

University of Tartu
Faculty of Mathematics and Computer Science
Institute of Computer Science

Nadežda Velikanova

**ANSWERS TO EQUATIONS FROM SCHOOL
TEXTBOOKS OFFERED BY COMPUTER
ALGEBRA SYSTEMS**

Master's Thesis

Supervisor: Eno Tõnisson

Tartu 2007

Contents

Introduction.....	4
1 Classification of school mathematic equations	9
1.1 Classification of equations	9
1.1.1 Introduction	9
1.1.1 Linear equations.....	13
1.1.2 Quadratic equations	14
1.1.3 Fractional equations.....	15
1.1.4 Equations containing the absolute value of expression.....	16
1.1.5 Irrational equations.....	18
1.1.6 Exponential equations.....	19
1.1.7 Logarithmic equations	20
1.1.8 Trigonometric equations.....	21
1.2 Examples of solved equations	23
2 Analysis of the answers.....	28
2.1 Computer algebra systems	29
2.2 Different commands for solving the equations	34
2.3 Types of obtained answers	36
2.4 Equations and CASs.....	40
2.4.1 Equations	40
2.4.2 Derive.....	45
2.4.3 Maple	46
2.4.4 Mathcad	48
2.4.5 Mathematica.....	50
2.4.6 MuPAD.....	52
2.4.7 TI-92 Plus.....	53
2.4.8 WIRIS.....	55
2.5 Unexpected answers and some possible actions	56
3 Web info system.....	59
3.1 Structure of the system	60
3.2 Structure of database	65
Conclusion.....	70

Resümee	73
References	75
Appendix 1. Answers given by Derive	79
Appendix 2. Answers given by Maple.....	86
Appendix 3. Answers given by Mathcad.....	92
Appendix 4. Answers given by Mathematica	99
Appendix 5. Answers given by MuPAD	105
Appendix 6. Answers given by TI-92 Plus	111
Appendix 7. Answers given by WIRIS	117
Appendix 8. Structure of web info system	123
Appendix 9. Structure of the database	124

Introduction

Computer algebra systems (CASs, e.g. Maple, Mathematica) are powerful systems for doing mathematics (algebra, trigonometry, calculus, etc); mainly they perform symbolic, but also numerical computations. In the beginning CASs were designed mainly to help professional users of mathematics, and were not directly intended for use in mathematics education, therefore in comparison with schools they may have different notions and standards. Nowadays this situation is improving and more systems are created or modified taking into consideration in particular school problems and needs, but there are still some differences. Therefore it is important to investigate how different computer algebra systems can be used at school.

Computer algebra systems are tools that automate the execution of algebraic computations [Kutzler, 2000]. This option is new for schools. Because of the novelty of this new tool a lot of traditional questions have to be posed in a new way [Weller, 2001]. Different researches, that analyze CASs from different points of view, base on various experiments, for example how changes the working style of students while using CASs, how the computer algebra systems can be used in teaching mathematics, or their general correctness is tested [Weller, 2001]. It is very useful, but teachers also need to know more how exactly CASs behave with ordinary school problems, because few researches and studies investigate problems namely from school textbooks. Therefore various tests should be made in order to find out how well one or another CAS handles school problems. For instance, teachers would like to know whether certain CAS can take into consideration the level of knowledge (for example, do we need answer on the set of real or complex numbers, as according to Estonian national curriculum complex numbers are not taught at school).

I analyzed one of the largest researches in this field done by Michael Wester from school point of view [Wester, 1999]. One chapter of his book is dedicated to comparing how 7 different computer algebra systems (Axiom, Derive, Macsyma, Maple, Mathematica, MuPAD, Reduce) deal with chosen set of problems. All the results are collected into one table showing the overall

tendencies, shortages and patterns. Totally 542 problems were solved. Only 68 of them are approximately like those as taught at Estonian schools and another 34 are similar to problems solved in advanced math classes. The answers given by CASs are middling, namely 33 problems out of 68 were solved more or less correctly (according to Michael Wester estimate 5 or more programs give correct or nearly correct answer, others produce wrong or partly wrong answer or can not solve given problem). And only 10 problems out of 34 for advanced math classes were solved correctly. Searching through the internet showed that there are not enough studies that concentrate on school exercises, and investigate how one or another CAS is especially suitable with school curriculum, therefore more detailed research should be made.

In this thesis I concentrate on the immediate solving (student enters the equation and program gives the answer), even though CASs allow to carry out “step-by-step” solutions. Solving problem in one step is the basic use of CASs. In this case student doesn’t have to think about the order of the steps in the correct solution and therefore concentrates more on the other important aspects like interpreting the results, reasoning or predicting the answer. CASs can do both symbolic and numerical calculations and these both options are used in the research.

Research, presented in this thesis, could be more detailed and thorough. Mainly it is an attempt to develop for the strategy and methods that allow to investigate and to compare the answers offered by CASs with the schools answers. Also this study helps to discover possible patterns and dependencies in the answers obtained by CASs. In this thesis I investigated **how well 7 computer algebra systems handle equations from school mathematics**. These systems are Derive, Maple, Mathcad, Mathematica, MuPAD, TI-92 Plus, WIRIS. I used commands that solve the equations in **one step** and as a “**black box**” when intermediate results are not shown. Didactic base for “white box / black box” principles can be found in the article [Buchberger, 1990]. Examples of the equations are taken from **Estonian textbooks** but they are quite analogical to the ones taught in other countries. Reasons, why specifically this field is important and worth attention, are given in the rest of the introduction.

As it is very difficult and laborious work to investigate all topics, the equations are at the centre of attention in this thesis, because equations play very important role in the national school curriculum and are taught from the third grade. Solving the equations is one field of mathematics that is convenient to teach with the help of computer [Mysovskikh, 2004]. There is a wide range of ways in which computer algebra systems can be used, helping better to understand this field of mathematics. Not all computer algebra systems handle equations equally and some of them are more suitable for using at school than others.

The first aim of this work is to develop the strategy for analyzing the answers to the equations offered by 7 computer algebra systems that the University of Tartu has access to (Derive, Maple, Mathcad, Mathematica, MuPAD, TI-92 Plus and WIRIS). Analysis is based on the obtained roots, the complexity of command that is needed for solving an equation, the inaccuracies done by computer algebra programs and other shortages. Such analysis would help to decide, how well CASs are suitable for using at school, because even CASs, which are very powerful and have a lot of functionality, can produce answers that are insufficient at school. Also it is interesting to know if systems work similarly for different types of equations and give same answers.

In order to determine, how well CASs can handle equations taught at schools, there should be some classification of equations. For making some conclusions about how well CAS deals with school curriculum the most striking representatives of each type of equations should be tested with each program. Unfortunately, the classification given in the school textbooks is lacking clear boundaries and criteria, by which equations could be divided into subtypes. Therefore more detailed classification was made up. It is based on the analysis of some textbooks, manuals and consulting with the teachers of mathematics (actually this classification would suit practically every school as it is simple and ordinary). Chapter 1 gives an overview of the classification that is used for investigating CASs. One of the goals of this thesis is to check, whether the classification of school equations is suitable for investigating computer algebra systems.

Information about the place of certain type of equations in the national curriculum, namely in which school level this topic is taught, is given below the description of this certain type. Totally there are four levels at school in Estonia that are described one by one in the national curriculum [RÕK; HM]. 12 school grades are divided into levels as follows:

- 1 level – grades 1-3, children are approximately from 7 till 9 years old
- 2 level – grades 4-6, children are approximately from 10 till 12 years old
- 3 level – grades 7-9, children are approximately from 13 till 15 years old
- 4 level – grades 10-12, children are approximately from 16 till 18 years old.

As the national curriculum is based on these levels, not on grades, I will use the same division to describe the place of particular type of equations in the curriculum.

Descriptions of CASs and possible explanations of obtained results, unexpected from the school point of view, are given in chapter 2, also there are recommendations for those people, who are going to use analyzed computer algebra systems at school. For example, which commands to prefer for certain type of equations or which program to purchase? In the second chapter results, given by CASs, are classified into four types and advices for teachers are given about how he or she should behave, when meeting one or another type. Answers are collected into the tables and from those tables it can be seen, that usually CASs handle school mathematics equations well (appendices 1-7). Obtained results can be used in mathematics and mathematical didactics, because they can make applying of the technology at school easier.

To make the results of the research available to as many people as possible it was decided to design web info system, which collects data (problems, comments, etc) from different people, like teachers or other researchers, and makes it possible for others to use this data for their own research or work. Only the main ideas are described in chapter 3, while the phase of implementation the structure can be expanded and maybe even changed. Such system should realize the possibility for teachers and researchers to exchange experience or use someone's already tested ideas in order to be more successful in their daily

work and to make mathematics more attractive to students. Description of the features, opportunities and functionality is given in chapter 3 and appendices 8, 9. The corresponding user-case diagrams and database schemes are done in notions of unified modeling language (UML).

1 Classification of school mathematic equations

Solving equations is a large and important field in school mathematics. It is important for children to understand the results and interpret them correctly. That is why the research about how well CASs handle school mathematics is bounded in this thesis by the equations. All the examples are taken from Estonian textbooks and manuals, which are currently used at schools. Probably textbooks from other countries have similar examples.

In this chapter next sources of information are used: [RÕK; Брусневская, 2001; Nurk, 7kl.; Nurk, 8kl.; Lepmann, 9kl.; Лепманн, 10кл.; Лепманн, 11кл.; Колмогоров, 10-11кл.; Тарасов, 1979; Уравнения1; Уравнения2; Уравнения3].

1.1 Classification of equations

1.1.1 Introduction

During the first level at school children deal with a simple equations, and they are not even explicitly called by the term “equation”. One component of a sum or difference is replaced with a picture or an empty box to be filled with a number. Child has to guess or calculate proper number using sum’s basic properties. In later grades, children start solving simple linear equations, then quadratic equations come into consideration and so on. Basic knowledge should be taught with paper and pencil in order to get basic skills for solving equations. But for interpreting the results and deeper understanding of mathematics computer algebra systems can be used.

One of the aims of this thesis is to know, how CASs handle equations from school curriculum, whether they are suitable for using during lessons. For investigating CASs there should be certain classification of equations, which are taught at school, in order to be able to test instances of each type of equations. Looking through school textbooks one may remark, that the classification is poor and not enough detailed for making valuable and valid conclusions. I also searched through the Internet and other available literature in order to find better

classification, but it seems that there is none. Only bigger differences within one type of equations are noticed. For example, for the quadratic equations only 3 subtypes are clearly distinguished in the textbooks. Quadratic equations in the school textbooks are divided by their form, namely to the first subtype belong equations in the form of $ax^2 + c = 0$, to the second subtype belong equations, which are in the form of $ax^2 + bx = 0$, the last subtype is described with general formula $ax^2 + bx + c = 0$. The classification proposed by textbooks and manuals turn out to be insufficient, as it is very interesting in which form CASs give the answer, when there are different number of real roots (in this thesis I divide last subtype, described by the general formula, into three separate subtypes, depending on the number of real roots). For more thorough tests another, a bit more detailed, classification is required.

The classification described below is simple, but it gives us more detailed overview about the types of equations, that are usually taught at school, and allows us to investigate the computer algebra systems more carefully, checking couple of representatives of each smaller subtype (one simpler and one more complicated equation). Of course, even investigating more detailed data, one can not be sure about the conclusions, but it still gives us more guarantees, that CAS really works properly within one smaller subtype. For example, if we test couple of arbitrary quadratic equations, we can not conclude that CAS works correctly for all quadratic equations. But if we check couple of equations from each smaller subtype, then more equation's types will be covered, and it will be easier to decide, whether CAS works properly for all quadratic equations or only for certain subtypes.

There are mixed equations that can not be easily classified to a certain type. For example, if their structure includes roots and signs of absolute values, then they are irrational equations containing absolute value and can not be classified to a concrete subtype, because mostly they differ from each other greatly (solving methods, algorithms). These mixed types of equations are rarely solved during class because of the lack of time and their complexity. Taking that into consideration, these more complicated equations are at first almost left out of the research. If equations, used in this thesis, can be classified to different

subtypes, they are assumed to belong to the subtype that is taught later. For instance, if equation includes roots and powers, then it is assumed that it belongs to the exponential equations, as exponential equations are taught later at school than irrational. It would be very interesting and important to know, how one or another CAS handles such exceptional cases, which methods uses for solving and which answers gives. Also there should be done more researches on how CASs deal with other fields of school mathematics, for example, simplifying polynomials, making operations with expressions and so on, but investigating these fields requires a lot of time and resources and, therefore, such fields are left out of sight in this thesis.

Obtained classification is based only on school mathematics, the possibilities of CASs were not taken into consideration. For making the classification more detailed and suitable for the research some Estonian and Russian textbooks and manuals were analyzed. Couple of teachers of mathematics had also helped with making sure, that this classification is sufficient for making conclusions. For instance, there are 6 subtypes of quadratic equations in this thesis, that are distinguished not only by their form, but also by the number of roots.

Looking through textbooks and national curriculum of Estonia 8 most important types of equations were distinguished. These types are: linear, quadratic, fractional, equations that contain absolute value of an expression, irrational, exponential, logarithmic and trigonometric equations. Every bigger type of equations was in turn fractured into several subtypes. As there is no certain classification in textbooks, there are also no criteria, which could be taken as a base for breaking equations into different subtypes. Therefore we used most logical division, which could be read from different textbooks and manuals, and criteria for dividing equations into subtypes are different depending on the type of equation. For example, in some cases in order to decide to which subtype certain equation belongs, one needs to look at the form of equation, its complexity or to know the number of roots (as it is for quadratic equations). If the classification is based on the form of equation, then it is supposed that equation is analogical to given parametrical form or can be easily transformed

to the required form by already taught mathematical operations. For instance, in order to get required form one must transfer numbers or expressions to the other side of the equation or divide/multiply both sides of the equation by the same number or use other already taught properties (involution, extracting the root).

Given classification is not complete and can be improved. It can be considered as a brief sketch for approximate delimitation and listing the problems, defining concrete boundaries requires much more thorough experiments and analysis. In this thesis some of the equations are practically left out of sight (mixed equations) and some equations can belong to 2 subtypes (for example pseudo quadratic equations, that after some operations can be transformed to linear and therefore can be classified as linear equation). But even such partial classification helps better investigate computer algebra systems. Teachers usually do the same work by dividing bigger types of equations into subtypes on their lessons in order to make understanding of certain topic easier for students. So this classification should be close and understandable to teachers. Probably, such classification is common for all schools in the world, the teaching may just differ in the amount of solved equations and their difficulty; some schools may go deeper in certain topic, which is considered to be more important. But the main list of equations, solved at school, still remains pretty much the same.

Next 8 parts of this chapter give overview of the classification. Each part describes one certain type of equations, taught at school. The division by subtypes is introduced in each part and criteria for this division are explained. The order of the descriptions of the subtypes corresponds to the order, in which such equations are taught at school. For each subtype description of possible solving methods is given. Overview, at which levels at Estonian schools are particular equations taught, is in the end of each part. The corresponding figures illustrate the classification. They show, how types are divided into subtypes, and give brief overview of the criteria. For each subtype there are one or two typical examples in the box, which visually helps understand, how equations were divided into subtypes.

Only linear and quadratic equations have general parametric form (correspondingly $ax + b = 0$ and $ax^2 + bx + c = 0$). Solving these equations begins with transforming the equation to the general parametric form with a help of already learned mathematical operations. When equation is transformed to the required form, then we can decide to which subtype this equation belongs.

1.1.1 Linear equations.

Textbook problems, concerning linear equations, are divided according to the form and complexity of the equation (figure 1.1). As this is the simplest type of equations we distinguish only three main subtypes.

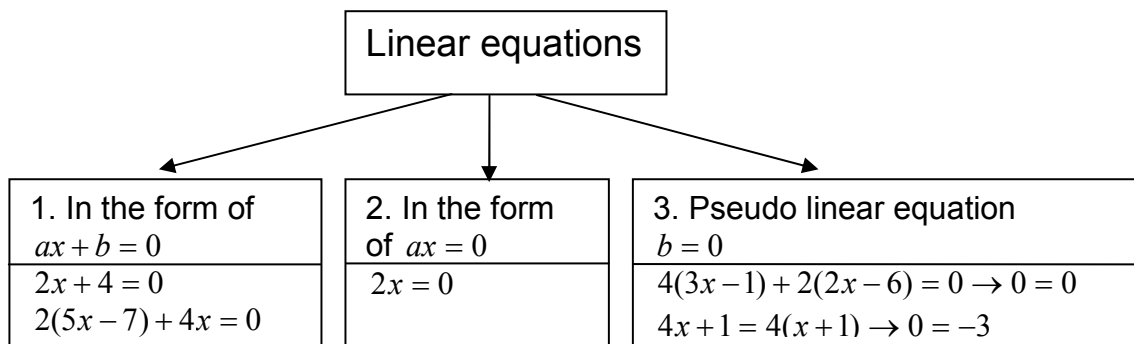


Figure 1.1. Classification of the linear equations

Linear equations that belong to the first subtype are in the form of $ax + b = 0$ (for example $2x + 4 = 0$). Under this subtype also fall equations, which are easily transformed to this form with a help of mathematical operations. For instance equation $2x = -4$ also belongs to this category. Second subtype of linear equation can be described by formula $ax = 0$. These equations are seen as separate subtype, as they do not need any calculations or operations in order to get answer, and thus are much simpler than the first subtype. For obtaining the answer student just has to analyze the equation, and that is why such equations are especially interesting for investigating. It is interesting to know, whether computer handles such problems in the same way that human does, and if it gives correct answer. Third subtype is so called pseudo linear equations, namely equations in form of $b = 0$ (the value of the first parameter a in general formula is zero). These pseudo equations do not have any unknowns in it. Such expressions can be the result of solving some complicated equation and student has to state the correct answer. Equations, that in result take this form,

also belong to this subtype. For example, if all unknowns are cancelled during solving (equation $4(3x-1)+2(2x-6)=0$ belongs to this subtype).

The notion of linear equation is introduced in the third school level [RÖK].

1.1.2 Quadratic equations

There are different criteria for dividing the quadratic equations into subtypes. Some books divide them according to their form, others take into consideration the number of roots. We try to combine both methods. In the beginning we look at the form of the quadratic equation, because the solving method greatly depends on it. Then, if quadratic equation's form is complete (equation is in the general parametric form of $ax^2+bx+c=0$, all parameters are non zero) we analyze the number of roots, as it is also logical division in many textbooks (figure 1.2).

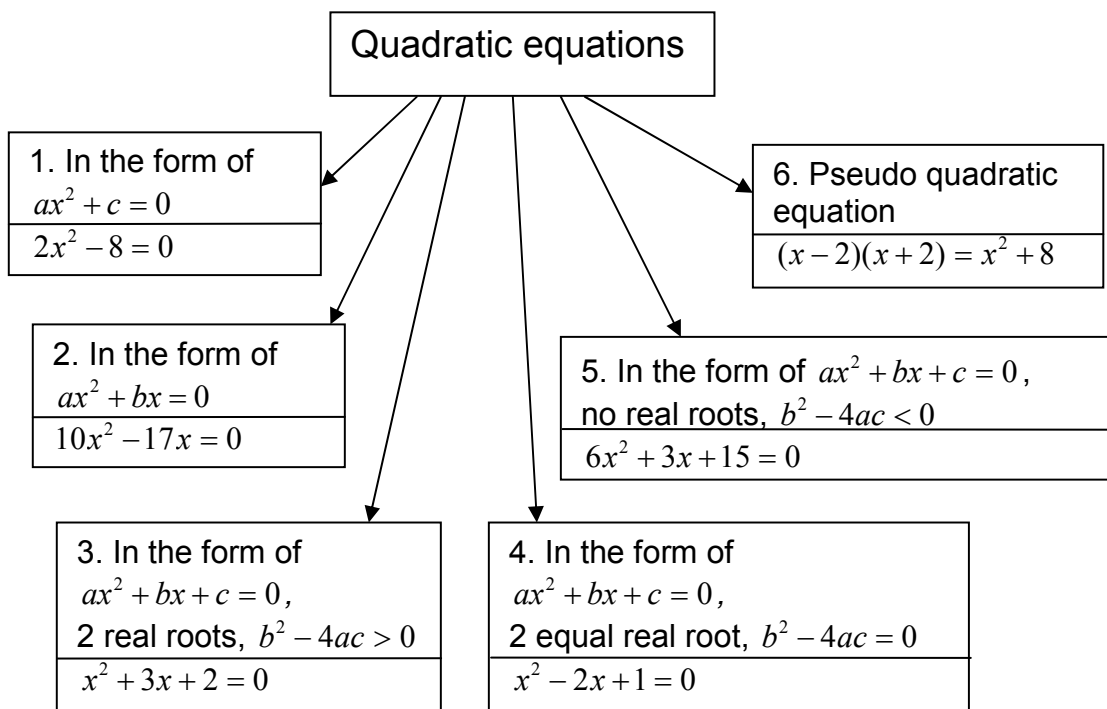


Figure 1.2. Classification of the quadratic equations

First subtype of quadratic equations is the simplest and is taught first of all. These are equations in the form of $ax^2+c=0$ (parameter b is zero) or the ones that can be transformed to this form. First subtype is solved by taking the number c to the other side of the equation (if needed) and extracting root (if

possible, namely if the number in the other part of the equation is not negative). The equations, that belong to the second subtype, are in the form of $ax^2 + bx = 0$ (parameter c is zero) and are solved by taking unknown out of parenthesis and analyzing each part of equation separately. Next three subtypes of quadratic equations are all solved by the basic formula $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The third subtype of the quadratic equations can be described with general parametric formula $ax^2 + bx + c = 0$ (it is complete quadratic equation with all parameters not equal to zero) and has 2 real roots ($b^2 - 4ac > 0$). The quadratic equations, that are in the form of $ax^2 + bx + c = 0$ and have 2 equal real roots (1 double real root, $b^2 - 4ac = 0$), belong to the fourth subtype [Zuckerman, 1976; Barnett, 1989]. The fifth subtype of the quadratic equations can be described with formula $ax^2 + bx + c = 0$ and these equations have no roots on the set of real numbers ($b^2 - 4ac < 0$). They have roots on the set of complex numbers, but this set of numbers is not taught at school and teachers usually say in such cases, that equation just has no (real) roots. Pseudo quadratic equations belong to the sixth subtype. These are equations that contain unknown in the square, but are reducible to the linear equation, so they could be also classified as linear equations (because they can be transformed into one subtype of the linear equations), but we decided to bring pseudo quadratic equations into separate subtype in quadratic equations as some textbooks propose problems like that in the chapter devoted to the quadratic equations.

Quadratic equations are taught in the third school level [RÖK].

1.1.3 Fractional equations

The classification of the fractional equations is based on the complexity of the form of given equation. One should look how many fractions there are and what is in the right side of the equation (figure 1.3). Equations, that are not included in the corresponding chapter in the school textbook because they contain more complicated functions, which are supposed to be taught later at school curriculum, are also excluded from classification (for example contain logarithmic or trigonometric functions).

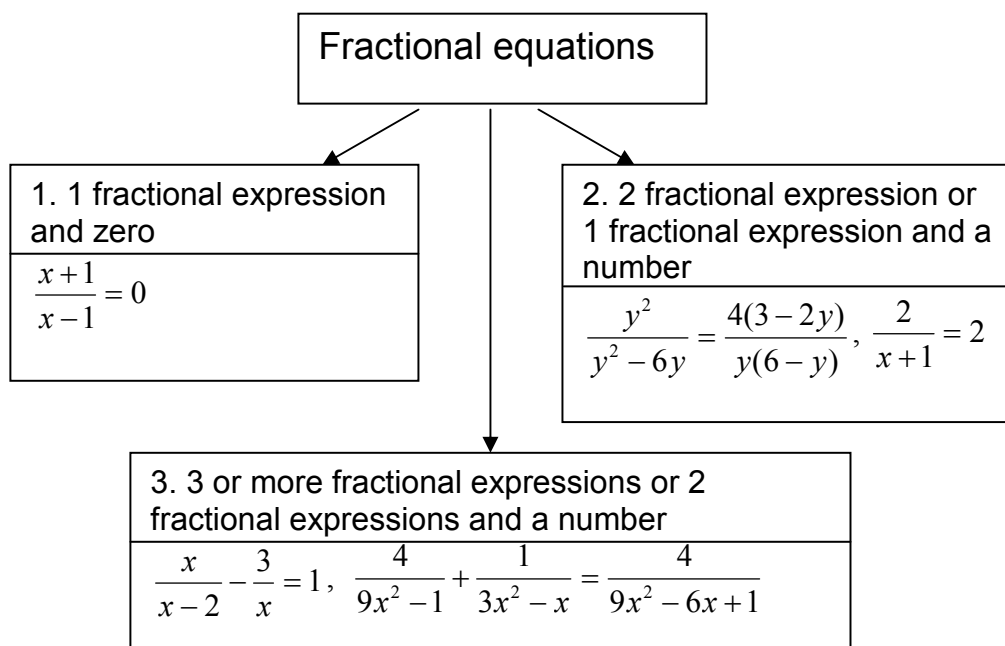


Figure 1.3. Classification of the fractional equations

The fractional equations, where one fractional expression is equal to zero, belong to the first subtype (one side of equation is fraction, containing unknown, and another side is zero). Solving this type of equations is reducible to finding roots of numerator separately and then checking, that denominator would not be zero at these points. The second subtype is described with equations, where fractional expression equals to another fractional expression or number (both sides are fractions, or one side is fraction and another side is number). The equations, containing 3 and more fractional expression or 2 fractional expressions and numbers, belong to the third subtype. The second and the third subtypes are both solved using the same algorithm. One must find the smallest common denominator, multiply corresponding expressions and then analyze numerator and denominator separately as it was for the first subtype of fractional equations.

Fractional equations are introduced in the third school level [RÖK].

1.1.4 Equations containing the absolute value of expression

Equations containing the absolute value of expression are divided into subtypes depending on the complexity of the equation. Namely one should look at the number of the absolute values in the expression, because methods for solving

this kind of equations differ depending on complexity of the equation (figure 1.4).

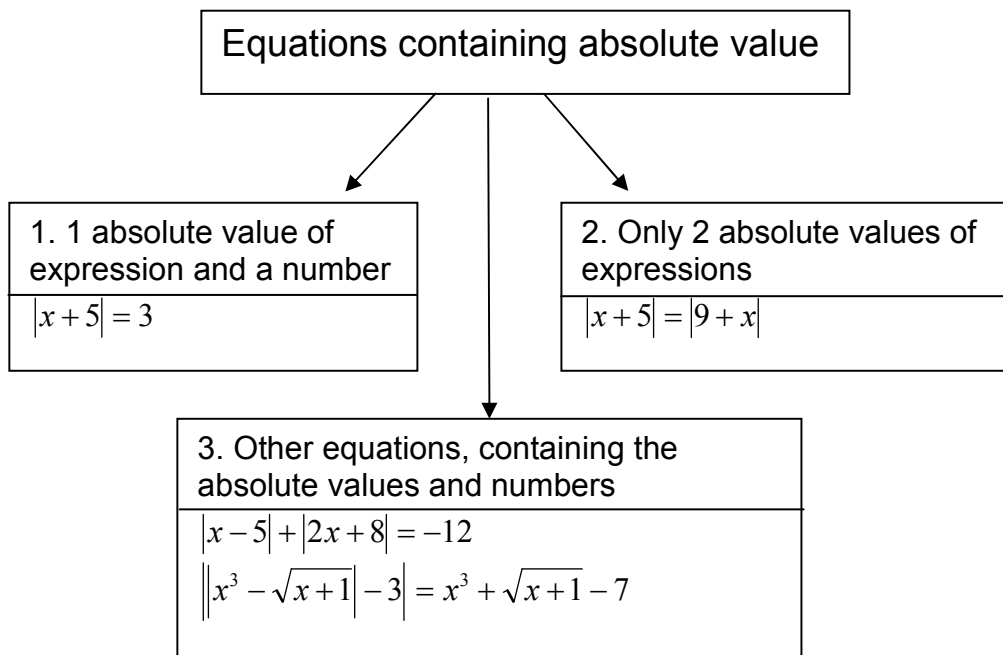


Figure 1.4. Classification of the equations containing absolute values of the expression.

