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BROADCAST SYSTEMS: ALAMOUTI INDEX CODING

Master's Thesis in Computer Engineering (30 ECTS)

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Tartu 2017

Broadcast Systems: Alamouti Index Coding

Abstract

Consider a wireless broadcast channel with a number of receivers, where each receiver possesses some side information. In an index coding problem, the transmitter aims at delivering different messages to different receivers. It is desirable to minimize the total number of message transmissions in order to improve the bandwidth efficiency. It is known that the minimum number of transmissions can be achieved by solving a minimum rank problem for a given side information graph, which is a known NP-hard problem.

In this thesis, the index coding problem over additive white Gaussian noise and Rayleigh fading channels is studied. Modulation techniques, such as phase shift keying and quadrature amplitude modulation, are tested, and it is shown that a careful choice of modulation can improve the performance.

It is also shown that a careful choice of a generator matrix can provide for further performance gains, in particular for "prioritized" receivers. A probabilistic soft information detection (PSID) is compared to a simple hard decision scheme, and the PSID performance is shown superior in achieving a lower error rate. Additional improvement in performance is achieved by using diversity, when employing two transmit and two receive antenna system in conjunction with the Alamouti code. All these techniques provide for the lower error rates and higher throughput when compared to the traditional schemes.

CERCS: T180 Telecommunication engineering; T121 Signal processing

Keywords: Index coding, bit error rate, Alamouti STBC, soft information, modulation.

Ülekande Süsteemi: Alamouti indeks-kodeerimine

Kokkuvõte

Kujutlegem juhtmevabat edastust mitme vastuvõtjaga, kus iga vastuvõtja valdab külje-informatsiooni. Indeks-kodeerimise eesmärk on toimetada ühest saatjast erinevad sõnumid erinevate vastuvõtjateni. Et ribalaiust kokku hoida, tuleb viia saatmiste arv miinimumini. On teada, et väikseim arv saatmisi on saavutatav lahendades minimaalse järgu probleemi vastuvõtja külje-informatsiooni graafist, mis on tuntud kui NP-raske probleem.

Selles töös on uuritud indeks-kodeerimist lisades Gaussi müra ja kasutades Rayleigh' hajumist. Samuti testitakse erinevaid modulatsiooni tüüpe, näiteks diskreet-faasmodulatsiooni ja kvadratuur-amplituudmodulatsiooni ning näidatakse, et hoolikas modulatsiooni valimine võib tulemust parandada.

Lisaks näidatakse, et õige generaator-matriksi valimine võib tulemust parandada veelgi enam, eriti prioritseeritud vastuvõtjatel. Tõenäosuslikku pehme teabe tuvastust võrreldakse lihtsakoelise raske otsustamise skeemiga, ning näidatakse et esimene neist töötab oluliselt väiksema veateguriga. Täiendav veateguri vähendamine saavutatakse hajusedastamise abil kasutades kahte saate- ja vastuvõtu antenni ning Alamouti kodeerimist. Kõik eelmainitud tehnikad aitavad võrreldes tavapäraste lahendustega vähendada veategurit ja tõsta läbilaskevõimet.

CERCS: T180 Telekommunikatsioonitehnoloogia T121 Signaalitöötlus

Keywords: indeks-kodeerimine, bitiveategur, Alamouti ruumilis-ajaline plokk-kood, pehme teave, modulatsioon

Acknowledgement

I wish to thank Dr Vitaly Skachek for suggesting the concept of index coding, his pieces of advice and supervision of this thesis. I also wish thank him for the opportunity he gave me to work in his lab, Coding and Information Transmission group, Institute of Computer science, University of Tartu.

I will like to appreciate the support of the European Regional Development Fund for the award of Dora Plus scholarship through Estonian Archimedes foundation towards the support of my study at the University of Tartu.

To Maryam my wife, I owe you a lot. Thanks so much for your support and encouragement both in time of ease and challenges. You are the only reason I am able to get to this stage. My best friend, together I believe, we shall make the best out of life.

To my parents, my in-laws, my brother, Abdulrafiu and sisters, Taibat and Maryam, my special Aunt, Mrs Idrees and all my family members, thanks for your moral support all through.

To my friends and well wishers, I appreciate your friendship. Thanks to Sajid Azad, Martin Põder, Dan and Jemila Prits, IOB, you guys are the best.

Thanks to Yauhen and other members of the Coding and Information Transmission group, Institute of Computer science, University of Tartu.

Unlimited appreciation to Dr Nalin Jayakody for the knowledge he impacted in me, as well as his loads of guidance and motivation during his stay at the University of Tartu. Dr Gholamreza Ambajafari (Shahab), thanks for your mentoring, support and guidance. Simple, strict, smart, responsive, young and very intelligent. Meeting you has been one of the finest moments of my life. A role model you are!. Deepest appreciation to my program Manager, Heiki Kasemägi, thanks for the coordination, curriculum guidance and your timely response to inquiries.

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¹Alamouti space-time block presented here is based on the earlier work done in the course, MTAT.03.309 - Special Assignment in Wireless Communication (University of Tartu).

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Abbreviation, constants

ICSI - Index coding with side information

\mathbb{F}_q - Field of size q

min-rank - minimum rank

AWGN - Additive white Gaussian noise

BER - Bit error rate

σ^2 - Variance

SNR - Signal-to-noise ratio

dB - Decibel

$\mathbf{P}(\mathbf{y} | \mathbf{x})$ - Conditional probability of y given x

PDF - Probability density function

iid - Independently identically distributed

PSID - Probabilistic soft information detection

PSK - Phase shift keying

BPSK - Binary phase shift keying

QPSK - Quadrature phase shift keying

QAM - Quadrature amplitude modulation

1 Introduction

1.1 Wireless communication channels

Channels are used by the transmitter(s) to convey message (information signal) to the receiver(s). In this thesis, we study wireless communication channel. Modulation maps the data onto the signals, which are sent through the communication channel. These signals are corrupted by the noise and other channel impairments. The receiver detects the corrupted messages, leading to error if the transmitted signals are significantly different to what the receiver detected. Channels are the medium through which data or information are sent, this medium

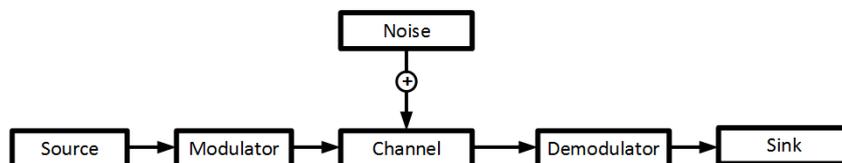


Fig 1.1: A basic communication model.

could be guided (wired) or unguided (wireless). Wired medium can be a cable such as fibre-optic, co-axial cable. Wireless transmission is broadcast in nature, example of these are microwave, satellite and radio broadcast channel, etc. The limit to which a channel can convey information is referred to as capacity of such channel measures by its bandwidth.

The channel is characterized by different parameters. In order to design a quality channel, factors such as bandwidth, impairments (noise, attenuation, interference, etc.) as well as cost have to be considered. Channel performance can be measured by its bit error rate (BER), packet error rate (PER), signal-to-noise ratio, etc.

1.1.1 Wireless channel model

Wireless channels are modelled statistically. The input signal is often assumed to be corrupted by noise and conditional probability of what is received given what is sent, can be estimated.

additive white Gaussian noise (AWGN) is a basic noise model used in communication network to imitate the natural random process that happen to signals in nature. The noise is additive because it is added to the transmitted signal and it is independent of the signal. The noise is white because it has a flat power spectral density which makes it's time domain autocorrelation to be zero for any non-zero time offset. The noise is Gaussian because its distribution is Gaussian (normal). A basic Gaussian channel can be modelled as in:

$$y = x + n, \quad (1.1)$$

where y is the received signal, x is the transmitted signal, and n is the Gaussian noise with mean μ and variance σ^2 , which has distribution $\mathcal{N}(\mu = 0, \sigma^2 = 1)$. The conditional probability of y received given that x is transmitted can be is expressed as:

$$P(y | x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-x)^2}{2\sigma^2}}. \quad (1.2)$$

1.2 Fading channel

1.2.1 Fading classification

In addition to noise, channel can also be modelled by random attenuation of the transmitted signal due to effect of its propagation environment. Channel of this nature is refer to as *fading* channel. Fading measure the changes in the signal power level over the course of transmission. In this work, we used a fading model which follows a Rayleigh distribution. A channel of this nature is termed as Rayleigh fading channel. It is modelled as:

$$y = hx + n, \quad (1.3)$$

where h is Rayleigh fading coefficient.

Wireless channels are random and time varying in nature. The signal strength diminishes due to various impairments such as path loss, shadowing and multipath fading. In a typical wireless communication environment, multiple propagation paths exist owing to the presence of reflectors. Signals experience fading, delay and phase shifts over these multiple paths. This causes the intensity of attenuation, delay and phase shift to be different for various paths. This phenomenon of fading can be reduced by increasing the signal power and bandwidth of the transmitted signal, [1], [2], [3], [4].

However, increasing the power poses various trade-offs in interference and cost. One efficient method of reducing the multipath fading is *diversity*, where the signals are transmitted and received using more than one antenna. We will talk more explicitly on diversity in Chapter 2.

Fading leads to fluctuation in amplitude and phase of the received signal. Fading can be analysed using different statistical models such as Rayleigh fading model, Rician Fading model, Nakagami Fading model, Weibull fading model and Log-normal shadowing model [3], [5]. We will only consider the Rayleigh fading model in this thesis.

Fading can be *fast* or *slow* according to Doppler spread. It can also be classified in terms of multipath time delay spread which are *frequency-selective* or *flat* fading channels. Fast fading channel occurs when the channel impulse response rapidly changes with symbol duration. In other word, the symbol period (T_s) of the transmitted symbol is greater than the coherence time (T_c) of channel. In terms of bandwidth or frequency domain, the transmitted signal bandwidth (B_s) is less than the Doppler spread of the channel (B_c). Fast fading occurs when

$$T_s > T_c,$$

$$B_s < B_c.$$

Slow fading occurs when the impulse response of the channel changes far less rapidly than the symbol duration of the transmitted signal. The Doppler spread is thus far significantly smaller than the bandwidth of the signal. There is slow fading if

$$T_s \ll T_c,$$

$$B_s \gg B_c.$$

Flat fading occurs when the fading coefficient is constant for the entire period of transmission and the channel bandwidth (B_c) is greater than that of the transmitted signal (B_s). In time domain, the signal symbol duration (T_s) is greater than the delay spread (σ_τ), i.e.

$$B_s \ll B_c,$$

$$\sigma_\tau \ll T_s.$$

Flat fading channel is an amplitude varying channel, and, therefore, its amplitude distribution is modelled as a Rayleigh distribution, as we will later show in Subsection 1.2.2.

Frequency selective fading occurs when the fading coefficient changes at every instant time throughout the period of the transmission. Due to this reason, it is difficult to model this type of fading. It occurs when the channel bandwidth (B_c) is less than that of the transmitted signal (B_s). In time domain, the signal symbol duration (T_s) is smaller than the delay spread (σ_τ), i.e.

$$B_s > B_c,$$

$$\sigma_\tau > T_s.$$

Fading can result in poor system performance and link degradation, as it reduces the signal power, which makes the noise level relatively large. Fading effects can however been mitigated by diversity techniques such as space time codes, Multiple-Input Multiple-Output (MIMO), etc.

1.2.2 Rayleigh fading

Rayleigh fading is represented as an impulse h :

$$h = \sum_{i=0}^{l-1} a_i e^{-j2\pi f_c \tau_i}, \quad (1.4)$$

where a_i is the amplitude, f_c the carrier frequency, τ_i is the respective symbol spread and j is a complex square root of -1 . Therefore, equation (1.4) can be expressed as:

$$h = \sum_{i=0}^{l-1} a_i \cos(2\pi f_c \tau_i) - j \sum_{i=0}^{l-1} a_i \sin(2\pi f_c \tau_i). \quad (1.5)$$

Let $h = X + jY$, where X and Y are real and imaginary components, which gives

$$X = \sum_{i=0}^{l-1} a_i \cos(2\pi f_c \tau_i)$$

and

$$Y = - \sum_{i=0}^{l-1} a_i \sin(2\pi f_c \tau_i),$$

where X and Y are assumed to be independently identically distributed (iid) Gaussian random variable $X \sim \mathcal{N}(0, \frac{1}{2})$, $Y \sim \mathcal{N}(0, \frac{1}{2})$ with probability density functions given as:

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu_x)^2}{2\sigma^2}} = \frac{1}{\sqrt{\pi}} e^{-x^2} \quad (1.6)$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu_y)^2}{2\sigma^2}} = \frac{1}{\sqrt{\pi}} e^{-y^2}, \quad (1.7)$$

where μ_x and μ_y are the means of random variables X and Y , respectively, which are equal to zero in both cases.

Since X and Y are independent, their joint probability distribution can be expressed as follow:

$$F_{X,Y}(x, y) = f_X(x)f_Y(y) = \frac{1}{\pi}e^{x^2+y^2}. \quad (1.8)$$

By transforming h into phase component, we obtain:

$$h = x + jy = ae^{j\varphi},$$

where φ is the phase angle. By transforming the PDF interms of phase components, we obtain:

$$f_{X,Y}(x, y) \rightarrow f_{A,\varphi}(a, \varphi),$$

$$f_{A,\Phi}(a, \varphi) = \frac{1}{\pi}e^{x^2+y^2}|J_{XY}|, \quad (1.9)$$

where $J_{X,Y}$ is the jacobian of the X and Y random variables X and Y whose determinant is a .

We obtain

$$f_{A,\Phi}(a, \Phi) = \frac{1}{\pi}e^{-a^2}|J_{XY}| = \frac{a}{\pi}e^{-a^2}. \quad (1.10)$$

We can find the marginal density of the amplitude and phase as thus:

$$f_A(a) = \int_{-\pi}^{\pi} f_{A,\varphi}(a, \varphi)d\varphi = 2ae^{-a^2}, \quad 0 < a < \infty, \quad (1.11)$$

it can be deduced from derivation in (1.11) that the amplitude a follows Rayleigh distribution.

Let us consider the phase distribution with marginal probability density function given as:

$$f_{\Phi}(\varphi) = \int_{-0}^{\infty} f_{A,\Phi}(a, \varphi)da = \frac{1}{2\pi}, \quad -\pi < \Phi < \pi, \quad (1.12)$$

it is shown in (1.12) above that the phase of h follows a uniform distribution. Therefore, the amplitude and phase of Rayleigh fading are independent random variables and can be expressed as

$$f_{A,\Phi}(a, \varphi) = f_A(a)f_{\Phi}(\varphi). \quad (1.13)$$

We show in Fig.1.2 the probability density function (PDF) plot for Rayleigh fading using different values of variance. From the plot, we observe that the higher the variance, the bigger the bell-shaped curved, and vice versa.

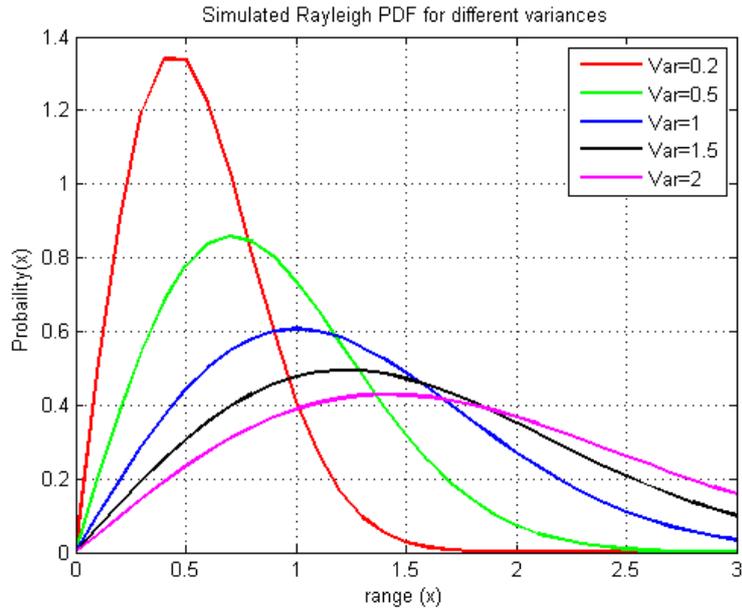


Fig 1.2: PDF for Rayleigh fading for different values of variance.

1.2.3 Bit error rate for Rayleigh fading

Consider a transmitted signal x over a Rayleigh fading channel, which is corrupted by the iid AWGN with distribution $\mathcal{N}(0, \sigma^2)$. The received signal can be expressed as

$$y = hx + n.$$

Let

$$|h| = a,$$

the received symbol power then is

$$P_y = |h|^2 p = a^2 p,$$

where p is the transmitted symbol power. The ratio of signal power to noise power is termed SNR. The fading SNR, SNR_F , is

$$SNR_F = \frac{a^2 p}{\sigma^2} = a^2 SNR,$$

where σ^2 is the variance. Bit error rate (BER), which is the number of errors divided by the total number of transmitted bits, is given as

$$BER = Q(\sqrt{SNR_F}) = Q(\sqrt{a^2 SNR}),$$

where $Q(\cdot)$ is *Q-function* which can be expressed as:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{x^2}{2}} dx.$$

1.3 Modulation

In communication systems, the information is sent in form different from its original form. Modulation helps to facilitate this transformation. Modulation is therefore the process by which digital information is converted or mapped into a suitable form so that it can be transmitted over a channel. It entails a varying carrier signal with respect to the message signal. Modulation could be either *coherent* or *non-coherent* [2]. Coherent modulation is used mainly by system which ensures a phase lock between the transmitter and the receiver carrier signals. In non-coherent modulation, the system has no knowledge of the changes in amplitude of the transmitted symbol, thus it does not maintain phase lock between the receiver and the transmitter. This type of modulation are differential in nature. All the modulation we considered in this work are coherent.

