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Elastse silindrilise kooriku optimiseerimine

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Sisukord

Sissejuhatus

Silindrilised koorikud on konstruktsioonielemendid, mis leiavad laialdast rakendamist praktikas. Silindriliste koorikutena võime kujutleda vee- ja gaasitrassi torusid, samuti survemahutite külgpinda. Silindrilised koorikud on ka rakettide kered. Seoses laia rakendusega on muutunud aktuaalseks silindriliste koorikute käitumise uurimine defektide (pragude) tekkimisel ning pragudega koorikute optimiseerimine.

Käesolevas töös käsitletakse ringsilindrilise telgsümmeetriliselt koormatud kooriku painet elastse materjali korral.

Esimeses paragrahvis tuuakse ringsilindrilise kooriku tasakaaluvõrrandid, arvestades koormuse telgsümmeetrilisust. Selliseks koormuseks võib olla näiteks välisrõhk. Samuti tuuakse kooriku deformatsiooni komponendid.

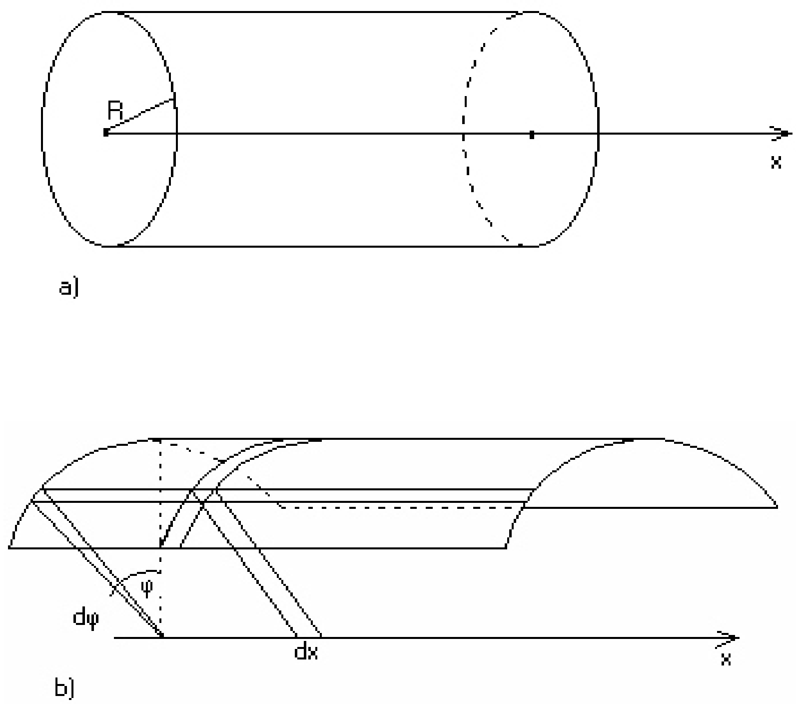
Teises paragrahvis vaadeldakse elastset ideaalselt kahekihilisest materjalist silindrilist koorikut, mille kandev kiht on tükiti konstantse paksusega. Tuletatakse selle kooriku painde võrrand ning leitakse vabale toetusele vastavad integreerimiskonstandid, arvestades tükiti konstantsest paksusest tulenevaid tingimusi.

Paragrahvis kolm uuritakse praoga elastset tükiti konstantse paksusega ideaalselt kahekihilist silindrilist koorikut. Pragu asub kandvas kihis paksuse muutumise kohas. Ka praoga juhul leitakse vabale toetusele vastav painde võrrand.

Praoga koorikuga tegeletakse edasi ka neljandas paragrahvis. Seal anname ette integraalse läbipainde ja leiame optimaalse kihi paksuse muutumise koha, kasutades paragrahvis kolm saadud konstantide avaldisi. Optimaalse projekti all mõistetakse miinimumkaaluga astmelist koorikut.

§ 1. Silindrilise kooriku põhiseosed

Vaatleme elastsest materjalist ringsilindrilist, telgsümmeetriliselt koormatud koorikut. Punkti asukoha määramiseks silindrilises koorikus kasutame koordinaate x ja φ, kusjuures x-telg suundugu paralleelselt kooriku moodustajaga (joon. 1.a). Eraldame ringsilindrilisest koorikust elemendi kahe moodustajasihilise ja kahe moodustajaga risti võetud põiklõikega, nii nagu näidatud joonisel 1.b.



Joon. 1. Silindriline koorik

Võib näidata (vt. [3], [4], [6], [12] ja [13]), et telgsümmeetrilise koormuse korral on kooriku tasakaaluvõrrandid järgmisel kujul:

$$\begin{cases} \dfrac{dN_x}{dx} = 0\,, \\[2ex] \dfrac{dQ_x}{dx} + \dfrac{1}{R} \cdot N_\varphi = -p\,, \\[2ex] \dfrac{dM_x}{dx} = Q\,. \end{cases} \tag{1}$$

Selle süsteemi teise ja kolmanda võrrandi võime ühendada järgmiseks võrrandiks:

$$\frac{d^2 M_x}{dx^2} + \frac{1}{R} N_\varphi = -p \, .$$

Siin w on kooriku seina keskpinna siire radiaalsuunas, M_x - paindemoment, N_φ - tangentsiaalsuunaline membraanjõud, p – jaotatud koormuse intensiivsus ning R – kooriku raadius. Saime tasakaaluvõrrandid kujul

$$\begin{cases} \dfrac{dN_x}{dx} = 0 \, , \\ \dfrac{d^2 M_x}{dx^2} + \dfrac{1}{R} N_\varphi = -p \, . \end{cases} \qquad (2)$$

Deformatsiooni kiiruse komponentide leidmiseks kasutame virtuaalkiiruste printsiipi, mille leiame pea igast mehaanikaõpikust, s.h ka raamatutest [3], [4], [6] ja [10],

$$D = A_{ext} \, ,$$

kus D on sisejõudude töö ja A_{ext} välisjõudude töö.

Lähtudes tasakaaluvõrranditest jõuame ringsilindrilise telgsümmeetriliselt koormatud kooriku deformatsiooni komponentideni järgmisel kujul:

$$\begin{cases} \varepsilon_x = \dfrac{du}{dx} \, , \\ \varepsilon_\varphi = -\dfrac{w}{R} \, , \\ \kappa_x = -\dfrac{d^2 w}{dx^2} \, . \end{cases} \qquad (3)$$

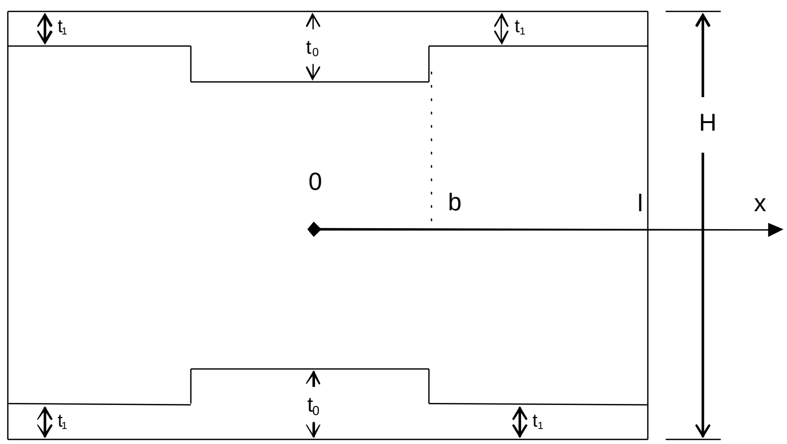
Erinevatel viisidel on samade tulemusteni jõutud ka raamatutes [3], [6] ja [13].

§ 2. Elastne tükiti konstantse paksusega silindriline koorik

Vaatleme ideaalselt kahekihilist silindrilist koorikut pikkusega 2*l*, mille kandva kihi paksus t on tükiti konstantne. Paksus muutub kohal $x = b$. Valime koordinaatide alguspunkti kooriku keskele. Siis

$$t = \begin{cases} t_0, \text{ kui } 0 \le x \le b, \\ t_1, \text{ kui } b < x \le l. \end{cases}$$

Ettekujutuse lihtsustamiseks vaatame joonist 1, millel on kujutatud kirjeldatud kooriku sein.



Joon. 2. Tükiti konstantse paksusega silindrilise kooriku sein

Läheme üle dimensioonita suurustele. Võttes $\xi = \frac{x}{l}$, $\beta = \frac{b}{l}$ ja $w = \frac{W}{H}$, saame suuruse $\gamma_j = \frac{t}{t_*}$, kus t_* on võrdluskooriku paksus, seega

$$\gamma_j = \begin{cases} \gamma_0 \ , & 0 \le \xi \le \beta, \\ \gamma_1 \ , & \beta < \xi \le 1. \end{cases}$$

Lähtume silindrilise kooriku tasakaaluvõrrandist (2) ja deformatsiooni komponentidest (3). Hooke´i seaduse kohaselt (vt. [13] ja [14])

$$N_\varphi = -2Et\frac{W}{R},$$

$$M_x = -D\frac{d^2W}{dx^2},$$

kus

$$D = \frac{EH^2}{2(1-v^2)} \cdot t.$$

Siin *R* on kooriku raadius, *H* - kooriku paksus, *p* – koorikule rakendatav koormus, *l* – pool kooriku pikkusest, *E* – Youngi moodul, *v* – Poissoni moodul, *D* – jäikustegur. Asetame nüüd toodud seosed tasakaaluvõrrandisse

$$-D\frac{d^4W}{dx^4} - 2Et\frac{W}{R^2} = -p\ .$$

Korrutades viimase võrrandi mõlemaid pooli *–1*-ga ning kasutades seoseid $W = w \cdot H$ ja $x = \xi \cdot l$, saame tulemuseks võrrandi

$$\frac{EH^3t}{2(1-v^2)l^4}\frac{d^4w}{d\xi^4} + 2\frac{EHt}{R^2}w = p\ ,$$

millest peale suurusega $\frac{2l^4(1-v^2)}{EH^3t_*\gamma_j} \neq 0$ võrrandi mõlemate poolte läbikorrutamist ja taandamist jääb järele

$$w^{IV} + 4\frac{l^4(1-v^2)}{R^2H^2}w = p\frac{2l^4(1-v^2)}{EH^3t_*} \cdot \frac{1}{\gamma_j}\ .$$

Tähistades viimases

$$a^4 := \frac{l^4(1-v^2)}{R^2H^2},$$

$$q := p\frac{2l^4(1-v^2)}{EH^3t_*},$$

saame elastse silindrilise kooriku painde võrrandi

$$w^{IV} + 4a^4w = \frac{q}{\gamma_j}. \tag{4}$$

Võrrandi (4) üldlahend avaldub

1) piirkonnas $\xi \in [0, \beta]$ kujul

$$w = w_0 + e^{a\xi}(C_1 \cos a\xi + C_2 \sin a\xi) + e^{-a\xi}(C_3 \cos a\xi + C_4 \sin a\xi)\ ,$$

2) piirkonnas $\xi \in [\beta, 1]$ kujul

$$w = w_1 + e^{a\xi}(B_1 \cos a\xi + B_2 \sin a\xi) + e^{-a\xi}(B_3 \cos a\xi + B_4 \sin a\xi)\ ,$$

kus w_0 ja w_1 on võrrandi (4) erilahendid vastavates piirkondades ning C_1,.., C_4 ja B_1, .., B_4 on kooriku ääretingimustest sõltuvad integreerimiskonstandid.

Vaatleme mõlemast otsast vabalt toetatud koorikut. Kooriku paksuse muutumise kohal $\xi = \beta$ peavad kehtima järgmised pidevuse tingimused:

$$[w(\beta)] = 0, \tag{5}$$

$$[w'(\beta)] = 0, \tag{6}$$

$$[M(\beta)] = 0, \tag{7}$$

$$[M'(\beta)] = 0. \tag{8}$$

Nurksulgudes tähistatakse siin vastava suuruse hüpet kohal $\xi = \beta$, s.t.

$$[y(\beta)] = \lim_{\xi \to \beta+} y(\xi) - \lim_{\xi \to \beta-} y(\xi),$$

ning M on moment, mille võib esitada seosena $M = \frac{EH^2 t}{2(1-v^2)} \cdot \kappa$, $\kappa = -\frac{d^2 W}{dx^2}$. Võttes

$m = -\frac{EH^2}{2\sigma_0 L^2} \cdot \frac{\gamma}{1-v^2} \cdot w''$, kus $w = \frac{W}{H}$, saame tingimuse (7) panna kirja

$$[\gamma \cdot w''(\beta)] = 0$$

ehk

$$\gamma_0 \cdot w''(\beta-) = \gamma_1 \cdot w''(\beta+). \tag{9}$$

Tingimus (8) on samaväärne seosega

$$\gamma_0 \cdot w'''(\beta-) = \gamma_1 \cdot w'''(\beta+). \tag{10}$$

Vaba toetuse korral kehtivad rajatingimused

$$w'(0) = 0, \tag{11}$$

$$w(1) = 0, \tag{12}$$

$$w''(1) = 0, \tag{13}$$

$$m'(0) = 0.$$

Viimane tingimus on samaväärne nõudega

$$w'''(0) = 0. \tag{14}$$

Edaspidi läheb meil vaja läbipainde esimest, teist ja kolmandat tuletist. Leiame need

$$\begin{aligned}
w &= w_0 + e^{a\xi}(C_1 \cos a\xi + C_2 \sin a\xi) + e^{-a\xi}(C_3 \cos a\xi + C_4 \sin a\xi), \\
w' &= a\left\{e^{a\xi}\left[(C_1 + C_2)\cos a\xi + (C_2 - C_1)\sin a\xi\right] + e^{-a\xi}\left[(C_4 - C_3)\cos a\xi + (-C_3 - C_4)\sin a\xi\right]\right\}, \\
w'' &= 2a^2\left[e^{a\xi}(C_2 \cos a\xi - C_1 \sin a\xi) + e^{-a\xi}(-C_4 \cos a\xi + C_3 \sin a\xi)\right], \\
w''' &= 2a^3\left\{e^{a\xi}\left[(C_2 - C_1)\cos a\xi + (-C_1 - C_2)\sin a\xi\right] + e^{-a\xi}\left[(C_3 + C_4)\cos a\xi + (C_4 - C_3)\sin a\xi\right]\right\}.
\end{aligned} \tag{15}$$

Raja- ja pidevuse tingimused (5), (6), (9) - (14) annavad meile peale sarnaste liikmete koondamist ja suuruse $a \neq 0$ või selle kordsetega vastava võrrandi mõlemate poolte läbijagamist ning mõningate elementaarsete teisenduste tegemist järgmise võrrandisüsteemi:

$$
\left\{
\begin{aligned}
& w_0 + e^{a\beta}(C_1 \cos a\beta + C_2 \sin a\beta) + e^{-a\beta}(C_3 \cos a\beta + C_4 \sin a\beta) = w_1 + e^{a\beta}(B_1 \cos a\beta + B_2 \sin a\beta) + \\
& + e^{-a\beta}(B_3 \cos a\beta + B_4 \sin a\beta), && (16) \\
& e^{a\beta}\left[(C_1 + C_2)\cos a\beta + (C_2 - C_1)\sin a\beta\right] + e^{-a\beta}\left[(C_4 - C_3)\cos a\beta + (-C_3 - C_4)\sin a\beta\right] = \\
& = e^{a\beta}\left[(B_1 + B_2)\cos a\beta + (B_2 - B_1)\sin a\beta\right] + e^{-a\beta}\left[(B_4 - B_3)\cos a\beta + (-B_3 - B_4)\sin a\beta\right], && (17) \\
& \gamma_0\left[e^{a\beta}(C_2 \cos a\beta - C_1 \sin a\beta) + e^{-a\beta}(-C_4 \cos a\beta + C_3 \sin a\beta)\right] = \\
& = \gamma_1\left[e^{a\beta}(B_2 \cos a\beta - B_1 \sin a\beta) + e^{-a\beta}(-B_4 \cos a\beta + B_3 \sin a\beta)\right], && (18) \\
& \gamma_0\left\{e^{a\beta}\left[(C_2 - C_1)\cos a\beta + (-C_1 - C_2)\sin a\beta\right] + e^{-a\beta}\left[(C_3 + C_4)\cos a\beta + (C_4 - C_3)\sin a\beta\right]\right\} = \\
& = \gamma_1\left\{e^{a\beta}\left[(B_2 - B_1)\cos a\beta + (-B_1 - B_2)\sin a\beta\right] + e^{-a\beta}\left[(B_3 + B_4)\cos a\beta + (B_4 - B_3)\sin a\beta\right]\right\}, && (19) \\
& C_1 + C_2 - C_3 + C_4 = 0, && (20) \\
& w_1 + e^{a}(B_1 \cos a + B_2 \sin a) + e^{-a}(B_3 \cos a + B_4 \sin a) = 0, && (21) \\
& e^{a}(B_2 \cos a - B_1 \sin a) + e^{-a}(-B_4 \cos a + B_3 \sin a) = 0, && (22) \\
& -C_1 + C_2 + C_3 + C_4 = 0, && (23)
\end{aligned}
\right.
$$

Tegemist on kaheksast võrrandist koosneva lineaarse võrrandisüsteemiga, mis sisaldab 8 tundmatut konstanti $B_1, \ldots, B_4, C_1, \ldots, C_4$. Lahendame süsteemi ja määrame konstandid.

