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Development and Performance Analysis of UKF Models for Satellite Position and Attitude Determination

Master's Thesis (30 EAP)
Robotics and Computer Engineering

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Resümee

Satelliidi asendi- ja orbiidi määramise süsteemi
UKF mudelite arendus ja testimine

Käesolev magistritöö keskendub satelliidi füüsika ja asendimääramise sensorite mudelite arendusele, keskkonnamudelite kohandamisele kasutamiseks optimeeritud UKF algoritmis ning kogu süsteemi täpsuse testimisele. Töö käigus seati üles töövahendite raamistik, mille abil kohandati UKF algoritm satelliidil töötamiseks sobivaks ning arendati välja tähekaamera, güro- ja päikesesensori mudelid. Mitmed vajalikud keskkonnamudelid kohandati UKF algoritmiga töötamiseks sobivaks. Orbitaaldünaamika osas pakutakse välja satellidi hinnatava asukoha täpsuse parandamise meetod, mida autorile teadaolevalt ei ole varem väikesatelliitidel kasutatud. Mainitud meetod rakendab satelliidi raadiosidesignaali leviku aja täpset mõõtmist, mille põhjal arvutatakse välja satelliidi asukoha parandusfaktor. Meetodi testimiseks lisati süsteemi maajaama ja satellidi omavahelist kaugust mõõtva ”sensori” mudel. Kõiki sensoreid ja UKF'i sensorite kombineerimise võimekust testiti erinevates olukordades, mis hõlmasid endas erineval hulgal sensorite kasutamist, vigaseid sensoreid ja erinevaid asendimanöövreid. Testimiseks vajalikud andmed genereeriti autori poolt arendatud simulatsioonikeskkonnaga, mis sisaldab endas kõiki antud ülesandeks vajalikke füüsika- ja keskkonnamudeleid. Testimise tulemusena leiti, et kõik väljatöötatud sensorite mudelid töötavad UKF algoritmiga ning tulemuste täpsus vastab sensorite konfigureeritud täpsusele. Näidati ka, et välja pakutud asukoha täpsuse parandamise meetod töötab ka oluliselt suuremate vigade puhul kui on tegelikul orbiidil oodata.

CERCS: T125 Automatiseerimine, robootika, control engineering

Keywords: kosmos, satelliit, asend, asendimääramine, orbiidi määramine, ukf

Abstract

Development and Performance Analysis of UKF Models for Satellite Position and Attitude Determination

This theses focuses on the development of models of satellite physics and attitude sensors, adaption of environmental models to be used in a highly optimised UKF algorithm for satellite attitude and orbit determination and the performance analysis of the system based on simulation data. During the writing of this theses a basic framework was set up to adapt the UKF algorithm for use in a satellite attitude determination environment and the star tracker, gyro and sun sensor models were developed. Several environmental models were adapted to work with the sensor models. Regarding orbital dynamics, a satellite communication ground station based orbital position correction method was proposed that, according to the knowledge of the author, has not been used on small satellites before. The method utilises the precise measurement of the communication signal return time to apply a correction factor to the orbital location of the satellite. To test the method, a ranging sensor model was devised. All sensors and the sensor fusion capability of the UKF were tested under varying operating conditions, which included different sensor combinations, sensor errors and different attitude manoeuvres. The required simulation data was generated using a simulation environment developed by the author. It contains the necessary physics and environmental models to produce realistic data. All models and the UKF algorithm were found to work well and the results match the accuracy configurations of the different sensors. The ground station ranging based orbital correction was shown to work well with considerably larger errors than would be encountered in actual orbit.

CERCS: T125 Automation, robotics, control engineering

Keywords: aerospace, satellite, attitude, orbit, determination, ukf

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EKF extended Kalman filter 8, 10

NORAD North American Aerospace Defence Command 14, 16

PSD photo-sensitive device 18

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std standard deviation 19, 20, 23

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1 Introduction

1.1 Satellite Location and Attitude Determination

Attitude determination and control is a crucial part of any satellite mission. It is required to point science experiments, communication antennas, solar panels, cooling radiators, etc in the correct direction. Most small satellites utilise several sun sensors, magnetometers and gyro sensors for attitude determination [1], [2], [3]. Some more advanced small satellites, requiring high precision attitude knowledge and control, have started to use star trackers as well [4]. The former are considerably simpler and more robust, however they require additional accurate environmental models to compare their measurements with. The latter is very accurate and self-sufficient, however requires good attitude control to keep the satellite in a stable attitude for it to be able to image the stars.

Whichever sensors are being used, to obtain a valid attitude solution, the different sensor measurements and model calculations need to be combined. This can be done using complex derivations and lots of complicated calculations or with statistical algorithms like extended Kalman filter (EKF) [5] or unscented Kalman filter (UKF) [1], [6]. This theses focuses on the development and performance testing of physics and sensor models for a fully implemented UKF that has been highly optimised to reduce the computational complexity allowing it to still run at sufficiently high iteration rates.

As mentioned above, both sun sensors and magnetometers require a location-based model to estimate the measured quantity in an Earth related reference frame. A widely used industry standard algorithm for spacecraft location and motion prediction is the Simplified General Perturbations (SGP4) algorithm by D. Vallado [7], [8]. For short time periods, up to a couple of days, its position accuracy for low Earth orbits is within a few tens of kilometres [9]. Based on ESTCube-1 experience, allowing a spacecraft to use a given set of orbital data for predictions with SGP4 for more than 2-3 days starts to significantly affect its ability to reliably determine its attitude.

A fairly common solution is to use a GPS receiver, however, on small satellites their power consumption may be prohibitive [8]. To avoid the extra power consumption while still maintaining an improved position estimation accuracy, this thesis also proposes and analyses a novel ”no extra power” orbital position correction method for small satellites.

1.2 Thesis Scope

The scope of this thesis includes the following:

- development of the satellite dynamics model,
- development of attitude sensor models,
- adaption of environmental algorithms to be used with the UKF (SGP4, Sun motion, etc.),
- analysis of the UKF attitude determination performance with different simulated datasets
- analysis of the possibility of using communication signal based radio ranging for improving on-board location estimation.

1.3 Notation

The following notation conventions are used throughout this thesis.

a	scalar
$\vec{a}$	vector (mostly 3-dimensional)
A	matrix
$\vec{a}_{ECI}$	suffixes ECI, ECEF, I, B denote different reference frames
$\vec{q}$	the symbol $\vec{q}$ specifically denotes a quaternion and not a vector
$\tilde{\vec{q}}$	quaternion conjugate
$\vec{a} \times \vec{b}$	vector cross product
$\vec{q_1} \otimes \vec{q_2}$	quaternion product
$[\vec{a}; 0]$	a vector represented as a quaternion, the scalar part is set to zero, used for vector rotations

2 Overview of the Used Tools

2.1 Unscented Kalman Filter

The UKF is a sensor fusion algorithm intended to predict the behaviour of a non-linear system based on a number of different measurements and models. The block diagram shown in figure 2.1 gives an overview of the operation of the UKF. The main idea behind a UKF is that instead of propagating the state vector through time using a linearised model, a number of noisy states (sigma points) are statistically sampled at each time step based on the augmented noise matrix and each of them is propagated. The expected sensor measurements are predicted for each sigma point separately and the results, then the results are combined with each other using predefined weights and eventually the predicted measurements are compared with the actual measurements taken by the sensors. From this the residuals (differences from the expectations) are calculated and covariance matrices updated which in turn are the basis for computing the updated state vector. [6]

A great advantage of a UKF is that its implementation does not require the complicated derivation of derivatives and linearisation of models as is the case for EKF. As it works on the statistical comparison of predicted and actual measurements, it only needs the models for propagating the state vector and predicting measurements based on the computed state vector [6]. This means that it is possible to incorporate additional parameters into the algorithm that can over time deduce sensor biases, orientation or other systematic errors, which is also implemented in this codebase.

The development of the base UKF algorithm is not part of this theses. The framework that implements the UKF mathematics used to conduct the work described in this theses was developed and is being actively updated by Milrem Robotics in cooperation with a Tartu Observatory workgroup.