There are different types of modulation (both analogue and digital) based on changing amplitude, frequency or phase. Analogue modulation are sensitive to noise, a draw-back which is overcome by digital modulation. Digital modulation makes communication more efficient. It's advantages over analogue are in terms of lower bandwidth, permissible power and high immunity to noise. In a digital modulation, information signals are converted to digital messages from the analogue and then modulated by a carrier wave.

For modulation to take place, the carrier signal has to be switched on and off in order to create pulses. The variation parameters (amplitude, frequency and phase) determine the type of modulation.

In this work, we use phase shift keying (PSK) and quadrature amplitude modulation (QAM).

1.3.1 Phase shift keying

In phase shift keying (PSK) modulation, the phase of the carrier wave is varied for each symbol. The PSK modulation is generally less sensitive to noise. Binary phase shift keying (BPSK) is the simplest form of digital phase modulation. It uses two phases separated by 180° apart to represent the two binary digits. It has a general form

$$S_k(t) = \sqrt{\frac{2E_s}{T}} \cos(2\pi ft + \pi(1 - k)), \quad k = \{0, 1\},$$

where f is the frequency of the carrier wave, E_s is energy per symbol and T is bit duration.

$$S_k(t) = \begin{cases} \sqrt{\frac{2E_s}{T}} \cos(2\pi ft + \pi) & \text{for binary symbol 0} \\ \sqrt{\frac{2E_s}{T}} \cos(2\pi ft) & \text{for binary symbol 1.} \end{cases}$$

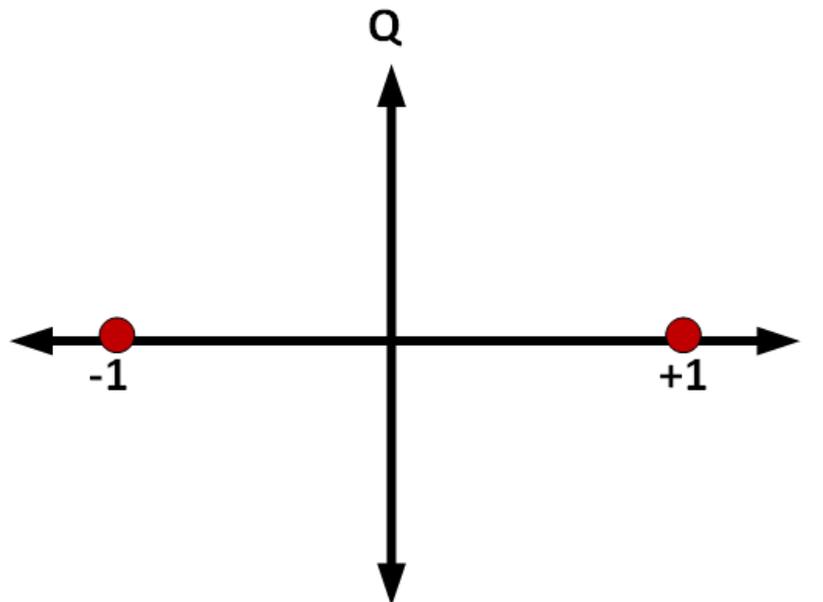


Fig 1.3: BPSK constellation diagram.

Fig. 1.3 shows the constellation mapping diagram for BPSK modulation. $S_k = +1$ for binary symbol 1 and $S_k = -1$ for binary symbol 0.

The *constellation points* are the set of points in the signal space that represent different signals. They serve as the decisions points required to recover each of the original binary symbols. Constellation points are labelled by assigning to all points a bit pattern with a goal that constant amplitude of the transmitted signal is maintained. The choice of the constellation influences the communication channel properties, such as BER. The labelling method chosen for mapping constellation points influences its performance. The number of constellation (N_c) points depends on the number of symbols (N_s) to be transmitted at once, N_c and N_s are related by $N_c = 2^{N_s}$.

The BPSK has only in-phase component. One inconvenience of BPSK is that the transmission bandwidth is twice the message bandwidth. However, it is a robust modulation scheme due to the phase difference which is usually used for long distance wireless communication.

Another phase shift keying modulation technique is the quadrature phase shift keying (QPSK) or 4 PSK. It offers a more efficient bandwidth usage as signals are represented by two bits which

are modulated at once. It uses phase shift in multiple of 90° with four constellation points, with corresponding angles: 45° , 135° , 225° and 315° . Unlike BPSK, QPSK has two components, in-phase (I) and quadrature (Q) components. The general representation for QPSK is given as:

$$S_k(t) = \sqrt{\frac{2E_s}{T}} \cos(2\pi ft + \frac{\pi}{4}(2k - 1)), \quad k = \{1, 2, 3, 4\},$$

which results in two-dimensional signal space.

$$S = I + jQ,$$

where $j = \sqrt{-1}$. The in-phase component is placed along the real axis and the quadrature component is placed along the imaginary axis. The constellation points for QPSK are shown in Fig. 1.4,

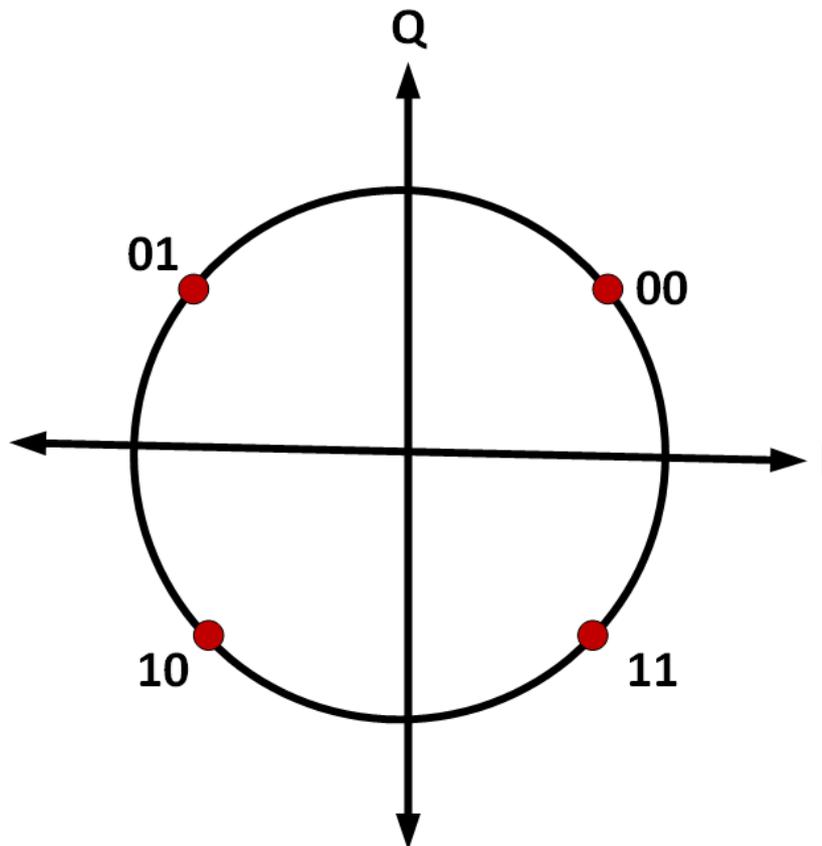


Fig 1.4: QPSK constellation diagram.

Table 1.1 shows how bit streams are converted to QPSK symbol. In order to normalize the modulated waveform, the complex values are multiplied by $\frac{1}{\sqrt{2}}$.

It is also possible to transmit 3-bit as one symbol at a time. This can be achieved by 8 PSK. It offers more data capacity than QPSK but its link degradation tolerance is lower. There are

$b(k)$	$b(k + 1)$	QPSK Symbol
0	0	$-1 - j$
0	1	$-1 + j$
1	0	$1 - j$
1	1	$1 + j$

Table 1.1: QPSK conversion.

eight constellation (decision) points. It is also a two-dimensional modulation techniques, with in-phase component and quadrature component. The general signalling for M -ary PSK (M -PSK) is given as:

$$s_k = \cos \frac{2\pi k}{M} + j \sin \frac{2\pi k}{M}, \quad k = 0, 1, 2, \dots, M - 1.$$

In the case of 8 PSK the expression becomes:

$$s_k = \cos \frac{2\pi k}{8} + j \sin \frac{2\pi k}{8}, \quad k = 0, 1, 2, \dots, 7.$$

The constellation mapping for this modulation is shown as *Gray code* ordering. *Gray code mapping* is an encoding scheme of binary number system in which successive numbers differ in one bit. It is useful in bit detection and helps in error correction. It offers a better performance by reducing BER compared to the standard binary encoding scheme. For instance, consider a natural ordering of the neighbouring points 011 and 100 in the binary representation, which respectively are 3 and 4 in decimal representation, respectively. An error in detecting 011 instead of 100 will cause three bit errors. Gray codes reduce the effect of this type of symbol error between neighbouring points by mapping them to, for example 010 and 110 which are 2 and 7 in the decimal representation respectively. Table 1.2 below shows the Gray mapping for 8 PSK and its output (in-phase and quadrature components).

Generally, 8 PSK Gray coded output can be expressed as:

$$\phi(z) = e^{\frac{jz\pi}{4}},$$

where z is the corresponding decimal value for Gray code mapping from Table 1.2.

The modulator output for 16 PSK can be expressed generally as $e^{\frac{jz\pi}{8}}$ where z is the corresponding decimal representation for the Gray code mapping in Table 1.3. Thus, 16PSK maps 4 bits on a single symbols which is converted to its in-phase and quadrature components. It has 16 constellation points and offer better throughput than lower prder PSK schemes.

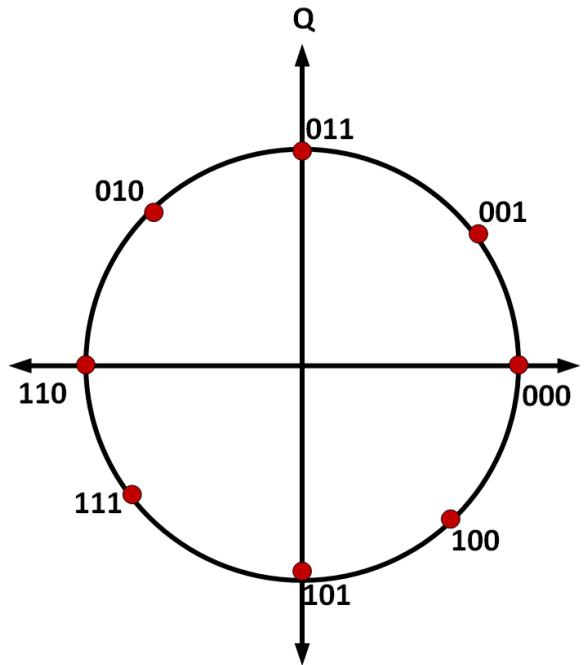


Fig 1.5: 8 PSK constellation diagram.

1.3.2 Quadrature amplitude modulation

Quadrature amplitude modulation (QAM) enables simultaneous changes both in amplitude and phase of the carrier signal. In other words, this modulation represents a combination of the amplitude shift keying (ASK) and PSK which increases transmission efficiency. It is more often used practical applications than PSK. Its data carrying capacity is better than that of the "ordinary" QPSK. It benefits rises from the fact that two orthogonal signals can be transmitted using the same carrier. This can be represented as follows:

$$S(t) = A_1(t) \cos(2\pi f_c t) + A_2(t) \sin(2\pi f_c t), \quad (1.14)$$

where A_1 and A_2 are the amplitude levels and one phase is at 90° to another. There are different modes of QAM techniques based on the number of constellations points, where 16, 64 and 256 QAM are the most popular. We will only discuss 16 QAM, since we use it in this thesis. In 16 QAM, the symbols are represented by four bits and each two of these four binary bits are mapped to either ± 1 or ± 3 . The Gray mapping for 16 QAM bits is shown in Table 1.4.

The constellation diagram for Gray coded 16 QAM is given in Fig. 1.7.

Decimal (d)	Binary	Gray code	Gray-coded Ordering(z)	Modulator output [I,Q]
0	000	000	0	[1,0]
1	001	001	1	$[\cos \frac{\pi}{4}, \sin \frac{\pi}{4}]$
2	010	011	3	$[\cos \frac{3\pi}{4}, \sin \frac{3\pi}{4}]$
3	011	010	2	[0,1]
4	100	110	7	$[\cos \frac{7\pi}{4}, \sin \frac{7\pi}{4}]$
5	101	111	6	[0,-1]
6	110	101	4	[-1,0]
7	111	100	5	$[\cos \frac{5\pi}{4}, \sin \frac{5\pi}{4}]$

Table 1.2: 8 PSK Gray code conversion.

1.4 Demodulation

Demodulation is the reverse process of modulation, i.e the process by which the receiver recovers the transmitted information. It involves detection and decision making based on the detected values. This task is done by demodulator or detector, which makes decision on each received symbol. The decision could either be *hard* or *soft* decisions depending on the amount of used information about each of the transmitted bits. Hard decision makes a threshold decision on the transmitted bit of being either a zero or one. For this reason, it is also referred to as a slicer. Soft decision uses the reliability of the received symbol to determine the likelihood of the transmitted bits as being zero or one. This additional information used by the soft demodulator helps to improve the system performance compared to the hard decision, thereby offering a lower BER. The common soft decision method is log-likelihood ratio (LLR). The LLR computes the natural logarithm ratio of the probability density functions of the likely received symbols.

Next, we analyze the conditional probability of receiving 16 QAM symbol soft information. As stated earlier, 16 QAM entails transmitting four bits with each two bits representing symbols ± 1 or ± 3 as it is shown in Table 1.4. Consider a symbol of 16 QAM consisting of four bits (a_1, a_2, a_3, a_4) . Bits a_1 and a_3 have the same probability based on the received signal, likewise bits a_2 and a_4 have the same probability distribution. We show this only for bits a_1 and a_2 , respectively but similar property applies to bits a_3 and a_4 . It is shown in Fig. 1.8, a_1 is either 0 or 1. By assuming that the received signal is y , the conditional probability that a_1 is 0, which

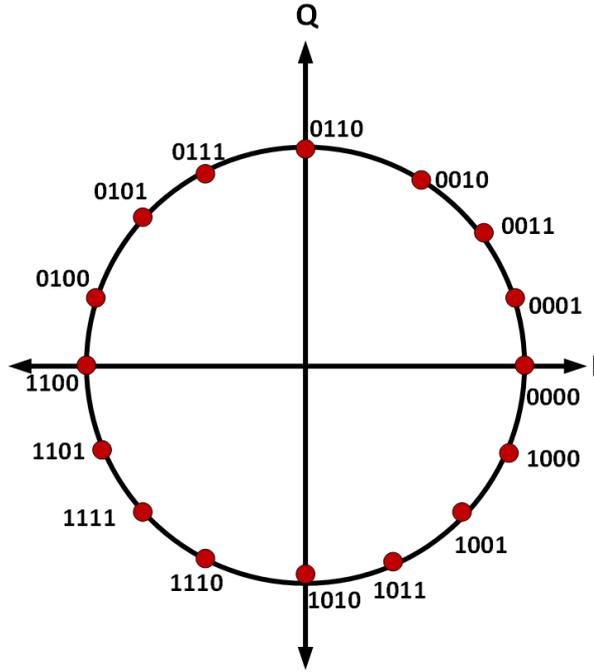


Fig 1.6: Gray coded 16 PSK constellation diagram.

means the transmitter symbol is either -1 or -3 is given as

$$P(y | a_1 = 0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y+3)^2}{2\sigma^2}} + \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y+1)^2}{2\sigma^2}}, \quad (1.15)$$

and when a_1 is 1 it means the transmitted symbol is either $+1$ or $+3$ as it is shown in regions 3 and 4 of Fig. 1.8, the conditional probability becomes:

$$P(y | a_1 = 1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-1)^2}{2\sigma^2}} + \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-3)^2}{2\sigma^2}}. \quad (1.16)$$

Therefore, $a_1 = 1$ when

$$\frac{P(y | a_1 = 1)}{P(y | a_1 = 0)} \geq 1, \quad (1.17)$$

which becomes

$$\frac{P(y | a_1 = 1)}{P(y | a_1 = 0)} = \frac{e^{-\frac{(y-1)^2}{2\sigma^2}} + e^{-\frac{(y-3)^2}{2\sigma^2}}}{e^{-\frac{(y+3)^2}{2\sigma^2}} + e^{-\frac{(y+1)^2}{2\sigma^2}}}. \quad (1.18)$$

Equation (1.18) can further be expressed as LLR simply by taking its natural logarithm. This approach is similar to the one used in [6], but some approximations were made which neglect some of the soft information. Thus, the decision is based on the regional thresholds, $y < -2$, $-2 \leq y < 0$, $0 \leq y < 2$ and $y \geq 2$ for regions 1, 2, 3 and 4 respectively. In this work, we use all available soft information for more robust detection.