Süsteemi lahendamist alustame võrranditest (20) ja (23). Liites nende võrrandite vastavad pooled omavahel ning koondades sarnased liidetavad, saame tulemuseks seose

$$C_2 = -C_4. \qquad (24)$$

Asetades saadud seose (24) võrrandisse (20), jääb peale koondamist kehtima võrdus

$$C_1 = C_3. \qquad (25)$$

Vaatleme eraldi ka võrrandeid (21) ja (22). Korrutame neist esimest suurusega $\cos a$ ja teist suurusega $\sin a$, seejärel liidame võrrandite vastavad pooled omavahel.

$$w_1 \cos a + e^{a}(B_1 \cos^2 a + B_2 \sin a \cos a) + e^{-a}(B_3 \cos^2 a + B_4 \sin a \cos a) +$$
$$+ e^{a}(B_2 \sin a \cos a - B_1 \sin^2 a) + e^{-a}(-B_4 \sin a \cos a + B_3 \sin^2 a) = 0$$

Koondades saadud võrrandis sarnased liidetavad ja kasutades trigonomeetriliste funktsioonide elementaarseid teisendamisvalemeid, saame viimasest võrrandist

$$w_1 \cos a + e^{a}(B_1 \cos 2a + B_2 \sin 2a) + e^{-a} B_3 = 0,$$

millest peale suurusega $e^{a} \neq 0$ võrrandi poolte läbikorrutamist saame avaldada B_3

$$B_3 = -w_1 \cdot e^{a} \cos a - e^{2a}(B_1 \cos 2a + B_2 \sin 2a). \qquad (26)$$

Nüüd korrutame võrrandit (21) $\sin a$ -ga ja võrrandit (22) $\cos a$ -ga ning lahutame esimesest võrrandist teise. Analoogselt eelmisega, kasutades trigonomeetrilisi teisendusi ja koondamist, saame avaldada suuruse B_4

$$B_4 = -w_1 \cdot e^{a} \sin a - e^{2a}(B_1 \sin 2a - B_2 \cos 2a). \qquad (27)$$

Kasutades seoseid (24) ja (25) ning seejärel koondades sarnased liidetavad, saame võrrandid (16) – (19) lihtsamal kujul

$$w_0 + C_1(e^{a\beta} + e^{-a\beta})\cos a\beta + C_2(e^{a\beta} - e^{-a\beta})\sin a\beta = w_1 + e^{a\beta}(B_1 \cos a\beta + B_2 \sin a\beta) +$$
$$+ e^{-a\beta}(B_3 \cos a\beta + B_4 \sin a\beta), \qquad (28)$$

$$C_1\left[(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin a\beta\right] + C_2\left[(e^{a\beta} - e^{-a\beta})\cos a\beta + (e^{a\beta} + e^{-a\beta})\sin a\beta\right] =$$
$$= e^{a\beta}\left[B_1\left(\cos a\beta - \sin a\beta\right) + B_2\left(\cos a\beta + \sin a\beta\right)\right] + e^{-a\beta}\left[B_3\left(-\cos a\beta - \sin a\beta\right) + \right. \qquad (29)$$
$$\left. + B_4\left(\cos a\beta - \sin a\beta\right)\right],$$

$$\gamma_0\left[C_1(-e^{a\beta} + e^{-a\beta})\sin a\beta + C_2(e^{a\beta} + e^{-a\beta})\cos a\beta\right] = \gamma_1\left[e^{a\beta}(B_2 \cos a\beta - \mathrm{B}_1 \sin a\beta) + \right.$$
$$\left. + e^{-a\beta}(-B_4 \cos a\beta + B_3 \sin a\beta)\right], \qquad (30)$$

$$\gamma_0\, C_1\left[-(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin a\beta\right] + C_2\left[(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin ab\right] =$$
$$= \gamma_1\left\{e^{a\beta}\left[B_1\left(-\sin a\beta - \cos a\beta\right) + B_2\left(\cos a\beta - \sin a\beta\right)\right] + e^{-a\beta}\left[B_3\left(\cos a\beta - \sin a\beta\right) + \right.\right. \qquad (31)$$
$$\left.\left. + B_4\left(\cos a\beta + \sin a\beta\right)\right]\right\}.$$

Edasi teisendame võrrandeid (28) – (31), asendades sinna B_3 ja B_4 avaldised (26) ja (27). Peale mahukat teisendamist saame nimetatud võrrandid järgmisel kujul:

$$w_0 + C_1(e^{a\beta} + e^{-a\beta})\cos a\beta + C_2(e^{a\beta} - e^{-a\beta})\sin a\beta = w_1\left\{1 - e^{a(1-\beta)}\cos\left[a(1-\beta)\right]\right\} +$$
$$+B_1\left\{e^{a\beta}\cos a\beta - e^{a(2-\beta)}\cos\left[a(2-\beta)\right]\right\} + B_2\left\{e^{a\beta}\sin a\beta - e^{a(2-\beta)}\sin\left[a(2-\beta)\right]\right\}$$

$$C_1\left[(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin a\beta\right] + C_2\left[(e^{a\beta} - e^{-a\beta})\cos a\beta + (e^{a\beta} + e^{-a\beta})\sin a\beta\right] =$$
$$= B_1\left\langle e^{a\beta}(\cos a\beta - \sin a\beta) + e^{a(2-\beta)}\left\{-\cos\left[a(2-\beta)\right] + \sin\left[a(2-\beta)\right]\right\}\right\rangle +$$
$$+B_2\left\langle e^{a\beta}(\cos a\beta + \sin a\beta) + e^{a(2-\beta)}\left\{\cos\left[a(2-\beta)\right] + \sin\left[a(2-\beta)\right]\right\}\right\rangle +$$
$$+w_1 e^{a(1-\beta)}\left\{\cos\left[a(1-\beta)\right] - \sin\left[a(1-\beta)\right]\right\}$$

$$\gamma_0\left[C_1(-e^{a\beta} + e^{-a\beta})\sin a\beta + C_2(e^{a\beta} + e^{-a\beta})\cos a\beta\right] = \gamma_1\left\langle w_1 e^{a(1-\beta)}\sin\left[a(1-\beta)\right] +\right.$$
$$\left.+B_1\left\{-e^{a\beta}\sin a\beta + e^{a(2-\beta)}\sin\left[a(2-\beta)\right]\right\} + B_2\left\{e^{a\beta}\cos a\beta - e^{a(2-\beta)}\cos\left[a(2-\beta)\right]\right\}\right\rangle$$

$$\gamma_0\left\{C_1\left[-(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin a\beta\right] + C_2\left[(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin ab\right] =\right.$$
$$= \gamma_1\left\{B_1\left\langle -e^{a\beta}(\cos a\beta + \sin a\beta) - e^{a(2-\beta)}\left\{\cos\left[a(2-\beta)\right] + \sin\left[a(2-\beta)\right]\right\}\right\rangle\right.$$
$$+B_2\left\langle e^{a\beta}(\cos a\beta - \sin a\beta) + e^{a(2-\beta)}\left\{\cos\left[a(2-\beta)\right] - \sin\left[a(2-\beta)\right]\right\}\right\rangle$$
$$\left.+\left\langle -w_1 e^{a(1-\beta)}\left\{\cos\left[a(1-\beta)\right] + \sin\left[a(1-\beta)\right]\right\}\right\rangle\right\}$$

Ülevaatlikkuse ja lühiduse mõttes tähistame

$$\begin{aligned} f &:= a(1-\beta), \\ g &:= a(2-\beta), \end{aligned} \tag{32}$$

siis saame viimasena toodud võrrandid panna kirja

$$\left\{\begin{aligned}
&w_0 + C_1(e^{a\beta} + e^{-a\beta})\cos a\beta + C_2(e^{a\beta} - e^{-a\beta})\sin a\beta = w_1\left(1 - e^{f}\cos f\right) + \\
&+B_1\left(e^{a\beta}\cos a\beta - e^{g}\cos g\right) + B_2\left(e^{a\beta}\sin a\beta - e^{g}\sin g\right) && (33)\\
&C_1\left[(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin a\beta\right] + C_2\left[(e^{a\beta} - e^{-a\beta})\cos a\beta + (e^{a\beta} + e^{-a\beta})\sin a\beta\right] = \\
&= B_1\left[e^{a\beta}(\cos a\beta - \sin a\beta) + e^{g}\left(-\cos g + \sin g\right)\right] + && (34)\\
&+B_2\left[e^{a\beta}(\cos a\beta + \sin a\beta) + e^{g}\left(\cos g + \sin g\right)\right] + w_1 e^{f}\left(\cos f - \sin f\right) \\
&\gamma_0\left[C_1(-e^{a\beta} + e^{-a\beta})\sin a\beta + C_2(e^{a\beta} + e^{-a\beta})\cos a\beta\right] = \gamma_1\left[B_1\left(-e^{a\beta}\sin a\beta + e^{g}\sin g\right) + \right. \\
&\left.+B_2\left(e^{a\beta}\cos a\beta - e^{g}\cos g\right) + w_1 e^{f}\sin f\right] && (35)\\
&\gamma_0\left\{C_1\left[-(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin a\beta\right] + C_2\left[(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin ab\right] =\right. \\
&= \gamma_1\left\{B_1\left[-e^{a\beta}(\cos a\beta + \sin a\beta) - e^{g}\left(\cos g + \sin g\right)\right] + B_2\left[e^{a\beta}(\cos a\beta - \sin a\beta) + e^{g}\left(\cos g - \sin g\right)\right] + \right. && (36)\\
&\left.+\left[-w_1 e^{f}\left(\cos f + \sin f\right)\right]\right\}
\end{aligned}\right.$$

Võrrandid (33) – (36) annavad meile nelja tundmatuga C_1, C_2, B_1, B_2 lineaarvõrrandisüsteemi. Lahendamist alustame võrrandist (33). Jagame võrrandi (33) mõlemaid pooli konstandi C_1 kordajaga $(e^{a\beta}+e^{-a\beta})\cos a\beta \neq 0$ ning avaldame C_1

$$C_1 = B_1 \frac{e^{a\beta}\cos a\beta - e^{g}\cos g}{(e^{a\beta}+e^{-a\beta})\cos a\beta} + B_2 \frac{e^{a\beta}\sin a\beta - e^{g}\sin g}{(e^{a\beta}+e^{-a\beta})\cos a\beta} - C_2 \frac{\sin a\beta\,(e^{a\beta}-e^{-a\beta})}{(e^{a\beta}+e^{-a\beta})\cos a\beta} +$$
$$+ \frac{w_0 - w_1\left(1-e^{f}\cos f\right)}{(e^{a\beta}+e^{-a\beta})\cos a\beta}.$$

Tähistades

$$\begin{aligned}
m_1 &:= \frac{e^{a\beta}\cos a\beta - e^{g}\cos g}{(e^{a\beta}+e^{-a\beta})\cos a\beta},\\
m_2 &:= \frac{e^{a\beta}\sin a\beta - e^{g}\sin g}{(e^{a\beta}+e^{-a\beta})\cos a\beta},\\
m_3 &:= -\frac{(e^{a\beta}-e^{-a\beta})\sin a\beta}{(e^{a\beta}+e^{-a\beta})\cos a\beta},\\
m_4 &:= \frac{w_0 - w_1\left(1-e^{f}\cos f\right)}{(e^{a\beta}+e^{-a\beta})\cos a\beta}
\end{aligned} \tag{37}$$

saame C_1 avaldise

$$C_1 = m_1 B_1 + m_2 B_2 + m_3 C_2 + m_4. \tag{38}$$

Järgmisena asetame seose (38) võrrandisse (35) ja koondame sarnased liidetavad

$$B_1\left[m_1\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta - \gamma_1\left(-e^{a\beta}\sin a\beta + e^{g}\sin g\right)\right]+$$
$$+B_2\left[m_2\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta - \gamma_1\left(e^{a\beta}\cos a\beta - e^{g}\cos g\right)\right]+$$
$$+C_2\left[m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta\right] + m_4\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta - w_1\gamma_1 e^{f}\sin f = 0$$

Viimasest võrrandist avaldame C_2

$$C_2 = -B_1 \frac{m_1\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta - \gamma_1\left(-e^{a\beta}\sin a\beta + e^{g}\sin g\right)}{m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta} -$$
$$-B_2 \frac{m_2\gamma_0\sin a\beta\,(-e^{a\beta}+e^{-a\beta}) - \gamma_1\left(e^{a\beta}\cos a\beta - e^{g}\cos g\right)}{m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta} -$$
$$-\frac{m_4\gamma_0\sin a\beta\,(-e^{a\beta}+e^{-a\beta}) - w_1\gamma_1 e^{f}\sin f}{m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta}$$

Tähistades

$$
\begin{aligned}
m_5 &:= -\frac{m_1\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta - \gamma_1\left(-e^{a\beta}\sin a\beta + e^{g}\sin g\right)}{m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta}, \\
m_6 &:= -\frac{m_2\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta - \gamma_1\left(e^{a\beta}\cos a\beta - e^{g}\cos g\right)}{m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta}, \\
m_7 &:= -\frac{m_4\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta - w_1\gamma_1 e^{f}\sin f}{m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta},
\end{aligned}
\tag{39}
$$

saame C_2 avaldise

$$
C_2 = m_5 B_1 + m_6 B_2 + m_7 \tag{40}
$$

ja (38) põhjal

$$
C_1 = \left(m_1 + m_3 m_5\right) B_1 + \left(m_2 + m_3 m_6\right) B_2 + m_3 m_7 + m_4 \tag{41}
$$

Asetades seosed (40) ja (41) võrrandisse (34), saame koondamise tulemusena

$$
\begin{aligned}
& B_1 \Big\{\left(m_1 + m_3 m_5 + m_5\right)(e^{a\beta} - e^{-a\beta})\cos a\beta + \left(-m_1 - m_3 m_5 + m_5\right)(e^{a\beta} + e^{-a\beta})\sin a\beta - \\
& -\left[e^{a\beta}(\cos a\beta - \sin a\beta) + e^{g}\left(-\cos g + \sin g\right)\right]\Big\} + \\
& + B_2 \Big\{\left(m_2 + m_3 m_6 + m_6\right)(e^{a\beta} - e^{-a\beta})\cos a\beta + \left(-m_2 - m_3 m_6 + m_6\right)(e^{a\beta} + e^{-a\beta})\sin a\beta - \\
& -\left[e^{a\beta}(\cos a\beta + \sin a\beta) + e^{g}\left(\cos g + \sin g\right)\right]\Big\} + \\
& + \left(m_4 + m_3 m_7 + m_7\right)(e^{a\beta} - e^{-a\beta})\cos a\beta + \left(-m_4 - m_3 m_7 + m_7\right)(e^{a\beta} + e^{-a\beta})\sin a\beta - \\
& - w_1 e^{f}\left(\cos f - \sin f\right) = 0,
\end{aligned}
$$