The equations, containing 1 absolute value of the expression in one side and some number in the other side, belong to the first subtype. Such equations are usually solved using the definition of the absolute value, namely we remove the sign of the absolute value with the help of the definition. In result we will have 2 simpler equations to solve. If there are 2 absolute values of the expressions (one in each side of the equation), then it is second subtype. Second subtype of the equation can be solved by raising to the second power, if both sides of the equation are not negative, or by using method of interval (student has to find zero-point of each expression under sign of absolute value, then analyze this expressions on each interval and, afterwards, solve equation taking into consideration the information from the done analysis) [Уравнения2; Уравнения3]. All the other equations containing absolute values belong to the third subtype and are solved by the method of interval. These are equations, which have 2 or more absolute values of the expression and numbers.

The notion of absolute value is introduced in the second school level. Deeper overview containing solving equations is taught in third and fourth school levels [RÖK].

1.1.5 Irrational equations

Irrational equations are classified according to the solving methods, used for the different types of equations. One should be able to determine, which solving method should be used to solve particular problem (figure 1.5). Usually it can be easily done by looking at the form of given equation. In complicated cases more thorough and detailed analysis can help.

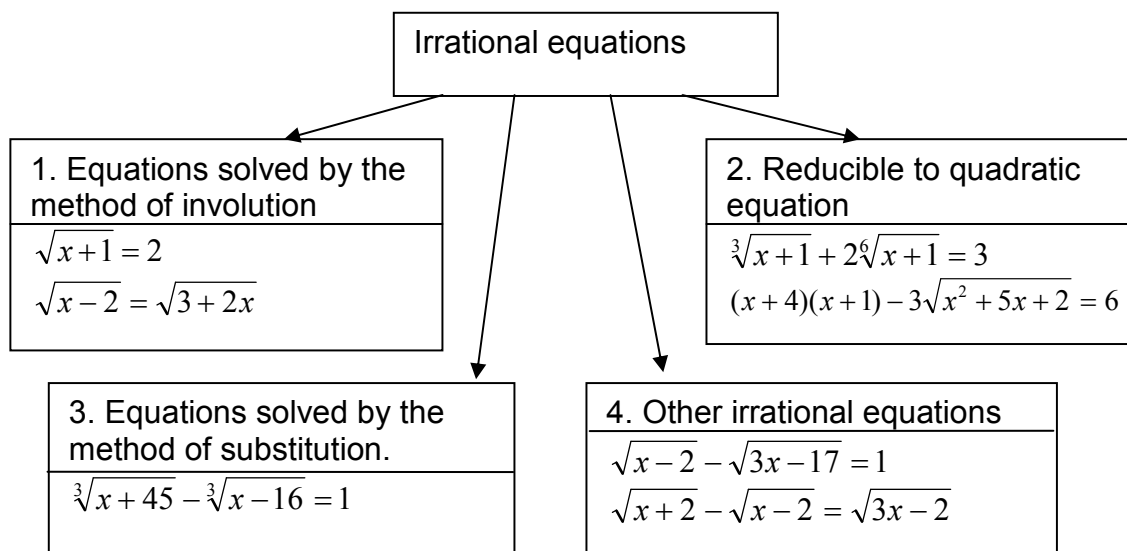


Figure 1.5. Classification of the irrational equations

The equations, belonging to the first subtype, are the equations that can be solved by the method of involution. These equations usually have 2 irrational expressions of the same power in both sides of the equation or an irrational expression in one side and a number in another side of the equation. The second subtype contains equations that can be reduced to quadratic and solved by the method of substitution. The equations, which belong to the third subtype, are also solved by the method of substitution. While solving this kind of equations one should express an unknown through a new variable. All the other irrational equations belong to the fourth type. They usually contain 2 or more irrational equations and numbers, and their solving methods depend on the particular problem.

Overview of irrational equations and methods are given in the fourth school level [RÖK].

1.1.6 Exponential equations

The classification of the exponential equations is based on the required solving method (figure 1.6). Usually in Estonian textbooks there are no more complicated equations than those, which fall under some category in classification given below so there is no such subtype as “Other equations”.

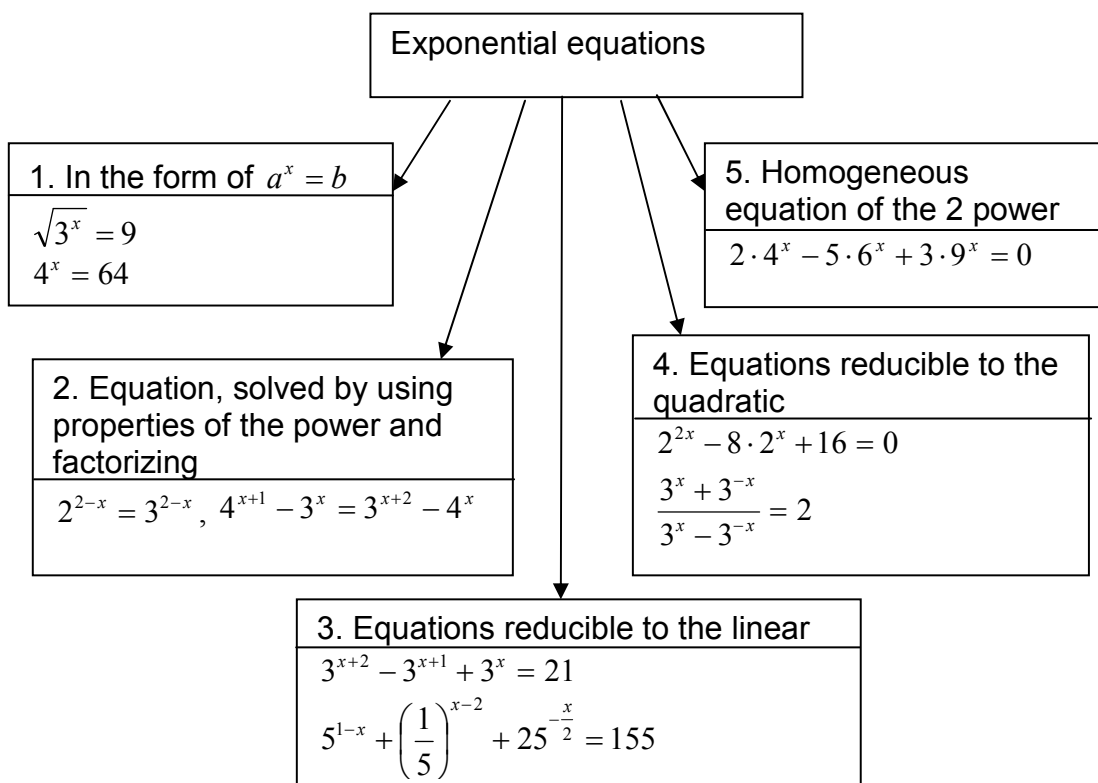


Figure 1.6. Classification of the exponential equations.

The first subtype is described with formula $a^x = b$. This kind of equations is solved by expressing both sides by the same base (expressing parameter b through some power with base a). The equations, belonging to the second subtype, are solved by using properties of the power and factorizing (for example, breaking the sum of powers into multiplication, breaking the difference of powers into division, etc.) and then expressing both sides of the equation by the same base. There are a lot of equations, which are reducible to some already known type, so there is the third subtype of equations, which are reducible to the linear equation, and the fourth subtype of equations, which are reducible to the quadratic. Both third and fourth subtypes are solved using method of substitution. To solve this equation student has to substitute expression containing power for a new variable and then solve obtained

equation for the new variable. The homogeneous equations of the second power belong to the last fifth subtype of exponential equations. These equations are solved by dividing both parts of the equation by certain expression. This expression should be chosen so, that after division the equation would be reducible to the quadratic equation by substituting expression, containing powers, for a new variable.

The notion of power and its properties are introduced in the third school level. Solving exponential equations is taught in the fourth school level [RÖK].

1.1.7 Logarithmic equations

Logarithmic equations are classified according to their complexity and form as it is usually done in ordinary Estonian textbooks and manuals (figure 1.7).

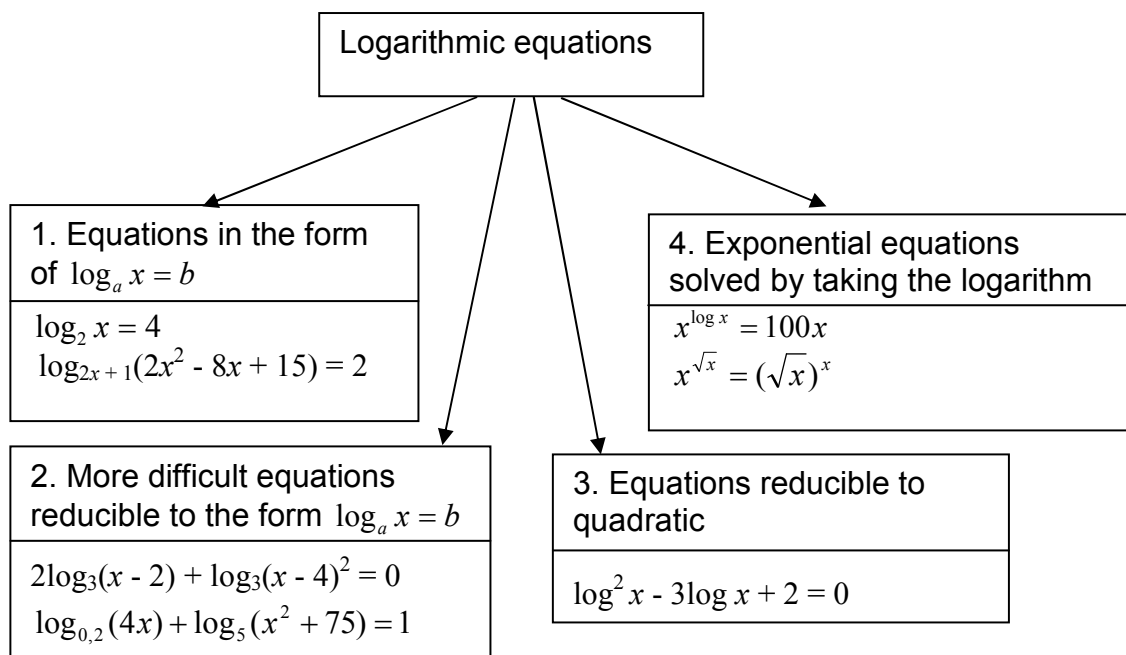


Figure 1.7. Classification of the logarithmic equations

The first subtype can be described with the formula $\log_a x = b$ (parameters a and b can be smaller or greater than zero). This subtype is solved by using the definition of the logarithm, namely student just has to express the unknown through other parameters, using the definition. Next subtype is like the first one, just the form of the equation is more complicated, and one should use properties of the logarithm to change the form of the equation. It is distinguished

as separate subtype because sometimes it requires a lot of effort from student to see, which properties of the logarithm and how should be applied for the most optimal result. Equations, belonging to the third subtype, are reducible to the quadratic equations by using the method of substitution. More difficult exponential equations, which are solved by taking the logarithm, belong to the fourth subtype. These equations are the most difficult in this type and usually are taught in advance classes of mathematics. Student has to determine which base to choose for taking the logarithm (usually it is 10 or constant e) and then transform obtained equation to one of the rest three subtypes, using the properties of powers and logarithm.

The notion of logarithm and the methods for solving logarithmic equations are introduced in the fourth school level [RÖK].

1.1.8 Trigonometric equations

Trigonometric equations are divided into subtypes according to the complexity of the equation and methods used for solving given equation (figure 1.8).

To the first subtype of trigonometric equations belong general trigonometric equations in the forms of $\sin x = a$, $\cos x = x$, $\tan x = a$, $\cot x = a$. This kind of equations is solved using circle graph or known general formulae. To solve these equations student must know the periods of every function and their domains of positive and negative values. Also one should know values of trigonometric functions in most often used points $\left(\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \pi, 2\pi\right)$. The simpler equations, that are reducible to the quadratic, belong to the second subtype. These equations are solved by the method of substitution. More complicated equations, which are reducible to the quadratic equations and contain such functions as tangent and cotangent, belong to the third subtype. These equations are usually solved by dividing both parts of the equation by function tangent or cotangent and substituting the other function for a new variable. The fourth subtype can be described with formula $a \sin x + b \cos x = 0$ (so called homogeneous equations of the first power). Equations, which can be reduced to the given form, also belong to this subtype. Such equations are solved by

dividing both parts of the equation with sine or cosine and then solving general trigonometric equation like those from the first subtype. The fifth subtype is like the fourth, but there is a number in the right side of the equation instead of zero. So the formula, describing the fifth subtype, would be $a \sin x + b \cos x = k$. These equations are solved at school by using the formula of the half-angle (substituting both functions with function of the half-angle and then applying required operations). Homogeneous equations of the second power, that contain functions sine and cosine, belong to the sixth subtype. The rest of the trigonometric equations, that are solved by using trigonometric formulae and usually are reduced to the first subtype, belong to the seventh subtype.

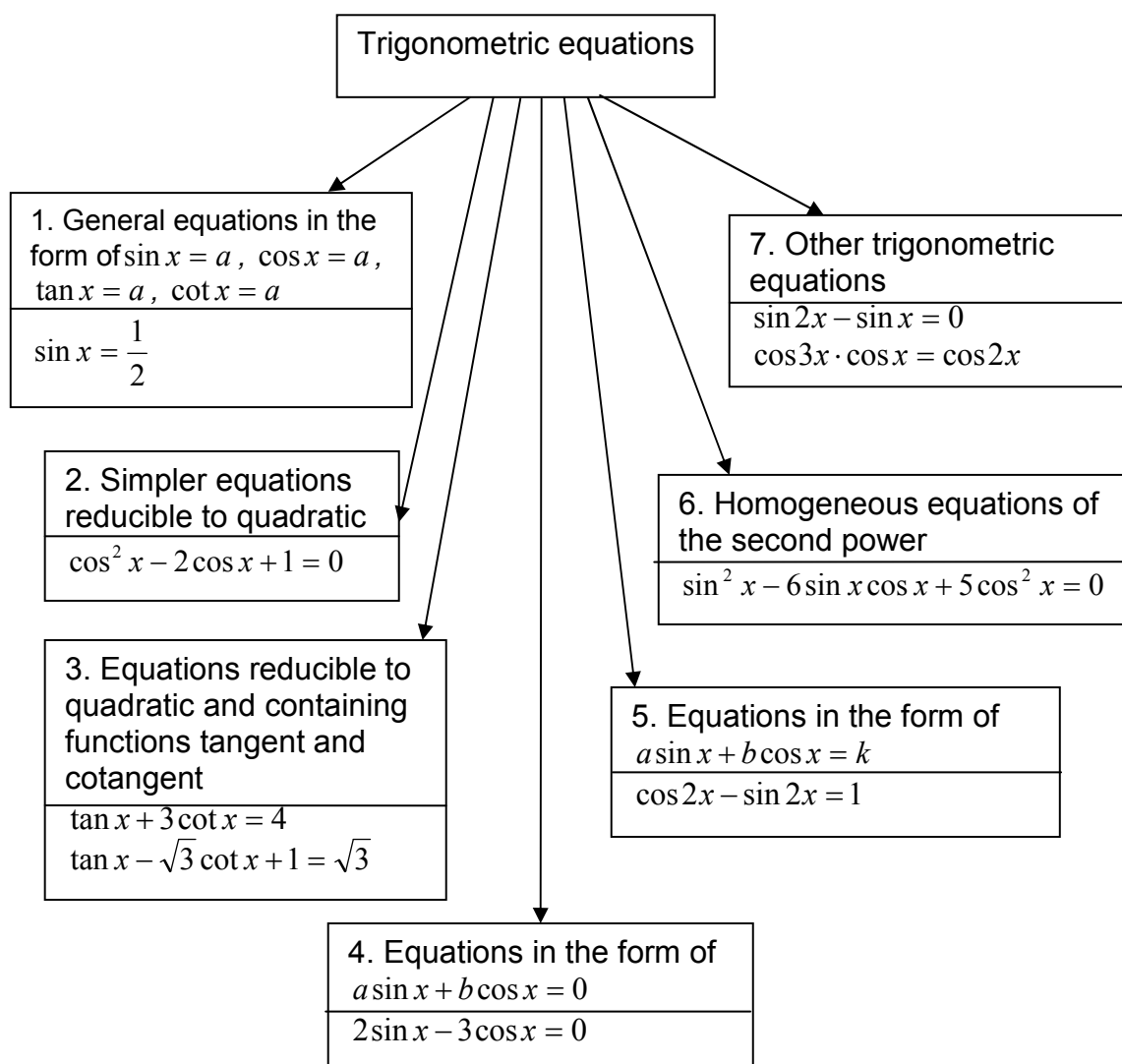


Figure 1.8. Classification of the trigonometric equations

Estonian textbooks usually use the same classification to divide problems into groups. Trigonometric functions are taught in the fourth level [RÕK].

1.2 Examples of solved equations

In order to investigate computer algebra systems 60 problems were solved with each of seven different CAS. Problems were chosen from Estonian school textbooks and manuals. Textbooks were analyzed taking into consideration the classification given above in chapter 1.1. It was easier to find suitable problems, when division of equations into subtypes repeated the one that is given in the school textbooks. But usually choice was made based on solving or the analysis of the form of some equation that seemed to suit our classification.

Answers, which are expected at school, are given in summary tables in the appendices 1-7. I took into consideration the most usual requirements to the form of the answer, as requirements can differ from school to school, and there might be some slight difference in the form of the result.

Aim was to find a couple of the representatives of each subtype – one simple and one more complicated. One problem was chosen from the beginning of each chapter that deals with certain topic in order to show the most typical representative of the subtype of equations. The second more complicated problem was chosen from the end of the chapter or even from manuals for advanced lessons. The difference in complexity between such two examples of the same subtype approximately shows the range of equations, belonging to the same subtype, and visually demonstrates the basic criteria, by which one can decide, whether equation belongs to this subtype or not. If solving the certain type of equation is very easy and all the equations belonging to this type are very similar, then only one representative was chosen (for example, as it is for the linear equations). Usually in this case equations of the same subtype differ only in the value of parameter and it does not influence the solving algorithm or the number of steps, required to solve this equation. For some subtypes more than 2 problems were chosen to make sure, that most of the possible cases within one subtype are covered. In the overview of the irrational equations of the first subtype there are described 2 possible forms of this kind of equations and, therefore, 4 problems are chosen in order to show simple and more complicated example (equation has 2 irrational expressions of the same power in both sides or an irrational expression in one side and a number in another side of equation).

As all equations, belonging to this subtype, are solved using the same algorithm, there was no need to break it into two separate subtypes.

Totally there are 60 problems and they are divided within subtypes of the equations as follows:

- Linear equations – 4 examples (table 1.1)

<i>Subtopic</i>	<i>Example 1</i>	<i>Example 2</i>
1. In the form of $ax + b = 0$	$5x - 11 = 0$	
2. In the form of $ax = 0$	$5x = 0$	
3. Pseudolinear equation in the form of $b = 0$	$4(x + 1) = 4x + 1$	$4(3x - 1) + 2(2 - 6x) = 0$

Table 1.1. Linear equations

- Quadratic equations – 7 examples (table 1.2)

<i>Subtopic</i>	<i>Example 1</i>	<i>Example 2</i>
1. In the form of $ax^2 + c = 0$	$2x^2 - 8 = 0$	
2. In the form of $ax^2 + bx = 0$	$10x^2 - 17x = 0$	
3. In the form of $ax^2 + bx + c = 0$ with 2 real roots, $b^2 - 4ac > 0$	$x^2 + 3x + 2 = 0$	$2x^2 - 4x - 5 = 0$
4. In the form of $ax^2 + bx + c = 0$ with 1 real double root, $b^2 - 4ac = 0$	$x^2 - 2x + 1 = 0$	
5. In the form of $ax^2 + bx + c = 0$, no real roots, $b^2 - 4ac < 0$	$6x^2 + 3x + 15 = 0$	
6. Pseudoquadratic equation	$(x - 2)(x + 2) = x^2 + 8x$	

Table 1.2. Quadratic equations

- Fractional equations – 6 examples (table 1.3)

<i>Subtopic</i>	<i>Example 1</i>	<i>Example 2</i>
1. One fractional expression and zero	$\frac{x + 1}{x - 1} = 0$	$\frac{2x^2 - 5x + 3}{10x - 5} = 0$
2. Two fractional expression or one fractional expression and a number	$\frac{2}{x + 1} = 2$	$\frac{y^2}{y^2 - 6y} = \frac{4(3 - 2y)}{y(6 - y)}$
3. Three or more fractional expressions or two fractional expressions and a number	$\frac{x}{x - 2} - \frac{3}{x} = 1$	$\frac{4}{9x^2 - 1} + \frac{1}{3x^2 - x} = \frac{4}{9x^2 - 6x + 1}$

Table 1.3. Fractional equations

- Equations, that contain absolute value of the expression – 4 examples (table 1.4)

<i>Subtopic</i>	<i>Example 1</i>	<i>Example 2</i>
1. One absolute value of the expression and number	$ x + 5 = 3$	
2. Only two absolute values of the expressions	$ x + 5 = 9 + x $	
3. Other equations containing the absolute value and numbers	$ x - 5 + 2x + 8 = -12$	$\left x^3 - \sqrt{x+1} - 3 \right = x^3 + \sqrt{x+1} - 7$

Table 1.4. Equations that contain absolute value of the expression

- Irrational equations – 10 examples (table 1.5)

<i>Subtopic</i>	<i>Example 1, (3)</i>	<i>Example 2, (4)</i>
1. Irrational equation solved by involution	$\sqrt{x+1} = 2$ $\sqrt{x^2 - 3x + 4} = 2$	$\sqrt{x-2} = \sqrt{2+x}$ $\sqrt{x-2} = \sqrt{3+2x}$
2. Irrational equation that is reducible to the quadratic	$\sqrt[3]{x+1} + 2\sqrt{x+1} = 3$	$(x+4)(x+1) - 3\sqrt{x^2 + 5x + 2} = 6$
3. Irrational equation solved by using method of substitution	$\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$	$\sqrt[3]{5x+7} - \sqrt[3]{5x-12} = 1$
4. Other irrational equations	$\sqrt{x-2} - \sqrt{3x-17} = 1$	$\sqrt{x+2} - \sqrt{x-2} = \sqrt{3x-2}$

Table 1.5. Irrational equations

- Exponential equations – 9 examples (table 1.6)

<i>Subtopic</i>	<i>Example 1</i>	<i>Example 2</i>
1. Equations in the form of $a^x = b$	$4^x = 64$	$\sqrt{3^x} = 9$
2. Exponential equation solved by using properties of the power and factorizing	$2^{2-x} = 3^{2-x}$	$4^{x+1} - 3^x = 3^{x+2} - 4^x$
3. Exponential equation reducible to the linear	$3^{x+2} - 3^{x+1} + 3^x = 21$	$5^{1-x} + \left(\frac{1}{5}\right)^{x-2} + 25^{-\frac{x}{2}} = 155$

4. Exponential equation reducible to the quadratic	$2^{2x} - 8 \cdot 2^x + 16 = 0$	$\frac{3^x + 3^{-x}}{3^x - 3^{-x}} = 2$
5. Homogeneous equation of second power	$2 \cdot 4^x - 5 \cdot 6^x + 3 \cdot 9^x = 0$	

Table 1.6. Exponential equations

- Logarithmic equations – 8 examples (table 1.7)

<i>Subtopic</i>	<i>Example 1</i>	<i>Example 2</i>
1. Equation in the form $\log_a x = b$	$\log_2 x = 4$	$\log_{2x+1}(2x^2 - 8x + 15) = 2$
2. More difficult equation that is reducible to the form $\log_a x = b$	$2 \log_3(x - 2) + \log_3(x - 4)^2 = 0$	$\log_{0,2}(4x) + \log_5(x^2 + 75) = 1$
3. Logarithmic equation reducible to quadratic	$\log^2 x - 3 \log x + 2 = 0$	$\log^2 100x + \log^2 10x + \log x = 14$
4. Exponential equation solved by taking the logarithm	$x^{\log x} = 100x$	$x^{\sqrt{x}} = (\sqrt{x})^x$

Table 1.7. Logarithmic equations

- Trigonometric equations – 12 examples (table 1.8)

<i>Subtopic</i>	<i>Example 1</i>	<i>Example 2</i>
1. General equation in the form of $\sin x = a$, $\cos x = x$, $\tan x = a$, $\cot x = a$	$\sin x = 0$	$\sin x = \frac{1}{2}$
2. Trigonometric equation reducible to the quadratic	$\cos^2 x - 2 \cos x + 1 = 0$	$2 \sin^2 x - \cos x - 1 = 0$
3. Trigonometric equation containing functions tangent and cotangent and reducible to the quadratic	$\tan x + 3 \cot x = 4$	$\tan x - \sqrt{3} \cot x + 1 = \sqrt{3}$
4. Equation in the form $a \sin x + b \cos x = 0$	$2 \sin x - 3 \cos x = 0$	
5. Equation in the form $a \sin x + b \cos x = k$	$\cos 2x - \sin 2x = 1$	

6. Homogeneous equation of second power	$\sin^2 x - 6 \sin x \cos x + 5 \cos^2 x = 0$	$6 \sin^2 x + \sin x \cos x - \cos^2 x = 2$
7. Other trigonometric equations	$\sin 2x - \sin x = 0$	$\cos 3x \cdot \cos x = \cos 2x$

Table 1.8. Trigonometric equations

2 Analysis of the answers.

During investigating 7 different computer algebra systems some results were noticed, that are unexpected from the school point of view and are common to the majority of tested programs. For example, one can see, that function's name is sometimes labeled otherwise in computer algebra systems than at school. Also usually teachers make stricter conditions for giving answers like reduction of the fraction or cancellation of the root in the denominator. CASs do not require such actions to be made in order to get correct answers and do not bring answer to a "nice form". Such inaccuracies can throw challenge for more talented students, and teachers can even use it in their lessons. For example, in the easier cases they can ask students to explain possible reasons of the "strange" behavior of the CAS. And of course errors, done by computer, can greatly influence the student's self-confidence in mathematics, as by hand he or she can solve certain equation correctly and computer can not. But there is another opposite possibility, that these inaccuracies can distract weaker students from thinking more about answers and analyzing them. Student can get confused and begin doubting, whether obtained result is correct, and that can make him lose the trust to the computer. Teachers should be informed about advantages and disadvantages of different computer algebra systems and basing on this information can determine by themselves, how and when to use the systems. This thesis gives such overview and helps teachers decide, which of investigated systems is more suitable for specific type of equations.

In this thesis I use some synonyms that indicate the same thing. If a program is solving equation differently, than it is done at school (giving an extraneous root or in opposite one root is missing), then I say that system has shortage, that some inaccuracies are noticed in the given answer, that the answer is unsuitable or insufficient for school. I say that the obtained answer is incomplete, when teachers have stricter requirements to the form of the answer and, in order to get satisfying answer, student has to do some additional simplifying steps. The boundaries are very indistinct, as teacher's requirements to the form of the answer can differ from school to school. Error means that if

student gave obtained answer to the teacher for checking during the ordinary lesson, then teacher would consider it as a wrong result (but such answers can be sometimes explained with the notions of the CASs). If there is no error, then the obtained answer is correct.

2.1 Computer algebra systems

In this research I investigated 7 computer algebra systems that are available in the University of Tartu and are popular in the world. The choice of programs is easily explainable. In this research such CASs were used, that the University of Tartu has a license for (some programs are accessible in the whole university, some in certain computer lab and some only from one certain computer). The release year is different for the programs, sometimes the used version is not even the newest. But actually with a help of developed strategy other CASs can be also easily compared, so it is not a problem. Reasons for the choice are simple: these versions of CASs are still used, obtained results show different trends, opportunities and patterns that probably are the same for newer versions, and also this research shows the variety of possible options in different CASs. 60 problems were solved with a help of each system. The results were analyzed, whether one or another system, can handle different types of equations. The largest comparative research done by Michael Wester also involved investigating 7 different CASs (Axiom, Derive, Macsyma, Maple, Mathematica, MuPAD, Reduce) [Wester, 1999]. Our choice of programs is slightly different, but the total number of used programs is the same, and we can suppose that the number of programs is sufficient for making reasonable decisions and conclusions.