We also similarly analyze the bit a_2 . From Fig. 1.9, when the bit $a_2 = 0$, the symbol ± 3 was transmitted. The conditional probability is given as

Decimal (d)	Binary	Gray code	Gray-coded Ordering (z)	Modulator output [I, Q]
0	0000	0000	0	[1,0]
1	0001	0001	1	$[\cos \frac{\pi}{8}, \sin \frac{\pi}{8}]$
2	0010	0011	3	$[\cos \frac{3\pi}{8}, \sin \frac{3\pi}{8}]$
3	0011	0010	2	$[\cos \frac{\pi}{4}, \sin \frac{\pi}{4}]$
4	0100	0110	7	$[\cos \frac{7\pi}{8}, \sin \frac{7\pi}{8}]$
5	0101	0111	6	$[\cos \frac{3\pi}{4}, \sin \frac{3\pi}{4}]$
6	0110	0101	4	[0,1]
7	0111	0100	5	$[\cos \frac{5\pi}{8}, \sin \frac{5\pi}{8}]$
8	1000	1100	15	$[\cos \frac{15\pi}{8}, \sin \frac{15\pi}{8}]$
9	1001	1101	14	$[\cos \frac{7\pi}{4}, \sin \frac{7\pi}{4}]$
10	1010	1111	12	$[\cos \frac{3\pi}{4}, \sin \frac{3\pi}{4}]$
11	1011	1110	13	$[\cos \frac{13\pi}{8}, \sin \frac{13\pi}{8}]$
12	1100	1010	8	[-1,0]
13	1101	1011	9	$[\cos \frac{9\pi}{8}, \sin \frac{9\pi}{8}]$
14	1110	1001	11	$[\cos \frac{11\pi}{8}, \sin \frac{11\pi}{8}]$
15	1111	1000	10	$[\cos \frac{5\pi}{4}, \sin \frac{5\pi}{4}]$

Table 1.3: 16 PSK Gray code conversion.

$$P(y | a_2 = 0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y+3)^2}{2\sigma^2}} + \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-1)^2}{2\sigma^2}}, \quad (1.19)$$

and when a_2 is 1, the transmitted symbol is either ± 1 . The conditional probability becomes

$$P(y | a_2 = 1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-1)^2}{2\sigma^2}} + \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y+1)^2}{2\sigma^2}}. \quad (1.20)$$

Therefore, $a_2 = 1$ when

$$\frac{P(y | a_2 = 1)}{P(y | a_2 = 0)} \geq 1, \quad (1.21)$$

which can further be expressed as:

$$\frac{P(y | a_2 = 1)}{P(y | a_2 = 0)} = \frac{e^{-\frac{(y+3)^2}{2\sigma^2}} + e^{-\frac{(y-3)^2}{2\sigma^2}}}{e^{-\frac{(y+1)^2}{2\sigma^2}} + e^{-\frac{(y-1)^2}{2\sigma^2}}}. \quad (1.22)$$

a_1a_2	QAM symbol	a_3a_4	QAM symbol
00	-3	00	-3
01	-1	01	-1
11	+1	11	+1
10	+3	10	+3

Table 1.4: 16 QAM symbol gray mapping.

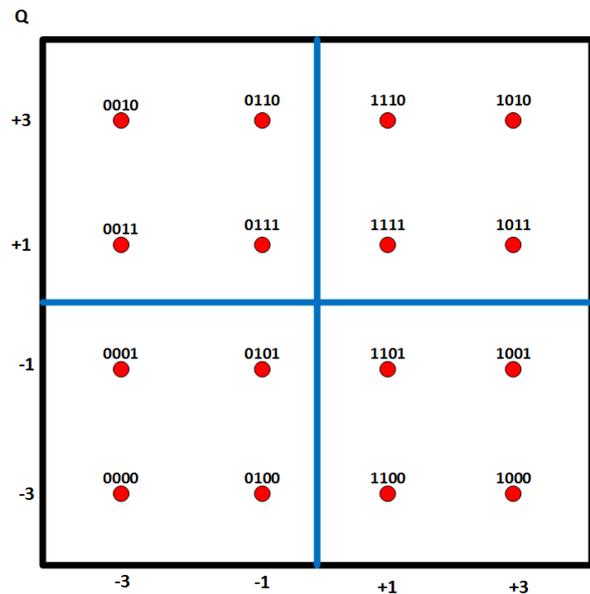


Fig 1.7: 16 QAM constellation diagram.

1.5 Coding

Generally, channel coding is used as a technique for error detection and correction [2]. Modern communication systems make use of various coding techniques in order to correct errors arising from noise as well as from fading effects.

In this work, we aim at reducing BER. We also consider space-time coding, which involves the use of multiple transmit and receive antennas for sending and receiving multiple copies of the same information in a number of time slots. We will discuss the Alamouti space-time code [7] in the sequel.

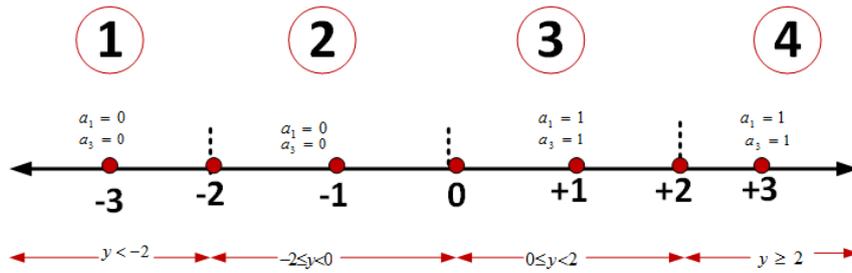


Fig 1.8: 16 QAM bit a_1 decision mapping.

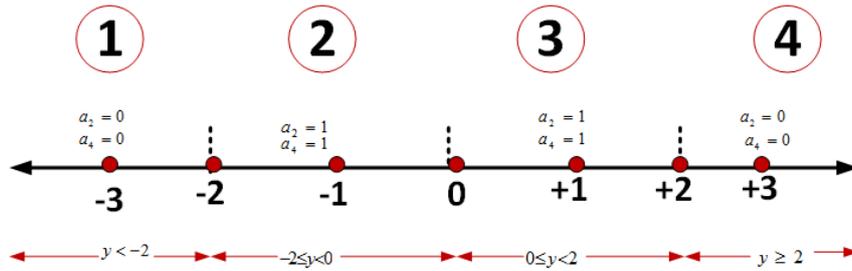


Fig 1.9: 16 QAM bit a_2 decision mapping.

1.6 Problem Overview

Index coding has attracted a lot of attention recently. Single source with one antenna has been used to transmit data in index coding scenario in the literature. This approach is vulnerable to transmission link failure due to channel impairment such as fading and noise. The use of more than one transmit and receive antennas allows for increased reliability and improved performance. This approach is known as diversity. We consider a special technique for transmit and received diversity, the Alamouti space-time block coding.

1.7 Goals

The goal of this thesis is to study the strategies for lowering BER in broadcast systems with side information. This is achieved by a probabilistic soft information detection (PSID) and recovery of multiple copies of the same message. The proposed detection method shows improved performance when compared with the straightforward method.

As min-rank of the of the binary matrices represents the minimum number of transmission of which the index coded data can be transmitted jointly to the receivers, we show how the columns of the coding matrix can be combined in order to recover each requested data in more

than one way. This provides additional reliability to the received data.

We also aim at combating fading through diversity. We use Alamouti space-time block coding which is a powerful diversity technique. We obtain improved performance compared to a system without space-time coding.

2 Diversity and space time block coding

2.1 Introduction

Diversity aims to mitigate deep fading effects in multipath propagation channel. One of the ways for achieving diversity is by coding a signal across space and time. It involves transmission and reception of the same copy of information message with multiple antennas over different channels. By combining the received signal, diversity gain can be achieved. Diversity lowers the probability that all statistically independent fading channels experiencing deep fading simultaneously, which results in a decrease in the bit error rate and improvement in the system performance. In reality, there is no total independence between these channels but correlation between them is low. Thus, this approach reduces the effective SNR (diversity gain).

The benefits of multi-input multi-output (MIMO) technology can be achieved through either diversity or spatial-multiplexing. The diversity techniques aim at receiving the same copy of information signals in multiple antennas or to transmit the same copies of information signals from multiple antennas. This helps to overcome deep fading effects and thus improves the reliability of transmission and reception.

Spatial-multiplexing techniques involve the simultaneous transmission of multiple independent data streams by the multiple transmission antennas, thus achieving a higher transmission rate. The number of streams or the maximum spatial diversity order N_s is given as

$$N_s = \min(N_t, N_r),$$

where N_t and N_r are the number of transmit and receive antennas, respectively. In general, MIMO technology offers great benefits such as improved system performance reliability, data rate.

Diversity can be achieved in any the following ways:

- **Space diversity:** Several copies of the information symbol are transmitted and received

using antenna(s) located at different positions to take advantage of the different radio uncorrelated multipath paths that exists in a typical environment. In space diversity, no extra power or bandwidth is required as the power and bandwidth are shared between the number of antennas.

- **Time diversity:** The same message may be transmitted using different timeslots which ensures repetition under independent fading conditions. If N time slots are used, then there will be spread in bandwidth by a factor of N . It however introduces a delay.
- **Frequency diversity:** This form of diversity uses more than one carrier for transmitting the same message. With frequency diversity, signals that have more spectral separation are free from channel impairments. Channel coherent bandwidth shows the correlation between different frequencies in the channel.
- **Angle diversity:** In angle diversity, multiple antennas with different directivity are used to receive or transmit copies of uncorrelated information signal at different angles.

Two diversity techniques can be combined in order to achieve better system performance. For example, combined space-time or space-frequency can be used.

For the receiver to detect the correct data, coding has to be used over different channels. This requires the utilization of MIMO spatial multiplexing.

In space-time coding, two or more copies of data are transmitted to compensate for unexpected impairments such as fading and noise in the channel. In this way, the receiver is less susceptible to noise because of the availability of multiple copies of the received signal.

With diversity, random nature of the signal propagation is exploited by finding the uncorrelated signal path for communication. Link improvement is also ensured at little cost. Brennan [9] and Khan [10] were the first to introduce the maximal ratio combining (MRC) scheme. Khan [10] gave the derivation for the optimal linear weights combiner of diversity of order two and Brennan [9] generalized his result to cover schemes of higher diversity order. Works on diversity schemes include techniques for correlated signal combiner by [11]. Equalization is suggested as a tool for inter symbol interference (ISI) mitigation as it is shown in [12]. The authors of [17] and [18] evaluate and compare different diversity combining methods in a Rayleigh fading channel. MRC is used mostly for coherent combining and it has been proven to be an optimal scheme. However, its complexity is a function of the branch signals located at the receiver. Equal gain combining (EGC) is the common scheme for non-coherent modulation.

2.2 Space-time block coding

While using the space-time diversity, the data stream is encoded in blocks. The data blocks are then sent using multiple antennas. This is called *Space-Time Block Coding* (STBC).

Space-time coding was introduced by Tarokh et al. [13] for achieving transmit diversity in a multi-antenna fading channel. Before that work, other diversity schemes such as the delay diversity scheme by Seshadri and Winters [23] and bandwidth efficient transmit diversity scheme by Wittneben [22] were presented in the literature. These results were generalized by the space-time coding scheme in [13] and [14] and used to design coding and modulation schemes for transmit and receive diversity. Space time block codes (STBC) and space time trellis codes (STTC) are the two main types of space time codes. STBC work on blocks of input symbols there by producing a matrix S_{ij} of i rows and j columns, which represent antennas and time slots, respectively. STBC provide full diversity and a very simple decoding, however, they do not provide coding gain [37].

STTC work on one input symbol at a time, thereby producing a vector symbol sequence with the same length as the number of antennas. They provide full diversity and coding gain, which gives them an edge over the STBC. However, they are difficult to design, and high complexity encoder and decoder are required. Foschini [16] considers a space-time architecture which are layer built of multiple antennas. The scheme achieves diversity either with or without coding [16], the disadvantage of such scheme is that the signals are non-orthogonal, which necessitate the need for interference cancellation scheme in the receiver because the arrival of the transmitted signal can corrupt signals of other layers. In this thesis, we only consider a special case of STBC, the Alamouti scheme.

2.3 Alamouti space-time block coding¹

Transmit beam forming requires the knowledge of the channel state information (CSI) of the system, which is a challenge in wireless communication. STBC do not require the knowledge of CSI, which advantageous from the practical perspective. Alamouti in his land mark paper [7] proposed a transmit diversity scheme, which operates without communication feedback. Alamouti code is a complex space-time diversity technique that can be used in two transmit

¹Alamouti space-time block presented here is based on the earlier work done in the course, MTAT.03.309 - Special Assignment in Wireless Communication (University of Tartu).

antenna and one receive antenna (2×1) in a multi input single output (MISO) mode or in a two transmit antenna and two receive antenna (2×2) MIMO mode. It is the only complex block code that achieve data rate of 1 while achieving maximum diversity gain. It is also called *Orthogonal STBC* because the columns in a specially designed matrix are orthogonal. The Alamouti code combines two diversity techniques: space and time transmit diversity. It is important for two main reasons:

1. It is an orthogonal space-time block code (we prove this in Appendix A).
2. The receiver does not require the CSI knowledge.

2.3.1 Alamouti scheme model

Alamouti scheme, proposed in [7] is either two transmit antennas and one receive antenna (2×1) or a two transmit antennas and two receive antennas (2×2). The implementation is in three stages:

1. Coding and transmission sequence.
2. Receiver combining method.
3. Symbol detection using maximum likelihood detector.

We present a model for a BPSK Alamouti code over an AWGN channel. We compare our analysis with single antennas at the transmitter and receiver with no diversity. The simulation results show improved performance as the diversity order increases. Each symbol is transmitted twice according to Table 2.1.

Time-slot	Antenna 1	Antenna 2
t	x_1	x_2
$(t + T)$	$-x_2^*$	x_1^*

Table 2.1: Alamouti scheme for two-branch transmit transmission sequence.

In the first time slot (t) x_1 is the symbol sent using antenna 1 and x_2 is sent using antenna 2. In the second time slot ($t + T$), $-x_2^*$ is the symbol sent using antenna 1 and x_1^* is the symbol sent using antenna 2.

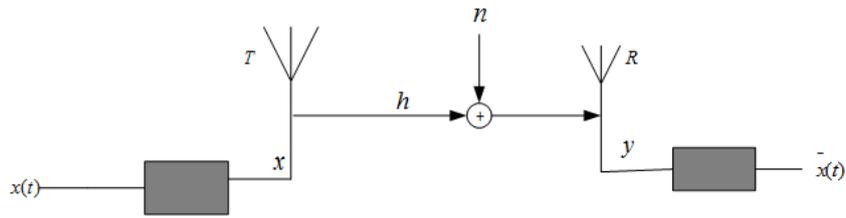


Fig 2.1: Single transmit and receive antennas (SISO).

2.3.2 Single transmit and receive antenna

Fig. 2.1 shows a single input single output scheme. This channel is of diversity order zero (without diversity).

It is shown in Fig. 2.1, x is the modulated symbol which suffers from a Rayleigh fading h , where h is the channel fading coefficient, which follows the Rayleigh distribution, n is the AWGN noise and y is the received symbol. The received symbol y can be expressed as:

$$y = hx + n.$$

In order to decode the transmitted symbol at the receiver with noise and channel effect, the estimate of symbol \hat{x} is given as:

$$\hat{x}(t) = \frac{y}{h} - \frac{n}{h}. \quad (2.1)$$

2.3.3 2×1 Alamouti scheme

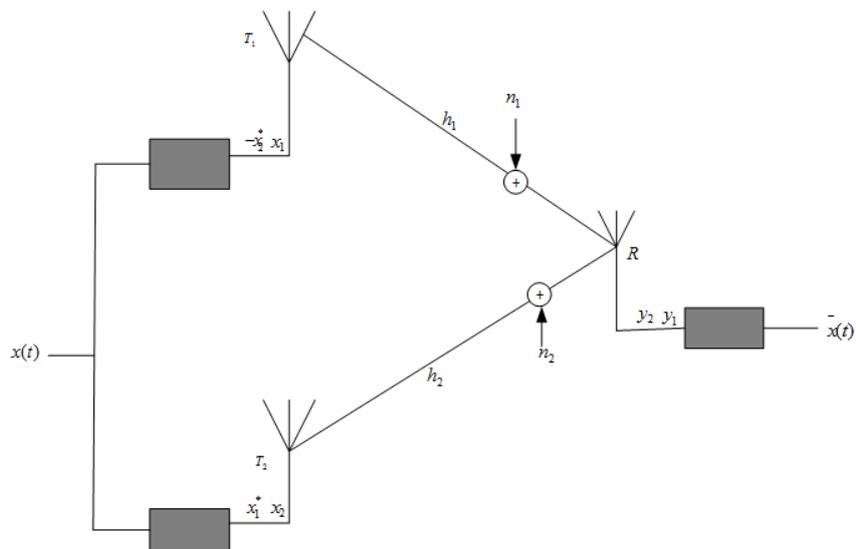


Fig 2.2: Alamouti two branch transmit diversity with one receiver.

