-values in the one case and uncertainty values in the other. Howson's definition of a consistent assignment of values is given in terms of solvability of those assignments subject to the relevant constraints. For traditional logic we have the clauses of a Tarskian truth-definition and for the logic of uncertainty Howson (2007, 2008) takes up the proposal of Cox (1961) and Good (1984) which starts with a quantitative notion and some general fundamental principles that any acceptable numerical measure of uncertainty should obey and reaches the probability axioms. The apparently distinct notions of logical inconsistency and probabilistic incoherence turn out to be subspecies of a more general concept of solvability of a set of equations—a set of equations is consistent if there is at least one single-valued assignment of values to its variables. Thus both deductive logic and probabilism offer rules that eliminate the possibility of overdetermination while assigning values to variables.

rationality. In other words, a set of degrees of belief can be considered irrational if these degrees of belief do not fit together, but it cannot be expected from real agents to recognize this incoherence. Coherence is thus a limiting case of one aspect of good thinking.<sup>18</sup>

Bringing in the concept of an ideally rational agent does not yet answer the accusation that the ideal posed by probabilism is not a suitable ideal. For example, it might be stressed that not all aspects of the ideal model are desirable for real agents—numerical precision of degrees of belief seems to be an idealization that is not held high by real agent's (Maher MS).<sup>19</sup> More importantly, the criticism can focus on the inapplicability of such standard of ideal rationality—one can acknowledge that probabilistic coherence would be a requirement for some logical super-being but still hold that this does not entail anything about rationality for creatures like us (e.g. Foley 1992, Talbott 2005, Howson 2007).<sup>20</sup>

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18 This might pose a question about other aspects of good thinking—it can be argued that empirical omniscience is as good an ideal and no interesting account of rationality should treat logical omniscience and empirical omniscience differently (Hacking 1967, Foley 1992, Talbott 2005). While differentiating between these two omnisciences is an interesting issue, it would lead too far from my goal of answering the redundancy question. I can only remark that I agree with Christensen's (2004) arguments that lead to the conclusion that “logical omniscience emerges naturally as the limiting case of one of the basic ingredients of good thinking, in a way that empirical omniscience does not” (Christensen 2004: 156).

19 An answer to such objection could point out that some aspects of idealization constitute a certain standard of ideal rationality and other aspects of idealization are made just for the benefit of clearness and fertility of the model. Numeric precision would then naturally belong to the latter class and logical omniscience to the first.

20 This criticism has also motivated attempts to develop models which reflect our actual logical abilities more accurately (see for example Garber 1983). But as the raised level of exactness is accompanied with significant decrease in fruitfulness, these probabilistic theories of what might called „limited logical omniscience“ are not considered to be serious alternatives to more traditional probabilism.

Indeed, it is far from clear how would the ideal of coherence be applicable to a real agent and how could one approximate to this ideal. This problem is taken up by Zynda (1996) who shows how belief functions can be ordered with regard to greater or lesser coherence and how we can thus make sense of the notion of closer approximations to coherence. He also makes a strong case for the more general claim that if unattainable ideals define a structure that enables the notion of better approximations then these ideals can be normatively relevant. Thus he argues that coherence provides a standard against which we can measure our opinions and thus the ideal of coherence has substantive normative significance.

That being said, one must agree that this ideal standard is so high that even the most elaborate thinkers are irrational compared to it. However, in everyday life we do not compare ourselves or others to the standards of absolute perfection—say of absolute morality—but rather use contextually appropriate standards (Christensen 2004). Also, we must not stress the irrationality of the agent, but can instead understand the requirement of coherence as a regulative ideal: violation of it opens the agent's state of opinion to legitimate criticism.

The fact that the agent's set of degrees of belief is defective and open to legitimate criticism does not automatically mean that the agent herself is therefore open to legitimate criticism, further information is needed to decide upon the irrationality of the agent. Such view is advocated by Kaplan (2003), who argues that the high standards—e.g. the demand of logical omniscience—nonetheless play an important role in our assessments of the rationality of actual persons since it offers a valuable contribution to the endeavor of understanding what constitutes a rational criticism of a state of opinion.

So we see that the cases when it is irrational to have or to act on coherent degrees of belief must not be disavowed by the probabilists—the probabilistic standard reflects only one aspect of rationality and does not claim to comprise all that we regard as relevant to rationality. Furthermore, it is also not claimed that having

coherent degrees of belief is of supreme importance among the complex intertwined multitude of considerations at work in our everyday life. The fact that one has to find context-dependent balance between different ideals is quite the usual way of things and does not speak against any of the ideals.

To sum up this section, our betting scenario turned out to be a model of ideal rationality, which was shown to be related to real rationality. The model's divergence from reality is again justified by its fruitfulness. Thus we have dealt with the question if probability axioms can be relevant to the question of rationality, but we have not yet taken into account the analysis of the previous chapters and the differences in interpreting inconsistency and thus irrationality.

#### 4.2.2 Different Interpretations of Rationality

The concept of rationality is rarely specified in the context of justifying probabilism and the link from Dutch Bookability to irrationality is typically left fuzzy. It is quite usual in this dispute that different authors are arguing with each other as if they were talking of the same thing, but in fact they are far from it. My intent is to unconfound the situation and uncover the different ways of using the basic concepts of this argument—consistency, rationality and fairness. Different types of inconsistencies were examined in Chapters 2 and 3 and the remainder of the present chapter will relate them to different concepts of rationality. When these matters are clarified, then we can discuss the disparate interpretations of fairness in the context of solving the value-additivity problem (Chapter 5). Thus although the keywords are inconsistency, irrationality and fairness, the reader should not search for any new rigorous definitions but rather be attentive to different usages that are identified.

The analysis in previous chapters has uncovered different defects that accompany the violation of the probability axioms and the corresponding concepts of inconsistency. A brief summary of the situation is offered by Table 1.

	<i>Classical DBA</i>	<i>RTA</i>	<i>Armendt's DBA</i>	<i>Christensen's DBA</i>	<i>Howson and Urbach's DBA</i>
<i>defect</i>	The agent loses money.	The agent's preferences violate the axioms of rational preference.	The pragmatically defective outcome shows that the agent gave different evaluations to the same options.	The agent's degrees of belief sanction as fair a set of bets that leads to a pragmatically defective outcome.	The agent takes all members of a set of bets to be fair although they cannot be fair together.
<i>inconsistency</i>	–	Axiom-violating-inconsistency of preferences	Inconsistency -in-valuing	Inconsistency of an epistemic kind (left unexplicated)	Inconsistency that is analogous to logical inconsistency

*Table 1: Defects and inconsistencies in RTA and several versions of DBA.*



Now, if we take the classical DBA, then the case is straightforward: Dutch Bookability is irrational in the sense of the possibility to be financially exploited and rationality is explicated as economic usefulness. Although there is a consensus that this solution is not satisfying, many opponents of probabilism still attacked DBA with counterexamples showing that in some certain circumstances it can be *useful* for the agent to have (or rather to use) incoherent degrees of belief.<sup>21</sup> However, such criticism loses its significance once it is appreciated that more contemporary DBA-s identify inconsistency as the defect that must be avoided.

Unfortunately the solution of relying on inconsistency is not straightforward: this concept is explicated in very different ways (see Table 1) and seems to be referring to different concepts of rationality. In the previous section we were referring to an analogy with classic logic, but in fact it is not even determined which kind of logic should be kept in mind.<sup>22</sup> Furthermore, it seems that in some cases no logic is kept in mind, since the term inconsistency is not even used in its logical meaning.

Armendt's argument seems to use the term in a non-logical sense: divided-mind inconsistency is a property of conflicting *evaluations* and hence it is essentially a preference defect. We have seen that there are problems with clearly identifying such defect but now it must be added that the link to rationality is also blurred.

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21 For a classical explication see Kennedy and Chihara (1979), for a more recent example see Eriksson and Hájek (2007).

22 Hájek (2009) for example notes that if the countable additivity axiom is formulated sententially then the logic has to be infinitary and that the omega-inconsistency arising in countable Dutch Books might not be of the troubling kind. He asks us to consider an infinite set of sentences that has as members  $F_n$  for every natural number  $n$ , but also  $\neg(\forall x)F_x$ .

Weatherson (2003) has argued that the logic suitable for the context of Dutch Book Arguments is intuitionistic, since the outcomes of the bets must be verified.

The argument seems to be referring to some rationality of evaluations, but this notion of rationality could use some further explication.