Computer algebra systems are powerful systems for doing symbolic and numerical mathematics, algebra, trigonometry, calculus and plotting graphs. Typical CAS deals with numbers, symbolic expressions, equations, inequalities, functions, vectors, and matrices that form the core components of algebra. For a teacher and a student, CASs are the tool for supporting the teaching and the learning of mathematics. Systems let a user work with updatable, interactive designs, allowing to capture the critical methods and values. More detailed

overview of CASs possibilities can be found from articles: [Mysovskikh, 2004; Nicaud, 2004; MacGregor, 2002; Connors, 2001; Weller, 2001; Kutzler, 2000].

Many school topics, that require extensive and laborious training at school, can be solved with a single keystroke using CAS: it eliminates the drudgery of performing long mathematical calculations. While CAS takes the burden of doing the mechanical parts of solving a problem, student can concentrate on the mathematical meaning of concepts. Instead of teaching and learning technical skills traditional way, teachers and students can concentrate on the useful techniques of problem solving and on the beauty of mathematics. Mostly such programs are easy to learn and use. Documentation and help usually make learning and using CASs an intuitive process.

Tested were 7 CASs. These systems are described in table 2.1:

CAS name	Invented by	Year	Sources
Derive 6	Texas Instruments	2003	[DER1, DER2]
Maple 8	Maplesoft	2002	[MAP1, MAP2]
Mathcad 2001	MathSoft	2001	[MTC1, MTC2]
Mathematica 5.2	Wolfram Research	2005	[MTH1, MTH2, MTH3]
MuPAD 3.1	SciFace Software	2004	[MUP1, MUP2, MUP3]
TI-92 Plus (OS 2.09)	Texas Instruments	2003	[TI1, TI2]
WIRIS	Maths for More	2007	[WRS]

Table 2.1 Used computer algebra systems

Obtained results and the form of the answer differ from system to system. It can be noticed, that programs have small shortages and sometimes need additional explanations about certain answers. For instance, systems Mathematica, MuPAD and WIRIS give answers in braces and that is different at school. Another example of inconvenience if used at school can be found while using system Maple. Namely, if there is no answer, then Maple doesn't show it in anyway, the line is left blank. If student has never used Maple before, program can make him or her a little bit confused. The TI-92 Plus differs from other

CASs, it is a programmable symbolic calculator made by Texas Instruments. It came with a computer algebra system based on Derive, and was one of the first calculators to offer 3D graphing. The TI-92 Plus has its “qwerty” keyboard, making entering problems simpler.

In the beginning CASs were not directly intended for use at school. Nowadays this situation is improving and systems are now taking into consideration school problems and needs. The CAS, with its symbolic manipulation capacity, can provide both an aid and a challenge to students, as they develop solid understanding of mathematical concepts and procedures. By allowing students to manipulate such mathematical entities as variable expressions, function rules, and equations, the CAS gives them access to the tools of mathematics. Therein lays the assistance. But the gift of true “power”, obtained through using the tools of mathematics, demands a deep-level understanding of the objects, on which the tools operate. The effectiveness of any tool depends entirely on the decisions made regarding its use. Therein lays the challenge.

There are at least three ways in which CAS can function as a cognitive technology [CAS, 2003]:

- *Students can use CAS for the repeated execution of the routine symbolic procedures and in rapid succession, without the diminished accuracy and increased fatigue usually associated with the repetitive execution of by-hand algebra routines.* For example students can rely on the CAS production, if they want to search for the patterns in symbolic results, like expanding polynomials of the form $(a + b)^n$ for different values of n to generate data for a search for a pattern in the coefficients.
- *Students can assign rote symbolic tasks to the CAS so that they can concentrate on making “executive decisions”.* For example, when solving a complicated problem, students can allocate the equation solving or the exact evaluation of complicated expression to the CAS so that they spend more time crafting an approach to solving the problem.
- *Students can use CAS to apply routine symbolic algorithms to complicated algebraic expressions, without the confusion students sometimes experience, when trying to apply a routine procedure to a*

complicated expression. Once a function is defined, students use the same keystroke to perform a procedure on an equation involving a simple function, as to perform a procedure on an equation involving a more complicated function (for example the same command “solve” solves simpler linear and more complicated logarithmic equations).

There are several tasks of CAS as pedagogical tool [CAS, 2003]:

- *Trivialization*. Before the technology, teachers had to carefully design examination questions and homework problems so that all intermediate and final results were “nice” (for example integer or simple fraction). Straightforward results were important so that students would not waste most of their time performing arithmetic operations. CAS has trivialized the process of performing arithmetic operations as there are no constraints to the form of the answer (answer can include complicated fractions or roots). CAS can be used to help students tackle problems that are more complex and more realistic.
- *Experimentation*. Mathematics was originally an experimental science that consisted of the phases of experimentation and application. A student should “locally” build an individual “house” of mathematics, so a substantial part of knowledge acquisition occurs during the phase of experimentation. Probably the reason, that so many students are at odds with mathematics, may be related to their lack of experimentation. CAS and interactive geometry software enable students to experiment with most topics taught in mathematics, because they produce proper results in a short time.
- *Visualisation*. Visualisation is primarily used for graphic illustrations of algebraic objects, numeric objects or facts, describing either the process or the result of the process. When using CASs immediate feedback is of central importance. The teacher can discuss the resulting picture immediately, and the student can associate the graph or illustration with the expression. This option can be widely used while plotting graphs or solving equations graphically. For example 5 of investigated CASs do not give general answer for trigonometric equations and in this case plotting the graph and reading the answer from it can help.

- *Concentration.* The treatment of almost any mathematical topic requires the mastery of the previously learned topics. A student, who follows the CAS-supported exercise, can fully concentrate on the higher-level skill and CAS performs the lower-level skill for the student. Thus students are able to direct their attention to more global algorithms, strategies and concepts rather than to the details of executing symbolic routines. For instance student has to choose appropriate equivalence transformation while solving equation and CAS performs the simplification.

Interesting opportunity of using CAS as a concentration tool is using it for checking the roots of the equation as it is usually taught at school, while solving by hand with paper and pencil. There are several possibilities in CASs. Firstly, there are special commands that substitute given value of the unknown into the expression saying, whether the expression is true or false. Secondly, when checking answers with tables, student can be encouraged to look at each side of an equation as a separate function. For example the expression $2x + 9 = -x + 3$ student can view $2x + 9$ and $-x + 3$ as distinct functions of x . In solving the equation, they will find input values that produce the same output for both functions (in our example, $x = -2$ produces same output 5). Alternative way is to check answers with a graph. Viewing simple equations as sets of simultaneous graphs fosters a visual interpretation of equation solving. By providing simultaneous graphs $2x + 9$ and $-x + 3$, teachers help students to link previously understood content, such as graphing lines, with symbolic methods of equation solving.

During solving the equations CASs implement all the tasks named above. CASs trivialize the process of solving simpler and more complicated equations by using the same commands; CASs allow students to experiment and explore by solving more representatives of each subtype of equations; CASs visualize the process or the result of the process (for example students can solve equations graphically); and finally CASs allow to concentrate on more important aspects, like interpreting the results, reasoning the answer, etc. *What teacher values in students' responses will change from computations and manipulation to analysis, setups, interpretations, and justifications* [CAS, 2003].

2.2 Different commands for solving the equations

Usually the commands and their syntax are similar for investigated CASs. Here will be described the commands that are sufficient for solving all types of equations described in the first chapter. Each CAS has ordinary solve command which calculates symbolically (leaves in the answer complex numbers, logarithms, etc). In most cases this command works well. If namely numerical computation is needed, then student has to use additional options for eliminating symbolic answer or some other commands that will produce numerical result.

Program **Derive** has different commands for solving equations. The most common command is “solve”. The syntax is *solve(equation, variable, [options])*, where equation is the equation needed to be solved and variable is the unknown for which to solve the equation. Sometimes this command is not narrow enough, for example in the case of quadratic equations if the equation has no roots on the set of real numbers, then command “solve” by default gives roots on the set of complex numbers. To eliminate this opportunity, we can use constraint and complement the command with word “real” in the options section. Then the produced answer will be calculated only on the set of real numbers, what is more suitable for the students. There is also command “nsolve” that gives all roots numerically, calculating approximate decimal values. This command eliminates constants like π or e , calculates the values of natural logarithm etc. Constrains can be indicated also from menu, which can be easier for students. Namely student has to choose “solve” in the menu and then in the appeared window indicate on the set of which numbers should be the equations solved.

The most common syntax for command for solving equation in **Maple** is *solve(equation, variable)*. This command leaves constants, natural logarithms, complex roots in the answer. For purely floating-point solutions command “fsolve” can be used. This command eliminates complex constant i from the answer, leaving the answer on the set of real numbers, or calculates value of expression, if there are some natural logarithms. Command “isolve” can be used to solve for integer solutions.

The system **Mathcad** is a bit more complicated. For example, in order to solve equation symbolically boolean equal signs can be used or operation “solve” followed by a comma and the variable for which to solve can be typed or student can just select the variable for which to solve, and choose Variable -> Solve from the Symbolics menu. Probably the easiest way is to use the menu as the computer calculates everything by itself and student does not have to worry and think about the right place of the command. If equation is a polynomial, student can find all roots at once using the “polyroots” function. If some constants or functions like natural logarithm are left in the answer, then some other commands from the symbolics toolbar can be used (for example evaluate).

Program **Mathematica** uses basic command “*solve[equation, variable]*” in order to solve equations. Command operates on the set of complex numbers. Command “*FindInstance[expression, variable, (domain)]*” can also be used in order to solve equations. Command “FindInstance” finds instances of the variable that make the statement expression to be true. Option “domain” can be used in order to narrow the set of numbers on which to search for the answer. If there are some constants left in the answer or the form of the answer should be changed to more used at school form, then command “*n[expression]*” can be used. This command gives the numerical values of the expression.

Different command can be used in program **MuPAD** in order to solve equation. Basic command is “solve” that has the same syntax, as in the previous programs. While solving quadratic equations with no real roots, this command operates on the set of complex numbers and gives roots with constant i . To narrow the set of numbers, another commands should be used. Command “*numeric::realroots(equation, variable)*” gives array of numerical roots of the equation eliminating constants. But it may be complicated to explain the meaning of the array of roots. So there are other possibilities for solving equations. Command “*numeric::solve(equation, variable)*” is analogical to the previous command and gives answer in the more understandable way. In some cases this command works better with an additional option, namely “*numeric::solve(equation, variable, AllRealRoots)*”. This command searches all

real roots of the particular equation and some times it gives better results. Obviously turning this option on makes command use different solving algorithm that works more effectively for some types of equations.

If with other systems by default answer is given on the set of complex numbers, then **TI-92 Plus** is different. It takes the set of real numbers by default and in order to get complex roots we need to use a bit more complicated command. The solve() function can be used to solve a variety of mathematical equations, it will return a real solution of the equation and it can also solve equations symbolically. Another function, csolve(), can be used to solve for complex solutions. The syntax of the solve function is solve(*equation, variable*). So the command “cSolve” is wider, it solves equations with real as well as complex solutions. There are also other commands, like “zeros()” (returns a list of real values that solve the equation) or “cZeros” (returns complex solutions) but command “solve” and “cSolve” are sufficient for solving equations taught at school. The difference in commands “solve” and “zeros” is in the form of the answer. Command “solve” gives answer in the form “ $x=n$ or $x=m$ ”, command “zeros” gives a list of numbers as the answer, like $\{n\ m\}$.

WIRIS has only two commands for solving the equations. First command is “solve(equation)”, which tries to find all solutions of the equation or system of equations by exact methods. When necessary, WIRIS tries to complete the task using numerical methods (always approximate). In this case it tries to find one solution starting from an initial point or interval. The act of searching for a unique solution to the equation means that the results obtained are different in nature from those obtained by the solve command. The solutions found are returned in braces in a list, which will be empty if none has been found.

2.3 Types of obtained answers

In order to use CASs safely teacher must beforehand give some instructions and prevent possible confusion by telling in advance about the differences in solving by hand and with the use of CAS. Probably it is possible to eliminate unexpected results with a help of built-in programming tools, which are included in the majority of computer algebra systems, but as it may be complicated for

students I do not consider this opportunity. Obtained results can be divided into groups basing on different dimension and criteria in more than one way. In this thesis I use classification that is simpler than the one proposed, for instance, in the article of David Stoutemyer, where the limitations are divided into four groups: bugs, theoretical, algorithmic and resource limitations [Stoutemyer, 1991]. As the results could be more interesting to teachers, it was decided to use such classification that would be clear and understandable for them.

I try to divide the obtained answers into four groups basing on two dimensions:

- correctness of the answer (obtained answer is equivalent with the answer required at school);
- possibility to eliminate the unexpected result (whether the obtained answer can be easily transformed to the required form by some additional steps, like reducing the fraction or simplifying obtained result).

Overview of four types, which can be described by the presence of listed above two dimensions, is given below. First sign in inverted commas stands for correctness of the answer, second sign – for the possibility to eliminate the unexpected result.

“-/-” Answers, which are not correct (not equivalent to the answer required at school, errors) and can not be transformed to the correct answer, belong to the **first type**. For example instead of two correct answers CAS gives only one and while solving at school student obtains 2 roots. Namely for the irrational equation $\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$ programs Mathcad, Mathematica and MuPAD give only root $x = 80$, while there are two roots: $x_1 = 80, x_2 = -109$. Errors are rare case and they may occur due to a bug in the program, but sometimes it can be made on purpose and can be easily explained by the standards of particular CAS. For instance, program Derive gives unexpected answer for the fractional equations, if there is a single unknown or multiplication of the unknown and some other expression in the denominator (for example $\frac{x}{x-2} - \frac{3}{x} = 1$). When solved at school, student gets root $x = 6$, but Derives gives $x = \pm\infty \vee x = 6$ as an answer. From school point of view it is a wrong answer. But Derive has own standards and assumptions, according to which expression

divided by the infinity is equal to zero. Taking into consideration such assumption this root really suits given equation and equation is solved correctly.

“+/-” Difference in the form of the answer given by CASs belongs to the **second type** (certain way of writing things down may differ from the one, used at school). Such answers are equivalent to the ones required at school (correct) but can not be easily eliminated or transformed to the desired form as we deal with built-in standards and notions. For example in some programs different kinds of parenthesis are used (program MuPAD gives answer like this: {-2, 2}) or minus sign is in the numerator, not in front of the whole fraction (programs Maple and Mathcad label negative fraction like this: $\frac{-1}{9}$). Usually it is enough to pay attention to such differences for one time and explain what they mean. Students get easily used to them and later such differences don't disturb anymore. Different programs have also different ways of saying student that equation has no solution on the set of real numbers. Program MuPAD gives answer in the same form that is used majority of schools ($x \in \emptyset$) and other 6 systems have their own message or way to show that they cannot find any real roots of the equations.

“+/+” Remarks regarding “incomplete” correct answers, which are given in a different way at school belong to the **third type**. These are answers that are equivalent with the school answer and can be easily transformed to the desired form with a help of some additional commands, and this is the main difference from the remarks of the second type. For example reduction of the improper fraction is usually required in answers at schools but is not required in CASs (for example reducing improper fraction will give required result). For instance, all 7 programs leave improper fraction $\frac{17}{10}$ in the answer, while at school it is expected from students to simplify this fraction.

“ok” Answers, that are suitable for school (correct answers) and do not need any additional steps, belong to the **fourth type**. As other types of obtained result are much more interesting to investigate, I will concentrate only on them in the further analysis. The results belonging to the third and fourth types are

quite relative because constrains to the form of the answer differ from school to school. For example some teachers may not require the fraction to be reduced. Another example is about the order of the roots. Some teachers require the roots to be sorted increasingly, others do not require that. In this thesis I take into consideration the most usual requirements of the Estonian school to determine, whether the obtained answer belong to the third type or not.

“-/+” Simplifying expression retains its equivalence with the original expression. Therefore such opportunity, that obtained result is not correct but can be transformed to the correct result, is impossible and option “-/+” does not give separate type.

Below is given a summary table that gathers information about researched topics of mathematic, programs and types of obtained answers in corresponding cell (Table 2.1). The fourth type (obtained result suits well schools requirements) is not marked in the table.

Programs	Derive	Maple	Mathcad	Mathematica	MuPAD	TI-92 Plus	WIRIS
Equations							
Linear	2,3	2,3	3	2,3	2,3	2,3	2,3
Quadratic	2,3	2,3	2,3	2,3	2,3	2,3	2,3
Fractional	1,2,3	2,3	2,3	2,3	2,3	3	2,3
Containing absolute value	2,3	2	2	2	1,2	3	2
Irrational	1,2	1,2	1,2	1,2	1,2	2	2
Exponential	2,3	3	3	1,2	1,2	2	2
Logarithmic	1,2	1	2	2	1,2	2	2
Trigonometric	1,2	1	1,2	1,2	1,2	1,2	1,2

Table 2.1. Summary table, containing information about equations, programs and types of shortages. 1- error, 2 – difference in the form of the answer, 3 – “incomplete” correct answer.

2.4 Equations and CASs

In the next parts of the second chapter I will give more detailed overview and more detailed explanation of the given above table 2.1. Firstly, in part 2.4.1 brief overview within each type of equations is given, namely how CASs handle one or another type of equations, studied at school (analysis of the rows of the table 2.1). Secondly, in parts 2.4.2 – 2.4.8 there are given descriptions of the seven systems, more detailed explanations and examples of answers belonging only to the first, second or third type (analysis of the columns of the table 2.1). Correctly solved examples are not included into the overview.

2.4.1 Equations

All studied computer algebra systems have some difference in writing data from the way how it is written at school. That is why there are many “2” in the cells in the table 2.1. Number “2” in this table represents difference in the form of the answer (answer is correct but can not be easily transformed to the required form). Such differences can not be easily corrected in CASs by the users, and teacher just has to notify student beforehand about such differences and give possible explanations. For example sign “v” (OR) in Derive is used when there are more than one answer to indicate, that unknown variable can take one or another value (for instance, $x = 1 \vee x = 2$). But in younger classes students are not familiar with this logical sign and teachers have to explain this sign before getting to work with Derive. Also there is number “3” in nearly all cells (“incomplete” correct answer – correct answer that is easily transformed to the required form), because in CASs requirements to the form of the answer are simpler, and students have to use some additional commands in order to get result that would be suitable for their teacher. For example all studied programs leave improper fractions in the answer, but usually teachers at school ask to give an answer the “nice form” and to do all possible operations with answer in order to get final result, leaving there only proper fractions (for instance, programs leave fraction $\frac{11}{5}$ in the answer, while usually teachers expect $2\frac{1}{5}$). If

there are remarks that belong to the second or third type, then we may say that particular CAS handles well certain type of equations, as such remarks do not

influence the quality of solving the equations. Teacher should be concerned only if there are remarks of the first type (answer is not correct from the school point of view). If CAS solves some equation not correctly from the school point of view, then mostly the remark is relevant within the whole subtype of equations. This doesn't take place for bigger subtypes like "others", because very different equations can belong to one subtype and CAS may solve wrongly only couple of them. More detailed information can be received if more equations of each subtype are solved. Students get used quickly to the notions and standards of particular CAS and also remarks concerning the answers of the second or third types can be surmountable, therefore answers of the first type are the focus of attention.

As we can see from the table, all programs handle very well linear equations. The most interesting aspect within this type regards the way of reporting student that equation has no roots or infinite number of roots. For instance MuPAD gives sign \emptyset if there is no roots and \mathbb{C} if there is infinite number of roots, Derive and TI-92 Plus give correspondingly *false* and *true*, Mathematica – $\{ \}$ and $\{ \{ \}$. No answers of the first type are obtained while solving linear equations and we can conclude that computer algebra systems handle linear equations the same way that school does and this way is correct from mathematical point of view. Quadratic equations are also handles very well by all 7 CASs. Sometimes CASs are even smarter than needed at school, namely for the quadratic equations with two equal roots ($x^2 - 2x + 1 = 0$), Maple, Mathcad and Mathematica give the root 1 twice, while teachers often say that equation has one root. Different textbooks have also different opinions about this question. Some say that equation has *only 1 root* [Zuckerman, 1976] or *since it came from two factories some can call it double root* [Barnett, 1989]. In this thesis I consider both variants correct, because mathematically it is correct to write down both roots even if they are equal, but on the other hand record with only one number might seem simpler and more logical to students, so it is up to teachers to decide which program to use and how to explain particular way of writing down the answer. Obtained results show that computer algebra systems deal with the quadratic equations practically in the same way as a school does, sometimes

answers proposed by CASs are more correct and precise than the ones proposed at school.

Fractional equations are solved correctly by computer algebra systems. One exception is program Derive, which gives sometimes sign of infinity ($\pm \infty$) as an answer, but taking in consideration the standards of Derive such answer is also correct. Namely Derive has assumption, that expression divided by the infinity is equal to zero. Taking such assumption into consideration, then according to Derive equation $\frac{x}{x-2} - \frac{3}{x} = 1$ really has roots $x = \pm\infty$ and $x = 6$. In the case of fraction equations school approach doesn't touch the mathematical analysis, like all other investigated systems except Derive. There are exceptional cases, when roots, obtained during solving the equations, do not fall into region of acceptability. For example after solving the equation $\frac{x^2}{x} = 0$ all investigated programs give $x = 0$ as a result, which is wrong as zero can not be in the denominator of the fraction (equation $\frac{x^2}{x} = 0$ is made up, not taken from the textbook). But such cases are very rarely mentioned at school, so they are also left out of sight in this research.

When dealing with equations that contain sign of absolute value, MuPAD doesn't give answer for one tested example $|x+5| = |9+x|$ (depending on the used command, MuPAD gives error message or sign of empty set). This problem can be solved if equation is entered using property $|x| = \sqrt{(x)^2}$, then MuPAD gives correct answer (equation is $\sqrt{(x+5)^2} = \sqrt{(x+9)^2}$). WIRIS doesn't solve equations including absolute value (if equation is entered with absolute values, then program gives always empty braces $\{\square\}$ as an answer). The only way to avoid this problem is always using above mentioned property and entering the equation in modified form. Other 5 CASs handle equations, including absolute values of the expression, in the same way, as it is done at school.

More interesting results are obtained for other equations. In the case of irrational equations some programs give partial answer for the third subtype of equations (equations containing cubic roots, that are solved by the method of substitution, for instance $\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$). Namely solving by hand student gets 2 roots ($x_1 = -109, x_2 = 80$) and programs Mathcad, MuPAD and Mathematica give only one (positive) root $x = 80$, ignoring the possibility of negative number under the sign of root with odd index. It may be noticed, that this problem occurs, if program does not allow to enter the equation in the same form, as the original given above, and instead of roots student enters equation with power index $\frac{1}{3}$. Solution methods differ greatly, because equations with roots and with fractional powers are treated differently. The rest of solved irrational equations are handled by CASs in the same way as it is done at school.

Exponential equations are well solved by the CASs. The bigger difficulty is with choosing the right command, because in the majority of systems ordinary command “solve” doesn’t give sufficient answer. Also homogeneous exponential equation of the second power turned out to be difficult type of equations to solve with a help of CASs (systems Maple, Mathematica, Mathcad, WIRIS didn’t give appropriate result). System Mathematica produced error for the equations $2 \cdot 4^x - 5 \cdot 6^x + 3 \cdot 9^x = 0$; WIRIS solved only the first subtype of exponential equations, giving empty braces for all the others; MuPAD gave different answers while using different commands and only in one case the answer is correct (*numeric::solve(equation, variable, AllRealRoots)*), but probably student would begin solving the equation with simpler commands, which unfortunately are not suitable in this case. That can make student stop trying other commands. Some programs leave natural logarithms in the answer (for example Mathcad) and student needs to do some additional evaluation in order to get result suitable at school. If appropriate command is chosen for solving exponential equations, then almost all CASs give the same result, as required at school.

All the programs solve correctly the simpler logarithmic equations but some of investigated CASs (Derive, Maple and MuPAD) give sometimes unexpected answer for more complicated equations. They either give one root instead of two or produce extraneous root. Mostly such error occurred while solving more complicated exponential equations, which are solved by taking the logarithm, and therefore are one subtype of logarithmic equations. For example Derive and MuPAD find 3 roots $x = 4 \vee x = 1 \vee x = 0$ for equation $x^{\sqrt{x}} = (\sqrt{x})^x$ and after check we can see that root $x = 0$ gives expression 0^0 , which is undefined in mathematics. Therefore at school student obtains only two roots ($x_1 = 1; x_2 = 4$). But programs Derive and MuPAD have assumption, that $0^0 = 1$ and for Derive and MuPAD the root $x = 0$ really suits given equation. Mathematica and WIRIS solve only the simplest logarithmic equations (Mathematica solved only the first and partially the third subtypes, others produced warning message in the result and only first subtype is solved by WIRIS and others produced empty braces as a result).

Trigonometric equations are difficult to teach with the help of computer algebra systems, because only 2 programs out of 7 (MuPAD and TI-92 Plus) take into consideration the periodicity of the trigonometric functions and only MuPAD describes the new variable n in the answer ($n \in \mathbb{Z}$). Others systems just calculate some exact values and give them as an answer, which is clearly insufficient at school (usually, these exact values in the case of solved trigonometric equations can be calculated from the general answer by substituting $n \in [-6, -1, 0, 1, 6]$ in Derive, $n \in [-1, 0, 1]$ in Mathcad, Mathematica and WIRIS; $n \in [0, 1, 2, 3]$ in Maple and different values from interval $n \in [-32, 26]$ in TI-92 Plus). Programs Mathematica and TI-92 Plus give warning that some roots may be lost but this information does not give much to a student. One way to overcome this difficulty is to solve trigonometric equations graphically (plot the graph of given trigonometric function and from the graph read correct roots). Analyzing obtained results we may conclude that school handles trigonometric equations more precisely and correctly from mathematical point of view, than majority of investigated CASs, because only

MuPAD gives practically always the right general answer with all necessary constraints for the new variable n).

2.4.2 Derive

The equations with no real roots are handled differently in Derive than at school. Namely if entered equation has no roots then program gives word “false” as a response. Such behavior of Derive is explained in its manual, namely the logical answer informs that *there is no value x that would make this equation valid*. If we look at equation as at identity, then Derive answer is true and correct. There are certain differences in the way things are written, that can be easily explained, and therefore such differences are not a big problem while using Derive at school. For example there is logical sign “v” (OR) used in the answers to indicate that there are several possibilities for the unknown ($x = 2 \vee x = -2$). Trigonometric function arctangent is labeled with “*atan*”, but students can also easily adjust to it.

Ordinary command “solve” will produce answer containing complex imaginary unit i for the quadratic equations of the fifth subtype (no real roots). For eliminating this possibility and obtaining result only on the set of real numbers, student should use command “solve(real)”. If one tries to solve exponential equation with the help of Derive, then he or she should use command “nsolve” or “solve(real)” in order to get proper result. Command “solve” gives answers through natural logarithm, what can be a result of some solving algorithm involving transforming the exponential equation into equation containing logarithms. Fractions in the answer are not reduced, so if Derive is to be used at school, then more operations should be performed and obtained answer should be changed into another, more suitable for teachers.

For one subtype of irrational equations, namely for the equations of the third subtype (equation containing roots of third power and is solved by the method of substitution) branch option should be changed in order to get the correct answer. By default the value of parameter *Branch field* is *principal* and in this case Derive gives only one root, as after extracting the odd root of negative number Derive gets complex expression with imaginary unit. For example after

solving equation $\sqrt[3]{5x+7} - \sqrt[3]{5x-12} = 1$ Derive gives only one root, $x = 4$, and actually there are two roots ($x_1 = 4, x_2 = -3$). If the value of *Branch field* is changed to *real*, then Derive is solving named type of equations correctly.