Fig. 2.2 shows a system with two transmit and one receive antennae (2×1). In it, h_1 and h_2 are fading channel coefficients of transmit antennas 1 and 2, respectively, and they both follow the Rayleigh distribution. The modulated symbols x_1 and x_2 are transmitted in the first time slot as:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Therefore, the symbol received at the first transmit instance is given as:

$$y_1 = y(t) = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1, \quad (2.2)$$

where n_1 is the additive white Gaussian noise (AWGN) with distribution $\mathcal{N}(0, 1)$.

In the second transmit period ($t + T$), the modulated symbols

$$\begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix}.$$

Here, x_k^* denotes a complex conjugate of x_k . The symbol received at the second transmit period is given as:

$$y_2 = y(t + T) = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + n_2. \quad (2.3)$$

By taking the conjugate of y_2 , we obtain:

$$y_2^* = \begin{bmatrix} h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2^*. \quad (2.4)$$

By combining (2.3) and (2.4), we obtain:

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}. \quad (2.5)$$

We can represent (2.5) as:

$$Y = HX + N, \quad (2.6)$$

where $H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$ is a 2×2 channel matrix, $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $N = \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}$

We are interested in finding the estimates of $\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$, firstly, we consider the noise property which is i.i.d. with expectation:

$$\mathbf{E} \left(\begin{bmatrix} n_1^* \\ n_2^* \end{bmatrix} \cdot \begin{bmatrix} n_1^* & n_2 \end{bmatrix} \right) = \begin{bmatrix} |n_1|^2 & 0 \\ 0 & |n_2|^2 \end{bmatrix}.$$

The pseudo-inverse of H is denoted as $H^+ = (H^H H)^{-1} H^H$ where H^H is *Hermitian* of H . Hermitian of a matrix M is a complex square matrix which is equal to the transpose of its conjugate.

$$(H^H H) = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} = \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix}, \quad (2.7)$$

and its inverse can be expressed as:

$$(H^H H)^{-1} = \begin{bmatrix} \frac{1}{|h_1|^2 + |h_2|^2} & 0 \\ 0 & \frac{1}{|h_1|^2 + |h_2|^2} \end{bmatrix}. \quad (2.8)$$

The estimate of the transmitted symbol is therefore:

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = H^+ \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}. \quad (2.9)$$

The combined signals are sent to the maximum likelihood (ML) detector, so that the decision can be made. We can express the decision as:

$$d^2(y_1, h_1 x_i) + d^2(y_2, h_2 x_i) \leq d^2(y_1, h_1 x_k) + d^2(y_2, h_2 x_k) \quad \forall i \neq k. \quad (2.10)$$

We used the PSID in our subsequent decision making. We can simplify (2.10) as follows:
chose symbol x_i iff

$$d^2(\hat{x}_1, x_i) \leq d^2(\hat{x}_1, x_k) \quad \forall i \neq k,$$

where $d^2(a, b)$ is the squared Euclidean distance between signals a and b and can be computed as follows:

$$d^2(a, b) = (a - b)(a^* - b^*).$$

The symbols are then compared to the transmitted symbol to find the bit error rate (BER). At this point, it will helpful to show some derivations as well as the properties of the noise in Alamouti scheme. This is given in Appendix A.

2.3.4 2×2 Alamouti scheme

Reception reliability is increased by using more than one receive antennas. This results in better performance in terms of the BER as compared to the 2×1 Alamouti scheme as we show later

by the simulation. Alamouti 2×2 scheme offers both transmit and receive diversity. The analysis for 2×2 Alamouti scheme is similar to 2×1 , the only difference being the number of receive antenna is now 2 and the diversity order is now 4.

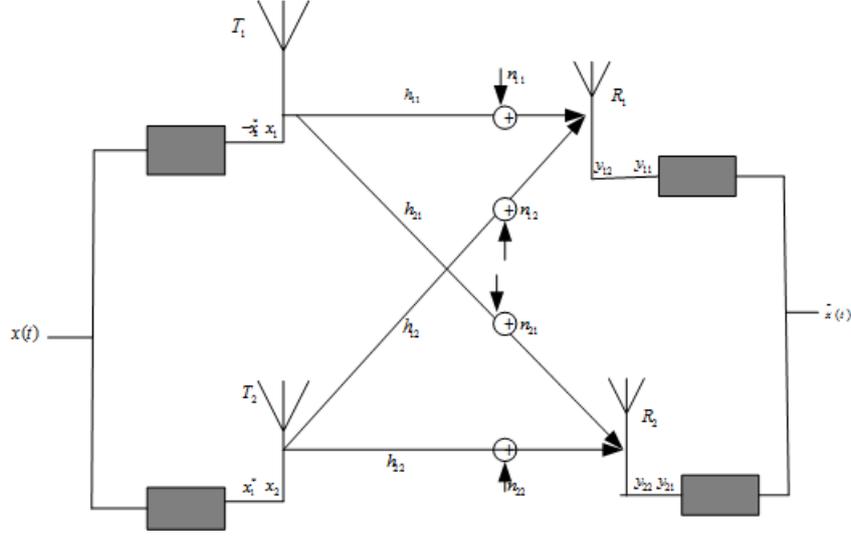


Fig 2.3: Alamouti two branch transmit diversity with two receivers.

Fig. 2.3 shows a two-transmit and two-receive antennas Alamouti scheme. Here, h_{11} and h_{12} are the fading channel coefficients between transmit antenna 1 to receive antennas 1 and 2, respectively, and h_{21} and h_{22} are the fading channel coefficients between transmit antenna 2 to receive antennas 1 and 2. The coding and transmitted sequence follows the same manner as that of 2×1 .

In the first time slot, the received vector is given as:

$$\begin{bmatrix} y_{11} \\ y_{12} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{12} \end{bmatrix}, \quad (2.11)$$

in the second time slot, the received symbol is given as:

$$\begin{bmatrix} y_{21} \\ y_{22} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + \begin{bmatrix} n_{21} \\ n_{22} \end{bmatrix}, \quad (2.12)$$

y_{ij} above represents the received symbol by the receive antenna j between transmit antenna i .

h_{ij} is the channel fading coefficient from j^{th} transmit antenna and i^{th} receive antenna.

Here, $\begin{bmatrix} n_{11} \\ n_{12} \end{bmatrix}$ and $\begin{bmatrix} n_{21} \\ n_{22} \end{bmatrix}$ are the noise vectors during the first and second time slots, respec-

tively. By combining (2.11) and (2.12), we obtain:

$$\begin{bmatrix} y_{11} \\ y_{12} \\ y_{21}^* \\ y_{22}^* \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{12} \\ n_{21}^* \\ n_{22}^* \end{bmatrix}. \quad (2.13)$$

We can represent (2.13) in a block form as follows:

$$Y = HX + N,$$

where $Y = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{21}^* \\ y_{22}^* \end{bmatrix},$

$$(H^H H) = \begin{bmatrix} |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2 & 0 \\ 0 & |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2 \end{bmatrix}, \quad (2.14)$$

and

$$(H^H H)^{-1} = \begin{bmatrix} \frac{1}{|h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2} & 0 \\ 0 & \frac{1}{|h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2} \end{bmatrix} \quad (2.15)$$

The estimate of the transmitted symbols which is sent to the ML detector is given as:

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = H^+ \cdot Y = (H^H H)^{-1} H^H \cdot \begin{bmatrix} y_{11} \\ y_{12} \\ y_{21}^* \\ y_{22}^* \end{bmatrix},$$

and the decoding is done in a similar way as the 2×1 case.

2.3.5 Simulation result

The result in Fig. 2.4 compares performances of single transmit and receive antennas, 2×1 and 2×2 Alamouti schemes. The 2×2 scheme has both transmit and receive diversity, thus achieving a better performance. The performance of the 2×1 scheme is also the same as that of 1×2 maximal receiver ratio combining (MRRC) scheme if full power is used at the transmitter as it is shown in [7]. Also, at full power, 2×2 Alamouti scheme gives the same performance as 1×4 MRRC scheme [7]. We show this later in Appendix A.

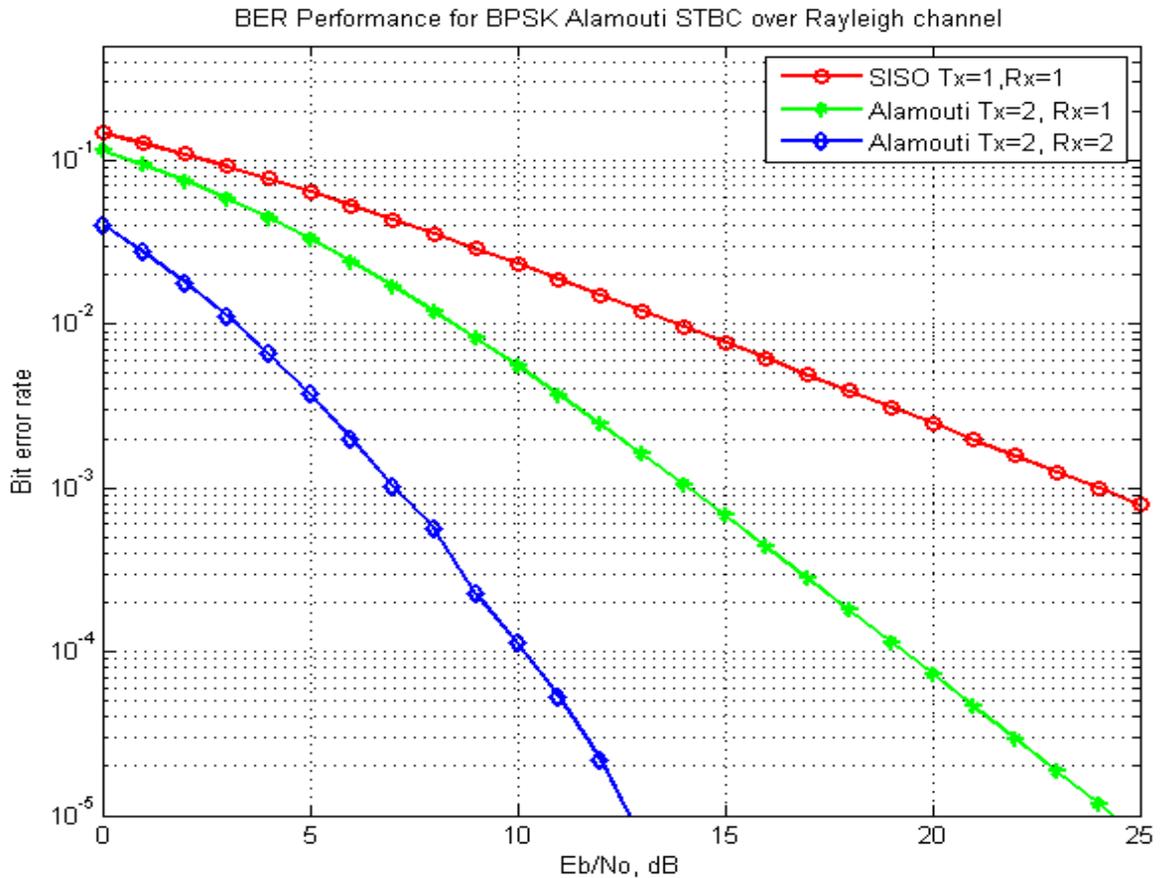


Fig 2.4: BER performance for Alamouti Scheme using BPSK modulation over Rayleigh fading channel.

2.4 Discussion

In the Alamouti scheme, an important thing to note is that, the total power is divided equally between the transmit antennas. This makes the the Alamouti scheme to lose 3 dB in performance compared to MRRC scheme of the same diversity order. The fading paths between the transmit and receive antenna follow Rayleigh distribution and are mutually uncorrelated.

The implementation issue with the Alamouti scheme are the power requirements as stated above, which could double the interference in the system and channel estimation errors, for which a perfect CSI was assumed. Decoding delay effects could also arise due to the number of distinct intervals in which the transformed signals are transmitted. For a 2-branch transmit diversity, the decoding delay is only 2 symbol periods. The distance of separation and correlation between antennas is also important in design consideration. The antenna are ought to be sufficiently uncorrelated. Antennas height and location also play a role in determining the sep-

aration distance between the transmit antennas. For diversity at the base station, antennas are at ten wavelength apart for sufficient decorrelation and three wavelength for antennas at remote location [7].

3 Index coding

3.1 Introduction

Recently, a lot of interest was attracted by index coding with side information (ICSI) problem. This concept was introduced by Birk and Kol in [19, 20] as an informed source coding on demand (ISCOD) over a noiseless broadcast channel. What necessitate this concept are video-on-demand, audio and newspaper delivery, where a sender (server) broadcasts a set of information (messages) to a set of receivers (clients). However, due to impairments such as interference, noise, fading, limited storage capacity, etc. the receiver might fail to receive some or all of the transmitted information. The sender has to find another means to get these failed message(s) across to the appropriate receiver(s). The server can send the missing message(s) to each of the receivers independently with the minimum number of transmissions (in order to save bandwidth). One approach that helps to achieve this is for the receivers to let the sender (server) know what message(s) they already have (side information) and what messages they request. By knowing this information, the total number of transmissions can be reduced [20].

ICSI problem is common in several other areas of communications. A good example is opportunistic wireless network [24, 25], where a node in the wireless network can eavesdrop a wireless communication channel and in this way, the node can obtain information which is not meant for it. This randomly collected information can serve as the side information later. System performance can be improved by taking full advantage of this side information.

For an integer n , denote $[n] = \{1, 2, \dots, n\}$. ICSI considers a scenario with one source \mathcal{S} that intends to send i messages from a set $X = \{x_1, x_2, x_3, \dots, x_i\}$, which belong to the finite field \mathbb{F}_2 , to a set of j receivers $R = \{R_1, R_2, R_3, \dots, R_j\}$.

Each receiver knows some of the messages in set X , these constitute the receiver's *side information*.

Two sets \mathcal{W}_m and \mathcal{H}_m are associated with every receiver $R_m \in R$. Here, $\mathcal{W}_m \subseteq X$ is the set of message requested by R_m and $\mathcal{H}_m \subseteq X$ is the set of messages R_m has. Based on the above notations, we can denote the index coding problem as $(X, R, \mathcal{W}_m, \mathcal{H}_m)$, where $\mathcal{W}_m = \{\mathcal{W}_m\}_m$ and $\mathcal{H}_m = \{\mathcal{H}_m\}_m$. A simple example of ICSI problem instance is given in Fig. 3.1, with

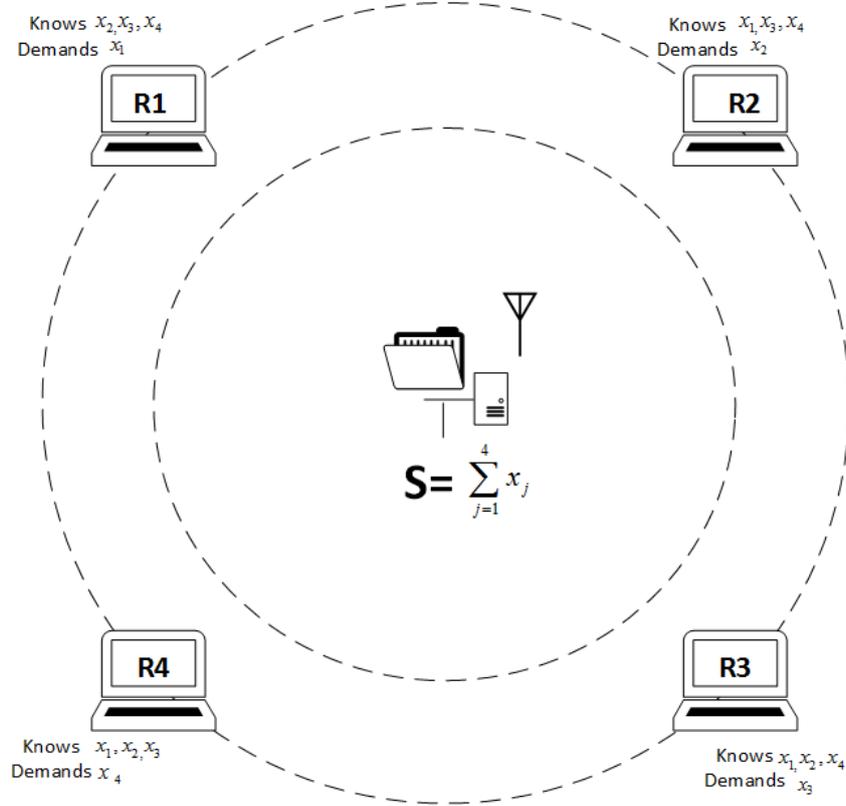


Fig 3.1: A simple example of index coding.

one sender S , the data base server and four receivers (clients). Each of these clients requests a different set of information symbols from the server. In the traditional approach, each of the requested information sets is sent in four different transmissions to the corresponding receivers. However, by exploiting the side information of these receivers, the server can send this information jointly in just one transmission and meet the demands of the clients, thereby saving bandwidth and improving the system throughput.

