

ANU PALU

Algklassiõpilaste matemaatikaalased teadmised,
nende areng ja sellega seonduvad tegurid

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SISUKORD

PUBLIKATSIOONIDE NIMEKIRI	6
SISSEJUHATUS.....	7
Matemaatika õpitulemuste hindamine ülesannete abil.....	7
Matemaatika õppimine ja õpetamine	8
Uurimisülesanded.....	11
EMPIIRILISED UURIMUSED	12
Meetod.....	12
Õpilaste uurimus I.....	12
Õpilaste uurimus II.....	13
Õpetajate uurimus	14
Andmeanalüüs.....	14
Tulemused	15
Õpilaste matemaatikaalased teadmised ja nende areng.....	16
Sagedamini esinenud vigade tüübid tekstülesannete lahendamisel	16
Verbaalsete võimete ja õpimotivatsiooni osa matemaatikaalaste teadmiste arengus.....	17
Õpetajate uskumused matemaatikaõpetuses ja õpetamismeetodite seos õpitulemustega	17
Järeldused ja ettepanekud.....	18
KIRJANDUS.....	21
SUMMARY	25
TÄNUSÕNAD	28
PUBLIKATSIOONID.....	29
CURRICULUM VITAE	93

PUBLIKATSIOONIDE NIMEKIRI

Väitekirja tugineb järgmistele publikatsioonidele, millele tekstis viidatakse rooma numbritega.

- I. **Palu, A., & Kikas, E.** (2007). Primary school teachers' beliefs about teaching mathematics. *Nordic Studies in Mathematics Education*, 12 (1), 5–21.
- II. Kikas, E., Peets, K., **Palu, A., & Afanasjev, J.** (2009). The role of individual and contextual factors in the development of maths skills. *Educational Psychology*, 29 (5), 541–560.
- III. **Palu, A., & Kikas, E.** (2010). The types of the most widespread errors in solving arithmetic word problems and their persistence in time. In A. Toomela (Ed.), *Systemic Person-Oriented Study of Child Development in Early Primary School* (pp.155–172). Frankfurt am Main: Peter Lang Verlag.

Väitekirja autori panus nende artiklite valmimisel oli järgmine:

- **I artikkel:** uurimuse kavandamine, küsimustiku koostamine, andmete kogumine ja analüüsimine ning artikli kirjutamine.
- **II artikkel:** uurimuse kavandamine, õpetajate küsimustiku koostamine, nii õpilaste kui õpetajate andmete kogumine, kirjeldavate analüüside läbiviimine, matemaatika didaktikaga seotud ülevaate ja järelduste tegemine ning arutelu kirjutamine. Mitmetasandilised kasvumudelid aitas koostada Kätlin Peets.
- **III artikkel:** uurimuse kavandamine, matemaatikatestide koostamine, andmete analüüsimine ja artikli kirjutamine.

SISSEJUHATUS

Läbi aegade on matemaatikat peetud keeruliseks nii õpetajate kui ka õpilaste seas. Riigisisene õpitulemuste hindamine näitab, et matemaatika on aine, milles õpilastel on kõige rohkem probleeme. Esimesed tõsisemad matemaatikaga seonduvad õpiraskused tekivad kooli keskastmes, kuid ka algklassides on märkimisväärne hulk selliseid õpilasi, kes ei saavuta riiklikus tasemetöös positiivset tulemust. Matemaatikas tekkivate probleemide ennetamiseks või nendest üle saamise abistamiseks on vaja teada, missugused on õpilaste raskused selle aine omandamisel ja millega on need seotud. Selle saab välja selgitada, uurides õpilaste matemaatikaalaseid teadmisi, nende arengut ja nendega seonduvaid tegureid.

Matemaatikaalaseid teadmisi võib hinnata aine- ehk sisuvaldkonnast või kognitiivsest valdkonnast lähtuvalt. Hinnates õpilasi ülesande lahendamiseks vajalike kognitiivsete tegevuste põhjal, saadakse põhjalikumalt tagasisidet õpilase teadmiste ja oskuste, tugevuste ja nõrkuste kohta. Nende tundmine aitab leida matemaatika õpiraskuste põhjusi. Senised kognitiivses valdkonnas läbi viidud matemaatikaalaste teadmiste uurimused on keskendunud põhiliselt keskmise ja vanema kooliastme õpilaste arengule, näiteks rahvusvahelised uuringud *Program for International Student Assessment* (PISA) ja *Trends in International Mathematics and Science Study* (TIMSS). **Väitekirja eesmärgiks oli hinnata algklassiõpilaste matemaatikaalaseid teadmisi ja nende teadmiste arengut, lähtudes ülesannete lahendamiseks vajalikest kognitiivsetest tegevustest.** Kuna õppeprotsessi mõjutavad nii õpetaja kui ka õpilane, siis oli eesmärgiks **uurida, kuidas õpetaja uskumused ja õpetamismeetodid ning õpilase verbaalsed võimed ja motivatsioon on seotud matemaatika õpitulemustega.** Tulemused on esitatud kolmes artiklis. Järgnevalt on põhjendatud valitud eesmärgid, tutvustatud uurimuste metoodikat ning antud ülevaade olulisematest tulemustest.

Matemaatika õpitulemuste hindamine ülesannete abil

Matemaatikaalaste teadmiste hindamise aluseks on riiklikus õppekavas esitatud õpitulemused. Kaasaegsed koolimatemaatika ainekavad esitavad õpitulemuste kirjeldustes enamasti kaks dimensiooni: sisulise ja kognitiivse ehk tunnetusliku. Sisuline valdkond hõlmab konkreetset temaatikat, kognitiivne valdkond aga toiminguid, mida õpilased peavad valdama. Kognitiivse valdkonna komponendid, mida tuntakse ka pädevuste või kompetentsuse nime all, määratletakse ja liigendatakse eri maade õppekavades väga erinevalt (vt nt *Programs of study*, 2007; *Principles and Standards for ...*, n.d.; *Bildungsstandards*, n.d.). Vaatamata erinevustele on enamike õppekavade üldtunnetuslike pädevuste jaotusel

lähtutud Bloomi taksonoomiast (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956), mille järgi on tunnetustegevused jaotatud kuueks põhikategooriaks: teadmine, mõistmine, rakendamine, analüüs, süntees ja hindamine. Samadel alustel on koostatud ka rahvusvaheline matemaatika ja loodusainete uuring TIMSS, kus on lihtsustamise eesmärgil mõned kategooriad koondatud ühte valdkonda. TIMSS 2003 uuringus jaotati õpilase kognitiivsed tegevused matemaatikaülesannete lahendamisel neljaks: faktide ja protseduuride tundmine, mõistete kasutamine, rutiinsete ülesannete lahendamine ning arutlemine (Mullis et al., 2003). TIMSS 2007 matemaatikatestis on need aga jagatud vaid kolme valdkonda: faktide ja protseduuride teadmine, teadmiste rakendamine ja arutlemine (Mullis et al., 2005).

Eesti riikliku õppekava (Põhikooli ja gümnaasiumi riiklik õppekava, 2002) matemaatika ainekavas ei ole arvestatud õppe-eesmärkide taksonoomiaga. Matemaatika õpitulemused on kooliastmeti määratletud väga üldiselt. Sellest tingitult on seniseid matemaatika tasemetööde ülesandeid koostatud sisu-, mitte kognitiivsest valdkonnast lähtuvalt. Selleks, et teada saada, millised on õpilaste raskused erinevate konkreetsete kognitiivsete oskuste rakendamisel, püstitas väitekirja autor **eesmärgi hinnata õpilaste matemaatikaalaseid teadmisi lähtudes mitte ainult aine sisust, vaid ka kognitiivsetest pädevustest**. Eesmärgiks oli hinnata matemaatika faktide ja protseduuride tundmist, teadmiste rakendamise oskust ja arutlemisoskust.

Tavaliselt hinnatakse riiklike tasemetööde, eksamite ja ka rahvusvaheliste testide abil õpilaste teadmiste hetkeseisu. Uurimuse **eesmärgiks oli jälgida õpilaste matemaatikaalast arengut mitme aasta jooksul (II ja III artikkel)**.

Matemaatika õppimine ja õpetamine

Matemaatikapädevuse all mõistetakse 1) matemaatiliste mõistete ja seoste tundmist; 2) üldist probleemi lahendamise oskust, mis sisaldab oskust probleeme püstitada, sobivaid lahendusstrateegiaid leida ja neid rakendada, lahendusideed analüüsida ning tulemuse tõesust kontrollida; 3) loogilise arutlemise ja põhjendamise oskust (Põhikooli riikliku õppekava eelnõu, 2009). Need pädevused ei teki iseenesest, vaid vajavad süstemaatilist arendamist. Õpilaste matemaatilisele loominguks tuleb hakata alust looma juba algklassides. Uue põhikooli riikliku õppekavaga taotletakse, et esimese kooliastme lõpuks saab õpilane aru õpitud reeglitest ja oskab neid täita; loeb, mõistab ja edastab eakohaseid matemaatilisi tekste; näeb matemaatikat ümbritsevas elus ja kirjeldab seda arvude või geomeetriliste kujundite abil (Põhikooli riikliku õppekava eelnõu, 2009). Nimetatud eesmärkide saavutamiseks ei saa matemaatika õppimine olla ainult valmis tõdede äraõppimine, vaid peab olema õpetaja poolt juhitud protsess, milles õpilane ise aktiivselt osaleb.

Matemaatikaalaste teadmiste omandamine on probleemide lahendamine, mille käigus õpitakse tundma uusi mõisteid ja seoseid, neid eelnevatega

seostama ja süstematiseerima ning siis uutes olukordades rakendama. Efektiivse õppimisega on seotud nii passiivsed kui ka aktiivsed õppimismehhanismid (vt nt Siegler, 2005). Passiivsed aitavad luua seoseid faktide ja strateegiate vahel, aktiivsed aga leida uusi strateegiaid ja konstrueerida uusi teadmisi. Neid kaht liiki õppimismehhanisme toetavad erinevad õpetamismeetodid.

Biheiviorism on suurimat mõju avaldanud traditsioonilistele õpetamismeetoditele. Matemaatika õpetamisel tähendab see rõhuasetust arvutamise- ja teisendamiskuse omandamisele, kus treening ja harjutamine on olulisemad kui arusaamine (Dionne, 1984; Pollard & Triggs, 1997). Lisaks võib matemaatikas eristada formalistlikku õpetamist, kus seatakse esikohale aine käsitlemise rangus (Dionne, 1984). Formalistliku õpetuse puhul peetakse oluliseks definitsioonide sõnasõnalist päheõppimist, täpset terminoloogiat jälgimist, korrektse keele ja sümboolika kasutamist ning kindlate vormistamisreeglite nõudmist. Tähtsustatakse ka sagedast süstemaatilist kontrollimist.

Algklassides on traditsiooniline õpetusviis omal kohal, sest selles vanuses õpetatakse niisuguseid matemaatikaalaseid teadmisi, mis vajavadki pidevat harjutamist ja treenimist (nt liitmine 20 piires ja korrutustabel). Lisaks faktide ja algoritmide tundmisele tuleb seal aga luua alus ka mõistete omandamisele, mida on siiski raske teha formalistlikku õpetamisviisi kasutades, st definitsioonide kaudu. Arusaamise tasemel mõistete õppimine ning probleemide lahendamine vajavad erinevat lähenemist ja teiste meetodite rakendamist, kui seda pakub biheivioristlik õpiteooria. Mõistete kujundamiseks algklassides tuleks kasutada induktiivset teed, kus lähtekohaks on aistingud ja kogemus.

Konstruktivistlik õpiteooria rõhutab õppimist kui protsessi, milles õppija omandab uusi teadmisi neid olemasolevate teadmiste põhjal ise konstrueerides ning kus õpetaja on õpilase toetaja ja suunaja (Pollard & Triggs, 1997; Shuell, 1996). Matemaatikaõpetuses innustatakse ülesannetele leidma erinevaid lahenduskäike ja nende üle ainetunnis arutlema. Tähtsustatakse ka kaaslaste mõju õppimisele. Oluline on koos tegutsemine.

Õpetaja käitumist klassis ja õpetamismeetodite valikut mõjutavad õpetaja uskumused (Thompson, 1992). Seni on uuritud peamiselt keskmise ja vanema astme matemaatikaõpetajate tõekspidamisi (nt Handel, 2003; Lepmann, 1998; 2004; Speer, 2005). Käesoleva väitekirja **eesmärk oli kaardistada ja analüüsida klassiõpetajate uskumusi tulemuslikust matemaatikaõpetusest (I artikkel) ning saada teada, milliseid õpetamismeetodeid õpetajad oma töös tegelikult kasutavad (II artikkel).**

Samuti oli väitekirja eesmärgiks selgitada **õpetamismeetodite mõju matemaatika õpitulemustele algklassides, sealjuures õpetaja tööstaaži mõju laste õpitulemustele matemaatikas (II artikkel).** Varasemad uurimused ei ole tuvastanud kuigi kindlaid seoseid erinevate õpetamismeetodite kasutamise ja õpilaste matemaatikaalaste saavutuste vahel. On täheldatud, et õpetajad, kes eelistavad õpilasekeskset õpetamist (konstruktivism), saavutavad paremaid tulemusi pigem õpilaste kontseptuaalses kui protseduurilises arusaamises (Walker, 1999). Põhiteadmiste ja protseduuride algoritmide omandamine

toimub efektiivsemalt biheivioristlikest alustest lähtuvate meetodite korral (Geary, 1994).

Õppimine sõltub ka õpilase arengutasemest ja võimetest. Kui õppeprotsessi läbiviimisel ei arvestata õpilase arengu eripärasid, pidurdatakse sellega tema õppimist. Efektiivse matemaatikaõpetuse tagamiseks on oluline teada, missugused võimed on seotud erinevate valdkondade õpitulemustega. Mitmetes uurimustes on õpilaste matemaatikatumusi seostatud üldise võimekusega (Hale, Fiorello, Kavanaugh, Hoepfner, & Gaitherer, 2001; Keith, 1999). Samas on leitud, et oluline roll on ka spetsiifilistel kognitiivsetel protsessidel ja võimetal – näiteks aritmeetikaülesannete lahendamine on seotud töömäluga (e.g., Geary, Brown, & Samaranayake, 1991; Wilson & Swanson, 2001) ja tähelepanuga (Fuchs et al., 2005).

Riiklike tasemetööde analüüsid on näidanud, et 3. klassi matemaatika ülesannetest valmistavad õpilastele enim raskusi tekstülesanded (Kaasik, 2004). Eelnevatest uuringutest on selgunud, et tekstülesannete lahendamine on seotud õpilaste keeleliste võimetega ja lugemisoskusega (Fuchs et al., 2005; 2006; Passolunghi & Siegel, 2004; Swanson & Sachse-Lee, 2001). Sellest lähtuvalt oli käesoleva töö üheks eesmärgiks **uurida õpilaste verbaalsete võimete mõju matemaatika õpitulemustele (II ja III artikkel).**

Lisaks sõltuvad õpitulemused õpilaste ootustest, motivatsioonist ning saavutuskäitumisest (Onatsu-Arvilommi, Nurmi, & Aunola, 2002). Kaasajal levinud saavutusmotivatsiooni teooria kohaselt keskendutakse uurimustes eesmärkidele, mille nimel õpilane tegutseb. Vaadeldakse kaht liiki (meisterlikkusele ja sooritusele suunatud) saavutuseesmäärke, mis omakorda jagunevad edule või ebaedu vältimisele suunatuiks (Elliot & McGregor, 2001; Midgley et al., 2000). Meisterlikkusele suunatud eesmärkidega õppija soovib tõsta oma pädevust ning õpitust aru saada. Puuduliku meisterlikkuse vältimisele suunatud õppijad püüavad aga vältida väärarusaamu, õpitava mittemõistmist ja ülesande valesti lahendamist (Elliot & McGregor, 2001; Guan, McBride, & Xiang, 2007). Kui edu saavutamisele suunatud eesmärkidega õppija soovib eksponeerida oma häid oskusi ning tõestada enda võimekust võrreldes teistega, siis ebaedu vältimisele suunatud eesmärkidega õppija tahab vältida negatiivseid hinnanguid ning endast rumala mulje jätmist (Kaplan, Middleton, Urdan, & Midgley, 2002). Uurimused on üheselt tõestanud, et ebaedu vältimisele suunatud saavutuseesmärgid on seotud pealiskaudsete õpistrateegiate kasutamise ja madalate õpitulemustega (Leondari & Gialamas, 2002; McGregor & Elliot, 2002). Saavutuskäitumine on erinevate saavutuseesmärkide puhul erinev. Eriti pärssivalt mõjub tulemustele nn vältiv käitumine – õpilased väldivad keerukamate ülesannete lahendamist, annavad alla ja hakkavad tegelema muuga (Onatsu-Arvilommi et al., 2002).

Saavutuseesmäärke ja -käitumist üheskoos on algklasside puhul vähe uuritud (kuid vt Onatsu-Arvilommi et al., 2002). Väitekirja **eesmärgiks oli niisiis ka tuvastada ja analüüsida õpilaste saavutuseesmäärke ja vältivat käitumist ülesannete lahendamisel ning selle seoseid matemaatikaalaste teadmistega (II artikkel).**

Uurimisülesanded

Eelnevat kokku võttes võib välja tuua väitekirja peamise eesmärgi: teada saada, millised on algklassiõpilaste matemaatikaalased teadmised ja kuidas on matemaatika õpitulemused seotud ühest küljest õpetajate uskumuste ja kasutatavate õpetamismeetoditega ning teisest küljest õpilaste verbaalsete võimete ja motivatsiooniga.

Vastavalt töö eesmärgile püstitati järgmised uurimisülesanded (sulgudes on artiklid, kus on esitatud tulemused):

1. Hinnata Eesti 1.–3. klassi õpilaste matemaatikaalaseid teadmisi ja uurida nende teadmiste arengut (II ja III artikkel). Välja selgitada, millised matemaatikaülesanded valmistavad algklassides enim raskusi (Palu & Kikas, 2007 ja III artikkel).
2. Analüüsida õpilastele enim raskusi valmistanud ülesannete lahendusvigu (III artikkel).
3. Selgitada õpilaste verbaalsete võimete ja motivatsiooni osa matemaatiliste teadmiste arengus (II artikkel).
4. Uurida klassiõpetajate uskumusi tulemuslikust matemaatikaõpetusest ning välja selgitada õpetajate poolt kasutatavate õpetamismeetodite ja õpitulemuste vahelised seosed (I ja II artikkel).

EMPIIRILISED UURIMUSED

Meetod

Selleks, et saada järelduste tegemiseks suure hulga õpilaste ja õpetajate kohta käivaid statistiliselt usaldusväärseid andmeid, valiti meetodiks kvantitatiivne uuring. Uurimuste läbiviimiseks kasutati kahte õpilaste ja ühte õpetajate valimit.

Õpilaste uurimus I

Antud uurimuse valimit, protseduuri ja mõõtevahendeid on täpsemalt kirjeldatud II artiklis. Kasutatud on rahvusvahelise projekti *International Project on Mathematical Attainment* (IPMA) raames väitekirja autori poolt kogutud Eesti laste andmeid. Rahvusvaheline projekt õpilaste matemaatikaalaste teadmiste uurimiseks, mille käivitas 1999. aastal Inglismaal Exeteri Ülikooli juures tegutsev matemaatikaõpetuse innovatsioonikeskus CIMT (*Centre for Innovation in Mathematics Teaching*) oli kavandatud pikaajalise uurimusena (IPMA Coordinators' manual, 1999). Eesti osales IPMA-projektis aastatel 2002–2005, jälgides õpilaste matemaatikaalast edenemist 1. klassi algusest kuni 3. klassi lõpuni. Projekti koordinaator ja läbiviija Eestis oli antud väitekirja autor.