Tähistades

$$
\begin{aligned}
m_8 &:= -\frac{\left(m_2 + m_3 m_6 + m_6\right)(e^{a\beta} - e^{-a\beta})\cos a\beta + \left(-m_2 - m_3 m_6 + m_6\right)(e^{a\beta} + e^{-a\beta})\sin a\beta - \left[e^{a\beta}(\cos a\beta + \sin a\beta) + e^{g}\left(\cos g + \sin g\right)\right]}{\left(m_1 + m_3 m_5 + m_5\right)(e^{a\beta} - e^{-a\beta})\cos a\beta + \left(-m_1 - m_3 m_5 + m_5\right)(e^{a\beta} + e^{-a\beta})\sin a\beta - \left[e^{a\beta}(\cos a\beta - \sin a\beta) + e^{g}\left(-\cos g + \sin g\right)\right]}, \\
m_9 &:= -\frac{\left(m_4 + m_3 m_7 + m_7\right)(e^{a\beta} - e^{-a\beta})\cos a\beta + \left(-m_4 - m_3 m_7 + m_7\right)(e^{a\beta} + e^{-a\beta})\sin a\beta - w_1 e^{f}\left(\cos f - \sin f\right)}{\left(m_1 + m_3 m_5 + m_5\right)(e^{a\beta} - e^{-a\beta})\cos a\beta + \left(-m_1 - m_3 m_5 + m_5\right)(e^{a\beta} + e^{-a\beta})\sin a\beta - \left[e^{a\beta}(\cos a\beta - \sin a\beta) + e^{g}\left(-\cos g + \sin g\right)\right]},
\end{aligned}
\tag{42}
$$

saame viimasest võrrandist avaldada B_1

$$
B_1 = m_8 B_2 + m_9 . \tag{43}
$$

Nüüd (40) ja (41) põhjal

$$
\begin{aligned}
C_2 &= \left(m_5 m_8 + m_6\right) B_2 + m_5 m_9 + m_7 \\
C_1 &= \left(m_8 m_3 m_5 + m_8 m_1 + m_3 m_6 + m_2\right) B_2 + m_9 m_3 m_5 + m_3 m_7 + m_9 m_1 + m_4
\end{aligned}
\tag{44}
$$

Järgmisena asetame seosed (43) ja (44) võrrandisse (36)

$$B_2\{\gamma_0\left(m_8m_3m_5+m_8m_1+m_3m_6+m_2\right)\cdot\left[-\cos a\beta\cdot(e^{a\beta}-e^{-a\beta})-\sin a\beta\cdot(e^{a\beta}+e^{-a\beta})\right]+\gamma_0\left(m_5m_8+m_6\right)\cdot$$
$$\cdot\left[\cos a\beta\cdot(e^{a\beta}-e^{-a\beta})-\sin a\beta\cdot(e^{a\beta}+e^{-a\beta})\right]-m_8\gamma_1\left[-e^{a\beta}(\cos a\beta+\sin a\beta)-e^{g}\left(\cos g+\sin g\right)\right]-$$
$$-e^{a\beta}(\cos a\beta-\sin a\beta)-e^{g}\left(\cos g-\sin g\right)\}+\gamma_0\left[m_9m_3m_5+m_3m_7+m_9m_1+m_4\right]\cdot$$
$$\cdot\left[-\cos a\beta\cdot(e^{a\beta}-e^{-a\beta})-\sin a\beta\cdot(e^{a\beta}+e^{-a\beta})\right]+\left[m_5m_9+m_7\right]\gamma_0\left[\cos a\beta\cdot(e^{a\beta}-e^{-a\beta})-\sin ab\cdot(e^{a\beta}+e^{-a\beta})\right]-$$
$$-m_9\gamma_1\left[-e^{a\beta}(\cos a\beta+\sin a\beta)-e^{g}\left(\cos g+\sin g\right)\right]+w_1e^{f}\gamma_1\left(\cos f+\sin f\right)=0.$$

Tähistades viimases võrrandis B_2 kordaja ja vabaliikme järgmiselt:

$$\begin{aligned}
m_{10}&:=\gamma_0\left(m_8m_3m_5+m_8m_1+m_3m_6+m_2\right)\cdot\left[-(e^{a\beta}-e^{-a\beta})\cos a\beta-(e^{a\beta}+e^{-a\beta})\sin a\beta\right]+\gamma_0\left(m_5m_8+m_6\right)\cdot\\
&\cdot\left[(e^{a\beta}-e^{-a\beta})\cos a\beta-(e^{a\beta}+e^{-a\beta})\sin a\beta\right]-m_8\gamma_1\left[-e^{a\beta}(\cos a\beta+\sin a\beta)-e^{g}\left(\cos g+\sin g\right)\right]-\\
&-e^{a\beta}(\cos a\beta-\sin a\beta)-e^{g}\left(\cos g-\sin g\right),\\
m_{11}&:=\gamma_0\left[m_9m_3m_5+m_3m_7+m_9m_1+m_4\right]\cdot\left[-(e^{a\beta}-e^{-a\beta})\cos a\beta-(e^{a\beta}+e^{-a\beta})\sin a\beta\right]+\left[m_5m_9+m_7\right]\gamma_0\cdot\\
&\cdot\left[(e^{a\beta}-e^{-a\beta})\cos a\beta-(e^{a\beta}+e^{-a\beta})\sin ab\right]-m_9\gamma_1\left[-e^{a\beta}(\cos a\beta+\sin a\beta)-e^{g}\left(\cos g+\sin g\right)\right]+\\
&+w_1e^{f}\gamma_1\left(\cos f+\sin f\right).
\end{aligned}\tag{45}$$

saame B_2 avaldada

$$B_2=-\frac{m_{11}}{m_{10}}.\tag{46}$$

Seostest (43), (44) ja (46) saame kõik neli konstanti süsteemile {(33), (34), (35), (36)}

$$B_1=-\frac{m_8m_{11}}{m_{10}}+m_9,$$

$$B_2=-\frac{m_{11}}{m_{10}},$$

$$C_1=-\left(m_8m_3m_5+m_8m_1+m_3m_6+m_2\right)\frac{m_{11}}{m_{10}}+m_9m_3m_5+m_3m_7+m_9m_1+m_4,$$

$$C_2=-\left(m_5m_8+m_6\right)\frac{m_{11}}{m_{10}}+m_5m_9+m_7,$$

kus $m_1,..,m_{11}$ on eespool toodud seostes (37), (39), (42) ja (45). Võttes lisaks veel konstantide C_3,C_4,B_3,B_4 avaldised (24), (25), (26) ja (27), saame kirja panna esialgse süsteemi {(16) – (23)} lahendi järgmisel kujul:

$$B_1=-\frac{m_8m_{11}}{m_{10}}+m_9,$$

$$B_2=-\frac{m_{11}}{m_{10}},$$

$$B_3=-w_1\cdot e^{a}\cos a-e^{2a}\left[\left(-\frac{m_8m_{11}}{m_{10}}+m_9\right)\cos 2a-\frac{m_{11}}{m_{10}}\sin 2a\right],$$

$$B_4 = -w_1 \cdot e^{a} \sin a - e^{2a}\left[\left(-\frac{m_8 m_{11}}{m_{10}} + m_9\right)\sin 2a + \frac{m_{11}}{m_{10}}\cos 2a\right],$$

$$C_1 = -\left(m_8 m_3 m_5 + m_8 m_1 + m_3 m_6 + m_2\right)\frac{m_{11}}{m_{10}} + m_9 m_3 m_5 + m_3 m_7 + m_9 m_1 + m_4,$$

$$C_2 = -\left(m_5 m_8 + m_6\right)\frac{m_{11}}{m_{10}} + m_5 m_9 + m_7,$$

$$C_3 = -\left(m_8 m_3 m_5 + m_8 m_1 + m_3 m_6 + m_2\right)\frac{m_{11}}{m_{10}} + m_9 m_3 m_5 + m_3 m_7 + m_9 m_1 + m_4,$$

$$C_4 = \left(m_5 m_8 + m_6\right)\frac{m_{11}}{m_{10}} - m_5 m_9 - m_7,$$

kus

$$f := a(1-\beta),$$

$$g := a(2-\beta),$$

$$m_1 := \frac{e^{a\beta}\cos a\beta - e^{g}\cos g}{(e^{a\beta} + e^{-a\beta})\cos a\beta},$$

$$m_2 := \frac{e^{a\beta}\sin a\beta - e^{g}\sin g}{(e^{a\beta} + e^{-a\beta})\cos a\beta},$$

$$m_3 := -\frac{(e^{a\beta} - e^{-a\beta})\sin a\beta}{(e^{a\beta} + e^{-a\beta})\cos a\beta},$$

$$m_4 := \frac{w_0 - w_1\left(1 - e^{f}\cos f\right)}{(e^{a\beta} + e^{-a\beta})\cos a\beta},$$

$$m_5 := -\frac{m_1\gamma_0(-e^{a\beta} + e^{-a\beta})\sin a\beta - \gamma_1\left(-e^{a\beta}\sin a\beta + e^{g}\sin g\right)}{m_3\gamma_0(-e^{a\beta} + e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta} + e^{-a\beta})\cos a\beta},$$

$$m_6 := -\frac{m_2\gamma_0(-e^{a\beta} + e^{-a\beta})\sin a\beta - \gamma_1\left(e^{a\beta}\cos a\beta - e^{g}\cos g\right)}{m_3\gamma_0(-e^{a\beta} + e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta} + e^{-a\beta})\cos a\beta},$$

$$m_7 := -\frac{m_4\gamma_0(-e^{a\beta} + e^{-a\beta})\sin a\beta - w_1\gamma_1 e^{f}\sin f}{m_3\gamma_0(-e^{a\beta} + e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta} + e^{-a\beta})\cos a\beta},$$

$$m_8 := -\frac{(m_2 + m_3 m_6 + m_6)(e^{a\beta} - e^{-a\beta})\cos a\beta + (-m_2 - m_3 m_6 + m_6)(e^{a\beta} + e^{-a\beta})\sin a\beta - [e^{a\beta}(\cos a\beta + \sin a\beta) + e^{g}(\cos g + \sin g}{(m_1 + m_3 m_5 + m_5)(e^{a\beta} - e^{-a\beta})\cos a\beta + (-m_1 - m_3 m_5 + m_5)(e^{a\beta} + e^{-a\beta})\sin a\beta - [e^{a\beta}(\cos a\beta - \sin a\beta) + e^{g}(-\cos g + \sin g}$$

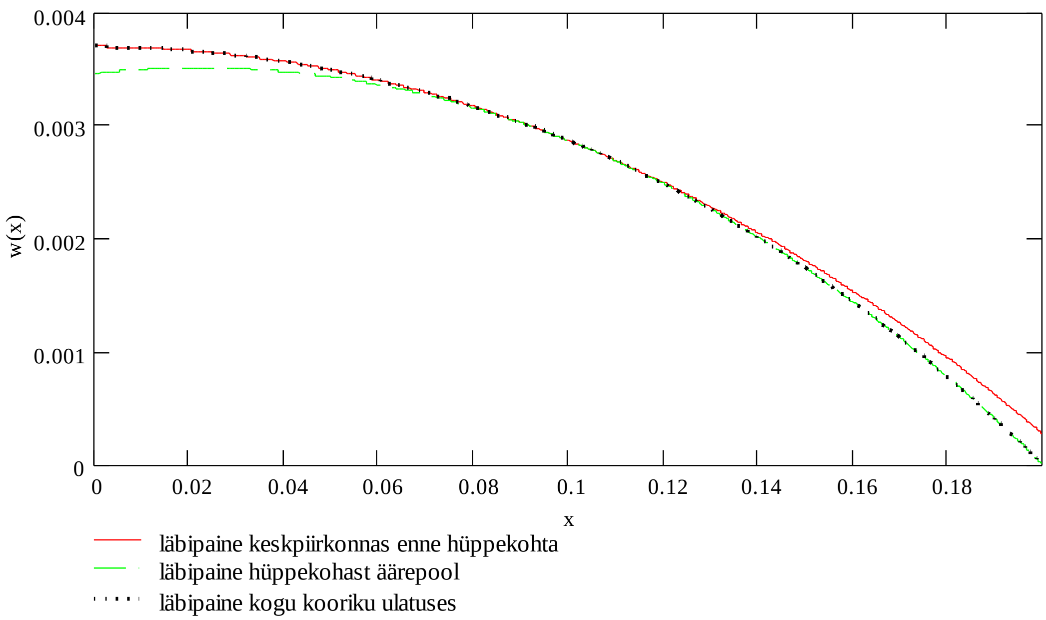
$$m_9 := -\frac{(m_4 + m_3 m_7 + m_7)(e^{a\beta} - e^{-a\beta})\cos a\beta + (-m_4 - m_3 m_7 + m_7)(e^{a\beta} + e^{-a\beta})\sin a\beta - w_1 e^{f}(\cos f - \sin f)}{(m_1 + m_3 m_5 + m_5)(e^{a\beta} - e^{-a\beta})\cos a\beta + (-m_1 - m_3 m_5 + m_5)(e^{a\beta} + e^{-a\beta})\sin a\beta - [e^{a\beta}(\cos a\beta - \sin a\beta) + e^{g}(-\cos g + \sin g}$$

$$m_{10} := \gamma_0\left(m_8 m_3 m_5 + m_8 m_1 + m_3 m_6 + m_2\right)\cdot\left[-(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin a\beta\right] + \gamma_0\left(m_5 m_8 + m_6\right)\cdot$$
$$\cdot\left[(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin a\beta\right] - m_8\gamma_1\left[-e^{a\beta}(\cos a\beta + \sin a\beta) - e^{g}\left(\cos g + \sin g\right)\right] -$$
$$- e^{a\beta}(\cos a\beta - \sin a\beta) - e^{g}\left(\cos g - \sin g\right),$$

$$m_{11} := \gamma_0 \left| m_9 m_3 m_5 + m_3 m_7 + m_9 m_1 + m_4 \right| \cdot \left[-(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin a\beta \right] + \left| m_5 m_9 + m_7 \right| \gamma_0 \cdot$$
$$\cdot \left[(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin ab \right] - m_9 \gamma_1 \left[-e^{a\beta}(\cos a\beta + \sin a\beta) - e^{g}\left(\cos g + \sin g\right) \right] +$$
$$+ w_1 e^{f} \gamma_1 \left(\cos f + \sin f\right).$$

Sellega on tükiti konstantse paksusega ideaalselt kahekihilise silindrilise kooriku läbipaine vaba toetuse korral analüütiliselt leitud.

Saadud tulemuse paremaks ettekujutuseks joonestame läbipainde võrrandit ja leitud konstantide avaldisi kasutades pehmest terasest valmistatud silindrilise kooriku läbipainde graafiku (joon. 3) Antud juhul on võetud terasest koorik pikkusega *20 cm*, raadiusega *5 cm* ja muutuva kihipaksusega $t_0=3$ *mm ja* $t_1=2$ *mm* ning sellele koorikule on rakendatud koormust *200* MN/m^2. Arvutustes on võetud $E=10^5$ MN/m^2 ning $\nu=0,3$. Kihi paksus muutub kohal $x=0,1$.

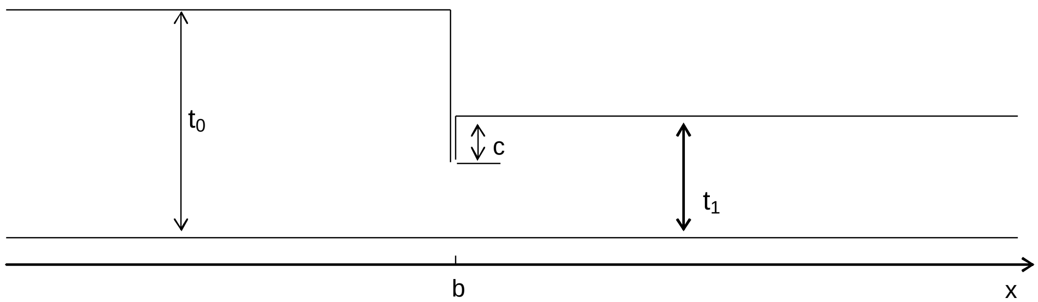


Joon. 3. Läbipaine tükiti konstantse paksuse korral

Tegemist vaba toetusega mõlemas otsas. Läbipainet kooriku keskpiirkonnas enne hüppekohta (x<0,1) kirjeldab katkendlik joon graafikul ja läbipainet hüppekohast äärepool (x>0,1) pidev joon. Hüppekohal x=1 need jooned ühtivad ja üleminek ühelt joonelt teisele on pidev. Ühendades need jooned vastavalt piirkonnale, saame punktiirjoone, mis näitab tegelikku läbipainet. Enne hüppekohta ühtib see katkendliku joonega ja pärast pideva joonega. Graafikult on näha, et läbipaine on maksimaalne kooriku keskel (kohal x=0) ja nulliga võrdne kooriku serval (x=0,2), mis on vaba toetuse korral loomulik.