Index coding has been extensively studied [21], [27], [28], [29], [30], [31], [32], [33], [35]. It is a special case of network coding (NC) problem which is a *non-multicast or unicast* [26]. It was shown in [33] that any NC problems can be reduced to the ICSI problem. That is, for an NC problem, a corresponding ICSI can be constructed if and only if the NC problem has a scalar linear network code and the corresponding ICSI problem has a perfect scalar index code. Bar-Yossef et al. in [21] consider a linear index coding problem, whose encoding and decoding

functions are linear. Furthermore it is shown in [21] that side information graphs can be used to represent such an index coding problem. This was later referred to as *single unicast* index coding problem in [28], where it was concluded that the length of optimal linear index code equals the *minrank* over the field \mathbb{F}_2 of its side information graph. Similarly, [32] considered error correction for index coding where the problem was extended to cover *minrank* over field of size q of the hypergraph corresponding to its side information.

Lattice based index codes, a special form of solution for index coding problem over the Gaussian broadcast channel in which the transmitter has K independent messages, each of the receivers requests all the messages at the source and has the knowledge of some subset of K messages a priori, were proposed in [34]. Lattice index codes help to convert the receiver side information into the SNR gains.

In [35], index coding was investigated over the Gaussian broadcast channel and an index code based on multi-dimensional quadrature amplitude modulation (QAM) constellation was proposed. It also introduced *side information gain* as a metric for measuring the efficiency to which index codes make use of the receiver side information but in [35] the number of transmissions was not minimized as in the case of [19]. Recently [39] extended [35] and proposed a golden-coded index coding which involves partitioning golden codes into a number of sub-codes, in that work, index coded was studied over MIMO fading channel.

In a recent work by Mahesh and Sundar Rajan [31], index coding was considered over AWGN. The authors give an algorithm that maps the index coded bits to appropriate sized PSK symbols based on the *minrank* of the index code problem. An index code of *minrank* N is mapped to 2^N PSK symbols. The authors also show that the number of side information symbols that the receiver knows has influence on its probability of error performance. Receiver with more symbols of side information has a low probability of error performance compared to the receivers with smaller amount of side information. This gain is denoted as PSK side information coding gain and PSK bandwidth gain.

One of the challenges in solving index coding problem is finding the minimum number of transmission (*minrank*) that satisfies all requests of the client. The difficulty in finding the optimal length of linear index code or its approximation for scalar index code was stated in [36]. Esfahanizadeh et al. in [38] proposed an algebraic solution for finding the optimal length of a linear

index code. The authors of [40] and [41] study the problem of finding the optimal length of index code. An estimate on the minimum number of optimal length codes is given therein.

In this work, we consider index coding over AWGN and Rayleigh fading channels used in conjunction with Alamouti STB code. We aim at improving the BER performance and reliability of the receivers by using the side information. We combine the columns of the encoding matrix while a probability soft information detection (PSID) was used for the decisions. In all experiments, we recover each receiver's request in several different ways while taking advantage of independent probability distribution each reception. We use different modulation schemes: PSK and QAM, in order to reduce the probability of error. We showed the results for both hard and soft decision. For the hard decision, majority voting gave the best performance. We also improve the reliability by combining index coding with a space-time block code (Alamouti code).

3.2 System model

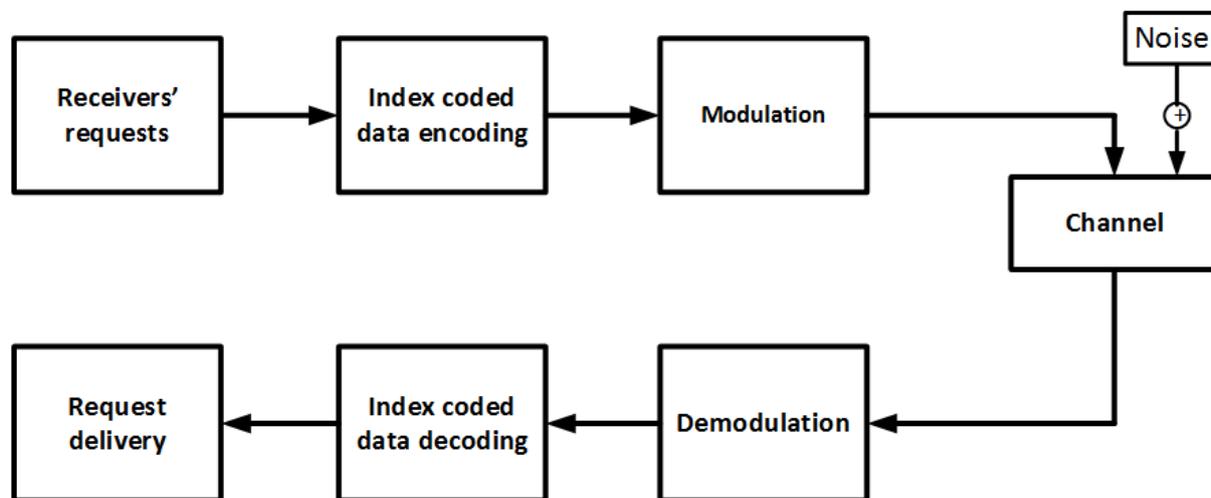


Fig 3.2: Index coding block diagram.

A block diagram for index coding is shown in Fig. 3.2. The receivers request for the message(s) they need, while making their side information known to the transmitter. The transmitter then determines how to satisfy these requests jointly while using the minimum number of transmissions. The messages are then encoded, modulated and transmitted over the channel. However, generally the requested messages can be corrupted by noise and other channel impairments. The requested signals are then demodulated and decoded. Each receiver then recover

the message it requested.

3.2.1 Algebraic representation

Consider an instance of index coding problem with side information, where there are a broadcast transmitter S and six receivers R_1, R_2, \dots, R_6 . Assume that S owns a binary vector $\mathbf{x} = (x_1, x_2, \dots, x_6) \in \mathbb{F}_2^6$. Each receiver R_i owns the subset (side information) of bits $\mathcal{X}_i \subseteq \{x_1, x_2, \dots, x_6\}$, $i \in [6]$, according to the following list of index sets:

$$\begin{aligned}\mathcal{X}_1 &= \{2, 3, 4, 5\} \\ \mathcal{X}_2 &= \{1\} \\ \mathcal{X}_3 &= \{1, 2\} \\ \mathcal{X}_4 &= \{1, 2, 3, 5\} \\ \mathcal{X}_5 &= \{6\} \\ \mathcal{X}_6 &= \{5\}.\end{aligned}$$

Consider the following family of binary matrix, which corresponds this index coding problem, where ‘•’ denotes an arbitrary symbol in \mathbb{F}_2 :

$$\mathbf{M} = \begin{pmatrix} 1 & \bullet & \bullet & \bullet & \bullet & 0 \\ \bullet & 1 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & 1 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & 1 & \bullet & 0 \\ 0 & 0 & 0 & 0 & 1 & \bullet \\ 0 & 0 & 0 & 0 & \bullet & 1 \end{pmatrix}.$$

Assuming that the receiver R_i demands the message x_i for $i \in \{1, 2, \dots, 6\}$, transmitter S could satisfy each receiver’s request in six independent transmissions, but that solution is not optimal. Instead, we find the minimum rank of the matrix \mathbf{M} which is equivalent to the minimum number of transmissions.

Assume that the transmitter S performs encoding operation $\mathbf{x} \cdot \mathbf{L} = \mathbf{y}$, where \mathbf{L} is the min-rank representation of \mathbf{M} and it is a $n \times N$ binary generator matrix of an index code, and $\mathbf{y} = (y_1, y_2, \dots, y_N)$ is the transmitted binary vector, $n = 6$, $N = 4$. It can be seen that the submatrix formed by the second, third, fourth and last rows contains a 4×4 lower triangular submatrix, and therefore the min-rank of any matrix of the form \mathbf{M} is at least 4.

The following matrix \mathbf{L} can be chosen as an optimal 6×4 encoding matrix:

$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \quad (3.1)$$

The four independent transmissions for the above index coding problem are given in equations (3.2)-(3.5) as follows:

$$y_1 = x_1 + x_2 \quad (3.2)$$

$$y_2 = x_2 + x_3 \quad (3.3)$$

$$y_3 = x_5 + x_6 \quad (3.4)$$

$$y_4 = x_2 + x_3 + x_4 + x_5. \quad (3.5)$$

We observe that each receiver R_i can recover the requested bit from $\mathbf{y} = \mathbf{xL}$ and from the side information indexed by \mathcal{X}_i using the following equations:

$$x_1 = y_1 - x_2 \quad (3.6)$$

$$x_2 = y_1 - x_1 \quad (3.7)$$

$$x_3 = y_2 - x_2 \quad (3.8)$$

$$x_4 = y_4 - x_2 - x_3 - x_5 \quad (3.9)$$

$$x_5 = y_3 - x_6 \quad (3.10)$$

$$x_6 = y_3 - x_5. \quad (3.11)$$

We then transmit the index coded bits $\mathbf{y} = (y_1, y_2, y_3, y_4)$ and recover each receiver R_i request $\mathcal{X}_i \subseteq \{x_1, x_2, \dots, x_6\}$ by using equations (3.6)-(3.11). We repeat this many times and compute bit error rate performance using different modulation techniques.

3.2.2 BPSK index coding with soft decision

Each of index encoded bits $\mathbf{y} = (y_1, y_2, y_3, y_4)$ can be transmitted independently by BPSK.

We can denote the received message under BPSK as:

$$y_{r_i} = y_i + w, \quad (3.12)$$

where y_i is the transmitted BPSK message, $i \in \{1, 2, 3, 4\}$, w is the AWGN and y_{r_i} is the received message. The SNR for each BPSK is given as:

$$SNR = \frac{E_b}{N_o},$$

where E_b is the energy per bit which we take as 1 and N_o is the noise power. We compute the noise power as follows:

$$N_o = \frac{1}{SNR}.$$

The SNR is in linear scale and computed from the SNR_{dB} in dB scale as follows:

$$SNR_{dB} = 10 \log SNR,$$

therefore,

$$SNR = 10^{\frac{SNR_{dB}}{10}}.$$

We can therefore express N_o as:

$$N_o = \frac{1}{10^{\frac{SNR_{dB}}{10}}} = 10^{-\frac{SNR_{dB}}{10}}.$$

The noise variance σ^2 is related to N_o as

$$\sigma^2 = \frac{N_o}{2},$$

and noise standard deviation σ is given as:

$$\sigma = \sqrt{\frac{N_o}{2}} = \sqrt{\frac{10^{-\frac{SNR_{dB}}{10}}}{2}}$$

Decision

In the BPSK simulation, we use probability soft information detection (PSID) for our decision.

For BPSK it is done as follows. We compute the total PDF (T_{PDF_i}) as:

$$T_{PDF_i} = P(y_i = 0) \cdot P(y_{r_i} | y_i = 0) + P(y_i = 1) \cdot P(y_{r_i} | y_i = 1), \quad (3.13)$$

where $P(y_i = 0) = P(y_i = 1) = \frac{1}{2}$ are equally probable.

$$T_{PDF_i} = \frac{1}{2\sqrt{2\pi\sigma^2}} e^{-\frac{(y_{r_i}+1)^2}{2\sigma^2}} + \frac{1}{2\sqrt{2\pi\sigma^2}} e^{-\frac{(y_{r_i}-1)^2}{2\sigma^2}}. \quad (3.14)$$

We then compute the normalized PDF of each transmitted message being zero or one. Normalized PDF for 0 being transmitted is

$$P(y_{r_i} | y_i = 0)_N = \frac{P(y_{r_i} | y_i = 0)}{P(y_{r_i} | y_i = 0) + P(y_{r_i} | y_i = 1)}, \quad (3.15)$$

and normalized PDF for 1 being transmitted is:

$$P(y_{r_i} | y_i = 1)_N = \frac{P(y_{r_i} | y_i = 1)}{P(y_{r_i} | y_i = 0) + P(y_{r_i} | y_i = 1)}. \quad (3.16)$$

In order to recover the requested information by the receiver R_i , we used these normalized PDF values to compare the $\mathbf{x} = (x_1, x_2, \dots, x_6) \in \mathbb{F}_2^6$ with the recovery in (3.6) - (3.11). We compute the bits that are in error and show by simulation in Fig. 3.3. The result shows the error rate performance of our system. We can improve this system by prioritizing the receivers i.e we can recover the bits in more than one way. We will discuss this prioritization, which is achieved by combining some columns of min-rank matrix \mathbf{L} in (3.1), in subsequent sections.

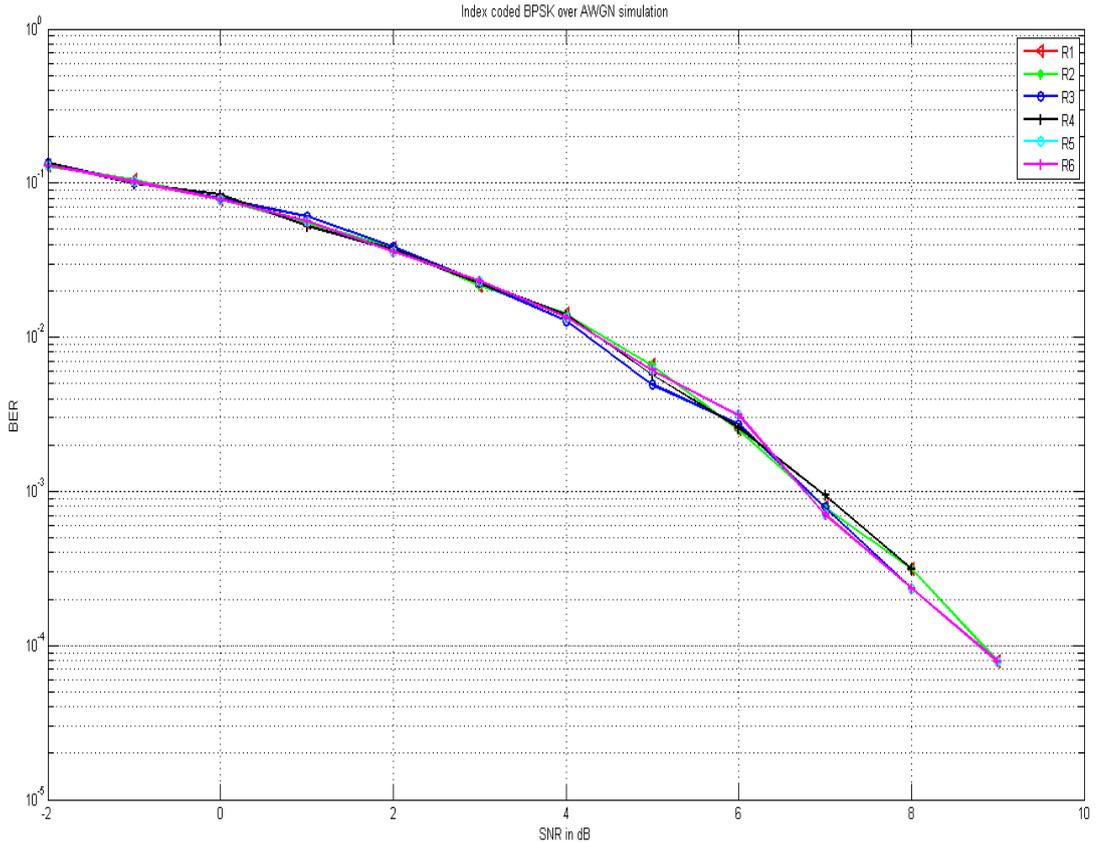


Fig 3.3: BER performance of index coded recovery BPSK simulation over AWGN.

3.2.3 16 PSK index coding with hard decision

Unlike BPSK, 16 PSK is not a bit detection scheme which makes bit by bit hard decision. The recovery of index coded data is possible by making hard decision, in which some data is prone to cut off abruptly and information lost which make recovery more difficult and more errors. We consider symbol mapping, where 4 index coded bits $\mathbf{y} = \mathbf{xL} = (y_1, y_2, y_3, y_4)$ are Gray coded, mapped to constellation as shown in Fig. 1.6 and modulated into the in-phase and quadrature components as shown in Table 1.3 then, they are transmitted as an in-phase and quadrature signals components. These signals are corrupted by AWGN which could influence the decision badly under the low SNR. By the hard decision, we need to decode the transmitted message and find the received signals estimates $\hat{\mathbf{y}} = (\hat{y}_1, \hat{y}_2, \hat{y}_3, \hat{y}_4)$ before we can use index coded recovery in equations (3.6) - (3.11). We present the simulation results in Fig. 3.4. It shows the BER of the receivers when recovering the requested bits. We show the BER performance in Fig. 3.4.