2.2 Simulation Environment

To analyse the performance of the UKF and its models, simulated spacecraft attitude and orbit data is required. To generate this data, a combined spacecraft attitude and orbit simulator, developed by the author for this purpose, was used. Note, parts of the system are still under development. The general aim of the simulator is to provide a versatile non-commercial tool to aid in the development and testing of spacecraft attitude and orbit determination and control systems and software. The simulation environment has been designed using Matlab and Simulink software packages. Both products are widely used in the engineering industry and greatly simplify the building of complex non-linear differential equation based systems. The Aerospace Toolbox and Blockset add-ons

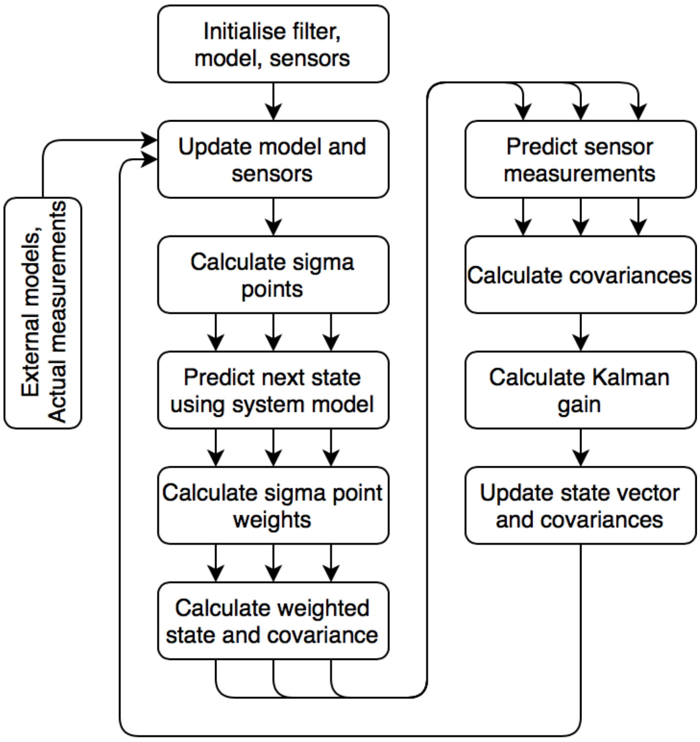

Figure 2.1: Overview of the UKF Algorithm

make the software especially useful for aerospace applications as these provide several complete and verified models of both Earth and celestial physics.

It is being considered by the author to re-design the whole system using free tools like Python or C++. This would completely avoid the need for any commercial software.

The main simulation has been designed as a Simulink model. The goal was to make it highly configurable and eventually extend it to allow any n-body simulations. In the current version only Earth orbiting simulations are supported. Matlab scripts are used to set up the simulation and configure the spacecraft and the environment in high detail (physical properties, orbital parameters, sensors, actuators, noise, random disturbances, etc).

Figure 2.2 shows the top level design of the Simulink model. The following sections describe the included components.

2.2.1 Spacecraft Attitude and Orbital Dynamics

The spacecraft is modelled as a rigid body using the standard Euler equation of rigid body rotation. The equation has been extended to include the effects of variable inertia and inertial actuators (reaction and momentum wheels) [11]. Configurable disturbances like gravity gradient torque, aerodynamic imbalance, residual magnetism, etc are added together with thruster outputs as external torques.

Orbital dynamics are calculated using continuous integration of Newton's equations of motion. The forces affecting the spacecraft include gravitational acceleration, atmospheric drag and thruster output. Currently there are some issues with the propagation of the orbital motion. Compared

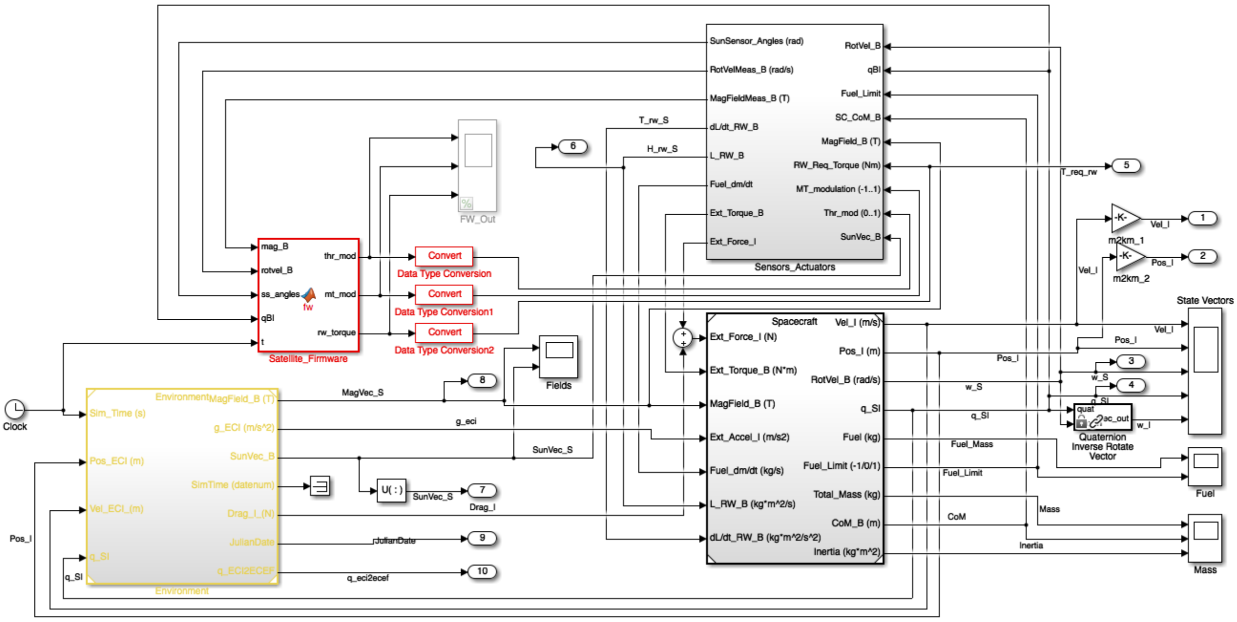

Figure 2.2: Top level Simulink diagram

to the fairly reliable SGP4 algorithm the orbital position and velocity diverge after a few hours. This is most likely due to the atmospheric model creating too little drag as the error remains within the orbital plane while the simulated position lags behind and remains higher than the SGP4 prediction. If the issue was with the gravity model, it should affect other orbital parameters as well, not just the motion in the orbital plane.

2.2.2 Sensors and Actuators

Models of sensors and actuators are included for software-in-the-loop testing of attitude determination and control algorithms. These include gyro and sun sensors, magnetometers, star tracker, thrusters, reaction wheels and magnetorquers. Any number, combination and configuration of sensors and actuators may be configured for a simulation run. Both types of devices have the following optional configurable parameters:

- Orientation with gaussian uncertainty
- Bias with gaussian uncertainty
- Gaussian output noise
- Output saturation limits
- Quantised input/output for digital interface simulation

The sensor part of the simulations was not used in this thesis. Attitude determination was run using the UKF framework as part of the performance testing of the developed models.

2.2.3 Environmental Models

Environmental models are used to compute inputs to satellite physics, disturbances, sensors and actuators. Most of the used models are directly available from the Simulink Aerospace Blockset. Currently implemented models include the following:

- Earth's rotation – Calculated based on Julian time and the average rotation rate. Used to convert spacecraft inertial location to Earth centred Earth fixed (ECEF) reference frame or to geodetic coordinates to be used as an input for all the other Earth-related models.
- Earth's gravity – EGM2008 Spherical Harmonic Gravity Model (based on the WGS-84 reference ellipsoid). Used for orbital dynamics and gravity gradient disturbance.
- Earth's magnetic field – IGRF-12, the International Geomagnetic Reference Field with its data file updated to the 2012 version. This is the industry standard magnetic field model with its data file being regularly updated. Used for calculating magnetic disturbances and magnetometer and magnetorquer inputs.
- Earth's atmosphere – NRLMSISE-00, 2001 United States Naval Research Laboratory Mass Spectrometer and Incoherent Scatter Radar Exosphere. Atmospheric model accurate from the surface up to 1000 km altitude, includes the effects of solar activity.
- Earth-Sun relative motion – Planetary ephemerides model based on the Chebysev coefficients provided by the NASA Jet Propulsion Laboratory. Computes the Earth-Sun vector used as an input for the Sun sensors.

2.2.4 Satellite Firmware

The satellite firmware block allows including external software written in Matlab or C/C++ in the simulation. The block is provided noisy measurements from the sensor models and absolute time as inputs and it may output actuator control signals back into the simulation. This allows testing of actual satellite attitude determination and control firmware before up-linking it to the orbit. This part of the simulation environment was used to simulate different attitude manoeuvres to generate varying data for the UKF performance testing part of this thesis.

3 Descriptions of the Developed Models

The following sections describe the system and sensor models developed by the author. Both background information and the mathematical calculations making up the models are provided. All the mathematically ideal models can be scaled, offset or re-oriented depending on their configuration. In addition to being part of the UKF algorithm the same models are also used for measurement generation from simulation data. In the latter case sensor uncertainties and measurement noise are added to the model output.

3.1 Satellite Dynamics

The satellite dynamics model is the non-linear system model the UKF algorithm attempts to propagate in time. Satellite dynamics divide into two main parts: orbital and attitude dynamics. These are mostly independent of each other and the differential equations describing the propagation of the states can be handled separately. In case a self-sufficient sensor (like a star tracker) is not being used, attitude determination system does require a decent knowledge of the satellite's current location to utilise other sensors.