While solving more complicated logarithmic equations of the fourth subtype (exponential equations that are solved by taking the logarithm) some extraneous roots may occur. For example for equation $x^{\sqrt{x}} = (\sqrt{x})^x$ Derive finds 3 roots ($x = 4 \vee x = 1 \vee x = 0$) and after solution check we can see that root $x = 0$ gives expression 0^0 , which has no certain value. Therefore at school student obtains only two roots ($x_1 = 1, x_2 = 4$). But Derive has its own standards by which $0^0 = 1$ and therefore for Derive the root $x = 0$ really suits given equation. Another remark is regarding the fractional equations. If there is an unknown in the denominator (or it is a part of the multiplication) or equation can be transformed to this form, then Derive gives $\pm\infty$ as one of the answers (assumption in Derive: expression divided by the infinity is equal to zero). It can be difficult to explain to students what this sign stands for in this situation.

For the simpler trigonometric equation program gives certain values. In the case of solved trigonometric equations Derive calculated some certain values for n from $[-6, -1, 0, 1, 6]$ in the general answer. For more complicated equations (for example homogeneous trigonometric equations of the second power) there is no useful answer. The equation is either left unchanged or there is only one root in radians as an answer (for $n = -2, -3$ in one of the general answers). That makes Derive a bit difficult to use at school during studying the trigonometry.

Detailed information about tested problems, obtained results and analysis of the results is in the Appendix 1.

2.4.3 Maple

Big inconvenience of Maple is in one of used notions. Namely if equation has no roots, then after pressing "enter" button, Maple doesn't give any reaction or warning. That is a shortage of Maple, because it makes student doubt whether program has done any operations at all. This may confuse even if a person is a

little bit familiar with this program. If using Maple at school teacher must at least at the first lessons remind about this special standard of Maple.

While solving quadratic equations some other questions may rise. For example for the quadratic equation of the fourth subtype $x^2 - 2x + 1 = 0$ (equation has two equal roots), Maple gives answer in the form of “1, 1”; program repeats the double root twice. It is correct, equation of the second power really must have 2 roots, sometimes they are equal, and Maple shows this explicitly. But at school at that level teachers usually say that quadratic equations, belonging to this subtype, have only one root. Only in later grades students find out and realize, that in such situation there are actually two roots that are equal. By that time when quadratic equations are taught, students do not have such knowledge and therefore this answer might confuse them. On the other hand as students learn more about equations with double roots in later grades, maybe it would be even reasonable to prepare them and let students get used to this way of writing down the answer. If quadratic equation has no real roots (but has complex roots), then Maple always firstly gives answer on the set of complex numbers. In order to get the answer that would be expected at school on that level, student has to use another command. For example command “fsolve”, which calculates floating-point roots, is giving blank line as an answer, what shows that particular equation doesn't have any real roots. Another difference from school in the way of how things are written down is connected with the fractions. Namely if fraction is negative, then the minus sign is in the numerator in front of the number like this: $\frac{-1}{9}$. At school if fraction is negative, the minus sign is always in front of the whole fraction. This can create some difficulties in understanding the results but again with a teacher's help this problem can be easily overcome. Another remark concerning fractions is that Maple does not reduce fractions in the answer. The answers to the trigonometric equations are not satisfying school requirements. Maple ignores the periodicity of functions and calculates only some certain values, that satisfy the general answer for some the n from $[0, 1, 2, 3]$ in the case of solved trigonometric equations ($n = 0$ is used more often than others). One way of showing that there are infinite

number of answers is to solve the equation graphically (plot the graph and with its help read the answers).

There are different commands that can be used in solving the equations. Command “solve” usually gives answers that need to be transformed in some another form in order to satisfy school requirements and conditions. Command “fsolve” is better in some way, because the result obtained after executing this operation is more like the one that is required at school. For example for logarithmic equation, command “solve” may produce answer with natural logarithms in it, even if the result is reducible to a real number. Command “fsolve” performs possible operations and calculations by itself, giving answer required at school.

The table containing all information about tested problems, received answers and remarks is given in the appendix 2.

2.4.4 Mathcad

When equation has no roots (not even on the set of complex numbers), Mathcad informs that there was *no solution found*, what is absolutely correct. This program gives answer always on the set of complex numbers, and there is no short command to get the result only on the set of real numbers. There is one more remark connected with the complex numbers, which are not included in the national curriculum of Estonia and therefore this remark should not bother teachers. Sometimes, the way of writing answer with constant i is a bit strange. Namely in the answer to the quadratic equation of the fifth subtype (equation has no real roots), the constant is written between multiplication signs and with coefficient “1” (... $\cdot 1i \cdot \dots$). But teachers always tell at school, that coefficient “1” is not written. Of course this is not a mistake, but just another thing, that student would have to get used to.

Some confusion can bring the form of the answers given in Mathcad. If there is only one root, then it is given as a number without any parenthesis. But if the number of roots is greater than 1 (2 and more), then program places the answers vertically in the parenthesis like this: $\begin{pmatrix} -2 \\ -8 \end{pmatrix}$. The minus sign in the

fraction is placed differently than at school. The minus sign is in the numerator, not in front of the whole fraction $\frac{-1}{9}$. Function arctangent is labeled in a different way, in Mathcad label “atan” is used. For the quadratic equations with one double root (two equal roots), Mathcad gives 2 equal numbers one under another. Weaker students may have troubles with understanding the meaning of such record. There are also some remarks of the third subtype (“incomplete” correct answer). For example fractions are not reduced in the result or answer is given with natural logarithm, while it can be transformed to a number. Usually at school teachers require some more actions in order to get final result. In Mathcad there are simple commands that can transform the answer to the “nice” form (for example “reduce” for reducing fractions or “evaluate” for simplifying natural logarithms).

Mathcad is widely used and has powerful functionality, but it still makes some mistakes. One subtype of irrational equation is not solved correctly in Mathcad. (equation containing the roots of the third power, for example $\sqrt[3]{5x+7} - \sqrt[3]{5x-12} = 1$) System gives only one root (the positive one $x = 4$) while there are two ($x_1 = 4, x_2 = -3$). Another problem occurred while solving logarithmic equation of the second type. Solving more complicated equation ($\log_{0,2}(4x) + \log_5(x^2 + 75) = 1$) gave unexpected result. While solving at school student should obtain 2 roots ($x_1 = 5, x_2 = 15$) and program gives only one. That is 4.9999. Probably Mathcad uses some approximate methods for solving more complicated equations and that could explain, where this inaccuracy come from, but one answer is still lost by Mathcad. Some types of the equations can not be solved easily by program Mathcad. For example solving exponential homogeneous equation of the second power ($2 \cdot 4^x - 5 \cdot 6^x + 3 \cdot 9^x = 0$) gives no plain answer (system just gives the same expression as a result). Differences from school also occur in trigonometry: program Mathcad calculates some values and gives them as an answer. Namely in the case of solved trigonometric equations we can get the result proposed by Mathcad if we substitute some n from $[-1, 0, 1, 3]$ into the general answer ($n = 0$ is used more often than others). Periodicity of trigonometric functions is not taken into

consideration and that makes it difficult to use this program while teaching or learning trigonometry.

Detailed overview of the advantages and disadvantages of system Mathcad is in the table in Appendix 3.

2.4.5 Mathematica

The standards of the answers are uncommon from school point of view. In the result braces are used in two ways. Firstly, they surround the whole answer and secondly braces surround each root separately. For example, if equation has two roots $x_1 = 1, x_2 = 2$, then answer will look like this: $\{\{x \rightarrow 1\}, \{x \rightarrow 2\}\}$. That can bring some confusion and be misleading in understanding the answer. If some function has arguments in it, then the argument is in the square brackets and that is also different at school (for example $\tan[x]$).

In case of the quadratic equation $x^2 - 2x + 1 = 0$ with double root (two equal roots) Mathematica gives the answer in form $\{\{x \rightarrow 1\}, \{x \rightarrow 1\}\}$. As noticed before, in some way this is more correct answer than the one given at school, because teachers usually say that such equations have only one real root. That is why it needs to be explained to the students. These were the remarks of the second type, now we will take a look at the remarks of the third type concerning “incomplete” correct answers.

Firstly, problem with fractions is also relevant here. Fractions in the answer are not reduced, as it is expected in some schools, and reduction of the fraction requires some additional steps. In some answers roots were left in the denominator, what is also correct from mathematical point of view but seems uncommon from school point of view $\frac{2}{\sqrt{13}}$. When quadratic equation has no real roots, then program by default gives answer that contains complex numbers. It is not very easy to put some constrains on the answer and to get the result only on the set of real numbers restricting answers containing imaginary unit i . There is a type of equations that is solved wrongly using Mathematica. These are irrational equations of the third subtype, which contain roots of the third power and are solved by the method of substitution. After

solving equation CAS gives only positive root, ignoring that there can be a negative number under the sign of root if the root index is an odd number (Mathematica gives only $\{x \rightarrow 80\}$ as the root for the equation $\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$, while there is one more root $x = -109$).

Only simpler logarithmic equations can be solved with a help of Mathematica. Namely, in case of more complicated logarithmic equations (second, third and fourth subtypes) Mathematica gives warning and no answer, or a very complicated answer that can not be used for next analysis. For example, if using command "solve" Mathematica gives warning that "*the equation appears to involve the variables to be solved for in an essentially non-algebraic way*" or that "*the expression involves unknowns in more than one argument, so inverse functions cannot be used*". If command "FindInstance" is used, then warning looks like this: "*The methods available to FindInstance are insufficient to find the requested instances or prove they do not exist*". Thus program Mathematica is inconvenient to use for teaching and learning logarithmic equations, because instead of solving equations student will spend time on searching the right command or understanding the meaning of the obtained warning.

Trigonometry is one more topic that can be difficult to teach at school using Mathematica, because program calculates some exact values. Namely in the case of solved trigonometric equations Mathematica calculates result for some n from $[-1, 0, 1]$ ($n = 0$ is used more often). Also for the trigonometric equations of the third subtype, that include functions tangent and cotangent and are reducible to quadratic, Mathematica gives only one answer, while at school student will receive two general roots. In all answers for the trigonometric equations Mathematica gives next warning: "*Inverse functions are being used by Solve, so some solutions may not be found*". So program admits, that not all the answers are obtained. But again in such situation solving equation graphically and reading needed answers from the graph can help.

More information is gathered in one table (problems, answers, analysis of the result) in Appendix 4.

2.4.6 MuPAD

This is the only CAS out of 7 that labels empty set in the same way as it is mostly used at school (with sign \emptyset). Teachers can be sure that if some equation has no roots, then the answer given by computer will be correctly interpreted and understood by the students.

There are several commands for solving equations. And in case of quadratic equations with no roots on the set of real numbers, command “solve” gives answer on the set of complex numbers. Command “numeric::realroots” excludes complex numbers from the answer leaving only real roots. If there are no real roots, then empty square brackets will appear in the answer line. Fractions can be mentioned, as an example of the remarks of the third type (“incomplete” correct answer). As other 7 investigated CASs, MuPad doesn’t reduce fraction in the answer. Transforming obtained result to desired form is not complicated, so this inconvenience can be eliminated by some extra steps. Unexpected behavior of program MuPAD was noticed while solving logarithmic equations and equations that contain absolute values of some expressions. Command “solve” solves equations in own manner giving answer in unordinary way. Namely, message about error can be received or only one root is obtained (while there can be two roots). More suitable answer required at school can be provided by commands “numeric::solve” or “numeric::solve(AllRealRoots)”. The irrational equations of the third subtype $\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$ (that contain root of the third power) are not solved correctly by this system. MuPAD gives as an answer only positive root ignoring negative numbers under the sign of root with odd index (root $x = 80$ is obtained and root $x = -109$ is lost). Exponential equations that are solved by taking the logarithm can produce extraneous roots if solved by MuPAD. For example in our case program gives three roots for equation $x^{\sqrt{x}} = (\sqrt{x})^x$, the roots are $\{0.0, 1.0, 4.0\}$, while there should be only two roots $x_1 = 1$, $x_2 = 4$. MuPAD has standards that are different from the ones at school, namely if we check the answer $x = 0$, then program gives the result that $0^0 = 1$ and therefore by its standards MuPAD solves equation correctly. There is no problem with other exponential equations.

Trigonometry is one field that can be successfully solved and taught with a help of MuPAD. Majority of the answers offered by MuPAD were complete, only 3 equations produce partial answers (exact examples are in Appendix 5). Maple takes into consideration the periodicity of trigonometric functions and gives correct constraints for the variable n (for example $\left\{\frac{3\pi}{4} + \pi k \mid k \in \mathbb{Z}\right\} \cup \{2\pi k \mid k \in \mathbb{Z}\}$).

The list of all tested problems, gained results and some remarks regarding the answers are given in the table in Appendix 5.

2.4.7 TI-92 Plus

The TI-92 Plus is a scientific calculator and it has small display. This is the reason, why TI-92 Plus uses linear method of writing data down. For simpler answers instead of writing fraction in ordinary way, when numerator is above the denominator and there is a division line between them, calculator TI-92 Plus writes the fraction in one line, where the numerator and denominator are separated by the slash (for example 1/9). Such way of writing may be unusual for students in the beginning but it doesn't take much time to get used to reading and understanding from the display. For more complicated formulas ordinary way of writing down the problems is used. Such option as "pretty mode" can be customized in order to turn the "pretty mode" off and leave only linear mode or turn the "pretty mode" on. In that case simpler fractions are written in one line and more complicated formulas are written in ordinary way. This feature becomes more intriguing, when student needs to solve and enter the equation it into the calculator. While entering the problem student has to use linear mode of writing. In some way it can be even very useful for student. This feature makes students think about the order of procedures, analyze where to put extra parenthesis in order to keep the original equation. Teachers may even use this option for teaching and training this topic at school with younger children. For example teacher can ask to write down some formula in linear mode or the opposite, write on the chalkboard the equation or some problem in linear mode and ask student to put it into "pretty mode".

One inconvenient standard is with decimal fractions. If there is no whole part in the fraction then zero is omitted. For example instead of writing “0,1” there would be “.1” on the screen. Student can easily get used to this feature, but if student’s working speed is very fast, then he or she can just not notice the dot in front of the number and then all further analysis concerning particular equation will be wrong. On the other hand this shows perfectly to the teacher, how much attention one or another student pays to the lesson.

The problem of the empty set is relevant for calculator TI-92 Plus. When equation has no solutions on the set of real numbers then, instead of some warning, word “*false*” appears in the answer area. As computer algebra system is based on Derive then it has similar standards. Namely such logical value shows, that no value of unknown makes the particular equation valid. This remark is not of a big concern, if teacher in the beginning of using calculator TI-92 Plus explains the meaning of the identity and gives instructions, how to behave and what to think in such situation.

Only the simpler trigonometric equations can be solved similarly as at school. Namely, for simpler equations TI-92 Plus gives result that is similar to the one expected at school. But here is also one remark. For each new answer a new variable is used. They are named $n1$, $n2$, $n3$, etc. and represent the periodicity of the functions. The problem is that these variables are not described, so one may think, that it can take every value, what is clearly incorrect. Nowhere is stated, that such variables are only whole numbers and belong to the set of whole numbers Z . For more complicated trigonometric equations some certain answers are given in radians. After solving the equation, calculator gives a list of roots and a warning that some roots could be lost during solving the equation. Such list of roots in radians is of small help to students at school, because answers in that form can not be analyzed later or illustrated with a graph. For example in the case of solve equations for calculating these certain answers from the general formula some values of parameter n from $[-32, -25, -21, -14, -10, -9, -8, -6, -3, -2, -1, 0, 1, 2, 3, 6, 9, 11, 12, 14, 22, 26]$ were used.

Calculator TI-92 Plus could be very well used at school. If student learns how to write down equation in linear mode, then using this calculator would be very

easy and student can make some decisions intuitively. As the display is small, it is tried to make everything as simple as possible. Most of the answers suit well school requirements. Calculator TI-92 Plus is small and compact but still has a great functionality that makes it very convenient in use. Some students, who do not have computer at home, may be a little bit afraid of using technology, but TI-92 Plus looks like ordinary calculator just bigger.

Exact problems used to investigate TI-92 Plus, answers required at school and proposed by calculator TI-92 Plus, some comments and remarks on shortages for each case are given in a table in appendix 6.

2.4.8 WIRIS

WIRIS is CAS that works through the internet and that makes it convenient to use. It does not need any installation, home web page www.wiris.com has link to free demo program. In this thesis I used browser Mozilla Firefox 1.0.7 to solve the equations.

The standards used in program WIRIS are similar with the ones used in Mathematica. Braces are used in the result, they surround the whole answer and each root separately, like this: $\{\{x = -2\}, \{x = 2\}\}$. Students have to get used to the way of writing the answers down and then it will no longer bother them. Fractions in the answer are not reduced and eliminating improper fractions requires additional step. WIRIS can't solve equations containing absolute values of the expressions. As a results program gives empty braces $\{\square\}$ and warning "*it is not possible to find a result or solution*". To avoid this situation, students have to enter such equation with the help of roots and powers (for example $\sqrt{(x+5)^2} = \sqrt{(x+9)^2}$). Answer given by program WIRIS is always on the set of real numbers and if there are no real roots, then program gives empty braces as a result. WIRIS can solve only simpler exponential and logarithmic equations (only first subtype of exponential equations and first and second subtypes of logarithmic). Other equations of these types cannot be solved and as a result student gets empty braces.

Program calculates some exact values of solving trigonometric equations. For example in the case of solved trigonometric equations listed by WIRIS roots can be obtained by substituting some n from $[-1, 0, 1]$ into the general formula. Sometimes in the same answer there are roots in degrees and in radians, also program calculates approximate values of function arctangent, which can be inconvenient while using at school. One way to avoid this problem is to solve trigonometric equations graphically and read needed answers from the graph.

More information is gathered in one table (problems, answers, analysis of the result) in Appendix 7.

2.5 Unexpected answers and some possible actions

It can be seen from the previous parts of the second chapter that CASs handle very well school equations. Only in the case of more complicated examples some inaccuracies were noticed. Other remarks relate mostly to the form of the answer, but it is not a big problem also as there is missing common standard of how exactly answer should look like. Should the fraction be reduced? Is it necessary to order the roots increasingly or not? Different teachers, schools and countries have different requirements to the answers. Such common remarks can be easily explained or changed with the help of some additional steps and that do not influence the quality of teaching and learning. For more useful and detailed advices thorough practical experiments at school should be made, afterwards feedback from teachers should be gathered and analyzed.

For instance, one field, that gives different opportunities, is trigonometry. Namely only two programs, MuPAD and partially TI-92 Plus, give general answer that takes into consideration the periodicity of the trigonometric functions, others calculates some certain values, sometimes even in radians (sometimes answers to more complicated trigonometric equations in TI-92 Plus are given in radians). This disadvantage can be used by teachers. For example teachers can ask students to find which values of parameter n from general answer correspond with the certain answers proposed by the CAS. This exercise helps students to understand the general answer better and learn to think abstractedly.

Table 2.2 gives advices what teacher could do, if he or she has noticed answer of some type described in chapter 2.3.

<i>Type of the remark</i>	<i>Possible actions</i>	<i>Command</i>
1 type - errors	<ul style="list-style-type: none"> • Solve equations and explain the reason of the error • Do not use equations, that can produce errors 	Commands for solving equations
2 type - difference in form of the answer (empty set, brackets, ...)	<ul style="list-style-type: none"> • Solve equations and explain the record • Do not use equations, that can produce such situations 	Commands for solving equations
3 type - "incomplete" correct answer (can be changed)	Solve equations and explain how to change the form to the required one	Commands for solving equations and simplifying the expression

Table 2.2 Shortages and possible actions of the teacher.

Example of the answer of the first type (error) is equation $\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$, which is solved wrongly by programs Mathcad, MuPAD and Mathematica. Such equations should not be solved by these programs at school, or teacher should explain, that programs treat root with index 3 and fractional exponent $\frac{1}{3}$ differently, though it is told at school that expressions are equivalent ($\sqrt[3]{x+45} = (x+45)^{\frac{1}{3}}$). Programs treat such expressions differently and choose solving algorithm according to the form of the equation. Thus if program has tools for entering the equation as the original given above, then probably it will give complete correct answer, otherwise if student has to enter the equation with the fractional exponents, then program probably ignores the negative root.

Answers of the second type are correct but sometimes it can be difficult for teacher to explain notions of certain program. For example it can be complicated to explain the meaning of the braces in the answer used by Mathematica in younger grades (e.g. $\{\{x \rightarrow 1\}, \{x \rightarrow 2\}\}$). Older students already

have some knowledge about the entities and it can be easier for them to get used to the uncommon standards. If CAS that is used at school has such differences in the way of writing things down, then teacher should often remind about them and their meaning, even if students are using particular CAS not at the first time.

Teacher should at the beginning of the lesson remind to students the requirements to the form of the answer (whether fraction should be reduced, roots sorted increasingly, etc), beforehand describe possible actions and suggest needed commands, in order for students not to waste time on the searching for the right command. Then if students get answer of the third subtype, then they know which steps to do next. For example, if there is improper fraction in the obtained answer, then students should already know which command to use in order to simplify the fraction.

One field that gives different opportunities is trigonometry. Namely only two programs, MuPAD and partially TI-92 Plus, give general answer that takes into consideration the periodicity of the trigonometric functions. One more remark about MuPAD that sometimes it loses one correct answer (this was noticed for 3 trigonometric equations). Others calculate some certain values, sometimes even in radians (for example answers to more complicated trigonometric equations in TI-91 Plus are given in radians). This disadvantage can be used by teachers. For instance, teachers can ask students to find, which values of the parameter n from the general answer correspond with the certain answers proposed by the CAS. This exercise helps students to understand the general answer better and learn to think more abstractedly.

3 Web info system

As teachers and researches of CASs are potential users of the results of this thesis, then one of my goals is to give more information about popular computer algebra systems and to provide base for other researches of the investigated topics and computer algebra systems. For accomplishing this goal web info system was designed. The system is not complete, only some thoughts and ideas are given. This chapter is oriented on the researches, who are interested in the development and realization of the web info system, but at the same time have some knowledge from the computer science. The structure of the system and its database are designed in close cooperation with supervisor Eno Tõnisson. There is no analogical system yet developed, all invented ideas regarding the structure and functionality of the system can be altered during further analysis.

A brief overview of the possible structure and features of the info system is given in this chapter (user case diagram describing the system can be found in appendix 8). Of course its functionality could be greater, but due to the lack of time I describe the most important parts of this system that could use and share the information from the research described above. Later realization of the system can bring changes into the structure and make it more complete. Such system should be available through the internet, as there are people all over the world who could be interested in it. As it is web based info system the security of the system should be well thought through. Such problems are solved during the realization and therefore are not discussed in this thesis.

System collects information about tested computer algebra systems, solved problems, answers, discovered errors or inaccuracies. It is possible to see description of certain computer algebra system and syntax of needed command for solving certain problem in given conditions. Also there is gathered information about sources, where more detailed descriptions or additional information can be found. This system can help for example with making decision, which program should school purchase or which command is preferable for particular type of problem. Teacher or the person, who is

responsible for making this decision can see, which program is handling school topics better, which advantages does it have in front of other programs. Another example is for teachers to know, what certain CAS can do and to compare it with some other systems relying on the other professional's opinion.

3.1 Structure of the system

Registered users can write comments, make their own notes, bookmark certain parts of the system in the private space or complete database, depending on their role. Different roles are described below in this chapter.

Spaces

All registered users have own private space on server, where they can store useful information, links to some sources, bookmarks from the same system, describe their own experience and save similar information on the system server (problems, tasks, description of computer algebra system and its errors etc.). It is possible for users to share their private space with other users in order to exchange information and experience. All original tables with data about investigated computer algebra systems and problems are in public space and thus are open for everybody. Also comments and remarks are accessible for everybody, even unregistered users.

Roles

The system is divided into several levels according to the information that can be accessed on the certain level. So there are four major levels and corresponding roles in this system (figure 3.1).

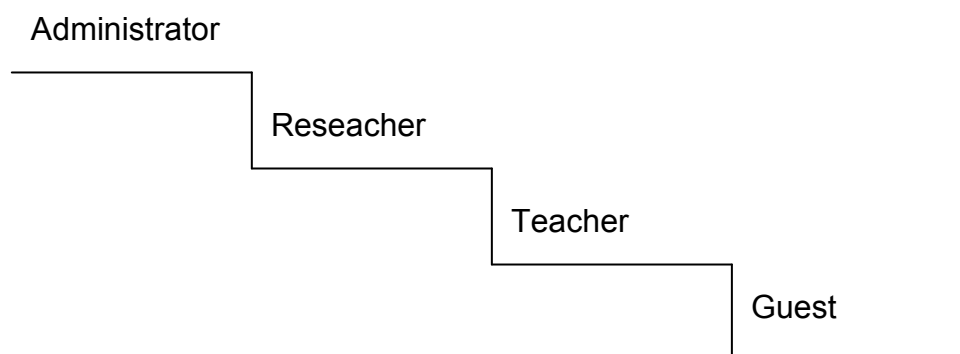


Figure 3.1. Roles in web info system

Web info system can be used by **unauthorized users (guests)**, but when user has not logged on, he or she can only passively use web info system. Unregistered user can see information about computer algebra systems, problems, read other user's comments, print out needed parts, search through the system, but he or she has no private space and therefore cannot store information on the server of the system. For example students would like to know, which programs are more suitable for solving certain problem or what do professionals think about one or other computer algebra system.

The second role is a **teacher** or observer. Teacher can write comments to different parts of the system, search through web info system, save bookmarks and links in the private space. Teacher has an opportunity to write new examples, to describe some error or strong side of the computer algebra system. This role is for ordinary school teachers, whose main interest is in how certain computer algebra system handles some subtype of equations. For example he or she would like to know, which errors could be done, which problems could arise while using certain computer algebra system in class with children. Also teacher can find detailed description of certain system, learn a bit more about used commands and their syntax, and surely exchange thoughts and experience with other users by writing comments and new problems.

Third level is less bounded. The role name is **researcher**. Researcher has all previous role opportunities plus some more. Users of this level can access all information excluding information about users, original database structure and other user's private spaces. This role is for people, who are more interested in computer algebra and are investigating similar fields, but it could be also teacher, who deals with investigating different computer algebra systems and therefore needs more information. Researcher has one great advantage: he or she can add attributes to the original database. For example, if researcher has done some very praiseful investigation and thinks, that the results should be reported to all the other users, he can add attributes in his private space and send request to administrator to complement original database with his attributes. After receiving this kind of request, administrator looks through this change and decides, whether it is remarkable and useful enough for other users

to know. Administrator can decline query or add these attributes into original database. Attributes from the original database can not be changed or deleted by researcher; he or she can only propose to add new columns. The reason, why teacher and researcher are separate roles, is that it makes the interface for teachers simpler, because unnecessary options, that ordinary teacher will probably not use, are eliminated. If teacher will have a wish to change the database, then he or she can be given the role of researcher at any moment.

Next level, that gives access to all the data, is **administrator** level. In the beginning there should be one person that maintains the system and users, distributes passwords, and analyzes the statistics. If the number of users is very big, there could be more administrators, each responsible for different tasks. In this case they could have also bounded access to the information in order for the system to be more secure (each one has access to the information he or she is responsible for and doesn't need access to other information that is maintained by someone else). The role of administrator is to make sure that the system works properly, all users behave and use system correctly. One of the administrator's tasks is to maintain and change the structure of the database (for example add new attributes), this option was described above. If the number of users is large, then the process of the registration and authentication should be automatized.

Functionality of the system

Some functions and opportunities were already described in previous parts, but here we will give more detailed overview.

First step for using the system is authentication. Of course it is not required for unregistered users. All other users, who want to add their comments or store some information on the system's server, are obliged to log in. In the beginning while there is a few number of users, administrator can manually distribute usernames and passwords. Later, when number of users will grow, automatic algorithm for registration should be applied. After logging in, user will have different opportunities. Firstly, he or she can manage own private space, store or delete some information or bookmarks, add information about computer

algebra systems. User can also add new interesting problems to existing tables and they can be shown only in his private space, where other users will not be able to see it, or in public space, where everyone can get access to the information. For instance, if teacher is giving lessons on exponential equations that can be reduced to quadratic by substitution and has found some examples, that computer algebra system can not solve or solves incorrectly, then teacher can save this problem in order to know in future, which program doesn't handle this topic very well. Also it is possible to make some notes about how different computer algebra systems behave in different situations with more complicated problems, than those given in textbooks.