What does it mean for preferences to be rational and how does it relate to epistemic issues? One can agree that divided-mind inconsistency (if established) is a shortcoming, but still question what precisely is the underlying clearly irrational property of beliefs which are traditionally considered to be epistemic entities. According to Joyce's (1998) diagnosis, Armendt's argument for probabilism would be convincing if it demonstrated that in case of incoherent credences the strength of beliefs varies when the same propositions are expressed in different ways. This would count as a defect of degrees of belief themselves, but instead Armendt's argument concentrates on preferences and leaves the epistemic entities aside. This of course is explained by reduction of degrees of belief to preferences, but this move does not clarify much but raises new questions instead (as seen in Section 2.2).

Similar questions can be raised about RTA although we have seen that it is otherwise quite a different argument. RTA offers quite another explication of the inconsistency of preferences—preferences are inconsistent if they do not satisfy the axioms of rational preference. Rationality demands conformity to the probability calculus, because otherwise the axioms of rational preferences are violated. Such precise explication also brings along criticism—how is this axiom-violating inconsistency connected to rationality?

Naturally, the axioms of rational preference differ a little from one axiomatization to another, but there is no uncontroversial set that would suffice for the purpose of the argument. Savage's (1954) system for example posits quite plausible normality and transitivity axioms but also a more doubtful Sure Thing Principle: if one would prefer doing  $f$  to doing  $g$  if a proposition  $X$  is true, and one would not otherwise prefer  $g$  to doing  $f$ , then one should prefer doing  $f$  to  $g$  in any case. By the proponents these axioms are defended as rationality principles but it is

very difficult to demonstrate that violation of any of those is irrational.<sup>23</sup> The axioms are surely not obvious and they generate some surprising and often strongly counterintuitive assumptions.

Allais and Ellsberg paradoxes are well-known decision problems demonstrating that most people have preferences that violate the Sure Thing Principle and they do not seem irrational because of it.<sup>24</sup> Even the innocuous principle of transitivity is rather hard to justify without begging the question. The most popular attempt is the so-called money-pump argument which shows that acting on an intransitive set of degrees of belief enables the opponent to construct a circular betting strategy and win from every circle.<sup>25</sup> The underlying feature of this practical deficiency of intransitivity is said to be the defect of giving two different evaluations to the same option. All this should already be familiar from the analysis of DBA which is indeed also a particular form of a money-pump argument. And similarly to DBA the money-pump argument for transitivity also assumes value-additivity, the feature supposedly proved by RTA.<sup>26</sup> It can be concluded that the necessary axioms of rational preference are not better established than the value-additivity principle itself.

We can conclude that neither divided-mind inconsistency nor axiom-violating inconsistency have some obvious connection to logic. Such labeling can be rather confusing and it should be emphasized that those arguments are based on defects

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23 See Maher (1993) for an overview of the posited preference axioms, corresponding problems and possible solutions .

24 See Gärdenfors and Sahlin (1988) for a debate over the relevant researches.

25 Suppose the agent prefers the outcome  $X$  to  $Y$ ,  $Y$  to  $Z$  and  $Z$  to  $X$ . Then the evaluation of  $X$  is that of  $Y + p$ , the value of  $Y$  is that of  $Z + q$  and the value of  $Z$  is that of  $X + r$ . As the combination of exchanges  $X$  for  $Y$ ,  $Y$  for  $Z$  and  $Z$  for  $X$  is an indirect way of the simple exchange  $X$  for  $X$ , such evaluations can easily be taken advantage of.

26 See Shick (1986) for an explication of the value-additivity assumption in the money-pump argument.

of preferences which appear quite removed from the familiar notion for full belief that constitutes an epistemic flaw (Vineberg 2001). Accordingly, the irrationality inherent to those preference defects must be an irrationality of preferences. Thus the results of preference-based arguments seem to be only relevant to decision-theoretic rationality and have no connection to rationality understood in an epistemic sense.

Admittedly, it is not fully intelligible what the definition of such epistemic rationality would be, but here it should be enough that it is not rationality of preferences.<sup>27</sup> Traditional epistemology does not define rationality of beliefs in terms of preferences, and the authors of depraematized DBA-s argue that we should talk of epistemic rationality also in the case of partial beliefs. Epistemic irrationality is manifested by an inconsistency among degrees of belief and here the degrees of belief are not defined or understood via preferences. Thus epistemic rationality is not dependent on some accompanying pragmatic irrationality of preferences, even if this could be proved.

We saw in the previous chapter that Christensen's depraematized DBA refrains from explicating the epistemic defect and thereby fails to assure a firm link to epistemic irrationality. Although he rejects reductionism, he continues to rely on the diagnostic device of a pragmatically defective outcome. Howson and Urbach (1989, 1993), on the other hand, indicate an inconsistency among degrees of belief which is in close analogy with the inconsistency of full beliefs.

As to the link between Howson and Urbach's specific inconsistency and epistemic irrationality, we can find support from the analysis offered in the previous section. We argued that probabilism takes one aspect of good thinking (coherence) to the limit and thus models ideally rational degrees of belief. As this argument draws on the analogy between logical consistency and probabilistic

<sup>27</sup> For a more thorough account of differentiating between decision-theoretic rationality and epistemic rationality, see Hellman (1997).

coherence, it is admissible only for Howson and Urbach's argument, which identifies an inconsistency that is very tightly connected to logical inconsistency as traditionally understood. Thus I conclude that Howson and Urbach's (1989, 1993) version of DBA is the best candidate for establishing a connection between Dutch Bookability and epistemic rationality.

#### 4.2.3 Redundancy of DBA

The following table sums up the results of the analysis and introduces an answer to the redundancy question.

	<u>Classical DBA</u>	<u>Preference-based arguments</u>		<u>Depragmatized arguments</u>	
		RTA	Armendt's DBA	Christensen's DBA	Howson and Urbach's DBA
<i>d e f e c t</i>	The agent loses money.	The agent's preferences violate the axioms of rational preference.	Relying on the diagnostic device of a pragmatically defective outcome  The pragmatically defective outcome shows that the agent gave different evaluations to the same options.	The agent's degrees of belief sanction as fair a set of bets that leads to a pragmatically defective outcome.	The agent takes all members of a set of bets to be fair although they cannot be fair together.
<i>i n c o n s i s t e n c y</i>	–	Inconsistency of preferences		Inconsistency of epistemic entities	
		Axiom-violating-inconsistency of preferences	Divided-mind inconsistency of preferences	Inconsistency of an epistemic kind that is left unexplicated.	Inconsistency that is analogous to logical inconsistency
<i>i r r a t i o n a l i t y</i>	Economic irrationality.	Irrationality of preferences i.e decision-theoretic irrationality		Irrationality of epistemic entities i.e epistemic irrationality	

Table 2: Relations between RTA and several versions of DBA.

We see that there are two completely different enterprises: establishing the probability axioms as the conditions of decision-theoretic irrationality and establishing the probability axioms as the conditions of epistemic irrationality. The fact that these goals are principally different from one another brings along significant consequences for the redundancy question.

The answer to this redundancy question depends on the fact if all the goals of DBA are achieved by the utility theory in a better way. Now, we have seen that certain versions of DBA purport to offer something desirable that is not available in the framework of utility theory—a demonstration that violating the probability axioms brings along epistemic irrationality. Thus DBA can strive at a genuine contribution to epistemology or even logic.

In contrast, the utility theory has different goals and another subject area: it operates in the field of prudential behavior and contributes to decision theory rather than epistemology. The decision-theoretic account of prudential behavior uses probabilities as a device for interpreting a person's preferences—it endorses a view that degrees of belief reduce to preferences or are definable in terms of them. From this point of view it is not possible to demonstrate some purely epistemic irrationality of incoherence—the discussion of a defect of degrees of belief cannot be separated from preferences, because degrees of belief are defined via preferences.

However, one can hold that “we should not need a general theory of rational preference in order to speak sensibly about estimates of uncertainty and the laws they obey” (Howson 2001). This is an alternative metaphysical view-point—degrees of belief are epistemic entities and interactions with preferences do not have to be important when we talk about their inconsistency. Thus we see that preference-based arguments are based on a metaphysical cornerstone that is incompatible with the corresponding cornerstone of de pragmatized arguments. I

thus claim that de pragmatized DBA-s can in principle not be made redundant by any preference-based argument, since the latter does nothing to fulfill the goal of de pragmatized arguments.