Õpilaste *valimi* moodustasid õpilased 20 erinevast Eesti koolist. Esimese aasta alguses osales projektis 330 ja esimese aasta lõpus 316 õpilast, teise aasta lõpus 330 ja kolmanda aasta lõpus 295 õpilast. Neid õpilasi, kes osalesid kõikides testides, oli 269 (119 poissi ja 150 tüdrukut). Õpilasi testiti kolme õppeaasta jooksul neli korda: esimese klassi alguses ja lõpus ning teise ja kolmanda klassi lõpus.

Mõõtevahendina kasutatud *matemaatikatestid* olid identsed IPMA testidega (IPMA Tests, 1999). Algtest (1. klassi alguses) koosnes kümnest ülesandest, järgmisest testi (1. klassi lõpus) lisati 10 uut ülesannet, 2. klassi testi 20 uut ülesannet ja 3. klassi testi veel 20 uut ülesannet. Iga eelmine test sisaldas järgmises. Viimane test koosnes 60 ülesandest, millega kontrolliti arvutuslikke, andmekäsitlemise ning algebra alusoskusi. Ülesannete sisu analüüs näitas, et testid vastasid Eesti riikliku õppekava matemaatika ainekavale. Sealhulgas olid vaid mõned ülesanded, mis testisid geomeetria teadmisi – ka Eesti I kooliastme ainekavas on geomeetria osa väga tagasihoidlik (õpitulemuste loetelus on kaheksast pädevusest vaid üks geomeetria-alane).

Tunnetuslike valdkondade järgi jagunesid testi ülesanded nii, nagu on soovitatud TIMSS 2007 teoreetilises raamistikus: teadmine, rakendamine ja arutlemine (Mullis et al., 2005). Vaadeldes ülesannete jaotust tunnetusliku valdkonna järgi, selgus, et proportsionaalselt oli fakti- ja protseduurilisi teadmisi nõudvaid ülesandeid ja rakendusülesandeid võrdselt, kuid arutlemisülesandeid nimetatutest vähem.

Analüüsitav testikomplekt oli valideeritud nii algklassiõpilaste matemaatika-teadmiste kui ka tunnetuslike üldoskuste hindamiseks. Testi põhjal järeldusi tehes tuleb siiski arvestada, et tunnetuslike tegevuste järgi ei olnud ülesannete arv tasakaalus.

Verbaalseid võimeid testiti Mairi Männamaa poolt koostatud *Mõistete äratundmise testiga*. Seda kasutatakse õpiraskuste hindamisel ja selle psühholoogilised näitajad on head (vt Kikas, Männamaa, Kumari, & Ulst, 2008; Männamaa, Kikas, & Raidvee, 2008). Test koosneb kuuest mõistatusest – õpilasele esitatakse objekti kirjeldus ning ta peab vastava sõna ära arvama ja kirja panema. *Mõistete äraarvamise test* mõõdab mõtlemisoskust, konkreetse mõiste leidmist etteantud vihjete alusel ja verbaalse informatsiooni integreerimise võimet.

Saavutusmotivatsiooni hindamisel lähtuti õpilase kolmest eesmärgist: meisterlikkuse saavutamine, edu saavutamine ja ebaedu vältimine matemaatikas. Testid koostas Katrin Mägi (vt ka Mägi, Lerkkanen, Poikkeus, Rasku-Puttonen & Kikas, 2010), kes toetus eelnevates uuringutes kasutatud skaaladele (Elliot & McGregor, 2001; Midgley et al., 2000; Skaalvik, 1997). Õpetajad hindasid õpilaste vältivat käitumist ülesande lahendamisel, kasutades Onatsu ja Nurmi skaalat (Onatsu & Nurmi, 1995; vt ka Aunola et al., 2003; Mägi, Häidkind, & Kikas, 2009; Onatsu-Arvilommi & Nurmi, 2000).

Õpilaste uurimus II

Antud uurimuse valimit, protseduuri ja mõõtevahendeid on täpsemalt kirjeldatud III artiklis. Uurimuse andmed koguti 2006.–2009. aastal läbi viidud Eesti põhikoolide uuringu “Eesti põhikooli efektiivsus” käigus. Uuring koosnes neljast etapist, mis viidi läbi vastavalt 2007. aasta kevadel (2. klass), 2007. aasta sügisel (3. klass), 2008. aasta sügisel (4. klass) ja 2009. aasta sügisel (5. klass). Igal aastal uuriti samu lapsi. Käesolevas töös on analüüsitud 2007. ja 2008. aasta sügisel kogutud andmeid.

Õpilaste *valimi* moodustasid 494 õpilast. Valik tehti 938 projektis osalenud õpilase seast, kes sooritasid kaks korda nii matemaatika kui ka eesti keele testi. Õpilasi testiti kahe kuu jooksul 3. ja 4. klassi alguses.

Mõõtevahendina kasutatud *matemaatikatestid* koostas käesoleva töö autor. Ülesannete sisu valikul lähtuti riiklikus õppekavas esitatud matemaatika õpitulemustest (Põhikooli ja gümnaasiumi, 2002). Kolmanda klassi testis oli 20 ülesannet arvude valdkonnast. Neljanda klassi testis oli 20 ülesannet (30 alaülesannet) kolmest ainevaldkonnast: arvud, geomeetria ja suurused. Testide koostamisel arvestati seda, et oleks võimalik kontrollida TIMSS 2007 teoreetilises raamistikus soovitatud kognitiivseid oskusi: faktide ja protseduuride tundmist, rakendamisoskust ning arutlemisoskust (Mullis et al., 2005). Võrreldes eelmises õpilaste uurimuses kasutatud testidega oli nendes testides ülesannete arv erinevate kognitiivsete oskuste kontrollimiseks tasakaalustatud.

Kolm ülesannet 3. klassi testist kordusid 4. klassi testis. Selles uurimuses analüüsiti neist kahe tekstülesande lahendamist.

Lugemise testid koostas Krista Uibu (vt ka Uibu, Kikas, & Tropp, 2010). Test eeldas tekstist arusaamist. Loetu põhjal pidid õpilased otsustama, millised antud kaheksast väitest on tõesed ja millised väärad.

Õpetajate uurimus

Antud uurimuse valimit, protseduuri ja mõõtevahendeid on täpsemalt kirjeldatud I ja II artiklis. *Valimi* moodustasid 103 klassiõpetajat 35 Eesti koolist ning 26 Tartu Ülikooli neljanda ja viienda aasta klassiõpetaja eriala üliõpilast. Õpetajate keskmine tööstaaž oli 19,7 aastat ($SD = 12,8$). Uuritavate hulgas oli ka 20 õpetajat, kelle õpilased osalesid esimeses õpilaste uurimuses.

Kahest osast koosneva *õpetajate küsimustiku* koostas käesoleva töö autor. Esimeses osas oli esitatud seitse matemaatika õpetamise eesmärki, mida paluti õpetajatel hinnata nende olulisuse seisukohalt. Mõned eesmärgid sisaldasid teadmiste omandamist, teised isiksuse arendamist. Loetletud eesmärgid ühtisid riiklikus õppekavas esitatud matemaatikaõpetuse üldiste eesmärkidega. Teises osas kirjeldati 26 aspekti matemaatika õpetamises ning paluti õpetaja hinnangut, kuivõrd tähtsaks nad neid oma töös peavad. Kirjeldati aspekte, mille olulisust rõhutatakse biheivioristliku ja konstruktivistliku õpikäsitluse järgi. Õpetajad pidid andma hinnangu 5-pallisel Likerti skaalal.

Esimese õpilaste uurimusega seotud 20 õpetajal paluti anda ka hinnang selle kohta, kui palju nad kasutavad matemaatika õpetamisel küsimustikus esitatud õpetamise meetodeid.

Andmeanalüüs

Uurimustes saadud andmete analüüsimisel kasutati kirjeldavat statistikat, faktoranalüüsi, mitmetasandilisi kasvumudeleid ja konfiguraalset sagedusanalüüsi.

Õpetajate küsitluse vastuste analüüsimisel kasutati peakomponentide meetodil varimaks pöördega faktoranalüüsi (vt I artikkel). Selle eesmärgiks oli leida antud küsitluse ühisosa omavad tunnused ja moodustada nende põhjal uued ühist laiemat aspekti kirjeldavad faktorid. Faktorite arvu üle otsustati omaväärtuse kriteeriumist ja teooriast lähtuvalt. Analüüsi tulemusena eristusid neli õpetamisviisi: traditsiooniline, formalistlik, sotsiaalne ja individuaalne.

Latentsed kasvumudelid võimaldasid hinnata laste teadmisi neljal ajahetkel ning analüüsida kasvu kaht komponenti – lõpptulemust (ingl *intercept*) ja kiirust (ingl *rate, slope, trend*). Mitmetasandiline modelleerimine oli vajalik, sest longituudse uurimuse (vt II artikkel) õpilased olid grupeeritud klassidesse ning andmed olid seega hierarhilised. Analüüs võimaldas teada saada, kuivõrd olid

teadmiste kasv ja kiirus seotud õpilaste individuaalsete omadustega (algteadmised, verbaalsed võimed, motivatsioon, õpikäitumine) ja kuivõrd klassiga seotud näitajatega (õpetaja staaž, rakendatavad õpetamismeetodid).

Konfiguraalne sagedusanalüüs CFA (vt Bergman, Magnusson, & El-Khoury, 2003; von Eye, 1990) võimaldas variaablikeskselt lähenemiselt edasi minna indiviidikesksele lähenemisele. Uuriti lugemise ja matemaatikatesti alusel moodustatud erinevatesse tasemegruppidesse kuuluvaid õpilasi, kes esitasid teatud tüüpi valesid vastuseid. CFA on χ^2 analüüsi edasiarendus ning on sobiv kategooriaalsete andmete analüüsimiseks ka juhul, kui mõnes grupis on vähe indiviide (või mitte ühtegi). CFA võrdleb oodatud ja mõõdetud (ingl *expected and observed*) sagedusi sagedustabeli igas lahtris. Analüüsi tulemusena eristatakse „tüübid“ (mõõdetud sagedus on oluliselt kõrgem kui oodatud sagedus) ning „antitüübid“ (mõõdetud sagedus on oluliselt madalam kui oodatud sagedus). Selle analüüsi kasutamine (vt III artikkel) võimaldas leida tekstülesannete lahendamisel saadud valede vastuste „tüübid“ (mõõdetud sagedus oli oluliselt suurem kui oodatud sagedus), samuti teada saada, millisesse eesti keele ja matemaatika tasemegruppi kuulusid konkreetsete valede vastuste andjad.

Tulemused

Empiiriliste uurimuste tulemused on koondatud kolme põhiartiklisse. Lisaks valmisid ka konverentside ettekanded, mis avaldati vastavates kogumikes. Tulemuste lühituvustuses on viited järgmistele konverentsiteesidele ja kogumikele:

1. Palu, A., & Kikas, E. (2007). Mathematical tasks causing difficulty for primary school students. In A. Andžans, D. Bonka & G. Lace (Eds.), *Teaching Mathematics: retrospective and perspectives* (pp.204–209). Proceeding of the 8th International Conference May 10–11, 2007. Riga: University of Latvia / Macibu gramata.
2. Palu, A., Afanasjev, J., & Vojevodova, K., (2007). Kolmanda klassi õpilaste matemaatikateadmistest rahvusvahelise uuringu IPMA testide põhjal. E. Abel (Toim), *Koolimatemaatika XXXIV* (lk 35–42): XXXIV Eesti matemaatikaõpetajate päevade ettekannete kogumik. Tartu: Tartu Ülikooli Kirjastus.
3. Afanasjev, J., & Palu, A. (2006). Esimese ja teise klassi õpilaste edenemine matemaatikas. E. Abel & L. Lepmann (Toim), *Koolimatemaatika XXXIII* (lk 35–42): XXXIII Eesti matemaatikaõpetajate päevade ettekannete kogumik. Tartu: Tartu Ülikooli Kirjastus.
4. Afanasjev, J., & Palu, A. (2005). First-Form Pupils' Learning Results and Progress in Mathematics. In A. Andžans, R. Kudžma, A. Monakov & E. Stankus (Eds.), *Teaching Mathematics: retrospective and perspectives* (pp.10–15). Proceeding of the 6th International Conference 13–14 May 2005. Vilnius: Vilnius University.

Õpilaste matemaatikaalased teadmised ja nende areng

IPMA testid (õpilaste esimene uurimus) olid Eesti kooliõpilastele jõukohased ning enamik ülesannetest lahendati väga hästi. Lõpptesti lahendatuse protsent oli 79,8 (Palu, Afanasjev, & Vojevodova, 2007). Selgelt oli näha korduv-ülesannete lahendatuse paranemine, kuid testides oli ka 60 ülesandest kuus sellist, mille lahendatuse protsent aastate jooksul paranes väga vähe või ei paranenud üldse (Afanasjev & Palu, 2006; Palu & Kikas, 2007). Niisugusteks raskusi valmistavateks ülesanneteks olid need, mille lahendamiseks oli vaja kahte liiki kognitiivseid oskusi: mõistete rakendamist ja arutlemist. Kõik halvasti lahendatud ülesanded olid seotud teksti mõistmisega ning nende lahendamiseks oli vaja tunda matemaatilisi mõisteid ja seoseid. Õpilastele ei valmistanud raskusi ülesanded, mis nõudsid fakti- ja protseduurilisi teadmisi. Halvasti lahendatud ülesannete seas ei olnud ühtegi puhtalt arvutamiseiga seotud ülesannet.

Teine õpilaste uurimus (**III artikkel**) kinnitas eelpool toodut. Nimelt selgus, et õpilased lahendavad kõige paremini arvutamises ülesandeid ning halvemini rakendamise- ja arutlemises oskust nõudvaid ülesandeid. Lahendatuse keskmised olid vastavates kognitiivsetes tegevustes järgmised (avaldamata andmed): teadmine 0,89 (SD = 0,17), rakendamine 0,62 (SD = 0,34) ja arutlemine 0,45 (SD = 0,31).

Longituudne uurimus (**II artikkel**) näitas, et matemaatiliste teadmiste areng 1. kuni 3. klassini oli positiivne ning matemaatika tulemused olid 3. klassi lõpus paremad neil õpilastel, kelle areng oli kiirem. Individuaalsel tasandil olid matemaatikatestide tulemused seotud õpilaste matemaatikaalaste eelteadmistega: paremate eelteadmistega õpilaste areng oli kiirem ja nad saavutasid paremaid tulemusi ka 3. klassis.

Sagedamini esinenud vigade tüübid tekstülesannete lahendamisel

Esimene õpilaste uurimus tõendas, et Eesti õpilased olid 3. klassi lõpuks väga hästi omandanud arvutamisoskuse, kuid raskusi valmistasid tekstülesanded (Palu & Kikas, 2007). Selles uurimuses kasutatud ülesannete komplekti (IPMA Tests, 1999) kuulunud tekstülesannete vähesuse tõttu ei olnud võimalik teha üldisemaid järeldusi ja seetõttu kasutati tekstülesannete lahendusraskuste põhjuste välja selgitamiseks teist õpilaste uurimust.

Uurides tekstülesannete valesid vastuseid selgus, et põhiliselt oli neid kaht tüüpi (**III artikkel**). Esiteks oli küllaltki suur hulk õpilasi (eriti 3. klassis), kes andsid vastuse, mis näitas, et nad on lahendanud ülesande vaid osaliselt. See tähendas, et õpilane oli lahendamisel valinud ja sooritanud ühe tehte õigesti. Teise suurema grupi moodustasid õpilased, kes olid lahendanud ülesande, kombineerides selles antud arve. Arvudega kombineerisid rohkem kolmanda klassi madala matemaatika- ja lugemistasemega õpilased.

Verbaalsete võimete ja õpimotivatsiooni osa matemaatikaalaste teadmiste arengus

Uurimus näitas, et õpilase verbaalsed võimed olid positiivselt seotud matemaatika lõpptulemustega 3. klassis (**II artikkel**). Vaadeldes erineva matemaatika ja lugemise tasemega õpilaste tekstülesannete lahendamise oskust selgus aga, et 3. klassis oli oodatust rohkem madala matemaatika- ja kõrge lugemistasemega õpilasi, kes ei osanud tekstülesannet lahendada (**III artikkel**). Ka hea lugemisoskusega õpilased olid raskustes matemaatilise teksti mõistmisega.

Longituudsest tööst (**II artikkel**) ilmnnes, et ebaedu vältimise eesmärgid ning vältiv käitumine olid negatiivselt seotud matemaatikaalaste teadmistega 3. klassis. Nendel õpilastel, kes ei taha näida rumalana ja kardavad saada negatiivset hinnangut, olid 3. klassi lõpuks matemaatikas halvemad õpitulemused. Seevastu aga meisterlikkusele ja sooritusele suunatud saavutuseesmärgid ei avaldanud matemaatika õpitulemustele erilist mõju.

Õpetajate uskumused matemaatikaõpetuses ja õpetamismeetodite seos õpitulemustega

Uurimusest selgus, et klassiõpetajad peavad matemaatikaõpetuses oluliseks nii teadmiste andmist kui ka õpilaste individuaalset arendamist, kuid tähtsustavad esimest siiski veidi rohkem (**I artikkel**).

Faktoranalüüsi tulemusel eristusid neli faktorit: sotsiaalne, individuaalne, traditsiooniline ja formalistlik õpetamine, millest klassiõpetajad hindasid kõige kõrgemalt traditsioonilist. Konstruktivistlikest õpetamisviisidest tähtsustati individuaalset õpetamist rohkem kui sotsiaalset. Erineva tööstaažiga õpetajate arvamuste võrdlusest ilmnnes, et sotsiaalset õpetamist pooldasid rohkem 21- kuni 30-aastase staažiga pedagoogid. Kõige madalamalt aga hinnati formalistlikule õpetamisele omaseid tunnuseid, näiteks definitsioonide ja seaduspärasuste sõnasõnalist päheõppimist ning õpetuse rangelt eesmärgistatud juhtimist õpiku baasil. Formalistliku õpetamisviisi pooldajaid oli kõige rohkem üle 30aastase staažiga õpetajate seas.

Longituudsest õpilaste uurimusest selgus, et õpetajate kogemus ja just formalistlik õpetamisviis olid positiivselt seotud 3. klassi matemaatika õpitulemustega (**II artikkel**). Teiste õpetamismeetodite olulist mõju matemaatika õpitulemustele ei täheldatud. Tulemuse interpreteerimisel tuleb mees pidada, et formalistlikku meetodit kasutati kõige vähem. Seega võiks väita, et 3. klassi lõpuks olid matemaatikaalased teadmised paremad neil õpilastel, kelle õpetaja kasutas traditsioonilise ning konstruktivistliku meetodi kõrval ka formalistlikku. Ilmselt on faktide ja reeglite tundmine vajalik eeldus ülesannete edukaks lahendamiseks.