There should be a possibility to share user's private space with other certain users. That is necessary for cases when researches or teachers want to exchange experience with each other, not showing the content of their private space to the rest of users. Probably later there could be organized some groups of users, for example "Friends", so when certain user wants to share his private space with the whole group, he or she doesn't have to share it with each user from the group separately, but with the whole group at once. That would be very convenient for example, if teachers from one school are using this web info system, then they can exchange useful information and ideas through it.

Researcher has one special opportunity to complement original database. For instance make new columns (attributes) or after making own series of experiments for some other computer algebra system, than described in this thesis, then he would like to share his knowledge with other users. In this case, researcher saves useful information in his own private space and sends a request to administrator to change (complement) original database. Administrator will decide, whether gained knowledge is worth being shown to all the users, and depending on the decision will update original database adding new attributes or decline the request, if it doesn't give much new information to the rest of the users.

The system permits to add comments to the different parts, for example users can comment computer algebra programs, certain problem or used command. All information can be printed out not only by registered users, but also by the

guests. There should be realized some algorithm for making printable information more convenient to use (for example converting it into the text format for later change) but this is one of the realization's problems.

Search options

System allows to search through its components, and this search can be very detailed. Some of these options are described in this thesis, but there could be also other opportunities in addition to those mentioned in this chapter (figure 3.2).

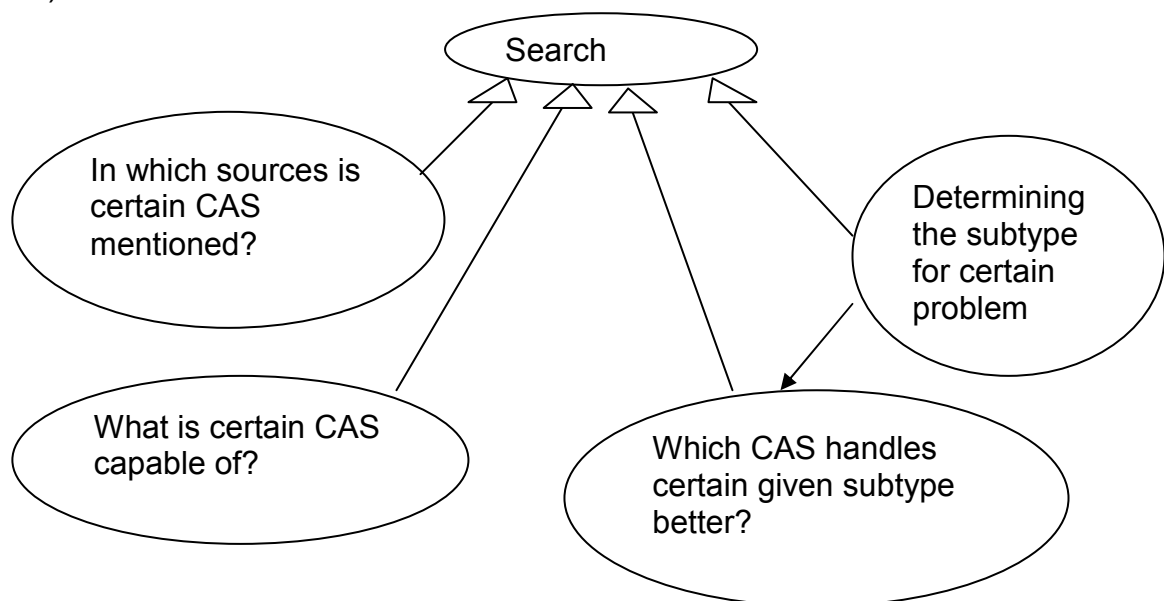


Figure 3.2. Search opportunities in web info system.

For example, if one would like to know, which computer algebra system handles better some certain topic, then he or she chooses type and subtype of equation and system will generate overview, which CAS can be a better help for the teacher or for the student, which problems can be solved, which errors can be done by one or another program, how the syntax of the commands should look like to get precise result. This overview should help teacher to choose the most appropriate computer algebra system, giving advises from different points of view, analyzing different aspects. Also web info system should give answer to another question, namely what can be done with the certain computer algebra system, which are its advantages and disadvantages, how it handles different types of equations, whether it gives correct answer for all school levels, which

commands are preferable, etc. Another question to which this system should give answer is about the used sources of information, gathered in the system. For example, in which sources one can find description of some problem or computer algebra system.

Search options for teachers and researcher were listed above, but they probably would not help much a student as he is not familiar with types and subtypes of equations. Student can be interested in solving certain problem, so that option could also be realized. Student (unregistered user) enters equation that he needs to solve, into the system and system after analyzing the form and composing functions of this equation should determine, which subtype it belongs to, and give corresponding information from database which programs to use in order to get correct result.

3.2 Structure of database

All gathered information should be stored in database that is placed on the server of the web info system. Information is divided into tables with certain attributes. In this thesis we will name only basic attributes and tables, detailed design and programming probably will require more exact database. There are omission points in the corresponding figures to show, that this structure of database is not final. In this thesis are described 5 main tables, which in their turn can be broken up into smaller tables in order to be more convenient to operate with. These basic tables are: Users, Problems, Programs, Answers and Sources (Appendix 9). Below in this chapter is given more detailed overview of these tables.

Users

This table contains information about all registered users. Besides compulsory information like username, password and role, here is also gathered personal information about users in order to get some statistics (name, surname, age, job title, etc.) On the figure 3.3 this information is not reflected.

Primary key of this table is user id, by which user is uniquely identified in the web info system and by which user is referenced in other tables. In the

beginning of functioning of the system, administrator could distribute usernames and passwords, but with growth of the number of users registration process should be automatized.

Users
Id Username Password Role Registration date Personal information ...

Figure 3.3. Part of the database, table "Users"

Computer algebra systems

Next table contains information about investigated computer algebra systems (figure 3.4). All possible information that can be useful for web info system's users should be stored in this table. Such information as name of the program, number of used version, year of release, authors and small description of program is obligatory.

Computer algebra systems
ID Name Version Year Authors Description Source id ...

Figure 3.4. Part of the database, table "Computer algebra systems"

One may wonder, which sources give more detailed information about certain computer algebra system, and that is why there should be such attribute as sources. Here is marked only the id of the source (foreign key of this table) and

all other information about certain source is in the table “Sources”. Primary key in this table is computer algebra system id, which uniquely identifies computer algebra system.

Problems

Table “Problems” gathers information about all problems, used in the web info system while investigating computer algebra systems (figure 3.5). Each problem is identified by its id (primary key). Stored is next information: type and topic, to which given problem belongs; source id (textbook, manual, internet etc.); school answer should be given, maybe with some explanations. For example students don’t learn about the set of complex numbers and therefore some quadratic equations for them do not have any roots. In this case teachers sometimes don’t add that those equations do not have roots only on the set of the real number in order not to confuse student with unknown terms.

Problems
Id Problem Type Topic Source id School answer User id, who added problem Date of input ...

Figure 3.5. Part of the database, table “Problems”

Also it would be convenient to know, who and when has added certain problem to the database in order to have statistics on who are the most active users of this web info system. So this table also has one foreign key, this is the id of the user who has added certain problem to the database.

Sources

As there is used a lot of materials to get all information together, like textbooks, articles, manuals, internet, it would be convenient for other users to know, which

sources are worth trust and are often used by other teachers or researchers. Therefore all information about the sources should be gathered in a separate table (figure 3.6).

Here should be kept such information as title of the source, its authors, volume, number, release date, publisher, and web link. Now a lot of data is accessible through the internet and even do not have a printed original, so some of the attributes in this table can be left blank. And again in order to have some statistics, information about user who added this certain source and about date when he or she did it, should be stored.

Sources
Id
Title
Authors
Volume
Number
Release date
Publisher
Web link
User id, who added source
...

Figure 3.6. Part of the database, table “Sources”.

Primary key in this table is source id and it is used in such tables as “Problems” and “Programs”. Foreign key is attribute user id from table Users.

Answers

Table “Answers” collects information from solving different problems and investigating computer algebra systems. Main results from this thesis are gathered in this table (figure 3.7).

Attributes of table “Answers” are: problem id that was solved and gave this answer; CAS id that produced this answer; exact syntax of the command used to solve problem; some special conditions and constraints, if there are any (for example, if answer should be given on some certain set of numbers, then it should be mentioned here); links to other answers for the same problem, that

are produced by other CASs (this should help users analyze which CAS gives more suitable and correct answers); evaluation from authors of the program or some other users who also investigated how well CAS handles certain problem; list of possible errors and inaccuracies; other remarks, that could be also useful for users (for example, if there are different commands to solve equation, then which to prefer); comments from other users; and general information for statistics such as date of input and id of the user, who has added this answer to the database.

Answers
Id
Problem id
Program id
Syntax
Special conditions
Answer
Link to other answers
Evaluation
Errors
Remarks
Comments
Date of input
User id, who added answer
...

Figure 3.7. Part of the database, table "Answers".

Table "Answers" connects different parts of the system together, therefore it has a lot of foreign keys. Primary key of this table is answer id.

Conclusion

In a CAS teaching and learning environment many indispensable manual skills and abilities will keep their importance. But some of today's skills will become less important, because CAS can perform such operations quicker and save spending time on routine or on the contrary complicated procedures [Kutzler, 2000]. Research described in this thesis shows, that CASs handle well school mathematic equations and therefore are quite convenient to use at the lessons. Research shows, which equations are solved differently in CASs than at schools and which result is suitable at school. Totally 60 problems were solved with a help of 7 CASs. Equations were divided into 8 types (35 subtypes). Bigger problems were noticed only with three types (irrational equations containing negative number under the root with odd root index, logarithmic equations solved by taking the logarithm and trigonometric equations). For example only two programs (partially MuPad and partially TI-92 Plus) give correct answer to trigonometric equations in the general form, others calculate values for some certain value of parameter n from the general answer. Other remarks differ from system to system and mostly depend on the notions of the particular system.

Main aim of this thesis was to **develop the strategy for researching and comparing CASs**, and afterwards basing on this strategy to investigate, how well 7 computer algebra systems handle equations from school textbooks. Done research shows, that **mostly systems are reliable and give reasonable answers**. Some remarks regarding the obtained answer can be easily explained by built-in standards and notions, which can differ from school assumptions. Investigated 7 programs (Derive, Maple, Mathcad, Mathematica, MuPAD, TI-92 Plus and WIRIS) **work similarly** for tested equations and give practically same results. For instance, the majority of systems treat similarly negative number under the root sign if the root index is odd number (it can be explained by different evaluation branches on the set of complex numbers). **The bigger differences are in the way of writing down the answers or indicating some known situations** (for example different is the reaction of programs, when equations have no solution or infinite number of solutions). If

the systems are not sure about obtaining all the roots for some kind of equations, then usually concurrently with giving the answer systems give message about the situations and reasons for possible loss of roots. Possibility of CASs to calculate numerically and symbolically can be widely used at all levels at school.

The classification of equations, which was made up for the research suits well primary aim of this thesis, namely developing methods for investigating and analysing how CASs handle school mathematics equations. Classification covers majority of the equations studied at school. Only more complicated mixed equations are left out of sight because of their variety and difficulty to determine certain subtype such equations belong to. Proposed classification is enough detailed and is suitable for beginning investigating the computer algebra systems and making conclusions about their coping with school mathematic equations. In future classification can be modified, namely more differences within one subtype can be distinguished. Now there are left such subtypes as “others”, which are bigger than others and their representatives can differ from each other greatly. So next step in this research could be fractioning such subtypes. Also mixed equations should be covered in next version of the classification. For more reliable results much more equations of each subtype should be solved (for this research only 60 problems were chosen). Some questions have arisen during this study concerning trigonometric equations and these questions should be explored. Namely, how does one or another CAS choose value of the parameter n from the general answer? Are these values always falling into some interval? As continuation of current research “step-by-step” solutions should covered and investigated.

Another way to develop this field is to investigate computer algebra systems more thoroughly and from more aspects. For example school students can take part in this research. Experiments could take place in computer labs during lessons of mathematics. Then student can say more, which system was easier and more comfortable to use, but such experiments should be well thought through, because if the same students solve the same equations with a help of different systems then the experience, they gain in the beginning of the

experiment, will influence the rest of it. Also teachers can be interviewed, which algebra systems they prefer to use and why.

All obtained results can be gained into one system, which should be available through the internet. In this thesis only the design of such system and main ideas about its functionality are described in the fourth chapter, but they give a good base for future realisation. During the implementation phase the structure can change and such system can gain much more additional possibilities.

Resümee

ARVUTIALGEBRA SÜSTEEMIDE VASTUSED KOOLIÕPIKUTEST PÄRIT VÕRRANDITELE

Nadežda Velikanova

Magistritöö

Tartu Ülikool

Arvutialgebra süsteemid (AASid, nt. Maple ja Mathematica) on programmid, mille abil saab tegeleda algebra, trigonomeetria, matemaatilise analüüsi jpm. Kasutusel on nii sümbol-, kui ka numbriline arvutus. Alguses olid AASid mõeldud peamiselt professionaalidele. Nüüd aga olukord on muutunud ning paljud programmid on uuendatud võttes arvesse ka kooli vajadusi. Erinevates uuringutes on analüüsitud AASide omadusi, mõju õpilastele ja õppekavale ning võimalikke viise kasutamiseks koolides. Käesolevas töös käsitletakse, milliseid vastuseid annavad erinevad süsteemid koolialgebra võrranditele ning mille poolest saadud tulemused erinevad koolis nõutavatest. Vaadeldakse käske, mille korral võrrandite lahendamine toimub ühe käsu täitmise tulemusena. Magistritöö tulemusena selgus, et 7 arvutialgebra süsteemi (Derive, Maple, Mathcad, Mathematica, MuPAD, TI-92 Plus ja WIRIS) töötavad üsna sarnaselt ning annavad sarnaseid tulemusi. Erinevused avalduvad aga lahendite vastuste andmise kujus – näiteks, mis tüüpi sulgusid kasutatakse, millisel kujul antakse tulemused, kui võrrandil puudub lahend jne. Selgub, et koolinõuded vastuse lõppkujule on rangemad AASi poolt pakutavatest – näiteks koolis on oluline murru taandamine ning irratsionaalsuse kaotamine nimetajas. Tundub küll, et koolis kehtivad nõudmised on muutumas leebemaks. Pealegi on praeguste nõuete täitmiseks vajalikud lisaoperatsioonid AASide puhul lihtsasti teostatavad ainult ühe või kahe käsu abil.

Töö põhieesmärk oli välja töötada meetodika, kuidas võiks kooli ja AASide vastuste vastavust uurida ja selle meetodika abil välja selgitada, kui hästi 7 arvutialgebra süsteemi tulevad toime koolialgebra võrranditega. Kokku on lahendatud magistritöö raames iga seitsme programmiga 60 võrrandit. Võrrandid esindavad 8 tüüpi, mis omakorda jaotatud 35 alamtüübiks. Uurimus

näitab, et enamasti on AASid usaldusväärsed ning annavad koolile sobivaid (või vähemalt seletatavaid) vastuseid. Tõsisemaid märkuseid sai teha ainult kolme võrranditüübi kohta (esiteks - irratsionaalsed võrrandid, mis sisaldavad negatiivset arvu juuremärgi all ja paaritut juurijat, teiseks - logaritmilised võrrandid, mis on lahendatavad logaritmime teel ning kolmandaks – trigonomeetrilised võrrandid). Näiteks ainult 2 AASi (osaliselt MuPAD ja osaliselt ka TI-92 Plus) lahendavad trigonomeetrilisi võrrandid koolialgebra seisukohalt korrektselt ning annavad lahendid üldkujul. Ülejäänud programmid aga arvutavad ainult mõningad konkreetsete väärtused. Samas isegi seda võivad õpetajad ära kasutada oma tundides. Näiteks paluda õpilastel uurida, millised üldise lahendi parameetri n väärtused vastavad AASide poolt arvutatud vastustele. See aitab õpilastel paremini mõista lahendeid üldkujul ning õpetab abstraktsemalt mõtlema. Ülejäänud märkused on süsteemiti erinevad ning enamasti seletatavad konkreetsete AAS'ide standarditega.

AASide kooliülesannetega toimetuleku uurimiseks on antud töö raames välja töötatud võrrandite klassifikatsioon, mis mõnevõrra täiendab liigitamist, mida pakutakse kooliõpikutes. Välja töötatud klassifikatsioon ei ole täielikult uudne, kuid arvestab siiski rohkemate aspektidega. Näiteks on võrrandite alamtüüpidesse jaotamisel arvesse võetud lahendamise meetodeid, lahendite arvu jne. Saadud klassifikatsioon katab koolis õpetatavatest võrranditest enamuse jättes välja vaid segavõrrandid, mis võivad kuuluda samaaegselt mitmesse erinevasse alamtüüpi.

Antud töö tulemuste kättesaadavaks tegemiseks inimestele, kes konkreetsetest tulemustest huvitatud oleksid, on kavandatud veebiinfosüsteem, mis võimaldab kasutajatel otsida vajalikku informatsiooni, kommenteerida teiste inimeste tulemusi, lisada endapoolseid näiteid ning isegi muuta esialgset andmebaasi. Süsteemi disaini kirjeldus on välja toodud magistr töö neljandas peatükis, mida illustreerivad skeemid on antud keele UML notatsioonis.

Töö võimalikuks edasiarenguks oleks veebiinfosüsteemi realiseerimine ning teiste koolimatemaatika valdkondade uurimine arvutialgebra süsteemide abil (segavõrrandid, lihtsustamine, jne). Samuti uuritud võrranditüüpide detailsem uurimine ning ka didaktiliste võtete väljatöötamine ja katsetamine.

References

[Barnett, 1989] R. Barnett, M. Ziegler, *College Algebra*, 4th edition, Mr-Graw-Hill Inc, USA, 1989.

[Buchberger, 1990] B. Buchberger, *Should Students Learn Integration Rules?*, ACM SIGSAM Nulletin, Volume 24, Issue 1, January 1990.

[CAS, 2003] The National Council of Teachers of Mathematics, INC., *Computer Algebra Systems in Secondary School Mathematics Education*, Reston, VA, 2003.

[Connors, 2001] M. Connors, K. Snook, *A Technology Tale: Integrating Handheld CAS into a Mathematics Curriculum*, Teaching Mathematics and Its Applications, Volume 20, Number 4, 2001.

[Kutzler, 2000] W. Herget, H. Heugl, B. Kutzler, E. Lehmann, *Indispensable Manual Calculation Skills in a CAS Environment*, October 2000, http://b.kutzler.com/article/art_indi/indisp.htm - last visited 25.02.2007

[MacGregor, 2002] M. MacGregor, K. Stacey, *Students' Understanding of Algebraic Notation*, Educational Studies in Mathematics, Springer Netherlands, Volume 33, Number 1, 2002.

[Mysovskikh, 2004] V. Mysovskikh, *Computer Algebra Systems and Symbolic Computations*, Journal of Mathematical Sciences, Volume 120, Number 4, 2004.

[Nicaud, 2004] J. Nicaud, D. Bouhineau, H. Chaachoua, *Mixing Microworld and CAS Features in Building Computer Systems that Help Students Learn Algebra*, International Journal of Computers for Mathematical Learning, 9:169-211, 2004.

[Stoutemyer, 1991] D. Stoutemyer, *Crimes and Misdemeanors in the Computer Algebra Trade*, Notices of the American mathematical society, Volume 38, Number 7, 1991.

[Weller, 2001] H. Weigand, H. Weller, *Changes of Working Styles in a Computer Algebra Environment – the Case of Functions*, International Journal of Computers for Mathematical Learning, Volume 6, number 1, May 2001.

[Wester, 1999] M. Wester, *Computer Algebra Systems, A Practical Guide*, Cotopaxi, USA, 1999.

[Zuckerman, 1976] M. Zuckerman, *Intermediate Algebra*, W. W. Norton & Company, New-York, 1976.

[Брусневская, 2001] Л. Брусневская, В. Гудинова, В. Кришталь, С. Шевченко, *Сборник Заданий по Математике для Гимназии*, Коолибри, Таллинн, 2001.

[Nurk, 7kl.] E. Nurk, A. Telgmaa, A. Undusk, *Matemaatika 7.klassile*, 3. ümbertöötatud trükk, Tallinn, Koolibri, 2000.

[Nurk, 8kl.] E. Nurk, M. Lepik, A. Telgmaa, A. Undusk, *Matemaatika VIII klassile*, Tallinn, Koolibri, 2000.

[Lepmann, 9kl.] T. Lepmann, A. Telgmaa, A. Undusk, K. Velsker, *Matemaatika IX klassile*, Tallinn, Koolibri, 2000.

[Лепманн, 10кл.] Л. Лепманн, Т. Лепманн, К. Вельскер, *Математика, учебник для 10 класса*, Коолибри, Таллинн, 2001.

[Лепманн, 11кл.] Л. Лепманн, Т. Лепманн, К. Вельскер, *Математика, учебник для 11 класса*, Коолибри, Таллинн, 2001.

[Колмогоров, 10-11кл.] А. Колмогоров, А. Абрамов, Ю. Дудницын, Б. Ивлев, с. Шварцбурд, *Алгебра и Начало Анализа 10-11*, Просвещение, Москва, 1993.

[Тарасов, 1979] Л. Тарасов, *Математический Анализ, Беседы об Основных Понятиях*, Просвещение, Москва, 1979.

[Уравнения1] *Квадратные Уравнения*,

<http://works.fio.ru/Samara/r5/index.html> - last visited 25.02.2007

[Уравнения2] *Уравнения, Содержащие Модуль*

<http://www.college.ru/mathematics/courses/algebra/content/chapter3/section1/paragraph8/theory.html> - last visited 25.02.2007

[Уравнения3] *Уравнения, Содержащие Модуль*

<http://som.fio.ru/resources/karpuhina/2004/8/schfpaykina/modulviatka.ppt> - last visited 25.02.2007

[HM] *Estonian Ministry of Education and Research*

<http://vana.hm.ee/uus/hm/client/index.php?13526230132223924> – last visited 25.02.2007

[RÕK] *Eesti Riiklik Õppekava*, <https://www.riigiteataja.ee/ert/act.jsp?id=1008388>

– last visited 25.02.2007

[DER1] *System Derive*, <http://www.derive.com> - last visited 25.02.2007

[DER2] B. Kutzler, V. Kokol-Voljc, *Introduction to Derive*,

http://education.ti.com/guidebooks/computer_sw/derive6/Derive6Intro.pdf - last visited 25.02.2007

[MAP1] *Introduction to Maple*,

<http://www.math.utah.edu/lab/ms/maple/mapleV4-intro.html#intro> – last visited 25.02.2007

[MAP2] *Web page of Maplesoft™*, <http://www.maplesoft.com/> - last visited

25.02.2007

[MTC1] *Web page of Mathcad*, <http://www.mathcad.com> - last visited

25.02.2007

[MTC2] *Web page of Mathsoft*, <http://www.mathsoft.com> - last visited

25.02.2007

[MTH1] *Wolfram Information Center*, <http://library.wolfram.com/> - last visited

25.02.2007

[MTH2] *What is Mathematica?*,

<http://www.wolfram.com/products/mathematica/introduction.html> - last visited 25.02.2007

[MTH3] *Mathematica – Computer Algebra System*,

<http://distance-ed.math.tamu.edu/Math696/Mathematica.html> - last visited 25.02.2007

[MUP1] *Information about MuPAD*, <http://www.sciface.com/majewski/edition1/> -

last visited 25.02.2007

[MUP2] *MuPAD Documentation*, <http://www.mupad.com/support/doc/40/en/> -

last visited 25.02.2007

[MUP3] *Mathematics Mastered with MuPAD*, <http://www.mupad.com> - last

visited 25.02.2007

[TI1] *TI-92 Plus tutorial*, <http://web.alfredstate.edu/quagliato/ti92/> - last visited

25.02.2007

[TI2] *TI-92 Plus series*, http://en.wikipedia.org/wiki/TI-92_series - last visited

25.02.2007

[WRS] *Web page of Wiris*, <http://www.wiris.com> - last visited 25.02.2007

Appendix 1. Answers given by Derive

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by DERIVE</i>	<i>Remarks</i>
Linear equations			
$5x - 11 = 0$	$x = 2\frac{1}{5}$	$x = \frac{11}{5}$	improper fraction
$5x = 0$	$x = 0$	$x = 0$	ok
$4(3x - 1) + 2(2 - 6x) = 0 \rightarrow 0 = 0$	$x \in \mathbb{R} \forall i x \in [-\infty, \infty]$	true	Derive standards
$4(x + 1) = 4x + 1 \rightarrow 3 = 0$	$x \in \emptyset$	false	Derive standards
Quadratic equations			
$2x^2 - 8 = 0$	$x_{1,2} = \pm 2$	$x = 2 \vee x = -2$	OR sign
$10x^2 - 17x = 0$	$x_1 = 0; x_2 = 1,7$	$x = \frac{17}{10} \vee x = 0$	OR sign, improper fraction
$x^2 + 3x + 2 = 0$	$x_1 = -2; x_2 = -1$	$x = -2 \vee x = -1$	OR sign
$2x^2 - 4x - 5 = 0$	$x_{1,2} = 1 \pm \frac{\sqrt{14}}{2}$	$x = 1 - \frac{\sqrt{14}}{2} \vee x = \frac{\sqrt{14}}{2} + 1$	OR sign
$x^2 - 2x + 1 = 0$	$x = 1$	$x = 1$	ok
$6x^2 + 3x + 15 = 0$	$x \in \emptyset$	solve $\rightarrow x = -\frac{1}{4} - \frac{\sqrt{39}i}{4} \vee x = -\frac{1}{4} + \frac{\sqrt{39}i}{4}$ solve(real) \rightarrow false	OR sign, imaginary unit i in the answer

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by DERIVE</i>	<i>Remarks</i>
$(x-2)(x+2) = x^2 + 8x$	$x = -\frac{1}{2}$	$x = -\frac{1}{2}$	ok
Fractional equations			
$\frac{x+1}{x-1} = 0$	$x = -1$	$x = -1$	ok
$\frac{2x^2 - 5x + 3}{10x - 5} = 0$	$x_1 = 1; x_2 = 1,5$	$x = \frac{3}{2} \vee x = 1$	OR sign
$\frac{2}{x+1} = 2$	$x = 0$	$x = 0$	ok
$\frac{y^2}{y^2 - 6y} = \frac{4(3-2y)}{y(6-y)}$	$y = 2$	$y = 2$	ok
$\frac{x}{x-2} - \frac{3}{x} = 1$	$x = 6$	$x = \pm\infty \vee x = 6$	infinity in the answer, OR sign
$\frac{4}{9x^2 - 1} + \frac{1}{3x^2 - x} = \frac{4}{9x^2 - 6x + 1}$	$x_1 = -\frac{1}{9}; x_2 = 1$	$x = \pm\infty \vee x = -\frac{1}{9} \vee x = 1$	infinity in the answer, OR sign
Equations containing absolute value of an expression			
$ x+5 = 3$	$x_1 = -8; x_2 = -2$	solve $\rightarrow x = 3\text{unit_circle}-5$; solve(real) $\rightarrow x = -8 \vee x = -2$	OR sign
$ x+5 = 9+x $	$x = -7$	solve $\rightarrow x = -7 - 2i \vee x = -7 + 2i \vee x = -7$ solve(real) $\rightarrow x = -7$	ok
$ x-5 + 2x+8 = -12$	$x \in \emptyset$	false	ok