However, the argument of different goals does not apply to all versions of DBA and thus there are also two different answers to the redundancy question. As the divided-mind DBA of Armendt (1993) is also preference based, it is possible to argue that it is made redundant by RTA. This conclusion is backed up by the fact that this version of DBA is not as rigorous as RTA, operates in the same domain and fails to identify the defect that is supposed to be its essence. As RTA also has its problems, the matter of preferring one to the other is not straightforward, but in general I agree that in the case of preference-based DBA the redundancy claim is plausible.

As already said, the redundancy claim about de pragmatized DBA-s can in principle not be made without deciding the metaphysical matter beforehand. The redundancy should remain an open question, since there is not much hope of conclusively establishing the nature of degrees of belief.<sup>28</sup> Nevertheless, one could still argue for the redundancy of de pragmatized DBA-s if it could be shown that they completely fail at establishing their result. This does not have to mean that the possibility of reaching the epistemological goal is denied, since it is enough to show that DBA is simply a defective argument which has no hope of reaching any of the named goals. This is something that the opponents of DBA have claimed to be true in relation to the value-additivity problem—the most stressed objection against DBA. The next chapter will deal with this problem and scrutinizes the solutions that have been offered.

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28 The different underlying views are mostly just presumed and not explicitly formulated and argued for. I also do not aim at contributing to this issue and my purpose is rather to draw attention to the role that the unformulated underlying views play. I can note, however, that I clearly incline towards the purely epistemological point of view.



## 5 VALUE-ADDITIVITY

Now we have reached the crucial part of the analysis—the value-additivity problem has been considered to be the most decisive factor for the conclusion of the redundancy. It has been claimed that DBA needs the additivity principle to reach its result, but has no perspective to prove it. The wide-spread reaction to the value-additivity problem is to regard DBA as redundant and turn to RTA for the justification of probabilism.

Since value-additivity is the main reason why DBA is rejected, this subject is of utmost importance to the thesis of this dissertation. In this chapter I will offer an analysis of the solutions that have been proposed by the proponents of DBA-s. this discussion relies heavily on the antecedent analysis, especially the distinction between different types of DBA-s. It is claimed that the threat posed by value-interference is not the same for different versions of the argument, since their dependence on value-related issues is different.

On the whole, two different approaches to value-additivity problem can be distinguished. One possibility is to confront the counter-examples and try to give a more sophisticated account of partial beliefs which makes allowances for utilities and valuing. This approach is motivated by the complex value situation that is working in reality. The second possibility is to dismiss the value-interference cases and develop the argument for only the cases where there is no need to worry about value-interference. The proponents of this view aim at giving a clear account of consistent degrees of belief and claim that in order to do this it is admissible to abstract away from the complexities of reality. In following, I will first exposit the problem of value additivity and then consider two above-mentioned approaches in turn.

## 5.1 Violations of Value-Additivity

Let us first take a closer look at the role that value-additivity assumption plays in DBA. As we already saw while questioning Armendt's divided-mind inconsistency (Section 2.1), it does not play some minor extraneous role, but is instead essential for the proof of the Dutch Book theorem. More precisely, the problematic assumption is needed in relation to the additivity axiom, to demonstrate the Dutch Bookability of degrees of belief that violate this axiom. To understand the need for the value-additivity assumption, let us examine the proof of the additivity axiom with an eye to values.

Suppose that we have two mutually exclusive propositions  $X_1$  and  $X_2$  and the respective bets  $BX_1 = '1 \text{ if } X_1, 0 \text{ if not}'$  and  $BX_2 = '1 \text{ if } X_2, 0 \text{ if not}'$ . We define  $B(X_1 \vee X_2) = '1 \text{ if } X_1 \vee X_2, 0 \text{ if not}'$  i.e. its payoff for each distribution of truth values over  $X_1$  and  $X_2$  is the sum of the payoffs from  $BX_1$  and  $BX_2$ . Let the agent's fair betting quotient on  $X_1$  be  $VBX_1$  (his fair price for  $BX_1$  is  $VBX_1$ ), on  $X_2$  be  $VBX_2$  (his fair price for  $BX_2$  is  $VBX_2$ ) and on  $X_1 \vee X_2$  be  $VB(X_1 \vee X_2) = V(BX_1 + BX_2)$  (fair price for  $X_1 \vee X_2$  is  $VB(X_1 \vee X_2) = V(BX_1 + BX_2)$ ).

The argument will assume that the additivity axiom  $V(BX_1 + BX_2) = VBX_1 + VBX_2$  does not hold and try to show that this enables the construction of a Dutch Book. So let us look at the betting deals that are enabled when the axiom is violated in one direction  $V(BX_1 + BX_2) > VBX_1 + VBX_2$  (the proof for the other direction is analogical). Now comes the crucial part: notice that if the bets on  $X_1$  and  $X_2$  are sold then this amounts to receiving  $VB(X_1 \vee X_2)$  and not necessarily  $VBX_1 + VBX_2$ . But in order to use the inequality and show the sure loss of the agent we would need to have the value of  $VBX_1 + VBX_2$ . The bet on the conjunction  $B(X_1 \vee X_2)$  is obviously equal to the sum of separate bets  $BX_1 + BX_2$  and therefore  $V(BX_1 \vee X_2) = V(BX_1 + BX_2)$ . But to reach the desirable result, we would need the additional  $V(BX_1 + BX_2) = V(BX_1) + V(BX_2)$  to hold. This is precisely the value-

additivity assumption: the value of the sum of the bets is the same as the value of those bets separately.

Thus the agent can violate the additivity axiom and nevertheless avoid the Dutch Book when she uses  $VBX_1$ ,  $VBX_2$  and  $V(BX_1 \vee X_2)$  consistently.<sup>29</sup> This reassuring might even be comforting since there are indeed many reasons why the agent should want to violate the additivity axiom. For example, the value of the bet can change according to the stakes in question. As the stakes are smaller for bets taken separately, the value of the sum of separate bets might be different than the value of a compound bet.

Let's say that Maria badly needs some money and wants to try her luck and gamble, but does not have much money to spend. She considers it fair to pay ten euros in the hope of getting twenty euros back if the coin lands heads. She also considers it fair to pay ten euros in the hope of getting twenty euros back if the coin lands tails. Nevertheless it is clear that the value of the compound bet is not equal to the values of separate bets. It must not be equal even if these bets are about different coin throws, because she does not have much money to spend and the possibility of losing twenty euros is already too frightening.

Dependence on stakes is tied to the utility function of money—DBA assumes it to be linear while it is not so in real life. Due to the diminishing marginal utility of money the fair price cannot be considered to be proportional to the stake. Twenty euros might be such a stake that the fair betting quotient of  $\frac{1}{2}$  seems fair to Maria, but after winning the gamble the other such bet does not necessarily have the

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<sup>29</sup> This surely does not mean that the formal correctness of the Dutch Book theorem is impugned. The mathematics is indisputable, but every theorem starts with setting the scene and making necessary presumptions. And this must of course be done in an explicit and rigorous way. Mathematics does not include interpretations, for example it might specify the formal properties of the function  $V$ , but the argument if it is appropriate to take  $V$  to be a value function is exterior to mathematics.

same value anymore. And after she has surprisingly succeeded to win many such gambles in a row and become quite rich, the possibility of winning 10 euros more no longer seems valuable to her. We see that values can interfere with each other in many ways and an impressive amount of plausible counterexamples is available to undermine the principle.

Although enough counterexamples have already been brought to show the implausibility of the value-additivity principle, one more special case should be mentioned. Namely, the principle is especially problematic when we are dealing with infinity. Although most probabilists contend themselves with finite additivity DBA is easily extended to cover countable additivity and this does not match our intuitions at all. The most famous and compelling example is the case of the infinite lottery. Already de Finetti (1972) noted that a criterion of consistency should not forbid a uniform distribution over a countable disjoint set. Or to take a more recent comment, Hájek (2009) concluded that “there simply need not be any tension between judging each of an infinite package of bets as favorable, and judging the whole package as unfavorable”. So the fact that DBA seems to establish not only finite but also countable additivity as a rationality constraint seems to speak against its validity as an argument. I will return to this issue in the analysis to come.

To conclude, DBA needs value-additivity principle in order to prove a theorem that is essential to the argument but it does not stand a chance to prove this principle. Naturally this has been often highlighted as its weak point. Without the principle DBA can only show that a rational agent has to have either coherent degrees of belief or her values have to violate additivity. Before turning to the attempts to solve the value-additivity problem, it is appropriate to end this section with a quote from Schick who was the first to point out the above-mentioned failure of DBA-s.