Järeldused ja ettepanekud

Uurides algklassiõpilaste matemaatikaalaseid teadmisi lähtuvalt kognitiivsest valdkonnast selgus, et Eesti 1.–3. klassi õpilastel on head fakti- ja protseduurilised teadmised, kuid vajaka jääb oskusest teadmisi rakendada ja arutleda. Võib oletada, et õpetajad on pööranud suuremat tähelepanu ülesannete lahendamise üksikoskuste, mitte üldoskuste arendamisele. Üksikoskused saavutatakse harjutusülesannete lahendamisel, mille eesmärk ongi lahendusalgorithmi treenimine. Üldoskuste alla aga kuuluvad üldised teadmised ülesande lahendamise käigu kohta ja need on vajalikud selleks, et õpilased suudaksid lahendada ülesandeid teadlikult ja sihikindlalt, mitte ainult matkimise ja analoogia põhjal. Mõtlemise arendamise seisukohalt on oluline õpilastel igasuguste matemaatikaülesannete lahendamise üldoskuste arendamine. Peamine on kujundada niisugune üldkäsitus ülesannete lahendamiseks, mille puhul ülesannet vaadeldakse kui analüüsimis- ja uurimisobjekti, ülesande lahendamist aga kui avastamist. Selline käsitus ei nõua tohutu hulga ülesannete lahendamist, vaid valitud ülesannete kiirustamata, tähelepanelikku ja asjalikku lahendamist.

Uurimus näitas, et klassiõpetajad pööravad suuremat tähelepanu õpetamismeetoditele, mis toetavad harjutamist ja treeningut, ning vähem nendele, mis tagavad õpitust arusaamise. Selline rõhuasetus võib olla tingitud asjaolust, et klassiõpetajad peavad väga oluliseks arvutamisoskuse omandamist, mille saavutamiseks on vajalik treening ja harjutamine. Rühmatöö ja projektide kasutamine, mis on konstruktivistlikule õpikäsitlusele omased õpetamismeetodid, ei leidnud klassiõpetajate suurt poolehoidu. Algoritmide ja tehnika drillimist on soodustanud ka aastast aastasse ühesugused tasemetööde ülesanded, mis on koostatud enamasti aine sisust, mitte kognitiivsete oskuste kontrollimisest lähtuvalt. Hilisemate õpiraskuste vältimiseks ei tohiks algklassides piirduda ainult algoritmide treeninguga, vaid tuleks lahendada ka probleemülesandeid. Oskuslikult valitud ülesannetega on võimalik juba varakult teha eeltööd üldistamisoskuse arendamiseks.

Õpilaste areng matemaatikas on seotud õpilaste eelteadmistega. Uurimusest ilmnes, et enne kooli omandatud teadmised olid positiivselt seotud hilisemate õpitulemustega: õpilased, kellel olid suuremad eelteadmised, olid kolmanda klassi lõpus teistest edukamad. Selgus ka, et oli küllaltki suur hulk selliseid nõrkade eelteadmistega õpilasi, kes juba esimese õppeaasta lõpuks liikusid tugevamasse rühma (Afanasjev & Palu, 2005). Esimesel õppeaastal toimus ka vastupidist liikumist: tugevast rühmast madalamasse. Võib järeldada, et sellised liikumised seavad kahtluse alla õpilaste edasise matemaatikaedukuse adekvaatse prognoosimise võimalused juba enne kooli astumist.

Uurimus näitas ka, et osa õpilasi omandas teadmised hilineumisega. Õppimise protsessi võib jagada kaheks etapiks. Esimesel etapil õpitakse uusi fakte, teises etapis toimub uute teadmiste integreerimine olemasolevasse süsteemi (Kikas, 2005). Üksikfaktide õppimiseks ja lihtsamate tegevuste meeldejätmiseks on võimalised enamik lastest. Kui aga õppimise teiseks etapiks (õpitava mõtesta-

miseks, aruteludeks) ei ole piisavalt aega, võib õppimine ebaõnnestuda. Käesolev uurimus tõendas, et osa õpilastest vajab rohkesti aega ka lihtsamate toimingute omandamiseks (õppimise esimene etapp). Ülesanded, mida need õpilased ei osanud lahendada 1. klassi lõpus, sooritati edukalt 2. klassi lõpus. Varasemate uuringute põhjal võib väita, et sellised õpilased kas jõuavad vanemates klassides teadmiste arenguga teistele õpilastele järele või omandavad vastavad teadmised pidevalt teistest üks kuni kaks aastat hiljem (Crown, 1990; Geary, 1994). See fakt rõhutab õppetöö individualiseerimise tähtsust ja parandusõppe vajadust.

Matemaatika õpitulemused on seotud õpilaste individuaalsete võimetega. Lähema vaatluse alla oli võetud seos verbaalsete võimetega. Selgus, et need mõjutavad positiivselt nii tekstülesannete lahendamisoskust kui ka üldist matemaatika õpitulemust. Samas ilmnes uurimusest, et ka hea lugemisoskusega õpilased võivad olla raskustes matemaatilise teksti mõistmisega. Õpilased, kes ei tajunud tekstis olevaid matemaatilisi seoseid, lahendasid ülesande vaid osaliselt või kombineerisid suvaliselt tekstis olevate arvudega. Nimetatud strateegia kasutamine tekstülesannete lahendamisel näitab, et õpilased piirduvad vaid nähtavaga – kasutavad arve, mis on tekstis kirjas ja asuvad ülesannet lahendama probleemi piisavalt analüüsimata. Osalisel lahendamisel olid õpilased valinud ja sooritanud ühe tehte õigesti. Ei saa väita, et nad jätsid ülesande pooleli seetõttu, et ei mõistnud kõiki ülesandes olevaid seoseid. Võib arvata, et need õpilased ei olnud suutelised korraga haarama kogu ülesande struktuuri. Taoliste matemaatiliste seoste nägemist mõjutab matemaatiline võimekus: võimekamad õpilased tajuvad mitmetehtelise ülesande struktuuri paremini (Krutetskii, 1976).

Tekstülesannete uurimuse tulemused kinnitavad, kui oluline on pühendada rohkem aega teksti analüüsile. Õpetajad peavad olema teadlikud, et vale vastus võib tuleneda lugemisel ja arusaamisel tehtud veast. Selleks, et ülesandes sisalduvaid matemaatilisi seoseid paremini mõista, tuleks algklassides kasutada ülesande kujundlikku esitlemist. Ülesande lahendusidee otsimisel peaksid õpetajad suunama õpilasi kasutama nii analüüsi kui ka sünteesi. Terviku nägemiseks on vajalik analüüs, milles liigutakse küsimuselt andmete poole: 1) mida ülesandes küsitakse; 2) mida peab teadma, et sellele küsimusele vastata; 3) kas me teame seda; 4) kuidas puuduvat leida ja kas meil on selleks andmeid. Kui õpetaja kasutab arutlemiseks vaid sünteesi (andmetest küsimuse poole), ei näe vähem võimekad õpilased kogu ülesande struktuuri ja püüavad olemasolevate arvudega kombineerida või lahendada ülesande vaid osaliselt.

Algklassiõpetajal on väga tähtis roll õpimotivatsiooni kujundamisel. Oluline on juba varakult uurida motivatsioonilist seost õpitulemustega ja õigeaegselt märgata ning vajadusel sekkuda õppeedukuse probleemidesse. Eelnevalt on teada, et õpilaste käitumist ja õppeedukust mõjutavad saavutuseesmärgid. Käesolev uurimus näitas, et meisterlikkusele ja sooritusele suunatud saavutuseesmärgid ei avaldanud erilist mõju matemaatika õpitulemustele algklassides. Küll aga ilmnes, et ebaedu vältimise saavutuseesmärgid on seotud madalamate

õpitulemustega selles õppeaines. Ebaedu vältijad kardavad eksida, mistõttu jätaavad sageli ülesande lahendamata või ei osale ühisaruteludes. Hirm vigade ees ei tohiks aga pärssida loomingulist lähenemist selles protsessis. Õpetajad peaksid julgustama õpilasi oma mõtteid ja tegevusi põhjendama ning erinevaid meetodeid või lahendusi otsima. Koostöös kaaslaste ja õpetajaga peaks õpilane saama täiendavat, julgustavat ja konstruktiivset tagasisidet oma tugevuste ja nõrkuste kohta. Eesmärgiks peaks olema huvi äratamine ja positiivse suhtumise loomine matemaatikaga tegelemisse.

Piirangud. Väitekirja uurimustel on ka mõningaid piiranguid. Tekstülesannete uurimuses analüüsiti lähemalt vaid kaht ülesannet. Järgnevates uuringutes peaks kasutama suuremat ülesannete hulka ja viima lisaks kirjalikule testile läbi ka intervjuusid õpilastega, et selgitada välja nende sügavamad arusaamist matemaatilistest seostest ja lahendusstrateegiatest. Matemaatika tulemustega seotud tegureid uuriti selles väitekirjas piiratud arvul, kuid mõjutajaid on rohkem. Lähemalt võiks uurida õpilase tähelepanuvõime mõju matemaatika õpitulemustele. Õpetajate uurimuses hinnati õpetamismeetodite kasutamist vaid õpetajapoolse enesehinnangu põhjal. Kuna selline hinnang võib olla subjektiivne, peaks edaspidi kasutama sarnaste uurimuste korral intervjuud ja tunnivaatlusi.

Kokkuvõtteks. Vaatamata piirangutele on tulemused siiski olulised nii teoreetilisest kui ka rakenduslikust aspektist. Need kinnitavad, et hinnates õpilaste matemaatikaalaseid teadmisi ja analüüsides ülesannete lahendamisel tehtud vigu on võimalik välja selgitada raskusi, millega õpilased matemaatika omandamisel kokku puutuvad.

Tulemused näitavad, et algklasside matemaatikaõpetuses on vaja muuta rõhuasetusi. Kuna väitekirja valmis paralleelselt uue riikliku õppekava koostamisega, oli väitekirja autoril võimalus tutvustada uurimuste tulemusi ja anda soovitusi matemaatika ainekava koostajatele. Uus õppekava rõhutabki matemaatikaülesannete lahendamise üldoskuste arendamist – õpitulemustesse on lisatud probleemülesande lahendamise üldise skeemi tundmine (Põhikooli riiklik õppekava, 2010). Uues õppekavas seatakse esikohale matemaatikast arusaamine, mitte faktide ja protseduuride tundmine. Matemaatikaõpetuse eesmärgid on orienteeritud õpilase tegevuse mõtestamisele ja toimetulekule erinevates situatsioonides, mitte suure hulga teadmiste ja oskuste omandamisele tüüpolekordades.

Selleks, et muutuks õpetamine, peab muutuma ka õpetajate arusaam matemaatika õpetamisest, mistõttu on vajalik välja töötada vastavad algklasside matemaatikaõpetuse täienduskoolituse programmid.

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SUMMARY

Mathematical knowledge of primary school pupils, its development and related factors

The objective of the dissertation at hand was to assess the mathematical knowledge of primary school pupils and its development not only in terms of the content of the subject, but cognitive competences as well. Normally, national achievement tests, examinations, as well as international tests are used to assess pupils' current knowledge; this study, however, followed pupils' development in mathematics over the course of several years. In addition, the influence of teaching methods as well as pupils' verbal abilities and motivation on maths learning results were studied.

The tasks of the study were as follows:

1. To study Estonian pupils' maths knowledge and its development in primary grades. To establish which maths problems pose the most difficulties.
2. To analyse the errors the pupils make in solving these problems.
3. To establish the role of pupils' verbal abilities and motivation in maths knowledge development.
4. To study class teachers' beliefs about productive maths teaching and the influence of the teaching methods employed by the teachers on learning outcomes.

Method

The study was conducted using two samples of pupils and one sample of teachers.

The first sample of pupils comprised the Estonian pupils who had participated in the IPMA project (International Project on Mathematical Attainment) (**Article II**). The pupils were tested four times over three years: at the beginning and end of Grade 1 and at the end of Grade 2 and Grade 3. The number of pupils who participated in all the tests amounted to 269 (119 boys and 150 girls). The *maths tests* used were identical to those of the IPMA tests (IPMA Tests, 1999). *Verbal abilities* were tested by an abridged version of the *Word Guessing* test. The test was developed by Mairi Männamaa; it is used to assess learning difficulties, and its psychometric indicators are good (see Kikas, Männamaa, Kumari, & Ulst, 2008; Männamaa, Kikas, & Raidvee, 2008). *Achievement motivation* was assessed based on three goals set by the pupil: mastery, performance-approach, and performance-avoidance orientation goals in maths. The tests have been prepared by Katrin Mägi (see Mägi, Lerkkanen, Poikkeus, Rasku-Puttonen, & Kikas, 2010), who relied on the scales used in previous studies (Elliot & McGregor, 2001; Midgley et al., 2000; Skaalvik, 1997). The teachers assessed the pupils' avoidance behaviour in problem

solving using the Onatsu and Nurmi scale (see Aunola et al., 2003; Mägi, Häidkind, & Kikas, 2009; Onatsu-Arvilommi & Nurmi, 2000).

The second sample of pupils comprised the pupils who had participated in the Ministry of Education and Research study “The efficiency of Estonian basic school” (**Article III**). The sample size was 494 pupils. They were selected from among 938 pupils who had participated in the project and who twice took the test in both mathematics and the Estonian language. The pupils were tested during a two-month period at the beginning of Grade 3 and 4. The *maths tests* used were prepared by the author of this dissertation. The tests were prepared with a view to enabling the testing of the cognitive skills recommended in the TIMSS 2007 framework: factual and procedural knowledge, application skills and reasoning skills (Mullis et al., 2005). The *reading tests* were prepared by Krista Uibu (see Uibu, Kikas, & Tropp, 2010). In this study, the functional reading part of these tests was used.

The study conducted among teachers involved 103 class teachers from 35 Estonian schools and 26 fourth- and fifth-year students training to be class teachers at the University of Tartu (**Articles I and II**). All those teachers whose pupils participated in the first pupils’ study (20 teachers) were also included in the study. The *teachers’ questionnaire* was prepared by the author of this paper. The questionnaire contained descriptions of 26 aspects of maths teaching and the teachers were asked to assess these aspects in terms of the importance they attach to them. The teachers whose pupils participated in the study were also requested to give an assessment of how much they used the teaching methods described in the questionnaire in maths teaching.

Results

1. The development of maths knowledge from Grade 1 to Grade 3 was positive (**Article II**). By the end of Grade 3, the pupils whose development had been faster scored better results in maths. The results of the maths tests correlated with the pupils’ preliminary knowledge of maths: the pupils with better preliminary knowledge developed faster and achieved better results. Preliminary knowledge correlated most strongly with the results of the third graders.
2. The pupils in Grades 1–3 in Estonia have good factual and procedural knowledge, but insufficient skills in applying the knowledge and in reasoning. Difficulties are experienced in solving word problems (**Article III**). This may be due to the fact that class teachers put more emphasis on the teaching methods that encourage rote learning, and less on those that encourage the actual understanding of the subject.
3. Two types of wrong answers were mainly given in the solutions of the word problems (**Article III**). First, there was a fairly large number of pupils (particularly in Grade 3) who gave answers that revealed that they had

solved the problem only partially. A second group of wrong answers included those which had been obtained by combining the numbers found in the text. Especially in Grade 3, pupils with low maths and reading skills did not perform the operations in compliance with the relationships given, but combined the numbers.

4. Class teachers preferred traditional teaching methods where rote learning and testing take precedence (**Article I**). This may be due to the fact that class teachers place high importance on the acquisition of calculation skills, which require drill and practice. Group work and projects which are intrinsic to the constructivist approach to teaching, did not find much favour with the class teachers.
5. The students whose teachers had longer work experience and made more use of formalist teaching methods, achieved better results in maths by the end of Grade 3. However, it should be stressed that teachers generally used the formalist methods the least. Respectively, the result may show that different methods are needed for successful development of pupils.
6. The pupils' verbal abilities had a positive effect on both the skills of solving word problems and the overall maths learning results (**Article II**). At the same time, the study also indicated that even pupils with high reading skills may experience difficulties in understanding mathematical texts. (**Article III**).
7. The pupils' performance-avoidance orientation and avoidance behaviour correlated negatively with maths knowledge in Grade 3 (**Article II**). The mastery and performance-approach orientations had no particular effect on the maths learning outcomes.

TÄNUSÕNAD

Tänan oma juhendajaid professor Eve Kikast ja dotsent Jüri Afanasjevit asjalike nõuannete ja suuniste eest. Eriline tänu professor Eve Kikasele, kelle abile toetudes said ületatud artiklite kirjutamisel ette tulnud raskused.

Tänan professor Aaro Toomelat võimaluse eest osaleda tema juhitud projektis. Uuringus „Eesti põhikooli efektiivsus“ osalemine andis mulle ideid ja mõtteid oma varasemate uurimuste jätkamiseks, võimaldades ühtlasi läbi viia minu poolt koostatud matemaatikatestid suurel õpilaste valimil.

PUBLIKATSIOONID

Palu, A., & Kikas, E. (2007). Primary school teachers' beliefs about teaching mathematics. *Nordic Studies in Mathematics Education*, 12 (1), 5–21.

Primary school teachers' beliefs about teaching mathematics

ANU PALU AND EVE KIKAS

The main aim of the study was to investigate the beliefs about the purposes and methods of teaching mathematics in primary school teachers with different teaching experience. The sample consisted of 103 practicing teachers and 26 pre-service teachers. It was shown that teachers with different teaching experience were concordant in their evaluations of the purposes of teaching mathematics – they evaluated the purpose of acquiring knowledge higher than the purpose of the development of personality. Also, all groups of teachers valued formalist teaching methods the least. However, teachers with different teaching experience held different beliefs about using traditional, formalist and social teaching methods.

Beliefs about the effectiveness of different teaching methods and their suitability to use in different age groups are related to conceptions of development and learning. It is widely acknowledged in developmental and cognitive psychology that already young children are active knowledge constructors, and that learning also includes the restructuring of current knowledge and changing concepts besides acquiring new knowledge (e.g. Carey, 2000; Chi & Roscoe, 2002). Additionally, children do not construct and re-construct their knowledge independently, but in cooperation with adults (mainly teachers and parents) and peers (e.g. Nelson, 2003; Valsiner, 2000). Teachers' beliefs are influenced by their own school experiences, theoretical knowledge received from university or college, their practical experience in classrooms, and feedback from their students' achievement. It is important to learn about teachers' beliefs because beliefs influence behaviour, and through this, the

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students' academic and social outcomes (Pehkonen & Törner, 1995; Thompson, 1992). Also, these beliefs have to be taken into account in in-service training and in designing new textbooks and other teaching materials because beliefs influence the way teachers interpret new information (Thompson, 1992). So far, there are several studies on the beliefs of mathematics teachers (e.g. Gales & Yan, 2001; Handal, 2003; Kupari, 1998; Pehkonen & Törner, 1995, 1998; Thompson, 1992). These studies have provided evidence that what teachers know and believe about mathematics is closely linked to their instructional decisions and actions (Thompson, 1992). Beliefs about teaching mathematics have been studied less in primary school teachers. Thus, the purpose of the current study was to investigate the beliefs about the purposes and methods of teaching mathematics held by Estonian primary school teachers with different teaching experience.

Concepts of learning and teaching mathematics

In understanding learning, two approaches can be distinguished – the behaviouristic (teacher-centred) and the constructivist (learner-centred) approaches (Pollard & Triggs, 1997; Shuell, 1996). The specifics and forms of both approaches for teaching mathematics are depicted in figure 1.