§ 3. Praoga elastne silindriline koorik

Selles paragrahvis uurime edasi elastset tükiti konstantse paksusega ideaalselt kahekihilist silindrilist koorikut. Eeldame, et kooriku seina kandvas kihis on pragu konstantse sügavusega *c* paksuse muutumise kohas $x = b$, nagu on näidatud joonisel 4.



Joon. 4. Pragu paksuse muutumise kohal $x = b$

Nagu eelmises paragrahvis, läheme siingi üle dimensioonita suurustele. Võttes $\xi = \frac{x}{l}$, $\beta = \frac{b}{l}$ ja $w = \frac{W}{H}$, võime esitada kooriku painde võrrandi kujul

$$w^{IV} + 4a^4 w = \frac{q}{\gamma},$$

kus $a^4 = \frac{l^4(1-v^2)}{R^2 H^2}$ ja $q = p\frac{2l^4(1-v^2)}{EH^3 t_*}$.

Selle võrrandi üldlahend avaldub

1) piirkonnas $\xi \in [0, \beta]$ kujul

$$w = w_0 + e^{a\xi}(G_1 \cos a\xi + G_2 \sin a\xi) + e^{-a\xi}(G_3 \cos a\xi + G_4 \sin a\xi),$$

2) piirkonnas $\xi \in [\beta, 1]$ kujul

$$w = w_1 + e^{a\xi}(H_1 \cos a\xi + H_2 \sin a\xi) + e^{-a\xi}(H_3 \cos a\xi + H_4 \sin a\xi),$$

kus w_0 ja w_1 on võrrandi erilahendid vastavates piirkondades ning $G_1, .., G_4$ ja $H_1, .., H_4$ on kooriku ääretingimustest sõltuvad integreerimiskonstandid.

Tegemist on vaba toetusega, seega rajatingimused (11) – (14) jäävad kehtima. Ka pidevuse tingimused (5), (9) ja (10) kehtivad. Muutub tingimus (6), sest läbipainde tuletis hüppekohal ei ole enam pidev. Kirjanduses [5] ja [11] on näidatud, et prao mõju kooriku käitumisele iseloomustab järgmine tingimus:

$$\begin{cases} K_T \cdot \left[\dfrac{dW}{dx}(b) \right] = -M(b), \\ K_T = K_T(c), \end{cases}$$

kus c on prao sügavus. Minnes ka siin üle dimensioonita suurustele, saame viimase tingimuse kujul

$$\left[w'(\beta) \right] = K\gamma_1 w''(\beta+). \tag{47}$$

Seega lahendada tuleb järgmine võrrandisüsteem:

$$\begin{cases} w_0 + e^{a\beta}(G_1 \cos a\beta + G_2 \sin a\beta) + e^{-a\beta}(G_3 \cos a\beta + G_4 \sin a\beta) = w_1 + e^{a\beta}(H_1 \cos a\beta + H_2 \sin a\beta) + \\ + e^{-a\beta}(H_3 \cos a\beta + H_4 \sin a\beta) \\ e^{a\beta}\left[(H_1 + H_2)\cos a\beta + (H_2 - H_1)\sin a\beta\right] + e^{-a\beta}\left[(H_4 - H_3)\cos a\beta + (-H_3 - H_4)\sin a\beta\right] - \\ - e^{a\beta}\left[(G_1 + G_2)\cos a\beta + (G_2 - G_1)\sin a\beta\right] - e^{-a\beta}\left[(G_4 - G_3)\cos a\beta + (-G_3 - G_4)\sin a\beta\right] = \\ = K\gamma_1 \cdot 2a\left[e^{a\beta}(H_2 \cos a\beta - \mathrm{H}_1 \sin a\beta) + e^{-a\beta}(-H_4 \cos a\beta + H_3 \sin a\beta)\right] \\ \gamma_0\left[e^{a\beta}(G_2 \cos a\beta - G_1 \sin a\beta) + e^{-a\beta}(-G_4 \cos a\beta + G_3 \sin a\beta)\right] = \\ = \gamma_1\left[e^{a\beta}(H_2 \cos a\beta - \mathrm{H}_1 \sin a\beta) + e^{-a\beta}(-H_4 \cos a\beta + H_3 \sin a\beta)\right] \\ \gamma_0\left\{e^{a\beta}\left[(G_2 - G_1)\cos a\beta + (-G_1 - G_2)\sin a\beta\right] + e^{-a\beta}\left[(G_3 + G_4)\cos a\beta + (G_4 - G_3)\sin a\beta\right]\right\} = \\ = \gamma_1\left\{e^{a\beta}\left[(H_2 - H_1)\cos a\beta + (-H_1 - H_2)\sin a\beta\right] + e^{-a\beta}\left[(H_3 + H_4)\cos a\beta + (H_4 - H_3)\sin a\beta\right]\right\} \\ G_1 + G_2 - G_3 + G_4 = 0 \\ w_1 + e^{a}(H_1 \cos a + H_2 \sin a) + e^{-a}(H_3 \cos a + H_4 \sin a) = 0 \\ e^{a}(H_2 \cos a - H_1 \sin a) + e^{-a}(-H_4 \cos a + H_3 \sin a) = 0 \\ -G_1 + G_2 + G_3 + G_4 = 0 \end{cases} \tag{48}$$

Seejuures on meil suuresti abiks eelmises paragrahvis lahendatud süsteem {(16) – (23)}. Tänu nimetatud süsteemi sarnasusele süsteemiga (48), saame kasutada paragrahvis 1 toodud vahetulemusi, kohandades neid praeguse süsteemiga.

Süsteemi (48) lahendamist alustame viiendast ja kaheksandast võrrandist. Liites nende võrrandite vastavad pooled omavahel ning koondades sarnased liidetavad, saame tulemuseks

$$G_2 = -G_4 . \tag{49}$$

Asetades saadud seose süsteemi (48) kuuendasse võrrandisse, jääb peale koondamist kehtima võrdus

$$G_1 = G_3 . \tag{50}$$

Vaatleme eraldi ka kuuendat ja seitsmendat võrrandit. Korrutame neist esimest suurusega $\cos a$ ja teist suurusega $\sin a$, seejärel liidame võrrandite vastavad pooled omavahel.

$$w_1 \cos a + e^a (H_1 \cos^2 a + H_2 \sin a \cos a) + e^{-a}(H_3 \cos^2 a + H_4 \sin a \cos a) +$$
$$+ e^a (H_2 \sin a \cos a - H_1 \sin^2 a) + e^{-a}(-H_4 \sin a \cos a + H_3 \sin^2 a) = 0$$

Koondades saadud võrrandis sarnased liidetavad ja kasutades trigonomeetriliste funktsioonide elementaarseid teisendamisvalemeid, saame viimasest võrrandist

$$w_1 \cos a + e^a (H_1 \cos 2a + H_2 \sin 2a) + e^{-a} H_3 = 0 ,$$

millest peale suurusega $e^a \neq 0$ võrrandi poolte läbikorrutamist saame avaldada H_3

$$H_3 = -w_1 \cdot e^a \cos a - e^{2a}(H_1 \cos 2a + H_2 \sin 2a) . \tag{51}$$

Nüüd korrutame seitsmendat võrrandit $\sin a$ -ga ja kuuendat $\cos a$ -ga ning lahutame esimesest võrrandist teise. Analoogselt eelmisega, kasutades trigonomeetrilisi teisendusi ja koondamist, saame avaldada suuruse H_4

$$H_4 = -w_1 \cdot e^a \sin a - e^{2a}(H_1 \sin 2a - H_2 \cos 2a) . \tag{52}$$

Kasutades seoseid (49), (50), (51) ja (52) ning tähistades

$$\begin{aligned} f &:= a(1-\beta), \\ g &:= a(2-\beta), \end{aligned} \tag{53}$$

seejärel koondades sarnased liidetavad, saame süsteemi (48) neli esimest võrrandit panna kirja lihtsamal kujul

$$\begin{cases} w_0 + G_1(e^{a\beta} + e^{-a\beta})\cos a\beta + G_2(e^{a\beta} - e^{-a\beta})\sin a\beta = w_1\left(1 - e^f \cos f\right) + \\ + H_1\left(e^{a\beta}\cos a\beta - e^g \cos g\right) + H_2\left(e^{a\beta}\sin a\beta - e^g \sin g\right), \\ G_1\left[(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin a\beta\right] + G_2\left[(e^{a\beta} - e^{-a\beta})\cos a\beta + (e^{a\beta} + e^{-a\beta})\sin a\beta\right] = \\ = H_1\left[e^{a\beta}(\cos a\beta - \sin a\beta) + e^g\left(-\cos g + \sin g\right) - 2K\gamma_1 a\left(-e^{a\beta}\sin a\beta + e^g \sin g\right)\right] + \\ + H_2\left[e^{a\beta}(\cos a\beta + \sin a\beta) + e^g\left(\cos g + \sin g\right) - 2K\gamma_1 a\left(e^{a\beta}\cos a\beta - e^g\cos g\right)\right] + \\ + w_1 e^f\left[\cos f - \sin f\left(1 + 2K\gamma_1 a \sin f\right)\right], \\ \gamma_0\left[G_1(-e^{a\beta} + e^{-a\beta})\sin a\beta + G_2(e^{a\beta} + e^{-a\beta})\cos a\beta\right] = \gamma_1\left[H_1\left(-e^{a\beta}\sin a\beta + e^g \sin g\right) + \\ + H_2\left(e^{a\beta}\cos a\beta - e^g\cos g\right) + w_1 e^f \sin f\right], \\ \gamma_0\left\{G_1\left[-(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin a\beta\right] + G_2\left[(e^{a\beta} - e^{-a\beta})\cos a\beta - (e^{a\beta} + e^{-a\beta})\sin ab\right]\right\} = \\ = \gamma_1\left\{H_1\left[-e^{a\beta}(\cos a\beta + \sin a\beta) - e^g\left(\cos g + \sin g\right)\right] + H_2\left[e^{a\beta}(\cos a\beta - \sin a\beta) + e^g\left(\cos g - \sin g\right)\right] + \\ + \left[-w_1 e^f\left(\cos f + \sin f\right)\right]\right\}, \end{cases} \tag{54}$$

Tegemist on nelja tundmatuga G_1, G_2, H_1, H_2 lineaarvõrrandisüsteemiga. Lahendamist alustame esimesest võrrandist. Jagame võrrandi mõlemaid pooli konstandi G_1 kordajaga $\cos a\beta(e^{a\beta} + e^{-a\beta}) \neq 0$ ning avaldame G_1

$$G_1 = H_1 \frac{e^{a\beta}\cos a\beta - e^g \cos g}{(e^{a\beta} + e^{-a\beta})\cos a\beta} + H_2 \frac{e^{a\beta}\sin a\beta - e^g \sin g}{(e^{a\beta} + e^{-a\beta})\cos a\beta} - G_2 \frac{\sin a\beta\,(e^{a\beta} - e^{-a\beta})}{(e^{a\beta} + e^{-a\beta})\cos a\beta} +$$
$$+ \frac{w_0 - w_1\left(1 - e^f \cos f\right)}{(e^{a\beta} + e^{-a\beta})\cos a\beta}.$$

Kasutades tähistusi (37) saame G_1 avaldise

$$G_1 = m_1 H_1 + m_2 H_2 + m_3 G_2 + m_4. \tag{55}$$

Järgmisena asetame seose (55) süsteemi (54) kolmandasse võrrandisse ja peale sarnaste liidetavate koondamist saame avaldada G_2

$$G_2 = -H_1 \frac{m_1\gamma_0(-e^{a\beta} + e^{-a\beta})\sin a\beta - \gamma_1\left(-e^{a\beta}\sin a\beta + e^g \sin g\right)}{m_3\gamma_0(-e^{a\beta} + e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta} + e^{-a\beta})\cos a\beta} -$$
$$-H_2 \frac{m_2\gamma_0(-e^{a\beta} + e^{-a\beta})\sin a\beta - \gamma_1\left(e^{a\beta}\cos a\beta - e^g \cos g\right)}{m_3\gamma_0(-e^{a\beta} + e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta} + e^{-a\beta})\cos a\beta} -$$
$$- \frac{m_4\gamma_0(-e^{a\beta} + e^{-a\beta})\sin a\beta - w_1\gamma_1 e^f \sin f}{m_3\gamma_0(-e^{a\beta} + e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta} + e^{-a\beta})\cos a\beta}$$

Kasutades tähistusi(39), saame G_2 avaldise

$$G_2 = m_5 H_1 + m_6 H_2 + m_7 \tag{56}$$

ja (55) põhjal

$$G_1 = \left(m_1 + m_3 m_5\right) H_1 + \left(m_2 + m_3 m_6\right) H_2 + m_3 m_7 + m_4 \tag{57}$$

Asetades seosed (56) ja (57) süsteemi (54) teise võrrandisse, saame koondamise tulemusena

$$\begin{aligned}
&H_1 \Big\{ (e^{a\beta} - e^{-a\beta})\left(m_1 + m_3 m_5 + m_5\right)\cos a\beta + (e^{a\beta} + e^{-a\beta})\left(-m_1 - m_3 m_5 + m_5\right)\sin a\beta - \\
&-\Big[e^{a\beta}(\cos a\beta - \sin a\beta) + e^{g}\left(-\cos g + \sin g\right) - 2K\gamma_1 a\left(-e^{a\beta}\sin a\beta + e^{g}\sin g\right)\Big]\Big\} + \\
&+H_2 \Big\{ (e^{a\beta} - e^{-a\beta})\left(m_2 + m_3 m_6 + m_6\right)\cos a\beta + (e^{a\beta} + e^{-a\beta})\left(-m_2 - m_3 m_6 + m_6\right)\sin a\beta - \\
&-\Big[e^{a\beta}(\cos a\beta + \sin a\beta) + e^{g}\left(\cos g + \sin g\right) - 2K\gamma_1 a\left(e^{a\beta}\cos a\beta - e^{g}\cos g\right)\Big]\Big\} + \\
&+\cos a\beta\,(e^{a\beta} - e^{-a\beta})\left(m_3 m_7 + m_4 + m_7\right) + \sin a\beta\,(e^{a\beta} + e^{-a\beta})\left(-m_3 m_7 - m_4 + m_7\right) - \\
&-w_1 e^{f}\Big[\cos f - \sin f\left(1 + 2K\gamma_1 a \sin f\right)\Big] = 0
\end{aligned}$$

Tähistades viimases võrrandis H_1 ja H_2 kordajad ning vabaliikme järgmiselt:

$$\begin{aligned}
&m_{12} := (e^{a\beta} - e^{-a\beta})\left(m_1 + m_3 m_5 + m_5\right)\cos a\beta + (e^{a\beta} + e^{-a\beta})\left(-m_1 - m_3 m_5 + m_5\right)\sin a\beta - \\
&\quad -\Big[e^{a\beta}(\cos a\beta - \sin a\beta) + e^{g}\left(-\cos g + \sin g\right) - 2K\gamma_1 a\left(-e^{a\beta}\sin a\beta + e^{g}\sin g\right)\Big], \\
&m_{13} := -\Big\{ (e^{a\beta} - e^{-a\beta})\left(m_2 + m_3 m_6 + m_6\right)\cos a\beta + (e^{a\beta} + e^{-a\beta})\left(-m_2 - m_3 m_6 + m_6\right)\sin a\beta - \\
&\quad -\Big[e^{a\beta}(\cos a\beta + \sin a\beta) + e^{g}\left(\cos g + \sin g\right) - 2K\gamma_1 a\left(e^{a\beta}\cos a\beta - e^{g}\cos g\right)\Big]\Big\}, \\
&m_{14} := -\Big\{ +\cos a\beta\,(e^{a\beta} - e^{-a\beta})\left(m_3 m_7 + m_4 + m_7\right) + \sin a\beta\,(e^{a\beta} + e^{-a\beta})\left(-m_3 m_7 - m_4 + m_7\right) - \\
&\quad - w_1 e^{f}\Big[\cos f - \sin f\left(1 + 2K\gamma_1 a \sin f\right)\Big]\Big\},
\end{aligned} \tag{58}$$

saame H_1 avaldada

$$H_1 = \frac{m_{13}}{m_{12}} H_2 + \frac{m_{14}}{m_{12}} \tag{59}$$

Seostest (56) ja (57)

$$\begin{aligned}
G_1 &= \left[\frac{\left(m_1 + m_3 m_5\right) m_{13}}{m_{12}} + m_2 + m_3 m_6 \right] H_2 + \frac{\left(m_1 + m_3 m_5\right) m_{14}}{m_{12}} + m_3 m_7 + m_4 \\
G_2 &= \left(\frac{m_5 m_{13}}{m_{12}} + m_6 \right) H_2 + \frac{m_5 m_{14}}{m_{12}} + m_7
\end{aligned} \tag{60}$$