“The values a person sets are typically not independent (or additive). Where his probabilities don't conform to the principles, prudence indeed requires that his values not be independent. This may be worth noting, but it does not take us far. We can never assume that a person's probabilities go against the principles. So we cannot argue that his values had better not be independent. The converse of this holds too: since we cannot assume that the agent's values are independent, we cannot argue that his probabilities must confirm to the principles.” (Schick 1986: 114, italics in original)

## 5.2 Sophisticated Model

We have seen that there are other interfering values that are not incorporated by monetary payoffs. This seems to lead us to the conclusion that the preference structure of the agent must be taken into account in a more systematic manner and “degrees of belief and utilities have to be elicited in concert” (Earman 1992). As Howson (1995) remarks, utility seems to be quietly assumed anyway, since the fair price must be independent of the stake and this is not true for stakes in money. If the concept of utility is implicit in the definition of the fair betting quotient then it is surely judicious to make this dependence explicit and see where it takes us.

Offhand it seems to solve all our problems: if we take the payoffs to be measured in quantities of value and thereby make allowance for the utilities, then the diminishing value of money, risk averseness, non-monetary goods and other interfering preferences are all taken into account. Of course, it is very complicated to calculate everything to utilities<sup>30</sup>, but the theoretical move of replacing money with utility still seems like a promising path that leads to an easy solution to the value-additivity problem.

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30 This was in fact De Finetti's (1937) reason for rejecting the utility-based approach. He proposed to concentrate on stakes in a certain range, not so small as to be completely uninteresting and not so large that the differences would start to lose their significance. See also Gillies (2000) for a more recent defence of such view.

Well, promising it might be, but easy it is not. As indicated by Maher (1993) and Kaplan (1996), when one adopts such solution then it is needed to explain the concept of utility and show that a rational person does indeed have such utility function. Furthermore, this simple step of replacing money with utility units unfortunately does not solve the value-additivity problem cause utilities do not have to be additive, as pointed out by Maher (1993). Thus turning to utilities can only be helpful if a befitting account of utilities is also offered.

### 5.2.1 Moderate approach to utilities

As a befitting account of utilities if offered by the general utility theory, the prevailing solution is to turn to RTA for a demonstration how value is measured and what are its properties. However, if DBA needs the utility theory—a theory which also establishes probability axioms as rationality constraints—then one might easily question if it does have any contribution at all. Furthermore, we have seen that RTA has its own presumptions and own problems and one can hesitate if it does in fact give a satisfying answer to the value additivity question.

So accounts of utility sufficient for DBA but different from the complete utility theory are welcome. It deserves to be noted that DBA does not claim to offer an account of all the aspects connected to degrees of belief, in fact its purpose is much more moderate since it deals with just a certain constraint on degrees of belief. Thus something less than a full-blown theory of utilities might suffice for its purposes.

A more moderate account of utilities is offered by Armendt (1993), whose answer to the value-additivity problem is twofold. On the one hand he advocates a simple model that simply presumes value-additivity to hold (see Section 5.3.1). However, on the other hand he admits the insufficiency of his simple model and outlines a more sophisticated model with payoffs in quantities of values. To this end he attempts to make enough sense of values to provide sufficient support for

the illustrative purposes of his divided-mind DBA. Therefore his approach does not claim to solve all the problems and complexities of values, since this is obviously not needed for DBA.

To start with, Armendt appeals to our intuitive familiarity of rough measures of value and ability to weigh the relative importance of ends that are not pure commodities. While this does certainly not amount to a rigorous formulation of the utility function, it serves as a reminder of the fact that our introspection supports positing it. We can suppose that it is enough to make it admissible to talk of bets with payoffs in quantities of values and idealize our model as to allow it to represent degrees of belief by ratios of those payoffs. If we grant that move, then there remains the question of additivity and how badly failures of it undermine the illustrative model that betting scenario offers.

Without a more thorough exposition of utilities there is no chance of proving any such general principle like value-additivity. Instead, Armendt (1993) proposes to look at the plausible violations of value-additivity and see if we can exclude them on some grounds. He argues that the additivity failures, which are due to the higher risk of compounded losses, are the most damaging for the needed assumption. It seems reasonable that the value of the sum bet, which carries a risk of compounded loss, is less than the sum of the values of the component bets.

The next move of Armendt (1993) is to draw attention to the fact that the above-mentioned failures assume that degrees of belief (represented as usual by fair betting quotients) are dependent on stakes: if the risk of compounded loss on a compound bet carries extra weight, then the doubled risk due to doubled stakes should do the same. If we deem degrees of belief to be stake-independent, then these violations of value-additivity are no longer possible. Of course, it can be said that dependence on stakes is one of the reasons why we turned to utilities in

the first place and we go round in circles if we are introducing the same presumptions again.

Indeed, the approach suggested by Armendt is not a new proof of anything and it does not pretend to be such. As a starting point it avowedly makes a presumption that degrees of belief are stake-independent. It goes on to build that assumption into a betting scenario, this time with payoffs in utilities. Then it points out that the most natural reasons for doubting additivity are already invalidated by this initial assumption. Armendt (1993) admits that there can be additivity failures in the other direction and that these are not dealt with so easily.<sup>31</sup> A possible approach to this is to claim that these possibilities are not so strong and pervasive that the *illustrative models* of the betting scenario and the DBA should be rejected.

Thus Armendt's (1993) solution is explicitly and entirely based on the assumption that what one believes is not a function of the importance of the matter at hand (although one's care in attending to it may be). The plausibility of this claim cannot be overlooked, but additional support is always welcome. Armendt (2008) tries to offer such reinforcement by arguing that a comparison with treatments of full belief suggests that the appeal to stake-invariance is not ad hoc. He draws attention to the fact that stake-invariance of belief is a widely accepted principle and even more so for stake-invariance of rational belief.

„When it is understood that the [Dutch Book] argument is used in connection with a model of stake-invariant belief, we are in a position to see that some objections to it, the ones I have labeled value-interaction objections, can be defused. The objections focus on deliberative phenomena that the model, incorporating a standard presupposition about belief, discounts. The phenomena may be often realized, but they arise from

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31 The inequality in the other direction is illustrated by the case where an agent is willing to overpay for insurance for broad coverage, compared to what one would pay for each of two narrower, non-overlapping policies alone. Armendt (1993) considers the reason to be the increased chance of winning, compared to that of the individual bets. He admits that no prior presumption of the argument helps to undermine these cases of additivity failures.



sources taken to be outside the doxastic inputs to deliberation, that is from sources other than belief.“ (Armendt 2008: 17)

Although Armendt's moderate approach illustrates and supports his position that the value-additivity problem is not an unsurmountable objection to DBA-s, it is still likely to be convincing only to those already favorably inclined. It is far too vague for achieving a more ambitious goal of solving the value-additivity issue. It draws on our relatively obscure understanding of utilities and measuring them, depends on a strong assumption of stake-invariance and leaves some failures of value-additivity unexplained. Hence it seems that such moderate approaches can only be of illustrative import and DBA simply does not have the means to take a well-founded stand on the general value-additivity question.

### **5.3 Dismissing the Problem**

It was noted at the outset of the chapter that one possible approach to the value-additivity issue is to admit the necessity of some theory of utility and the other to simply evade the problem. There is reason to think that DBA stands a much better chance of success if the latter course is pursued, since taking up the first means competing with the full-blown utility theory in its own field. DBA is after all a specific argument about rational degrees of belief and therefore has bleak prospects in such a comparison.

Current proponents of DBA-s have indeed mainly aimed at giving a simple and clear model and resisted the temptation to use utilities. In fact, all the versions of DBA we have been looking at—reckoning without long-abandoned behavioral DBA—refer to the possibility of analyzing degrees of belief without giving full account of preferences, utilities and values. All of them suppose that dealing with inconsistency of degrees of belief does not demand taking a stand on the general validity of value-additivity.

In what follows, it is argued that this move is not equally eligible to different versions since the relevancy of their argument might suffer too much. Furthermore, the admissibility of ignoring value-interference depends directly on the view that is held about separability of degrees of belief from preferences. But let us move step by step and see what exactly is offered to convince us of a simplified but nevertheless relevant explication of rational degrees of belief.