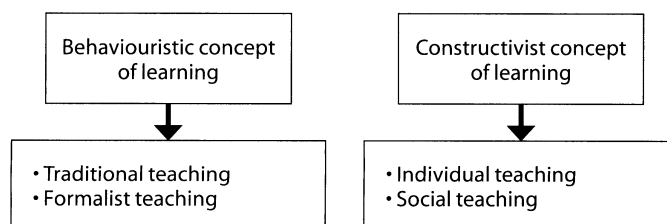


Figure 1. *Two concepts of learning*

When learning is conceptualized as accumulation of knowledge, the role of the teacher is that of active knowledge provider and the role of the learner passive knowledge receiver (metaphorically, as an 'empty vessel' to be filled with knowledge). In line with this conception, behaviouristic teacher-centred teaching methods were developed. In teaching mathematics, this so-called traditional teaching means a focus on acquiring skills of calculating and variation, and stress on practice rather than comprehension (Dionne, 1984; Pollard & Triggs, 1997). In

mathematics, additionally, formalist teaching is distinguished from traditional (Dionne, 1984). Here, the strictness of the subject is set foremost. Formalist teaching values the verbatim acquisition of definitions, using the terminology, correct use of language and symbols and it presupposes strict rules of formulation. Another feature of this method is frequent systematic assessment of learning results.

When learning is conceptualized as active knowledge construction, it also means that learners' preliminary knowledge, values, motivation and personality influence their activity, and, accordingly, the learning process and the achievements (Aronson, 2002; Carey, 2000; Covington, 2000; Merenluoto & Lehtinen, 2004). Accordingly, the roles of the teacher change, and she/he acts more as the students' supporter and supervisor (Dionne, 1984; Pollard & Triggs, 1997; Shuell, 1996). In line with this conception, constructivist learner-centred teaching methods were developed. In mathematics, individual constructivist teaching emphasizes independent raising, analysing, and solving problems, finding different solutions, and creative thinking. Social constructivism has brought about group work, research projects and the overall use of project learning. Students should experience that the result may be attained in various different ways, they are encouraged to find different ways of finding the solution, and discuss these during lessons. Co-operation is valued as well as using elements of games (see Geary, 1994; Handal, 2003). Both constructivist approaches value the development of the student's personality and knowledge comprehension instead of gaining 'pure' (factual) knowledge. Contemporary methods of teaching mathematics focus on the process of acquiring knowledge and skills in mathematics, not giving packaged knowledge; it means that constructivist approach is more valued than behaviouristic one (Geary, 1994; NCTM, 2006).

However, using pure constructivist child-centred methods did not gain hoped results (e.g. Geary, 1994). Without enough time for discussions and teacher's guidance in this process, misconceptions may arise (e.g. Bergqvist & Säljö, 1994). It is now acknowledged that behavioristic methods are specifically important for building basic skills and procedures in mathematics (see Geary, 1994). Just learning of the procedures requires extensive practice on variety of problems. Learning basic skills and procedures starts from the first grades, i.e., this form of practicing is of importance already in primary grades. Conceptual understanding also requires experience, although not so much drill (Geary, 1994). For a deep understanding of the ways of solving problems and their theoretical background, using constructivist methods (discussions, encouraging to use different ways of solving the same problem etc.) has additional value besides traditional methods.

Teachers' beliefs

Despite many educational reforms, which have been driven by conceptualising learning as active and have stressed the value of active learner-centred teaching methods, several studies have shown that a large number of teachers still perceive teaching mathematics in behaviouristic (traditional) rather than in constructivist terms. Handel (2003) gave an overview of studies about the beliefs of the mathematics teachers of various countries. He found that students attending teacher education institutions held beliefs mostly in accordance with traditional (behaviouristic, formalistic) approach. For example, they thought that mathematics learning in school should be based on memorising facts and rules. Beliefs of in-service teachers showed more variety. Some studies and teachers showed the preference for traditional, others for constructivist methods. Still, studies showed quite concordantly that more teachers favoured the traditional than constructivist model.

In Estonia, beliefs of mathematics teachers have been studied by Lepmann (1998, 2004). She differentiated between three approaches to teaching mathematics: the traditional, the formalist and the constructivist approach (she did not differentiate between individual and social constructivist learning, see figure 1). In accordance with Handel (2003), she found that although mathematics teachers valued constructivist teaching methods to some extent, they still did not fully favour these. The strictness in mathematical facts and formulae, and the high level of students' procedural and factual knowledge were also important to these teachers.

In accordance with trends in other countries, the constructivist teaching methods have been promoted in Estonia at least in the past 15 to 20 years. This has been done by means of national curricula design (Põhikooli ja gümnaasumi riiklik õppekava, 2002). Today, the new Estonian national curriculum is being developed. However, the working group of the mathematics curriculum has found that Estonian school mathematics in general and teaching techniques especially have primarily been based on the behaviourist approach to learning. The stereotypes of teaching and assessment tend to put extremely strict demands on all of the students (Ainevaldkond "Matemaatika", 2006).

The aims and hypotheses of the study

The main aim of the study was to investigate the beliefs about the purposes and methods of teaching mathematics in primary school teachers with different teaching experience. So far, mainly the beliefs and opinions of middle and high school mathematics teachers have been studied (e.g. Handel, 2003; Lepmann, 1998, 2004). The beliefs of primary school

teachers may be different due to the aims of primary education, but also due to the peculiarities of the educational system of primary grades and the children's age. The role of the primary school teacher is crucial in building the base for acquiring and comprehending knowledge of mathematics further, in helping to develop students' views on mathematics as a science, and in forming their attitudes towards studying mathematics. Primary school teachers who teach several subjects have better opportunities for integrating mathematics with other subjects than middle and high school teachers who teach mathematics only. Using problems from daily life and other school subjects enables them to show students the possible area of applying mathematics. First-grade students are generally interested in learning (e.g. Stipek & Ryan, 1997). However, children of this age are not always able to work independently for a long time, to find and compare different ways of solving problems. Also, due to their limited skills of group work, teacher's guidance and help is of specific importance (e.g. Azmitia, 1996).

Firstly, we studied the purposes of teaching mathematics. Namely, we analysed to what extent primary school teachers stress the importance of acquiring knowledge and to what extent they value the individual development of personality. According to the current Estonian national curriculum, the main objectives of teaching mathematics are to develop the creativity of the students on the basis of intuition and logical thinking as well as to provide the students with sufficient mathematical skills necessary in everyday life (Põhikooli ja gümnaasumi riiklik õppekava, 2002). Lepmann (1998, 2004) has shown that middle and high school mathematics teachers value the accumulation of knowledge, but we assumed that primary school teachers value individual development of students at least as highly as knowledge acquisition.

Secondly, we studied teachers' beliefs about the importance of using specific methods for the effective teaching of mathematics. A questionnaire was developed to represent traditional, formalist, individual, and social teaching methods (see figure 1). We expected that behaviouristic methods (traditional and formalist) are higher evaluated than constructivist methods.

Thirdly, we compared the beliefs of teachers with different teaching experience. We hypothesized that students and novice teachers would evaluate the elements of constructivist learning higher and teachers with long teaching experience would value behaviouristic methods more than students and novices. Students and novices study or have studied in university in the time when constructivist theory has been valued and teachers with long experience when the behaviouristic approach was taught. We also expected that teachers with medium experience would value different teaching methods.

Method

Sample and procedure

The sample consisted of 103 practicing teachers from 35 schools in different parts of Estonia and 26 fourth or fifth year university students of primary teacher education. Twenty-nine teachers had taught in school for less than 10 years, 32 teachers for 11 to 20 years, 21 teachers for 21 to 30 years, and 21 teachers for over 30 years. All the participants were female.

Questionnaires were distributed to practicing teachers by contact persons. Teachers filled in the questionnaires at home and returned to the contact persons. Of the 120 questionnaires distributed, 103 were returned. Students filled in the questionnaires in the university during a lecture. All the students returned the completed questionnaires.

Questionnaire

The questionnaire was developed in order to assess how relevant are different purposes and methods of teaching mathematics as considered by teachers. The introductory text of the questionnaire read as follows:

The curriculum of basic school mathematics should be treated as one system. Topics studied in primary grades form the basis of school mathematics. It depends greatly on the primary school teacher, how strong a foundation is built. There are different ways of gaining and sharing knowledge, and no exact recipe for achieving the best results. We would like to know your opinion about the effective teaching of mathematics in primary grades.

Next, descriptions of different purposes of teaching mathematics (part I) and methods of teaching (part II) were described. Teachers had to assess their importance on the five-point Likert scale (1 – not important, 2 – not very important, 3 – neither important nor unimportant, 4 – important, 5 – very important).

In the first part, 7 purposes of teaching mathematics were described. In selecting items, we based on the Estonian national curriculum (Põhikooli ja gümnaasumi riiklik õppekava, 2002). Three of the descriptions were focused on personality development, four on knowledge acquisition (see table 1).

In the second part, methods in teaching were described. The items were formulated to represent four approaches to teaching mathematics – the traditional, the formalist, the individual and the social approach

(see figure 1), in developing descriptions we based on the earlier studies (Lepmann, 1998, 2004) and theoretical assumptions (Dionne, 1984; Pollard & Triggs, 1997; Shuell, 1996). At first, the questionnaire included six descriptions of traditional, formalist, and individual constructivist teaching methods, and seven descriptions of social teaching methods. As the preliminary factor analyses showed that three of the items (two of the formalist and one of the traditional methods) loaded on several factors, three items were excluded from further analyses. Consequently, the second part contained 22 items (see table 2).

Data analysis

First, we carried out exploratory factor analyses separately on two parts of the questionnaire, with the Principal Component Method and Parallel analyses for determining the number of factors both for purposes and methods. Second, differences between evaluations of different constructs were compared with paired-samples *t*-tests. Third, differences between evaluations of teachers with different teaching experience were compared with the ANOVA and the LSD test in post-hoc analyses.

Results

Purposes of teaching mathematics

Exploratory factor analysis was run for seven items. Parallel analysis showed two factors. The loadings of all the items are given in table 1. All in all, 48.5 per cent of the variance was explained by the variables. Internal consistencies (Cronbach's α of the scales were .53 and .56, the item-total correlations were higher than .30 for both scales. As scales contained only three and four items, these values may be considered acceptable (Field, 2005).

As expected, the first factor (loadings higher than .57) describes purposes related to the development of the individuality of students, i.e. *personality development*. The purposes characterising this factor were developing students' creativity and problem cognition, the individual development of every student, moulding independence, persistence and patience. The second factor (loadings higher than .43) describes *knowledge acquisition*. This factor includes the following purposes: acquiring basic skills of calculating with natural numbers, guaranteeing the ability to use mathematical knowledge and skills in everyday life, developing general skills and abilities, as well as promoting interest in learning

Table 1. *Evaluations of the purposes of teaching mathematics*

Item No	Purpose	Personality development	Knowledge acquisition
7	Developing students' independence, profoundness, persistence and discipline	.85	-.14
6	Developing students' creativity and problem cognition	.67	.25
4	Individual development of each student	.57	.27
1	Acquiring the basics skills of calculating with natural numbers (algorithms of mental and written calculation, basic characteristics of arithmetic operations)	.12	.71
2	Developing ability of using mathematical knowledge and skills in daily life	-.09	.71
5	Developing general abilities and skills (comparing, systematizing, classifying, logical thinking)	.32	.60
3	Raising students' interest in studying mathematics and maintaining pleasure in their work	.29	.43
Cronbach's alpha		.56	.53
Mean		4.53	4.76
Standard deviation		.31	.43

mathematics. Mean scores of the evaluations of personality development and knowledge acquisition were compared. In general, teachers valued knowledge acquisition significantly more than personality development, $t(128) = 3.11, p = .002$.

Approaches to teaching mathematics

Exploratory factor analysis was run for 22 items. In accordance with theoretical assumptions, parallel analysis showed four factors. All in all, 43.6 per cent of the variance was explained by the variables. The loadings of all the items are given in table 2. The internal consistencies of the scales were .70, .68, .66, .66, and the item-total correlations were higher than .30 for all four scales. These values may be considered acceptable (Field, 2005).

The two first factors characterise the constructivist concept of learning and teaching. Factor 1 (loadings higher than .36) characterizes *social*

Table 2. *Evaluations of the teaching methods*

Item No	Method	Social teaching	Individual teaching	Traditional teaching	Formalist teaching
4	Learning with concrete materials and through concrete activities	.75	-.09	.05	.13
5	Using the elements of games	.68	-.01	.02	-.20
3	Learning with visual aids	.66	-.03	.07	.06
21	Using group work	.62	.08	.22	.08
22	Using project work	.52	.27	-.05	.13
7	Motivating students	.40	.23	.16	.04
15	Taking into account students' experience, knowledge and skills	.36	.21	-.00	.12
8	Raising and analysing problems	.12	.71	.11	-.06
10	Differentiated and versatile practicing	.00	.68	-.09	.06
20	Showing the possibilities of using mathematical knowledge in daily life	.27	.58	.12	-.13
11	Guiding students to self-control when solving problems	-.11	.54	.26	-.01
14	Developing ability to work independently (e.g. working with literature, instructions)	.07	.52	.21	.22
16	Aiding multiple ways of solving problems	.12	.50	-.07	.20
6	Teacher's instruction and explanations	.09	.04	.71	.09
9	Intensive practicing and repetition of basic knowledge and skills	-.04	.22	.68	-.01
12	Assessing students' knowledge and skills regularly	-.01	.05	.61	.23
2	Sequential raising of the level of difficulty	.21	.06	.56	-.06
13	Systematic repetition of the material learnt earlier	.16	-.01	.56	.24
18	Teaching mathematical definitions and rules	.07	.20	.23	.73
17	Solving problems which develop skills of proving	.15	.18	-.02	.69
19	Learning definitions or rules verbatim by heart	.15	.00	.06	.68
1	Guiding learning exactly and rigidly according to the textbook	-.16	-.14	.23	.58
Cronbach's alpha		.70	.68	.66	.66
Mean		4.15	4.39	4.48	3.05
Standard deviation		.43	.43	.37	.64

teaching in which the teacher encourages children to participate in different activities and playing games, utilises group and project work while motivating children and taking into account their current knowledge and experience. Factor 2 (loadings higher than .50) characterises more *individual teaching* in which differentiated practice along with self-control is important, developing independent working skills, directing students into finding problems and analysing them as well as finding different ideas of solutions.

The next two factors characterise the behaviouristic concept of learning and teaching. Factor 3 (loadings higher than .56) characterises *traditional teaching*, in which the student is left a passive role and the teacher directs and explains, mediating knowledge in small bits as the level of difficulty is raised. Basic knowledge is practiced, systematically repeated and regularly assessed. Factor 4 (loadings higher than .58) characterises *formalist teaching*, valuing the teaching of definitions and regularities, acquiring definitions verbatim, developing the ability to prove theorems. The teaching process is guided strictly by aims on the basis of the textbook.

Mean scores of the evaluations of social, individual, traditional, and formalist teaching were compared by pairs. Paired-samples *t*-tests showed that teachers evaluate traditional teaching significantly higher than all the other approaches ($p < .001$), individual teaching higher than social and formalist teaching ($p < .04$), and social teaching significantly higher than formalist teaching ($p < .001$).

Teaching experience, purposes and approaches to teaching

An analysis of variance was carried out to determine differences between groups of teachers with different teaching experience. Five groups of teachers were compared: students, novices (with a teaching experience of 1 to 10 years), younger experts (with a teaching experience of 11 to 20 years), experts (with a teaching experience of 21 to 30 years), older experts (with a teaching experience of more than 30 years). Mean evaluations of teaching approaches in different groups are given in figure 2. The main effect of experience was significant for social teaching, $F(4,124) = 3.17$, $p = .016$, traditional teaching, $F(4,124) = 3.35$, $p = .012$, and formalist teaching $F(4,124) = 2.24$, $p < .001$, but nonsignificant for individual teaching and purposes (knowledge acquisition and personality development).

Post-hoc analyses with an LSD test showed that all groups valued traditional teaching and individual teaching higher than social teaching and formalistic teaching. However, just older experts valued traditional teaching significantly more than students, novices, and experts ($p < .05$), and formalist teaching higher than all the other groups.

Students valued social teaching significantly higher than experts and older experts ($p < .005$). Novices and younger experts valued formalist teaching significantly higher than students ($p < .04$).

Discussion

The beliefs about the purposes and methods of teaching mathematics held by Estonian primary school teachers were studied. We found that teachers with different teaching experience were concordant in their evaluations of the purposes of teaching mathematics – they evaluated the purpose of acquiring knowledge higher than the purpose of the development of personality. Also, all groups of teachers evaluated the formalist teaching methods the least. Teachers with different teaching experience held different beliefs about using traditional, formalist and social teaching methods.

Besides teaching subject knowledge and skills, primary school teachers have greater role in socializing students than middle and high school teachers. According to Handal's review (2003) and Lepmann's study (1998, 2004), middle and high school mathematics teachers valued the accumulation of knowledge highly. In contrast, we expected that primary

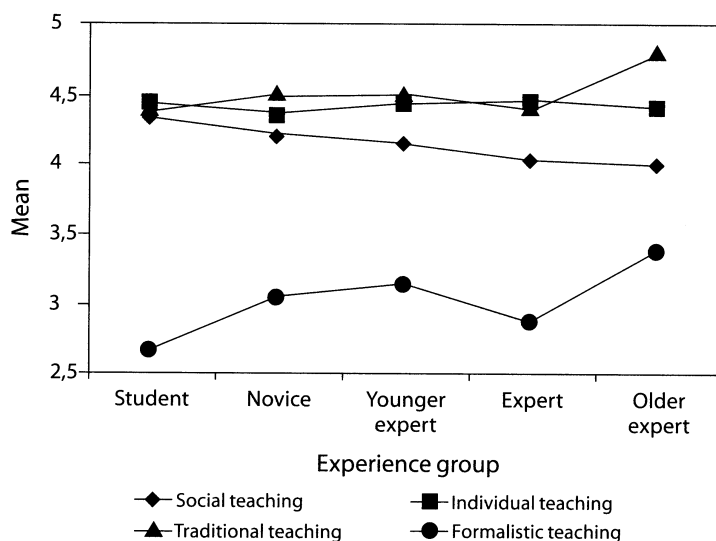


Figure 2. Evaluations of teaching approaches in different experience groups

school teachers evaluate the purpose of individual development of students at least as highly as knowledge acquisition. The results showed that although teachers – in spite of their experience – evaluated the acquisition of knowledge higher, another purpose – the development of personality – was evaluated highly as well.

While sometimes two – behaviouristic and constructivist – broad approaches of teaching are differentiated (see Handel, 2003), we could differentiate between four approaches – traditional, formalist, social and individual constructivist – which were evaluated differently in the whole group and in groups of teachers with different teaching experience. Our expectation that primary school teachers evaluate behaviouristic (traditional and formalist) teaching methods higher was partly confirmed. Great differences were found between evaluations of traditional as compared with formalist methods. Formalist methods – the verbatim acquisition of definitions, strict terminology, correct use of language and symbols, strict rules of formulation – were least evaluated in all experience groups. These methods seem to be more important to use in middle and high school (Handel, 2003; Lepmann, 1998, 2004). In contrast, traditional methods – focus on acquiring skills of calculating and variation, stress on practice – were evaluated the highest. The latter result is similar to earlier studies with primary, basic and high school teachers (Handel, 2003; Lepmann, 1998; 2004). Intensive practicing, repetition, sequential raising of difficulty as well as regular assessment seem to be an important part of teaching mathematics at all ages (cf. Geary, 1994).