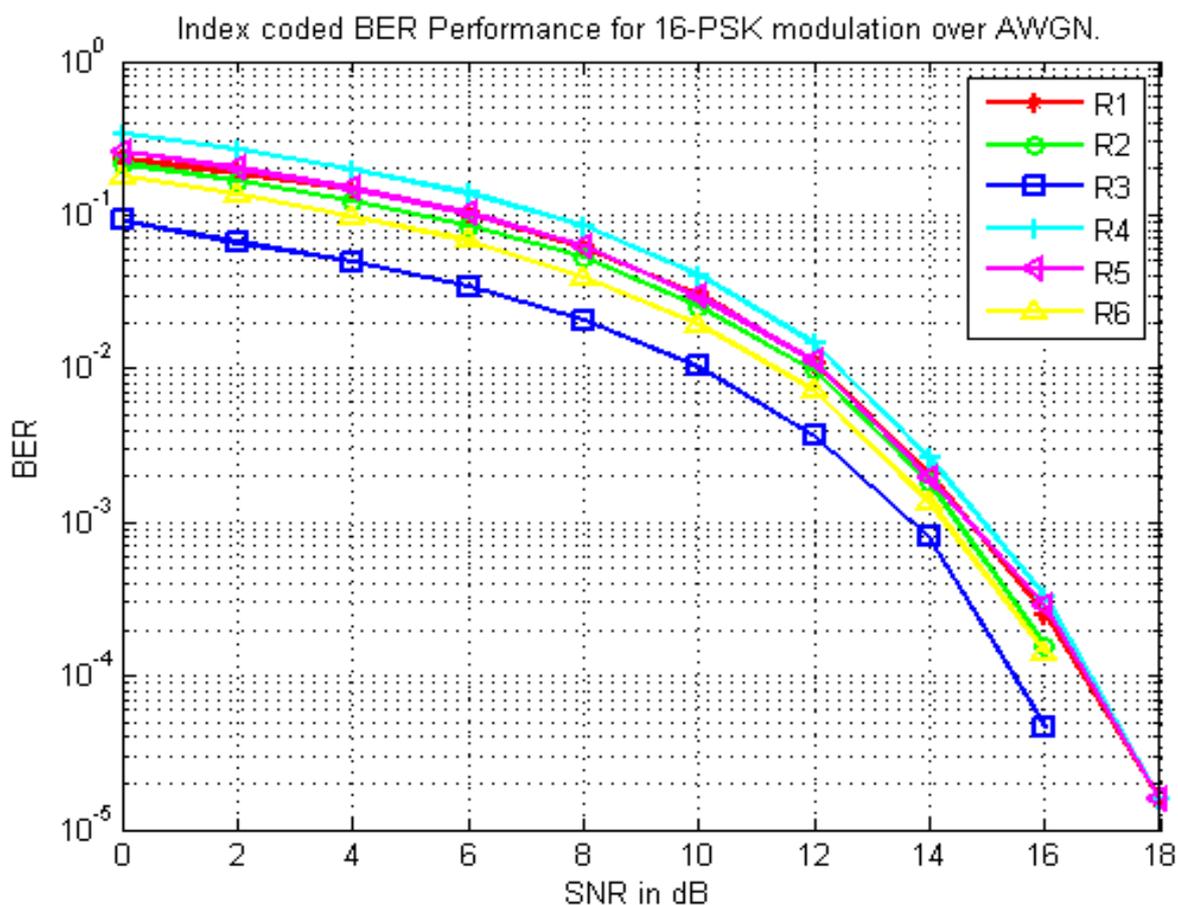


Fig 3.4: BER performance for index coded data in for 16 PSK modulation over AWGN.

3.2.4 Prioritized index coding

Suppose now that R_1 is a prioritized receiver, either because it has some priority in the system, or simply because the link from \mathcal{S} to R_1 is less reliable than the other links (we can prioritize any of the receivers R_i as well). Then, we can choose an alternative matrix to \mathbf{L} which is given in (3.1). We choose a matrix \mathbf{L}_1 ,

$$\mathbf{L}_1 = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad (3.17)$$

whose columns span the same vector space as the columns of \mathbf{L} and also has the same rank of 4.

Matrix \mathbf{L}_1 in (3.17) is constructed from the columns of matrix \mathbf{L} in (3.1) as follows:

$$\mathbf{L}_1 = \begin{pmatrix} y_1 & y_1 + y_2 & y_3 & y_1 + y_2 + y_3 \end{pmatrix}, \quad (3.18)$$

It can be easily seen that each receiver $R_i, i = \{2, 3, \dots, 6\}$, can recover the requested bit from $\mathbf{y} = \mathbf{x}\mathbf{L}_1$ and from the side information indexed by \mathcal{X}_i using (3.19) - (3.23):

$$x_2 = y_1 - x_1 \quad (3.19)$$

$$x_3 = y_2 - x_1 \quad (3.20)$$

$$x_4 = y_4 - x_1 - x_2 - x_5 \quad (3.21)$$

$$x_5 = y_3 - x_6 \quad (3.22)$$

$$x_6 = y_3 - x_5. \quad (3.23)$$

Additionally, x_1 can now be reconstructed in three independent ways:

$$x_1 = y_1 - x_2 \quad (3.24)$$

$$x_1 = y_2 - x_3 \quad (3.25)$$

$$x_1 = y_4 - x_2 - x_4 - x_5. \quad (3.26)$$

We use the matrix \mathbf{L}_1 to reduce the error rate and to improve the system reliability of bit x_1 . This is demonstrated by the simulation results.

3.2.5 Prioritized 16 PSK index coding using majority voting

In 16 PSK, we simply generate index coded bit from L_1 , similarly to approach in subsection 3.2.3, and recover R_1 in three independent ways. We then make the decision by *majority voting* for x_1 's. The majority decision is inefficient for two bit detection. Simulation shows that both one and two bit detection has the same performance. However, we observe improved performance for the three bit detection as it is shown by simulation in Fig.3.5.

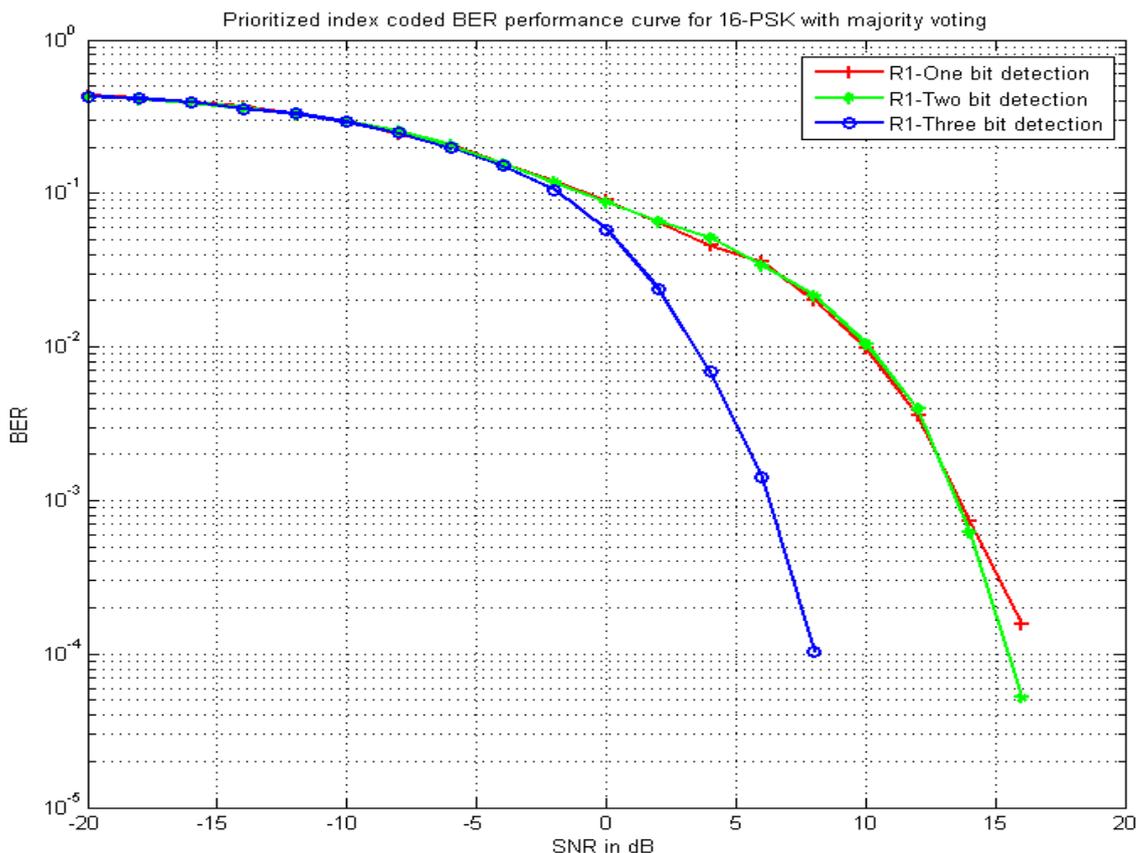


Fig 3.5: BER performance for prioritized index coded 16 PSK with hard decision using majority voting.

3.2.6 Prioritized index coding with BPSK and soft decision

In prioritized BPSK, we recover R_1 in three independent ways, using L_1 as encoded matrix, by computing probabilities as we did in Subsection 3.2.2. We combine two and three independent probabilities, respectively, to make the decision. From the law of probability, for any two independent events A and B with probabilities $P(A)$ and $P(B)$, respectively, the probabilities

that they both occur together is given as:

$$P(A \cap B) = P(A) \cdot P(B), \quad (3.27)$$

by applying the same probability law to our scheme, we multiply the probabilities of each independent way of recovering R_1 and make the decision, this is what we refer to as two bit detection. If we also have another independent event C with probability $P(C)$, the probability of A, B and C occurring together is given as:

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C). \quad (3.28)$$

We compare the performance to one bit decision, we obtain improved performance, the simulation results are shown in Fig. 3.6.

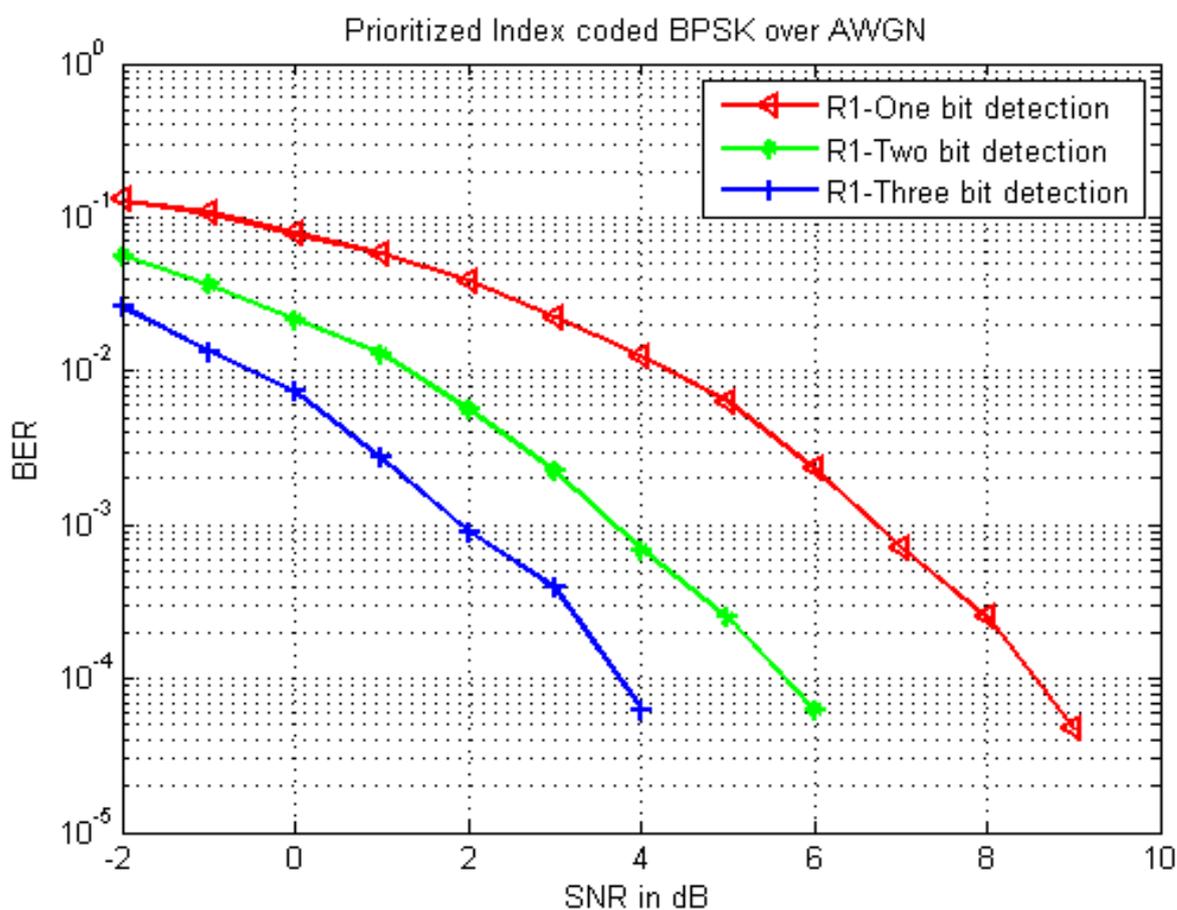


Fig 3.6: Prioritized index coded BER performance for BPSK.

3.2.7 Prioritized index coding with QPSK soft decision

In order to prioritize R_1 using QPSK modulation, the index coded bits $\mathbf{y} = (y_1, y_2, y_3, y_4)$ are mapped to QPSK symbol as shown in Table 1.1. We have two independent transmissions, each

can be viewed as a symbol in $(\mathbb{F}_2)^2$, y_1y_2 and y_3y_4 . To show the PSID under more practical scenario, we consider the index coded bit under the influence of Rayleigh fading. The noise standard deviation for this scheme is given as:

$$\sigma = \sqrt{\frac{10^{-\frac{SNR_{dB}}{10}}}{2}}. \quad (3.29)$$

The received signal is:

$$y_q = ht_i + n_i, \quad (3.30)$$

where t_i is a transmitted symbol and $i \in \{1, 2\}$, n_i is the AWGN and h is the Rayleigh fading coefficient. Similar approach can be taken as for BPSK with soft decision in Subsection 3.2.6. The results of the simulation in Fig. 3.7 shows little improvement in the one, two and three bit detection using soft information. This can be explained by the influence of Rayleigh fading. However, we will combat fading in this system by using Alamouti diversity scheme in Chapter 4.

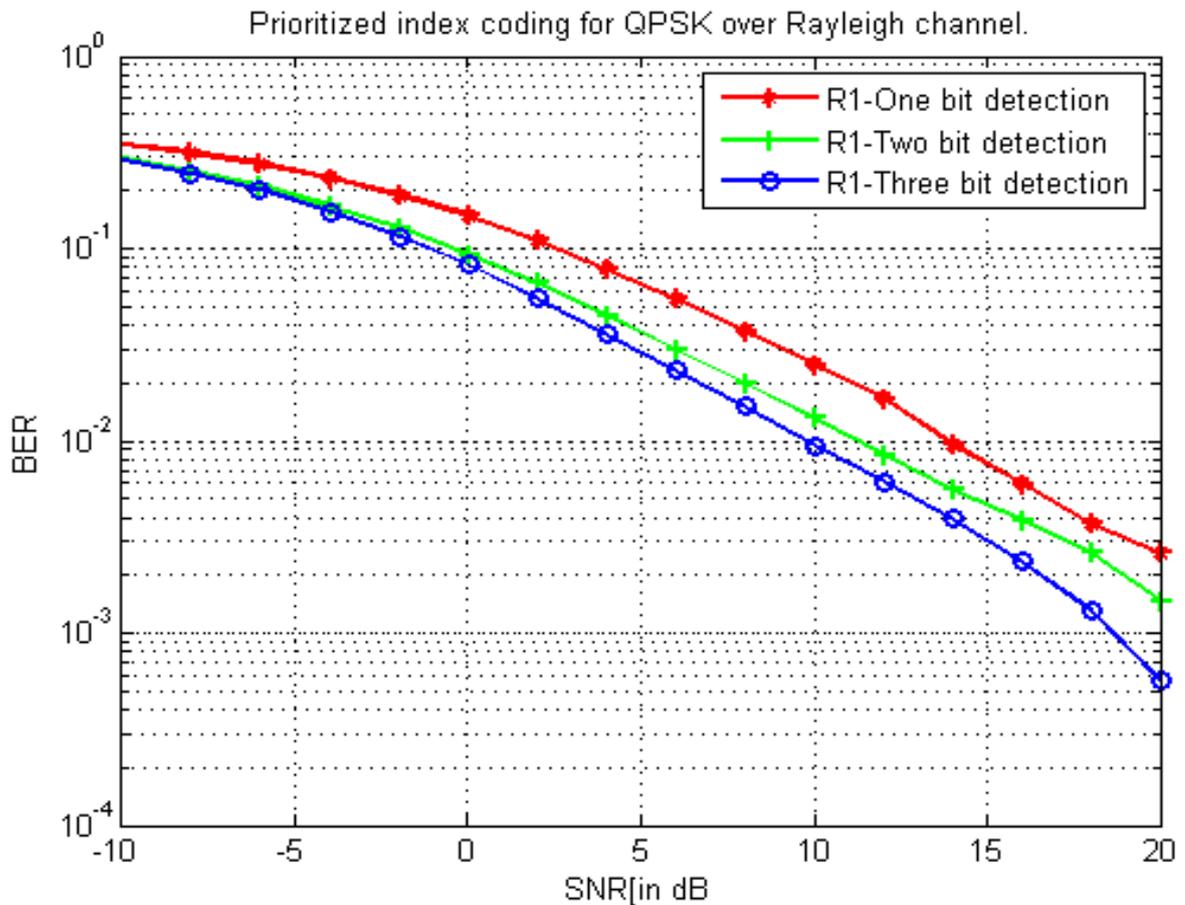


Fig 3.7: Prioritized index coded BER performance for QPSK.