### 5.3.1 Simple models

The simple-model approach finds it admissible to assume additivity for the purposes of the argument. Let us start with Armendt (1993) who takes the presuppositions of linear utility of money and of values being represented by payoffs as the starting point of his inconsistency-of-valuing DBA. The point is to give a simple model that illustrates partial beliefs with a manageable stand-in for the complexities of real life situations:

“Actions are represented by wagers, ends by their payoffs. The advocate of the DBA does not ask us to believe that all choices are bets, nor that actions cannot carry inherent value. He does suppose that we are familiar with choices and actions that are like this, enough so that the illustration makes sense.” (Armendt 1993: 9)

Thus making those presuppositions does not mean arguing that these conditions are fulfilled in reality or that they should be, they merely serve the purpose of offering a clear model. Of course this raises the question whether the simple situation is not too remote from reality to be significant:

“If the criticism is made by saying that the DBA has failed to prove that the betting scenario illustrates something about us, then it shows misunderstanding of the point of the argument; worse than that, it may amount to an impossible demand to prove something, while assuming nothing” (Armendt 1993: 6)

However, problematic assumptions cannot be erased by merely lifting them to the status of a presumption. This move results with having to deal with the same problem on the other level and somehow establish the relevancy. But if an

argument about degrees of belief wants to leave the features of preferences and values out of its account then it must presume that degrees of belief are—at least to an extent—separable from preferences and values. While considering the stake-invariance presumption, Armendt in fact alludes to separability:

“...the present question is whether what I believe (to what extent I believe it) varies according to the importance of the interests at hand. The view embodied in the betting scenario on this issue is that, while we may adjust our decision-making when the importance of the decision changes [...] those adjustments are in our utility function and decision-making, rather than variation in our belief.” (Armendt 1993: 8)

He goes on with noting that it is far from clear that *degrees of belief* vary according to the size of stakes, although the latter affects our attitude towards bets and influences the values. I fully agree with this, but I find that Armendt can not find much support from this fact—suggestions of separability cannot be taken seriously if degrees of belief are reduced to preferences.

We have seen in previous chapters that the divided-mind DBA takes the defect of degrees of belief to *be* inconsistent valuing and therefore a defect of preferences. Leaning on separability certainly looks dubious as part of the argument that assumes such a tight connection between degrees of belief and preferences. In fact, Armendt (1993) partly admits the weakness of his simple-model approach—although he does not seem to recognize the reason for it—and sees the necessity of a utility-based DBA that would nevertheless not completely depend on the utility theory.

It was claimed above that if incoherence is taken to be a defect of valuing, then it is difficult to make out a case that different value complexity does not make a difference. Hence it is appropriate to turn to Christensen's version of DBA that was found to be an upgrade of inconsistency-of-valuing DBA improving on exactly the reduction matter (Section 3.3). Similarly to Armendt's (1993) argument Christensen (2001) takes the presuppositions of linear utility of money

and of values being represented by payoffs as the starting point of the argument and separates the argument itself from the discussion if this situation can be considered to say anything on the whole. In fact, he explicitly reformulates his argument so that it applies only to simple agents—agents who value money in a linear way and do not value anything else. After this move the opponents have no reason to reject the argument itself but they certainly would not admit it to be significant.<sup>32</sup>

To the question of relevance, Christensen (2001) answers that the restricted scope of the argument does not deprive it of its interest, since the point of the argument is not dependent on the peculiarly simple values assumed. This seems to be the main relative merit of Christensen's modification of Armendt's DBA:

“For if the basic defect were in the simple agent's preferences, then it would be unclear why we should think that the problem would generalize to agents with very different preference structures. But the basic defect diagnosed in the simple agent is not a preference-defect. In severing the definitional or metaphysical ties between belief and preferences, the de pragmatized Dutch Book Argument frees us from seeing the basic problem with incoherent beliefs as a pragmatic one, in any sense. The simple agent's problematic preferences are functioning here as merely a diagnostic device, a device that discloses a purely epistemic effect.” (Christensen 2001: 373-374)

But replacing the metaphysical connection with a normative one is not tantamount to separating degrees of belief from preferences. And the fact that Christensen grounds his argument on a connection between degrees of belief and preferences does by far not establish its independence of preference patterns.

Although the basic defect Christensen hints at is not a preference-defect, preferences play an important role in his argument. He can call it a 'mere diagnostic device' but his whole argument consists of nothing much except the diagnostic device. Recall the discussion of his argument in Section 3.3 where we saw that Christensen leans heavily on the defective evaluations that make the

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<sup>32</sup> Such attitude is taken by Maher (1997) for example.

pragmatically defective outcome possible; identification of the basic defect in under-lying degrees of belief is either not done or it is highly preference-dependent. For this reason it is unconvincing that the lesson of the argument is not even restricted to agents who actually have the preferences sanctioned by their beliefs as Christensen claims. So while he sees “no reason to think that the defect is somehow an artefact of the imagined agent's unusually simple value structure”, I in turn see no reason to think that the defect is present when it does not give rise to defective preferences.

Thus the relevancy of Christensen's model is still questionable, although his simple model is more credible than Armendt's. It remains unclear though, how much more credible it is, because Christensen does not express an opinion on the separability issue. He rests his case on the normative connection and it seems that he does not deem it possible to theorize about degrees of belief without—either explicitly or inexplicitly—bringing preferences into the picture. In any case, the argument he offers is not independent of preferences, which sheds doubt on its putative escape from the value-additivity problem. I conclude that restricting the argument to simple agents can well be argued to limit its relevancy and ignoring value-interference would be much more admissible, if it would not depend on preferences.

The only such value-independent DBA that I have accounted and found worth advocating is due to Howson and Urbach (1989, 1993). Ironically, the main problem of this view is exactly the extreme separation of degrees of belief from preferences. Betting is a highly value-related context and it might be that one's assessment of fair odds is a judgment, that cannot be divorced from considerations of one's gain or loss and how one values these. Notice that there are two sides to this problem.

Firstly, the de pragmatized account is continuously misinterpreted—and as a result also undervalued—because of the value-implications brought forward by the terminology of DBA. So much so, that Howson, who continued to follow up on his de pragmatized DBA for years, has now denounced his former epistemological ambitions of modeling rational degrees of belief and turned to the logical view—a context that makes the preference-independence more clear and admissible.<sup>33</sup>

Secondly, opponents can simply decline the separation and claim that partial belief is necessarily connected to preferences and values and every bit of theorizing about them must take this into account. It was already stressed in Section 4.2.3, that these are incompatible metaphysical cornerstones and the confrontation does not seem to be solvable by conclusive arguments. Offering examples and counterexamples to illustrate one's underlying conviction might be about all that there is to do. To that end I will later present Hellman's (1997) illustration of Howson and Urbach's argument. But my main interest and contribution lies in the first subject-field and I hope to clarify how the separation enables to overcome the value-additivity problem.

### 5.3.2 Value-Independent Additivity

For a long time I was puzzled that the matter which urgently needs attention is treated as unproblematic in the context of de pragmatized DBA. It was long-known that value-additivity is the main problem of DBA-s, but the original presentation of Howson and Urbach and the later works of Howson still left the

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<sup>33</sup> Logic is not about rational belief or action as such, rather it provides the conditions regulating coherent truth value assignments (deductive logic) or coherent betting quotients (probabilism). The logical view that Howson advocates continues to use fair betting quotients to represent degrees of belief, but does not need to appeal to Dutch Books or other betting-related transactions (see also Section 4.2). Howson stresses that “the features which prevent the standard Bayesian model from being a plausible theory of rationality—sharp values, closure under the Boolean operations—are strikingly features it has in common with the models of ordinary deductive logic” (Howson, 2003: 4).

question unanswered. After admitting the necessity of presupposing additivity, the corresponding principle is simply claimed to be plausible without further explication (Howson and Urbach 1989, 1993, Howson 1997b, 1997c, 2001, 2003).

It was only after I fully realized the crucial disparity between de pragmatized DBA and other versions of the argument that I started to recognize small pieces of the puzzle in different sources. I resolved that the principle is indeed plausible in the context of de pragmatized DBA, but this plausibility demands to be thoroughly accounted for. Subsequently I will present the essence of what I have sorted out about Howson's approach to value-additivity.

As might be expected, the answer is rooted in the extreme separation of degrees of belief from preferences (hereon I will abbreviate it to extreme separation). Such separation is certainly not an isolated statement, but is instead inherent to every argument, claim and definition in the theory. If the extreme separation is left unnoticed—as it tends to be—every argument, claim and definition of the de pragmatized DBA is misinterpreted. Once again, our usual conception of betting and usage of the corresponding terminology certainly upholds this mistake. This might explain Howson's wish to turn away from DBA-s, but it does not justify his antecedent neglect of the value-additivity question. In my view, it is crucial to draw attention to the extreme separation on every level of the argument. I will subsequently try to do that.