3.1.1 Orbital Dynamics

The orbital position and velocity of the satellite are modelled using the SGP4 algorithm [7]. The algorithm is widely used in the industry to estimate the current and future locations and motion of spacecraft and space debris. It predicts the orbit of a desired object for a given time instance based on its orbital parameters known at another time instance. The parameters are published by North American Aerospace Defence Command (NORAD) in the industry standard two line elements (TLE) format [10]. For low Earth orbiting satellites the TLEs are updated roughly 3 times per day.

To use the SGP4 algorithm with the satellite model inside the UKF its interface needed some adaptation. It was also slightly optimised by removing redundant and unused variables and calculations. In the future it needs further optimisations to make it as light weight as possible for running on an embedded system.

As mentioned in the Introduction section, the problem with SGP4 is that the reference parameters need to be updated on the satellite at least every other day, better every day. If the update is delayed much longer, the accuracy of attitude determination will suffer. Section 3.2 describes an attempt to improve the location estimation reliability by introducing a measurement-linked correction factor to the UKF state vector.

3.1.2 Attitude Dynamics

The attitude dynamics are based on the standard Euler equation of rigid body rotation (eq. 3.1). The equation has been modified to include the total torque and angular momentum of inertial actuators (reaction wheels) [11]. These will be provided as external inputs to the UKF model. All quantities in the equation are given in the satellite body frame.

$$\dot{\vec{\omega}}_s = I^{-1} \cdot (\vec{T}_{ext} - \vec{T}_{rw} - \vec{\omega}_s \times (I \cdot \vec{w}_s + \vec{h}_{rw}) \tag{3.1}$$

$\dot{\vec{\omega}}_s$	Angular acceleration of the satellite
I	The inertia matrix of the satellite
$\vec{T}_{ext}$	External torque (thrusters, gravity gradient, etc)
$\vec{T}_{rw}$	Total torque applied to the reaction wheels
$\vec{h}_{rw}$	Total angular momentum of all reaction wheels

The change in angular velocity is calculated directly from the angular acceleration and time step size using equation 3.2

$$w_s(t+1) = w_s(t) + \dot{w} \cdot dt \tag{3.2}$$

The satellite's attitude is stored in quaternion form. It represents a rotation from the inertial reference frame to the satellite body frame. To propagate the attitude based on the angular velocity there are two options.

The naive approach would be similar to equation 3.2 by calculating the quaternion derivative from the angular velocity using equation 3.3. This works accurately only for analytical and continuous time solutions as quaternions are not additive. When dealing with discrete systems where the time step is not infinitely small, the resulting quaternion would be incorrect as the derivative would have already changed during the time step.

$$\vec{dq} = 0.5 \cdot \vec{q} \otimes [\vec{\omega}; 0] \tag{3.3}$$

$\vec{dq}$	Quaternion derivative
$\vec{q}$	The quaternion the derivative is calculated for
$\vec{\omega}$	Current angular velocity

The better approach makes use of the fact that rotations can be combined together to produce a single compound rotation. In quaternion form this can be done using equation 3.4.

$$\vec{q}_c = \vec{q}_2 \otimes \vec{q}_1 \tag{3.4}$$

$\vec{q}_c$	Combined rotation
$\vec{q}_2$	Second rotation
$\vec{q}_1$	First rotation

To propagate the attitude numerically correctly the following steps are taken:

1. Calculate the rotation axis. A unit vector aligned with the angular velocity vector ($\hat{w}_{xyz}$).
2. Calculate the angle rotated during the given time step. The norm of the angular velocity vector multiplied by the length of the time step ($\alpha = |\vec{w}| \cdot dt$).

3. Convert from axis-angle to quaternion form using equation 3.5.
4. Combine the resulting delta rotation with the original attitude using equation 3.4.

$$\begin{bmatrix} \vec{q}_x \\ \vec{q}_y \\ \vec{q}_z \\ \vec{q}_w \end{bmatrix} = \begin{bmatrix} \hat{\omega}_x \cdot \sin(\alpha/2) \\ \hat{\omega}_y \cdot \sin(\alpha/2) \\ \hat{\omega}_z \cdot \sin(\alpha/2) \\ \cos(\alpha/2) \end{bmatrix} \tag{3.5}$$

3.2 Ground Station Ranging

As described in section 3.1.1, the TLEs combined with the SGP4 algorithm have an initial accuracy of a few kilometres which increases to a few tens of kilometres within a few days. The analysis presented in [1] shows that a worst case position error of 6 km (SGP4 with up to date TLEs) contributes approximately 0.1 degrees of attitude uncertainty when a star tracker is not being used. Specific analysis for larger position errors was not found, but based on the practical experience from the ESTCube-1 satellite it is known to the author that the attitude error grows significantly if the TLEs are not updated frequently enough.

A GPS receiver can be a relatively simple solution to improve the position knowledge accuracy. However, its high power consumption often prohibits its use on small satellites [8]. NORAD and other institutions use radio ranging to track the vast majority of objects (both satellites and debris) orbiting the Earth. A similar method is proposed for small satellites where instead of a radar a standard ground station and the satellite communication signal are used. By measuring the satellite response time and accounting for delays in the software, it is possible to calculate the line-of-sight distance of the satellite from the receiving ground station. In addition the radial velocity of the satellite relative to the ground station can be determined by measuring the Doppler shift of the signal transmitted by the satellite. To reduce the delay between obtaining the measurement and utilising it in the position determination system, the time measurement functionality could be built into the satellite's communication software. This would eliminate the need to up-link ageing measurements from ground.

To use this ranging information to improve the accuracy of the predicted position, an additional correction factor is added to the satellite's position model and the UKF state vector. This allows the UKF algorithm to continuously update the correction value to find the statistically best solution to match the ranging measurements. This is in principle similar to determining a sensor bias. However, as orbital motion is elliptical, ie. the velocity vector is continuously changing its direction, the correction can't be a constant offset. Therefore the solution shown by equation 3.6 could be used. This allows the correction factor to behave similar to an offset while the velocity vector ties the correction to the orbital motion. If k is a scalar, it has the effect of advancing or retarding the position of the satellite within its orbital path. If k is a vector, it can also be used to make out of orbital plane corrections. Note, this solution might not work well for highly elliptical orbits as the linear and angular velocities change significantly over the orbit in this case.

$$\vec{R}_{corr} = \vec{R}_{SGP4} + k \cdot \vec{V}_{SGP4} \cdot dt \tag{3.6}$$

$\vec{R}_{corr}$ Corrected position vector
$\vec{R}_{SGP4}$ Position vector produced by the SGP4 algorithm

$\vec{V}_{SGP4}$ Velocity vector produced by the SGP4 algorithm
dt Length of the time step
k The correction coefficient determined by UKF

Theoretically it would also be possible to let the UKF adjust the actual orbital parameters used by the SGP4 algorithm. In practice this would create a lot of additional degrees of freedom, increase the computational complexity and reduce the likelihood of the algorithm converging on the correct solution. Also, considering the way the SGP4 algorithm is designed, it would need to be reinitialised after every time step adding significant computational overhead.

3.3 Sensor Models

3.3.1 Star Tracker

A star tracker is essentially a digital camera which is designed specifically for imaging stars. To determine spacecraft attitude, the star tracker needs to capture an image containing multiple asymmetrically located stars. The geometric pattern of the imaged stars is matched against an on-board database and the attitude is calculated from the obtained results.

The main advantage of a star tracker is its accuracy. With high quality optics, engineering and a large database of stars, accuracies down to a few arcseconds can be achieved. Its downside is that it requires low angular velocities to ensure the capture of sharp images and that the star geometry remains uniquely identifiable. Datasheets of commercially available devices quote maximum angular velocities from a few deg/s up to not above 10 deg/s depending on the resulting accuracy.

In terms of the UKF model, the star tracker is the simplest attitude sensor there can be. As it provides a direct and accurate measurement of the spacecraft attitude, it is directly linked to the attitude field in the UKF state vector. The following calculations are used to compute the sensor measurement prediction:

1. Verify that the UKF angular velocity is not above the maximum limit.
2. Rotate the UKF attitude quaternion to sensor reference frame.
3. Increase the estimated measurement noise provided to the UKF proportional to the angular velocity

The last step is meant to slightly decrease the measurement certainty as the angular velocity increases. Several commercial providers quote the ability to operate the star tracker with high accuracy at very low rotation rates and with slightly reduced accuracy at higher speeds.

An additional restriction on the usage of the star tracker is the fact that the sensor cannot obtain a valid attitude if it is pointed towards any bright object (the Sun, the Earth, the Moon). This constraint has not been included in the UKF sensor model at the moment. As determining the locations of the mentioned objects requires considerable amounts of additional calculations this constraint will be implemented only if deemed necessary by further analysis or practical testing.

3.3.2 Sun Sensor

A sun sensor is in principle a fairly simple sensor which detects the direction of the sun relative to the sensor. It generally consists of one or more photo-sensitive devices (PSDs) which are covered by a dark matte mask. For each separate PSD the mask has a very small opening (a pinhole or a slit in the order of 0.1 mm) above the sensing element which allows a light spot to form on the PSD. Sensor readings are created by the locations of these light spots which move as the relative direction of the Sun changes.