Examples	Answers required at school	Answers given by DERIVE	Remarks
$\left x^3 - \sqrt{x+1} - 3 \right = x^3 + \sqrt{x+1} - 7$	$x = 3$	$x = 3$	ok
Irrational equations			
$\sqrt{x+1} = 2$	$x = 3$	$x = 3$	ok
$\sqrt{x^2 - 3x + 4} = 2$	$x_1 = 0; x_2 = 3$	$x = 3 \vee x = 0$	OR sign
$\sqrt{x-2} = \sqrt{2+x}$	$x \in \emptyset$	false	Derive standards
$\sqrt[3]{x+1} + 2\sqrt{x+1} = 3$	$x = 0$	$x = 0$	ok
$(x+4)(x+1) - 3\sqrt{x^2 + 5x + 2} = 6$	$x_1 = -7; x_2 = 2$	$x = -7 \vee x = 2$	OR sign
$\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$	$x_1 = -109; x_2 = 80$	$x = 80 \vee x = -109$	OR sign
$\sqrt[3]{5x+7} - \sqrt[3]{5x-12} = 1$	$x_1 = -3; x_2 = 4$	$x = 4 \vee x = -3$	OR sign
$\sqrt{x-2} - \sqrt{3x-17} = 1$	$x = 6$	$x = 6$	ok
$\sqrt{x+2} - \sqrt{x-2} = \sqrt{3x-2}$	$x = 2$	$x = 2$	ok
Exponential equations			
$4^x = 64$	$x = 3$	$\text{solve} \rightarrow x = 3 + \frac{3 \cdot \pi \cdot i}{LN(2)} \vee x = 3 - \frac{2 \cdot \pi \cdot i}{LN(2)}$ $x = 3 + \frac{2 \cdot \pi \cdot i}{LN(2)} \vee x = 3 - \frac{\pi \cdot i}{LN(2)}$ $x = 3 + \frac{\pi \cdot i}{LN(2)} \vee x = 3;$ nsolve, solve(real) $\rightarrow x = 3$	imaginary unit i in the answer

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by DERIVE</i>	<i>Remarks</i>
$\sqrt{3^x} = 9$	$x = 4$	solve $\rightarrow x = 4 - \frac{4 \cdot \pi \cdot i}{LN(3)}$ v $x = 4 + \frac{4 \cdot \pi \cdot i}{LN(3)}$ v $x = 4$; nsolve, solve(real) $\rightarrow x = 4$	imaginary unit i in the answer
$2^{2-x} = 3^{2-x}$	$x = 2$	solve $\rightarrow x = 2 - \frac{2 \cdot \pi \cdot i}{LN\left(\frac{3}{2}\right)}$ v $x = 2 + \frac{2 \cdot \pi \cdot i}{LN\left(\frac{3}{2}\right)}$ v $x = 2$; nsolve $\rightarrow x = 2$	imaginary unit i in the answer
$4^{x+1} - 3^x = 3^{x+2} - 4^x$	$x = \frac{1}{2 - \log_2 3}$	solve $\rightarrow x = -\frac{\ln(2)}{\ln\left(\frac{3}{4}\right)} - \frac{2 \cdot \pi \cdot i}{LN\left(\frac{3}{4}\right)}$ v $x = -\frac{\ln(2)}{\ln\left(\frac{3}{4}\right)} + \frac{2 \cdot \pi \cdot i}{LN\left(\frac{3}{4}\right)}$ v $x = -\frac{\ln(2)}{\ln\left(\frac{3}{4}\right)}$; nsolve $\rightarrow x = 2.46942$; solve(real) $\rightarrow x = -\frac{\ln(2)}{\ln\left(\frac{3}{4}\right)}$	imaginary unit i in the answer
$3^{x+2} - 3^{x+1} + 3^x = 21$	$x = 1$	solve $\rightarrow x = 1 - \frac{2 \cdot \pi \cdot i}{LN(3)}$ v $x = 1 + \frac{2 \cdot \pi \cdot i}{LN(3)}$ v $x = 1$; solve(real) $\rightarrow x = 1$	imaginary unit i in the answer

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by DERIVE</i>	<i>Remarks</i>
$5^{1-x} + \left(\frac{1}{5}\right)^{x-2} + 25^{-\frac{x}{2}} = 155$	$x = -1$	solve -> $x = -1 - \frac{2 \cdot \pi \cdot i}{LN(5)}$ v $x = -1 + \frac{2 \cdot \pi \cdot i}{LN(3)}$ v $x = -1$; nsolve -> $x = -0.99999$; solve(real)-> $x = -1$	imaginary unit i in the answer
$2^{2x} - 8 \cdot 2^x + 16 = 0$	$x = 2$	solve -> $x = 2 - \frac{2 \cdot \pi \cdot i}{LN(2)}$ v $x = 2 + \frac{2 \cdot \pi \cdot i}{LN(2)}$ v $x = 2$; nsolve -> $x = 2.000000476$; solve(real) -> $x = 2$	imaginary unit i in the answer
$\frac{3^x + 3^{-x}}{3^x - 3^{-x}} = 2$	$x = \frac{1}{2}$	solve-> $x = \frac{1}{2} + \frac{3 \cdot \pi \cdot i}{LN(3)}$ v $x = \frac{1}{2} - \frac{2 \cdot \pi \cdot i}{LN(3)}$ v $x = \frac{1}{2} + \frac{2 \cdot \pi \cdot i}{LN(3)}$ v $x = \frac{1}{2} - \frac{\pi \cdot i}{LN(3)}$ v $x = \frac{1}{2} + \frac{\pi \cdot i}{LN(3)}$ v $x = \frac{1}{2}$; nsolve-> $x = 0.499999$; solve(real)-> $x = \frac{1}{2}$	imaginary unit i in the answer
$2 \cdot 4^x - 5 \cdot 6^x + 3 \cdot 9^x = 0$	$x_1 = -1; x_2 = 0$	solve(real) -> $x = -1$ v $x = 0$; nsolve -> $x = 0$	OR sign
Logarithmic equations			
$\log_2 x = 4$	$x = 16$	$x = 16$	ok
$\log_{2x+1}(2x^2 - 8x + 15) = 2$	$x = 1$	$x = 1$	ok
$2 \log_3(x - 2) + \log_3(x - 4)^2 = 0$	$x = 3$	$x = 3$	OR sign
$\log_{0,2}(4x) + \log_5(x^2 + 75) = 1$	$x_1 = 5; x_2 = 15$	$x = 5$ v $x = 15$	OR sign
$\log^2 x - 3 \log x + 2 = 0$	$x_1 = 10; x_2 = 100$	$x = 100$ v $x = 10$	OR sign

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by DERIVE</i>	<i>Remarks</i>
$\log^2 100x + \log^2 10x + \log x = 14$	$x_1 = \frac{\sqrt{10}}{100000}; x_2 = 10$	$x = \frac{\sqrt{10}}{100000} \vee x = 10$	OR sign
$x^{\log x} = 100x$	$x_1 = 0,1; x_2 = 100$	solve, solve(real) $\rightarrow e^{LN(x)^2 / LN(10)} - 100x = 0$; nsolve $\rightarrow x = 0.1$	partial answer
$x^{\sqrt{x}} = (\sqrt{x})^x$	$x_1 = 1; x_2 = 4$	$x = 4 \vee x = 1 \vee x = 0$	Derive standards
Trigonometric equations			OR sign, Computer algebra program Derive always gives insufficient answer; Derive doesn't take into consideration the periodicity of the trigonometric functions. These calculated certain roots can be obtained from general answer for $n \in [-6, -1, 0, 1, 6]$.
$\sin x = 0$	$x = \pi n, n \in Z$	$x = -\pi \vee x = \pi \vee x = 0$	
$\sin x = \frac{1}{2}$	$x = (-1)^n \frac{\pi}{6} + \pi n, n \in Z$	$x = -\frac{7\pi}{6} \vee x = \frac{5\pi}{6} \vee x = \frac{\pi}{6}$	
$2\sin^2 x - \cos x - 1 = 0$	$x_1 = \pi + 2\pi n, x_2 = \pm \frac{\pi}{3} + 2\pi n, n \in Z$	$x = -2\pi \vee x = 2\pi \vee x = \frac{\pi}{3} \vee x = -3\pi \vee x = -\pi \vee x = \pi$	
$\tan x + 3 \cot x = 4$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 3 + \pi n, n \in Z$	$x = \frac{3\pi}{2} - \operatorname{atan}\left(\frac{1}{3}\right) \vee x = -\operatorname{acot}\left(-\frac{1}{3}\right) \vee x = \operatorname{acot}\left(\frac{1}{3}\right)$ $\vee x = \frac{5\pi}{4} \vee x = -\frac{3\pi}{4} \vee x = \frac{\pi}{4}$	
$\tan x - \sqrt{3} \cot x + 1 = \sqrt{3}$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \frac{\pi}{3} + \pi n, n \in Z$	$x = -\frac{5\pi}{4} \vee x = \frac{3\pi}{4} \vee x = -\frac{\pi}{4} \vee x = \frac{4\pi}{3} \vee$ $x = -\frac{2\pi}{3} \vee x = \frac{\pi}{3}$	

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by DERIVE</i>	<i>Remarks</i>
$2\sin x - 3\cos x = 0$	$x = \arctan \frac{3}{2} + \pi n, n \in \mathbb{Z}$	$x = \operatorname{atan}\left(\frac{1}{5}\right) + \frac{5 \cdot \pi}{4} \vee x = \operatorname{atan}\left(\frac{1}{5}\right) - \frac{3 \cdot \pi}{4} \vee$ $x = \operatorname{atan}\left(\frac{1}{5}\right) + \frac{\pi}{4}$	
$\cos 2x - \sin 2x = 1$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \pi n, n \in \mathbb{Z}$	$x = \frac{3\pi}{4} \vee x = -\frac{\pi}{4} \vee x = 0$	
$\sin^2 x - 6\sin x \cos x + 5\cos^2 x = 0$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 5 + \pi n, n \in \mathbb{Z}$	solve, solve(real) -> $4\cos(x)^2 - 6\sin(x)\cos(x) = -1$; nsolve-> $x = -5.497787$; solutions-> []	
$6\sin^2 x + \sin x \cos x - \cos^2 x = 2$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \arctan \frac{3}{4} + \pi n, n \in \mathbb{Z}$	solve, solve(real) -> $\sin(x)\cos(x) + 7\sin(x)^2 = 3$; nsolve -> $x = -8.781276$; solutions -> []	
$\cos 3x \cdot \cos x = \cos 2x$	$x = \frac{\pi n}{3}, n \in \mathbb{Z}$	$x = -\frac{\pi}{3} \vee x = \frac{\pi}{3} \vee x = -2\pi \vee x = 2\pi \vee x = 0$	
$\sin 2x - \sin x = 0$	$x_1 = \pm \frac{\pi}{3} + 2\pi n, x_2 = \pi n, n \in \mathbb{Z}$	solve, solve(real) -> $\cos(x)\cos(3x) - \cos(2x) = 0$; nsolve -> $x = 0$; solutions -> []	

Appendix 2. Answers given by Maple.

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MAPLE</i>	<i>Remarks</i>
Linear equations			
$5x - 11 = 0$	$x = 2\frac{1}{5}$	$\frac{11}{5}$	improper fraction
$5x = 0$	$x = 0$	0	ok
$4(3x - 1) + 2(2 - 6x) = 0 \rightarrow 0 = 0$	$x \in \mathbb{R} \quad \forall \delta i \quad x \in [-\infty, \infty]$	x	Maple standards
$4(x + 1) = 4x + 1 \rightarrow 3 = 0$	$x \in \emptyset$		no answer
Quadratic equations			
$2x^2 - 8 = 0$	$x_{1,2} = \pm 2$	2,-2	ok
$10x^2 - 17x = 0$	$x_1 = 0; x_2 = 1,7$	solve -> $0, \frac{17}{10}$; fsolve -> 0., 1.70000	improper fraction
$x^2 + 3x + 2 = 0$	$x_1 = -2; x_2 = -1$	-1,-2	ok
$2x^2 - 4x - 5 = 0$	$x_{1,2} = 1 \pm \frac{\sqrt{14}}{2}$	$1 + \frac{\sqrt{14}}{2}, 1 - \frac{\sqrt{14}}{2}$	ok
$x^2 - 2x + 1 = 0$	$x = 1$	1,1	same number twice
$6x^2 + 3x + 15 = 0$	$x \in \emptyset$	solve -> $-\frac{1}{4} + \frac{1}{4}I\sqrt{39}, -\frac{1}{4} - \frac{1}{4}I\sqrt{39}$; fsolve ->	complex roots

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MAPLE</i>	<i>Remarks</i>
$(x-2)(x+2) = x^2 + 8x$	$x = -\frac{1}{2}$	$-\frac{1}{2}$	minus in the numerator
Fractional equations			
$\frac{x+1}{x-1} = 0$	$x = -1$	-1	ok
$\frac{2x^2 - 5x + 3}{10x - 5} = 0$	$x_1 = 1; x_2 = 1,5$	$\frac{3}{2}, 1$	improper fraction
$\frac{2}{x+1} = 2$	$x = 0$	0	ok
$\frac{y^2}{y^2 - 6y} = \frac{4(3-2y)}{y(6-y)}$	$y = 2$	2	ok
$\frac{x}{x-2} - \frac{3}{x} = 1$	$x = 6$	6	ok
$\frac{4}{9x^2 - 1} + \frac{1}{3x^2 - x} = \frac{4}{9x^2 - 6x + 1}$	$x_1 = -\frac{1}{9}; x_2 = 1$	$1, -\frac{1}{9}$	minus in the numerator
Equations containing absolute value of an expression			
$ x+5 = 3$	$x_1 = -8; x_2 = -2$	-2, -8	ok
$ x+5 = 9+x $	$x = -7$	-7	ok
$ x-5 + 2x+8 = -12$	$x \in \emptyset$		no answer

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MAPLE</i>	<i>Remarks</i>
$\left x^3 - \sqrt{x+1} - 3 \right = x^3 + \sqrt{x+1} - 7$	$x = 3$	3	ok
Irrational equations			
$\sqrt{x+1} = 2$	$x = 3$	3	ok
$\sqrt{x^2 - 3x + 4} = 2$	$x_1 = 0; x_2 = 3$	0,3	ok
$\sqrt{x-2} = \sqrt{2+x}$	$x \in \emptyset$		no answer
$\sqrt[3]{x+1} + 2\sqrt[6]{x+1} = 3$	$x = 0$	0	ok
$(x+4)(x+1) - 3\sqrt{x^2 + 5x + 2} = 6$	$x_1 = -7; x_2 = 2$	2, -7	ok
$\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$	$x_1 = -109; x_2 = 80$	80, -109	ok
$\sqrt[3]{5x+7} - \sqrt[3]{5x-12} = 1$	$x_1 = -3; x_2 = 4$	4, -3	ok
$\sqrt{x-2} - \sqrt{3x-17} = 1$	$x = 6$	6	ok
$\sqrt{x+2} - \sqrt{x-2} = \sqrt{3x-2}$	$x = 2$	2	ok
Exponential equations			
$4^x = 64$	$x = 3$	solve -> $\frac{\ln(64)}{\ln(4)}$; fsolve -> 3.0000	answer with natural logarithms
$\sqrt{3^x} = 9$	$x = 4$	solve -> $\frac{\ln(81)}{\ln(3)}$; fsolve -> 4.0000	answer with natural logarithms
$2^{2-x} = 3^{2-x}$	$x = 2$	2	ok

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MAPLE</i>	<i>Remarks</i>
$4^{x+1} - 3^x = 3^{x+2} - 4^x$	$x = \frac{1}{2 - \log_2 3}$	$-\frac{\ln(2)}{\ln\left(\frac{3}{4}\right)}$	ok
$3^{x+2} - 3^{x+1} + 3^x = 21$	$x = 1$	1	ok
$5^{1-x} + \left(\frac{1}{5}\right)^{x-2} + 25^{-\frac{x}{2}} = 155$	$x = -1$	fsolve -> -1; solve -> $RootOf(5^{(1-Z)} + \frac{1}{5}^{-Z-2} + 25^{-\frac{Z}{2}} = 155);$	operation "solve" doesn't give appropriate answer
$2^{2x} - 8 \cdot 2^x + 16 = 0$	$x = 2$	solve -> $\frac{\ln(4)}{\ln(2)}, \frac{\ln(4)}{\ln(2)}$; fsolve -> 2.0000	answer using natural logarithm
$\frac{3^x + 3^{-x}}{3^x - 3^{-x}} = 2$	$x = \frac{1}{2}$	$\frac{1}{2}$	ok
$2 \cdot 4^x - 5 \cdot 6^x + 3 \cdot 9^x = 0$	$x_1 = -1; x_2 = 0$	fsolve -> 0; solve -> $RootOf(2 \cdot 4^{-Z} - 5 \cdot 6^{-Z} + 3 \cdot 9^{-Z});$	operation "solve" doesn't give appropriate answer
Logarithmic equations			
$\log_2 x = 4$	$x = 16$	16	ok
$\log_{2x+1}(2x^2 - 8x + 15) = 2$	$x = 1$	1	ok
$2 \log_3(x - 2) + \log_3(x - 4)^2 = 0$	$x = 3$	3	ok
$\log_{0,2}(4x) + \log_5(x^2 + 75) = 1$	$x_1 = 5; x_2 = 15$	5, 15	ok
$\log^2 x - 3 \log x + 2 = 0$	$x_1 = 10; x_2 = 100$	100, 10	ok
$\log^2 100x + \log^2 10x + \log x = 14$	$x_1 = \frac{\sqrt{10}}{100000}; x_2 = 10$	solve -> powers of e ; fsolve -> 10.0000	partial answer
$x^{\log x} = 100x$	$x_1 = 0,1; x_2 = 100$	solve -> power of e ; fsolve -> 100.0000	partial answer

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MAPLE</i>	<i>Remarks</i>
$x^{\sqrt{x}} = (\sqrt{x})^x$	$x_1 = 1; x_2 = 4$	1,4	ok
Trigonometric equations			Maple always gives insufficient answer; it doesn't take into consideration the periodicity of the trigonometric functions. These calculated certain roots can be obtained from general answer for $n \in [0, 1, 2, 3]$ ($n = 0$ is used more often). Operation "fsolve" gives one answer in radians.
$\sin x = 0$	$x = \pi n, n \in \mathbb{Z}$	0	
$\sin x = \frac{1}{2}$	$x = (-1)^n \frac{\pi}{6} + \pi n, n \in \mathbb{Z}$	$\frac{\pi}{6}$	
$\cos^2 x - 2 \cos x + 1 = 0$	$x = 2\pi n, n \in \mathbb{Z}$	0,0	
$2 \sin^2 x - \cos x - 1 = 0$	$x_1 = \pi + 2\pi n, x_2 = \pm \frac{\pi}{3} + 2\pi n, n \in \mathbb{Z}$	$\pi, \frac{\pi}{3}, -\frac{\pi}{3}$	
$\tan x + 3 \cot x = 4$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 3 + \pi n, n \in \mathbb{Z}$	$\arctan(3), \frac{\pi}{4}$	
$\tan x - \sqrt{3} \cot x + 1 = \sqrt{3}$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \frac{\pi}{3} + \pi n, n \in \mathbb{Z}$	solve -> $\frac{\pi}{3}, -\frac{\pi}{4}$; fsolve -> -2.04944	
$2 \sin x - 3 \cos x = 0$	$x = \arctan \frac{3}{2} + \pi n, n \in \mathbb{Z}$	$\arctan\left(\frac{3}{2}\right)$	
$\cos 2x - \sin 2x = 1$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \pi n, n \in \mathbb{Z}$	solve -> $-\frac{\pi}{4}, 0$; fsolve -> 0	
$\sin^2 x - 6 \sin x \cos x + 5 \cos^2 x = 0$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 5 + \pi n, n \in \mathbb{Z}$	solve -> $\arctan(5), \frac{\pi}{4}$	
$6 \sin^2 x + \sin x \cos x - \cos^2 x = 2$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \arctan \frac{3}{4} + \pi n, n \in \mathbb{Z}$	solve -> $\arctan\left(\frac{3}{4}\right), \arctan\left(\frac{3}{4}\right), -\pi, -\frac{\pi}{4}, \frac{3\pi}{4}$; fsolve -> 0.6435	

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MAPLE</i>	<i>Remarks</i>
$\cos 3x \cdot \cos x = \cos 2x$	$x = \frac{\pi n}{3}, n \in \mathbb{Z}$	solve -> $\pi, 0, \frac{\pi}{3}, \frac{2\pi}{3}$; fsolve -> 0.	
$\sin 2x - \sin x = 0$	$x_1 = \pm \frac{\pi}{3} + 2\pi n, x_2 = \pi n, n \in \mathbb{Z}$	solve -> $\pi, 0, \frac{\pi}{3}, -\frac{\pi}{3}$; fsolve -> 0.	

Appendix 3. Answers given by Mathcad

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MATHCAD</i>	<i>Remarks</i>
Linear equations			
$5x - 11 = 0$	$x = 2\frac{1}{5}$	$\frac{11}{5}$	improper fraction
$5x = 0$	$x = 0$	0	ok
$4(3x - 1) + 2(2 - 6x) = 0 \rightarrow 0 = 0$	$x \in \mathbb{R} \quad \forall i \quad x \in [-\infty, \infty]$	x	Mathcad standards
$4(x + 1) = 4x + 1 \rightarrow 3 = 0$	$x \in \emptyset$	no solution found	ok
Quadratic equations			
$2x^2 - 8 = 0$	$x_{1,2} = \pm 2$	$\begin{pmatrix} 2 \\ -2 \end{pmatrix}$	form of the answer
$10x^2 - 17x = 0$	$x_1 = 0; x_2 = 1,7$	$\begin{pmatrix} 0 \\ \frac{17}{10} \end{pmatrix}$	improper fraction, form of the answer
$x^2 + 3x + 2 = 0$	$x_1 = -2; x_2 = -1$	$\begin{pmatrix} 2 \\ -1 \end{pmatrix}$	form of the answer

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MATHCAD</i>	<i>Remarks</i>
$2x^2 - 4x - 5 = 0$	$x_{1,2} = 1 \pm \frac{\sqrt{14}}{2}$	$\begin{pmatrix} 1 + \frac{1}{2}\sqrt{14} \\ 1 - \frac{1}{2}\sqrt{14} \end{pmatrix}$	form of the answer
$x^2 - 2x + 1 = 0$	$x = 1$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	same number twice, form of the answer
$6x^2 + 3x + 15 = 0$	$x \in \emptyset$	$\begin{pmatrix} -\frac{1}{4} + \frac{1}{4} \cdot i \cdot \sqrt{39} \\ -\frac{1}{4} - \frac{1}{4} \cdot i \cdot \sqrt{39} \end{pmatrix}$	complex roots, form of the answer
$(x-2)(x+2) = x^2 + 8x$	$x = -\frac{1}{2}$	$-\frac{1}{2}$	minus in the numerator
Fractional equations			
$\frac{x+1}{x-1} = 0$	$x = -1$	-1	ok
$\frac{2x^2 - 5x + 3}{10x - 5} = 0$	$x_1 = 1; x_2 = 1,5$	$\begin{pmatrix} 1 \\ \frac{3}{2} \end{pmatrix}$	improper fraction, form of the answer
$\frac{2}{x+1} = 2$	$x = 0$	0	ok
$\frac{y^2}{y^2 - 6y} = \frac{4(3-2y)}{y(6-y)}$	$y = 2$	2	ok

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MATHCAD</i>	<i>Remarks</i>
$\frac{x}{x-2} - \frac{3}{x} = 1$	$x = 6$	6	ok
$\frac{4}{9x^2-1} + \frac{1}{3x^2-x} = \frac{4}{9x^2-6x+1}$	$x_1 = -\frac{1}{9}; x_2 = 1$	$\begin{pmatrix} -1 \\ 9 \\ 1 \end{pmatrix}$	minus in the numerator, form of the answer
Equations containing absolute value of an expression			
$ x+5 = 3$	$x_1 = -8; x_2 = -2$	$\begin{pmatrix} -2 \\ -8 \end{pmatrix}$	form of the answer
$ x+5 = 9+x $	$x = -7$	-7	ok
$ x-5 + 2x+8 = -12$	$x \in \emptyset$	no solution found	ok
$ x^3 - \sqrt{x+1} - 3 = x^3 + \sqrt{x+1} - 7$	$x = 3$	3	ok
Irrational equations			
$\sqrt{x+1} = 2$	$x = 3$	3	ok
$\sqrt{x^2 - 3x + 4} = 2$	$x_1 = 0; x_2 = 3$	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$	form of the answer
$\sqrt{x-2} = \sqrt{2+x}$	$x \in \emptyset$	no solution found	ok
$\sqrt[3]{x+1} + 2\sqrt{x+1} = 3$	$x = 0$	0	ok

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MATHCAD</i>	<i>Remarks</i>
$(x+4)(x+1) - 3\sqrt{x^2+5x+2} = 6$	$x_1 = -7; x_2 = 2$	$\begin{pmatrix} 2 \\ -7 \end{pmatrix}$	form of the answer
$\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$	$x_1 = -109; x_2 = 80$	80	partial answer
$\sqrt[3]{5x+7} - \sqrt[3]{5x-12} = 1$	$x_1 = -3; x_2 = 4$	4	partial answer
$\sqrt{x-2} - \sqrt{3x-17} = 1$	$x = 6$	6	ok
$\sqrt{x+2} - \sqrt{x-2} = \sqrt{3x-2}$	$x = 2$	2	ok
Exponential equations			
$4^x = 64$	$x = 3$	$\frac{\ln(64)}{\ln(4)}$, evaluate -> 3	answer with natural logarithms, required additional evaluation
$\sqrt{3^x} = 9$	$x = 4$	$\frac{\ln(81)}{\ln(3)}$, evaluate -> 4	answer with natural logarithms, required additional evaluation
$2^{2-x} = 3^{2-x}$	$x = 2$	2	ok
$4^{x+1} - 3^x = 3^{x+2} - 4^x$	$x = \frac{1}{2 - \log_2 3}$	$\frac{\ln(2)}{\ln\left(\frac{4}{3}\right)}$, evaluate -> 2.409	answer using natural logarithm, required additional evaluation
$3^{x+2} - 3^{x+1} + 3^x = 21$	$x = 1$	1	ok
$5^{1-x} + \left(\frac{1}{5}\right)^{x-2} + 25^{-\frac{x}{2}} = 155$	$x = -1$	-1.0000-19.5198 <i>i</i>	operation "solve" doesn't give appropriate answer
$2^{2x} - 8 \cdot 2^x + 16 = 0$	$x = 2$	$\frac{\ln(4)}{\ln(2)}$, evaluate -> 2	answer using natural logarithm, required additional evaluation
$\frac{3^x + 3^{-x}}{3^x - 3^{-x}} = 2$	$x = \frac{1}{2}$	$\frac{1}{2}$	ok

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MATHCAD</i>	<i>Remarks</i>
$2 \cdot 4^x - 5 \cdot 6^x + 3 \cdot 9^x = 0$	$x_1 = -1; x_2 = 0$	$x = \text{fsolve}(2 \cdot 4^x - 5 \cdot 6^x + 3 \cdot 9^x, x, \text{complex})$	operation "solve" doesn't give appropriate answer
Logarithmic equations			
$\log_2 x = 4$	$x = 16$	16	ok
$\log_{2x+1}(2x^2 - 8x + 15) = 2$	$x = 1$	1	ok
$2 \log_3(x - 2) + \log_3(x - 4)^2 = 0$	$x = 3$	3	ok
$\log_{0,2}(4x) + \log_5(x^2 + 75) = 1$	$x_1 = 5; x_2 = 15$	4.9999	operation "solve" doesn't give appropriate answer
$\log^2 x - 3 \log x + 2 = 0$	$x_1 = 10; x_2 = 100$	$\begin{pmatrix} 100 \\ 10 \end{pmatrix}$	form of the answer
$\log^2 100x + \log^2 10x + \log x = 14$	$x_1 = \frac{\sqrt{10}}{100000}; x_2 = 10$	$\begin{pmatrix} 3.162 \times 10^{-5} \\ 10 \end{pmatrix}$	form of the answer
$x^{\log x} = 100x$	$x_1 = 0,1; x_2 = 100$	$\begin{pmatrix} 100 \\ 0.1 \end{pmatrix}$	form of the answer
$x^{\sqrt{x}} = (\sqrt{x})^x$	$x_1 = 1; x_2 = 4$	$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$	form of the answer
Trigonometric equations			Mathcad gives insufficient answer; it doesn't take into consideration the periodicity of the trigonometric functions. Calculated roots can be obtained from general answer for $n \in [-1, 0, 1]$ ($n = 0$ is used more often).
$\sin x = 0$	$x = \pi n, n \in \mathbb{Z}$	0	
$\sin x = \frac{1}{2}$	$x = (-1)^n \frac{\pi}{6} + \pi n, n \in \mathbb{Z}$	$\frac{1}{6} \cdot \pi$	
$\cos^2 x - 2 \cos x + 1 = 0$	$x = 2\pi n, n \in \mathbb{Z}$	0	