While Handel (2003) in his overview found that it was students who valued traditional teaching methods and that more experienced teachers hold different views, in our study, students evaluated the constructivist approach as highly as the traditional approach, and gave specifically low grades to the formalist approach. They evaluated individual teaching specifically highly. Novice teachers (with a teaching experience of 1 to 10 years) stressed the importance of traditional methods the most but both constructivist methods as well. Younger experts and experts (i.e. teachers with a teaching experience of 11 to 30 years) evaluated traditional and individual teaching the highest but social teaching quite highly as well. So, social constructivist methods – group work, research projects, discussions, elements of games – were evaluated higher by students and teachers with less experience. Beliefs of teachers with teaching experience more than 30 years (i.e. older experts) differed from those of other groups. In particular, both formalist and traditional teaching approaches were evaluated higher by these teachers than by teachers with less teaching experience or by students. These results may be explained, taking into account that teachers' beliefs are influenced by their own school

experiences, theoretical knowledge studied in university, and their practical experience in classrooms. Actually, the participating students have studied in the university according to the curriculum where individual constructivist methods (e.g. discovery learning, project work) have been introduced and practiced specifically for teaching science. These experiences might have influenced their beliefs about the methods of teaching mathematics as well. Also, social constructivist methods are stressed as valuable for usage in school. Additionally, social constructivist methods are used in university teaching and these are also popular in in-service training courses. Older experts in particular have studied both in school and in the university at a time when traditional and formalist teaching methods were highly valued. If they do not take an active part in in-service training courses, their practical skills of using child-centred methods may even not be very high. These personal experiences and low skills in using constructivist methods might have influenced their preferences.

Across arithmetic and problem solving skills, conceptual and procedural competencies must be acquired (Geary, 1994). These skills must be taught and practiced; children need encouragement but also possibilities and time for the construction and reconstruction of knowledge. In the different stages of teaching, or in teaching different topic areas, various methods are used. Therefore, it is no surprise that Estonian teachers evaluated almost all the methods and approaches highly. However, teaching methods based on the behaviouristic approach (specifically, traditional methods) are more favoured (for older grades' teachers see Lepmann, 1998, 2004). The members of the committee engaged in developing the new national curriculum have also revealed reasons why constructivist methods are not widely used in Estonian schools (Ainevaldkond "Matemaatika", 2006). According to their analysis, teaching methods based on problem solving take too much time to be effective. When little time is left for discoveries, discussions, and – specifically – for group work, it is students with lower skills and knowledge who are not able to gain understanding (cf. Azmitia, 1996). Additionally, constructivist teaching puts higher demands on teachers' knowledge and skills: the teacher has to integrate subject matter knowledge, pedagogical knowledge, student characteristics and the environmental context of learning (Leino, 1994). It has also been stressed earlier that learning some mathematical concepts and algorithms is more effective especially by means of behaviouristic methods. For example, when considering how to teach the memorisation of the multiplication facts, behaviourism may be a better option than constructivism since the aim is to remember rather than to understand (Zevenbergen, Dole & Wright, 2004).

It should be stressed that this investigation studied beliefs not behaviour. To determine which methods teachers really use in their teaching, classroom research is needed. As a limitation of the study, it should be mentioned that the sample size was quite small. In the future, teachers of different cultural backgrounds should be studied. Also, differences in the in-service training of teachers must be taken into account. Also, longitudinal studies have to be carried out to find out possible changes in beliefs due to experience.

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Sammanfattning

Huvudsyftet med studien var att undersöka uppfattningar (beliefs) om syftet med och metoder för matematikundervisning hos grundskollärare (primary school teachers) med olika undervisningserfarenhet. Undersökningsgruppen bestod av 103 verksamma lärare och 26 lärarstudierande. Resultatet gav att lärarna, trots olika undervisningserfarenhet, var samstämmiga beträffande syftet med att undervisa i matematik – de värderade syftet att utveckla kunskaper högre än den personliga utvecklingen. Dessutom värderades formalistiska undervisningsmetoder lägst av samtliga. Däremot visade det sig att lärare med olika undervisningserfarenhet hade skilda uppfattningar beträffande traditionella, formalistiska och sociala undervisningsmetoder.

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The role of individual and contextual factors in the development of maths skills

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In this study, we examined the development of maths skills in 269 Estonian primary school children (119 boys and 150 girls; 20 classes). Testing was carried out over a three-year period (Grade 1–Grade 3). Before the last testing session, children's verbal skills and motivational orientations were also tested. In addition, teachers evaluated children's learning behaviour and provided information about their own teaching methods. The data were analysed using multilevel growth curve modelling. We found that children with higher levels of pre-maths skills developed at a faster rate. At the individual level, pre-maths skills and verbal ability were positively associated with maths achievement in Grade 3, and avoidance orientations (self-reported) and task-avoidant behaviour (teacher-reported) were negatively associated with maths achievement in Grade 3. At the classroom level, formalist teaching methods and teacher experience had a positive effect on students' maths performance.

Keywords: academic achievement; primary; maths; teaching practices

Introduction

Although maths is usually regarded as a difficult subject, there is a considerable amount of heterogeneity between individuals, classrooms, and countries (Mullis, Martin, & Foy, 2005; Tatsuoka, Corter, & Tatsuoka, 2004). There is evidence that different individual-level factors – such as children's general and specific abilities, prior maths knowledge, motivational goals, and learning behaviour – influence maths achievement (e.g., Aunola, Nurmi, Lerkkanen, & Rasku-Puttonen, 2003; Fuchs et al., 2006). Classroom-level factors have been studied to a lesser extent. There are debates around more and less effective teaching methods, but empirical results have been contradictory (Campbell, Kyriakides, Muijs, & Robinson, 2004; Walker, 1999).

Moreover, relatively little is known about the simultaneous effect of individual and teacher-related factors on the development of maths skills and knowledge in elementary grade students. Thus, we conducted a longitudinal study to examine the development of maths skills during the elementary school years. In addition, we were interested in studying the influence of individual (verbal skills, motivational orientations, learning behaviour, and preliminary knowledge) and contextual (teacher's working experience, and use of different teaching methods) factors on maths performance.

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Prior knowledge, verbal skills, motivational orientations, learning behaviour, and maths performance

Although maths skills develop rapidly in the elementary grades, individual differences are relatively stable throughout these years, indicating that the level with which children enter the school plays an important role in determining future success (e.g., Burchinal, Peisner-Feinberg, Pianta, & Howes, 2002; Lerkkanen, Rasku-Puttonen, Aunola, & Nurmi, 2005). Over the years, there has been a debate over whether higher achievers become better and lower achievers become worse (also called the Matthew effect). Longitudinal studies have shown a more diverse picture (e.g., Phillips, Norris, & Osmond, 2002), but also opposite trends. For instance, Burchinal et al. (2002) studied maths development (using maths subtests from the Woodcock-Johnson Tests of Achievement-Revised) in four- to eight-year-old children, and found that children with lower initial scores showed slightly greater gains than children with higher scores (however, this was not the case for reading). Also, a large-scale study conducted in Dutch elementary schools showed that highly able students showed the greatest declines in terms of language and arithmetic performance from Grade 2 to Grade 4 (Mooij & Driessen, 2008). However, Luyten, Cremers-van Wees, and Bosker (2003), using partly the same sample as in the latter study, showed that difference in language and arithmetic performance increased between students with poorly and well-educated parents.

The role of general intelligence in maths achievement is also widely acknowledged (Hale, Fiorello, Kavanaugh, Hoepfner, & Gaitherer, 2001). Recently, more emphasis has been laid on studying the role of specific cognitive skills in maths achievement across different areas. For example, various studies have demonstrated the role of working memory (e.g., Geary, Brown, & Samaranayake, 1991; Passolunghi & Siegel, 2004; Wilson & Swanson, 2001), as well as verbal and reasoning skills (Delgado & Prieto, 2004; Fuchs et al., 2006; Nunes, Bryant, & Evans, 2007).

In addition, children's achievement-related beliefs, goal orientations, and behaviours play an important role in their academic performance. Children who expect success and are persistent in the face of obstacles do better at school than those children who are afraid of failure, deploy self-handicapping behaviours, and are not persistent in learning situations (e.g., Onatsu-Arivilommi & Nurmi, 2000). Studies carried out within the framework of achievement goal theory have differentiated between mastery and performance orientations, which have been further divided into approach and avoidance components (Elliot & McGregor, 2001; Midgley et al., 2000). Being mastery-oriented is associated with understanding the material learnt at school, and also with studying more intensively. Mastery avoidance is conceptualised as an orientation to avoid misunderstanding or failing to master a task (Elliot & McGregor, 2001; Guan, McBride, & Xiang, 2007). Those with a performance-approach orientation strive to demonstrate superior ability and outperform other students, whereas those with a performance-avoidance approach avoid the demonstration of inability and being negatively judged by others (Kaplan, Middleton, Urdan, & Midgley, 2002). While studies have shown that performance avoidance is associated with non-effective learning strategies and low achievement (Leondari & Gialamas, 2002; McGregor & Elliot, 2002; Wolters, 2004), results concerning the mastery and performance-approach orientations are mixed (Kenney-Benson, Pomerantz, Ryan, & Patrick, 2006; Midgley, Kaplan, & Middleton, 2001). Such variation in results may be explained by differences in the general success level of a student, his/her age, and various contextual factors (e.g., Midgley et al., 2001).

Few studies have examined the role of motivational orientation and task-focused (vs. task-avoidant) behaviour in maths performance in elementary grade students. Few studies with older students have generally shown the supportive effect of mastery goals and task-focused behaviour and the distractive effect of avoidance beliefs and task-avoidant behaviour (e.g., Galloway, Leo, & Rogers, 1995). Among younger children, the role of maladaptive motivational orientations and learning behaviour is less clear. For instance, Onatsu-Arvilommi, Nurmi, and Aunola (2002) found that achievement strategies had an impact on the development of maths skills during the first school year, in a study using children's self-reports. However, when utilising teacher reports of children's maladaptive achievement strategies (task-irrelevant behaviour, helplessness beliefs, and lack of persistence), no effect of task-avoidant behaviours on maths skills was found (Onatsu-Arvilommi & Nurmi, 2000).

Results concerning gender differences in maths skills have been mixed. One of the factors that seems to matter is students' age. Studies conducted in elementary school have either not found a gender difference (e.g., Burchinal et al., 2002; Lachance & Mazzocco, 2006) or demonstrated a negligible difference favouring girls (see Halpern et al., 2007). Among older children, boys outperform girls in some types of tasks. For instance, boys have shown better achievement in tasks demanding visuospatial skills (see Halpern et al., 2007).

Teaching methods and maths performance

Nowadays, it is acknowledged that both passive and active mechanisms are involved in effective learning (e.g., Siegler, 2005). Passive processes include building associations among strategies and facts, while active mechanisms involve searching for and constructing new strategies and ways of problem solving. In order to be able to construct new strategies and synthesise knowledge, associations among strategies and knowledge need to be built and strengthened to free working memory capacity (see also Paas, Renkl, & Sweller, 2003). These two types of learning benefit from different teaching methods. Generally, two major approaches to teaching have been differentiated: behaviourist or teacher-centered teaching, with an emphasis on acquiring calculation and variation skills, and on stressing practice rather than comprehension; and constructivist or learner-centered teaching (Ackerman, 2003; Hermans, van Braak, & Van Keer, 2008). Whereas the first approach is efficient for building associations (i.e., when a child learns basic skills), the latter becomes more important when active construction of strategies and knowledge is required (i.e., when learning advanced skills).

In maths, the development of procedural skills and conceptual knowledge are intertwined (see Rittle-Johnson, Siegler, & Alibali, 2001). As learning maths procedures requires extensive practice on a variety of problems, behaviourist methods that stress making associations are considered important for building basic skills (see Geary, 1994; Siegler, 2005). Some researchers have differentiated between practicing procedures and learning about maths symbols, rules, and definitions. For instance, Dionne (1984) has identified three types of methods: traditional, formalist, and constructivist. This differentiation has been used in some northern European countries, including Estonia (see Lepmann, 1998; Palu & Kikas, 2007b). Proponents of the formalist teaching approach value the verbatim acquisition of definitions, the use of exact terminology and symbols, and adherence to strict rules of formulation. Another feature of this method is frequent and systematic assessment of learning results.

In reality, usage of the various methods is related to the teacher's beliefs (Handal, 2003) and the demands of the curriculum (Pihlap et al., n.d.).

In addition, teacher experience influences which teaching methods are preferred, and thereby has an effect on the achievement of students. For instance, Palu and Kikas (2007b) studied elementary school teachers' preferences for formalist, traditional, and individual and social constructivist teaching methods, and found that teachers valued the formalist teaching methods the least. However, teachers with more teaching experience (i.e., more than 30 years) placed a higher value on the formalist and traditional teaching approaches than teachers with less teaching experience. Several studies have shown that children who are taught by more experienced teachers display better maths achievement (e.g., Clotfelter, Ladd, & Vigdor, 2007; Klecker, 2002; Meijnen, Lagerweij, & de Jong, 2003).

The case of Estonia

The sample for the present study comes from Estonia. Comparative surveys have shown that Estonian children perform better on maths tests than same-age students from other countries (Trends in International Mathematics and Science Study [TIMSS], 2003, n.d.). Also, their IPMA (International Project on Mathematical Attainment) test results are among the best (Palu, Afanasjev, & Vojevodova, 2007). However, we reasoned that there must be a great deal of variability in maths achievement across different classrooms and schools in Estonia (*Tasemetööd 3. Klass*, 2006, 2007), and this would be, at least partly, explained by the different methods teachers use.

The cultural-historical background of Estonian society and its current educational system provide a good context for studying the role of teacher and teaching methods. Teacher-centred and formalist teaching methods (at least in maths education) prevailed in Estonian elementary schools until the 1990s, when Estonia was part of the Soviet Union. Although a child-centred constructivist approach has been promoted in Estonia during recent decades, many teachers and parents still hold values and beliefs more characteristic of the former educational system. There is evidence that middle and high school maths teachers place a high value on the accumulation of knowledge and formalist teaching methods (see Lepmann, 1998, 2004). In addition, Palu and Kikas (2007b) showed that Estonian elementary school teachers with more teaching experience prefer using behaviourist and formalist methods.

Another unique feature that characterises the Estonian elementary school system is that one teacher is responsible for teaching almost all the subjects (except for music and gym) for (at least) the first three years. This means that in Estonia, the style and methods that a specific teacher prefers may have a greater impact on students' outcomes than in countries where children are taught by a different teacher every year (e.g., the US).

The present study

Although studies have consistently shown the facilitative effect of preliminary knowledge and abilities on the development of maths skills in children of various ages, the role of motivational goals and behaviours has not been fully explored at the elementary school level. Also, the effectiveness of using different methods to teach basic maths in the elementary grades needs further clarification. Thus, the aims of this study were:

- (1) to examine maths development during the elementary school years (Grade 1–Grade 3)
- (2) to analyse the individual (verbal skills, motivational orientation, learning behaviour, preliminary knowledge, and gender) and contextual (teacher experience and preference for using certain teaching methods) factors that play a unique role in maths achievement.

Students' performance in maths was assessed three times – at the end of each grade. Pre-maths skills were assessed at the beginning of the first grade. This study was carried out as part of the International Project on Mathematical Attainment (*IPMA Coordinators' Manual*, n.d.). IPMA tests mainly assess children's basic factual and procedural knowledge and skills, the development of which is important during the elementary school years. This knowledge base is in turn crucial for the development of application and reasoning skills later on (see *Mathematics Framework*, n.d.). The data were analysed using multilevel growth curve modelling (using *Mplus 5.0*; Mythén & Mythén, 1998–2007).

First, we expected that children would improve their maths skills overall over time. Second, regarding the rate of development, we reasoned that there were two possibilities: children with poorer pre-maths skills could catch up with children who had better initial skills (e.g., Burchinal et al., 2002), or initial differences between low and high performers would widen over time (e.g., Luyten et al., 2003). As our data were hierarchical (students nested in classrooms), specific hypotheses were formed at the student and classroom level.

At the student level, we anticipated that preliminary knowledge (e.g., Burchinal et al., 2002; Lerkkanen et al., 2005), verbal reasoning (e.g., Delgado & Prieto, 2004; Fuchs et al., 2006; Lerkkanen et al., 2005), and mastery orientations would be positively associated with maths achievement, with the opposite being true for avoidance orientations (as assessed by student reports) and task-avoidant learning behaviour (assessed by teacher reports) (Aunola et al., 2003; Onatsu-Arviolommi & Nurmi, 2000; Onatsu-Arviolommi et al., 2002; Van den Broeck, Opdenakker, & Van Damme, 2005). With regard to performance-approach orientations, we did not formulate a specific hypothesis because earlier results have been contradictory. We also explored the effect of gender. Previous studies of gender differences in maths achievement have had mixed results (Ai, 2002; Lachance & Mazzocco, 2006). In elementary school, some studies (e.g., Burchinal et al., 2002; Lachance & Mazzocco, 2006) have not found differences, whereas others (see Halpern et al., 2007) have demonstrated a negligible difference favoring girls.

At the classroom level, we expected teacher experience to have a positive effect on students' maths achievement (Clotfelter et al., 2007; Meijnen et al., 2003). As Estonian schools employ teachers who received their education both before and after Estonia gained its independence, we anticipated that teachers would vary in the kinds of methods they use. In addition, we were interested in examining which teaching methods would explain the variation in children's maths achievement across different classrooms.

Method

Subjects and procedure

Elementary school students' maths skills were tested three times within a three-year period: at the end (the last week of May) of the first grade (Time 1 respectively), at

the end of the second grade (Time 2), and at the end of the third grade (Time 3). Pre-maths skills (Time 0) were assessed at the beginning of the first grade (the first week of September).

Twenty classes of students were included in the study. The number of participating students at the different time points was as follows: 330 students (mean age 7.4, $SD = .36$) at Time 0, 316 students at Time 1, 330 students at Time 2, and 295 students at Time 3. The 269 students (119 boys and 150 girls; 20 classrooms) who participated throughout the study were included in the analyses. The average class size was 13.45. Pre-maths skills were significantly higher ($F[1,328] = 12.874$, $p < .001$) in children who were included in our final sample ($M = 6.14$, $SD = 2.28$), compared to children who had missing data points ($M = 4.97$, $SD = 2.46$).

Twenty class teachers who had taught these children during the whole three-year period also participated in the study. Their teaching experience varied from one to 34 years, with an average of 19.43 years ($SD = 8.66$). All students were tested during regular maths lessons by their maths teachers. Teachers had been informed about the research, and they had received written instructions about conducting the tests as well as photocopied test questions. In order to guarantee the objectivity of the results, teachers did not check the results and were not allowed to give correct solutions to the tasks.

Shortly before the last testing session, children's verbal skills and motivational orientations were assessed using a written test during a regular school lesson. At the same time, teachers evaluated children's motivational strategies and provided information about their teaching methods and working experience.

Measures

Maths performance

Pre-maths skills and maths performance were assessed using tests from the International Project on Mathematical Attainment (Centre for Innovation in Mathematics Teaching, n.d.). The IPMA tasks can be classified into four types by content: whole numbers; fractions and decimals; number sentences; and data reading and interpreting. Also, these tasks cover three cognitive domains (see Appendix for examples) as recommended in the Trends in International Mathematics and Science Study (TIMSS) framework: knowing, applying, and reasoning (see *Mathematics Framework*, n.d.). The importance of these cognitive domains for Grades 1–3 is stressed in the Estonian National Curriculum (*Põhikooli ja Gümnaasiumi Riiklik Õppekava*, 2002).

Pre-maths skills (Time 0) were assessed by 10 calculation tasks. Ten new tasks were added into Test 1 (Time 1), 20 into Test 2 (Time 2), and 20 into Test 3 (Time 3). Test 1 included 14 calculation tasks, three reasoning tasks, and three verbal tasks; Test 2 included 26 calculation tasks, seven reasoning tasks, and seven verbal tasks; Test 3 included 37 calculation tasks, 14 reasoning, and nine verbal tasks. Each test contained all the tasks from earlier tests. Accordingly, the last test included 60 tasks. One point was given for each correct solution. Each wrong solution or unsolved task earned 0 points. Only sum scores were used in all analyses.