The following calculations are used to implement the Sun sensor measurement prediction model.

1. The Earth-Sun vector in the inertial reference frame is calculated using an algorithm based on the mean motion of the Earth [12]. (Comparing this to the model provided in Simulink Aerospace Toolbox resulted in a maximum difference of 0.5 deg.)
2. The inertial Sun vector obtained in step 1 is rotated to the sensor reference frame using attitude from the current UKF state vector and the sensor orientation relative to the satellite body frame.
3. A Sun sensor can only produce a measurement if it is illuminated by the Sun. This is verified by checking that the z-axis (sensor normal) coordinate is positive.
4. If the sensor is illuminated by the Sun, the angles relative to sensor z-axis are calculated using the following equations:
   $\alpha_x = \mathrm{atan2}(sunvec.x, sunvec.z)$
   $\alpha_y = \mathrm{atan2}(sunvec.y, sunvec.z)$
5. Bias, if configured, is added to the final result.
6. If the sensor is not illuminated, both angles are set to π radians and the measurement noise indicated to the UKF is increased to 1e5 rad (an arbitrary large value).

4 Analysis of Simulation Results

To test the UKF performance using the simulated data, noise and sensor placement uncertainties were added to produce quasi-realistic (ie. noisy) measurements. For each enabled sensor the following fixed uncertainties were added (where applicable):

- fixed orientation error with added gaussian uncertainty
- fixed placement error with added gaussian uncertainty
- fixed bias with added gaussian uncertainty

In this context "fixed" means fixed for a single simulation run. The gaussian uncertainty is re-calculated each time the simulation is initialised. In addition, random gaussian noise was added to each measurement input to the UKF to simulate sensor noise.

For all the test cases shown in the next sections a spacecraft with the following parameters was used:

- Dimensions [0.1, 0.1, 0.3] m
- Initial mass 5 kg
- Principal moments of inertia (based on the 5 kg mass) [0.04, 0.04, 0.01] $\mathrm{kg} \cdot \mathrm{m}^2$

4.1 Attitude Dynamics

This section shows the performance of the UKF attitude determination models in different configurations. The case studies range from basic slow rotation with a few sensors to complex attitude manoeuvres and fusion of several, including noisy and erroneous, sensors. The sensor noise configuration was based on commercially available sensors. In most cases the measurement noise was increased to analyse the UKF behaviour under more difficult circumstances.

The star tracker, included in all cases below, was configured with a linearly increasing Gaussian noise with standard deviation (std) 33 arcsec (0.00928 deg) at 0 deg/s and at 330 arcsec at 10 deg/s per axis. Above 10 deg/s rotation rates star tracker measurements were not provided to the UKF.

4.1.1 Slow unstable rotation, single gyro and star tracker

Figure 4.1 shows the results for a simple rotation around a non-stable spacecraft axis. A single star tracker and gyro sensor were enabled. The following configuration was used:

- Initial angular velocity vector: [1, 2, 3] deg/s
- Gyro measurement noise std: 0.34 deg/s per axis
- No orientation or bias errors

The following results were obtained:

- Attitude error mean: 0.049705 deg
- Attitude error std: 0.021144 deg
- Angular velocity error norm mean: 0.674564 deg/s
- Angular velocity error norm std: 0.299331 deg/s

The means are calculated from the norms of the errors, not per axis. This is why they are not close to zero as expected with zero bias. The standard deviations meet the expectations based on the sensor measurement noise configurations. As the angular velocity is very close to the gyro noise level, the relative error is proportionately large.

The spikes visible before 100 seconds are caused by approximately 20 measurements being missing due to simulation time rounding errors.

4.1.2 Slow unstable rotation, additional noisy gyro

This case has the same setup as the previous one, except a second gyro sensor was added. The second sensor is configured to be erroneous as follows:

- Random orientation error with std 10 deg per axis
  Actual error quaternion: [0.121353, 0.056257, 0.008598, 0.990977]
  Total rotation error 15.4 deg
- Random bias with std 1 deg/s per axis
  Actual bias: [1.02, 1.68, -0.95] deg/s
- Measurement noise std: 1.03 deg/s per axis (triple of the first gyro)

Figure 4.2 illustrates the following results with the same set of plots as in the previous case.

- Attitude error mean: 8.48549e-11 deg
- Attitude error std: 6.57778e-11 deg
- Angular velocity error mean: 0.004924 deg/s
- Angular velocity error std: 0.001318 deg/s

As can be seen from these values, an extra sensor, even if it is considerably more noisy and erroneous, significantly improves the overall results. This demonstrates how the UKF can statistically reduce the uncertainties of the measurements and the state vector if it is provided with more data.

On figure 4.3 it can be seen how the UKF learns the orientation and bias errors of the second gyro. The scalar component of the orientation quaternion (approaching 1.0) has been excluded from the plot for clarity. The mean orientation and bias error of the second gyro during the last 200 seconds are [0.435838, -0.016267, 0.522618] deg and [-0.005735, 0.013576, -0.002252] deg/s respectively.