Howson and Urbach start their argument with constructing the thought-experiment and explicitly stating that it does not bring along any consequences for the actions of the agent. While it is necessary to stress the difference from the behavioral DBA, it would be as important to state that all considerations of preferences and values are also dismissed. This remark would be especially helpful while explaining fairness. If attention is not drawn to it, then it is natural

for the reader to follow the tradition established by other versions of DBA-s and understand fairness in a value-related manner. Since fairness holds the key to the plausibility of additivity principle, let us now look closer at the notion of fairness that Howson and Urbach have in mind.

Howson and Urbach (1989) define subjective fairness as a property of conferring no advantage to either side of the bet (from the subjective point of view of the agent). Naturally, that raises the question how should we understand the notion 'advantage', which is left undefined by Howson and Urbach. However, they point out that the informal notion of advantage is mathematically representable as expected value relative to the person's subjective probability function.

It can be argued though that this formalization is not admissible—it is not suitable to represent advantage via expected value. Maher (1997) looks 'advantage' up in a dictionary, finds it to mean “benefit, profit or gain” and argues that such advantage is certainly not stake-independent. Moreover, there is an intelligible rationale for letting one's advantage depend on the size of a stake: consider an even money bet on a proposition that a fair coin lands heads; if the stake is relatively low (say 1 euro), then the advantage is zero (one would not regard buying or selling the bet as a benefit, profit or gain), but if the stake is very high (say 10000 euros) then the bet has a negative advantage because buying it is a liability and selling it “benefit, profit or gain”. Thus our pretheoretically intelligible notion of advantage is not suitable for Howson and Urbach's purposes since it is not as divorced from preference as they need it to be.

While Maher's (1997) objection advocates a certain alternative understanding of advantage, Vineberg (2001) offers a more neutral analysis. She sees the use of non-epistemic terms as a weakness of the de pragmatized DBA and concludes that the success of Howson and Urbach's argument depends on the meaning of



'advantage'. If advantage turns out to reduce to preferences, then the inconsistency of partial beliefs would be a pragmatic notion after all.

The missing piece—definition of advantage that does not depend on preferences—can be found in Howson (1997b). Here, in the context of refuting Neyman and Pearson's logic and substituting it with probabilism, Howson finally explains that for him 'advantage' is “a bias in a bet, so that the advantage to you in a bet against me is equal to the advantage to me if and only if the advantage to both is zero” (1997b: 277). He notes once more, that although the explication of advantage as expected value is not presupposed, it is still “an illuminating way of giving mathematical expression to the idea, that a bet at even money on heads with a throw of a fair coin is fair, independently of the stake and the fortunes of players” (1997b: 277).

In contrast, if advantage is understood as “benefit, profit or gain” then the advantage can be negative for both sides of the bets. Conveniently, this is demonstrated by Maher's above example where a bet with a very high stake is disadvantageous for both sides. In Howson's interpretation this bet would indeed be fair no matter what the stake would be, since the agent assesses unbiasedness.<sup>34</sup> An even more befitting example of conflating two interpretations of advantage is the famous St. Petersburg paradox concerning a game of chance, which is allegedly disadvantageous although its expected value converges to infinity.<sup>35</sup> This paradox also begs the question that a bet cannot be fair if it would

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34 Strictly speaking it is not correct to say, that this bet is necessarily fair for the agent, since this depends on the particular agent and her background knowledge (if the particular coin does not seem fair to her then she might judge quite different betting quotients to be unbiased).

35 In this game of chance the agent pays a fee to enter and then a fair coin will be tossed repeatedly until a tail first appears, ending the game. The pot starts at 1 dollar and is doubled every time a head appears. The agent wins whatever is in the pot after the game ends i.e.  $2^{k-1}$  dollars if the coin is tossed  $k$  times until the first tail appears.

be highly imprudent for one side (or both sides) to accept. Hence fairness is implicitly connected with prudential matters again and the extreme separation inherent to deprivatized DBA-s is not taken into account.

We saw that assessments of biasedness are to be considered independent from the preferences and values of the agent. This can be made more comprehensible by slightly modifying the thought experiment: the agent should imagine a bet between two other agents so that she cannot consider herself as an interested party in this deal (Howson 2003). This change is not substantial and the point is exactly the same: one's own preferences are irrelevant; the agent must not prefer a larger amount of the substance to a smaller quantity (or the other way around) in order to try to balance the amounts. An alternative way to add credibility to value-independence is to state the condition in counter-factual terms: “were it possible to repeat a fair (to you) gamble arbitrarily often in what you deem the same relevant conditions (i.e. those on which you based your estimate of the odds), your odds should be very high that the average gain or loss should eventually differ very little from 0” (Howson 2007: 501). These are just different attempts to lend plausibility to the idea that a condition of fairness can be independent of any consideration of preference.

Although value-independent interpretation of fairness also has a strong tradition<sup>36</sup>, nowadays the context of justifying probabilism brings along the silent assumption of the relevance of preferences. But the disparity between two interpretations of advantage, equity and preference, follows already from the obvious fact that one may prefer a gamble that one acknowledges to be disadvantageous, as pointed out by Howson (2007). He continues with a charge:

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36 The zero-expected gain condition has been the condition of fairness in games of chance from the early eighteenth century. Motivation for this is provided by the fact that under suitable general conditions in repeated gambles whose expectation exists the average gain tends to the common expectation.

„Nevertheless the conflation is routinely made. Explicit in Savage’s claim that ‘to ask which of two “equal” betters has the advantage is to ask which of them has the preferable alternative’ (1954: 63), it has been a systematic feature of discussions of subjective probability throughout the twentieth century, undoubtedly assisting the acceptance of the orthodox view that defining a fair bet in terms of expected gain commits the error of assuming that money is linear in value.” (Howson 2007: 500)

If one is already in search of such complaints about conflating two different interpretations of advantage, then they can in fact be found scattered all over Howson's work. What is missing is an integral whole, that would explicitly connect the distinguishing feature of de pragmatized DBA (extreme separation) to the main problem of DBA-s (value additivity).

So now we have a preference-independent definition of advantage at hand and it remains to be pointed out how it is relevant to the value-additivity problem. As usual, Howson (1997b) calls the needed principle a reasonable assumption and then neglects the issue. I propose that the needed value-additivity principle is plausible in the context of de pragmatized DBA exactly because it ceases to be about values. If it is granted that unbiasedness does not depend on values or preferences then the necessary principle should similarly not be considered to be about values. This is a missing piece that I have found nowhere in Howson and Urbach's or Howson's writings. While it might seem obvious, the confusion created by absence of a clear account shows that it must be emphasized. Extreme separation is so uncustomary in the context of DBA-s that its implications are not easily recognized.

Hence, according to my account of Howson's presumable view on additivity, no general or moderate claim about values being additive is needed. Rather, if bets are judged to be unbiased (give no calculable advantage to either side) separately then those bets are also unbiased when taken in conjunction. Values are of no relevance to the argument; they have only seemed relevant because gambles are traditionally paid in money. But the substance that is imagined for payoffs does

not affect the unbiasedness and the value attached to payoffs is not relevant to the intellectual judgment of biasedness.

If such clean separation of advantage—and therefore fairness and therefore in turn degrees of belief—from preferences is allowed, then the principle does indeed seem much more plausible than the principle that demands values to be additive. However, considering the long tradition of preference-based betting behavior and the present success of the utility theory, this approach might be incomprehensible at first and one might need some help in shifting the viewpoint. So it is worth to dwell on this question for a moment and offer a helpful illustration of the underlying idea.

One of the attempts to illustrate how degrees of belief can be thought of as completely separable from values and utilities is due to Hellman (1997a). Hellman endorses the approach of Howson and Urbach (1989) and purports to represent their de pragmatized DBA in its pure form so, that it is completely free from questions of value and utility. To say it in his words, “the only role of “preference” is one perfectly appropriate for scientific rationality: we prefer not to contradict ourselves” (Hellman 1997a: 194). To provide this illustration he proposes a more abstract setting, that does not include gambling and is not committed to any course of action.

Hellman (1997a) claims that if the real role of the betting scenario is to help the agent to pinpoint a numerical degree of belief in some proposition, then this can be performed by a number of other scenarios, that are not about betting at all. And some other more abstract scenario might be more befitting from the point of value-independence. He finds the essence of the scenario that the agent tries to estimate the “expected flow” of some “test quantity” in the direction of truth or falsity of the proposition.