Verbal reasoning skills

These were assessed with six tasks from the 'Word Guessing' test used by Estonian school psychologists to screen for specific learning difficulties (see Kikas, Männamaa, Kumari, & Ulst, 2008; Männamaa, Kikas, & Raidvee, 2008). This subtest is also part

of several intelligence tests (e.g., Kaufman & Kaufman, 1983). Children were provided with a written description of an object and their task was to infer the relevant concept and write it down. As an example, the description of 'staircase' was: 'What has several steps? You can go up and down on it and it sometimes has handrails.' The tasks were complex, demanding the ability to integrate information (three features of the concepts), draw conclusions about the concept, and write down the correct word – thus the tasks assessed both verbal reasoning skills and working memory capabilities. The answer was considered correct if the child provided the exact word, even if there were spelling or grammatical errors. The sum verbal reasoning score was used in the analyses. The internal consistency of the test was good (Cronbach $\alpha = .82$).

Motivational orientation

On the basis of earlier scales and studies (Elliot & McGregor, 2001; Midgley et al., 2000; Skaalvik, 1997), we developed a new scale for measuring four achievement goals (mastery-approach, mastery-avoidance, performance-approach, and performance-avoidance) in maths in the elementary grades. All items were assessed with regard to maths. Students rated each statement on a three-point Likert scale (1 = 'I don't agree', 2 = 'I agree a bit', 3 = 'I totally agree'). Exploratory factor analysis (using the principal component method, varimax rotation, and parallel analyses for deciding the number of factors) showed that all avoidance items loaded on one factor. Thus, in our analyses, we used three scales: mastery (e.g., 'I want to learn many new things in maths', seven items, $\alpha = .76$), performance approach (e.g., 'I want to show the teacher that I'm smarter than other children in maths', five items, $\alpha = .83$) and avoidance (e.g., 'I'm afraid that the others think I'm silly in maths', nine items, $\alpha = .86$) orientations. Altogether, these three factors explained 54% of the original variance in the items. All items had a loading greater than .64 on their main factor and all cross-loadings were below .21.

Teachers' ratings of children's task-avoidant learning behaviour

Five items from the Behavioural Strategy Rating Scale (see Aunola et al., 2002; Onatsu-Arvilommi & Nurmi, 2000) were used to assess children's avoidance behaviour. Teachers were first asked to think about and recall how each student typically behaved in classroom learning situations (e.g., 'If the activity or task is not going well, does the student lose his/her focus?'), and then to rate his or her behaviour on a five-point rating scale (1 = 'not at all' to 5 = 'to a great extent'). The internal consistency of the test was good (Cronbach $\alpha = .94$).

Teachers' use of different teaching methods

Teaching methods were assessed via a modified version of the questionnaire used by Palu and Kikas (2007b). First, teachers were given the following introduction:

The curriculum of basic school math should be treated as one system. Topics studied in elementary grades form the basis for school math. The elementary school teacher is largely responsible for establishing a strong math foundation. There are different ways of gaining and sharing knowledge, and there is no exact rule how to achieve the best results. We would like to obtain information about your ways of teaching of math in elementary grades.

Next, items describing different teaching methods (seven for social constructivist, six for individual constructivist, five for traditional, and four for formalist teaching) were provided and teachers were asked to assess the importance of each on a five-point Likert scale (1 = 'not important' to 5 = 'very important'). In an addition to the original version (Palu & Kikas, 2007b), teachers were asked to rate how frequently they actually endorsed each method (1 = 'never' to 5 = 'always'). We expected that if teachers could first rate the importance of each method (reflecting also their knowledge about its usefulness), their later answers would be less influenced by social desirability (i.e., objective factors like overcrowded classrooms or overloaded curricula may impede the real implementation of the methods). Only the frequency data were analysed in the current study. In addition, due to a high correlation between the individual and social constructivist scales ($r = .79, p < .001$), we combined these two scales. The internal consistencies of the scales were good (Cronbach $\alpha > .70$).

Data analysis

The data were analysed using multilevel growth curve modelling. Latent growth curve modelling allowed us to examine two growth components – final status (intercept) and rate of development of maths skills (trend or slope). In addition, as students were nested in classrooms and our data were thus hierarchical, we employed multilevel modelling. More specifically, we were interested in examining the degree to which the variance of the intercept and trend was due to the heterogeneity of individuals (i.e., individual-level variance) or to differences between classrooms (i.e., between-level variance). Finding significant variance of the intercept and slope at the individual level would suggest that individuals varied with regard to their final maths status (intercept variance), and the rate at which they developed across time (trend variance). At the classroom level, significant growth component variances would indicate that different classrooms varied in their final status and rate of change. Moreover, by employing multilevel modelling, we were able to investigate individual-level and classroom-level predictors that could potentially explain the variance in the growth components at both levels.

All analyses were conducted using the software *Mplus 5.0* (Mythén & Mythén, 1998–2007). Throughout the analyses a robust estimator MLR (normality-based maximum-likelihood estimation with robust standard errors) was used to correct for the violation of normality assumption (Muthén & Muthén, 1998–2007). The fit of the models was evaluated using the following indices: chi-square test (χ^2), comparative fit index (CFI), and root mean error of approximation (RMSEA). The cut-off criteria for the fit indices were CFI $> .90$ and RMSEA $< .08$ (Kline, 2005). As the chi-square test is sensitive to sample size and deviations from underlying assumptions (e.g., multivariate normality), other fit indices are considered more adequate to assess the model fit (e.g., Hu & Bentler, 1998; Kline, 2005).

Results

Descriptive statistics

Means and standard deviations of the different child-level variables are provided in Table 1. Children had the highest scores on mastery orientation and the lowest scores on avoidance orientation. Girls had significantly higher levels of verbal reasoning skills ($F[1,255] = 6.46, p < .05$), and were more mastery-orientated ($F[1,253] = 5.53$,

$p < .05$) as well as avoidance-orientated ($F[1,253] = 8.06, p < .01$). In contrast, boys has higher scores on avoidance behaviour, according to teacher reports ($F[1,253] = 6.87, p < .01$).

In addition, a repeated measures ANOVA showed that teachers used certain methods more frequently than others ($F[2,38] = 48.48, p < .001$). All post-hoc comparisons (with Bonferroni adjustment) were significant. More specifically, traditional teaching methods ($M = 4.34, SD = .42, \text{min.} = 3.40, \text{max.} = 5.00$) were used the most frequently, followed by constructivist teaching methods ($M = 3.79, SD = .46, \text{min.} = 3.16, \text{max.} = 4.70$). Formalist teaching methods ($M = 3.05, SD = .59, \text{min.} = 2.00, \text{max.} = 4.50$) were used the least frequently.

Multilevel growth curve modelling

First, we examined intraclass correlations and variance estimates for the observed maths scores at four time points (pre-maths skills then maths achievement at three time points). Intraclass correlations indicate the proportion of variance between higher-level units (i.e., classrooms in the present study). Results showed that observed maths scores varied significantly between individuals as well as between classrooms (see Table 2).

Next, we constructed an unconditional growth curve model to explore the means and within-level and between-level variance estimates of the growth components. Observed maths scores from the three time points (Time 1, Time 2, and Time 3) were used when building the unconditional model. More specifically, maths scores at each time point were fixed to 1 on the intercept factor and to $-2, -1$, and 0 on the slope factor. As we fixed the loading of the slope factor at a value of 0 at Time 3, the mean of the intercept factor represented the average maths performance at the end of Grade 3 (see Duncan, Duncan, & Strycker, 2006). This was primarily done for the purpose of being able to include covariates that were assessed only in Grade 3. In the first model, the means and variances of the two growth components, as well as their covariances, were freely estimated. In addition, the residual variances of the observed maths scores were freely estimated. Then, we modified the model. As the variance of the slope factor was nonsignificant at the classroom level, indicating that different classrooms did not differ significantly in the rate at which maths skills developed, this parameter was fixed to 0 . In addition, due to the negative between-level residual variance of the observed maths score at Time 1, this parameter was fixed to 0 . Also, the fit of the model improved when the intercept of the first maths score was freely estimated. The fit indices of the final model were: $\chi^2(3, n[\text{within}] = 269, n[\text{between}] = 20) = 10.64, p < .05, \text{CFI} = .98$, and $\text{RMSEA} = .10$. Parameter estimates are presented in Table 3.

We found that both intercept and trend means were significantly different from 0 . On average, children achieved a score of 46.98 in Grade 3. In addition, there was a significant positive linear change in maths skills over a two-year interval (between the end of the first and third grade) ($M = 13.62$). The significant within-level variance around the intercept mean indicated that children also differed in their final maths skills. Although the within-level variance of the slope factor did not reach statistical significance, the fit of the model would have dropped tremendously if we had fixed this parameter to 0 . Moreover, there was considerable overlap between the two growth factors at the individual level, indicating that children who had a more positive change in maths skills over time also had higher final maths skills.¹ Finally, classrooms varied in their final level of maths skills.

Table 1. Means and standard deviations of different child-level variables.

	Overall				Boys				Girls			
	<i>M</i>	Min.	Max.	<i>SD</i>	<i>M</i>	Min.	Max.	<i>SD</i>	<i>M</i>	Min.	Max.	<i>SD</i>
Verbal reasoning	5.37	0	6	1.05	5.18 _a	0	6	1.22	5.51 _a	1	6	.89
Mastery	2.62	1.29	3.00	.34	2.56 _b	1.29	3.00	.37	2.66 _b	1.57	3.00	.30
Performance approach	2.01	1.00	3.00	.57	2.01	1.00	3.00	.52	2.01	1.00	3.00	.60
Avoidance	1.62	1.00	2.89	.44	1.53 _c	1.00	2.67	.35	1.68 _c	1.00	2.89	.49
Avoidance behaviour	2.49	1.00	5.00	1.19	2.71 _d	1.00	5.00	1.16	2.32 _d	1.00	5.00	1.19
Maths test 0	6.14	0	10	2.28	6.22	0	10	2.24	6.09	0	10	2.32
Maths test 1	16.97	4	20	3.07	17.10	9	20	2.65	16.87	4	20	3.37
Maths test 2	34.12	14	40	4.22	34.50	26	40	3.42	33.82	14	39	4.75
Maths test 3	47.89	27	58	5.95	47.96	29	58	5.23	47.83	27	58	6.44

Note: Subscripts with the same letter indicate a significant difference between boys and girls at $p < .05$.

Table 2. Sample correlation matrix of observed maths scores, intra-class correlations, and variance estimates.

	Time 0	Time 1	Time 2	Time 3	ICC	WL variance	BL variance
Time 0		.50*	.39	.52*	.13	4.53 (.50)	.71 (.30)
Time 1	.48***		.81***	.84***	.32	6.63 (1.24)	3.15 (1.35)
Time 2	.43***	.57***		.92***	.28	13.21 (2.49)	5.14 (2.44)
Time 3	.48***	.60***	.69***		.30	26.02 (3.40)	11.01 (4.19)

Note: Within- and between-level correlations are presented below and above the diagonal, respectively. Standard errors are presented in brackets. ICC = intraclass correlation; WL = within level; BL = between level. *** $p < .001$.

Table 3. Unstandardised estimates (and standard errors) from unconditional model.

	Means	WL variance	WL covariance	BL variance	BL covariance
Intercept	46.98 (.84)	18.26 (3.84)	5.12 (1.39)	3.03 (1.36)	0*
Slope	13.62 (.44)	1.22 (.80)		0*	

Note: WL = within level; BL = between level. Although the within-variance of the slope factor was non-significant, the fit of the model would have decreased considerably if we had fixed this parameter to 0. *The parameter was set to 0.

We next examined which individual characteristics would account for the variance in the two growth factors at the individual level, and which teacher characteristics would explain differences in the final level of maths skills across different classrooms. Gender, teacher-reported avoidance, self-reported avoidance, mastery approach, performance approach, and verbal reasoning were specified as within-level variables, while teaching methods and years of teaching experience were treated as classroom-level covariates. In addition, we investigated whether pre-maths skills would have an influence on the growth components at the individual level and on final maths skill status at the classroom level. As most of the covariates were measured in Grade 3, we were only able to estimate the paths from these variables to the final status of maths skills. Only pre-maths skills and gender were included as predictors of the slope at the within level. The within- and between-level associations were estimated simultaneously. Nonsignificant paths were eliminated from the final model. As pre-maths skills did not have any effect on the final status at the between level, it was specified as a within-level variable. The fit indices of the final model were: $\chi^2(14, n[\text{within}] = 253, n[\text{between}] = 20) = 38.68, p < .001$, CFI = .95, and RMSEA = .08. The final model is presented in Figure 1.

At the individual level, children showing higher levels of pre-maths skills developed their maths skills at a faster rate and had higher maths scores at the end of Grade 3. In addition, children with higher levels of verbal reasoning skill and lower levels of self- and teacher-reported avoidance had higher final maths scores. Individual-level predictors explained 24% ($p < .01$) and 46% ($p < .001$) of the variance in the slope and intercept, respectively. At the classroom level, teaching experience and formalist teaching methods were positively associated with the intercept factor. Thus, children had higher levels of final maths skills in classrooms where teachers had more teaching experience and used formalist teaching methods more frequently. These two

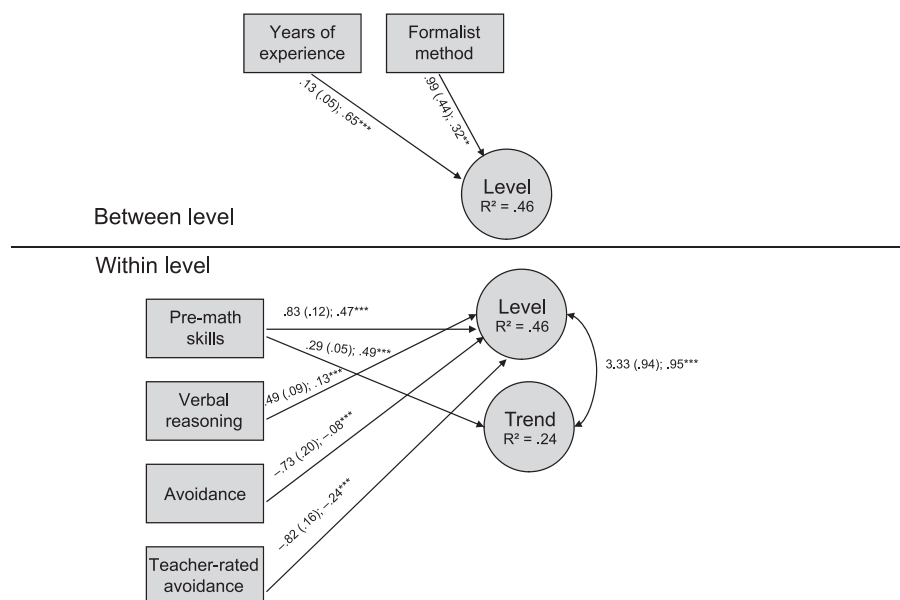


Figure 1. Final model.

predictors explained altogether 46% of the variance ($p < .01$) in the intercept. Combined and traditional teaching methods did not make any unique contribution to the final status of maths skills.

Discussion

The purpose of the current study was to explore the development of maths skills in elementary school children over the course of three years. As expected, maths development was positive from Grade 1 to Grade 3. Moreover, children whose maths skills developed faster also had better maths performance at the end of Grade 3. In addition, at the individual level, whereas preliminary knowledge and verbal reasoning were positively linked to final maths achievement, avoidance orientations (as assessed by student reports) and behaviours (assessed by teacher reports) were negatively associated with maths achievement in Grade 3. At the classroom level, teacher experience and preference for using formalist teaching methods had a positive effect on students' maths performance.

At the individual level, preliminary maths skills was the strongest predictor of maths achievement in Grade 3. This result is concordant with earlier studies showing that the level of skill with which children start the school plays an important role in influencing future performance (Burchinal et al., 2002; Lerkkanen et al., 2005). However, in contrast to findings by Burchinal et al. (2002), who showed that children with lower levels of pre-maths skills (at age four) developed at a faster rate (until age

eight), we found that children showing higher levels of pre-maths skills developed their maths skills at a faster rate and had higher maths scores at the end of Grade 3. This suggests that children who have better initial skills tend to improve at a faster rate than children who have lower initial skills, thereby increasing the gap between high and low performers over time. It is possible that some children may progress slowly and acquire maths skills and knowledge with a time lag. Geary (1994) has described a group of children with maths disabilities (related to using immature procedures) who are characterised by great difficulties at the beginning of the school, but who catch up with others within some years. Similarly, Crown (1990) has proposed that some children acquire the skills and knowledge one to two years after the material has been taught to them.

One of the reasons why children with higher pre-maths skills developed faster may be due to the nature of the tasks. Among the IPMA tests, the most difficult are the problems that require application of conceptual knowledge and reasoning skills (see Palu & Kikas, 2007a). Whereas more skillful students are independently able to understand the formal structure of these tasks, other students with poorer skills are not (Krutetskii, 1976). These less skillful students need individualised special support from their teachers. In general, Estonian education is achievement-oriented, with high academic standards (including fluent reading and calculating) set at the beginning of the first grade. Also, as the maths curriculum is so full (see Pihlap et al., n.d.), there is no time for students with lower abilities to practice enough. Estonian education is oriented toward children with average or above-average preliminary skills and knowledge. The supportive role of individualised teaching has been discussed, but has not been put into practice in classrooms.

It is also possible that children with higher pre-maths skills had received additional teaching, either in special pre-school groups or at home. In both cases this means that parents had been intensively involved in their child's educational process before school. The parents of these children might continue to help their children. This proposition that children's faster development is related to family background should be more thoroughly explored by future studies. Jordan, Kaplan, and Hanich (2002) have found that parental income level (which is related to parental education levels) predicted general development in maths among second- and third-grade children with learning difficulties. Also, Luyten et al. (2003) found that the initial difference in arithmetic performance increased between students with poorly-educated parents and students with well-educated parents.

In addition to pre-math skills, verbal reasoning was positively associated with maths achievement. Verbal reasoning abilities are especially necessary to solve word problems, and also other tasks. Maths has a very particular language and being able to read, interpret, and respond in that language is central to being an effective learner (see Krutetskii, 1976). The tasks used to assess students' verbal ability were complex, demanding the ability to integrate information, draw conclusions about a concept, and write down the correct word – which means that they assessed both verbal reasoning skills and working memory capabilities. Earlier studies using different verbal and reasoning measures also demonstrated the importance of these abilities in solving maths tasks (Delgado & Prieto, 2004; Fuchs et al., 2006; Nunes et al., 2007). Moreover, the role of working memory in solving different tasks is widely known (Bull, Espy, & Wiebe, 2008; Passolunghi, Mammarella, & Altoè, 2008; Passolunghi & Siegel, 2004; Wilson & Swanson, 2001). Future studies could shed light on the associations between working memory and other individual as well as contextual factors.

Finally, an avoidance orientation (self-reported) and avoidance behaviour (teacher-reported) were inversely related to third-grade maths achievement. Such an effect has been demonstrated in several earlier studies that have examined achievement in maths (Aunola et al., 2003; Onatsu-Arviolommi & Nurmi, 2000; Onatsu-Arviolommi et al., 2002; Van den Broeck et al., 2005) and in other areas (Leondari & Gialamas, 2002; McGregor & Elliot, 2002; Middleton & Midgley, 1997; Pajares, Britner, & Valiante, 2000; Wolters, 2004). In contrast to a study by Onatsu-Arviolommi et al., (2002), we found that both teacher-ratings and self-ratings had an independent (negative) effect on maths achievement. Mastery and performance orientation (collected via self-reports) did not have any unique effect on maths achievement in Grade 3. Earlier results regarding associations between achievement (or grades) and mastery and performance orientation have been mixed (Midgley et al., 2001; Pajares et al., 2000; Pintrich, 1999). However, it has been argued that if the relative level of mastery orientation is high (which was the case in the current study), performance orientation might not necessarily have a negative effect on achievement.