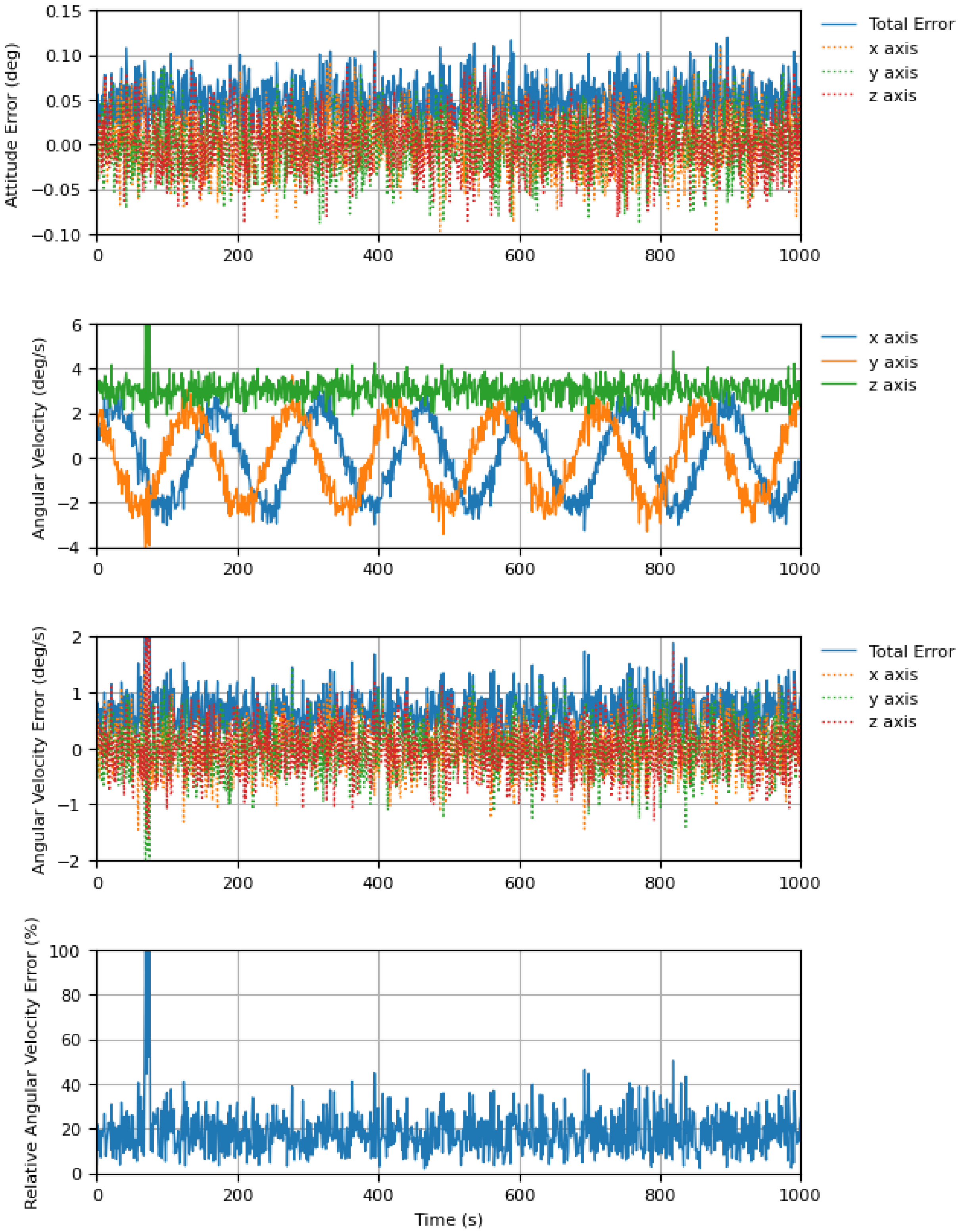

Figure 4.1: Slow rotation, single star tracker and gyro

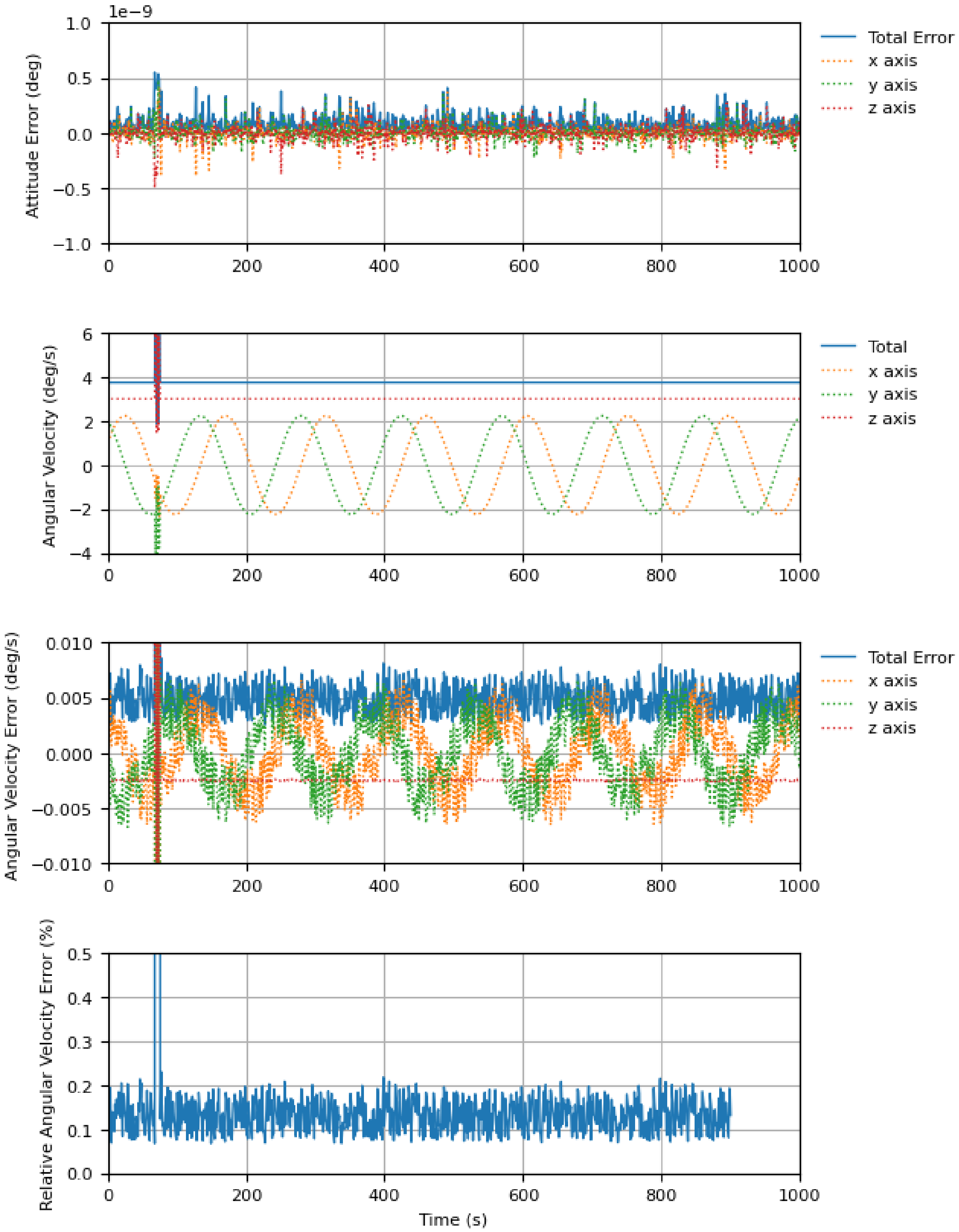

Figure 4.2: Slow rotation, single star tracker, 2 gyros

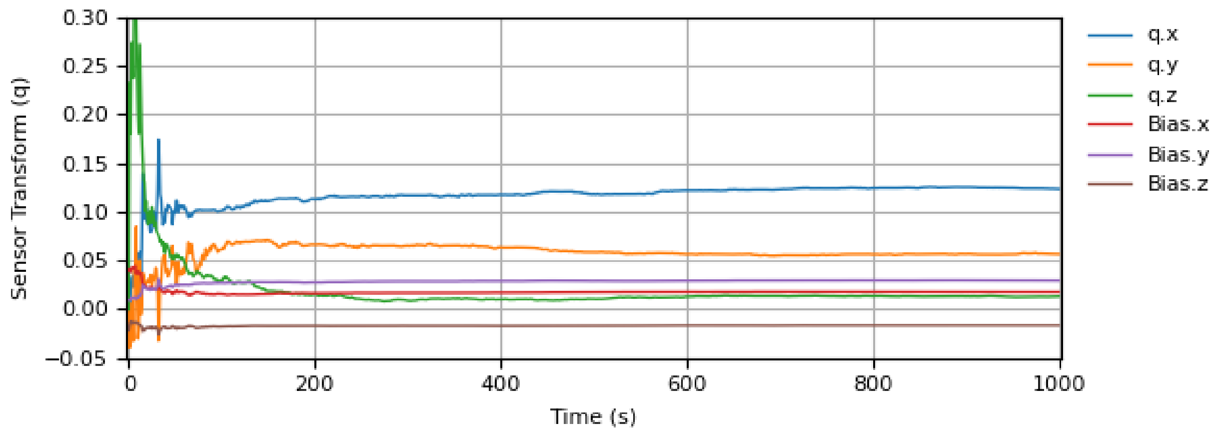

Figure 4.3: Slow rotation, 2nd gyro transform error

4.1.3 High angular velocity tumbling

The following sections demonstrate the performance of the UKF during a high angular velocity three-axis spin-up and detumbling manoeuvre. For the first case the simple configuration of a star tracker and a single gyro is used again. The results are illustrated in figure 4.4. Numerical statistics are not provided here as there are many different operational modes covered.

In the beginning, when the rotation is still slow, attitude and angular velocity are correctly measured by the two sensors. As soon as the angular velocity goes above the star tracker limit, the attitude knowledge drops drastically as expected. The angular velocity error rises somewhat as well as the UKF does not have the star tracker to cross-verify the rotation. When the angular velocity drops below the star tracker limit again, the attitude is immediately recovered as the star tracker is configured with very low uncertainty. When the angular velocity is relatively high, the relative error of the UKF state is visibly lower when compared to figure 4.1. This is due to the fact that the sensor is now operating clearly outside its noise range. The last 200 seconds of the error are cut out for clarity as they just show the sensor noise being comparable to the actual angular velocity. The case was also looked at with the erroneous gyro included, but without a decent knowledge of the state vector the UKF is unable to learn the sensors' errors and therefore it does not improve the overall accuracy.

4.1.4 High angular velocity tumbling, 1 sun sensor

In this configuration one 2-axis sun sensor was added. Sun sensors are not meant for attitude determination on their own as they can determine only the direction of the sun vector, not the rotation around it. However, here it once again demonstrates how the UKF algorithm can make use of the extra information. The sensor was configured with measurement noise std 0.29 deg on both axes (based on ESTCube-1 experience) and to indicate an invalid measurement (sun not visible) by setting the UKF measurement noise value very high. The results are shown in figure 4.5.

The massive error in the z-axis has reduced considerably and even though some of the uncertainty has transferred over to the other axes, the overall attitude error is reduced. The added noise on the angular velocity state is most likely due to the fact that the sun model implemented for the