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MATHCAD</i>	<i>Remarks</i>
$2\sin^2 x - \cos x - 1 = 0$	$x_1 = \pi + 2\pi n, x_2 = \pm \frac{\pi}{3} + 2\pi n, n \in \mathbb{Z}$	$\begin{pmatrix} \pi \\ \frac{1}{3} \cdot \pi \\ -\frac{1}{3} \cdot \pi \end{pmatrix}$	The form of the answer is different from the one used at school.
$\tan x + 3 \cot x = 4$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 3 + \pi n, n \in \mathbb{Z}$	$\begin{pmatrix} \frac{1}{4} \cdot \pi \\ \arctan(3) \end{pmatrix} (**)$	
$\tan x - \sqrt{3} \cot x + 1 = \sqrt{3}$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \frac{\pi}{3} + \pi n, n \in \mathbb{Z}$	$\begin{pmatrix} -\frac{1}{4} \cdot \pi \\ \frac{1}{3} \cdot \pi \end{pmatrix}$	
$2\sin x - 3\cos x = 0$	$x = \arctan \frac{3}{2} + \pi n, n \in \mathbb{Z}$	$\arctan\left(\frac{3}{2}\right) (**)$	
$\cos 2x - \sin 2x = 1$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \pi n, n \in \mathbb{Z}$	$\begin{pmatrix} -\frac{1}{4} \cdot \pi \\ 0 \end{pmatrix}$	
$\sin^2 x - 6\sin x \cos x + 5\cos^2 x = 0$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 5 + \pi n, n \in \mathbb{Z}$	$\begin{pmatrix} \frac{1}{4} \cdot \pi \\ \arctan(5) \end{pmatrix} (**)$	

$6 \sin^2 x + \sin x \cos x - \cos^2 x = 2$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \arctan \frac{3}{4} + \pi n, n \in \mathbb{Z}$	$\left(\begin{array}{c} \arctan\left(\frac{3}{4}\right) \\ \arctan\left(\frac{3}{4}\right) - \pi \\ -\frac{1}{4} \cdot \pi \\ \frac{3}{4} \cdot \pi \end{array} \right) (**)$	
$\cos 3x \cdot \cos x = \cos 2x$	$x = \frac{\pi n}{3}, n \in \mathbb{Z}$	$\left(\begin{array}{c} 0 \\ \pi \\ \frac{1}{3} \cdot \pi \\ -\frac{1}{3} \cdot \pi \end{array} \right)$	
$\sin 2x - \sin x = 0$	$x_1 = \pm \frac{\pi}{3} + 2\pi n, x_2 = \pi n, n \in \mathbb{Z}$	$\left(\begin{array}{c} 0 \\ \frac{2}{3} \cdot \pi \\ \frac{1}{3} \cdot \pi \\ \pi \end{array} \right)$	

(**)Actually program Mathcad in all answers uses label atan instead of label arctan, what makes it a bit difficult to interpret the answer by the students

Appendix 4. Answers given by Mathematica

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MATHEMATICA</i>	<i>Remarks</i>
Linear equations			
$5x - 11 = 0$	$x = 2\frac{1}{5}$	$\{\{x \rightarrow \frac{11}{5}\}\}$	improper fraction
$5x = 0$	$x = 0$	$\{\{x \rightarrow 0\}\}$	ok
$4(3x - 1) + 2(2 - 6x) = 0 \rightarrow 0 = 0$	$x \in \mathbb{R} \quad \forall i \quad x \in [-\infty, \infty]$	$\{\{\}\}$	Mathematica standards
$4(x + 1) = 4x + 1 \rightarrow 3 = 0$	$x \in \emptyset$	$\{\}$	empty set
Quadratic equations			
$2x^2 - 8 = 0$	$x_{1,2} = \pm 2$	$\{\{x \rightarrow -2\}, \{x \rightarrow 2\}\}$	ok
$10x^2 - 17x = 0$	$x_1 = 0; x_2 = 1,7$	$\{\{x \rightarrow 0\}, \{x \rightarrow \frac{17}{10}\}\}$	improper fraction
$x^2 + 3x + 2 = 0$	$x_1 = -2; x_2 = -1$	$\{\{x \rightarrow -2\}, \{x \rightarrow -1\}\}$	ok
$2x^2 - 4x - 5 = 0$	$x_{1,2} = 1 \pm \frac{\sqrt{14}}{2}$	$\left\{ \left\{ x \rightarrow \frac{1}{2}(2 - \sqrt{14}) \right\}, \left\{ x \rightarrow \frac{1}{2}(2 + \sqrt{14}) \right\} \right\}$	ok
$x^2 - 2x + 1 = 0$	$x = 1$	$\{\{x \rightarrow 1\}, \{x \rightarrow 1\}\}$	same number twice
$6x^2 + 3x + 15 = 0$	$x \in \emptyset$	$\left\{ \left\{ x \rightarrow \frac{1}{4}(-1 - i\sqrt{39}) \right\}, \left\{ x \rightarrow \frac{1}{4}(-1 + i\sqrt{39}) \right\} \right\}$	imaginary unit i in the answer

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MATHEMATICA</i>	<i>Remarks</i>
$(x-2)(x+2) = x^2 + 8x$	$x = -\frac{1}{2}$	$\{\{x \rightarrow -\frac{1}{2}\}\}$	ok
Fractional equations			
$\frac{x+1}{x-1} = 0$	$x = -1$	$\{\{x \rightarrow -1\}\}$	ok
$\frac{2x^2 - 5x + 3}{10x - 5} = 0$	$x_1 = 1; x_2 = 1,5$	$\{\{x \rightarrow 1\}, \{x \rightarrow \frac{3}{2}\}\}$	improper fraction
$\frac{2}{x+1} = 2$	$x = 0$	$\{\{x \rightarrow 0\}\}$	ok
$\frac{y^2}{y^2 - 6y} = \frac{4(3-2y)}{y(6-y)}$	$y = 2$	$\{\{y \rightarrow 2\}\}$	ok
$\frac{x}{x-2} - \frac{3}{x} = 1$	$x = 6$	$\{\{x \rightarrow 6\}\}$	ok
$\frac{4}{9x^2 - 1} + \frac{1}{3x^2 - x} = \frac{4}{9x^2 - 6x + 1}$	$x_1 = -\frac{1}{9}; x_2 = 1$	$\{\{x \rightarrow -\frac{1}{9}\}, \{x \rightarrow 1\}\}$	ok
Equations containing absolute value of an expression			
$ x+5 = 3$	$x_1 = -8; x_2 = -2$	$\{\{x \rightarrow -8\}, \{x \rightarrow -2\}\}$	ok
$ x+5 = 9+x $	$x = -7$	$\{\{x \rightarrow -7\}\}$	ok
$ x-5 + 2x+8 = -12$	$x \in \emptyset$	$\{\}$	empty set

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MATHEMATICA</i>	<i>Remarks</i>
$\left x^3 - \sqrt{x+1} - 3 \right = x^3 + \sqrt{x+1} - 7$	$x = 3$	$\{\{x \rightarrow 3\}\}$	ok
Irrational equations			
$\sqrt{x+1} = 2$	$x = 3$	$\{\{x \rightarrow 3\}\}$	ok
$\sqrt{x^2 - 3x + 4} = 2$	$x_1 = 0; x_2 = 3$	$\{\{x \rightarrow 0\}, \{x \rightarrow 3\}\}$	ok
$\sqrt{x-2} = \sqrt{2+x}$	$x \in \emptyset$	$\{\}$	empty set
$\sqrt[3]{x+1} + 2\sqrt{x+1} = 3$	$x = 0$	$\{\{x \rightarrow 0\}\}$	ok
$(x+4)(x+1) - 3\sqrt{x^2 + 5x + 2} = 6$	$x_1 = -7; x_2 = 2$	$\{\{x \rightarrow -7\}, \{x \rightarrow 2\}\}$	ok
$\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$	$x_1 = -109; x_2 = 80$	$\{\{x \rightarrow 80\}\}$	partial answer
$\sqrt[3]{5x+7} - \sqrt[3]{5x-12} = 1$	$x_1 = -3; x_2 = 4$	$\{\{x \rightarrow 4\}\}$	partial answer
$\sqrt{x-2} - \sqrt{3x-17} = 1$	$x = 6$	$\{\{x \rightarrow 6\}\}$	ok
$\sqrt{x+2} - \sqrt{x-2} = \sqrt{3x-2}$	$x = 2$	$\{\{x \rightarrow 2\}\}$	ok
Exponential equations			
$4^x = 64$	$x = 3$	$\{\{x \rightarrow 3\}\}$	ok
$\sqrt{3^x} = 9$	$x = 4$	$\{\{x \rightarrow 4\}\}$	ok
$2^{2-x} = 3^{2-x}$	$x = 2$	$\{\{x \rightarrow 2\}\}$	ok
$4^{x+1} - 3^x = 3^{x+2} - 4^x$	$x = \frac{1}{2 - \log_2 3}$	$\left\{ \left\{ x \rightarrow \frac{\text{Log}[2]}{\text{Log}\left[\frac{4}{3}\right]} \right\} \right\}$	ok

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MATHEMATICA</i>	<i>Remarks</i>
$3^{x+2} - 3^{x+1} + 3^x = 21$	$x = 1$	$\{\{x \rightarrow 1\}\}$	ok
$5^{1-x} + \left(\frac{1}{5}\right)^{x-2} + 25^{-\frac{x}{2}} = 155$	$x = -1$	$\{\{x \rightarrow -1\}\}$	ok
$2^{2x} - 8 \cdot 2^x + 16 = 0$	$x = 2$	$\{\{x \rightarrow 2\}\}$	ok
$\frac{3^x + 3^{-x}}{3^x - 3^{-x}} = 2$	$x = \frac{1}{2}$	$\{\{x \rightarrow \frac{1}{2}\}\}$	ok
$2 \cdot 4^x - 5 \cdot 6^x + 3 \cdot 9^x = 0$	$x_1 = -1; x_2 = 0$	Solve->(#) FindInstance->{{x→0}}	error
Logarithmic equations			
$\log_2 x = 4$	$x = 16$	$\{\{x \rightarrow 16\}\}$	ok
$\log_{2x+1}(2x^2 - 8x + 15) = 2$	$x = 1$	$\{\{x \rightarrow 1\}\}$	ok
$2 \log_3(x - 2) + \log_3(x - 4)^2 = 0$	$x = 3$	Solve -> (#) FindInstance->(∅)	other commands needed
$\log_{0.2}(4x) + \log_5(x^2 + 75) = 1$	$x_1 = 5; x_2 = 15$	Solve -> (#) FindInstance -> (∅)	other commands needed
$\log^2 x - 3 \log x + 2 = 0$	$x_1 = 10; x_2 = 100$	$\{\{x \rightarrow -10\}, \{x \rightarrow 100\}\}$	ok
$\log^2 100x + \log^2 10x + \log x = 14$	$x_1 = \frac{\sqrt{10}}{100000}; x_2 = 10$	Solve -> (#) FindInstance-> complicated expression, containing logarithms and constant e	other commands needed
$x^{\log x} = 100x$	$x_1 = 0,1; x_2 = 100$	Solve-> The expression involves unknowns in more than one argument, so inverse functions cannot be used. FindInstance -> complicated expression, containing logarithms and constant e	other commands needed

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MATHEMATICA</i>	<i>Remarks</i>
$x^{\sqrt{x}} = (\sqrt{x})^x$	$x_1 = 1; x_2 = 4$	$\{\{x \rightarrow 1\}, \{x \rightarrow 4\}\}$	ok
Trigonometric equations			<p>Mathematica gives insufficient answer; it doesn't take into consideration the periodicity of the trigonometric functions. Calculated certain roots can be obtained from general answer for $n \in [-1, 0, 1]$ ($n = 0$ is used more often). The form of the answer is unused to students at school.</p>
$\sin x = 0$	$x = \pi n, n \in \mathbb{Z}$	$(\Delta)\{\{x \rightarrow 0\}\}$	
$\cos^2 x - 2 \cos x + 1 = 0$	$x = 2\pi n, n \in \mathbb{Z}$	$(\Delta)\{\{x \rightarrow 0\}\}$	
$2 \sin^2 x - \cos x - 1 = 0$	$x_1 = \pi + 2\pi n, x_2 = \pm \frac{\pi}{3} + 2\pi n, n \in \mathbb{Z}$	$(\Delta)\{\{x \rightarrow -\pi\}, \{x \rightarrow -\frac{\pi}{3}\}, \{x \rightarrow \frac{\pi}{3}\}, \{x \rightarrow \pi\}\}$	
$\tan x + 3 \cot x = 4$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 3 + \pi n, n \in \mathbb{Z}$	$(\Delta)\{\{x \rightarrow \frac{\pi}{4}\}\}$	
$\tan x - \sqrt{3} \cot x + 1 = \sqrt{3}$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \frac{\pi}{3} + \pi n, n \in \mathbb{Z}$	$(\Delta)\{\{x \rightarrow -\frac{\pi}{4}\}\}$	
$2 \sin x - 3 \cos x = 0$	$x = \arctan \frac{3}{2} + \pi n, n \in \mathbb{Z}$	$(\Delta)\{\{x \rightarrow -\text{ArcCos}\left[-\frac{2}{\sqrt{13}}\right]\}, \{x \rightarrow \text{ArcCos}\left[\frac{2}{\sqrt{13}}\right]\}\}$	
$\cos 2x - \sin 2x = 1$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \pi n, n \in \mathbb{Z}$	$(\Delta)\{\{x \rightarrow 0\}, \{x \rightarrow -\pi\}, \{x \rightarrow -\frac{\pi}{4}\}, \{x \rightarrow \frac{3\pi}{4}\}, \{x \rightarrow \pi\}\}$	

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MATHEMATICA</i>	<i>Remarks</i>
$\sin^2 x - 6 \sin x \cos x + 5 \cos^2 x = 0$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 5 + \pi n, n \in \mathbb{Z}$	$(\Delta) \left\{ \left\{ x \rightarrow -\frac{3\pi}{4} \right\}, \left\{ x \rightarrow \frac{\pi}{4} \right\}, \left\{ x \rightarrow -\text{ArcCos} \left[-\frac{1}{\sqrt{26}} \right] \right\}, \right. \\ \left. \left\{ x \rightarrow \text{ArcCos} \left[\frac{1}{\sqrt{26}} \right] \right\} \right\}$	
$6 \sin^2 x + \sin x \cos x - \cos^2 x = 2$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \arctan \frac{3}{4} + \pi n, n \in \mathbb{Z}$	$(\Delta) \left\{ \left\{ x \rightarrow -\frac{\pi}{4} \right\}, \left\{ x \rightarrow \frac{3\pi}{4} \right\}, \left\{ x \rightarrow -\text{ArcCos} \left[-\frac{4}{5} \right] \right\}, \right. \\ \left. \left\{ x \rightarrow \text{ArcCos} \left[\frac{4}{5} \right] \right\} \right\}$	
$\cos 3x \cdot \cos x = \cos 2x$	$x = \frac{\pi n}{3}, n \in \mathbb{Z}$	$(\Delta) \left\{ \left\{ x \rightarrow 0 \right\}, \left\{ x \rightarrow -\frac{\pi}{3} \right\}, \left\{ x \rightarrow \frac{\pi}{3} \right\} \right\}$	
$\sin 2x - \sin x = 0$	$x_1 = \pm \frac{\pi}{3} + 2\pi n, x_2 = \pi n, n \in \mathbb{Z}$	$(\Delta) \left\{ \left\{ x \rightarrow 0 \right\}, \left\{ x \rightarrow -\pi \right\}, \left\{ x \rightarrow -\frac{2\pi}{3} \right\}, \left\{ x \rightarrow -\frac{\pi}{3} \right\}, \right. \\ \left. \left\{ x \rightarrow \frac{\pi}{3} \right\}, \left\{ x \rightarrow \frac{2\pi}{3} \right\}, \left\{ x \rightarrow \pi \right\} \right\}$	

(#) Warning in Mathematica: “Solve::tdep: The equation appears to involve the variables to be solved for in an essentially non-algebraic way.”

(⊠) Warning in Mathematica when used function FindInstance: “The methods available to FindInstance are insufficient to find the requested instances or prove they do not exist.”

(Δ) In all equations containing trigonometric functions, Mathematica gives warning: „Inverse functions are being used by Solve, so some solutions may not be found.”

Appendix 5. Answers given by MuPAD

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MUPAD</i>	<i>Remarks</i>
Linear equations			
$5x - 11 = 0$	$x = 2\frac{1}{5}$	$\left\{\frac{11}{5}\right\}$	improper fraction
$5x = 0$	$x = 0$	$\{0\}$	ok
$4(3x - 1) + 2(2 - 6x) = 0$ $\rightarrow 0 = 0$	$x \in \mathbb{R} \quad \forall i \quad x \in [-\infty, \infty]$	\mathbb{C}	set of complex numbers
$4(x + 1) = 4x + 1 \rightarrow 3 = 0$	$x \in \emptyset$	\emptyset	ok
Quadratic equations			
$2x^2 - 8 = 0$	$x_{1,2} = \pm 2$	$\{-2, 2\}$	ok
$10x^2 - 17x = 0$	$x_1 = 0; x_2 = 1,7$	$\left\{0, \frac{17}{10}\right\}$	improper fraction
$x^2 + 3x + 2 = 0$	$x_1 = -2; x_2 = -1$	$\{-2, -1\}$	ok
$2x^2 - 4x - 5 = 0$	$x_{1,2} = 1 \pm \frac{\sqrt{14}}{2}$	$\left\{1 - \frac{\sqrt{7} \cdot \sqrt{2}}{2}, \frac{\sqrt{7} \cdot \sqrt{2}}{2} + 1\right\}$	ok
$x^2 - 2x + 1 = 0$	$x = 1$	$\{1\}$	ok
$6x^2 + 3x + 15 = 0$	$x \in \emptyset$	solve-> $\left\{-\left(\frac{1}{4}i\right)\sqrt{39} - \frac{1}{4}, \left(\frac{1}{4}i\right)\sqrt{39} - \frac{1}{4}\right\}$ numeric::realroots -> []	imaginary unit i in the answer

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MUPAD</i>	<i>Remarks</i>
$(x-2)(x+2) = x^2 + 8x$	$x = -\frac{1}{2}$	$\left\{-\frac{1}{2}\right\}$	ok
Fractional equations			
$\frac{x+1}{x-1} = 0$	$x = -1$	$\{-1\}$	ok
$\frac{2x^2 - 5x + 3}{10x - 5} = 0$	$x_1 = 1; x_2 = 1,5$	$\left\{1, \frac{3}{2}\right\}$	improper fraction
$\frac{2}{x+1} = 2$	$x = 0$	$\{0\}$	ok
$\frac{y^2}{y^2 - 6y} = \frac{4(3-2y)}{y(6-y)}$	$y = 2$	$\{2\}$	ok
$\frac{x}{x-2} - \frac{3}{x} = 1$	$x = 6$	$\{6\}$	ok
$\frac{4}{9x^2 - 1} + \frac{1}{3x^2 - x} = \frac{4}{9x^2 - 6x + 1}$	$x_1 = -\frac{1}{9}; x_2 = 1$	$\left\{-\frac{1}{9}, 1\right\}$	ok
Equations containing absolute value of an expression			
$ x+5 = 3$	$x_1 = -8; x_2 = -2$	solve -> $\{3e^{(2i)\pi \cdot X1} - 5 X1 \in [0,1]\}$ numeric::solve-> $\{-2.0\}$ numeric::solve(AllRealRoots)-> $\{-8.0, -2.0\}$	complicated command
$ x+5 = 9+x $	$x = -7$	solve-> $\{i \cdot \text{lambda} - 7 \text{lambda} \in \mathbb{R}\}$ numeric::solve-> \emptyset solved correctly if entered: $\sqrt{(x+5)^2} = \sqrt{(9+x)^2}$	ok, if entered with a help of powers and roots

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MUPAD</i>	<i>Remarks</i>
$ x-5 + 2x+8 =-12$	$x \in \emptyset$	\emptyset	ok
$ x^3 - \sqrt{x+1} - 3 = x^3 + \sqrt{x+1} - 7$	$x = 3$	solve -> error, numeric::solve-> {3.0}	ok
Irrational equations			
$\sqrt{x+1} = 2$	$x = 3$	{3}	ok
$\sqrt{x^2 - 3x + 4} = 2$	$x_1 = 0; x_2 = 3$	{0,3}	ok
$\sqrt{x-2} = \sqrt{2+x}$	$x \in \emptyset$	\emptyset	ok
$\sqrt[3]{x+1} + 2\sqrt[6]{x+1} = 3$	$x = 0$	solve->{0.728} U {D344 ³ -1 D344€ RootOf(X56 ⁴ - 8·X56 ³ +46·X56 ² -72·X56+81, X56)} numeric::solve-> {0.0}	ok
$(x+4)(x+1) - 3\sqrt{x^2+5x+2} = 6$	$x_1 = -7; x_2 = 2$	{-7,2}	ok
$\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$	$x_1 = -109; x_2 = 80$	solve->{80}, numeric::solve (AllRealRoots)->{80.0} (#)	partial answer
$\sqrt[3]{5x+7} - \sqrt[3]{5x-12} = 1$	$x_1 = -3; x_2 = 4$	solve->{4}, numeric::solve (AllRealRoots)->{4.0} (#)	partial answer
$\sqrt{x-2} - \sqrt{3x-17} = 1$	$x = 6$	{6}	ok
$\sqrt{x+2} - \sqrt{x-2} = \sqrt{3x-2}$	$x = 2$	{2}	ok

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MUPAD</i>	<i>Remarks</i>
Exponential equations			
$4^x = 64$	$x = 3$	solve-> $\left\{ \frac{3 \cdot \ln(2) + i\pi k}{\ln(2)} \mid k \in \mathbb{Z} \right\}$ numeric::solve->{3.0}	ok
$\sqrt{3^x} = 9$	$x = 4$	numeric::solve->{4.0}	ok
$2^{2-x} = 3^{2-x}$	$x = 2$	numeric::solve-> {2}	ok
$4^{x+1} - 3^x = 3^{x+2} - 4^x$	$x = \frac{1}{2 - \log_2 3}$	solve->solve(5·4 ^x - 10·3 ^x = 0, x) numeric::solve->{2.40942084} numeric::realroots-> fail	no exact root
$3^{x+2} - 3^{x+1} + 3^x = 21$	$x = 1$	numeric::solve->{1.0}	ok
$5^{1-x} + \left(\frac{1}{5}\right)^{x-2} + 25^{-\frac{x}{2}} = 155$	$x = -1$	numeric::solve->{-1.0}	ok
$2^{2x} - 8 \cdot 2^x + 16 = 0$	$x = 2$	solve-> $\left\{ \frac{2 \cdot \ln(2) + i\pi k}{\ln(2)} \mid k \in \mathbb{Z} \right\}$ numeric::solve->{1.9999999} numeric::solve(AllRealRoots)->{2.0}	ok
$\frac{3^x + 3^{-x}}{3^x - 3^{-x}} = 2$	$x = \frac{1}{2}$	numeric::solve->∅ numeric::solve(AllRealRoots)->{0.5}	ok
$2 \cdot 4^x - 5 \cdot 6^x + 3 \cdot 9^x = 0$	$x_1 = -1; x_2 = 0$	numeric::solve->{0.0} numeric::solve(AllRealRoots)->{-1.0, 0.0} (#)	ok
Logarithmic equations			
$\log_2 x = 4$	$x = 16$	{16}	ok

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MUPAD</i>	<i>Remarks</i>
$\log_{2x+1}(2x^2 - 8x + 15) = 2$	$x = 1$	solve-> solve (log ₁₀ (2x ² - 8x + 15) - 2·log ₁₀ (2x + 1) = 0, x) \ {0} numeric::solve-> ∅ numeric::solve(AllRealRoots)->error	error
$2 \log_3(x - 2) + \log_3(x - 4)^2 = 0$	$x = 3$	numeric::solve-> {3.0}	ok
$\log_{0,2}(4x) + \log_5(x^2 + 75) = 1$	$x_1 = 5; x_2 = 15$	numeric::solve-> {15.0} numeric::solve(AllRealRoots)-> {5.0, 15.0} (#)	ok
$\log^2 x - 3 \log x + 2 = 0$	$x_1 = 10; x_2 = 100$	{10, 100}	ok
$\log^2 100x + \log^2 10x + \log x = 14$	$x_1 = \frac{\sqrt{10}}{100000}; x_2 = 10$	{10, $\frac{\sqrt{10}}{100000}$ }	ok
$x^{\log x} = 100x$	$x_1 = 0,1; x_2 = 100$	numeric::solve-> {100.0} numeric::solve(AllRealRoots)-> error	error
$x^{\sqrt{x}} = (\sqrt{x})^x$	$x_1 = 1; x_2 = 4$	solve-> solve(x ^{√x} - √x ^x = 0, x) numeric::solve-> {0.0} numeric::solve(AllRealRoots)-> {0.0, 1.0, 4.0}	MuPAD standards
Trigonometric equations			some answers are partial
$\sin x = 0$	$x = \pi n, n \in \mathbb{Z}$	$\{\pi k k \in \mathbb{Z}\}$	
$\sin x = \frac{1}{2}$	$x = (-1)^n \frac{\pi}{6} + \pi n, n \in \mathbb{Z}$	$\left\{ \frac{\pi}{6} + 2\pi k k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{6} + 2\pi k k \in \mathbb{Z} \right\}$	
$\cos^2 x - 2 \cos x + 1 = 0$	$x = 2\pi n, n \in \mathbb{Z}$	$\{2\pi k k \in \mathbb{Z}\}$	
$2 \sin^2 x - \cos x - 1 = 0$	$x_1 = \pi + 2\pi n, x_2 = \pm \frac{\pi}{3} + 2\pi n, n \in \mathbb{Z}$	$\left\{ 2\pi k - \frac{\pi}{3} k \in \mathbb{Z} \right\} \cup \left\{ \frac{\pi}{3} + 2\pi k k \in \mathbb{Z} \right\}$ partial answer, x ₁ is missing	