As regards gender differences, girls and boys showed similar development and final level of math skills. Gender differences might become more evident when children get older (e.g., Ai, 2002). Studies conducted in elementary school have either not found a gender difference (e.g., Lachance & Mazzocco, 2006) or demonstrated a negligible difference favouring girls (Halpern et al., 2007). Also, no gender differences have been found in Estonian state-level achievement tests carried out in Grade 3 (*Tasemetööd 3. Klass*, 2006, 2007).

At the classroom level, there was significant variance in final maths status, but not in the rate of development. Also, inter-classroom differences in pre-maths skills were not predictive of differences in maths achievement in Grade 3. This might reflect the relative homogeneity of our final sample: children who were excluded from the final sample due to not having complete data had lower pre-maths skills. Although there might be different reasons for not completing the tests (e.g., being ill when the testing took place or having changed school), this group also includes children who repeated the class. Although it is not common, there are instances of class repetition in Estonian elementary grades (in general, 2% of elementary grade students repeat class).

With regard to the classroom-level predictors, only formalist teaching methods were positively associated with third-grade maths achievement. Constructivist (combining individual and social constructivism) and traditional teaching methods did not explain any unique between-classroom variance in final achievement. Thus, children had higher levels of final maths skills in classrooms where teachers used formalist teaching methods more frequently. This positive effect of formalist teaching may be due to the peculiarities of elementary school maths, where the emphasis is on studying the basics of maths. Mastering basic skills and procedures is frequently related to extensive practicing and the memorising of formulae (e.g., Geary, 1994). In elementary grades, social constructivist methods (e.g., group work) may not be appropriate because of the age of the children (Azmitia, 1996). These methods might be more beneficial in higher grades. In addition, our results might not be so surprising if we consider how maths performance was measured in the current study: the majority of the IPMA tasks assess children's factual and procedural knowledge and skills.

In line with other studies (Clotfelter et al., 2007; Kleckler, 2002; Meijnen et al., 2003), children's final maths performance was higher in classrooms where teachers had a longer history of teaching. Compared to novice teachers, experienced teachers have certain priorities and aims, and they are able to choose adequate methods for

gaining these goals and supporting the development of each student (Berliner, 1994). With practice, some teaching procedures become automatised, which gives more time for dealing with more difficult problems and paying attention to students' individual needs.

We should also highlight some of the limitations of the current study. First, verbal skills, motivational orientations and behaviours, and teacher preferences were studied only once (before the last testing in Grade 3). Thus, we were not able to examine their effects on maths development. Second, as children with incomplete data had lower pre-maths skills than children who were included in our final sample, our results are not generalisable to the more heterogeneous population. In addition, the relative homogeneity of our sample potentially influenced variance estimates at the classroom level. Third, total maths scores were used in the analyses; however, maths skills are hierarchical, and recent studies have shown differential effects of cognitive abilities on different maths skills (e.g., Fuchs et al., 2006). Fourth, the children's family background was not solicited; however, parental involvement in children's pre-school and elementary school studies might be related to their pre-maths skills and their school achievement. Fifth, information about teaching methods was solely based on teachers' self-reports. Although we tried to minimise the possibility that teachers would give socially desirable answers, future research should employ diverse methods (such as observations or interviews) in order to validate information obtained via questionnaires. Furthermore, we should point out that as teaching methods may also reflect teachers' general beliefs, classroom management strategies, and styles (Stipek, Givvin, Salmon, & MacGyvers, 2001), more research is needed before drawing any conclusions about which teaching methods are effective *per se*.

In summary, the goal of the current study was to examine the development of maths skills in Estonian children from Grade 1 to Grade 3. We found that pre-maths knowledge had the strongest effect on maths skills in Grade 3, and that children with higher pre-maths skills developed at a faster rate than children with lower pre-maths skills. It seems that more attention should be paid to teaching maths skills and knowledge to pre-school children. Otherwise, a group of children can stay behind from early on. Also, individualised teaching should be used in the elementary grades, which can be especially helpful for children who have poorer initial skills. As both avoidance orientations and avoidance behaviours were related to lower maths performance, teachers should also pay more attention to children who are afraid of negative feedback and challenges, and who do not persist when tasks become difficult. In general, teachers used traditional teaching methods most frequently, followed by constructivist and formalist methods. At the classroom level, children performed better when teachers used formalist teaching methods more frequently. During the elementary school years children begin their studies in maths, and need to practice how to use formal maths language and procedures (cf. Geary, 1994; Siegler, 2005). It seems that elementary grade teachers should not be afraid of using formalist methods alongside traditional and constructivist ones. In maths, the development of procedural skills and the development of conceptual knowledge are intertwined (see Rittle-Johnson et al., 2001). Future research could reveal how changes in motivational factors and verbal skills relate to changes in maths development. Also, it is important to follow development over a longer time period and explore how different maths skills unfold with age. Finally, future work could shed light on the long-term impact of different teaching methods on maths performance.

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Note

1. We also ran the analysis where the intercept reflected the initial status of maths skills (instead of the final status). Similarly, we found a high overlap between the two growth factors at the individual level.

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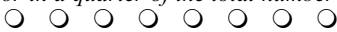
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Appendix. Examples of tasks according to cognitive domain

Knowing

1. *Fill in the missing numbers.*
(a) $27 + 12 = \dots$ (b) $35 - 3 = \dots$
(c) $15 + 17 = \dots$ (d) $46 - 18 = \dots$
(e) $73 + \dots = 99$ (f) $43 - \dots = 27$
2. *Write the following numbers in digits:*
(a) *seven hundred and sixty one.*
(b) *three hundred and nine.*

Applying

1. *Color in a quarter of the total number of circles.*

2. Mary buys two sweets costing 20p and 23p. What is her change from 50p?

Reasoning

1. *Fill in the missing number 3, 9, 27,*
2. *Peter thinks of a number. He multiplies it by three, takes away 2 and gets 25. What was his number?*

III

Palu, A., & Kikas, E. (2010). The types of the most widespread errors in solving arithmetic word problems and their persistence in time. In A. Toomela (Ed.), *Systemic Person-Oriented Study of Child Development in Early Primary School* (pp.155–172). Frankfurt am Main: Peter Lang Verlag.

Chapter 7

The Types of the Most Widespread Errors in Solving Arithmetic Word Problems and Their Persistence in Time

Anu Palu and Eve Kikas

Arithmetic or the teaching of numbers and operations constitutes the lion's share of the primary school math. Word problems play a significant role in arithmetic in several ways. First, word problems make arithmetic more realistic and meaningful to pupils and, therefore, should make math easier to learn (Dowker, 2005). Second, these problems support the development of students' thinking processes in general as their solving includes the same stages as when solving other types of problems (see Gick, 1986). Although learning to solve word problems is essential in children's math education, many children struggle with it (Geary, 2006). Children tend to exclude real-life information from problems and thus, the idea of bridge-building is not realized in school (see Verschaffel, de Corte, & Lasure, 1994; Xin, Lin, Zhang, & Yan, 2007). Also, as for other problems (see literature about expert-novice differences, e.g., Chi, Feltovich, & Glaser, 1981; Larkin, McDermott, Simon, & Simon, 1980), children tend to see only the surface structure of the problem, pay little attention to its identification, and use inadequate strategies to solve math problems.

Although previous studies have largely concentrated on the comprehension of word problems (e.g., Clarkson, 1991; Kintch & Greeno, 1985), surprisingly little attention has been paid to the types of errors that children with different levels of math and reading skills make when solving multistep arithmetic word problems, and also on their persistence in time. The analysis of mistakes enables to find false strategies behind the solution (Fleischner & Manheimer, 1997; Woodward & Howard, 1994), which, in turn, helps teacher to modify the teaching methods. Methodically proper arrangement of solving word problems is instrumental in improving children's skills of generalization as well as analysis and synthesis. It is therefore important to understand how children develop problem-solving skills and identify the sources of problem-solving difficulty. Thus, the aim of the study was to examine the types of mistakes that children make when solving two different arithmetic word problems, and their persistence over one year.

Definition of an Arithmetic Word Problem

In mathematics education, the term *word problem* is often used to refer to any mathematical exercise where significant background information of the problem is presented as text rather than in mathematical notation. Word problems are defined as verbal descriptions of problem situations. Each problem embeds one or more questions that can only be answered by first constructing an understanding of the mathematical relationships in the text (Verschaffel, de Corte, & Greer, 2000). A word problem, or verbal problem, is simply a question which requires the application of mathematics in order to achieve a solution, but in which the required procedure has to be extracted initially from within sentences. These sentences are often intended to provide a real-life setting for a simple task (Orton, 2005). To be able to solve a word problem a student must identify the question embedded in the text, choose an appropriate method and conduct necessary calculations.

Arithmetic word problems are defined as linguistically presented problems requiring arithmetic solution. An arithmetic word problem embeds a question which can be answered by performing arithmetic operations (addition, subtraction, multiplication and division). Any arithmetic word problem consists of the number(s) to be found and the given numbers. The peculiarity of the word problem lies in not directly indicating the required operation(s) in the exercise. The word problem presents relationships between the given numbers and the number(s) to be found, on the basis of which one has to select the arithmetic operation(s). The problem is solved by detecting the link between the data and what is to be found, and by conducting the necessary calculations according to the appropriate arithmetic operation.

Difficulties with Solving Arithmetic Word Problems

When a student solves a problem incorrectly, an error may be made at different stages, and it does not necessarily represent a mathematical error. The following four stages in solving word problems may be differentiated (see Dockvell & McShane, 1995; Newman, 1983, quoted in Zevenbergen & Wright, 2004): 1) reading and comprehending the text; 2) carrying out a mental transformation from the words of the question by building a mental representation of the problem; 3) selecting of an appropriate mathematical strategy; 4) applying the strategy and encoding the answer in an acceptable written form. This model indicates that reading, as well as mathematical, competence is needed for solving the problem. Similarly to other areas (e.g., Chi et al., 1981; Larkin et al., 1980), the main difficulties lie in comprehending the task and selecting the appropriate strategy. For instance, Ellerton and Clements (1996, quoted in Clements & El-

lerton, n.d.) found that 80% of errors occurred when reading, comprehending, and selecting the strategy and only 6% when applying the strategy. Therefore, special attention needs to be paid to these difficulties.

Comprehending text and representing the problem. At these stages, student should be able to read and comprehend the text (Dockvell & McShane, 1995; Newman, 1983, quoted in Zevenbergen & Wright 2004). Several studies have established the connection of comprehending word problems with the pupils' reading skills (e.g., Mercer & Sams, 2006; Thurber, Shinn, & Smolkowski, 2002; Verschaffel et al., 2000).

The main difficulty with solving math word problems is that students are unable to recognize the structure of the exercise, that is, to comprehend the relationships between the elements of the exercise and to express them mathematically. Instead of mathematical relationships, they see concrete items to be dealt with (Krutetskii, 1976). It means that students see only visible features and surface, not deep structure of the problems (cf. Chi et al., 1981; Silver, 1981). They also tend to spend time on familiar procedures without making sure they followed a correct solution plan (see Schoenfeld, 1992).

Research has shown that semantic structure of word problems influences children's ability to solve these problems (e.g., Carpenter & Moser, 1983; de Corte & Verschaffel, 1991; Stern, 1993). Most arithmetic word problems can be classified into four general categories: change, combine, compare or equalize (e.g., Geary, 1994). In different types of problems on, the content elements have been presented with varying clarity and, respectively, cause comprehension difficulties in children to a different extent. For example, "more flowers" may mean adding when the problem is of change-add-to type ("*Mary has three flowers. She buys two more flowers. How many flowers does she have now?*"), but it means subtracting when the problem is of change-take-from type ("*Mary has five red flowers. She has two more than Jane. How many flowers does Jane have?*"). However, children tend to connect the key word *more* with adding. Thus, the ability to understand word problems is influenced by key words, such as *more* or *less*.

The degree of complexity of a word problem is also determined by its formulation. Some studies have found that simply rewording the problems makes them more accessible to students (e.g., Vicente, Orrantia, & Verschaffel, 2007). Laborde et al. (1990, quoted in Orton 2004) found that the order of information, the relations between the known and the unknown and transition from the known and the unknown influence understanding of a word problem in younger learners. Problems where the unknown set is the solution are more frequently solved correctly than when the set is at the start of the problem as the problem's model should be constructed first (e.g., Kintsch & Greeno, 1985).

Beside reading skills and formulation of the problem itself, comprehension of relationships can be influenced by the math abilities of the pupil (Krutetskii,

1976), a component of which is the formalized perception of mathematical material. Krutetskii claims that children with lesser abilities do not see mathematical relationships but concrete items to be dealt with, so that they proceed to solving the problem right after having read it and without much consideration.

Selecting an appropriate strategy and implementing this strategy. In solving the problems, the strategy is selected according to how the exercise has been comprehended in the first stage. Additionally, however, errors may emerge in selecting the strategy. One of the strategies that pupils use is mechanic adding, subtracting, multiplying, or dividing whatever numbers are given in a problem. Schoenfeld (1991) has shown that students similarly combine random operations from the numbers given in the text. Sowder (1992) described some strategies that children use to decide which operation is appropriate. These included 1) finding the numbers and adding them; 2) guessing the operation; 3) calculating all possibilities and selecting the most plausible of these; 4) looking for keyword to signal the correct operation; 5) inferring the operation from the size of the numbers. Such non-semantic strategies are often successful, specifically when solving routine one-step problems. In case of multistep problems, strategies for solution are harder to select, therefore presenting a halfway solution is often the case. Ryan and Williams (2007) who studied errors of 4-15-year-old children in solving mathematical exercises found that when presented with a problem requiring two steps, children often respond by performing just one step.

Aims and Hypotheses

As in other countries (see e.g., Geary, 2006), Estonian primary school children struggle with word problems. The results of national achievement tests at the end of the third grade have shown that this is the most difficult part of math (Kaasik, 2004). The research conducted during the international project IPMA showed that Estonian third grade children have acquired calculation skills very well but are not able to implement them in solving word problems (Palu & Kikas, 2007). When a child runs into difficulties, in terms of teaching it is important to know what the specific causes for poor results are. Analysis of errors helps to find reasons for difficulties in problem solving (Fleischner & Manheimer, 1997) and support teachers in planning their activities and developing word problem solving skills. Previous research has paid a lot of attention on comprehending the content of word problems, but not so much on the types of mistakes that children make when solving multistep arithmetic word problems.

The purpose of this research is to find out the errors made in solving word problems and assess their persistence in one year. The same set of pupils is tested twice—in third and fourth grades. The specific aims and hypotheses of this study were the following.

First, to describe the most frequent mistakes by third and fourth grade children in solving two word problems. It can be assumed that some pupils find it difficult to comprehend the structure of the exercise and the relationships presented in it (Carpenter & Moser, 1983; de Corte & Verschaffel, 1991; Krutetskii, 1976). Such pupils are likely to make two types of mistakes: 1) they pay no attention to the relationships between numbers and mechanically add, subtract, multiply, or divide whatever numbers are given in a problem (Schoenfeld, 1991); 2) they can comprehend the text partially and only perform some operations in a multistep problem (Ryan & Williams, 2007).

Second, to examine the distribution of mistakes among children with poor, average, and good results in math and reading tests in both grades. As the comprehension of word problems is influenced by reading skills (e.g., Geary, 1994; Verschaffel et al., 2000) and mathematical abilities (Krutetskii, 1976), it can be suggested that pupils who are weaker in reading as well as in math have trouble finding operations appropriate to relationships and make different errors.

Third, to analyze if children give similar types of answers to different problems. Earlier studies have observed difficulties related to types of exercises (e.g., Carpenter & Moser, 1983). Teaching focus on particular type of exercises presents the danger of leaving general skills of problem solving undeveloped. However, if it is known that some mistakes are made by pupils regardless of the type of exercise, developing problem solving skills can be better organized. Although problems used in this study differ in their semantic structure, we expect that children tend to use the same strategy in solving different problems.

Fourth, to examine how stable is the making of a specific mistake over one year period. In a year, children's reading skills as well as problem solving skills are improved. As children's conceptual knowledge in math develops, they become more flexible in their choice of solution (Carey, 1991) and see the whole structure of the problem (Krutetskii, 1976). Therefore we assume that some types of errors will change over time.

Method

Sample and Procedure

Primary school students from Estonia were tested during two months twice within one-year period: at the beginning of third grade and at the beginning of fourth grade. In the current paper, we analyze the data from 494 students who participated twice in math and reading tests.

The math tests were carried out in writing during math lessons, and reading tests during native language lessons, each lasting about 45 minutes. The tests

were administered by the class teacher. The results were not assessed by the teacher.

Tests

Math tests. The tests were developed by the first author of the paper. When choosing the tasks, the learning outcomes of third grade math prescribed by the Estonian National Curriculum (Põhikooli ja gümnaasiumi..., 2002) were taken into consideration. The third grade test included 20 tasks from the Numbers domain. Also, these tasks cover three cognitive domains as recommended in the TIMMS (Trends in International Mathematics and Science Study) framework: knowing, applying, and reasoning (see Mathematics framework, n.d.). To check the facts and procedures (knowing) there were two different types of tasks: computing (seven tasks e.g., $37 + 4 = \dots$; $43 - \dots = 37$; $250 \text{ kg} - 50 \text{ kg} + 700 \text{ kg} = \dots$) and recalling (two tasks e.g., “Write the preceding and following number: ... 709 ...”). To check the applying skills, five tasks were included, all of which were word problems. For example, “One pie costs 4 kroons. How many pies can be bought for 15 kroons?” and also “Andy spends 10 minutes for walking 1 km. The stadium is 3 km away. How much time will Andy spend to get to the stadium?” The reasoning tasks were assessed by two non-routine tasks which required ability to integrate learnt information (for example “Write two equations, using the numbers 2, 16 and 8”).

The fourth grade test included 20 tasks (30 subtasks) from three domains: Numbers (12 tasks), Geometry (five tasks) and Measurement (three tasks). By cognitive domains, the fourth grade tasks divided into the following: knowing (seven tasks), applying (six tasks) and reasoning (seven tasks). Examples:

Knowing: “Calculate the value of the expression $48 - 36 : 3$ ”;

Applying “The film showing at the cinema begins at the following times: session 1 at 2 pm; session 2 at 3.30 pm; session 3 at 5 pm. At what time does session 4 begin?”;

Reasoning: “John wanted to add 1379 and 243 on his pocket calculator. He mistyped it as 1279 and 243. What can be done to correct the error? a) subtract 100, b) add 1, c) subtract 1, or d) add 100.”

Each answer was coded as correct or wrong and the mean scores of the tests (sum of the correct answers divided by the number of subtasks) were calculated for both years. The internal reliability of the tests was good (Cronbach $\alpha = .75$, .82, respectively for Grade 3 and Grade 4).

Children were categorized into three ability groups according to their score in the math test both in third and fourth grades. In the Low group were students whose score in the math test was lower than mean minus 0.5 standard deviation (in third grade $< .66$ and in fourth grade $< .60$) and in the High group were stu-

dents whose test result was higher than mean plus 0.5 standard deviation (in third grade $> .84$ and in fourth grade $> .77$).