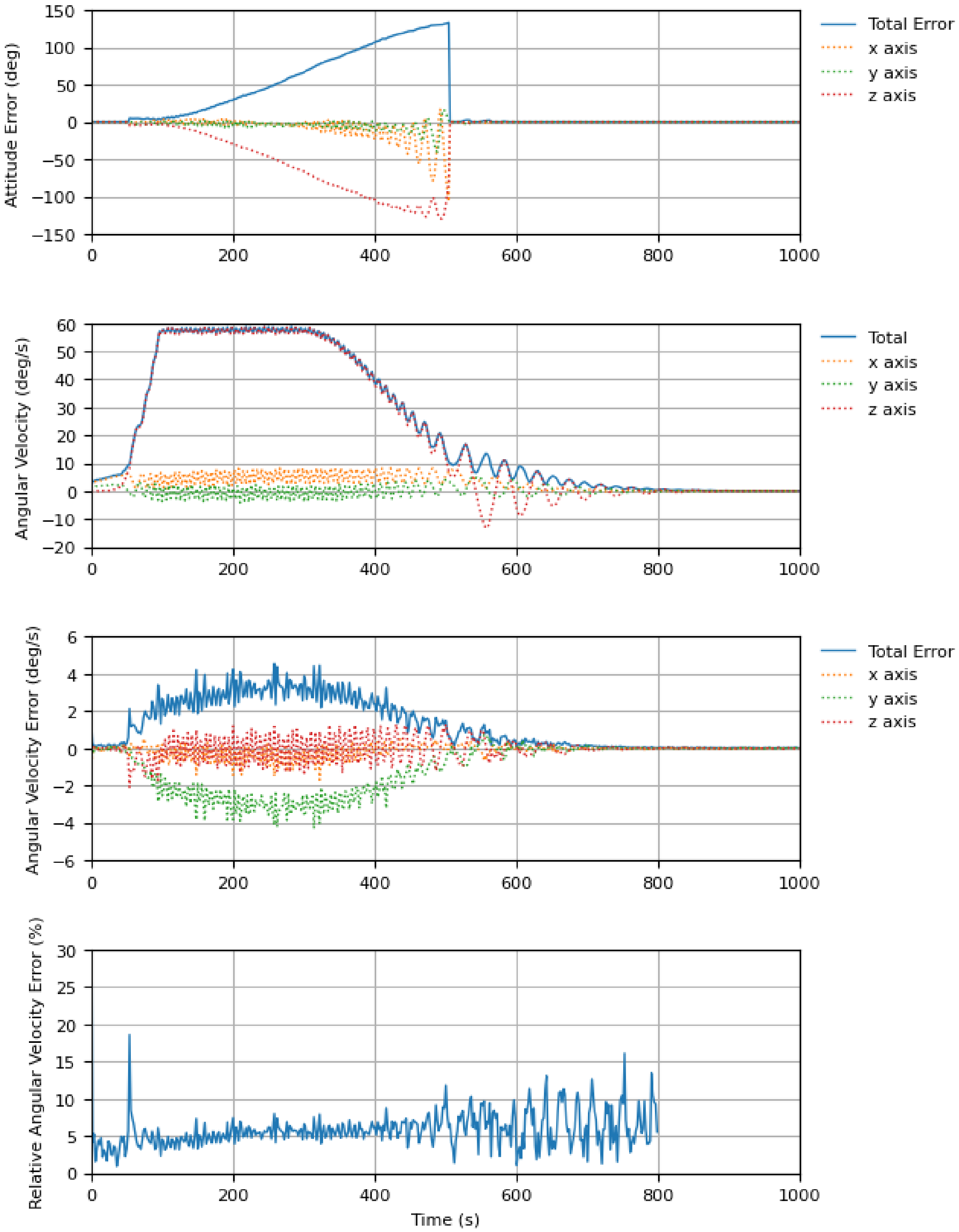

Figure 4.4: Tumbling satellite rotation performance, star tracker, one gyro

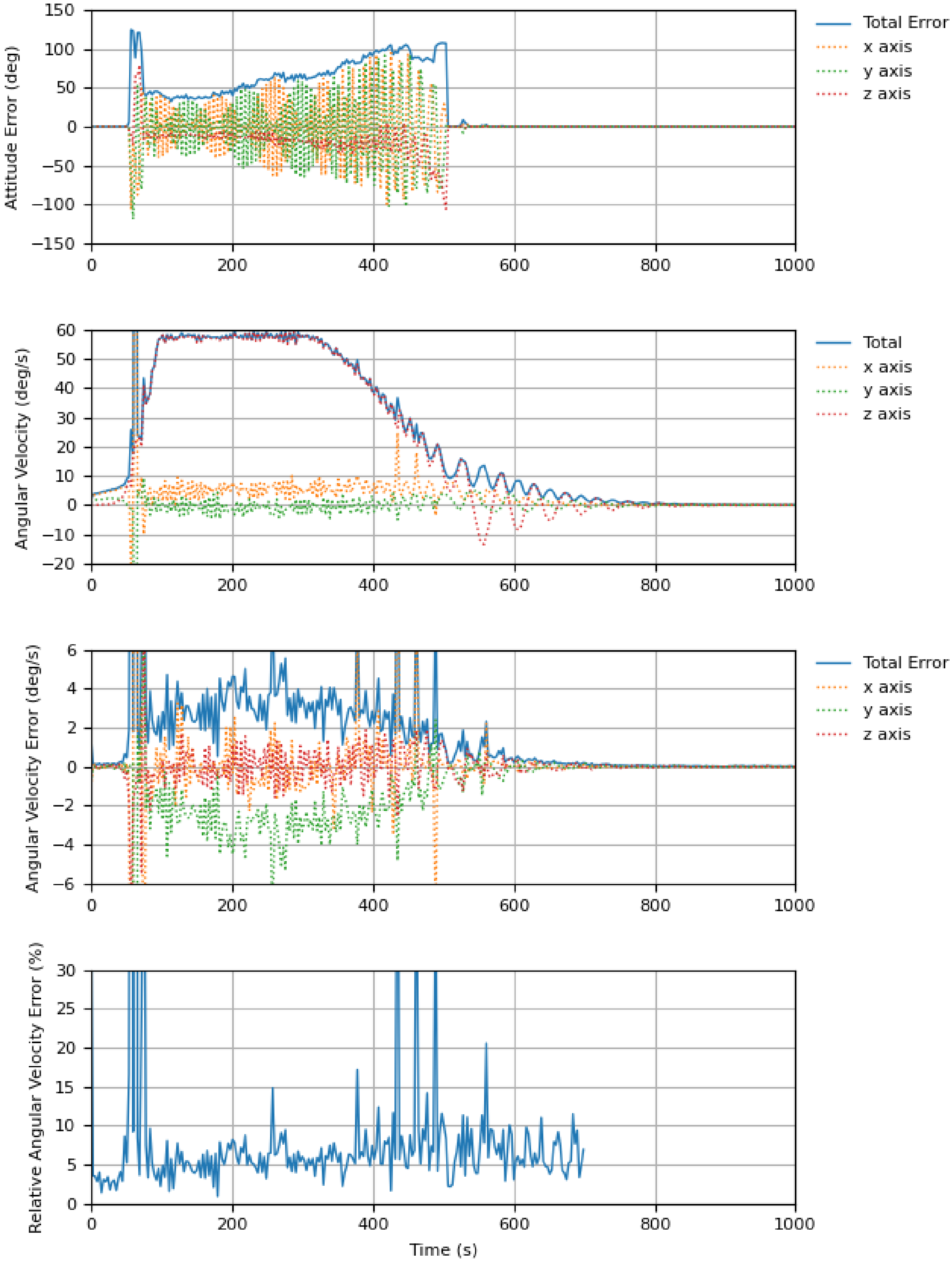

Figure 4.5: Tumbling satellite rotation performance, 1 sun sensor

UKF is not ideal and as the algorithm has no other good sources to verify the accuracies of the measurements, the noise on the state increases.

Figure 4.6 shows the difference between the expected sun sensor measurements and the actual measurements. From this it can be said that the pointing knowledge is actually very good as most measurements are within 2 degrees of the expectation. The spikes may be ignored as those are erroneous measurements reported by the sensor where a measurement was not expected. The reported variance of these measurements was set very high so they are effectively discarded by the UKF algorithm. This means that the attitude error shown in the previous figure comes from the rotation around the sun vector that cannot be determined using sun sensors. To solve the unknown rotation an additional measurement vector, eg. magnetic field, is required.

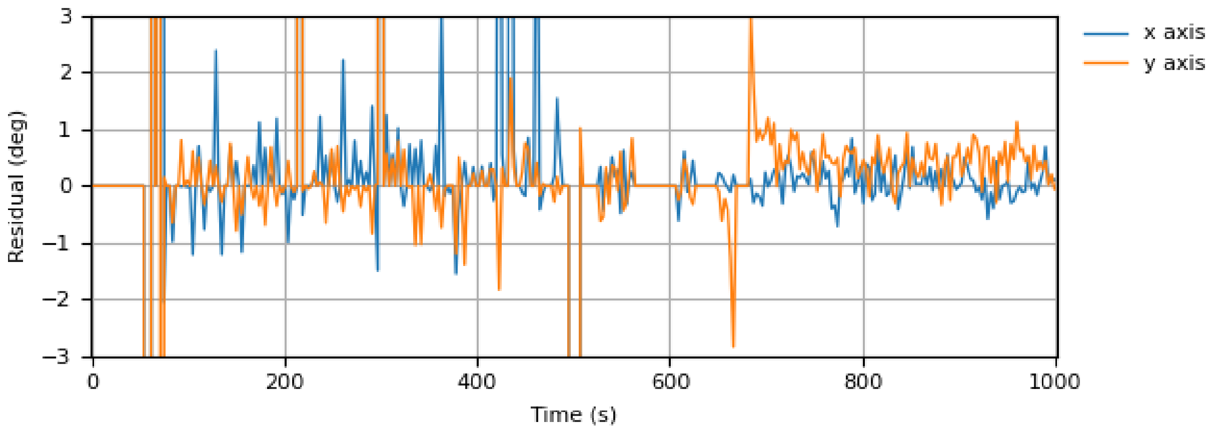

Figure 4.6: Tumbling satellite, sun sensor measurement residual

4.1.5 High angular velocity tumbling with 6 sun sensors

For the last case a full set of 6 sun sensors was configured. The results are shown in figure 4.7. This time the attitude error takes considerably longer to build up and is limited to a lower, albeit still a large value. As there is otherwise little change in the error distribution between the axes, it suggests the satellite has an edge pointing towards the sun which means there are two axes which are affected by the rotation around the sun vector uncertainty.