Asetame nüüd võrdused (59) ja (60) süsteemi (54) neljandasse võrrandisse

$$
H_2\left\{(e^{a\beta}-e^{-a\beta})\left[-\frac{(m_1+m_3m_5)m_{13}}{m_{12}}+\frac{m_5m_{13}}{m_{12}}-m_3m_6+m_6-m_2\right]\gamma_0\cos a\beta-\right.
$$

$$
-(e^{a\beta}+e^{-a\beta})\left[\frac{(m_1+m_3m_5)m_{13}}{m_{12}}+m_2+m_3m_6+\frac{m_5m_{13}}{m_{12}}+m_6\right]\gamma_0\sin a\beta-
$$

$$
\left.-\frac{m_{13}}{m_{12}}\gamma_1\left[-e^{a\beta}(\cos a\beta+\sin a\beta)-e^g\left(\cos g+\sin g\right)\right]-\gamma_1\left[e^{a\beta}(\cos a\beta-\sin a\beta)+e^g\left(\cos g-\sin g\right)\right]\right\}+
$$

$$
+(e^{a\beta}-e^{-a\beta})\left[-\frac{(m_1+m_3m_5)m_{14}}{m_{12}}-m_3m_7-m_4+\frac{m_5m_{14}}{m_{12}}+m_7\right]\gamma_0\cos a\beta-
$$

$$
-\left(e^{a\beta}+e^{-a\beta}\right)\left[\frac{(m_1+m_3m_5)m_{14}}{m_{12}}+m_3m_7+m_4+\frac{m_5m_{14}}{m_{12}}+m_7\right]\gamma_0\sin ab-
$$

$$
-\frac{m_{14}}{m_{12}}\gamma_1\left[-e^{a\beta}(\cos a\beta+\sin a\beta)-e^g\left(\cos g+\sin g\right)\right]+w_1e^f\gamma_1\left(\cos f+\sin f\right)=0.
$$

Tähistades viimases võrduses H_2 kordaja ja vabaliikme järgmiselt:

$$
m_{15}:=\left\{(e^{a\beta}-e^{-a\beta})\left[-\frac{(m_1+m_3m_5)m_{13}}{m_{12}}+\frac{m_5m_{13}}{m_{12}}-m_3m_6+m_6-m_2\right]\gamma_0\cos a\beta-\right.
$$

$$
-(e^{a\beta}+e^{-a\beta})\left[\frac{(m_1+m_3m_5)m_{13}}{m_{12}}+m_2+m_3m_6+\frac{m_5m_{13}}{m_{12}}+m_6\right]\gamma_0\sin a\beta-
$$

$$
-\frac{m_{13}}{m_{12}}\gamma_1\left[-e^{a\beta}(\cos a\beta+\sin a\beta)-e^g\left(\cos g+\sin g\right)\right]-\gamma_1\left[e^{a\beta}(\cos a\beta-\sin a\beta)+e^g\left(\cos g-\sin g\right)\right],
$$

$$
m_{16}:=-(e^{a\beta}-e^{-a\beta})\left[-\frac{(m_1+m_3m_5)m_{14}}{m_{12}}-m_3m_7-m_4+\frac{m_5m_{14}}{m_{12}}+m_7\right]\gamma_0\cos a\beta+ \tag{61}
$$

$$
+\left(e^{a\beta}+e^{-a\beta}\right)\left[\frac{(m_1+m_3m_5)m_{14}}{m_{12}}+m_3m_7+m_4+\frac{m_5m_{14}}{m_{12}}+m_7\right]\gamma_0\sin ab+
$$

$$
+\frac{m_{14}}{m_{12}}\gamma_1\left[-e^{a\beta}(\cos a\beta+\sin a\beta)-e^g\left(\cos g+\sin g\right)\right]-w_1e^f\gamma_1\left(\cos f+\sin f\right),
$$

võime viimasest võrrandist avaldada H_2

$$
H_2=\frac{m_{16}}{m_{15}}. \tag{62}
$$

Võrdustest (59), (60) ja (62)

$$
G_1=\left[\frac{(m_1+m_3m_5)m_{13}}{m_{12}}+m_2+m_3m_6\right]\frac{m_{16}}{m_{15}}+\frac{(m_1+m_3m_5)m_{14}}{m_{12}}+m_3m_7+m_4,
$$

$$
G_2=\left(\frac{m_5m_{13}}{m_{12}}+m_6\right)\frac{m_{16}}{m_{15}}+\frac{m_5m_{14}}{m_{12}}+m_7,
$$

$$H_1=\frac{m_{13}m_{16}}{m_{12}m_{15}}+\frac{m_{14}}{m_{12}},$$

$$H_2=\frac{m_{16}}{m_{15}}.$$

Oleme saanud süsteemi (54) lahendi milles kasutatud tähised on toodud seostes (37), (39) , (58) ja (61).

Nüüd saame kirja panna ka süsteemi (54) lahendi täielikul kujul, selleks on:

$$G_1=\left[\frac{(m_1+m_3m_5)m_{13}}{m_{12}}+m_2+m_3m_6\right]\frac{m_{16}}{m_{15}}+\frac{(m_1+m_3m_5)m_{14}}{m_{12}}+m_3m_7+m_4,$$

$$G_2=\left(\frac{m_5m_{13}}{m_{12}}+m_6\right)\frac{m_{16}}{m_{15}}+\frac{m_5m_{14}}{m_{12}}+m_7,$$

$$G_3=\left[\frac{(m_1+m_3m_5)m_{13}}{m_{12}}+m_2+m_3m_6\right]\frac{m_{16}}{m_{15}}+\frac{(m_1+m_3m_5)m_{14}}{m_{12}}+m_3m_7+m_4,$$

$$G_4=-\left(\frac{m_5m_{13}}{m_{12}}+m_6\right)\frac{m_{16}}{m_{15}}-\frac{m_5m_{14}}{m_{12}}-m_7,$$

$$H_1=\frac{m_{13}m_{16}}{m_{12}m_{15}}+\frac{m_{14}}{m_{12}},$$

$$H_2=\frac{m_{16}}{m_{15}},$$

$$H_3=-w_1\cdot e^{a}\cos a-e^{2a}\left[\left(\frac{m_{13}m_{16}}{m_{12}m_{15}}+\frac{m_{14}}{m_{12}}\right)\cos 2a+\frac{m_{16}}{m_{15}}\sin 2a\right],$$

$$H_4=-w_1\cdot e^{a}\sin a-e^{2a}\left[\left(\frac{m_{13}m_{16}}{m_{12}m_{15}}+\frac{m_{14}}{m_{12}}\right)\sin 2a-\frac{m_{16}}{m_{15}}\cos 2a\right],$$

kus

$$f:=a(1-\beta),$$

$$g:=a(2-\beta),$$

$$m_1=\frac{e^{a\beta}\cos a\beta-e^{g}\cos g}{(e^{a\beta}+e^{-a\beta})\cos a\beta},$$

$$m_2=\frac{e^{a\beta}\sin a\beta-e^{g}\sin g}{(e^{a\beta}+e^{-a\beta})\cos a\beta},$$

$$m_3=-\frac{(e^{a\beta}-e^{-a\beta})\sin a\beta}{(e^{a\beta}+e^{-a\beta})\cos a\beta},$$

$$m_4=\frac{w_0-w_1\left(1-e^{f}\cos f\right)}{(e^{a\beta}+e^{-a\beta})\cos a\beta}$$

$$m_5 = -\frac{m_1\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta - \gamma_1\left(-e^{a\beta}\sin a\beta + e^{g}\sin g\right)}{m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta},$$

$$m_6 = -\frac{m_2\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta - \gamma_1\left(e^{a\beta}\cos a\beta - e^{g}\cos g\right)}{m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta},$$

$$m_7 = -\frac{m_4\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta - w_1\gamma_1 e^{f}\sin f}{m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta + \gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta},$$

$$m_{12} = (e^{a\beta}-e^{-a\beta})\left(m_1 + m_3m_5 + m_5\right)\cos a\beta + (e^{a\beta}+e^{-a\beta})\left(-m_1 - m_3m_5 + m_5\right)\sin a\beta -$$
$$-\left[e^{a\beta}(\cos a\beta - \sin a\beta) + e^{g}\left(-\cos g + \sin g\right) - 2K\gamma_1 a\left(-e^{a\beta}\sin a\beta + e^{g}\sin g\right)\right],$$

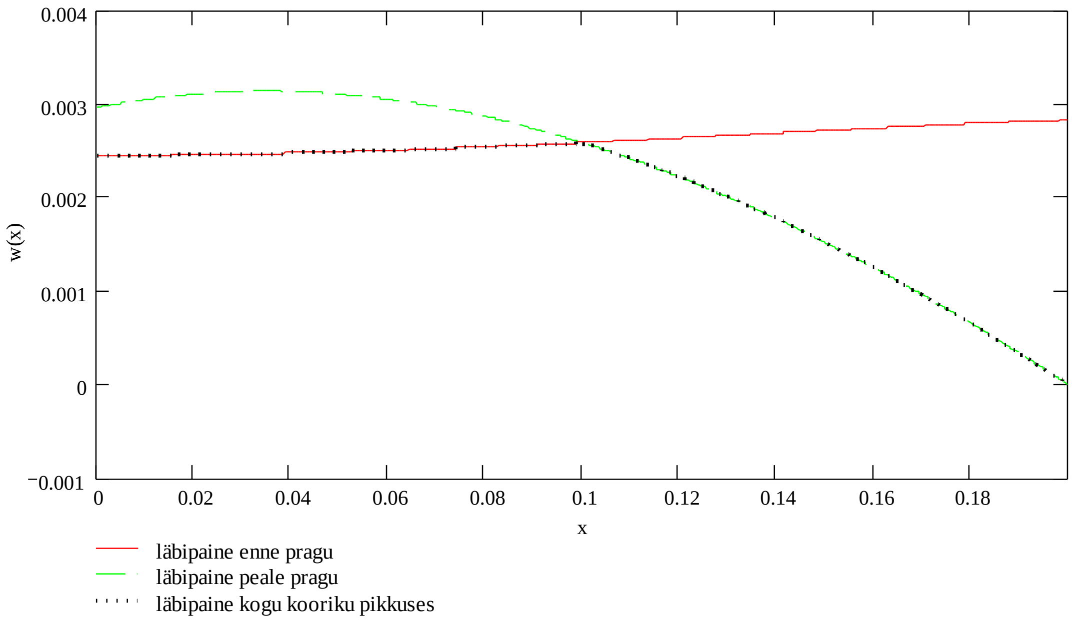
$$m_{13} = -\Big\{(e^{a\beta}-e^{-a\beta})\left(m_2 + m_3m_6 + m_6\right)\cos a\beta + (e^{a\beta}+e^{-a\beta})\left(-m_2 - m_3m_6 + m_6\right)\sin a\beta -$$
$$-\left[e^{a\beta}(\cos a\beta + \sin a\beta) + e^{g}\left(\cos g + \sin g\right) - 2K\gamma_1 a\left(e^{a\beta}\cos a\beta - e^{g}\cos g\right)\right]\Big\},$$

$$m_{14} = -\Big\{+\cos a\beta\,(e^{a\beta}-e^{-a\beta})\left(m_3m_7 + m_4 + m_7\right) + \sin a\beta\,(e^{a\beta}+e^{-a\beta})\left(-m_3m_7 - m_4 + m_7\right) -$$
$$- w_1 e^{f}\left[\cos f - \sin f\left(1 + 2K\gamma_1 a\sin f\right)\right]\Big\},$$

$$m_{15} = \Bigg\{(e^{a\beta}-e^{-a\beta})\left[-\frac{\left(m_1 + m_3m_5\right)m_{13}}{m_{12}} + \frac{m_5m_{13}}{m_{12}} - m_3m_6 + m_6 - m_2\right]\gamma_0\cos a\beta -$$
$$-(e^{a\beta}+e^{-a\beta})\left[\frac{\left(m_1 + m_3m_5\right)m_{13}}{m_{12}} + m_2 + m_3m_6 + \frac{m_5m_{13}}{m_{12}} + m_6\right]\gamma_0\sin a\beta -$$
$$-\frac{m_{13}}{m_{12}}\gamma_1\left[-e^{a\beta}(\cos a\beta + \sin a\beta) - e^{g}\left(\cos g + \sin g\right)\right] - \gamma_1\left[e^{a\beta}(\cos a\beta - \sin a\beta) + e^{g}\left(\cos g - \sin g\right)\right],$$

$$m_{16} = -(e^{a\beta}-e^{-a\beta})\left[-\frac{\left(m_1 + m_3m_5\right)m_{14}}{m_{12}} - m_3m_7 - m_4 + \frac{m_5m_{14}}{m_{12}} + m_7\right]\gamma_0\cos a\beta +$$
$$+\left(e^{a\beta}+e^{-a\beta}\right)\left[\frac{\left(m_1 + m_3m_5\right)m_{14}}{m_{12}} + m_3m_7 + m_4 + \frac{m_5m_{14}}{m_{12}} + m_7\right]\gamma_0\sin ab +$$
$$+\frac{m_{14}}{m_{12}}\gamma_1\left[-e^{a\beta}(\cos a\beta + \sin a\beta) - e^{g}\left(\cos g + \sin g\right)\right] - w_1 e^{f}\gamma_1\left(\cos f + \sin f\right).$$

Paragrahvi lõpetuseks joonestame läbipainde võrrandit ja leitud konstantide avaldisi kasutades pehmest terasest valmistatud silindrilise kooriku läbipainde graafiku (joon. 5). Et tulemus oleks eelmisega võrreldav, on algandmed võetud samad: tegemist on terasest koorikuga, mille pikkus on *20 cm*, raadius *5 cm* ja muutuv kihipaksus $t_0=3$ *mm ja* $t_1=2$ *mm* ning sellele koorikule on rakendatud koormust *200* MN/m^2. Arvutustes on võetud $E=10^5$ MN/m^2 ning $\nu=0{,}3$. Kihi paksus muutub kohal $x=0{,}1$. Sellel kohal asub koorikus ka pragu, mille sügavuseks on võetud 1 mm.



Joon. 5. Praoga kooriku läbipaine

Nagu jooniselt näha, on läbipaine kõige suurem prao kohal (x=0,1), mis ei ole ka üllatav, sest arvestades prao sügavust, mis on pool õhema kihi paksusest, peakski seal olema läbipaine suurim, s.t. prao koht on kõige nõrgem, see on tõenäoline purunemiskoht piirpinge saavutamisel. Kui prao sügavus moodustab vaid kümnendiku või sajandiku õhema kihi paksusest, siis see läbipaindele suuremat mõju ei avalda ja läbipainde graafiku kuju on sarnane joonisel 3 tooduga.

§ 4. Praoga kooriku optimiseerimine

Vaatleme jällegi ideaalselt kahekihilist silindrilist koorikut, mille kandva kihi paksus t on tükiti konstantne (vt. joon. 1). Paksuse muutumise kohal $\xi = \beta$ on koorikus pragu.