Three of the tasks were the same at both testing times. In the current paper, we analyze the specific answers of two word problems that were used in both grades. Problem A had the following instruction: „*Write a number which includes six ones digits, two more tens digits, and twice less hundreds digits than ones digits.*” Problem B was the following: „*Four pies cost 20 kroons and 1 bun costs three kroons. How much do 1 pie and 1 bun cost together?*” As seen, Problem A was a multistep problem which included two one-step compare type word problems where one and the same number (six) was the basis for comparison for finding out both ones digits and tens digits. Problem B was a multistep word problem where it was impossible to perform the second operation without performing the first: before finding the sum, one had to perform a division. The information in the Problem B was presented in different ways: some numerical data in words and some in digits (e.g., four pies, 1 bun).

We analyzed all the answers to both problems and differentiated between five categories (see examples in Table 1).

1. Correct answers.
2. Partial: the child applies the relationships in the task only partially.
3. Numbers: the child uses numbers provided in the text but does not perform any operations (problem A) or adds these numbers (problem B).
4. Other: the rest of incorrect answers.
5. Missing answers: the child did not solve the problem.

Reading tests. The Estonian language tests were developed by Krista Uibu (see Uibu, Kikas, & Tropp, this book). In the current paper, we analyze answers of a reading comprehension task. Children had to read a text (a poem in third grade and parable in fourth grade) and answer to the questions about the text. There were eight statements after the text. Pupils had to decide on the basis of the text which of the statements were true and which were false. All the answers were coded as correct or wrong and the mean scores of the tests (sum of the correct answers divided by number of subtasks) were calculated for both years. Children were categorized into three ability groups according to their score in the reading test both in third and fourth grades. In the Low group were students whose score in the reading test was lower than mean minus 0.5 standard deviation (in third grade $< .79$ and in fourth grade $< .59$) and in the High group were students whose test result was higher than mean plus 0.5 standard deviation (in third grade $> .96$ and in fourth grade $> .88$).

Results

Types of Mistakes in the Word Problems

The first task of our research was to find out which are the most common mistakes in the solutions by pupils. As we assumed, the mistakes divided into two groups in both kinds of exercises.

Table 1 provides the percentage of children in different categories of answers for Problem A in grade 3 and 4. Problem A was solved correctly by 16.0 % of the pupils in third grade and by 39.6 % in fourth grade. The most common wrong answers to Problem A can be grouped in two: 1) Partial: answers that are partially right (the pupil performs one of the operations correctly) and 2) Numbers: answers that have been provided without performing any operations (the pupil lists the numbers in the text).

Table 2 provides the percentage of children in different categories of answers for problem B in grade 3 and 4. Problem B was solved correctly by 57.4 % of pupils in third grade and 74.2 % in fourth grade.

TABLE1 Different Types of Answers and Percentage of Children Giving These Answers for Problem A

Answer	Type	Description	Grade 3	Grade 4
386	Correct	Problem solved correctly	16.0%	39.6%
No answer	Other	Problem unsolved	7.5%	4.3%
486	Partial	Applies the relationships presented in text only partially correctly (uses the notion “x less” rather than “x times less”)	36.5%	24.0%
286 or 686	Partial	Applies the relationships presented in text only partially correctly (unable to use the notion “x times less”)	10.0%	3.3%
622 or 226	Numbers	Does not perform operations. Just lists the numbers in the text.	6.7%	1.9%
426	Partial	Applies the relationships presented in text only partially correctly (only detects ones digits).	0.8%	3.3%
Other answers	Other		22.5%	23.5%

TABLE 2 Different Types of Answers and Percentage of Children Giving These Answers for Problem B

Answer	Type	Description	Grade 3	Grade 4
8	Correct	Problem solved correctly	57.7%	74.2%
No answer	Other	Problem unsolved	5.4%	2.4%
23 or 21	Numbers	Adds the numbers in the text: $20 + 1 + 1 + 1 = 23$ or $20 + 1 = 21$	11.5%	6.5%
6 or 7	Partial	Performs the operation on the first relationship correctly but then adds the numbers in the text: $5 + 1 = 6$ or $5 + 1 + 1 = 7$	7.2%	4.1%
Other answers	Other		18.4%	12.8%

The most common wrong answers to Problem B can also be grouped in two: 1) Partial: answers which are partially right (the pupil only performs the first operation in a multistep task) and 2) Numbers: answers that have been provided without performing any operations (the pupil adds the numbers in the text).

Mistakes and Correct Answers in Different Achievement Groups

The second task of our research was to examine the distribution of mistakes among children with poor, average, and good results in math and reading tests. For that, we used Configural Frequency Analysis (CFA). CFA is an extension of χ^2 -analysis and it examines patterns in categorical variables (see Bergman, Magnusson, & El-Khoury, 2003; von Eye, 1990). The answers are written into a table, and CFA tests whether in each cell the observed frequency differs significantly from the expected frequency. The goal of this comparison is to determine whether the difference between the observed and the expected frequency for a given configuration is larger than some critical value and is statistically significant. The results of the analysis reveal types (observed frequency is significantly higher than expected frequency) and antitypes (observed frequency is significantly lower than expected frequency). An exact test for the comparison of the observed frequency with expected frequency is the binomial test. The analysis was performed with the CFA module of the program SLEIPNER 2.1 (Bergman & El-Khoury, 2002). Alpha levels are adjusted with Bonferroni's adjustment.

We used achievement groups (math and reading: low, average, high), and four types of answers and carried out analyses separately for both grades and

TABLE 3. Statistically Significant Types or Antitypes in Problem A

Grade	Math achievement group	Reading achievement group	Type	Observed	Expected	<i>p</i>
Third	Low	Low	Correct	0	5.97	.0887
Third	Low	Low	Other	27	10.90	< .001
Third	Low	Low	Numbers	11	2.71	.004
Third	Low	Average	Correct	0	6.67	.043
Third	Low	High	Correct	0	10.59	< .001
Third	Low	High	Other	38	19.34	.003
Third	High	High	Correct	45	15.82	< .001
Third	High	High	Other	10	28.87	.001
Fourth	Low	Low	Other	25	6.53	< .001
Fourth	Low	Low	Partial	6	.45	< .001
Fourth	Low	Average	Other	60	31.72	< .001
Fourth	High	Average	Correct	90	58.47	.001
Fourth	High	Average	Other	18	41.17	.001
Fourth	High	Average	Partial	24	45.31	.010
Fourth	Low	High	Correct	0	20.06	< .001
Fourth	High	High	Correct	66	26.04	< .001
Fourth	Low	Average	Correct	14	45.05	< .001

Note. Types in **bold**

problems. The four types of answers (see Tables 1 and 2) were the following: 1) correct; 2) partial (partially correct solution, the relationships in the text are only partially applied); 3) numbers (simple writing or adding of numbers); 4) other (other mistakes or no answer). Below, only the types and antitypes will be brought out and only types for mistakes described; the observed frequencies, expected frequencies and the *p* values and exact distribution of answers are provided in Tables 3 (for Problem A) and 4 (for Problem B). CFA revealed 9 types and 8 antitypes for Problem A (see Table 3) and 9 types and 11 antitypes for Problem B (see Table 4).

CFA revealed that in both grades there were more pupils than expected who gave different incorrect answers for both problems in groups with low math and reading scores and low math and average reading scores. In third grade, there were also more pupils than expected with different incorrect answers in low math but high reading group. In both grades, more students with low math and reading scores than expected did not perform operations according to given relationships but combine numbers (in both problems). In fourth grade, such combination answers were revealed as types only for problem B and in groups of children with low math and average reading score.

The third task of our research was to analyze if children give similar type of answers to problem A and B. We carried out CFA on both problems with four types of answers (see Table 5). CFA revealed that there were more pupils

TABLE 4 Statistically Significant Types and Antitypes in Problem B

Grade	Math achievement group	Reading achievement group	Type	Observed	Expected	<i>p</i>
Third	Low	Low	Other	22	8.47	.002
Third	Low	Low	Numbers	13	3.89	.007
Third	Low	Low	Correct	8	22.22	.015
Third	High	Low	Other	2	12.65	.010
Third	Low	Average	Correct	5	24.82	< .001
Third	Low	Average	Other	21	9.46	.026
Third	High	Average	Other	1	14.13	< .001
Third	Low	High	Correct	12	39.42	< .001
Third	Low	High	Other	29	15.03	.026
Third	High	High	Correct	102	58.85	< .001
Third	High	High	Other	1	22.44	< .001
Third	High	High	Numbers	0	10.30	< .001
Fourth	Low	Low	Other	26	3.59	< .001
Fourth	Low	Average	Correct	59	84.34	.050
Fourth	Low	Average	Other	39	17.39	< .001
Fourth	Low	Average	Numbers	20	7.39	.003
Fourth	High	Average	Other	4	22.57	< .001
Fourth	Low	High	Correct	14	37.64	< .001
Fourth	High	High	Correct	94	48.86	< .001
Fourth	High	High	Other	2	10.07	.090

Note. Types in **bold**

than expected by chance who 1) solved both problems correctly, 2) made different mistakes (not the typical mistakes detected by us) in both problems, 3) combined numbers in both problems, and 4) combined numbers in Problem A and made other mistakes in Problem B and vice versa.

TABLE 5 Types and Antitypes in Problem A and B

Type		Observed	Expected	<i>P</i>
Problem A	Problem B			
Correct	Correct	363	281.08	< .001
Other	Correct	221	287.69	< .001
Numbers	Correct	14	41.67	< .001
Correct	Other	31	82.40	< .001
Other	Other	136	84.34	< .001
Numbers	Other	32	12.22	< .001
Correct	Numbers	11	37.82	< .001
Other	Numbers	60	38.71	< .012
Numbers	Numbers	15	5.61	< .011

Note. Types in **bold**

The Persistence of Types of Mistakes in Time

The fourth task of our research was to examine the persistence of types of mistakes in time in different math achievement groups. Different types and antitypes emerged for problem A (Tables 6) and problem B (Table 7). For problem A, there were more pupils than expected with low math scores in both grades who 1) combined numbers in grade 3 and made other mistakes in grade 4, and 2) made different mistakes in Grade 3 and solved the problem partially in Grade 4. In high math score groups, there were more pupils who solved the problems par

TABLE 6 Types or Antitypes in Problem A Third and Fourth Grade Achievement Groups

Type		Math achievement group		Observed	Expected	<i>p</i>
Grade 3	Grade 4	Grade 3	Grade 4			
Other	Other	Low	Low	35	3.00	< .001
Numbers	Other	Low	Low	12	.63	< .001
Other	Numbers	Low	Low	19	3.29	< .001
Other	Partial	Low	Low	5	.13	< .001
Partial	Correct	High	Low	1	12.63	< .006
Partial	Partial	Average	Average	32	14.26	< .004
Other	Correct	High	Average	3	14.42	< .044
Correct	Correct	High	High	48	6.30	< .001
Partial	Correct	High	High	50	18.40	< .001

Note. Types in **bold**

TABLE 7 Types or Antitypes in Problem B Third and Fourth Grade Achievement Groups

Type		Math achievement group		Observed	Expected	<i>p</i>
Grade 3	Grade 4	Grade 3	Grade 4			
Other	Other	Low	Low	27	1.17	< .001
Partial	Other	Low	Low	9	.53	< .001
Other	Numbers	Low	Low	10	.60	< .001
Correct	Correct	Average	Low	7	22.15	< .021
Correct	Correct	High	Low	12	28.24	< .057
Other	Correct	High	Low	1	11.15	< .023
Correct	Correct	Low	Average	15	33.12	< .039
Other	Other	Low	Average	9	2.28	< .084
Correct	Correct	High	Average	81	54.80	< .044
Other	Correct	High	Average	4	21.64	< .001
Correct	Correct	Low	Low	2	24.86	< .001
Other	Correct	High	High	2	16.25	< .002

Note. Types in **bold**

tially in Grade 3 but gave correct answer in Grade 4. For problem B (see Table 7), there were more pupils than expected with low math scores in both grades who gave different incorrect answers in Grade 3 but combined numbers in Grade 4.

Discussion

The purpose of the current study was to analyze the types of mistakes that children with different levels of math and reading skills make when solving multistep arithmetic word problems, to examine their consistency when solving different problems, and their persistence in time. Primary school students were tested twice within about one-year period, in third and fourth grades.

First, we analyzed most frequent mistakes children make when solving two multistep word problems in third and fourth grades. As expected, two types of errors, made by several children, were detected. However, their frequency was different for two problems.

Partial answers were widely given for Problem A but less for Problem B. In these answers, the pupil had selected and performed one operation correctly. Either s/he did not know all the notions (such as “x times less”) or did not apply the knowledge correctly (was satisfied with one operation and considered the problem solved). In case of the last type of pupils, it cannot be stated that they leave the problem halfway solved because of not comprehending all relationships given in the problem. We can presume that the pupils were unable to comprehend the structure of the problem in its entirety. It also indicates that they have not analyzed the problem sufficiently but have proceeded directly to solving it (cf. Schoenfeld, 1992). It is possible that in case of a one-step problem, these pupils would be able to solve it. Solving a multistep problem is also connected to mathematical abilities: more skilful students are independently able to understand the formal structure of problems; other students with poorer skills are not (Krutetskii, 1976).

Simple combination of numbers was a widely used strategy in solving Problem B, but also in Problem A. In case of Problem A, the pupil listed the numbers in the text and in case of Problem B, the pupil added the numbers in the text. This indicates that the children are indeed satisfied merely with the obvious – using the numbers written in the text of the problem (cf. Silver, 1981). One of the misleading factors in Problem B was also that some of the numbers were written in words, others in digits. The number combiners mostly used the numbers written in digits. Another kind of numerical presentation would have yielded different results (Vicente et al., 2007).

In both grades and tests, the reasons for quite a number of errors remained unspecified. Those could have been simple calculation errors, mistakes in writ-

ing digits etc., the reasons for which are hard to guess solely on the basis of a written answer. Several children gave such divergent answers.

Second, we examined the distribution of mistakes among children with poor, average, and good results in math and reading tests. Carrying out Configural Frequency Analysis, we found typical combinations of answers, math and reading score groups. For both problems and grades, children in the low math score group tended to solve problems incorrectly. Although for most types, children belonged to low reading score groups, in some cases, their reading scores were either average or high. So, both math and reading skills play a role in solving word problems; however, math-related skills may be of greater importance. Krutetskii (1976) has specifically stressed the importance of math skills, a component of which is a formalized perception of mathematical material. Relations between reading comprehension and word problem solving skills have been shown by earlier studies as well (Mercer & Sams, 2006; Thurber et al., 2002; Verschaffel et al., 2000). Specifically in third grade, there were pupils in low math and reading achievement groups who did not perform operations according to the given relationships but combined the numbers. Apparently, such children do not understand the deep structure of the problem (cf. Krutetskii, 1976) but do not want to leave the problem unsolved, whereby they combine the numbers or add them mechanically. The pupils are motivated for a similar solution by the occasional success of this line of action: one of the randomly picked four arithmetic operations may prove appropriate. Such strategy of combination has also been indicated by earlier research (Schoenfeld, 1991; Sowder, 1992).

Third, we analyzed if children give similar type of answers to different problems. Our assumption that same mistakes may be made in solving problems of different semantic structure proved correct in terms of the so-called combinatorics. As previously shown by Schoenfeld (1991), it was also revealed by our research that there are pupils who solve the problem by combining the numbers independently from the structure of the exercise. However, many pupils solved just one problem by combining the numbers but made a different type of mistake in solving the other (the reasons for which could not be found out by us). Previous research with one-step problems has shown that semantic structure of arithmetic word problems influences the children's ability to solve these problems (e.g., Carpenter & Moser, 1983; de Corte & Verschaffel, 1987). Therefore it can be assumed that even if the child has some kind of inclination to prefer a strategy, the selection of the strategy is also influenced by the semantic structure of the problem.

Fourth, we examined how stable the making of specific type of mistake is in consecutive years, also taking into account the mathematical ability of the child. The results were different between the two problems, referring to the possible influence of the semantic structure of the problem (e.g., Carpenter & Moser, 1983; de Corte & Verschaffel, 1987). The results could also have been influ-

enced by the degree of complexity of the task—Problem A was generally solved with poorer results than Problem B. Problem A was a two-step problem in which the second relationship was not connected to the first. One typical combination was that the low math achievement group's pupils who combined the numbers in third grade tended to give either different incorrect answers or did not answer in fourth grade. Whereas in third grade the weaker pupils combined the numbers of that problem, in fourth grade they were evidently more analytical (better experts) and left the problem unsolved or made other errors (which could also be simple calculation errors). Another typical combination was that the low math achievement group's pupils who either did not answer or made different mistakes in third grade tended to solve the problem partially in fourth grade.

Problem B was also a two-step problem but unlike in Problem A, the second operation in Problem B could not be performed without performing the preceding operation. At the same time, numerical and verbal significations were mixed up. In problem B, the low math achievement group's pupils gave either different incorrect answers or did not answer to the problem in third grade, but in fourth grade they added the numbers mechanically. It is possible that Problem B seemed too difficult for the weaker pupils in third grade so that they left it altogether unsolved. In fourth grade, the pupils took notice of the obvious only, failing to see the deep structure (Krutetskii, 1976) and adding the numbers mechanically (Schoenfeld, 1991; Sowder, 1992).

Limitations

The study has also some limitations. First, only two word problems with different degrees of complexity were under survey. In future studies, it is worth using a greater variety of tasks. Second, written tests do not reveal the precise reasons for many mistakes, so that the reasons for the studied mistakes can also only be guessed. In further studies, there is a need for an interview between the teacher and the pupil which would help to find out about the pupil's deeper understanding of mathematical relationships and solution strategies. Also, the solving process of the pupils could be surveyed in order to find out about time management and strategies.

Conclusions

Analysis of errors helps to find reasons why some of the pupils run into difficulties with solving word problems. Teachers have to be aware that the wrong answer may result from a mistake made in reading the text of the problem, comprehending it and modeling the problem. They could also be familiar with strategies which the pupils use in solving word problems. Our research revealed that misunderstanding the text and the misguided selection of a solution strategy cause two significant types of errors in solving arithmetic word problems: combining the numbers (a child uses numbers provided in the text but does not perform any operations or adds these numbers) and partial solution of a multistep problem (a child applies only one relationship in the task). These errors are linked to the comprehension of the mathematical relationships given in the problem.

In order to prevent both types of errors, the teachers should pay greater attention to the consideration of the problem. Seeing the whole presumes analysis whereby it is proceeded from the question to the data: 1) what is asked in the problem; 2) what needs to be known to answer the question; 3) do we know it; 4) how to find what is missing and do we have data for it. If a teacher only uses synthesis for consideration (from the data to the question), the pupils with lesser abilities fail to see the whole structure of the problem. They try to do something with the given data, combining the numbers or solving the problem only partially.

It is often emphasized that the difficulties with solving word problems are primarily related to poor reading skills, especially to difficulties with comprehending the text. Earlier studies (Mercer & Sams, 2006; Thurber et al., 2002; Verschaffel et al., 2000) as well as our results also refer to such links. At the same time, the research revealed that also the pupils with good skills of functional reading can run into difficulties with comprehending a mathematical text. In order to solve the problem, one needs to see the deep structure, mathematical relationships (whether one has to add or subtract, multiply or divide, etc.), not the superficial, conspicuous features (e.g. numbers) (Krutetskii, 1976; Silver, 1981). Better comprehension of mathematical relationships in a problem is fostered by a figurative presentation of the problem. One of the keys to improving the word problem solving skills might lie in a greater emphasis on the modeling of problems. By using a sketch, the pupils can visualize the problem. Using a chart of systematized data also helps the pupil to see the relationships and find the missing information by developing a certain system.

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