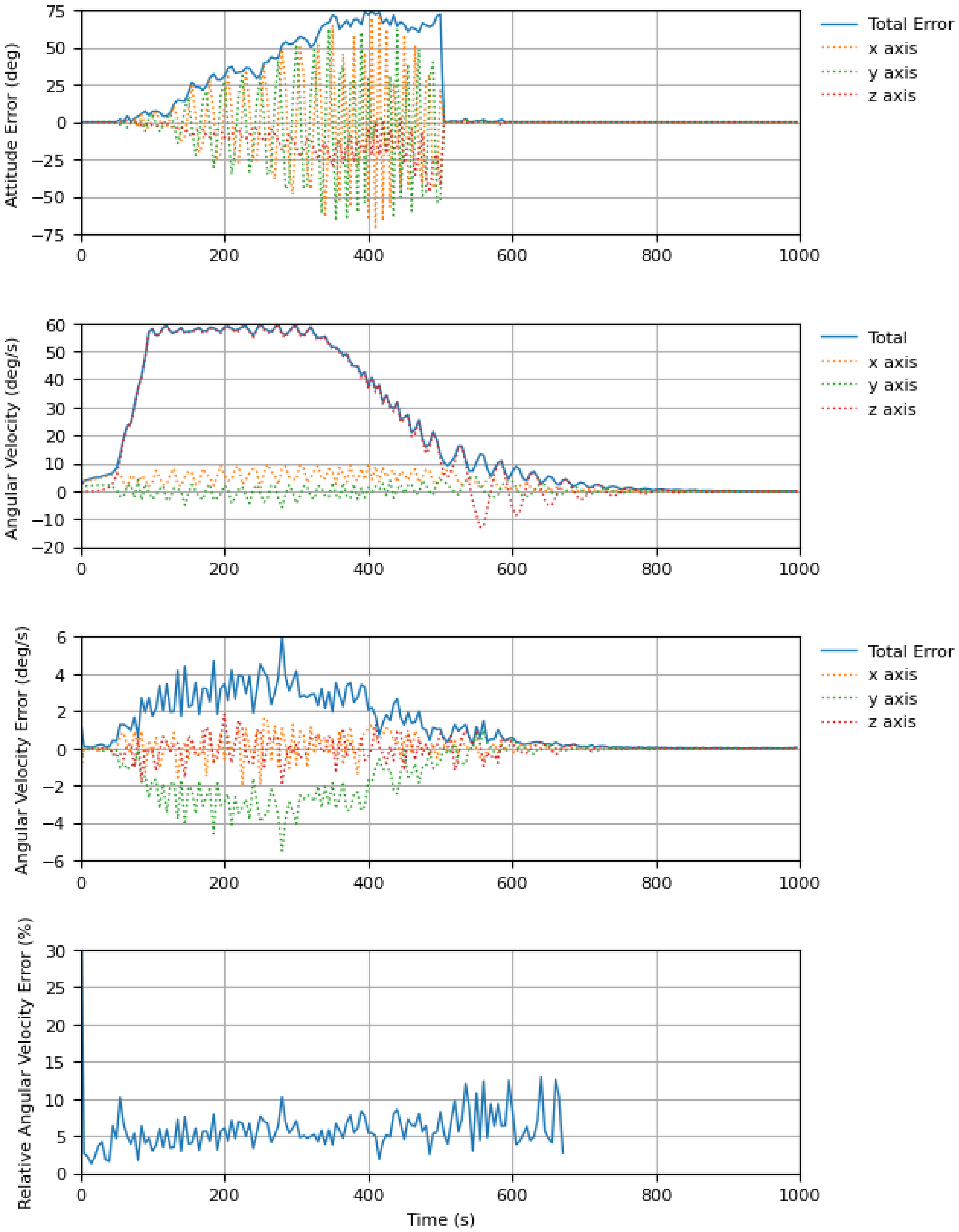

Figure 4.7: Tumbling satellite rotation performance, 6 sun sensors

4.2 Orbital Dynamics

As was mentioned in section 2.2, the simulation environment used to generate the data to test the UKF has an error in the long term propagation of spacecraft position and velocity. However, as the error remains within the orbital plane, the data can still be used to verify if the proposed ground station ranging based orbital position correction method can work. If the method can coerce the UKF algorithm to follow the simulation data it should definitely be able to correct the errors between the SGP4 algorithm and true orbital motion, which are expected to be a lot smaller.

The uncorrected development of the difference between the simulation data and the SGP4 algorithm is shown in figure 4.8. As can be seen, there are no major issues during the first 20000 seconds (5.5 hours). The error vector oscillates a bit but converges again. After that time period the simulation data clearly diverges.

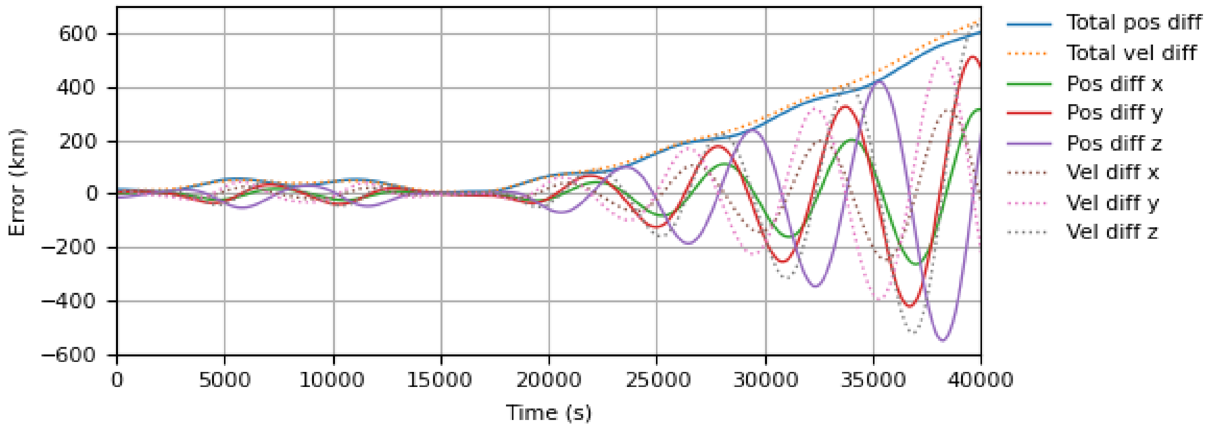

Figure 4.8: Orbital motion error, no ranging

4.2.1 Position Correction with 1 Ground Station

Figure 4.9 shows the satellite position error with 1 ground station configured for ranging measurements. The ground station is located at 60 degrees north latitude to make it frequently visible on a polar orbit. As can be seen, the position correction works very well when the ground station is in sight. Between the measurement sessions the error creeps up again as the simulation data lags further behind the SGP4 prediction. This is also illustrated by the correction factor, shown on the last plot, getting larger with every new measurement.

4.2.2 Position Correction with 2 Ground Stations

Figure 4.10 shows the same situation with two ground stations configured for ranging. The second ground station is located at 50 degrees south latitude with a 90 degree shift in longitude from the first station. The locations were chosen to be somewhat reasonable considering the continental geography of the Earth.

Similarly to the previous case the correction works very fast when a ground station is in view. The second station gives more opportunities for correction updates but the divergence is still visible.