Leiame miinimumkaaluga projekti tingimusel, et integraalne läbipaine

$$\int_0^1 w\, d\xi = A$$

on ette antud. Minimiseerime suurust

$$J = \gamma_0 \beta + \gamma_1 (1-\beta) \quad \to \min.$$

Tükiti konstantse paksusega elastse silindrilise kooriku painde võrrand on esitatav kujul

$$w^{IV} + 4a^4 w = \frac{q}{\gamma_j}, \tag{63}$$

kus $a^4 = \dfrac{l^4(1-v^2)}{R^2 H^2}$, $q = p\dfrac{2l^4(1-v^2)}{EH^3 t_*}$ ja $\gamma_j = \dfrac{t_j}{t_*}$,

kus t_* on võrdluskooriku paksus, seega $\gamma_j = \begin{cases} \gamma_0, & 0 \le \xi \le \beta \\ \gamma_1, & \beta < \xi \le 1 \end{cases}$

Kui nüüd tähistada

$$\begin{aligned} y_1 &:= w, \\ y_2 &:= w', \\ y_3 &:= -m, \\ y_4 &:= -m', \end{aligned} \tag{64}$$

kus *m* on dimensioonita suurustele üleminekul saadud moment ja *w* on läbipaine, siis elastse silindrilise kooriku põhivõrrandid on esitatavad süsteemina

$$\begin{cases} y_1' = y_2, \\ y_2' = \dfrac{y_3}{D_j}, \\ y_3' = y_4, \\ y_4' = -4 \cdot d \cdot a^4 \gamma_j y_1 + d \cdot q, \end{cases} \tag{65}$$

piirkonna $S_j = (a_j,\ a_{j+1}),\quad j = 0;1$ jaoks. Sandwich – tüüpi kooriku korral teatavasti

$D_j = d \cdot \gamma_j$, kus $d = \dfrac{EH^2 t_*}{2(1-\nu^2)}$.

Kasutades toodud tähistust, võime süsteemi (65) ümber kirjutada kujul

$$\begin{cases} y_1' = y_2, \\ y_2' = \dfrac{y_3}{d \cdot \gamma_j}, \\ y_3' = y_4, \\ y_4' = -4 \cdot d \cdot a^4 \gamma_j y_1 + d \cdot q, \end{cases} \tag{66}$$

Vaatleme mõlemast otsast vabalt toetatud koorikut, koordinaatide alguse valime kooriku keskele. Kooriku pikkus on $2l$ ja raadius R. Rajatingimused põhisüsteemi (66) jaoks on

$$\begin{aligned} &y_1(1) = 0, \\ &y_2(0) = 0, \\ &y_3(1) = 0, \\ &y_4(0) = 0. \end{aligned} \tag{67}$$

Koorikule vastavad pidevuse tingimused on

$$\begin{aligned} &\left[y_1(\beta)\right] = 0, \\ &\left[y_2(\beta)\right] = \frac{1}{K_t} y_3(\beta), \\ &\left[y_3(\beta)\right] = 0, \\ &\left[y_4(\beta)\right] = 0, \end{aligned} \tag{68}$$

kus $K_t = \dfrac{1}{\gamma_1 \cdot K}$, $K = 6\pi f\left(1-\nu^2\right)$, kus f on prao sügavusest c sõltuv funktsioon $f = f(c)$.

Eeldame, et teada on integraalne läbipaine

$$\int_0^\beta y_1 d\xi + \int_\beta^1 y_1 d\xi = A, \tag{69}$$

kus A on etteantud arv.

Püstitatud ülesanne on parameetritega optimaalse juhtimise ülesanne. Optimaalsuseks tarvilike tingimuste saamiseks moodustame abifunktsionaali (vrd. [1], [2], [8], [9] ja [15])

$$
\begin{aligned}
J_* = {} & \gamma_0 \beta + \gamma_1 (1-\beta) + \int_0^\beta \left[\psi_0 y_1 + \psi_1 \left(y_1' - y_2 \right) + \psi_2 \left(y_2' - \frac{y_3}{d \cdot \gamma_0} \right) + \psi_3 \left(y_3' - y_4 \right) + \right. \\
& \left. + \psi_4 \left(y_4' + 4 \cdot d \cdot a^4 \gamma_0 y_1 - d \cdot q \right) \right] dx + \int_\beta^1 \left[\psi_0 y_1 + \psi_1 \left(y_1' - y_2 \right) + \psi_2 \left(y_2' - \frac{y_3}{d \cdot \gamma_1} \right) + \psi_3 \left(y_3' - y_4 \right) + \right. \\
& \left. + \psi_4 \left(y_4' + 4 \cdot d \cdot a^4 \gamma_1 y_1 - d \cdot q \right) \right] dx + \mu \left[y_2(\beta +) - y_2(\beta -) - K \gamma_1 y_3(\beta) \right],
\end{aligned}
$$

kus μ on Lagrange´i kordaja ja $\psi_1,\ \psi_2,\ \psi_3,\ \psi_4$ – kaasmuutujad.

Arvutame selle funktsionaali täisvariatsiooni ja võrdsustame selle nulliga. Nii saame võrrandi

$$
\begin{aligned}
\Delta J_* = {} & \Delta\gamma_0 \beta + \gamma_0 \Delta\beta + \Delta\gamma_1 (1-\beta) + \gamma_1 (-\Delta\beta) + \int_0^\beta \left\{ \psi_0 \delta y_1 + \psi_1 \left(\delta y_1' - \delta y_2 \right) + \right. \\
& + \psi_2 \left(\delta y_2' - \delta y_3 \frac{1}{d \cdot \gamma_0} + \frac{y_3}{d \cdot \gamma_0^2} \Delta\gamma_0 \right) + \psi_3 \left(\delta y_3' - \delta y_4 \right) + \left. \psi_4 \left[\delta y_4' + 4 \cdot d \cdot a^4 \left(\Delta\gamma_0 y_1 + \gamma_0 \delta y_1 \right) \right] \right\} dx + \\
& + \int_\beta^1 \left\{ \psi_0 \delta y_1 + \psi_1 \left(\delta y_1' - \delta y_2 \right) + \psi_2 \left(\delta y_2' - \delta y_3 \frac{1}{d \cdot \gamma_1} + \frac{y_3}{d \cdot \gamma_1^2} \Delta\gamma_1 \right) + \psi_3 \left(\delta y_3' - \delta y_4 \right) + \right. \\
& \left. + \psi_4 \left[\delta y_4' + 4 \cdot d \cdot a^4 \left(\Delta\gamma_1 y_1 + \gamma_1 \delta y_1 \right) \right] \right\} dx + \mu \left[\Delta y_2(\beta +) - \Delta y_2(\beta -) - \frac{\Delta y_3(\beta)}{K_t} - y_3(\beta) \cdot \Delta (K \cdot \gamma_1) \right] = 0.
\end{aligned}
\tag{70}
$$

Integreerime viimases liikmeid $\delta y_i',\ i = 1,..,4,$ ositi, siis võrdus (70) saab kuju

$$
\begin{aligned}
\Delta J_* = {} & \Delta\gamma_0 \beta + \gamma_0 \Delta\beta + \Delta\gamma_1 (1-\beta) + \gamma_1 (-\Delta\beta) + \mu \left[\Delta y_2(\beta +) - \Delta y_2(\beta -) - \Delta y_3(\beta) K \gamma_1 - y_3(\beta) \cdot (K \cdot \Delta\gamma_1 + \gamma_1 \cdot \Delta K) \right] + \\
& + \left(\psi_1 \delta y_1 + \psi_2 \delta y_2 + \psi_3 \delta y_3 + \psi_4 \delta y_4 \right) \Big|_0^\beta + \left(\psi_1 \delta y_1 + \psi_2 \delta y_2 + \psi_3 \delta y_3 + \psi_4 \delta y_4 \right) \Big|_\beta^1 + \int_0^\beta \left\{ \psi_0 \delta y_1 - \psi_1' \delta y_1 - \right. \\
& - \psi_1 \delta y_2 - \psi_2' \delta y_2 - \psi_2 \delta y_3 \frac{1}{d \cdot \gamma_0} + \psi_2 \frac{y_3}{d \cdot \gamma_0^2} \Delta\gamma_0 - \psi_3' \delta y_3 - \psi_3 \delta y_4 - \psi_4' \delta y_4 + \left. \psi_4 4 \cdot d \cdot a^4 \left(\Delta\gamma_0 y_1 + \gamma_0 \delta y_1 \right) \right\} dx + \\
& + \int_\beta^1 \left\{ \psi_0 \delta y_1 - \psi_1' \delta y_1 - \psi_1 \delta y_2 - \psi_2' \delta y_2 - \psi_2 \delta y_3 \frac{1}{d \cdot \gamma_1} + \psi_2 \frac{y_3}{d \cdot \gamma_1^2} \Delta\gamma_1 - \psi_3' \delta y_3 - \psi_3 \delta y_4 - \psi_4' \delta y_4 + \right. \\
& \left. + \psi_4 4 \cdot d \cdot a^4 \left(\Delta\gamma_1 y_1 + \gamma_1 \delta y_1 \right) \right\} = 0.
\end{aligned}
\tag{71}
$$

Integraali märgi all võrdsustame $\delta y_1,\ \delta y_2,\ \delta y_3,\ \delta y_4$ kordajad nulliga, saame võrdused

$$
\begin{cases}
\psi_1{}' = \psi_0 + 4d \cdot a^4 \gamma_j \psi_4, \\
\psi_2{}' = -\psi_1, \\
\psi_3{}' = -\dfrac{1}{d \cdot \gamma_j} \psi_2, \\
\psi_4{}' = -\psi_3.
\end{cases}
\tag{72}
$$

Süsteemi (72) näol on meil tegemist kaassüsteemiga iga piirkonna $S_j,\ j = 0;1$ jaoks.

Vaadeldes põhisüsteemi (66) ja kaassüsteemi (72), näeme, et ülesanne on enesekaasne, kusjuures

$$
\begin{aligned}
\psi_1 &= \frac{\psi_0}{d \cdot q} y_4 \\
\psi_2 &= -\frac{\psi_0}{d \cdot q} y_3 \\
\psi_3 &= \frac{\psi_0}{d \cdot q} y_2 \\
\psi_4 &= -\frac{\psi_0}{d \cdot q} y_1
\end{aligned}
\tag{73}
$$

Asetades seosed (73) süsteemi (72), saavad kaasvõrrandid kuju

$$
\begin{cases}
\dfrac{\psi_0}{d \cdot q} y_4{}' = \psi_0 + 4d \cdot a^4 \gamma_j \left(-\dfrac{\psi_0}{d \cdot q} y_1 \right), \\
-\dfrac{\psi_0}{d \cdot q} y_3{}' = -\dfrac{\psi_0}{d \cdot q} y_4, \\
\dfrac{\psi_0}{d \cdot q} y_2{}' = -\dfrac{1}{d \cdot \gamma_j} \left(-\dfrac{\psi_0}{d \cdot q} y_3 \right), \\
-\dfrac{\psi_0}{d \cdot q} y_1{}' = -\dfrac{\psi_0}{d \cdot q} y_2,
\end{cases}
$$

mis peale lihtsustamist annab meile süsteemi (66). Seega kaassüsteem langeb tõepoolest kokku põhisüsteemiga, mistõttu võime edaspidi kaassüsteemi vaatluse alt välja jätta. Lihtne on näha, et täidetud on ka transversaalsuse tingimused, kui on rahuldatud faasikoordinaatidele peale pandud rajatingimused.

Nüüd vaatame $\Delta\gamma_0$ ja $\Delta\gamma_1$ kordajaid võrrandis (71). Et muudud $\Delta\gamma_0$ ja $\Delta\gamma_1$ on suvalised parameetrid, mitte faasimuutujad, siis saame need tuua integraali märgi alt välja ja kordajad võrdsustada nulliga järgmiselt:

$$\beta + \int_0^\beta \left(\psi_2 \frac{y_3}{d \cdot \gamma_0^2} + \psi_4 4 \cdot d \cdot a^4 y_1 \right) dx = 0,$$
$$1 - \beta + \int_\beta^1 \left(\psi_2 \frac{y_3}{d \cdot \gamma_1^2} + \psi_4 4 \cdot d \cdot a^4 y_1 \right) dx - \mu \left(K + \frac{\partial K}{\partial \gamma_1} \gamma_1 \right) y_3(\beta) = 0. \tag{74}$$

Nüüd saab võrrand (71) kuju

$$\Delta J_* = \Delta\beta(\gamma_0 - \gamma_1) + \mu \left[\Delta y_2(\beta+) - \Delta y_2(\beta-) - \Delta y_3(\beta) \frac{1}{K_t} \right] + \sum_{j=1}^4 \left(\psi_j \delta y_j \Big|_0^\beta + \psi_j \delta y_j \Big|_\beta^1 \right) = 0. \tag{75}$$

Et

$$\delta y_j(\beta \pm) = \Delta y_j(\beta \pm) - y_j'(\beta \pm) \cdot \Delta\beta$$

ja $y_j(\beta)$, $j = 1,3,4$, on pidev, seega

$$\Delta y_j(\beta -) = \Delta y_j(\beta +) = \Delta y_j(\beta), \quad j = 1,3,4,$$

siis

$$\delta y_j(\beta \pm) = \Delta y_j(\beta) - y_j'(\beta \pm) \cdot \Delta\beta, \quad j = 1,3,4, \tag{76}$$

ja võrrand (75) saab kuju

$$\Delta J_* = \Delta\beta(\gamma_0 - \gamma_1) + \mu \left[\Delta y_2(\beta+) - \Delta y_2(\beta-) - \Delta y_3(\beta) \frac{1}{K_t} \right] + \sum_{j=1}^4 \Big\{ \psi_j(\beta-) \left[\Delta y_j(\beta-) - y_j'(\beta-) \cdot \Delta\beta \right] -$$
$$- \psi_j(0) \left[\Delta y_j(0) - y_j'(0) \cdot \Delta 0 \right] + \psi_j(1) \left[\Delta y_j(1) - y_j'(1) \cdot \Delta 1 \right] - \psi_j(\beta+) \left[\Delta y_j(\beta+) - y_j'(\beta+) \cdot \Delta\beta \right] \Big\} = 0$$

ehk

$$\Delta J_* = \Delta\beta(\gamma_0 - \gamma_1) + \mu \left[\Delta y_2(\beta+) - \Delta y_2(\beta-) - \Delta y_3(\beta) \frac{1}{K_t} \right] + \sum_{j=1}^4 \Big\{ \psi_j(\beta-) \left[\Delta y_j(\beta-) - y_j'(\beta-) \cdot \Delta\beta \right] -$$
$$- \psi_j(0) \Delta y_j(0) + \psi_j(1) \Delta y_j(1) - \psi_j(\beta+) \left[\Delta y_j(\beta+) - y_j'(\beta+) \cdot \Delta\beta \right] \Big\} = 0.$$

Kirjutame summa pikemalt lahti, arvestades tingimusi (76)

$$\Delta J_* = \Delta\beta(\gamma_0 - \gamma_1) + \mu \left[\Delta y_2(\beta+) - \Delta y_2(\beta-) - \Delta y_3(\beta) \frac{1}{K_t} \right] \Big\} +$$
$$+ \psi_1(\beta-) \left[\Delta y_1(\beta) - y_1'(\beta-) \cdot \Delta\beta \right] - \psi_1(0) \Delta y_1(0) + \psi_1(1) \Delta y_1(1) - \psi_1(\beta+) \left[\Delta y_1(\beta) - y_1'(\beta+) \cdot \Delta\beta \right] +$$
$$+ \psi_2(\beta-) \left[\Delta y_2(\beta-) - y_2'(\beta-) \cdot \Delta\beta \right] - \psi_2(0) \Delta y_2(0) + \psi_2(1) \Delta y_2(1) - \psi_2(\beta+) \left[\Delta y_2(\beta+) - y_2'(\beta+) \cdot \Delta\beta \right] +$$
$$+ \psi_3(\beta-) \left[\Delta y_3(\beta) - y_3'(\beta-) \cdot \Delta\beta \right] - \psi_3(0) \Delta y_3(0) + \psi_3(1) \Delta y_3(1) - \psi_3(\beta+) \left[\Delta y_3(\beta) - y_3'(\beta+) \cdot \Delta\beta \right] +$$
$$+ \psi_4(\beta-) \left[\Delta y_4(\beta) - y_4'(\beta-) \cdot \Delta\beta \right] - \psi_4(0) \Delta y_4(0) + \psi_4(1) \Delta y_4(1) - \psi_4(\beta+) \left[\Delta y_4(\beta) - y_4'(\beta+) \cdot \Delta\beta \right] = 0.$$

Võttes arvesse rajatingimused (67), saame

$$\Delta y_2(0)=0, \qquad \Delta y_1(1)=0,$$
$$\Delta y_4(0)=0, \qquad \Delta y_3(1)=0,$$

seega nulliga võrduvad korrutised $\psi_2(0)\Delta y_2(0)$, $\psi_4(0)\Delta y_4(0)$, $\psi_1(1)\Delta y_1(1)$, $\psi_3(1)\Delta y_3(1)$. Tulenevalt $\Delta y_1(0)$, $\Delta y_3(0)$, $\Delta y_2(1)$ ja $\Delta y_4(1)$ suvalisusest, ka korrutised $\psi_1(0)\Delta y_1(0)$, $\psi_2(1)\Delta y_2(1)$, $\psi_3(0)\Delta y_3(0)$, $\psi_4(1)\Delta y_4(1)$ on võrdsed nulliga.