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by MUPAD</i>	<i>Remarks</i>
$\tan x + 3 \cot x = 4$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 3 + \pi n, n \in \mathbb{Z}$	$\left\{ \frac{\pi}{4} + \pi k \mid k \in \mathbb{Z} \right\} \cup \left\{ \arctan(3) + \pi k \mid k \in \mathbb{Z} \right\}$ (#)	
$\tan x - \sqrt{3} \cot x + 1 = \sqrt{3}$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \frac{\pi}{3} + \pi n, n \in \mathbb{Z}$	$\left\{ \frac{\pi}{3} + \pi k \mid k \in \mathbb{Z} \right\} \cup \left\{ \pi k - \frac{\pi}{4} \mid k \in \mathbb{Z} \right\}$	
$2 \sin x - 3 \cos x = 0$	$x = \arctan \frac{3}{2} + \pi n, n \in \mathbb{Z}$	$\left\{ 2 \arctan \left(\frac{3}{\sqrt{13}+2} \right) - \pi + 2\pi k \mid k \in \mathbb{Z} \right\} \cup \left\{ 2 \arctan \left(\frac{\sqrt{13}-2}{3} \right) + 2\pi k \mid k \in \mathbb{Z} \right\}$	
$\cos 2x - \sin 2x = 1$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \pi n, n \in \mathbb{Z}$	$\left\{ \frac{3\pi}{4} + \pi k \mid k \in \mathbb{Z} \right\} \cup \left\{ 2\pi k \mid k \in \mathbb{Z} \right\}$ partial answer, x_2 is not complete	
$\sin^2 x - 6 \sin x \cos x + 5 \cos^2 x = 0$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 5 + \pi n, n \in \mathbb{Z}$	$\left\{ 2 \arctan \left(\frac{5}{\sqrt{26}+1} \right) - \pi + 2\pi k \mid k \in \mathbb{Z} \right\} \cup \left\{ 2 \arctan \left(\frac{\sqrt{26}-1}{5} \right) + 2\pi k \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{\pi}{4} + \pi k \mid k \in \mathbb{Z} \right\}$	
$6 \sin^2 x + \sin x \cos x - \cos^2 x = 2$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \arctan \frac{3}{4} + \pi n, n \in \mathbb{Z}$	$\left\{ -2 \arctan(3) - \pi k \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{3\pi}{4} + \pi k \mid k \in \mathbb{Z} \right\}$	
$\cos 3x \cdot \cos x = \cos 2x$	$x = \frac{\pi n}{3}, n \in \mathbb{Z}$	$\left\{ \frac{\pi k}{3} \mid k \in \mathbb{Z} \right\}$	
$\sin 2x - \sin x = 0$	$x_1 = \pm \frac{\pi}{3} + 2\pi n, x_2 = \pi n, n \in \mathbb{Z}$	$\left\{ \frac{2\pi}{3} + \pi k \mid k \in \mathbb{Z} \right\} \cup \left\{ -\frac{2\pi}{3} - \pi k \mid k \in \mathbb{Z} \right\} \cup \left\{ 2\pi k \mid k \in \mathbb{Z} \right\}$ partial answer, x_2 is not complete	

Appendix 6. Answers given by TI-92 Plus

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by TI-92 Plus</i>	<i>Remarks</i>
Linear equations			
$5x - 11 = 0$	$x = 2\frac{1}{5}$	$x = 11/5$	linear record, improper fraction
$5x = 0$	$x = 0$	$x = 0$	ok
$4(3x - 1) + 2(2 - 6x) = 0 \rightarrow 0 = 0$	$x \in \mathbb{R} \quad \forall \delta i \quad x \in [-\infty, \infty]$	true	TI-92 Plus standards
$4(x + 1) = 4x + 1 \rightarrow 3 = 0$	$x \in \emptyset$	false	TI-92 Plus standards
Quadratic equations			
$2x^2 - 8 = 0$	$x_{1,2} = \pm 2$	$x = 2$ or $x = -2$	ok
$10x^2 - 17x = 0$	$x_1 = 0; x_2 = 1,7$	$x = 17/10$ or $x = 0$	linear record, improper fraction
$x^2 + 3x + 2 = 0$	$x_1 = -2; x_2 = -1$	$x = -1$ or $x = -2$	ok
$2x^2 - 4x - 5 = 0$	$x_{1,2} = 1 \pm \frac{\sqrt{14}}{2}$	$x = \frac{\sqrt{14} + 2}{2}$ or $x = \frac{-(\sqrt{14} - 2)}{2}$	ok
$x^2 - 2x + 1 = 0$	$x = 1$	$x = 1$	ok
$6x^2 + 3x + 15 = 0$	$x \in \emptyset$	false	TI-92 Plus standards

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by TI-92 Plus</i>	<i>Remarks</i>
$(x-2)(x+2) = x^2 + 8x$	$x = -\frac{1}{2}$	$x = -1/2$	linear record
Fractional equations			
$\frac{x+1}{x-1} = 0$	$x = -1$	$x = -1$	ok
$\frac{2x^2 - 5x + 3}{10x - 5} = 0$	$x_1 = 1; x_2 = 1,5$	$x = 3/2$ or $x = 1$	linear record, improper fraction
$\frac{2}{x+1} = 2$	$x = 0$	$x = 0$	ok
$\frac{y^2}{y^2 - 6y} = \frac{4(3-2y)}{y(6-y)}$	$y = 2$	$y = 2$	ok
$\frac{x}{x-2} - \frac{3}{x} = 1$	$x = 6$	$x = 6$	ok
$\frac{4}{9x^2 - 1} + \frac{1}{3x^2 - x} = \frac{4}{9x^2 - 6x + 1}$	$x_1 = -\frac{1}{9}; x_2 = 1$	$x = 1$ or $x = -1/9$	linear record
Equations containing absolute value of an expression			
$ x+5 = 3$	$x_1 = -8; x_2 = -2$	$x = -2$ or $x = -8$	ok
$ x+5 = 9+x $	$x = -7$	$x = -7$	ok
$ x-5 + 2x+8 = -12$	$x \in \emptyset$	false	TI-92 Plus standards

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by TI-92 Plus</i>	<i>Remarks</i>
$\left \left x^3 - \sqrt{x+1} \right - 3 \right = x^3 + \sqrt{x+1} - 7$	$x = 3$	$x = 3$	ok
Irrational equations			
$\sqrt{x+1} = 2$	$x = 3$	$x = 3$	ok
$\sqrt{x^2 - 3x + 4} = 2$	$x_1 = 0; x_2 = 3$	$x = 3$ or $x = 0$	ok
$\sqrt{x-2} = \sqrt{2+x}$	$x \in \emptyset$	false	TI-92 Plus standards
$\sqrt[3]{x+1} + 2\sqrt{x+1} = 3$	$x = 0$	$x = 0$. (*)	warning
$(x+4)(x+1) - 3\sqrt{x^2 + 5x + 2} = 6$	$x_1 = -7; x_2 = 2$	$x = 2$ or $x = -7$	ok
$\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$	$x_1 = -109; x_2 = 80$	$x = 80$. or $x = -109$.	ok
$\sqrt[3]{5x+7} - \sqrt[3]{5x-12} = 1$	$x_1 = -3; x_2 = 4$	$x = 4$. or $x = -3$.	ok
$\sqrt{x-2} - \sqrt{3x-17} = 1$	$x = 6$	$x = 6$	ok
$\sqrt{x+2} - \sqrt{x-2} = \sqrt{3x-2}$	$x = 2$	$x = 2$	ok
Exponential equations			
$4^x = 64$	$x = 3$	$x = 3$	ok
$\sqrt{3^x} = 9$	$x = 4$	$x = 4$	ok
$2^{2-x} = 3^{2-x}$	$x = 2$	$x = 2$	ok
$4^{x+1} - 3^x = 3^{x+2} - 4^x$	$x = \frac{1}{2 - \log_2 3}$	$x = \frac{-\ln(2)}{\ln(3/4)}$	ok
$3^{x+2} - 3^{x+1} + 3^x = 21$	$x = 1$	$x = 1$	ok

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by TI-92 Plus</i>	<i>Remarks</i>
$5^{1-x} + \left(\frac{1}{5}\right)^{x-2} + 25^{-\frac{x}{2}} = 155$	$x = -1$	$x = -1$	ok
$2^{2x} - 8 \cdot 2^x + 16 = 0$	$x = 2$	$x = 2$	ok
$\frac{3^x + 3^{-x}}{3^x - 3^{-x}} = 2$	$x = \frac{1}{2}$	$x = 1/2$	linear record
$2 \cdot 4^x - 5 \cdot 6^x + 3 \cdot 9^x = 0$	$x_1 = -1; x_2 = 0$	$x = 0$ or $x = -1$	ok
Logarithmic equations			
$\log_2 x = 4$	$x = 16$	$x=16$	ok
$\log_{2x+1}(2x^2 - 8x + 15) = 2$	$x = 1$	$x=1$	ok
$2 \log_3(x-2) + \log_3(x-4)^2 = 0$	$x = 3$	$x = 3$	ok
$\log_{0.2}(4x) + \log_5(x^2 + 75) = 1$	$x_1 = 5; x_2 = 15$	$x=15.$ or $x=5.$	ok
$\log^2 x - 3 \log x + 2 = 0$	$x_1 = 10; x_2 = 100$	$x = 100$ or $x = 10$	ok
$\log^2 100x + \log^2 10x + \log x = 14$	$x_1 = \frac{\sqrt{10}}{100000}; x_2 = 10$	$x = 10$ or $x = \frac{\sqrt{10}}{100000}$	ok
$x^{\log x} = 100x$	$x_1 = 0,1; x_2 = 100$	$x = 100.$ or $x = .1$ (*)	decimal fraction, warning
$x^{\sqrt{x}} = (\sqrt{x})^x$	$x_1 = 1; x_2 = 4$	$x = 4$ or $x = 1$	ok
Trigonometric equations			
$\sin x = 0$	$x = \pi n, n \in Z$	$x = n1 \cdot \pi$	

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by TI-92 Plus</i>	<i>Remarks</i>
$\sin x = \frac{1}{2}$	$x = (-1)^n \frac{\pi}{6} + \pi n, n \in \mathbb{Z}$	$x = 2 \cdot n2 \cdot \pi + \frac{5\pi}{6}$ or $x = 2 \cdot n2 \cdot \pi + \frac{\pi}{6}$	<p>Sometimes calculator TI-92 Plus gives exact answer, using variables $n1, n2$ etc.</p> <p>The values of these new variables are not described, nowhere is marked that variables ni must be whole number. In more complicated cases calculator gives some values in radians but also gives warning, that some roots may be lost. These calculated certain roots can be obtained from general answer for $n \in [-32, -25, -21, -14, -10, -9, -8, -6, -3, -2, -1, 0, 1, 2, 3, 6, 9, 11, 12, 14, 22, 26]$.</p>
$\cos^2 x - 2 \cos x + 1 = 0$	$x = 2\pi n, n \in \mathbb{Z}$	1) $x = 2 \cdot n3 \cdot \pi$	
$2 \sin^2 x - \cos x - 1 = 0$	$x_1 = \pi + 2\pi n, x_2 = \pm \frac{\pi}{3} + 2\pi n, n \in \mathbb{Z}$	$x = 86.9174$ or $x = 7.33038$ or $x = 5.23599$ or $x = 3.14159$ or $x = 1.0472$ or $x = -1.0472$ or $x = -3.14159$ or $x = -5.23599$ or $x = -7.33038$ or $x = -86.9174(*)$	
$\tan x + 3 \cot x = 4$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 3 + \pi n, n \in \mathbb{Z}$	1) $x = 38.9482$ or $x = 38.4845$ or $x = 7.53223$ or $x = 7.06858$ or $x = 4.39064$ or $x = 3.92699$ or $x = 1.24905$ or $x = .785398$ or $x = -1.89255$ or $x = -2.35619$ or $x = -5.03414$ or $x = -5.49779$ or $x = -77.2908$ or $x = -77.7544(*)$	
$\tan x - \sqrt{3} \cot x + 1 = \sqrt{3}$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \frac{\pi}{3} + \pi n, n \in \mathbb{Z}$	$x = 68.3296$ or $x = 7.33038$ or $x = 5.49779$ or $x = 4.18879$ or $x = 2.35619$ or $x = 1.0472$ or $x = -.785398$ or $x = -2.0944$ or $x = -3.92659$ or $x = -5.23599$ or $x = -25.9181$ or $x = -64.9262$	
$2 \sin x - 3 \cos x = 0$	$x = \arctan \frac{3}{2} + \pi n, n \in \mathbb{Z}$	$x = n3 \cdot \pi - \tan^{-1}(2/3) + \frac{\pi}{2}$	
$\cos 2x - \sin 2x = 1$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \pi n, n \in \mathbb{Z}$	$x = \frac{(4 \cdot n4 - 1)\pi}{4}$ or $x = n4 \cdot \pi$	

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by TI-92 Plus</i>	<i>Remarks</i>
$\sin^2 x - 6 \sin x \cos x + 5 \cos^2 x = 0$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 5 + \pi n, n \in \mathbb{Z}$	$x = 35.9309$ or $x = 35.3429$ or $x = 7.65659$ or $x = 7.06858$ or $x = 4.51499$ or $x = 3.92699$ or $x = 1.3734$ or $x = .785398$ or $x = -1.76819$ or $x = -2.35619$ or $x = -4.90978$ or $x = -5.49779$ or $x = -77.1664$ or $x = -77.7544(*)$	
$6 \sin^2 x + \sin x \cos x - \cos^2 x = 2$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \arctan \frac{3}{4} + \pi n, n \in \mathbb{Z}$	$x = 80.896$ or $x = 6.92669$ or $x = 5.49779$ or $x = 3.78509$ or $x = 2.35619$ or $x = .643501$ or $x = -.785398$ or $x = -2.49809$ or $x = -3.92699$ or $x = -5.63968$ or $x = -32.2013$ or $x = -99.8875(*)$	
$\cos 3x \cdot \cos x = \cos 2x$	$x = \frac{\pi n}{3}, n \in \mathbb{Z}$	$x = 6.28319$ or $x = 3.14159$ or $x = 2.0944$ or $x = 1.0472$ or $x = 0.$ or $x = -1.0472$ or $x = -2.0944$ or $x = -3.14159$ or $x = -6.28319(*)$	
$\sin 2x - \sin x = 0$	$x_1 = \pm \frac{\pi}{3} + 2\pi n, x_2 = \pi n, n \in \mathbb{Z}$	$x = 42.9351$ or $x = 28.2743$ or $x = 5.23599$ or $x = 3.14159$ or $x = 1.0472$ or $x = 0.$ or $x = -1.0472$ or $x = -3.14159$ or $x = -5.23599$ or $x = -28.2743$ or $x = -42.9351(*)$	

(*) In some cases calculator TI-92 Plus gives warning that some roots may be lost. For trigonometric equations this warning is appropriate but for irrational and logarithmic equations calculator gives all roots and such warning only confuses student.

Appendix 7. Answers given by WIRIS

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by WIRIS</i>	<i>Remarks</i>
Linear equations			
$5x - 11 = 0$	$x = 2\frac{1}{5}$	$\{\{x = \frac{11}{5}\}\}$	improper fraction
$5x = 0$	$x = 0$	$\{\{x = 0\}\}$	ok
$4(3x - 1) + 2(2 - 6x) = 0 \rightarrow 0 = 0$	$x \in \mathbb{R} \quad \forall \text{i} \quad x \in [-\infty, \infty]$	$\{\{x = x\}\}$	WIRIS standards
$4(x + 1) = 4x + 1 \rightarrow 3 = 0$	$x \in \emptyset$	$\{\square\}$	empty set
Quadratic equations			
$2x^2 - 8 = 0$	$x_{1,2} = \pm 2$	$\{\{x = -2\}, \{x = 2\}\}$	ok
$10x^2 - 17x = 0$	$x_1 = 0; x_2 = 1,7$	$\{\{x = 0\}, \{x = \frac{17}{10}\}\}$	improper fraction
$x^2 + 3x + 2 = 0$	$x_1 = -2; x_2 = -1$	$\{\{x = -2\}, \{x = -1\}\}$	ok
$2x^2 - 4x - 5 = 0$	$x_{1,2} = 1 \pm \frac{\sqrt{14}}{2}$	$\left\{ \left\{ x = \frac{\sqrt{14}}{2} + 1 \right\}, \left\{ x = -\frac{\sqrt{14}}{2} + 1 \right\} \right\}$	ok
$x^2 - 2x + 1 = 0$	$x = 1$	$\{\{x = 1\}\}$	ok
$6x^2 + 3x + 15 = 0$	$x \in \emptyset$	$\{\square\} (*)$	empty set

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by WIRIS</i>	<i>Remarks</i>
$(x-2)(x+2) = x^2 + 8x$	$x = -\frac{1}{2}$	$\{\{x = -\frac{1}{2}\}\}$	ok
Fractional equations			
$\frac{x+1}{x-1} = 0$	$x = -1$	$\{\{x = -1\}\}$	ok
$\frac{2x^2 - 5x + 3}{10x - 5} = 0$	$x_1 = 1; x_2 = 1,5$	$\{\{x = 1\}, \{x = \frac{3}{2}\}\}$	improper fraction
$\frac{2}{x+1} = 2$	$x = 0$	$\{\{x = 0\}\}$	ok
$\frac{y^2}{y^2 - 6y} = \frac{4(3-2y)}{y(6-y)}$	$y = 2$	$\{\{y = 2\}\}$	ok
$\frac{x}{x-2} - \frac{3}{x} = 1$	$x = 6$	$\{\{x = 6\}\}$	ok
$\frac{4}{9x^2 - 1} + \frac{1}{3x^2 - x} = \frac{4}{9x^2 - 6x + 1}$	$x_1 = -\frac{1}{9}; x_2 = 1$	$\{\{x = 1\}, \{x = -\frac{1}{9}\}\}$	ok
Equations containing absolute value of an expression – solved only if entered through roots and powers using property $x = \sqrt{(x)^2}$			
$ x+5 = 3$	$x_1 = -8; x_2 = -2$	$\{\{x = -8\}, \{x = -2\}\}$	ok
$ x+5 = 9+x $	$x = -7$	$\{\{x = -7\}\}$	ok
$ x-5 + 2x+8 = -12$	$x \in \emptyset$	$\{\square\}$	empty set

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by WIRIS</i>	<i>Remarks</i>
$\left \left x^3 - \sqrt{x+1} \right - 3 \right = x^3 + \sqrt{x+1} - 7$	$x = 3$	message: "Lack of time"	no answer
Irrational equations			
$\sqrt{x+1} = 2$	$x = 3$	$\{\{x = 3\}\}$	ok
$\sqrt{x^2 - 3x + 4} = 2$	$x_1 = 0; x_2 = 3$	$\{\{x = 0\}, \{x = 3\}\}$	ok
$\sqrt{x-2} = \sqrt{2+x}$	$x \in \emptyset$	$\{\square\}$	empty set
$\sqrt[3]{x+1} + 2\sqrt[6]{x+1} = 3$	$x = 0$	$\{\{x = 0\}\}$	ok
$(x+4)(x+1) - 3\sqrt{x^2 + 5x + 2} = 6$	$x_1 = -7; x_2 = 2$	$\{\{x = -7\}, \{x = 2\}\}$	ok
$\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$	$x_1 = -109; x_2 = 80$	$\{\{\{x = -109\}, \{x = 80\}\}\}$	ok
$\sqrt[3]{5x+7} - \sqrt[3]{5x-12} = 1$	$x_1 = -3; x_2 = 4$	$\{\{x = -3\}, \{x = 4\}\}$	ok
$\sqrt{x-2} - \sqrt{3x-17} = 1$	$x = 6$	$\{\{x = 6\}\}$	ok
$\sqrt{x+2} - \sqrt{x-2} = \sqrt{3x-2}$	$x = 2$	$\{\{x = 2\}\}$	ok
Exponential equations			
$4^x = 64$	$x = 3$	$\{\{x = 3.\}\}$	no answer
$\sqrt{3^x} = 9$	$x = 4$	$\{\{x = 4.\}\}$	no answer
$2^{2-x} = 3^{2-x}$	$x = 2$	$\{\square\}$ (*)	no answer
$4^{x+1} - 3^x = 3^{x+2} - 4^x$	$x = \frac{1}{2 - \log_2 3}$	$\{\square\}$ (*)	no answer
$3^{x+2} - 3^{x+1} + 3^x = 21$	$x = 1$	$\{\square\}$ (*)	no answer

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by WIRIS</i>	<i>Remarks</i>
$5^{1-x} + \left(\frac{1}{5}\right)^{x-2} + 25^{-\frac{x}{2}} = 155$	$x = -1$	{ <input type="checkbox"/> } (*)	no answer
$2^{2x} - 8 \cdot 2^x + 16 = 0$	$x = 2$	{ <input type="checkbox"/> } (*)	no answer
$\frac{3^x + 3^{-x}}{3^x - 3^{-x}} = 2$	$x = \frac{1}{2}$	{ <input type="checkbox"/> } (*)	no answer
$2 \cdot 4^x - 5 \cdot 6^x + 3 \cdot 9^x = 0$	$x_1 = -1; x_2 = 0$	{ <input type="checkbox"/> } (*)	no answer
Logarithmic equations			
$\log_2 x = 4$	$x = 16$	{{x = 16.}}	ok
$\log_{2x+1}(2x^2 - 8x + 15) = 2$	$x = 1$	{ <input type="checkbox"/> } (*)	no answer
$2 \log_3(x - 2) + \log_3(x - 4)^2 = 0$	$x = 3$	{ <input type="checkbox"/> } (*)	no answer
$\log_{0,2}(4x) + \log_5(x^2 + 75) = 1$	$x_1 = 5; x_2 = 15$	{ <input type="checkbox"/> } (*)	no answer
$\log^2 x - 3 \log x + 2 = 0$	$x_1 = 10; x_2 = 100$	{ <input type="checkbox"/> } (*)	no answer
$\log^2 100x + \log^2 10x + \log x = 14$	$x_1 = \frac{\sqrt{10}}{100000}; x_2 = 10$	{ <input type="checkbox"/> } (*)	no answer
$x^{\log x} = 100x$	$x_1 = 0,1; x_2 = 100$	{#}	no answer
$x^{\sqrt{x}} = (\sqrt{x})^x$	$x_1 = 1; x_2 = 4$	{#}	no answer

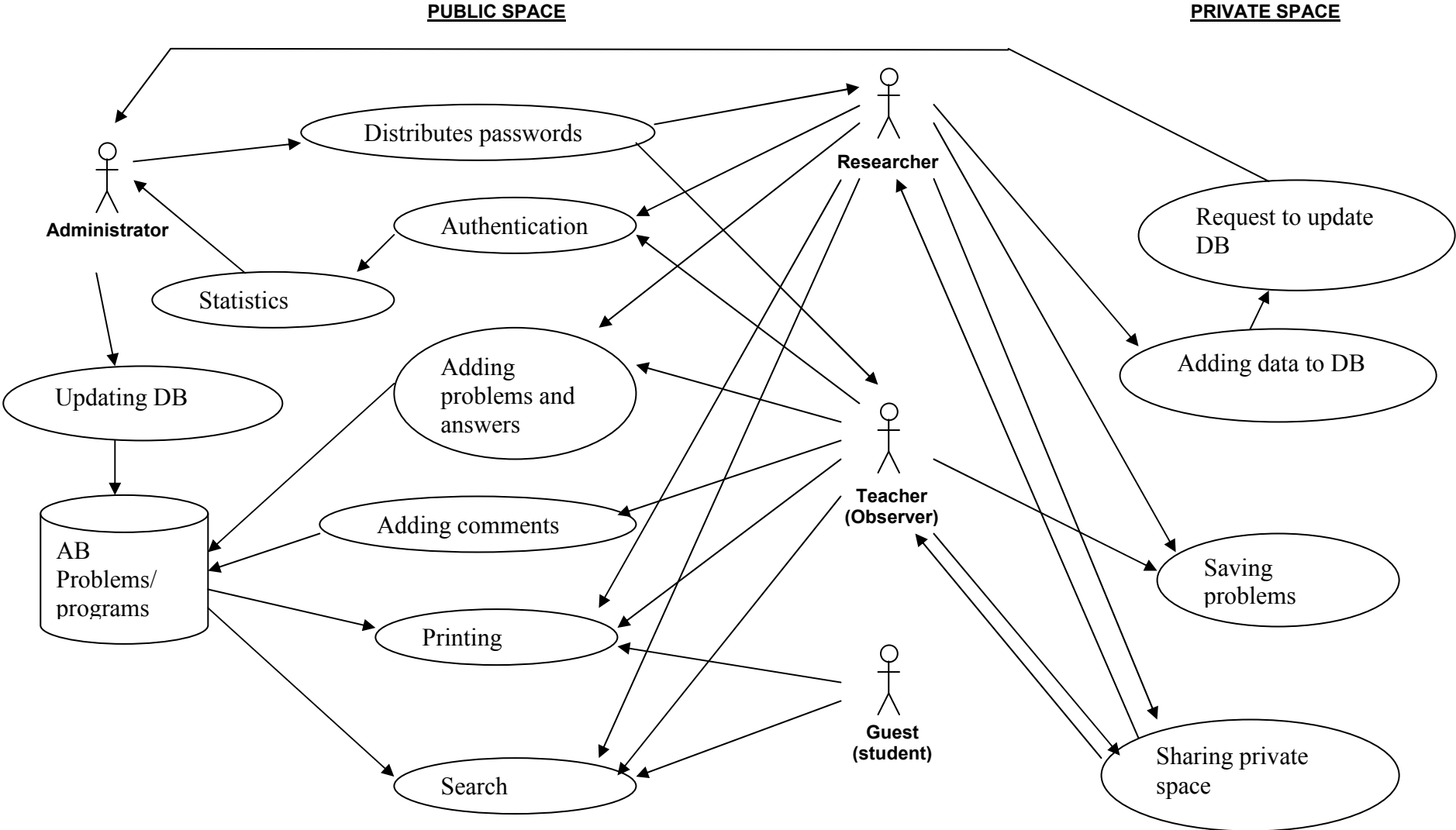
<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by WIRIS</i>	<i>Remarks</i>
Trigonometric equations			<p>WIRIS gives insufficient answer; it doesn't take into consideration the periodicity of the trigonometric functions. Calculated certain roots can be obtained from general answer for $n \in [-1, 0, 1]$. The form of the answer is unused to students at school.</p>
$\sin x = 0$	$x = \pi n, n \in \mathbb{Z}$	$\{\{x = 0\}, \{x = \pi\}\}$	
$\cos^2 x - 2\cos x + 1 = 0$	$x = 2\pi n, n \in \mathbb{Z}$	$\{\{x = 0\}\}$	
$2\sin^2 x - \cos x - 1 = 0$	$x_1 = \pi + 2\pi n, x_2 = \pm \frac{\pi}{3} + 2\pi n, n \in \mathbb{Z}$	$\{\{x = -\pi\}, \{x = \pi\}, \{x = -1.0472\}, \{x = 1.0472\}\}$	
$\tan x + 3\cot x = 4$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 3 + \pi n, n \in \mathbb{Z}$	$\{\{x = \frac{\pi}{4}\}, \{x = \frac{5\pi}{4}\}, \{x = 1.249\}, \{x = 4.3906\}\}$	
$\tan x - \sqrt{3}\cot x + 1 = \sqrt{3}$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \frac{\pi}{3} + \pi n, n \in \mathbb{Z}$	$\{\{x = -\frac{\pi}{4}\}, \{x = \frac{\pi}{3}\}, \{x = \frac{3\pi}{4}\}, \{x = \frac{4\pi}{3}\}\}$	
$2\sin x - 3\cos x = 0$	$x = \arctan \frac{3}{2} + \pi n, n \in \mathbb{Z}$	$\{\{x = -2.1588\}, \{x = 0.98279\}\}$	
$\cos 2x - \sin 2x = 1$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \pi n, n \in \mathbb{Z}$	$\{\square\} (*)$	
$\sin^2 x - 6\sin x \cos x + 5\cos^2 x = 0$	$x_1 = \frac{\pi}{4} + \pi n, x_2 = \arctan 5 + \pi n, n \in \mathbb{Z}$	$\{\{x = \frac{\pi}{4}\}, \{x = -\frac{3\pi}{4}\}, \{x = -1.7682\}, \{x = 1.3734\}\}$	
$6\sin^2 x + \sin x \cos x - \cos^2 x = 2$	$x_1 = -\frac{\pi}{4} + \pi n, x_2 = \arctan \frac{3}{4} + \pi n, n \in \mathbb{Z}$	$\{\{x = -\frac{\pi}{4}\}, \{x = \frac{3\pi}{4}\}, \{x = -2.4981\}, \{x = 0.6435\}\}$	

<i>Examples</i>	<i>Answers required at school</i>	<i>Answers given by WIRIS</i>	
$\cos 3x \cdot \cos x = \cos 2x$	$x = \frac{\pi n}{3}, n \in \mathbb{Z}$	{ } (*)	
$\sin 2x - \sin x = 0$	$x_1 = \pm \frac{\pi}{3} + 2\pi n, x_2 = \pi n, n \in \mathbb{Z}$	{ } (*)	

(*) Warning, difficulty: It is not possible to find a result or solution,

(#)Error, general error: Index out of the object.

Appendix 8. Structure of web info system



Appendix 9. Structure of the database