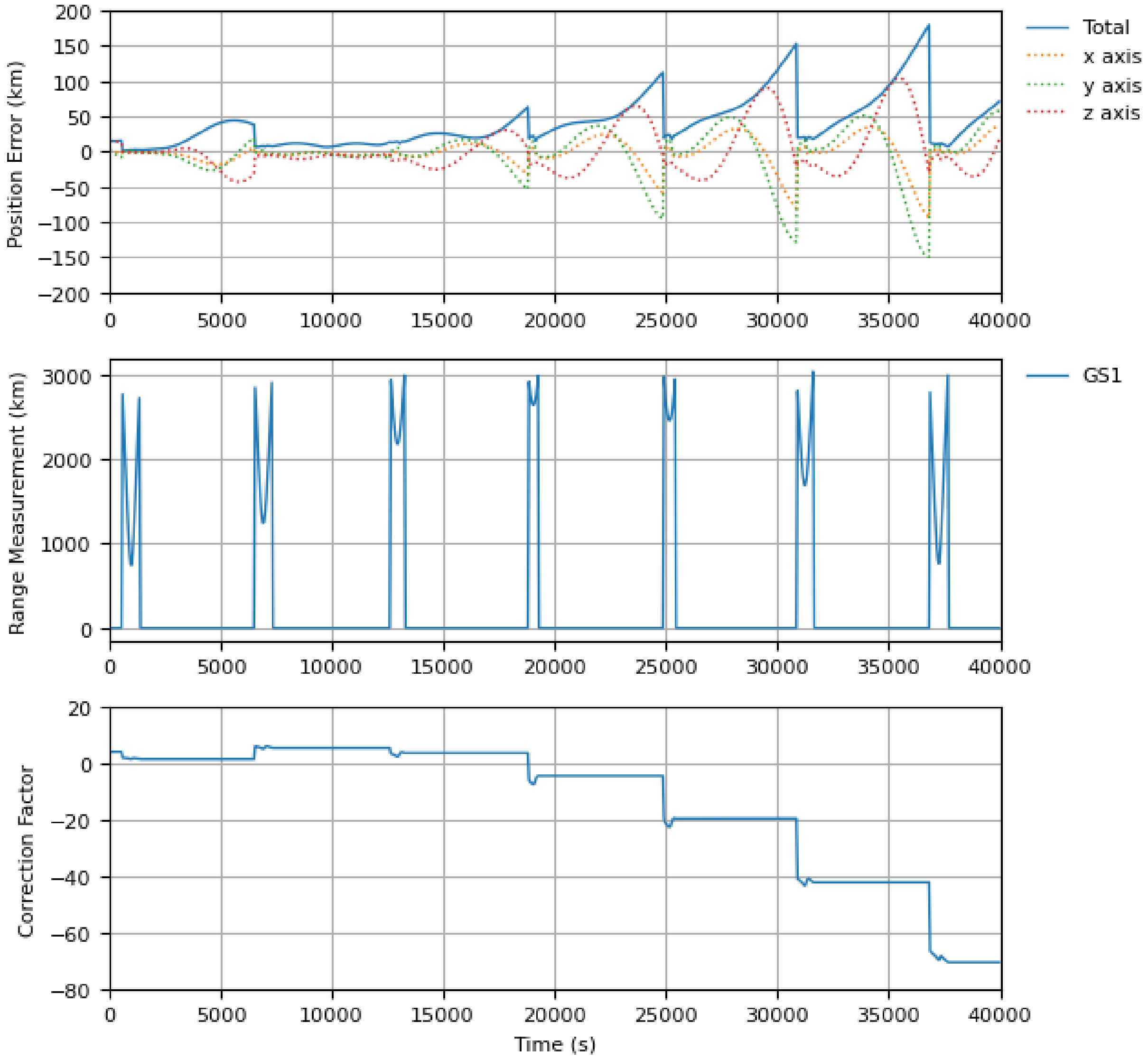

Figure 4.9: Position error, single ground station ranging

However, considering the fact that the accuracy of the SGP4 predictions over a period of a few days remains within a few tens of kilometres, such a correction method can easily be applied in the case of true orbital motion.

As future work the possibility to replace the linear velocity vector based correction with angular mean motion based correction should be investigated. Even though the velocity vector ties the correction to the orbital motion, it remains linear in its nature. Implementing the angular correction would involve modifying the interface of the SGP4 algorithm as in its current form only the linear position and velocity vectors are accessible in its output.

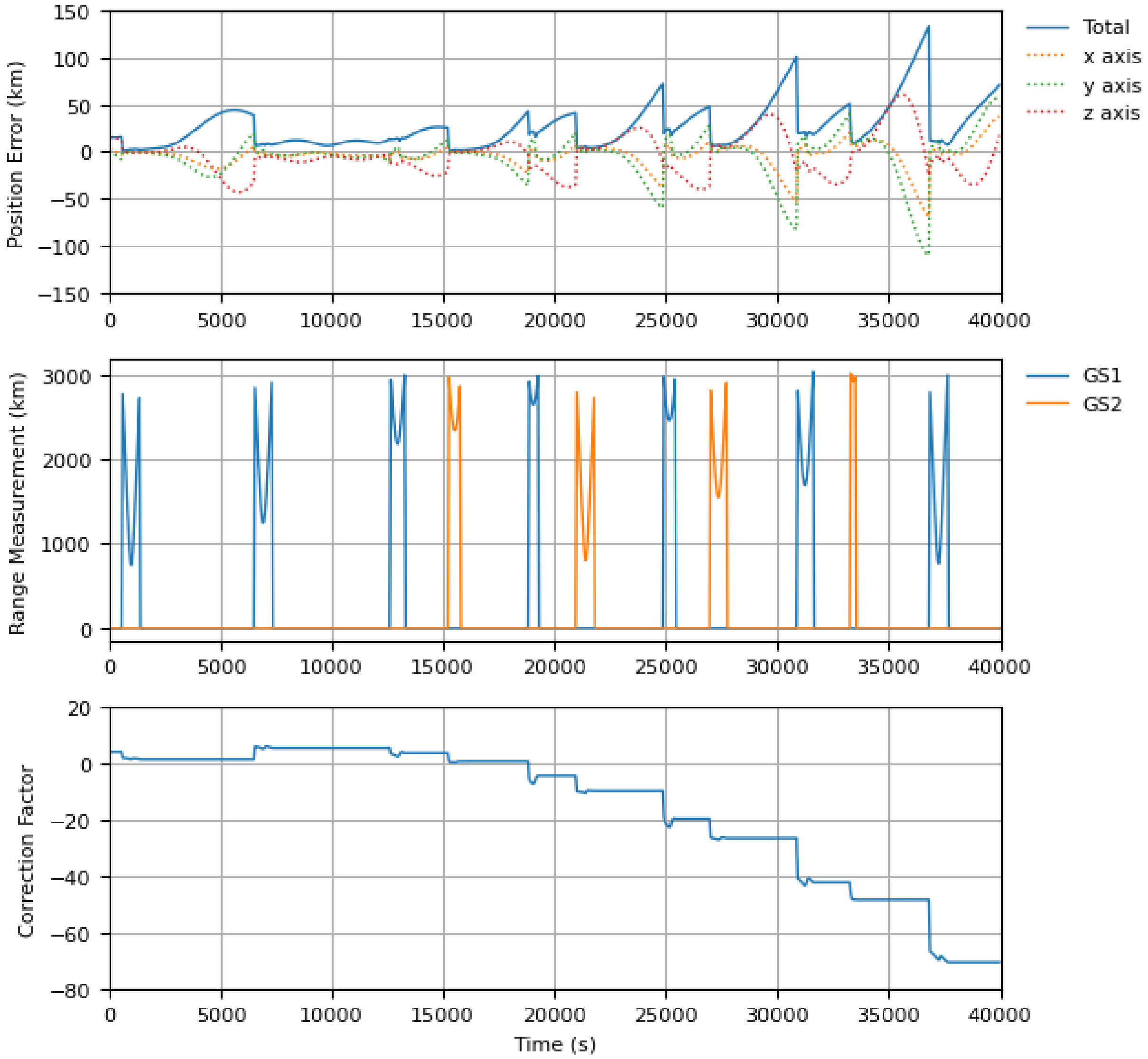

Figure 4.10: Position error, two ground stations ranging

Conclusion

This theses focuses on the development of physics, sensor and environmental models for an UKF and their performance analysis. During the writing of this theses the following models were developed or adapted for use with the UKF algorithm:

- The satellite physics model
- Star tracker with varying accuracy and angular velocity constraints
- Gyro sensor
- Sun sensor with limited field of view
- Ground station ranging for orbital position correction
- SGP4 orbital position and velocity prediction model
- Sun position model
- Earth rotation model

In addition, a method for orbital position correction, not used on small satellites before and including the ranging sensor mentioned above, was proposed. To develop and test the different models a framework was set up which includes the UKF algorithm and allows relatively convenient new model incorporation and results analysis.

All sensors were tested in different configurations and under different operating conditions. These include accurate and noisy sensors, sensors with unknown orientation or bias errors, missing measurements, simple low rate attitude motion and complex high-rate attitude manoeuvres.

To prepare testing data for the UKF to work on, a simulation environment, developed by the author using Matlab, Simulink and their associated add-ons, was used. The simulated data includes all the necessary physical and environmental details required for the attitude and orbit determination system to work successfully.

The attitude sensor models were found to work well with the UKF and the results corresponded to the noise configuration of the sensors. Regarding the orbital dynamics model, it was found that the simulated data was somewhat erroneous. However, the principle of the ground station ranging method could still be verified and was shown to be capable of correcting the SGP4 algorithm calculation errors.

The following is a list of tasks that are still outstanding before the system can be used on a satellite. The author intends to continue working on these tasks.

- Implement models of missing attitude sensors and actuators
- Characterise the sensors and models to provide accurate noise values to the UKF
- Further test the system with different combinations of several sensors and with limited number or broken sensors
- Refactor the UKF code to work on an embedded system

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