Jättes viimasena saadud võrrandis nulliga võrduvad liikmed vaatluse alt välja, võime võrrandi kirjutada

$$\begin{aligned}
\Delta J_* &= \Delta\beta(\gamma_0-\gamma_1)+\mu\left[\Delta y_2(\beta+)-\Delta y_2(\beta-)-\Delta y_3(\beta)\frac{1}{K_t}\right]\Bigg\}+\\
&+\psi_1(\beta-)\left[\Delta y_1(\beta)-y_1'(\beta-)\cdot\Delta\beta\right]-\psi_1(\beta+)\left[\Delta y_1(\beta)-y_1'(\beta+)\cdot\Delta\beta\right]+\\
&+\psi_2(\beta-)\left[\Delta y_2(\beta-)-y_2'(\beta-)\cdot\Delta\beta\right]-\psi_2(\beta+)\left[\Delta y_2(\beta+)-y_2'(\beta+)\cdot\Delta\beta\right]+\\
&+\psi_3(\beta-)\left[\Delta y_3(\beta)-y_3'(\beta-)\cdot\Delta\beta\right]-\psi_3(\beta+)\left[\Delta y_3(\beta)-y_3'(\beta+)\cdot\Delta\beta\right]+\\
&+\psi_4(\beta-)\left[\Delta y_4(\beta)-y_4'(\beta-)\cdot\Delta\beta\right]-\psi_4(\beta+)\left[\Delta y_4(\beta)-y_4'(\beta+)\cdot\Delta\beta\right]=0.
\end{aligned}$$

Vaatleme $\Delta y_j(\beta)$, $j=1,3,4$, ning $\Delta y_2(\beta-)$, $\Delta y_2(\beta+)$ kordajaid. Võrdsustades need nulliga, saame

$$\psi_1(\beta-)-\psi_1(\beta+)=0,$$
$$\psi_3(\beta-)-\psi_3(\beta+)-\frac{\mu}{K_t}=0,$$
$$\psi_4(\beta-)-\psi_4(\beta+)=0,$$
$$\psi_2(\beta-)-\mu=0,$$
$$-\psi_2(\beta+)+\mu=0.$$

Viimasest kahest võrdusest saame $\psi_2(\beta-)=\psi_2(\beta+)$ ja $\mu=\psi_2(\beta)$. Saadud võrdustest nähtub, et ψ_1, ψ_2 ja ψ_4 on pidevad ja ψ_3 on katkev kohal $\xi=\beta$, mis on kooskõlas ka seostega (73). Samuti oleme saanud tingimuse kordaja μ määramiseks: $\mu=\psi_2(\beta)$.

Kuna $\Delta\beta$ on suvaline, siis peab kehtima seos

$$\begin{aligned}
&\gamma_0-\gamma_1-\psi_1(\beta-)y_1'(\beta-)+\psi_1(\beta+)y_1'(\beta+)-\psi_2(\beta-)y_2'(\beta-)+\psi_2(\beta+)y_2'(\beta+)-\\
&-\psi_3(\beta-)y_3'(\beta-)+\psi_3(\beta+)y_3'(\beta+)-\psi_4(\beta-)y_4'(\beta-)+\psi_4(\beta+)y_4'(\beta+)=0.
\end{aligned} \tag{77}$$

Et meil ψ_1, ψ_2 ja ψ_4 olid pidevad, siis

$\psi_k(\beta -)=\psi_k(\beta -)=\psi_k(\beta)$, k=1,2,4

ja võrrandis (77) võime koondada sarnased liidetavad

$$\gamma_0-\gamma_1-\psi_1(\beta)\left[y_1'(\beta -)+y_1'(\beta +)\right]-\psi_2(\beta)\left[y_2'(\beta -)+y_2'(\beta +)\right]-\psi_3(\beta -)y_3'(\beta -)+$$
$$+\psi_3(\beta +)y_3'(\beta +)-\psi_4(\beta)\left[y_4'(\beta -)+y_4'(\beta +)\right]=0.$$

Ka $y_1(\beta)$, $y_3(\beta)$ ja $y_4(\beta)$ on pidevad ning põhivõrrandite süsteemi (66) põhjal $y_3'=y_4$, seega ka y_3' on pidev ja viimane võrrand saab kuju

$$\gamma_0-\gamma_1-\psi_1(\beta)\left[y_1'(\beta -)-y_1'(\beta +)\right]-\psi_2(\beta)\left[y_2'(\beta -)-y_2'(\beta +)\right]-y_3'(\beta)\left[\psi_3(\beta -)-\psi_3(\beta +)\right]-$$
$$-\psi_4(\beta)\left[y_4'(\beta -)-y_4'(\beta +)\right]=0.$$

Asetades siia veel seosed süsteemist (66), saame viimase võrrandi kujul

$$\gamma_0-\gamma_1-\psi_1(\beta)\left[y_2(\beta -)-y_2(\beta +)\right]-\psi_2(\beta)\left[\frac{1}{d\cdot\gamma_0}y_3(\beta -)-\frac{1}{d\cdot\gamma_1}y_3(\beta +)\right]-y_4(\beta)\left[\psi_3(\beta -)-\psi_3(\beta +)\right]-$$
$$-\psi_4(\beta)\left[-4d\cdot\gamma_0a^4y_1(\beta -)+d\cdot q+4d\cdot\gamma_1a^4y_1(\beta +)-d\cdot q\right]=0.$$

Arvestades $y_1(\beta)$, $y_3(\beta)$ ja $y_4(\beta)$ pidevust, võime viimase võrrandi kirjutada

$$\gamma_0-\gamma_1-\psi_1(\beta)\left[y_2(\beta -)-y_2(\beta +)\right]-\psi_2(\beta)y_3(\beta)\left(\frac{1}{d\cdot\gamma_0}-\frac{1}{d\cdot\gamma_1}\right)-y_4(\beta)\left[\psi_3(\beta -)-\psi_3(\beta +)\right]-$$
$$-4a^4d\cdot y_1(\beta)\psi_4(\beta)\left[-\gamma_0+\gamma_1\right]=0. \qquad (78)$$

Arvestades seoseid (73), saame (78) panna kirja järgmiselt:

$$\gamma_0-\gamma_1-\frac{\psi_0}{d\cdot q}y_4(\beta)\left[y_2(\beta -)-y_2(\beta +)\right]+\frac{\psi_0}{d\cdot q}y_3^2(\beta)\left(\frac{1}{d\cdot\gamma_0}-\frac{1}{d\cdot\gamma_1}\right)-y_4(\beta)\left[\frac{\psi_0}{d\cdot q}y_2(\beta -)-\frac{\psi_0}{d\cdot q}y_2(\beta +)\right]-$$
$$-4a^4d\cdot\frac{\psi_0}{d\cdot q}y_1^2(\beta)\left[-\gamma_0+\gamma_1\right]=0,$$

mis peale koondamist annab

$$\gamma_0-\gamma_1+\frac{\psi_0}{d\cdot q}\left\{2y_4(\beta)\left[y_2(\beta +)-y_2(\beta -)\right]+y_3^2(\beta)\left(\frac{1}{\gamma_0 d}-\frac{1}{\gamma_1 d}\right)+4a^4d\cdot y_1^2(\beta)(\gamma_1-\gamma_0)\right\}=0 \qquad (79)$$

Edasi vaatleme elastse silindrilise kooriku põhivõrrandeid kujul (66). Teisendades seda süsteemi, jõuame tükiti konstantse paksusega elastse silindrilise kooriku painde võrrandini

$$y_1^{IV} + 4a^4 y_1 = \frac{q}{\gamma_j}, \tag{80}$$

kus $a^4 = \dfrac{l^4(1-v^2)}{R^2H^2}$, $q = p\dfrac{2l^4(1-v^2)}{EH^3t_*}$ ja $\gamma_j = \dfrac{t_j}{t_*}$,

kus t_* on võrdluskooriku paksus, seega $\gamma_j = \begin{cases} \gamma_0, & 0 \le \xi \le \beta \\ \gamma_1, & \beta < \xi \le 1 \end{cases}$.

Tegemist on hariliku 4. järku konstantsete kordajatega diferentsiaalvõrrandiga, mille üldlahend esitub

1) piirkonnas $\xi \in [0, \beta]$ kujul

$$y_1 = y^0 + e^{a\xi}(D_1 \cos a\xi + D_2 \sin a\xi) + e^{-a\xi}(D_3 \cos a\xi + D_4 \sin a\xi), \tag{81}$$

2) piirkonnas $\xi \in [\beta, 1]$ kujul

$$y_1 = y^1 + e^{a\xi}(E_1 \cos a\xi + E_2 \sin a\xi) + e^{-a\xi}(E_3 \cos a\xi + E_4 \sin a\xi),$$

kus y^0 ja y^1 on võrrandi (80) erilahendid ning $D_1, D_2, D_3, D_4, E_1, E_2, E_3, E_4$ on integreerimiskonstandid vastavas piirkonnas.

Konstantide määramiseks kasutame raja- ja pidevuse tingimusi (67) ja (68)

$$y_1(1) = 0,$$
$$y_2(0) = 0,$$
$$y_3(1) = 0,$$
$$y_4(0) = 0,$$
$$[y_1(\beta)] = 0,$$
$$[y_2(\beta)] = \frac{1}{K_t} y_3(\beta),$$
$$[y_3(\beta)] = 0,$$
$$[y_4(\beta)] = 0.$$

Arvestades tähistusi (64), näeme, et püstitatud ülesanne on identne paragrahvis 3 püstitatud ülesandega. Seega võime kasutada seal saadud tulemust. Konstandid $D_1, D_2, D_3, D_4, E_1, E_2, E_3, E_4$ avalduvad kujul

$$D_1 = \left[\frac{(m_1 + m_3 m_5)m_{13}}{m_{12}} + m_2 + m_3 m_6\right]\frac{m_{16}}{m_{15}} + \frac{(m_1 + m_3 m_5)m_{14}}{m_{12}} + m_3 m_7 + m_4,$$

$$D_2 = \left(\frac{m_5 m_{13}}{m_{12}} + m_6\right)\frac{m_{16}}{m_{15}} + \frac{m_5 m_{14}}{m_{12}} + m_7,$$

$$D_3=\left[\frac{(m_1+m_3m_5)m_{13}}{m_{12}}+m_2+m_3m_6\right]\frac{m_{16}}{m_{15}}+\frac{(m_1+m_3m_5)m_{14}}{m_{12}}+m_3m_7+m_4,$$

$$D_4=-\left(\frac{m_5m_{13}}{m_{12}}+m_6\right)\frac{m_{16}}{m_{15}}-\frac{m_5m_{14}}{m_{12}}-m_7,$$

$$E_1=\frac{m_{13}m_{16}}{m_{12}m_{15}}+\frac{m_{14}}{m_{12}},$$

$$E_2=\frac{m_{16}}{m_{15}},$$

$$E_3=-w_1\cdot e^a\cos a-e^{2a}\left[\left(\frac{m_{13}m_{16}}{m_{12}m_{15}}+\frac{m_{14}}{m_{12}}\right)\cos 2a+\frac{m_{16}}{m_{15}}\sin 2a\right],$$

$$E_4=-w_1\cdot e^a\sin a-e^{2a}\left[\left(\frac{m_{13}m_{16}}{m_{12}m_{15}}+\frac{m_{14}}{m_{12}}\right)\sin 2a-\frac{m_{16}}{m_{15}}\cos 2a\right],$$

kus

$$f:=a(1-\beta),$$
$$g:=a(2-\beta),$$

$$m_1=\frac{e^{a\beta}\cos a\beta-e^g\cos g}{(e^{a\beta}+e^{-a\beta})\cos a\beta},$$

$$m_2=\frac{e^{a\beta}\sin a\beta-e^g\sin g}{(e^{a\beta}+e^{-a\beta})\cos a\beta},$$

$$m_3=-\frac{(e^{a\beta}-e^{-a\beta})\sin a\beta}{(e^{a\beta}+e^{-a\beta})\cos a\beta},$$

$$m_4=\frac{w_0-w_1\left(1-e^f\cos f\right)}{(e^{a\beta}+e^{-a\beta})\cos a\beta}$$

$$m_5=-\frac{m_1\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta-\gamma_1\left(-e^{a\beta}\sin a\beta+e^g\sin g\right)}{m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta+\gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta},$$

$$m_6=-\frac{m_2\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta-\gamma_1\left(e^{a\beta}\cos a\beta-e^g\cos g\right)}{m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta+\gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta},$$

$$m_7=-\frac{m_4\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta-w_1\gamma_1e^f\sin f}{m_3\gamma_0(-e^{a\beta}+e^{-a\beta})\sin a\beta+\gamma_0(e^{a\beta}+e^{-a\beta})\cos a\beta},$$

$$m_{12}=(e^{a\beta}-e^{-a\beta})(m_1+m_3m_5+m_5)\cos a\beta+(e^{a\beta}+e^{-a\beta})(-m_1-m_3m_5+m_5)\sin a\beta-$$
$$-\left[e^{a\beta}(\cos a\beta-\sin a\beta)+e^g(-\cos g+\sin g)-2K\gamma_1a\left(-e^{a\beta}\sin a\beta+e^g\sin g\right)\right],$$

$$m_{13}=-\Big\{(e^{a\beta}-e^{-a\beta})(m_2+m_3m_6+m_6)\cos a\beta+(e^{a\beta}+e^{-a\beta})(-m_2-m_3m_6+m_6)\sin a\beta-$$
$$-\left[e^{a\beta}(\cos a\beta+\sin a\beta)+e^g(\cos g+\sin g)-2K\gamma_1a\left(e^{a\beta}\cos a\beta-e^g\cos g\right)\right]\Big\},$$

$$m_{14}=-\left\{+\cos a\beta\,(e^{a\beta}-e^{-a\beta})\left(m_3m_7+m_4+m_7\right)+\sin a\beta\,(e^{a\beta}+e^{-a\beta})\left(-m_3m_7-m_4+m_7\right)-\right.$$
$$\left.-w_1e^f\left[\cos f-\sin f\left(1+2K\gamma_1 a\sin f\right)\right]\right\},$$

$$m_{15}=\left\{(e^{a\beta}-e^{-a\beta})\left[-\frac{(m_1+m_3m_5)m_{13}}{m_{12}}+\frac{m_5m_{13}}{m_{12}}-m_3m_6+m_6-m_2\right]\gamma_0\cos a\beta-\right.$$
$$-(e^{a\beta}+e^{-a\beta})\left[\frac{(m_1+m_3m_5)m_{13}}{m_{12}}+m_2+m_3m_6+\frac{m_5m_{13}}{m_{12}}+m_6\right]\gamma_0\sin a\beta-$$
$$-\frac{m_{13}}{m_{12}}\gamma_1\left[-e^{a\beta}(\cos a\beta+\sin a\beta)-e^g\left(\cos g+\sin g\right)\right]-\gamma_1\left[e^{a\beta}(\cos a\beta-\sin a\beta)+e^g\left(\cos g-\sin g\right)\right],$$

$$m_{16}=-(e^{a\beta}-e^{-a\beta})\left[-\frac{(m_1+m_3m_5)m_{14}}{m_{12}}-m_3m_7-m_4+\frac{m_5m_{14}}{m_{12}}+m_7\right]\gamma_0\cos a\beta+$$
$$+\left(e^{a\beta}+e^{-a\beta}\right)\left[\frac{(m_1+m_3m_5)m_{14}}{m_{12}}+m_3m_7+m_4+\frac{m_5m_{14}}{m_{12}}+m_7\right]\gamma_0\sin ab+$$
$$+\frac{m_{14}}{m_{12}}\gamma_1\left[-e^{a\beta}(\cos a\beta+\sin a\beta)-e^g\left(\cos g+\sin g\right)\right]-w_1e^f\gamma_1\left(\cos f+\sin f\right)$$

Sellega oleme määranud konstandid ja saanud kätte erilahendi süsteemile (66), mis rahuldab etteantud tingimusi.