More precisely, in Hellman's scenario the test quantity is said to display positive, negative, or zero expected flow according as the quantity  $xPr(h)$  is  $>$ ,  $<$ , or  $= yPr(\neg h)$  where  $x$  is the quantity of the fluid that flows in an arbitrarily chosen positive direction if  $h$  turns out to be true, and  $y$  is the quantity that flows in the opposite direction if  $h$  turns out false. When the expected flow is zero i.e.  $xPr(h) = yPr(\neg h)$  then we take  $y/(y + x)$  to be the neutral belief-test quotient. Now, analogously to Howson and Urbach's DBA, we have the Dutch Flow Argument establishing that a set of belief-test quotients can consistently be called neutral iff they satisfy the axioms of probability.

The test quantity might be thought of as ideal fluid and the agent who estimates the flow does not need to attach any value to it. Thus the estimator's preferences are completely irrelevant to assessing the expected flow of the fluid to be zero. Hellman hopes that this relabeling also helps to demonstrate value-independence of the needed additivity principle:

“All that is being required is that the expected flow of a (finite) *plurality* or *set* of simultaneous belief-tests each having zero expected flow be zero. Since we are imagining a conserved quantity and measuring degrees of belief by correlated expected *amounts* of the quantity, and since the amounts in each separate test are discrete from one another, one would generally compute the amount expected from a plurality or set of tests by adding the individual expected amounts;” (Hellman, 1997b: 319, italics in original)

This setup has the advantage of not referring to bets and thus avoiding the confusion created by our ordinary prudential interpretation of betting-concepts. If it is taken to be an independent argument for showing that degrees of belief ought to satisfy the axioms of probability, it can be accused of several deficiencies.<sup>37</sup> But it is rather aimed at bringing out the value-independent nature of Howson's and Urbach's argument.

The main contribution of Hellman's abstraction is illustrating the irrelevancy of preferences and the plausibility of value-independent additivity principle. He

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<sup>37</sup> See Maher (1997) for such criticism.

seems to be the first one to point out that the bet imagined by the agent is nothing personal i.e. the agent does not have to imagine herself as an interested party of the bet. Another important observation concerns the test quantity: no particular value has to be attributed to it by the agent while trying to estimate the flow.

If we talk of betting and payoffs then it is natural to calculate the payoffs in money, but this is actually very misleading and prompts the conflation of two interpretations of advantages. It would be befitting for the abstract nature of the hypothetical set-up to deal with a neutral quantity which has no connotation of value whatever (like some abstract ideal continuous fluid), but this is also not needed. Being inspired by Hellman, Howson (2001) suggests that one could even imagine coprophiliacs betting with stakes of manure. Then we may indeed have values present, but the agent's supposed disdain towards the bettors and disgust towards manure would in fact not affect judging the unbiasedness of the bet.

The above remarks should help to clarify that the reasonableness of the additivity principle is not necessarily affected by the shape of utility-of-money functions or other such considerations. It might even be that a Hellman-style thought experiment arouses the intuition that our judgments of uncertainty (degrees of belief) are separable from our preferences to a great extent but that is left for everyone to decide for themselves.

Now, after arguing that values and their additivity do not necessarily have to be of concern to DBA, it is noteworthy to observe that this renders value-additivity to be more of a problem for the utility theory than DBA. To be sure, measurements in terms of utility solve a good deal of the counterexamples presented in the beginning of the chapter. However, as already noted in Section 4.2.2, the axioms of rational preference—leading to the expected utility principle—generate some strongly counterintuitive evaluations and their status is thus controversial. But

these problems do not undermine a principle that is not about values at all. Let us now see what can be said about this value-independent additivity principle?

After separating the additivity principle from values it ceases to be a substantial problematic presumption that is not always satisfied in the cases of rational behavior, evaluations or judgments. The fact remains, that it is an additional assumption and it is not provable, but this objection has lost its destructiveness. Instead, several observations suggest at its plausibility. For example, it can be found “so fundamentally constitutive of the ordinary notion of a fair game that we are entitled to adopt it as a desideratum to be satisfied by any formal explication” (Howson 2001: 147).

In addition, the additivity principle is also supported by the close analogy of probabilism and formal logic: logic deals with extensional properties and the exchanges under question are extensionally equivalent (Howson 1995: 7). Or similarly, after seeing the kinship of fairness and truth, “it seems intuitively correct that fairness should behave with respect to a sum of fair bets analogously to the way that truth behaves with respect to a conjunction” (Howson 2003). Thus we see that there are reasons to argue for the principle that the class of fair (unbiased) bets is closed under sums.

If the assumption of value-independent additivity is raised to the status of an independent presumption, then it allows to differentiate between DBA-s for finite additivity and their extensions for countable additivity. For we have seen that DBA can reveal an inconsistency only in conjunction with the additivity assumption, by itself it is silent on that issue. Now if this principle is not extended over suitably convergent infinite sums then DBA does not establish countable additivity but merely demonstrates that the sum of an infinite set of fair bets is not necessarily fair.

Howson (2007, 2008) argues, that although finite additivity is intuitively very plausible, countable additivity should be rejected for a simple reason: if denumerable sums are allowed then the principle in question does not seem to be true. Recall the case of infinite lottery and the conclusion that the assumption of countable additivity does not fit our intuitions at all. Howson takes these observations seriously and argues that:

“...finite additivity can be seen as in effect producing a *compact* logic of uncertainty, which countable additivity does not. With finite additivity, if an assignment is inconsistent then some finite subset is: with countable additivity, on the other hand, the countable lottery above is an inconsistent assignment every finite subset of which is consistent, so compactness fails.”  
(Howson 2008: 8, italics in original)

Thus Howson rejects DBA-s that have been extended to cover countable additivity, since he does not grant the additivity principle necessary for those extensions.

In conclusion, the de pragmatized DBA offered by Howson and Urbach holds a special place among DBA-s. It is the only version of the argument that advocates extreme separation and therefore also gathers the harvest of resulting merits. If that is not recognized, then it seems to be very similar to simple models that presume value-additivity and thereby restrict their scope. But it turned out that extreme separation has substantial consequences for the value-additivity assumption. In fact it can be plausibly argued that no claim about the features of values is needed for this kind of DBA.

The de pragmatized DBA of Howson and Urbach and the simple-model approach have the common feature of not taking a stand on the universal validity of the value-additivity principle. But while the simple models refrain from doing so for the price of presuming a special simple-value context and therefore throwing doubt upon the relevancy of their model, de pragmatized DBA does not need to restrict its scope. It indeed holds a special place because it is the only value-



independent DBA and it is therefore left untouched by the value-based criticism against DBA-s, the criticism that is considered to be so destructive that all DBA-s should be abandoned.

This chapter has demonstrated that the de pragmatized DBA of Howson and Urbach stands a good chance at reaching its epistemological goal, *if* we allow it to separate the discussion of beliefs from considerations of preferences. Thus the solution clearly rests on value-independence and this is clearly not admissible if one takes the reductionist standpoint to degrees of belief and claims that they are necessarily connected to values. But I have argued that separability and reductionism are two incompatible metaphysical views and neither should be assumed while assessing the redundancy of the arguments in question. This should not be taken to mean that the metaphysical question should be ignored, quite the contrary, explication of its role is necessary and helpful for everyone to take a well-founded stand or knowingly refrain from doing it.

## CONCLUSION

DBA in its many forms tries to demonstrate that rational degrees of belief should be constrained by the probability axioms. The utility theory offers RTA as an argument that claims to prove the same result and a lot more. Nowadays it is widely held, that DBA serves at best as an illustration of deeper truths that are in fact proved by more successful and rigorous RTA. In this dissertation I do not take a stand on the issue if RTA is more successful in achieving its goal than DBA or the other way around—both approaches have their own problems which were also indicated here. But I do underline that DBA can have a different goal than RTA and thereby purport to achieve something desirable that is not available with the help of decision-theoretic approach. As this is not true for all kinds of DBA, the answer to the question of redundancy depends on the kind of DBA we are looking at.

The most abstract kind of DBA, Howson and Urbach's de pragmatized DBA, is certainly the most promising kind of DBA, if we have the redundancy question in mind. It is distinctly distinguished from other kinds by identifying an *epistemic* defect in incoherent beliefs and therefore being relevant to probabilism construed as a thesis in *epistemology*. This is clearly not achievable within utility theory where degrees of belief are essentially a device for interpreting a person's preferences and used for explaining behavior. The context of the utility theory is necessarily prudential and the only defect that it can demonstrate is the defect of preferences. To my mind, it has not been emphasized enough that although the utility theory may be fruitful for many purposes, it can not answer the purpose of identifying an epistemic defect.