3.2.8 Prioritized index coding with 16 QAM soft decision

We also consider 16 QAM modulation, together with soft decision with improved reliabilities, there, we map the index coded bits $\mathbf{y} = (y_1, y_2, y_3, y_4)$ into Gray-coded QAM symbol as shown in Table 1.4. Symbols (pairs of bits) y_1y_2, y_3y_4 are mapped as in-phase and quadrature components respectively. For 16 QAM, we can express the received signal as follows. Let y_1y_2 be mapped to 16 QAM symbol I and y_3y_4 be mapped to 16 QAM symbol Q where $I, Q \in \{\pm 1, \pm 3\}$, the received signal for the in-phase component is:

$$y_{r_I} = I + n_I, \quad (3.31)$$

and for the quadrature component, the received signal is:

$$y_{r_Q} = Q + n_Q, \quad (3.32)$$

where n_I and n_Q are the AWGN of the in-phase and quadrature components, respectively. The noise variance is computed as follows. The average energy for QAM modulation is given by the relation:

$$E_Q = \frac{2}{3}(M - 1), \quad (3.33)$$

where M is the number of constellation points of the QAM modulation. In the studied case, $M = 16$, therefore the average energy per 16 QAM symbol is 10. This can also be shown by taking the sum of the squares of all the QAM symbol divided by the square root of the division of the number of constellation points and the number of quadrant. For 16 QAM, we obtain:

$$E_{16Q} = \frac{1^2 + 1^2 + 3^2 + 3^2}{\sqrt{\frac{16}{4}}} = 10$$

$$SNR = \frac{E_{16Q}}{N_o},$$

$$N_o = \frac{10}{SNR}$$

$$N_o = \frac{10}{10^{\frac{SNR_{dB}}{10}}},$$

$$\sigma^2 = \frac{N_o}{2} = \frac{5}{10^{\frac{SNR_{dB}}{10}}}$$

$$\sigma = \sqrt{\frac{N_o}{2}} = \sqrt{\frac{5}{10^{\frac{SNR_{dB}}{10}}}}$$

Decision

We use soft values of y_{r_I} and y_{r_Q} to compute the likelihood of the bits (y_1, y_2) , (y_3, y_4) respectively. The analysis for likelihood of y_1 is the same as for y_3 , similarly y_2 and y_4 are subjects to the same likelihood analysis. The total probability for y_{r_I} is expressed as:

$$T_{PDF_I} = P(I = +1) \cdot P(y_{r_I} | I = +1) + P(I = -1) \cdot P(y_{r_I} | I = -1) + P(I = -3) \cdot P(y_{r_I} | I = -3) + P(I = +3) \cdot P(y_{r_I} | I = +3), \quad (3.34)$$

where $P(I = +1) = P(I = -1) = P(I = +3) = P(I = -3) = \frac{1}{4}$ are equally probable. To determine if $y_1 = 1$, we compute the normalized PDF as follows:

$$P(y_{r_I} | y_1 = 1)_N = \frac{P(y_{r_I} | I = +1) + P(y_{r_I} | I = +3)}{T_{PDF_I}}, \quad (3.35)$$

and to determine if the transmitted index coded bit is $y_1 = 0$, we compute

$$P(y_{r_I} | y_1 = 0)_N = \frac{P(y_{r_I} | I = -1) + P(y_{r_I} | I = -3)}{T_{PDF_I}}. \quad (3.36)$$

In the case of index coded bit y_2 , to determine that the transmitted index coded bit $y_2 = 1$, we compute

$$P(y_{r_I} | y_2 = 1)_N = \frac{P(y_{r_I} | I = +1) + P(y_{r_I} | I = -1)}{T_{PDF_I}}, \quad (3.37)$$

and for the transmitted index coded bit being $y_2 = 0$, we have:

$$P(y_{r_I} | y_2 = 0)_N = \frac{P(y_{r_I} | I = +3) + P(y_{r_I} | I = -3)}{T_{PDF_I}}. \quad (3.38)$$

In order to recover R_1 , we then use the decision values to compare $\mathbf{x} = (x_1, x_2, \dots, x_6) \in \mathbb{F}_2^6$ with the recovery equations in (3.19) - (3.23). We compute the bits that are in error using one, two and three bit recovery. We compare the performance by simulation as it is shown in Fig. 3.8. The result shows the error rate performance. We can see clearly that the prioritization yields improved performance. Therefore, by carefully selecting the matrix, we can recover the bits in more than one way, thus leading to improved performance.

3.2.9 Discussion

As we have shown by simulation, recovery can be improved by prioritizing the receivers. However, we need to ensure more reliability in getting these messages to the receivers as well as to combat fading. We introduced diversity by transmitting these index coded bits using two transmit antennas. We will discuss this in Chapter 4.

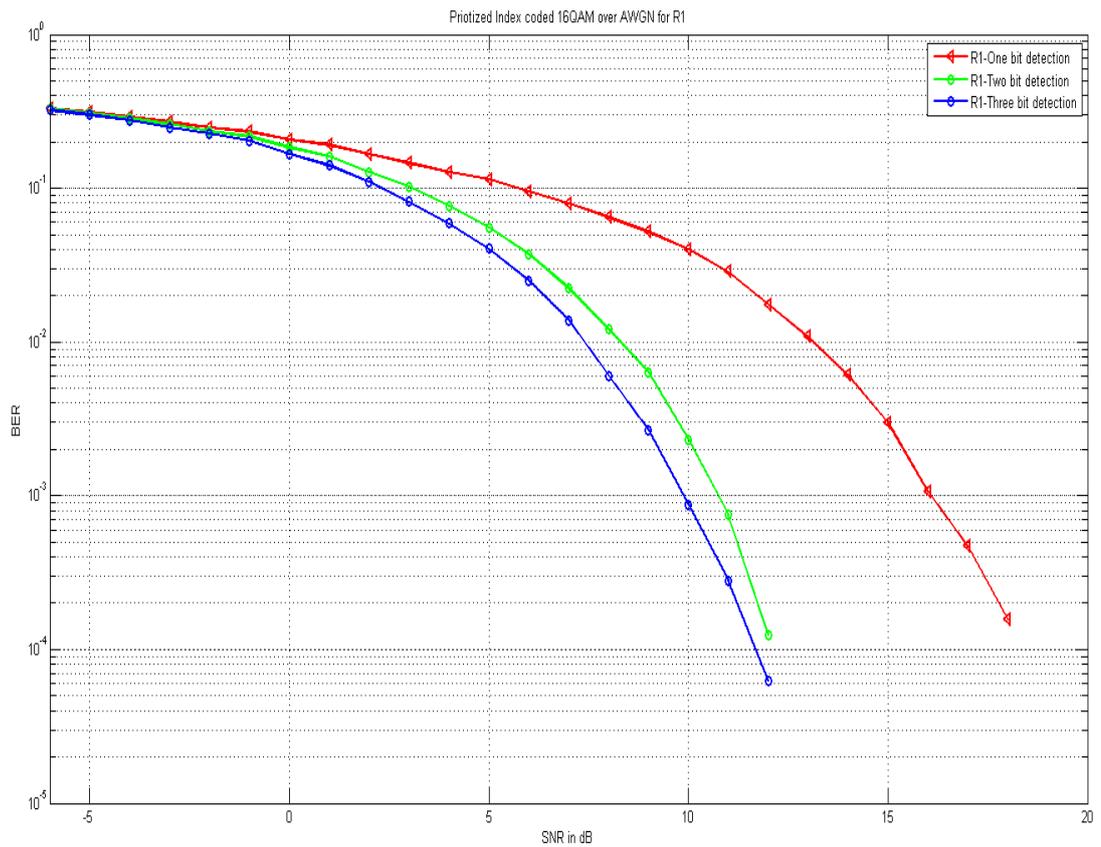


Fig 3.8: BER performance for prioritized index coding using 16 QAM.

4 Alamouti index coded systems

4.1 Introduction

In this chapter, we consider a combined scheme, which employs index coding in conjunction with Alamouti codes. The block diagram of the system under consideration is shown in Fig. 4.1. The index coded bits are mapped and modulated. Space time encoding with two antennas is then applied. The reverse process is used to recover the requested bit. We base our analysis on prioritized index coding, as in Chapter 3. We consider QPSK with Rayleigh fading. The simulation results shows improved performance when compare it to the similar modulation scheme for single antenna transmission in Subsection 3.2.7.

4.2 2×1 Alamouti index coding using QPSK modulation

In this section, we consider a system where index coded bits are mapped into QPSK symbol as in Table 1.1 and then Alamouti scheme in Chapter 2 is applied. We employ transmit diversity in order to obtain more transmission reliability and bandwidth gain. Fig. 4.2 shows the 2×1 Alamouti system model for index coded system. The index coded bits are transmitted in two different time slot. Antennas T_1 and T_2 transmit $y_1 + jy_2$ and $y_3 + jy_4$ in the first time slot, respectively. In the second time slot, T_1 and T_2 transmit $-(y_3 + jy_4)^*$ and $(y_1 + jy_2)^*$, respectively. The received signal in the first time slot is given as:

$$y_{T_1} = h_1(y_1 + jy_2) + h_2(y_3 + jy_4) + n_1, \quad (4.1)$$

where h_1 and n_1 are the Rayleigh coefficient and AWGN respectively. In the second time slot, the received signal is:

$$y_{T_2} = -h_1(y_3 + jy_4)^* + h_2(y_1 + jy_2)^* + n_2, \quad (4.2)$$

where h_2 and n_2 are the Rayleigh coefficient and AWGN respectively. By combining equations (4.1) and (4.2) into Alamouti STBC we obtain:

$$\begin{bmatrix} y_{T_1} \\ y_{T_2}^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} y_1 + jy_2 \\ y_3 + jy_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}. \quad (4.3)$$

The estimates of $y_1 + jy_2$ and $y_3 + jy_4$ are given by:

$$\begin{bmatrix} \hat{y}_1 + j\hat{y}_2 \\ \hat{y}_3 + j\hat{y}_4 \end{bmatrix} = H^+ \begin{bmatrix} y_{T_1} \\ y_{T_2}^* \end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix} y_{T_1} \\ y_{T_2}^* \end{bmatrix}, \quad (4.4)$$

where $H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$ is a 2×2 orthogonal channel matrix and H^+ is the pseudo-inverse of H .

We apply PSID to these received estimates and obtain the performance as shown in Fig.4.3 when we prioritize R_1 . We compare this result to a similar QPSK scheme with a single antenna in Fig.3.7, and observe that there is an improvement in the error performance. This improvement stresses the usefulness of diversity.

4.3 2×2 Alamouti index coding using QPSK modulation

In this section, we provide additional diversity to the system in Fig. 4.2 for reliability enhancement on the receiver end as well bandwidth gain and improved performance. Analysis is based on index coded QPSK symbol over Rayleigh fading channel. The schematics is shown in Fig. 4.4. The transmission is performed using two time slots, in the first time slot, the received vector is:

$$\begin{bmatrix} y_{T_{11}} \\ y_{T_{12}} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} y_1 + jy_2 \\ y_3 + jy_4 \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{12} \end{bmatrix}. \quad (4.5)$$

The received vector in the second time slot is

$$\begin{bmatrix} y_{T_{21}} \\ y_{T_{22}} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} -(y_3 + jy_4)^* \\ (y_1 + jy_2)^* \end{bmatrix} + \begin{bmatrix} n_{21} \\ n_{22} \end{bmatrix}, \quad (4.6)$$

By combining (4.5) and (4.6), we obtain:

$$\begin{bmatrix} y_{T_{11}} \\ y_{T_{12}} \\ y_{T_{21}}^* \\ y_{T_{22}}^* \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{bmatrix} \begin{bmatrix} (y_1 + jy_2) \\ (y_3 + jy_4) \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{12} \\ n_{21}^* \\ n_{22}^* \end{bmatrix}. \quad (4.7)$$

The estimates of $y_1 + jy_2$ and $y_3 + jy_4$ are given by:

$$\begin{bmatrix} \hat{y}_1 + j\hat{y}_2 \\ \hat{y}_3 + j\hat{y}_4 \end{bmatrix} = H^+ \begin{bmatrix} y_{T11} \\ y_{T12} \\ y_{T21}^* \\ y_{T22}^* \end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix} y_{T11} \\ y_{T12} \\ y_{T21}^* \\ y_{T22}^* \end{bmatrix}. \quad (4.8)$$

We then apply PSID on these estimates in order to recover the requested bits in the receivers using one-bit, two-bit and three-bit detection. As we stated earlier, we prioritize receiver R_1 and obtain the improved BER performance in it. We show the improved performance as shown in Fig. 4.5 when compared to similar schemes in Fig. 4.3 and Fig.3.7.

4.4 Discussion

Table 4.1 shows the performance measurement for single antenna (1×1), 2×1 , and 2×2 Alamouti scheme using QPSK modulation, for one bit, two bit and three bit detection. From the result, there are relative improvement in each scheme, both in terms of bit detection and diversity order. Error rate of 1×10^{-3} is the target rate for the measurement. The gain in performance stresses the benefit of diversity. In order to eliminate the worry about cost of power and for fair comparison, the total power through all antenna elements is equal to that of a single antenna system. The relation between the power of a single antenna system and that of a multi-transmit antenna system is given as

QPSK scheme	One bit detection 1	Two bit detection	Three bit detection
1×1	23 dB	20 dB	19 dB
2×1	15 dB	11.2 dB	9.7 dB
2×2	7.5 dB	4 dB	2.5 dB

Table 4.1: Performance comparison for various antenna schemes using QPSK modulation.

$$\sum_i^N p_i = P, \quad (4.9)$$

where N is the number of transmit antenna, p_i is the power of antenna i , and P is the power of a single transmit antenna.

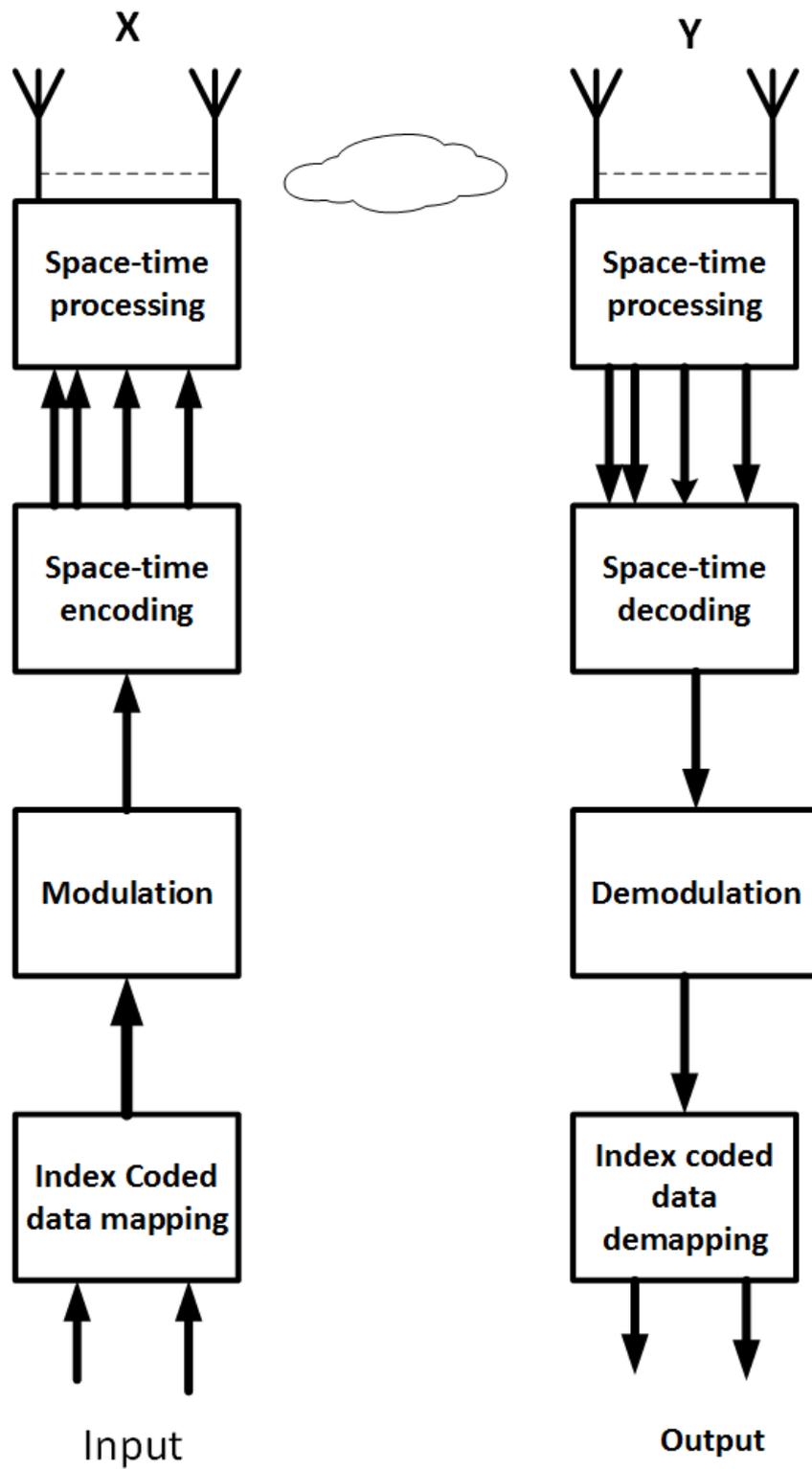


Fig 4.1: Block diagram of the Alamouti index coded scheme.