Pöördume tagasi võrrandite (74) juurde. Võttes suuruse $\gamma_0=1$, siis esimene võrrand võrranditest (74) langeb vaatluse alt välja. Uurime lähemalt $\Delta\gamma_1$ kordajat ehk (74) teist võrrandit

$$1-\beta+\int_\beta^1\left(\psi_2\frac{y_3}{d\cdot\gamma_1^2}+\psi_4 4\cdot d\cdot a^4y_1\right)dx-\mu\left(K+\frac{\partial K}{\partial\gamma_1}\gamma_1\right)y_3(\beta)=0\,.$$

Kasutades võrdusi (73), saame selle kirja panna kujul

$$1-\beta-\frac{\psi_0}{d^2q\cdot\gamma_1^2}\int_\beta^1 y_3^2dx-\frac{4a^4\psi_0}{q}\int_\beta^1 y_1^2dx-\mu\left(K+\frac{\partial K}{\partial\gamma_1}\gamma_1\right)y_3(\beta)=0\,. \tag{82}$$

Tähistame

$$S:=\int_\beta^1 y_3^2dx,$$
$$T:=\int_\beta^1 y_1^2dx. \tag{83}$$

Leiame eraldi integraalid S ja T. Selleks on meil kõigepealt vaja y_1 ja y_3 avaldisi (81)

$$y_1=y^1+e^{ax}(E_1\cos ax+E_2\sin ax)+e^{-ax}(E_3\cos ax+E_4\sin ax) \tag{84}$$

ja põhisüsteemist (66) võime avaldada y_3

$y_3 = y_2{}' d \cdot \gamma_1 = y_1{}'' \gamma_1 d$.

Kasutades läbipainde teise tuletise avaldist (15), võime y_3 kirja panna

$$y_3 = 2a^2\gamma_1 d\left[e^{ax}(E_2\cos ax - E_1\sin ax) + e^{-ax}(-E_4\cos ax + E_3\sin ax)\right],$$

millest peale ruutuvõtmist, kahekordse nurga siinuse valemi rakendamist ja koondamist saame

$$y_3{}^2 = 4a^4\gamma_1{}^2 d^2\Big[E_2{}^2e^{2ax}\cos^2 ax + E_4{}^2e^{-2ax}\cos^2 ax + E_1{}^2e^{2ax}\sin^2 ax + E_3{}^2e^{-2ax}\sin^2 ax -$$
$$-E_1E_2e^{2ax}\sin 2ax - E_3E_4e^{-2ax}\sin 2ax - 2E_2E_4\cos^2 ax - 2E_1E_3\sin^2 ax + \left(E_2E_3 + E_1E_4\right)\sin 2ax\Big].$$

Paneme kirja integraali S avaldise

$$S = \int_\beta^1 y_3{}^2dx = 4a^4\gamma_1{}^2d^2\left[E_2{}^2\int_\beta^1 e^{2ax}\cos^2 ax\ dx + E_4{}^2\int_\beta^1 e^{-2ax}\cos^2 ax\ dx + E_1{}^2\int_\beta^1 e^{2ax}\sin^2 ax\ dx +\right.$$
$$+E_3{}^2\int_\beta^1 e^{-2ax}\sin^2 ax\ dx - E_1E_2\int_\beta^1 e^{2ax}\sin 2ax\ dx - E_3E_4\int_\beta^1 e^{-2ax}\sin 2ax\ dx - 2E_2E_4\int_\beta^1 \cos^2 ax\ dx -$$
$$\left.-2E_1E_3\int_\beta^1 \sin^2 ax\ dx + \left(E_2E_3 + E_1E_4\right)\int_\beta^1 \sin 2ax\ dx\right].$$

Tähistades viimases

$$I_1 := \int_\beta^1 e^{2ax}\cos^2 ax\ dx,$$

$$I_2 := \int_\beta^1 e^{-2ax}\cos^2 ax\ dx,$$

$$I_3 := \int_\beta^1 e^{2ax}\sin^2 ax\ dx,$$

$$I_4 := \int_\beta^1 e^{-2ax}\sin^2 ax\ dx,$$

$$I_5 := \int_\beta^1 e^{2ax} \sin 2ax\, dx, \tag{85}$$

$$I_6 := \int_\beta^1 e^{-2ax} \sin 2ax\, dx,$$

$$I_7 := \int_\beta^1 \cos^2 ax\, dx,$$

$$I_8 := \int_\beta^1 \sin^2 ax\, dx,$$

$$I_9 := \int_\beta^1 \sin 2ax\, dx,$$

saame

$$S = 4a^4 \gamma_1^{\,2} d^2 \left[E_2^{\,2} I_1 + E_4^{\,2} I_2 + E_1^{\,2} I_3 + E_3^{\,2} I_4 - E_1 E_2 I_5 - E_3 E_4 I_6 - 2E_2 E_4 I_7 - 2E_1 E_3 I_8 + \right.$$
$$\left. + (E_2 E_3 + E_1 E_4) I_9 \right]. \tag{86}$$

Leiame integraalid $I_1,..., I_9$ ükshaaval. Kasutades korduvalt ositi integreerimist ning poolnurga ja kahekordse nurga trigonomeetrilisi seoseid, saame $I_1,.., I_9$ kujul

$$I_1 = e^{2a}\left[\frac{\sin 2a}{4a} + \frac{1}{4a} - \frac{1}{8a}(\sin 2a - \cos 2a)\right] - e^{2a\beta}\left[\frac{\sin 2a\beta}{4a} + \frac{1}{4a} - \frac{1}{8a}(\sin 2a\beta - \cos 2a\beta)\right],$$

$$I_2 = e^{-2a}\left[\frac{\sin 2a}{4a} - \frac{1}{4a} + \frac{1}{8a}(-\sin 2a - \cos 2a)\right] - e^{-2a\beta}\left[\frac{\sin 2a\beta}{4a} - \frac{1}{4a} + \frac{1}{8a}(-\sin 2a\beta - \cos 2a\beta)\right],$$

$$I_3 = e^{2a}\left[-\frac{\sin 2a}{4a} + \frac{1}{4a} + \frac{1}{8a}(\sin 2a - \cos 2a)\right] - e^{2a\beta}\left[-\frac{\sin 2a\beta}{4a} + \frac{1}{4a} + \frac{1}{8a}(\sin 2a\beta - \cos 2a\beta)\right],$$

$$I_4 = e^{-2a}\left[-\frac{\sin 2a}{4a} - \frac{1}{4a} - \frac{1}{8a}(-\sin 2a - \cos 2a)\right] - e^{-2a\beta}\left[-\frac{\sin 2a\beta}{4a} - \frac{1}{4a} - \frac{1}{8a}(-\sin 2a\beta - \cos 2a\beta)\right],$$

$$I_5 = \frac{1}{4a} e^{2a}(\sin 2a - \cos 2a) - \frac{1}{4a} e^{2a\beta}(\sin 2a\beta - \cos 2a\beta),$$

$$I_6 = \frac{1}{4a} e^{-2a}(-\sin 2a - \cos 2a) - \frac{1}{4a} e^{-2a\beta}(-\sin 2a\beta - \cos 2a\beta), \tag{87}$$

$$I_7 = \frac{1-\beta}{2} - \frac{\sin 2a\beta - \sin 2a}{4a},$$

$$I_8 = \frac{1-\beta}{2} + \frac{\sin 2a\beta - \sin 2a}{4a},$$

$$I_9 = \frac{1}{2a}(-\cos 2a + \cos 2a\beta).$$

Sellega oleme leidnud integraali S kujul (86), kus tähistused on toodud (87). Integraali T leidmist alustame jällegi y_1 ruututõstmisest. Et

$$y_1 = y^1 + e^{ax}(E_1 \cos ax + E_2 \sin ax) + e^{-ax}(E_3 \cos ax + E_4 \sin ax)\,,$$

siis

$$y_1^{\,2} = E_1^{\,2} e^{2ax} \cos^2 ax + E_3^{\,2} e^{-2ax} \cos^2 ax + E_2^{\,2} e^{2ax} \sin^2 ax + E_4^{\,2} e^{-2ax} \sin^2 ax + E_1 E_2 e^{2ax} \sin 2ax +$$
$$+ E_3 E_4 e^{-2ax} \sin 2ax + 2E_1 E_3 \cos^2 ax + 2E_2 E_4 \sin^2 ax + \left(E_2 E_3 + E_1 E_4\right) \sin 2ax + \left(y^1\right)^2 +$$
$$+ 2y^1 E_1 e^{ax} \cos ax + 2y^1 E_2 e^{ax} \sin ax + 2y^1 E_3 e^{-ax} \cos ax + 2y^1 E_4 e^{-ax} \sin ax$$

ja

$$T = \int_\beta^1 y_1^{\,2} dx = E_1^{\,2} \int_\beta^1 e^{2ax} \cos^2 ax\; dx + E_3^{\,2} \int_\beta^1 e^{-2ax} \cos^2 ax\; dx + E_2^{\,2} \int_\beta^1 e^{2ax} \sin^2 ax\; dx +$$
$$+ E_4^{\,2} \int_\beta^1 e^{-2ax} \sin^2 ax\; dx + E_1 E_2 \int_\beta^1 e^{2ax} \sin 2ax\; dx + E_3 E_4 \int_\beta^1 e^{-2ax} \sin 2ax\; dx + 2E_1 E_3 \int_\beta^1 \cos^2 ax\; dx$$
$$+ 2E_2 E_4 \int_\beta^1 \sin^2 ax\; dx + \left(E_2 E_3 + E_1 E_4\right) \int_\beta^1 \sin 2ax\; dx + \int_\beta^1 \left(y^1\right)^2 dx + 2y^1 E_1 \int_\beta^1 e^{ax} \cos ax\; dx +$$
$$+ 2y^1 E_3 \int_\beta^1 e^{-ax} \cos ax\; dx + 2y^1 E_2 \int_\beta^1 e^{ax} \sin ax\; dx + 2y^1 E_4 \int_\beta^1 e^{-ax} \sin ax\; dx.$$

Tähistame eelnevas veel

$$I_{10} := \int_\beta^1 e^{ax} \cos ax\; dx,$$

$$I_{11} := \int_\beta^1 e^{-ax} \cos ax\; dx,$$

$$I_{12} := \int_\beta^1 e^{ax} \sin ax\; dx,$$

$$I_{13} := \int_\beta^1 e^{-ax} \sin ax\; dx,$$

siis

$$T = E_1^{\,2} I_1 + E_3^{\,2} I_2 + E_2^{\,2} I_3 + E_4^{\,2} I_4 + E_1 E_2 I_5 + E_3 E_4 I_6 + 2E_1 E_3 I_7 + 2E_2 E_4 I_8 + \left(E_2 E_3 + E_1 E_4\right) I_9 +$$
$$+ 2y^1 E_1 I_{10} + 2y^1 E_3 I_{11} + 2y^1 E_2 I_{12} + 2y^1 E_4 I_{13} + \left(y^1\right)^2 (1 - \beta).$$

Suuruse T lõplikuks avaldamiseks on meil nüüd vaja veel leida integraalid I_{10}, ..., I_{13}.

Tehes seda, saame tulemuseks

$$I_{10} = \frac{1}{2a}\left[e^{a}\left(\sin a + \cos a\right) - e^{a\beta}\left(\sin a\beta + \cos a\beta\right)\right],$$
$$I_{11} = \frac{1}{2a}\left[e^{-a}\left(\sin a - \cos a\right) - e^{-a\beta}\left(\sin a\beta - \cos a\beta\right)\right],$$

$$I_{12} = \frac{1}{2a}\left[e^{a}\left(\sin a - \cos a\right) - e^{a\beta}\left(\sin a\beta - \cos a\beta\right)\right], \tag{88}$$

$$I_{13} = \frac{1}{2a}\left[e^{-a}\left(-\sin a - \cos a\right) - e^{-a\beta}\left(-\sin a\beta - \cos a\beta\right)\right].$$

Sellega oleme leidnud T väärtuse ja saame nüüd võrrandi (82) ilma integraalideta.

Lisades siia võrrandi (79), saame kahest võrrandist koosneva süsteemi

$$\begin{cases} 1-\beta - \dfrac{\psi_0}{d^2 q\cdot\gamma_1^2}\cdot S - \dfrac{4a^4\psi_0}{q}\cdot T - \mu\dfrac{\partial}{\partial\gamma_1}\left(\dfrac{\gamma_1}{K_t}\right)\cdot y_3(\beta) = 0, \\ 1-\gamma_1 + \dfrac{\psi_0}{d\cdot q}\left\{2y_4(\beta)\left[y_2(\beta+) - y_2(\beta-)\right] + y_3^2(\beta)\left(\dfrac{1}{d} - \dfrac{1}{\gamma_1 d}\right) + 4a^4 d\cdot y_1^2(\beta)(\gamma_1-\gamma_0)\right\} = 0, \end{cases} \tag{89}$$

kus

$$S = 4a^4\gamma_1^2 d^2\left[E_2^2 I_1 + E_4^2 I_2 + E_1^2 I_3 + E_3^2 I_4 - E_1E_2I_5 - E_3E_4I_6 - 2E_2E_4I_7 - 2E_1E_3I_8 + \right.$$

$$\left. + \left(E_2E_3 + E_1E_4\right)I_9\right],$$

$$T = E_1^2 I_1 + E_3^2 I_2 + E_2^2 I_3 + E_4^2 I_4 + E_1E_2I_5 + E_3E_4I_6 + 2E_1E_3I_7 + 2E_2E_4I_8 + \left(E_2E_3 + E_1E_4\right)I_9 +$$

$$+2y^1E_1I_{10} + 2y^1E_3I_{11} + 2y^1E_2I_{12} + 2y^1E_4I_{13} + \left(y^1\right)^2(1-\beta),$$

ja $I_1,\ I_2,\ \dots,\ I_{13}$ on toodud seostes (87) ja (88).

Süsteem (89) kujutab endast kahte algebralist võrrandit, mida kasutame koos etteantud integraalse läbipaindega (69) optimiseerimisülesande lahendamiseks. Nendest võrranditest saame leida β, γ_1 ning ψ_0. Sellega oleme lahendanud lõpuni ka püstitatud optimiseerimisülesande.

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Optimization of an Elastic Cylindrical Shell

Tiina Tõkke

Summary

Elastic cylindrical shells subjected to uniformly distributed external pressure are considered.

The equilibrium equations for axisymmetrically loaded circular cylindrical shells are presented in the first section of the study. Geometrical relations corresponding to these equations are presented herein, as well.

An elastic circular cylindrical shell of an ideal sandwich shell wall is considered in the second section of the paper. It is assumed that the thickness of carrying layers is piece wise constant whereas the total thickness is constant. Making use of appropriate boundary conditions the solution of governing equations is derived for a simply supported shell.

In the third section a cylindrical shell with a part - through crack is considered. The crack has a constant depth and it is located at the re-entrant corner of the step.

Optimization of stepped cylindrical shells with cracks emanating from corners of steps are considered in the fourth section of the study. Making use of the variational methods of the theory of optimal control necessary conditions for optimality are derived. The obtained set of algebraic and differential equations admits to define the constants of integration as well as the design parameters.