For similar reasons, DBA-s that identify the defect as an inconsistency of preferences are not suitable to achieve this goal. These approaches are certainly a

step forward from the standard DBA that advocates avoidance of monetary loss. Nevertheless, it remains unclear what kind of irrational property of beliefs underlies the inconsistency of preferences and therefore this kind of argument has no clear connection to epistemology. Its metaphysical background is similar to the utility theory and the context stays prudential. In addition the utility theory establishes the principle of value-additivity while this version of DBA can at best argue that the problem of value-additivity is not so destructive as generally thought. Thus the widespread conclusion of redundancy is justified for divided-mind DBA: it is fair to say that if DBA-s illustrate certain truths about preferences, then it is superseded by the axiomatic expected utility theory upon which the case for probabilism can rather be grounded.

This answer to the redundancy question can also be formulated in terms of rationality, as shown by my analysis of the rationality issue. While all justifications of probabilism try to establish conditions of ideal rationality, it helps for addressing the redundancy matter to further distinguish between epistemological and decision-theoretical rationality.

RTA and preference-based DBA both try to accomplish that coherence guarantees rationality understood in the prudential decision-theoretic sense. Although both of these arguments refer to inconsistency, they explicate it in a different way. Both explications bring along a specific obstacle: while RTA has some trouble defending its principles of rational preference, divided-mind DBA has problems with identifying the existence of giving two different evaluations to the same thing. Since the latter demonstrations of inconsistency remain problematic for all three axioms, this DBA can be considered redundant in relation to the success of the utility theory.

But the conclusion is completely different if we consider depraematized DBA-s that relate to the epistemic concept of rationality. The inconsistency of degrees of

belief that is identified by Howson and Urbach's DBA is analogous to the notion of inconsistency in formal logic and such kind of inconsistency is related to epistemic rationality. These two concepts of rationality are significantly different and as a result the arguments strive to different objectives: the utility theory discloses a defect of preferences and de pragmatized DBA identifies a defect of degrees of belief. If this is recognized, then it becomes clear that the widely embraced attitude to redundancy must be adjusted.

The novelty of de pragmatized DBA stems from its different metaphysical base. De pragmatized DBA-s are an attempt to disengage the argument from the prudential context and talk about degrees of belief as intellectual judgments belonging to the epistemological domain. This approach presumes that it is possible and meaningful to separate the discussion of degrees of belief from the discussion about preferences. It holds that the connection between degrees of belief and preferences is not constitutive and preferences are irrelevant to the epistemic defect among incoherent degrees of belief. This underlying view cannot be conclusively justified as can not the opposite metaphysical approach.

The metaphysical question, separability of preferences from degrees of belief, is a main line in the dissertation that helps to explicate many questions. In the beginning it plays a role in distinguishing the preference-based view from de pragmatized arguments and in the end it leads to an answer to the redundancy question. It also plays a grand role in distinguishing between two main de pragmatized DBA-s and assessing the destructiveness of the value-additivity accusation: according to my analysis only Howson and Urbach's DBA escapes the problem.

Value-additivity has been the main motivation for renouncing DBA-s in favor of the utility theory. The opponents see it as a fatal flaw for all kinds of DBA, while the proponents of de pragmatized DBA do not see the question of values as

relevant. Depragmatized DBA-s are not so new on the scene that it would be justified to refrain from forming an opinion. In order to form a well-grounded opinion, it is needed to examine them in detail and not just assume that a reference to unproved value-additivity assumption is enough to cast them aside with other kinds of DBA-s. This is something that I hope to have done in the present dissertation.

I resisted the temptation to call Christensen's depragmatized argument similar to Howson and Urbach's as is usually done in the literature and on the closer look it turned out not to be so similar. In particular, it draws heavily on the dramatic device of preferences and consequently does not achieve independence of valuing. As a result, it does not explain the essence of the epistemic defect and also fails to make a clean escape from the value-additivity problem.

Howson and Urbach's argument, in contrast, really takes full advantage of the metaphysical separability and cuts off all relations between the argument and several concerns about preferences or values. It follows that this version of DBA does not assume the problematic value-additivity principle but a much more plausible assumption about additivity. Surprisingly, a clear explication of this was missing from the presentations of the argument and this certainly helped to uphold the common misinterpretation. My analysis of depragmatized additivity is intended to shed light on the significant consequences of the metaphysical separation for the additivity question.

As for other versions of DBA, values remain to be an issue: in order not to fall back on the utility theory, value-additivity is assumed in the framework of simple models. These arguments establish a connection between rationality and coherence in a simple value-situation and this result can indeed be granted to them. But the assumption of value-additivity clearly cannot be proved by DBA and hence the relevance of the model becomes questionable.

Thus simple models are admissible to the extent that separation of preferences and degrees of belief is allowed in the framework of the argument. Relevance is more plausible if one argues for separability of degrees of belief from preferences—as Christensen does to deny the restricted scope—but is very dubious if degrees of belief are reduced to preferences as in Armendt's divided-mind DBA. Thus the separability issue also determines the comparative success of simple models.

But while the value-independence of Christensen's argument remains dubious, Howson and Urbach's argument uses separation to the full. This fully de pragmatized DBA no longer needs values to be additive and thus it is significantly different from simple models that presume it. Until now this disparity has either been neglected or inadequately explicated and this dissertation contributes to its recognition. It must be acknowledged that neither is DBA necessarily bound to the restricted scope of the simple argument nor inevitably inferior to RTA. If the option of theorizing about degrees of belief in isolation from preferences and values is acknowledged as a possibility, then DBA can be developed in a way that eludes the destructive criticism and thus amounts to a powerful defense of probabilism.

Thus we have seen that it all boils down to the underlying metaphysical convictions. DBA-s and the utility theory both offer a justification, that rational degrees of belief should conform to the probability axioms. Although they can lend support to one another on some level, their apparent similarity should not be presumed for all kinds of DBA. In fact, the most promising version of it, Howson and Urbach's de pragmatized DBA, takes degrees of belief to be separable from preferences and thus emanates from a diametrically different metaphysical view. As a result, it has different goals, it operates in a different domain and refers to different concept of rationality. These two defenses of probabilism clearly appeal to different audiences and it is not eligible to presume one's general framework

for the criticism of the other. Thus my answer to the redundancy question is: DBA cannot be rendered otiose by RTA as long as their understanding of degrees of belief is disparate; favoring one to the other implicitly means deciding the nature of the connection between preferences and degrees of belief.

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## SUMMARY

This dissertation “The Analysis of the Redundancy of the Dutch Book Argument: Separability of Degrees of Belief and Preferences” disputes the general consensus that the Dutch Book Argument (DBA) is made redundant by the utility theory because of the value-additivity issue. It is argued that such a view does not do justice to a fully de pragmatized version of DBA, which does not depend on the features of values or preferences. To reach this conclusion, several versions of DBA are distinguished, the focus being on the role of preferences and their putative connection to degrees of belief. Convictions about the latter connection are in fact metaphysical and not solvable by conclusive arguments, however, they bring along significant consequences about redundancy. The offered analysis shows that de pragmatized versions can escape the value-additivity problem, which was the main motivation for the redundancy claim. Even more importantly, de pragmatized DBA-s strive towards a contribution to epistemology, a result that is completely unachievable for the utility theory.

## RESÜMEE

Käesolev väitekirj „Hollandi kihlveo argumendi tarbetuse analüüs: uskumuse astmete lahutamine eelistustest” vaidlustab üldlevinud seisukohta, et probabilismi õigustav Hollandi kihlveo argument (HKA) on seoses väärtuste aditiivsuse probleemiga vigane ja seeläbi ka tarbetu, sest kasulikkuse teooria (*utility theory*) annab meile nii probabilismi kui ka lahenduse eelmainitud probleemile. Antud töös väidetakse, et selline hinnang ei võta arvesse argumendi depraigmatiseeritud versiooni eripära, milleks on uskumuse astmete täielik lahutus väärtustest või eelistustest. Väite kaitsmiseks analüüsitakse mitmesuguseid versioone HKA-st, nii et tuleb esile, millist uskumuse astmete ja eelistuste vahelist seost neis eeldatakse. Näidatakse, et valik erineva tugevusega seoste vahel on metafüüsiline ja kuigi ükski valik pole argumentidega lõplikult õigustatav, annab probleemi selline püstitus vastuse tarbetuse küsimusele: lahutades uskumuse astmed väärtustest, saavad depraigmatiseeritud HKA-d võimaluse pääseda väärtuste aditiivsuse probleemist, mis oli peamine argument tarbetuse kasuks. Veelgi enam, depraigmatiseeritud HKA-d seavad eesmärgiks epistemoloogilise panuse, mis on kasulikkuse teooria jaoks põhimõtteliselt saavutamatu.