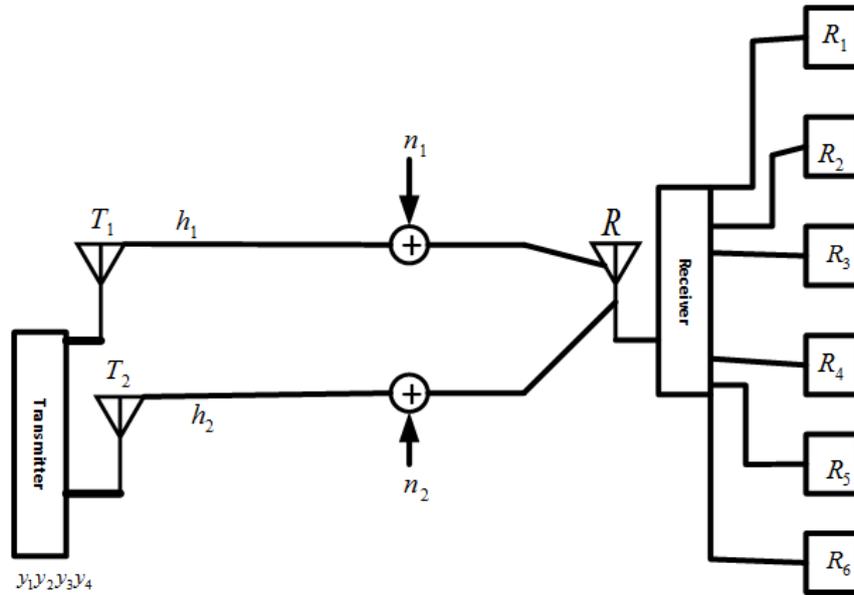


Fig 4.2: System model for 2×1 Alamouti index coding.

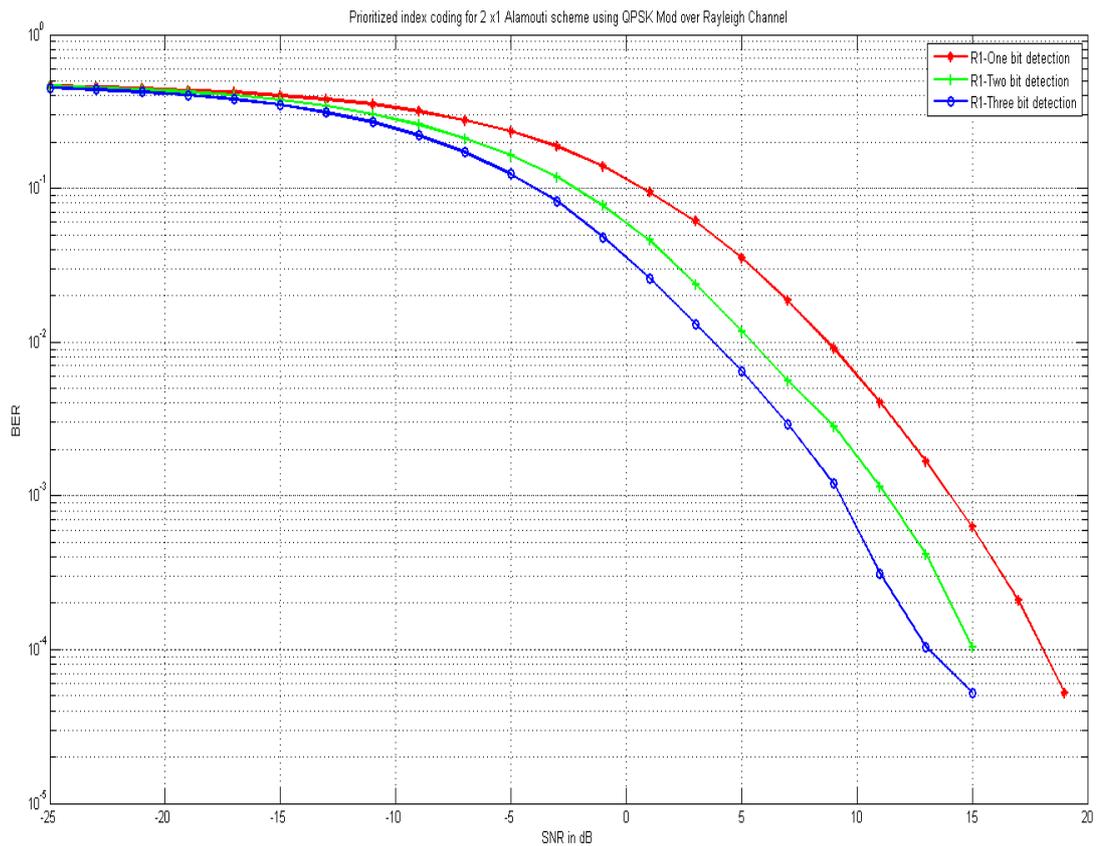


Fig 4.3: BER performance for prioritized index coding with 2×1 Alamouti scheme using QPSK modulation.

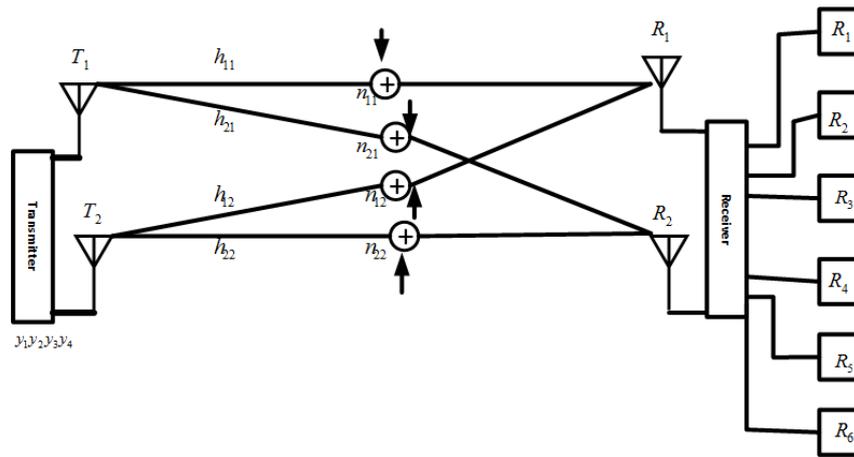


Fig 4.4: System model for 2×2 Alamouti index coding.

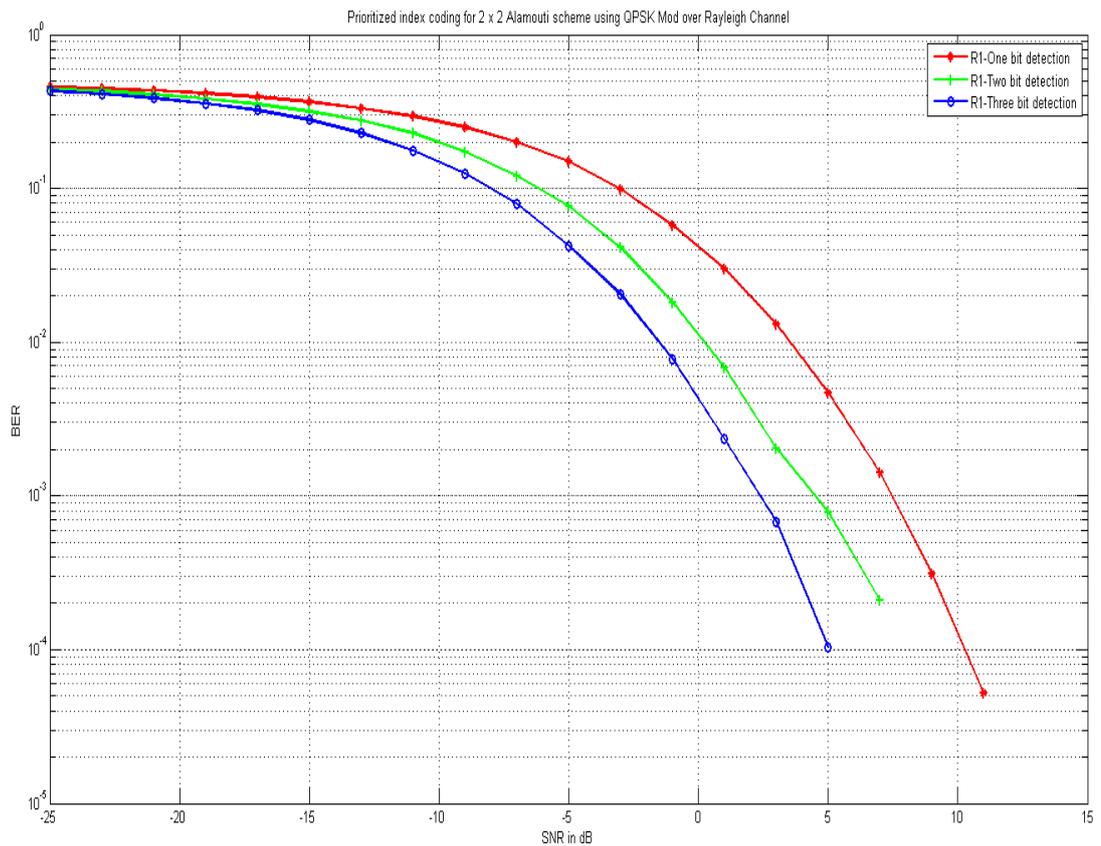


Fig 4.5: BER performance for prioritized index coding with 2×2 Alamouti scheme using QPSK modulation.

5 Conclusions and future research

5.1 Conclusions

In this thesis, we studied index coding problem with side information under realistic conditions, i.e when transmission is done over the AWGN and Rayleigh fading channels. We also consider an index coding problem, where the transmitter and receivers have multiple antennas. By contrast, in the traditional scheme a single antenna transmission is used.

We improve the traditional scheme by prioritizing the receivers and recovering the requested bits in more than one way. We perform two and three bit detection and obtain improved performance for all the modulation schemes and decision types (hard and soft decisions) we considered. However, one needs to combat deep fading that causes distortion to the transmitted index coded bits, and to ensure additional reliability as well as improved performance. To achieve that, we studied index coding under space-time diversity scheme, and we considered a special case of space time block coding, Alamouti code, in which we used two antennas and two time-slots for the transmission. We obtained improved performance at the receivers. Fading can be combated by using this scheme as two copies of each index coded symbols are sent, thereby reducing the probability for simultaneous failure of two transmissions.

5.2 Future research

- Index coding can be enhanced by using several transmit and receive antennas in order to achieve improved BER performance.
- For different scenarios and reliability requirements, it remains an open question how to construct good generator matrices.

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A Appendix

Alamouti Scheme¹

Below, we show derivations of properties of Alamouti scheme. We use the example of 2×1 scheme, however, this can be extended to higher diversity order schemes. As it is stated in equation (2.5),

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}, \quad (\text{A.1})$$

where x_i is the transmitted signal by the antennas, y_i is the received signal in each time-slot and n_i is the AWGN in time-slot i . we denote, $\bar{y} = \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}$, $H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$, and $\bar{w} = \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}$.

We can denote the columns of H as $\bar{C}_1 = \begin{bmatrix} h_1 \\ h_2^* \end{bmatrix}$ and $\bar{C}_2 = \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix}$. Therefore, we can write:

$$\bar{y} = \bar{C}_1 x_1 + \bar{C}_2 x_2 + \bar{w}, \quad (\text{A.2})$$

By taking the inner product of the columns of H , we obtain:

$$\bar{C}_1^H \cdot \bar{C}_2,$$

where A^H is the Hermitian of the matrix A .

$$\begin{bmatrix} h_1^* & h_2 \end{bmatrix} \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix},$$

$$h_1^* h_2 - h_2 h_1^* = 0.$$

¹Alamouti properties presented here is based on the earlier work done in the course, MTAT.03.309 - Special Assignment in Wireless Communication (University of Tartu).

We observe that,

$$\bar{C}_1^H \cdot \bar{C}_2 = 0. \quad (\text{A.3})$$

Since the inner product is zero, the columns of the channel matrix, H , are orthogonal to each another. For this reason, Alamouti code is called *orthogonal space time block code* (OSTBC).

Receiver

We perform beam forming at the receiver in equation (2.5) in order to decode the symbols x_1 and x_2 . For the decoding of x_1 , we make a projection along C_1 as follow:

$$\frac{\bar{C}_1^H}{\|\bar{C}_1\|} \bar{y} = \frac{\bar{C}_1^H}{\|\bar{C}_1\|} (\bar{C}_1 x_1 + \bar{C}_2 x_2 + \bar{w}), \quad (\text{A.4})$$

$$\frac{\bar{C}_1^H}{\|\bar{C}_1\|} \bar{y} = \frac{\bar{C}_1^H}{\|\bar{C}_1\|} \bar{C}_1 x_1 + \frac{\bar{C}_1^H}{\|\bar{C}_1\|} \bar{C}_2 x_2 + \frac{\bar{C}_1^H}{\|\bar{C}_1\|} \bar{w}. \quad (\text{A.5})$$

As we have shown earlier in equation (A.3), by simplification, we obtain:

$$\frac{\bar{C}_1^H}{\|\bar{C}_1\|} \bar{y} = \|\bar{C}_1\| x_1 + \frac{\bar{C}_1^H}{\|\bar{C}_1\|} \bar{w}. \quad (\text{A.6})$$

Similar analysis is done to decode x_2 . In this case, we make a projection along \bar{C}_2 and we obtain:

$$\frac{\bar{C}_2^H}{\|\bar{C}_2\|} \bar{y} = \|\bar{C}_2\| x_2 + \frac{\bar{C}_2^H}{\|\bar{C}_2\|} \bar{w}. \quad (\text{A.7})$$

We will show the properties of the noise component $\frac{\bar{C}_1^H}{\|\bar{C}_1\|} \bar{w}$.

Properties of the noise

Noise is a Gaussian random variable which has undergone some mathematical transformations as it is shown in (A.3) and (A.7). We show the relations between the transformed noise and the original noise vector \bar{w} which is i.i.d. Gaussian noise (AWGN) with Gaussian distribution $\mathcal{N}(0, \sigma^2)$. In the analysis, we take variance $\sigma^2 = 1$, therefore, we define:

$$\tilde{w} = \frac{\bar{C}_1^H}{\|\bar{C}_1\|} \bar{w}.$$

We know that:

$$\mathbf{E}\{\tilde{w} \tilde{w}^H\} = \sigma^2 I, \quad (\text{A.8})$$

where I is an identity matrix.

$$\mathbf{E}\{\tilde{w} \tilde{w}^H\} = \frac{\bar{C}_1^H}{\|\bar{C}_1\|} \mathbf{E}\{\bar{w} \bar{w}^H\} \frac{\bar{C}_1}{\|\bar{C}_1\|} \quad (\text{A.9})$$

$$= \frac{\bar{C}_1^H}{\|\bar{C}_1\|} \sigma^2 I \frac{\bar{C}_1}{\|\bar{C}_1\|} \quad (\text{A.10})$$

$$= \frac{\bar{C}_1^H \bar{C}_1}{\|\bar{C}_1\|^2} \sigma^2 I, \quad (\text{A.11})$$

which shows that

$$\mathbf{E}\{|\bar{\mathbf{w}}|^2\} = \mathbf{E}\{\tilde{\mathbf{w}}\tilde{\mathbf{w}}^*\} = \sigma^2. \quad (\text{A.12})$$

In (A.12), we show that the noise property remains the same despite all the transformations.

The signal to noise ratio, SNR, is given as:

$$SNR = \frac{\|\bar{C}_1\|^2 \mathbf{E}\{|x_1|^2\}}{\sigma^2}. \quad (\text{A.13})$$

Power

The two transmitted symbols, x_1 and x_2 , each has half of the power,

$$\mathbf{E}\{|x_1|^2\} = \mathbf{E}\{|x_2|^2\} = \frac{P}{2}. \quad (\text{A.14})$$

$$\|\bar{C}_1\|^2 = \|\bar{C}_2\|^2 = \sqrt{|h_1|^2 + |h_1|^2} = \|\bar{h}\|. \quad (\text{A.15})$$

$$SNR = \frac{\|\bar{h}_1\|^2 P}{2\sigma^2}. \quad (\text{A.16})$$

The SNR in the Alamouti scheme is half of that of the MRRC scheme. On the $10 \log(\frac{P_o}{P_i})$ scale, this is equivalent to a 3 dB penalty for transmitting with half of the power at each of the transmitters. However, if full power is available at each transmit antenna of the Alamouti scheme, we will obtain the same performance as that of the MRRC scheme.

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